MODELING THE SATURATED
BIPOLAR TRANSISTOR

by

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ABSTRACT

In this paper an analysis of the saturated bipolar transistor is presented, in the situation where the dc collector current is zero or close to zero. In this particular case, the device is referred to as a zero-biased transistor (ZBT). The analysis is based on a version of the integral charge-control model of Gummel and Poon.

Through analytic solution of the model equations and computer simulation, the dc collector-emitter voltage, small-signal collector-emitter conductance, and second and third-order nonlinear distortion are examined over a wide range of base currents. Both the ac and dc characteristics are found to be dominated by different physical phenomena at different levels of $I_B$. The effects of the parasitic substrate transistors on device operation are also examined. Criteria for acceptable performance are established with a view towards practical applications.

A number of applications for the ZBT have been suggested, particularly where a low-power small-signal variable-impedance element is required. Performance of the device in a small-signal attenuator and a feedback amplifier are discussed. The effects of a fixed parallel resistance and a finite base-drive impedance on device performance are investigated.
The results are found to describe the measured dc behaviour at low and moderate base currents for a family of lateral p-n-p transistors having symmetric forward and reverse-active properties. Similarly, low-frequency ac behaviour is well described at low and moderate base currents. Suggestions for future work are included.
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\[ b \]
\[ \text{Ratio } \frac{\beta_F}{\beta_R} \]
\[ C \]
\[ \exp \left( \frac{v_{CB}}{V_T} \right) \]
\[ C_2 \]
\[ \text{Forward low-current nonideal base current coefficient} \]
\[ C_4 \]
\[ \text{Reverse low-current nonideal base current coefficient} \]
\[ C_{JC}(0) \]
\[ \text{Zero-bias collector-base junction capacitance} \]
\[ C_{JE}(0) \]
\[ \text{Zero-bias emitter-base junction capacitance} \]
\[ C_{JS}(0) \]
\[ \text{Zero-bias substrate-base junction capacitance} \]
\[ c_n \]
\[ \text{Nonlinear small-signal capacitance due to } q_E \]
\[ c_p \]
\[ \text{Nonlinear small-signal capacitance due to } q_C \]
\[ c_{\text{SUB}} \]
\[ \text{Nonlinear small-signal capacitance due to } q_{\text{SUB}} \]
\[ E \]
\[ \exp \left( \frac{v_{EB}}{V_T} \right) \]
\[ e_0 \]
\[ \text{Value of } E/C \text{ for } I_C = 0 \]
\[ g_{eco} \]
\[ \text{Small-signal collector-emitter conductance for } I_C = 0 \]
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\[ g_{sr}, g_{sf} \]
\[ \text{First-order small-signal conductances due to the parasitic substrate transistors} \]
\[ g_2 \]
\[ \text{Second-order coefficient of the Taylor series expansion for } i_C \]
\[ g_3 \]
\[ \text{Third-order coefficient of the Taylor series expansion for } i_C \]
\[ H_1 \]
\[ \text{First-order small-signal transfer function} \]
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<td>$I_B$</td>
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<td>Forward knee current</td>
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<tr>
<td>k</td>
<td>Boltzmann's constant</td>
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<td>m_c</td>
<td>Collector-base grading coefficient</td>
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<td>m_e</td>
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<td>m_sub</td>
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<tr>
<td>n_e</td>
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<tr>
<td>n_c</td>
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<td>q</td>
<td>Electronic charge</td>
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<td>q_b</td>
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<td>r_b</td>
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<td>s</td>
<td>( \exp \left( \frac{v_{SB}}{V_T} \right) )</td>
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<td>t</td>
<td>Temperature</td>
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<td>v_a</td>
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<td>v_b</td>
<td>Reverse Early voltage</td>
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<td>v_eb</td>
<td>DC base-emitter voltage</td>
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<td>v_{eb*}</td>
<td>External dc base-emitter voltage</td>
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$V_{EC}$: DC collector-emitter voltage

$V_{ECO}$: DC collector-emitter voltage with the collector open-circuited

$V_T$: Thermal voltage

$V_{CB}$: Large-signal base-collector voltage

$V_{EB}$: Large-signal base-emitter voltage

$V_{SB}$: Large-signal base-substrate voltage

$v_{ec}$: Small-signal collector-emitter voltage

$v_{eco}$: Small-signal collector-emitter voltage for $I_C = 0$

$x$: Ratio $v_{ec}/V_T$

$\beta_F$: Ideal forward current gain

$\beta_R$: Ideal reverse current gain

$\beta_{RS}$: Ideal reverse current gain of the parasitic substrate transistors

$\phi_C$: Base-collector potential

$\phi_E$: Base-emitter potential

$\phi_{SUB}$: Base-substrate potential

$\tau_F$: Forward transit time

$\tau_R$: Reverse transit time
INTRODUCTION

One of the criteria for a good model of the bipolar junction transistor is that it successfully represent the device in all modes of operation. Such a model, based on a clear physical understanding of device behaviour, is an invaluable tool for evaluating performance and exploring new, possibly unconventional uses.

This paper focuses on the modeling and understanding of the bipolar transistor in the saturation mode, and in particular in the situation where the dc collector current is zero or close to zero. In this particular case the device is referred to as a zero-biased transistor (ZBT). Both dc and ac performance are examined, with a view towards practical applications.

The ZBT is of primary interest as a current-controlled small-signal variable impedance element. Other active devices, namely p-n junction diodes, Schottky barrier diodes, junction gate field-effect transistors and metal-oxide semiconductor transistors have been used for this purpose [1], [2]. To date, however, the possibilities of bipolar transistors as variable gain elements seem to have remained uninvestigated.

Variable gain elements are not easily realized in low-voltage micropower integrated circuits. On such single-chip devices space is at a premium, and the ability to obtain several decades of continuously variable small-signal resistance with a single transistor is of enormous
benefit. Furthermore, it can be easily fabricated alongside other bipolar circuitry.

The following chapter discusses the structure of the devices used in this study and the model used to represent them. Chapter Two outlines the techniques used to obtain the model parameters. Due consideration must be given to the dc properties of the ZBT and possible dc interaction with external circuitry; these topics are discussed in the third chapter. Chapter Four examines the ac properties of the ZBT, including small-signal resistance and nonlinear distortion.
CHAPTER 1

Device Structure and Models

The devices studied in this paper reflect their intended application as variable gain elements. Of central interest is a type of lateral p-n-p transistor having closely matched forward and reverse-active mode characteristics. These devices, designated Q009, were specially manufactured by Linear Technology Inc. The advantages of their symmetry will be made apparent in Chapter Four. In Chapters Three and Four theoretical study is made of a hypothetical device Q1 having parameters selected to be typical of ordinary (i.e. $g_T \gg g_R$) integrated p-n-p transistors (such as the RCA CA3096E p-n-p).

1.1 Device Structure

The topology of the Q009 is shown in Figures 1.1(a) and (b). The grid structure, consisting of alternating collector and emitter segments, provides the desired symmetry between forward and reverse mode characteristics. This differs from conventional lateral p-n-p devices, where the emitter is surrounded by a ring-shaped collector region. As in any lateral p-n-p, additional parasitic p-n-p structures are formed between the collector, n-type base region, and the p-type substrate, and between the emitter, base, and substrate. These structures can affect device operation, and must be considered in any complete model.
Figure 1.1. Topology of the symmetric lateral p-n-p transistor, Q009. (a) Top view. (b) Cross-section.
1.2 Three-Terminal Device Model

The bipolar transistor model used in this study is a modified version of the integral charge-control model of Gummel and Poon [3], as contained in the circuit simulation program SPICE [4], [5]. With reference to Figure 1.2, the effective transported component of the collector current is defined by

\[ I_{CT} = \frac{1}{q_b} (I_{CC} - I_{EC}) \]  \hspace{1cm} (1.1)

where

\[ I_{CC} = I_S \left[ \exp \left( \frac{V_{EB}}{V_T} \right) - 1 \right] \] \hspace{1cm} (1.2)

\[ I_{EC} = I_S \left[ \exp \left( \frac{V_{CB}}{V_T} \right) - 1 \right] \] \hspace{1cm} (1.3)

\[ q_b = \frac{q_1}{2} + \left[ \frac{q_1^2}{2} + q_2 \right]^{1/2} \] \hspace{1cm} (1.4a)

\[ q_1 = 1 + \frac{V_{CB}}{V_A} + \frac{V_{EB}}{V_B} \] \hspace{1cm} (1.4b)

\[ q_2 = \frac{I_S}{I_K} \left[ \exp \left( \frac{V_{EB}}{V_T} \right) - 1 \right] \] \hspace{1cm} (1.4c)

\[ + \frac{I_S}{I_{KR}} \left[ \exp \left( \frac{V_{CB}}{V_T} \right) - 1 \right] \]

\( V_T \) is the thermal voltage \( kT/q \) and \( I_S \) is the saturation current parameter; \( V_A \) and \( V_B \) characterize forward and reverse mode base width modulation (the Early effect); \( I_K \) and \( I_{KR} \) are parameters ("knee"
Figure 1.2. Large-signal model of the bipolar junction transistor (shown for a p-n-p transistor).
characterizing high-level injection effects in forward and reverse mode operation; \( q_b \) is the normalized base majority charge (Gummel charge). The total collector terminal current is then given by

\[
i_C = \frac{I_S}{q_b} \left[ \exp \left( \frac{V_{EB}}{V_T} \right) - \exp \left( \frac{V_{CB}}{V_T} \right) \right] - \frac{I_S}{\beta_R} \left( \exp \left( \frac{V_{CB}}{V_T} \right) - 1 \right) - C_H I_S \left( \exp \left( \frac{V_{CB}}{n_e V_T} \right) - 1 \right),
\]

(1.5)

while the total base terminal current is given by

\[
i_B = \frac{I_S}{\beta_F} \left[ \exp \left( \frac{V_{EB}}{V_T} \right) - 1 \right] + C_2 I_S \left( \exp \left( \frac{V_{EB}}{n_e V_T} \right) - 1 \right) + \frac{I_S}{\beta_R} \left( \exp \left( \frac{V_{CB}}{V_T} \right) - 1 \right) + C_H I_S \left( \exp \left( \frac{V_{CB}}{n_e V_T} \right) - 1 \right),
\]

(1.6)

where \( \beta_F \) and \( \beta_R \) are the ideal forward and reverse current gains; \( n_e \) and \( C_2 \) characterize a nonideal diode modeling the effects of carrier recombination within the emitter-base space-charge region and at the surface, which dominates the base current under conditions of low forward bias [7], [3]. Similarly, \( n_c \) and \( C_H \) characterize recombination inside, and at the surface of, the collector-base space-charge region. Three ohmic resistances \( R_C \), \( R_B \), and \( R_E \) are included in the model, and are taken to be constant.

Time and frequency-dependent device behaviour are modeled by the
nonlinear charge-storage elements \( q_E \) and \( q_C \) [9], given by

\[
q_E = \tau_F I_S \left[ \exp \left( \frac{v_{EB}}{v_T} \right) - 1 \right] + C_{JE}(0) \int_0^{v_{EB}} \frac{dv}{m_E (1 - v/\phi_E)}
\]

\[
q_C = \tau_R I_S \left[ \exp \left( \frac{v_{CB}}{v_T} \right) - 1 \right] + C_{JC}(0) \int_0^{v_{CB}} \frac{dv}{m_C (1 - v/\phi_C)}
\]

where \( \tau_F \) and \( \tau_R \) are the forward and reverse transit times, respectively, and \( C_{JE}(0) \) and \( C_{JC}(0) \) are the emitter-base and collector-base small-signal depletion layer capacitances, respectively, for zero dc junction voltage. Parameters \( m_E, \phi_E, m_C \) and \( \phi_C \) characterize the variation of these capacitances as a function of the junction voltages; \( \phi_E \) and \( \phi_C \) are the barrier potentials of the emitter-base and collector-base junctions, respectively; \( m_E \) and \( m_C \) depend on the impurity doping profiles of the respective junctions.

The model diagram in Figure 1.2 is drawn for a p-n-p transistor. Terminal currents flowing out of the device are taken to be positive. For an n-p-n transistor the voltage polarities of the internal elements and the reference current directions are reversed. In both cases all model parameter values are considered to be positive.
Models of the type used here enable the required parameters to be obtained directly from measurements at the transistor terminals. Devices can also be modeled by assigning separate diodes to the various internal injection currents. This approach can be used to examine arbitrary two or three dimensional structures, such as merged transistor logic (MTL) devices [10]. While providing valuable insight into the physical mechanisms responsible for device behaviour, the utility of such models is limited by the inaccessibility of the required input parameters, which must be determined from measurements of internal geometry and doping levels or of specially fabricated test devices. In the case of lateral p-n-p transistor, a one dimensional model similar in form to that of Gummel and Poon has been found to adequately describe dc active mode behaviour [11]. In cases where two dimensional effects are encountered, they can usually be treated by adding elements to the basic model. Such a case is considered in the next section.

