THE SOCIAL ORIGINS OF EARLY MODERN MECHANISM

BY

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THE SOCIAL ORIGINS OF EARLY MODERN MECHANISM

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ABSTRACT

It is argued that the fundamental concepts of early modern, mechanistic science are in part socially constituted. Mechanism is here understood as a conception of nature wherein natural objects are abstractly reduced and homogenised such that they come to be viewed as comprised of one primary material. Sensually intuitable events are then seen as explicable in terms of the mathematical relation between qualitatively similar particles.

This abstraction is gained by analogy to a society which is becoming similarly abstract. When the pivotal relation in society becomes that between owners of exchangable commodities, a similar abstraction occurs in the commodities. Qualitatively different goods come to be seen as commensurate in terms of "value". The mathematics and record-keeping which develop to keep track of "value", understood as an expression of a social relation, become the basis for a similarly abstract science of nature.

Of the major contributors to the early modern mechanistic view of nature, the work of many is seen to be in some way commercially inspired. Although no direct links are found for Galileo, Vieta, Descartes or Bradwardine, for Tartaglia, Bombelli, Oresme, Pacioli and Stevin, a rather strong connection exists. Concepts in early modern mathematics and mechanics thereby bear reference to a more abstract and homogeneous object.

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Chapter One - Introduction

The scientific status of sociology has been a contentious issue for decades. The sociology of knowledge, that discipline devoting itself to formulating the relationships between social existence and thought, has been particularly reluctant to discuss scientific thought in that context. Politics, ideology or religion may readily be seen as related to social existence in some fashion, but science has, for the most part, been seen as independent.

Science, it is argued, amounts to the study of nature according to its own principles. In this view it is thus wholly unnecessary and in fact mistaken to attempt to relate scientific thought to a social context. To the extent that politics, ideology, religion or any other aspect of social reality affects science, the results of this effect are errors or misperceptions. The sociology of knowledge is thus relegated to the explanation of error.

The only socio-historical development which is deemed relevant here is the emergence in early modern Europe of a movement which liberated thought from the "fetters of theology" and thus allowed for the practical and theoretical exercise of human rationality with respect to nature. One sociologist of knowledge, Peter Hamilton, recognizes this as a social and historical development, but since the values developed were scientific ones, the social link involved represents no problem for the rational and empirical purity of science's self-understanding.[1] Hamilton goes on to suggest that the attempt to understand the knowledge-society relationship should itself be made more scientific.[2]

In our view, however, the social and historical character of science's emergence have more far-reaching consequences. Whereas many commentators are willing to describe these social factors which either help or hinder science's institutional development, we are suggesting in addition that the very concepts of early-modern science are in some measure socially constructed. There are many potential candidates for the factors which influence the emergence of science, some of which are social. Religious disputes, new technical requirements and occasional discoveries, a renewed interest in ancient texts and the flourishing set of both amateur and professional virtuosos all vie as factors for understanding the rather dramatic rise of scientific thought in early-modern Europe.

For our view, we take the lead from Franz Borkenau's pathbreaking work, Der Uebergang vom Feudalen zum Buergerlichen Weltbild.[3] Although the greater part of Borkenau's book is devoted to an attempt to characterise changes in philosophy from a theological to an anthropological mode by tracing the changes in the meaning of the term "lex naturalis" from Aquinas to Pascal as a result of class conflict, he does state near the beginning of the work that the result of this development is what he terms the mathematical-mechanistic world-view. Both Galileo and Descartes suggest, for example, that we may understand the differences between sensually intuitable events in terms of differences in the speed, shape and relation between qualitatively similar "bodies".[4]

For Galileo, various sense impressions are explicable in terms of the movement of particles producing different sensations where the movement and relation are different. Different tastes, for example, may be explained with reference to the "various shapes, numbers and speeds of the particles" on the tongue. Descartes, in turn, explains differences in colour in terms of differences in figure possessed by each of the objects of a different colour.

Borkenau suggests what the similarity or parallel might be between a form of social relations, on the one hand, and this mathematical-mechanistic world-view, on the other. This abstract view of nature is in some measure a result, he suggests, of the following social process of abstraction:

> On the one hand, the extreme division of labour creates an abstract, general substrate of labour, the chemical and other qualities of which are ignored as much as possible and which is to be viewed only as 'stuff in itself', as pure matter. On the other hand, it creates the completely unqualified worker, who is considered only as labour power in itself, whose function is labour in the abstract, pure physical movement. The greatest classic physicist of the manufacture period, Galileo, deals in his main work, the Discorsi, with the laws of abstract work.5

The specific division of labour referred to by Borkenau is that which brings about and, in turn is brought about by the advance of commodity production in early modern Europe. The notions of commodity and abstraction used here are taken from Marx' development of the labour theory of value. Briefly, the exchange of dissimilar commodities produced by dissimilar labours results in an abstraction from the specific qualities of both the commodities and the labour which produces each. Each is reduced to something which allows them to be commensurable - value. This process is predicated on the social relations between the producers. (cf. Chapter III infra for a more complete development of this.)

A society develops in which the needs and wants of members are more and more satisfied by exchange and commodity production. In the process of exchange, furthermore, what we have is a comparison of dissimilar goods. The bread that I have made will exchange on the market for a certain number of pairs of shoes which I require. If it happens, for example, that twenty-five loaves of bread exchange for two pairs of shoes, then the twenty-five loaves of bread are seen to contain something in equal quantity to the two pairs of shoes - value. The substance of both the bread and the shoes has been abstracted from their sensible qualities; their property of being exchangeable with each other receives attention at the expense of other sensually intuitable properties of the bread and the shoes. This property of exchangeability is abstracted - "pulled away from" - the bread as an integrated whole, an object of our everyday perception.

This leaves us with the question of what allows those goods to exchange in a given proportion. If we take as our answer the amount of labour time expended in their production, then we have another abstraction. The labour of the baker and that of the shoemaker in our example are qualitatively different. In the act of exchange these labours are also made commensurable; they become human labour, pure and simple. Hence, Marx terms the labour producing the exchange-value of a commodity abstract labour - labour expended without regard to the mode of its expenditure.

In the social realm, then, we have abstraction from the specific qualities of the objects exchanged on the market and from the specific qualities of the labour involved in the production of each. Similarly, in terms of mechanistic science or natural philosophy as it was then called, we have a process of abstraction from the specific qualities of the objects of our perception. If, for example, for Galileo or Descartes the shape and speed of particles happens to be important in formulating a mathematical expression for accounting for their sensually intuitable qualities, we abstract from those qualities themselves to obtain measurable "operationalizations" of shape and speed. These properties are "pulled away from" the sensually intuitable properties of the objects considered. The particles on Galileo's tongue no longer appear as red, soft, delicate and so forth, but simply as triangular and moving at a certain speed in relation to each other.

The tradition of thought about nature in the West has not always been of this character. Much of Greek science was distinguishable by its concern for hierarchy and the essential differences between things in the world. This is reflected in their mathematics. Number and magnitude are separate entities and the principle of homogeneity is maintained - proportions may obtain only between quantities of like kind.

In the more modern case, number and magnitude are reunited under the rubric of "general magnitude" and, owing to the abstraction described above, the homogenisation of the world makes the principle of homogeneity superfluous. Since, by abstraction, the world is reduced to one like substance, one need not worry about comparing quantities of unlike kind since their reduction to this like substance becomes an easy matter mathematically. Whereas the Greeks had no notion of velocity as a single quantity, the early moderns could suggest that v=ks/t and thus relate time and distance, unlike quantities, in the same expression.

We are thus claiming here that the history of science is less than continuous, that a break in the conception of nature occurred in the early modern period and that the new proliferation of commodity relations which is partly responsible for the break is partly responsible at the same time for the constitution of some of the most fundamental concepts of early modern science. That the content of science can be influenced by social factors is a notion coming to be entertained by modern sociologists of science.[6] Barry Barnes suggests that the role of "external factors" in the scientific revolution has not yet been adequately dealt with. Barnes suggests, furthermore, that

> (t)he social context of sixteenth- and seventeenth- century natural philosophy and the ethos within which important figures worked may prove impossible to reconstruct. The gap between internal and external history may prove technically unbridgeable.7

The present work is intended as a contribution to this bridge.

In his analysis of the social context of scientific discovery, Brannigan alludes to a shift in world-view which in his view changes the very meaning of "discovery". In attempting to characterise the peculiar nature of modern discovery he suggests that it

> may derive from the shift of attention from the scholastic 'world' of the middle ages to the unknown

> 'nature' of the Renaissance, from the world of common-sense knowledge and belief, to the mathematical nature of existence. The shift is nicely reflected in the change of attitude regarding the formal representation of the world. In Cusanus, the arithmetical models of nature are referred to as 'De Conjecture' - conjectures. With Galileo, the shoe is on the other foot: the real world is the 'mathesis universalis', and the world of everyday life is elusive and 'conjectural'. 8

We are characterising the shift as a shift to a mathematical-mechanistic world-view as defined above and characterise it as partly socially produced by the development of commodity relations. The socially produced abstraction in commodity relations makes certain aspects of social existence, particularly exchange, calculable. Pieces of value measured in money can be reckoned in order to size-up one's life chances, once such relations proliferate in society. This provides an analogy, we are arguing, for a similarly abstract and mathematical interpretation of nature on the part of those committed to the advance of commodity relations in the formulation of a more complete and consistent world-view.

We have thus claimed that exchange relations and the attendant social form of labour are responsible, by analogy, for the rise of early-modern mechanistic conceptions of nature. The notion here that a certain set of social relations "gives rise to", "determines", or "gives shape to" a certain conception of nature needs a more complete formulation.

It is our view that no logical, transhistorical relationship between social relations and thought may be formulated. We still maintain, however, that they are connected. The character of this connection is not something physical or natural. To think of it in this way would be to presuppose a mechanistic interpretation of nature and the unity of human and natural history. We could study zoology to reach conclusions about epistemology. It is an inappropriate assumption in this work in any case, since we are concerned to display the extent to which a given form of thought is connected with a given set of social relations.

Modern science has, in any case, superceded mechanism in many respects. Werner Heisenberg points out the relative character of images of nature as a result of his own inquiries in physics:

> It has been pointed out before that in the Copenhagen interpretation of quantum theory we can indeed proceed without mentioning ourselves as individuals, but we cannot disregard the fact that natural science is formed by men. Natural science does not simply explain and describe nature; it is part of the interplay between nature and ourselves; it describes nature as exposed to our method of questioning. This was a possibility of which Descartes could not have thought, but it makes the sharp separation between the world and the I impossible.9

For Borkenau, a specific set of social relations is not the "cause" of the mechanistic world-view. His analysis is concrete and historically specific. The abstract aspects of a given set of social relations provide a "preconception" for a view of nature. The development of abstract social relations and an abstract view of nature are moments of the same complex, highly nuanced social and historical process. Borkenau does believe, however, that in all periods of history, periods defined by their mode of production, there is some theoretical generalisation of the specific social relations of that period. He warns us at the same time that it is not simply technology which gives rise to conceptions of nature "(b)ut what arises from the thought content of technology depends on the relationships of human beings with one another."[10]

We take the lead here from Marx who, although opposed in most respects to generalising tendencies in interpreting history, decided that the most fundamental human, social activity is that activity in which people provide for their existence. We must look at how people live an everyday life to understand other aspects of their society. At the same time, we may understand differences between elements of different cultures, including science, on the grounds that different peoples lead different lives.

Thus, what we have been calling an analogy is for Borkenau a preconception and it arises from the relations of humans with one another, especially relations of production. Gerald Holton attempts to make slightly more specific this notion of preconception. Holton uses the term "analogon" to describe an imaginatively constructed model for the explanation of puzzling events. In the case of Plato's cosmology a geometric, kinematical system of circles is hypothesised.[11]

The most important part of this process, however, for Holton, is what he terms "preselection". The process of preselection operates in such a way as to place constraints on which facts are selected, on the hypothesis proposed and on the kind of correspondence deemed necessary to relieve puzzlement and, hence, to produce "understanding". Holton's account of the Platonic example is that preselection operates there so as to assure the appropriate knowledge of order in the world deemed necessary for a magistrate; conceptions of nature here turn out to be ancillary to moral philosophy.[12]

Holton's more formal account of this process is that when one makes one's musings public one often

> smuggles the style, motivation, and commitment of his individual system and that of his society into his supposedly neutral, value-indifferent luggage. And it is at this point that the concept of projection will help us to understand how the style of contemporary personal and social thought introduces itself into scientific work. 13

Furthermore,

> Social and artistic processes and productions have often served as explanations by analogy for the universe as a system - in short, by projecting outward into the universe conceptual images from the domain of social and productive action. 14

Holton mentions in the context of the notion of projection that certain aspects of relativity theory, for example, require an egalitarian social philosophy as opposed to an Aristotelian hierarchical one. These social analogies which have been projected onto nature may also be "retrojected" back onto human society.

Jacob Klein, whose work figures very centrally in our analyses later on, also suggests, following Husserl, that scientific ideas are intentional products - they are the objects of consciousness. As such they are anticipated and accomplished. Once accomplished, their initial anticipation and construction can be forgotten and the "sedimented" notions and techniques simply used. For the phenomenological analyst, the task is to deconstruct the sedimentations to uncover the intention and anticipation initially giving rise to them.[15]

Thus, if we are seriously to raise the question of the origins of mechanistic science, a mechanistic or naturalistic presupposition will not be appropriate. Such intellectual products as the concepts of early modern mechanistic science are the products of spontaneous human beings living in an intersubjective, historical, everyday world. We cannot thus formulate any universal relation between society and idea; the relation is a historically specific one and not a universally logical one. Thus our view is not in keeping with that which suggests that science is wholly a response to the technological requirements of a particular age. Following Borkenau we suggest that mechanistic thought is the result of a "mechanistic" society and proves, once fully constructed, to have a certain applicability to the aims and purposes of various interests in that society at a later date.

In general, then, our methodology is to put forth an analogon, viz. that of analogy, and survey the work of some likely contributors to early modern mechanism for any sort of evidence of entailment with commerce or for direct analogies to the social, commercial world in the texts themselves. We too must "puzzle out" the development of early modern science to see, by our preselection criteria, if there is an understandable connection with commerce. We do not, however, lay any claim to absoluteness here. We acknowledge, from the outset, the efficacy of other factors. What motivates us here is that we find very little in the history and sociology of science which attempts to relate the content of science to social or other external factors. We intend this work not simply as a contribution to the bridge between external and internal factors in the history of science, but also as a counter to positivistic and naturalistic assumptions in sociology. The scientific status of sociology is for us unproblematic.

Our method for demonstrating the entailment of mechanistic science with commerce and commodity production is to focus, first of all, on the development of aspects of early modern algebra and mechanics. This involves demonstrating that there is a difference between ancient and modern mathematics. The ancient Greeks separated number and magnitude and prohibited the comparison of quantities of unlike kind. Both principles, the concept of number and the principle of homogeneity, make anything like modern mechanics and mathematics impossible.

Ancient discussions of motion, such as Aristotle's Physics, for example, never compare quantities of unlike kind in the same expression. The time in one motion will be compared to the time in another; similarly for force and distance. This was the case in dynamics and kinematics until the late middle ages.

In another work however, the Nichomachean Ethics, Aristotle already provides a hint at the context in which quantities of unlike kind may be made commensurable. Although he states that it is strictly mathematically incorrect to compare quantities of unlike kind, in the case of exchange of commodities money may be said to make them commensurate for practical purposes. The scientific texts of some of the major contributors to these aspects of mathematics and mechanics will be discussed with a view to determining the possible extent to which their concepts may be said to have commercial origins. Our contributors are John Philoponos, Leonardo Fibonacci, the Italian abacists, Thomas Bradwardine, Nicole Oresme, Luca Pacioli, Rafael Bombelli, Nicolo Tartaglia, Galileo Galilei, Francois Vieta, Rene Descartes, Simon Stevin, Robert Recorde, John Dee, and the English founders of the Royal Society with a focus on William Petty. Eight of the fifteen are seen to have strong commercial interests, many having written about commerce and bookkeeping as well. In the case of Pacioli, Oresme and Stevin the connections are very strong. Eventually the algebraic formula becomes a way of interpreting nature: V=k s/t. That technique taken over from the Arabs into European commerce which was known as the Rule of Three provides a starting point for this modern development.

First of all, the position of the sociology of knowledge must be formulated. In Chapter Two the works of Gruenwald, Durkheim, Weber, and Mannheim will be discussed to help us formulate the perspective we shall use. Gruenwald asks about the position of a sociology of knowledge in the system of the sciences but considers that system as unassailable and also unexaminable by the sociology of knowledge. Durkheim wants to place society between mind and nature in the formation of knowledge but does so in a way which is ultimately reductionist. The basic properties of organised matter appear to determine, for Durkheim, the nature of all three entities in this process. Weber is at least capable of seeing modern science as helped along by social motives and developments, especially religious ones, but capitulates in the end to the purposive-rational action and instrumental reason about which he has ambivalent feelings. In Mannheim's relationism he is willing to consider any form of thought as socially constituted except science. Its quantitative aspect is precisely what makes it inaccessible to the sociology of knowledge for Mannheim.

We then discuss the work of two more modern sociologists of knowledge, Alfred Sohn-Rethel and David Bloor. Both, although coming part way toward our analysis, espouse a naturalistic conception of the relationship between society and knowledge. Both are quite willing to see the relationship of society to the content of science but do so in a way which presupposes that the notions of the science we are investigating are adequate to describe the society-knowledge relation.

Chapter Three begins with a discussion of Marx' contribution to the sociology of knowledge. Because of his notion of historical specificity, Marx provides a way of understanding intellectual development without postulating transhistorical laws or relationships. He also provides an account of why these historically specific relations are not visible to participants. Relations between people appear as relations between things. The properties of physical things appear to do the acting on the market.

Next we develop Marx' analysis of the commodity abstraction and the social relations which lie behind it. Value, a social relation, apparently becomes a property of physical things. His development of the labour theory of value takes us to the point where we may speak of a set of social relations which are capable, by analogy, of being used to describe nature mechanistically. The last part of Chapter Three is devoted to the evidence that the social relations of which Marx speaks actually existed. We draw here on the work of economic historians.

Chapter Four begins by discussing the various extant interpretations of the scientific revolution. Many different factors are cited as having helped or hindered the progress of a scientific community, but the only view which really attends the formation of fundamental concepts is that view which states that they are received from recently arrived ancient Greek texts. Because of this we then, drawing on the work of Jacob Klein, develop the argument that there is a significant rupture in meaning and intention between ancient and modern mathematics. Greek mathematics was ontological in concern and early modern mathematics was instrumental. Klein's argument is that early modern mathematicians who claimed to find "general magnitude" or symbolic algebra in those recently arrived or recently translated texts were reading them through the filter of an already semi-constituted modern symbolic algebra. A form of calculation was developed in late medieval and early modern Europe which allowed the development of the "variable". The construction of symbolic, general magnitude made it possible to compare unlike quantities in an equation such that relations in nature could be formulated generally in terms of variables rather than allowing simply a given solution to a given practical problem. The variable was an impossibility in ancient mathematics since the formulation of a problem allowed only one determinate solution, not a number of possible values for a variable. The chapter concludes with a brief overview of the history of mechanics to the early modern period.

In Chapter Five we analyse the writings of our fifteen exemplars with a view to demonstrating the possible conceptual connection between commerce and science. Of course, the work of each and the possible connection of each with commerce is not something uniform. The story is a highly nuanced one. For many of our protagonists, however, the connection is rather clear. The reinterpretation and violation of ancient concepts found in Euclid is carried out in many cases by thinkers concerned, at least at some point, with commercial arithmetic and bookkeeping. Through this, it is argued that the modern technique of applying algebraic formulae involving the concept of general magnitude is derived from techniques initially developed to calculate the values of commodities in exchange and thus is predicated on the social relations which lie behind commodity production and exchange.

Naturally, there is more than this to modern science. Many historians and philosophers have produced cogent and interesting interpretations of the rise of different sciences. Some more general ones claim other factors such as religious disputes and the reception of Greek texts as contributing to modern science's early development. We have no general quibble with these authors. Our aim here is to present a case for the possible influence of social factors on the conceptual development of early modern science which have received scant attention until now. We claim that most other interpretations pay less attention to the <u>content</u> of science and in particular to the possible role of social relations in the formation or constitution of that content.

In Chapter Six we conclude generally that social relations may be seen as influencing the conceptual development of science in a constitutive manner and, furthermore, of helping to constitute these concepts which we regard as more or less adequate or fruitful. We claim this, however, in a way which refrains from the formulation of any transhistorical postulation of the relationship society-knowledge. Any connections to be found are specific, historically occurring ones. We regard science as a human, social creation and see history as produced by human beings. We infer, thus, that many fruitful interpretations of nature are possible. We line up on the side of freedom for human social and intellectual development.

Notes

1. Peter Hamilton Knowledge and Social Structure Routledge and Kegan Paul, London 1974, 2.
2. ibid. 150-51.
3. Wissenschaftliche Buchgesellschaft, Frankfurt, 1976.
4. Galileo Galilei "The Assayer" in Discoveries and Opinions of Galileo translated by Stillman Drake, Anchor, New York, 1957, 237-8.

   Rene Descartes "Rules for the Direction of the Mind" in The Philosophical Works of Descartes translated by Elizabeth S. Haldane and G.R.T.Ross, Cambridge University Press, Cambridge, 1931, 37.
5. Franz Borkenau "Zur Soziologie des mechanistischen Weltbildes" in Zeitschrift fuer Sozialforschung I,3, p.
6. cf. Michael Mulkay Science and the Sociology of Knowledge George Allen and Unwin, London, 1979, Chapter four passim.
7. Barry Barnes Scientific Knowledge and Sociological Theory Routledge and Kegan Paul, London, 1974, 107.
8. Augustine Brannigan The Social Basis of Scientific Discoveries Cambridge University Press, Cambridge, 1981, 3.
9. Werner Heisenberg Physics and Philosophy Harper and Row, New York, 1962, 81.
10. Borkenau loc. cit. 13.
11. Gerald Holton "Thematic and Stylistic Interdependence" in Thematic Origins of Scientific Thought Harvard University Press, Cambridge, Mass., 1973, 92.
12. Ibid. 93-4.
13. ibid. 101.
14. ibid. 102.
15. Jacob Klein "Phenomenology and the History of Science" in Philosophical Essays in Memory of Edmund Husserl ed. Marvin Farber Harvard University Press, Cambridge, Mass., 1940, 152-163.

Chapter Two Science and the Sociology of Knowledge

We have already mentioned that the sociology of knowledge has been reluctant for the most part to consider science as socially constituted. The classical sociology of knowledge has, however, considered the possible connections between social life and other forms of thought. We shall look here, to begin with, at the work of four classical sociologists of knowledge, Ernst Grüenwald, Emile Durkheim, Max Weber, and Karl Mannheim, to establish our position with respect to the classical literature. We shall consider them with reference to the perspective and method employed and to their formulation of the relationship between society and knowledge.

Although they do not, for the most part, consider science in their analyses, they do provide problem areas and frameworks which have later proven useful for those who have considered science as subject matter for the sociology of knowledge. We shall look next at the work of Alfred Sohn-Rethel and David Bloor. Each considers science in relation to society but ultimately in ways of which we are critical. This leads us then to our consideration of Marx' contribution in Chapter Three.

The Classical Sociology of Knowledge - Perspective and Method

Each of the four thinkers discussed here has a set of assumptions about the nature of social reality and thus about the means of access to this reality. Assumptions made about the object and our knowledge of it thus have a bearing on conceptions of the relation between the character of social reality and its possible influence on other forms of knowledge.

For Ernst Gruenwald it is precisely the scientific character of such a discipline which is at issue. His inquiry focuses on the prerequisites which the sociology of knowledge must have to be considered a science. His way of asking this question is to ask what position it might occupy in the system of the sciences.

With respect to knowledge, Gruenwald is concerned to avoid any conflation of origin and validity. The social origin of a proposition can tell us nothing about its validity.[1] The system of the sciences turns out to be unassailable for Gruenwald and his prime concern is with the unprovable metaphysical assumptions necessary for a sociology of knowledge. This is the assumption of social reality as an absolute layer of reality, an <u>ens realissimum</u>, which manifests itself in some way in knowledge and cognition.

Emile Durkheim sees society as a moral entity kept together by the moral authority of society itself. At the same time, however, he exhorts us to consider social facts as things; society is much like physical reality. He negates the significance of subjectivity in history and claims that moral facts represent phenomena "whose conditions must be sought in the essential properties of organised matter".[2] In a nutshell Durkheim's formulation of social reality is the following: conduct is rooted in thought, thought is rooted in social reality, social reality is rooted in "the nature of things".[3]

In this picture, society appears as an entity completely external to individuals making them do what they do. Objectified sentiments which are the forces making individuals behave, the forces constituting the moral authority of society, are reduced by Durkheim to the propert-ies of organised matter.

> The fundamental relations that exist between things - just that which it is the function of the categories to express - cannot be essentially dissimilar in the different realms. [4]

> (T)hat which is at the foundation of the category of time is the rhythm of social life; but if there is a rhythm in collective life one may rest assured that there is another in the life of the individual, and more generally in the life of the universe. [5]

Moral force and the authority of society are responsible for belief, but this force and this authority are further reduced to quasi-physical entities. Thus, for Durkheim, the relation of logical thought to social origin does not debase it, but rather relates it "to a cause which implies it naturally".[6] There is, furthermore, an evolutionary hierarchy in Durkheim's scheme such that the collective representations which develop follow nature more and more closely.

Whereas Durkheim is concerned with quasi-physical, social facts and moral authority, Max Weber is concerned with meaningful social action and rationality. Meaningful action becomes understandable for Weber in terms of the irrational influence of ultimate values on the purposive-rational type of social action. If one does not choose the most appropriate, efficient means of achieving a given end, this result must be interpreted as the interference of absolute values.[7]

Meanings and motives are organised into complexes, these in turn into value spheres and these further are embodied in corporate groups.

This is said to comprise a legitimate order when there is a high probability that action so oriented will in fact take place.

His substantive concern is to account for the rise of purposive-rational action in the West. He has ambivalent feelings about this development, but capitulates to it in the consideration that the world has become disenchanted to the extent that everything is masterable by calculation.[8] Thus, for Weber, not only does this become a feature of the modern world but constitutes his methodology as well.

Karl Mannheim is also concerned with rationality. Whereas for Weber, value spheres are in irreconcilable conflict, Mannheim explores the possibility of a rational, scientific politics. In this Mannheim is concerned with how people in fact think, rather than with criteria of validity which are external to everyday life. "The principle thesis of the sociology of knowledge is that there are modes of thought which cannot be adequately understood as long as their social origins are obscured."[9]

Attempts to debunk the views of opponents where there are many conflicting groups can in itself lead to the view that most knowledge is perspectival; we thereby learn to see a perspective as such. In any case, Mannheim sees social reality as constantly changing. The knowledge of any group is thus seen, not as illusion, but as knowledge which is not absolute. Because of this, however, Mannheim also tends to see social existence as "a reality which is the outcome of constant reorganization of the mental processes which make up our worlds".[10]

Society and Knowledge

Gruenwald explores the problems which ensue from the assumption that knowledge is the "manifestation" of social existence, which in turn is seen as absolute reality. He is concerned that this assumption is scientifically unprovable and is worried, lest one conflate origin validity. To resolve this problem, Gruenwald suggests dividing a proposition up into proposition material and claim to validity. In this way the claim to validity may be left intact while the proposition material may be related to the absolute layer of social existence. Social being or existence can be seen as having "selective relevance" for the utterance of a proposition while the claim to validity is left independent.[11] In this way "meta-empirical entities", such as the proletariat, may be seen as responsible for the utterance of a proposition, but not for its validity.[12]

In raising the problematic matter of the metaphysical assumption necessary for the sociology of knowledge, however, Gruenwald makes problematic the status, not only of the sociology of knowledge, but also of the system of the sciences. The metaphysics necessary for the operation of other sciences is also scientifically unprovable. Gruenwald thus sets for himself a dilemma. The world considered as a unified whole presents the possibility of a system of sciences whose research may be considered adequate to the character of its objects. When the necessary sort of unity is accepted for something like the sociology of knowledge, however, the possible results fly in the face of a formal, transcendental unity required by some other sciences.

As Kurt Wolff puts it,

> Inspecting our results, we find that Gruenwald wants - in the two-fold sense of this term - a cosmology, universal truth, cognitive features common to all men, identical objects, points of view that could be understood with reference to an order of culture and history, and a science that can make testable claims about the origin of a given thought, about the relations between thought and social class, and about the existentiality of knowledge. His world is a world that exhibits these wants. 13

This is the problem of attempting to demonstrate transhistorical relations between society and knowledge. Although at an extremely general level something like that may be possible, we believe that the assumption of uniformity and regularity in the social world, necessary for such a position as Gruenwald's, is untenable.

Durkheim also attempts to formalise the society-knowledge relationship in his notion of collective representations, his term for most thought. Unlike Gruenwald, however, Durkheim sees no problem for validity in relating thought to social context. The categories of thought have a social origin.[14] In Durkheim's scheme social reality occupies a position midway between mind and nature. It is able to exercise an unproblematic influence precisely because it is organised according to the same principles as nature. It is thus neither mysterious nor epistemologically dangerous for Durkheim that socially produced categories be capable of explaining natural phenomena.

> It is true that since collective sentiments can become conscious of themselves only by fixing themselves upon external objects, they have not been able to take form without adopting some of their characteristics from other things: they have thus acquired a sort of physical nature; in this way they have come to mix themselves with the life of the material world, and then have considered themselves capable of explaining what passes there. But when they are considered only from this point of view and this role, only their most superficial aspect is seen. In reality, the essential elements of which these collective

> sentiments are made have been borrowed by the understanding. It ordinarily seems that they should have a human character only when they are conceived under human forms; but even the most impersonal and the most anonymous are nothing else than objectified sentiments. 15

Durkheim also introduces an evolutionary hierarchy of objectified sentiments whereby, over the course of history, sentiments become more like that which is their foundation and that which they depict,[16] such that in primitive societies things are taken as true because they are collective, whereas in modern societies beliefs become collective because they are true; the combinations of consciousnesses and their products obey laws of their own.[17]

Durkheim thus assumes a unique status for his own thought and at the same time implies extra-collective criteria for the validity of modern collective representations. He has thus placed an abstract notion of social reality midway between mind and reality. This results in a psychologistic epistemology as well. One commentator who terms this epistemology the "constancy hypothesis", the idea of the correspondence between a given sensation and a given concept, comments on Durkheim's sociology in this regard.

> Whatever the criticisms that may be made of Durkheim's sociology and its method, the sociological systems substituted for the natural attitude's conception of man in relation to fellow man yield a primacy of sociology and sociological theory where, e.g., scientific belief turns out to be what religious belief should be. But that scientific belief belies that very origin it suppresses under the specifying assumption of the constancy hypothesis. This reduction of what people 'know' as 'reality' to the capacity to believe, transforms a concern with what people 'know' into a concern <u>for</u> what people 'know', in such a way that what is 'real' for members of society is replaced by an underlying 'collectivity' constructed by scientific thinking. 18

Thus, although Durkheim has gone a long way toward formulating the possible influence of society on adequate beliefs or knowledge, the "society" which he has in mind here is an extremely abstract construction and the knowledge which he really values is not seen as related very closely to collective representations.

Weber, on the other hand, is less abstract in his interpretation of social reality and is not so anxious as Durkheim to embrace a picture of the dove-tailing aspects of mind, society and nature. Because of the attention paid to meaning in his perspective he develops a notion of "adequacy on the level of meaning". Concepts used in the explanation of social action must be meaningful to those members whose actions are being explained. His actual researches are thus more concrete than Durkheim's.

In his studies of religion Weber is concerned to understand differences in forms of rationality in terms of differences in irrational motivation. In fact he understands scientific, purposive rationality as having origins in irrational, religious motivations. His analysis augurs well in some ways for our own analysis. Weber depicts an affinity between Protestant religious concerns, modern capitalist enterprise and modern western rationality. There is, nonetheless, a tension between the causality which he ascribes to religious world-views and the instrumental reason and causal adequacy which he hopes to make a part of his social science. He describes the above "affinities" in the following manner:

> Obviously, the mere existence of capitalism of some sort is not sufficient by any means, to produce a uniform ethic, not to speak of an ethical congregational religion. Indeed, it does not automatically produce any uniform consequences. For

> the time being, no analysis will be made of the kind of causal relationship subsisting between a rational religious ethic and a peculiar type of commercial rationalism, where such a connection exists at all. At this point, we desire only to establish the existence of an affinity between economic rationalism and certain types of rigoristic ethical religion, to be discussed later. This affinity comes to light only occasionally outside the Occident, which is the distinctive seat of economic rationalism. In the West, the phenomenon is very clear and its manifestations are the more impressive as we approach the classic bearers of economic rationalism. 19

What Weber is trying to account for here is the overcoming of the "serious inner resistence" to economic rationality provided by the other-worldly asceticism of medieval Catholicism.[20] Capitalism, economic rationality, is, furthermore, characterised as comprised by the following three elements: 1) rational industrial organisation, 2) the separation of business from the household and 3) rational bookkeeping.[21] After suggesting that sums of capital in themselves are not sufficient to produce such rationalisation, he goes on to describe the process itself in socio-economic terms:

> What happened was, on the contrary, often no more than this; some young man from one of the putting out families went out into the country, carefully chose weavers for his employ, greatly increased the rigor of his supervision of their work, and thus turned them from peasants into laborers. 22

Weber's story here is well-known. Through the notions of "calling" and "predestination" Calvinism was able to overcome the resistence of Catholic absolute values and effect a rigourous supervision of everyday life. It is here, however, where Weber has some difficulty with his notion of cause; he argues, on the one hand, the virtual impossibility of formalising causal connections between religious and economic variables, but nonetheless claims as well to have developed a causal interpretation of the rise of economic rationality "with the

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hope of attaining even a tolerable degree of approximation".[23] Borkenau criticises Weber on this score claiming that Weber's methodology is an isolating-causal one. Borkenau's position here is that it is really impossible to abstract and separate out as variables aspects of people's lives which form an integrated whole in any given period in history. Religions make processes of adaptation to changing conditions less difficult and are not so easily separated from those other aspects to which they may be seen as an adaptation.[24]

Thus, although Weber's treatment of rationality in connection with the rise of modern capitalism augurs well for our argument in some ways, his position is one in which the calculability of separable forces in the social world is assumed. A condition which is the explicandum of his sociology of religion becomes a cannon, in some respects, of his methodology. The purposive rationality which he explains by means of his historical studies of religion becomes the cornerstone of his own instrumental reason.

Mannheim, on the other hand, regards Weber's instrumentalist method as having only limited validity. For Mannheim, the terms or perspective used in a particular inquiry must go beyond the interest motivating the investigation.[25] He wants to make the "interpretation of meaning a vehicle of precision"[26] so that the irrational elements of politics do not get out of hand, that they may be subjected to the influence of "intellectual control and self-criticism".[27]

He wants thereby to be able to relate the psychological motivations and collective thought forms which make up ideologies. The first aspect of this relationism is basically a debunking function. Any particular view of reality is shown to be perspectival and motivated by a particular interest. The collective thought forms, however, are not individual matters and the nature of collective life is partly determinant of thought forms for Mannheim. Out of political conflict, analysis of the roots of a group's outlook becomes possible.[28]

Mannheim anticipates more rational decision-making out of the ability to view the origins of conflicting forms of thought. A particular position is related to its roots and motivations, hence, particularised; all positions are seen as thus situated, hence totalised; the situations are seen as socio-historically related, hence Mannheim's version of relationism. In this context, however, Mannheim has difficulty grounding his own perspective; he needs an ontology himself.[29] With respect to science, he suggests that its attention to quantity rather than quality requires no ideological analysis.[30] An implicit ontology lies behind this proposition. It is decided that aspects of the world may be adequately accounted for in this scientific attention to quantity. It is our argument, of course, that this attention to quantity, although perfectly adequate in Mannheim's sense of relationism, has motivations and origins which are similar in kind to the origins of other forms of thought. His relationism is a useful concept but needs to be applied to quantitative pursuits as well.

There are two modern sociologists of knowledge in particular who have attempted to relate science's conceptual development to social origins. Both present much bolder analyses of the relation of science to social origin than any of the above commentators but do so in a way which is naturalistic. Alfred Sohn-Rethel uses certain of the concepts which we shall develop further in the next chapter but eventually falls back on an identification of human and natural history. David Bloor begins with a very strident relativism, but ends up arguing a rather positivistic naturalism very reminiscent of our discussion of Durkheim above.

Alfred Sohn-Rethel

Sohn-Rethel produces a characterisation of the similarity between the commodity abstraction and mechanism which is quite close to Borkenau's, although Borkenau is nowhere cited. He states:

> The economic concept of value resulting from (the commodity abstraction) is characterised by a complete absence of quality, a differentiation purely by quantity and by applicability to every kind of commodity and service which can occur on the market.

This concept and this abstraction bear, he claims, a "striking similarity with fundamental categories of quantifying natural science." About the character and source of these abstractions and concepts he writes,

> While the concepts of modern science are thought abstractions, the economic concept of value is a real one. It exists nowhere other than in the human mind but it does not spring from it. Rather it is purely social in character, arising in the spatio-temporal sphere of human interrelations. It is not people who originate these abstractions but their actions. 31

Thus far we are in broad agreement with Sohn-Rethel. After suggesting, however, that the abstractions and concepts he is analysing do not spring from the mind but from social relations, he procedes to give both mind and conceptions of nature an autonomy and natural foundation which contradict his notion of their social relativity.

In a nutshell his argument is as ~~follows.~~ In exchange, thought and action are separated. Action on the market abstracts from the natural qualities of the objects exchanged. In exchange use is banished from the activity but not from the mind;[32] thus, the appearance of an independent intellect left on its own to contemplate use, i.e., the physical character of the world. The abstractness of the activity is not accessible owing to the business at hand of exchange and the desirability of the commodity dealt with. Thus exchange activity, for Sohn-Rethel, mitigates against an interpretation of nature based on any direct perception or appreciation of nature through manual labour.[33]

Abstract social relations produce an independent intellect which in Greek society based on slave-labour gives rise to philosophy and in European society based on wage-labour gives rise to modern science.[34] This independent intellect, the independent mind, receives its formal elements from the commodity abstraction and provides the foundations for both Greek philosophy and modern science. In the case of modern science its categories differ from Greek philosophy inasmuch as they are required to effect a measure of control over a labour-process peopled with essentially equal beings, beings with human qualities. The ideal abstraction provides the form of thought but not the content.[35]

> These contents are nothing but the basic features of the physical act of commodity transfer between private owners. It is this physical event which is abstract (this is precisely why we have called it the 'real abstraction'). It is a compound of the most fundamental elements of nature such as space, time, matter, movement, quantity and so on. The concepts which result from the identification of these elements are thus in their origin concepts of nature.[36]

This is precisely to beg the question. If we are to raise the question as to the possible social origin of science and its concepts, it will not do to blithely claim that they are simply concepts of nature in their origin. Although there may be some account here of the origins of specialised abstract thought and division of intellectual and manual labour generally, there is no historical or social account of the origin of the concepts of mechanistic science.

Sohn-Rethel, in his desire to claim scientific abstraction for capitalist social relations, creates the problem for himself of having to separate out the natural foundations of the content of the concepts in a relation with nature. The mathematics is for him abstract; the concepts are themselves natural. His orthodox Marxism prevents him from relating the content of the concepts to historically specific conditions. This too, we are arguing, is a possibility consistent with Marx' thought, although there are tensions in Marx' writings on this issue as well.

Sohn-Rethel marks mathematics as the dividing line between intellectual and manual labour[37] but provides no account of the possible root of changes in mathematical thought with the rise of modern science. He cites the rise of coinage in ancient Greece as marking the beginnings of theoretical mathematics and, although we may be in broad agreement here, Sohn-Rethel does little to analyse its content beyond remarking that it amounts to a "generalisation" from monetary commensuration.[38] He remarks that the Renaissance craftsmen needed mathematics (for controlling a new social environment technologically) but provides no account of how the new concepts of this mathematics were derived.[39]

Sohn-Rethel bases his argument here on a correlation, but does not really provide an explanation of the connections. "Capital and mathematics correlate: the one wields its influence in the fields of economy, the other rules the intellectual powers of social production." [40] His understanding of the mechanism of Hobbes and Descartes is that their notion of the world as a self-operating mechanism is based on the postulate of the "self-acting property of the labour process."[41]

Mathematics appears as the logic guiding "the human mind in its socialised form."[42] What we have, thus, with Sohn-Rethel, is the proposition that, first of all, only the formal properties of science can be dealt with, since they are the product of an abstract, private intellect, and that its content is natural. He claims that an understanding of human cognitive faculties demands a formal analysis of commodity exchange "in complete methodological separation from any consideration of the magnitude of value and the role of human labour associated with it."[43]

For to do so would force Sohn-Rethel away from a rather crude materialism which he wants to espouse very dearly. Here the form of thought is considered social, the content natural. Sohn-Rethel wants to explain in social and historical terms only the abstractness of scientific thought. The following passage is a telling one:

> Commodity exchange, when attaining the level of a monetary economy, gives rise to the historical formation of abstract cognitive concepts able to implement an understanding of abstract primary nature from sources other than manual labour. It seems paradoxical, but is nevertheless true, that one has first to recognise the non-empirical character of these concepts before one can understand the way in which their indirect natural origin through history achieves their validation. One might

> speak of science as a self-encounter of nature blindly occurring in man's mind. 44

What is social here is simply the independence of the intellect. Sohn-Rethel is concerned to demonstrate that concepts which are not directly empirical can bear a "necessary reference to nature at all."[45] He states that the capacity of our intellectual knowledge of nature has exceeded that accessible to handicraft production; true, but this does not necessarily mean that social abstractions constitute adequate tools having indirect reference to nature. He identifies social and natural history.

