FAST NEUTRON SCATTERING METHOD

FOR

LOCAL VOID FRACTION DISTRIBUTION MEASUREMENT

TWO-PHASE FLOW

IN

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A new method is presented for measuring local void-fraction and its distribution in gas-liquid two-phase flow. The method is based on reconstructing the physical distribution of steam and water (phase distribution) from the energy spectrum of fast neutrons scattered in the test section. The unique relationship between neutron energy and angle of single scatterings in the liquid phase is utilized to reconstruct the phase distribution.

ABSTRACT

Monte Carlo simulations of scattering in the test sections have been used in developing the method. The validity of the Monte Carlo methodology for this purpose is discussed within the logic of the MORSE computer code employed in this work. The usage of the code is verified by simulating actual experiments during the course of the work.

The neutron scattering problem is formulated into non-linear mappings of the phase distribution to the measured neutron spectrum via matrices. The phase distribution is reconstructed by solving the inverse problem posed by the mappings. The non-linearity is overcome by

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applying a series of successive approximations to the mappings. The mathematical foundations of the inverse problem are discussed and its implementation in the form of a numerical algorithm is shown.

The energy spectra of the scattered neutrons are measured for a few test sections, and the results are shown to compare favourably with those obtained from Monte Carlo simulations. The techniques used in the measurements are also discussed.

The conclusion of this study is that the proposed new method can be applied successfully for measuring phase distributions of gas-liquid flows. The method in contrast to other radiation methods, requires only a single exposure of the test section to radiation. The inverse problem of reconstructing the phase distribution is solvable and the required neutron energy spectrum can be measured using available techniques. The only restriction is that the test section diameter has to be of order of one mean-free-path of the source neutrons in the scattering medium (about 100 mm in water for 14 MeV neutrons). This is necessary to reduce the contributions from neutron rescattering that complicate the

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reconstruction problem.

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CHAPTER 1

INTRODUCTION

1.0 <u>SCOPE</u>

The objective of the work presented in this thesis is to develop a new non-intrusive technique for measuring local void fraction. distribution in gas-liquid two-phase flow. A technique is proposed based on reconstructing the liquid phase distribution from information carried by fast neutrons (particularly 14 MeV neutrons) scattered in the section of flow under investigation. The technique, as distinct to normal methods of tomography, requires only a single exposure of the flow section to the radiation source. If the thickness of the scattering material in the object is of the order of one mean-free-path of the incident neutrons in this material (10 cm for 14 MeV neutrons in water), the contribution of neutrons that suffer more than one collision will be small. The energy spectrum of the scattered neutron will then provide information regarding the number and energy of neutrons that suffered single collisions in the object. The number of scattered neutrons is related to the amount of the scattering material in the object. The energy of the neutrons scattered is uniquely related to the angle of scattering and, therefore, provides information about the direction from

which the scattered neutrons were emitted and consequently, the location of the scattering material. Therefore, the energy spectrum carries information about the amount and location of the scattering material. The concentration and distribution of the scattering material in the scattering object may be extracted from the energy spectra of the scattered neutrons. In steam-water two-phase flow, the scattering material is mainly hydrogen and therefore, if the concentration and distribution of water is determined, the concentration of steam (local void fraction) and void distribution is known.

Local void fraction distribution (or flow regime) is an important two-phase flow parameter. It affects interfacial and wall transfer of mass, momentum and heat, and affects the structure of the conservation equations through distribution coefficients required to generate a solvable set of equations. The role played by void distribution in two-phase flow studies is further discussed in Section 3 of the present Chapter, following a section explaining the importance of these studies in industrial applications, particularly in the nuclear industry.

The importance of the new fast neutron scattering technique of L local void fraction distribution presented here is that it does not have the drawbacks associated with existing techniques. The proposed

technique is non-intrusive, requires only a single radiation exposure of the test section under investigation, and can handle flows contained in thick metallic pipes required for high pressure systems. In Section 4 of this Chapter, the existing techniques for void fraction distribution are reviewed, and Section 5 discusses the need for developing a new technique. The discussion illustrates the lack of a technique based on fast neutron scattering, despite the advantages of such a technique.

Fast neutrons, particularly 14 MeV neutrons, are chosen from among different radiation types since they easily penetrate metallic pipe walls, have a reasonably high interaction probability, and because of their mono-energetic nature their path into the material can be traced. Moreover, 14 MeV neutrons can be produced using compact neutron generators, rather than large acceleration facilities such as cyclotrons or Van de Graaf accelerators. Therefore, phase distribution measurements can be done in the laboratory. Section 6 discusses these and other advantages of using fast neutrons, in particular 14 MeV neutrons, for phase distribution measurements.

Before proceeding to the rest of the Chapter, the structure of the thesis is outlined. To develop a new method of measurement, it is desirable to have an experimental tool that can be used to test and investigate different ideas and approaches. A numerical stochastic

experiment (Monte Carlo simulation) was used in the present work. This experimental tool is explained in Chapter 2. Verification of Monte Carlo simulations as being equivalent to experimental tools is discussed in Chapters 2 and 5, by comparing simulations to laboratory measurements. The train of thought that led to the development of the single scattering-single exposure fast neutron technique for the measurement of local void fraction is traced in Chapter 3. As explained before, this technique is based on the information carried by the scattered neutrons' energy spectra. The methods used to extract the distribution of local void fraction from the measurements (the inverse problem) are discussed in Chapter 4. The methods that could be used to measure these energy spectra are presented in Chapter 5. Recommendations for future work to improve the performance of the technique and enhance its capabilities are discussed at the end of Chapter 4, in view of the results of the phase reconstruction process.

2.0

IMPORTANCE OF TWO-PHASE FLOW IN NUCLEAR INDUSTRY

Boiling two-phase flow is an important phenomenon in a wide range of engineering applications. Early two-phase flow studies were aimed at the petroleum industry, mainly to determine the pressure drop of two-phase flows through pipelines. As space and nuclear technologies flourished, the study of boiling heat transfer received great impetus because boiling is an extremely efficient mode of transferring heat for rocket combustion chambers and nuclear reactors. Recent nuclear safety research also provides incentives for the study of transient phenomena in boiling two-phase flow.

Two-phase flow dominates the coolant circuit of boiling water reactors (BWR) during normal and abnormal conditions. In a BWR the coolant is permitted to boil in the core and the vapour produced is used to drive a turbine. A detailed study of two-phase flow of the coolant in the reactor is required for proper design of these reactors. In a pressurized water reactor (PWR), the reactor coolant is kept under pressure high enough to prevent boiling and the coolant flow is normally single-phase. However, boiling two-phase flow may occur during abnormal conditions. For example, a depressurization of the reactor coolant due to a break in the pipe, a failure of a coolant pump or certain other malfunctions may result in boiling of the coolant. Boiling two-phase flow is, however, the normal mode of operation of PWR steam generators where the reactor coolant exchanges its heat with water at lower temperature and pressure in a secondary loop, causing the latter to boil, and hence, generating steam. The steam generated is used to produce useful power by expanding in a turbine. Two-phase flow studies are therefore required for designing the secondary loop of a PWR and predicting the behaviour of the reactor coolant during accidents that are accompanied by boiling.

5.

The above discussion illustrates the importance of two-phase systems in the nuclear industry. Safe operation of a nuclear reactor depends strongly on the behaviour of the reactor coolant. This coolant may undergo phase change during normal and upset conditions of the reactor. Therefore, a good understanding of boiling two-phase coolant flow is essential to safe operation of water cooled reactors.

IMPORTANCE OF PHASE DISTRIBUTION

3.0

The presence of moving internal interfaces makes the analysis of two-phase flows immensely more difficult than in the case of single-phase flows. In particular, the shape and motion of internal interfaces profoundly affect heat, mays and momentum transfer between phases and duct walls. For accurate prefliction of quantities of engineering interest like pressure drop and heat transfer, one has to find appropriate correlations for friction factors and heat transfer These correlations are strongly flow-regime dependent and coefficients. an accurate prediction of the flow regimes is necessary, in order to select the proper correlations for the system under consideration. Flow regimes also have a large effect on quantities like interfacial area, interfacial friction and void fraction (fraction of gas volume in the duct in comparison with total volume). The determination of these quantities is essential for accurate modelling of two-phase flow. In

order to further illustrate the above facts, we consider a set of averaged conservation equation of mass, linear momentum and energy of a two-phase flow.

Since averaged quantities are of engineering interest, one of the main approaches to two-phase modelling has been to average (in time, space, over an ensemble, or in some combination of these) the original instantaneous conservation equations (see for examples References 1 to 6). Let us consider the volume averaged one dimensional conservation equations of mass, linear momentum and energy, as described by Banerjee and Chan⁽⁶⁾.

Mass:

$$\frac{\partial}{\partial t} \alpha_{k} < \rho_{k} > + \frac{\partial}{\partial z} \alpha_{k} < \rho_{k} u_{k} > = - \langle \dot{m}_{k} \rangle_{i} \qquad (3.1)$$

Linear momentum:

$$\frac{\partial}{\partial t} \alpha_{\mathbf{k}} < \rho_{\mathbf{k}} u_{\mathbf{k}} > + \frac{\partial}{\partial z} \alpha_{\mathbf{k}} < \rho_{\mathbf{k}} u_{\mathbf{k}}^{2} > - \alpha_{\mathbf{k}} \frac{\partial \mathcal{D}_{\mathbf{k}}}{\partial z} - \frac{\partial}{\partial z} \alpha_{\mathbf{k}} < \tau \tilde{z}_{z}$$

$$- \Delta P_{\mathbf{k}i} \frac{\delta \alpha}{\delta z} \mathbf{k} = \alpha_{\mathbf{k}} < \rho_{\mathbf{i}} F_{z,\mathbf{k}} - \langle \tilde{\mathbf{m}}_{\mathbf{k}} u_{\mathbf{k}} \rangle - \langle \Delta P_{\mathbf{k}i} \langle \tilde{\mathbf{n}}_{\mathbf{k}} \rangle - \langle \Delta P_{\mathbf{k}i} \langle \tilde{\mathbf{n}}_{\mathbf{k}} \rangle - \langle \tilde{\mathbf{n}}_{\mathbf{k}} u_{\mathbf{k}} \rangle - \langle \tilde{\mathbf{n}}_{\mathbf{k}} u_{\mathbf{k}} \rangle - \langle \Delta P_{\mathbf{k}i} \langle \tilde{\mathbf{n}}_{\mathbf{k}} \rangle - \langle \Delta P_{\mathbf{k}i} \langle \tilde{\mathbf{n}}_{\mathbf{k}} \rangle - \langle \tilde{\mathbf{n}}_{\mathbf{k}} u_{\mathbf{k}} \rangle - \langle \tilde{\mathbf{n}}_{\mathbf{k}} u_{\mathbf{k}} \rangle - \langle \Delta P_{\mathbf{k}i} \langle \tilde{\mathbf{n}}_{\mathbf{k}} \rangle - \langle \Delta P_{\mathbf{k}i} \langle \tilde{\mathbf{n}}_{\mathbf{k}} \rangle - \langle \Delta P_{\mathbf{k}i} \langle \tilde{\mathbf{n}}_{\mathbf{k}} \rangle - \langle \tilde{\mathbf{n}}_{\mathbf{k}} u_{\mathbf{k}} \rangle - \langle \tilde{\mathbf{n}}_{\mathbf{k}} u_{\mathbf{k}} \rangle - \langle \Delta P_{\mathbf{k}i} \langle \tilde{\mathbf{n}}_{\mathbf{k}} \rangle - \langle \Delta P_{\mathbf{k}i} \langle \tilde{\mathbf{n}}_{\mathbf{k}} \rangle - \langle \tilde{\mathbf{n}}_{\mathbf{k}} u_{\mathbf{k}} u_{\mathbf{k}} u_{\mathbf{k}} \rangle - \langle \tilde{\mathbf{n}}_{\mathbf{k}} u_{\mathbf{k}} u_{\mathbf{k}} \rangle - \langle \tilde{\mathbf{n}}_{\mathbf{k}} u_{\mathbf{k}} u_{\mathbf{k}} u_{\mathbf{k}} u_{\mathbf{k}} u_{\mathbf{k}} u_{\mathbf{k}} u$$

Energy:

$$\frac{\partial}{\partial t} \alpha_{\mathbf{k}} < \rho_{\mathbf{k}} E_{\mathbf{k}} > + \frac{\partial}{\partial z} \alpha_{\mathbf{k}} < \rho_{\mathbf{k}} E_{\mathbf{k}} + \frac{\partial}{\partial z} \alpha_{\mathbf{k}} < \rho_{\mathbf{k}} E_{\mathbf{k}} + \frac{\partial}{\partial z} \alpha_{\mathbf{k}} < q_{\mathbf{z},\mathbf{k}} > + \leq P_{\mathbf{k}} > \frac{\partial \alpha_{\mathbf{k}}}{\partial z} + \frac{\partial}{\partial z} \alpha_{\mathbf{k}} < q_{\mathbf{z},\mathbf{k}} > + \sum_{\mathbf{k}} P_{\mathbf{k}} > \frac{\partial \alpha_{\mathbf{k}}}{\partial z} + \frac{\partial}{\partial z} \alpha_{\mathbf{k}} < q_{\mathbf{z},\mathbf{k}} > + \sum_{\mathbf{k}} P_{\mathbf{k}} > \frac{\partial \alpha_{\mathbf{k}}}{\partial z} + \frac{\partial}{\partial z} \alpha_{\mathbf{k}} < q_{\mathbf{z},\mathbf{k}} > + \sum_{\mathbf{k}} P_{\mathbf{k}} > \frac{\partial \alpha_{\mathbf{k}}}{\partial z} + \frac{\partial}{\partial z} \alpha_{\mathbf{k}} < q_{\mathbf{z},\mathbf{k}} > + \sum_{\mathbf{k}} P_{\mathbf{k}} > \frac{\partial \alpha_{\mathbf{k}}}{\partial z} + \frac{\partial}{\partial z} \alpha_{\mathbf{k}} < q_{\mathbf{z},\mathbf{k}} > + \sum_{\mathbf{k}} P_{\mathbf{k}} > \frac{\partial}{\partial z} + \frac{\partial}{\partial z} \alpha_{\mathbf{k}} < q_{\mathbf{z},\mathbf{k}} > + \sum_{\mathbf{k}} P_{\mathbf{k}} > \frac{\partial}{\partial z} + \frac{\partial}{\partial z} \alpha_{\mathbf{k}} < \frac{\partial}{\partial z} - \frac{\partial}{\partial z} + \frac{\partial}{\partial$$

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The sympol $\langle \cdot \rangle$ in the above equations represents the volume average of the variable considered, while $\langle \cdot \rangle_i$ is its value averaged over the interfacial area, $\langle \cdot \rangle_i$ is the variable average over the wall surface with which the phase considered is in contact. The subscript k refers to the value of the variable for the phase k; i to its value at the interface between the phases, and w refers to the variable's value at the wall surface, while the subscript z refers to the variable's value at the wall surface, while the subscript z refers to the direction of the flow. The phases volume fraction is denoted by α , density by α , velocity by \mathfrak{h} , pressure by \mathfrak{p} , energy by E and enthalpy by h. The interphase mass transfer is denoted by \mathfrak{m}_k ; \underline{F} is the external force, \underline{i} , is the stress tensor; \underline{n} is the unit normal vector; E is the total energy; \underline{q} is the heat flux and Q is the external heat source. Δp_{ki} is the difference between the average interfacial and average phase pressure and Δp_{ki} is the difference between the local and average interfacial pressures.

 $- \langle [\hat{\mathbf{m}}_{k} (\mathbf{h}_{k} + \frac{\mathbf{u}^{2}}{2}\mathbf{k})_{k}^{+} + \underline{\mathbf{n}} \cdot \underline{\mathbf{v}}_{k} \Delta \mathbf{p}_{k} + \underline{\mathbf{n}}_{k} \cdot \underline{\mathbf{q}}_{k} - \underline{\mathbf{n}}_{k} \cdot \underline{\mathbf{v}}_{k} \cdot \underline{\mathbf{t}}_{k} \rangle$

 $\frac{\langle (\underline{n} \cdot \underline{q}) \rangle}{kw k w k w k k k k k k}$ (3.3)

To obtain a working set of equations, the averages of products of the dependent variables must be related to the product of averages.

This requires considering the distribution effects. It is usual to define average velocities and enthalpies as (5):

$$\overline{\langle \mathbf{u} \rangle}_{\mathbf{k}} = \alpha_{\mathbf{k}} \langle \mathbf{u}_{\mathbf{k}} \rangle$$

$$\overline{\langle \mathbf{u}_{\mathbf{k}} \rangle}_{\mathbf{k}} = \alpha_{\mathbf{k}} \overline{\langle \mathbf{u}_{\mathbf{k}} \rangle}_{\mathbf{k}}$$

$$(3.4)$$

$$(3.4)$$

$$(3.5)$$

where the overbar denotes time averaging. Definition (3.4) allows the mass conservation equation to be written in terms of $\frac{1}{1}k'$ $\overline{\rho}_{k}$, and $\overline{\langle u_{k} \rangle}_{k}$. However, for the momentum and energy equations, it is required to define various distribution coefficients. The momentum conservation equation requires distribution coefficients of the form:

$$= \frac{\alpha_{k} < u_{k} ^{2} \times k}{\overline{\alpha}_{k} < \overline{u}_{k} \times k}$$

and the energy equation requires:

$$\frac{\alpha_{k} < h_{k}u_{k} > k}{\overline{\alpha}_{k} < \overline{h}_{k} > k < \overline{u}_{k} > k}}$$

$$= \frac{\alpha_{k} < u_{k}^{3} > k}{\overline{\alpha}_{k} < \overline{u}_{k} > k} \qquad (3.8)$$

$$\frac{\alpha_{k} < u_{k}^{3} > k}{\overline{\alpha}_{k} < \overline{u}_{k} > k} \qquad (3.8)$$

(3.6)

If the forms of these distribution coefficients are known then the left-hand-sides of the conservation coefficients can be written in terms of $\bar{\rho}_k$, $\bar{\alpha}_k$, $\overline{\langle u_k \rangle}_k$ and $\overline{\langle h_k \rangle}_k$.

At the present stage of knowledge, very little data on distribution coefficients are available, so they are generally set equal to unity. A proper evaluation of these distribution coefficients requires velocity and phase volume fraction profiles. The technique presented in the present work is aimed at providing the phase volume fraction distribution of the system considered and consequently enabling a more adequate formulation of a working set of averaged conservation equations.

The phase volume fraction distribution also plays an important role in determining the terms in the right-hand-side of the mass, momentum and energy conservation equations. Estimation of these terms requires interphase relationships phrased as functions of the average dependent variables $\frac{\pi}{k}$, $\frac{\sigma}{k}$, $\frac{\langle u_k \rangle}{\langle u_k \rangle}$, and $\frac{\langle h_k \rangle}{\langle h_k \rangle}$. These relationships are supplied largely on the basis of empirical information. The empirical expressions are based on relationships requiring concepts like friction factors, drag coefficients and heat transfer coefficients. Consider for example, the interfacial friction term $\frac{\langle n_k \rangle}{\langle h_k \rangle}$ is that appears on the right-hand-side of the momentum equation, (being time averaged here). This term is sometimes modelled by a friction term of the form $\langle a | B_k(u_1 - u_2) \rangle$, where a_i is the local interfacial area, B_k is a coefficient involving densities, perhaps local velocity, and a characteristic local length scale, $(u_1 - u_2)$ is the time average local velocity difference. Clearly the proper evaluation of the coefficient B_k requires the knowledge of the velocity and <u>phase volume fraction profiles (or distributions)</u>. Similar relationships are used for the interfacial heat transfer q_{ik} in terms of a local temperature difference and heat transfer coefficients. The proper estimation of the velocity and phase volume fraction profiles, Relationships involving a phase-wall interface also requires the knowledge of the phase distribution.

To summarize, the phase distributions (or flow regime) affect interfacial and wall transport of mass, momentum and heat, and affect the structure of the conservation equations through distribution coefficients. Therefore, determination of local phase distribution is necessary for adequate modelling of two-phase systems.

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LOCAL VOID FRACTION MEASUREMENT

Measuring phase volume fraction distribution is equivalent to measuring local void fraction distribution in gas-liquid flows. Various techniques have been developed for local void fraction measurements. A

comprehensive review of most of the techniques is available in the book by Hewitt⁽⁷⁾ and the review article by Banerjee and Lahey⁽⁸⁾. A brief summary is given here.

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In the present short review of the different methods used for local void fraction measurements, techniques will be classified either as interfering (probe based) or non-interfering (radiation or optical methods). By an interfering method it is meant that the measurement device interferes with the fluid flow.

4.1 Interfering: Point Contact Based

Interfering, methods depends on the insertion of small probes directly into the fluid(s). These probes come in touch with the fluid and electrical, thermal or optical properties of the fluid can be measured. If these properties depend on the concentration of the phases, the void fraction at the contact point can be deduced from the measurements. The following is a summary of some of the methods used for local void fraction measurements using contact probes.

4.1.1 Local Impedance Probes

Since the electrical impedance of a two-phase flow depends on the concentration of the phases, the impedance measurement can be used for virtually instantaneous estimates of the void fraction. Impedance probes built as co-axial devices are used for two-phase flow impedance measurements. These probes can be constructed by using either direct current (D.C.) or alternating current (A.C.) electric fields. In D.C. excitation conductivity dominates, while in high frequency A.C. excitation capacity dominates.

The easiest way of exciting a local impedance probe is with D.C. For this type of excitation, more current flows when the probe tip is in liquid (the conductive medium) than in vapour. This technique has been extensively used in steam/water experiments (9,10). Unfortunately, ions in the liquid phase will cause polarization unless the probe voltage is very small. Thus D.C. excitation requires low voltages. This results in problems with the processing of low level signals. Moreover, even at low voltages, electrochemical attack and deposits will age and damage the probes. This technique is also very sensitive to water resistivity changes due to temperature, and/or water purity, changes.

A.C. excitation at fairly low frequency (in the Hz to kHz range) has the advantage of relatively low cost and can eliminate probe damage and polarization problems. However, the signal must be carefully processed because the frequency of the void fraction fluctuations may be comparable to the excitation frequency.

While A.C. excitation is suitable for many situations, radio frequency (R.F.) excitation may also be considered where accuracy and speed of response are important. R.F. excitation senses the dielectric constant of the fluid and is, therefore, insensitive to liquid phase resistivity changes. Thus, when the temperature dependence of the liquid phase dielectric constant is known, the probe can be compensated during transients, if a transient fluid temperature reading is simultaneously taken.

4.1.2 Hot Film Anemometry

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Several investigations, such as Hsu et al⁽¹¹⁾ and Delhaye⁽¹²⁾, have used hot wire and hot film anemometry to study two-phase flows. The basic principle of the device is based on Joule heating and conductive cooling of the probe element. Usually the probe is run as a constant temperature device. In, this case, the probe temperature is maintained constant by varying the current through the resistive heating element to compensate for the convective heat losses. A histogram showing probability of current signal level as a function of signal level shows two peaks, one representing the liquid phase flow and the other gas flow. The void fraction can be obtained by directly integrating the areas in the histogram occurring for the two regions.

4.1.3 <u>Microthermocouples</u>

The classical microthermocouple enables measurement of some statistical characteristics of the temperature, and if combined with an 'electrical phase indicator, data regarding the local void fractions may be obtained. The work done by Delhaye et al⁽¹³⁾ is based on the possibility of separating the temperature of the liquid phase from the temperature of the vapour phase, and of giving the statistical properties of each phase as well as the local void fraction. These workers use an insulated microthermocouple both as a temperature measuring instrument and as an electrical phase indicator by using an electrical bridge to sense the presence of a liquid conductor between the noninsulated junction and the ground.

4.1.4 Optical Probes⁽¹⁴⁾

An optical probe is sensitive to the change in the refractive index of the surrounding medium and is thus responsive to interfacial passages enabling measurements of local void fraction. Usually, the probe is designed to have total internal reflection when the probe tip is surrounded by air; whereas when it is surrounded by liquid, the light is lost by refraction.

Though the principle of operation of optical probes is straightforward, the use of optical probes in high temperature systems is fraught with many difficulties. High-temperature probes require the use of either a high-temperature fiber (e.g., Sapphire) or an appropriate tip (e.g., Diamond). Moreover, the ray tracing analysis has to take into account the fact that, due to the properties of fiber optic wave guides, none of the light rays will be travelling in a coaxial direction when they strike the tip. Using a diamond tip, while it looks promising, faces numerous technical problems. A satisfactory diamond-to-sheath seal, thermal expansion matching between various components, a suitable material for handling the glass fibers and a suitable optical bonding between the fiber optic bundle and the diamond tip are among the formidable but not impossible, difficulties facing the development of high temperature bi-material local optical probes.

4.2 Non-Interfering: Radiation Based

4.2.1 Radiation Transmissions

By using a finely collimated beam and crossing it through the channel, it is possible to deduce the diametral distribution of void fraction. In round tubes, this type of measurement is often called "chordal mean measurement". By unfolding a series of chordal-averaged measurements the local void fraction can be obtained. Neutron transmission measurements were utilized by Zakaib et al ⁽¹⁵⁾ for investigating two-dimensional void distribution. Gamma-ray attenuation (absorption) techniques have been used ^(16,17) for chordal mean measurements. X-ray absorption was also employed for void fraction measurements by Lahey ⁽¹⁸⁾ and many others. Light attenuation is particularly effective at both low and high void fraction of known particle size, where gamma and x-ray attenuation methods are not sensitive. Infrared absorption methods are attractive at high qualities (Barschdorff et al ⁽¹⁹⁾). Beta-ray attenuation is also a sensitive method for determining average void fraction in the beam path (Zirnig ⁽²⁰⁾).

While attenuation of various types of radiation is by far the most commonly used non-interfering method for void fraction measurements, it estimates only a chordal averaged void fraction. Therefore, a set of radiation attenuation measurements at different angles is required in order to result to a local void fraction measurement. This requires a rotation of either the radiation source or the test section, a tedious procedure that is neither preferred by the radiation shielding requirements nor the time response of fast transient systems, not to mention the mechanical difficulties. X-ray computer assisted tomography (CAT) ⁽²¹⁾ is a good example of this tedious process. X-ray exposures of the section under investigation are recorded at different directions. Every exposure would represent a group of chords set parallel to the direction of exposure. The integrated void fraction along each chord is then estimated, resulting in a set of chordal void fractions for this
specific projection. Similar sets are obtained for the other different projections, resulting in a group of projective data. The void fraction at every point is determined, using an unfolding computer algorithm, from this group of projection data.

Gamma rays are also used by Hau and Banerjee⁽¹⁷⁾ for these computerized assisted techniques instead of x-rays. However, x-rays systems are in some cases superior to the gamma systems since high source intensities can be obtained. Therefore, they are suitable for very fast transients if the shielding required for the equivalent gamma source cannot be accommodated. However, they are not particularly stable and fairly sophisticated analysis of the the signals is necessary to compensate for fluctuation and drift in the photon flux.

Light attenuation is sensitive for both very thin and very thick hydrogeneous media, particularly for bubbly or droplet flow. This is because for these flows the light refraction is minimum and the light attenuation is directly proportional to the effective interfacial area. Nevertheless, light attenuation is used only in transparent (low pressure) ducts. Optical digital interferometery has been used for phase distribution measurements in bubbly and droplet flows by Lahey and co-workers⁽⁸⁾. Infrared absorption techniques are useful at high void fractions in steam-and-water systems⁽¹⁹⁾. For these systems, there are infrared wave lengths which are not affected by the air or steam; others which are strongly absorbed by steam, and a third group which are strongly absorbed by water. Therefore infrared absorption techniques can yield information on the steam/air fraction, steam fraction and liquid fraction. However, these techniques are limited to high void fractions because of the small value of the infrared absorption coefficients.

Microwave absorption is also used⁽²²⁾ for void fraction measurement in steam-water systems. The resonant frequency of a microwave cavity, which forms part of the flow line, is related to the mean hydrogen density.

Beta-particle attentuation is also sensitive to hydrogeneous materials. Beta particles have a short range in hydrogeneous media compared to photons. Therefore, they are more sensitive than photons for the diagnosis of hydrogeneous media. However, the high attenuation suffered by beta particles in heavy metals restricts their usage to non-metallic containers.

Neutrons are attractive for void measurement in flows containing hydrogeneous materials, i.e., steam water flows. In general neutrons are less affected than gamma or x-rays by the (usually) metal walls containing the flow, and more by the materials comprising the flow itself (provided they are hydrogeneous). However, this strong sensitivity affects the counting statistics of the attenuated neutron beam, especially if thermal neutrons are used. Better counting statistics can be obtained if epithermal or fast neutrons are used.

Regardless of the type of radiation used in the attenuation (absorption) techniques, sets of exposures at different angles of views are required for a detailed void fraction distribution. This requires a tedious procedure of rotating either the radiation source or the object under investigation. This procedure might not be possible for very fast transient measurements.

4.2.2 Radiation Scattering

The analysis and use of side-scattering of gamma beams has been discussed by Zielke et al⁽²³⁾, Kondic and Lassahn⁽²⁴⁾, and by Kondic and Hahn⁽²⁵⁾ among others. The basic principle used is Compton scattering, in which a unique relationship between the energy of the scattered photon and its direction exists for each element. With detectors that have directional as well as energy resolution, the intensity of the scattered radiation can be measured and can be interpreted to give the local void fraction. Hours of counting are

required in order to get good accuracy, even with strong radiation sources. This is because of the very small macroscopic cross section for Compton scattering by hydrogeneous materials (low electron density). When metallic pipe walls are involved, Compton scattering by the metal will dominate the scattering, (since metals have higher electron density), and the scattering by water will be difficult to interpret.

Scattering of other types of radiation may also be used to measure local void fraction. Light scattering can be utilized if there is a small amount of scattering material in the beam path. Otherwise multiple scattering occurs and makes interpretation of scattering data extremely difficult.

Fast neutron scattering is a very promising technique for local void fraction measurements. This is because hydrogen nuclei are very effective neutron scatterers and because of the unique relationship that exists between the energy and direction of a neutron scattered by a proton. Though neutron scattering has been used for global void fraction measurements, (see for example Banerjee and co-workers ^(26,27,28)), no use of fast neutron scattering for local void fraction measurements has been yet reported. The present work investigates a local void fraction measurement method using fast neutron scattering by hydrogen. The merits and difficulties associated with this technique will be discussed throughout the present report.

Another possible use of radiation for local void fraction measurements in two-phase flow is to activate one of the phases. Then by counting the emitted radiation using two collimated detectors with intersecting fields of view, the density of the emitting radionuclide in the intersection volume can be determined. For a vapour-water system, oxygen can be activated using high energy neutrons (around 10 MeV), resulting in N¹⁶ nuclei which emit a strong hard (1.6 MeV) gamma ray with a short (7.11 sec) half-life. In order to obtain a direct estimation of the phase density, a uniform activation of the phases is required. Non-uniform activation causes non-equal contribution of the activated zones, resulting in a complicated interpretation of the emitted radiation intensity. To our knowledge, no use of the emission technique for phase dist ibution measurements has been reported, though neutron activation has been used for flow measurements and for flow pattern

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4.2.4 <u>Ultrasolic Methods</u>

Ultrasonic techniques also appear to be promising for local void fraction measurements. When an emitted ultrasonic wave reaches a vapour/liquid interface, there is a large acoustic impedance (density times sound speed) mismatch. This mismatch causes a signal to be

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4.2.3

Radiation Emission

returned to the transducer. The time difference between the emitted wave and the returning wave yields the location of the gas-liquid interface. Therefore, bubble or droplet location and size can possibly be determined, which makes the technique useful for droplet and bubbly flows.

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The application of acoustic methods (including ultrasonic) is inherently limited by interference of the reflected signals and by low signal-to-noise ratio. Ultrasonic transducers for example, respond to pressure pulses in the system, adding noise to the original reflected signal. Pulses due to reflections by the container wall also contribute to the noise and the interference.

5.0 NEED FOR NEW DEVELOPMENT IN LOCAL

VOID FRACTION MEASUREMENT TECHNIQUES

To appreciate the need for new development in local void fraction measurement techniques, the difficulties associated with the existing techniques are summarized below.

Point contact probes, though they have the advantages of fast response and relative simplicity of operation, cause considerable flow disturbance. Further, their useful lifetime in high temperature or corrosive environments is usually limited. Since the probes are intrusive, hydrodynamic and surface tension effects lead to signal

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disturbance, which results in the need for proper processing of the signal to recognize the phases. This requires calibration of the probe using a radiation based method as a standard.

Light transmittance and scattering methods are now widely used in two-phase flow systems ⁽¹⁴⁾. However, there are still optical access and signal processing problems which make these techniques expensive and difficult to apply.

From a practical point of view, gamma-ray and x-ray attenuation devices are the most highly developed. They essentially measure chordal void fraction and a set of measurements is required in order to obtain local void fraction distribution. This requires rotation of the radiation source around the test section, or perhaps multiple sources and detectors; a cumbersome operation which requires massive radiation shields and very fast scanning systems in order to respond to fast transients in the two-phase flow, famma-ray side scattering methods do not need this rotation procedure, however their obvious disadvantages lies in the long counting time necessary to obtain adequate accuracy.