1.3 Inclusion of the Substrate

The parasitic p-n-p transistors can be readily incorporated into the existing device model [6], [12]. With reference to Figure 1.3(a), QA represents the lateral p-n-p device, QSE is the parasitic p-n-p formed between the emitter, base, and substrate, and QSC is that formed between the collector, base, and substrate. The substrate is defined as the collector of QSE and QSC, as in most circuit applications it is at a lower potential than the other three terminals. Note that the emitter-base junction of QSC and the collector-base junction of QA are
Figure 1.3. The p-n-p device model with the parasitic substrate transistors included. (a) Three-transistor equivalent model. (b) Components added to existing device model.
physically the same junction, and must not be modeled twice. When using
the program SPICE this is readily accomplished by giving $R_T$ a very large
value (say $10^7$) for QSC, effectively eliminating its contribution to the
base current. The same comments apply to the emitter-base junctions of
QA and QSE.

Due to the symmetric structure of the Q009, the parameters of QSE
and QSC are taken to be identical. Referring to Figure 1.3(b), the
effective transported components of the substrate current $i_{SUB}$ are

$$I_{STE} = \frac{I_{SS}}{q_{bSE}} \left[ \exp \left( \frac{V_{EB}}{V_T} \right) - \exp \left( \frac{V_{SB}}{V_T} \right) \right] \quad (1.9)$$

$$I_{STC} = \frac{I_{SS}}{q_{bSC}} \left[ \exp \left( \frac{V_{CB}}{V_T} \right) - \exp \left( \frac{V_{SB}}{V_T} \right) \right] \quad (1.10)$$

with

$$q_{bSE} = \frac{I_{SS}}{I_{KS}} \left[ \exp \left( \frac{V_{EB}}{V_T} \right) - 1 \right]^{1/2} \quad (1.11)$$

$$q_{bSC} = \frac{I_{SS}}{I_{KS}} \left[ \exp \left( \frac{V_{CB}}{V_T} \right) - 1 \right]^{1/2} \quad (1.12)$$

$I_{SS}$ is the saturation current parameter for the parasitic transistors;
$I_{KS}$ models high-level injection for these devices. The currents $I_{SBE}$
and $I_{SBC}$ represent the contribution of the substrate to the base
terminal current. Each consists of an ideal component and a low-current
recombination component, similar in form to those of (1.5). For
example, the ideal component of $I_{SBE}$ and $I_{SBC}$ is given by
\[ I_{SBE(\text{ideal})} = I_{SBC(\text{ideal})} = \frac{I_{SS}}{\beta_{RS}} \left( \exp \left( \frac{V_{SB}}{V_T} \right) - 1 \right) \]  \hspace{1cm} (1.13)

where \( \beta_{RS} \) is defined as the ideal reverse current gain of the parasitic transistors. Similarly, parameters can be defined to characterize the recombination component. The contribution of \( I_{SBE} \) and \( I_{SBC} \) to the total base current will be negligible when the substrate-base junction is reverse biased. Finally, an ohmic resistance \( R_{SUB} \) and a charge-storage element \( q_{SUB} \) are included; the latter is given by

\[ q_{SUB} = C_{JS}(0) \int_{0}^{v_{SB}} \frac{dv}{(1 - v/\phi_{SUB})^{m_{SUB}}} \]  \hspace{1cm} (1.14)

where \( C_{JS}(0) \) is the substrate-base depletion layer capacitance at zero junction voltage, \( \phi_{SUB} \) is the barrier potential, and \( m_{SUB} \) is the junction grading coefficient.

It should be noted that the symmetry between the parasitic devices does not limit the generality of the model outlined in Figure 1.3(b). Where warranted, different parameters can be ascribed to the two substrate \( p-n-p' \)'s, and determined using the terminal measurement techniques outlined in the next chapter.
CHAPTER 2

Measurement of Model Parameters

Many of the techniques used to measure model parameters closely follow those outlined by Getreu [6]. Devices under test were placed in a shielded enclosure; ambient temperature was held to 22.5 ± 1°C.

The dc forward mode characteristics, with the exception of the Early voltage, were obtained using the circuit of Figure 2.1, with \( v_{CB} = v_{SB} = 0 \). Under this condition, equations (1.5), (1.6), (1.9) – (1.13), and Figure 1.3(b) yield

\[
i_C = I_C = \frac{I_S}{q_b} \exp \left( \frac{v_{EB}}{V_T} \right)
\]

\[
i_B = I_B = \frac{I_S}{q_F} \left[ \exp \left( \frac{v_{EB}}{V_T} \right) - 1 \right]
\]

\[
+ C_2 I_S \left[ \exp \left( \frac{v_{EB}}{n_E V_T} \right) - 1 \right]
\]

\[
i_{SUB} = I_{SUB} = \frac{I_{SS}}{q_{BSE}} \left[ \exp \left( \frac{v_{EB}}{V_T} \right) - 1 \right].
\]

For low-level injection \( q_B = 1 \), and the intercept of \( \ln(I_C) \) versus \( V_{EB} \) curve is simply \( \ln(I_S) \). Similarly, the intercept of the \( \ln(I_{SUB}) \) versus \( V_{EB} \) curve yields \( I_{SS} \). At low currents, \( I_B \) is dominated by the second term of (2.2); from the \( \ln(I_B) \) measurements in this region \( C_2 \) and \( n_E \) are obtained. As \( I_B \) increases, the first term of (2.2) dominates, yielding
\[ V_{SB} = V_{CB} = 0 \]

Figure 2.1. Test circuit used to measure the dc model parameters.
\( \beta_F \). Measured \( I_B \) characteristics in this region often have slopes close to, but not identical to, \( q/kT \). This may be due to current crowding in the base region \([13]\) or the onset of other high-current effects not incorporated in the SPICE version of the Gummel-Poon model. In general, a very close fit of the measured characteristics to (2.2) can be obtained by setting \( \beta_F \) equal to the maximum value of \( I_C/I_B \). At high currents the externally measured emitter-base voltage \( V_{EB}^* \), defined by

\[
V_{EB}^* = V_{EB} + I_B R_B + I_E R_E
\]

must be corrected to allow for the ohmic voltage drops across \( R_B \) and \( R_E \) \([6]\). When this is done, the intercept of the high and low-current asymptotes of \( \ln(I_C) \) define the knee current \( I_K \). Similarly, when the \( \ln(I_{SUB}) \) versus \( V_{EB}^* \) curve is corrected for the voltage drops across \( R_E \) and \( R_C \), the high and low-current asymptotes define \( I_{KS} \).

Figure 2.2 depicts the \( \ln(I_C) \), \( \ln(I_B) \), and \( \ln(I_{SUB}) \) versus \( V_{EB}^* \) characteristics of a typical sample of the Q009 device. All terminal voltage and current measurements were made using a Keithley Model 610 electrometer, except at very high currents, where they were displayed on the CRT of a Tektronix Type 576 curve tracer. Below 10 nA base currents were provided by a dc power supply in series with \( 10^{10} \) or \( 10^{11} \) ohm 1% resistors; above 10 nA the calibrated base current source of the Tektronix Type 576 curve tracer was used. The latter was operated in a pulsed mode at very high current levels in order to avoid internal heating in the device; the pulses were of 300 \( \mu \)s duration. From a similar set of measurements, made with the positions of the collector
Q009 Symmetric lateral P-N-P Transistor  $T=22.5^\circ C$

$V_{SB} = V_{CB} = 0$

- $I_C$ measured
- " corrected
- $I_{Sub}$ measured
- " corrected
- $I_B$ measured

$slope = \frac{q}{kT}$

$slope = \frac{q}{n_{E} kT}$

\[ I_C \quad I_{Sub} \quad I_B \]

\[ V_{EB} \]

**Figure 2.2.** Measured ln ($I_C$), ln ($I_B$) and ln ($I_{Sub}$) versus $V_{EB}$ characteristics for the Q009 transistor.
and emitter terminals in Figure 2.1 interchanged, the parameters $C_R$, $n_C$, 
$\beta_R$, and $I_{KR}$ were obtained.

With the emitter and collector grounded and a positive bias 
applied to the substrate, the parameters modeling the contribution of 
the substrate to the base terminal current can be determined. For 
example, if $I_{SUB}$ is sufficiently large that the recombination term of 
this contribution can be neglected, equations (1.5), (1.6), (1.9) - 
(1.13) and Figure 1.3(b) yield, for $v_{EB} = v_{CB} = 0$,

$$I_C = \frac{I_{SS}}{q_{bSE}} \left[ \exp \left( \frac{V_{SB}}{V_T} \right) - 1 \right]$$  \hspace{1cm} (2.5)

$$I_B = \frac{2I_{SS}}{\beta_{RS}} \left[ \exp \left( \frac{V_{SB}}{V_T} \right) - 1 \right] .$$  \hspace{1cm} (2.6)

In the absence of high-level injection, $q_{bSE} = 1$, and the ratio of $I_C$ to 
$I_B$ is $\beta_{RS}/2$. Under the same conditions at very low currents, $I_B$ is 
dominated by the recombination terms of the parasitic transistors; the 
$\ln(I_B)$ versus $V_{SB}$ characteristic in this region yields the corresponding 
model parameters. The latter were not determined for the Q009 device, 
since the measured base currents were found to follow (2.6) down to 
values of 1 nA.

The emitter series resistance is frequently measured using an 
open-circuited collector method, with a related forced-beta method used 
to measure the collector series resistance [14]. This technique 
requires measurement at very high base currents, often in excess of 10 
mA, in order to obtain these parameters. For many of the transistors 
tested this was not found to be practical, due to problems associated
with internal heating and potential device destruction.

A computer-aided technique has been introduced by Incencik [15] which avoids these difficulties. However, it assumes that the Gummel-Poon model accurately predicts the open collector device behaviour at current levels where the voltage drops across \( R_C \) or \( R_E \) become significant. This was not found to be true for the Q009 device, and application of the computer-aided method resulted in strongly current-dependent resistance values. These ranged from over 100\( \Omega \) at base currents below 100 \( \mu A \) to about 10\( \Omega \) at 50 mA. Physically, such a variation cannot be correct since the series resistances of the highly doped emitter and collector regions of the symmetric p-n-p are virtually constant.

Although the computer-aided results asymptotically approached constant resistance values at high currents, a more definitive assessment of \( R_E \) and \( R_C \) was made using the small-signal circuit illustrated in Figure 2.3 (the operational amplifier is a National Semiconductor LF355N). At sufficiently high frequencies the blocking capacitor \( C_B \) behaves as a short circuit, and the voltage gain is given by

\[
\frac{v_o}{v_1} = 1 + \frac{R_F}{r_{eeco}},
\]

where \( r_{eeco} \) is the small-signal collector-emitter resistance of the transistor with \( I_C = 0 \). It consists of the internal device resistance, which is a function of base current, in series with \( R_E \) and \( R_C \), which are assumed constant. As shown in Chapter Four, the internal component
Figure 2.3. Test circuit used to measure the small-signal collector-emitter resistance $r_{eco}$. 
becomes negligible at high currents, and \( r_{\text{eco}} \) asymptotically approaches \( R_E + R_C \). Then, due to the symmetry of the Q009,

\[
R_E = R_C = \frac{1}{2} r_{\text{eco}} \left|_{I_B \to \infty} \right.
\]

(2.8)

The \( r_{\text{eco}} \) versus \( I_B \) characteristics required for determining \( R_E \) and \( R_C \) were readily obtained using (2.7) by monitoring the voltage gain and \( R_F \) at each operating point. Grounding the substrate and biasing the emitter at \(-2V\), \( R_C + R_S \), and hence \( R_S \), were obtained after a similar fashion. In all cases the input voltage \( v_i \) was maintained near 4 mV rms.

The base series resistance \( R_B \) is generally very difficult to measure. An estimate can be made using (2.4):

\[
R_B = \frac{V_{EB}^* - V_{EB} - I_E R_E}{I_B} \ ;
\]

(2.7)

the internal voltage drop \( V_{EB} \) lies on the ideal \( \ln(I_B) \) characteristic of Figure 2.2 extrapolated into the high-current region.

Standard techniques were employed in measuring the Early voltages \( V_A \) and \( V_B \), the forward and reverse transit times \( \tau_F \) and \( \tau_R \), and the parameters modeling the junction small-signal capacitances \( C_{JC}(v_{CB}) \), \( C_{JE}(v_{EB}) \) and \( C_{JS}(v_{SB}) \). Details are given by Getreu [6].