> Thus my derivation of the concepts a priori of science is a natural one, not relating, it is true to the external nature but to the historical nature of man himself. 46

It is our argument here, of course, that Sohn-Rethel has developed only one side in his use of Marx' analysis of the commodity relation. Marx was in fact most concerned to display the opposition of the social and natural in his discussion of the peculiarities of the equivalent form of value and the concept of commodity fetishism. Natural and social properties, concrete and abstract labour and the social and private character of labour are made commensurate in the commodity exchange. This accounts for a lack of consciousness, on the part of participants to whom only the physical properties of commodities are visible, of the social character of the relation between things. (cf. Chapter III infra)

Sohn-Rethel cites this lack of consciousness and makes it responsible for the separation of the independent intellect.[47] In this, the argument may have some cogency. Mathematical-mechanistic science, however, was not constituted by consciousnessless consumers. It was begun, in part, we are arguing, by resolute people who were very conscious of the calculable aspects of social relations which needed keeping track of and who helped develop concepts and techniques for carrying out this social accounting. It is extended by analogy to nature by those whose mathematical interests are academically influenced by commercial concerns and traditions.

David Bloor

Although Sohn-Rethel gives some expression to the notion that the commodity relation is in some sense 'responsible' for early modern mechanistic conceptions of nature, he does so in a way which wants to ensure the naturalistic grounding of the resultant concepts. David Bloor gives expression to a superficially strident relativism claiming that there are no limits to investigation by the sociology of knowledge which lie in the character of scientific knowledge, rationality, validity, truth or objectivity.[48] Sohn-Rethel claims:

> From our viewpoint, however, these economic and sociological changes are not the main focus of interest. They are not the ones that can explain the logical and historical steps leading to the formation of science. 49

He goes on to formulate an answer in terms of the technological requirements of the new social forms.

Bloor's relativism, however, is rooted in the formulation of a "strong Programme" for the sociology of knowledge which takes as special and transcendent the very form of thought which it is our purpose here to interpret in social and historical terms. The sociologist, Bloor suggests, is concerned with knowledge "purely as a natural phenomenon".[50]

Generally, Bloor suggests that any thought, belief or knowledge may be related to social life as long as it is collective. He is concerned here with the partly social causes which bring about certain beliefs or states of knowledge. In addition to this tenet he develops three others which constitute the core of his strong programme, impartiality, symmetry and reflexivity. Each side of the dichotomies, truth-falsity, rationality-irrationality, and success-failure would require explanation. Each side would, furthermore, require the same sort of explanation, hence, the principle of symmetry. Sociology itself would have to be open to such explanation, hence, the tenet of reflexivity.[51]

For Bloor these are not new tenets but represent "an analgam of Durkheim, Mannheim and Znaniecki".[52] His claim that this represents a relativistic position rests on his attack on notions of the autonomy or transcendence of special kinds of knowledge. These areas are related to social causes for Bloor. The relation that Bloor has in mind, however, is one in which, <u>à la</u> Durkheim, society can produce more or less adequate concepts because it, like the rest of the world, is organised on rational, scientifically accessible principles. His notion of causality here is one which seems to violate his principle of symmetry. Although we are in general agreement with his concerns about the transcendent and timeless character of certain kinds of knowledge, Bloor refers these kinds to a "physicalistic" brand of causality.

> It is as if men can transcend the directionless push and pull of physical causality and harness it, or subordinate it, to quite other principles and let these determine their thoughts. If this is so then it is not the sociologist or the psychologist

> but the logician who will provide the most important part of the explanation of belief. 53

This, for Bloor, leads to a relegation of the sociology of knowledge to a consideration of the causes only of errors and accidents, and the attack on this position is one which we, of course, can embrace. Theoretical elements give us the terms in which we see the world.[54] Social norms and conventions, argues Bloor, define those experiences deemed admissable in the formation of reliable knowledge. Different societies or cultures will provide different "tribunals" to adjudicate the admissability of experience.

If misperception were to become the focus of a sociology of knowledge, it would then fail to come to terms with the "reliability, repeatability and dependability of science's empirical basis. They will fail to show the role within science of experimental procedures, controls and practices".[55] The reliability of sense experience, however, is more or less a given for Bloor, even though he also claims that "(t)here is a social component in all knowledge."[56] The social component, for Bloor, appears as authority or power,[57] the source of "reinforcement schedules"[58] or the approval of "well-tried and successful routines and established techniques of enquiry"[59] which allow for a particular adequate organisation of perceptions. He suggests here that perception and thinking are two different faculties and that the perceptive faculty influences the thinking faculty more than vice-versa.[60] The nature of society and world are presupposed in Bloor's metaphysics. "Materialism and the reliability of sense experience are thus presupposed by the sociologist of knowledge and no retreat from these assumptions is possible."[61]

Bloor takes scientific method for granted.[62] In doing so he selects one of a possible number of socially interpretable ontologies for his own ontology. Again this is asymmetrical in his own terms. Whereas he claims initially that there is no limit to the subject matter of the sociology of knowledge, his empiricism and scientism contain an implicit limit. Although he claims in his tenet of reflexivity that the sociology of knowledge must also be admissable subject matter for itself, we run the risk of affirming the consequent by allowing only such an empiricism or scientism. Just as Kuhn cannot tell us when an anomoly is full-blown enough to allow a deviation from the conduct of normal science, so Bloor cannot tell us when our mechanistic society is in enough danger to allow a non-mechanistic interpretation either of nature or our ideas of it.

Although he is against making knowledge special or considering some of it to be guided teleologically by principles not of this world, and warns that making it so would lead to a lack of control over theorising about its nature, he himself ends up doing a good bit of "special pleading". If this were the case, he claims, accounts of knowledge would be at the mercy of social metaphors. Although social factors may be invoked in the production of adequate knowledge, they may not be seen as metaphors at root. If these factors remained metaphors they would not, in Bloor's view, be causally adequate in the production of useful knowledge. By following his form of special knowledge we may prevent the occurrence of an account of knowledge "doomed to finish its life in as great a state of bondage to (social metaphors) as it began it".[63] Of course, we also are anxious to prevent spurious self-understandings. This is why our topic here becomes the failure of a mechanistic understanding of mechanism. It is only by Bloor's special knowledge (naturalistic sociology) that we may prevent such acts of bondage, claims Bloor.

It seems that there are only certain collectivities whose power and authority or "reinforcement schedules" Bloor is willing to accept. We can see the reason for this in his use of Durkheim as part of his amalgam. Just as religion is socially based for Durkheim, so is science for Bloor. As with Durkheim, however, in Bloor's scheme this appears as the result of a society organised more or less on rational, mechanistic principles intervening between nature and the thought about it. Nature and society are organised according to similar principles.

For us, of course, the Marxian notions of historical specificity and a materialism which notes the mutual influence between relations in society and relations to nature, provide for accounts of ideologies, religions or sciences which are more open-ended and historical. Bloor is worried here about the possible destruction of all claims to knowledge if sceptical attempts to use the sociology of knowledge to explain error remove the "natural limit to the scope of causal explanation".[64] The principles guiding the causal account of knowledge are themselves timeless for Bloor.

In Bloor's account thus far we have a position which in some ways augurs well for our argument. He states that the difference between knowledge and belief is society,[65] the logic of a concept is a social residue and not vice-versa[66] and refers to the social component of all knowledge. On the other hand, however, we have an implicit metaphysic which suggests that some social components are better than others, namely, those giving rise to naturalistic accounts of knowledge.

Bloor does, however, treat as examples those forms of knowledge taken to be most special, primarily mathematics. After discussing the possible contributions of Mill and Frege to an account of mathematics which sees mathematics as both experience-bound (Mill) and objective (Frege), Bloor claims that the reality to which mathematics refers is not special but is rather physical objects and society.[67] He wants to produce a causal account for uniformity and consensus.

He then discusses precisely those developments in mathematics which concern us in Chapter Four, the "intention" or concept of number and the split between number and magnitude. His use of and agreement with Jacob Klein parallel ours here. He mentions the difference in cognitive style and the intention of number between ancient and modern mathematics, the use of ligatures in Diophantus to refer only to one specific solution rather than to a possible value for a variable, and Stevin's justification of the status of 'one' as a number as presupposing the homogeneity and continuity of number.[68] Bloor explains these differences in terms of the fact that these brands of mathematics exist "both in their social setting and against a backdrop of natural, psychological propensities".[69]

He understands the ancient number concept as "a symbolic exemplification of the order and hierarchy of Being".[70] The modern concept as it appears in Stevin he understands as founded on technological requirements. For Bloor, "Numbers come to perform a new function by indicating the properties of moving, active processes of change";[71] true, but numbers had to be formulated in such a way as to allow for this applicability. The ability of numbers to perform this function, we argue, is developed gradually in commercial practice from Fibonacci to Stevin. The use of a new concept of number in understanding nature is by analogy from commerce, not by technological applicability.

The character of Bloor's empiricist tenets requires that he find an engineering rather than a strictly social "backdrop" for this new character of number. To be fair to Bloor, he does contend that "all knowledge is relative to the local situation of the thinkers who produce it".[72] He relates knowledge here to ideas, conjectures, problems, assumption and criticism, purposes and aims, experiences, standards and meanings. These are, he claims, naturalistic determinants of belief. It is this naturalism with its empiricism and physicalist causality with which we take issue.

We are in agreement with Bloor that one of the criteria for beliefs to become knowledge is that they become collective. Neither we nor Bloor can dream an interpretation of nature and expect its status to become knowledge without its collective reinforcement. We cannot select any belief at will and give it an equal status as potentially knowledge with any other belief, even though their status may also be socially relative.

Bloor's answer to this is naturalistic. We cannot defy certain of the regular responses of nature to our actions. In this we are in general agreement. This obviously limits greatly the number of beliefs which can become candidates for the status of knowledge. It is another matter entirely, however, to suggest in advance that nature is a rational, all-encompassing nexus of causality which can be comprehended by one view. Although the number of possible beliefs is limited here, the number is still very large. There are no real grounds for supposing that any particular view has a monopoly on the correct interpretation of nature, whether it be "ours" or anyone else's.

Bloor's ultimate justification for his view is that it represents our culture and our form of knowledge.[73] We are back now to the authority and power of a collectivity; it is truly, however, "an underlying collectivity constructed by scientific thinking". Just which we we are allowed to have membership in to do the sociology of knowledge is determined in advance and ungrounded by Bloor.

All six of the thinkers discussed here have an unexamined belief or faith in modern science. Even Weber, who has ambivalent feelings about it in some respects, capitulates to this belief. In Durkheim's use of the notion of collective representations, there is a positivistic, mechanistic world-view in operation. Mannheim explicitly refuses to consider science as socially or ideologically rooted.

In the cases of Sohn-Rethel and Bloor, analysts who thematise science itself in the sociology of knowledge, their scientism becomes all the more curious. A relativism which endeavours to discover the extent to which science is a social construction, cannot stop short at "scientistic" understandings of relativism.

To use "materialist and empiricist" tenets to understand the question of science's possible social origins, is barely to raise the question at all. It seems to be part of an attempt to leave science's self-understanding intact at the same time that social roots for ideas are hinted at in a way which also fits that self-understanding in most ways.

We fail to see the need for the sociology of knowledge to apologise to science in this fashion. The mechanistic world-view, although it has had a highly nuanced history, still constitutes something of a Zeitgeist. All six of our commentators here pay some sort of homage to science. In Weber's terms, they view all things, including social relations, as masterable by calculation. Social life is probably that realm most recalcritrant to calculation. The efforts of sociologists to approach it in this manner have had only very low-level results. To subject science as subject matter to a questioning of its possible social origin or construction is thereby also to raise the spectre of the recalcitrance to calculation ~~of~~ this relation itself. Our view is thus more radically relative than that of Bloor.

Sohn-Rethel has also used Marx for this kind of purpose. There is another side of Marx, however, which we intend to use for our own relativism. In his notions of historical specificity, the mutual influence of relations in society-relations to nature, the labour theory of value and commodity fetishism, Marx provides a way of understanding historically situated intellectual developments at the same time as he provides an understanding of why those roots are not visible to contemporary participants. We draw here from that side of Marx' work which emphasises social relations. Not much modern commentary is directed at that side of Marx' work, with the exception of the work of Lawrence Krader.[74] It is from this side too that Borkenau draws his notion of abstraction. We turn now to our development of this part of our framework in Chapter Three.

Notes

1. Ernst Gruenwald "Systematic Analyses" in James E. Curtis and John W. Petras eds. The Sociology of Knowledge Duckworth, London, 1970, 203.
2. Emile Durkheim The Division of Labour in Society translated by George Simpson Free Press, New York, 1964, 41.
3. Durkheim The Elementary Forms of the Religious Life translated by Joseph Swain Free Press, New York, 1965, 18, 21, 22.
4. ibid. 31
5. ibid. 32 fn.
6. ibid 493
7. Max Weber The Theory of Social and Economic Organization translated by Talcott Parsons Free Press, New York, 1947, 87-112
8. Weber "Science as a Vocation" in Hans Gerth and C.Wright Mills eds. From Max Weber Oxford University Press, New York, 1946, 139.
9. Karl Mannheim Ideology and Utopia translated by Louis Wirth and Edward A. Shills, Harcourt, Brace and World, New York, 1936, 2.
10. ibid. 94.
11. Gruenwald op. cit. 212.
12. Gruenwald op. cit. 215.
13. Kurt Wolff "Ernst Gruenwald and the Sociology of Knowledge" in Trying Sociology Free Press, New York, 1974, 604.
14. Durkheim op. cit. 28, 493.
15. Durkheim op. cit. 466-7.
16. Durkheim op. cit. 31.
17. Durkheim op. cit. 471, 485-8.
18. Frederick I. Kersten "The Constancy Hypothesis in the Social Sciences" in Lester E. Embree ed. Life-World and Consciousness Northwestern, Evanston, 1972, 537.
19. Weber The Sociology of Religion translated by Ephraim Fischoff, Free Press, New York, 1964, 94.
20. Weber The Protestant Ethic and the Spirit of Capitalism translated by Talcott Parsons Scribners, New York, 1958, 27.
21. ibid. 21.
22. ibid. 67.
23. ibid. 27.
24. Borkenau Uebergang 159.
25. Mannheim op. cit. 103.
26. Mannheim op. cit. 50.
27. Mannheim op. cit. 2.
28. Mannheim op. cit. 59.
29. Mannheim op. cit. 89-90.
30. Mannheim op. cit. 290-91.
31. Alfred Sohn-Rethel Intellectual and Manual Labour translated by Martin Sohn-Rethel MacMillan, London, 1978, 20.
32. ibid. 25-6
33. ibid. 27, 75.

34. ibid. 28.
35. ibid. 118.
36. ibid. 70.
37. ibid. 101.
38. ibid. 102.
39. ibid. 112.
40. ibid. 112.
41. ibid. 123.
42. ibid. 130.
43. ibid. 34.
44. ibid. 75.
45. ibid. 128.
46. ibid. 129.
47. ibid. 27.
48. David Bloor <u>Knowledge and Social Imagery</u> Routledge and Kegan Paul, London, 1976, 1.
49. Sohn-Rethel op. cit. 112.
50. Bloor op. cit. 2.
51. Bloor op. cit. 4-5.
52. Bloor op. cit. 5.
53. Bloor op. cit. 5.
54. Bloor op. cit. 18.
55. Bloor op. cit. 20.
56. Bloor op. cit. 28.
57. Bloor op. cit. 46.
58. Bloor op. cit. 68.
59. Bloor op. cit. 72.
60. Bloor op. cit. 28.
61. Bloor op. cit. 29.
62. Bloor op. cit. 39.
63. Bloor op. cit. 70.
64. Bloor op. cit. 72.
65. Bloor op. cit. 72.
66. Bloor op. cit. 66.
67. Bloor op. cit. 93.
68. Bloor op. cit. 99-104.
69. Bloor op. cit. 104.
70. Bloor op. cit. 104.
71. Bloor op. cit. 104.
72. Bloor op. cit. 142.
73. Bloor op. cit. 144.
74. Lawrence Krader <u>Dialectic of Civil Society</u> Van Gorcum, Assen, 1976. <u>A Treatise of Social Labour</u> Van Gorcum, Assen, 1979.

Chapter Three - Marx and the Sociology of Knowledge and Science

Like the other classical sociologists of knowledge, Marx did not comment directly very often about science. Unlike the others, however, he did not begin with positivistic, empiricist or mechanistic assumptions about society itself. Basically Marx saw society as a more or less integrated whole, the nature of which changes over time. The most fundamental aspect of society for Marx is the way in which a particular people provide for their existence. This, claims Marx, gives shape to everything else which they do and think.

This "giving shape to" is not a mechanically deterministic process. There is some debate, even within the context of the sociology of science, as to whether or to what extent, Marx would support the notion of the autonomy or independence of the knowledge-claims of science.[1] We are making no attempt to resolve this issue. Our discussion of Marx here is limited to the present purpose of formulating those aspects of Marx' thought which are capable of contributing to our perspective on the social constitution of the context of early modern science. Mulkay points out that academic sociology has, for the most part, not read Marx in this way. He cites Merton's interpretation of Marx, which is also Merton's own view, as more typical and as "granting science a status quite distinct from that of ideology".[2]

It is in The German Ideology that Marx formulates some of the concepts of which we make use. The discussion in this work concerns primarily the social mediation of the human relationship with nature. Marx criticises the universal formulation on the part of the young

Hegelians of the relationship of the human being with the world. Although there may be a few obvious sine qua nons of this relationship, what is important for Marx is an understanding of the different roots of different relationships to the world.

In the context of religion, Marx criticises the young Hegelians' criticism of concepts. The old Hegelians had viewed such concepts (religious) as the true bonds of society whereas the young Hegelians saw them as the chains of men. Marx points out that both view the religious concept as "a universal principle existing in the world".[3] The young Hegelians, says Marx, are exhorting others to relinquish their own consciousness in favour of a universal one. It is the young Hegelians' own consciousness, however, which is in need of explanation.

Marx has a different conception of the relation between consciousness and reality; this is based on the notion of the social mediation of the human relation with nature.

> The first premise of all human history is, of course, the existence of living human individuals. Thus the first fact to be established is the physical organization of these individuals and their consequent relation to the rest of nature.... The writing of history must always set out from these natural bases and their modification through the course of history through the action of men.[4]

The human being is thus a part of nature and distinct from other species of animals. As philosophers we could distinguish human beings from other animals by any means we wish, consciousness, religion, etc. But:

> They themselves begin to distinguish themselves from animals as soon as they begin to produce the means of subsistence, a step which is conditioned by their physical organization. By producing their means of subsistence men are indirectly producing their actual material life.[5]

To make a very long story short, a division of labour, civil society, classes and the state ensue. Marx periodises history according to the "mode of production" found in each epoch, which , in turn, is characterised by the particular social form of labour found in it. Labour may be bound, unfree in some particular fashion, or it may be formally free, as in the case of wage-labour in a market society.

The way in which the life of a people is produced is thus seen to give shape to other aspects of their activity. Human beings actively produce their means of existence, their relations with each other, and their conceptions, but always with regard to a specific set of material conditions which circumscribe that activity. It is the practical activity of people which underlies all such production and practical life-activity is always the practical activity of a particular group of people in a particular period in history.

> The production of ideas, of conceptions, of consciousness, is at first directly interwoven with the material activity and the material intercourse of men, the language of real life. Conceiving, thinking, the mental intercourse of men, appear at this stage as the direct efflux of their material behaviour. The same applies to mental production as expressed in the language of politics, law, morality, religion, metaphysics, etc. of a people. Men are the producers of their conceptions, ideas, etc., - real, active men as they are conditioned by a definite development of their productive forces and of the intercourse corresponding to these, up to its farthest forms. Consciousness can never be anything else but conscious existence, and the existence of men is their actual life process.6

According to Marx, then, we cannot understand historically occurring products or relations in terms of universal categories or principles. The place of philosophy for Marx is a summing up of the general results of such development.

An important notion thus emerges which distinguishes Marx' conception of social relations, the notion of historical specificity. We must look at how people live in order to understand their relations and products, including intellectual products. In another work, Marx uses the examples of the notion of authority and individualism and is worth quoting at length:

> Each principle has had its own century in which to manifest itself. The principle of authority, for example, had the eleventh century, just as the principle of individualism had the eighteenth century. In logical sequence, it was the century that belonged to the principle, and not the principle that belonged to the century. In other words it was the principle that made the history, and not the history that made the principle. When, consequently, in order to save principles as much as to save history, we ask ourselves why a particular principle was manifested in the eleventh or in the eighteenth century rather than in any other, we are necessarily forced to examine minutely what men were like in the eleventh century, what they were like in the eighteenth, what were their respective needs, their productive forces, their mode of production, the raw materials of their production - in short what were the relations between man and man which resulted from all these conditions of existence. To get to the bottom of all these questions - what is this but to draw up the real, profane history of men in every century and to present these men as both the authors and the actors of their own drama?[7]

In the Grundrisse , furthermore, Marx criticises eighteenth century thinkers not so much for their individualism per se, although this too is involved, but primarily for asserting the universality of that principle. He criticises the eighteenth century "Robinsonades" both for positing the contracting individual as universal and for using as that universal category what was in fact a condition specific to their own society.[8] Production and property receive similar analyses in the same context.[9]

Thus, not only are the abstractions from sensible objects in a society of commodity production abstract from sense data; they are used abstractly in another sense when they are seen as categories capable of interpreting what passes in other periods of history.

For Marx the most highly developed form in the development of these abstract measures is money. This development is a direct result of the reciprocal relations of exchange and the division of labour. This objectification is that of a relation between persons, relations of labour and relations in the market-place as owners of commodities. Although money proves not to be the bottom line in this development it is an interesting place to begin our analysis because of its abstract character. This abstract character, furthermore, can be shown to have been derived from "congealed" social relations and can provide an analogy for a similar analysis of science. Marx' choice of analogies in the following passage augurs well for our argument.

> Money, however, as the individual of general wealth, as something emerging from circulation and representing a general quality, as a merely social result, does not at all presuppose an individual relation to its owner; possession of it is not the development of any particular essential aspect of his individuality; but rather possession of what lacks individuality, since this social relation exists at the same time as a sensuous, external object which can be mechanically seized, and lost in the same manner. Its relation to the individual thus appears as a purely accidental one; while this relation to a thing having no connection with its individuality gives him at the same time, by virtue of the thing's character, a general power over society, over the whole world of gratifications, labours, etc.. It is exactly as if, for example, the chance discovery of a stone gave me mastery over all the sciences regardless of my individuality. The possession of money places me in exactly the same relationship towards wealth (social) as the philosopher's stone would toward the sciences.10

Marx then cites Shakespeare's definition of money as the equation of the incompatible. There is no greed, for example without money. This section of the Grundrisse harks back to the Paris Manuscripts where Marx is concerned with the relation between estrangement and the money system and looks forward to Capital where the abstract character of money becomes understood through an analysis of commodities, value, surplus-value, and fetishism.

If Mannheim deems the quantitative immune from the "idols of the market place", Marx understands the most quantified area of modern life, the economic, as predicated on certain historically specific kinds of social relations. It is our contention, here, that quantitative science is understandable as a social phenomenon in much the same way as Marx attempts to understand the "power" of commodities and markets in social terms, by means of his labour theory of value.

We are drawing, again, upon the work of Franz Borkenau, who, in turn, draws his inspiration from Marx' citation in Capital I of the following passage from Descartes,

> (T)hey caused me to see that it is possible to obtain knowledge which is very useful in life, and that, instead of that speculative philosophy which is taught in the schools, we may find a practical philosophy by means of which, knowing the force and the action of fire, water, air, the stars, heavens and all other bodies that environ us, as distinctly as we know the different crafts of our artisans, we can in the same way employ them in all those uses to which they are adopted, and thus render ourselves the masters and possessors of nature.12

Marx mentions here that Descartes is looking at the world with the eyes of the manufacture period and anticipates new processes of production as a result of new modes of thought. Borkenau's notion here is that a view of the conditions of manufacture gets projected onto all of nature and eventually human society as well; manufacture provides an analogy or preconception for the mechanistic view of nature. The irony here is that a socially derived analogy is used to interpret nature, which view of nature is then used to interpret human society. A conception of human society as regular is derived from a mechanically conceived nature.

In his analysis Borkenau refrains almost entirely from attempting to derive rules for the relationship, society-idea, and instead provides a historically specific account of the role of manufacture and its world view in the class struggles which bring about modern bourgeois Europe.

What we have here are a division of labour, and process of exchange which are said in turn to produce a similar conception of nature. We must first analyse the social relations said to encompass these abstractions. For this we require a study of the problem of value, and in particular we require the labour theory of value.

The theory of value has its beginning with Aristotle who condemned usury but saw exchange as useful as long as need placed a limit on the accumulation of goods. He saw exchange as a relation of equivalence but failed to identify the common element which made dissimilar goods commensurable. The modern expression of the theory of value begins in the sixteenth century with Machiavelli and Luther and labour here plays a significant part . Labour is dealt with in terms of two components: the relations of labour in society, and the relation of labour to nature. These two components were not treated together by Marx' forerunners.

They were treated together only with the treatment of use-value/exchange value and abstract labour/concrete labour. The labour theory of value "is bound in the capitalist historical period to the science of nature and natural relations on the one hand, and to the science of society and social relations on the other; the two sides were developed in their separation pari passu."[13]

Machiavelli contributed to the relation to nature component in arguing that wealth comes from labour of the hands, not of the head. In the History of Florence (1535) he states that goods come only indirectly from nature and are shaped by human industry. He is treating here of the creation of useful things and therefore of concrete labour.

> The value of labour in production and exchange was grasped by the human mind during the sixteenth and seventeenth centuries in either of two ways: Either it was labour in society in general, or it was labour in relation to nature. At this time labour was considered both concretely, in relation to the production of useful things, and abstractly, in relation to exchange of things produced. In order to effect such exchanges a value had to be put upon the labour. The act of exchange and its expression in abstract measure of value were not introduced for the first time; but the theoretical expression of the exchange was now promulgated, at first slowly, and only much later became widespread, in the later seventeenth and eighteenth centuries, with the full development of the capitalist mode of production in western Europe.[14]

But whereas Machiavelli grasped labour in relation to nature and not labour in society, Luther did precisely the opposite. According to Luther the merchant is paid according to the time and effort required to acquire a specific item and such time and effort is to be expressed in units of a day-labourer's labour, and the merchant's earnings in terms of a day-labourer's wages. He did not, however, treat of any relation of labour to nature. He condemned usury and saw goods as the gifts of God which were only distributed by human labour.

For Thomas Hobbes, wealth depends on labour and the industry of men. Born the son of a vicar he met Galileo and decided to apply mechanistic philosophical principles to human society. The wants of men in commonwealth are satisfied by commodities, commodities which are free, in nature, or purchased for labour. Utility determines exchangeability. The labour for which commodities are purchased is also a commodity, and is contracted for by means of the same kind of contract as that involved in the formation of society. Production and exchange are connected for Hobbes since he maintains that it is the surplus product which is exchanged. Just as commodities have as their abstract expression money, so the worth of a man is his price, i.e. the price of his labour-power.

John Locke saw only the relation of labour to nature and represents a regression from Hobbes in this regard. In his Second Treatise on Government Locke sets out to distinguish between common and private right and argues that labour is that which distinguishes common from private property. In this regard he distinguishes between the labour of the body and the work of the hands (and thus between abstract and concrete labour). By virtue of this labour we make ours what was formerly common.

> (T)he grass my horse has bit, the turfs my servant has cut, the ore I have digged in any place, where I have a right to them in common with others, become my property without the assignation or consent of anybody. The labour that was mine, removing them out of that common state they were in, hath fixed my property in them.15

William Petty[16], although he used the term price instead of value, is the original figure in the history of the labour theory of value proper, particularly in respect of the measure of value by labour time. Petty wrote,

> Let a hundred men work 10 years upon corn, and the same number of men the same time, upon silver; I say that the net proceed of the silver is the price of the whole net proceed of the corn, and like parts of the one, the price of like parts of the other.17

Implicit in Petty's equation is the notion of human equality, at least where labour is concerned. If labour-power can be seen as quantifiable in relation to the power and product of other labours, then the labourers must be seen as equal.

The similarity between individual labours, which began to have intellectual cogency at the end of the middle ages, is only a formal one and, for Marx, comes to mean simply that where value is concerned one man's labour is as good as another's. This formal or abstract equality is a function of the social form of labour in society and for this point Marx takes his cue from a reading of Aristotle.

In his commentary on the problem of value Aristotle found it odd that various goods seemed to exchange with each other in rather definite proportions. Having reduced the money form of commodities to the simpler relation of one commodity to another, Aristotle used the example of five beds = one house. What he found odd in this relation stemmed from his inability to decide what is the common substance contained in equal proportions in five beds and one house. Of course, Marx' answer to this is human labour in the abstract. The character of that congealed labour and its root in the social relations of the producers, however, is the most important consideration in Marx' theory of value and in discussing Aristotle he goes on to explicate this point by suggesting what it was that prevented such a thinker as Aristotle from arriving at similar conclusions to his own.

> There was, however, an important fact which prevented Aristotle from seeing that, to attribute value to commodities, is merely a mode of expressing all labour as equal human labour, and consequently as labour of equal quality. Greek society was founded on slavery, and had, therefore, for its natural basis, the inequality of men and their labour-powers. The secret of the expression of value, namely, that all kinds of labour are equal and equivalent, because, and so far as they are human labour in general, cannot be deciphered, until the notion of human equality has already acquired the fixity of a popular prejudice. This, however, is possible only in a society in which the great mass of the produce of labour takes the form of commodities, in which, consequently, the dominant relation between man and man, is that of owners of commodities. The brilliancy of Aristotle's genius is shown by this alone, that he discovered, in the expression of the value of commodities a relation of equality. The peculiar conditions of the society in which he lived, alone prevented him from discovering what, "in truth", was at the bottom of this equality.18

Thus Marx begins his analysis with the nature and form of the commodity and its relations. This is especially relevant here not only because this form is related to social relations but also because the relation between nature and society and its reification are contained in it. In this way, the relation between relations of production and thought, particularly mechanistic thought, are also brought forth. This is most relevant for the sociology of knowledge because, in the first chapter of Capital, Marx provides an account not only of the workings and constitution of the commodity form but also simultaneously an account of why that constitution is not visible to us.

Marx begins his analysis by telling us that the commodity is a thing of nature, it is a material thing. By virtue of its natural properties it is able to satisfy various human wants. The creation of such a product demands a material with certain properties and a specific form of labouring activity performed upon that material. The fact, however, that commodities can be exchanged in certain proportions appears to have nothing to do with their natural properties or the specific form of the labour producing them. Since these are qualitatively different things we are at this point left to discover what is common to both commodities by virtue of which they may be exchanged in definite proportions.

To determine this common thing contained in both, then, we abstract from the natural properties which make them useful and from the specific character of the labour which makes them useful, from the qualities of the material and from the quality of the labour. Just as the commodity appears to have a two-fold character, use-value and exchange-value, so is the labour producing it of a two-fold nature. A specific, useful, concrete form of labour helps make the commodity a use-value and homogeneous, abstract labour, "labour-power expended without regard to the mode of its expenditure", makes it a value. But Marx continues to add that when commodities are looked at as values, manifested as exchange-values, they are crystals of a <u>social substance</u>.

The labour,abstract labour, which forms the value and hence exchange-value of commodities has its measure of quantity in the form of time. Since this labour is homogeneous, however, another important consideration comes to light, a consideration which helps to explicate the social character of this substance. The relation of the various labours from which abstraction is made is such that it results in the constitution of a unit of homogeneous labour. The product of any given individual would thus be seen as containing the number of units normally contained in that product. This unit is the socially necessary labour-time for the production of a given class of article. A certain social division of labour is thus necessary for the existence of products as commodities since the producers must stand in a certain relation to each other.

For the products of labour to confront each other as commodities in an act of exchange, they must have been produced by individuals performing <u>different</u> concrete labours and standing in a certain relation to one another in order to produce such an abstraction from the differences in the concrete forms of their labour. The differences between the useful forms of labour carried on by independent producers develops into a complex social division of labour.

The commodity, then, is a mixture of social and natural components. A natural, law-like explanation cannot be produced for social relations or ideas because, owing to human activity, that relation is always a changing one. This comes out more clearly in the notion of the social character of value and the abstraction of homogeneous labour and the reduction to simple, unskilled labour. "The different proportions in which different sorts of labour are reduced to unskilled labour as their standard are established by a social process that goes on behind the backs of the producers, and, consequently, appear to be fixed by custom."19 In other words, the constitution of the value of commodities is not a natural relation.

> If, ..., we bear in mind that the value of commodities has a purely social reality, and that they acquire this reality only insofar as they are expressions or embodiments of one identical social substance, viz., human labour, it follows as a matter of course, that value can only manifest itself in the social relation of commodity to commodity.20

The commodity appears to have value by virtue of its relation to another commodity. The value of a commodity is something materially different from the body of that commodity and yet appears to have objective existence. In fact, it appears to have this existence in all commodities. This peculiar situation becomes even more curious when we consider that in the act of exchange, the value of one commodity is expressed in the body of another. For example, if x commodity A = y commodity B, the value of A is expressed as the body of B.

> By means, therefore, of the value relation expressed in our equation, the bodily form of commodity B becomes the value-form of commodity A, or the body of commodity B acts as a mirror to the value of commodity A. By putting itself in relation with commodity B, as value in propria persona, as the matter of which human labour is made up, the commodity A converts the value in use, B, into the substance in which to express its, A's, own value. The value of A, thus expressed in the use-value of B, has taken the form of relative value.21

Commodity A in this case is seen as having value by virtue of the fact that B is equated with it without B assuming a form different from its bodily form. One commodity expresses its value in the use-value of another commodity of a different kind. This other commodity thus has impressed upon it the equivalent form of value. The value of the equivalent in this equation finds no quantitative expression. Even in the relative form the value may remain constant while the relative value varies owing to changes in the productivity of labour. In the equivalent form, however, an even more curious fact results. "(U)se-value becomes the form of manifestation, the phenomenal form of its opposite, value.[22] Thus, owing to the social relation of the labourers and the subsequent entry of their products into a relation of exchange, the social property of one is represented in the natural properties of another. In this form, then, even though the two commodities can stand in a relation of equivalence because they contain the same quantities of human labour, the equivalent in the relation does not receive a quantitative expression of that social substance which is contained in it. To use Marx' example of linen = coat, the value of the linen is expressed in terms of the use-value of the coat, "but the coat, in the expression of value of the linen, represents a non-natural property of both, something purely social, namely, their value."[23]

Marx' analysis of the enigmatical character of commodities is thus directed primarily at unveiling the mystified character of the social relations which lie behind it. As long as there is a value relation the fact that a material commodity can express value gives value the appearance of something natural. The relation between commodities thus appears as a relation between things.

> Since, however, the properties of a thing are not the result of its relations to other things, but only manifest themselves in such relations, the coat seems to be endowed with its equivalent form, its property of being directly exchangeable, just as much by Nature as it is endowed with the property of being

heavy, or the capacity to keep us warm.24

Just as the two-fold character of the commodity has its root in the two-fold character of the labour embodied in it, so the peculiar expression of the value of one commodity in the use-value of another has its root in the manifestation of abstract human labour in its opposite, concrete, useful labour.

Because of the necessity of this "opposition" in the form of exchange, the labour of private individual commodity producers is social in character, since it is only by virtue of the "opposition" of their labours that we have a relation of value. This becomes clearer when we realize the multiplicity of forms of labour which are thus related in the world of commodities. Just as we would not exchange one commodity for another of the same kind, so we require a multiplicity of various concrete labours, socially related such that when their products stand in a relation of exchange the abstraction is made from the concrete form of the labour embodied in them, we have a relation of value. The social determination of the magnitude of value determines the exchange proportions of commodities.

This leads to the notion of fetishism. Marx takes the lead here from an early French anthropologist, Charles de Brosses, who used the term to describe the fact that in animistic religions the products of peoples' consciousness come to dominate them in the form of the spirits inhabiting natural objects. For Marx this notion refers to the products of labour coming to control the producers. Consciousness of the social character of the relations between these products is difficult to achieve because people are related through their products. When a table is considered simultaneously as wood and as a commodity, "It not only stands with its feet on the ground, but, in relation to all other commodities, it stands on its head, and evolves out of its wooden brain grotesque ideas, far more wonderful than 'table-turning' ever was".[25] In the reduction of all labour to its common denominator in which the social character of the labour manifests itself, the relations between producers take the appearance of a social relation between things. When all that is observable, owing to the private status of labour, is the relation of objects, "There it is a definite social relation between men, that assumes, in their eyes, the fantastic form of a relation between things".[26] The social character of their labour appears as an objective property of the object. In exchange the objects acquire this social status as distinct from their existence as use-values. The labour thus embodied, however, acquires a peculiar social form not simply because goods happen to get exchanged with some frequency. This is of importance for the relation between commodity relations and the mechanistic world-view.

> This division of a product into a useful thing and a value becomes practically important, only when exchange has therefore to be taken into account, beforehand, during production. From this moment the labour of the individual producer acquires socially a two-fold character. On the one hand, it must, as a definite useful kind of labour, satisfy a definite social want, and thus hold its place as part and parcel of the collective labour of all, as a branch of the social division of labour that has sprung up spontaneously. On the other hand, it can satisfy the manifold wants of the individual producer himself, only so far as the mutual exchangeability of all kinds of useful private labour is an established social fact, and therefore the private useful labour of each producer ranks on an equality with that of all others.27

Thus not only do the reduced and abstracted units of labour and matter appear as properties of natural objects but since the establishment of socially necessary labour-time is a process which goes on behind the back of the producer and appears to be the action of the objects, the producers also appear ruled by the products. So in the relation of commodity to commodity what is really compared, but hidden, is the labour of the private producer with the labour of society.

But when does all this begin to happen? Just about every commentator places it at about the beginning of the sixteenth century and Marx is no exception. "The modern history of capital dates from the creation in the sixteenth century of a world-embracing commerce and a world embracing market."[28] This period corresponds to the manufacture period described by Borkenau. According to Marx in manufacture we have 1) an extension of the division of social labour, 2) a concentration of the implements of labour and of workmen, and 3) the development of colonial markets. Capital's self-valorization could take place on this basis until the middle of the nineteenth century when machines were made by machines and so the manual instrument was reproduced on a cyclopean scale.[29]

Marx has thus proposed a set of social relations which produce the abstractions requisite for our analogy between social abstraction, on the one hand, and a conception of nature as abstract and homogeneous on the other. The social and historical process involved here, written large, is the transition from feudalism to capitalism.

By the beginning of the sixteenth century the process was gaining momentum. Some of its most important elements were already contained <u>in embryo</u> in fourteenth century Flanders and Tuscany. Owing to the size of the political territory involved, as well as other factors, these areas did not sustain development in a specifically capitalist direction. It is in Holland and England in the late sixteenth and early seventeenth century that the die is finally cast.

We are looking then, for developments in commerce and, for this, must draw on the work of economic historians. We are looking for the development of exchange and commodity production. This involves, above all, a new form of daily life. Although it is beyond the scope of this work to carry out a proper social history, we wish to point to evidence of changes toward modern forms in some pivotal social relations.

We expect to see the division of labour expand, the specialization of tasks begin, more of wants and needs satisfied by exchange and hence evidence of political pressure toward the opening of markets and, hence, advances in the profits of the merchant class.

Economic historians bring a set of modern categories to historical subject matter. We are doing that as well. Whereas most economic historians view economic changes in terms of growth or decline in output, productivity or G.N.P., we are looking for changes in social relations which may not necessarily involve any immediate change in productivity or output. We shall attempt now to tease an account of such changes out of the accounts of the period by economic historians.

The Concrete Existence of Abstract Relations

In general, in the 200 years from 1500 - 1700, local economies in Europe burst their bounds giving rise to many changes in social relations. Renard and Weulersse summarize the period in England:

> After the Wars of the Roses there followed, in the sixteenth century, a period of peace and internal development under the despotism of the Tudors. The country became Protestant and in the reign of the Virgin Queen Elizabeth entered on a period of splendid and rapid growth. In the seventeenth century, fighting at the same moment absolute monarchy and the renewed offensive of Catholicism, England became the revolutionary

> center of Europe. Charles I was beheaded (1649), a Republic was proclaimed, and under Cromwell's dictatorship new advances were made. After a short-lived Restoration (1660), a second revolution took place, which gave the crown, in 1688, to a Protestant constitutional monarch.30

Political pressure in the direction of commerce was most clearly manifest in England but it was certainly not alone in this regard.

> The merchants of Amsterdam and London, the administrators of Louis XIV's France, Gustavus Adolphus' Sweden, and Frederick William's Prussia, the military recruiters of the new standing armies all in their ways were working to open up the bound, local economy.31

All of these efforts represent the move from a Stadtswirtschaft to a Volkswirtschaft where initially the city economy exerts its influence over a wider area to become a 'national economy.'[32] Naturally at the beginning of this period much resistance is encountered from conservative theories and forces. J. D. Mackie mentions that 'various logical devices' were used to reconcile the new expansion with the orthodox view.[33] In Mackie's terms business became wholesale rather than simply local and retail. In Tudor England there was a confidence shared between Crown and Parliament. "(T)he Tudors developed the system in which Crown and commons were in alliance."[34] As Crown and Church, however, ceased to be in alliance, and the process, which in one aspect was fulfilled with the dissolution of the monasteries in 1536 and 1539 was completed, "clerical wealth was attracting the eyes of a laity thinking more and more in terms of trade and industry".[35] Commercial expansion arose most in and was enhanced by those nations having a strong central state. Thus in the fifteenth and sixteenth centuries mercantile objectives were foisted upon towns and principalities breaking down protectionist barriers.[36]

What is here termed commercial expansion or minor industrial revolution, in short, the bursting of its local bounds by the economy;

does not necessarily mean vast changes in productivity.