The main attraction of radiation-based method is their non-intrusive nature. A detailed measurement of the local phase concentration density can be measured without any interference with the fluid flow. However, a detailed estimation of the **rece**l void fraction requires either a set of different chordal void measurements, (multiple radiation exposure), or a long counting time, (in order of hours), of side-scatter photons. Transient two-phase measurements might not afford the time required for multiple radiation exposures or long counting times necessary to obtain adequate accuracy in side-scatter photon technique: Therefore, a single exposure of strongly scattered radiation is desirable for transient two-phase flow local phase concentration density measurements.

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In conclusion, existing local void fraction measurement techniques have their drawbacks. Point contact probes cause flow disturbance. Radiation attenuation (absorption) techniques, though non-intrusive, require more than one exposure of the object to the radiation, a process that is neither convenient nor suitable for fast transient measurements. Radiation scattering techniques do not require more than one exposure but only gamma scattering techniques have been considered. Keeping in mind that gamma scattering techniques are based on Compton scattering, one can directly conclude that it is not wise to consider these techniques for steam-water system contained in thick

metallic pipes.

If a new method of local void fraction measurement is to be developed, it appears from the above discussion that it should concentrate on developing radiation scattering techniques. This is because radiation techniques have no mechanical interference with the flowing medium and therefore they do not influence the two-phase flow phenomena under investigation. Also scattering of radiation provides the main advantage of requiring only a single exposure. This advantage gives scattering techniques a potential applicability in fast transient system and also reduces considerably the radiation shielding requirement in comparison to attenuation techniques, where more than one radiation exposure is required. Apparently, a new development in radiation scattering techniques should consider finding another type of radiation instead of gamma radiation, since the latter suffers from very low probability of scattering with hydrogeneous media.

One can consider acoustic and optical methods, since these methods, in addition to being non-intrusive, (in contrast to point contact methods), are also easier and safer to handle than radiation methods. However, optical methods are not suitable for use with flows inside thick metallic components. Also, acoustic methods seem to be influenced by the physical phenomena occurring in the flow under investigation, especially the dispersion, damping and propagation of finite amplitude pressure

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waves. For these reasons the development in the present work is concentrated on radiation methods, particularly fast neutrons, rather than optical or acoustic methods. The following section summarizes the advantages that fast neutrons enjoy over other radiation types.

FAST NEUTRONS: A DIAGNOTIC TOOL

6.0

If a radiation scattering method for local void fraction measurement in steam-water system is to be developed, a suitable radiation particle has to be used. By a-suitable radiation particle, it is meant that the particle has a high probability of being scattered by water and the capability of penetrating the container walls without being severely attenuated. Charged particles (beta rays, protons and alpha particles) and gamma and x-rays are sharply attenuated by metallic media (system containers). Thermal neutrons have a high probability of scattering in water and are not attenuated severely by metals. However, thermal neutrons are subject to multiple scatterings in water, making it difficult to interpret the scattering data and relate them to the void distribution. The scattering of thermal neutrons in hydrogeneous media was studied by Archer and Harms (30) and the difficulty of interpreting, the scattering of thermal neutrons is shown in their study, Fast neutrons have proved to be useful for global void fractions (26,27,28)

The scattered thermal neutrons resulting from the scattering of fast neutrons were measured. The flux of these scattered thermal neutrons was proved to be proportional to the water content and consequently the void fraction. This is because thermalization of fast neutrons is caused by the water (not the void) in the system and the amount of thermalization, (and consequently the scattering thermal flux) depends on the amount of water present. However, thermal neutrons are subject to multiple scattering and consequently they lose memory of the details of their passage, making scattering data hard to interpret and relate to the void distribution.

Neutrons have unique characteristics when scattered by the hydrogen nucleus, since both particles have almost the same mass. A neutron scattered by a proton might lose between nothing and its whole energy. The amount of energy loss is uniquely determined by the angle of scattering. A proper measurement of the angle and energy of the scattered neutrons should reflect; in principle at least, the amount of hydrogen in the chord facing the detector. A single exposure of the whole test section to fast neutrons should be sufficient in order to obtain the local distribution of water or hydrogen in the test section. Therefore, fast neutron scattering could be used for local void measurements in fast transient flows. In conclusion, fast neutrons are the most suitable particles for radiation scattering techniques for local void fraction measurements in steam-water systems. If one exposes the flow test section to fast neutrons and records the scattered neutrons, instead of thermal neutrons, one may be able to relate the fluence of the scattered neutrons to the phase distribution. This is because scattered fast neutrons suffer few collisions in the test section and their path in the flow material can be traced back. In this study, we restrict ourselves to monoenergetic neutron sources, since one can directly relate the energy of the scattered neutron to its direction of scattering. Because of this restriction the use of radioactive isotopic sources is discarded, since they produce multienergetic neutrons. For the same reason neutrons produced by nuclear reactors are not considered. Monoenergetic neutrons can be produced by accelerator sources using a suitable nuclear reaction.

Neutron-producing reactions involve the interaction of a bombarding particle (or nucleus) and a target nucleus (or particle). The result is an excitation of the target or a temporary coalescence of the two bodies. Depending on the energies and atomic species, various radiations and/or particles emerge, leaving a residual hucleus which may or may not be radioactive. Two of the most prolific and common neutron production reactions are:

The D-D reaction:

$${}^{2}_{H} + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + {}^{1}_{0}n$$
, Q value = + 3.26 MeV

The D-T reaction:

 $\frac{2}{1}H + \frac{3}{1}H + \frac{4}{2}He + \frac{1}{0}n$., Q value = + 17.6 MeV

The reactions do not need acceleration of the bombarding deuterons to a very high energy in order to create a significant neutron field. A 1 ma béam of deuterons produces about 10^9 neutrons/second from a tritium target ⁽³¹⁾. These reactions are widely used in "neutron generators" in which deuterium ions are accelerated by a potential of about 100-300 keV. The energy of the emerging neutrons is a function of incident ion energy and observation angle with respect to the ion beam direction. For an observation angle of 90°, 2.47 MeV neutrons are produced from the D-D reaction and 14.1 MeV neutrons for the D-T reaction ⁽³²⁾.

A number of other charged particle reactions are also applied to neutron generation. Some common examples are 9 Be (d,n), 7 Li (p,n), and 3 H(p,n) reactions. In these reactions, a high incident neutron energy is required, and large accelerator facilities such as cyclotrons or Van de Graaff accelerators are needed to produce the incident particle beam.

The D-T reaction was chosen for use in the present study. Since this reaction results in a neutron energy of 14 MeV, test sections in the order of 100 mm in diameter can be considered by the phase distribution measurement technique proposed here. This technique depends on the measurement of the energy of the scattered neutrons. Scattering of 14 MeV neutrons produces neutrons in the MeV range. In this range, measurement of the neutron energy can be achieved using small detectors, consequently enabling one to directly relate the energy to the angle of scattering. Also, neutron energy measurement in the MeV range is relatively less complicated as compared with that in the keV range, since in the latter, discrimination against gamma-rays that usually accompany neutrop, production is more difficult.

The D-T reaction was also chosen since compact neutron generators consisting of a sealed tube containing the ion source and target, together with a portable high-voltage generator, can be utilized. This facilitates the use of the neutron generator in two-phase flow laboratories. D-T neutron generators have been already used for measuring the flow rate, making use of the gamma-ray field resulting from the ${}^{16}O(n_7p){}^{16}N$ reaction in a steam water mixture, see, for example Reference (29).

CHAPTER 2

MONTE CARLO NUMERICAL EXPERIMENTS

1.0 INTRODUCTION

cost.

In order to study the feasibility of using fast neutron. scattering for local void fraction measurements in steam-water flows, a neutronic experiment is required. Setting up a laboratory neutronic experiment is quite expensive, though practical, especially if a parametric study is to be done. Neutron sources of different energies have to be available, in order to study the effect of source neutron energy on the phenomenon. Different types of neutron detectors also have to be available to detect neutrons of various energies. Each of these detectors may require its own special set of electronics. An electronic system capable of discriminating between gamma-rays and neutrons is needed to neutralize the effect of.gamma-ray background. All these requirements make it hard to test different concepts at a reasonably low-

Fortunately a numerical experiment can be used to simulate neutronic systems by using Monte Carlo methods. The Monte Carlo method is a stochastic numerical experiment in which mathematical and physical

problems are solved by the sampling of random quantities. In this method, it is possible to simulate completely the neutron source, medium geometry and neutron detectors. Insight into the behaviour of the neutron field in the system can be obtained since the method keeps track of neutron energy, position, direction, collisions, etc. Alterations in the characteristics of the system can be easily implemented in the Monte Carlo experiment.

One does not have to develop his own Monte Carlo computer program. Computer codes are now available for Monte Carlo experiments. Probably, MORSE⁽¹⁾ of Oak Ridge National Laboratory and SAM-CE⁽²⁾ of the Mathematical Applications Group Inc.; of Elmsford, New York, are the best known Monte Carlo codes. Recently, Los Alamos Scientific Laboratroy has released its general Monte Carlo code for neutron and photon transport - MCNP⁽³⁾, and the CEA/CEN SERMA Shielding laboratory, Saclay, France has contributed the TRIPOLI II code⁽⁴⁾.

A brief description of each of these codes is given in the following.

. The MORSE code solves multi-group neutron, gamma ray or coupled neutron-gamma ray transport problems. It contains a three dimensional combinatorial geometry package that can conveniently handle different

types of complicated geometries. MORSE is an extremely sophisticated code that is usually modified for each problem analyzed. A user of MORSE must tailor the code to meet his own specifications.

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SAM-CE, is a Monte Carlo code designed to solve the time-dependent neutron and gamma ray transport equation in complex three-dimensional geometries. A special advantage of SAM-CE is its tabulation and use of neutron cross section data in detailed precise point energy meshes. Such treatment avoids the ambiguities associated with multigroup codes (MORSE for example), which use flux-averaged cross sections based on assumed flux distributions which may or may not be appropriate.

MCNP is a continuous-energy, generalized geometry, coupled neutron-photon transport code. MCNP treats an arbitrary three-dimensional configuration of arbitrary materials in geometric cells bounded by first and second degree surfaces and some fourth-degree. surfaces (elliptical torr), (unlike MORSE and SAM-CE where combinatorial geometry techniques are used). MCNP is widely used at Los Alamos Scientific Laboratory, basically for military purposes. It has been

recently released for general use.

TRIPOLI II is also a general purpose Monte Carlo code. It treats geometry as an arrangement of homogeneous volumes, defined by portions of plane and quadratic surfaces. This geometry configuration can be made repetitive by relocation, rotation or symmetry. The cross? sections are represented either as fine multigroup meshs (250-300 groups), or in a pointwise fashion.

A few years ago, in choosing the appropriate Monte Carlo code, the selection was limited to MORSE and SAM-CE, because the others were not available yet. Since the main material to be handled was water, where violent resonances in cross-sections do not exist, a multigroup gode was sufficient. Therefore it was decided to use the MORSE code. If at the time of choice, all the four codes were available, one would discard SAM-CE and MCNP codes due to their needless time consuming pointwise cross-section representation. The MORSE code would be preferred to the TRIPOLI II code, since the former code uses the handy combinatorial geometry formalization while the later uses a tedious analytic geometry representation. The main features of the MORSE code are given in the following section, followed by an example showing how the code can be used for simulating laboratory experiments.

THE MORSE COMPUTER CODE

The Multigroup Oak Ridge Stochastic Experiment MORSE, is a Monte Carlo neutron and photon transport code. It can solve multigroup transport problems utilizing neutron or gamma ray cross-section libraries. A three dimensional combinatorial geometry package ⁽¹⁾ that can conveniently handle all types of geometries is used. Other features of MORSE are:

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Solution of either the forward or adjoint radiation transport problems

An albedo (radiation reflection) option at any material surface Time dependence provided by keeping track of the chronological age of each particle

Several importance sampling (biasing) options available.

The following illustrates how the MORSE code solves the Boltzmann transport equation.

The Boltzmann Transport Equation

The goal of the transport theory is to determine the particle • distribution at any location, energy, time and direction. With several fundamental assumptions conservation of neutrons is expressed by the

2.0

2.1

linear Boltzmann transport equation⁽⁵⁾. Briefly, it is assumed (a) that neutrons flow without change of direction or speed until they either interact with atomic nuclei in the considered domain or escape, (b) that neutrons do not appreciably alter the medium with which they interact within the time interval considered and (c) that quantum mechanical effects are unimportant. The Boltzmann transport equation is

derived using a bookkeeping process that sets the net storage of particles within a differential element of phase space ($drdEd \Omega$) equal to the particle gains minus particle losses in ($drdEd \Omega$) leads to the following familar general time-dependent integro-differential form of the Boltzmann transport equation.

$$\mathbf{1} = \phi(\mathbf{r}, \mathbf{E}, \mathbf{\overline{\Omega}}, \mathbf{t}) + \mathbf{\overline{\Omega}} \cdot \nabla \phi(\mathbf{\overline{r}}, \mathbf{E}, \mathbf{\overline{\Omega}}, \mathbf{t}) + \Sigma_{+}(\mathbf{\overline{r}}, \mathbf{E}) - \phi(\mathbf{\overline{r}}, \mathbf{E}, \mathbf{\overline{\Omega}}, \mathbf{t}) -$$

$$= [\mathbf{S}(\vec{\mathbf{r}},\mathbf{E}, | \vec{\Omega}, \mathbf{t}) + \mathcal{J}\mathcal{J} | \mathbf{d}\mathbf{E}^{\dagger} \mathbf{d}\vec{\Omega}^{\dagger} | \boldsymbol{\Sigma}_{\mathbf{c}}(\vec{\mathbf{r}},\mathbf{E}^{\dagger} + \mathbf{E}, \vec{\Omega}^{\dagger} | \boldsymbol{M}) | \boldsymbol{\psi}(\vec{\mathbf{r}},\mathbf{E}^{\dagger}, \vec{\Omega}^{\dagger}, \mathbf{t})]$$
(2.1)

where

 $(\bar{r}, E, \bar{\Omega}, t)$ = denotes the general seven-dimensional phase space,

= /position variable,

the particle's kinetic energy,

=, the particle's speed corresponding to its kinetic

energy E,

a unit vector which describes the particle's direction of motion

time variable,

 $\phi(\mathbf{r}, \mathbf{E}, \Omega, \mathbf{t})$ $\phi(\mathbf{r}, \mathbf{E}, \mathbf{\Omega}, \mathbf{t}) \mathbf{d} \mathbf{E} \mathbf{d} \mathbf{\overline{\Omega}}_{=}$

 $\frac{\partial}{\partial t} \Phi(\bar{r}, E, \bar{\Omega}, t)$

d Ed Ω

. ν φ (r, Ε, Ω,

 $dEd\overline{\Omega}$

the time-dependent angular flux, the number of particles that cross a unit area normal to the $\overline{\Omega}$ direction per unit time at the space point \overline{r} and time t with energies in dE about E and with directions that lie within the differential solid angle $d\Omega$ about the unit vector \overline{a} ,

net storage (gains minus losses) per unit volume and time at the space point \vec{r} and time t of particles with energies in dE about E and with directions which lie in d $\overline{\Omega}$ about $\overline{\Omega}$

net convective loss per unit volume and time at the space pointer and time t of particles with energies in . dE about E and directions which lie in $d\overline{u}$ about $\overline{\Omega}$, $\Sigma_{\pm}(\bar{r}, E)$ the total cross sections at the space point $\widetilde{\mathbf{r}}$ for particles of energy E,

> collision loss per unit volume and time at the space point r and time t of particles with energies in dE about E , directions which lie in d $\overline{\Omega}$ about $\overline{\Omega}$, the differential scattering cross section which describes the probability per unit path that a particle with an initial energy E and an initial

 $z_{\pm}(\mathbf{r},\mathbf{E})$. $\phi(\overline{r}, \varepsilon, \overline{\Omega}, t) ded\overline{\Omega}$

 $\Sigma_{\rm e}(\bar{r}, E) \rightarrow E_{\rm e}$ Ω̃'→Ω) dedΩ

direction $\frac{1}{2}$, undergoes a scattering collision at which places it into a direction that lies in $d\overline{u}$ about \overline{u} with a new-energy in dE about E.

 $\{ \text{If } \Sigma_{\mathbf{s}}(\overline{\mathbf{r}}, \mathbf{E}^{\dagger} \rightarrow \mathbf{E}, \overline{\Omega}^{\dagger}, \overline{\Omega}^{\dagger}) \mid \phi(\overline{\mathbf{r}}, \mathbf{E}^{\dagger}, \overline{\Omega}^{\dagger}, \mathbf{t}) \det[d\overline{\mathbb{N}}^{\dagger}] \mid d\overline{\text{ed}} \overline{\mathcal{I}} = 1 \}$

in - scattering gain per unit volume and time at the space point \bar{r} and time t of particles with energies $\frac{1}{2}n$ dE about E and directions which lie in d \bar{z} about \bar{z} , $S(\bar{r}, E, \bar{\Omega}, t) dEd\bar{\Omega}$ = source particles emitted per unit volume and time at the space point \bar{r} and time t with energies in dE about E and directions which lie in d $\bar{\Omega}$ about $\bar{\Omega}$.

An effect of interest, such as biological dose, energy deposition, or particle flux (denoted by λ) for a given problem can be expressed in terms of the flux field $\phi(\overline{r}, E, \overline{n}, t)$ and an appropriate response function $P^{\phi}(\overline{r}, E, \overline{n}, t)$ due to a unit angular flux and is given

 $\lambda = \iiint \mathbf{p}^{\phi}(\mathbf{r}, \mathbf{E}, \overline{\Omega}, \mathbf{t}) \quad \phi(\mathbf{r}, \mathbf{E}, \overline{\Omega}, \mathbf{t}) \quad \texttt{d}\mathbf{r} \texttt{d}\mathbf{E} \texttt{d}\mathbf{\bar{d}}\mathbf{\bar{d}}^{\dagger}$

by:

(Fluence is defined by the above integral if $p^{\frac{1}{2}}=1^{-1}$)

(2.2)

Consistent with the MORSE code, the energy dependence of Equation (1) will be represented in terms of energy groups which are defined such that:

- ΔE_{g} = energy width of the gth group
 - = 1 corresponds to the highest energy group.
 - -= G corresponds to the lowest energy group.

with the obvious constraint that

 $\begin{array}{ccc} G & E & E \\ \Sigma & \Delta E & = & \int & 0 \\ g=1 & g & 0 \end{array}$, the maximum particle energy

A "group" from Equation (2.1) is obtained by integrating each term with respect to the energy variable over the energy interval ΔE_{q} resulting in the group form of the Boltzmann equation:

- $\frac{1}{v_{\rm d}} \frac{\partial}{\partial t} \phi_{\rm g}(\bar{r}, \bar{\Omega}, t) + \bar{\Omega} \cdot \nabla \phi_{\rm g}(\bar{r}, \bar{\Omega}, t) + \Sigma_{\rm t}(\bar{r}) \phi_{\rm g}(\bar{r}, \bar{\Omega}, t)$
 - $= s_{q}(\overline{r},\overline{\Omega},t) + \sum_{g'=q}^{1} f_{-d\Omega'} \sum_{s}^{q' \to q} (\overline{r},\overline{\Omega}' \to \Omega) \phi_{q'}(\overline{r},\overline{\Omega}',t)$ (2.3)

41 where the summation over energy groups could be expanded over all g to allow for upscattering and $\hat{\phi}_{q}(\bar{r},\bar{\Omega},t) = time - dependent group angular flux,$ $\int \phi(\tilde{\mathbf{r}}, \mathbf{E}, \tilde{\mathbf{W}}, \mathbf{t}) d\mathbf{E}$ ΔĒ $\Sigma_t^g(\bar{r})$ energy-averaged total cross section for the gth group, $\int \Sigma_{t}(\vec{r}, E) \phi(\vec{r}, E, \vec{\Omega}, t) dE / \int \phi(\vec{r}, E, \vec{\Omega}, t) dE / \Delta E_{g}$ $(r_1 \overline{\Omega} - \overline{\Omega}!) = \text{group g' to group g scattering cross section,}$ $\frac{f}{AE_{q}} = \frac{\sum_{i} (\bar{r}, E^{*}, E, \bar{\Omega}, t)}{\int \bar{\phi}(\bar{r}, \bar{E}^{*}, \bar{\Omega}, t) dE^{*}}$ Ţ , E) dE! dE ΔE g Λe_g,



The integro-differential form of the Boltzmann transport equation can be transformed to an integral form⁽¹⁾ resulting the Integral-Flux Density Equation:

$$\phi_{g}(\vec{r}, \vec{\Omega}, t) = \int dR \exp\{-\beta_{g}(\vec{r}, R, \vec{\Omega})\} [S_{g}(\vec{r} - R\vec{\Omega}, \vec{\Omega}, t - R/\nu) \\ 0 \\ \frac{1}{2} \int d\vec{\Omega}^{\dagger} \Sigma^{g^{\dagger} + g}(\vec{r} - R\vec{\Omega}, \vec{\Omega}^{\dagger}, \vec{\Omega}) \phi_{g}(r^{\dagger}, \Omega^{\dagger}, t^{\dagger})]$$
(2.4)

$$g^{\dagger} = g \qquad (2.4)$$
where $R_{i} = \sum_{j=1}^{\infty} p_{j} t_{j} d\vec{\Omega}^{j} \nabla_{j} d\vec{\Omega}^{j} d\vec{\Omega$

 (\bar{r}) to an arbitrary point (\bar{r}) such that

$$\mathbf{r} = \mathbf{r} - \mathbf{R}\mathbf{\bar{\Omega}}$$
$$\mathbf{r} = \mathbf{t} - \mathbf{R} / \mathbf{v}$$

r, t 3. Integral Event Density Equation

Q.

 $\beta_{g}(\bar{r},R,\bar{\Omega}) = \int_{\Omega}^{R} \Sigma_{t}^{g}(\bar{r}-R^{\dagger}\bar{\Omega}) dR^{\dagger}$

The "event density" $\widehat{\Psi}_{q}(\overline{r}, \overline{r}, \overline{t})$ describes the density of particles going into a collision and is related to the group angular flux in the following manner:

 $\psi_{g}(\bar{r},\bar{\Omega},t) \equiv \Sigma_{t}^{g}(\bar{r}) \phi_{g}(\bar{r},\bar{r},t)$

where

 $\Psi_{\mathbf{q}}(\mathbf{r},\Omega,\mathbf{t})\mathrm{d}\Omega$

and

= the number of collision events per unit volume and time at the space point r and time t

> experienced by particles having energies within the gth energy group and directions in d $\overline{\Omega}$ about

The defining equation for the event density is obtained by multiplying both sides of Equation (2.4) by the group total cross section $\Sigma_{r}^{g}(\bar{r})$ and identifying the product $(f(\bar{r})+g(\bar{r},\bar{r},t))$ as the event density $\psi_{g}(\bar{r},\bar{\Omega},t)$:

$$\psi_{\mathbf{q}}(\mathbf{r},\overline{\Omega},\mathbf{t}) = \int d\mathbf{R} \ \Sigma_{\mathbf{t}}^{\mathbf{q}}(\overline{\mathbf{r}}) \ \exp\{-\beta_{\mathbf{q}}(\overline{\mathbf{r}},\mathbf{R},\overline{\Omega})\} [S_{\mathbf{q}}(\overline{\mathbf{r}}-\mathbf{R}\overline{\Omega},\overline{\Omega},\mathbf{t}-\mathbf{R}/\mathbf{v})]$$

$$\int d\vec{\Omega}! = \frac{\sum_{s=1}^{g^{+} \neq g} (\vec{r} - R\vec{\Omega}, \vec{\Omega}^{+} \rightarrow \vec{\Omega})}{\sum_{s=1}^{g^{+}} (\vec{r}^{+})} \quad \forall_{g^{+}} (r^{+}, \Omega^{+}, t^{+})$$

Integral Emergent Particle Density Equation

Define the emergent particle density $\chi_{g}(\bar{r},\bar{n},t)$ as the density of particles leaving source or emerging from a real collision with phase space coordinates (group g, \bar{r}, \bar{n} (t),

$$\chi_{g}(\mathbf{r},\Omega,t) = S_{g}(\mathbf{\bar{r}},\overline{\Omega},t) + \Sigma (d\overline{\Omega} \cdot \Sigma_{s}^{q' \rightarrow \overline{\Omega}}(\mathbf{\bar{r}},\overline{\Omega} \cdot \rightarrow \overline{\Omega}) \phi_{g'}(\mathbf{r},\Omega',t)$$
(2.6)

$$g' = 1 4\pi$$
Equation (2.4) can be written as

 $\phi_{\mathbf{g}}(\mathbf{\bar{r}},\mathbf{\bar{\Omega}},t) = \int d\mathbf{R} \exp\{-\beta_{\mathbf{g}}(\mathbf{\bar{r}},\mathbf{R},\mathbf{\bar{\Omega}})\}_{\mathcal{T},\mathbf{g}}(\mathbf{\bar{r}}',\mathbf{\bar{\Omega}},t)$

The "Integral Emergent Particle Density Equation" is obtained by substituting Equation (2.7) into Equation (2.6):

 $\chi_{\mathbf{g}}^{*}(\overline{\mathbf{r}},\overline{\Omega},\mathbf{t}) = \tilde{S}_{\mathbf{g}}^{*}(\overline{\mathbf{r}},\overline{\Omega},\mathbf{t}) + \sum_{\mathbf{g}^{*}=\mathbf{g}^{*}=\mathbf{d}^{*}}^{1} \int d\overline{\Omega}^{*} \times \sum_{\mathbf{g}^{*}=\mathbf{g}^{*}=\mathbf{d}^{*}}^{\mathbf{g}^{*}=\mathbf{g}^{*}} (\overline{\mathbf{r}},\overline{\Omega}^{*},\overline{\Omega}) + \int d\mathbf{R} \exp\{-\beta_{\mathbf{g}^{*}}(\overline{\mathbf{r}},\mathbf{R},\overline{\Omega}^{*})\},$ $\chi_{\mathbf{g}^{*}}^{*}(\overline{\mathbf{r}}^{*},\overline{\Omega}^{*},\mathbf{t}^{*})$

 $= S_{g}(\bar{r},\bar{\Omega}_{i}t) + \sum_{q'=q'}^{i} f d\bar{\Omega}^{i},$ $= S_{g}(\bar{r},\bar{\Omega}^{i},\bar{\Omega}^{i}) + \sum_{q'=q'}^{i} f d\bar{\Omega}^{i},$ $= S_{g}(\bar{r},\bar{\Omega}^{i},\bar{\Omega}^{i}) + \sum_{q'=q'}^{i} f d\bar{\Omega}^{i},$ $= S_{g}(\bar{r},\bar{\Omega}^{i}) + \sum_{q'=q'}^{i} f d\bar{\Omega}^{i},$

(2.8)

(2.9)

(2.10)

Operator Notation and Summary of Equations

Define the transport integral operator

 $T_{g}(\vec{r}' \rightarrow \vec{r}, \vec{\Omega}) = \int_{0}^{\infty} dR' \Sigma_{t}^{g}(\vec{r}) \exp\{-\beta_{g}(\vec{r}, R, \vec{\Omega})\},$

and the collision integral operator

 $C_{q^{*}\rightarrow q}(\bar{r},\bar{\Omega}^{*}\rightarrow\bar{\Omega}) = \sum_{q^{*}=q}^{1} \int d\bar{\Omega}^{*} \left[\frac{\Sigma_{q^{*}}^{q^{*}\rightarrow q}(\bar{r},\bar{\Omega}^{*}\rightarrow\bar{\Omega})}{\Sigma_{q}^{q^{*}}(\bar{r})} \right] \left[\frac{\Sigma_{q}^{q^{*}}(\bar{r})}{\Sigma_{q}^{q^{*}}(\bar{r})} \right]$

where

$$\Sigma_{\mathbf{s}}^{\mathbf{g}^{\dagger}}(\mathbf{\bar{r}}) = \Sigma_{\mathbf{s}}^{\mathbf{1}} f d\mathbf{\bar{\Omega}} \Sigma_{\mathbf{s}}^{\mathbf{g}^{\dagger} \rightarrow \mathbf{g}}(\mathbf{\bar{r}}, \mathbf{\bar{\Omega}}^{\dagger} \rightarrow \mathbf{\bar{\Omega}})$$

In Equation (2.10), $\Sigma_{s}^{g^{\dagger} \rightarrow g}(\bar{r}, \bar{\Omega}^{\dagger} \rightarrow \bar{\Omega}) / \Sigma^{g^{\dagger}}(\bar{r})$ is a normalized probability density function from which the selection of a new energy and direction can be accomplished and $\Sigma_{s}^{g}(\bar{r}) / \Sigma_{t}^{g^{\dagger}}(\bar{r})$ is the nonabsorption probability.

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Using the transport and collision integral operators, Equation (2.5) can be rewritten as

$$\psi_{\mathbf{g}}(\mathbf{\bar{r}},\mathbf{\bar{\Omega}},\mathbf{t}) = \pi_{\mathbf{g}}(\mathbf{\bar{r}'},\mathbf{\bar{r}},\mathbf{\bar{\Omega}}) \left[\mathbf{s}_{\mathbf{g}}(\mathbf{\bar{r}'},\mathbf{\bar{\Omega}},\mathbf{t'}) + \mathbf{c}_{\mathbf{g'},\mathbf{q}}(\mathbf{\bar{r}'},\mathbf{\bar{\Omega}'},\mathbf{\bar{\Omega}}) \psi_{\mathbf{g}}(\mathbf{\bar{r}'},\mathbf{\bar{\Omega}'},\mathbf{t'}) \right]$$

The term $T_g \xrightarrow{T} S_g$ can be identified as the "firstcollision source" and denoted by

 $\psi_{\mathbf{g}}(\bar{\mathbf{r}},\bar{\Omega},\mathbf{t}) = \mathbf{s}_{\mathbf{c}}^{\mathbf{g}'}(\bar{\mathbf{r}},\bar{\Omega},\mathbf{t}) + \mathbf{T}(\bar{\mathbf{r}}^{\dagger},\bar{\mathbf{r}},\bar{\Omega}) - \mathbf{C}_{\mathbf{g}^{\dagger},\mathbf{g}}(\bar{\mathbf{r}}^{\dagger},\bar{\Omega}^{\dagger},\bar{\Omega}) + \psi_{\mathbf{g}}(\bar{\mathbf{r}}^{\dagger},\bar{\Omega}^{\dagger},\mathbf{t}^{\dagger}) - \mathbf{C}_{\mathbf{g}^{\dagger},\mathbf{g}}(\bar{\mathbf{r}}^{\dagger},\bar{\Omega}^{\dagger},\bar{\Omega}) + \psi_{\mathbf{g}}(\bar{\mathbf{r}}^{\dagger},\bar{\Omega}^{\dagger},\mathbf{t}^{\dagger}) - \mathbf{C}_{\mathbf{g}^{\dagger},\mathbf{g}}(\bar{\mathbf{r}}^{\dagger},\bar{\Omega}^{\dagger},\bar{\Omega}) + \psi_{\mathbf{g}}(\bar{\mathbf{r}}^{\dagger},\bar{\Omega}^{\dagger},\mathbf{t}^{\dagger}) - \mathbf{C}_{\mathbf{g}^{\dagger},\mathbf{g}}(\bar{\mathbf{r}}^{\dagger},\bar{\Omega}^{\dagger},\bar{\Omega}) + \mathbf{C}_{\mathbf{g}^{\dagger},\mathbf{g}}(\bar{\mathbf{r}}^{\dagger},\bar{\Omega}^{\dagger},\mathbf{t}^{\dagger}) - \mathbf{C}_{\mathbf{g}^{\dagger},\mathbf{g}}(\bar{\mathbf{r}}^{\dagger},\bar{\Omega}^{\dagger},\bar{\Omega}) + \mathbf{C}_{\mathbf{g}^{\dagger},\mathbf{g}}(\bar{\mathbf{r}}^{\dagger},\bar{\Omega}^{\dagger},\mathbf{t}^{\dagger}) - \mathbf{C}_{\mathbf{g}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger}) - \mathbf{C}_{\mathbf{g}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger}) - \mathbf{C}_{\mathbf{g}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger}) - \mathbf{C}_{\mathbf{g}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger}) - \mathbf{C}_{\mathbf{g}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf{t}^{\dagger},\mathbf$

(2.11)

$$S_{c}^{g}(\bar{r},\bar{r},t) = T_{g}(\bar{r}' \rightarrow \bar{r},\bar{\Omega}) S_{g}(\bar{r}',\bar{\Omega},t')$$

and the "Integral Event Density Equation" becomes

Using the relationship

 $\Psi_{\mathbf{g}}(\mathbf{\bar{r}},\mathbf{\bar{\Omega}},\mathbf{t}) = \Sigma_{\mathbf{t}}^{\mathbf{g}}(\mathbf{\bar{r}}) \ \phi_{\mathbf{g}}(\mathbf{\bar{r}},\mathbf{\bar{\Omega}},\mathbf{t}), ...$

Equation (2.11) can be transformed into the "Integral Flux

Density Equation:"

 $\phi_{\mathbf{g}}(\bar{\mathbf{r}},\bar{\Omega},\mathbf{t}) = \mathbf{s}_{\mathbf{c}}^{\mathbf{g}}(\bar{\mathbf{r}},\bar{\Omega},\mathbf{t}) / \Sigma_{\mathbf{t}}^{\mathbf{g}}(\bar{\mathbf{r}}) + \mathbf{T}_{\mathbf{g}}(\bar{\mathbf{r}}' \rightarrow \bar{\mathbf{r}}) \phi_{\mathbf{g}' \rightarrow \mathbf{g}}(\bar{\mathbf{r}}',\bar{\Omega}' \rightarrow \bar{\Omega}).$

(2.12)

 $[\Sigma_{t}^{g'}(\overline{r}') \neq \Sigma_{t}^{g}(\overline{r})] \Rightarrow_{g}^{f}(\overline{r}', \overline{r}', L').$

Finally, the integral operators are introduced into Equation (2.8) and the following form for the "Integral Emergent Particle

Density Equation" is obtained

 $\chi_{\mathbf{g}}(\overline{\mathbf{r}},\overline{\mathbf{0}},\overline{\mathbf{t}}) = S_{\mathbf{g}}(\overline{\mathbf{r}},\overline{\mathbf{0}},\mathbf{t}) + C_{\mathbf{g}} \otimes_{\mathbf{g}} (\overline{\mathbf{r}},\overline{\mathbf{0}}) \otimes_{\mathbf{g}} (\overline{\mathbf{r}}) \otimes_{\mathbf{g}} (\overline{\mathbf{r}) \otimes_{\mathbf{g}} (\overline{\mathbf{r}}) \otimes_{\mathbf{g}} (\overline{\mathbf{r})} \otimes_$

An examination of Equations (2.11), (2.12), and (2.13) would reveal that either the "Integral Event Density Equation" or the "Integral Emergent Particle Density Equation" would provide a reasonable basis for a Monte Carlo random walk. Equation (2.13) was selected for the MORSE code since the source particles would be introduced according to the natural distribution rather than the distribution of first collisions. However, it is noted that after the introduction of the source particle, the subsequent random walk can be regarded in terms of either Equation (2.11) or Equation (2.13) with the particle's weight at a collision site being the weight before collision or the weight after collision respectively. The weight is a real number associated with a particle random walk; normally the source particle is assigned a weight of unity.