The parameters obtained for a representative sample of the Q009 transistor are listed in Table 2.1. Parameters relating to the parasitic transistors in the same device are shown separately in Table 2.2. Table 2.3 lists the model parameters of the hypothetical device
Q1, discussed in Chapter One. These were selected to be typical of conventional general-purpose lateral p-n-p's, and were obtained from manufacturers' specification sheets and actual measurements of a variety of transistors.
Table 2.1  
Model Parameter Values for the Q009 Symmetric Lateral p-n-p Transistor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Measured Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_F )</td>
<td>14.58</td>
</tr>
<tr>
<td>( P_R )</td>
<td>15.14</td>
</tr>
<tr>
<td>( I_S )</td>
<td>( 1.34 \times 10^{-15} ) A</td>
</tr>
<tr>
<td>( R_B )</td>
<td>7 Ω</td>
</tr>
<tr>
<td>( R_C )</td>
<td>4 Ω</td>
</tr>
<tr>
<td>( R_E )</td>
<td>4 Ω</td>
</tr>
<tr>
<td>( V_A )</td>
<td>60 V</td>
</tr>
<tr>
<td>( V_B )</td>
<td>60 V</td>
</tr>
<tr>
<td>( I_K )</td>
<td>( 6.5 \times 10^{-4} ) A</td>
</tr>
<tr>
<td>( I_{KR} )</td>
<td>( 6.5 \times 10^{-4} ) A</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>142</td>
</tr>
<tr>
<td>( n_E )</td>
<td>1.56</td>
</tr>
<tr>
<td>( C_4 )</td>
<td>137</td>
</tr>
<tr>
<td>( n_C )</td>
<td>1.60</td>
</tr>
<tr>
<td>( \tau_F )</td>
<td>37.9 ns</td>
</tr>
<tr>
<td>( \tau_R )</td>
<td>31.8 ns</td>
</tr>
<tr>
<td>( C_{JE}(0) )</td>
<td>0.909 pF</td>
</tr>
<tr>
<td>( m_E )</td>
<td>0.389</td>
</tr>
<tr>
<td>( \phi_E )</td>
<td>0.5 V</td>
</tr>
<tr>
<td>( C_{JC}(0) )</td>
<td>0.981 pF</td>
</tr>
<tr>
<td>( m_C )</td>
<td>0.347</td>
</tr>
<tr>
<td>( \phi_C )</td>
<td>0.5 V</td>
</tr>
</tbody>
</table>
Table 2.2

Model Parameter Values for the Parasitic Substrate Transistors of the Q009 Device

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Measured Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{RS}$</td>
<td>$1.95 \times 10^{-2}$</td>
</tr>
<tr>
<td>$I_{SS}$</td>
<td>$1.75 \times 10^{-16}$ A</td>
</tr>
<tr>
<td>$R_S$</td>
<td>13 $\Omega$</td>
</tr>
<tr>
<td>$I_{KS}$</td>
<td>$9.0 \times 10^{-5}$ A</td>
</tr>
<tr>
<td>$C_{JS}(0)$</td>
<td>4.11 pF</td>
</tr>
<tr>
<td>$m_{SUB}$</td>
<td>0.235</td>
</tr>
<tr>
<td>$\phi_{SUB}$</td>
<td>0.5 V</td>
</tr>
</tbody>
</table>
Table 2.3

Model Parameters Selected for the Q1
p-n-p Transistor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Measured Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_f$</td>
<td>50</td>
</tr>
<tr>
<td>$\beta_R$</td>
<td>1</td>
</tr>
<tr>
<td>$I_S$</td>
<td>$1.0 \times 10^{-16}$ A</td>
</tr>
<tr>
<td>$R_B$</td>
<td>50 $\Omega$</td>
</tr>
<tr>
<td>$R_C$</td>
<td>5 $\Omega$</td>
</tr>
<tr>
<td>$R_E$</td>
<td>1 $\Omega$</td>
</tr>
<tr>
<td>$V_A$</td>
<td>100 V</td>
</tr>
<tr>
<td>$V_B$</td>
<td>100 V</td>
</tr>
<tr>
<td>$I_K$</td>
<td>10 mA</td>
</tr>
<tr>
<td>$I_KR$</td>
<td>100 mA</td>
</tr>
<tr>
<td>$C_2$</td>
<td>100</td>
</tr>
<tr>
<td>$n_E$</td>
<td>1.75</td>
</tr>
<tr>
<td>$C_4$</td>
<td>5</td>
</tr>
<tr>
<td>$n_C$</td>
<td>1.75</td>
</tr>
<tr>
<td>$\tau_F$</td>
<td>1 ns</td>
</tr>
<tr>
<td>$\tau_R$</td>
<td>50 ns</td>
</tr>
<tr>
<td>$C_{JE}(0)$</td>
<td>1 pF</td>
</tr>
<tr>
<td>$m_E$</td>
<td>0.333</td>
</tr>
<tr>
<td>$\phi_E$</td>
<td>0.7 V</td>
</tr>
<tr>
<td>$C_{JC}(0)$</td>
<td>3 pF</td>
</tr>
<tr>
<td>$m_C$</td>
<td>0.333</td>
</tr>
<tr>
<td>$\phi_C$</td>
<td>0.5 V</td>
</tr>
</tbody>
</table>
CHAPTER 3

DC Characteristics of the Zero-Biased Transistor

In this chapter the dc characteristics of the ZBT are discussed; namely, the variation of the dc collector-emitter voltage $V_{EC}$ as a function of the base current. A detailed knowledge of this behaviour is useful in predicting the dc interaction of the device with the concomitant circuitry, and permits the establishment of criteria for improved dc performance, based on the physical device properties.

3.1 Analytic Formulation

An exact solution for $V_{EC}$ using the model equations of Chapter One requires numerical methods. While this is readily accomplished with the aid of a computer, it is possible, by making some simplifying assumptions, to obtain analytic expressions for $V_{EC}$. In addition to providing computational simplicity, they afford a means whereby the influence of the various device parameters on the observed behaviour can be directly assessed. As such they increase the understanding of the device and assist in determining design criteria for improved performance.

From (1.5) $i_C = 0$ implies
\[ \exp \left( \frac{v_{EB}}{V_T} \right) = -q_b \left( \frac{1}{\beta_R} + C_4 \right) + \left( 1 + \frac{q_b}{\beta_R} \right) \exp \left( \frac{v_{CB}}{V_T} \right) \]

\[ + q_b C_4 \exp \left( \frac{v_{CB}}{n_C V_T} \right) \]  \hspace{1cm} (3.1)

The first term on the right hand side of (3.1) can be neglected, except when the junction voltages are close to \( V_T \) in magnitude. Note that \( v_{EC} = v_{EB} - v_{CB} \) and let

\[ C = \exp \left( \frac{v_{CB}}{V_T} \right) \]  \hspace{1cm} (3.2)

\[ E = \exp \left( \frac{v_{EB}}{V_T} \right) \]  \hspace{1cm} (3.3)

\[ v_{ECO} = v_{EC} \bigg|_{i_C = 0} \]  \hspace{1cm} (3.4)

\[ e_o = \frac{E}{C} \bigg|_{i_C = 0} = \exp \left( \frac{v_{ECO}}{V_T} \right) \]  \hspace{1cm} (3.5)

Equation (3.1) can then be written as

\[ e_o = \left( 1 + \frac{q_b}{\beta_R} \right) + \frac{q_b C_4}{(1-1/n_C)} \]  \hspace{1cm} (3.6)

The terms dominating (3.6) vary according to the base current level. At moderate levels the recombination term is insignificant, and, with the assumption of low-level injection, (3.6) reduces to
\[ e_o = 1 + \frac{1}{\beta R} \]  \hspace{1cm} (3.7)

or

\[ V_{ECO} = V_T \ln \left( 1 + \frac{1}{\beta R} \right) \]  \hspace{1cm} (3.8)

This is the result obtained by Choma [14]. Note that \( V_{ECO} \) is independent of \( I_B \). Similarly, with the device inverted and the emitter terminal open,

\[ V_{CEO} = V_T \ln \left( 1 + \frac{1}{\beta_F} \right) \]  \hspace{1cm} (3.9)

where \( V_{CEO} \) is defined as the emitter-collector voltage with \( I_E = 0 \).

At increased currents the term \( q_b \) becomes significant, and (3.6) becomes

\[ V_{ECO} = V_T \ln \left( 1 + \frac{q_b}{\beta R} \right) \]  \hspace{1cm} (3.10)

The voltage drop across the emitter series resistance \( R_E \) must also be considered and hence the external emitter-collector voltage \( V_{ECO}^* \) across the device terminals is given by

\[ V_{ECO}^* = V_{ECO} + I_B R_E \]  \hspace{1cm} (3.11)

with \( V_{ECO} \) given by (3.10). If it is assumed that \( q_b \) correctly models high-level injection, it can be shown that the slope of the \( V_{ECO}^* \) versus \( I_B \) characteristic approaches \( R_E \) as \( I_B \) is increased, thereby affording a technique for measuring the emitter series resistance [14].

At low current levels the term \( C^{(1-1/n_C)} \) of (3.6) becomes small, marking the onset of recombination effects. Hence, \( V_{ECO} \) rises above the
constant mid-current value given by (3.8). It is useful at this stage to analyze separately conventional and symmetric devices, for each of which a different set of simplifying approximations apply. For the conventional device, $\beta_F \gg \beta_R$ and $C_4 \gg C_2$; under these conditions (1.6) reduces to

$$I_B = I_B = \frac{I_S}{\beta_R} C + C_4 I_S C_{1/n_C}^C,$$  (3.12)

with $C \gg 1$. A comparison of (3.6) with (3.12) yields

$$e_o = 1 + \frac{I_B}{I_S C},$$  (3.13)

with $q_b$ taken to be unity. For most devices $n_C$ is found to be approximately 2; this is consistent with the physical theory for carrier recombination in the space-charge layer [7], [16]. With $n_C=2$, (3.12) permits $C$ to be expressed as a function of $I_B$:

$$C = \frac{\beta_R I_B}{I_S} \left[ 1 + \frac{I_S C_4}{2I_B} \left( 1 + \frac{4I_B}{\beta_R I_S} \right)^{1/2} \right].$$  (3.14)

Let the break-point current $I_{BL}$ be defined by

$$I_{BL} = I_S \beta_R C_4^2.$$  (3.15)

(3.13) now becomes

$$e_o = 1 + \frac{1}{\beta_R} \left[ 1 + \frac{I_{BL}}{2I_B} \left( 1 + \frac{4I_B}{I_{BL}} \right)^{1/2} \right]^{-1}.$$  (3.16)
Note that $I_{BL}$ is half the value of $I_B$, as given by (3.12), at which the recombination and ideal components are equal. For $I_B \gg I_{BL}$, (3.16) collapses to (3.7), as expected. For $I_B \ll I_{BL}$, expansion of $(1 + \frac{4I_B}{I_{BL}})^{1/2}$ yields

$$e_0 = \frac{1}{\beta_R} \frac{I_{BL}}{I_B},$$

(3.17)

or$$V_{ECO} = V_T \left[ \ln \left( \frac{I_{BL}}{I_B} \right) + \ln \left( \frac{1}{\beta_R} \right) \right].$$

(3.18)

Thus, at very low currents the $V_{ECO}$ versus $\ln I_B$ curve has a slope of $-V_T$. This rise in $V_{ECO}$ with decreasing $I_B$ does not continue indefinitely. When the junction voltages become comparable in magnitude to $V_T$, the $-\left(1/\beta_R + C_4\right)$ term of (3.1), neglected in this analysis, becomes significant, and eventually $V_{ECO}$ is zero when $I_B = 0$.

For the symmetric device, the approximations made in obtaining (3.12) are no longer valid, and a simple analytic expression for $e_0$ cannot be obtained. However, when the recombination terms dominate the base current, (1.6) reduces to

$$I_B = I_B = C_2 I_S E^{-1/n_E} + C_4 I_S C^{-1/n_C},$$

(3.19)

with $C_2 = C_4 = C_B$ and $n_E = n_C = n$, this becomes

$$I_B = I_S C_B C^{1/n} \left[ 1 + \left( \frac{E}{C} \right)^{1/n} \right],$$

(3.20)

which for $I_C = 0$ becomes
\[ I_B = I_S C_B C^{1/n} (1 + e_0^{1/n}). \] (3.21)

Under the same conditions (3.6) may be written:

\[ e_0 = 1 + \frac{C_B}{C(1-1/n)}. \] (3.22)

From (3.21) and (3.22) the following expression is obtained:

\[ e_0 = 1 + C_B \left( \frac{I_S}{I_B} \right)^{n-1} \left( 1 + e_0^{1/n} \right)^{n-1}. \] (3.23)

This is readily solved for \( e_0 \), and hence \( V_{ECO} \), using iterative techniques. A solution for the Q009 device, with \( C_B = 140 \) and \( n = 1.56 \), is plotted in Figure 3.1. Again, \( V_{ECO} \) is seen to rise with decreasing base currents. If \( n=2 \) and \( I_B \ll I_S C_B^2 \), (3.23) reduces to

\[ e_0 = (I_S C_B^2/I_B)^2. \] (3.24)

The slope of the \( V_{ECO} \) versus \( \ln I_B \) characteristic is then \(-2V_T\), twice that which was obtained for the conventional device at very low currents.