> The unfolding of economic life, even in the absence of dramatic new inventions or conquests, is shaped by irreversible processes. Modern economic theory relies heavily on the concept of stable equilibria toward which economic life tends to converge, even though nudged away by some impulse. But, in fact, the varied impulses that affect an economy more often move it, however slightly, to new positions from which it is impossible to recapture precisely the former position. 37

Various struggling groups wanted access to increased supplies of resources. This required not so much technical innovation as an alteration in the structure of society. It involved foremost the liberation of labour, foodstuffs, raw materials and capital from the bound, local economy.[38]

The economy of seventeenth century Europe was still, however, a peasant economy.[39] The commercial-industrial expansion experienced in the sixteenth century ran its course in the middle of the seventeenth when economic crises demanded a political revolution to complete the transition from feudalism to capitalism. Hobsbawm sees the hint of bourgeois and industrial revolutions in fourteenth century Tuscany and Flanders, and Germany in the early sixteenth century, but from the middle of the seventeenth century, particularly in England and the Low Countries, the die is cast.[40] In seventeenth century Tuscany, however, money went back into the land or into the purchase of offices, palaces or works of art. The institutional changes required for competitive industry were not carried out in Milan.[41]

John Nef accounts for England's rise during the last half of the sixteenth century in terms of a more nationally economic and centralised authority in England than on the continent. In the reign of

Henry VIII which ended in 1547, England was economically backward compared to the continent, but in the period from 1575-1620 England reversed its position relative to France.[42] For Nef, "The relations between industrial and constitutional history provide one instructive chapter concerning the rise of capitalist industry and of representative government in western Europe."[43] The development of constitutional monarchy went hand in hand, for Nef, with the development of industry. The judicial and administrative machinery for enforcing royal will became effectively abolished in England.

Whereas in England the holding of successful political power depended on good relations with merchants and gentry as well as other forces in society, in France the crown had to deal with people who were office holders for life, nobles, churchmen and lawyers but not with capitalists with investment in industry. Although the French crown had been weakened by religious wars from 1560, it was stronger than ever by the 1650's.

In the case of both nations the crown was attempting to expand its powers in the sixteenth century in a process extending from medieval constitutionalism to modern absolutism. At the beginning of the sixteenth century the French crown was more powerful than the English.[44] In this case the Tudors were trying to be as strong as the Valois but the difference was that in England the administrative set-up inherited from the middle ages made it more difficult not to take into consideration the wishes of a greater number of subjects including those with commercial and industrial interests. In England one basically had to please the country and the city represented by landed property and wealthy merchants respectively. In France one had to please the Church and the nobility. In England the gentry and city merchants had interests in the putting-out system in the country and in large-scale production in the country and towns of salt, glass, beer, soap, metals, sugar and alum.

The sixteenth century is typically characterised in economic terms as a mercantile period. Commercial expansion was based on exploiting the price differences between different areas for a given item. This was coupled with and enhanced by colonial expansion, first east and then west. The Portuguese and Spanish reigned first, later to be replaced by the Dutch and the English. In particular it was the Spanish who brought large amounts of gold and silver back to Europe which practice peaked in the years 1608-1610 and steadily declined from 1622.[45] This complicated the price-revolution and helped lead to the crisis mentioned above.

It is not the case, however, that this 'mercantile' explosion is a matter simply of increased market expansion and plunder. The organisation of labour went through great changes during this period, changes affecting the populations of Europe, Africa, the Americas, and to a much lesser extent, Asia. Goods had to be produced in Europe to ~~exchange~~ (even if the exchange were unequal) for goods such as slaves or silver and in the Americas the goods produced were products of slave labour, sugar and tobacco. According to at least one commentator on the gold and silver phenomenon, the period from 1450-1530 was marked by many changes not produced by silver and gold.

> The demographic and economic revival of Europe predated the influx of gold. The great economic changes of the sixteenth century were not caused by the annual arrival of 1,000 or 1,500 kilos of gold on the western tip of Europe, they were the result of a long evolution in demography, agriculture, technology and industry, of the exploration of the European silver mines, of communications, of commercial and financial techniques, and of the organization of national economies by their rulers. 46

So we have a situation with more people engaged in wage-labour than ever before providing the textiles traded on expanding markets. Although many of these people also work agriculturally, a small revolution in agriculture is required to feed a growing population of 'industrial' workers. Coupled with this was the increase in inflation beginning around the middle of the sixteenth century which made labouring for a wage even more necessary. Here are a few examples: At the end of the fifteenth century a hundredweight of wheat cost sixty hours labour, while after the price-revolution of the sixteenth century the same wheat cost 200 hours labour.[47] The wages per hour did not rise at all while this was happening. After 1550 food prices rose while wages remained constant.[48] For the following years the number of weeks of work required to purchase a year's supply of bread is as follows: 1495 -10, 1533 - 14-15, 1564 - 20, 1593 - 40, 1653 -43, 1684 - 48, 1726 - 52.[49]

> The age when all must needs work was beginning, and when the poor were, indeed, driven to work by law. Those who had no capital must give up all hope of becoming masters and resign themselves to becoming forever the servants of the more fortunate. Moreover, they began to suffer from legal disabilities. In many cases then mechanics, as they were scornfully called, were excluded from municipal offices. The proletariat of the future was in sight. 50

A certain relation of labour gradually becomes a necessity for the sake of survival, partly because of its enshrinement in law. The requisite agricultural revolution took place in Holland. Conditions were ripe there for peasants to commercialize agriculture whereas this could not take place in Italy because the urban markets actually declined.

> In the Low Countries, which since the 1580's had been divided into the Dutch Republic and the Southern or Spanish Netherlands, many of these trade flows found their focal point: here the most profound and far-reaching specialization process operated to reorganize the agrarian structure and raise up the most thoroughly commercialized agriculture of Europe. 51

Following this development in Holland, the English Crown, during the Civil War, lost its ability to resist the imposition of capitalist relations on the land.[52] So beginning in Holland we have an agriculture which is capable of supplying the needs of a relatively 'industrial' work force and a commercial sector trading industrial goods on a world-wide market. What agricultural needs could not be met locally were met by importing grain from Poland where the nobles saw an opportunity of extracting a surplus arising from a forced return of the labourers to serfdom.[53] The main hindrance to the Dutch was the small size of their country. With the price of grain kept high, many were forced to labour longer than they otherwise might have and contemporaries were certainly not unaware of this.[54] When a clothier is keeping the grain price high to ensure a steady supply of labour, we can also rest assured that he is aware of labour's profit-producing qualities. These processes reach fulfillment following the crisis of the seventeenth century, but "Long before the seventeenth century the form of industrial organization characteristic of medieval municipalities had been forced from its dominant position."[55]

William Petty uses the Dutch as a model. Based on such processes and indigenous advantages England caught and surpassed France and the Low Countries. In general the private economy boomed because of a symbiotic relationship between the state, military power and the private economy in this age of absolutism.[56]

DeVries has summarized the changes in production occurring during the period ~~and is~~ worth quoting at length in this connection.

> Another major field of production was in the organization--in contrast to technology--of production. The principle of division of labour did not have to await the rise of the factory system to be made operative. In our period many regions felt the impact of agricultural specialization or witnessed the elaboration of a putting-out system in rural industry. These organizational changes can be seen as the result of two interacting pressures. Relative price changes and demographic movements created opportunities which were seized upon by merchants, industrial producers, landlords, and portions of the peasantry wherever the legal and institutional environment gave sufficient rein to their profit motives. Under these combined pressures an elaborate stratification of commercial farmers, cotters, wage-dependent labourers, plus a variety of artisans and service-sector workers came to populate the countryside. 57

We have, now, a situation where Italy and Spain have declined, France has become stagnant and not seized opportunities and the Low Countries and England have gone a long way toward developing the kind of abstract social relations we have been talking about. The remaining discussion will be divided into two main parts, commercial and financial changes, and the organization of labour and labour time.

<u>Changes in Commerce, Finance, Credit and the Extension of the Money Economy</u>

According to Carlo Cipolla it makes sense to use the term 'commercial revolution' to describe England and Holland in the period 1550 - 1700,[58] and many specialized services developed to meet changing business needs.[59] Adam Smith claims that the most important reasons for the changing world economy are the explorations and discoveries of daGama and Columbus. He also believed that it was the application of the compass which was, in turn, responsible for the success of their voyages.[60] Here, in a nutshell, is the problem of this thesis. Does the commercial revolution mentioned above and its attendant reorganization of the relations of labour in society and to nature result in the mechanisation of the world picture? Or, as Adam Smith would have it, is it the other way around, with technical devices slowly acquiring the ability to reproduce themselves and increase productivity? In short, which is the vehicle for the 'conquest' of nature, and its mechanistic interpretation, the mind of man or social labour? Cipolla's introduction cited above augurs well for our argument here. In a work devoted to economic history, Cipolla characterizes the period he is dealing with in intellectual terms, as that of the battle between the ancients and moderns with their mechanised world view.[61] He characterizes the period further by noting the connection between these areas in citing William Petty's letter to his friend Robert Southwell, "My virtue and vanity lies in prating of numbers, weight and measure."[62]

Petty sees, here, the beginning of that very discipline which wants to ask only quantitative questions of his environment. It is our argument that he sees here as well, symbolically at least, the beginning of other quantitative disciplines. This development, we are arguing, is not simply a turn of mind. In the midst, however, of our difficulty in finding works by economic historians where issues of organizational change are studied and seen as potentially significant economically and socially even when they do not directly result in vast changes in output or profit, we find the almost-back-door admission that our period marks one of the first in history where events may be looked at in this fashion because they were thus looked at by contemporaries.

> The major difficulty [concerning records] is that so much that went to sustain the life of the people was imperfectly separated from the home, and much, perhaps most, production was still part of a non-market organisation. Economists, of course, are familiar with the position: at the beginning of most nations' movement towards a 'modern' economic structure comes a time when there is a semi-illusory leap ahead as a higher proportion of the country's production starts for the first time to flow in channels in which it can be counted. 63

This writer, Sybil Jack, in the course of her critique of the 'early industrial revolution' thesis, even though she is motivated to search primarily for evidence showing as little economic advance as possible, still admits, and she is unique in this regard amongst recent writers in economic history, that other kinds of changes in work and economic life may be singled out as highly significant.

> Significant change may have been structural, that is, concerned with a growing integration of industry, rather than with large-scale growth in itself. What was occurring concerned the changes which are necessary before revolution can come rather than revolution itself. 64

Significant change and advance begins, nonetheless, in the commercial area. Exchange between communities has an effect within the community. The advent of formal equality between persons with respect to labour which Marx referred to begins with a market and a market relationship between nations; "the whole World as to Trade, is but one Nation or People, and therein Nations are as Persons."[65] As this trade develops so does the money economy and wage-labour.

> The spread of the money economy meant that a growing proportion of the European labour force, particularly in the developing trades, was recruited as wage labour." 66

In the sixteenth century the expansion of these markets was striking.

The profitability to some merchants was even more striking. For example, in the Levant trade, £50,000 - £60,000 of cloth per year would be exported, but the imports in one shipment represented a money value of £70,000. This increased until 1630 when complaints about a balance of trade problem were voiced.[67] Even though production advances were made to supply the cloth and metals used in this trade, the changes in the relative power of nations during this period stemmed from commerce and finance.[68] Robert Brenner notes the changes in the form of commercial expansion which involve different forms of production. The transition from the Turkey and Venice companies to the Levant company represents growth in a strictly mercantile sense. A small group of merchants hoped to restrict the trade in a large area to themselves. Twelve merchants in the Turkey company had the entire mid-East market. Of the twelve patentees in 1581, six were London aldermen by the 1580's and three were M.P.'s. Two of them, Thomas Smy[illegible] Richard Martin, were mediators between crown and city.[69] When the Turkey Co. merged in 1588 with the Venice Co. to form the Levant Co. the attempt was made to restrict the trade of the whole Mediterranean to forty-one merchants. When the Levant Co. was chartered in 1592, fifty-two merchants were allowed to conduct business in the area. In other ways, such as expensive apprenticeships, access to merchant status was restricted as well.

The Americas trade, however, was differently organized by different people. Trade was developed outside the companies by voluntary partnerships. These were colonial operations. "In marked contrast to the es-blished London trades, colonial operations required investment in commodity production not merely in commodity exchange."[70] This production was carried out at first on plantations producing tobacco and sugar. The men organizing these operations were not from the overseas merchant operations but were, rather, from a lower socio-economic stratum, the smaller gentry or prosperous yeomanry.[71]

The merchant still inspired and symbolized the new forms of enterprise. In the words of one seventeenth century writer,

> All other callings receive their vigour, life, strength and increase from the merchant...to whose extravagant and hazardous, as well as prudent and cautious, undertaking this nation chiefly owes all its wealth and glory. 72

This new view of social life was found to be inspiring to the poor and middling and cause for concern to the wealthy. According to Bernard Palissy the ploughman wants his son to be a burgher and the workman wants to eat meat like the rich whereas in Germany in the early sixteenth century "the clodhopper aspires to equality with the noble, citizens wished to look like gentry, the gentry aped princes".[73] This universalizing, levelling and equalizing trend continued as the state played a role in standardizing weights and measures and removing internal trade restrictions.[74]

For the new men who benefitted from these developments the main concerns were money flow and capital turnover. Order was now to be kept for the sake of commercial viability and differential tariffs and laws were set up between nations. Although specialized services were developing, they were rather a far cry from modern institutions. Here again, however, we see the curious bias of modern economists trying to be historians.

> Joined to the fact of imperfect communication and slow transportation was the existence of discontinuous and limited markets. And it is these two elements in the economic framework which explain the outstanding characteristics of commercial and financial enterprise: the absence of functional specialization and the need to decentralize the making of entrepreneurial decisions. 75

Thus, only the relative inefficiency of early modern commerce and industry is attended. What is really at stake, however, is the extent of changes in its structure and social organization.

Hermann von der Wee, however, makes a much better attempt to describe for us the nature and organization of enterprise in this period. Von der Wee reports that the urban expansion of the late middle ages brought the money economy to a pervasive status. This is our most important cue to the existence of abstract social relations. Not only was the money economy growing but it had penetrated the countryside; rural production, agricultural and industrial, was more commercialized, but "Urban industry stimulated internal and external demand through specialization."[76] Through the advance of money and markets economic activity was being integrated into a system of relations.

> The market, designed to bring supply and demand together more efficiently, catalysed the new impulses of the sixteenth, seventeenth and eighteenth centuries. Urban and rural markets, weekly markets and fairs, multiplied in Europe or intensified their activity, assisting the penetration of the local economy by money and credit in many forms. 77

Money use was extended primarily in the form of silver coins, gold coins being seen primarily as merchandise rather than as money of account, and this was made possible by a vast increase in the mining of precious metals from the middle of the sixteenth century. "In principle", however, "token money with limited legal validity did not yet exist in the sixteenth and seventeenth centuries."[78] This is partly responsible for money's apparently mystical nature and the search for value in the physical properties of precious metals. The function of money, however, was in practice performed by silver coins of either domestic or foreign origin, which had been circulated so much they were known as black money.[79] Owing to the lack of legal control and standardization and international agreement money could be debased by means of a decrease in the silver content of a given coin, with the older, higher content coins remaining the same in terms of money of account. This situation is what gave rise to Gresham's Law: bad money drives out good. The older coins in this case would be driven back to the mint and melted down after speculation by merchants, money-changers, and the authorities themselves. If two countries already had an affinity of account between them, one country could flood another country with lower content coins thus making the other country's currency of decreased usefulness in terms of account and driving it out of circulation. Even with these problems standardization was still taking place with the taler becoming increasingly the standard multiple for current money of account in the middle of the sixteenth century.[80]

The development of local credit also helped to integrate the European economy during this period. This we take as evidence of the consciousness of the value being produced in the production process.

> Consumer credit underwent great expansion in the local economy during the period under discussion. Merchants from the town or country regularly bought up agricultural produce from the farmers before it had been harvested, against partial or complete cash payment; the rural population was paid cash for linen or cloth that was to be woven later during the winter months. Where the putting-out system occurred in the towns, similar practices were common. Thus farmers and workers received credit guaranteed by their future harvest or future labour output; sometimes merchants supplied the materials and implements themselves. 81

The most common form of credit, however, was the extension of payment which was bound up with the development of retail business. To keep track of this credit an elementary form of bookkeeping called the tally or current account, was kept by the shopkeeper, brewer, innkeeper or craftsman.

These accounts would be kept open and "set-offs" worked out between two or more parties. A payment might also be made to a third party by means of the transfer of an IOU or scripta obligatoria.[82]

Loans at interest were also becoming more common and governments were setting the legal limits on them. In 1541 Charles V made loans at interest (maximum 12%) legal for commercial purposes. In England such loans were also permitted as long as interest remained below the legal maximum. Triumphs on the part of the Counter-Reformation made such practices difficult in some countries. In general, in the sixteenth century, local economic crises could be successfully bridged by means of the growth of retail trade and with it the custom of the tally or current account. With the price revolution and Thirty Years War local depressions became more severe and could not be solved by those means.[83] Later the sale of annuities became the usual form of credit at the local level. Craftsmen or small businessmen would buy annuities on land, lay foundations for houses on the land and sell annuities on the houses. The urbanization of Antwerp in the sixteenth century was accomplished in this manner.[84] The significance of labour and production in this scheme increased,

> Normally short term credit in local farming, industry or trade was obtained through extension of payment by the suppliers of raw materials or by advance payment by the purchasers. In the putting-out system the farm labourer himself was not infrequently the "involuntary supplier of short-term capital" when his employer applied the long-pay principle. Sometimes the employer paid his workers in IOU's, vouchers on shops or other tokens which the worker could use for his purchases; in such a situation not only the workers but also the local tradespeople became the employer's bankers. 85

We now turn to a specific examination of the character of production and labour.

Production, Labour and Labour Time

As mentioned above the primary stimuli to new forms of production were provided by the expansion of markets and the general shift from a city to a national economy. Within this new sphere manufacturers were protected by prohibitions on the import of goods which were later replaced by a duty. In seventeenth century France Colbert united the northern and central parts of the country by abolishing internal tariffs and using unified import restrictions. Supplies of raw materials for home industry were also insured by placing restrictions on their export from the sixteenth to the eighteenth centuries. This applied to skins, iron ore, copper, wood, potash, flax, and talc.

Necessities were provided for in the medieval city by ensuring that everything necessary was produced there. This was often accomplished by importing the master rather than the goods. The territorial states of the seventeenth century did the same for the trades involved in the production of silk, cotton, carpets, tools, lace, glass, mirrors, porcelain, tobacco, clocks, paper, tapestries and soap.[86] New industries would be granted tax concessions and money advances. The move to the territorial state began in the fifteenth century in England, the Italian territories, in the following centuries in the Netherlands and France and still later in Prussia and Austria. France was fully unified economically only at the time of the revolution and the state generally had a tough battle with cities, gilds and small country princes who defended their former rights.

In the continuing struggle between merchants and gilds, cities, at first, could bring in unavailable goods from another city. The cloth trade was the first to make further inroads in this direction.

In Strassburg in 1477 twenty-two kinds of Rhein cloth were named.[87] Different cloths found their way there from Flanders, Brabant, France, England and Italy.

With different sorts of goods being produced in different places the geographic division begins early in the sixteenth century. At first products would be allowed into a city only from very far away but gradually each city or region attracted different industries resulting in developmental differences in the extension of different trades. This beginning specialization in production tended to improve the quality of many sorts of goods and thereby met the rising standards of taste of the Renaissance. In order then to meet the requirements imposed by foreign competition a given trade, initially practised everywhere, would find itself restricted to a given area where the best workmen and materials were available. Many specialised industries, for example, English cloth, Lyon silk and Solingen metal, were thereby producing for much larger markets than the local one.[88] Those industries which were specifically looking for an expanded marketing area took on different forms of operation such as manufacture, a centralized undertaking in which goods were produced in the workshop of the entrepreneur and under his direction. Much more common, however, was another form of trade production-cottage industry or the putting-out system. Although medieval handicraft was still the basis of this system it differed from handicraft in the mode and size of its organization. This manner of producing goods marks the transition to a wholly different form of society.

Different writers characterize this period of history differently. We are concerned here primarily with changes in scientific thought from

1500 to 1700. For our purposes, the period roughly corresponds to the period from the end of Giordano Bruno to Isaac Newton, from the beginning of the proliferation of mechanism to its formalization and establishment. Donald Coleman,[89] an economic historian, carves up the period from 1485 - 1714, according to political criteria. Christopher Hill,[90] whose concerns are primarily intellectual, uses political criteria as well to demarcate the second half of this period from 1603 -1714. The economic historian Domenico Sella,[91] however, characterizes the period in economic terms but the dates still correspond closely enough; he terms it the period from Columbus to the Bank of England, from the time of a man from the country which is industrially superior at the beginning of our period to the beginning of a financial institution in the country which is predominant at the end of the period.

Even though industrial technique then may not have been highly developed, the labour process was rapidly changing and more and more needs were fulfilled through a market. Most writers comment on the satisfaction primarily of basic needs through industry. The most notable expansion in this early period was in the textile industry, so much so that contemporary statesmen and political writers equated the success and spread of that industry with general prosperity. Construction and metals also were booming; "for the everyday necessities of life...iron is as essential as bread."[92] Another historian, who focuses primarily on the lower classes in his study of the period, entitled his book The Iron Century.[93] Changes in industry began to happen around 1500 and even where the basic structure of production was not changing its geographic distribution certainly was.[94]

Where production itself was changed it was usually changed from handicraft to the putting-out or the Verlagssystem. Even Max Weber attests to the profound significance of this change,[95] although he terms its effects "rationalization" rather than abstraction or the socialization of labour. The expansion in markets domestic and foreign brought about this change in the production process. The basis was still handwork but with two important distinctions. First, the handwork gradually became the prime occupation of the handworker, as opposed to agriculture (even in agriculture the labourer began to be paid cash wages with the proliferation of the money economy and became dependent upon markets). Second, it brought about a gradual specialization with the control and supervision of the whole process by the entrepreneur.

These processes

> meant a notable dichotomizing of the activity of the handworker, which originally encompassed production and marketing, into two independent occupations gradually separated from one another. 96

Some workers would make the move from selling their own products to selling the products of others. Those who had money and schooling gradually oriented themselves toward marketing the products of the trades and became putters-out in the cottage industry system. Those without money and schooling were gradually reduced to producers of goods and gave up selling their own wares to produce solely for the putter-out. Here we see the beginnings of the modern relations of labour.

> The subordination of production to capital, and the appearance of this class relationship between capitalist and producer is, therefore, to be regarded as the crucial watershed between the old mode of production and the new, even if the technical changes that we associate with the industrial revolution were needed both to complete the transition and to afford scope for the full maturing of the capitalist mode of production and of the great increase in the productive power of the human labour associated with it. 97

This process of handwork becoming employed chiefly in cottage industry was completed in the seventeenth century when the handworkers generally worked to the specifications of the business people. The process arises from the end of the middle ages where a process would be broken down into a few parts, each part being carried out at a different place with the goods going through each workshop in succession. Those who ended up as marketers were usually those involved in the final stage of production. The situation changed from one in which the handworker produced for the consumer to one in which he produced for the putter-out.

The system dates from luxury textile production of the fourteenth century.[98] In these export producing cities a class of putters-out developed. Its further extension depended on the development of markets. It was not so much a new productive technology in the modern sense which led to changes in production as it was a way of organizing the market on a large scale.

In the ensuing battle with the gilds the notion of human equality was given its initial boost. The gilds' main restrictions on its members in this regard were the monopoly on selling the finished goods and the firing of workers. Only gildmasters were allowed to sell and the numbers of apprentices and journeymen were fixed so that no one other than these could be hired. A gildorder in Paris in 1575 proclaimed that only those gildmembers who were poor and with no means to open their own workshop were allowed to work in the shops of other masters. Generally, handworkers came to be employed by former colleagues who had risen to putting-out status. This in fact happened in most cloth-producing cities when production exceeded the requirements of the local market.

Many masters had difficulty marketing their own products and ended up having to deal with the putters-out. Small masters and home workers found it to their advantage to sell their wares to business people so they did not have to spend time going from door to door. The number of putters-out continued to grow. Still in connection with the battles between merchants and gilds, once trade in a variety of products was opened to guildmasters and also to those who no longer belonged to gilds, it was a small step from there to the hiring of untrained people for putting-out in the cottage industry.[99] Production and selling became completely separate functions.

The putters-out bought up all the raw materials, went to the countryside to avoid conflict with the gilds, required references from workers and thus made the workers dependent on them, formed new cartels and hired women and children.

Remember that all this is taking place during the age of mercantilism. In this fashion the goods traded abroad were produced and their importance did not go unnoticed. A contemporary French merchant was quoted as saying that linen fabrics are the true gold and silver mines of the realm.[100] Jean Bodin noticed as well that most of Spain's trading goods came from France. And this took place in a situation where an occupational change from plough to loom did not require moving to a town.

Nor did it require many technical improvements since "human muscles were still the great prime mover of industry."[101] The expansion of trade during this period was far more striking than technological improvement. In the putting-out system we have a combination of improved markets (demand) plus the application of home techniques (supply). Once the system began in England it experienced a rapid development since it provided the link between production and the market.

> So it is usually merchant-entrepreneurs who are to be found exploiting the labour of a rural workforce by putting out wool for spinning or yarn for weaving, or iron for making up into small metal wares. Their role was essentially to organize and finance. 102

The system made use of simple skills usable in the household, used a financial structure whose main function was the provision of working credit to cover the completion of a process to the point of sale of the product, and made for the development of an elaborate system of debit and credit. It made use of a situation of underemployed labour, so that even new industries tended to be more labour intensive.[103] This resulted in a three-fold increase in cloth export from 1450-1550, and, in rough figures, a fifteen-fold increase in textile exports from 1485- 1714.[104] This system was particularly suited to textile-production.

> (T)he whole sequence of manufacture could be split up into separate processes, thus achieving that greater degree of division of labour which facilitated the lowering of costs and raising of productivity. 105

Here we have a craft, penetrated by commercial enterprise which suited labour within a family structure. During the reign of Henry VIII the number of undyed cloths sent to Flanders increased from 85,000 to 120,000. J.D.Mackie describes the prosperity desired from it and the various means employed.

> A business so widespread and so prosperous soon burst forth from the simple organisation of the gild and the narrow limit of the town. The 'clothier' became a producer on a large scale who employed men engaged upon all the processes of manufacture. Factories were not unknown. The most famous was that established in the reign of Henry VIII by John Winchombe whose achievements,

> even if they did not reach the magnificence of the ballads, were real. 'Jack of Newbury' was not alone in his enterprise, William Stumpe was a considerable figure too, who not only bought the abbey of Malmesbury from the king, but rented Osney Abbey near Oxford in 1546 and installed, in each of the conventual buildings, a large number of looms. Tucker of Barford, who employed 500 workers, sought in 1538, to establish himself in the abbey of Abingdon and all over the country from Kendall in the north to the Cotswolds, Somerset and Wiltshire, and East Anglia, wealthy clothiers began to flourish. From the evidence it is plain that their looms were in their houses, but often too, perhaps usually, workmen still lived in their own homes each doing his proper work upon the material as it was delivered to him with apparatus supplied by the capitalist. 106

In the centralised form of manufacture the labour process was under the control of the entrepreneur to a greater degree and the production and marketing aspects united even further. Most operations were still done by hand. In terms of language usage, in the seventeenth century "manufacture" either referred to all kinds of production or to textiles, while "factory" referred to means of production using fire and hammer, i.e., metals.[107] Centralised manufacture arose in the sixteenth century and became ubiquitous in the seventeenth.

John Nef argues that this development was great enough to constitute what he termed an industrial revolution in the period from 1540-1640. Sparked by the rise of the British coal industry, Nef began to investigate other areas of industrial expansion. He claims, furthermore, that in the Europe of the Renaissance no single form of industrial organisation was predominant.[108] It was a time when the average tiller of the soil needed to buy only cloth and leather and when there were two - three million workers in Europe. The papal alum works at Tolfa employed a large number of people and the Venetian arsenal employed from 1,000 - 2,000 workers.[109]

> The largest establishments were nearly all owned or vigorously controlled by some public authority -- the pope, a king, a prince, a bishop, a duke or a town council. An increasing proportion of those mines and quasi-factories which employed from a dozen to fifty workers were coming under the direct supervision of such authorities. 110

Nef claims that in addition to the tremendous expansion of the domestic or putting-out system there was also a rapid growth in works using water or horse power because in both there is "a concentration ... on inventive objectives primarily aimed at the reduction of labour costs in the interest of quantitative production",[111] unprecedented in previous history.

Around 1600 there were about 10,000 merchants, 100,000 shopkeepers, tradesmen and artisans in England; in all about 500,000 or just under 20% of the population made their living from trade or manufactures.[112] Although to Sybil Jack this number is small, for our purposes it is sufficiently large to be significant. She, along with D.C.Coleman, argues against Nef's thesis of the earlier industrial revolution. The debate here revolves around technology and output whereas we are more concerned with changes in production relations. As far as the new centralised labour process is concerned it is necessary simply to establish that it was not an utter rarity. There are reports of a Glasgow woolen miller in 1700 employing 1400 employees and one at Saptes in France employing 800,[113] silk manufacturers with 700, a saltmaker with 1,000 and sail cloth makers with up to 600 employees.[114] In industries needing large machines or where the product itself was large (such as a ship) manufacture would be used. This was the case in mining, smelting, metal finishing, alum, fulling, brewing, paper, soap and glass.

All these relations of exchange lying behind the industrial developments gave rise to particular relations of labour and to an acute examination of those relations, at least inasfar as cost and profit were concerned. In many cases these manufactories used a new form of unfree labour in the form of those confined in mental institutions, jails, workhouses and poorhouses. Different sorts of wanderers and the infirm provided a reserve army of the unemployed. Burghers began to give money to factories for employing those who would normally have received the money in the form of alms.[115]

If the conditions were better outside the workhouses, the attitude towards labour was about the same. Christopher Hill quotes Bernard Mandeville,

> In a free nation where slaves are not allowed of, the surest wealth consists in a multitude of laborious poor. ... To make the society happy and the people easy under the meanest circumstances, it is necessary that great numbers of them should be kept ignorant as well as poor. ... We have hardly poor enough to do what is necessary to make us subsist. ... Men who are to remain and end their days in a laborious, tiresome and painful station of life, the sooner they are put upon it, the more patiently they'll submit to it for ever after. 118

Coleman mentions that seventeenth century writers were in general agreement about the 'laborious poor' on three points. 1) There were many of them and should be more, 2) they should be kept properly employed, 3) the poor should remain poor.[119] He cites the contemporary Peter Chamberlen to the effect that the poor could be "the richest treasure of a nation if orderly and well employed".[120] Charles Davenant spoke to the notion that the people are "the first matter of power and wealth", and similar views were put forward by Child, Petty, Pollexfen and the author of <u>Brittania Languens</u>.[121] Wage setting was the setting of maximum not minimum wages, and John Cary, Daniel Defoe, and Dudley North were unique in their arguments for higher wages. The common view lasted for a long time, according to Coleman. A collection of value-potential was to be kept above starvation. Everyone was indeed forced to work and, as Andrew Yarranton said in the 1670's of children who did not work, "He who has most is poorest".[122]

> To Bacon, labourers and cottagers were 'but housed beggars'; to a writer of the 1640's it seemed reasonable to suppose that 'the fourth part of the inhabitants of most of the parishes of England are miserable poor people, and (harvest time excepted) without any subsistance. The comprehensive and well-known investigations of Gregory King in the 1680's and 1690's tell an even grimmer tale. He classed 23 per cent of the national population as 'labouring people and out-servants' and a further 24 per cent as 'cottagers and paupers', estimating that both groups had annual family expenditures greater than income. 123

This places one quarter to one half of the population below the poverty line and includes most labourers. Coleman's whole analysis, however, concentrated on 'long-term forces' and one writer takes him to task for the fact that there is little difference between Coleman's and the seventeenth century preacher's attitude toward labour except in Coleman's search for exogenous influences.[124]

Not only was labour seen as the source of wealth, but it was beginning to be calculated in units of time. The Statute of Artificers in 1563 was an attempt to regulate hours.[125] In Bleiberg (Corinthia) there was a work schedule calling for a 58½ hour week divided into 6½ shifts.[126] E.P.Thompson reports that in early urban England there was a division between court time and merchants' time, which Jacques

LeGoff opposes to the time of the medieval church.[127] Time is reduced to money from the employer's point of view and the distinction between worker's time and employer's time is ushered in and as soon as hands are employed, labour is timed.[128] Sextons were paid to ring bells, horns were sounded and knockers-up made their rounds to wake-up labourers and send them to rest. With increased specialisation of tasks time became even more important because operations had to be synchronised.

> It is by no means clear how far the availability of precise clock time extended at the time of the industrial revolution. From the fourteenth century onwards church clocks and public clocks were erected in the cities and large market towns. The majority of English parishes must have possessed Church clocks by the end of the sixteenth century. 129

Whereas beforehand life had been allowed to ebb and flow from periods of intense labour to periods of rest, it now became more disciplined and controlled. Time was now important for the worker as well since "it was in the sixteenth century that domestic clocks and watches became much less of a rarity".[130] At the end of the seventeenth century Sir Ambrose Crowley, in his iron works, controlled his workers' hours, watched over their children's education and checked on their off-duty behaviour.[131]

We now have a Europe which is establishing colonies, expanding its internal markets, changing from a hereditary to an occupational society in which people are forced to work for a wage in order to live. Industry was developing and industrial capital, although not predominant, was there and operative.Ronald Meek refers to the analyses of Nicholas Barbon in this connection.

Barbon's artificers, as he himself makes quite clear, are assumed

> to 'cast up Profit and Loss' with reference solely to time. It is only the merchants who 'cast up Profit and Loss' with reference to interest. 132

Commerce and industry thus generally expanded in Europe and expecially in Holland and England, from 1500 - 1700. Wage-labour was on its way to becoming the dominant form of earning a living; exchange was becoming the dominant way of acquiring necessities; the money economy was thus also growing. These are the relations analysed by Marx in Capital I, where he describes the abstractions resulting from them. Calculation of many aspects of everyday life could then take place to reckon up one's life chances if one were in a position to deal on the market with more than one's own labour-power as a commodity.

It has been widely accepted that the early foundations of modern science were laid in this period as well. It is more than coincidental, we argue, that these relations and this form of science developed together. These relations have been largely unattended by interpreters of the scientific revolution. These relations were abstradt and by analogy helped produce a similarly abstract view of nature. The entailment, furthermore, of the social abstraction with science, took place through the calculation and record-keeping practices of this early capitalism. (We shall deal specifically with this, case by case, in Chapter Five) For the moment we shall turn to a brief examination of the work of interpreters of the scientific revolution so that we may then find the place of our own perspective in that general picture.

Notes

1. Michael Mulkay Science and the Sociology of Knowledge George Allen and Unwin, London, 1979, 5-11.
2. ibid. 8.
3. Karl Marx and Friedrich Engels The German Ideology edited by C. J. Arthur International, New York, 1970, 41.
4. ibid. 42.
5. ibid. 42.
6. ibid. 47.
7. Marx "The Poverty of Philosophy" in Karl Marx, Friedrich Engels Collected Works 6 International, New York, 1976, 170
8. Marx Grundrisse translated by Martin Nicholas Vintage, New York, 1973, 83-4.
9. ibid. 85-6; 87-8
10. Karl Marx Grundrisse New York 1973 p. 222
11. Karl Marx Economic and Philosophic Manuscripts of 1844 Progress, Moscow, 1974, 61-75.
12. Descartes, Rene "Discourse On Method" in Works translated by E. Haldone and G. R. T. Ross, Cambridge University Press, Cambridge, 1936, 119.
13. Lawrence Krader Dialectic of Civil Society Van Corcum, Assen, 1976, p. 164.
14. ibid. 165-6
15. John Locke Second Treatise of Government In Earnest Barker ed. Social Contract
16. Petty figures much larger in the argument here than the space allotted him here would indicate. See Chapter V below.
17 William Petty A Treatise of Taxes and Contributions London, 1679, p. 24 quoted from Krader, op. cit. 172.
18. Karl Marx Capital I translated by Samuel Moore and Edward Aveling Progress, Moscow n.d., p. 65-66.
19. ibid. 51-52.
20. ibid. p. 54.
21. ibid. 59.
22. ibid. p. 62.
23. ibid. p. 63.
24. ibid. 63.
25. ibid. 76.
26. ibid. 77.
27. ibid. 78.
28. ibid. 145.
29. ibid. 363.
30. G. Renard & G. Weulersse Life and Work in Modern Europe translated by Margaret Richards Routledge and Kegan Paul, London, 1968 (originally 1926) p. 56.

31. Jan DeVries The Economy of Europe in an Age of Crisis 1600-1750 Cambridge University Press, Cambridge, 1976, p. 3.
32. Joseph Kulisher Allgemeine Wirtschaftsgeschichte des Mittelalters und der Neuzeit Oldenburg, Munich, 1968 p. 102 (originally 1930).
33. J. D. Mackie The Early Tudors 1485-1558 Oxford, London, 1952 p. 3.
34. ibid. p. 7.
35. ibid. p. 21.
36. Karl Polanyi The Great Transformation Beacon, Boston 1957 (originally 1944) p. 65.
37. DeVries op. cit. 2.
38. ibid. 3.
39. ibid. 30.
40. E. J. Hobsbawm "The Crisis of the Seventeenth Century" in Trevor Aston ed. Crisis in Europe, 1560 - 1660. Routledge and Kegan Paul, London, 1965, p. 5.
41. DeVries op. cit. 27.
42. John Nef Industry and Government in France and England 1540-1640 American Philosophical Society, Philadelphia 1940 p. 1.
43. ibid. 2.
44. ibid. 5-6.
45. DeVries op. cit. 17.
46. Pierre Vilar A History of Gold and Money 1450 - 1920 translated by Judith White NLB, London, 1976 p. 69.
47. ibid. 70.
48. Renard and Weulersse op. cit. 46.
49. Kulisher op. cit. 188.
50. Renard & Weulersse op. cit. 13-14.
51. DeVries op. cit. 70.
52. DeVries op. cit. 75-76.
53. DeVries op. cit. 46.
54. DeVries op. cit. 179.
55. DeVries op. cit. 94.
56. DeVries op. cit. 242.
57. DeVries op. cit. 243.
58. Carlo Cipolla his introduction to The Fontana Economic History of Europe III ed. Carlo M. Cipolla Collins, Glasgow, 1974, p. 11.
59. Walter Minchinton "Patterns and Structure of Demand" in Cipolla ed. The Fontana Economic History of Europe III Glasgow, 1974, pp. 157-8
60. Kulisher op. cit. 198.
61. Cipolla op. cit. 8, 9.
62. Cipolla op. cit. 8.
63. Sybil Jack Trade and Industry in Tudor and Stuart England George Allen and Unwin, London, 1977, p. 26.
64. ibid. p. 115.
65. Dudley North in Discourses (1691) from Early English Tracts on Commerce ed. J. R. McCulloch quoted from Charles Wilson "Cloth Production and International Competition in the Seventeenth Century" in Economic History Review. V. 20, 1972, 209.

66. Minchinton op. cit. 153.
67. Robert Brenner "The Social Basis of English Commercial Expansion 1550-1650" Journal of Economic History XXXII (1972) 364.
68. Barry Supple "The Nature of Enterprise" in The Cambridge Economic History of Europe V: The Economic Organization of Early Modern Europe Cambridge University Press, Cambridge, 1977, p. 394-5.
69. Brenner op. cit. 369.
70. Brenner op. cit. 377.
71. Brenner op. cit. 379.
72. Supple op. cit. 399 quoted from E. Lipson Economic History of England II (London, 1948) 190.
73. Renard and Weulersse op. cit. 8.
74. Supple op. cit. 399.
75. Supple op. cit. 408.
76. Herman von der Wee "Monetary, Credit and Banking Systems" in The Cambridge Economic History of Europe V: The Economic Organization of Early Modern Europe Cambridge U.P., Cambridge, 1977 p. 290.
77. ibid. 290.
78. ibid. 295.
79. ibid. 295.
80. ibid. 291.
81. ibid. 300.
82. ibid. 301.
83. ibid. 302.
84. ibid. 303-4.
85. ibid. 305-6.
86. Kulisher op. cit. 104.
87. Kulisher op. cit. 108.
88. Kulisher op. cit. 109.
89. Donald C. Coleman Industry in Tudor and Stuart England MacMillan, London, 1975.
90. Christopher Hill The Century of Revolution 1603-1714 Cardinal, London, 1974.
91. Domenico Sella "European Industries 1500-1700" in Carlo M. Cipolla ed. The Fontana Economic History of Europe Collins. Glasgow, 1974.
92. ibid. 356 quoted from G. Botero The Reason of State (1589) ed. D. P. Waley, London 1956 p. 152.
93. Henry Kamen Cardinal, London 1971.
94. Domenico Sella op. cit.
95. Max Weber The Protestant Ethic and The Spirit of Capitalism New York, 1958, p. 67 also ch. II supra.
96. Kulisher op. cit. 113.
97. Maurice Dobb Studies in the Development of Capitalism International, New York, 1963 p. 143.
98. Kulisher op. cit. 117.
99. Kulisher op. cit. 120.
100. Sella op. cit. 363.
101. Coleman op. cit. 16.
102. Coleman op. cit. 24.
103. Donald C. Coleman "An Innovation and Its Diffusion: 'The New Draperies'"

Economic History Review XXII 3, 1969.
104. Coleman Industry 25, 26.
105. Coleman Industry 26.
106. Mackie op. cit. 462-63.
107. Kulisher op. cit. 146.
108. Nef The Conquest of the Material World University of Chicago Press, Chicago, 1964, p. 110.
109. ibid. 112.
110. ibid. 112.
111. ibid. 122.
112. Jack op. cit. 24-25.
113. Frederick L. Nussbaum The Triumph of Science and Reason 1660-1685 Harper & Row, New York, 1962, 224.
114. Hill op. cit. 231.
115. Kulisher op. cit. 150.
116. Kulisher op. cit. 152.
117. Kulisher op. cit. 154.
118. Hill op. cit. 232.
119. Donald C. Coleman "Labour in the English Economy of the Seventeenth Century" from E. Carus-Wilson ed. Essays in Economic History II Edward Arnold, London, 1962, p. 291.
120. ibid. 291 from Peter Chamberlen The Poor Man's Advocate 1649 p. 30.
121. ibid. 291.
122. ibid. 298.
123. ibid. 294-95
124. Ian Blanchard "Labour Productivity and Work Psychology in the English Mining Industry, 1400-1600" Economic History Review 2nd ser. XXXI 1978 p. 1.
125. Coleman op. cit. 303.
126. Herman Kellenbenz "The Organization of Industrial Production" in The Cambridge Economic History of Europe V: The Economic Organization of Early Modern Europe, p. 492.
127. E. P. Thompson "Time, Work-Discipline and Industrial Capitalism" in M. W. Flinn and T. C. Smout eds. Essays in Social History Oxford, London, 1974 p. 39.
128. ibid. 42-3.
129. ibid. 44-5.
130. Carlo M. Cipolla Clocks and Culture 1300-1700 Norton, New York, 1977 p. 49.
131. Nussbaum op. cit. 223.
132. Ronald L. Meek Studies in the Labour Theory of Value Monthly Review, New York, 1956 p. 17.