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The random walk based on the "Integral Emergent Particle Density Equation" would introduce a particle into the system according to the source function. The particle travels to the site of its first collision as determined by the transport kernel. Its weight is modified by the non-absorption probability and a new energy group and flight direction kernels are applied successively determining the particle's emergent phase space coordinates corresponding to the second, third,, collision sites until the random walk is terminated due to the reduction of the particle's weight below some cut-off value or because the particle escapes from the portion of phase space associated with a particular problem (for example, escapes the system or slows down below an energy cut-off).

2.6 Random Walk Procedure

The actual implementation of the random walk procedure is accomplished by approximating the integrals implied in the collision and transport integral operators by the sum

$$\chi_{g}(\vec{r},\vec{\Omega},t) = \sum_{\substack{n=0 \\ n \neq 0}}^{\infty} \chi_{g}^{n}(\vec{r},\vec{\Omega},t),$$

where

 $\chi_{g}^{n}(\bar{r},\bar{\Omega},t)d\bar{\Omega} = ...$ the emergent particle density of particles emerging from its nth collision and having phase space coordinates (group g, \bar{r} , $d\bar{\Omega}$, about $\bar{\Omega}$, timet),

 $\chi^0_{\rm q}(\bar{\mathbf{r}},\bar{\bar{\Omega}},{\tt t})={\tt s}_{\rm q}(\bar{\mathbf{r}},\bar{\Omega},{\tt t})\ ,$ $\chi^{\mathbf{n}}_{\mathbf{g}}(\overline{\mathbf{r}},\overline{\Omega},\mathbf{t}) = c_{\mathbf{g}^\dagger \to \mathbf{g}}(\overline{\mathbf{r}},\overline{\Omega}^\dagger \to \overline{\Omega}) \ \mathbf{T}_{\mathbf{g}^\dagger}(\overline{\mathbf{r}}^\dagger \to \overline{\mathbf{r}},\overline{\Omega}^\dagger) \ \chi^{\mathbf{n-1}}_{\mathbf{g}^\dagger}(\mathbf{r}^\dagger,\overline{\Omega}^\dagger,\mathbf{t}^\dagger)$

Thus the source coordinates (group g_0 , \bar{r}_0 , $\bar{\Omega}_0$, time t_0) are selected from $S_q(\bar{r}, \bar{\Omega}, t)$ and a flight distance R is picked from $\Sigma_{t0}^{q}(r)e^- g_{t0}^{(\bar{r},R,\bar{\Omega}_0)}$ to determine the site from the first collision \bar{r}_1 , and the particles $t_1 = t_0 + R/v_{g_0}$. The probability of scattering is $\Sigma_s^{q}\rho(\bar{r}_1)/\Sigma_t^{q}$ all particles are forced to scatter and their weight is modified with this probability. A new group g is

selected according to the distribution

$$\frac{\int d\bar{\Omega} \sum_{s}^{g_{0} \rightarrow g} (\bar{r}_{1}, \bar{\Omega}_{0} \rightarrow \bar{\Omega})}{\Sigma^{g_{0}} (\bar{r}_{1})}$$

and then a new direction $\[b]{\Omega}$ is determined from



The process is repeated until the particles history is terminated. Contributions to the quantity of interest are estimated at appropriate points in the random walk (boundary crossing, before or after collisions, etc.) using the particle's weight and the appropriate response function.

Figure 1 shows the code modules. The "source" module generates a source particle. After a particle is generated, "random walk" routines generate the histories and do the major bookkeeping. The "combinatorial geometry" routine is required for tracking from collision to collision. The geometry is described as unions and intersections (OR and AND Boolean algebra operations) of primary geometrical bodies (e.g., spheres, cubes).



to compose complicated geometries. The "collision" module determines energy loss, change of direction and secondary generation at a collision. The "cross section" module is needed to provide information both in the collision and tracking processes. The "analysis" module is used to estimate the contribution of a random walk to the parameters of interest. "Analysis" of events may be obtained during the tracking process (track length/unit volume or boundary crossing estimators) and at. the collision sites (collision density or next event estimators). The "analysis" may also make use of the "cross section" module (as required by the used estimates.

The MORSE code, through the "combinatorial geometry" module, is capable of handling complicated geometries. Since the code utilizes multigroup cross sections and a scattering matrix (P_n Legendre coefficients) the results obtained depend on how fine are the energy groups used and the order of Legendre approximations used. The MORSE code, being based on the Monte Carlo method, is best suited for the analysis of geometrically complex assemblies for which analytical

approaches are virtually impossible and numerical schemes are extremely time-consuming. On the other hand, because of its inherently statistical nature, it is poorly suited for predicting accurate values of fluence and fluence-like quantities over extended ranges of space, energy or time. 0 SIMULATION OF NEUTRON SCATTERING TECHNIQUE

FOR GLOBAL VOID FRACTION MEASUREMEN'TS

To show the usefulness of the Monte Carlo Experiment in simulation of real experiments, the use of the MORSE code to simulate experimental technique for global void fraction measurements is presented. The results of this study and the significance of these results are reported elsewhere ⁽⁶⁾.

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The objective of the simulation was to find a sound theoretical bases for the technique investigated by Banerjee et al⁽⁷⁾. In this technique, thermal neutrons produced due to scattering of epithermal and fast neutron by water were measured and then fluence was found to be linearly proportional to the amount of water that cause the scattering. A description of the experimental setup of the technique will be presented and then the Monte Carlo simulation of this set up will be given. Further verification of the Monte Carlo method is presented in Chapter 5.

.]. Experimental Setup

The experimental setup used for global void fraction measurements is shown schematically in Figure 2. The setup consisted mainly of:


Neutron beam port and neutron collimator Test sections for which void fraction is to be measured Neutron detector(s)

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Neutron and gamma ray shielding

(a)

(b)

(c)

(d)

Beam port number 2 of the McMaster University Nuclear Reactor was used for this experiment. Two sequential collimators were aligned within the beam port in order to produce a 1 x 5 cm rectangular beam. Each collimator consisted essentially of a perspex box contained within an aluminum tube. The space between the aluminum tube and perspex box was filled with a mixture of wax and borax. The geometrical configuration of the two collimators is shown in Figure 3.

The spectrum of the neutrons produced was expected to be composed of fast and epithermal and thermal components. Since only the fast and epithermal components were needed, the neutron beam was passed through cadmium sheets to remove thermal neutrons before being made incident on the test section. Cadmium is a good filter of thermal neutrons since it absorbs heutrons with energies. Delow 0.5 eV.

Aluminum-water cylindrical test sections of 152.4 mm in height and 50.8 mm in diameter shaped to represent various typical flow patterns (Figure 4) were used in the experiment. The gas (or vapour) phase was





simulated by aluminum because it has a small thermalization effect on neutrons compared to water (low scattering and absorption cross sections)

To measure the scattered thermal neutrons, 55 mm diameter Helium-3 counters at 2 bar internal pressure were used. Helium-3 counters were selected because of their high efficients for thermal neutron detection and low sensitivity to gamma-rays.

Shielding against neutrons and gamma rays was provided mainly by wax/borax blocks surrounding the test section (Figure 5). A secondary shield of water tanks on the beam sides was also used to provide protection against scattered and stray radiation. A beam catcher was utilized in order to stop the beam penetration outside the experimental zone.

Typical Experimental Results (7)

3.2

To determine how the scattered neutron flux was related to void fraction and phase distribution a series of experiments was done with aluminum water mixtures. The aluminum was meant to simulate the gas phase and was shaped into pieces that represented inverted annular and stratified flow patterns as shown in Figure 4. The value fraction of aluminum (i.e. the void fraction) was known and compared with the void fraction calculated from the scattered neutron count rate using



$$S_{E_{1}} = \frac{N(0) - N(\alpha)}{N(0) - N(1)}$$

where $\alpha_{\rm E}$ is the estimated void fraction, N(α) is the count rate for a test section of actual void fraction of α , and N(0) and N(1) are the count rates for zero and one void fraction, respectively.

(3.1)

Results for a 50.8 mm diameter tubular test sections are shown in Figure 6. The void fraction calculated from the scattered count rate using Eq. (3.1) agrees reasonably well with the actual void fraction, but there is some effect of flow pattern. This is especially evident for the stratified cases. This effect can evidently be corrected, to a great extent, by placing two detectors on either side of the test section and arithmetically averaging the scattered neutron count rate. The experiments were repeated with a 25.4 mm tubular test section and the results were similar.

These results are purely experimental and do not give insight into the effect of the energy spectrum of the neutron beam, temperature effects in the test section, optimum placement of detectors and detector response functions. Numerical simulation appeared necessary for an evaluation of these effects. The first step, however, was to establish the accuracy of the simulation and interpret the scattering experiments already completed.





3.3

Simulation of Experiments

To simulate the experiments numerically, the neutron transport equation must be solved. For sufficiently accurate calculations, one had to select between the discrete ordinates (S_n) and the onte Carlo Methods.

The Monte Carlo method was selected because it is more capable of handling complex geometries resulting from the various two-phase flow patterns. Furthermore, the neutron flux was to be determined in specified. regions where detectors were located, and not everywhere in the problem space. The Monte Carlo method is also more suitable for this purpose.

The calculational framework for the simulation was provided by the multigroup Monte Carlo radiation transport code MORSE⁽¹⁾. For these simulations, the following aspects had to be modelled:

(a) neutron beam geometry, angular distribution and energy spectrum;
(b) neutron-cross sections of all materials present in the system;
(c) system geometry;
(d) representation of the system boundaries;
(e) importance sampling techniques to be used;

(f) scouring techniques and detecting media.

Since the adequacy of the calculations depends on these specifications they are wriefly discussed below.

3.3.1 Representation of Neutron Beam

The collimated neutron beam in the experiments had a rectangular cross-section. To represent this in the numerical simulations, source neutrons were picked randomly from a rectangle of the beam cross-sectional dimensions and given directions of motion normal to the cross-sectional plane.

The neutron spectrum was based on a similar measurement for the McMaster Nuclear Reactor by McCormack⁽⁸⁾. The spectrum is shown in Figure 7. It consists of the fission component, the intermediate component and the Maxwell-Boltzmann thermal component. The fission component can be represent by Watt's formula⁽⁹⁾. The intermediate component varies as 1/E, where E is the neutron energy. The thermal portion is not shown in detail since the epi-cadmium neutrons were used in the experiments.

.3.3.2 Cross Sections

Since accurate calculations were required at least in the Initial simulations, a reasonably large number of energy groups and Legendre



coefficients were used with an appropriate weighting spectrum for averaging over each energy group.

A set of 100-fine-group P_8 cross-sections was generated from the ENDF/B-IV library using SUPERTOG⁽¹⁰⁾. A Maxwellian thermal spectrum and a 1/E function joined to a fission spectrum were used as weighting functions for the integration of pointwise cross-section data given in the ENDF/B-IV library. This is justified because the McMaster Reactor has a typical thermal reactor spectrum and a relatively small amount of hydrogeneous material was present in the experimental system. Therefore the energy spectrum in the system is expected not to be too much different from that of incident neutrons. The maximum order (8) of Legendre coefficients available in SUPERTOG was used. This was because of the importance of hydrogen scattering anisotropy.

Sets of cross-sections for the system and detector elements 1 H, 3 He, 10 B, 14 N, 16 O, 27 Al and 112 Co were generated. Due to computer memory limitations, the cross-section data were collapsed to standard $^{(11)}$ 33 broad group sets. ANISN $^{(12)}$, a one-dimensional discrete ordinate radiation transport code, was used to provide the spatial fine group fluxes necessary for the weighting process. The one-dimensional representation in ANISN was considered a reasonable approximation for weighting purposes. This is because the flux was not expected to change very much in the experimental setup due to the relatively small amount of water present.

3.3.3 System Geometry

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The system geometry for the calculations is shown in Figure 8. This geometrical representation of the experimental setup took into consideration the collimator, shields and detector configurations. The neutron beam collimator walls were represented by reflecting surfaces (albedos). Also the water shielding tanks surrounding the experiment were replaced by reflecting surfaces. Some escape windows (completely absorbing media) were used in describing the three dimensional geometry of the system in order to avoid tracking escaping neutrons.

3.3.4 Reflecting Surfaces (Albedos)

In the experiments, different materials were used for collimating the neutron beam and radiation shields. These materials consisted mainly of wax, borax, concrete or water. Tracking neutrons through these materials would waste computer time, since the reflected neutrons were the only



contributor to the detector. These reflecting surfaces were therefore represented by albedos (reflectors) in the Monte Carlo simulation. The reflectors were mainly hydrogenous. Therefore, when a neutron collided with the reflector, its scattering and total cross-sections (for the appropriate energy group) were picked from the hydrogen cross-section set, and the corresponding reflection coefficients were determined. These coefficients were taken from the coefficients listed by Maynard⁽¹³⁾. The coefficients were orignally derived by solving the transport equation and were tested against experiments. Using these reflection coefficients **()** and the hydrogen cross-sections the non-absorption probability and reflection angles were determined, and the neutron weight and direction were then modified. Use of the albedo concept substantially decreased the number of neutrons that had to be tracked in the system simulation.

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3.3.5 Importance Sampling Techniques

Importance sampling techniques are used in Monte Carlo simulations to intervene with the random walk in such a way that the variance of the quantity of interest is reduced. In the problem considered, the use of importance sampling techniques to improve the standard deviation of the results was not necessary. Regions in which neutrons do not significantly contribute to the considered detectors were either replaced by reflectors or complete absorbers. Escape windows (complete absorbers) were used in appropriate regions to function as sinks for neutrons leaving the system. Neutrons in the lowest energy group also had the chance to escape, and were not trapped in this group, since the limited amount of water (scattering medium) in the test section allows these neutrons to escape.

The only biasing technique necessary in the calculations wat energy biasing for the source neutrons. This was done because thermal neutrons were expected to be absorbed by the cadmium sheet placed in the beam path. Source biasing was therefore used such that the probability of thermal neutrons being picked from the source spectrum was reduced. This also saved computer time that would otherwise have been used in tracking thermal neutrons through the cadmium sheets.

3.3.6 Scoring Techniques

The next event estimator, available in SAMBO⁽¹⁴⁾, and the track length estimates were used to estimate the neutron fluence and fluence-like quantities (such as ³He and BF₃ detector counts) in the detection regions. The next event estimator is a point detector estimator which scores, from each collision point, the probability of the next event being at the detector. This estimator does improve computational efficiency since it does not require the neutron to reach the detector site. However, the next event estimator is still expensive, since scoring is required for every real collision. To reduce computational cost only one point detector was used in the present calculations. The fractional standard deviation for the fluence estimated by this point detector estimator could be calculated in the SAMBO package.

Since a 50 mm diameter detector was used in the experiments, the track length estimator was also used to evaluate the fluence in the detection cavity representing the actual detector. The track length estimator is based on the fact that the neutron fluence can be defined as the number of neutron path lengths per unit volume per second. In the Monte Carlo simulations neutron path lengths are identical to neutron track lengths. Fractional standard deviations for the fluence estimated by the track length estimator could not be calculated in the MORSE package. Therefore, runs with different numbers of particles were used to determine whether the results were converging.

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3.3.7 Results

All the results of the Monte Carlo method are statistical in , nature. Therefore, in order to set confidence limits, estimates of the errors had to be made.

For the point detector fluences, the fractional standard deviations were calculated with SAMBO⁽¹⁴⁾. However, no option to estimate the statistical error for the track length estimated fluences, was available in the MORSE code. For these fluences some additional random walk processes had to be run using increasing numbers of source neutrons to test for dependence of results on number of tracked neutrons.

The reference cases were taken to be void fractions of zero and unity. For these reference cases it is desirable to minimize the statistical errors associated with the scattered neutron fluences. Table 1 shows the number of source neutrons tracked, and the fractional standard deviations obtained for the scattered neutron fluences of the point detector. The increase in the fractional standard deviations for the BF₃ and ³He fluences is obvious due to the 1/y behaviour of the response function of the BF₃ and ³He detectors.

Fractional Standard Deviation of the Neutron Scattering Fluences for Different Void Fractions "Annular Flow Regime"(a).

Actual	Number of histories	F.S.D. for total neutron fluence(b)	F.S.D. for thermal fluence(c)	F.S.D. for BF ₃ fluence	F.S.D. for ³ He fluence
				······	
0.6	9 481	0.01775	0.028	0.02612	0.02612
0.25	2 690	0.03303	0.057	0.05383	0.05384
0.50	3 210	0.03125	0.058	0.05360	0.05360
0.75	4 258	0.02766	0.071	0.06205	0.06205
1.00	11 877	0.01682	0.188	0.0561	0.0564
· · ·					· · · ·

a_(a)

(b)

(c)

For the other flow regime errors of about the same size were obtained.

Fast, epithermal and thermal neutrons.

Neutrons of energy 0.04 eV.

TABLE

The track length estimated fluences were tested for dependence on number of histories in typical cases. As an example, when the number of histories tracked was changed from 4,618 to 9,864 for the case of $\alpha = 0$, less than 4% variation in the magnitude of the fluences resulted. Nonetheless, the track length estimator was used as a supplementary estimator.

For the annular flow regime, statistical errors for the void fractions calculated for a ³He point detector at a 68% confidence level, are shown by the bars in Figure 9. The corresponding errors associated with the void fractions determined in the experiments are denoted by the lines between the small bars in the Figure. The same magnitude of statistical error is obtained for the other flow regimes. Good agreement is obtained, as evident from the Figure 9 between the Monte Carlo calculations and the experimental results. Some other calculational results are also shown in Figure 9 which we will discuss later.

Even though the main objective of the present simulation was to verify the experimental results, advantage was taken of the extra information provided by the MORSE code. Detailed energy spectra of the scattered neutrons, estimations of the neutron fluence in different geometric regions, and the number of neutron collisions with each material present in the system, were also obtained in the calculations. This





information, which cannot be obtained easily in the experiments, is extremely useful in the design of future experiments and portable neutron densitometers.

Investigation of the scattered neutron spectrum was necessary to show which part of the spectrum could (or could not) provide a good estimate of the amount of water in the test section. Figure 10 shows for different void fractions, the calculated lethargy spectra of the scattered neutrons counted by the point detector. The spectra shown for the annular flow regime, are typical for the other flow regimes. The higher energy portion of the spectra consisted mainly of neutrons scattered by the aluminum in the test sections. Therefore the Figure shows that the neutron fluences per unit lethargy increase with void fraction for the high-energy portion of the spectrum. Comparison with the spectrum α = 1.0 (all aluminum) immediately indicates that most of the for epithermal neutrons were produced by the water, moderation of the faster neutrons, rather than by the direct scattering of the epithermal neutrons aluminum. A greater effect of the amount of water in the test section on the epithermal part of the spectra can be observed as the energy decreases. This effect obviously becomes strongest in the thermal component of the spectra. In the thermal component, the scattered neutron fluence is directly proportional to the amount of water in the test section. This is, of course, because the thermal neutrons in the source



beam were cut off by the cadmium sheets. Therefore, thermal neutrons \Im could only be produced by moderation due to the water in the test section.

7.

It is important to mention that the spectra shown in Figure 10 were evaluated at the point detector using the next event estimator. This estimator scored whenever a real collision was encountered, and consequently did not score for the albedo collisions. This implies that the thermal neutrons shown in the spectra could not include those produced by the direct moderation of the reflecting walls.

The scattered neutron spectra investigated above showed that the amount of water (and consequently void) could be predicted by measuring the thermal components of the spectra. Practically, thermal neutrons could be measured using BF_3 or ³He detectors. These detectors count the thermal neutrons and part of the epithermal neutrons. According to the calculated behaviour of the neutron spectra shown above, they are ideal for our purposes, since these detectors are not sensitive to the fast neutrons (due to their 1/v activation cross-sections).

Another useful result obtained from the calculations was the effect of the method of collecting the scattered neutrons on void fraction prediction. In the experimental setup the scattered neutrons were collected by housing the detectors within a wax borax collimator with a field of view that just included the test section. This was simulated by the detection cavity shown in Figure 8 (region 7). In the calculations the non-collimated scattered neutrons could also be counted inside regions 5 and 6 of Figure 8. A track length estimator was used to evaluate the neutron fluence inside these regions. In addition, of course, the point detector in the calculations saw a well collimated cone of the scattered neutrons. Therefore, regions 5, 6 and 7, and the point detectors, represented different methods of collecting the scattered neutrons.

Figure 11 shows the void fractions estimated by using the different methods of collecting the scattered neutrons for the particular stratified flow regime illustrated in the Figure. It can be observed that, within the statistical errors, all these methods predicted the same void fractions. However, region 6 (far from the water in the test sections), determined higher void fractions than region 5. Averaging the void fractions obtained in these regions could give better predictions.

The mirror image calculations are shown for stratified flow in Figure 13. In this case region 6 shows a somewhat lower void fraction because it is closer to the water. Also the point detector in this case determined slightly higher void fractions. This means that for this particular flow regime few neutrons were directed towards the centre of the detecting cavity as compared to off-centre regions.



FIGURE 11 Void-Fraction Predicted for Stradified Flow. Regime # 1 (w.t.l.f denotes weighted track length fluence)











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Figure 11 and 12 demonstrate the effect of the flow regime on void fraction determination. Smaller void fractions were obtained with the detector positioning for the stratified flow in Figure 11, whereas higher void fractions were obtained for the mirror image case in Figure 12. This also corresponds to the experimental observations. It was found both in the experiments and the calculations that the position of the detector relative to the position of the water in the test section was of some importance for the stratified flow regimes shown in these Figures.

By examining the void fraction predicted for the other flow regimes shown in Figures 9, 13, 14 and 15 one can claim that the void fraction predictions are almost, but not entirely, independent of flow regime. Within the statistical errors the different methods of collecting the scattered neutrons determined the same void fractions, although the point detector predicted slightly higher void fraction for the stratified flow regime in Figure 15.

The number of neutron collisions with the water in a test section was also obtained in the calculations. This number (normalized to the number of source neutrons) can also be used to predict the amount of water in the test sections, since the neutron scattering is caused by collisions with the water. Void fractions calculated using the average number of collisions with water are shown in Figures 9 and 11 to 15. For the annular flow regime (Figure 9) the neutrons encountered a thinner layer of water than in other flow regimes of the same water content. Consequently, neutrons could be scattered outwards from the test section after the first few collisions, rather than suffering more collisions further in. This could explain, in physical terms, the higher void fractions determined by the number of collisions in the annular flow regime. For the stratified flow regimes of Figures 11 and 12 one could also expect a lower number of collisions, because the surface areas of the water directly exposed to the beam neutrons are generally smaller than the other flow regimes of the same water content. This again explains why the void fractions calculated from the collisions are always higher than from scattering. For the other flow regimes (Figures 13, 14 and 15) the calculations of void fraction based on scattering and collisions lead to almost identical results.

4.0 CONCLUSIONS

The Monte Carlo method is a powerful method of simulation. It takes into account all the experimental aspects of the setup and reflects honestly the phenomena encountered in real life. Moreover, the Monte Carlo method provides supplementary information that gives insight into the phenomena and enables better design and understanding of the system. Some parameters that can not be measured experimentally, because of the impossibility or impracticability of measurement (such as the number of collisions in the problem discussed above), could be evaulated using the Monte Carlo Method. Such parameters might be of great importance for designing, developing or establishing some ideas.

The exercise of simulating a neutron scattering method for measuring global void fraction in two-phase flow, shows that the Monte Carlo method is a valuable numerical experiment. This method could be used to test and develop new techniques, without carrying out any laboratory experiments before the establishment of sound theoretical bases for the technique. This, of course, saves a lot of time and effort that might be spent on laboratory experimentation. It also enables the experimentalist to have a clear understanding of the system and to solve the problems he might encounter.

Monte Carlo experiments, like laboratory experiments, need experience and knowledge of the tools used and their limitations. A misuse of a fluence estimator, or a misapplication of a biasing technique, could be as destructive in a Monte Carlo experiment as a faulty laboratory experimental device. Therefore, the user of a Monte Carlo experiment must have not only good understanding of the method but also a reasonable amount of experience in running the experiment. This simulation of a neutron scattering method for measuring void fraction in two-phase flow, as well as the simulation of an irradiated fuel bay $^{(15)}$, using Monte Carlo method, gave the author the required experience. The textbooks of Greenspan et al $^{(5)}$, Schaeffer $^{(16)}$, Spanier and Gelbard $^{(17)}$ and others were very helpful in understanding of the different aspects of the Monte Carlo method. The seminar workshop on Theory and Application of Monte Carlo Methods, held in Oak Ridge of April 21-23, 1980 $^{(19)}$, and attended by the author, was a good chance for exchanging experience with other attendees and for confirming the author's understanding of the method.

The use of the Monte Carlo experiment to develop a new technique for local void fraction measurements in two-phase flow is discussed in the following chapter.

DEVELOPMENT OF A SINGLE EXPOSURE

CHAPTER 3

NEUTRON TECHNIQUE FOR LOCAL VOID FRACTION MEASUREMENT

INTRODUCT ION

The objective of this Chapter is to investigate the feasibility of utilizing the information carried by neutrons scattered in a two-phase flow test section exposed to a fast (14 MeV) neutron beam, for reconstructing the phase distribution in the test section. First, the dependence of the fluence of the scattered neutrons on the phase distribution is shown. Then, it is shown how this dependence leads to the concept of utilizing single scattering for phase distribution determination. The mathematical formation of the single scattering phenomenon into a set of algebraic equations that can be utilized for finding the phase distribution is discussed. This formulation is based on a single scattering approximation. Therefore, this approximation is compared to Monte Carlo simulation to show the situations under which

this approximation correctly represents the physical phenomenon. The methodology of the phase reconstruction process is then generally discussed. The different detector arrangements that can be used for recording the fluence of scattered neutrons are discussed at the end of the Chapter.

Knowing the void distribution is useful in studying the behaviour of a two-phase flow system, since the void distribution is an important modelling factor, as explained in Chapter 1. This distribution could be determined by obtaining a phase Tomography. The word Tomography is a combination of the Greek word "tomos", which means a piece cut off, and the word graphy. Tomography was first used to name a technique of " x-ray photography in which a single plane was to be photographed, with the outline structure in other planes eliminated. However, there is no reason to restrict the usage of the term Tomography to x-ray photography

only.

Different radiographical methods could be suggested for vapour-water Tomography. X-ray, gamma-ray or neutron transmission techniques could be used. However these techniques require multiple exposure of the object to the radiation source. This can be achieved by rotating either the source or the object or by having multiple radiation sources to which the test section is exposed simultaneously. The former
method is not appropriate for fast transient two-phase flows, unless a very fast scanning method is employed. Also using multiple radiation sources is not practical because of expenses, shielding requirements, the limited number of projection produced and the difficulties caused by the interference of the scattered radiation. Therefore, a single exposure radiographical method is very desirable.

Fast neutrons are selected for this single exposure because of their small total cross-section in the material of the object, which would reduce the self-shielding attenuation effect of the object and allow neutron scattering to be the dominant criterion. Fast neutrons scatter by hydrogen into a wide energy range. This enables the measurement of the scattered-neutron energy spectrum to be made with a reasonable resolution. Also, neutron energy spectrum measurement could be avoided in the range below 1 MeV and above the thermal energy, where practical difficulties are encountered. The measurement of the energy of the scattered neutrons would be helpful in unscrambling the measurements, in order to reconstruct the void-water pattern.

Fast neutrons, in contrast to gamma-rays or x-rays, are much less affected by thick metal walls. Therefore fast neutrons could be used for Tomography of flows contained inside thick (high pressure) ducts. Fast neutrons are also preferred over gamma-rays in scattering experiments because of their higher (hydrogen) scattering

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cross-sections. This increases the count rates of the scattered neutrons and consequently the counting statistics.

2.0 NEUTRON SCATTERING DEPENDENCE ON VOID-WATER DISTRIBUTION

Several Monte Carlo experiments were performed in order to investigate the relationship between the scattering information and the void-water distribution. Monte Carlo simulations were used as statistical analog experiments in order to gain insight into the technique and to provide data that can be used to test the phase reconstruction algorithms. These Monte Carlo simulations are later verified against laboratory experiments, as shown in Chapter 5. A 100 mm wide line source of 14 MeV parallel neutrons was used. Cylindrical ducts of 100 mm in diameter of different void-water configurations were exposed to this stream of neutrons. The 100 mm diameter was selected since it

corresponds to one mean free path of 14 Mey neutrons. This would decrease the probability of multiple scattering and hence reduce the problem to a single scattering problem.

In these Monte Carlo simulations, the energy group structure of 33 groups, used in the simulation discussed in Chapter 2, was applied. A

P scattering-matrix was used in order to produce reasonably accurate scattering data. No biasing techniques were implemented.

The first set of Monte Carlo experiments investigated the relationship between the total fluence of the scattered neutrons and the void-matter distribution. The object was surrounded by detectors, in order to get a good feeling for the effect of the void-water distribution on neutron scattering. Figure 1 shows the geometrical setup, used in these Monte Carlo experiments and Figure 2 shows the total neutron fluences obtained at each detector site, (except transmission

detectors). By total fluence, it is meant the fluence contains neutrons of all possible energies and directions. The total scattering fluence, as shown in Figure 2 for different flow regimes, gives a clear indication of the shape in which water is distributed inside the object affects neutron scattering fluences. If a single scattering is assumed and neutron attenuation through the object is neglected, the relationship between the void-water pattern and the neutron scattering fluence can be explained by using pure geometrical considerations. These assumptions were reasonably achieved in this set of experiments, since objects of diameters less than one mean-free-path of source neutrons were used. The geometrical argument to be used is the simple fact that the closer the

detector to, the scattering medium (water), the more neutron fluence is to be scored by the detector. This simple fact will be considered later to obtain a rough idea of the flow regime.



FIGURE 2 Neutron Scattering Fluences for Different Flow Regimes :

- A : Annular Flow Regime
- B : Inverted Annular Flow Regime
- C : Stratified Flow Regime # 1 .
- D : Stratified Flow Regime # 2









. (D)

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The transmitted fluences are shown in Figure 3 for different flow regimes. These fluences are mainly uncollided fluences, since the collided neutrons have insignificant effect on transmission fluences. Again, this is due to the fact that the amount of water present in the object has at the most a thickness equal to one mean free path of source neutrons. Therefore, the transmission fluences give a good estimation of the water thickness seen by each detector. The information given by the transmitted fluences is considered supplementary, since their interpretation is straightforward. A prediction of the void-water distribution based only on the scattered fluences will be investigated.

As mentioned above, if neutron removal only by single scattering is assumed, a simple geometrical argument can be used to give a rough idea of the flow regime. This geometrical argument will be discussed in the following.

If it is only required to identify the flow regime by two zones, an empty zone (void) and a full zone (water), one is trying to obtain two pieces of information. Therefore, only two detectors are necessary and sufficient to obtain the required information. Any pair of detectors, located symmetrically with respect to the beam axis, can be used.