From the circuit design point of view, the rise in \( V_{ECO} \) at low base currents is of major consequence. First, modulation of the base current, such as that which might occur if the ZBT were employed as a small-signal variable gain element in a compression amplifier, produces a corresponding modulation of the collector-emitter voltage. Special precautions, such as capacitive filtering, must then be taken to prevent such spurious fluctuations of \( V_{ECO} \) from affecting circuit performance.
Figure 3.1. DC open-circuited collector-emitter voltage $V_{ECO}$ of the Q009 transistor as a function of base current (substrate biased at -2V with respect to the emitter).
Second, a large dc voltage between the emitter and collector is in itself undesirable in cases where supply voltages are limited. Values of $V_{ECO}$ commonly reach 100 - 200 mV at low currents. In hearing-aid or pacemaker circuits, for example, this represents a large fraction of the supply voltage, which is usually of the order of one or two volts.

From the discussion of conventional and symmetric devices at low base currents, it is clear that the break-point value of $I_B$, at which $V_{ECO}$ begins to rise, is roughly proportional to $C_B^2$ ($C_H^2$ in the conventional device) for values of $n$ near 2. Device processing efforts should be concentrated on ensuring that recombination effects are sufficiently low that this break-point current does not lie within the desired range of operation. At moderate current levels, where recombination effects are negligible, $V_{ECO}$ is reduced by maintaining $\beta_R$ (or $\beta_F$ in the open-emitter case) as large as possible, as is clear from (3.8) and (3.9). The inevitable rise in external collector-emitter voltage at high currents, implicit in (3.10) and (3.11) is tamed somewhat by keeping the emitter series resistance to a minimum. In sum, with proper fabrication it should be possible to produce a ZBT having low, reasonably constant values of $V_{ECO}$ over several decades of $I_B$.

Up until this point the effects of the parasitic substrate p-n-p's have been ignored. With recombination and high-current effects neglected, (1.5) and (1.10) enable the collector current to be written as

$$i_C = I_S (E-C) - \frac{I_S}{B_R} C - I_{SS} (C-S) , \quad (3.25)$$
where

\[ S = \exp \left( \frac{V_{SB}}{V_T} \right) \]  

(3.26)

With the substrate reverse-biased \( S = 0 \); then for \( i_C = 0 \), (3.25) yields

\[ V_{ECO} = V_T \ln \left( 1 + \frac{1}{\beta R} + \frac{I_{SS}}{I_S} \right). \]  

(3.27)

It can be seen that the substrate will substantially increase \( V_{ECO} \) if the ratio \( I_{SS}/I_S \) is comparable in magnitude to \( 1/\beta_R \). In the case of the Q009 device, for which \( I_{SS}/I_S = 0.131 \) and \( 1/\beta_R = 0.066 \), (3.27) yields \( V_{ECO} = 4.58 \text{ mV} \). Without the parasitic p-n-p action (3.27) reduces to (3.8) and \( V_{ECO} \) is then only 1.63 mV. In these and all subsequent calculations \( T \) is taken to be 22.5\( ^\circ \text{C} \). Parasitic action also results in a power drain due to \( I_{SUB} \). For a symmetric device (1.6), (1.9) and (1.10) yield, under the conditions assumed in deriving (3.27),

\[ \left| \frac{I_{SUB}}{I_E} \right|_{i_C=0} = \left( 1 + \frac{1}{\beta} \right)^{-1} \left( \frac{I_S}{I_{SS}} \right), \]  

(3.28)

where \( \beta = \beta_R = \beta_F \). For the Q009 device this ratio is calculated to be 0.663, an undesirable figure.

With reference to Figure 1.1, the parasitic effects can be reduced by extending the \( n^+ \) base contact diffusion down to the buried layer, thus suppressing the collection of holes at the junction between the \( n \)-type epitaxial layer and the \( p^{++} \) isolation wall. Incorporation of such a deep \( n^+ \) diffusion into conventional lateral p-n-p's has been found to reduce the open-collector ratio of \( I_{SUB} \) to \( I_E \) by a factor
ranging from over 4 at low currents to about 2 at $I_E = 1\text{mA}$ [17].

It is useful to briefly consider a parallel configuration of two identical conventional asymmetric devices, with the two base terminals connected together, and the emitter terminal of one device connected to the collector terminal of the other and vice versa. Such a configuration might conceivably be employed in place of a single symmetric device. With the substrate, high-current, and recombination effects ignored, an analysis similar to that used to obtain (3.8) yields

$$V_{ECO} = V_T \ln \left[ 1 + \frac{1}{2} \left( \frac{1}{\beta_F} + \frac{1}{\beta_R} \right) \right].$$

(3.29)

Here one of the connected emitter-collector terminals has been arbitrarily defined as the emitter, and the other as the collector, with the latter left open. For the Q1 device, having parameters listed in Table 2.3, $\beta_F = 50$ and $\beta_R = 1$. Substitution of these values into (3.29) yields $V_{ECO} = 10.5 \text{mV}$. For a single symmetric device with $\beta_F = \beta_R = 50$, (3.8) yields $V_{ECO} = 0.501 \text{mV}$. The parallel configuration is clearly inferior to the latter so far as dc performance is concerned.

3.2 Computer Simulation

The program SPICE, which employs the BJT model presented in Chapter One, was used to simulate the dc open-collector behaviour of the Q009 device. This simulation was performed both with and without the parasitic substrate devices, over a range of base currents of from 1nA to 10 mA. All parameters were taken from Tables 2.1 and 2.2. The
results are displayed in Figure 3.1, and clearly exhibit the features discussed in the preceding section.

Computer simulation was also carried out with the device operated with a fixed resistor connected in parallel with the collector and emitter terminals. Such resistors might be employed in cases where it is desirable to limit the maximum small-signal impedance between the collector and emitter terminals, for example in the feedback loop of an amplifier. As indicated by the results displayed in Figure 3.2, such parallel resistance substantially reduces the dc collector-emitter voltage at low base currents. This desirable consequence may be explained qualitatively by noting that a small current flows through the resistor into the collector terminal, which is at a negative voltage with respect to the emitter. This can only be due to an increase in the voltage $V_{CB}$ across the forward-biased collector-base junction. Since $V_{EC}$ is the difference $V_{EB} - V_{CB}$, it decreases. A quantitative account for this change using small-signal perturbation techniques is included in Chapter Four.

Open-collector and open-emitter simulations were also performed using the parameters listed for the conventional Q1 device. The relatively small recombination components of $I_B$ caused $V_{ECO}$ and $V_{CEO}$ to remain close to the constant values predicted by (3.8) and (3.9), respectively, at moderate base currents.
Figure 3.2. DC collector-emitter voltage of the Q009 device in parallel with a fixed resistance (substrate biased at -2V with respect to the emitter).
3.3 Experimental Results

The experimentally obtained $V_{ECO}$ versus $I_B$ data are shown in Figure 3.1. The base current was provided by the calibrated current source of a Tektronix Type 576 curve tracer; above 1 mA this was used in the pulsed mode. Below 1 mA $V_{ECO}$ was measured with an electrometer (Keithley Type 610) having an input impedance of $10^{14}$ Ω. Above 1 mA $V_{ECO}$ was displayed on the CRT of the curve tracer. The substrate was biased at -2V with respect to the grounded emitter. With the device inverted, a virtually identical set of measurements was obtained for $V_{CEO}$, as expected for a symmetric transistor.

It can be seen from Figure 3.1 that the measured and simulated results are in excellent agreement at low and moderate base currents. Above 100 μA the discrepancy noted in Chapter Two becomes apparent. Since precautions were taken to avoid errors caused by internal heating, this must be attributed to the onset of high-current effects not incorporated in the model. These include conductivity modulation in the active base region [13]. This can be important insofar as the voltage drop across the active base region can affect the $V_{ECO}$ versus $I_B$ characteristic at high base currents. Because of high power consumption and rising values of $V_{ECO}$, the ZBT is not likely to be used in this region in micropower applications. Figure 3.1 also illustrates the importance of modeling the substrate. When this is not done the simulated results are markedly inconsistent with the measurements in the mid-current region.

The results presented in Figure 3.1 establish the validity of
many of the expressions derived in Section 3.1. The mid-current value of $V_{ECO}$ predicted by (3.27), 4.58 mV, is in close agreement with the measured and simulated values. The values obtained by solving (3.23) account quite well for the rise in $V_{ECO}$ at low base currents.

Measurements of $V_{ECO}$ were also made with the Q009 transistor in parallel with 100 kΩ and 100 MΩ resistors. The results are presented in Figure 3.2. Once again, excellent corroboration is found between modeled and experimentally observed behaviour in the low and medium base current range.
CHAPTER 4

AC Characteristics of the Zero-Biased Transistor

This chapter is concerned with the modeling and understanding of the ac behaviour of the ZBT. Emphasis is placed on the small-signal collector-emitter device characteristics, including nonlinear distortion.

The Volterra-series approach is often employed in nonlinear small-signal analyses of the bipolar transistor [18], [19]. This consists of expanding each of the nonlinear elements of the large-signal model equations as a Taylor series, with the corresponding junction voltage as the independent variable. A perturbation method of analysis results in a set of complex linear node equations, one for each order. These are then solved separately and successively, beginning with the first order. Such a procedure is incorporated in the program SPICE, which computes solutions up to the third order.

4.1 Analytic Formulation

4.1.1 Small-Signal Device Characteristics

In the case of the ZBT it is possible to treat the device as a single nonlinearity at low frequencies by solving the large-signal model equations to obtain $i_C$ in terms of $I_B$ and $v_{EC}$. The result is then expanded as a Taylor series about the operating point $I_C = 0$, with $v_{EC}$ as the independent variable and $I_B$ as a parameter. This permits the
small-signal equations to be expressed in a tractable form.

From (3.6) and (1.5), for \( E, C \gg 1 \),

\[
i_C = \frac{I_S C}{q_b} \left( \frac{E}{C} - e_0 \right)
\]  

(4.1)

With \( I_C = 0 \), \( i_C \) is then simply the small-signal current \( i_C \). Similarly,

\[ v_{EC} = v_{ECO} + v_{ec} \]

where \( v_{ec} \) is the small-signal collector-emitter voltage. Let \( x \) be defined by

\[ x = \frac{v_{ec}}{V_T} \]

(4.2)

(4.1) now becomes

\[
i_C = \frac{I_S C}{q_b} e_0 (e^x - 1)
\]

(4.3)

It is useful to consider separately the small-signal behaviour at moderate levels of base current, where recombination is negligible, and at low levels, where recombination dominates. In both cases \( q_b \) is assumed to be unity, with the effects of high-level injection left to the section on computer simulation.

At moderate base currents (1.60 yields

\[
i_B = i_B = \frac{I_S C}{a_R} \left( 1 + \frac{1}{b} e_0 e^x \right)
\]

(4.4)

with

\[
b = \frac{\alpha}{\beta_R}
\]

(4.5)

(4.3) and (4.4), with \( q_b = 1 \), are combined to yield \( i_C \) as a function of \( v_{ec} \) and \( i_B \):
\[ i_c = \frac{\beta_R}{1 + \frac{1}{b} e_0 e^x} (e^x - 1) \]  

(4.6)

When the exponential terms in the numerator and denominator are expanded as Taylor series about the point \( v_{EC} = V_{ECO} \) (i.e. \( I_C = 0 \)), and the mid-current expression (3.7) for \( e_0 \) utilized, (4.6) becomes

\[ i_c = g_{eco} (v_{ec} + \frac{g_2}{g_{eco}} v_{ec}^2 + \frac{g_3}{g_{eco}} v_{ec}^3) \]  

(4.7)

where

\[ g_{eco} = \frac{I_B}{V_T} \frac{\beta_F (\beta_R + 1)}{(\beta_F + \beta_R + 1)} \]  

(4.8)

\[ g_2/g_{eco} = \frac{1}{V_T} \left[ \frac{1}{2} - \frac{\beta_R + 1}{\beta_F + \beta_R + 1} \right] \]  

(4.9)

and

\[ g_3/g_{eco} = \frac{1}{V_T^2} \left[ \frac{1}{8} - \frac{\beta_F (\beta_R + 1)}{(\beta_F + \beta_R + 1)^2} \right] \]  

(4.10)

The term \( g_{eco} \) is simply the small-signal collector-emitter conductance; the second and third-order nonlinearities in \( i_c \) are represented by \( g_2 \) and \( g_3 \), respectively. The ratios \( g_2/g_{eco} \) and \( g_3/g_{eco} \) are extremely useful in comparing the distortion-producing capabilities of two different devices for particular values of \( g_{eco} \). Note that they are constant. Terms above the third order have been omitted; under small-signal conditions they will generally be negligible. For inverted operation, expansion of \( i_c \) yields expressions identical to (4.8) - (4.10) with \( \beta_R \) and \( \beta_F \) interchanged.