Chapter Four - From Economic to Intellectual Abstraction

Interpretations of The Rise of Science

Many interesting and cogent interpretations of the scientific revolution have been produced. The rise of modern science is a highly nuanced story with many factors contributing to its rapid growth in early modern Europe. Generally, however, its interpreters may be divided into two main groups, Marxists and liberals. Benjamin Farrington was one of the first Marxists to interpret the rise of science and his writings, as well as those of a few others, sparked a good deal of liberal response. The Marxists argue that science arose from technique once theology receded from prominence, whereas the liberals generally argue a classical Greek source for the concepts of science and democratic institutions for its establishment.

The Marxists to be examined here comprise George D. Thomson, Benjamin Farrington, J. D. Bernal and Stephen Mason. The liberals include Ludwig Edelstein, Alexandre Koyré, John Herman Randall, Marie Boas, A. Rupert Hall, A. C. Crombie, Herbert Butterfield and Hugh Kearney.

The Marxist Interpretation: Technique

George D. Thomson employs an argument which superficially resembles our own but, as signalled in his long association with Alfred Sohn-Rethel, the similarity proves to be quite superficial. His use of Marx' notion of fetishism and abstraction does not proceed as radically as it might but in the course of his discussion touches upon many suggestive possibilities. Thomson sees scientific ideas as having social origins but chooses to see correct scientific ideas as having an origin in technique, and alienated, abstract thought as fetishized, as the beginning of a feudal suppression of correct, mechanical ideas. The Babylonian, Hebrew and Greek cosmologies represent the projection of the structure of society onto nature but in Greece and Judea theogony already bears less relation to the creation than in Babylon.[1] With the Milesians, beginning with Thales who is said to have had interests in astronomy, geometry and engineering as well as commercial and philosophical interests, the structure of the world changed from genetic to self-regulating.[2] The three basic principles, furthermore, of the common origin of all things, perpetual motion, and the conflict of opposites are all derived from primitive thought.

For Thomson, pre-Socratic truth was suppressed under feudalism, and even before that by Plato's and Aristotle's abstraction, finally to be recovered after a long struggle against feudalism.

Benjamin Farrington is not concerned with fetishism or abstraction so much as with the positive contribution of early technique to scientific thought. In order to claim roots in the pre-Socratics, Aristotle distorts their meaning, claims Farrington. Their origin was much more in the realm of technique than Aristotle will allow and,

> In Egypt and Babylon the control over nature exercised in the techniques threw little light on the processes of nature as a whole. Practice did not pass beyond the domain of practice. The domain of nature was already occupied by mythology. Mythology and technology constituted two entirely different fields of knowledge. With the Milesians technology drove mythology off the field. The central illumination of the Milesians was the notion that the whole universe works in the same way as the little bits of it that are under man's control. 3

In the figures of Parmenides, Socrates and Plato, however, these early advances began to be opposed.

Plato was born in the year Anaxagoras is supposed to have

> died. In the interval that separated the two men the attitude of Athens to Ionian science had become more clearly defined and the antagonism had deepened. It was not only that Socrates had begun his powerful movement of revolt against Ionian materialism; the technique of government through religion was also better understood as well as the threat to this technique inherent in the spread of Ionian rationalism. 4

For J. D. Bernal, "It was the condition of the rise of capitalism that made that of experimental science possible and necessary."[5] His prognosis is that science's productivity ends up making capitalism unnecessary.

For Bernal, the edifice inherited from the Greeks was overthrown. Although the Renaissance partly bridged the gap between theory and praxis, the rendering of a new science from the old was accomplished by a new set of revolutionaries, the bourgeoisie. During the Renaissance and Reformation there was a movement toward the buying and selling of commodities and labour and away from hereditary status, which movement led at a later date to a heightened conflict between ancients and moderns.[6] A new respect was now won for artists and artisans since they were now essential to the making as well as the spending of money. Great developments were produced in the areas of perspective and engineering.

Although he is correct in paying attention to the development of capitalism he pays too much attention to it as technology rather than as a set of relations. His conception of science, subsequently, is also very naive. Speaking of Copernicus and Vesalius he states, "They were the first pictures of how the heavenly spheres or the human body would appear to those who had eyes clear enough to see for themselves and not through the spectacles of ancient authority."[7] In a similar vein he sees the issue around Galileo as one of science versus religious dogma, views the Middle Ages as barbarous and sees in the development of science a continuous ridding of the evils of idealism.[8]

Mason's position is basically that an amalgam of intellectual and craft traditions during which time Protestant modernity and universalism were battling Catholicism, Thomism and Aristotelianism. Craftsmen such as William Gilbert had as heirs in an earlier craft tradition Pierre Maricourt, Agricola and Robert Norman.[9] This does not, however, constitute mechanism and Mason mentions the sixteenth century tradition in mechanics which culminates in Galileo.

> He took geometry away from its subject matter of lengths, areas and volumes and applied it to other measurable properties, namely time, motion and amount of matter, in order to discover the connection between them and to deduce the consequences of those connections. 10

Descartes, the son of a counsellor in the Bretagne parlement with Vieta, generalised the mathematical method and built up a mechanical picture of the operations of nature. He describes Descartes as the mechanist par excellence.[11]

Scientists and Protestant reformers come together to attack cosmological and theological elements of the old world-view and occasionally these functions were combined in the same person , e.g., John Wilkins. In general the interests of technique and crafts, Protestantism, rationalism and experimental science coincide for Mason.

In general the Marxist view is that a true knowledge of nature comes through production technique which can only really take off once theology and metaphysics are driven off the stage.

The Liberal View

Although both A. C. Crombie and Herbert Butterfield come close to confessing to be at a loss to explain why science took off so dramatically in the early modern period, Wiener and Noland have no doubt about what it is that develops. They claim that it is defined by its rejection of absolute premises, self-evident axioms, essences, substances and first principles. "It prefers to seek constant relations among events whose functional covariation can be expressed mathematically without absolute metaphysical assumptions."[12]

Many commentators mention the conditions that allow science to develop: Edelstein's Renaissance institutions which helped to establish science, Crombie's late medieval artisanry and technology, Randall's marriage of a this-worldly commercial culture to Aristotelianism, Butterfield's complicated set of social conditions allowing science to rise with the middle class.

Just about all commentators see the content of scientific concepts as constituted, to varying degrees, by the recovery of ancient texts. The atomists, Plato, Aristotle, Euclid, Pappus, Diophantus and Archimedes are seen, either separately or in some combination, as prime contributors of the basic concepts of early modern science. Even the Marxists have a hard time dealing with Archimedes in their fashion. His own work is primarily contemplative, not technique oriented. In fact, mathematics causes problems for both views and that is why we make it our focus in the present work.

Ludwig Edelstein attacks Farrington on the grounds that analogies from craft or technique would not constitute any science.[13] Aristotelians, Platonists and Stoics were the real contributors for Edelstein but science needed Renaissance institutions to establish itself.[14]

Marie Boas claims that modern mechanism, like that of Robert Boyle can have little connection with Greek atomism. The notion of corpuscularity can and was imported from Greek atomism[15] and the mathematical part from Platonic and Pythagorean sources.[16]

The development of a "dynamic mechanical philosophy" was, however, a "triumph of seventeenth century science".[17] She describes the essential difference between atomism and mechanism as follows:

> A true mechanical philosophy, however, required the introduction of another concept, the concept that the motion of the particles might affect the properties of the matter they composed. Ancient atomism had conceived of the atoms as in continuous motion, like motes in a sunbeam; but variation in this motion no more changed the nature of matter than variation in the motion of the motes would change the properties of the dust. 18

Crombie, on the other hand, sees the transition from a metaphysical to a mathematical-physical interpretation of nature as focusing on a recovery of Aristotle's logic along with Greek and Arabic mathematics.[19] These elements were then wedded to commercial and industrial concerns for measurement. Its external influences then are from the artisan and commercial traditions.[20]

John Randall also sees the development of modern science hinging on Aristotle. For Randall, the marriage of a this-worldly commercial culture to the self-criticism of the Aristotelians is largely responsible for the scientific revolution.[21] From 1400 on the medieval Aristotelian tradition was continued at Padua and many Renaissance natural philosophers studied there. He emphasizes the influence of the Paris Ockhamites as against Leonardo da Vinci because most of da Vinci's writings were not available until the nineteenth century.[22]

Alexandre Koyré, on the other hand, prefers Plato to Aristotle, claiming that Galileo's mathematical approach reflects a Platonic philosophical perspective.[23] Although his attack on empiricist accounts, based on the notion that much of early modern science amounted to anti-common-sense developments, is cogent, his attack on Borkenau and others is unfounded. He attacks Borkenau on the grounds that the middle ages already had a power technology and a quest for wealth and power represented by alchemy.[24] This is not Borkenau's position. The possible influence of relations of production receives no critique from Koyré. Koyré's whole position is basically that where mathematics is primary the philosophical position is at least implicitly Platonic.[25]

Herbert Butterfield coined the term 'scientific revolution' in 1948. He does, perhaps, overestimate the significance of the event.[26] He emphasizes the position echoed by many others that the defining characteristic of modern science is the mathematical treatment of motion.[27] Translations of Euclid and Archimedes provided the means for Galileo to carry out this task.[28] He also mentions, however, the influence was of trade and religion, the development of a new world of finance, the rise of commerce and the middle class and a process of secularisation.[29]

Although A. Rupert Hall believes science to have developed out of human reason with some input from medieval science and Greek philosophy and mathematics,[30] a system of ideas developed by force of their reasonability and predictive validity.[31]

Hugh Kearney, on the other hand, sees modern science as developing about equally from organic, magical and mechanical traditions. In the organic tradition regularity is explained with aspect to final causes.[32] In the magical tradition nature is viewed generally as art, specifically as beauty, contrivance, surprise or mystery.[33] In the mechanical tradition, mathematics is emphasised with the dominant analogy that of a machine or of God as an engineer.

Kearney describes social and intellectual changes primarily in response to Marxism. He mentions the attractiveness of machine and productive analogies to Marxists. This, however, is not what Borkenau is arguing. Kearney mentions noone in particular in voicing this criticism.

Thus neither view really pays attention to the possible influence of the nature of social relations as analogy. The mention by many commentators of financial, commercial, industrial and craft traditions touch on our thesis and we have drawn on their inspiration here. Few of these commentators are true internalists. At the same time we wish to draw attention to the fact, in opposition to the views of Kearney and Koyre in particular, that there is another kind of machine providing analogies for nature than the kind used in power technology. This machine is the social relations of the early modern period and the "technique", if there is a relation to technique here, is the organisational and record-keeping practices commensurate with those social relations.

All are attempting to describe and explain an early modern event - the scientific revolution as a shorthand term. Many provide cogent accounts of those early modern practices and institutions which help to establish science. Most give various examples of Greek texts as providing the concepts which eventually received institutional affiliation in the early modern period. Since we are arguing that the concepts of modern science have a modern social origin, we shall argue that there is a marked difference between the ancient and modern concepts themselves and that the social relations and their concomitant practices can help to explain this difference. The works of Euclid, Archimedes, Diophantus and Pappus were relied upon, to be sure, but they were transformed by the early modern thinkers. This transformation of meaning was performed by participants in the above-mentioned social conditions of abstract labour. We are not arguing here that the translations were bad but simply that they were reading through the spectacles of another period.

The present chapter will end with a brief discussion of those areas of mechanics upon which we shall focus and why we shall focus on them. First, however, it will be necessary to develop the notion of the difference between ancient and modern concepts using the work of Jacob Klein. The reference of 'variables' in modern formulas to elements of nature, argues Klein, is made possible by a rather marked shift in the meaning and intention of mathematical concepts.

The Ontological Shift to Modern Mathematics -- Jacob Klein

Klein takes a clue for his analysis of this conceptual shift from the difficulties encountered by other modern analysts in rendering ancient mathematical texts. Although for more modern mathematical purposes the algebraic treatment of the problems posed in the works of the ancients presents no real problem in itself, those who have carried out such a treatment have frequently encountered what they refer to as cultural or ethnic peculiarities or disappointments. Klein's understanding of this difficulty is that the modern interpreters from Vieta to the present are reading certain ancient texts through modern spectacles constituted by more modern mathematics and so fail at certain points to comprehend the meaning and intention of those ancient authors.

It is important for our purposes, as well, to establish the conceptual differences between ancient and modern mathematics in order to establish the point that, although the reading by the early moderns of ancient texts was important, the science of those early moderns was not simply a matter of re-discovery. Klein sets himself "the limited task of recovering to some degree the sources, today almost hidden from view, of our modern symbolic mathematics."[34]

This development hinges in some respects on the use of the work of the Alexandrian arithmetician, Diophantus. Vieta, argues Klein, actually modified the work of Diophantus in a significant way in the last quarter of the sixteenth century, although his text was known as early as the fifteenth.[35]

Generally, for Klein, the concepts of Greek science are abstractions from everyday experience and "the meaning of this abstraction, ... , is the pressing ontological problem of antiquity."[36] Modern science, for Klein, is distinguished by its polemical attitude to ancient science and by its rejection of concern with "immediate insight" and preference for a concern with the "mutual relatedness of concepts."[37]

Klein's argument concerns the absence in the Greeks and presence in the moderns of a notion of general magnitude. Ancient mathematics contains a tension between object and method. The ancients were concerned with the ontological meaning of mathematical concepts. In modern mathematics method determines the being of the objects.[38] In the work of Euclid or Diophantus determinate numbers of units are always intended in their solutions. Two things are here lacking which are central to modern symbolic procedure: The intention of determinate solutions means that

> It does not identify the object represented with the means of its representation, and it does not replace the real determinateness of an object with the possibility of making it determinate, such as would be expressed by a sign which, instead of illustrating a determinate object, would signify possible determinacy.[39]

Hence, Euclid wrote different books of his Elements concerning number and magnitude. Owing to the incommensurability of certain geometric magnitudes in terms of discrete measures (numbers), different theories of proportion are found to obtain for numbers, on the one hand, and magnitudes, on the other. There is, thus, no such concept as "general magnitude" which could allow the numerical commensurability of irrational magnitudes and, hence, his division of books on proportion into these two kinds appears to some modern commentators as a curiosity.

For Klein, the transition to the modern concept of "general magnitude" represents a transformation in the understanding of the kinds and material of number. The concept of kinds of number "undergoes a universalising extension while preserving its tie to the realm of numbers,"[40] while the result of this process is that

> The 'material' of this universal and fundamental science is no longer furnished by 'pure' units whose mode of being may be subject to dispute, since they can be conceived either as independent beings, or as obtained by 'abstraction' (ἀφαίρεσις); the 'material' is now rather constituted by -- 'numbers' whose being no longer constitutes any problem since, as the products of symbol-generating abstraction, they can be immediately grasped in the notation. 41

Part of the reason for this development, Klein argues, rests with the difficulty encountered by Plato and various neo-Platonists in developing a satisfactory ontological understanding because, Klein states, of Plato's fundamental distinction between practical and theoretical sciences. In addition to this distinction they developed the following others: numbers in themselves -- numbers in relation to others, kind -- material, arithmetic -- logistic.

The basic problem here concerns the being or material of pure units and their relation to objects of sense; the problem is one of the relation between the 'numbers' with which things are counted and the things which are counted. One appears to presuppose the other.

'One' as a pure unit becomes indivisible because it becomes the unit according to which any collection of similarly constituted things can be counted, whereas the objects of sense themselves can, of course, be partitioned. Fractions, thus, caused a great deal of difficulty for the ontology of 'arithmos' which these thinkers were attempting to clarify. Whereas Plato wanted to establish a separate realm for number, Olympiadorus and the Gorgias scholiast wanted to regard the material of number in terms of "sensible units, since only these are amenable to the partitioning which exactitude of calculation requires."[42] This conception is no longer consonant with Plato's.

As is well-known, Aristotle departs from Plato's attempt to keep form and matter separate and thus pose the problem of 'participation'. Aristotle attempts to retain the character of such objects as 'arithmos' as noetic (objects of thought) while maintaining their connection with objects of sense. Noetic objects suitable for contemplation by science, episteme, are dependant on, but separable from, objects of sense by abstraction. Mathematical objects have their being by abstraction and the objects from which they are detached become mere items or bodies. Numbers, for Aristotle, are derived from sense objects,[43] from counting multitudes. Number is precisely multitude, more than one, measured by a unit.

The being of number is simply that number of units. The discovery of incommensurability thus forced a thorough geometrisation of Greek mathematics.

> (T)he crucial difficulty of theoretical logistic as the theory of those mutual relations of numbers that provide the basis of all calculation lay in the concept of the monad, insofar as it is understood as an independent and, as such, simply indivisible object. Aristotle's criticism obviates this difficulty by showing that this 'indivisibility' does not accrue to the monas as a self-subsisting hen, but by virtue of the measuring character of any such unit, be it of an aisthetic or noetic nature. Only when the Aristotelian critique has taken effect can a whole series of 'applied' sciences, such as were cultivated in the Alexandrian school, be justified as 'sciences'. 44

Diophantus calculates with fractional parts of the unit of measure and his work has an 'algebraic' appearance because he uses ligatures to denote the number which is sought in a particular problem. There are Platonic and Aristotelian components in Diophantus' theoretical logistic since he can, at the same time calculate with fractional parts of the unit and consider the material of numbers to be pure monads.

There is still as yet, however, no 'general magnitude' in Diophantus. He is concerned with "the possible relations that numbers, ... , can bear to one another" and not with "types of equations and methods of solution -- which is what modern interpreters usually look for".[45] The 'unknown' in each problem is a number "which is about to be determined in its multitude".[46]

The question now for Klein becomes

> What transformation did a concept like that of arithmos have to undergo in order that a 'symbolic' calculating technique might grow out of the Diophantine tradition.[47]

This transformation developed through a collection of contributors including Fibonacci, Tartaglia and Bombelli, among a few others, through to Stevin, Vieta and Descartes, all of whom will be treated in some detail below. In the latter three we have the symbolic realisation of general magnitude.[48]

In his treatment of Diophantus' Arithmetic, Sir Thomas Heath uses modern symbols in rendering his equations as indeed do most others.[49] Even Heath, however, mentions Diophantus' definition of the unknown as πλῆθος μονάδων ἀόριστον -- an undefined number of units -- and the middle term here is in fact 'monad'. His sign, Heath mentions, was verbally called ὁ ἀριθμός -- "the number par excellence of the problem in question".[50] Here Heath does not miss the determinate nature of Diophantus' inquiry.

In terms of Diophantus' method, he cites Hankel to the effect that after careful study of all of the extant six books of his Arithmetic over fifty type problems may be discerned with no real inherent classification in Diophantus' order of presentation except that the earlier ones help in the solution of the later ones. Although Heath disagrees with Hankel he cites him at length and is worth repeating here.

> Almost more different in kind than the problems are their solutions, and we are completely unable to give an even tolerably exhaustive review of the different turns which his procedure takes. Of more general comprehensive methods there is in our author no trace discoverable; every question requires a quite special method, which often will not serve even for the most closely allied problems. It is on that account difficult for a modern mathematician even after studying 100 Diophantine solutions to solve the 101st problem; and if we have made the attempt, and after some vain endeavours read Diophantus' own solution we shall be astonished to see how suddenly he leaves the broad highroad, dashes into a side path and with a quick turn reaches the goal, often enough a goal with reaching which we should not be content; we expected to have to climb a toilsome path, but to be rewarded at the end by an extensive view; instead of which our guide leads by narrow, strange, but smooth ways to a small eminence; he has finished! 51

Heath also quotes the Charmides scholiast to the effect that the ancient methods of calculation were usually intended to serve "utility for contracts".[52] Thus it is this curious mixture in Diophantus of attending two modes of being, a Peripatetic and a Platonic conception, together with a view to the poristic (practical) aspect of calculation which allows him to open up the way for a "genuine sign-language".[53] Tropfke here also presents one of the few plain language renderings of a Diophantine sample equation as, "Also 10 Zahlen (und) 30 Einheiten sind gleich 11 Zahlen (und) 15 Einheiten" (therefore 10 numbers and 30 units are equal to 11 numbers and fifteen units).[54] He mentions also in this context that the abbreviations, ligatures or signs used are, by ingenious scripts, even grammatically declined in sentence structures thus supporting the notion further that Diophantus' arithmetic was not symbolic.

The practical aspects of calculation often derived from commerce. Plato had distinguished between the arithmetic of the market place and that of lovers of wisdom. The Arabs as well, from whom Leonardo of Pisa (Fibonacci) brought 'algebra' to Europe in the early thirteenth century, had a well-developed commerce around the Mediterranean when that of the Greek Ionians had fallen off. This seems to have had an effect ~~on~~ the symbolisation involved since according to Tropfke,

> Unfortunately after Diophantus leading minds were missing who progressed in his path. The Arabs, to whom fell otherwise such a high role in mathematics, remained, themselves after they had come to know Diophantus' work -- about 970 A.D. Abu' lwafu (940-998 Baghdad) authored a commentary on it - almost thoroughly with its rhetorical form. Only an anonymous ~~Arabic~~ writing in Latin translation is known which uses letter-abbreviations (C=census=x^2;r=radix=x; d=dragma=coins, constants). The simultaneous application of a point with negative numbers and the writing of fractions suggests Indian influence. 55

The decline of such developments in mathematics until the later middle ages is coincident with the decline and subsequent rise of commerce in Europe. Although commodity production was present in antiquity it was not dominant and fell off only to develop through the growth of medieval towns and begin to take off in northern Italy in the fourteenth century. The Arabs were decisive for the arithmetical thought of Europeans from Fibonacci through Jordanus de Nemore up to Regiomantanus.[56]

Mathematics and Mechanics -- The Homogenisation of Nature

The application of formulae to the interpretation of nature and the use of variables in these formulae are recent developments. Ancient science regarded nature qualitatively. Although ancient astronomy and music, for example, were mathematical, there was no use of variables.

Before this could take place, both mathematics and mechanics had to undergo some conceptual changes. As hinted at earlier, these changes have to do with the commensurability of unlike quantities and the split between number and magnitude. For Aristotle, only like quantities could be compared in a proportion and numbers were discrete whereas magnitudes were continuous. We shall begin our brief overview of the history of mechanics with Aristotle in order to identify those developments wherein the principle of homogeneity and the distinction between number and magnitude came to be violated in an attempt to account mechanically for natural processes. We shall thus be looking for instances where unlike quantities, such as time and distance, are compared in the same expression and where continua are arithmetised. Toward the end of our story thinkers such as Tartaglia, Galileo and Stevin were engaging in these practices with some frequency.

The history of mechanics goes back, at least in terms of its available documentation, to Aristotle's (384-322 B.C.) physics. To begin with, however, the overview of this development has been aptly summarised by René Dugas.

> As Mr. Joseph Peres has remarked, to speak of the miracle of Greece or of the night of the middle ages in the evolution of mechanics is not possible. Correctly speaking, Archimedes was able to conquer statics and knew how to construct a rational science in which the precise deductions of mathematical analysis played a part. But Hellenic dynamics is now seen to be quite erroneous. It was, however, in touch with everyday observation. But, being unable to recognize the function of passive resistances and lacking a precise kinematics of accelerated motion, it could not serve as a foundation for classical mechanics. 57

For Aristotle, motion refers to many sorts of natural changes; changes in quality were considered motion as well as changes in place, "local motion".[58] Local motions are divided into species according to their path[59] and the chief problem here is to decide precisely in terms of what criterion or dimension different states are to be judged.[60] We encounter here the same problem as with value where Marx cites Aristotle. How do we compare unlike species or how do we divide species?

> But we have no general term like 'unlike' or 'unequal' to express the relation between 'different' primary beings in generations and destructions and no pair of terms corresponding to 'more' and 'less' in the case of qualities differing intensively or extensively or to 'greater' and 'smaller' in the case of different quantities. 61

Here, in the case of physical bodies, we have the same problem as in the case of commodities. **Aristolle has again adequately** posed the problem without being able to solve it. What we need for modern science is the existence of objects merely as bodies so that space, time, distance etc., may become variables in equations, just as the reduction of the commensurability of exchangeables to value encapsulates the fact that they are exchanged in certain proportions on the market. In both cases Aristotle recognises a relation of equality without knowing what it is that might be contained in equal quantities.

Aristotle does become quantitative, however, with respect to the relation between force, distance and time. He states that it must take a certain amount of time for a certain force to move an object a certain distance. His 'formula' here is basically that the distance moved varies directly as the force applied and the time taken varies inversely as the force. He adds the important proviso that the time taken does not increase indefinitely as the force decreases because when the force becomes small enough, no motion at all will result.

As for the rest of antiquity, not much attention was paid to the problem of motion let alone steps taken toward the comparison of dissimilar quantities. Archimedes of Syracuse (287-212 B.C.),often referred to as the world's greatest mathematician, concentrated his mechanical efforts on statics and Pappus of Alexandria (fourth century A.D.) incorrectly formulated the problem of the inclined plane, although his mathematics did have an influence on Renaissance thinkers.

The use in antiquity of Eudoxus' method of exhaustion has been cited by some as anticipating approximation techniques or even calculus. It was, however, a method of proof and not of calculation.

> Accordingly, we always find, in proofs of the method of exhaustion, a demonstration that an impossibility is involved by any other assumption than that which the proposition holds. 62

There always results a double reductio ad absurdum in every instance of the use of the method of exhaustion. These characteristics of the method are also found in Archimedes' use of it says Heath.

Archimedes used this method extensively in his own theorems but was much more theoretically than practically concerned.[63] In his Mechanics, the problems deal almost exclusively with equilibria and centers of gravity and the proportions obtaining between the areas or volumes of different figures. The mechanical comparison of physical objects was used only to suggest theorems.

Mechanics for Archimedes became theoretically independent from cosmology. The mathematical thought of the Renaissance and scientific revolution hinges largely on the reception of Archimedes.[64]

For Pappus mechanics is both science and art and in this respect he is more modern than Aristotle or Archimedes. He is most important for his discussion of the mathematical procedures of analysis and synthesis and this part of his work was important for early modern mathematics (cf. Chapter Five infra).

His mechanics is also a matter of equilibrium. His formulation of the problem of the inclined plane results in an absurdity; an infinite force would be required to pull a weight straight up. He is said to be the compiler of 700 years of Greek achievement.[65]

The development upon which we focus here is best described by Marshall Clagett. The Greeks, he claims, always compared like quantities and so could not develop metric definitions of the quantities they described.

> That is to say they compared the <u>distances</u> traversed in two uniform movements when the times <u>are assumed</u> to be the same, or the times when the distances are the same, or the ratios of the distances on the one hand and the times on the other. Hence we find no metric definition like V=k(s/t). The comparisons of Autolycus and other Greek authors were thus true proportions in the Euclidean sense as being between like quantities. It ~~is not~~ surprising, then, that none of the

> Greek authors arrived at the idea of velocity itself as a number or a magnitude representing a ratio of unlike quantities, namely distance and time. On the other hand, Gerard [of Brussels] in his treatise of the thirteenth century, while not yet defining velocity as a ratio of unlike quantities, seems to assume that speed or motion can be assigned some magnitude not simply identical with the quantity of time or the distance alone, although it is measured by either one with the other considered constant. 66

This is characteristic of Aristotle's physics which receives its first real criticism at the hands of John Philoponos who criticised the antiperistasis theory. He criticised the notion that the mover had to be in constant conjunction with the moved. The throwing of a projectile caused a problem for this theory since the projectile kept moving after it left the hand. Aristotle's answer to this was that the medium (air) rushed around behind the projectile to fill the void space and that this movement pushed the projectile further. This hinges as well on Aristotle's distinction between natural and violent motion. Basically throwing is a violent motion because it represents motion away from the natural place of an object. Philoponos counters Aristotle with the notion of the transferability of force, an early version of impetus (cf. Chapter Five).

The notion of impetus receives more complete expression in the middle ages. Ockham had defined motion as a relation rather than an entity, hence, no continuous cause was required. Pierre Olivi, Duns Scotus, Franciscus de Marchia and Nicholas Bonetus entertained notions of impetus.

The Oxford Mertonians and Paris Ockhamites were the first real exponents of the development we are describing. Velocity, a ratio, became a quantity in itself. Clagett outlines the significance of the work done by Bradwardine, Swineshead, Heytesbury and Dumbleton.

> From the discussions of these four men of Merton emerged some very important contributions to the growth of mechanics: 1) a clear-cut distinction between dynamics and kinematics, expressed as a distinction between the causes of movement and the spatio-temporal effects of movement. 2) A new approach to speed or velocity, where the idea of an instantaneous velocity came under consideration, perhaps for the first time, and with it a more precise idea of "functionality". 3) The definition of a uniformly accelerated movement as one in which equal increments of velocity are acquired in any equal periods of time. 4) The statement and proof of the fundamental kinematic theorem which equates with respect to space traversed in a given time a uniformly accelerated movement and a uniform movement where the velocity is equal to the velocity at the middle instant of the time of acceleration. It was this last theorem in a somewhat different form that Galileo states and which lies at the bottom of his description of the free fall of bodies 67

Although the main hurdle of comparing dissimilar quantities is at least partly overcome, the treatment of motion was not experimentally amenable and so no constants were found. With Bradwardine's Tractatus de proportionibus in 1328, velocity became a mathematical quantity in itself. The new definition is there but we still have no variables.

The situation is much the same for the Paris Ockhamites, Jean Buridan, Nicole Oresme, Albert of Saxony and Marsilius of Inghen. The most important figure for our purposes is Nicole Oresme. He is usually considered the inventor of the treatment of motion according to the framework of the intension and remission of forms. Although there is some resemblance here to analytic geometry there is in Oresme's work no algebraic expression and thus no translation into curves and vice versa. For motion, extension was time and intension velocity. A rectangle thus represented a uniform quality and a right triangle represented a uniformly difform quality. (Uniformly difform means increasing or decreasing at a uniform rate.) Oresme proved the mean speed theorem by proving that the rectangle area (the mean speed) was equal to the area of a right triangle (the velocity during the whole increment).[68]

It does not appear to be until the sixteenth century that actual measurements were made, numerical treatment given the problems and algebraic formulation really took off. In the work of Tartaglia, Galileo, Descartes and Stevin, in particular, we discern the growing homogeneity of nature such that unlike quantities are compared; general magnitude became a property of nature. The important development here is the abstraction and homogenisation of nature. Developments occur in mathematics which, when applied to phenomena such as the velocity of a motion or the weight required to balance a beam at a given point, allow for and in fact require, the comparison of unlike quantities in a single expression, the arithmetisation of continua and, in fact, the calculation of numerical values for mechanical or geometric problems which the ancient mechanicians and mathematicians rarely, if ever, did.

Neither Archimedes nor Euclid was interested in the area of a particular circle; neither Archimedes nor Pappus had definitions of velocity nor compared quantities of unlike kind.

The ability to carry out these more modern mechanical practices is, we are arguing, predicated on mathematical developments for which commerce is largely responsible. We shall thus look at some of the major contributors to these mechanical and mathematical developments to judge the extent to which they may be indebted to their own or others' commercial activity or commentary. We shall look at the contributions to mathematics or mechanics of each of our contributors and attempt to assess the extent to which the contribution of each, in terms of modern mechanical and mathematical developments, may be accounted for by involvement with commerce and its parallel practices. We begin with an examination of the changes toward the requisite mathematics.

Notes

1. George D. Thomson Studies in Ancient Greek Society II: The First Philosophers Lawrence and Wishart, London, 1955, 153-4.
2. ibid. 158-9.
3. Benjamin Farrington Head and Hand in Ancient Greece Watts, London, 1947, 3.
4. Farrington Science and Politics in the Ancient World George Allen and Unwin, London, 1939, 87.
5. J. D. Bernal Science in History II Watts, London, 1954, p. 252.
6. ibid. 258, 347.
7. ibid. 262.
8. ibid. 346-7.
9. Stephen F. Mason A History of the Sciences Collier, New York, 1962 140.
10. ibid. 157.
11. ibid. 168.
12. Phillip Wiener and Aaron Noland intro. to their edition of Roots of Scientific Thought Basic Books, New York, 1957, p. 18.
13. Ludwig Edelstein "Recent Trends in the Interpretation of Greek Science" in Wiener and Noland, p. 105.
14. ibid. 115.
15. Marie Boas "The Establishment of the Mechanical Philosophy" Osiris 1952, p. 415-18.
16. Boas The Scientific Renaissance 1450-1630 Harper & Row, New York, 1962.
17. ibid.
18. Boas op. cit. 521.
19. A. C. Crombie Augustine to Galileo II 292-93.
20. ibid. 133.
21. John Herman Randall Jr. "Scientific Method in the School of Padua" in Wiener and Noland p. 145, 146.
22. Randall "The Place of Leonardo da Vinci in the Emergence of Modern Science" in Wiener and Noland 208-209.
23. Alexandre Koyre "Galileo and Plato" in Wiener and Noland 174.
24. ibid. 148 also Galilean Studies translated by John Mepham, Sussex, Harvester, 1978, p. 39 n. 8.
25. Koyre op. cit. 168.
26. Herbert Butterfield The Origins of Modern Science 1300-1800 Clarke, Irwin and Co., Toronto, 1968, p. vii
27. ibid. 1-4.
28: ibid. 13-15.
29. ibid. 182-87.
30. A. Rupert Hall The Scientific Revolution 1500-1800 Beacon, Boston, 1954, p. 9.
31. ibid. 32-3.
32. Hugh Kearney Science and Change 1500-1700 McGraw-Hill, New York, 1971, 23.
33. ibid. 24.
34. Jacob Klein Greek Mathematical Thought and the Origin of Algebra translated by Eva Brann M.I.T., Cambridge Mass. 1968, (originally 1934) p. 4.

35. ibid. 5.
36. ibid. 120.
37. ibid. 120-21.
38. ibid. 123.
39. ibid. 123.
40. ibid. 165-6.
41. ibid. 223-4.
42. ibid. 60.
43. ibid. 110.
44. ibid. 112.
45. ibid. 135.
46. ibid. 140.
47. ibid. 147.
48. ibid. 175.
49. Diophantus of Alexandria Arithmetik Goettingen, Vandenhoeck and Ruprecht, 1952 translated by Arthur Czwalina, for example
50. Sir Thomas L. Heath Diophantos of Alexandria Dover, New York, 1964, (originally 1885) p. 32.
51. Hankel Zur Geschichte der Mathematik im Altertum und Mittelalter Leipzig, 1874, pp. 164-5 in ibid. p. 54-5.
52. Heath op. cit. 111.
53. Johannes Tropfke Geschichte der Elementar-Mathematik: Zweiter Band Allgemeine Arithmetik Walter de Gruyter, Berlin, 1933, 7.
54. ibid. 5.
55. ibid. 7.
56. ibid. 8.
57. Rene Dugas A History of Mechanics translated by J. R. Maddox Routledge and Kegan Paul, London, 1957, 11.
58. Aristotle Physics translated by Richard Hope University of Nebraska Press, 1961 p. 42.
59. ibid. 140.
60. ibid. 140-41.
61. ibid. 141.
62. Heath in his introduction to Archimedes Works Cambridge U. P., Cambridge, 1897, cxliii.
63. Sir Thomas L. Heath, History of Greek Mathematics I Oxford, London, 1921, 17.
64. Markus Fierz Vorlesungen zur Entwicklungsgeschichte der Mechanik Springer, Berlin 1972, 20.
65. Dugas op. cit. 35.
66. Marshall Clagett The Science of Mechanics in the Middle Ages University of Wisconsin Press, Madison, 1959, 167.
67. Clagett op. cit. 205.
68. Clagett op. cit. 343.

Chapter Five - The Science of Business and the Business of Science

Mathematics in History - Contentious Issues

It has been apparent for some time that there are potential difficulties in the standard interpretations of Greek mathematics and the notion of its continuity with modern-day practice. In a slightly different context Christopher Hill comments, "It is always easy to construct chains of causes once you know what you have to explain"[1]. The history of mathematics is particularly recalcitrant to this criticism.

> The situation is particularly scandalous in the history of ancient and medieval mathematics. It is in truth deplorable and sad when a student of ancient or medieval culture and ideas must familiarize himself first with the notions and operations of modern mathematics in order to grasp the meaning and intent of modern commentators dealing with ancient and medieval mathematical texts. With very few and notable exceptions, Whig history is history, in the domain of the history of mathematics; indeed it is still, largely speaking, the standard, acceptable, respectable, "normal" kind of history, continuing to appear in professional journals and scholarly monographs. 2

This statement from Sabetai Unguru in 1975 has its truth born out in the fact that it is only the third strong statement about even the symbolic anachronisms, in the writing of the history of mathematics, let alone the problems concerning the intentions which the ancients actually had. The other two, cited by Unguru are Jacob Klein in 1934 and Arpad Szabo in 1969.[3] In the replies by B. L. van der Waerden, the primary target, and Hans Freudenthal, the traditional position is simply restated with no attempt to demonstrate that modern symbols may express the intention of the ancients. The argument presented is a modern mathematical one and not a historical one at all.[4]

As a careful scholar, nevertheless, van der Waerden mentions that the actual tool of Greek mathematics was the language of proportions. Unguru notes van der Waerden's characterization of this language as cumbersome and we quote here the passage in full from van der Waerden:

> Equations of the first and second degree can be expressed clearly in the language of geometric algebra and, if necessary, also those of the third degree. But to get beyond this point, one has to have recourse to the bothersome tool of proportions.
>
> Hippocrates, for instance, reduced the cubic equation
>
> $$x^3 = V$$
>
> to the proportion
>
> $$a:x = x:y = y:b$$
>
> and Archimedes wrote the cubic
>
> $$x^2(a-x) = bc^2$$
>
> in the form
>
> $$(a-x): b = c^2 : x^2$$
>
> In this manner we can get to equations of the fourth degree; examples can be found in Apollonius (e.g. in Book V). But one can not get any further; besides, one has to be a mathematician of genius, thoroughly versed in transforming proportions with the aid of geometric figures, to obtain results by this extremely cumbersome method. Any one can use our algebraic notation, but only a gifted mathematician can deal with the Greek theory of proportions and with geometric algebra. 5

It is our opinion, of course, for reasons outlined in Chapter IV, that this is not algebra at all. Most importantly, there is no real attempt to understand the ancient texts in any context other than that of modern mathematics. For Unguru, this is a function of modern mathematics' self-understanding; it would palliate its own knowledge-claims to see its own development as having a history.

> Such an approach ... stems from the unstated assumption that mathematics is a <u>scientia universalis</u>, an algebra of thought containing <u>universal ways of inference</u>, everlasting structures, and timeless, ideal patterns of investigation which can be identified throughout the

> history of civilized man and which are completely independent of the form in which they happen to appear at a particular juncture in time. In other words, such an interpretation takes it for granted that form and content do not constitute an integrated whole in mathematics, that, as a matter of fact, content is independent of form, and that one can, therefore, transcribe with impunity ancient mathematical texts by means of modern symbolic algebraic notation in order to gain an "insight" into their otherwise "cumbersome" content. 6

Our story, however, begins not with everlasting structures but with the origin and career of the very concept which proves to be the basis of our own interpretation, analogia, proportion, same ratio. Social relations provide the basis for these analogies. The problem is to describe how this happened in particular historically specific situations and provide what documentation is available.

Our story is one of how the formal, academic tradition in mathematics is changed from the double reductio ad absurdum of Euclid and Archimedes to the "analytic" techniques of the modern period beginning with Stevin and Vieta. This happens by means of development of those forces initially giving rise to proportion in mathematics, commerce. The para-Euclidean tradition develops into a way of using proportionality to represent profit, price, partnerships, etc. The Babylonians, Myceneans, Egyptians, Phoenicians, Byzantines and Arabs (not to mention Indians and Chinese) all used proportional mathematics for business and financial calculation. When in Greece the separation of mental and manual labour receives conscious expression in the works of philosophy we have the "battle between ancients and moderns" taking place already in ancient times.