If the response of one of the two detectors to a collision at a point P is denoted by S_1 and that of the other detector by S_2 , then

$$S_1 = W \exp(-\Sigma_1 x) P_{\text{scatl}} \exp(-\Sigma_{011}) \frac{1}{2.\pi R_1^2}$$
 (1)

 $S_2 = W \exp(-\Sigma_i x) P_{Bcat2} \exp(-\Sigma_0 2Y_2) \frac{1}{2 \pi R_2^2}$, (2)

and

where W is the weight of a source neutrons undergoing collision at point P, x is the distance a source neutron travels inside the object before reaching the point P, Σ_{i} is the total cross-section of source neutron, P_{scatl} and P_{scat2} are the probabilities of scattering from the point P to the first and second detector, respectively, y_{1} and y_{2} are the distances a neutron travels inside the object in its way towards the first and second detector respectively, Σ_{01} and Σ_{02} are the total cross-sections of neutrons in their way out to the first and second detector, respectively and R_{1} and R_{2} are the distances from the scattering point P to the sites of the first and second detectors, respectively, (see Figure 4).

In Equations (1) and (2), it is assumed that the neutron source is a well collimated parallel beam. Therefore W will be 1/N, if the neutron beam is assumed to be uniform, where N is the total number of neutrons. Since the first detector is opposite to the second



one, then a neutron that scatters from the point P to either of the two detectors sustains the same angle of scattering. Therefore P_{scat1} is equal to P_{scat2} , and Σ_{o1} is equal to Σ_{o2} (equal say to Σ_{o}), since the neutron is being scattered to the same energy. Then

$$s_1/s_2 = exp - \sum_{o} [(y_1 - y_2)] \cdot (R_2^2/R_1^2)$$
 (3)

If the removal of outgoing neutrons is ignored, or if this removal is equal for neutrons going towards the detectors, then the exponential term in Equation (3) can be dropped. The removal of outgoing neutrons can be neglected, if the detection energy is high. However, an equal removal of neutrons outgoing to both detectors could be achieved, if the point P lies in the centre of the water zone encountered between the two detectors, this is a rare event.

By neglecting the removal of outgoing neutrons, Equation (3) can be rewritten as

$$1/R_2 = (s_1/s_1)^{1/2}$$
 (4)

where, S_1 and S_2 are the detector responses due to a scattering undergone at the point P. If we imagine that the water inside the object is concentrated in point P, or in other words, the point P is the centre of scattering of neutrons by water, then S_1 and S_2 would be the

detector responses; due to scattering by all the water in the object.

Equation (4) will be then the equation of the locus of all the points that can act as centres of scattering. This locus would coincide with the water zone axis of symmetry. In other words, Equation (4) defines a line around which water exists.

Using the above simplified concept, the water loci were generated for different flow regimes using each pair of corresponding detectors in the Monte Carlo experiments. These loci are shown in Figure 5 together with the actual water content. The Figure shows that the loci strongly reflect the region at which water exists. However, these loci cannot discriminate between water in an annular or inverted annular regime, since both have the same axis of symmetry. All what we can get is a line that reflects the general feature of the flow regime. This is the kind of information one could expect from two pieces of data (two detectors), i.e. two pieces of data would give only two pieces of information (the location of the water concentration and the location of the void concentration). No more could be expected.

3.0 SINGLE SCATTERING PHENOMENON

3.1 Unique Relationship between Energy and Angle of Scattering

An insight into the simple geometrical argument discussed in the previous chapter, leads to a more profound method of phase

reconstruction. The contribution S, defined by Equation (1), in fact,

FIGURE 5 Water Loci Obtained from Neutron Scattering of Different Flow Regimes : A : Annular Flow Regime B : Inverted Annular Flow Regime C : Stratified Flow Regime #1

D : Stratified Flow Regime #2









gives the response of the detector to a scattering that occurred at the point P. If there is no water in the point P (i.e., there is void), the

detector would not respond, i.e., it would score zero. Also, S.

reflects a scattering of at a specific angle, defined by the line joining the point P and the detector site (outgoing neutron direction together with the line defining the incoming neutron direction). Since there is a unique relationship between the angle of scattering and the energy of scattering, S₁ would correspond to a specific energy. Therefore Equation (1) can be more elaborately written as:

$$S_{E}(E) = W \exp \left[-\Sigma(E_{e}) \times \right]$$
. $P(0)$

$$\exp \left[-\sum (E_{1}) y\right] / 2 \pi R_{1}^{2}$$

where again single scattering is assumed and E is the source neutron energy, E_{θ} is the energy of scattering, θ is the corresponding angle of scattering and Σ (E) is the total cross-section at the energy E:

The relationship between the angle of scattering and the incoming and outgoing energy of scattering is defined, for elastic scattering, by the relationship, (see for example Reference 1).

$$\cos \theta = \frac{A \cos \psi + 1}{\{A^2 + 2 A \cos \psi + 1\}^{1/2}},$$

where A is the ratio of the target nucleus mass to the neutron mass and ψ is the scattering angle in the centre of mass system, (corresponding to θ in the laboratory system), defined by the equation:

$$(\theta) = 0.5 \times E_{\beta}$$
 $(1+\alpha) + (1-\alpha) \cos \psi$,

where

$$\left\{\begin{array}{c} \frac{\mathbf{A}-\mathbf{1}}{\mathbf{A}+\mathbf{1}}\right\}^{2}$$

Considering a scattering by hydrogen (A=(1) and $\alpha = 0$) then Equations (6) and (7) can be combined to give;

$$\cos \theta = \{ E(\theta), /, E_{g}, \}$$
 (8)

This equation shows the unique relationship between the angle and energy of scattering by hydrogen. Therefore, in principle, if the energies of neutrons that suffered one collision by the hydrogen in the target are recorded at specific position, one can determine the directions from which these neutrons have been emanated. Scattering by materials other than hydrogen, reflects a complete different angle-energy relationships. Figure 6 shows the relationship between the energy and angle of scattering for different elements in the laboratory system. One



FIGURE 6 . Relationship Between Energy and Angle

of Scattering

can see from this Figure that single scatterings by hydrogen always result in foward scattering. The Figure also shows that by recording forward scattering at energies below $(15/17) E_s$, neutrons scattered only by hydrogen in water will be displayed, while those scattered by oxygen will not appear. This fact implies also that scattering by any material heavier than oxygen will have no effect on neutron recording and consequently metal barriers can be tolerated.

A very important conclusion can be drawn from the above discussion. That is, if the energies of neutrons forwardly scattered by water are measured for energies below (15/17) of the neutron source energy, the direction of scattering can be determined, provided that the direction of the neutron source is known. Moreover, the fluence of neutrons scattered to a specific energy carries information about the amount of hydrogen, and consequently water, in its path: To show how the energy spectrum of the scattered neutrons is related to the amount of water in the scattering medium, Monte Carlo experiments for different flow regimes were performed. In the first Monte Carlo experiment, the hypothetical flow regime shown in Figure 7, was used. This flow regime was selected for its simplicity which enables a direct interpretation of the scattering data. Figures 8 shows the scattered neutron fluences of neutrons scattered into the four detectors illustrated in Figure 7 each in the angle bins $(30^\circ-40^\circ)$, $(40^\circ-50^\circ)$ and $(50^\circ-60^\circ)$. These bins correspond



to the energy ranges (10.5-8.2), (8.2- -5.8) and (5.8 - 3.5) MeV respectively, if a 14 MeV monoenergetic neutron beam was used. Figure 8 shows clearly how the fluences of the scattered neutrons reflects the water distribution in the scattering medium. Figure 9 shows the energy spectra for the four detectors illustrated in Figure 7. These energy spectra strongly reflect the distribution of water inside the test section.

Monte Carlo experiments were carried out for different typical flow regimes. Figures 10-13 shows the scattered neutron fluences at the detector sites illustrated in Figure 7 for the different flow regimes. These Figures show the strong dependence of the scattered neutron spectra and the water distribution inside the scattering media. If neutrons that suffered only a single scattering were considered, one can, at least in principle; reconstruct the water distribution in the scattering object. This is because single scattered neutrons carry direct information regarding the amount and location of the water they have passed through.

One has, now, a very important information carrier, i.e., neutrons that have suffered a single scattering by hydrogen. Before showing how the above facts are formulated into a mathematical model describing the system, Equation (5) must be examined carefully. This is FIGURE 8 Neutron Scattering Energy-Dependent Fluences for the Hypothetical Flow Regime at Different Angles of Scattering

A : 30-40° of Scattering (Average 35°)
B : 40-50° of Scattering (Average 45°)
C : 50-60° of Scattering (Average 55°)











FIGURE 10 Neutron Scattering Energy-Dependent
Fluences for Annular Flow Regime at
Different Angles of Scattering :
A : At 30-40° of Scattering (Average 35°)

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B : At 40-50° of Scattering (Average 45°)
C : At 50-60° of Scattering (Average 55°)





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(C)

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FIGURE 11 Neutron Scattering Energy-Dependent Fluences for Inverted Annular Flow Regime at Different Angles of Scattering A : At 30-40° of Scattering (Average 35°) B : At 40-50° of Scattering (Average 45°) C : At 50-60° of Scattering (Average 55°)






FIGURE 12 Neutron Scattering Energy-Dependent Fluences for Stratified Flow Regime at Different Angles of Scattering :

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A : At 30-40° of Scattering (Average 35°) B : At 40-50° of Scattering (Average 45°) C : At 50-60° of Scattering (Average 55°)







FIGURE 13 Neutron Scattering Energy-Dependent Fluences for Single-Phase (Water) Flow : A : At 30-40° of Scattering (Average 35°) B : At 40-50° of Scattering (Average 45°) C : At 50-60° of Scattering (Average 55°)

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because Equation (5) is the link between the information carried by the scattered neutron, i.e., its fluence, energy and angle, and the water content in its path.

2 Calculation of Single Scattering Fluonces

Equation (5) gives the fluence of a neutron of weight W and energy E_g that travelled in the scattering medium (water) a distance x before colliding with a hydrogen dom and consequently changing its energy to E and its direction by an angle 6 and then travelling a distance y_1 in the scattering medium on its way outwards a detector that lies at a distance R_1 from the collision site (see Figure 4). Attenuation by materials other than the scattering medium (water) is not considered. These materials are assumed to exist in a fixed known quantity (such as pipe walls) that can be taken in consideration or has a negligible effect (such as air). The weight W is the weight of the neutron considered in the equation with respect to all source neutrons. This weight can be easily estimated, if the total number of source neutrons is known and if the geometry of the source is given. The scattering probability P(0) can be also written for generality as

 $P(E_s \rightarrow E; 0)$ which denotes, the probability that a neutron of energy E_s scatters to energy E and change its direction by an angle 0. There

is a unique relationship between E/E_{c} and θ , as given by

Equation (7), if and only if the scattering is elastic. Scattering of neutrons by hydrogen is always elastic. The threshold energy of inelastic scattering by oxygen is about 6 MeV, however its probability compared to that of elastic scattering is too small and can be ignored⁽²⁾.

The probability $P(E \rightarrow E; \phi)$ is approximated with a truncated s Legendre Polynomial expansion.

$$\mathbf{P}(-0;) = \frac{\Sigma_{\mathbf{s}}(\mathbf{E}_{\mathbf{s}} \rightarrow \mathbf{E})}{\Sigma_{\mathbf{s}}(\mathbf{E}_{\mathbf{s}})} \frac{\Sigma_{\mathbf{s}}}{\varepsilon} = \frac{\Sigma_{\mathbf{s}}(\mathbf{E}_{\mathbf{s}})}{\varepsilon} \frac{\varepsilon}{\varepsilon} \frac{\varepsilon}$$

where $(E_{s} \rightarrow E) = transfer cross-section from Energy E to Energy S$

 (E_s) = scattering cross-section at energy E

= cosine of the angle of scattering

and $P_{\mu}(\mu)$ = the value of the 1th Legendre Polynomial at N.

The cross-sections and Legendre coefficients are customarily reported in cross-sections libraries. The factor $(1/2^{-1})$ that appears in Equation (5) is introduced to take into consideration the azimuthal

direction within which isotropic scattering is assumed.

3.3 Single Scattering Conditions:

Equation (5) Gs valid if and only if single stattering is assumed. Considering a thickness of scattering material equivalent to one mean free path of source neutrons penetrating this material, the probability that a single neutron interaction occurs is $(1-e^{-1})$ (.e., 0.632. Neutrons scattered by this first interaction might be exposed to further interactions, resulting in multiple scatterings. The probability of multiple scatterings increases as the energy of neutrons emerging from the first collisions decreases. This is because the lower the neutron energy, the shorter the mean free path and consequently the larger the collision probability. Therefore, if one intends to use Equation (5) to estimate the scattering fluence, the contribution of multiple scattering must be considered. Such contribution may be insignificant if the amount of scattering materials is only equivalent to one mean free path (or less) of the source neutrons. Multiple scattering dependent free path (or less) of the source neutrons. Multiple scattering dependent for neutrons of the scattering material is only equivalent to one mean free path (or less) of the source neutrons. Multiple scattering dependent for neutrons of the scattering material for the scattering fo

the effect of multiple scattering must be justified experimentally.

In order to investigate the contribution of multiple scattering as compared to that of single scattering, neutron scattering for test sections of different diameters were simulated using the Monte Carlo method. In Table 1, the weight of rescattered neutrons is compared to that of first scattered neutrons. The Table shows that the contribution of rescattering increases with increasing test section diameter; however, it is generally small even for the test section of 100 mm diameter (onemean-free-path of 14 MeV neutrons). Since these contributions were estimated for energy groups of different widths, the numbers in the Table do not reflect the change of rescattering contribution with energy.

Also, one has to note that not all scattered neutrons contribute to the detectors considered. Therefore, one has to compare a detector response to single-scattered neutrons to its response to multiscattered neutrons. In the next. Section it is shown how the detector response to single scatterings is, calculated and in the Section after this response is compared to Monte Carlo simulations, where multiscattering is considered.

SINGLE SCATTERING APPROXIMATION

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The cross-sections sets prepared for the Monte Carlo simulation discussed in Chapter 2 were used for single scattering calculations. These sets provide for different elements, total, scattering, energy transfer cross-section and also the Legendre coefficients required for Table 1: Contribution of Neutron Rescattering

	LOC DOGCECTINGS	5
Prior to		

Rescattering		Tès	st Section	Diameter,	mm
(E) MeV	12.7	23.9	45,8	70.7	100
11.1	1.1	2.1	.3.9	5.0	5.7
9.1	, 1.2	1.4	2.9	4.6	5.8
7.4	0.7	0.8	1.9	-2.4	• 3.2
6.1	0.5	0.8	2:3	3.7	5.2
4-4	0.5	0.8	2.0	3.0	3.9
3.9	0.7	,1.6	3.7	4.9	7.5
2.9	1.7	2.2	2.8	5.3	5-8
i.7	0.4	1.0	1.9	2.8	3.0

.0.5 1.1 2.2 4.1

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the scattering probability calculation. The cross section data was provided for 33 standard energy groups and N, the number of Legendre coefficients used in Equation (9), was eight. This number of coefficients is thought to be large enough to provide reasonably accurate scattering probabilities.

Consider a 100 mm diameter pipe exposed to a parallel beam of 14 MeV neutrons as shown in the schematic diagram of Figure 14. A 4 x 4 mesh is constructed, as shown in the Figure, since neutrons can be tracked conveniently through their passage in the pipe while being

scattered towards any of the eight detectors shown in the Figures. These dotectors were set to detect neutrons forwardly scattered by the hydrogen of the water in the pipe. As is shown in Figure 6, neutrons that scatter 45° by hydrogen carry half of their original energy after scattering ie., 7 MeV, while those elastically scattered by oxygen carry 0.964 of their initial energy i.e., 13.50 MeV. Therefore, if the detectors were set to measure neutrons of energy of 7 MeV, they will be recording only scatterings caused by hydrogen, since a single scattering of neutrons by.

oxygen cannot degrade neutrons to half of their energy. The 45° angle of scattering was chosen here just for sake of illustration. Any other forward angle could be considered.



Scattering cells were defined as the squares, or portions of squares, that exist inside the pipe. Water and void inside a scattering cell j are homogenized to produce a mixture of density . where

$$\overline{p}_{j} = (1 - \alpha_{j}) \quad p_{f} + q_{j} \quad q \quad (10)$$

where α_j is the void fraction in cell j , ρ_f is the water density and ρ_g is the void density. ρ_g is zero for pure void or equal to vapour density in a vapour-water system. However, ρ_g is always much

less than ρ_f , and the second term in the above equation can be neglected.resulting in

$$= (1 - \alpha_j) \rho_f$$

and the water fraction $(1 - ia_i)$ will be given by

$$\rho_{j} = 1 - \alpha_{j} = \bar{c}_{j} / c_{f}$$
(12)

To calculate the response of a point detector i to source neutrons that are scattered 45° to reach that detector, the following

systematic procedure is traced, guided by Figure 14. A ray representing

drawn from the detector towards the scattering object making an angle of 45° with respect to the source neutron beam. The scattering cells that this ray intersects are determined. The coordinates of all points at which the scattering ray intersects with the boundaries of the cells are determined. Scatterings are assumed to occur in a cell, on the average, "at the middle of the segment of the scattering ray that exists inside the cell. These middle points define the centres of scattering inside each cell. From each scattering centre, a ray parallel to the source neutron beam is drawn. This ray will be called the source ray. The cells that the source ray intersect are then determined. The coordinates of the points at which the source ray interacts with boundaries of cell, in its path towards the scattering cell are also determined.

The contribution of each cell that the scattered ray intersects is denoted by $C_{ji}(\theta)$, i.e., the contribution of neutron scattered by an angle θ from cell j to detector i. Of course, cells that the scattering ray does not intersect do not contribute to the detector under consideration. $C_{ji}(\theta)$ is calculated from an equation similar to Equation (5), as follows:

 $\mathbf{c}_{1} = \mathbf{w}_{1} \exp \left(\mathbf{c}_{2} - \mathbf{y}_{1} \right) + \mathbf{c}_{2} \exp \left(\mathbf{c}_{2} - \mathbf{y}_{2} \right) + \mathbf{c}_{2} \exp \left(\mathbf{c}_{2} - \mathbf{c}_{2} \right) + \mathbf{c}_$

M

* $\exp \left\{ \frac{1}{2} \sum_{j=1}^{n} \frac{1}{2} + \frac{1}{2}$

 $\mathbf{r} = \mathbf{r} + \mathbf{m} \mathbf{n} \mathbf{n} \mathbf{n}$, where $\mathbf{r} = \mathbf{n} \mathbf{n} \mathbf{n}$, we have the second state of the sec

°j

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where W = weight of source neutron entering cell j

. total cross-section of source neutron in cm⁻¹,

water fraction in cell m,

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×_{ej}

Ymi

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R.

distance that source ray travels in cell before being scattered in cell j

distance that scattering ray travels in cell m after being exattered in cell j

total cross-section of scattered neutrons,

P(0) = probability that source neutron scatters by angle by water,

distance between the centre of scattering and the

detector,

total number of cells encountered by a source neutron before being scattered in cell j: f_{1} = total number of cells encountered by a neutron after

being scattered in cell j.

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In Equation (12), W_{j} is estimated for a line source by the ratio of the area of the cell exposed to source neutrons to the total area of the source. The first exponential term in Equation (13) estimates the source neutron removal probability, while the second one estimates that of scattered neutrons. The term [p_{j} , P (0)] is the probability that source neutrons that reach cell j scatter by an angle

 θ . Note that if $\rho_j = 0$ (cell j is full of void), the scattering probability will be zero and if $\theta_j = 1$ (full of water) the scattering probability equal to P(0). The distances x_{1j} are calculated as

follows:

For cells in the path of the source ray (except the scattering cell):

intersects the boundaries of cell number j,

For the scattering cell:

distance between the point at which the source ray intersects the scattering coll and the position of the cell scattering centre, otherwise, ...x_{1j} = 0⁻¹

The distances y_{mj} are estimated as follows:

For cells in the way of the source ray (except the scattering cell):

y_{mj} = distance between the two points at which the scattering ray intersects the boundaries of cell number m;

For the scattering cell:

 \neq distance between the point at which the scattering ray leaves the scattering cell and the position of the cell scattering centre, otherwise, $y_{mj} = 0$.

The cross sections and scattering probability in Equation (13)

are obtained at the appropriate energy groups using the cross-section library prepared for the Monte Carlo experiment. The angle of scattering θ that corresponds to the desirable energy of scattering is obtained

using Equation (8), which consider scattering by hydrogen.

The response of detector i to neutrons of energy g (scattered by an angle θ) is then given by,

(14)

S (E)

where $C_{ij} = 0$ if cell j is not contributing to detector j.

Due to the energy grouping used in the multi-group cross section library, it is not possible to estimate detector responses at specific neutron energy. Instead the responses would be estimated within a specific energy range (energy group). Every energy group would then according to Equation (8), correspond to a range of scattering angles. Therefore, a band of scattering angles corresponding to the energy range of the specific energy group is to be considered. Although neutron scattering within an angle band would be assigned the same total cross section as that given to neutrons having the average energy of the group, the probability of scattering by these angles would slightly differ from angle to another. This is because the probability that neutrons in the energy group g (source neutrons group) to an energy group g with an angle " is given by: (similar to Equation (9)):

$P(\theta) = \frac{\Sigma(q + q^{4})}{\Sigma(q)} \frac{\sum_{k=0}^{N} \frac{(2k+1)}{2} f_{k}(q^{4}+q) T_{k}(\mu)}{\xi=0} + \frac{1}{2} \frac{1}$

where the notation used here is similar to that used in Equation (9). Therefore, a variation in the angle , within the range that corresponds to the energy range of the group g, would result in no variation in the right hand side of the above equation, except for the Legendre Polynomial (P (μ), ($\mu = \cos^{-1} - 0$). This change in the value of P F(μ) due to the change in -0 might not be very significant, but the path of a scattering ray through the scattering material would change by changing θ . The change in the path of the scattering rays would result in h change in neutron attenuation through their passage in the scattering material. Such a change in the scattering ray, and consequently the source ray, could result in a significant change in the detector response, if a slight change in θ cause the scattering ray, or the source ray, to move from a very high density area to a very low density

area (or vice versa).

In order to compensate for the discrepancy in angle of scattering due to the energy grouping, several scattering rays, instead of one ray are traced, as shown in Figure 14. The angles associated with these rays are equivalent to those corresponding to the energy range of the energy group considered. That is, if is the angle corresponding to the upper energy of the group, and L is the one corresponding to the lower energy, then the ith ray of an N ray

scattering band would assigned an angle 0, such that

$\theta_{i} = (0_{i} - 0_{L})(i/N) + 0_{L}$

If only one scattering ray is to be traced, then it would be assigned the average angle of scattering ie., $(\bigcirc_L^+ \oslash_U^-)/2$. The procedure of tracing N scattering rays will be called N-Folding, since N scattering rays are traced to obtain one detector response.

A listing of a computer program used for estimating detector

responses assuming single scattering is available as indicated in Appendix A. This program can estimate the detector responses due to 14 MeV.mono-directional neutrons scattered to different angles by water, Different pipe diameters are allowed and any mesh size for scattering cells can be assigned. Point detectors as well as wide detectors can be considered in this program.

The Single Scattering Algorithm considers a backward walk of the scattered neutron. In other words, in order to estimate the detector response $S_1(\theta)$ produced by a source neutron that suffered a single scattering of angle θ , a neutron ray that makes an angle θ with the

source neutron beam is drawn from the detector site, and the scattered neutron is tracked backward through this ray. All the cells that this ray goes through contribute to the detector response. From the mid-point of the scattering ray intersection with each cell, a source ray, parallel to the source beam is drawn and the source neutron is tracked backwardly through this ray. Figure 14 filustrates this backward neutron walk. The weight $W_{ji}(\theta)$, of neutrons enteding cell j are to be scattered by an angle θ towards detector i, is then estimated as

$W_{ji}(0) = y_{ji} / D \cos \theta$,

where y_{ji} = distance neutrons scattered to detector i travel in cell j.

and

D = length of target exposed to the neutron beam, usually the target diameter. If D is less than the pipe diameter, cells outside the range of the neutron beam officient dessigned a zero weight.

The other parameters required for the neutron walk are calculated as

illustrated before.

Establishing a simple and fast computer algorithm for detector responses calculations is very important in the process of reconstructing the phase distribution inside the object. One can provide a guess for the distribution, calculate simply and cheaply the corresponding detector

responses, compare the calculation with the corresponding experimental data, readjust the water distribution accordingly and repeat the procedure until a reasonable agreement between the calculations and the

experimental data is achieved. This procedure of reconstructing the phase distribution from the experimental data will be discussed in the

next Chapter, while the next Section will show a comparison between the

detector responses obtained using the simple single scattering approximation and those obtained in a Monte Carlo experiment. This comparison is important in order to provide confidence in the single scattering approximation, since it will be the workhorse of the phase reconstruction process.

SINGLE SCATTERING APPROXIMATION

VERSUS MONTE CARLO EXPERIMENTS

5.0

A Monte Carlo Experiment (MCE) as shown in Chapter 2 takes into account the different aspects of the physical phenomenon considered. Therefore, multiple scattering and inelastic scattering are considered in a MCE while ignored in a Single Scattering Approximation (SSA) of the same experiment. SSA allocates the scattering events in the centre of a neutron path in a homogenized cell, while MCE considers the real scattering positions/without any homogenization of the material present in the neutron path. However, since SSA is restricted to only one mean-free-path (or less) of the scattering material, the discrepancy between SSA and MCE is expected to be insignificant. This, though, has to be shown.

In order to test the correctness and accuracy of the SSA, the hypothetical Water-Void distribution, shown in Figure 7, was considered. Because of the simplicity and distinctiveness of this distribution, any errors in the computer program can be corrected.

In using the SSA, as mentioned before, a one fold or more than one fold can be used to estimate the scattering fluence. By a fold it is meant that one scattering ray has been used to estimate a detector fluence. Since one normally estimates the fluence within a wide angular (and consequently energy) range, i.e. in between angles θ_1 and θ_2 . (or energies E, and E₂), one scattering ray taken at the average

angle or any other angle will not be sufficient to estimate an accurate fluence. This is because if the scattering ray happens to pass at the edge of a cell, then the contribution of that cell will be minimum. Therefore, for an accurate estimation of the scattered neutron fluence at a specific detector, more than one scattering ray (fold) has to be considered for that detector. The question, now, is how many folds are required. Table 2 shows the fluences estimated using an increasing

number of folds, as compared to the MCE for different detectors. The detector numbers in this Table correspond to those shown in Figure 8. The error, e, associated with the SSA fluences F_{SSA} , is estimated relative to the fluences obtained in the MCE, F_{MCE} , as follows

$e = | (F_{MCE} - F_{SSA})/F_{MCE} |$

while the errors shown in the table associated with the MCE fluences are the statistical errors expressed as fractional standard deviation, (standard deviation/mean).

Ignoring the MCE poor fluences, (fluences with large statistical error), Table 2 shows that about 21 folds are required for the SSA to result in fluences that lie within the statistical error band associated with the MCE fluences. Since the fluences listed in Table 2 cover angular ranges of 10°, then for 21 scattering rays (21 folds), each scattering ray covers an angular range of 1/2°. More than 15 folds will result in a better resolution. By defining a satisfactory resolution as the resolution that results in a SSA fluences within the statistical error band of the MCE fluence, then 1/2° per a scattering ray is a

			•	•					
	2:	Monte	Carlo	and	Single	Scatterin	۱ġ.	Approxis	ation
÷		Fluer	es for	e Pha	ase Dist	ribution	of	Figure	8

	Average	Detector Responses [neutrons/cm				m ² /sourc	2/source neutron) x 10 ⁶				
stactor	Scatterin	g Monte "Carlo	<u>Numbe</u> 1	<u>r of Fol</u> 11.	<u>ds in Sin</u> 21	gle Scat	101	401	.2239		
2NLL-E1			·					<u></u>			
	55*	7.487	8 224	7.010	7.604	7.507	7.489	7.524	7.517		
•		5.6*	9-8**	6 1	1.6	0.3	0.03	.05	• 0.4		
	559	8,959	10.13	8 866	9.015	8.854	8.929	8.904	8.896		
anta de la		6-3	13.1	10	0.63	1.2	0.3	0.6	0.7		
		0 770	n n	A 681	1.671	81.854	8.929	8.904	8.895		
		95.		4.001				,			
								·			
	45*	5.888	00,	7.200	8 308	7.174	7.353	0.16.10	1-23/		
		21.	10	22.2	41.1	21-8	. 28.	24+2	43,13		
•	.45*	11.18	14.12	11.93	11 37	.11.45	11.43	11.43	7.257		
		4.7	26.3	6.71	1.7	2.2	2.2	2.2	2.3		
· • • •	4 C #	14.19	15.79	11.70	13.71	11.89	13.91	13.95	13.98		
	•••	5.0	11.3	3.5	3.4	2.1	2.0	1.7	1.7		
•				5 ¹	۰.	, • • ·	•• •				
· · ·	45*	0.050	0.0	5.371	5.743	3.713	3.512	3.676	3.7.		
•		BC.		•	· · · ·		• • •	• • .			
. · · ·	J5°	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		
· · · ·	750	10.86	12-62	11.63.	11.29	11.02	11.05	11.02	11.01		
	· •	5-4	16.2	7.1	4.0	1.5	1.7	1.5	1.4		
· · · · ·		· · ·			• • • •						
-	35	13.28	1.49	12.79	12:91	12.97	13.00	13.02	13.07.		
	•	6•Z	1.6	·· 3·/:	2.0	2 3	2.1	•	× , u .		

* Practional standard deviation, N. *

Practional error relative to Monte Carlo,

satisfactory resolution. A better resolution, in fact, is of no value. That is because the confidence level that we have in the MCE fluences is bounded by the standard deviation, (for 68% confidence level), and therefore all the SSA folds that result in a fluence within that confidence level are equally valuable.

The resolution required for a satisfactory SSA is problem dependent. The further the detector from the scattering object, the better the resolution required. This is because the further the distance, the bigger is the region covered by the angle bin between two adjacent scattering rays. Also, a better resolution might be required if the fluences against which the SSA fluences are to be compared have a better statistical error.

For the setup illustrated in Figures. 8 to 13, it was found that a resolution of 1/2° per scattering ray, (20 folds for a 10° detector angular bin); is sufficient to produce SSA fluences that lie within the error band associated with MCE fluences of around 0.05 fractional standard deviations (fsd's). MCE fluences of less than 0.05 fsd's were

not sought since in order to reduce the fsd by half, roughly a quadruplication of the number of particles tracked, and consequently quadruplication of the computer time, is required. It should be noted that the 0.05 fsd mentioned above is the fsd associated with the fluence recorded at an angle Bin of 10°. When the fluence was recorded for all the angular range, (total fluence), a fsd of 0.002 was obtained.

It is interesting to note that while tracking of around 9000 source neutrons was required to result in a MCE fluence within a 0.05 fsd, only around 20 scattering rays were required to produce a SSA fluence that lies within the error band associated with the MCE fluence. In fact, not all the 9000 source particles contribute to the fluence of detectors under consideration, but only those neutrons that suffer collisions. Also, not every collision contributes to the detectors fluence. For example, for the phase distribution of Figure 8, only 2239 of 9000 source particles are scattered by the water and the probability that one of these scattered neutrons will reach one of the detector within one of the angle bins of interest is about 2.4%. This probability includes neutron scatterings that lie in the field of view of the angular bin under consideration. Other scatterings outside this view do not contribute to the detector fluence within this angular bin .. Therefore, it is not surprising that 20 scattering rays in the SSA produce the same contribution 9000 source neutrons produce in a MCE. These scattering rays are in fact the effective contributors to the angular bin of the detector for which the fluence is to be estimated.

Another factor that could affect the accuracy of the SSA is the number of cells used. In the fluences listed in Table 2, a 4 x 4 grid (Figure 15) was used. Tables 3, 4, 5 and 6 show the SSA fluences calculated using a 4 x 4 grid and an 8 x 8 grid (Figure 16) for the Water-Void distribution shown Figures. 10 to 13 as compared to the MCE fluences. Table 3, for annular flow, shows that in general, increasing the number of cells tends to bring the SSA fluences closer in value to those of MCE. Since most of the SSA fluences for the 4 x 4 grid and the

8 x 8 grid lie within the statistical error band of the MCE fluences, it is hard to tell if the increase in the grid size produces better

results. However, for the detectors 3 with 55°, and 4 with 45° average degree of scattering, the 4 x 4 grid SSA produces fluences that lie outside the error band of the corresponding MCE fluences, while those of 8 x 8 grid lie within the error band. This shows that increasing twe number of cells tends to improve the accuracy of the SSA. The reason for that is in a larger grid, the cell size is smaller and the error introduced by homogenizing the water inside the cell is reduced. This is especially true for the detectors that record scattering from the edges of the water zone (like the detectors named above).