From (4.9) it is clear that second-order nonlinear distortion
vanishes for $\beta_F = \beta_R + 1$. In practice this condition is closely realized if $\beta_F = \beta_R >> 1$. It is here that the advantages of a symmetric device become apparent. For conventional devices, where $\beta_F >> \beta_R$ and $\beta_F >> 1$, $|g_2/g_{eco}|$ approaches its maximum value of $1/2 V_T$, which also holds true for $g_2/g_{eco}$ with the device inverted. The third-order term, given by (4.10), is found to vanish for $\beta_F = 3.72 (\beta_R + 1)$. Under small-signal conditions third-order nonlinearities are generally of less concern than those of the second order; efforts are therefore primarily devoted to minimizing the latter. For a symmetric device with $\beta_F = \beta_R >> 1$, $g_3/g_{eco} + 1/12 V_T^2$, a factor of two lower than the value obtained when $\beta_F >> \beta_R$.

It is interesting to compare the small-signal behaviour of the bipolar transistor to that of an ideal diode. The current-voltage relationship for the latter is given by

$$i_D = I_S \left[ \exp \left( \frac{v_D}{V_T} \right) - 1 \right], \quad (4.11)$$

where $i_D$ and $v_D$ are the large-signal diode current and voltage respectively. Charge-storage effects have been neglected. With (4.11) expanded about the operating point $i_D = I_D$, the small-signal current $i_d$ may be expressed as

$$i_d = \frac{I_D}{V_T} \left( v_d + \frac{v_d^2}{2V_T} + \frac{v_d^3}{6V_T^2} \right) \quad (4.12)$$

Terms above the third order have again been omitted. Comparison of the small-signal diode conductance $I_D/V_T$ to that of a symmetric transistor
(for which \( g_{\text{eco}} + I_B \beta/2V_T \) when \( \beta = \beta_F = \beta_R >> 1 \) reveals that the dc current drawn by the latter is only about \( 1/\beta \) that required by the diode at a given conductance. Furthermore, the magnitudes of the nonlinear terms in (4.12) are only approached by the transistor under worst-case conditions (i.e. \( \beta_F >> \beta_R; \beta_F >> 1 \), or \( \beta_R >> \beta_F; \beta_R >> 1 \)). Thus, the transistor would appear to be the obvious choice of the two devices for small-signal variable-impedance applications.

It is also interesting to compare the symmetric transistor to the parallel configuration of two identical conventional devices discussed in Chapter Three. An analysis of this configuration at moderate current levels yields

\[
g_{\text{eco}} = \frac{2I_B}{V_T} \frac{\beta_F \beta_R (2\beta_F \beta_R + \beta_F + \beta_R)}{(\beta_F + \beta_R)(4\beta_F \beta_R + \beta_F + \beta_R)} \tag{4.13}
\]

\[
g_2/g_{\text{eco}} = \frac{1}{V_T} \left[ \frac{1}{2} - \frac{2\beta_F \beta_R + \beta_F + \beta_R}{4\beta_F \beta_R + \beta_F + \beta_R} \right] \tag{4.14}
\]

\[
g_3/g_{\text{eco}} = \frac{1}{V_T} \left[ \frac{1}{2} - \frac{2\beta_F \beta_R (2\beta_F \beta_R + \beta_F + \beta_R)}{(4\beta_F \beta_R + \beta_F + \beta_R)^2} \right] \tag{4.15}
\]

As a numerical example, let \( \beta_F = 50 \) and \( \beta_R = 1 \) (the parameters of the Q1 device listed in Table 2.3), with \( T = 22.5^\circ \text{C} \). Then (4.14) and (4.15) yield, respectively, \( g_2/g_{\text{eco}} = -3.99 \) and \( g_3/g_{\text{eco}} = -112 \). For a symmetric device having \( \beta_F = \beta_R = 50 \), (4.9) and (4.10) yield, respectively, \( g_2/g_{\text{eco}} = -0.194 \) and \( g_3/g_{\text{eco}} = -128 \). While the third-order term \( g_3/g_{\text{eco}} \) is similar in each of the two cases, the parallel pair is seen to exhibit significantly greater second-order nonlinearity.
than the comparable symmetric device. This is true in general when the reverse current gain $\beta_R$ of each element of the parallel pair is close to unity and $\beta_F \gg \beta_R$ (as is typically the case for conventional devices). Since second-order nonlinearities are usually of greater concern, the parallel-pair configuration has little to recommend it, especially in view of its inferior dc performance.

Plots of $r_{eco} (=1/g_{eco})$, $g_2/g_{eco}$, and $g_3/g_{eco}$ for the Q009 device under moderate-current conditions are shown in Figures 4.1-4.3.

In the interest of analytic simplicity, the low-current small-signal behaviour of symmetric and conventional devices are considered separately. For the latter, the conditions $\beta_F \gg \beta_R$ and $C_4 \gg C_2$ (and $q_b = 1$) are assumed to hold. Then (3.13) and (4.3) yield

$$i_C = (I_S C + I_B) (e^x - 1) . \quad (4.16)$$

From (3.14), derived for $n_C = 2$, $C$ is seen to vary as a function of $I_B$ only. Thus

$$i_C = I_B (\beta_R + 1) \left[ 1 + \frac{I_{BL}}{2I_B} \left[ 1 - \left(1 + \frac{4I_B}{I_{BL}}\right)^{1/2} \right] \right] (e^x - 1) , \quad (4.17)$$

with $I_{BL}$ defined by (3.15). With $e^x$ expanded up to the third order, (4.17) can be expressed in a form identical to (4.7), with

$$g_{eco} = \frac{I_B (\beta_R + 1)}{V_T} \left[ 1 + \frac{I_{BL}}{2I_B} \left[ 1 - \left(1 + \frac{4I_B}{I_{BL}}\right) \right] \right] , \quad (4.18)$$
\[ \frac{g_2}{g_{e0}} = \frac{1}{2V_T} \] \hspace{1cm} (4.19) \\
\[ \frac{g_3}{g_{e0}} = \frac{1}{6V_T^2} \] \hspace{1cm} (4.20)

These expressions encompass both the low and moderate-current regions of operation. For \( I_B \gg I_{BL} \), (4.18) reduces to (4.8), when the condition \( \beta_T \gg \beta_R \) is applied to the latter. For \( I_B \ll I_{BL} \), expansion of \( (1 + 4I_B/I_{BL})^{1/2} \) in (4.18) gives

\[ g_{e0} = \frac{I_B^2}{I_{BL} V_T} \left( \beta_R + 1 \right) \] \hspace{1cm} (4.21)

Thus \( g_{e0} \) varies from a linear function of \( I_B \) at moderate currents to a parabolic function of \( I_B \) at low currents.

With the device inverted, the expression for the resulting small-signal emitter-collector conductance \( g_{e0} \) is identical to (4.18), except that the term \( (\beta_R + 1) \) is replaced by \( \beta_R \). The second and third-order terms are \(-1/2V_T\) and \(1/6V_T^2\), respectively.

In discussing the symmetric device, it is assumed that \( I_B \) is dominated by the recombination terms, and that \( C_2 = C_4 = C_B \) and \( n_C = n_E = n \). Recalling (3.20), \( C \) may then be expressed directly as a function of \( I_B \) and \( v_{EC} \):

\[ C = \left( \frac{I_S C_B}{I_B} (1 + e_0^{1/n} e^{x/n}) \right)^{-n} \] \hspace{1cm} (4.22)

Combination of (4.22) and (4.3), and expansion of the exponential terms in \( x \) as Taylor series about \( v_{EC} = V_{EC0} \) enables \( I_o \) to be expressed in a form identical to (4.7), with
\[
g_{\text{eco}} = \frac{I_B}{V_T} \frac{e_0}{(e_0 - 1)(1 + e_0^{1/n})} \tag{4.23}
\]
\[
g_2/g_{\text{eco}} = \frac{1}{V_T} \left[ \frac{1}{2} - \frac{e_0^{1/n}}{1 + e_0^{1/n}} \right] \tag{4.24}
\]
\[
g_3/g_{\text{eco}} = \frac{1}{V_T} \left[ \frac{1}{6} - \frac{1}{2} \left( \frac{n + 1}{n} \right) \left( \frac{e_0^{1/n}}{1 + e_0^{1/n}} \right)^2 \right. \\
+ \left. \frac{1}{2} \left( \frac{n + 1}{n} \right) \left( \frac{e_0^{1/n}}{1 + e_0^{1/n}} \right) \right] . \tag{4.25}
\]

The expression (3.23) is used to solve for \( e_0 \). Plots of \( r_{\text{eco}} \) and \( g_2/g_{\text{eco}} \) under low-current conditions for the Q009 device are shown in Figures 4.1 and 4.2. In general \( g_2/g_{\text{eco}} \) rises slowly with decreasing base current, due to the increase in \( e_0 \) discussed in Chapter Three. The third-order term \( g_3/g_{\text{eco}} \) is positive at very low base currents, where the second and third terms in the bracketed quantity on the right-hand side of (4.25) tend to cancel. It is interesting to note that the mid-current value of \( g_3/g_{\text{eco}} \), given by (4.10), is always negative for a symmetric device. Thus, a null in \( g_3/g_{\text{eco}} \) will be observed as \( I_B \) passes from the mid-current to the low-current region.

When \( I_B \ll I_S C_B^2 \) and \( n = 2 \), (4.25) reduces, with the aid of (3.24), to
\[
g_{\text{eco}} = \frac{I_B^2}{I_S C_B^2} \frac{1}{V_T}. \tag{4.26}
\]
Figure 4.1. Small-signal collector-emitter resistance of the Q009 transistor as a function of base current.
Figure 4.2. Second-order nonlinearity $g_2/g_{eco}$ of the Q009 transistor as a function of base current.
Figure 4.3. Third-order nonlinearity $g_3/g_{ec0}$ of the Q009 transistor as a function of base current.
This is very similar to the expression (4.21) obtained for the conventional device, with \( I_{\text{BL}}/(\beta_R + 1) \) replaced by \( I_S C_B^2 \). Once again \( g_{\text{eco}} \) changes from a linear to a parabolic function at low currents. The break-point current marking this transition, \( I_B = I_S C_B^2 \), is identical to that at which \( V_{\text{EO}} \) begins to rise. From a practical point of view this "corner" is undesirable, as it disrupts the linear relationship between the small-signal impedance and the controlling base current provided by (4.8). Thus the remarks made in Chapter Three about reducing recombination effects also apply here. Suppressing these effects also avoids the increase in \( g_2/g_{\text{eco}} \) and \( g_3/g_{\text{eco}} \) described above.

4.1.2 Effects of the Substrate

Thus far the effects of the substrate on the small-signal device behaviour have been neglected. They are most easily treated by first reverting to the Volterra-series approach discussed at the beginning of this chapter, in which each nonlinearity of the large-signal model equations is individually expanded as a Taylor series about the operating point. The first-order terms of the resulting small-signal model equations define the linear small-signal model, shown in Figure 4.4 for a three-terminal p-n-p device. The currents are defined to be positive when flowing out of the device.

In Figure 4.4,

\[
\frac{\partial g_f}{\partial v_{EB}} = \left. \frac{\partial i_C}{\partial v_{EB}} \right|_Q
\]  

(4.27a)
\[ g_r = \frac{\partial i_C}{\partial v_{CB}} \quad Q \quad (4.27b) \]

\[ g_\pi = \frac{\partial i_B}{\partial v_{EB}} \quad Q \quad (4.27c) \]

\[ g_\mu = \frac{\partial i_B}{\partial v_{CB}} \quad Q \quad (4.27d) \]

The low-frequency first-order model equations may then be written as

\[ i_C = g_f v_{eb} + g_r v_{cb} \quad (4.28) \]

\[ i_B = g_\pi v_{eb} + g_\mu v_{cb} \quad (4.29) \]

First-order small-signal charge-storage effects are modeled by the capacitors \( c_\pi \) and \( c_\mu \):

\[ c_\pi = \frac{\partial q_E}{\partial v_{EB}} \quad Q \quad (4.30a) \]

\[ c_\mu = \frac{\partial q_C}{\partial v_{CB}} \quad Q \quad (4.30b) \]

The ohmic series resistances \( R_E, R_C, \) and \( R_B \) are added at the external terminals. They are omitted here, since their effect on the ZBT is assumed negligible outside the high-current region.