The history of the decline of trade and of civilization from Greece westward presents a story of the borrowing and redevelopment of mathematics. When the separation of mental and manual labour intensifies in late medieval and Renaissance Europe the inputs into mathematics are the rediscovered ancient academic texts and the contemporary Rechenbücher of merchants who had borrowed some and developed other techniques of calculating profit and loss. The reading of Diophantus, Pappus, Euclid and Al-Khwarizmi is coupled to a practical tradition stretching from Babylon, to Leonardo of Pisa to the Abaci manuscripts to Tartaglia Cardano and Bombelli. Social relations provide part of the input into the development of a science which sees unlike quantities as commensurable.

Unguru claims that the above debate stems from "an unstated assumption". We would like, however, to situate this assumption in social context. Sohn-Rethel has in fact attempted to do this but, in our view, unsuccesfully. It is our position here that a practical calculation tradition developed theoretically in Europe, primarily Northern Italy at first, after its adoption from the Arabs from 1200 to 1600 during which time it joined with the medieval theoretical traditions and culminated in a distinctively modern and western mathematics.

Even in its ancient development, mathematics received impetus from commercial and financial calculation. Thekla Horowitz examines the origins of Greek proportion theory in the financial and administrative practice of Mycenaean culture.[7] Kurt Vogel addresses the Babylonian interest in the calculation of prices and labour time.[8] In the Mycenaean example, proportion was used to keep a constant value in taxes paid in kind even though different goods might be used to achieve the value of taxes owed.[9]

According to Aristoxenus, however, Pythagoras took arithmetic out of the realm of the marketplace.[10] The Pythagoreans developed mathematics into a theoretical science from the examination of problems in cosmology and music. The universe and its order were, for the Pythagoreans, understandable in terms of number. Their own investigations, however, led to one discovery which proved anathema to this perspective, the irrational, the incommensurable. The ratio of certain magnitudes cannot, sometimes, be expressed by numbers, for example, the ratio of the diagonal of a square to its side. No unit, no matter how small, may be selected which is capable of measuring both side and diagonal without a remainder.

The concepts ἀριθμός (number) and μέγεθος (magnitude) went separate ways thereafter. Eudoxos' theory of proportions as compiled in Euclid's *Elements* maintains this separation. The brilliance of Eudoxos consists in the fact that he was able to define a ratio obtaining between non numerical quantities.

The resulting definitions concerning the proportionality of magnitudes are given by Euclid in the definitions to Book V of the elements.

> 3. A *ratio* is a sort of relation in respect of size between two magnitudes of the same kind.
>
> 4. Magnitudes are said to *have a ratio* to one another which are capable, when multipled, of exceeding one another.
>
> 5. Magnitudes are said to be in the same ratio, the first to the second and the third to the fourth, when, if any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order.

These represent criteria of homogeneity. With respect to the definitions of number and its proportionality, however, no such criterion is required.

> 1. A *unit* is that by virtue of which each of the things that exist is called one.

> 2. A number is a multitude composed of units ....
>
> 20. Numbers are proportional when the first is the same multiple, or the same part, or the same parts, of the second that the third is of the fourth. 11

Here we see that since number is a multitude composed of units, the unit itself cannot be a number since it is not a multitude. This unity is the eidos of any numbering. Since it is the basis of numbering or counting it cannot be fractionalised; hence, the difference in the following definitions from Euclid:

> 1. A magnitude is a part of a magnitude, the less of the greater, when it measures the greater.
>
> 2. The greater is a multiple of the less when it is measured by the less. 12

Even though these definitions refer obviously to commensurable magnitudes, the corresponding definitions for similar relations between numbers are very different.

> 3. A number is part of a number, the less of the greater, when it measures the greater;
>
> 4. but parts when it does not measure it.
>
> 5. The greater number is a multiple of the less when it is measured by the less. 13

Again we have further evidence of the homogeneity, hence, to anticipate Aristotle, infinite divisibility of magnitude and the discreteness and indivisibility of the unit of calculation. The "part - parts" distinction shows this clearly.[14] For Jones, this shows the irreducibility of the arithmetic of Euclid to the geometry. They are each irreducible constituents of contemporary mathematical and philosophical traditions.

> In summary we have argued that Euclid organized the Elements on two principles; one philosophical and the other mathematical. The philosophical principle was the split between number and magnitude as found in Aristotle's analysis of the category of Quantity. The distinction is based on ontology and finds it's roots in problems presented by the existence of incommensurables. 15

The existence of incommensurables created problems for proportion theory. There could be no "part or parts" referred to. The old proportion theory is transformed on its way into Euclid's Elements by way of Theaetetus Eudoxus and Aristotle, and by this route the two sets of distinctions are introduced. The philosophical distinction is between number and magnitude where numbers are discrete and magnitudes continuous. The mathematical distinction is that between commensurability and incommensurability, where for commensurables there exists a common measure and for incommensurables there does not. Both these distinctions have a common root in the notion of "unity".[16] Unity is that possessed by the individuals of a certain eidos which allows them to be numbered whereas no such unit is present in magnitudes. Unity is that property possessed by the common measure of commensurables.

Unity is a property of magnitude in another sense. In his Physics (VI, 1), Aristotle "characterises continua as those totalities in which the ends of juxtaposed parts fall together into one."[17] For Waschkiess, Aristotle's notion of continuity and infinite divisibility has three roots, two philosophical and one mathematical and the mathematical root lies in the work of Eudoxus. About Definition V, 4 of the Elements, Waschkiess says:

> This determination of the concept contains a criterion of homogeneity, which divides the multitude of extended magnitudes up into

certain classes, whose members can be compared with one another according to the theory of proportion. 18

Aristotle, Proportionality and Commerce

In his Nichomachean Ethics, however, Aristotle softens his criteria of proportionality in his discussion of justice. Justice as a virtue has no precise contrary; it is characterised as relative virtue.

> Again, all other unjust acts are ascribed invariably to some particular kind of wickedness, e.g., adultery to self-indulgence, the desertion of a comrade in battle to cowardice, physical violence to anger; but if a man makes gain, his action is ascribed to no form of wickedness but injustice. Evidently, therefore, there is apart from injustice in the wide sense another, "particular", injustice which shares the name and nature of the first, because its definition falls within the same genus; for the significance of both consists in a relation to one's neighbour, but the one is concerned with honour or money or safety - or that which includes all these, if we had a single name for it - and its motive is the pleasure that arises from gain; while the other is concerned with all the objects with which the good man is concerned. 19

Aristotle then wants to grasp the genus and differentia of the former form of justice. He also distinguishes at this point distributive from commutative justice; distributive being that kind which effects a relative equality, relative to a predetermined share; commutative being that kind which restores an intermediate condition in transactions. Aristotle is concerned here primarily with voluntary transactions, examples of which he gives as "sale, purchase, loan for consumption, pledging, loan for use, depositing, letting."[20] Distributive justice is that proportionality respecting the nature of different persons according to the constitution. The just in this case is intermediate, equal and relative. It involves two people and two things, their respective shares. "The just, therefore, involves at least four terms; for the persons for whom it is in fact just are two, and the thing in which it is manifested, the objects distributed are two."[21] As the things are related, so are the people. Things are here distributed according to merit and Aristotle here mentions that democrats, oligarchists and aristocrats will have different conceptions of merit, freedom, wealth and excellence respectively. Without having specific units in mind, Aristotle is nonetheless able to formulate generally his notion of distributive justice and its foundation in mathematical proportionality.

> The just, then, is a species of the proportionate (proportion being not a property only of the kind of number which consists of abstract units but of number in general). For proportion is equality of ratios and involves four terms at least (that discrete proportion involves four terms is plain but so does continuous proportion, for it uses one term as two and mentions it twice; e.g. 'as the line A is to the line B, so is the line B to the line C'; the line B, then, has been mentioned twice, so that if the line B be assumed twice, the proportional terms will be four); and the just, too, involves at least four terms, and the ratio between one pair is the same as that between the other pair; for there is a similar distinction between the persons and between the things. As the term A, then, is to B, so will C be to D and, therefore, alternando, as A is to C, B will be to D. Therefore also the whole is the same ratio to the whole; and this coupling the distribution effects, and, if the terms are so combined, effects justly. The conjunction, then, of the term A with C and of B with D is what is just in distribution, and the species of the just is intermediate, and the unjust is what violates the proportion; for the proportionate is intermediate, and the just is proportional. (Mathematicians call this kind of proportion geometrical; for it is in geometrical proportion that it follows that the whole is to the whole as either part is to the corresponding part.) This proportion is not continuous; for we cannot get a single term standing for a person or a thing. 22

Aristotle's notions of continuity, divisibility and homogeneity make difficult any sort of calculation when it comes to nature or physics. The stringent criteria laid down for continuity which makes it impossible to compare time and distance as 'variables', for example, lead Aristotle to an awareness of the difficulties of a quantitative physics, if, indeed, it makes any sense to use this term at all. He uses proportionality similarly in his Physics.[23] The inner terms of the four term proposition of proportionality are interchanged in the case of justice as in the case of local motion.

Before continuing our discussion of the problems of continuity, proportionality, profit and acceleration, we shall return to the second type of proportionality mentioned by Aristotle. It is commutative, and, mathematically speaking, is arithmetic. It effects equality in transactions between man and man. A judge attempts to equalise an inequality by imposing a penalty on a wrongdoer in a transaction. The wrongdoer will then have subtracted from his goods that amount by which they exceeded the intermediate. Each is said to have his own when he has what is equal, and Aristotle uses a diagram of linear magnitudes to display this principle.[24] The judge here rectifies loss and gain.

> These names, both loss and gain, have come from voluntary exchange; for to have more than one's own is called gaining, and to have less than one's original share is called losing; e.g., in buying and selling, and in all other matters in which the law has left people free to make their own terms; but when they get neither more nor less but just what belongs to themselves, they say that they have their own and that they neither lose nor gain.
>
> Therefore the just is intermediate between a sort of gain and a sort of loss, viz., those which are involuntary; it consists in having an equal amount before and after the transactions 25

This form of reciprocity is not in effect, for Aristotle, when the personal relation involved is between people of different status. Distributive justice is, however, precisely that which holds men together - reciprocity in accordance with a proportion and not on the basis of precisely equal return.[26] This is the basis on which, for Aristotle, the city holds together. Labour is seen here as social but not as equal in determining value; in fact, the labour per se does not appear as a concept in Aristotle's Ethics, but the significant term in the equation is the status of the producer relative to other producers. The notion of proportionality and the trend toward the production of a common quality are touched upon here by Aristotle and this most important passage from the Nichomachean Ethics must be quoted at length.

> Now proportionate return is secured by cross-conjunction. Let A be a builder, B a shoemaker, C a house, D a shoe. The builder, then, must get from the shoemaker the latter's work, and must himself give him in return his own. If, then, first there is proportionate equality of goods, and then reciprocal action takes place, the result we mention will be effected. If not, the bargain is not equal, and does not hold, for there is nothing to prevent the work of the one from being better than the work of the other; they must therefore be equated. (And this is true of the other acts also; for they would have been destroyed if what the patient suffered had not been just what the agent did, and of the same amount and kind.) For it is not two doctors that associate for exchange, but a doctor and a farmer, or in general people who are different and unequal; but these must be equated. This is why all things that are exchanged must be somehow comparable. It is for this end that money has been introduced, and it becomes in a sense an intermediate; for it measures all things and therefore the excess and the defect - how many shoes are equal to a house or to a given amount of food. 27

The ratio of the products must "correspond" to the ratio between the producers. Shoes are to houses, for example, as builder is to shoemaker. The unit allowing for the necessary "equalization" of goods is need, for it is need which makes for exchange in the first place, which establishes the relation. Money, for Aristotle, has by convention become a representative of this need. Its conventionality is reflected in its name (nomesma) because it is a product not of nature but of law (nomos) and may be humanly altered. Aristotle adds the proviso here that the terms must be brought into proportion only before the exchange has been transacted, when each still has his own goods. Money, furthermore, acts as surety for future needs in exchange. He goes on to discuss the sufficiency for purposes at hand of commensurability by money.

> Money, then, acting as a measure, makes goods commensurate and equates them; for neither would there have been association if there were not exchange, nor exchange if there were not equality, nor equality if there were not commensurability. Now in truth it is impossible that things differing so much should become commensurate, but with reference to demand they may become so sufficiently. There must, then, be a unit, and that fixed by agreement (for which reason it is called money); for it is this that makes all things commensurate, since all things are measured by money. Let A be a house, B ten minae, C a bed. A is half of B, if the house is worth five minae or equal to them; the bed, C, is a tenth of B; it is plain, then, how many beds are equal to a house, viz., five. That exchange took place thus before there was money is plain; for it makes no difference whether it is five beds that exchange for a house, or the money value of five beds. 28

It is clear that equality here is defined with reference to proportionality, not the other way around, for he states that men may be either proportionately or arithmetically equal. We have, by virtue of money's existence as that unit allowing commensurability, a method of taking stock in transactions. Aristotle is speaking here of "positive law" which for him can be defective because it is impossible to make universal statements about the changing.

Here we find in Aristotle a hint of the social abstraction described in chapters one and three. The analogous situation presented by Aristotle in the Physics and Nichomachean Ethics are treated differently.

In the Physics no comparison of dissimilar quantities is admitted. In the Ethics, however, money is admitted as representative of those abstract properties of exchanged objects, in terms of which they may exchange in a given proportion. In order for the proportions to hold the goods measured and compared in a transaction must have a common property abstracted from the goods as sensually intuitable objects which property is then measured in money.

Aristotle comments however that mathematically speaking this proportionality between dissimilar quantities is incorrect and thus makes no attempt in his physics to use similar techniques. The world of nature is still a place of quality, genus and difference. The story we are trying to tell, however, is one in which those precise conditions which Aristotle illustrates in the Ethics become far more proliferate and thus are able to constitute an analogy for a similarly abstract interpretation of nature. As the mathematical proportion techniques which Aristotle describes develop, a theoretical mathematics develops from it which is capable of application to nature. In later periods the questions raised by Aristotle in his Physics are dealt with by precisely those kinds of techniques which he illustrates in his Ethics. If ten candies cost twenty-one cents, how much do thirteen cost? If I travel thirteen miles in forty-nine minutes, how far would I travel in one hour? Aristotle would have approved of the first question; the second, however, might have been meaningless to him. Both cases require the comparison of quantities of unlike kind.

John Philoponos

One of Aristotle's first critics was John Philoponos. Michael Wolff has presented us with an argument concerning the social, economic and theological context of Philoponos' critique of Aristotle's antiperistasis theory. Wolff's argument also concerns the notion of value. Wolff claims that Philoponos has a notion of the transferability of force and cause, a precursor of the concept of impetus.

The socio-economic analogy is based first of all on the fact that Philoponos represented urban artisans. They, unlike the slaves in the countryside, became the owners of their products. Units of their "force" were imparted to their products which were then sold on the market. Theologically, he and his followers were monophysites; God gave his nature to man through Christ. The diophysites argued that Christ had two natures, godly and human. The monophysites used the notion of causality transfer to suggest that the human and godly characteristics are in all men.

The overcoming of Aristotelian physics could not be done empirically until the notion of causality transfer could be presupposed. "Nature is a work of God, δημιουργία. It is therefore, in a manner of speaking something artificial of a higher order, of which human art is something like a copy."[29] In the realm of physical science this world view, for Wolff, leads in Philoponos to the first mechanistic interpretation of heavenly motion.

Just as God gave man his power, so the human producer imparts force, value to his product. Just as the producer imparts a force which becomes a property of the product, so the thrower imparts a force which becomes a property or quality of the projectile. An alternative to the antiperistasis theory is thus constructed from social and theological context.

Transfer of force entails transfer of value.

> The idea of the transfer of force which lies at the basis of Philoponos' theory of physical motion must be set in connection not only with a specific idea of the production of force but rather likewise with a specific idea of 'economic' exchange. It is transferred forces which are understood as the genetic content of "production", of "completion", through motion and the altering of things. Similarly it is transferred forces which are viewed as the actual content of the exchange of things, as the object of the externalization of value. 30

The Early European Arithmetic and Record Keeping Traditions.

In the earlier middle ages arithmetic is represented in Europe primarily in the figures of Bede, Alcuin, and Gerbert, whose central project in this regard was the calculation of the date of Easter and other holidays. Although numeracy was generally not a prerequisite for a career in Church or state bureaucracies, the use of commerce made familiarity with arithmetic imperative if one were to seek one's fortune in this direction.

Alexander Murray suggests that the possibility of such careers is the result of the growth of a money economy spurred on by greater and greater trade of surplus between communities. Money makes this exchange more fluid. The desire for this fluidity and, hence, for money, requires special skills, labour, mining and minting, for which , in turn active royal authority and organization were required.

> The factors I have identified interacted. Money lubricated exchanges, reducing their cost as against other means of distributing property. A reduction in the cost of exchange increased its volume. That increase in volume not only, as we saw just now, made the market more fluid, reducing the creditor's preference for payment in kind; it also raised the demand for money - for people in a market of growing fluidity tend to find themselves short of cash. A higher demand for money, in turn lowered the unit-cost of mining and minting, making money cheaper, and thereby available to facilitate even more exchanges, formerly unprofitable without it; and so on. 31

This is a statement of the basic condition hinted at by Aristotle above. This comparison of unlike quantities begins again in central-late medieval Europe and the necessary arithmetic was as mysterious for the average European as were the properties of silver or gold which seemed to give it value as money. This monetary economy began to boom with the advent of long distance trade. In the early and central middle ages it was the case that the richer you were, the less money you had. Money was an instrument of traders only. The break-up of the Carolingian empire left a trade legacy but royal authority was lacking. Coinage in the ninth century was left to local authorities.

Meanwhile in Eastern Islam the world's most active currency was being created. The seventh century Islamic conquests left Byzantium and Persia joined so that "The new empire thus linked the most ancient centres of civilization - north-west India, Mesopotamia and Egypt - into a vast subcontinent of technological and commercial interchange."[32] Explosive economic change began to occur in eleventh century Europe. Native silver was mined and German and English coinage replaced the oriental by the year 1000; it was joined with that trade in the Italian cities so that by 1600 money was established in the centers of Europe.

Although we follow Murray's argument in outline, here he seems to fall prey to the same technical mechanism which we take as his purpose to counteract. After outlining the social conditions giving rise to money economies, he goes on to suggest that money creates these social conditions.[33] No doubt, the institutionalization of this result of social relations had an effect back on those relations, but it was not without struggle. One result of these new conditions, in any case, was that new habits of calculation were developed. One could plan and understand one's fate by arithmetically taking stock of wealth, wealth now measurable in money.

Arithmetic proper has its history embedded in the history of commerce and record-keeping. With Otto III, a pupil of Gerbert, mathematics takes hold of Europe. The basic process initially was the replacement of the dust-table and abacus methods with the new method of place-value numerals which allowed the keeping of a record of the calculations. The new numerals themselves, however, did not spark this trend.

> In the European phase of its history, if one certainty stands out, it is its refutation of the idea that inventions by themselves start revolutions. The new numerals were available, complete with instructions, to any educated persons who wanted them by 1200. It was only c. 1400 that they began an effective conquest of all literate culture. This delay is our opportunity. The pattern of the numerals' adoption will reflect, not any foreign technological bombardment, but native aspirations and pressures. 34

The Latins were suspicious of arithmetic and also of anything non-Latin. The English Exchequer did not use Arabic numerals until the sixteenth century, and many of the Italian cities did not use them in official accounts until the fifteenth. The Italian word "abaco" became the term for written arithmetic, indicating its origins outside the mainstream of officialdom or academia (cf. the discussion of the "abacus manuscripts" below). "In the legal treatise known as 'Glanvill', compiled in 1187, the capacity to count money and measure cloth is said to be the test of legal majority for a burgher's son."[35]

The "skill of numbers", furthermore, receives both inspiration and content from social conditions. As concerns modern mathematics and its continuity with the ancient, there are problems for internalist interpretation. Sàlomon Eochner, also in the internalist tradition, has ambivalent feelings about this period in the history of mathematics because of the absence of a continuous, identifiable academic tradition.

> As far as is known to me the only rationalization ever attempted is a socio-economic one. It may be strange, and even painful, to contemplate that our present-day mathematics, which is beginning to control even the minutest distances between elementary particles and the intergalactic vastness of the universe, owes its origination to counting house needs of "money changers" of Lombardy and the Levant. But, regrettably, I do not know by what arguments to disagree, when economic determinists, from the right, from the center, and from the left, all in strong unison agree. 36

Leonardo Fibonacci, with his Liber abaci in 1202, marks the earliest of modern European mathematics. His father was a representative of the Republic of Pisa and from 1192 was directing the trading colony of Bugia in what is now Algeria. Leonardo was **supposed to become a merchant** and learn the art of calculation. With his father, Leonardo travelled to Egypt, Syria, Greece, Sicily and Provence and studied and disputed with native scholars. In the course of this study he decided that calculation with Indian numerals was best. About 1200 he returned to Pisa and composed his written works, which were a mixture of practical solutions to problems as well as theoretical arithmetic and geometry. Of special interest is that with regard to the second and tenth books of Euclid's Elements he gives not only proofs in Euclid's manner, but also treatment in numerical form. According to Kurt Vogel, he went beyond his predecessors in indeterminate analysis and number theory.

He spent the latter part of his life as a friend of Frederick II and the last bit of information we have about his life is from the Pisa archives mentioning an honorarium to be paid him of twenty denarii.

He evidently had advised the city and its officials, without payment, on matters of accounts, a service the city expected him to continue. This decree of the city, which was inscribed on a marble tablet in the Pisa city archives in the nineteenth century, is the last information we have on Leonardo's life. 37

Also in the court of Frederick II was Michael Scotus who had made his way from Balwearie, Scotland (1190) through Paris, and Toledo (1217). The second edition of the Liber abaci in 1228 was dedicated to Michael Scotus. The book is divided into fifteen sections. The first five sections are on the numerals and the practice of calculating with them. The next two are on fractions. The next four are on business problems, pricing, exchange, partnerships, and the mixture of coins respectively. The twelfth is on manifold exercises, the thirteenth on the rule of Elchatayn, the fourteenth on roots and the last on geometry, algebra and almucabala.[38] He was the first to make breaking a fraction into parts a task in itself. He wrote a separate treatise on commercial arithmetic in Italian, the Di minor guisa, which appears to be lost.

The Italian abacist tradition spans the period from Fibonacci to Luca Pacioli who published the first printed book on mathematics in 1494. Warren Van Egmond has studied the manuscripts belonging to this tradition and suggests that everyday reckoning of this type not only improved and grew during the commercial revolution of the late middle ages but also had a substantial effect on mathematics of a higher academic order. These manuscripts are a more everyday form of Fibonacci's Liber abaci. The tradition represented by both Fibonacci and the abacists had social roots and supports outside the universities. The manuscripts are made up almost entirely of practical, commercial problems. For Van Egmond the period from 1200 to 1600 in Europe forms a distinct period in the history of mathematics.

> During this time the West was incorporating those elements it had taken from Greece and Islam into its own consciousness, absorbing their meaning and adapting them to its own purposes. We might call this the nascent period of Western mathematics, for it was a time when a native Western mathematics was being born from a mixture of Greek and Arabic elements under the shaping influence of the West's own needs. 39

A sedentary merchant whose power was growing posed new problems for mathematics, which development proved to have formative influence on western algebra.[40] In this context, Kurt Vogel echoes Alexander Murray's assertion about the co-nurturing of money, industry, and arithmetic. He cites Agricola's De re metallica to the effect that the use of coins is much more pleasant than simple exchange of goods.[41] The exchange of goods provided the arithmetic of the period immediately preceding Agricola, the period of Van Egmond's concern, with its typical problem. It is here that proportion becomes the language of gain; in the words of Luca Pacioli, "Exchange is nothing other than giving one good for another with the intention of getting more."[42] The setting of a cash price and a higher goods price for their wares by each of two merchants made the arithmetic involved quite complicated.

In the simplest situation, A offers a selling price (cash) b_1, and a good value S_1, while B offers b_2 and S_2 respectively. The exchange is valid and none is betrayed when $b_1:S_1 = b_2:S_2$. Countless examples of this exist in the reckoning books of the fourteenth to sixteenth centuries. From the Algorismus ratiobonensis we have the example of wax exchanged against ginger . Where the corresponding values are, for example, b_1 = 8 fl., S_1 = 9 fl. and b_2 = 19 fl., S_2 = 21-3/8 fl. But where all values are given the exchange is not correct.[43] Vogel finds also that a problem in an anonymous Luca manuscript and in the works of Fridericus and Adam Riess is solved with the same numerical values.

The problems get more complicated when one or both parties want part of their end of the bargain in cash and part in goods. Vogel claims that the following "ungrounded" formula is applied:

$$\left(b_1 - \frac{S_1}{n}\right) : \left(S_1 - \frac{S_1}{n}\right) = b_2 : S_2 \quad (F_1)$$

The luca manuscript offers the following example in which merchant A who exchanges cloth against the wool of B wants one-fifth in cash:

b_1 = 45 pfennig for 1 Channa (4 ells) of cloth;

S_1 = 50 pfennig and b = 35 gulden for 1 hundredweight of wool; S is sought. The formula states:

$$45 - \frac{50}{5} : 50 - \frac{50}{5} = 35 : S_2, \; S_2 \text{ thus is } 40 \text{ florins.}$$

When referring to B, F_1 becomes:

$$b_2 + \frac{S_2}{n-1} : S_2 + \frac{S_2}{n-1} = b_1 : S_1 \quad (F_2)$$

where B offers A not 1/n in cash but 1/n-1, Vogel multiplies both sides by n-1/n yielding:

$$\left(\frac{n-1}{n} \cdot b_2 + \frac{S_2}{n}\right) : S_2 = b_1 : S_1 \quad (F_3)$$

and states that this is found in the work of Francesco Pellas in 1492.

Vogel uses an example from Tartaglia to show the application of F_3:

> An example from Tartaglia should show how an individual exchange transaction of this kind is carried out. A has 2640 pounds of wool at 40 and 48 Ducats a hundred weight respectively. He wants to exchange the wool against cloth which costs 20 groschen (1/24 Ducat) per ell. A wants half in cash besides. First the exchange price is calculated according to F_1, namely, (20-24):(48-24) = 20:S_2, therefore S_2 = 30 groschen. Now 2640 pounds of wool cost 1267-1/5 ducats in exchange, from which A wants 633-3/5 ducats in cash; for the other 633-2/5 ducats he gets 506-22/25 ells of cloth (at 30 groschen/ell, S_2). The trial (Probe) shows that A received from B 633-3/5 ducats cash and 506-22/25 ells of cloth at the cash value of 442-2/5 ducats, thus altogether 1056 ducats. For this B gets 2640

> pounds of wool, at 40 ducats per hundred weight, thus likewise 1056 ducats.
>
> If all four values are given then the transaction can only be in order, if one demanded from his partner a part in cash. 44

The abacist tradition is very important for us here because it represents new directions in mathematics. Although traditions are never made of whole cloth, the cloth trade in fourteenth century Tuscany gave new impetus to mathematical thought. Van Egmond describes a typical manuscript.

There is an extant syllabus from a Pisan master, Christofano di Gherardo diDino, which lists seven topics in order of descending difficulty; 1. multiplication, 2. one digit division, 3. two digit division, 4. three digit division, 5. fractions, 6. rule of three. Concerning the latter:

> This rule was the basic method of problem solving in abacus arithmetic. Its study thus set the foundation for solving most of the problems the student would likely encounter in his life. It is based on the principle that the product of the means equals the product of the extremes in a simple proportion of the form a/b = c/d, so that given three terms it is possible to find the fourth from the relation a = bc/d. The use of the rule of three is best seen in simple pricing problems like if eight yards of cloth are worth eleven florins, how much are 97 yards worth. 45

These manuscripts were written by professionals who, although spending most of their time teaching or solving complex business or construction problems, found some time to get together in small tutorial sessions to try new problems and new methods. Their histories and contracts, claims Van Egmond, display a "flow of ideas and challenges between teachers and amateurs,..., that provided the intellectual challenge and stimulus to further progress."[46] Although some of the manuscripts paid considerable attention to special theoretical subjects like algebra, number theory, tariffs or astrology, they are distinguishable by their collection of problems. In ninety-one separate works for Florence alone averaging 200 - 300 problems each there were thus, 20,000 - 25,000 problems with very few duplicates. There was contribution to both business and mathematics.

With respect to mathematics, the most abstract and theoretical result of the abacus manuscripts are the sections on algebra. Arising from the pricing and double barter problems are more abstract ones such as "find me a number..." problems. Different rules or regule were developed for each type of problem. For example:

> When a censo and a cosa are equal to a number, we must divide by the censo and then divide the cosa in half, and this half multiplied by itself, added to the number, and the square root of this sum less half of the cosa so much is the cosa worth. 47

This reminds us of Nesselmann's description of Diophantus' work as syncopated algebra. Although the authors of the abacus manuscripts did not have access to Diophantus, they partook of activities and traditions similar to those which perhaps constituted Diophantus' development. The specialized algebra portions of the manuscripts use the term cosa for the unknown number and in this respect only are the problems dealt with different from the commercially practical problems. According to Van Egmond, the abacus manuscripts represent the only algebra tradition in Europe of the relevant period. Almost all treatments of it take place in the work of the abacists.

The tripartite historical division of periods of Algebra of Nesselmann, rhetorical, syncopated and symbolic, may also, claims Van Egmond, be seen in the 200-year career of the abacus tradition itself.

> The development of arithmetical problem-solving can reasonably be divided into the same three steps with the abaci obviously belonging to the first, though towards the end of the fifteenth century, there is some use of abbreviations and a primitive symbolism that is the mark of the syncopated step. 48

There are two other practical traditions to be considered here, cossist algebra and bookkeeping. The term cosa, mentioned above for unknown, occurs in both and seems to mean something like "thing".

The name which stands out in the linking of the bookkeeping and algebraic traditions is that of Luca Pacioli whose Summa de Arithmetica Geometria Proportioni et Proportionalita was a culmination of the abacist tradition in algebra and the first published treatise on bookkeeping. The bookkeeping portions have been published separately as De Computis et Scripturis. The Summa has been given more attention by accounting historians than historians of mathematics probably because of his status as father of their science. Grammateus, Cardano and Stevin are others to have combined the activities and traditions to name only a few of the most prominent. Although Pacioli's Summa is the first printed book on both algebra and bookkeeping, he is not the inventor of either. Some claim that double-entry bookkeeping is as much as 300 years older than Pacioli.[49] De Roover places its development realistically as taking place on a trial and error basis in several northern Italian cities between 1250 and 1400.[50] The earliest extant account books are similar records of partnerships. Between 1155-1164, Giovanni Scriba, a Genoese notary, kept the contract records of an account between Ingo da Volta and Ansaldo Baiallardo for purposes of dividing after each contracted journey the appropriate proportion of profits following deductions of expenses and allowing for unsold goods.[51] This business of partnership proves to be a necessary ingredient in the development of double-entry. Littleton mentions as pre-conditions and essential elements writing, arithmetic, private property, money, credit, commerce and capital while De Roover boils it down to the three big factors of partnership, credit and agency.[52] This is an accountant's eye view of the relations of commodity production. Littleton explicitly lays out the socio-economic requisites for the emergence of this form of record keeping in opposition to the notion of its emergence as simple technical development in the narrower sense. So although duality of form emerges as a characteristic of double-entry bookkeeping, and because of the requisite social contexts, this outward form is not sufficient cause to regard it as criteria for the existence of double-entry.

> It is evident, also, that a considerable degree of duality of record probably existed long before double entry bookkeeping was completely formulated. In fact, instead of being the <u>sine qua non</u> of bookkeeping, this duality of form is quite <u>probably a mere</u> reflection or result of a deeper, more basic characteristic. 53

For Littleton, bookkeeping records facts about property and property rights and for this calculation requires the widespread use of money as common denominator. In addition, credit is necessary for the deferring of balances. The trend toward double entry involves the trend toward balances and away from remainders. "Perhaps equilibrium of results may be the keynote of double entry rather than duality of form."

The calculation of positive and negative properties involved presupposes the existence of such rights in law. The ultimate purpose involved is, within this social context, to enable a proprietor to assess the relative merit of different specific risks. Although accounting historians would not use this language, double entry depends on the existence of certain relations of labour.

> When commercial proprietorship drew to itself and adapted to its own requirements the account-keeping methods of banking agents and trading factors, modern double entry bookkeeping emerged. 'Factor's bookkeeping' had become 'proprietor's bockkkeeping'. 54

This occurs when the sedentary merchant mentioned by Van Egmond becomes the norm and especially when he employs others and gives and receives credit. This was indeed growing in northern Italy in the fourteenth century. By the thirteenth century money changers were becoming merchants of exchange. By 1338 there were eight banking houses in Florence and by 1400 there were 120.[55] The firm became an entity in itself whose property can be measured by calculating the legally established gains and losses in goods measured by money. By 1400 accounting practices were already a tool of management and control with the rudiments of cost-accounting having been developed. One account running from 1296 - 1305 displays double entry practice in paragraph form, that of the firm of Rivieri Fini and brothers, bankers to Philip the Fair of France (1285 - 1314). Another, Farolfi Co., Florentine merchants in Languedoc and Provence, shows operating results with cross referenced debits and credits and prepaid rent as a deferred expense, but there was as yet no cohesive system.[56]

The next closest thing to modern double entry results from the books of the del Bene Co. in Florence from 1318 - 22. This company imported undyed woolen cloth from Flanders and finished it in Florence and kept one set of books for industrial activities. The production, earnings and advances of dyers and finishers were kept track of.

> The reason why the del Bene company kept such elaborate records may be due in part to the fact that it was engaged in manufacturing, which always calls for more detail and stricter control. The great amount of duplication may also have been a clumsy way of providing internal checks, a result which could have been achieved more efficiently by a simpler and more rational system of accounting. However, we should not blame the del Bene bookkeeper. His was still a period of experimentation. 57

In this period the term ragione referred to a statement of account of a firm or one of its branches. Other derivatives of the term refer to bookkeeping and its practices or practitioners. Similar terms are also used to describe mathematical practitioners of the abacist or algebraic variety; also from the same period Paolo Gerardi's abacus book with the first recorded attempt at an algebraic solution of a cubic equation, the Libro di ragioni, was written in 1328.[58] The first systematic use of double entry comes from the Datini firm in about 1340. The records go from 1335 - 1410 and display the transition from single to double entry. Double entry is used exclusively after 1383 and after 1390 was used in all branches. Yamey decides that double entry was about two centuries old when Pacioli wrote his treatise. He rejects the notion that it resulted from the application of science and scientific measurement to the world of trade. Although he is worried about the direction of influence, he does not doubt the connections. Nor does double entry respond to a technical business need.[59] It, along with European algebra and mechanics, we are arguing, is part of the intellectual reflex, and this itself is not meant in any "mechanical" or "derived" sense, of rapidly changing social relations, effecting both everyday life on the street and in the offices and palaces of clergy, nobility and royalty.

Thus a new mathematics developed in a Europe whose social foundations were beginning to change very rapidly. The commodity and wage-labour were becoming more wide-spread and the calculation methods for keeping track of these relations were becoming more abstract in keeping with the relations between the objects of exchange. We are now at the point where we may begin to see the application of this new mathematics to nature. Although at first the connections are difficult to trace and document, with some of the later figures the task is ~~much~~ easier. We shall now look at some of those thinkers who were concerned with problems of mechanics, velocity, equilibrium and so forth, whose work violates certain Aristotelian and Euclidean notions even though certain of those figures would claim to be Aristotelian themselves. We shall begin with Thomas Bradwardine and Nicole Oresme.

<u>Thomas Bradwardine</u> (d. 1349)

Bradwardine was one of the first to compare unlike physical quantities in the same expression. In his physics Bradwardine is essentially an Aristotelian but the source of his mathematics is not really known. He was an Augustinian as well who preached to the English forces after their victory over the French at Crecy in the Hundred Years War, lectured to the matriculi at Merton College, was called to the Curia in Avignon to await a decision about the orthodoxy of his views but fled to the protection of Louis of Bavaria in Pisa and became Archbishop of Canterbury for a few days until be died of the plague.[60]

Bradwardine differs from Aristotle in some important respects in comparing quantities of unlike ~~kind and in~~ basing the continuity of motion on the properties of the continuum. He cites only one source for his new mathematics.

> According to Ahmad ibn Jusuf (in his letter On Proportion and proportionality) they differ in another and most important way in that, in the case of continuous proportionality, all the terms must be of the same kind, whereas in discontinuous, or disjunct, proportionality, some of the terms may be of different kinds. For example, as the length of one musical string is to that of another, so is the pitch of one to the pitch of another - and as one moving power is to another, so is the speed of the one motion to the speed of the other. [61]

There are two problems here. Physical properties, which are continuous, are made numerical, discrete; second, forces are related to velocities and thus terms of different genus are related. Ahmad ibn Jusuf's answer to the second question (Bradwardine does not bother with the first at all), is that although continuous proportionality must be between terms of the same genus, terms of different genus may be related in discontinuous proportionality. Rationals are proportional immediately by number, irrationals immediately by proportion and mediately by number. Older writers like Jusuf were attended with enthusiasm for the growing application of proportions. We may suggest at this point the possible association of this kind of proportionality with commercial practice since in the same Arab culture the rule of three and rules of single and double false position appeal to a similar base in proportionality, in the commercial realm between quantities of unlike kind. "Ametus (Ahmad ibn Jusuf) extols the utility of proportionality for finance, statics, optics and music."[62]

After introducing the notion that proportionality, strictly speaking, obtains only between quantities of the same kind, Bradwardine goes on to speak of denomination by number, a term missing in the Euclidean discussion of proportions. He develops his notion of qualitative velocity as a response to an objection raised against his dynamics. The objection raised is that if equal velocities are to result from equal proportions of forces (Bradwardine's geometric law), then the space traversed by a larger body should be equal to that traversed by a smaller body, which of course cannot be the case if velocities are equal. In the words of Lamar Crosby,

> This dilemma thus raises the problem of relating force to distance, dynamic to kinematic functions, and brings out the ambiguity inherent in Aristotle's remark that the "weight" of a body is a factor in its velocity in free fall. Bradwardine offers in solution of the above dilemma, a distinction between "qualitative" and "quantitative" proportionality as applied to motion. Qualitatively, the moving force bears the same proportion to any and all fractions of the impeding medium, and the qualitative proportion determines qualitative velocity. Quantitatively, however, the proportion is between the times of the two motions. 63

The proportionality arrived at here is basically that to double the velocity we double not the motive force simply but double the proportion of motive force to resistance.[64] Bradwardine and his immediate followers never did any experimental work (that is known about) so the problem of units and the development of constants never really drove home the question of commensurability in all its ramifications. Nothing in the whole of the <u>Tractatus de proportionibus</u> is measured or given any numerical value. Only proportions are identified as halved, doubled or tripled etc. The definition of one quantity in terms of two others was used by Bradwardine's followers in a whole range of theological, philosophical and other problems.[65] A ratio is now a quantity in itself, contrary to the notion of the Greeks. Although for many medievals the universe was ordered by number, weight and measure, there is a large jump from the notion that the cosmos displays numerical properties to the use of numbers to measure continuous magnitudes in proportion. Bradwardine's rules of proportion seem for the most part in line with Boethius and Euclid and the subsequent composition tradition except for his comparison of unlike quantities, for which he uses Ahmad ibn Jusuf as authority, and for his notion of continuity. His rule continues to be used whenever growth is a function of size.[66]

The only sources cited by Bradwardine are Boethius and Ahmad ibn Jusuf. John Murdoch has carried out an analysis of these medieval proportion traditions but is unable to establish their sources. Bradwardine's notion of denomination was at the center of new uses of proportions. It had been well known that a rational proportion could be denominated by a number. Bradwardine says "immediately" denominated to distinguish it from the following:

> The second order comprises those proportions which are called "irrational". These are not immediately but only mediately denominated by a given number, for they are immediately denominated by a given proportion, which is, in turn, immediately denominated by a number. Of this sort is the square root of the proportion of two to one, which is the proportion of the diagonal of a square to its side, and the square root of the proportion of nine to eight, which constitutes a musical half-tone. 67

We have, then, quantities which are incommensurable in two senses; they are irrational and are quantities of unlike kind. For Stillman

Drake, however, Bradwardine is still a long way from modern algebraic treatments of dynamics and kinematics.

There is, nonetheless, a move toward the unification of number and magnitude in the notion of denomination and this move has as its only cited source the work of one who recommends the use of proportionality for finance.[68] Murdoch cites John Wallis' comment, "'...the whole fifth book of Euclid's Elements is arithmetic'. If the medievals were historically no stimulus for Wallis' determined position, still their speculations pointed in his direction."[69] Murdoch mentions the five basic medieval renderings of the Elements as those of Boethius, al-Nairizi, Gerard of Cremona, Adelard of Bath and Paris Manuscript BN. 10257. The latter two mention continuous proportionality, a concept foreign to Euclid and likely of Arabic origin although no source has been found.

Bradwardine at least hints at the numerical treatment of magnitudes and engages in the comparison of quantities of unlike kind. The only available analogy would seem to be commercial calculation, in which the Arabs were also involved, as Aristotle describes it in the Ethics. We have, thus, in Bradwardine a hint of the abstraction of nature along the lines we have indicated.