The same conclusion can be withdrawn from the fluences of Table 4 for the core flow regime of Figure 11. A significant improvement in the SSA fluences of the detectors one with 35°, detector three with FIGURE 15 Cells as seen by Detectors at Different Scattering Angles A : At 35° of Scattering B : At 45° of Scattering C : At 55° of Scattering

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Table 3: Monte Carlo' and Single Scattering Approximation fluences for Phase Distribution of Figure 10 (Annular Flow Regime)

X

	Average	10 ⁶ x 1	Wence (neutr	ons/cm ² /sourc	e neut <i>r</i> on)
Detector	Scatteri	ng Monte	4 x 4 Grid	4 x 4 Grid	8 x 8 Gri
Number	Angle	Carlo	21 Folds	41 Folds	_ 21 Folds
•				.	
1	55°	14.052	14.41	14.39	14.08
		. (6.7)*	(2.55)+	(2-41)+	(.20)+
-	c = 0	15 11	16 16	16 14	1 5 0 7
4		10.211	10.10	14 111	10.52. IA GGN
		• • (0• /)	(0.1/)	(0-11)	(4.00)
3:	55.°	18.014	16.49	16.57	18.87
- " 7 .		. (6.5)	(8.46)	T (8.02)	(4.75)
	`				
1.	45 °	29.782	27.97	29.07	30.39
		(4.8)	(6.08)	(5.75)	(2.04)
2	45°	233.9	24.20	24.18	23.11
•		(6.4)	(3.82)	(3.74)	(0.05)
	-43				
3	45°	24.945	25.82	25.83	24.82
		(6,.5)	(3.51)	(3.55)	(0.50)
A	45°	26 623	23.28	23.39	26.35
4	45	(5 3)	(12 5)	(12 1).	(1.025)
	•	(3:3)			(1.023)
2	· 35 °	29.071	27.74	2787	30.15
· · · ·		(1.8)	(4.5B)	(4.13)	(3.71)
		, , , , , , , , , , , , , , , , , , , ,	21 70	21 70	21 45
. د	35	21.394	21.72	21.70	21.4J (0.36)
•		(6.5)	(1.54	(1+43)	(0.20)
4	35.°	21/22	21.62	21.61	21.19
-		(8.7)	(1.89)	(1.84)	(0.14)

Fractional standard deviations (1)

+ Fractional error relative to Monte Carlo (%)

Table 4: Monte Carlo, and Single Scattering Approximation Fluences for Phase Distribution of Figure 11 (Core Flow Regime)

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	Average	<u>10⁶ x Flue</u>	nce (neutrons/	cm ² /source neut	tron)
erector	angle	Carlo	21 Folds	21 Folds	
unider					
i si	55°	21 563	20,23	20.15	
		(5.17)*	(6.18)+	(6.55)+	
	55°	18.462	16.46	17.28	
		(5.65)	(10.84)	(6.4)	
		A 977	. 7 532	· · · · · · · · · · · · · · · · · · ·	
	22	4.022	* (50 A7)	(3.49)	
		(10.0)			
	45°	· 13.271 \	17.40	13.35	- -
		(6.25)	(31.11)	(0.60)	
	45°	.33.929	33.14	34-00	
		(3.9)	(2.33)	(0.21)	•
		(1, 1)		• • •	
•	45°	27.537		27.28	
• • • • •		(4.5)	(4.42)	(0.93)	
		3 .			· ·
	45°	9.126	.11.19	8.232	•
··· · ,		(7.4)	(21.84)	(9.8)	·
***	· •	· · · · · · · · · · · · · · · · · · ·	10.20	11.00	
	35 .	10.55	10,-38	(12.90)	
	· · · · · ·	(7.+0)	(25.20) •		
	350	28.26	29.07	30.42	
		(5.0)	(2.807)	7.64	~~
			*		
	35°	24.839	23.70	24.08	
		(4.4)	(4.59)	(3.06)	1. •
	•			- · · · ·	· · ·

Fractional standard deviations (%)

Fractional error relative to Monte Carlo (%)-

Table 5: Monte Carlo Experiment Single Scattering Approximation Fluences for Phase Distribution of Figure 12 (Stratified Flow)

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	Average	10 ⁶ x Fluence (neutrons/cm ² /source neutron)					
Detector	Scattering	Monte	4 x 4 Grid	8 x 8 Grid			
Number	Angle	Carlo	21 Folds	21 Folds			
			10.11	10.70			
1	55	19.093	19.21	12+12			
•	· · · · · · · · · · · · ·	(5.0)-	(10+01)	- (3.03)			
2	55	13.802	14.17	14.39			
-		(3.16)	(2.67)	(2.26)			
• • •				• 🗍			
3	. 55	3.449	3.286	. 3.313.			
•		-(7.9)	(4.73)	(3.95)			
• • •			· · · · · · · · · · · · · · · · · · ·				
1 . <u></u>	45	35.656	(34.82	33.51			
		(3.64)	(2.34)	(0+41)			
2	45°	33.838	33.31	33.93			
~ ~		(4.1)	(1.56)	(0.27)			
	•		· · · · · · · · · · · · · · · · · · ·				
3	45°	18.68	19.06	19.34			
•		(5.57)	(2.03)	(3.55)			
		•					
4	45°	2.411	22.58	22.60			
•		(8.5)	(6,36)	(6.28)			
,	35.0	34 246	. 35.2	35.73			
1	, CC [,]	(4.1)	(2.79)	(4-33)			
		(/	_ · · · · · ·				
2	35°	30.803	30.37	30.67			
•••	ال. •	(5.0)	(1.41)	(0.45)			
· · · ·	•		• • •				
3	. 35°	15 15	15.02	15.19 -			
		(8.2)	(2.25)	(1.64)			
• • • *		11 00	11 37	א רו			
6.	\$55" "	11.88	11,427	(1,7)+			
		(0+00)	(3.137)	(Joint			
7	550	12,191	11.57	11.72			
•		(4.67)	(4.36)	(3.12)			
	· .						
8	55°	9.829	9.427	9.534			
		(4.7) "	(4.09)	(3.00)			
				Х. .			
5	45°	19.95	19.24	19.44			
\sim	•	(4.2)-	(3.56)	(2.57)			
20	•			1			

Table 5 Continued

	Average	.10 ⁶ x Fluence (neutrons/cm ² /source neutron)						
Detector Number	Scattering Angle	Monte Carlo	4 x 4 Grid 21 Folds	8 x 8 Gri 21 Folds	d .			
5.	45°	20,767	19.97	20.15	• • •			
		(3.8)	(4.80)	(7.97)				
, .	. 45°	13.74	13.87	13.97	· ·			
		(4.0)	(9.5)	(1.67)				
8	45°	3.350 (8.6)	3.377 (8.1)	3.391 (1:23)	\sim			
5	35°	18.028 (4.3)	18.46 (2.4)	18.58 (3.06)				
5 ,	; 35°	16.665 (5.2)	11.18 4.16	11.24 (3.64)	, 1			
7	35°	1.004 (5.5)	1.013 (0.88)	1.013 (.88)				

ì. -

*. Fractional standard deviations (%)

Fractional error relative to Monte Carlo.(%)

Table	6: 1	Monte (Carlo	and	Sing	le Scat	Lering
Approxi	matic	on Pluc	inces	for	Phas	e Distr	ibution
	of	Figure	13 (Full	of	Water)	÷.,

	Average	10 ⁶ x Fluence (neutrons/cm ² /source neutron)					
Detector	Scattering	Monte	4 x 4 Grid .	8 x 8 Grid			
Number	Angle	Carlo	21 Folds	21 Folds			
1	55°	22,96 . (5.66)	22.89 (0.3) +	123.51 (2.4)+-			
2	55°	20.584 (6.4)	20.09 (2.45)	20.38 (0.99)			
3	55°	13.672 (7.88)	12.96 (5.21)	13.09 (4.26)			
1	45°	37.413 (4.7)	36.46 (2.55)	37.04 (1.0)			
2	45°	42.654 .(5.1)	40.16 (5.85)	.40.81 (4.32)			
3	45°	33.797 (4.5)	31.19 (4.75)	32.57 (3.63)			
4	45°	20.791 (3.6)	20.52 (1.30)	20.65 (0.68)			
1	3 5°	36.424 (5.6)	35.77 (.80)	36.19) (0.64)			
2	35°	38.341 ' (5.7)	36.20 (1.58)	36.51 (4.78)			
3	35°	28.696	28.04 ; (2.29)	28.29 (2.32)			

Fractional standard deviations (%) *

Fractional error relative to Monte Carlo (%) ÷

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55° and detector four with 45° average angle of scattering. These detectors record scatterings from the edges of the water zone. Homogenizing these edges over the volume of the cell they occupy results in a significant error if the cell size is large.

One can imagine that the homogenization process causes spreading of the water zone. In the annular flow, the water annulus tends to spread towards the centre of the annulus, while in the core flow the spreading is outwards. This spreading results in scatterings from a volume that is full of void and should not cause scattering at all. Spreading also reduces the density of the fluid on the edges of the water zone from which swelling starts. This reduction in the density decreases the scattering probability from these edges, and it also decreases the attenuation of neutrons, interducing non-realistic neutron tracks and. consequently errors in the estimated scattering fluences. Spreading tends to fill the cells that were partially filled with water.

In Tables 5 and 6, the fluences of a stratified flow regime (of Figure 12) and a full water flow regime (of Figure 13) are listed. In these two flow regimes no homogenization is necessary because the water fills completely the cells it occupies. The effect of reducing the cell volume by increasing the mesh size is to reduce the error introduced in the SSA by assuming that the scattering events occur on the centre of the section of the scattering ray crossing a cell. The smaller the cell volume, the greater the number of cells that are intersected by a specific scattering ray and consequently the more are the source rays

backwardly tracked for that scattering ray. In fact, the maximum number of cells a scattering ray can intersect is (2N-1) for an NXN mesh. This number of intersections is also the number of the scattering sites and consequently the number of source rays corresponding to the scattering ray that crosses the maximum number of cells. In other words an NXN mesh SSA corresponds at most to (2 N-1) folds, in addition to the number of folds specified for each neutron scattering ray. As mentioned before, increasing the number of folds tend to increase the resolution and consequently the accuracy of the SSA. Since the MCE fluences accuracy is limited by their statistical errors, a resolution that produces fluences that lie within the MCE fluences error bound is considered satisfactory.

For this reason, in Tables 4 and 5, the effect of increasing the mesh size from 4×4 to 8 x 8 is not significant, since both of these mesh sizes produce SSA fluences that lie within the error band of the MCE

fluences.

In conclusion, a better resolution in the SSA neutron walk-can be achieved by reducing the cell size or increasing the number of folds, or both. When the SSA is used to simulate an expériment, like the MCE, the size of mesh and the number of folds that result in fluences that lie within the statistical error band of the experiment fluences, are considered to result in a satisfactory resolution.

6.0 SINGLE EXPOSURE NEUTRON TOMOGRAPHY (SENT)

The restriction of the neutron scattering phenomenon to single scatterings makes the scattered neutrons, as indicated before, very, useful information carriers. A neutron in its path from the source beam to the detector collects information about the material it passes through. This information is then deposited into a detector as a message. Since this process of information collection can be represented by a simple formulation using the Single Scattering Approximation, one should be able to reconstruct back these information from the messages, provided by the detectors. A Single Exposure Neutron Tomography of the flow phases is then possible.

In a Single Exposure Neutron Tomography (SENT); the object for which the phase distribution is to be determined, would be exposed to a neutron beam and the forwardly scattered neutrons would be detected at different energies (corresponding to specific angles). Only one exposure is necessary in this process. If the object contains at most the equivalent of one mean free path of the source neutrons in water, single scatterings will be the most probable events and the Single Scattering Approximation can be used; provided that lower energy bins are not considered in order to reduce contribution of neutron rescattering. Then, one could suggest the following unfolding procedure:

- provide a guess for the phase distribution in the object by assigning a water fraction to each cell. (the object is to be divided to fictitious cells)

b - calculate the detector responses corresponding to that guess, using the Single Scattering Approximation

compare the calculated detector responses to the measured ones and accordingly readjust the phase distribution guess by correcting the water fraction in each cell

repeat the above procedure until an agreement within the measurement error is achieved between the calculated and the measured detector responses. The phase distribution guess that achieves this

agreement will be the reconstructed distribution;

d

In fact the problem of finding the phase distribution from experimental measurements is the inverse problem encountered in a variety of fields. The formulation of this inverse problem and the method of solving it are investigated in the next Chapter. The problems of the existence and uniqueness of the solution and the error associated with it are also explained in the following Chapter.

7.0 DETECTOR ARRANGEMENT

As indicated earlier, scattered neutrons with specific energies and angles of scattering are to be detected. One can detect neutrons scattered by a specific angle by mounting a long narrow detector in the direction of scattering, and setting the energy band on the detector to correspond to the energy of scattering. The scattered neutrons can also be collimated such that the neutrons arriving at a detector are only those that went through the collimator. With these arrangements, only one direction of scattering and consequently one energy is recorded per detector. Therefore, if it is desired to determine the phase distribution in m cells at least m detectors are to be used. This is generally a large number even for the modest case of a 4 x 4 grid, where at least 16 detectors are required. However, detectors that measure the intensity of fast neutrons as a function of energy are usually capable of measuring their energy spectrum over a wide energy range. Therefore, by

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utilizing one energy band per detector, as in the above arrangements, one would not be taking advantage of the full capacity of these detectors. Therefore, the arrangement of using long narrow detectors or collimating the scattered neutrons, though precise in detecting a specific energy and angle of scattering, is not efficient since the capabilities of the detectors are not fully utilized. Moreover, the cost of the detection system associated with these arrangements would be high because of the large number of detectors used and consequently the number of electronic circuits required.

The alternative economical arrangement is to use a non-directional detector; that is a detector that can record neutrons incident on it from any direction. For such a detector, every energy bin recorded corresponds to a specific direction of scattering, provided that only <u>single</u> scatterings are encountered in the test section. Therefore, one can significantly reduce the number of detectors required since every detector would provide m detector responses, if its resolution permits recording in m energy bins. Consequently, the better the resolution of the detector, the less the number of detectors required. In fact one can, in principle, obtain all the required detector responses using one detector with a resolution that is good enough to provide the required

data.

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The detector arrangement mentioned above has the advantage of fully utilizing the detector capability of measuring the energy.spectrum, and consequently reducing the number of detectors and electronic systems required, and in turn the cost of the detection system. However, the detectors used - being non-directional - observe not only neutrons that suffered a single collision but also multiscattered neutrons. However, this problem can be solved by limiting the size of the test section considered to a dimension equivalent to one mean-free-path of the source neutrons, consequently reducing the probability of neutron rescattering. One can also set an energy threshold on the detector used in order to discard low energy bins which contain the major contribution from multiscattering.

Table 1 shows the relative weight of higher order scattering compared to the first scattering as obtained from the Monte Carlo simulations. The Table shows that the number of multiscatterings in each energy group represents a small fraction of the total number of scatterings. However, as the amount of water increases, i.e. for a larger test section, the fraction of multiscattering increases. One can reduce the contribution of multiscatterings to the measured detector responses, as shown in Table 7, by setting a lower energy threshold below which the measured responses are not recorded. This reduction is caused.

by the fact that the rescattered neutron, having an energy E before

Table 7 Reduced Contribution of Neutron

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Rescattering as a Function of Threshold Energy

	Rescattering	Reduced Contr	ibution	5	•
	Threshold En	ergy (Ets), Me	V*		
0.0	1.5	2.5	3.5		4.5
100	93	89	. 84		80
100	92	86	81		75
100	90	83	,76		63
100	89	80.	71		
100	83	72	56		
100	71	68	28	•••	- :
100 .	64	.38	·		-
1. 1.		· · · · ·		·. ·	

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100

Reduction due, to hydrogen rescattering - Ets/E.

Reduction due to oxygen rescattering = Ets $\rightarrow (15/17)^2 E / (E-Ets)$ for.

E < (17/15)²Ets, otherwise there is no reduction.

scattering, could carry any energy between E and zero if it has been scattered by hydrogen and any energy from E to $(15/17)^2$ E if it has scattered by oxygen. Therefore, it arrives to the detector with energy within this range. By setting an energy threshold on the detected energy, one gets,rid of the contribution of some of multiscattered neutrons. If the contribution is less than 5 percent, which is in the range of the expected measurement error, one can claim that the contribution of multiscattering is insignificant. Of course, one can further reduce the effect of multiple scattering by increasing the threshold energy. But keeping in mind that an 'upper energy threshold of $(15/17)E_s$ has to be set to avoid measuring first order oxygen scattering, the available detected energy range would be reduced, and in turn more detectors or detectors with finer energy resolution would be required.

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RECONSTRUCTION OF LOCAL VOID FRACTION DISTRIBUTION

CHAPTER 4

FROM SCATTERED NEUTRON FLUENCES

INTRODUCTION

1.0

As indicated in Chapter 3, the energy spectra of neutrons scattered by a sample containing a vapour-water depend on the local void fraction distribution in the object. Therefore, if the energy-dependent fluences of the scattered neutrons are measured, one should be able to reconstruct the local void fraction distribution. This reconstruction problem is, in fact, an inverse problem in which a set of experimental data (scattered neutron fluences) are used to reconstruct a set of sought parameters (local void fractions). A formal definition of the inverse problem and its solution is given in this chapter. The formulation of the inverse problem for the reconstruction of the local void distribution and the methods of solving this problem are discussed. These methods are then applied to the problem of

reconstructing the local void fraction distribution.

The abstract analysis given at the beginning of this Chapter provides guidelines to solving an inverse problem. An excellent layout of the inverse problem and its solution is given by the French mathematician Pierre C. Sabatier in his introduction to the lecture on Applied Inverse Problems presented through the "Recherche Co-opérative sur Programme No. 264: Etude Interdisciplinaire des Problèmes Inverse"; reported in Reference 2. Sabatier mentioned that in order to address an inverse problem, one has to consider the extension, existence, construction and error aspects associated with it. We analyze Sabatier's definition of these aspects in Section 3 using modern abstract analysis The introduction of this abstract analysis was necessary in theories. order to provide a systematic approach to tackling the inverse problem encountered in the present work, as well as other inverse problems. This analysis, though based on well established theories of modern abstract analysis, is in itself new. The novelty is in compiling and arranging these theories to serve our purpose of providing abstract guidelines to solving the inverse problem.

Based on the general abstract problem, the inverse problem of the reconstruction of the local void distribution is considered in Section 4. That is, the abstract aspects of the inverse problem are applied to our specific problem. However, due to the non-linear nature of our problem, one more aspect ought to be added to the aspects of the inverse problem defined by Sabatier. That is, the non-linearity problem. This aspect is discussed in an abstract form and then applied specifically to our particular problem. The analysis of the non-linear problem by itself is not new; since well established abstract theories of modern analysis are used. The new contribution is in recognizing these theories and utilizing them to solve our problem. In fact, what happened in this particular instance that the idea of solving the non-linear problem using the method of successive approximations was first developed, then we performed an abstract analysis of this method to find the conditions under which the method leads to a unique solution. This led to the utilization of the abstract concepts of "contraction mapping" and the "fixed point theorem", as will be shown later. Also, when the strong non-linearity of the problem is being removed, usually at the late stages of the successive approximation, we make use of this near-linearity to increase the efficiency of the solution procedure.

7

In Section 5 of this Chapter, the computational procedure for solving the void fraction distribution inverse problem is outlined. Numerical examples illustrating solutions of the inverse problem are given in Section 6. In these examples, calculated single scattering data (error-free), Monte Carlo simulations data, and laboratory measured data are utilized. The examples show that the inverse problem of constructing the phase distribution is solvable for different flow regimes and types of available data. A review of the parameters contributing to the problem is given in Section 7, showing their effect on the solution and the rational behind choosing the proper parameters. A conclusion statement is given at the end of the Chapter.

2.0 INVERSE PROBLEM

Definition

To define an inverse problem, one has to define first the forward or direct problem. For practical applications, the direct problem is the "mapping" of a set of theoretical parameters that one is interested to determine, into a set of experimentally measurable results. Mapping is an abstraction of the concept of a function. While a function assigns to a given number another number, a mapping assigns to a given element of one set an element of another set. In other words, a mapping m from a set C to a set E assigns to each element c c C an element e $_{E}$ E ($_{E}$ reads belongs to). Then if C is a set of theoretical parameters and E is the set of corresponding measurable results, then a mapping

(2.1)

(2.2)

represents a forward problem. In fact, the forward problem is the theoretical modelling of the physical phenomenon.

The inverse problem is the problem of reconstructing the theoretical parameters from the measured data. In other words, it is the estimation of the parameters sought, C, from the measurement data, E, given the mapping m.

Obviously, the equation

mc = e , $c \in C$ and $e \in E$

 $m: C \rightarrow E$

has a solution on C only for those elements e that belong to the set $mC \subseteq E$. The element e is usually known only approximately, since it is obtained from measurements. Let e denote its approximate value. In such cases, one can speak only of finding an approximate solution (that is close to c found from mc = e) of the equation

(2.3)

Here e usually does not belong to the set E = mC. Frequently, the mapping m is such that the inverse mapping m⁻¹, even if defined, is not continuous, that is, it contains singularities. Under these conditions, one cannot take for the approximate solution the exact solution of Equation (2.2) with approximate right-hand member, that is, one cannot take for the approximate solution the element $c = m^{-1}$ for two

reasons: First, such a solution may not exist on the set C since e may not belong to the set mC \cong E. Second, even if such a solution does exist, it will not possess the property of stability if the inverse mapping m⁻¹ is not continuous, that is, a small change in e results (rather unpredictably) in a large change in c. In order to define the property of stability, let us assume that each of the sets C and E are equipped with a well-defined single-valued, nonnegative, real function distance (or "metric") d(a,b), which provides a measure of the distance between two elements a and b in the set. Then the set and the associated distance define a metric space. The problem $C = m^{-1}E$ is then said to be stable⁽¹⁾ on the spaces (C,E) if, for every positive number ℓ , there exists a positive number $\delta(\hat{\ell})$ such that the inequality $d(e_1, e_2) \leq \delta(\ell)$ implies $d(c_1, c_2) \leq \ell$, where $c_1 = m^{-1}e_1$ and $c_2 = m^{-1}e_2$. with e_1 and e_2 in E and c_1 and c_2 in C. The meaning of stability is essentially that there should be an upper bound to the extent to which a change in any element of E can be amplified by the inverse mapping to C.

Stability of a solution of the problem (2.2) is usually a consequence of its physical determinacy and an approximate solution must possess this property. Otherwise, the problem is ill-posed, as defined in the following.

2.2 Quasi-Solutions

Inverse problems, in many practical cases, do not satisfy all conditions of well-posed problems, namely⁽¹⁾ for the problem of determining the solution c in the space C from the "initial data" e in the space E, the conditions:

(1) for every element e_{ε} E there exists a solution c in the space C;

(Z) the solution is unique;

must be satisfied in order for the problem to be well-posed on the pair of metric spaces (C, E). Problems that do not satisfy these conditions are said to be ill-posed.

Conditions (1) and (2) characterize the mathematical determinacy of the problem, while condition (3) is connected with the physical determinacy of the problem and also the possibility of applying numerical methods to solve it on the basis of approximate initial data. Violation of condition (3) means that certain small disturbances in the initial data can produce large changes in the solution, and the problem of finding an approximate solution is, in practice, ambiguous. However, with the aid of available supplementary information concerning the solution, one could select an appropriate solution. This supplementary, or "a priori", information could be of either quantitative or qualitative nature. The attempt to use supplementary information of a quantitative nature requires only the "extension" of the set E to include both computed results, m.c, and results of measurements, e. This leads to the concept of a "quasi-solution". (1) The use of supplementary information of a qualitative nature (for example, smoothness of the solution) necessitates a different approach to the "construction" of approximate solutions.

(3)

General Solution Procedure

From the above discussion, one can say that, to solve the inverse problem, one must: (2)

(1)

extend the set E so that it contains both computed results and results of measurements, .(extension problem);

(2)

show, for any element e of the extension of E, the existence (or not) of a non-void subset of C that m maps into e (existence

question);

(3) define mappings of the extension of E into C (construction problem), with the intention of inverting m approximately or, in the best possible case, exactly; and

(4) estimate level of confidence in the resulting solution (problem

of errors).

The existence question is to be addressed simply because no solution can be found if the inverse mapping of the extended set into C does not exist. The problem of errors is to be investigated to establish a level of confidence in the solution, given a certain level of confidence in the experimental data and possible errors in the inverse mapping construction. The different aspects of the inverse problem, as listed above, are discussed after the following brief review of the fields in which inverse problems arise.

2.4 Applications of the Inverse Problem

Inverse problems are important applied problems which are is studied in many fields. They are encountered in geophysics where gravity, seismic and geomagnetic wave data are inverted in order to find the earth structure or to determine the position and time of origin of an earthquake source $^{(3)}$. In planetary and stellar atmospheres, inverse problems are used to deduce the nature of space media on the basis of measurements of the radiation field $^{(4)}$. Inverse problems are also involved in the analysis of microwave observations from earth's orbiting satellites to map humidity, temperature profile, ice and other physical parameters $^{(5)}$. Image extrapolation and object restoration in optics are also inverse problems. Computer assisted tomography for image reconstruction from x-ray transmission imaging is an important inverse problem in medical technology. Inverse problems are also important in scattering theory in order to determine the potential of a scattering field and the location of its boundaries. Experimentalists encounter the inverse problem when experimental measurements yield a result which is different from what they initially intended to measure due to distortions introduced by the instrument. Some familiar examples of distortions are those occurring:

(1) in a neutron or gamma-ray spectrum due to the pulse-height distribution of a multichannel scintillation spectrometer (see Chapter 5);

(2) when scanning with a telescope or directional antenna, due to the finite width of the response pattern,

and in many other science and engineering applications.

In many of these cases; the measurements may be characterized to a very good approximation by a linear operator through the Fredholm integral equation (of the first kind):

 $e(x) = \int K(x,y) c(y) dy, e_{\varepsilon} E and c_{\varepsilon} C$

where e(x) is the measurement taken at the point x, c(y) is the desired parameter estimation at the point y, K(x,y) is the response function of the measurement device; also called kernel, impulse transfer function, apparatus function or Green's function, Ω is the space which contains the points y. When the uncertainty in the measurements is ignored, the determination of c from e is referred to as "unfolding". This stems from the German usage of "Faltung" (folding) for the above integral equation. For systems exhibiting translational invariance, the response function takes the form K(x-y) and the integral equation is referred to as the convolution integral, and the process of extracting C from E is referred to as "deconvolution". The term "unscrambling" is also used for the inversion of the Fredholm integral, especially when any "a priori" information that may be known are made use of in the inversion process. Regardless of the terminology used, "unfolding", "deconvolution" or "unscrambling", the fact remains that all are inverse problems which are encountered in many important applications.

From the above discussion, one can see that inverse problems are very important applied problems. How are they solved? That is the subject of the following Section.

3.0 SOLVING THE INVERSE PROBLEM

As mentioned in the previous section, solving the inverse problem requires: (1) tackling the extension problem in order to take into account the uncertainty in the measured data, (2) addressing the question of the existence of an inverse mapping, (3) examining the problem of inverse mapping construction to guarantee the stability of the solution under small changes in the data, and (4) considering the problem of errors to estimate the errors associated with the solution due to propagation of the data errors and the errors that could accompany the construction of the inverse operator. All these four aspects are encountered in many inverse problems. However, in some problems one or more of these aspects may dominate the others because of the special nature of the problem considered. In the following subsections, these aspects of the inverse problem are discussed, each followed by an example in which the aspect considered is important.

3.1 Extension Problem

The extension problem, as defined before, is to extend the set of "results" E such that it contains both computed results (using the forward mapping), and the results of measurements. The extension is necessary because the errors usually accompany the measurements and lead to non correspondence of some measurements to any of the points of the theoretical parameters set C: Apparently, if the observable results are free of errors, one could look for solving an "exact" inverse problem. In other words, one would be expecting to find a mapping m⁻¹ that exactly inverts the original mapping, which is The inverse mapping m⁻¹, in an exact inverse problem, should satisfy the identity

 $m(m^{-1}(e)') = e$

for all admissible data e, assuming for the moment that there is no error associated with m. Uniqueness would require that

$m^{-1}(m(C)) = C$

for all admissible model parameter sets C. The continuity of the inverse mapping is to be investigated in order to achieve stability of the solution of the exact problem.

For most inverse problems, exact solutions cannot be found because of the errors associated with the measurements, and the extension problem is to be addressed. This extension can be achieved as follows: If there are N measured parameters, one can represent them by N-component vectors, and define the set of "results" $E \subseteq R_N$, which implies E is a subset of the set R_N of all N-dimensional real vectors. Consequently, E is a metric space. Since this definition of E depends on the number N of measurements, one is led, instead of describing the forward problem by the mapping m of C into E, to describe it by means of several mappings m_i , into open sets E_i , each mapping corresponding to one (one type of) measurement. A set is said to be open if its points are, all interior points. In each set E_i , let E_i be a suitably small open subset of our extension of E_i that contains the value given by the measurement and those which are allowed by error appraisals, (within error bounds); see Figure 1 for illustration. Then a true solution can be defined as any point c whose ith measurement lies in E_i , i = 1, ...N. Let d_i be the distance in E_i , a true solution C annihilates the function

$$D_{o} = \sum_{i=1}^{n} d_{i} \left(\frac{E_{i}}{i}, m_{i}(c) \right)$$

$$(3.1.1)$$

where \underline{E}_{i} an open ball, centred at e_{i} (the measured value), and of, radius Δ_{i} , $d_{i}(\underline{E}_{i}, \underline{m}_{i}(c))$ vanishes when $d\{e_{i}, \underline{m}_{i}(c)\}$ is, smaller than or equal to Δ_{i} (error associated with the measurement) The term open ball indicates that for every point $\underline{e}_{i} \in \underline{E}_{i}$, $d(\underline{e}_{i}, \underline{e}_{i}) < \Delta_{i}$. One can consider Δ_{i} as the size of the worst error



associated with the measurement e_i . For a more elaborate error analysis, one could use a weighted Euclidean distance and rewrite (3.1) as

(3.1.2)

$$D_{2} = \{ \sum_{i=1}^{N} w_{i} d_{i}^{2} (e_{i}, m_{i}(c)) \}$$

where w_i 's are convenient weights, (usually the data variances and covariances) D_2 is called the "cost*function"⁽²⁾ or the "objective function"⁽⁶⁾ and "a good fit" is then defined as "the parameters" c which minimizes the cost function. The minimizing element is often called a "quasi-solution".⁽¹⁾

3.1.1 Cost Function

i=l

The cost function (3.1.2) is the weighted sum of squares of the residuals of the solution. The statistical interpretation of this cost function is as follows. Because of the random nature of the data errors, a test for the acceptability of a given solution has ultimately to be based on a probabilistic model of the errors. Suppose one has a candidate solution, \hat{C} , and one proposes that it is, in fact, the true solution, C. In this case, the residuals $(E - m \hat{C})$ must be equal to the

random errors. A test of these residuals against the probability density function (pdf) for the data errors will then provide a consistency test of the hypothesis that the candidate solution is correct. Such an approach is aimed primarily at eliminating from considerations those solutions which are inconsistent with the data. Assume that the data errors are Gaussian with zero mean. The general pdf for mean free Gaussian errors is

$$p(\xi) = (2 \pi^{n} D)^{-1/2} \exp \left(-\frac{1}{2} \xi^{T} \nabla_{=}^{-1} \xi\right)$$
(3.1.3)

where n is the number of data, $\underline{\underline{r}}$ is the error vector, $\underline{\underline{v}}$ is the covariance matrix for data errors $\underline{\underline{\epsilon}}$, and D is the determinant of $\underline{\underline{v}}$. The hypothesis test for a given solution is made by computing the residuals vector \underline{r} , $(\underline{r}_i = \underline{d}_i(\underline{e}_i, \underline{m}_i(c))$, then substituting these for $\underline{\underline{s}}$ in Equation (3.1.3). However, the computation is simplified by the fact that the residuals enter only in the form $\underline{r}^T \underline{\underline{v}}^{-1} \underline{\underline{r}}$, which is equivalent to D_2 , the cost function (3.1.2) when the weights are represented by the covariance function. Furthermore, if the regiduals are presumed to satisfy (3.1.3), then D_2 is a chi-squared variable with

n degrees of freedom. (The chi-squared distribution arises when the squares of several independent random variables, each of which has a normal distribution, zero mean, and unity variance are added together.) Then, it is not even necessary to evaluate (3.1.3), since a chi-squared test can be used to measure the acceptability of the solution. The solution (quasi-solution) that maximizes (3.1.3), or equivalently, minimizes the cost function (3.1.2), deservedly carries the name "maximum likelihood solution"⁽⁶⁾. One can see that the problem is, in fact, the famous least squares problem.

The cost function (3.1.2) when weighted with the covariance matrix of the data, is based on the assumption of Gaussian statistics. The differences between Gaussian statistics and some other variety may be negligible for many problems. The primary role of statistics is to determine the weighting to be given to the various data. If the data are relatively complete and mutually consistent, the weighting would not matter much. (6)

One might be wondering: " what does the introduction of the cost function have to do with the extension problem.?" As mentioned earlier, the answer to the extension problem is achieved by surrounding each element e_i of the data set by an open ball centred at e_i and of a radius equal to the error bound associated with e_i . For a true solution, the distance $d_i(e_i, m_i(c))$ lies within this bound. The cost function provides a measure of the closeness of a solution vector to the centres of the extension balls, and consequently to the true
solution. Minimization of the cost function, then, gives the closest possible consistent solution (quasi-solution) of the problem.