The higher-order terms of the nonlinear model equations can be represented as equivalent current sources added to the linear model of Figure 4.4 [18], [19]. These are used in solving the higher-order node
Figure 4.4. Linear small-signal model of the p-n-p transistor.
equations, following the perturbation method. In the case of the ZBT it is simpler at low frequencies to solve the first-order equations to obtain \( i_c \) in terms of \( v_{ec} \), then expand the result as a power series in \( v_{ec} \) to include the higher-order terms. This reduces the number of equivalent current sources required at each order to one.

Before proceeding it is useful to obtain an expression for \( i_c \) in terms of the low-frequency first-order elements (4.7a)-(4.7d). With the assumption of an ideal current source supplying \( I_B, i_b \) is zero. Then (4.29) and Kirchhoff's laws enable \( v_{eb} \) and \( v_{cb} \) to be expressed in terms of \( v_{ec} \):

\[
v_{eb} = \frac{g_{\mu}}{g_{\mu} + g_\pi} v_{ec}
\]

\[
v_{cb} = \frac{-g_\pi}{g_{\mu} + g_\pi} v_{ec}
\]

This enables (4.28) to be expressed as

\[
i_c = \frac{g_f - g_r g_\pi/g_{\mu}}{1 + g_\pi/g_{\mu}} v_{ec}
\]

With the right-hand side expanded as a Taylor series with \( v_{ec} \) as the independent variable, (4.33) may be expressed in a form identical to (4.8), with

\[
g_{eco} = f(v_{eb}, v_{cb}) \bigg|_{I_c = 0} \quad (4.34)
\]

\[
g_2 = \frac{1}{2} \frac{d}{d v_{ec}} f(v_{eb}, v_{cb}) \bigg|_{I_c = 0}
\]

\[
(4.35)
\]
\[ g_3 = \frac{1}{6} \frac{d^2}{v_{ee}} f(v_e, v_{eb}) \bigg|_{I_C = 0} \]  \hspace{1cm} (4.36)

where

\[ f(v_e, v_{eb}) = \frac{g_r - g_r \frac{g_m}{g_m}}{1 + g_m/g_m} \]  \hspace{1cm} (4.37)

Assume recombination and high-current effects to be negligible.

Then from (1.5) and (1.6), the low-frequency first-order elements are

\[ g_f = \frac{I_S E}{V_T} \]  \hspace{1cm} (4.38a)

\[ g_r = - \left( 1 + \frac{1}{\beta_R} \right) \frac{I_S E}{V_T} \]  \hspace{1cm} (4.38b)

\[ g_m = \frac{1}{\beta_r} \frac{I_S E}{V_T} \]  \hspace{1cm} (4.38c)

\[ g_m = \frac{1}{\beta_r} \frac{I_S C}{V_T} \]  \hspace{1cm} (4.38d)

Substitution of (4.38a)-(4.38d) into (4.34) yields, with the aid of (3.7),

\[ g_{eco} = g_{fo} \]  \hspace{1cm} (4.39)

where \( g_{fo} \) is the value of \( g_f \) at \( I_C = 0 \). Note that \( g_{fo} \) is simply the quantity on the right-hand side of (4.8).

The parasitic substrate devices are readily incorporated into the nonlinear small-signal model. Under the assumptions stated in the preceding paragraph, and with the substrate reverse-biased, (1.5), (1.6), (1.9), (1.10) and Figure 1.3 enable \( I_C, I_B \) and \( I_E \) to be written
\[ i_C = I_S (E - C) - \frac{I_S}{\beta_R} C - I_{SS} C \quad (4.40) \]

\[ i_B = \frac{I_S}{\beta_F} + \frac{I_S}{\beta_R} \quad (4.41) \]

\[ i_E = -I_S (E - C) - \frac{I_S}{\beta_F} E - I_{SS} E \quad (4.42) \]

Inspection of (4.40) and (4.42) reveals that two new elements must now be included in the first-order small-signal model, in addition to those given by (4.38a)–(4.38d). These are defined by

\[ g_{sr} = \frac{\partial i_C}{\partial v_{CB}} \quad (4.43a) \]

\[ g_{sf} = \frac{\partial i_E}{\partial v_{EB}} \quad (4.43b) \]

Then from (4.40) and (4.42),

\[ g_{sr} = - \frac{I_{SS} C}{V_T} \quad (4.44a) \]

\[ g_{sf} = - \frac{I_{SS} E}{V_T} \quad (4.44b) \]

The first-order expressions for \( i_C \) and \( i_E \) may then be written as
\[
\begin{align*}
    i_c &= g_f v_{eb} + g_r v_{cb} + g_{sr} v_{cb} \quad \text{(4.45)} \\
    i_e &= -g_f v_{eb} - g_r v_{cb} \\
    &\quad - g_{\pi} v_{eb} - g_{\mu} v_{eb} + g_{sf} v_{eb} \quad \text{(4.46)}
\end{align*}
\]

At high frequencies the capacitor \( c_{\text{SUB}} \), defined by
\[
c_{\text{SUB}} = \frac{\partial q_{\text{SUB}}}{\partial v_{SB}} |_{Q}
\]

is added across the base-substrate junction. The linear small-signal p-n-p model with the parasitic substrate devices included is shown in Figure 4.5.

If the base current supply is assumed to be ideal, the expressions (4.31) and (4.32) are found to hold true even with the parasitic substrate devices included in the model. This enables (4.45) to be expressed as
\[
    i_c = \left[ g_f - g_r \frac{g_{\mu}}{g_{\pi}} - g_{sr} \left( \frac{g_{\pi}/g_{\mu}}{1 + g_{\pi}/g_{\mu}} \right) \right] v_{ec} \quad \text{(4.48)}
\]

When \( I_C = 0 \) the bracketed term is equal to \( g_{ec0} \). Using (4.48), (4.38a)-(4.38d) and (4.44a), one obtains, with the aid of (3.27) and (4.41),
\[
g_{ec0} = \frac{I_B}{V_T} \left[ \frac{\beta_F (\beta_R + 1) + \beta_F \beta_R (I_{SS}/I_S)}{\beta_F + \beta_R + 1 + \beta_R (I_{SS}/I_S)} \right] \quad \text{(4.49)}
\]

Note that for \( I_{SS} = 0 \) (4.49) reduces to (4.8). The influence of the parasitic substrate transistors on \( g_{ec0} \) is small for \( I_{SS}/I_S \ll 1 \). In the case of the Q009 device, for which \( I_{SS}/I_S = 0.131 \), the values of
Figure 4.5. Linear small-signal model of the p-n-p transistor including the substrate, with the substrate reverse-biased.
$g_{eO}$ obtained using (4.49) differ by 5% at any given base current from the values of $g_{eO}$ calculated using (4.8). Contrast this with the dramatic effect of the substrate on $V_{E0}$ at moderate base currents for the Q009 device, discussed in Chapter Three.

The higher-order terms of $i_c$ are obtained by expanding (4.48). The effects of the parasitic substrate transistors on device nonlinearity are more readily determined by means of computer simulation. This is discussed in Section 4.2.

4.1.3 Effects of a Parallel Resistance

As was stated in the preceding chapter, the ZBT is likely in many applications to be employed in parallel with a fixed resistance between the collector and emitter terminals. The dc collector current is no longer zero in this case; if $R_p$ is the fixed resistance, then

$$i_c = \frac{V_{EC}}{R_p}$$

$$= \frac{V_{EC}}{G_p}$$

where $G_p = 1/R_p$. Under small-signal conditions, $i_c$ can be expressed in the form of (4.7), which is repeated here for reference:

$$i_c = g_{eO} v_{ec} + g_2 v_{ec}^2 + g_3 v_{ec}^3$$

with $v_{ec} = V_{EC} - V_{E0}$. Assume that $i_c$ is sufficiently small that it can be treated as a small-signal perturbation. Then if Kirchhoff's law is applied at the collector terminal, (4.50) and (4.51), with the higher order terms neglected, yield
\[ V_{EC} = \frac{1}{1 + \frac{G_p}{g_{eco}}} V_{ECO} \]  \hspace{1cm} (4.52)

With \( g_{eco} \) obtained using (4.8) or (4.23) (depending on the base current level), the deviation in \( V_{EC} \) from its open-collector value due to \( G_p \) is readily obtained with the aid of (4.52). When this is done for the Q009 device, with values of \( V_{ECO} \) taken from the measured points of Figure 3.1, the resulting \( V_{EC} \) versus \( I_B \) characteristics for \( R_p = 10^5 \, \Omega \) and \( R_p = 10^7 \, \Omega \) are found to be in very close agreement with the measured characteristics of Figure 3.2.

Assuming once again that \( i_C \) can be treated as a small-signal perturbation, the change in small-signal collector-emitter conductance due to \( G_p \) may be estimated. Let this change be defined as \( \Delta g_{ec} \). Then, using (4.51),

\[
\Delta g_{ec} = \left. \frac{\partial g_{eco}}{\partial V_{EC}} \right|_{I_C = 0} (V_{EC} - V_{ECO}) = 2 g_2 (V_{EC} - V_{ECO}) \]  \hspace{1cm} (4.53)

Then with the aid of (4.52)

\[
\frac{\Delta g_{ec}}{g_{eco}} = -2 \left( \frac{g_2'}{g_{eco}} \right) \frac{V_{ECO}}{1 + g_{eco}/G_p} \]  \hspace{1cm} (4.54)

In general this change will be quite small. In the case of the Q009 device, with \( G_p = 10^{-5} \) mhos, \( \Delta g_{ec}/g_{eco} \) is found, using (4.8), (4.9) and (4.54), to be \( 2.5 \times 10^{-9} \) when \( I_B = 10^{-8} \) A, decreasing to \( 1.1 \times 10^{-5} \) when \( I_B = 10^{-5} \) A. A similar type of analysis can be used to show that the changes in the higher-order terms \( g_2 \) and \( g_3 \) due to the parallel
resistance $R_p$ are also very small, and can generally be neglected.

With reference to (4.6), the small signal expansion for $i_c$ is valid only for $|x| << x^2/2$; that is, $|v_{EC} - V_{ECO}| << 2 V_T$. Then from (4.51) and (4.52) the small-signal approximation for the collector current is valid only for

$$\frac{R_p}{r_{SCO}} + 1 \gg \frac{V_{ECO}}{2 V_T} \quad (4.55)$$

In practice this condition will always hold, except possibly at very low base currents, when $r_{SCO}$ is very large and $V_{ECO}$ may exceed $2 V_T$.

4.1.4 Effects of a Finite Base-Drive Impedance

Under real operating conditions the current source supplying the base current of the ZBT has a finite output impedance $R_S$. The equivalent small-signal representation of this current source is simply a conductance $G_S$ between the base and emitter terminals. Then with reference to Figure 4.1, (4.28), and (4.29), the resulting circuit equations are

$$i_c = g_f v_{eb} + g_r v_{cb} \quad (4.55a)$$

$$G_S v_{eb} + g_\pi v_{eb} + g_\mu v_{cb} = 0 \quad (4.55b)$$

Solution for $i_c$ in terms of $v_{ec}$ yields

$$i_c = \frac{g_f g_\mu - g_r g_\mu - g_r G_S}{g_\mu + g_\pi + G_S} v_{eco} \quad (4.56)$$
Evaluation of the bracketed quantity for $I_C = 0$ yields, with the aid of (3.7) and (4.38a)-(4.38d),

$$g_{eco} = g_{fo}^*$$  \hspace{1cm} (4.57)

This is identical to (4.39), the result obtained under the assumption of an ideal base current source. Expanding (4.56) in terms of $v_{ec}$, one obtains, again with the aid of (3.7) and (4.38a)-(4.38d),

$$g_2 / g_{eco} = \frac{1}{G_S V_T} \frac{1}{2} \frac{1}{V_T} \left( \frac{1}{1 + \frac{G_S V_T}{I_B}} \right) \left( \frac{\frac{G_S V_T}{I_B}}{2} - \frac{\frac{\beta_R + 1}{\beta_R + \beta_F + 1}} \right)$$

$$- \frac{1}{2} \frac{G_S V_T}{g_{eco}} \frac{A^2 \left( 1 + \frac{G_S V_T}{I_B} \right) - 1}{A^2 \left( 1 + \frac{G_S V_T}{I_B} \right)} - \left( \frac{\beta_R + 1}{\beta_R + 1} \right)$$

where

$$A = \frac{\beta_F + \beta_R + 1}{\beta_F}$$  \hspace{1cm} (4.59)

Note that as $G_S \rightarrow 0$ (4.58) reduces to (4.9).