Nicole Oresme (d. 1382)

In terms of the connection between the expanding commercial relations and the new mathematics of motion Oresme's predecessors were Pierre

Jean Olivi and Franciscus de Marchia. Both Olivi and de Marchia were Franciscans and as such condemned the use of money within the order but also provided an early defense of bourgeois wealth and financial practices as long as these practices were not engaged in by the Church. Olivi was a hero to the people and anti-Christ to the Church. In 1304 all of Olivi's works were ordered destroyed. In economic terms Olivi's writings concern the concept of money rather than usury specifically. In the thirteenth century in a work entitled "De contractibus usurariis", Olivi conceived of money as capital. Capital (quandam seminalem rationem lucrosi) was opposed to simple money (simplex pecunia). Olivi uses the Latin "capitale" and apparently got it from economic life. Ethically Olivi considers the profit from the loan of money to be legitimate when the same profit could have been made from the money in a legitimate business. Charging interest was non usurious when the rates of interest were set according to the profit rates in mercantile businesses. Value was added through labour or industry whether a good or money was involved. Money becomes, for Olivi, a transporter of industry. The Franciscans and Paris Ockhamites retained Olivi's economics and notion of motion.

Michael Wolff argues that there is a close parallel between Olivi's notions of both motion and capital as forms.[70] The justification of profit on capital provides the motivation for a justification of the impetus concept. Money is analogous to any other tool, not just an instrument of exchange. The activities of handworker, merchant and banker are all of the same kind.

From about the mid-thirteenth century the national coinage in France was bimetallic and the ratio of gold to silver was nominally established in the interest of the state coffers. It was soon realised, however, that this damaged trade, causing problems like inflation. Pierre du Bois argued against devaluing money in his "*De recuperatione terrae sanctae*". Oresme convinced Charles V against the monetarist policies of his predecessors. Both Buridan and Oresme wrote tracts on "economics". The upshot of their considerations is that, as opposed to Aristotle, money was not considered simply conventional; its value could not be wished away; money had a real value.

In his *De Moneta* Oresme argues basically the position that money has a real rather than simply a nominal value. It belongs to the community and not the prince and its value is determined by the labour expended in producing it, although this determination receives no quantitative expression in Oresme. Wolff's argument concerning freedom, force and ownership is expressed clearly in a passage from the *De Moneta*:

> For if a man gives bread or bodily labour in exchange for money, the money he receives is as much his as the bread or bodily labour which he (unless he were a slave) was free to dispose. For it was not to princes alone that God gave freedom to possess property, but to our first parents and to all their offspring, as it is in Genesis. 71

In addition to the assertion of the real value of money what this hints at is that there is an actual substance contained in certain proportions in dissimilar objects in virtue of which they may be compared. Value, it appears, occurs in Oresme as something like a natural property of gold and silver, owing partly to their scarcity. Whereas Aristotle had argued that money is strictly conventional and could [illegible] in truth, provide a basis for comparing proportionally the quantities of objects of dissimilar kind, thus making alternation of the inner terms of a proportion problematic, Oresme sees 'value' as a real property of quantities thus enabling them to be viewed at least in one sense as quantities of like kind. Buridan questioned cross-conjunction both in economics and in kinematics.[72] Both Buridan and Oresme were concerned with the physical properties that coins must have to be bearers of value. Oresme, for example, determines that the value of gold should exceed that of silver by a definite proportion owing to its natural scarcity.[73]

Oresme was countering a practice engaged in by many princes of his day: that of producing a debased, new coinage which upon circulation would drive out the older, purer one, thus enabling the prince, bearer of the older, driven-out coins, to enrich himself. This produced inflation as well as other financial problems. In arguing that the coinage belonged to the community, to the subjects and not to the prince, Oresme was arguing thereby not only that proportionality was <u>in truth</u> possible mathematically between quantities of unlike kind, but also, since money had a real value, the proportions correspond to some real determinable property of the objects considered. The units into which this value is divided are nominal, pence, shilling, mark, and if for any reason the units are changed the proportions of different coins to each other will change since the prince cannot legislate the value relation of the metals concerned but only the name of the units. Oresme points to the origin of the term moneta and emphasises thereby its difference from "nummisma". He quotes Uguccione of Pisa's "Derivationes magnae" to the effect that "Moneta is so called from moneo (to warn) because it warns us against fraud in metal or weight." Oresme agrees with Aristotle, Ovid and others in arguing that it is unnatural for money to increase itself "by changing itself into itself, as changing one form of it for another". He also argues, however, that money may realise profit "by laying it out in the purchase of natural wealth".[74] Hector Estrup, in his discussion of fourteenth century economic thought, calls Oresme's theory in this regard the theorie marchandise as opposed to the more predominant theorie signe.[75]

To return to kinematics and mathematics, the medieval part of our story begins essentially with Bradwardine and, although we can only speculate about his involvement in economics or finance, it is here that we must begin our argument about the relation between the three realms. William Wallace makes the following observation:

> Bradwardine would undoubtedly have identified himself as an Aristotelian, for the problem to which he set himself was to save the rules given by Aristotle for comparing motions and deciding on their commensurability. Yet in defining motion he and other fellows at Merton College implicitly abandoned Aristotle's analysis in favour of that furnished by William of Ockham. A close study of their writings shows that, rather than conceive motion as the actus entis in potentia inquantam huiusmodi, as Aristotle had done, the Mertonians regarded motion essentially as a ratio.

To this Wallace adds the following footnote:

> How this transition came about is not easy to explain, involving as it did a rejection of such basic principles as the Euclidean condition for any ratio, viz., that it must be between entities of a single kind. 76

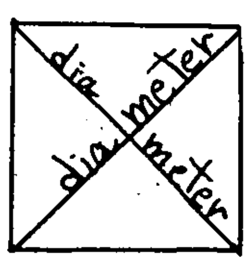
With Oresme this question takes on new and suggestive dimensions.

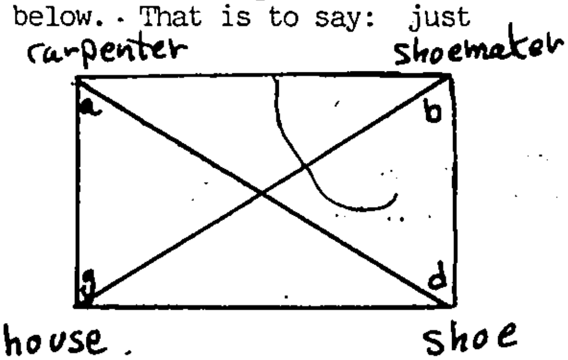
We do not know that Oresme formulated economic or mathematical notions in direct responce to business practice but Johnson reports that he was "'bursar' of the College of Navarre from 1348 to 4 October 1356, when he became Master".[77] His De Moneta was written in 1355, most of his scientific writings between 1360 and 1370 and finally he produced translations of Aristotle's Ethics and Politics.[78] His connection of concerns with economics, proportionality and commensurability comes out clearly in his translation and gloss of the Ethics. His translation of the passage reads:-

> And a diametrically organised combination causes a proportional remuneration, as would happen if one arranged four things in a square diagram, which would have four corners or angles named a,b,g,d; and put the carpenter in angle a , the shoemaker in b, the house built by the carpenter in g and the shoe made by the shoemaker in d. Now then the proper procedure would be for the carpenter to hire the services of the shoemaker and to remunerate him by offering his services in return. And then if what the carpenter did were proportionally equal to what he received from the other, things would be as we said.

To this he adds the following notes:

> Diameter is a line which crosses a square diagram from one corner to the other; as shown below. That is to say: just





> commutation in kind, not according to the equality of things but according to the proportion of their value; for a house has more value than a shoe and a pound of twimbic more value than a pound of candles. 79

That his familiarity with business was generally recognised is evidenced by the fact that he was sent by the Dauphin in 1360 to secure a loan from the municipality of Rouen.[80]

In his kinematics and mathematics, Oresme is concerned with whether ratios can be part or parts of each other and appears as well to produce violations of Euclid's and Aristotle's concerns about the admissibility of ratios between quantities of unlike kind. In admitting such ratios Oresme is preparing the ground, by creating a more abstract, homogeneous mathematical object, for the coalescence of number and magnitude and hence also for reducing the problems created by incommensurability. His propositions in the De proportionibus proportionum all point to which ratios are commensurable with one another, which ratios are aliquot parts of other ratios.

With respect to dynamics, Oresme was able to argue that most ratios of velocities are likely to be irrational. This inexactitude did not bother him. He operated by treating ratios as numbers, thus supporting further our notion of his contribution of more abstract and homogeneous mathematical objects.

> I offer an example formulated in numbers, because if ratio B is parts of A then each [i.e. A and B] can be treated as a number, by the fifth [proposition] of the tenth [Book of Euclid]. Let B be 3/5 of A, then by the preceding proposition, it is necessary that C be 2/5 of A. By subtracting C from B, 1/5 remains which is part of A because it is 1/5 of A. 81

Edward Grant sees the problem here of the number - magnitude distinction.

> We see that Oresme, in coping with the various problems arising from his treatment of ratios of ratios, frequently applied to the same case propositions from the arithmetic books of Euclid and the more general books embracing both number and magnitude. It seems that Oresme, perhaps unknowingly, was ignoring the traditional distinctions between number and magnitude in general and bridging the gap that artificially separated them. Such

moves were essential before mathematics could advance to the development of analytic geometry. 82

Most of the De proportionibus is devoted to showing which types of ratios are commensurable and which are not. In the process, however, Oresme succeeds in obtaining, in our view conditioned by his economic activities and writings, an abstract enough mathematical object to be able to reconcile number and magnitude. Number is used to calculate with continuous magnitudes.

Rafael Bombelli (d. 1573)

The work of Rafael Bombelli displays quite graphically the kind of use and reading of Diophantus which occurred in the sixteenth century. With Bombelli modern mathematics came of age. The older literary and anecdotal style disappeared with him and more symbolism and formalism was ushered in. Olschki argues that the use of Diophantus and Archimedes by Bombelli and Tartaglia put mathematics into a more formal language by which the early modern Europeans were only then able to appreciate Greek mathematics.[83]

We are arguing, of course, that the new culture did allow an appreciation of Diophantus but that it was a new rendering. It was precisely a commercial culture which allowed this new, more abstract rendering. The career of Bombelli's Algebra displays this nicely.

Jayawardene places Bombelli at the end of a tradition beginning with Leonardo Fibonacci and running through Luca Pacioli. Bombelli came in contact with the work of Diophantus and this contact had a tremendous effect on the formalisation of western mathematics. The published version of his Algebra contains no commercial examples.

> In fact, in the introduction to Book III he said that he had deviated from the practice of the majority of contemporary authors of arithmetics who stated their problems in the "guise of human actions": (buying, selling, barter, exchange, interest, defalcation, coinage, alloys, weights, partnership, profit and loss, games and other numerous transactions and operations relating to daily living). 84

Bombelli claimed to teach the higher arithmetic in the manner of the ancients. An unpublished version of the Algebra, however, reveals that Bombelli too had originally conceived it in practical terms. In 1923 a manuscript version was found in a library in Bologna. According to Jayawardene it precedes Bombelli's reading of Diophantus. Bombelli did not have formal mathematical training. His father was a wool merchant and his mother the daughter of a tailor. His professional activity was primarily that of an engineer and he enjoyed the patronage of a rich nobleman to engage in the writing of his Algebra.

In the manuscript Book I dealt with elementary concepts like powers and roots, Book II with powers and notations for the actual operations. Book III dealt with practical examples .

> The introduction to Book III of the manuscript is quite different from that of the printed work: there is no reference whatever to the contemporary authors of arithmetics who stated their problems in the "guise of human actions:" The absence from the manuscript of the 143 problems borrowed from Diophantus found in the printed version suggest that Bombelli had not seen the Vatican Codex when he first wrote the Algebra. 85

Book III of the original manuscript, however, contains 156 practical problems. The problems in the manuscript version follow the problem classification of Kurt Vogel, which includes exchange, partnership, interest and labour time.[86]

Antonio Maria Pazzi, a mathematics teacher at the University of Rome, showed Bombelli a codex of the Arithmetica of Diophantus, which exerted a wide influence. In place of cosa and censo Bombelli used the Diophantine tanto and potenza, and excluded the practical problems replacing them with 143 taken from Diophantus. In the introduction to the revised Book III Bombelli dissociates himself from the tradition he followed in writing the initial version.[87] He also introduces a new symbolism.

> He represented the powers of the unknown quantity by a semicircle inside which the exponent was placed: (1) for the modern x, (2) for x^2, and 5 (1) for 5x. In the printed work the semicircle was reduced to an arc. 88

This is the best sort of circumstantial evidence for our argument that it was a particular social and economic context which allowed a modern, symbolic reading of Diophantus. The creation of an abstract mathematical object to be of use in physical calculation was developed in commercial calculation in the context of a burgeoning commercial culture and enabled what Klein has termed the innovation in modern algebra. We have a man with no formal mathematical training, whose father was a merchant and who essentially conceived of his algebra in most respects before reading Diophantus. Without further revision he was able to insert into Book III the "more abstract" problems of Diophantus in place of commercial problems. We can thus safely conclude that the reading of Diophantus did not really constitute Bombelli's algebra.

Niccolo Tartaglia (d. 1557)

Tartaglia fixed some general expressions of calculation in formulas. He was consulted on problems of arithmetic by engineers, artillerers, merchants, architects and others and taught mathematics to business people in Verona. While his writings "preserved the tone of the Venetian workshop, store and arsenal",[89] emphasizing the connection of daily life with technical and mathematical tasks, he read Archimedes and translated Euclid with a commentary so "that every mediocre head would be capable of conceiving it with ease without pre-knowledge or the help of other sciences."[90] He was a teacher of Benedetti who, in turn, influenced Galileo.

Tartaglia is one of the most prominent mathematicians of the sixteenth century and is an extension of the practical abacist tradition. His opponents, however, attempted to draw him into peripatetic debate. He introduced geometric problems into arithmetic[91] and influenced scholarly mathematical methods.[92] The style of his work, for Olschki, is that of the burghers and not of the court.

Olschki argues as well that "the new skills, methods and experiences made possible an immediate understanding of the Diophantive book."[93] Tartaglia also read Diophantus and began to use some of the new mathematics in mechanics. Whereas the ancients had been concerned primarily with statics, Tartaglia was more involved with ballistics. He concluded, for example, that a 45° elevation gives the longest cannon shot.

In the first sections of his Nova Scientia Tartaglia does not engage in much algebra, constructing Archimedian proportionalities between magnitudes of like kind. Times are compared with times, distances with distances and angles with angles. With proposition IX of the second book, however, we begin to see some algebraic calculation but a very careful attempt is made not to calculate with magnitudes of unlike kind. He does, however mix areas and lines in order to deduce the length of a 45° shot compared to a "point-blank" shot.

On the thirty-ninth question, twelfth proposition of the Quesiti there is no doubt that quantities of unlike kind are compared in the same expression. The form of calculation here is algebraic. He begins with a question having a history in Jordanus and the medieval Archimedean tradition.

> If there is a solid rod, beam, or staff, as in the two preceding [propositions], which is similar and equal in thickness, breadth, substance, and heaviness in every part and of which the heaviness as well as the length is known, and if it be divided into two unequal parts which are also known, it is possible to find a weight which, when suspended at the end of its shorter part, will make the said solid rod, beam, or staff stay horizontal. 94

In this work, which is written as a dialogue, Tartaglia has his interlocuter, the "Duke of Mendoza", the Spanish ambassador to Venice, request a proof in the form of a "material" example. We cite the example in full from Tartaglia.

> Tartaglia: For example, let there be the solid rod (beam or staff) AB as proposed, that is equal and similar in breadth, thickness, substance, and heaviness on every side or in every part; and let us assume the heaviness of the said solid rod to be known, that is, let the length be two paces or ten feet; and let us also assume that the rod is divided into two unequal parts at the point C and that the [lengths of] said parts are known, it being assumed that the shorter part AC is two feet and the longer CB is eight feet. Now I say that it is possible to find how many pounds that body must be which, suspended at the point A (end of the

> shorter part), will make the said rod or beam stand parallel to the horizon. For (by the things demonstrated in the two previous propositions) it is manifest that the ratio of the heaviness of that body to the heaviness of that difference which exists between the longer part CB and the shorter AC (which difference becomes DB) will be as the length of the whole rod or beam AB (which is ten feet) to the double of the shorter part AC (which is two feet), and this double comes to be four feet. Let us call this AD. Then the heaviness of that Body [at A] will be to the heaviness of the partial rod DB as the whole length of AB (which is ten feet) is to the length of AD, which is four feet. Whereby, arguing, conversely, let us say that the ratio of AD (which is four feet) to the whole AB (which is ten feet) will be as the heaviness of the partial rod DB which (at the rate of 40 pounds to all AB) is 24 pounds. Now the weight of the body we seek is that which hung at the point A, should maintain the rod or beam parallel to the horizon. Whence in order to find this, we shall proceed by the rule ordinarily called the rule of three, founded on Euclid VII.20; multiplying ten by 24 gives 240, and then we shall divide by four, obtaining 60. I say that that weight which I called the body F will be 60 pounds; and this is the proposition. 95

Proposition thirteen does much the same thing. The problem here, of course, is the citation of Euclid as authority. Although Tartaglia cites VII.20, we are assuming here that this is the same proposition rendered in modern editions of Euclid as VII.19.[96] Euclid had reasons for writing separate books on proportion for magnitudes and numbers. In VII.19 Euclid is referring to pure numbers only, not to numbers as measures of physical or geometric magnitudes. The rule of three, furthermore, is an Arabic rule with primarily commercial applications. Feet and pounds may now occur in the same expression just as cloth and ducats may be measures of each other in a commercial sense. The abstract concept of number derived from commercial calculation was used by Tartaglia to produce a view of nature as homogeneous. He believed, furthermore, that he was simply applying Euclid, because Euclid for him was reconstituted through Tartaglia's own sixteenth century eyes.

Galileo Galilei (1564 - 1642)

Galileo is typically the figure most closely associated with the advent of a mathematical science of nature. It is also rather commonly held, thanks to the work of Stillman Drake, his foremost modern commentator, that his only real predecessors are Euclid and Archimedes. We argue, however, that there is reason to believe that his allegiance to Euclid and Archimedes is incomplete and that the influences on him came from the same tradition in which Tartaglia had participated.

It is true that Galileo attempts to remain within the Euclidean-Archimedean tradition but he does violate their principles in one respect. Drake argues that he does not violate Euclid's or Archimedes' principles. Galileo brings a different sort of concern from that of Archimedes to his attempt to mathematise natural phenomena. Galileo was more interested in motion and change whereas Archimedes was interested in statics; Archimedes was more concerned with general, mathematical truth and Galileo with a different sort of knowledge of the physical and alterable. Where reality and certainty cannot overlap completely, Galileo prefers reality and Archimedes prefers certainty.

In Galileo's case there is a more practical, less contemplative inspiration. The social form of labour makes itself felt early on in his Two New Sciences. He is viewing processes and accomplishments carried out by those with less contemplative concern than Galileo himself.

> Salviati. Frequent experience of your famous arsenal, my Venetian friends, seems to me to open a large field to speculative minds for philosophizing, and particularly in that area which is called mechanics, inasmuch as every sort of instrument and machine is continually put in operation there. And among its great number of artisans there must be some who, through observations handed down by their predecessors as well as those which they

> attentively and continually make for themselves, are truly expert and whose reasoning is of the finest.
>
> Sagredo. You are quite right. And since I am by nature curious, I frequent the place for my own diversion and to watch the activity of those whom we call "key men" [Proti] by reason of a certain preeminence that they have over the rest of the workmen. Talking with them has helped me many times in the investigation of the reason for effects that are not only remarkable, but also abstruse, and almost unthinkable. Indeed, I have sometimes been thrown into confusion and have despaired of understanding how some things can happen that are shown to be true by my own eyes, things remote from any conception of mine. Nevertheless, what we were told a little while ago by that venerable workman is something commonly said and believed, despite which I hold it to be completely idle, as are many other things that come from the lips of persons of little learning, put forth, I believe, just to show they can say something concerning that which they don't understand. 97

Here we have a partial union of ancients and moderns, socially and intellectually. Galileo was both well-versed in Aristotle and was, at the same time, concerned to explain nature in a way connected with productive activity. Galileo's interest in the labour-process is evidenced a few pages later where he discusses the resistance of human bones to different forces.[98] These conditions are precisely the conditions under which human knowledge grows for Galileo. Olschki argues that this demonstrates Galileo's embeddedness in a context of goal-directed labour.[99] Peripatetics and artisans thus join forces in the person of such a scientific virtuoso as Galileo.

Owing partly to this new and growing context Galileo is able to eradicate some older distinctions. The distinction between natural and violent motion is removed when the analogy to human labour is applied to account for a good deal of what passes in many realms of nature. The "levelling" quality of the new context also homogenizes the objects to which mathematics is applied so that proportion theory applies now to qualitatively similar bodies. As early as 1584-90 Galileo adumbrates these developments in his early notebooks,

> since comparisons are properly made between things of the same species and not between things of different kind, it also results that we can properly compare such qualities among themselves in activity and resistance only by comparing the activity of one with the activity of the other, and similarly the resistance of one with that of the other, for this alone is comparison in the proper sense. In an improper way, however, we can compare activity with resistance even though they are of a different kind, and this in two ways: first, absolutely; second, not absolutely but in its kind, by seeing which of them is closer to the highest and most perfect of its kind. For example, the heavens can be compared with a fly even though they are of a different kind, in two ways: first, absolutely, by inquiring which of these is more perfect, and in this way it is certain that the fly, being an animal, is more perfect absolutely than the inanimate heavens; second, not absolutely, and in this way the heavens are more perfect than the fly, because in the genus of simple body, the heavens are closer to the highest degree of perfection since they are the most perfect simple body; on the other hand the fly, in the genus of animal, is farthest away from the highest perfection of animal. So in either way we can also improperly compare resistance with the activity of a quality; and each of these two ways can further be divided into two additional ways. 100

Although there are here Aristotelian vestiges which are not present in Two New Sciences, we can see that even early on Galileo was concerned with the technical and philosophic means of homogenizing and mathematizing nature. After displaying its application to uniting heaven and earth in a defense of Copernicus, Galileo's method reaches its peak in Two New Sciences. This again is our problem of the comparison of magnitudes of unlike kind. Although Galileo is very careful to stick to Archimedean and Euclidean principles, he does perform some operations which are close to those of Tartaglia, although there is no mention of the rule of three or its source in Euclid.

Stillman Drake, although he, of course, takes the opposite position to ours, has already done much of our work for us. On the third day of the discourse, Proposition IV, Theorem IV, Galileo is concerned to demonstrate that the ratio of spaces traversed by uniformly moving bodies in unequal times is a ratio compounded from the ratio of the times and the ratio of the speeds.

> If two moveables are carried in equable motion but at unequal speeds, the spaces run through by them in unequal times have the ratio compounded from the ratio of speeds and from the ratio of times. 101

Galileo sets up the solution as follows:

> Let two moveables E and F, be moved in equable motion, and let the ratio of the speed of moveable E be to the speed of moveable F as A is to B, while the ratio of the time in which E is moved, to the time in which F is moved, is as C is to D; I say that the space run through by E at speed A in time C has, to the space run through by F at speed B in time D, the ratio compounded from the ratio of speed A to speed B and from the ratio of time C to time D.

A -------------[speed]
[moveable] E
C --------- [time]

B ------------ [speed]
[moveable] F
D ---------- [time]

G --------------------------- [space]

I -------------------- [space derived from speeds and times]

L --------------- [space]

> Let G be the space run through by E at speed A in time C, and let G be to I as speed A is to speed B, and let I be to L as time C is to time D. It follows that I is the space through which F is moved in the same time as that in which E is moved through G, since spaces G and I are as speeds A and B. Since I is to L as time C is to time D, and I is the space that is traversed by moveable F in time C, then L will be the space traversed by F in time D with speed B. Hence the ratio of G to L is compounded from the ratios of G to I and of I to L, that is, from the ratio of speed A to speed B and of time C to time D; Therefore the proposition holds. 102

There are problems here with the use of Archimedian compound ratios and Drake is alerted to them. For Drake, "The Archimedean concept of compound ratios is essential here, as elsewhere in Galileo's application of mathematics to physics."[103] It appears to us that quantities of unlike kind are being compared in Galileo's illustration. The notion of speed already does this; if there were any experiments involved the actual numerical calculations would require it. In any case Drake is keenly aware of the potential difficulty in arguing that there is a pure or unmediated use of Archimedes here. In an earlier passage dealing with the resistance to fracture of prisms and cylinders of differing lengths and diameters Galileo also employs compound ratios. Drake adds the following note:

> This is probably the first expression of a strictly physical property in terms of two independent variables. Archimedes had used the compounding of ratios in a similar way, but only for mathematical relationships. 104

Drake directs us in this connection to Heath's discussion of Archimedes' terms in his translation of Archimedes' works. The term used in Greek for "compound ratio" is λόγος συνημμένος (logos synemmenos), literally "ratio tied together". Heath translates the Greek καί, "and", as "multiplied by".[105] Archimedes' term for multiply is given by Heath as πολλαπλασιάζω (pollaplasiazo); it seems reasonable ~~to~~ expect Archimedes to use this term if he meant "multiplied by". In ~~any~~ case, Archimedes did not apply compound ratios to physics.

Drake remains steadfast in his assertion that Galileo ~~rigorously~~ applies the Archimedean and Euclidean mathematical traditions. We can now write formulas for physical relationships whereas Galileo could only compare physical quantities "in pairs alike in kind".[106] Drake follows here the considerations we outlined in the beginning of this chapter and he is certainly correct in maintaining that Galileo was anxious to follow the Eudoxean definitions for the proportions of magnitudes.

To begin with, however, there seems to be some équivocation on Drake's part. He states that in compounding the ratios v_1/v_2 and t_1/t_2 into v_1t_1/v_2t_2 , Galileo is careful not to perform an algebraic operation on them such as would result in the terms v_1/t_2 and v_2/t_1. What is "v" if not the comparison of dissimilar quantities? Drake mentions that Galileo "was able to express certain functional relationships without assuming or implying that any particular distance was the product of a particular speed and a particular time".[107] Although Galileo might have had other reasons for avoiding explicitly algebraic calculation, he was, nonetheless, interested in expressing "functional relationships". For Drake, Galileo avoided algebraic calculation because for him it would have implied "bad metaphysics".

Although we cannot tie Galileo directly with commercial endeavour, Ludovico Geymonat has Galileo's father as both a musician and a merchant. His teachers had also been pupils of Tartaglia.[108] "(I)t was by reason of economic problems that Vincenzio was constrained to take up trade in addition to music".[109]

From the consideration of the alteration of elements to an interest in and inspiration from practical problems to the mathematisation of nature, a new philosophy emerges, a philosophy which, if not strictly mechanical itself, helps to usher in the mechanical philosophy.

> Philosophy is written in the grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed.

> It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these one wanders around in a dark labyrinth. 110

This leads, in the end, to a philosophy which is pretty clearly mechanical. In discussing an account of our senses, Galileo states referring to the taste on the tongue:

> here the tiny particles are received, and mixing with and penetrating its moisture, they give rise to tastes, which are sweet or unsavory according to the various shapes, numbers, and speeds of the particles. And those minute particles which rise up may enter by our nostrils and strike upon some small protuberances which are the instruments of smelling; here likewise their touch and passage is received to our like and dislike according as they have this or that shape, are fast or slow, and are numerous or few. 111

Francois Vieta (d. 1603)

The process of homogenization of the world was also carried out in mathematics and Francois Vieta is, for Jacob Klein, one of the keys in the development of abstract mathematical objects and hence of symbolic algebra. Although he had no known connections with commerce, he was associated with the law, that realm governing and reflecting the social relations of the day. His father, Etienne, was an attorney and notary and Francois received his bachelor's degree in 1560 from the University of Poitiers. In 1564 he entered the service of Catherine of Parthenay and became her tutor and life-long friend. Catherine was an ardent Huguenot and married René of Rohan, a member of a prominent Breton family, her first husband having been killed at the Massacre of St. Bartholomew. One of their children, Henri, became a famous leader of the Huguenots.[112] After a brief three year stay in Paris he became counselor to the parliament of Brittany at Rennes for six years and for the next four years maître de requetes at Paris and royal privy counselor.

For the next five years political enemies had him banished from the court but he was called back by Henry III when he moved the parliament to Paris. He was intermittently in Henry IV's service being finally dismissed by him in 1602, the year before his death.[113]

This is most significant for his algebra because there is a contentious issue about whether Vieta got his term "specious arithmetic" from Diophantus or from legal usage. If from legal usage, it helps greatly to make our argument about abstraction and homogeneity being based on social analysis because it is precisely the calculation by species rather than numbers which marks the distinctive turn to modern mathematics in terms of the ontological status of mathematical objects and may help to connect with "actual" history Klein's concerns about commensurate developments in "intentional" history. We argue here, with Klein, that although Vieta claims to be a renovator, he is every bit as much an innovator.

Although he claims to be renovating an ancient art, Vieta himself also has a sense of his innovative talents, especially in terms of developing a new language. This comes out in the dedication of his Isogoge to Catherine of Parthenay.

> O princess most to be revered, these things which are new are wont in the beginning to be set forth rudely and formlessly and must then be polished and perfected in succeeding centuries. Behold, the art which I present is new, but in truth so old, so spoiled and defiled by the barbarians, that I considered it necessary, in order to introduce an entirely new form into it, to think out and publish a new vocabulary, having gotten rid of all its pseudo-technical terms (pseudo-categorimatis) lest it should retain its filth and continue to stink in the old way, but since till now ears have been little accustomed to them, it will be hardly avoidable that many will be offended and frightened away at the very threshold. And yet underneath the Algebra or Almucababa which they lauded and called "the great art", all Mathematicians recognized that incomparable gold lay hidden, though they used to find very little. There were those who vowed hecatombs and made sacrifices to the Muses and Apollo if anyone

> would solve some one problem or other of the order of such problems as we solve freely by the score, since our art is the surest finder of all things mathematical. 114

We are now thus dealing with an art; the intent is now instrumental calculation rather than concern with the status of the objects. Even though this is an art, Vieta cautions us that neither rhetoric nor the "pleadings of lawyers" are of any use, for the gold to be found is in nature. "(F)or I am not one to fight against nature."[115] To this end Vieta divides his analytic art into three parts, zetetic, poristic and a third called rhetic or exegetic. The zetetic finds an equation or proportion between the magnitude sought and those given, the poristic investigates the truth of a theorem from the equation or proportion thus set up and in the exegetic the magnitude sought is produced from the equation or proportion. These three constitute the "science of right finding in mathematics."[116] The zetetic art, furthermore, is applied indifferently to numbers and magnitudes although Vieta is still concerned to obey the law of homogeneity, a principle no longer adhered to immediately afterwards in Harriot and Descartes.

> In the zetetic art, however, the form of proceeding is peculiar to the art itself, inasmuch as the zetetic art does not employ its logic on numbers - which was the tediousness of the ancient analysts - but uses its logic through a logistic which in a new way has to do with species. This logistic is much more successful and powerful than the numerical one for comparing magnitudes with one another in equations, once the law of homogeneity has been established; and hence there has been set up for that purpose a series or ladder, hallowed by custom, of magnitudes ascending or descending by their own nature from genus to genus; by which ladder the degrees and genera of magnitudes in equations may be designated and distinguished.117

After this follows a short collection of common notions and propositions from Euclid. Thereafter comes a section outlining the rungs (powers) of magnitudes in order that the law of homogeneity may be observed. The problem he deals with here is basically that of determining what must be done with a magnitude of one rung to make it commensurable with a magnitude of another. This is relevant primarily for addition and subtraction, which operations may only take place between magnitudes of the same order. Magnitudes may be multiplied or divided but the resulting product is of a different genus from the initial magnitudes and when added to another magnitude must be of the same genus.

Vieta next describes the laws of zetetics. If it is a length, plane or solid which is sought, the unknown will be represented by a side, square or cube respectively. In accordance with the conditions dictated by the problem, the given magnitudes and the unknown will be compared "by adding, subtracting, multiplying, and dividing, the constant law of homogeneity being everywhere observed."[118] Ultimately, something will be found which is equal to the magnitude sought. An equation may be formed as long as the powers on both sides are the same (same genus). The unknowns are represented by vowels and the givens by consonants. In other words if A is a cube it might be equated with a square multiplied by a length C. In Vieta's terms this would be A cube in C.[119] In modern notation this would be $x^3=ax^2$.

Three propositions then follow which are familiar to us as typical manipulations of equations. Vieta asserts that equations are not changed by antithesis, hypobibasm or parabolism which are changing sides of terms by a change of sign, dividing both sides by the unknown and dividing both sides by a given quantity respectively.

He next discusses poristic which for him is the possible finding of a new theorem by retracing in reverse order the steps of the initial analysis. Then, in his discussion of the rhetic art, Vieta makes clear his disregard of the number - magnitude distinction through his use of the new term "species", and is worth quoting at length here.

> When the equation of the magnitude which is being sought has been set in order, the rhetic or exegetic ... art, which is to be considered the remaining part of the analytical art and as one which pertains principally to the application of the art (since the two others are concerned more with general patterns than with precepts, as one must by right concede to the logicians), performs its function both in regard to numbers if the problem concerns a magnitude that is to be expressed by numbers, and in regard to lengths, surfaces, and solids if it is necessary to show the magnitude itself. And, in the latter case, the analyst appears as a geometer by actually carrying out the work in imitation of the like analytical solution; in the former case, he appears as a logistician by resolving whatever powers have been presented numerically, whether simple powers or conjoined. Whether it be in arithmetic or geometry, he produces some specimens of his own [analytic] art according to the conditions of the equation that has been found or of the proposition that has been found in an orderly way from it. 120

The "reckoning by species" thus sets up equations yielding much more formal and general solutions than those yielding concrete individual answers to concrete, individual questions. We are well on the way to Wallis' general theory of ratios and to Klein's interpretation that this depends on a symbolic reinterpretation of the concept of arithmos. Thus Vieta says:

> Therefore analysis, whether with a view to arithmetic or to geometry, discloses the mystery, known hitherto by no one, of the division of angles, and it teaches how:
>
> When the ratio of the angles is given, to find the ratio of the sides.

To make an angle to be in the same ratio to an angle that a number is to a number. 121

Vieta makes the final claim for his art that it "appropriates to itself" the ultimate problem—"TO LEAVE NO PROBLEM UNSOLVED."[122] It is our argument here that this abstracting, homogenising and universalising tendency takes place in a social context in late sixteenth century France where commoditisation and the proliferation of markets were taking place along with the struggles attendant to that development and the position of a jurist attempting to make sense of it all. It can be argued that at least three of Vieta's terms are of legal origin. The most important of these terms is "species". John Wallis, in his A Treatise of Algebra (1685), argues that Vieta's use of this term derives from a common legal usage whereby the names for types of cases would be represented by person's names to indicate any person in such circumstances and even on occasion simply by letters.

> Now with respect hereunto, Vieta (accustomed to the language of the Civil Law) did give, I suppose, the name of Species to the letters A, B, C etc. made use of by him to represent indefinitely any Number or Quantity, so circumstanced as the equation required. And accordingly, the accomodation of Arithmetical Operations to Numbers or other Quantities thus designated by Symbols or Species, was called Arithmetica Speciosa or Specious Arithmetic, the word Species signifying what we otherwise call Notes, Marks, Symbols or Characters made use of for the compendious expressing or designation of Numbers or other Quantities. 123

Thus the language of contracts, conditions and stipulations finds its way as a universalising tendency through a jurist into early modern mathematics. What was unwieldy, tedious or lacking in generality in the ancients, is for Vieta by means of these judicial terms turned into something far more universal and formal. In rendering Euclid's and Diophantus' terms for the conditions of possibility of a problem and its solution Vieta also uses judicial terms where Euclid "stipulates" conditions for a proportion, by "symbolism". "Vieta here understands a 'contractual stipulation' which corresponds to the judicial concept of the Greek symbolon".[124] Where Diophantus calls the condition of possibility a prosdiorismos Vieta uses the judicial term cautio (security).[125] Whereas Xylander translates Diophantus' eide as "species", J. Winfree Smith concludes that

> It may be, of course, that the word "species" as used by Vieta is meant to contain something of the meaning of the Diophantine eide and also something of the judicial meaning. 126

Vieta's reinterpretation of Diophantus in this light thus shows the laying of foundations for operations which immediately afterward in Harriot and Descartes no longer needed to be outlined. For Klein the symbolic techniques at that point become "opaque"; for Vieta, the hidden has become something obvious.

> Diophantus in those books which concern arithmetic employed zetetics most subtly of all. But he presented it as if established by means of numbers and not also by species (which, nevertheless, he used), in order that his subtlety and skill might be the more admired; inasmuch as those things that seem more subtly and more hidden to him who uses the reckoning by numbers (logistice numerosa) are quite common and immediately obvious to him who uses the reckoning by species (logistice speciosa). 127

René Descartes (1596 - 1650)

As regards science, the thought of Rene Descartes is a formulation of the world which allows corporeality to be understood as extension and thus as calculable in accordance with his mathesis universalis. This universal mathematics has its ground not only in a concern with nature, per se, but in a desire to systematically formulate human social life.

He wants to interpret contingency optimistically and to this end constructs a rationalistic system, able to comprehend nature and thus also to provide a ground for human conduct in a world where outer event is beyond human control. According to Borkenau this leads to a conception of motion, for example, where commodity relations and the concomitant conception of impetus play a role. The bottom line here proves to be the good of God and equivalence in exchange.

> Descartes discovers a law of the constancy of motion and grounds it in the good of God. The special completeness of this law of constancy, which he claimed as self-evident without grounding it, lies in truth in the exchange of equivalents in the transfer of motion from one one body to another. Bourgeois exchange equivalence thus proves to be the basic category of nature. [128]

The development of a national economy and provision for a provisory morality of obeying the laws of the fatherland go hand in hand. Neither of these developed in Descartes' France; he spent the second half of his life in Holland to avoid the sort of persecution to which Bruno and Galileo were subject. Colbert and the bourgeoisie suffered a telling blow with the Edict of Nantes. In non-Anglo-Saxon countries capitalism became the preserve of a small section of the nation. The struggles over these issues took a religious form. For Borkenau, the work of Descartes represents the first attempt to build a unified world-view from the categories determining the life of the individual within capitalism. [129] He took up anew the problem of grounding a new morality on a new natural science. The new universal science was to be enlightening and normative.

Commerce and gentry come together in Descartes' family. There are commercial bourgeoisie, parliamentarians and intellectuals in his ancestry.[130] His father was a member of the Bretagne parliament at Rennes with Vieta. The unity of nature and morality, which is to be grounded in this new universal science, orients itself to the requirements of practice both in moral theory and in science. The control of nature, both inner and outer, is at a premium. The purpose of his Principia is the control of nature and of the moral life. Throughout his works he attempts to reconstruct the world in terms about which we may be absolutely certain. He wants to apply proportions to everything but stops short of constructing blueprints for a new society. Unlike More, Bacon and Comte he never constructed a utopia. In fact he did not really suggest a new morality but rather wanted to provide grounds for one which was fast coming into existence. His was at most a contemplative perspective attempting to interpret fate optimistically. A new philosophy was needed for controlling natural fate.

The production of insight into the cause of fate, furthermore, is to be brought about by achieving the identity of an "I" reduced to pure thought with an outer world reduced to pure reason. We have knowledge, to begin with, only of ourselves. Those ideas which are clearly and distinctly perceived and require no induction for their verification are the building blocks and starting points in Descartes' method. The imagination then connects this naked intellect with reality by way of "seal impressions". Mind and body are connected by the imagination to allow knowledge of reality. Impressions are connected in sequences and mathematics is that which moulds the intelligence enabling it to perceive connected sequences. The point here is that out of these motivations came a universal science whose general object is identified with the substance of the world; corporeality is defined as extension. For this purpose Descartes developed a symbolism of figures which he later restricted to lines.

> For Descartes this conception of a figure-symbolism connects two different trains of thought: (1) the conception of algebra as a 'general' theory of proportions whose object, only symbolically comprehensible, acquires its specific characteristics from the numerical realm, and (2) the identification of this 'symbolic' mathematical object with the object of the 'true physics.' 131

Although Descartes' mathematics are not practical in the direct sense of enabling us to build a better bridge or mouse trap, they most certainly are motivated by everyday concern. They do so, furthermore, in a way which reveals the levelling, homogenising tendencies occurring in a society well on its way to being predominantly a commodity producing, market society. In one of his earlier works Descartes compares and equates activities which most, for other practical reasons having to do with the division of labour and specialisation of tasks, would not consider commensurable. Although various operations must be divided up amongst different functionaries, various processes and objects may be viewed theoretically by the same science.

> Whenever men notice some similarity between two things, they are wont to ascribe to each, even in those respects in which the two differ, what they have found to be true of the other. Thus they erroneously compare the sciences, which entirely consist in the cognitive exercise of the mind, with the arts, which depend upon an exercise and disposition of the body. They see that not all the arts can be acquired by the same man, but that he who restricts himself to one, most readily becomes the best executant, for it is not so easy for the same hand to adapt itself both to agricultural operations and to harp-playing, or to the

> performance of several such tasks as to one alone. But this is certainly wrong. For since the sciences taken all together are identical with human wisdom, which always remains one and the same, however applied to different subjects, and suffers no more differentiation proceeding from them than the light of the sun experiences from the variety of the things which it illumines, there is no need at all for minds to be confined within limits, for neither does the knowing of one truth have an effect like that of the acquisition of one art and prevent us from finding out another, it rather aids us to do so. 132

Here in one passage we have reference to the division of labour and specialisation of tasks and the creation of homogeneous, abstract objects, all formulated in a way which firmly distinguishes between intellectual and manual labour. This is no longer the science of genus and difference but one which deals with a pretty homogeneous universe, comprehensible by one human wisdom, by human beings now in possession of formal equality since "Good sense or Reason, is by nature equal in all men."[133] That which distinguishes us from brutes is found complete in each individual,[134] but depends on being guided by means of reason alone.[135] That his notion of science has origins by analogy in a particular set of social relations and that his intention is that it have practical application back into that context is evident from the defense of his method in the <u>Discourse</u>.[136]

It is the particular set of social relations in which we find crafts and labours so organised which makes possible such a world-view in the first place. This enables, furthermore, the construction of a method which reduces the "I" to thought and the world to body. Body becomes graspable mathematically by the construction of an abstract, homogeneous mathematical object, general magnitude. An intellect operating by Descartes' method is capable of separating out these abstract objects, objects whose very existence is predicated on the proliferation in society of the commodity abstraction and the concomitant accounts of social relations and of the legal thought and activity designed to govern them. Objects acquire specific properties from the numerical realm but are seen as "representing" concretely existing things. There are two highly significant developments here: 1) the constitution of general magnitude, 2) the reference of this general magnitude to objects of the real world and therefore its significance for physics.