3.1.2 Approximate Mapping

When the mapping m is not known exactly, instead an approximation is available, one has to expand the extension ball further to allow for the error in m. In this case, one would be searching for a solution that minimizes a "cost function" that may be expressed as

$$D_{2} = D_{2} + \sum_{i=1}^{d} (\delta_{i}(c), e_{i}), (3.1.4)$$

where D_2 is as defined in Equation (3.1.2) and δ_1 is the error in m. The second term in Equation (3.1.4) represents the expansion in the extension ball and can be considered as the "cost function" evaluated for a problem in which the measurements are free of errors while the

mappings m, are approximate.

3.1.3 Example⁽²⁾

This simple example shows how the concepts discussed above are used to extend the measurement set E to include the results of measurements together with the computed results.

Consider a simple physical model leading to a linear relationship y = ax + b between the physical quantity y and the space coordinates x. The set C is then the set of points y calculated from y = ax + b at every point x. Suppose that two measurements $y_i(x_i)$, i = 1, 2 with "error bounds" $\pm \Delta_i$, i = 1, 2 are available. With the simple understanding of $\pm \Delta_i$ as being the worst error, let us try to construct the set of measurements E.

A tentative definition of E would be the set of couples of points that are consistent with the measurements. But any $y(x_i)$ between $y_i(x_i) - \Delta_i$ and $y_i(x_i) + \Delta_i$ is consistent with the measurements. Therefore, the two measurements result in an infinite number of couples. If a third measurement were available, two other sets of couples can be defined in conjunction with the first two

measurements. Then in order for the problem to have a solution, there should be couples in the three different sets belonging to the same

straight line. One must have a rule that makes these couples

mathematically identical, otherwise they would result in redundant measurements in E. But how to implement such a rule into E, if it consists of sets of infinite couples? Obviously, this tentative definition of E is not satisfactory.

One has to try another definition of E. If there are N , measurements, one can represent them by an N-component vector and ECR_N. Now, if instead of describing the model by the mapping m of C into E, one describes it by several mappings m_i into open sets E_i, each mapping corresponding to one measurement, then the measurement y_i at one point x_i corresponds to one mapping m_i. Let E_i be the smallest possible open subset that contains the measurement y_i(x_i) and those allowed by the error bounds, i.e. the interval $[y_i(x_i) - \Delta_i, y_i(x_i) + \Delta_i]$. Now a true solution of the problems can be defined as any point in C whose ith measurement lies in E_i, i = 1, ..., N. Then a true solution annihilates the cost function (3.1.1) with the understanding of Δ_i as being the worst error, or arbihilates the cost function (3.1.2) for the probabilistic understanding of the measurement model y = ax + b can be obtained using the well known least-squares

fitting.

3.2 Existence Question

The existence question as stated earlier: "for any element e of the measurement set E, show the existence (or not) of a non-void subset of the parameters c that m maps into e". This is equivalent to the question of the existence of at least one right-inverse mapping m_R^{-1} (and uniqueness makes this right inverse a genuine inverse). In order to approach this question, we start with a careful examination of the definition of mappings and their inverse (see Reference (7)).

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3.2.1 Inverse Mapping.

If C and E are arbitrary sets, then a rule associating a Unique element $e = m(C) \in E$ with each element $C \in C$, is said to define a mapping m of C into E and m is said to map C into E (and c into e). The element e = m(C) is called the image of C (under the mapping m) and C is called the pre-image of e. In general, e may have several pre-images. Moreover, E may contain elements with no pre-images at all. If e has a unique pre-image, the pre-image is denoted by $m^{-1}(e)$. A mapping m is said to map C "into" E if m(C), e E, (i.e., if m(C) is a subset of E), as is always the case, and m is said to map C "onto" E, if m(C) = E. Thus, every "onto mapping" is an "into mapping". Duit not conversely. When m maps C to E, m is said to be one-to-one if each element $e \in E$ has a unique pre-image $m^{-1}(e)$. In this case, m is said to establish a one-to-one correspondence between C and E and the mapping m^{-1} associating $m^{-1}(e)$ with each e ε E is called the "inverse" of m. The mapping m is said to have a "domain" C and a "range" m(C) = E, consequently, the inverse mapping m^{-1} has a domain E and a range

From the above examination of the definition of a mapping and its inverse, one can say that a mapping m is invertible (i.e., m^{-1} exists) if the equation

in(c) = c

_m-1

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has a unique solution for every $e \in E$.

If m is invertible, we can associate the unique solution of the above equation with each e $\varepsilon \in E$. This gives a mapping, with domain E, which the inverse of m, i.e., m⁻¹.

Therefore, two conditions are to be met if the inverse mapping, , is to be defined:

m(C) = C, or in other words, m is an onto mapping, also called surjective⁽⁸⁾,

m is a one-to-one mapping, also known as injective

(8) mapping

(2)

In other words, for m^{-1} to be defined, m has to be surjective and injective (it is then called bijective).

Failure of m^{-1} to exist would result in no solution for some e $_{E}E$ or more than one solution for e $_{E}E$ of the equation mc = e. In most practical problems, the set E of measured results is "slightly" perturbed by unavoidable measurement errors. Consequently, a one-to-one mapping of C into the measurement set may not be observed, or in other words, the correspondence between C and E is not a mapping. That is because repeating the experiment may result in a different set of results. However, if the error is reasonably small and bounded, (which is usually the case), a perturbed result, e' would exist in the neighbourhood of the unperturbed result e = m(c) and an extended set containing the perturbed and unperturbed results can be defined, as shown before. Through this extension, a quasi-solution can be found. The existence and uniqueness of such a quasi-solution are discussed in the following.

3.2.2 Existence of a Solution

Recalling the problem to be solved (for c),

mc = e

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let e belong to a metric space \check{U} and c belong to a metric space F. The operator m maps F into U. It is assumed that m has an inverse operator m^{-1} though the latter is not, in general, continuous (the conditions for the existence of m^{-1} are just discussed above). Further, suppose that the class of possible solutions, C ϵ F is compact, that is, every sequence lying in C has a limit in C (in metric spaces)⁽⁸⁾. The requirement of the compactness of the set C is based on the following topological lemma:⁽¹⁾

Lemma: Suppose that a compact (in itself) subset C of a metric space F is mapped onto a subset E of a metric space U. If the mapping C \rightarrow E is continuous and one-to-one, the inverse mapping E \rightarrow C is also continuous.

Then, if we are seeking a solution on a compact set C of F and the right-hand member e of the equation mc = e belongs to the set E = mC, it is possible to use the formula

to construct a solution of the equation mc = e, that is stable with

 $c = m^{-1} e$

respect to small changes in e. However, in practical problems, we often do not know the exact value of the right-hand member e, but an approximate value of it; \tilde{e} , which may not belong to the set E = mC. In these cases, we cannot use the formula $m^{-1}e$ to construct a solution since the symbol $m^{-1}e$ may be meaningless. Therefore, the concept of a quasi-solution was introduced. An element c of C minimizing, for given e, the functional $d_U(mc,e)$ on the set C is called a quasi-solution of the equation mc = e on C:

The term inf denotes the infimum, which is the greatest lower bound of d_U and is equivalent to the minimum of d_U if the set C is compact. Note that $d_U(mc,e)$, the distance between mc and e on the metric space U is the cost function introduced before in discussing the extension problem.

If C is a compact set, a guasi-solution obviously exists for every $e \in U$, (since the compact set C contains its limits, including, of course, the minimum \tilde{c}). If the right-hand member e is known exactly: $e = e_t$, and if the desired exact solution c_t belongs to the set C, then

 $\inf d (mc, e) = 0$

сЕС

and this infimum is attained with the exact solution c.

There may be more than one quasi-solution. In such a case, and quasi-solution will mean any element of the set D of quasi-solutions. However, it is possible to exhibit sufficient conditions for a quasi-solution to be unique and depend continuously on the right-hand member e.

3.2.3 Uniqueness of Solution

Let us discuss the following Theorem⁽¹⁾:

<u>Theorem</u>: Suppose that the equation mc = e is linear, that the homogeneous equation mc = 0 has only the zero solution, that the set C is convex, and that every sphere in the space U is strictly convex. A sphere S (a,r) consists of all points that are at a distance r from the center of the sphere a. Then, a quasi-solution of the equation mc = e on the compact set C is unique and it depends continuously on the right-hand member e. Since in any normed space, every closed ball is convex, (15)the above Theorem guarantees the uniqueness and continuity of the quasi-solution, if the spaces F and U be normed and U is strictly normed (i.e. non-degenerate triangles satisfy strict triangle inequality). This is in addition to the linearity condition and to the condition that mc = 0 has only the zero solution. These conditions are easily met in practical problems and the quasi-solution is widely obtained in these problems. In fact, the well known least squares solution is just a quasi-solution with the cost function defined as a weighted sum of the

squares of the residuals (mc - e).

To summarize, no solution can be found if the inverse mapping cannot be defined for error-free results, or in other words, if the mapping of the problem is not bijective. If the mapping is bijective, a solution can be found even if errors are associated with the measurements. The existence of errors in an over-determined problem where the number of available measurements is larger than the number of unknowns, is overcome by finding a quasi-solution that minimizes a given cost function. This quasi-solution can be proven to be unique for a given cost function under certain conditions. If the problem is well-determined, i.e., the number of measurements is equal to the number

of unknowns, the results have to be consistent in order to find a solution. Otherwise, there is no solution, unless the set of admissible measurements is arbitrarily reduced leading to an under-determined

problem. For under-determined problems, there is more than one solution, however, one solution or more may be singled out according to some physical grounds. Stability of the inverse mapping, in all kinds of inverse problems, has to be checked (the stability question). An inverse mapping to a linear mapping would be stable if it is continuous, and it is continuous if it is bounded, and the converse is true⁽¹⁾. This stability problem is addressed in more detail when the construction problem is discussed.

3.2.4 Example⁽¹⁾

This example shows an inverse problem for which no solution may exist. Consider the problem of designing an optical system with given transmittance that can be obtained with the aid of coating deposited in the form of a thin film on the surface of the "backing". The direct problem in such a system consists of finding the transmittance $T(\lambda)$ for a monochromatic light of wave length λ from given wave amplitude A, index of refraction n(z) as a function of film points (z) and the film thickness H. The inverse problem is to determine from a given $T(\lambda)$ the system $\{n(z), H\}$, that is, finding the index of refraction n(z) and the film thickness H. For this inverse problem a solution may not exist, since there may not exist a system $\{n(z), H\}$ having the prescribed

transmittance $T(\lambda)$. In other words, there may not be a one-to-one

mapping of $\{n(z),H\}$ to $T(\lambda)$. It is also possible that more than one system will have that transmittance, that is, the mapping is not one-to-one.

3.3 Construction Problem

The existence of the inverse mapping, though necessary, is not sufficient to obtain a solution (or a quasi-solution) of the inverse problem. The inverse mapping has to be bounded, and consequently, continuous. If the inverse mapping is unbounded, arbitrary small variations of the measurement data (such as experimental error) may produce large variation in the sought parameters and, consequently, the set of theoretical parameters, C, will not be compact (see Figure 2). The problem in that case is called an ill-posed problem in the sense of Hadamard⁽¹⁾ and its solutions are unstable under small variations in the data: It is absolutely necessary to find a way to restore stability in order to be able to obtain any solution of the inverse problem. This can be achieved by constructing "regularizing" mappings that produce approximate solutions that are stable under small changes in the data.

There are standard methods for the construction of regularizing mappings. One can find, for instance, a very good study of them in the treatise by A. Tikhonov and V. Arsenin⁽¹⁾. The basic features of all proposed methods are guite similar. All try to restore stability by



means of supplementary constraints on the set of admissible solutions, based on some "a priori" knowledge about the solution. We will present one of these methods to illustrate the basic concept. However, one could consult to the references 1, 3, 4 and 9 for other methods.

3.3.1 Tikhonov-Miller Method for Constructing (1,10)

Regularized Mappings

Let us write our basic equation as follows:

m c + ℓ '= e

. (3.3.1)

where c is the unknown vector, e is the data vector, $\boldsymbol{\ell}$ is the experimental error, and m is a linear continuous mapping from the solution space F to the data space 0, which are both Banach spaces, i.e., complete normed linear spaces. Besides, we assume the existence of the inverse mapping m⁻¹. The idea of the method is to restore continuous dependence on the data, when m⁻¹ is unbounded, by means of supplementary constraints on the set of admissible solutions. For

instance, stability is guaranteed when searching for a solution in a

compact set⁽¹⁾. This follows from an elementary theorem of functional analysis, which states that the inverse of a continuous operator with a compact domain is continuous⁽¹⁾. The constraint on the solution are, provided by an "a priori" knowledge about the solution. The "a priori" knowledge is the price to be paid for controlling error propagation in the inversion procedure. Hence, we presume to have some "a priori"

knowledge of c under the form of the constraint:

|| В с ||_Е <u><</u> Н

where H is some positive given number and B is the "constraint mapping" having dense domain in F, range in the Banach space U, and bounded inverse B^{-1} . By a dense domain in F it is meant that every point in F is the limit of a suitable sequence in the domain. We also suppose the a priori knowledge of the following bound on the errors:

(3.3.2)

$|| \mathcal{E} ||_{E} = || mc - e ||_{E} \leq G$ (3.3.3)

Now, consider any c satisfying both constraints (3.3.2) and (3.3.3) as an approximation to the unknown c. Let us combine quadratically the two constraints in a single one:

$\left| \left| \begin{array}{c} mc - e \end{array} \right| \left| \frac{2}{E} + \left(\frac{G}{H} \right)^2 \right| \left| \begin{array}{c} Bc \end{array} \right| \left| \frac{2}{E} \le 2G^2 \end{array} \right|$ (3.3.4)

If the mappings are presented in a matrix form and the data in a vector form, then the vector c, which minimizes the left-hand side of the above equation, satisfies:

$$= \left\{ m^{T}m + \left(\frac{G}{d} \right)^{2} B^{T}B \right\}^{-1} m^{T} e$$
 (3.3.5)

If we choose some norm |[.|] in C for measuring the accuracy of the approximation, it can be proved that (10)

$$||\hat{c} - \tilde{c}|| < 2^{1/2} M(G, H)$$
 (3.3.6)

where c is any vector satisfying (3.3.2) and (3.3.3), and M(G,H) is the following quantity:

$$M(G,H) = \sup_{c \in C} || (3.3.7)$$

such that

The problem is stable with respect to the chosen norm ||.|| when M(G,H) tends to zero with G, for fixed H. Then M(G,H) is called the best-possible stability estimate and c is a nearly-best-possible approximation, since the error ||c - c|| is bounded by M(G,H) up to an irrelevant numerical factor (10). The solution (3.3.5) is called a

regularized solution of the inverse problem (1)

This method is also known as constrained linear inversion method⁽⁴⁾ and is a simple extension of the least squares methods for solving linear problems. For a minimum constraint, the constraint mapping B would be the identity mapping as it is clear from Equation (3.3.2). However, B would provide a useful solution if it isframed to be in some sense a measure of the smoothness of c. In the above analysis, it is assumed that both G and H are to be "a priori" known. This, however, is not necessarily known, since only their ratio appears in the solution. A method in which the identity mapping is used for B and a single factor, γ , is used for (G/H) in Equation (3.3.4) is called the Twomey-Phillips method⁽¹²⁾. This method takes the form:

 $\hat{c} = c^{\circ} + m \{m^{T}m + \gamma I\}^{-1} (e - mc^{\circ})$ (3.3.8)

where c° is an initial approximation to the solution and γ is called the smoothing (or regularization) parameter. As γ decreases, the error magnification increases. Therefore, the magnitude of γ

determines how much error magnification can enter into the solution.

Regularizing methods, such as the one just reviewed, are also useful in obtaining a solution for an under-determined problem, where the number of available data is less than the number of sought parameters. Clearly, in such a problem, one needs to pick out of the family of possible solutions, one which is "appropriate" in some sense. This appropriateness can be introduced through the "constraint mapping". B, in the regularization process. Phillips⁽¹²⁾ showed the form of B when the smoothest solution is desired, while Tewarson⁽¹³⁾ illustrated how

available information regarding the desired solution can be incorporated in constraint mappings similar to B.

3.3.2 <u>Example</u>

To show the importance of the construction problem, let us consider the abel problem⁽²⁾ that appeared in 1826. In a vertical plane, define a profile's(x) as a function of the altitude x; for a particle moving in the gravity field and starting from a point of altitude a, let the time of arrival to altitude x be t(a). Abel integral relates s(x) to t(a), i.e., the mapping $t \rightarrow s$, by

 $G^{-1} \int t(a)/(a-x)^{\frac{1}{2}} da = s(x) , 0 \le x \le a$, (3.3.9)

where G is a positive constant. Consider the inverse problem of finding, a solution for t(a) giving approximate values of s(x), obtained with the aid of a recording device. Since only an approximation of s(x) is known, one can speak only of finding an approximate solution that is close to the exact solution $t_1(a)$. This exact solution can be obtained from the

Abel "direct" problem given by:

$$t_1'(a) = \frac{1}{2}G \int_{Q}^{a} ds_1(x) / (a-x)^{\frac{1}{2}}, 0 \le x \le a$$
, (3.3.10)

where $s_1(x)$ is the exact value of s(x). If one attempts to obtain a solution of the inverse problem (3.3.9) using the approximately known $s(x) \neq s_1(x)$, one may fail, since such a solution may not exist. This is because the problem as posed by equation (3.3.9) does not possess the property of stability, that is, small changes in s(x) may result in large changes in t(a).

To show that the problem may not be stable, consider the function

$$t_2(a) = t_1(a) + A \sin \omega a$$
 (3.3.11)

where A and ω are arbitrary constants. This function t₂(a) is a solution ⁽¹⁾ of (3.3.9) with the right-hand side

$$s_2(x) = s_1(x) + AG^{-1} \int_0^x \sin(\omega a) / (a-x)^{\frac{1}{2}} da$$
, (3.3.12)

0**<x**<a

The second term in the right-hand side of the above equation can be considered as the noise associated with the measurements. Now, if we measure the change in s(x) by the distance:

$$d(s_1, s_2) = \left\{ \int_{0}^{1} \{s_1(x) - s_2(x)\}^2 dx \right\}^{\frac{1}{2}},$$
 (3.3.13)

using Equation (3.3.12), then

$$(s_1, s_2) = [A]G^{-1}[f] \{\sin(\omega a)/(a-x)^{\frac{1}{2}}da\}^{\frac{1}{2}}dx\}^{\frac{1}{2}},$$
 (3.3.14).

Note that the value of this integral increases as ω decreases, as can be shown if the integration is performed. Let us measure the change in t(a) by the distance

$$d(t_1, t_2) = \max |t_1(a) - t_2(a)|$$
 (3.3.15)

Using (3.3.11), then

$$d(t_1, t_2) = \max |\hat{A} \sin \omega a| = |A|$$
 (3.3.16).

Obviously, for any (value of A, if the values of ω are sufficiently large, one can make the change $d(s_1, s_2)$ arbitrarily small (i.e. small measurement errors) without preventing the change in the corresponding solutions $d(t_1, t_2)$ from being arbitrarily large. The same conclusion can be reached if the distances $d(s_1, s_2)$ and $d(t_1, t_2)$ are defined using a different norm ^(1,2). Therefore, the problem is not stable since arbitrarily small changes in the measurements can cause arbitrarily large changes in the solution.

In order to solve the Abel inverse problem, one needs an algorithm for constructing an approximate solution that will be stable under small changes in the initial data s(x). This algorithm can be obtained, for example, by regularizing the mapping of the problem such that the resulting solution t(a) is a smooth function of a. Then one can use a regularization method such as that of Phillips⁽¹²⁾.

3.4 Problem of Errors

The error associated with the measurements implies that an error is combined with the "estimated" solution of the inverse problem, regardless of the method of solution. The problem of errors is to investigate the amount of error the solution carries. This error can be measured by the deviation of the "estimated" (quasi- or regularized) solution, c, and the "correct" solution c. The "correct" solution is assumed to exist and to be unique, otherwise there is no solution for the problem. It satisfies the Equation (3.3-1),

 $e = m c + \xi$ (3.4.

while the estimated solution satisfies the equation

(3.4.2)

where m⁻¹ is the "constructed" inverse mapping.

 $\hat{c} = m_o^{-1} e$

From (3.4.1) and (3.4.2), one may now compute the errors of the estimated solution,

 $\hat{\mathbf{c}} - \mathbf{c} = (\mathbf{m}_{o}^{-1}\mathbf{m} - \mathbf{I})\mathbf{c} + \mathbf{m}_{o}^{-1}\boldsymbol{\xi}$

From this equation, one can see that the estimation errors are of two types: a "bias" or resolving error, represented by the first term of the right-hand side of the equation, and a "random" error which depends on the data errors. Since the "correct" solution c is not known, the resolving error can be approximated by $(m_0^{-1} m - I)$ c. The random nature of the data errors necessitates a probabilistic treatment of the "random" error ξ . In many cases, the probability density function (pdf,) of the data errors can be described by a Gaussian distribution, in terms of a vector of means and a covariance matrix. If it is assumed that the data errors have zero mean and covariance matrix \underline{M}_{\pm} , then the estimation random error, $\underline{m}_{0}^{-1} \boldsymbol{\xi}$, will have zero mean and covariance matrix $\underline{m}_{0}^{-1} \underline{M}_{\boldsymbol{\xi}}$ (\underline{m}_{0}^{-1})^T, where the mappings are represented by matrices. When the data errors are statistically independent, then the covariance elements of $\underline{M}_{\boldsymbol{\xi}}$ are zero and $\underline{M}_{\boldsymbol{\xi}}$ reduces to a diagonal matrix, with the data variance as its elements. It is then called the variance

3.4.2 Example⁽²⁾

matrix.

An example where the problem of errors was considered important appeared when physicists tried to determine the constant R in the Boyle-Mariatte law pV = RT. Very few number of experimental results (e.g. one measure of gas pressure p, volume V and temperature T), were enough to determine the parameter R of the theoretical curve. If other experimental results were available, physicists either fitted the theoretical curve inside the range of possible errors, or they did not, and then it was legitimate to question the validity of the model.

Therefore, the inverse problem is reduced to only one of its aspects: determining the range of errors for the unique parameter .R.

5 Aepthetics in the Inverse Problem

As a way of closing this section about the solution of the inverse problem, the author would like to quote the comment of

Dermendjiam (Reference 4, page 138) about the word "aesthetics" since one should be seeking for an aesthetic solution of the problem.

"I like it because aesthetics derives from a Greek verb meaning 'to perceive with the senses'. It is a scientist's prerogative to introduce - and to be governed a little - by aesthetics in his work. This implies things like restraint, non-exaggeration, non-reliance on innumerable assumptions, criteria of data banks, and so on. I would like to make an analogy, if I may, between sailing, about which I know some. things, and the use of mathematical inversion techniques, about which I know very little. Some people want us to use more instruments and electronic gadgetry which are supposed to help us sail better, on the assumption that we have no senses - seeing, hearing, sensory feeling - or judgment or 'sea-sense'. A good sailor does use all these things to . advantage for a successful voyage. So, in analogy to this, I feel that sometimes we tend to resort to inversion techniques too blindly, without. using our judgment or 'feel' about handling a given problem, which may lead to 'anti-aesthetic excesses'. This is what one should keep in mind. when he is attempting to solve an inverse problem".

The use of aesthetics in the present work appears in navigating the inverse problem within the closed interval [0,1] of void fraction, as

shown in Section 6 of the present Chapter.

NEUTRON SCATTERING TOMOGRAPHY AS AN INVERSE PROBLEM

As shown in Chapter 3, the Fast Neutron - Single Exposure

Neutron Scattering Technique for local-void fraction Tomography can be abstracted from the physical world into mathematics, using a single scattering approximation, through the mathematical formulation:

$$\underline{S} = \underline{\underline{A}}(\underline{\rho}) \underline{\rho}$$

where,

4.0

S is the detector response vector (measurement results),

 ρ is the water fraction vector (sought theoretical parameters);

and

 $\underline{A}(\underline{\rho})$ is the system response matrix (mapping), which is a function of $\underline{\rho}$.

A typical element A_{ij} of the matrix A is:

 $\begin{bmatrix} -\sum & \sum p_{m} & y_{mj} \\ m=1 & m & mj \end{bmatrix} \cdot (1/2 \pi R_{ji}^{2})$

where, (see Pig 3)

- . weight of a source neutron entering cell j,
- = total cross section of a source neutron,
- Σ' = total cross section of scattered neutron,
- ρ_1 = water fraction in cell 1;
- L = total number of cells encountered by a source neutron before being j
 - scattered in cell j.
- y = distance a scattered neutron travels in cell m after being scattered in cell j,
- M = total number of cells encountered by a neutron after being scattered in cell j,
- $P(\theta)$ = probability of scattering of a source neutron in water by an
 - angle θ , where θ is the angle of scattering of a source neutron in cell j to detector i ,

and



ji = distance between the centre of scattering of cell j and the site of detector i

If a cell j is not seen by the scattering ray of detector i, the element A is set equal to zero.

If N_d detector energy-dependent fluences are available and the water fraction in N_c cells is sought, then the matrix $\underline{A}(\rho)$ will be an N_d x N_c matrix while S and ρ will be N_d and N_o dimensional vectors, respectively. We will be considering solving Equation (4.1), for , when

Nd [>] Nc

i.e., the problem is overdetermined. Underdetermination, as shown before, requires an a priori supplementary information regarding the void fraction sought is in order to find an appropriate solution. This information can be obtained from other physical criteria regarding the local void fraction or flow regime. However, we will assume that no such a priori data is available, and we will seek a solution of an '

overdetermined problem.

Before solving the inverse problem (4.1) according to the procedure discussed in the previous Section, we have first to treat the apparent non-linearity of the mapping of ρ into S by the non-linear mapping $\underline{A}(\rho)$. Therefore, the problem of non-linearity will be added to the extension problem, construction problem, the existence question and problem of errors that one has to investigate in order to solve the inverse problem. These problems are discussed in the following. From now on, the inverse problem of (4.1) will be referred to as SENT (Single Exposure Neutron Tomography) inverse problem.

4.1 <u>Non-linearity Problem</u>

The major obstacle facing the SENT inverse problem is that the mapping $\underline{A}(\underline{\rho})$ itself depends on the sought solution $\underline{\rho}$. Therefore, one cannot directly address the inverse problem in the systematic way illustrated in the previous section, unless the problem of non-linearity is overcome. In the following, we will show how to handle this problem. We will consider an error-free set of data E. The effect of errors in E will be discussed when the extension problem is addressed.

By examining Equation (4.2) which presents a typical element of

the matrix $\underline{A}(\rho)$, one can conclude that the mapping $\underline{A}(\rho)$ is continuous. That is because R_{ii} is never zero, (detector is not located inside the test section), and the cross sections and the scattering probabilities of neutrons in water are continuous. The components of the solution of the problem are sought inside the closed interval [0,1], (there is neither a negative water fraction nor a fraction greater than one). Therefore, the set of solutions C is a closed set that is included in the real space. The diameter of this set is

$$D(C) = \sup d(x,y) = 1;$$

• x y y c C

where d denotes the (largest absolute component) distance between the points x and y (both belong to the set) and sup denotes the supremum of the set, i.e., the least upper bound. Since C exists in the real space and D(C) is finite, then C is bounded⁽⁷⁾. In the real space any closed bounded set is automatically a compact set⁽¹⁵⁾. Therefore, the set C is a compact set. Since $\underline{A}(\underline{\rho})$ is continuous and we are seeking a solution on a compact set C and the right-hand member of S of Equation (4.1) belongs to the set $\underline{E} = \underline{A}(\underline{\rho}) \cdot \underline{\rho}$, (E is error-free), it is possible to use the following formula⁽¹⁾ (provided \overline{A}^{-1} exists):.

(4.3)

$$\rho = A^{-1}(\rho) \cdot S$$

to construct a solution of equation (4.1) that is stable with respect to small changes in S. However, the mapping $\underline{A}^{-1}(\underline{\rho})$ is not linear, since it depends on $\underline{\rho}$. Therefore, a direct construction of a solution of Equation (4.1) using the formula (4.3) is not possible. Nevertheless, one can proceed by starting with an initial guess of the sought parameters, $\underline{\rho}_0$, in the compact set C and evaluate $\underline{A}(\underline{\rho})$ accordingly, then evaluate an intermediate solution $\underline{\rho}_1$, use this solution to update $A(\underline{\rho})$ and repeat the process until the sequence: $\underline{\rho}_0, \underline{\rho}_1, \underline{\rho}_2, \ldots, \alpha$ converges. The question will be then, does this sequence converge, and if it does, does it converge to a unique solution? To answer this question, let us write the proposed iterative process as:

$$= \frac{1}{N} \frac{\rho}{\rho_{k}} = \frac{1}{N} \frac{-1}{p_{k}} (\rho_{k-1}) \cdot \frac{1}{2} , \quad k=1,2,\ldots, \quad (4.4)$$

where k is the iteration numbers and N is the dimension of the vector ρ . In fact, Equation (4.4) represents a mapping of ρ into ϕ .

.1.1 Contraction Mapping

In order to prove that the sequence $[\underline{\rho}_k]$ converges, one has to prove that the mapping m of (4.4) is a contraction mapping. (14) To

define a contraction mapping, let us define the N- dimensional vector space

$$\mathbb{R}^{N} \equiv \left\{ \rho = (\rho_{1} \rho_{2} \cdots \rho_{N})^{T} \right\}, \rho_{1} \varepsilon \text{ (the real numbers).} (4.5)$$

Let us associate with this space, the norm:

$$||\underline{\rho}|| = \max |\rho_i|$$
 (4.6)

(4.7)

This space is a normed linear space. Since ρ exists in the closed set [0,1], then $||\rho|| < 1$. The mapping m is contraction (14) if there exists an α , $0 \le \alpha \le 1$, such that

$$||\underline{\phi}_{\mathbf{k}}-\underline{\phi}_{\mathbf{k}+1}|| < \alpha ||\underline{\rho}_{\mathbf{k}-1}-\underline{\rho}_{\mathbf{k}}||$$

for all p, ϕ , R^N . (This implies that m is uniformly continuous, which is the case as shown before.)

The condition of (4.7) of contraction mapping implies that the distance between two consecutive iterations decreases as the number of iterations increases, that is $||\rho_{k-1} - \rho_{k}|| \rightarrow 0$ as $k \rightarrow \infty$. Therefore, the iterative procedure of (4.4) converges, if condition (4.7) is satisfied. Using the mean value theorem⁽⁸⁾, then:

$$\frac{\Phi_{k+1}}{\rho_k} = \underline{\Phi}(\rho_k) \cdot (\rho_k - \rho_{k-1}) ,$$

$$\frac{\rho_k}{\rho_k} = [\rho_k, \rho_{k-1}]$$

where $\underline{\phi}$ is the Jacobi matrix (of partial derivatives) of $\underline{\phi}$ with respect to $\underline{\rho}$, estimated at a mean value $\underline{\rho}$. Therefore, the condition (4.7) for contraction mapping can be rewritten as:

$$\left|\left|\underline{\phi}^{*}\left(\underline{\rho}_{k}\right)-\left(\underline{\rho}_{k}-\underline{\rho}_{k-1}\right)\right|\right| \leq \alpha \left|\left|\underline{\rho}_{k}-\underline{\rho}_{k-1}\right|\right|$$

$$(4.9)$$

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But,

$$||\underline{\phi}^{*}(\hat{\underline{\rho}}_{k}) \cdot (\underline{\rho}_{k} - \underline{\rho}_{k-1})|| \leq ||\underline{\phi}^{*}(\hat{\rho}_{k})|| \cdot ||\underline{\rho}_{k} - \underline{\rho}_{k-1}|| \quad (4.10)$$

By comparing (4.9) and (4:10), then the condition for . contraction mapping can be written as:

$$|\{\underline{\phi}, (\underline{\rho}_{k})\}| \leq \alpha < 1$$
 (4.11)

Hence, if $||_{\oint} ||_{for any} = \frac{\rho_{k}}{k} \in \{0,1\}$ is less than unity, the mapping of (4.4) will be a contraction mapping. Let us see under which circumstances this condition can be satisfied.

From the definition of ϕ in Equation (4.4), then the jth column of ϕ is

$$\phi_{1}^{*} = -\frac{1}{N} A_{\pm}^{*-1} A_{1}^{*} A_{\pm}^{-1} S_{\pm}^{*}$$
(4.12)

where \underline{A}_{j}^{i} is the partial derivative of the matrix \underline{A} with respect to ρ_{j}^{i} , but from Equation (4.3) $\underline{\phi} = \frac{1}{N} \underline{A}^{-1} \cdot \underline{S}^{i}$, then

$$\underline{\phi}_{j}^{\dagger} = -\underline{A}^{-1} \underline{A}_{j}^{\dagger} \underline{\phi} \qquad (4.13)$$

Now let us consider $\underline{A}_{j}^{*}(\underline{\rho})$. From the structure of $\underline{A}(\underline{\rho})$, shown in Equation (4.2), a typical element of $\underline{A}_{j}^{*}(\underline{\rho})$ is such that:

$$1p = -\frac{\partial A_{1p}}{\partial \rho_{j}} = -\beta_{j1p} A_{1p}, \beta_{1jp} \geq 0, \quad (4.15)$$

 $A_{10} \geq 0$

Note that an element A_{lp} of A represents the contribution of cell p to detector 1. A_{lp} is an exponential function of p; if cell p is seen by detector 1; otherwise it is zero.