It is apparent from (4.58) that the effect of $G_S$ on second-order nonlinear distortion is negligible under the condition

$$\frac{G_S V_T}{I_B} \ll 1.$$  \hspace{1cm} (4.60)

When this condition is violated $\left| g_2 / g_{eco} \right|$ increases dramatically, approaching $1/(2V_T)$ as $R_S \rightarrow 0$. Similarly, third-order nonlinear distortion rises as $R_S$ decreases.
4.1.5 ZBT Attenuator Circuit

Figure 4.6 shows a simple small-signal attenuator circuit employing the ZBT. The blocking capacitor $C_B$ will not affect performance above 50 Hz, even with $r_{ec}$ as low as 10 Ω. The output terminal is shown unloaded; loads of finite impedance will appear in parallel with $R_p$. Note that $R_p$ has been connected directly across the transistor. As discussed in Section 4.1.3, the small-signal expressions developed for $I_C = 0$ will continue to be valid. Furthermore, direct connection of $R_L$ to the transistor has the beneficial effect of suppressing the rise in $V_{ECO}$ at low base currents.

With the output assumed to be unloaded, the nonlinear small-signal circuit equation can be written

$$G_L (v_i - v_o) - G_P v_o - i_c = 0,$$  \hspace{1cm} (4.61)

where $i_c$ is the nonlinear small-signal collector current. At low frequencies $i_c$ can be written as an expansion in the form of (4.51), with $v_{ec}$ as the independent variable. Noting that $v_o = v_{ec}$, (4.61) becomes

$$G_L (v_i - v_o) - G_P v_o - g_{ec} v_o - g_2 v_o^2 - g_3 v_o^3 = 0.$$  \hspace{1cm} (4.62)

At high frequencies the expansion for $i_c$ will include time-dependent terms which are due to the nonlinear charge-storage elements $q_E$ and $q_C$.

Assume that the small-signal output voltage $v_o$ may be written as

$$v_o(t) = \sum_{m=1}^{3} H_m v_i^m(t),$$  \hspace{1cm} (4.63a)
with
\[
\begin{vmatrix}
H_1 v_1(t) \\
H_2 v_1^2(t) \\
H_3 v_1^3(t)
\end{vmatrix} \gg \begin{vmatrix}
H_2 v_1^2(t) \\
H_3 v_1^3(t)
\end{vmatrix} \quad (4.63b)
\]

\(H_1, H_2,\) and \(H_3\) are, respectively, the first, second, and third-order transfer functions of the circuit. Substitution of (4.63a) into (4.62) yields
\[
v_1(t) \left[ G_L (1 - H_1) - \left( G_p + g_{\text{eco}} \right) H_1 \right] + v_1^2(t) \left[ -G_L H_2 - \left( G_p + g_{\text{eco}} \right) H_2 - g_2 H_1^2 \right] + v_1^3(t) \left[ -G_L H_3 - \left( G_p + g_{\text{eco}} \right) H_3 - 2 g_2 H_1 H_2 - g_3 H_1^3 \right] = 0 \quad (4.64)
\]

Terms above the third order have been omitted. Following the perturbation method [18], [20], the second and third-order terms are neglected and the first-order term solved for \(H_1\). This yields
\[
H_1 = \frac{G_L}{g_{\text{eco}} + G_p + G_L} \quad (4.65a)
\]

Note that \(H_1\) is simply the linear transfer function of the circuit. Similarly, with the third-order term neglected, (4.64) is solved for \(H_2\). This yields
\[
H_2 = \frac{-g_2 H_1^2}{g_{\text{eco}} + G_p + G_L} \quad (4.65b)
\]

Finally, solution for \(H_3\) yields
\[
H_3 = \frac{-2 g_2 H_1 H_2 - g_3 H_1^3}{g_{\text{eco}} + G_p + G_L} \quad (4.65c)
\]
Consider an input signal of the form

\[ v_1(t) = v_1 \cos \omega t \quad , \quad (4.66a) \]

where \( v_1 \) is a constant. Then

\[ v_1^2(t) = \frac{v_1^2}{2} (\cos 2\omega t + 1) \quad (4.66b) \]

\[ v_1^3(t) = \frac{v_1^3}{4} (\cos 3\omega t + 3 \cos \omega t) \quad (4.66c) \]

With reference to (4.63a), the dc term of \( v_1^2(t) \) appears in \( v_0(t) \), and can cause a small change in the dc operating point of the ZBT. However, when the condition (4.63b) is satisfied its effect is negligible. Similarly, \( v_1^3(t) \) gives rise to a term \( \frac{3}{4} H_3 v_1^3 \cos \omega t \) appearing in \( v_0(t) \). This adds to the term \( H_1 v_1 \cos \omega t \), giving rise to expansion or compression of the fundamental frequency component of \( v_0(t) \). This too is negligible when (4.63b) is satisfied. Then (4.63a) may be written as

\[ v_0(t) = H_1 v_1 \cos \omega t + \frac{1}{2} H_2 v_1^2 \cos 2\omega t \]

\[ + \frac{1}{4} H_3 v_1^3 \cos 3\omega t \quad . \quad (4.67) \]

Define the fractional second harmonic distortion \( HD_2 \) as the ratio of the amplitude of the second harmonic at the output to the amplitude to the fundamental at the output. Then from (4.67)

\[ HD_2 = \frac{1}{2} \frac{H_2}{H_1} v_1 \quad . \quad (4.68a) \]
Similarly, define the fractional third harmonic distortion $HD_3$ as the ratio of the amplitude of the third harmonic at the output to the fundamental at the output. From (4.67)

$$HD_3 = \frac{1}{4} \frac{H_3}{H_1} v_1^2 .$$

(4.68b)

These quantities are readily evaluated for the attenuator circuit using (4.65a)-(4.65c):

$$HD_2 = -\frac{1}{2} \frac{g_2}{G_L} H_1^2 v_1$$

(4.69a)

$$HD_3 = \frac{1}{4} \frac{H_1^3}{G_L} \left[ \frac{2g_2^2 H_1}{G_L} - g_3 \right] v_1^2 .$$

(4.69b)

It is apparent from (4.65a) that $H_1$ approaches unity for $g_{eco} \ll g_p$. Since $g_2$ and $g_3$ decrease with $g_{eco}$, second and third harmonic distortions are minimized for $H_1 = 1$.

Intermodulation distortion, which is usually of greater consequence in practical applications, is readily analyzed by considering an input signal of the form

$$v_1(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t .$$

(4.70)

This is substituted into (4.63a) to obtain the second and third-order intermodulation terms of $v_0(t)$. Under small-signal conditions these are found to be constant multiples of the corresponding harmonic distortion terms [20].
4.1.6 Variable-Gain Feedback Amplifier

The techniques used in the preceding section can be applied to circuits of arbitrary complexity. As an illustration, consider the circuit of Figure 4.7, where the ZBT is employed in the feedback loop of an amplifier. The gain thus depends on $g_{eco}$, which is controlled by $I_B$. If the open-loop gain of the amplifier is very large, $v_f = v_i$, and Kirchhoff's law yields simply

$$G_1 v_i = i_c$$

$$= g_{eco} (v_o - v_i) + g_2 (v_o - v_i)^2$$

$$+ g_3 (v_o - v_i)^3.$$  \hspace{1cm} (4.71)

When $v_o$ is expressed in the form of (4.63a), the first-order circuit equation becomes

$$G_1 v_i = -g_{eco} (H_1 - 1) v_i,$$  \hspace{1cm} (4.72)

which yields

$$H_1 = 1 + G_1 / g_{eco}.$$  

This is simply the small-signal gain of the amplifier. The second-order equation is

$$g_{eco} H_2 + g_2 (H_1^2 - 2H_1 + 1) = 0,$$  \hspace{1cm} (4.73)

which when solved for $H_2$ yields

$$H_2 = -\frac{g_2}{g_{eco}} (H_1 - 1)^2.$$  \hspace{1cm} (4.74)

Similarly, solution of the third-order circuit equation yields
Figure 4.7. Variable-gain feedback amplifier employing the ZBT.
\[ H_3 = \frac{1}{G_1 + g_{eco}} \left[ 2g_2 H_2 (H_1 - 1) - g_3 (H_1 - 1)^3 \right] \]  

(4.74)

The fractional second harmonic distortion, as defined by (4.68a) is

\[ HD_2 = -\frac{1}{2} \frac{g_2}{g_{eco}} \left( \frac{H_1 - 1}{H_1} \right)^2 v_1 \]  

(4.76)

which for \( H_1 \gg 1 \) reduces to

\[ HD_2 = -\frac{1}{2} \frac{g_2}{g_{eco}} H_1 v_1 \]  

(4.77)

As expected, the second harmonic distortion rises when the gain of the amplifier is increased.

4.2 Computer Simulation

The program SPICE [4] [5], which employs the BJT model discussed in the first chapter, was used to simulate the ac behaviour of the Q009 device. The input parameters used are those listed in Table 2.1; the device operating temperature was taken to be 22.5°C. Simulation was performed both with and without the parasitic substrate devices; the parameters of the latter are listed in Table 2.2.

The small-signal resistance \( r_{eco} \) was obtained with the device operated in parallel with a \( 10^{12} \) \( \Omega \) resistor between the collector and the grounded emitter, with an ideal ac voltage source coupled to the collector terminal through a large blocking capacitor. This is in essence an open-collector configuration. The results obtained at a frequency of 1 kHz, with the substrate biased at -2V with respect to the
emitter, are shown in Figure 4.1. At low base currents $r_{\text{eco}}$ approaches the asymptote predicted by (4.23). As $I_B$ is increased, $r_{\text{eco}}$ begins to approach the asymptote predicted by (4.8). At very high base currents the high-level injection term $q_b$ becomes important, and $r_{\text{eco}}$ deviates from this line. Eventually $r_{\text{eco}}$ approaches the constant value $R_E + R_C$, which is $8 \Omega$ in the case of the Q009 device.

With the substrate omitted the simulated values of $r_{\text{eco}}$ were found to be about 5% higher than those shown in Figure 4.1, as was calculated in Section 4.1.2. With the device operated in parallel with fixed resistances of 100 kΩ and 10 MΩ, the simulated small-signal collector-emitter resistance of the device remained virtually unchanged from its open-collector value $r_{\text{eco}}$, supporting the conclusion reached in Section 4.1.3.

High-frequency effects, which are difficult to analyze through manual computation, are easily simulated with the aid of a computer. The small-signal resistance $r_{\text{eco}}$ was found to remain constant for the Q009 device up to a frequency of 5 MHz, above which point it decreased, asymptotically approaching a slope of 6dB/octave. When both the emitter-base and collector-base junctions are forward-biased, as is the case with the ZBT, this decrease is due chiefly to the small-signal capacitance associated with the diffusion component of the stored charge, modeled by $\tau_F$ and $\tau_R$.

The small-signal nonlinear device behaviour was simulated with the ZBT incorporated into the attenuator circuit of Figure 4.6. Again the substrate was biased at $-2V$. Simulation was carried out at
frequencies of 1 kHz and 10 kHz. The fractional second and third-harmonic distortions for a constant output signal of 10 mV rms at the fundamental frequency are plotted in Figures 4.8 and 4.9, respectively. Note the marked increase in HD₂ at 10 kHz, especially at low base currents. This is due to the second-order nonlinearities associated with the charge-storage terms qₑ and qₑ. The program SPICE lists the contributions of each element of the BJT model to the total harmonic distortion. The contributions of qₑ and qₑ to HD₂ are negligible at 1 kHz, but become significant at 10 kHz. Third harmonic distortion remains virtually unchanged at 10 kHz. Note also the null in HD₃ at Iₑ = 6 x 10⁻⁷ A. This is due to the null in g₃, discussed earlier, which occurs during the transition from the low base current to the moderate base current region of operation.

With the aid of (4.65a)-(4.65d), g₂/gₑ and g₃/gₑ were calculated from the low-frequency (1 kHz) values of HD₂ and HD₃. They are shown in Figures 4.2 and 4.3. As base current is reduced, g₂/gₑ increases, closely following the curve predicted by (4.25). At moderate base currents g₂,gₑ is virtually independent of Iₑ. Note that the value of g₂/gₑ in this region is approximately twice that calculated using (4.9). This is due to the parasitic substrate transistors; when these are removed, the simulated value of g₂/gₑ decreases by a factor of two at moderate base currents. With reference to Figure 4.3, it can be seen that g₃/gₑ is strongly influenced by the recombination terms of the model equations, even at relatively high base currents. Only when Iₑ > 10 μA does g₃/gₑ approach the mid-current value predicted by
Figure 4.8. Fractional second harmonic distortion of the ZBT small-signal attenuator.
Figure 4.9. Fractional third harmonic distortion of the ZBT small-signal attenuator.
(4.10).