The final goal in this endeavour, however, is a science of human, social relations, a moral science.

> Thus philosophy as a whole is like a tree whose roots are metaphysics, whose trunk is physics, and whose branches, which issue from this trunk, are all the other sciences. These reduce themselves to three principal ones, viz. medicine, mechanics and morals - I mean the highest and most perfect moral science which, presupposing a complete knowledge of the other sciences, is the last degree of wisdom. 137

This last degree of wisdom, we are arguing, has its roots not only in metaphysics but in forms of human, social relations which are already in existence and fast coming to dominance - exchange relations. Its roots are in social analogies and influences. An analogy having its roots in social relations is used to reinterpret nature mechanistically. This interpretation of nature is then proposed as a basis for understanding and guiding social relations. Since the morality proposed by Descartes is only a provisory one and involves no real changes in conduct, the goal of his philosophy turns out, as Borkenau has argued, to be the optimistic interpretation of fate, of outer event. From Aquinas to Cusa to Descartes, we view the change from an understanding of nature in human terms, to an understanding of humans in natural terms. The latter understanding is, however, also an understanding which has the social realm as its root.

The cornerstone of this attempt on the part of Descartes is his development of general magnitude and it is here that the social realm makes itself felt most directly. In the Discourse Descartes mentions his pleasure in finding mathematics to be a suitable foundation for all knowledge because of the certainty and self-evidence of its proofs.[138] Interestingly enough, in the Discourse, in order to justify his move away from logic and letters and toward mathematics, Descartes spends a few pages describing what amounts to a curious relationship between intellectual and manual labour. He describes having found the study of letters to be empty and useless and leaves it in favour of travelling in order to discover how people reason in "matters that specially concern" them.[139] Although he was much more taken with this in some respects, he found the diversity in manners disconcerting and so resolved to study himself in isolation.

He wants to argue here on behalf of a greater unity and consistency in human experience and judgement than existed at the time and justifies this on the grounds that a cathedral conceived by one man will be more beautiful than one which has a multitude of architects. This conclusion bears the mark of a world whose distinctions have been considerably levelled and in which universally applicable laws are desired. Descartes would like to remove the spontaneous and thus haphazard character of such historical developments.

Descartes now lives in a world where questions of more or less occur "only in the sphere of the accidents and [do] not affect the forms or notions of the individuals in the same species."[140] It is thus, however, a homogenised and abstract world, a world where questions of more or less provide Descartes with a method of grounding philosophical truth. He now lives in a world where mathematics can provide the basis for loftier edifices than mechanical arts. He thus chooses to work with logic, analysis of the ancients and algebra of the moderns in a way which simplifies them and puts them on firmer foundations, just as a state is better ruled when having a few, strictly observed laws.[141] To this end all objects are reduced to lines in order to view the proportions obtaining between them. This involves "borrow(ing) all that is best in Geometrical Analysis and Algebra and correct(ing) the errors of the one by the other".[142] This method, furthermore, is to assure us of correct reasoning in all things. This certainty, in turn, makes us masters and possessors of nature.

Those things in nature about which we can be certain are, in effect, primary qualities because they may be clearly and distinctly perceived, are magnitude or extension in length, breadth or depth, figure, movement, substance, duration and number.[143] To all of this, Descartes first establishes his existence as thinking substance. Although we have no specific quarrel with this, his grounding of each quality as clearly and distinctly perceived is nowhere in his work very clear or distinct. The whole edifice in the end is grounded on the completeness of God. Other ideas may be contained in him eminently and "diligent attention" points the way to God's existence. The knowledge of God's existence, furthermore, allows the acquisition of knowledge of corporeal things inasmuch as they are the objects of pure mathematics.[144] Since the clarity and distinction of the perception of these qualities is never really grounded, however, (in fact Descartes evokes them in the first instance and only then proceeds to wonder how they may be seen as clearly and distinctly perceived), we take their origin to lie in some source other than the completeness of God or naked intellect.

The reduction of the world to such a substrate is, rather, the result of social processes, division of labour and exchange. Borkenau cites Adam to the effect that Descartes' family displayed an example of the growing together of mercantile and intellectual classes into the parliamentary bourgeoisie, that he was a member of the gentry whose capital for the purchase of offices came from the maternal grandmother, and his family thus were members of the strongest of the bourgeois strata. His family thus also, however, viewed his studies as meaningless, failing to realise that his thought constituted a justification of their interests.[145]

The path toward such studies represented also a reaction against medieval values in the form of a rejection of Jesuit education. The medieval world view did not suffice in a world without harmoniously divided estates and with this went also its view of nature. The Molinism encountered by Descartes amongst the Jesuits was grounded in the theory of estates.[146] The unity of nature and morality had to be established anew. Physics, metaphysics, geometry and morality are all interdependent in Descartes' system. A natural-moral life (under the new conditions) is only possible with a certain measure of control of nature."[147] This is the meaning of universal science for Descartes. As alluded to earlier, however, there is no revolutionary intent on Descartes' part regarding a new social-moral order. The key here is adaptation to an alien world. "Therein consists precisely the reification of social processes."[148] The attitude here is a contemplative one.

Laws of nature, furthermore, are seen by Descartes as regular and universal owing to the constancy, perfection and unchangability of God. A world thus conceived is not simply thereby capable of fitting with a happy, harmonious human life. The use of this conception lies in its intellectual penetrability, which Descartes procures for us by stating and attempting to ground its quantifiability and the constancy of motion. The world simply must have an abstractable substrate and the ground for this is the constancy of motion. There are echoes here of Wolff's treatment of the notion of impetus in Oresme, Olivi and du Bois.

Borkenau claims that there is here an equivocation on Descartes' part. Two different elements are artificially forced together in the concept of the constancy of motion which Descartes himself takes to be quite simple. There is a formal equality built upon the notion of constancy which is unrelated to the context of norms.

> In this unchangeability of eternal norms in nature there is reflected the basic general acceptability of the modern maxim of law which suffers no individual exception in opposition to the law of the middle ages which is based throughout on individual privileges. 149

Borkenau, citing a letter to Mersenne, states that Descartes is aware of the analogy between the two realms but does not realise that he is interpreting nature according to social analogy. The calculability of the process of law became a requirement. The calculability of law, exchange equivalence, the breakdown of labour to its simplest movements became basic categories of nature and are manifest in Descartes' notion of natural regularity. The general applicability of law under capitalism as opposed to feudalism, appears to be a crucial inspiration in Descartes' work.[150]

Borkenau, however, finds it difficult to see how the application of proportion could solve these difficulties for Descartes and embarks on a philosophical discussion of the meaning of the "enthusiasm" which Descartes felt for this application. In our view this excursus and diversion on Borkenau's part was totally unnecessary.

In the theory of proportions, more accurately, in geometry and algebra, Descartes finds an expression of clearly and distinctly perceived rules and objects which correspond to a world thus homogenised. Although the substrate thus apprehended is not what is directly perceived by the senses, it is that which is most real. The rules of mathematics and the method of radical doubt, and clear and distinct perception, thus collapse into one for Descartes. "Arithmetic and Geometry alone are free from any taint of falsity or uncertainty."[151]

We also see this method as new, for according to him the ancients grudged the secret to posterity.

> At the present day also, there flourishes a certain kind of Arithmetic, called Algebra, which designs to effect, when dealing with numbers, what the ancients achieved in the matter of figures. 152

The ancient world only had traces of this science in Pappus and Diophantus but they suppressed it by a "low cunning, deplorable indeed."[153]

Descartes hoped to simplify and universalise this method and to include in it astronomy, music, optics, mechanics and several other sciences. Already in the Rules ... Descartes conceived mechanistically of the relation of primary quality to sense impression and measured primary quality in terms of general magnitude.

> It is exceedingly helpful to conceive all these matters thus, for nothing falls more readily under sense than figure, which can be touched and seen. Moreover, that nothing false issues from this supposition more than from any other, is proved by the fact that the concept of figure is so common and simple that it is involved in every object of sense. Thus whatever you suppose colour to be, you cannot deny that it is extended and in consequence possessed of figure. Is there then any disadvantage, if, while taking care not to admit any new entity uselessly, or rashly to imagine that it exists, and not denying indeed the beliefs of others concerning colour, but merely abstracting from every other feature except that it possesses the nature of figure, we conceive the diversity existing between white, blue and red, etc., as being like the difference between the following similar figures? The same argument applies to all cases; for it is certain that
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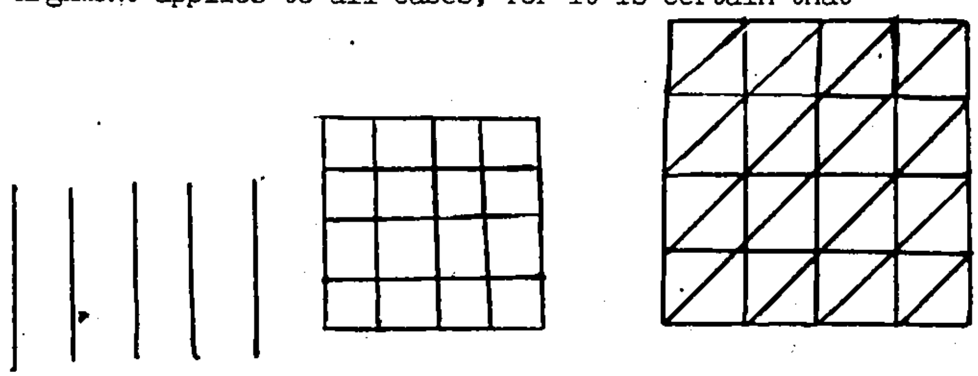
> 
>
> the infinitude of figures suffices to express all the differences in sensible things. 154

We are to compare things in terms of the common element in them; we are to construct ratios and transform them into equations. Once the terms are compared so that a uniformity can be said to exist between what is sought and what is known,

Next we must mark that nothing can be reduced to this uniformity,

> save that which admits of a greater and a less, and that all such matter is included under the term magnitude. Consequently when, in comformity with the previous rule, we have freed the terms of the problem from any reference to a particular subject, we shall discover that all we have left to deal with consists of magnitude in general. 155

This is grounded metaphysically in the law of constancy of motion as stated in the Principia.[156]

In this way we may apply number to continuous magnitudes. Anything measurable is countable.[157] Mathematics deals solely with dimension but "It falls rather to Physics to inquire whether they are founded on anything real."[158] But figure is here identified with extension and extension with the extended body itself.[159] While Descartes states this, Klein points out that this problem of body-mind mediation is the insoluble problem of the Cartesian doctrine.[160] The treatment given "Arithmos" by Vieta is now carried out by Descartes on geometric objects and related inherently to the physical world.

The mathematical objects are abstracted. They become an independent being, even if only in thought. They are in scholastic language second intentions.

> When now - and this is of crucial importance - the ens rationis as a"second intention" is grasped with the aid of the imagination in such a way that the intellect can, in turn, take it up as an object in the mode of a "first intention", we are dealing with a symbol, either with an "algebraic" letter-sign or with a "geometric" figure as understood by Descartes. This is the sense in which we spoke earlier of "symbol-generating abstraction". 161

Descartes states in Rule XIV that what is true of general magnitude is also true of each particular instance. The imagination itself becomes a real body, possessing extension and figure.[162]

A socially abstracted world where exchange, markets, universally applicable laws, extended division of labour and specialisation of tasks are coming to the fore is thus reflected in a conception of nature in which things reduced to primary qualities and the mathematically conceived relations between them may be said to account for all that is sensually intuitable, for all that is touched, tasted, smelled, heard or seen may be accounted for in this manner,[163] all phenomena are explicable by mathematics,[164] and the variety or forms of matter depends on motion.[165]

Simon Stevin (1548 - 1620)

Stevin was a polymath who wrote treatises on numerous topics, amongst them bookkeeping, finance, mechanics, mathematics, and astronomy. Some of these works were written as textbooks for Prince Maurice of Nassau to whom he was tutor and who made Stevin quartermaster - general, a position which he held until his death in 1620. The author of the biographical sketch of Stevin in the Dictionary of Scientific Biography, M. G. J. Minnaert, asserts that his work is part of a revival of science in the sixteenth century owing to commercial and industrial prosperity in the cities of the Netherlands and northern Italy.[166]

Rather little, unfortunately, is known of his early development. Struik has him simply "trained as a bookkeeper";[167] Dijksterhuis mentions his position in the financial administration of the city of Bruges and concludes from a casual remark in one of Stevin's works that he had earlier been a bookkeeper and cashier in Antwerp.[168]

DeWaal, however, suggests as well that he was himself involved in commerce.[169]

At any rate, Stevin's early experience involved many years in various kinds of commerce and finance. His first published work, in 1582, was Tables of Interest. This work gave rise to problems having to do with calculations using fractions and in De Thiende (Dime) Stevin introduces his innovations for calculation with decimal fractions. Problems uncovered here alerted Stevin to theoretical problems having to do with the concept of number which he analysed in his L'Arithmetique. Here he challenged the ancient number concept on the grounds of the divisibility of the unit of calculation, the ontological separation of number and magnitude and the discontinuous character of number.

Stevin wrote his Tables of Interest for use by those with "only a little experience in the rule of proportions (which some call the rule of three)." He hoped by this means they would "be able to solve offhand any question of interest that may commonly occur in practice."[170] In the initial definitions in this work we get a clue as to Stevin's later theoretical concern with certainty and commensurability. After defining principal, interest and time he continues:

> Principal, interest and time are three inseparable things, i.e. Principal does not exist unless in respect to a certain interest, and interest does not exist unless in respect of a certain Principal and time.[171]

The rate of interest is here seen as a ratio.[172] To avoid cumbersome fractions in the construction of the tables Stevin takes their root to be 10,000,000.[173] To address this problem Stevin then wrote De Thiende. In his preface he mentions those for whom the book is intended.

> To astronomers, Land-meters, Measurers of Tapestry, Gaugers, Stereometers in general, Money-Masters and to all Merchants, SIMON STEVIN wishes health.[174]

Also in the preface, he finds it necessary to apologize that in this short work there is no "conversion of proportion." The matter of The Dime is in fact preparatory to a reconsideration of such problems. He goes on to suggest that "the matter of this Dime is number."[175] In his Tables of Interest the sample problems involved common fractions. His purpose in The Dime was to invent a method of calculation without fractions or "broken numbers."[176] Although Stevin's notation is inferior to Napier's, it is not in the area of notation per se where Stevin made his real contribution but rather in the area of a conceptual shift. His notion of decimal fractions is based on the decimal character of the number system. "It was Stevin who understood the central role of the number concept and the necessity of treating 1 as a number before a true theoretical grounding could be given for fractions."[177]

In Stevin's time and immediately afterward, analysis became standard procedure in mathematics in the work of Vieta, Descartes and Fermat. This analysis required a broadened number concept.

In order to proceed with

> 'the assumption of what is sought as though it were granted' (as Theon of Alexandria described as the key step in analysis) and then drawing conclusions that lead to an already accepted truth (Klein, 154-5), one first needs to recognize a generic

> relationship among all possible solutions ~~to~~ certain kinds of mathematical problems in order to make the initial assumption. In other words, one must ignore the fact that some solutions may be numbers and others magnitudes, some positive and others negative, rational and irrational, etc. In the post Renaissance period, this approach to problem solving occurred and we are of the opinion that Stevin's new number concept contributed to its wide-spread use.[178]

Hence Stevin's awareness of and need to account for the continuity of number and the status of unity as number. To calculate with fractional parts of the unit, we need to establish that unity is number, and indeed, in his L'Arithmetique Stevin writes in block letters "Que L'Unité Est Nombre."[179]

To justify this he provides the following argument:

> The part is the same matter that is its whole,
> Unity is a part of the multitude of units
> Therefore unity is the same matter as the multitude of units;
> But the matter of the multitude of units is number,
> Therefore the matter of unity is number.[180]

To deny this, claims Stevin, would be to deny that a piece of bread is bread. Jones points out that, philosophically, the analogies of Stevin break down. To this end Jones concludes his work with a discussion of the work of a near contemporary critique of Stevin from a philosophical point of view by Jacques-Alexandre Le Tenneur.[181]

Thus Stevin has constructed a new number concept which works in practice, provides a methodology for mathematics, but the justification for which is frought with philosophical difficulties. Stevin goes on, in addition to his bread analogy, to contend that number is in magnitude as wetness is in water. He also writes in block letters that number is not discontinuous[182] and there are no absurd, irrational, incommensurable or surd numbers.[183]

In the work of Stevin, then, a commercially inspired concern with the number concept allows the development of a mathematical methodology which makes irrelevant the ontological separation of number and magnitude without being able to really justify this philosophically. We have as well, in his mechanics, the appearance of quantities of unlike kind in the same calculation.[184] His thought begins and ends with everyday life. He writes in The Dime:

> We should like to give examples in all the common rules of Arithmetic occurring often in man's actions, such as the rule of society, of interest, of exchange, etc. showing how they can all be expedited by integer numbers, as well as by easy use of counters; but we shall leave it at that because it is clear from the preceeding.[185]

Although Stevin's early bookkeeping works are fairly inaccessible and are in Dutch, several commentators have reproduced parts of his sample accounts and provided explanations of his procedures as well as indicating his original contributions. The procedures, we are arguing, already employ notions of general magnitude and a broadened number concept which are carried over into theoretical mathematics. Stevin, as Jones argues, is very conscious of the need to broaden the number concept for theoretical mathematics. Commentators suggest that Stevin's innovations in accounts are quite modern, having been followed in their essentials for nearly 300 years. Littleton suggests that the differences between Stevin and his predecessors were more marked than those between him and his successors for that period. After giving a sample ledger account, Littleton states,

> Here for the first time the account takes on a modern

> arrangement - to have less the appearance of narrative paragraphs and more of the effect of tabulations.[186]

Geijsbeek provides a most interesting discussion of the possible entailment of bookkeeping and algebra in Stevin's bookkeeping writings. After referring to an explanation by Stevin to the Prince of a particular bookkeeping procedure and of its preferability in government accounts Geijsbeek adds:

> The prince then asks if bookkeeping ever had been worthy of such consideration that books were published on it. Stevin replied that numerous writers had taken up the subject, and that while doubtless the double-entry system was originated in olden times, yet in Italy where it is said to have been executed first, it is considered an art of which no other is so honourable and worthy.
>
> The prince (apparently floored by Stevin's lucid arguments) thereupon agrees to take up the study with the view of installing double-entry bookkeeping in governmental departments as soon as Stevin and he were through with their studies of algebra.[187]

Geijsbeek continues to suggest that in the theoretical chapter of his bookkeeping work Stevin eliminates the proprietors' account by use of an algebraic formula.[188] That Stevin's bookkeeping, in turn, reflects social and legal developments is also clear. "His fundamental principle for bookkeeping refers to the beginning and end of 'property'."[189] The property of the commodity ends when the property of the receivable begins. This involves the notion of a rather strict formal equality between the participants in a transaction. It is this, as well, we are arguing, which allows the development of a more abstract concept of number. In the fourteenth century, however, this would have been much less possible. In a fourteenth century South German account book, written in paragraph form, "customers are classified according to social rank and grouped in different sections: noblemen (the first thirty folios), clergy (the next six), and burghers (the remaining twelve)."[190]

The History of Trades and the Establishment of the Royal Society

Stevin's influence extended beyond Holland. His De Thiende was translated by Robert Norton, himself the author of several mathematical works. Norton did much to popularise decimal fractions in his appendix to the 1615 edition of Robert Recorde's Arithmetick.[191] There are several references as well to Stevin's influence on bookkeeping through Richard Dafforne who referred to Stevin as "my good friend."[192]

English mathematical practitioners were generally receptive to Stevin's ideas. A contemporary of Norton's, Henry Lyte, wrote The Art of Tenths or Decimal Arithmetic which is an

> introduction to decimal computation which contains a summary of Stevin's treatment of fractions, followed by a series of exercises illustrating the value of the decimal principle for calculations in various spheres of commercial mathematics."[193]

This whole development is predicated on the development of commerce. Until the rise of commerce "scientific treatises addressed to advanced students contemplated the likelihood of their not being able to do simple division."[194] The English translator of Sacrobosco's De Arte Numerandi in the fifteenth century already reveals some confusion as well regarding the form and matter of number.

> Sothely .2. manere of nombres ben notifiede; Formalle, as nombre is vnitees gadred to-gedres; Materialle, as nombre is a colleccioun of vnitees.

Steele adds a footnote to this passage indicating that Sacrobosco's manuscript has the terms formal and material reversed.[195] Steele ends his edition of early arithmetics with examples of pricing and wages problems from the 1543 editon of Robert Recorde's <u>Arithmetic</u>.

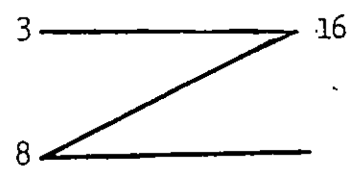
In the 1542 edition Recorde claims that for Aristotle no one ignorant of arithmetic can do any science and that for Plato geometry cannot stand without arithmetic. In the epistle dedicatory to Rycharde Whalley he states that his work is

> for all privat welaes of Lordes and all possessioners, all merchauntes and all other occupyers, and generally, for all estates of men besydes, auditeurs, treasorers, receivers, stewardes, baylyffes and such like whose offices without Arithmetick is nothynge.[196]

He defines "sence and Wytte" in terms of a knowledge of arithmetic. The work is written in the form of a dialogue with Master and Scholar as interlocutors. When the scholar proclaims his diligence and willingness to believe all, Recorde has the master declare that "That is to much and meete for no man, to be believed in all things, with out shewing of reason."[197] He continues to teach positional notation with Arabic numerals but includes also a section on calculation in pounds, shillings and pence since they are decimalised like the number system.

In a way he comes close, notationally, to decimal fractions by illustrating division by ten. He suggests, for example, to divide 3648 by 10 one should note the result thus: 364|8. He declines to get into this, however, declaring that 8 is the remainder which cannot be divided except by breaking it into fractions "wherewyth 1 wyl not medle yet."[198] In a section entitled Reduction, Recorde deals with money, the value of English coins, French coins, Flanders coins, weights, liquid measure, dry measure, time.

In a section on proportion, Recorde refers to the rule of three as the Golden Rule, "whose use is by the numbres knowen to fynde out an other unknowen."[199] Most of his examples here involve questions of pricing and expenses; for example, if three months board cost 16 shillings, how much do eight months cost? He employs the following scheme and notation for solving the problem:



> I muste multiply the lowermost on the left side, by that on the right syde, and the sum that amounteth, I must divide by the highest on the left syde.[200]

He mentions that the first and third quantities must be of the same denomination and likewise for the second and fourth. That these quantities which are multiplied together are not of the same demonination, however, does not seem to bother him at all. His work continues with questions of expenses, partnership shares and a section for merchants on the use of counters for calculating.

His First Principles of Geometry (1551) uses the analogy of a ship to demonstrate the usefulness of geometry. It is useful, he claims for everyone from the merchant who owns the ship to the makers of instruments and of the ship itself.[201] Here he adds that

Aristotle needed "proportion geometricall" to teach or execute moral philosophy.[202] In The Castle of Knowledge (1556) he advocates the use of the Golden Rule for calculating unequal hours. Since night and day were each divided into twelve hours, the varying lengths of night and day during the year demanded the calculation of the relative lengths of the hours.[203]

In The Whetstone of Witte (1557), after dedicating the work to the governor and councilors of the Muscovy Company, he mentions that his first work was "set forth for the Merchaundes trade."[204] He becomes more theoretical in this last work which contains an interesting juxtaposition of orthodox and modern notions. He introduces here the art of the Cossists, but insists still that unity is indivisible.[205] He mentions the part/parts distinction, that between absolute and contract numbers (contract numbers expressing the value of some definite quantity of something) and between whole and broken numbers. A proper fraction, he says, is less than unity but is not a number. One itself is also not a number.[206]

He defines numbers as commensurable when they have a common multiple, but in his discussion of "diametrialle" numbers, numbers whose square root approaches a whole number with "unspeakable nerenesse" may be considered diametralle. Squares with sides of 2, 5, 12, 29, 70, 169, and 408, eg. may be considered as having "rational" diagonals. In general, though, one cannot have a broken number as a root.[207] In a later section on cossic numbers he mentions that all the preceeding referred to numbers absolute. He next proposes to deal with 1) numbers contract or denominate, 2) numbers denominate vulgarly or cossically, 3) radical, irrational or surde.[208] Cossic signs, furthermore, can refer to both rational and irrational numbers.[209]

The signs he uses are φ, absolute number, Ze, root of any number, ȝ square number, ce, cubic number. This notation can be extended infinitely to include sursolids, squares of square cubes etc. The cossic sign, furthermore can stand for any number.[210] For the use of these signs he advocates the following formulation of Algebers rule:

> The somme of the rule of equation:
>
> When any question is propounded, apperteinyng to this rule, you shall imagin a name for the nomber, that is to bee soughte, as you remember, that you learned in the rule of false position. And with that nomber shall you procede, accordyng to the question, until you find a Cossike nomber, equall to that nomber, that the question expresseth, which you shall reduce ever more to the leaste nombers. And then divide the nomber of the lesser denomination, by the nomber of the greateste donomination, and the quotient doeth aunswere to the question. Except the greater denomination, doe bear the signe of some rooted nomber. For then must you extract the roote of that quotiente, accordyng to that signe of denomination.[211]

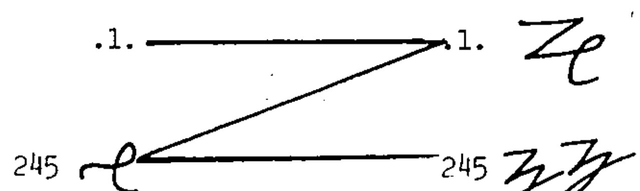
In answer to the scholar's question, Master suggests that this is superior to the rule of false position because here one takes a true number "before he knoweth resolutely, what he hath named."[212] He does not like the notion of "false" position anyway and suggests calling it "the rule of darke position, or of straunge position: but not of false position"; "we do commonly name that darke position .1.Ze."[213] Recorde then introduces the modern equals sign

214

His problems now become more abstract and he still uses a Golden Rule sort of calculation. He sets forth the following question:

> A Bricklaier had a pile of Brike, which he sold by the yarde. The lengthe of is was 7/2 to the bredthe, that is <u>Triplasesquialtera</u>. And the heighte was five tymes so moche as the lengthe. This pile the owner sold for .980. crounes. By such rate that he had for every yarde so many Crounes, as the pile had yardes in bredthe. Now is the question, what was the lengthe, bredthe, and heighte of this pile?[215]

The master poses this question to the scholar and the latter answers that he will let the bredth be 1 ℨ. The length would then be 3½ ℨ and the height 17½ ℨ. The number of yards then becomes $\frac{245}{4}$ ꝛℯ. Since each yard cost 1 ℨ of crowns, then the question becomes "If .1. yarde coste .1. ℨ of Crounes, what shall $\frac{245}{4}$ ꝛℯ coste."[216] Applying the "Golden Rule" the equation is



Since 980 crowns equal this figure, i.e., the total number of yards, 245 ʒʒ ═══ 3920 ʓ. Thus 2 is the breadth, the length 7 and the height 35 yards (a rather unlikely pile of bricks in sixteenth century England). There are echoes here of the "Find me a number" problems which Van Egmond found amongst the abacists. Although Recorde was a physician, he, like the abacists, was first in a position to advise merchants mathematically and developed this profession to the point where he treated mathematics theoretically. For the remainder of his examples he dispenses with Z.

and uses __________. These cossic numbers, for Recorde, are not quite so abstract as the unknowns of modern algebra. He also expressly forbids the use of cossic signs to represent absolute numbers.[217]

Recorde's immediate successor is John Dee who edited and augmented Recorde's Grounde of Artes. The enlargement in 1561 included Dee's "The Second Part of Arithmetike Touching Fractions, briefly set forthe," and Peter French takes this work on Recorde as preparation for Dee's preface to Billingsley's English translation of Euclid.[218] In 1582 he prepared another edition of the same work with John Mellis, a Southwark schoolmaster and author of a work on bookkeeping. This edition contains verses by Dee explaining his interpretation of the relationship between arithmetic and geometry.

Although he studied at Cambridge from 1542 he spent most of his time in London and at his home in Mortlake. In London there was an "amorphorus third university" organized by Henry Percy, the ninth Earl of Northumberland, at Syon House. There Dee was associated with Thomas Harriot, Walter Warner, Nathaniel Torporley as well as with Christopher Marlowe, John Donne and Walter Ralegh. The latter did favours at court for Dee. On the continent he knew Gemma Frisius, known to historians of bookkeeping, Pedro Nuñez, Gerard Mercator and Oronce Finé.

He became an adviser to the Muscovy Company, formed to further the interests of the Duke of Northumberland and the London wool merchants,[219] and envisioned the formation of an "Incomparable

B R Y T I S H I M P I R E".[220] For French,

> Dee was most desirous of improving England's economic situation. As early as 1570 he outlined (British Museum, Cotton Charter XIII, act. 39) a plan to 'MAKE THIS KINGDOME FLOURISHING, TRIUMPHANT, FAMOUS AND BLESSED'. Among other things he suggests that it would be wise 'to make England both abroad and at home to be Lord and ruler of the Exchange', a task that Sir Thomas Gresham accomplished and that produced great benefits for England. The plan is quite thorough and covers most of the areas of England's economy, such as tin production and doth trade.[221]

Dee's most important work for our purposes is his preface to Billingsley's translation of Euclid (1570). He opens with a Platonic conception of the objects of mathematics. There are Supernatural, natural and mathematical realms. The objects of mathematics are in between; they "are thinges immateriall: and nonetheless, by materiall thinges hable somewhat to be signified."[222] We are trained, he claims, to see in corporeal objects a likeness to number "and to use Arte in them to our pleasure and proffit."[223]

In one place he states that "Of Number, an Unit, and of Magnitude, a Poynte, doo seeme to be much like Originall causes: But the diversitie neverthelesse, is great."[224] Numbers are made of indivisible units; magnitudes are infinitely divisible. Numbers consist of units but lines do not consist of points. He suggests also changing the name of geometry to megethelogia to indicate a more theoretical science.

Elsewhere he suggests that practice has produced new sciences of number in the areas of fractions and proportionality. The definition of units, for example, may alter in practice and number may be applied to magnitude. These developments Dee attributes to the work of the "reckenmasters". In all of this he appears to keep theory and practice pretty well separate as far as his definitions are concerned but

> Consider: the infinite desire of knowledge, and incredible power of mans Search and Capacitye: how they, ioyntly have waded farder (by mixtyng of speculation and practise) and have found out, and atteyned to the very chief perfection (almost) of Numbers Practicall use. Which thing, is well to be perceived in that great Arithmeticall Arte of Aequation: commonly called the rule of Coss or Algebra. The Latines termed it, Regulamrei et census, that is the Rule of the thyng and its value.[225]

Those benefitting most from such developments, claims Dee, are merchants, primarily owing to their use of the "golden rule." They need it, he states, for partnerships with or without time, relations between merchants and their factors, the rule of bartering both in wares alone and in combination with money and the exchange of currency.[226] The rest of this Preface discusses the use of these sciences in other artes and practices from music and statics to navigation, a science to which he made important contributions himself, and Archemastrie, his name for what was to become experimentation.[227] In Dee's words

> This Arte, teacheth to bryng to actuall experience sensible, all worthy conclusions by all the Artes Mathematicall proposed, and by true Naturall Philosophie concluded: and both addeth to them a farder scope, in terms of the same Artes, and also by hys propre Method, and in pecular termes procedeth, with helpe of the foresayd Artes, to the performance of complete Experiences, which of no particular Art, are hable (Formally) to be challenged.[228]

Dee was adviser to the Muscovy Company for thirty years and Billingsley himself was a successful London merchant.[229] We have already mentioned another London merchant, Sir Thomas Gresham (1518-

79), who "built the Royal Exchange and left the revenue from shops there jointly to the City of London and the Mercers' Company to endow a new college."[230] In fact, many merchants endowed grammar schools, as well, in the sixteenth century.

According to Francis Johnson, historian of Gresham College

> As a coterie of scientific workers maintaining active co-operation among themselves, providing instruction for others, and keeping in close touch with scientific activity abroad, the group centering about John Dee must be ranked as the earliest ancestor of the Royal Society to contribute significantly to its patrimony.[231]

Another mathematical practitioner who deserves attention at this point is the figure of Thomas Harriot. Dee mentions him in his diary.[232] Harriot received a pension from the Earl of Northumberland and other times was befriended by Sir Walter Ralegh. He was one of the Earl's "Three Magi" in the tower of the "Wizard Earl."[233] Although Harriot published nothing during his lifetime, his friend Walter Warner, published part of his papers on algebra ten years after his death in 1631 as <u>Ars Analiticae Praxis</u>. It is significant that the development of this art is supported by nobility who are participating in new economic forms. Robert Kargon mentions that the Percys entered a more capitalist form of landownership, extracting rents in money from their estates.[234] Harriot was also involved in the Earls's financial affairs in this regard.[235] In fact, it has been suggested that Harriot acted as Sir Walter **Raleigh's** accountant. There is mention in Harriot's will that these papers should be destroyed.[236]

After Harriot finished his studies at Oxford, he moved to

London. It was there that he eventually entered the service of Raleigh but it is possible that, in the mean time, he opened a school for the training and advising of merchants and artisans. Shirley mentions this as a distinct possibility noting as well that such bookkeepers as Hugh Oldcastle and John Mellis had done the same.[237] Harriot's friend, Walter Warner, to whom the publication of his algebra was entrusted, "did much work on money and exchange, no doubt at the instigation of Sir Thomas Aylesbury who was Master of the Mint."[238] Jacquot suggests as well that Warner's philosophy was one which deals mechanistically with a homogenised world, much after the fashion of Galileo and Descartes.[239]

The development of science in seventeenth century England, however, is most marked by its association with trades. Francis Bacon, William Petty, John Evelyn and Robert Boyle all developed schemes for a history of trades. Bacon alluded to a history of trades but Petty was actually charged with the duty of writing one, a task which he never completed.

The idea, however, can be seen most clearly in the work of Petty in "The Advice of W.P. to Mr. Samuel Hartlib for the Advancement of some particular Parts of Learning." This work is publicity for Hartlib's proposed Office of Public Address, a communication office for those working on scientific and economic projects, and concerns itself with the "History of Art and Nature" and their relationship. Essentially the relation boils down to the conception of God as the "Master-Builder".[240]

The thrust of the argument is for educational reform in the form of literary workhouses where essentially equal but unfortunate people might be put to better use, since "many ... are now holding the Plough, which might have been made to steer the state."[241] The opposite is also true since he recommends "That all Children, though of the highest Rank, be taught some genteel Manufacture in their minority." But old distinctions still hold as well since "They will certainly bring to pass most excellent Works being, as Gentlemen, ambitious to excell ordinary workmen."[242]

His most earnest quest here, however, is for the establishment of a Gymnasium Mechanicum which would afford the following:

> From this Institution we may dearly hope, when the Excellent in all Arts are not only Neighbours, but intimate Friends and Brethren, united in a common Desire and Zeal to promote them, that all trades will miraculously prosper, and new inventions would be more frequent, than new Fashions or Cloaths and Household-stuff. Here would be the best and most effectual Opportunities and Means, for writing a History of Trades, in Perfection and Exactness; and what Experiments and stuff would all those Shops and Operations afford to active and philosophical Heads, out of which, to extract that interpretation of Nature, whereof there is so little, and that so bad, as yet extant in the world.[243]

Three books are important for this task: 1) a compilation of all the useful information from extant works, 2) John Pell's three mathematical works and 3) descriptions of practice as observed in the Gymnasium Mechanicum. Petty's description of this last is worth quoting at length:

> All the practical Ways of getting a subsistence, and whereby

Men raise their Fortunes, may be at large declared. And among these, we wish that the History of Arts or Manufactures might first be undertaken as the most pleasant and most profitable of all the rest, wherein should be described the whole Process of Manual Operations and Applications of one natural thing (which we call the Elements of Artificials) to another, with the necessary instruments and Machines, whereby every Place of Work is elaborated and made to be what it is. 244

This is to establish the prospect of experiments "all being equally luciferous, although not equally lucriferous".[245] It is easy to see that this is the same man responsible for beginning the labour theory of value, particularly in its quantitative aspect. (cf. Ch. III supra) The history of trades, for Petty, would then make it possible as well "to demonstrate Axioms in philosophy, the Value and Dignity whereof cannot be valued or computed".[246]

This history, claims Petty, is also a history of nature, but of nature"vexed and disturbed". The next project should be a history of nature free. It is this latter notion which in fact took over from the idea of the history of trades with the development of experimental laboratories but it is a development building on notions derived from the history of trades.

An associate of Petty in the Hartlib circle, Robert Boyle, the chemist, displays the transition from history of trades to laboratories. In a work entitled That the Goods of Mankind, may be much increased by Naturalists Insight into Trades, Boyle says that he has

often wished, that some ingenious friends to experimental philosophy would take the pains to enquire into the mysteries, and other practices of trades, and give us an account, some of one trade, and some of another, ... towards the motivation of the professions they write of. 247

In the same piece Boyle compared and contrasted the tradesmen in their shops and the virtuosi in their laboratories.

What we have in England by the 1640's is a situation much more favourable to the advancement of science once the development of its basic concepts had taken place. Once the development of western mathematics had reached a certain point and its application to mechanics and other sciences had been suggested, it remained to see modern science embodied as an institution. It had to be seen as the most appropriate way to view the world. This task was accomplished by the same men associated through commercial and scientific interests. They were also usually Puritans, Baconians and Parliamentarians.

Charles Webster presents a convincing case for the argument that the advancement of science is carried out under the aegis of a Puritan world-view.[248] He mentions fifteen men, John Wilkins, William Petty, John Pell, Samuel Hartlib and John Dury among them, who all got good posts, all scientifically oriented and got positions of advantage during the Puritan revolution. A £200 per annum stipend was usual.[249]

These were men who formed the nucleus of the group which formed the Royal Society, which received its first charter in July, 1662. They were, furthermore, involved in other, less formal, groups in the 1640's and 1650's, such as Gresham college, the "Hartlib Circle", the Oxford Experimental Philosophy Club and the London based "1645 group". Their focus was on experiment and mathematics.[250]

Notes

1. Christopher Hill Intellectual Origins of the English Revolution Panther London, 1972, 2.
2. Sabetai Unguru "On the Need to Rewrite the History of Greek Mathematics" Archive for History of Exact Sciences, V.15, 1975, 68.
3. Klein was cited in Chapter Four, Szabo's work is Anfaenge der Griechischen Mathematik R.Oldenbourg, München, 1969.
4. "When I speak of Babylonian or Greek or Arab algebra, I mean albebra - in the sense of Al-Khawarizmi, or in the sense of Cardano's Ars Magna or in the sense of our school algebra. Algebra, then, is:

   the art of handling algebraic expressions like $(a+b)^2$ and of solving equations like $x^2+ax = b$.

   If this definition is applied to any Babylonian or Arab text it is unimportant what symbolism the text uses. Our relation $(a+b)^2=a^2+b^2+2ab$ can be stated in words thus:

   'The square of a sum of the squares of the terms and twice their product.'
   The statement in words says exactly the same thing as the formula. Instead of 'product' one may also say 'area' (of a rectangle), as the Babylonians did, or just 'rectangle', as the Greeks did."
   B.L. van der Waerden "Defense of a Shocking Point of View" Archive for History of Exact Sciences V. 15, 1975-76, 199-200.

   Hans Freudenthal makes reference to the "hidden spirit of the proof" and reproduces Archimedes' statement of two propositions but not his proofs; he uses Heath's algebraic ones. "What is Algebra and What has it been in History" Archive for History of Exact Sciences V. 16, 1976-77, 189-200.
5. B.L. van der Waerden Science Awakening translated by Arnold Dresden, P. Noordhoff, Groningen, 1954, 266.
6. Unguru, op. cit., 73.
7. ibid. 10.
8. Kurt Vogel Vorgriechische Mathematik II Hermann Schroedel, Hannover, 1959, 41-45 and "Bemerkungen zur Vorgeschichte des Logarithmus" Beiträge zur Geschichte der Arithmetik Minerva, München, 1978, 55.
9. Thekla Horowitz Vom Logos zur Analogie Hans Rohr, Zürich, 1978, 27.
10. Sir Thomas L. Heath A History of Greek Mathematics I Clarendon Press, Oxford, 1921, 66.
11. Euclid The Thirteen Books of Euclid's Elements II, translated by Sir Thomas L. Heath Cambridge U. P., Cambridge, 1908, 114,277,278.
12. ibid. 113.
13. ibid. 277.
14. "The 'part-parts' terminology might make it appear that

Definition 3 suggests that the unit cannot be a part (since it is not a number) but in proposition VII, 4, the unit is explicitly called a "part" of a number. It is sometimes helpful in understanding this terminology if the part-parts terms are understood as fractions* with the part or parts as numerator and the number as denominator: the 'part-fraction' would be reducible to a unit fraction, whereas the 'parts-fraction' could not be so reduced. 3 is part of 12 since 3/12 =1/4; 8 is parts of 12 since 8/12 = 2/3, not a unit fraction. That is to say, if the lesser number is not one part of the greater, then it must be several of some part of the greater, hence parts." Charles V.Jones The concept of One as a Number Ph.D. Thesis, University of Toronto, 1978,106-107. *To the notion of considering them as fractions Jones adds the following footnote: "Which Euclid neither does nor suggests be done, contrary to Heath's remark at Euclid II, 280.