Using Figure (4),

 $\beta_{j,lp} = \Sigma d$ when cell j lies completely in the path of a source ray,

 $\beta_{j1p} = \Sigma'd / \sin \theta$ when cell j lies completely in the path of scattering ray ($0 \neq 0$);

 $\beta_{jlp} = \frac{d}{2} (\Sigma + \Sigma' / sin0)$ when cell j is the scattering cell, (intersection of a source ray and a

scattering ray), (4.16)

where d is the cell width


FIGURE 4 Schematic Diagram for Evaluating the Derivative of the Response Matrix : $A'_{ji} = \beta_j A_{ji}$

Let us take

$$j = \max_{j,p} \beta_{j,p}$$

then one can write

$$-\underline{A}_{j}^{!} \leq \beta_{j} \underline{A}_{j}$$

Substituting Equation (4.18) in Equation (4.13), then

$$\frac{\phi}{j} \leq \underline{A} - \beta_{j} \underline{A} = \beta_{j}$$

Let $\beta = \max \beta$

since ϕ'_{j} is the jth column of ϕ'_{j} , then one can write

where Φ is a matrix such that an element $\phi_{ij} = \phi_j$.

Subsequently;

$$|| \leq |\beta| || || ||$$

If $\phi \in [0, 1/N]$, then

$$\left| \left| \phi \right| \right| = \max \sum_{i=1}^{N} \left| \phi_{i} \right| = \max \sum_{i=1}^{N} \left| \phi_{j} \right| \le 1$$

(4.19)

(4.17.)

18)

In other words, the condition (4.11) for the mapping of Equation (4.4) to be a contraction mapping is simply

<u>∳</u>'

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if $\phi \in [0, 1/N]$, or equivalently if $\phi_k \in [0, 1]$. An illustration of the above proof is given in Appendix C.

From the definition of β , and relationships (4.16) one can see that condition (4.21) implies that the cell width has to be smaller than the value of d that satisfies the inequalities:

 $(1) \qquad d < 1/\Sigma$

d

Therefore,

if:

and

(3)

(2) $d \leq \sin(\theta) / \Sigma^{1}$

 $d < 2 \sin(\theta) / (\Sigma' + \Sigma \sin \theta)$

(4.21)

1 .

(4.22)

The first condition is always satisfied as long as the SENT problem is restricted to a single scattering problem, implying that at most one mean free path, $1/\Sigma$, of water is allowed. Therefore, if an n x n cell mesh is constructed, (nd < $1/\Sigma$) or (.d < $1/n\Sigma$), the first condition will be satisfied when n > 1.

In order to show the maximum size of cell, d, to be allowed in order to satisfy the first and second conditions of Equation (4.22), let us suppose that the largest angle of scattering to be recorded is 60. For a 14 MeV neutron source, an energy of scattering of 3.5 MeV corresponding to 60° of single scattering. The total cross section corresponding to this energy is 0.2208 cm⁻¹. Then in order to satisfy the second condition of (4.22), one has to have

and to satisfy the third condition

d < 3.92 cm

< 5.62 cm

(4.24)

(4.23)

Thus, if one is examining a test section of a diameter equivalent to one mean free path of a 14 MeV neutron, that is 10 cm, one should have a mesh of n x n cells where n > (10/3.92), i.e., n > .2.6. If n has to be integer, for convenience, then n = 3 will satisfy the condition of contraction mapping. This condition is not physically a severe

condition, since the technique was developed to determine the local void fraction; maturally in a cell of size less than $3.92 \times 3.92 \text{ cm}^2$.

Having shown the condition for the iterative procedure of (4.4) to converge, the question now is: does it always converge to a unique solution? Let us quote the Contraction Mapping Theorem⁽¹⁶⁾ (also called the Fixed Point Theorem).^(7,8,17)

4.1.2 Fixed Point Theorem

Let $(X, ||\cdot||)$ be a <u>Banach</u> space and $m: X \to X$ be a contraction mapping. Then, there is a unique $x^* \in X$, such that $mx^* = x^*$, x^* is

called the fixed point of m. Moreover, if x_0 is any point in X, and we define the sequence $[x_n]$ by

$$x_1 = m x_0$$
, $x_2 = m x_1$, \dots $x_n = m x_{n-1}$

then $x_n \rightarrow x^*$ as $n_n \rightarrow \infty^*$, and

$$||x_n - x_n|| \le \frac{\alpha^n}{1 - \alpha} ||x_1 - x_0||$$
 (4.25)

where $0 < \alpha < 1$, the constant that satisfies the contraction mapping condition:

$$|| \mathbf{m} \mathbf{x} - \mathbf{m} \mathbf{y} || \leq \alpha || \mathbf{x} - \mathbf{y} ||$$
 for all $\mathbf{x}, \mathbf{y} \in \mathbf{x}$

In order to make use of the Contraction Mapping Theorem, the space we are dealing with has to be a Banach space. A Banach space is a complete normed linear space, and the space \mathbb{R}^N , we are considering, is a Banach space. ⁽¹⁶⁾ Therefore, the successive approximations. (contraction mappings) of (4.5) have a unique fixed point. This means that regardless of the initial guess, one would end up with a unique solution. The Fixed Point Theorem also implies the existence of the solution through the existence of the unique fixed point.

4.1.3 Method of Successive Approximation

The next question is that, does that unique fixed point exist inside the closed interval [0,1], since $0 \le p \le 1$. Let us quote the local version of the Contraction Mapping Theorem: (16)

Let B be a closed subset of a Banach space X and $m:B \rightarrow B$ be such that

 $||mx - my|| \le K ||x - y||$, $0 \le K < 1$

for all x, y c B

where

Then, there exists a unique vector x^* in B, such that

 $m(\mathbf{x}^{\star}) = \mathbf{x}^{\star}$

Moreover, x^* may be obtained as the limit of the sequence $[x_n]$

 $\mathbf{x}_n = \mathbf{m} (\mathbf{x}_{n-1})$, $\mathbf{x}_0 \in \mathbf{B}$

According to this Theorem, it can be said that if one starts with the "zeroth approximation" with a vector $\underline{\rho}_{0}$ of elements inside the closed interval [0,1], the successive approximation

$$= \frac{1}{N} \rho_{k} = \frac{1}{N} A^{-1} (\rho_{k-1}) S$$

converges to a unique vector $\underline{\rho}$ * such that

if

$$\underline{\rho}^{*} = \underline{A}^{*} (\underline{\rho}^{*}) \cdot \underline{S}$$

 $|| \underline{\phi}_{k+1} - \underline{\phi}_{k}| | \leq \alpha || \underline{\rho}_{k} - \underline{\rho}_{k+1} ||, \quad 0 \leq \alpha < 1$ (4.26)

for all $\phi_k \in B = [0, 1/N]$, or equivalently $\rho_k \in [0, 1]$. As shown before, the condition (4.26) is satisfied if conditions (4.22) are satisfied, which is the case in practical situations. Therefore, the local version of the Contraction Mapping Theorem guarantees the conversion of the successive approximations to a unique fixed point, even in our case where the solution is sought inside a closed interval.

However, one should keep in mind that in order to make use of the local version of the Contraction Mapping Theorem, the mappings used should map the closed subset into itself. Therefore, if a mapping results in an element outside this subset, the local conversion, that is,

the conversion to a limit inside the closed subset; will not be

guaranteed. Nevertheless, a fixed point will be obtained inside the space, according to the general Contraction Mapping Theorem. Therefore, if in the kth approximation, an element or more the vector ρ_k goes beyond the closed interval [0,1], one should adjust this vector such that all its elements exist inside [0,1] and consider the "adjusted" vector as the zeroth approximation and start the successive approximation procedure all over again. Eventually, one should reach the unique fixed point that exists inside the closed interval [0,1]. The "adjustment" procedure is an arbitrary process; however, a proper "adjustment" will lead to a faster convergence of the successive approximation to the fixed point. The "adjustment" procedure used for the SENT problem is discussed in Section 5.

4.1.4 Local Linearity

In the successive approximation:

 $\underline{\phi} = \frac{1}{N} (\underline{\rho}) \cdot \underline{S}$

one is mapping: $\{ \ \underline{\rho} \ \}$ into $\{ \ \underline{\phi} \ \}$. Nowever, during any stage k of the successive approximation one can say that the problem is equivalent to mapping S into $\underline{\rho}$ using the "locally" linear mapping $\underline{\Lambda}^{-1}(\underline{\rho}_{k-1})$. By "locally" linear it is meant that the mapping is linear only during that stage of the contraction mapping. Linearity stems from the fact that the mapping $\underline{\Lambda}^{-1}(\underline{\rho}_{k-1})$ does not defend on its domain $\underline{\rho}_{k}$, since the mapping has been constructed using $\underline{\rho}_{k-1}$. it is locally linear. To illustrate that, let us consider two solutions

$$\underline{\rho}_1 = \underline{\underline{A}}^{-1}(\underline{\rho}_1) \cdot \underline{\underline{S}}_1$$

 $\underline{\rho}_2 = \underline{\lambda}^{-1}(\underline{\rho}_2) \cdot \underline{s}_2$

Apparently, the linear combination

1. 1. 1. 1.

$$\int - \frac{\alpha_1 \rho_1 + \alpha_2 \rho_2}{\alpha_1 \rho_1 + \alpha_2 \rho_2}$$

where a_1 and a_2 are constants, is not a solution since

$${}^{\alpha}1^{\underline{\rho}}1^{+\alpha}2^{\underline{\rho}}2 \stackrel{=}{=} \underline{\lambda}^{-1} (\alpha_1 \underline{\rho}_1) \cdot \underline{s}_1 + \underline{\lambda}^{-1} (\alpha_2 \underline{\rho}_2) \cdot \underline{s}_2$$

and

and '

$$\alpha_1 \underline{\rho}_1^{+\alpha} \underline{2\rho}_2^{-\gamma} \neq \underline{\Lambda}^{-1} (\alpha_1 \underline{\rho}_1^{+\alpha} \underline{2\rho}_2) \cdot (\alpha_1 \underline{s}_1^{+\alpha} \underline{s}_2^{-\gamma})$$

However, for the two solutions of the local problem

$$\underline{\rho}_{k,1} = \underline{\underline{A}}^{-1} (\underline{\rho}_{k-1}) \cdot \underline{S}_{1}$$

and

$$\underline{\rho}_{k,2} = \underline{A}^{-} (\underline{\rho}_{k-1}) \cdot \underline{S}_{2}$$

One has :

 $\alpha_{1 \stackrel{\rho}{-} k, 1} + \alpha_{2 \stackrel{\rho}{-} k, 2} = \alpha_{1 \stackrel{p}{=}} (\frac{\rho_{k-1}}{2}) \cdot \frac{S_{1}}{2} + \alpha_{2 \stackrel{p}{=}} (\frac{\rho_{k-1}}{2}) \cdot \frac{S_{2}}{2}$

thus,

 $\alpha_{1} \frac{\rho_{k,1}}{2} + \alpha_{2} \rho_{k,2} = \underline{\Lambda}^{-1} (\underline{\rho}_{k-1}) \cdot (\alpha_{1} \frac{S}{2} + \alpha_{2} \frac{S}{2})$

that is, any linear combination of two solutions still is a solution, and the problem is linear.

The "local" linearity of the successive approximation reduces the kth approximation to a conventional linear problem for which the extension problem existence question, construction problem and the problem of error, can be addressed, as shown before. That is, in every step of the successive approximation, one is solving a linear inverse problem and if the solution of the local inverse problem is kept inside the closed interval [0,1], the conditions required in the local version of the Contraction Mapping Theorem will be satisfied and the "global" problem will converge to a unique fixed point, providing that a unique solution of the problem exists. We will call the successive approximation procedure as the "global" inverse problem, while the linear

inverse problem at any of the stages of this procedure as the "local" there problem. One should mention that the "global" problem is just a, in of successive "local" problems and if the extension, existence, construction and error problems of every-element of that set is solved, the corresponding problems for the "global" problem are in turn solved.

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In fact, the last "local" problem in the set of successive "local" problem provides the solution of the "global" problem. The "local

problem is discussed after the following one-dimensional demonstration of the "global" problem.

4.1.5 One-Dimensional Illustration

The behaviour of the "global" problem of SENT can be illustrated without loss of generality if the one-dimensional case is considered. That is if a one detector response S is known and a single water fraction x is sought. In this case one can write:

$$= x \exp(-\beta x), \quad x \in \{0,1\}$$

27)

where x is the water fraction, β is the total neutron path length in mean free paths. The exponential term in the above equation represents the only element of the response matrix \underline{A} . Equation (4.3.7) can be

solved for x using the successive approximations:

 $x_{k} = S \exp(\beta x_{k-1})$, k=1,2,3 (4.28)

This is equivalent to solving the two transcendental equations:

$$f = S \exp(\beta x)$$

(4.30)

(4.29)

Since f is non-negative, (S is non-negative), then f and g can intersect at most twice within the closed interval [0,1]; depending on the value of ${}^{2}\beta$. That Ts, there is at most two solutions for the problem. The problem has only one solution within [0,1] if and only if $\beta \leq 1$ (see Figure 5 to 7). Also it can be shown that solving the transcendental Equations (4.29) and (4.30) using the successive approximation method (4.28) leads only to the solution at which $(\partial f / \partial x) \leq 1$. This condition also implies that $\beta \leq 1$. If the problem has two solutions, $(\partial f/\partial x)$ at the largest solution has to be greater than unity, in order for the function f to intersect the straight line g once again (see Figure 7). The successive approximation method rapidly diverges if $(\partial f/\partial x)$ is greater than unity, then no solution can be

reached.

and



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FIGURE 6 Solution of the One-Dimension Problem : $x = S \exp(Ex)$, $\beta = 1.0$, $x \in [0,1]$ (Note Uniqueness of Solution). N



FIGURE 7 Solution of the One-Dimension Problem : x = S exp(8x) , S = 1.5 , x c [0,1] (Note Non-Uniqueness of Solution)



From the above discussion one can conclude that in order for the solution of the problem (4.37) to be unique, β has to be at most equal to unity. Also if this unique solution is to be reached using the successive approximation method (4.38), β has to be at most unity.

However, if β is exactly equal to unity and the problem solution is near unity, the convergence of the successive approximations will be extremely slow.

The above conclusions agree with the fixed point theorem discussed above, which generalizes the problem into the multidimensional space.

2 Extension Problem of SENT

The extension problem, as defined in the subsection 3.1, is to extend the set of experimental results, E, such that it contains both computed results and results of the measurement, the latter having errors associated with. It was also shown in that subsection that this

extension can be achieved by surrounding each element e_i in the set E by an open ball, centred at e_i , and of a radius equivalent to the error

associated with e. One then would search for a solution, (quasi-solution), that minimizes a function, (cost function), that measures how far the solution vector is from the centres of the balls. When the data errors are assumed to be random in nature and have a Gaussian distribution with zero mean, it was shown that the minimization problem is the least squares problem. That is the problem of minimizing the cost function

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$$D_{2} = \{ \sum_{i=1}^{N} w_{i} d_{i}^{2} (e_{i} - m_{i} c_{i}) \}^{l_{2}}$$

$$(4.31)$$

where w_i's are the data variances, (assuming that data errors are 'statistically independent), and n is the number of available data.

For solving the problem

$$\underline{\underline{A}}_{=}^{-1}(\underline{\rho}_{k-1}) \cdot \underline{\underline{S}} = \underline{\rho}_{k}$$
(4.32)

in the kth successive approximation, where ρ_k is an N-dimensional vector (unknown), S is an M-dimensional (measurement data) and $\underline{A}(\rho_{k-1})$ is an M x N matrix. Since the problem is linear (locally) at

 $A \stackrel{[0]}{=} k-1$ is an M x N matrix. Since the problem is linear (locally) every step k, the subscripts will be dropped and the problem will be

expressed as

The cost function, D2, of this problem can be written as

(4.33)

(4.36)

$$M = \frac{1}{\sigma^2} \left(\frac{s_i}{s_i} - \sum_{j=i}^{N} \frac{A_{ij}}{j} \right)^2$$
(4.34)
i=1 i j=i

where σ_1^2 is the variance of the ith measurement. The cost function is weighted by the inverse of the variance so that measurements with higher statistical variances have less effect on the cost function. To find the value of \hat{p} (quasi-solution) that minimizes D_2 , the derivative of D_2 , with respect to each element of \hat{p} ; is set equal to zero. Then

$$\frac{\partial D_2}{\partial \rho_1} = \sum_{i=1}^{\infty} \frac{2}{\sigma_i^2} + \sum_{ij=1}^{A_{ij}} (s_i - \sum_{i=1}^{N_{ij}} A_{ij} = 0)$$
(4.35)

for i = 1,2,...,N

$$\hat{\underline{\rho}} = (\underline{\underline{A}}^{\mathrm{T}} \underline{\underline{W}} \underline{\underline{A}})^{-1} \underline{\underline{A}}^{\mathrm{T}} \underline{\underline{W}} \underline{\underline{S}}$$

 $S = \rho$

where \underline{A}^{+} = transpose of the matrix A

and

$$W_{ij} = \delta_{ij} / \sigma_i^2$$

The validity of the formal solution clearly requires that the matrix $\underline{A^TWA}$, called the "normal matrix" be non-singular, which will be assumed until further notice. The question to be answered next is about the existence and uniqueness of the quasi-solution.

.3 Existence Question of SENT

Before addressing the question of the existence of the quasi-solution, we will examine the existence of a solution for the error-free problem. In order to get a solution, the matrix <u>A</u> has to be invertible, that is, zero is not an eigenvalue of <u>A</u>, (non-singular). If <u>A</u> is invertible, then the inverse matrix <u>A</u>⁻¹ is uniquely defined and there is a unique solution of the error free problem: $\underline{p} = \underline{A}^{-1}S$. Since <u>A</u> is basically a mapping, then two conditions are to be met in order for \underline{A}^{-1} to exist (see Section 3.3).

(1) $\underline{A} \rho = S$, that A is surjective (onto) mapping, which is the case for an error-free problem, and

2) <u>A</u> is one-to-one mapping, (injective).

The latter condition means each vector \underline{S} has a corresponding unique vector $\underline{\rho}$. In the SENT problem, this means that every water distribution, $\underline{\rho}$, gives rise to a unique vector of detector responses \underline{S} . This is true physically, since a duplication of the detector responses requires a

duplication of the same neutron path in the same medium, that is, the same water content and distribution. However, this uniqueness can be only justified for the "global" problem. What is considered here is a "local" problem, that is the kth successive approximation. Here, the

matrix \underline{A} is evaluated using the values of $\underline{\rho}$ at the previous approximation, k-1. Nevertheless, these values represent a physical

distribution, (provided that $\rho \in [0,1]$,) though not the distribution one is after. Therefore, the matrix <u>A</u> in any successive approximation is non-singular, since it corresponds to a physically valid pattern, and for every physical pattern, ρ , the associated matrix is non-singular. Thus, the inverse matrix A^{-1} exists at every step of the successive

approximation process and a corresponding unique solution for the error free problem exists.

Practically, we do not know the exact value of <u>S</u>, instead an approximate value of it, <u>S</u>; (because of the experimental errors), is provided. The vector <u>S</u> may not belong to the set $\underline{S} = \underline{A} \underline{\rho}$. In this case,

the symbol $\underline{A} \stackrel{1}{\underline{S}}$ may be meaningless, since the inverse mapping may magnify the error propagation. The attempt to avoid this difficulty leads, as shown before, to the concept of a quasi-solution.

A quasi-solution is a vector $\underline{\rho}$ that minimizes a given cost function. The cost function expresses the discrepancy between \underline{A} and \underline{S} , that is, the residuals. As shown in subsection 3.3 of the present chapter, a quasi-solution is unique (and depends continuously on \underline{S}) if:

(1) the equation $\underline{A} \rho = S$ is linear

- (2) the homogenous equation $\Delta q^2 = 0$ has only the zero solution
- .(3) the set that contains the solutions is convex and every sphere in the space of the data is strictly convex
- (4) the solution is sought over a compact set

The first condition is satisfied since the local problem is linear, as shown before. The second condition is met by the physics of the problem since zero detector response results only if there is no

scattering medium; that is, $\underline{\rho}' = \underline{0}$. Since $\boldsymbol{\ell}_2^n$ spaces are considered for the solution space and the data space, the third condition is satisfied.

To consider the last condition, regarding the compactness of the set over which the solution is sought, let us recall the fact that any closed bounded subset of the Euclidean n-space, ℓ_n^n , is compact. (7) This in fact, requires that the normal matrix A WA be non-singular. In this case, the set of solutions will be bounded and it is also closed since it results in a solution within closed bounds. The invertibility of the normal matrix, in turn, requires that the matrices <u>A</u> and <u>W</u> are nonsingular. The non-singularity of the matrix <u>A</u> is necessary for the problem to be solvable. The matrix W, matrix of error, is a diagonal matrix with the inverse variances of the measurements as its elements, if the measurements are independent. This matrix will be singular if any of its élements is zero, that is, if a measurement has a variance of infinity; a measurement that must be discarded to begin with. Therefore, from a practical point of view, the normal matrix should be non-singular in order to find a solution of the problem. Singularity of the normal matrix implies that the problem is unsolvable. However, practical problems arise quite often when the normal matrix has eigenvalues sufficiently close to zero, that is, the matrix is ill-conditioned. In : these cases, the problem becomes unstable, in the sense that arbitrary small variations in the data vector S may produce arbitrary large

variation on the solution vector ρ . One, then, has to restore stability using a suitable regularization method.

In our present "local" inverse problem, we want, in addition to guaranteeing the stability of the solution, to confine the solution

around the interval [0,1], in order to guide the global problem towards convergence, as discussed before. This requires a careful treatment of, the regularization problem. This is, the subject of the following section.

.4 Construction Problem of SENT

The construction problem here is the problem of stabilizing the "local" problem, if it tends to be unstable, and of confining the solution around the interval [0,1], in order to keep the successive approximation problem ("global" problem) as a contraction mapping.

In subsection (3.3) of the present Chapter, Tikhonov -Miller method for constructing a stable (regularized) mapping was discussed. Applying this method on the local SENT problem, a regularized solution is given by:

$\hat{\rho} = \left[\underline{A}^{\mathrm{T}}\underline{W},\underline{A} + \left(\frac{G}{H}\right)^{2}\underline{B}^{\mathrm{T}}\underline{W},\underline{B}\right]^{-1}\underline{A}^{\mathrm{T}}\underline{W} \leq (4.4.1)$

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where B is a "constraint matrix" of a bounded inverse and G and H are

- constraints such that

anđ

||<mark>₿</mark>ρ'|| ≤ H

| **ξ** | = | **Δ**ρ-

(4:4.2)

(4.4.3)

That is, H is a constraint on the solution and G is a constraint on the solution residuals (errors). The constants H and G are assumed to be "a priori known". The main difficulty with this regularization method is the determination of the "constraint matrix" \underline{B} . For a minimum constraint, the "constraint matrix" is the identity matrix and,

consequently, H can be choosen to be the upper bound of the solution. However, in the "local" SENT problem, the minimum constraint is not enough, since it is desired to keep the solution as close as possible to the interval [0,1] in order to guide the global problem towards convergence. Therefore; a more elaborate analysis of the problem is required to determine the proper "constraint matrix". In the following, we will go through a fundamental analysis of the problem in order to find the proper "constraint matrix". This analysis is guided by the work of Burrus ^(18,19,20) on the utilization of "a priori" information in unfolding measurements. 4.4.1 Solution Construction

Let us recall the "local" problem:

 $\underline{\underline{A}} \underline{\rho} = \underline{\underline{S}}$

j

2 .

where a solution for \underline{p} is sought. Let an estimate of the desired solution be given by:

$$= \sum_{j=1}^{N} u_{j} s_{j}, j=1,2,...,N$$
 (4.4.5)

where M and N are the dimensions of the vectors S and ρ , respectively, and u_{ij} 's form a set of coefficients. In principle, one needs to find for each unknown ρ_j , a set of coefficients that gives the best result for the estimate ρ_j . Each set of coefficients for each unknown ρ_l is essentially independent of the other unknowns, ρ_l , $l=1,2,\ldots,N, l \neq j$.

Therefore, only one unknown will be considered in the analysis. After the analysis has been developed for one unknown, the analysis can be applied, in turn, for each of the N unknowns. Since ρ is an estimate of ρ , it has an uncertainty because of the statistical error in S. The statistical error in ρ is given by the customary formula for the standard deviation of a sum of terms:

(4.4.6)

(4.4.7)

(4.4.8)

where σ_i^2 is the variance of the measurement S_i .

М

l=1

 $\sigma^{2}(\hat{\rho}) = \Sigma u_{ij}^{2} \sigma^{2}$ $\cdot i=1$

However, there is another source of uncertainty if

 $\sum_{\substack{u_{ij} \\ ji = 1}}^{u_{ij}} A_{ji} - 1 \neq 0$

This uncertainty may result during the successive approximation process in which one is forced to use an approximate matrix, estimated at the previous step.

If we can find a weak upper bound δ , such that

= upper bound $\Sigma \mid I_{jl} - \Sigma \quad u_{ji} \quad A_{il} \mid \rho_{l}$

iąl

where I_{j1} is an element of the identity matrix, then the final error

$$\hat{\rho}_{\mathbf{i}}^{\mathbf{\rho}} = \hat{\rho}_{\mathbf{j}} - \sigma(\hat{\rho}_{\mathbf{j}}) - \delta_{\mathbf{j}}$$

$$\hat{\rho}_{\mathbf{i}}^{\mathbf{\rho}} = \hat{\rho}_{\mathbf{j}} - \sigma(\hat{\rho}_{\mathbf{j}}) + \delta_{\mathbf{j}} \quad (4.4.9)$$

Understandably, one is looking for a solution that has a confidence level as high as possible. This requires finding a suitable expression for δ_j and the determination of the coefficients u_i 's so that the error band is as narrow as possible.

4.4.2 Weak Upper Bound

The first step in finding an expression for the weak upper bound δ is the inequality:

$$\mathbf{I}_{ij} = \sum_{i=1}^{M} \mathbf{v}_{ij} \mathbf{A}_{ij}^{*} | \leq \sum_{\substack{k=1 \\ k \in =1}}^{N} |\mathbf{I}_{jk} - \sum_{\substack{i=1 \\ i=1}}^{M} |\mathbf{i}_{k}| \mathbf{A}_{i'j}^{*} / \mathbf{A}_{i'k}$$
(4.4.10)

where A^* is an element of the correct matrix A^* , that is

$$\underline{A}^{*}(p_{1}) \cdot \underline{\rho}_{1} = \overline{g}$$
(4)

4.11

(4.4.12)

(4.4.13

where ρ_t is the true solution of the problem and the vector <u>S</u> is free of errors. One should re-emphasize that the matrix <u>A</u> considered in the present analysis is the "local" matrix, that is, the matrix evaluated at a specific step of the successive approximation process. Therefore <u>A</u> is an approximate matrix, not the correct matrix. The matrices <u>A*</u> and <u>A</u> an non-negative matrices because of the physical nature of the SENT problem. The inequality (4.4.10) holds weakly at values at l(=i'), (i = 1,2,...,N), such that A_{1i}=0. This weak inequality allows us to obtain an upper bound δ ', such that:

ρj

$$\mathbf{j} = \sum_{\substack{\mathcal{L} = 1 \\ \mathcal{L} = 1$$

n

But, from Equation (4.4.11)

Σ

(for some values of $l(=i^*)$, $i = 1, 2, \dots, N$).

The inequality in the above statement is due to the non-negativity of the matrix \underline{A}^* and the vector ρ . From Equations (4.4.12) and (4.4.13),

$$\mathbf{y} = \sum_{\substack{k=1 \\ j \in \mathbf{I}}}^{N} |\mathbf{I}_{jk} - \sum_{\substack{i=1 \\ i=1}}^{M} \mathbf{u}_{ji} \mathbf{A}_{ik} | (\tilde{\mathbf{S}}_{i}, / \mathbf{A}_{i\rho}) ; \mathbf{A}_{i\rho} \neq 0 \quad (4.4.13)$$

is an upper bound to the magnitude of

$$\sum_{\ell=1}^{N} |\mathbf{I}_{j\ell} - \sum_{i=1}^{M} \mathbf{u}_{j\ell} | \rho_{\ell}$$

Since S_i , error free measurement, is not available, we use $(S_i + \sigma_i)$ instead and choose i to minimize $(\frac{S_i + \sigma_i}{A_i \cdot l})$ over all values of 1 for each l. Let

$$Q_{l,l} = \min_{i} \frac{S_{i}^{+\sigma}}{A_{il}}; A_{il} \neq 0 \qquad (4.4.14b)$$

then (4.4.13) becomes



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(4.4.15).

(4.4.16

Then the confidence interval is given by:



and ρ_{j}^{up} , (expressed as ρ_{j}^{low} , but with errors being added instead of subtracted). The final step is to choose the coefficients u (i = 1,2,...,M) such that the sum of the second and third terms in the right-hand side of the above equation is minimum, . consequently obtaining the "constraint matrix" we are seeking. Let us denote this sum by χ_{i}^{2} .

4.4.3 Constraint Matrix

Since, χ_j^2 is somewhat complicated, let us choose a simpler expression χ_j^2 to minimize, which is always larger than χ_j^2 . By choosing the u's to minimize χ_j^2 , one tends to reduce χ_j^2 also, though χ_j^2 may not be at its minimum value. The basis for choosing $\chi_j^*^2$ is the chain inequality.⁽⁸⁾

N M $\Sigma | I_{jl} - \Sigma | u_{jl} | Q_{i'l}$

i=1-

= $(\Sigma u_{ji}^2 \sigma_{i2})^{1/2}$

٤·=i

.X

 $\leq \left(\sum_{ij}^{2} \sigma_{i2}^{2} \right)^{1/2} + N^{1/2} \left[\sum_{j\ell}^{2} | \tau_{j\ell}^{2} - \sum_{ij\ell}^{2} u_{j\ell}^{2} A_{i\ell}^{2} \right]^{1/2}$ $= 1 \qquad i = 1 \qquad i = 1$

 $\stackrel{M}{\leq} (1 + \tau^{2}) \left[\sum_{j \in \mathcal{I}} u_{j1}^{2} - \frac{2}{\tau^{2}} + \frac{N}{\tau^{2}} \sum_{\ell=1}^{2} |I_{j\ell} - \sum_{i=1}^{2} |I_{i\ell}|^{2} Q_{i\ell}^{2} \right]$ $i=1 \qquad i=1 \qquad i=1$

• (4.4.17)

The first inequality in this chain follows from the well known

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Schwartz inequality:

Σ

£ =1

$$| \alpha_{\chi} | \leq n^{1/2} (-7.1 - \alpha_{\chi} |^2)^{1/2}$$

 $g = 1$

The second inequality follows from the general inequality:

$$(1+\tau^2)^{1/2}$$
 $(\alpha^2 + \beta^2 / \tau^2) \ge |\alpha| + |\beta|$

where the factor ($1 + \tau^2$) has the smallest value that will allow the

.). - u² ji

. M

Σ

i=1

e =D

 σ_{i}^{2}

 $v_{j\ell}A_{i\ell} | 2 Q_{i\ell}^2$

inequality to hold for all α and β .