In order to determine the possible effects of a parallel resistance on $g_2$ and $g_3$, the 100 kΩ resistor of Figure 4.6 was replaced by a $10^{12}$ Ω resistor, effectively creating an open-collector condition. The simulated results for $g_2$ and $g_3$ were not found to change.

Computer simulation was also performed for the Q1 device, having parameters listed in Table 2.3. Due to the relatively small recombination components of $I_B$, the values of $r_{ec}$ and $g_2/g_{ec}$, and $g_3/g_{ec}$ were found to be very close to those predicted by (4.8)-(4.10), at low and moderate base currents.

4.3 Experimental Results

For the circuit of Figure 2.3, $r_{ec}$ was measured in the manner described in Chapter Two. The base current was supplied by the calibrated current source of a Tektronix Type 576 curve tracer. The input signal was maintained at 4.24 mV rms.

The $r_{ec}$ versus $I_B$ characteristic measured for the Q009 device at 1 kHz is shown in Figure 4.1. Note the excellent agreement between the computer simulation and experimental results, even at high base currents. Similar measurements were made at 10 kHz; no change was found. Measurements were also made with the device operated in parallel with a fixed resistor $R_p$ across the collector and emitter terminals. For values of $R_p$ of 100 kΩ and 10 MΩ, the small-signal resistance of the combination was found, within the limits of experimental error, to be simply $(1/R_p + 1/r_{ec})^{-1}$. This supports the conclusion of Section
4.1.3 — that $R_p$ does not affect the small-signal collector-emitter resistance of the ZBT if $I_C$ is small.

Nonlinear distortion measurements were made with the Q009 transistor connected in the attenuator circuit of Figure 4.6. The substrate was biased at -2V with respect to the emitter. The input signal was supplied by a low-distortion Tektronix SG-505 oscillator; the base current was supplied by a Tektronix Type 575 curve tracer. Distortion products at the output were measured using a Hewlett-Packard 3580A Spectrum Analyzer. $R_p$ was adjusted so that the parallel combination of $R_p$ and the input impedance of the spectrum analyzer was 100 kΩ. The input signal was adjusted as $I_B$ was varied, in order to maintain a constant value of $v_o$ at the fundamental frequency. The values of $HD_2$ and $HD_3$ measured with an output signal of 10 mV rms at the fundamental are shown in Figures 4.8 and 4.9, respectively.

In order to ascertain the contribution of the base current source or the oscillator to the measured distortion, the ZBT was removed from the circuit, and the output of the current source fed directly into $R_L$. The value of $v_o$ at the fundamental frequency was maintained at 10 mV rms as the base current was varied from $1 \times 10^{-3}$ A to $1 \times 10^{-4}$ A. No distortion products were detected within -90 dB of the fundamental component of $v_o$, the noise floor of the spectrum analyzer. Hence the results in Figures 4.8 and 4.9 can be attributed to the device itself.

The analysis of nonlinear distortion presented in this chapter assumes that the small-signal condition, as given by (4.63b), is satisfied. When this condition no longer holds, compression or
expansion of the fundamental component is observed. In the test circuit of Figure 4.6, \( v_1 \) was increased in 10 dB steps and the corresponding change in \( v_o \) measured. This was found to be \( 10 \pm 0.5 \) dB for values of \( v_o \) up to 31.7 mV. Similarly, the corresponding changes in HD_2 and HD_3 were \( 10 \pm 0.5 \) dB and \( 20 \pm 1 \) dB, respectively. Thus, the experimental data of Figures 4.8 and 4.9 are within the realm of nonlinear small-signal theory.

With reference to Figure 4.8, good agreement exists at 1 kHz between the simulated and measured values of HD_2 at moderate base currents. At lower base currents a discrepancy arises; at \( I_B = 1 \times 10^{-8} \) A the measured value of HD_2 is almost 20 dB higher than the simulated value. This discrepancy is even greater at 10 kHz, where the theory and experiment are in agreement only above 10 \( \mu \)A. It is felt that this discrepancy is due chiefly to inaccurate modeling of the emitter-base and collector-base small-signal junction capacitances. From (1.7) and (1.8) these are, respectively,

\[
C_{JE}(v_{EB}) = \frac{C_{JE}(0)}{m_C} \frac{1}{1 - \frac{v_{EB}}{\Phi_E}} \quad (4.78)
\]

and

\[
C_{JC}(v_{CB}) = \frac{C_{JC}(0)}{m_C} \frac{1}{1 - \frac{v_{CB}}{\Phi_C}} \quad (4.79)
\]

Ordinarily the junction capacitance is important only under reverse-bias and low forward-bias conditions, since under greater forward bias it is
much smaller than the small-signal diffusion capacitance. The three parameters associated with each junction capacitance are therefore usually obtained by fitting the capacitance versus voltage curve, measured with the junction reverse biased, to an expression of the form of (4.78) and (4.79) [6]. This can lead to very inaccurate results for the junction potentials \( \phi_E \) and \( \phi_C \). In the case of the Q009 device \( \phi_E \) and \( \phi_C \) were found to be 0.5 V using this technique, whereas in actual fact they are of the order of 0.85 V for silicon devices.

Equations (4.78) and (4.79), which are included in SPICE, predict infinite capacitance at the point where the junction voltage equals the junction potential. Chawla and Gummel [21] have shown that (4.78) and (4.79) are not valid near this point. Instead, the junction capacitances reach a finite maximum, then decrease sharply for junction voltages greater than the junction potential. The program SPICE avoids the problem of infinite capacitance by approximating \( C_{JE} \) and \( C_{JC} \) as linear functions for \( v_{EB} > N\phi_E/2 \) and \( v_{CB} > N\phi_C/2 \), where \( 0 < N < 2 \). The parameter \( N \) is set by the user. This is generally acceptable in first-order small-signal analyses, since under forward bias the junction capacitances are small in comparison with the diffusion capacitances. However, nonlinearities in \( C_{JE} \) and \( C_{JC} \) are completely ignored. Since \( C_{JE} \) and \( C_{JC} \) are in actual fact highly nonlinear functions of the junction voltages, their contribution to device nonlinearity is expected to be significant at high frequencies.

In the simulation performed for the Q009 device, \( N \) was set equal to one. As a result, the contributions of the junction capacitances to
HD₂ and HD₃ were neglected for junction voltages greater than 0.25V. While it is possible to increase N, this is of little value if φ₂ and φ₁ are not known with accuracy. Even if N = 2, the nonlinear behaviour of the junction capacitors of the Q009 device is neglected for junction voltages greater than 0.5 V. For vᵥC < Nφ₂/2 and vᵥE < Nφ₁/2 the predicted nonlinear behaviour is correct only if φ₂ and φ₁, and hence the capacitance-voltage relationships (4.78) and (4.79), have been accurately determined. Precise measurements of φ₂ and φ₁ can be made using procedures more elaborate than the reverse-bias curve-fitting technique. It is felt that this would result in better prediction of the frequency-dependent nonlinear behaviour of the ZBT.

The effect of a finite base-drive impedance on device nonlinearity, discussed in Section 4.1.4, must also be considered as a possible explanation for the discrepancy between the measured and simulated values of HD₂ at low base currents. At IᵥB = 1 x 10⁻³ A the output impedance of the base current source was found to be approximately 50 M Ω. Through computer simulation the magnitude of HD₂ was found to increase by 5 dB at Iᵥ₅ = 1 x 10⁻³ A over the simulated value shown in Figure 4.8, which was obtained for an ideal current source. This is insufficient to account for the discrepancy of almost 20 dB between the measured and simulated curves. At higher base currents, the simulated effect of the current source on HD₂ was found to be negligible.

From Figure 4.9 it is apparent that the measured HD₃ versus IᵥB curves are nearly identical at 1 kHz and 10 kHz, except at low base
currents. Hence the offset between the theoretical and simulated curves cannot be attributed solely to inaccurate modeling of the junction capacitances. It is likely also due to the sensitivity of $g_3/g_{e0}$ to the recombination parameters of the model equations. With reference to (4.25), even very small errors in $n$ and $C_B$ result in a significant error in $g_3/g_{e0}$, and hence in $HD_3$. The effects of the finite output impedance of the base current source on $HD_3$ were simulated, and found to be negligible over the range of base currents for which $HD_3$ was measured.
CONCLUSION

A modified version of the integral charge control model of Gummel and Poon has been used to investigate the ac and dc characteristics of the saturated bipolar transistor, in the situation where $I_C$ is zero or close to zero. Analytic expressions have been obtained for the dc collector-emitter voltage, the ac small-signal conductance, and second and third-order small-signal nonlinear distortion in terms of the base current and the device parameters. The validity of these expressions has been established through computer simulation.

At low base currents the ac and dc characteristics are found to be dominated by carrier recombination within the junction space-charge layers and at the surface. At high base currents high level injection and the collector and emitter series resistances dominate device behaviour. From the standpoint of practical applications, optimum performance for a given device is achieved at moderate base currents, where recombination and high-current effects are negligible. In this region $V_{ECO}$ and the small-signal nonlinear terms $g_2/g_{eco}$ and $g_3/g_{eco}$ are minimized, and are independent of $I_B$, while the small-signal conductance $g_{eco}$ varies linearly with $I_B$. With proper fabrication it should be possible to produce a ZBT having these characteristics over several decades of $I_B$. It is shown that a device having symmetric forward and reverse-active mode parameters exhibits substantially less second and third-order nonlinearity than a conventional device, for which $\beta_F \gg \beta_R$. 

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and $C_2 \gg C_4$. Thus, the optimum structure for a ZBT is markedly different from that required for typical high-gain transistors. The symmetric device is also found to be superior to a parallel configuration of two conventional devices, in terms of both ac and dc performance.

The parasitic substrate transistors are shown to have a significant effect on the behaviour of the ZBT, and must be included in the model if accurate results are to be expected. In general, the effects of the substrate can be reduced through appropriate processing techniques.

The behaviour of the ZBT in various circuit configurations has also been examined. The results of Section 4.1.3 show that a fixed resistance in parallel with the ZBT has no significant effect on its small-signal characteristics, if the dc collector current is small. The dc collector-emitter voltage decreases with respect to its open-circuited value at low base currents. The results of Section 4.1.4 indicate that the output resistance of the current source supplying $I_B$ can increase the small-signal device nonlinearity if it is sufficiently low.

A number of applications for the ZBT have been suggested, particularly in situations where a small-signal variable-impedance element is required. Its small size and low current requirements make it ideally suited for low-voltage micropower circuits. A perturbation method has been used to analyze a small-signal attenuator and a variable-gain feedback amplifier incorporating the ZBT. Expressions
have been obtained for the second and third harmonic distortion components of the output signal in terms of the circuit parameters, the input voltage, and the small-signal characteristics of the ZBT. This technique can be applied to circuits of arbitrary complexity.

Experimental data were obtained for a family of symmetric lateral p-n-p transistors. The measured and theoretical values of $V_{ECO}$ are in excellent agreement at low and moderate base currents. At very high base currents a discrepancy is observed. It is felt that this can be attributed to high-current effects not included in the model, in particular conductivity modulation in the active base region. A detailed investigation of these effects would undoubtedly lead to a better understanding of the dc behaviour of the ZBT in the high-current region. The predicted values of the small-signal resistance are in excellent agreement with the measured data, even at high base currents. Thus, the phenomena giving rise to the discrepancy noted above do not appear to have a significant effect on the small-signal behaviour of the ZBT.

At moderate base currents the predicted and measured values of small-signal second-order distortion are in good agreement, with a discrepancy arising at lower currents. This is more marked at higher frequencies. It is felt that more accurate measurements of the junction potentials would result in better agreement, since these parameters critically influence the behaviour of the nonlinear small-signal junction capacitances.

The third-order small-signal characteristics of the ZBT have been
found to be less sensitive to frequency-dependent nonlinearities than those of the second order. The theoretical curve of $HD_3$ versus $I_B$ qualitatively accounts for the features of the measured curve, although there is a significant offset between the two. This is felt to be a result of the sensitivity of $g_3/g_{ec0}$ to small errors in the recombination parameters.

The results presented in this paper have provided a sound basis for future study of the ZBT. Possible areas of further research include an investigation of the effects of base current modulation on the ac and dc device behaviour, and an examination of the noise characteristics of the ZBT. A better understanding of the device would undoubtedly encourage its application in a wide variety of circuits.
REFERENCES