"By the expression Parts Euclid denotes what we should call a proper fraction. To call them proper fractions we feel, is misleading in the context of Greek mathematics. Perhaps Heath's use of 'relative magnitude' leads to this lapse." also: for Eratosthenes, for example, we do not speak of 11/83 of a unit arc of the earth's meridian, but rather, eleven parts of which the meridian has eighty-three, further indication that the unit of calculation even as late as Eratosthenes, was not fractionalised. Jones ibid. 82.

15. Jones ibid. 200-201.
16. Jones ibid. 204.
17. Hans-Joachim Waschkiess Von Eudoxos zu Aristoteles B.P.Grüner, Amsterdam, 1972, 7.
18. ibid. 22.
19. Aristotle Nichomachean Ethics V, 1, 1130 a29 - 1130 b5.
20. ibid. V,2, 1131a 3-4
21. ibid. V,3, 1131a 18-20.
22. ibid. V,3,1131a30 - b16.
23. Physics VII, 5, 249b1 - 250a3 and 250a25-27.
24. Ethics V, 4, 1132b 10.
25. ibid. V,4, 1132b 11-20.
26. ibid. V,5, 1132b 32-34.
27. ibid. V,5, 1133a 5-21.
28. ibid. V,5, 1137b 18-29.
29. Michael Wolff Geschichte der Impetustheorie. Surkamp, Frankfurt, 1978, 68.
30. ibid. 129-30.
31. Alexander Murray Reason and Society in the Middle Ages, Oxford, New York, 1978,40.
32. ibid, 59-60.
33. ibid. 123.
34. ibid. 181.
35. Epist. ad Leutherium (MGH Auctores Antiquissimi 15, 477, lines 12-18) from ibid. 148.
36. ibid. 194.

37. Moritz Cantor Vorlesung über Geschichte der Mathematik II Teubner, Leipzig, 1894, 7-8.
38. ibid. 53.
39. Warren Van Egmond The Commercial Revolution and the Beginnings of Western Mathematics in Renaissance Florence, Ph.D. Thesis, Indiana U., 1976, 39, 44, 324-25, 339-40.
40. Kurt Vogel "Überholte Arithmetische Kaufmännische Praktik aus dem Mittelälter" in Beiträge zur Geschichte der Arithmetik Minerva, München, 1978, 67.
41. Luca Pacioli Summa de Arithmetica, Venice, 1494, fol. 161 r, from Vogel ibid. 68.
42. ibid. 69.
43. ibid. 69.
44. ibid. 70-71.
45. Van Egmond op. cit. 127-8.
46. Van Egmond op. cit. 135.
47. Van Egmond op. cit. 220.
48. Van Egmond op. cit. 233.
49. R. Emmett Taylor No Royal Road: Luca Pacioli and His Times The University of North Carolina Press, Chapel Hill, 1942, 62.
50. Raymond De Roover, " The Development of Accounting Prior to Luca Pacioli" in A.C. Littleton and B.L. Yamey eds., Studies in the History of Accounting Homewood, Illinois, Richard D. Irwin, 1956, 117.
51. Florence Edler De Roover "Partnership Accounts in Twelfth Century Genoa" In Littleton and Yamey 88-90.
52. A.C.Littleton, Accounting Evolution to 1900 American Institute, New York, 1933, 12. and De Roover op.cit. 115.
53. ibid, 25.
54. ibid. 27.
55. ibid. 34.
56. R. De Roover op. cit. 117-20.
57. " " 128.
58. " " 131 fn. and Van Egmond op. cit. 262.
59. B.S.Yamey, introduction to Littleton and Yamey, op. cit. 2-3, 5-6.
60. H.A.Oberman Archibishop Thomas Bradwardine, A Fourteenth Century Augustinian Hermink and Zoon, Utrecht, 1957, 3.
61. H.Lamar Crosby introduction to his translation of Bradwardine Tractatus de proportionibus University of Wisconsin Press, Madison, 1955, 44. We object of course to the term "function" as anachronistic, in fact wrong. On this point cf. Stillman Drake "Bradwardine's function, mediate denomination and multiple continua" in Physis 12, 1970.
62. For the simplest explanation of this procedure cf. John E.Murdoch and Edith Sylla "The Science of Motion" in David C.Lindberg, ed. Science in the Middle Ages University of Chicago Press, Chicago, 1978, 225-26.
63. Crosby op. cit. 13, Murdoch and Sylla, 227.
64. Crosby, op. cit. 13.
65. Bradwardine in Crosby op. cit. 67. the last part of this in the original reads: "sicut medietas duplae proportionis, quae est proportio diametri ad custam et medietas sesquioctavae proportionis, quae toni mediatatem constituit." 66.

66. Drake, op. cit. 54.
67. Murdoch in "The Medieval Language of Proportions" 240-42.
68. Alfred Sohn-Rethel suggests the obscuring of the connections between the teaching at oxford and the finances of the Norman overlords as the reason why Bradwardine's connections with finance are not known. Sohn-Rethel cites no source for this however. Intellectual and Manual Labourtranslated by Martin Sohn-Rethel, Macmillan, London, 1978, 108.
69. Murdoch op.cit. 256.
70. Wolff op. cit. 190.
71. Nicole Oresme The De Moneta of Nicholas Oresme ed., Charles Johnson, Thomas Nelson, London, 1956, 10-11
72. Wolff. op. cit. 210-11.
73. Oresme op. cit. 15.
74. " " 25.
75. Hector Estrup "Oresme and Monetary Theory" Scandinavian Economic History Review, 14, 1966, 102.
76. William A. Wallace "Mechanics from Bradwardine to Galileo" Journal of the History of Ideas XXXII, 1, 1971, 18.
77. Charles Johnson, introduction to his edition of Oresme, op. cit. ix.
78. Edward Grant, in his introduction to Oresme's De proportionibus proportionum disagrees with this dating. University of Wisconsin Press, Madison, 1966, 5.
79. Nicole Oresme Le Livre D'Ethiques D'Aristote edited by Albert D. Menut, Stechert, New York, 1940, 99. (translations by Jean-Louis Picot).
80. Edward Grant his introduction to Nicole Oresme De porportionibus... op. cit., 6.
81. Oresme De proportionibus , 190.
82. Grant his notes to Oresme De proportionibus , 339.
83. Leonardo Olschki Geschichte der neusprachlichen wissenschaftlichen Literatur III Galilei und seine Zeit Halle, Niemeyer, 1927,107.
84. S.A.Jayawardene "The Influence of Practical Arithmetics on the Algebra of Rafael Bombelli" Isis V.64, December 1973, 511. Bombelli Algebra (1572), 414.
85. ibid. 513.
86. ibid. 514-520.
87. ibid. 522.
88. Jayawardene "Rafael Bombelli" Dictionary of Scientific Biography II Scribners, New York, 1970, 281.
89. Olschki op.cit. 82.
90. Olschki op.cit. 79-80. Originally from the title page from the 1543 Venice edition of Tartaglia's Opera Archemede Syracusani etc. per Nicolaum Tartaleam etc. in lure posita, etc.
91. Olschki op. cit. 98.
92. Olschki op. cit. 100.
93. Olschki op. cit. 106.
94. Niccolo Tartaglia "Nova Scientia" translated and edited by Stillman Drake in Drake and Drabkin eds. Mechanics in Sixteenth Century Italy Wisconson, Madison, 1968, 138.
95. ibid. 139.
96. T.L.Heath, ed. The Thirteen Books of Euclid's Elements II , 318-20.

97. Galileo Galilei Two New Sciences translated by Stillman Drake, University of Wisconsin Press, Madison, 1979, 11.
98. ibid. 14.
99. Olschki op. cit. 156.
100. Galileo Galilei "Tractate on the Elements" in William A. Wallace ed. Galileo's Early Notebooks University of Notre Dame Press, South Bend Indiana, 1977, 246.
101. Galileo Galilei op. cit. 151.
102. Galilei op. cit. 151.
103. Drake's not in Galileo op. cit. 151 fn.
104. Drake in Galileo op. cit. 121 fn.
105. I am indebted to Professor W.A Slater of the Department of Classics, McMaster University, for these translations. Sir Thomas L. Heath, The Works of Archimedes, Cambridge University Press, Cambridge, 1897, from Heath's introduction, clxxix.
106. Drake's introduction to Two New Sciences, xxi.
107. ibid. xxv.
108. Ludovico Geymonat, Galileo Galilei translated by Stillman Drake, McGraw-Hill, New York, 1965, 6-7.
109. ibid. 5. To this Drake adds the following note: "Tradition makes Galileo's father a wool merchant as well as a musician. The researches of Professor Favaro throw grave doubt on this tradition, which appears to be no older than the beginning of the nineteenth century. It may have originated from no better evidence than from the fact that Vincenzio accepted a part of Julia's dowry in cloths. Contemporary documents which referred to his occupation consistently called him a lutenist." ChI. note 1, 227.
110. Galileo Galilei "The Assayer" translated by Stillman Drake in Discoveries and Opinions of Galileo translated by Stillman Drake, Anchor, New York, 1957, 237-8.
111. ibid. 276.
112. J.Winfree Smith, footnote to his translation of F. Vieta, Introduction to the Analytic Art as appendix to J.Klein, Greek Mathematical Thought and the Origin of Algebra, translated by Eva Brann, M.I.T. Press, Cambridge, Mass., 1968, 315-16 fn.
113. H.L.L.Brussard, "Francois Viete" Dictionary of Scientific Biography Scribners, New York, 1976.
114. Francois Vieta Introduction to the Analytic Art, translated by J.Winfree Smith in Klein op.cit.
115. ibid. 319.
116. ibid. 321.
117. ibid. 321-2.
118. ibid. 339.
119. ibid. 341 and Smith's note 341 fn.
120. ibid. 346-7.
121. ibid. 352.
122. ibid. 353.
123. ibid. John Wallis, A Treatise of Algebra (London, 1685), 66. quoted in ibid., 321-2 fn. Harriot also shares this opinion. His Algebra, published in 1631 by his friend Walter Warner, states that Vieta

derives the term species ex uso forensi (from legal usage). from Klein op. cit. 281.
124. Klein, op. cit., 264.
125. Klein, op. cit., 156-7, 261
126. Smith in Klein op. cit., 322 fn.
127. Vieta, in Klein op. cit., 345.
128. Franz Borkenau "Zur Soziologie des mechanistischen Weltbildes" in Zeitschrift für Sozialforschung III,2, 1932,
129. Borkenau Der Übergang vom feudalen zum bürgerlichen Weltbild Wissenschaftliche Buchgesellschaft, Darmstadt, 1976, (originally 1934) 268.
130. ibid. 269-70.
131. Jacob Klein op. cit., 198.
132. René Descartes "Rules for the Direction of the Mind" in The Philosophical Works of Descartes I translated by Elizabeth S. Haldane and G.R.T.Ross, Cambridge University Press, Cambridge, 1931, 1-2.
133. Descartes, "Discourse on the Method of Rightly Conducting the Reason and Seeking for Truth in the Sciences" in WorksI 81.
134. ibid., 82.
135. ibid. 88.
136. cf. Chapter Three p. 52 for the full passage from Descartes.
137. Descartes preface to "Principles of Philosophy" in Works I, 211.
138. Descartes,"Discourse..." Works , 85.
139. ibid. 86.
140. ibid. 82.
141. ibid. 91-2.
142. ibid. 93.
143. Descartes "Meditations on First Philosophy" Works, 164.
144. ibid. 185.
145. Borkenau, op. cit. 270-71.
146. ibid. 275.
147. ibid. 277.
148. ibid. 314.
149. ibid. 363.
150. ibid. 365-74.
151. Descartes "Rules..." Works I, 4.
152. ibid. 10.
153. ibid. 12.
154. ibid. 37.
155. ibid. 56.
156. "principles..." in Works I , 267.
157. "Rules..." Works I, 61.
158. ibid. 62.
159. ibid. 56-7.
160. Klein, op. cit. 203.
161. ibid. 208.
162. "Rules..." Works I, 56.
163. "Principles..." Works I, 291-3.
164. ibid. 269.

165. ibid. 265.
166. M.G J.Minnaert, "Simon Stevin" Dictionary of Scientific Biography, XIII, Scribners, New York, 1976, 47.
167. Dirk J. Struik, The Land of Stevin and Huygens Reidel, Dordrecht, 1981, 53-4.
168. E.J. Dijksterhuis, Simon Stevin, Martinus Nijhoff, The Hague, 1970, 4. and Principle Works of Simon Stevin I Mechanics ed. Dijksterhuis, C.V.Swets and Zeitlinger, Amsterdam, 1955, 5.
169. P.G.A.de Waal, Van Paciolo tot Stevin, J.J.Romen and Zonen, Roermond, 1927, 260.
170. Simon Stevin "Tables of Interest" Principle Works of Simon StevinIIA Mathematics, ed. Dirk Struik, S.V.Swets and Zeitlinger, Amsterdam, 1958, 27.
171. ibid. 31.
172. ibid. 31.
173. ibid. 53. also Struik's introduction 21.
174. Stevin, "The Dime" translated by Richard Norton (1608) in PWSS op. cit. 389.
175. ibid. 393.
176. ibid. 397.
177. Charles V.Jones op. cit. 220. We are much indebted to Jones' work on Stevin's La Disme and L'Arithmetique.
178. ibid. 221-2 fn.
179. Stevin L'Arithmetique in PWSS II B , 495.
180. From Jones, op. cit. 233.
181. ibid. 234-35, 271-299.
182. Stevin, op. cit. 501.
183. ibid. 532.
184. Stevin, "The Practice of Weighing" in PWSS I , 365.
185. Stevin "The Dime" loc. cit. 45.
186. A.C.Littleton, Accounting Evolution to 1900 New York, Russell and Russell, 1966, 97.
187. John B. Geijsbeek Ancient Double-Entry Bookkeeping Scholars Book Co., Houston, 1914, 114.
188. ibid., 115.
189. Onko ten Have, The History of Accounting, translated by A. von Seventer, Bay Books, Palo Alto, 1976, 64.
190. Raymond de Roover, "The Development of Accounting Prior to Luca Pacioli According to The Account Books of Medieval Merchants" in A.C.Littleton and B.S.Yamey eds., Studies in the History of Accounting Richard Irwin, Homewood Ill., 1956,170.
191. cf. Charles Webster, The Great Instauration,Holmes and Meier, New York, 1976, 413.
192. Onko ten Have "Simon Stevin of Bruges" in Littleton and Yamey op. cit., 242.
193. Webster op. cit., 413.
194. Robert Steele ed., The Earliest Arithmetics in English, Oxford University Press, London, 1922, v.
195. ibid. 33.

196. Robert Recorde The Ground of Artes (London, 1542), Da Capo Press, New York, 1969, Bii v.
197. ibid. Biii r.
198. ibid. L i v.
199. Liii r.
200. Liii r.
201. Robert Recorde, First Principles of Geometry (London, 1551) Theatrum Orbis Terrarum, Amsterdam, 1974, i.
202. ibid. ii.
203. Robert Recorde The Castle of Knowledge (London, 1556) Theatrum Orbis Terrarum, Amsterdam, 1975, 242-3.
204. Robert Recorde The Whetstone of Witte (London, 1557) Da Capo Press, Amsterdam, 1969, biii.
205. ibid. Ai v.
206. ibid Aiii r-v.
207. ibid. Dii r - Diii r.
208. ibid. Si r.
209. ibid. Si v.
210. Si v - Siii r.
211. Eeiiii v - Ffi r.
212. ibid. Ffi r.
213. ibid. Ffi r.
214. ibid. Ffi v.
215. ibid. Hhi v.
216. ibid. Hhi r.
217. ibid. Kkiii r.
218. Peter M. French, John Dee: The World of an Elizabethan Magus Routledge and Kegan Paul, London, 1972, 165.
219. ibid. 178.
220. ibid. 180.
221. ibid. 180.
222. John Dee The Mathematicall Preface to the Elements of Geometrie of Euclid of Megara (1570) Science History Publications, New York, 1976, i.
223. ibid. iv.
224. ibid. ai v - aii r.
225. ibid. ii v.
226. ibid. ii v - iii r.
227. ibid. bi r.
228. ibid. Aiii r.
229. Christopher Hill, Intellectual Origins of the English Revolution Panther, London, 1972, 18, Marie Boas, The Scientific Renaissance 1450-1630 Harper, New York, 1962, 197.
230. Hill 34.
231. Francis R. Johnson "Gresham College: Precursor of the Royal Society" in Wiener and Noland, eds., Roots of Scientific Thought Basic Books, New York, 1957, 340.
232. John Dee The Private Diary of Dr. John Dee ed. James O. Halliwell, Johnson Reprint, New York, 1968, 41.
233. John W.Shirley Thomas Harriot, Oxford U.P., Oxford, 1983,364.

234. Robert Kargon *Atomism in England from Harriot to Newton* Oxford University Press, Oxford, 1966, 5-6.
235. Shirley op. cit. 210-12.
236. Shirlèy "Sir Walter Raleigh's Guiana Finances" *Huntington Library Quarterly* 18, 1, San Marino, 1949, 55-6, from John W. Shirley ed. *Source Book for the Study of Thomas Harriot* Arno Press, New York, 1981.
237. Shirley Thomas Harriot 73.
238. Jean Jacquot "Harriot, Hill, Warner and the New Philosophy" in John W. Shirley ed. *Thomas Harriot: Renaissance Scientist*, Clarendon, Oxford, 1974, 117.
239. ibid. 118-19.
240. William Petty "The Advice of W.P. to Mr. Samuel Hartlib, for the Advancement of some particular Parts of Learning" in *The Harleian Miscellany VI* , London, T.Osborne, 1745,
241. ibid. 3.
242. ibid. 4.
243. ibid. 5.
244. ibid. 9.
245. ibid. 11.
246. ibid. 13.
247. Boyle, Works III 449, in Walter E. Houghton, fn. "The History of Trades: Its Relation to Seventeenth Century Thought" in Wiener and Noland op. cit. 378.
248. Charles Webster op. cit. , passim.
249. ibid. 84.
250. ibid. 95.

Chapter Six -- Conclusion

The development of mathematical and mechanical concepts which helped form the basis of the new mechanistic science was thus in part commercially inspired and constituted. While there are many factors which help to account for the advance of science (religious, political, technological) there is really only one other argument which attempts an account of the concepts of early modern science. This is the argument that certain Greek texts, either alone or in some combination with late medieval and early modern artisanry and technology, constituted these concepts. In the case of mathematics, the concepts are seen as formed almost entirely by the reading of Euclid, Archimedes, Pappus and Diophantus; hence, our emphasis on the work of Klein, Unguru and Jones and our attempt to find the extent to which commercial social relations are responsible for the new meaning of mathematical concepts which these authors describe. We argue that commercial practice had an effect on definitions of number, magnitude, homogeneity and commensurability.

The origin of early modern scientific concepts does not lie in technology in the narrower sense. It does lie in part in the recovery of Greek texts and the already active Arabo-Latin tradition. The substance of these traditions, however, was read through the spectacles of a different mentality, a mentality constituted by a different set of social relations. Nature came to be viewed as mere "stuff", matter in motion, when the dominant form of relation in society between person and person was rapidly becoming the relation between owners of commodities (often only one's capacity to labour) which were commensurate in terms of their value. Qualitatively different natural objects were made commensurate in terms of "stuff"; qualitatively different commodities were made commensurate in terms of value.

We have examined the cases of a number of the major contributors to the concepts of the mathematical - mechanistic world-view. These concepts were developed, at least in part, by thinkers also involved in exchange relations and the techniques developed to deal with them. In general, there is a growing abstraction and homogenisation in scientific concepts in times and places where commercial relations begin to take hold. While for Bradwardine, Galileo, Vieta and Descartes no convincing case can be made for the origin of their concepts in commercial activity and thought, for Philoponos, Fibonacci, Oresme, Pacioli, Tartaglia, Stevin, Recorde, Dee and Petty at least a strongly suggestive case can be made. For Fibonacci, Oresme, Pacioli, Stevin and Petty there is a very strong case for the connection between commerce and science.

We have taken our lead from the work of Franz Borkenau who, unfortunately, is rarely cited in the history or sociology of science literature; Koyre[1] completely misunderstands him and Leiss[2] makes favourable but only passing mention of his work. Although we believe there are difficulties and anachronisms in Borkenau's book, his version of the sociology of knowledge and science is one which we support for the following reasons: 1) it is historically specific, refraining from the attempt to state transhistorical relations, 2) although taking the lead from Marx his argument does not, unlike most Marxist arguments in the history of science, depend on the notion of a purely empirical and rational human mind confronting nature directly once the fetters of theology have been overthrown. Relations of production receive emphasis in Borkenau's account over means of production.

We have had to rely on Borkenau, Klein and Marx, as well as certain of the historians of science mentioned in Chapter Four, for our account of mechanistic science which is not in itself mechanistic. Modern capitalist society developed in such a way that, according to Max Weber, the world was disenchanted and all things became masterable by calculation. This calculability demanded abstraction and homogenisation of the world. In science this involved the mathematisation of nature. In economic life a sphere of calculation was opened up where one could, within limits, calculate one's success based on a knowledge of risks and other similar factors. This calculation cannot, however, succeed in producing an understanding of society as a whole nor of its historical movement. The understanding of the development of mechanistic science cannot itself be carried out on mechanistic principles. The sociology of knowledge and science would do well to pick up on the lead of Marx, Borkenau and Klein.

Notes

1. Alexandre Koyré "Galileo and Plato" in Wiener and Noland Roots of Scientific Thought New York, Basic Books, 1957, 148.
2. William Leiss The Domination of Nature Beacon, Boston, 1972.

Bibliography

Archimedes The Works of Archimedes. edited and translated by Sir Thomas L. Heath, Cambridge: Cambridge University Press, 1897.

Aristotle Nichomachean Ethics translated by H.Rackham Cambridge, Mass.: Harvard University Press, 1958.

.... Physics translated by Richard Hope Lincoln: University of Nebraska Press, 1961.

Barnes, Barry Scientific Knowledge and Sociological Theory London: Routledge and Kegan Paul, 1974.

Barrett, William Aristotle's Analysis of Movement: Its Significance for Its Time. New York: Columbia University Press, New York, 1938.

Bernal, J.D. Science in History II. London: Watts, 1954.

Blanchard, Ian "Labour Productivity and Work Psychology in the English Mining Industry 1400-1600" Economic History Review second series, XXXI, 1978.

Bloor, David Knowledge and Social Imagery London: Routledge and Kegan Paul, 1976.

Boas, Marie "The Establishment of the Mechanical Philosophy" Osiris. 10, 1952.

..........The Scientific Renaissance 1450-1630 New York: Harper and Row, 1962.

Bochner, Salomon The Role of Mathematics in the Rise of Science. Princeton: Princeton University Press, 1966.

Borkenau, Franz "Zur Soziologie des mechanistischen Weltbildes" Zeitschrift für Sozialforschung I,3,1932.

.... Der Ubergang vom feudalen zum bürgerlichen Weltbild Darmstadt: Wissenschaftliche Buchgesellschaft, 1976.

Bradwardine, Thomas Tractatus de proportionibus translated and edited by Lamar H. Crosby, Madison: University of Wisconsin Press, 1955.

Brannigan, Augustine The Social Basis of Scientific Discoveries Cambridge: Cambridge University Press, 1981.

Brenner, Robert "The Social Basis of English Commercial Expansion 1550-1650" Journal of Economic History. XXXII, 1972.

Brussard, H.L.L. "Francois Viete" Dictionary of Scientific Biography New York: Scribners, 1976.

Butterfield, Herbert The Origins of Modern Science 1300-1800. Toronto: Clarke, Irwin and Co., 1968.

Cantor, Moritz Vorlesungen über Geschichte der Mathematik I,II. Leipzig: Teubner, 1894.

Cipolla, Carlo M. Clocks and Culture 1300-1700. New York: Norton, 1977.

......., ... editor, The Fontana Economic History of Europe III. Glasgow: Collins, 1974.

Clagett, Marshall Archimedes in the Middle Ages I The Arabo-Latin Tradition. Madison: University of Wisconsin Press, 1964.

......., ... The Science of Mechanics in the Middle Ages. Madison: University of Wisconsin Press, 1959.

Coleman, Donald C. Industry in Tudor and Stuart England. London: MacMillan, 1975.

.......,......... "An Innovation and its Diffusion: 'The New Draperies'" Economic History Review XXII, 3, 1969.

......., ... "Labour in the English Economy of the Seventeenth Century" in E. Carus-Wilson Essays in Economic History II. London: Edward Arnold, 1962.

Crombie, A.C. Augustine to Galileo. Cambridge, Mass.: Harvard University Press, 1953.

......., ... "From Rationalism to Experimentalism" in Phillip Wiener and Aaron Noland editors, Roots of Scientific Thought New York: Basic Books, 1957.

Dee, John The Mathematicall Preface to the Elements of Geometrie of Euclid of Megara (1570) New York: Science History Publications, 1976.

..., ... The Private Diary of Dr. John Dee. edited by James O. Halliwell, New York: Johnson Reprint, 1968.

De Roover, Florence Edler "Partnership Accounts in Twelfth Century Genoa" in Littleton and Yamey editors Studies in the History of Accounting Homewood, Ill.: Richard Irwin, 1956.

De Roover, Raymond Business, Banking and Economic Thought in Late Medieval and Early Modern Europe . Chicago: University of Chicago Press, 1974.

........., ... "The Development of Accounting Prior to Luca Pacioli" In A.C.Littleton and B.S.Yamey editors, Studies in the History of Accounting Homewood, Ill.: Richard Irwin, 1956.

Descartes, Rene The Philosophical Works of Descartes I. translated by Elizabeth S. Haldane and G.R.T.Ross, Cambridge: Cambridge University Press, 1931.

De Vries, Jan The Economy of Europe in an Age of Crisis 1650-1750. Cambridge: Cambridge University Press, 1976.

Dijksterhuis, E.J. Simon Stevin The Hague: Martinus Nijhoff, 1970.

Diophantus of Alexandria Arithmetik translated by Arthur Czwalina, Göttingen: Vandenhoeck, 1952.

Dobb, Maurice Studies in the Development of Capitalism New York: International, 1963.

Drake, Stillman "Bradwardine's Function Mediate Denomination and Multiple Continua". Physis 12, 1970.

....., ... and I.E.Drabkin editors Mechanics in Sixteenth Century Italy. Madison: University of Wisconsin Press, 1962.

Durkheim, Emile The Elementary Forms of the Religious Life translated by Joseph Swain, New York: Free Press, 1965.

Dugas, Rene A History of Mechanics translated by J.R.Maddox London: Routledge and Kegan Paul, 1957.

Edelstein, Ludwig "Recent Trends in the Interpretation of Greek Science" in Phillip Wiener and Aaron Noland editors, Roots of Scientific Thought. New York: Basic Books, 1957.

Estrup, Hector "Oresme and Monetary Theory" Scandinavian Economic History Review 14, 1966.

Euclid of Alexandria The Thirteen Books of Euclid's Elements. translated by Sir Thomas L. Heath, Cambridge: Cambridge University Press, 1908.

Fierz, Marcus Vorlesungen zur Entwicklungsgeschichte der Mechanik. Berlin: Springer, 1972.

Franklin, Allan The Concept of Inertia in the Middle Ages. Boulder: Colorado Associated University Press, 1976.

French, Peter J. John Dee: The World of an Elizabethan Magus. London: Routledge and Kegan Paul, 1972.

Freudenthal, Hans "What is Algebra and What has it been in History" Archive for History of Exact Sciences. V.16, 1976-7.

Galilei, Galileo "The Assayer" in Discoveries and Opinions of Galileo. translated by Stillman Drake, New York: Anchor, 1957.

......., ... "Tractate on the Elements" in Galileo's Early Notebooks. translated by William A. Wallace South Bend: Notre Dame University Press, 1977.

......., ... Two New Sciences. translated by Stillman Drake, Madison: University of Wisconsin Press, 1979.

Geijsbeek, John Ancient Double-Entry Bookkeeping. Houston: Scholars Book Company, 1914.

Geymonat, Ludovico Galileo Galilei . translated by Stullman Drake, New York: McGraw-Hill, 1977.

Grant, Edward Physical Science in the Middle Ages. Cambridge: Cambridge University Press, 1977.

....., ... A Source Book in Medieval Science. Cambridge, Mass.: Harvard University Press, 1974.

Grünwald, Ernst "Systematic Analyses" translated by Rainer Koehne in The Sociology of Knowledge. edited by James E.Curtis and John W. Petras, London: Duckworth, 1970.

Hall, A. Rupert The Scientific Revolution 1500-1800. Boston: Beacon, 1954.

Hamilton, Peter Knowledge and Social Structure. London: Routledge and Kegan Paul, 1974.

ten Have, Onko The History of Accounting. translated by A. von Seventer, Palo Alto: Bay Books, 1976.

........, ... "Simon Stevin of Bruges" in A.C.Littleton and B.S.Yamey editors, Studies in the History of Accounting Homewood, Ill.: Richard Irwin, 1956.

Heath, Sir Thomas L. Diophantus of Alexandria. New York: Dover, 1964.

....., ... A History of Greek Mathematics I, II. Oxford: Clarendon Press, 1921.

Hill, Christopher The Century of Revolution 1603-1714. London: Cardinal, 1974.

...., ... Intellectual Origins of the English Revolution. London: Panther, 1972.

Hobsbawm, E.J. "The Crisis of the Seventeenth Century" in Trevor Aston editor, Crisis in Europe 1560-1660. London: Routledge and Kegan Paul, 1965.

Horowitz, Thekla Vom Logos zur Analogie. Zürich: Hans Rohr, 1978.

Houghton, Walter E. Jr. "The History of Trades: Its Relation to Seventeenth Century Thought" in Wiener and Noland editors, Roots of Scientific Thought . New York: Basic Books, 1957.

Jack, Sybil Trade and Industry in Tudor and Stewart England. London: George Allen and Unwin, 1977.

Jacquot, Jean "Harriot, Hill, Warner and the New Philosophy" in John W. Shirley editor, Thomas Harriot: Renaissance Scientist. Oxford: Clarendon, 1974.

Jayawardene, S.A. "The Influence of Practical Arithmetics on the Algebra of Rafael Bombelli" in Isis. 64, December, 1973.

........., ... "Rafael Bombelli" Dictionary of Scientific Biography New York: Scribners, 1970.

Jeannin, Pierre Merchants of the Sixteenth Century. translated by Paul Fittingoff, New York: Harper and Row, 1972.

Johnson, Francis "Gresham College: Precursor of the Royal Society" in Wiener and Noland editors, Roots of Scientific Thought. New York: Basic Books, 1957.

Jones, Charles V. The Concept of 'One' as a Number. Phd. Thesis, University of Toronto, 1978.

Kamen, Henry The Iron Century London: Cardinal, 1971.

Kargon, Robert Atomism in England from Harriot to Newton. Oxford: Oxford University Press, 1966.

Kearney, Hugh Science and Change 1500-1700. New York: McGraw-Hill, 1971.

Kellenbenz, Hermann "The Organization of Industrial Production" in The Cambridge Economic History of Europe V. Cambridge: Cambridge University Press, 1977.

Kersten, Frederick "The Constancy Hypothesis in the Social Sciences" in Lester E. Embree editor, Life-World and Consciousness. Evanston: Northwestern, 1972.

Klein, Jacob Greek Mathematical Thought and the Origin of Algebra. translated by Eva Brann, Cambridge, Mass.: M.I.T. Press, 1968.

....., ... "Phenomenology and the History of Science" in Philosophical Essays in Memory of Edmund Husserl. Cambridge, Mass.: Harvard University Press, 1940.

Koyré, Alexandre Galilean Studies. translated by John Mepham, Sussex: Harvester, 1978.

....., ... "Galileo and Plato" in Wiener and Noland editors, Roots of Scientific Thought. New York: Basic Books, 1957.

Krader, Lawrence Dialectic of Civil Society. Assen: Van Gorcum, 1976.

......, ... A Treatise of Social Labour. Assen: Van Gorcum, 1979.

Kulisher, Joseph Allgemeine Wirtschaftsgeschichte des Mittelalters und der Neuzeit. München: Oldenbourg, 1968.

LeGoff, Jacques "Merchant's Time and Church's Time in the Middle Ages" in Time, Work and Culture in the Middle Ages. translated by Arthur Goldhammer, Chicago: University of Chicago Press, 1980.

Littleton, A.C. Accounting Evolution to 1900. New York: American Institute, 1933.

Locke, John Second Treatise of Government in Earnest Barker ed. Social Contract. London: Oxford University Press, 1947.

Mackie, J.D. The Early Tudors 1485-1558. London: Oxford University Press, 1952.

Mannheim, Karl Ideology and Utopia. translated by Louis Wirth and E.A. Ahills, New York: Harcourt, Brace and World, 1936.

Marx, Karl Capital I. translated by Samuel Moore and Edward Aveling, Moscow: Progress, n.d.

.... Economic and Philosophic Manuscripts of 1844 Moscow: Progress, 1974.

...., ... Grundrisse translated by Martin Nicholas, Moscow: Progress 1973.

...., ... and Friedrick Engels The German Ideology. New York: International, 1970.

...., ... "The Poverty of Philosophy" in Karl Marx, Friedrich Engels, Collected Works VI. New York: International, 1976.

Mason, Stephen F. A History of the Sciences. New York: Collier, 1962.

Meek, Ronald L. Studies in the Labour Theory of Value. New York: Monthly Review Books, 1956.

Minnaert, M.G.J. "Simon Stevin" Dictionary of Scientific Biography XIII New York: Scribners, 1976.

Minchinton, Walter "Patterns and Structure of Demand" in Carlo Cipolla editor, The Fontana Economic History of Europe III. Glasgow: Collins, 1974.

Mulkay, Michael Science and the Sociology of Knowledge. London: George Allen and Unwin, 1979.

Murdoch, John E. "The Medieval Language of Proportions: Elements of the Interaction with Greek Foundations and the Development of New Mathematical Techniques" in A.C.Crombie editor, Scientific Change. London: Heinemann, 1963.

......., ... and Edith Sylla "The Science of Motion" in David C. Lindberg editor, Science in the Middle Ages. Chicago: University of Chicago Press, 1978.

Murray, Alexander Reason and Society in the Middle Ages. Oxford: Oxford University Press, 1978.

Nef, John U. The Conquest of the Material World. Chicago: University of Chicago Press, 1964.

..., ... Industry and Government in France and England 1540-1640. Philadelphia: American Philosophical Society, 1940.

Nussbaum, Frederick The Triumph of Science and Reason 1660-1685. New York: Harper and Row, 1962.

Oberman, H.A. Archbishop Thomas Bradwardine: A Fourteenth Century Augustinian. Utrecht: Hermink and Zoon, 1957.

Olschki, Leonardo Geschichte der neusprachlichen wissenschaftlichen LiteraturI.III. Heidelberg: Niemeyer, 1919,1927.

Oresme, Nicole The De Moneta of Nicholas Oresme. edited and translated by Charles Johnson, London: Thomas Nelson, 1956.

......, ... Le Livres D'Ethiques D'Aristote. edited by Albert D. Menut, New York: Stechert, 1940.

......, ... De proportionibus proportionum. edited and translated by Edward Grant, Madison: University of Wisconsin Press, 1966.

Pappus of Alexandria Mathematical Collection Viii 1-5. in Morris R. Cohen and I.E.Drabkin Source Book in Greek Science. New York: McGraw-Hill, 1948.

Petty, William "The Advice of W.P. to Mr. Samuel Hartlib for the Advancement of some particular Parts of Learning" in The Harleian Miscellany. London: T. Osborne, 1745.

Polanyi, Karl The Great Transformation. Boston: Beacon, 1957.

Randall, John Herman Jr. "The Place of Leonardo da Vinci in the Emergence of Modern Science" in Wiener and Noland editors, Roots of Scientific Thought. New York: Basic Books, 1957.

Recorde, Robert The Castle of Knowledge. (London, 1556) Amsterdam: Theatrum Orbis Terrarum, 1975.

......., ... First Principles of Geometry (London 1551) Amsterdam: Theatrum Orbis Terrarum, 1974.

......., ... The Ground of Artes (London 1542) New York: Da Capo Press, 1969.

......., ... The Whetstone of Witte (London 1557) Amsterdam: Da Capo Press, 1969.

Renard, G. and G. Weulersse Life and Work in Modern Europe. translated by Margaret Richards, London: Routledge and Kegan Paul, 1968.

Sella, Domenico "European Industries 1500-1700" in Carlo Cipolla editor, The Fontana Economic History of EuropeIII. Glasgow: Collins, 1974.

Shirley, John W. "Sir Walter Raleigh's Guiana Finances" in Huntington Library Quarterly. 18, 1, San Marino, 1949, in Source Book for the Study of Thomas Harriot. New York: Arno Press, 1981.

......., ... Thomas Harriot. Oxford: Oxford University Press, 1983.

Sohn-Rethel, Alfred Intellectual and Manual Labour. translated by Martin Sohn-Rethel, London: MacMillan, 1978.

Steele, Robert The Earliest Arithmetics in English. London: Oxford University Press, 1922.

Stevin, Simon Principle Works of Simon Stevin I. Mechanics. edited by E.J. Dijksterhuis, Amsterdam: C.V.Swets and Zeitlinger, 1955.

......, ... Principle Works of Simon Stevin II A and B, Mathematics. edited by Dirk Struik, Amsterdam: C.V.Swets and Zeitlinger, 1958.

Struik, Dirk The Land of Stevin and Huygens. Dordrecht: Reidel, 1981.

Supple, Barry "The Nature of Enterprise" in The Cambridge Economic History of Europe V. Cambridge: Cambridge University Press, 1977.

Szabo, Arpad Anfänge der Griechischen Mathematik. München, R.Oldenbourg, 1969.

Tartaglia, Niccolo "New Science" translated by Stillman Drake in S. Drake and I.E.Drabkin editors, Mechanics in Sixteenth Century Italy. Madison: University of Wisconsin Press, 1968.

........., ... "Questions and Inventions" translated by S. Drake in S.Drake and I. E. Drabkin editors Mechanics in Sixteenth Century Italy Madison: University of Wisconsin Press, 1968.

Taylor, R.Emmett No Royal Road: Luca Pacioli and His Times. Chapel Hill: University of North Carolina Press, 1942.

Thompson, E.P. "Time, Work-Discipline and Industrial Capitalism" in M.W.Flinn and T.C.Smout editors, Essays in Social History. London: Oxford University Press, 1974.

Tropfke, Johannes Geschichte der Elementar-Mathematik II. Berlin: Walter De Gruyter, 1933.

Unguru, Sabetai "On the Need to Rewrite the History of Greek Mathematics" in Archive for History of Exact Sciences. 15, 1975.

Van Egmond, Warren The Commercial Revolution and the Beginnings of Western Mathematics in Renaissance Florence, 1300-1500. Phd. Thesis, Indiana University, 1976.

Vieta, Francois "Introduction to the Analytic Art" translated by J. Winfree Smith, in Jacob Klein Greek Mathematical Thought and the Origin of Algebra. Cambridge, Mass.: M.I.T. Press, 1968.

Vilar, Pierre A History of Gold and Money. translated by Judith White, London: New Left Books, 1976.

Vogel, Kurt "Bemerkungen zur Vorgeschichte des Logarithmus" in Beiträge zur Geschichte der Arithmetik. München, Minerva, 1978.

...., ... "Fibonacci" in Dictionary of Scientific Biography. New York: Scribners, 1971.

....., ... "Überholte kaufmännische Praktik aus dem Mittelalter" in Beiträge zur Geschichte der Arithmetik. München : Minerva, 1978.

....., ... Vorgriesche Mathematik II. Hannover: Hermann Schroedel, 1959.

de Waal, P.G.A. Van Paciolo tot Stevin. Roermond: Romen and Zonen, 1927.

van der Waerden, B.L. "Defense of a Shocking Point of View" in Archive for History of Exact Sciences. 15, 1975-76.

... ..., ... Science Awakening. translated by Arnold Dresden, Groningen: P. Noordhoof, 1954.

Waschkiess, Hans-Joachim Von Eudoxos zu Aristoteles. Amsterdam: B.P. Gruner, 1972.

Weber, Max The Protestant Ethic and the Spirit of Capitalism. translated by Talcott Parsons, New York: Scribners, 1958.

....., ... "Science as a Vocation" in Hans Gerth and C.Wright Mills editors, From Max Weber. New York: Oxford University Press, 1946.

....., ... The Sociology of Religion. translated by Ephraim Fischoff, New York: Free Press, 1964.

....., ... The Theory of Social and Economic Organization. translated by Talcott Parsons, New York: Free Press, 1947.

Webster, Charles The Great Instauration. New York: Holmes and Meier, 1976.

von der Wee, Hermann "Monetary, Credit and Banking Systems" in Cambridge Economic History of Europe V. Cambridge: Cambridge University Press, 1977.

Wiener, Philip and Aaron Noland editors Roots of Scientific Thought. New York: Basic Books, 1957.

Wilson, Charles "Cloth Production and International Competition in the Seventeenth Century" in Economic History Review. V. 20, 1972.

Wolff, Kurt "Ernst Grünwald and the Sociology of Knowledge" in Trying Sociology New York: Free Press, 1974.

Wolff, Michael Geschichte der Impetustheorie. Frankfurt: Suhrkamp, 1978.