 $\chi_{j}^{2} \leq \chi_{j}^{2} = (1 + \tau^{2})$

 Σ



N ____2 (4.4.18)

The value of the parameter τ^2 is free and can be chosen for best results, as it will be shown later. Finally, χ_j^2 may be readily minimized by setting the derivatives with respect to u_{ij} (i = 1,2,...,M) equal to zero and solving the resulting equations. In matrix notation, the solution is

$$\underline{A} = [\underline{A} \ \underline{Q}^2 \ \underline{A}^T + (\ T^2/N) \ \underline{W}^{-1}] \qquad \underline{A} \ \underline{Q} \ \underline{J} \qquad (4.4.19)$$

.(4.4.20)

where \underline{U}_{j} is the vector of coefficients corresponding to the solution U_{j} , \underline{Q} is a diagonal matrix with

$$Q_{ll} = \min [(S_i + \sigma_i) / A_{il}] ; A_{il} \neq 0$$

 $\hat{\rho}_{j} = \underline{\underline{u}}_{j}^{\mathrm{T}} \underline{\underline{s}}$

 \underline{W} is a diagonal matrix with $W_{\ell\ell} = \sigma_{\ell}^{-2}$, and \underline{J} is a unit vector, $J_{\ell} = I_{\ell\ell}$ Now the estimated solution for ρ_{ℓ} is given by

Therefore, the solution can be constructed using the matrix:



In order to show how the matrix \underline{U} is related to the conventional "normal" matrix that results from the conventional least squares method and to see how regularization has been introduced, we consider the equivalent expression for \underline{U}_{-} :

$$\underline{U}_{j} = \underline{\underline{W}} \underline{\underline{A}} \begin{bmatrix} \underline{\underline{A}}^{\mathrm{T}} \underline{\underline{W}} \underline{\underline{A}} + (1^{2}/N) & \underline{\underline{Q}}^{-2} \end{bmatrix}^{-1} \underline{\underline{J}}$$
(4.4.22)

This expression can be shown to be equivalent to that of Equation (4.4.20) by premultiplying the transformation matrices in both expressions by

$$[\underline{A} \underline{Q}^2 \underline{A}^T + (1 - \tau^2 / N) \underline{W}^{-1}]$$

and postmultiplying them by

$$[\underline{A} \underline{W} \underline{A}^{\underline{T}} + (\tau^2/N) \underline{Q}^{-2}]$$

For $\tau \neq 0$, both the expressions of (4.4.20) and (4.4.22) give the same result. But in the limit $\tau \rightarrow 0$, Equation (4.4.22), agrees with the conventional least squares result of

when $M \ge N$. The "a priori" information is contained in the matrix \underline{Q} which serves to restrict the magnitude of u_{ji} . As $Q_{jj} \ge \infty$, the solution of Equation (4.4.22) also approaches the conventional least squares solution. Comparing the transformation matrix of Equation (4.4.22) with Tickhnov-Miller regularized transformation matrix of Equation (4.4.1), one can see that the "constraint matrix" B is such that

$$(G/H)^{2} \underline{B}^{T} \underline{W} \underline{B} = (\tau^{2}/N) \underline{Q}^{-2}$$
 (4.4.23)

Since both \underline{O} and \underline{W} are diagonal matrices, \underline{B} is also a diagonal matrix. Let us consider one element in Equation (4.4.23), then

$$(G/H)^{2}B_{ii}W_{ii}B_{ii} = (\tau^{2}/N)Q_{ii}^{-2}$$

 $W_{ii} = 1/\sigma_i^2$ and

$$Q_{ii} = \min \left[(S_{\ell} + \sigma_{\ell}) / A_{\ell i} \right] = \left[(S_{m} + \sigma_{m}) / A_{m i} \right] ; A_{\ell i} \neq$$

(4.4.24)

then

• but

$$(G/H)^{2}B_{ii}^{2}/\sigma_{i}^{2} = (\tau^{2}/N) \Lambda_{mi}^{2}/(S_{m}+\sigma_{m})$$

If one takes

$$A_{ii} = A_{mi} / (S_m + \sigma_m)$$

$$^{2}/N = (G/H)^{2}/\sigma_{i}^{2}$$

Recalling that

then

and

$$G \geq || \underline{A}\underline{\rho} - \underline{S} ||$$
$$B \geq || \underline{B}\underline{\rho} ||$$

and taking, just for the sake of illustration,

H

$$G = r_i^2 = largest residual$$

and

$$= B_{ii}\rho_{i} = A_{mi}\rho_{i}/(S_{m}+g_{m})$$

then

$$r^{2}/N = (r_{i}/\sigma_{i}^{2}) [(s_{m}+\sigma_{m})^{2}/A_{mi}\rho_{i}]^{2}$$
 (4.4.26)

(4.4.25)

From the above equation, one can see that the parameter τ^2 controls error propagation. As τ^2 increases, the error propagation increases.
Another way of looking into the regularized solution, as constructed using Equation (4.4.22), is to consider the fact that this solution minimizes:





The above equation results in the conventional least squares procedure, which has a zero residual vector, when r^2 is set equal to zero. In the conventional least squares method, one is after a solution that has a

zero residual inside the solution error ball. The solution error ball isa closed ball that has the true solution (error-free solution) as its centre, and the norm of the error vector, associated with the estimated

quasi-solution, as its radius (see Figure 8). In a regularized solution, with $\tau^2 > 0$, a solution is found inside a larger ball. The size of the



 $\underline{\underline{C}} = \{\underline{\underline{A}}^{\mathrm{T}} \ \underline{\underline{W}} \ \underline{\underline{A}}\}^{-1} \ \underline{\underline{A}}^{\mathrm{T}} \ \underline{\underline{W}}$ $\underline{\underline{\underline{H}}} = \{\underline{\underline{A}}^{\mathrm{T}} \ \underline{\underline{W}} \ \underline{\underline{A}} + (\tau^{2} / N) \ \underline{\underline{Q}}^{-2}\}^{-1} \ \underline{\underline{A}}^{\mathrm{T}} \ \underline{\underline{W}}$

FIGURE 8 · Solution Error Balls Associated with Solving $\underline{A} \ \underline{\rho} = \underline{S}$, for $\underline{\rho}$ using Different Methods new ball is determined by the value of r^2 , as shown in Figure 8. By increasing the size of the solution error ball, one can guide the solution towards the desired goal, that is, $\underline{\rho} \in [0,1]$.

4.4.4 Regularization Parameter

a'nd

The basis upon which the value of the parameter. τ^2 , (from now on it will be called the regularization parameter), can be explained, without loss of generality, if the one-dimensional case is considered. For this case, where only one measurement is available and one solution is to be found, the matricies \underline{C} and \underline{H} defined in Figure 8 become

$$C = (a \frac{1}{\sigma^2} a)^{-1} a \frac{1}{\sigma^2} = a^{-1}$$

 $\underline{H} = \{a \ \frac{1}{\sigma^2} \ a + (\tau^2 \ / \ 1) \ a^2 \ (S+\sigma)^{-2}\}^{-1} a \ \frac{1}{\sigma^2}$

where "a" is the only element of the matrix \underline{A} , S is the available measurement and σ^2 is the variance associated with this measurement. The radii of the balls, or in this case the circles, containing the solution, as defined in Figure 8, become:

$$R_{1} = ||a^{-1}S|| = |a^{-1}S| = a^{-1}S, a \ge 0, S \ge 0$$

$$R_{2} = ||a^{-1}S + (1 - \underline{\mu} a) (S + \sigma) a^{-1}||$$

i.e. the fractional standard deviation in the measurement, then

$$R_2 = \left[\left| \underline{a}^{-1} \mathbf{S} + (\mathbf{1} - \underline{\mathbf{H}} \mathbf{a}) \right| (\mathbf{1} + \mathbf{f}) \cdot \mathbf{a}^{-1} \mathbf{S} \right]$$

assuming that f << 1 , then

Let

$$1 - \frac{H}{2} a = || \tau^{2} f^{2} || = |\tau^{2} f^{2}| = \tau^{2} f^{2}, \text{ and}$$

$$R_{2} - R_{1} = (a^{-1} S) \cdot (\tau^{2} f^{2})$$

This equation shows that the regularization process increases the radius of the ball within which the solution 1s to be found by $\tau^2 f^2$ of $a^{-1}s$. If τ^2 is taken equal to N, number of unknowns, (that is, τ^2 =1, for this particular case), then the size of the ball is to be expanded by one f i.e. one fractional standard deviation. If the error in the measurements is given as one standard deviation, then it is reasonable to expand the solution ball by one f, i.e. to choose τ^2 equal to N. Since the one-dimensional case is just a reduction of the general multidimensional case, then one can generalize the above rule by choosing τ^2 equal to the number of unknowns, N, if the solution ball is to be expanded by one fractional standard deviation, i.e. by an amount equivalent to the confidence interval of the measurements. Further numerical verifications are given in Section 7.

Problem of Errors in SENT

.5

The solution obtained for the inverse problem carries two kinds of errors, statistical errors and resolving errors. The statistical error associated with the estimated solution $\hat{\rho}_{i}$ is given by

std $(\hat{\rho}_{j}) = \begin{bmatrix} M & \hat{\nu}_{ij} & 2 \\ 1 = 1 & ij & 2 \end{bmatrix}$ (4.5.1)

while the resolving error is

In the SENT problem, there is another source of error in addition to the above errors. This error results from the fact that the physical process, representing the problem, is approximated by a single collision phenomenon, ignoring multiple scattering. Although this error is small, as shown in Chapter 3, it results in an error in the final solution. In order to estimate the effect of multiple scattering, we introduce the buildup matrix <u>B</u>. Thus, the response matrix of the

system, considering multiple scattering, is given by the matrix AB. Then one will obtain a solution $\underline{\rho}_{o}$ for the problem

when multiple scattering is considered and a solution ρ , for single scattering approximation such that:

$$\mathbf{A} \rho = \mathbf{S}$$

<u>I + A</u>

 ≤ 1

Let

and

 $\underline{\varepsilon}$ is the error associated with ρ due to the single scattering then approximation. The matrix Δ represents the deviation of the problem from the single scattering phenomenon. The errors Δ and will ε be called the approximation errors. Since no more than one mean free path of the source neutron in water is to be considered, then the effect of rescattering is small compared to that of the first scattering, that is

 $\underset{=}{\overset{A}{=}} \overset{B}{=} \rho_{0} = \underbrace{S}_{0}$

(4.5.4)

(4.5.4)

(4.5.5)

(4.5.6)

$$\underline{\mathbf{e}} = \left(\mathbf{I} + \underline{\mathbf{b}} \right)^{-1} \cdot \underline{\mathbf{b}} \mathbf{\mathbf{e}}$$

Using Equation (4.5.6), $\underline{\varepsilon}$ can be approximated by

$$\underline{\underline{\varepsilon}} \approx \left(\underline{\underline{\mathbf{I}}} - \underline{\underline{\delta}} \right) \quad \underline{\underline{\delta}} \quad \underline{\underline{\delta}}$$

Then

$$\frac{||\underline{e}||}{||\underline{e}||} \leq \frac{||\underline{1} - \underline{A}||}{||\underline{A}||}$$

Thus, the relative approximation error in $\underline{\rho}$ is at most as large as the relative approximation error in \underline{A} , i.e., $(\underline{\Delta})$. Therefore, one does not have an error in $\underline{\rho}$ due to the single scattering approximation more than the error introduced in the system response matrix due to that approximation.

To summarize, three types of errors are associated to an estimate ρ of the true solution ρ . The first is the statistical error, Equation (4.5.1), the second is the resolvant error due to the solution construction process, Equation (4.5.2), and the third is the

(4.5.7)

(4.5.8)

(4.5.9)

approximation error, Equation (4.5.7), caused by the approximation introduced into the matrix \underline{A} by the single scattering assumption.

Near-Linear Inverse Problem

 $\underline{\rho}_{\mathbf{k}} = \underline{\mathbf{A}}^{-1} (\underline{\rho}_{\mathbf{k}-1}) =$

4.6

The solution of the SENT problem, as explained above, is basically through the successive approximations procedure:

This procedure, which requires the inversion of the response matrix \underline{A} , (or a corresponding transformation matrix) was necessary in order to overcome the strong non-linearity in the problem. However, when the values of the elements of the matrix \underline{A} do not change significantly from one approximation to the other, the problem becomes nearly linear. Let K by the approximation at which

(4.6.1)

$$|\underline{A}^{-1}(\underline{\rho}_{k})|\underline{A}(\underline{\rho}_{k+1})| - \underline{I}|| \leq \gamma$$
 (4.6.2)

where I is the identity matrix and γ is a non-negative fraction reasonably close to zero. Then, one can consider the problem (4.6.1) to be nearly linear and can switch to the conventional iterative techniques used in linear problems, in which no direct inversion is required.

4.6.1 The Bayesian Procedure

The Bayesian procedure is based on Bayes' theorem which is basic to the concept of conditional probability. The theorem published posthumously in 1763⁽²³⁾ says essentially, in self-evident notations, that:

Prob (Hypothesis) Datum)

Prob (Datum Hypothesis) Prob (Hypothesis)

Prob (Datum)

when division by zero is not entailed. Provided that the "a priori" probability, Prob (Hypothesis), can be ascertained, the theorem is completely valid. However when the "a priori" probability is not known, an accepted practice (24,25) in the field of decision process is to use an estimate of the "a priori" probability; then the above relationship is known as the Bayes' postulate. This postulate is used here to develop a procedure for solving the "local" problem. The details are given in the following paragraphs.

Iterative methods, in linear problems, are basically successive approximation methods in which a first guess (approximation) for the unknown vector is taken and then modified so as to reduce the discrepancies between the measured vector and the vector calculated, based on that guess. The modification process does not require the inversion of the response matrix - instead, the given approximation is adjusted by an additive (or a multiplicative) correction, that is, by adding to the approximation (or multiplying it by) a correction proportional to the difference (or the ratio) between the measured vector and the calculated vector. The additive correction suffers from the drawback of permitting negative excursions in the solution vector, (similar to the direct inversion method). On the other hand, the multiplicative correction has a positive constraint inherent in it. However, if the initial guess includes zero values, these values will remain zero and will never be adjusted by using the multiplicative correction.

In the SENT problem, when the successive approximation process (based on direct inversion) approaches linearity, one can claim that the problem is quite close to the final solution. That is because the response matrix will not change significantly if the process (4.6.1) is continued. Consequently, the solution vector does not differ greatly from one approximation to the other. Thus, if one uses the vector $\underline{\rho}_{\rm K}$

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that satisfies Equation (4.6.2) as the initial guess for a linear iterative method, this guess will be quite close to the final solution. Consequently, a multiplicative correction can be used in that iterative process, since an additive correction may result in a negative solution. The fact that the multiplicative correction does not change the value of an initial guess if it has the zero value is overcome by the closeness of that guess to the final solution. (The correction process is applied only when direct inversion approaches final solution.) That is, a zero in the initial guess is most probably a correct value. However, a zero in the initial guess will be replaced by a very small non-zero value and the iterative process will either bring this value closer to zero, if the correct solution is zero, or increase it, if the actual solution is not exactly zero.

The factor by which an approximation will be multiplied in order to correct its value is derived in the following paragraphs using a probabilistic (Bayesian) approach based on the work of Kennett et al. (21,22)





This mapping can be rewritten in a normalized form as



where



 $T_{i} = C_{i} \rho_{i}$

and

 $C_{j} = \sum_{\substack{k=1}^{N} A_{jk}}^{N}$

The matrix R is normalized, since:

 $\begin{array}{c} M \\ \Sigma \\ i=1 \end{array} = 1$

(4.6.3)

(4.6.4)

(4.6.5)

(4.6.6)

(4.6.7).

...,М

1,...,N

j =

In fact, one can look to any element of <u>R</u>, R_{ij} , as the probability of obtaining a measurement S_i given the normalized water fraction T_i , that is

$$P(S_{i} | T_{j}) = R_{ij}$$
 (4.6.8)

The conditional probability $P(S_i | T_j)$ can be related to the conditional probability $P(T_j | S_j)$ by the Bayes' postulate:

$$P(T_{i} | S_{i}) = P(S_{i} | T_{j}) P(T_{j}) / P(S_{i})$$

or equivalently, making use of relationship (4.6.4),

$$P(T_{j} | S_{i}) = P(S_{i} | T_{j}) / \sum_{\ell=1}^{N} P(S_{i} | T_{\ell}) P(T_{\ell})$$
 (4.6.9)

In this postulate $P(T_j)$ is termed the "a priori" and $P(T_j|S_j)$ the "a posteriori" probability. $P(T_j)$, the probability of occurrence of T_j , is related to $P(S_j)$, the probability of occurrence of S_j , by the rule of elimination:

$$P(T_{j}) = \sum_{i=1}^{M} P(T_{j} | s_{i}) P(s_{i})$$
(4.6.10)

Since the probabilities $P(T_j)$ and $P(S_j)$ are not known, complete ignorance can be proclaimed and an equipropable distribution can be assumed. That is,

$$P(T_i) = T_j / \sum_{\ell=1}^{M} T_{\ell}$$

(4.6.11)

6.12

(4.6.13)

$$P(S_{i}) = S_{i} / \sum_{l=1}^{M} S_{l}$$

However

and

To prove that, consider Equations (4.6.7) and (4.6.6) and (4.6.3), then

 $\sum_{\substack{\ell=1}}^{N} \mathbf{T}_{\ell} = \sum_{\substack{\ell=1}}^{M} \mathbf{C}_{\ell} \rho_{\ell} = \sum_{\substack{\ell=1}}^{M} \sum_{\substack{m=1}}^{M} \mathbf{A}_{m\ell} \rho_{\ell} = \sum_{\substack{m=1}}^{M} \sum_{m=1}^{M} \mathbf{S}_{m}$

 $T_{\ell} = \sum_{\ell=1}^{N} S_{\ell}$

Σ

L = 1

Now the rule of elimination (4.6.10) can be rewritten, using \checkmark Equations (4.6.11) to (4.6.13), as

using Bayes' postulate for
$$P(T_1|S_1)$$
, then

 $T_{j} = \sum_{i=1}^{M} R_{ij}T_{j}S_{i} / \sum_{\ell=1}^{M} R_{i\ell}T_{\ell}$

 $\underbrace{\mathbf{T}_{j}}_{\mathbf{T}_{j}} = \underbrace{\sum_{i=1}^{M} \mathbf{P}(\mathbf{T}_{j} \mid \mathbf{S}_{i}) \mathbf{S}_{i}}_{\mathbf{i} = 1}$

$$\mathbf{T}_{j} = \sum_{i=1}^{N} \mathbf{P}(\mathbf{S}_{i} \mid \mathbf{T}_{j}) \mathbf{P}(\mathbf{T}_{j}) \mathbf{S}_{i} \neq \sum_{\ell=1}^{N} \mathbf{P}(\mathbf{S}_{i} \mid \mathbf{T}_{\ell}) \mathbf{P}(\mathbf{T}_{\ell})$$

(4.6.14)

From Equation (4.6.8), and Equations (4.6.11) to (4.6.13),

Using Equation (4.6.4), then

ij.



$$S_{i}^{C} = \sum_{\ell=1}^{M} R_{i\ell} T_{\ell}$$
(4.6.1)

5)

is a calculated value and is different from the corresponding measured value S,, if T is an approximation.

Equation (4.6.14) can be used to construct a successive approximation process by using T_j in the right-hand side as the value of T_j in the kth approximation, and using for S_j the measured value. Then T_j in the left-hand side of the equation is the value of T_j at the (k+1)th approximation. Thus, one can rewrite Equation (4.6.15) as

$$T_{j}^{(k+1)} = T_{j}^{(k)} \sum_{i=1}^{n} R_{ij} S_{i} / S_{i}^{c}$$

Using Equations (4.6.5) and (4.6.7) for R_{ij} and T_{j} , then

$$\binom{(k+1)}{j} = \rho_{j}^{(k)} \sum_{i=1}^{M} A_{ij} (s_{i}/s_{i}^{c}) / c_{j}$$
 (4.6.16)

This equation introduces the multiplicative correction implied on the given approximation. We will call the successive approximations (4.6.16) the Bayesian procedure, to distinguish it from the direct

inverse successive approximation process of Equation (4.6.1).

In the following, we will show the conditions under which the successive approximations procedure of Equation (4.6.16) represents a contraction mapping. If it is a contraction mapping, then it will converge to a unique fixed point as shown before in subsection (4.1).

Let $\phi_{j}^{(k)} = \rho_{j}^{(k+1)}$, then (4.6.16) represents a contraction mapping if

for all $j = 1, \dots, N$ and all k.

Using Equation (4.6.16), then

$$\left|\frac{\partial \phi_{j}^{(k)}}{\partial \rho_{j}^{(k)}}\right| = \left|\frac{M}{i j} \left[A_{i j} s_{i} / c_{j} s_{i}^{c}\right]\right|$$

 $\left|\frac{\partial \phi_{j}^{(k)}}{\partial \rho_{j}^{(k)}}\right| < \alpha \qquad 0 \leq \alpha < 1$

 $-\rho_{j}\lambda_{ji}s_{i}(\partial s_{j}^{c}/\partial \rho_{j}) / c_{j}(s_{i}^{c})^{2})$

4.6.17)

Using Equation (4.6.6), then condition for contraction mapping is

$$\begin{vmatrix} \frac{\partial \phi_{j}^{(k)}}{j} \\ \frac{\partial \phi_{j}^{(k)}}{j} \end{vmatrix} = \begin{cases} \frac{M}{\Sigma} A_{ji} S_{i} (S_{i}^{c} - A_{ij} \rho_{j}) / C_{j} (S_{i}^{c}) \\ i = 1 \end{cases}$$

Note that in deriving this condition, the problem is considered as a linear non-singular problem, that is, the matrix \underline{A} is not a function of the solution and $C_j \neq 0$ for any $j = 1, \dots, N$. Non-linearity in \underline{A} was not considered since it is intended to use the Bayesian procedure only when the problem approaches linearity.

4.6.18

Condition (4.6.18) for contraction mapping is hard to analyze and to ensure its satisfaction at all times. However, since the Bayesian procedure will be implemented after using the direct inversion procedure until the strong non-linearity of the problem has been reduced, one can claim that the final solution of the problem is not too far from the approximation at which the Bayesian procedure will be used. In such a case, s_j^c should close to s_j , that is, s_j/s_j^c is close to unity. Then the condition (4.6.18) can be approximated by:

$$\sum_{ij}^{M} A_{ij} (1 - A_{ij} A_{j} S_{i}^{c}) / C_{j} < 1$$
 (4.6.19)

Taking into account the definitions of C_j, and the non-negativity of A_{ij} and ρ_j , it is clear condition (4.6.19) is satisfied except if $\rho_j = 0$ or A_{ij} = 0 for all i = 1,...,N. In the latter case, the matrix <u>A</u> is singurar and no solution exists, while in the former case the Bayesian procedure does not succeed, as indicated. before; unless the zero is replaced by a very small negative number, consequently satisfying condition (4.6.19).

The above discussion illustrates that at the late stages of the direct inverse successive approximation, when the matrix \underline{A} does not change significantly, the computation cost involved in the inversion process can be reduced by switching to the Bayesian procedure. This procedure, which does not involve direct inversion, is essentially a continuation of the contraction mapping, established by the direct inversion. The guarantee of the contractive nature of the Bayesian mapping stems from the fact that it will be implemented at the late stages of the successive approximation process where one is very close to the final solution. Use of the Bayesian mapping at early stages of the mapping.

COMPUTATIONAL ALGORITHM

5.0

In the previous Sections, the different theoretical aspects of the inverse problem were discussed. The implementation of these aspects into a computational algorithm is illustrated in this Section.

As indicated before, the inverse problem of SENT is composed of two major problems. The first one is the "global" problem, where the non-linear aspects of the problem are considered through the

implementation of a series of approximations. The second problem is the "local" problem which concerns with obtaining the local solution of the problem at each stage of the successive approximation process.

The computational procedures required to implement the "global problem" are outlined in the flowchart shown in Figure 9, and are discussed in more detail after the following brief explanation of the flowchart. For every approximation, k-1, the local problem is solved in order to find the water fraction of the next approximation, k. If a local water fraction produced at this approximation lies outside the interval [0,1], this fraction is adjusted such that all elements of the vector

 $\underline{\rho}$ lie within [0,1]. This adjusted water fraction vector is used to calculate the corresponding detector responses through the forward problem. Consequently, the acceptability of the kth approximation as the



FIGURE 9 Solution of the "Global" Problem

final solution is examined. The criteria used for determining the solution acceptability are discussed later. If this (adjusted) kth approximation is proved to be unacceptable, the successive approximation process is continued. However, if the kth approximation is considered as an acceptable solution of the problem, then errors associated with this solution are estimated and the problem is terminated. In the following subsections; the different computational procedures illustrated in Figure 9 are explained. The solution of the "local problem" is, therefore, discussed in the subsection corresponding to its block in the flowchart of Figure 9. A complete listing of a FORTRAN IV program is available as indicated in Appendix A.

Zeroth Approximation

In order to start the successive approximation process, an initial guess is required. This guess is considered as the zeroth order approximation. The initial guess can be chosen arbitrarily anywhere within the interval [0,1]. The choice of the zeroth approximation does not generally affect the solution procedure as will be shown in the numerical examples to follow. However, it is useful to apply reasoning in setting the zeroth approximation. For example, as experience builds up, one can make use of some of the characteristics of the distribution

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of the detector responses to guess a zeroth approximation closer to the actual solution of the problem; consequently reducing the computational effort.

One possible approach to chosing the zeroth approximation is to consider the solution previously obtained for a coarse mesh as the zeroth approximation for a problem of a finer mesh. However, as indicated earlier, the cell width should not be greater than 39.2 mm, in order to satisfy the conditions of contraction mapping. For a pipe of 100 mm in diameter, the coarsest mesh for which a solution is possible is a 3×3 mesh. Therefore, the solution obtained for a 3 x 3 mesh could be used, for example, as the zeroth approximation for obtaining the phase distribution in a 6 x 6 mesh. This may reduce the computational effort for the 6 x 6 mesh problem. However, the error caused by the homogenization of the phases inside a large cell in the coarse mesh problem may result in a large error in the solution of this problem. Then, the use of this solution as the zeroth approximation for the finer mesh problem may be as good as using any other approximation. Therefore, the use of a solution as the zeroth approximation for a finer mesh problem is only recommended when the solution is obtained for a reasonably fine mesh structure (say an 8 x 8 mesh), and one is trying to

obtain a solution within a finer mesh structure.

5.2 Response Matrix Construction

Once an approximation of the solution vector is obtained, the elements of the response matrix can be calculated using the relationship (4.2); as previously explained in Chapter 3. As clear from this relationship the elements of the response matrix depend on the solution vector. Therefore, the response matrix has to be updated following each approximation of the solutions to provide the corresponding approximation of the response matrix. When the successive approximations procedure converges to the solution of the problem, the response matrix reaches its actual value.

5.3

Forward Calculations

Once the response matrix for a given solution approximation is. constructed, the corresponding detector responses are calculated. These detector responses are compared later to the given measured detector responses. This comparison provides a measurement of the acceptability of the given approximation as a solution of the problems as shown in the next step of solving the global inverse problem.

Solution Acceptability

5.4

In order to determine if a given approximation; $\rho_{\rm k}$, is acceptable as a solution of the global inverse problem the following norms are defined.

Percentage Error Norm:

$$||\underline{s} - \underline{s}^{c}|| = \{\sum_{i=1}^{M} (s_{i} - s_{i}^{c})^{2}\}^{\frac{1}{2}} / \sum_{i=1}^{M} s_{i} \times 100$$
(5.1)

where, s_i and s_i^c are ith elements of the measured detector response vector, \underline{s} , and the calculated detector response vector, \underline{s}^c , respectively, and M is the number of available measurements, \underline{s}^c is calculated by the forward calculations discussed above. This norm indicates the deviations of the calculations from the measurements.

Euclidean Distance Between Last Two Approximations

$$||\underline{\rho}_{k} - \underline{\rho}_{k-1}|| = \frac{1}{N} \{\sum_{i=1}^{M} (\rho_{i,k} - \rho_{i,k-1})^{2}\}^{\frac{1}{2}}$$
(5.2)

where $\rho_{i,k-1}$ and $\rho_{i,k}$ are the ith elements of the (k-1)th and kth solution approximation vectors, ρ_{k-1} and ρ_{k} ; respectively, and N is the dimension of the solution vector. This norm is useful in studying the convergence of the global problem:

Euclidean Norm of Greater Than One

$$\| \cdot | \cdot |_{p} = \frac{1}{N} \left\{ \sum_{i=1}^{N} (\rho_{i}^{+} - 1)^{2} \right\}^{\frac{1}{2}}$$
(5.3)

where ρ_1^+ 's are the elements of a solution vector ρ_1^+ , which are greater than unity. This norm indicates the deviation of a solution vector from the upper bound of the solution interval $\{0,1\}$.

Euclidean Norm of Negativity

$$||.<0|| = \frac{1}{N} \{\sum_{i=1}^{N} (p_i^{-} - 0)^2\}$$

where ρ_i 's are the negative elements of the solution vector $\underline{\rho}$. This norm indicates the deviation of the solution vector from the lower bound

of the solution interval [0,1].

An approximation \underline{P}_{K} is considered to be an acceptable solution,

a) $||\underline{s} - \underline{s}^{C}|| \leq \varepsilon_{m}$

b) $||\underline{\rho}_{k} - \underline{\rho}_{k-1}||$, $k = K, K-1, \ldots, k$ reflects a contraction mapping, within at least the previous few iterations,...

c) $||.>1|| \leq \epsilon_1$

and

if:

d) ||.<0||`<u><</u>`.c₀

where, $\varepsilon_{\rm m}$, $\varepsilon_{\rm l}$, and $\varepsilon_{\rm 0}$ are presassigned error limits. Condition (a) which indicates that $\rho_{\rm K}$ results in detector response ${\rm S}^{\rm C}$ that agrees; within error bounds, with the given S. Though this condition is necessary for the solution acceptability, it is not sufficient. That is because more than one solution can satisfy this condition. However, the fixed point theorem (the local version), as previously shown, indicates that the problem has a unique solution if the successive approximation procedure represents, locally at least, a contraction mapping. Therefore, condition (b) is to be satisfied in order to ensure that $\rho_{\rm K}$ is the sought unique acceptable solution. In addition to these mathematical justifications, the acceptable solution should exist in the interval [0,1]. This is tested by conditions (c) and (d) where some error is tolerated because of the error in S and the approximations used in constructing the matrix A. Justifications of conditions (c) and (d) is also necessary to guarantee the uniqueness of the solution; since the fixed point theorem requires that the successive approximations are mapped within the closed interval [0,1].

5.5 Solution of the "Local Problem"

has not been accepted. If a solution approximation $\frac{\rho}{-k-1}$ according to the conditions defined above, a new approximation ρ_k is sought. This new approximation is obtained by solving the "local. problem". The "local" inverse problem can be solved, as shown in Figure 10, using either a regularized direct inversion, or the Bayesian. procedure. If the user does not specify the method of solution, the regularized direct inversion method is used as long as the "global" problem is far from linearity. When the "global" problem reaches near-linearity, the Bayesian procedure is then used to obtain the next The near-linearity of the problem is solution approximation determined according to the criterion given at the end of this subsection. First, the calculational algorithms used in the regularized direct inversion method and the Bayesian procedure are discussed.







$$= \{ A = (\tau^2/N) \cdot Q^{-2} \}^{-1} = A^T = S$$
(5.4)

where $\mathbf{A}^{\mathrm{T}} = \mathbf{A}$ (transpose)

W_11

$$Q_{jj} = \min \left(\frac{S_{i} + \sigma_{i}}{A_{ij}} \right), A_{ij} \neq 0$$

N = dimension of vector. p

and τ^2 is a regularization parameter. In the RHS of the above equation, the matrix \underline{A} which is function of ρ , has been evaluated using the available approximation ρ_{k-1} , as shown before. The regularization parameter, τ^2 , is chosen to be equal to N as previously explained at the end of subsection 4.4.3. The bulk of the numerical work in Equation (5.4) is the computation of the transformation matrix:

$$\underline{\underline{H}} = \{\underline{\underline{A}}^{\mathrm{T}} \ \underline{\underline{W}} \ \underline{\underline{A}} + (\tau^{2}/N) \ \underline{\underline{Q}}^{-2}\}^{-1} \ \underline{\underline{\underline{A}}}^{\mathrm{T}} \ \underline{\underline{\underline{W}}}$$
(5.5)

The computation of this matrix may proceed formally. However, an indirect method is used. This method is based on the observation that the least-squares solution of the equations

with a weight matrix M, that is

$$= (\underline{\underline{B}}^{T} \underline{\underline{M}} \underline{\underline{B}})^{-1} \underline{\underline{B}}^{T} \underline{\underline{M}} \underline{\underline{T}}$$

is identical to Equation (5.4), where

and

$$H - M + N - n$$

$$\frac{W^{-1}}{2} \qquad Q$$

Then an orthonormalization least-squares method,

Gramm-Schmidt-Hilbert method, adopted from Reference 20, is used to obtain the transformation matrix \underline{H} ; without the necessity for forming the explicit product

 $\underline{\underline{A}} \underline{\underline{W}} \underline{\underline{A}}.$

5.5.2 The Bayesian Procedure

In the Bayesian solution of the local problem, the new approximation $\underline{\rho}_k$ is obtained from $\underline{\rho}_{k-1}$ using the relationship:

 $\rho_{j,k} = \rho_{j,k-1} \frac{\sum_{i=1}^{M} A_{ij} (s_{i} / s_{i}^{c}) / C_{j}}{i=1}$ (5.6)

where

 $s^{c} = \sum_{i=1}^{N} a_{ij} \rho_{j,k-1}$

 $C_{j} = \sum_{i=1}^{M} A_{ij}$

and

The Bayesian procedure is more efficient than the direct

inversion method from the calculation point of view; since no matrix. inversion is required. However the Bayesian procedure is ineffectives for

the zero elements of ρ_{k-1} , since according to relationship (5.6) zero

values stay zero. This problem is overcome by using a very small value, equal to the permissible error in \underline{v} , rather than the zero value. Moreover, the Bayesian procedure is a very slow method since it results in small changes in the given approximation, as will be shown in the numerical examples given later. This slowness in convergence is substituted by the efficiency of the calculational process. Nevertheless, as will be shown later in the numerical examples, in most cases using the direct inversion for the first few approximations then using the Bayesian procedure for the rest of the approximations helps in reaching the problem solution using a small number of successive approximations.

5.5.3 Determination of Near-Linearity

As explained in Section 4.6 of this Chapter, the successive approximation is considered nearly linear if

$$\left\| \left\| \underline{A}^{-1}_{k-1} \cdot \underline{A}_{k} - \underline{I} \right\| \leq \gamma$$

where $\underline{\underline{A}}_{k}$ is the response matrix constructed at the kth successive approximation, $\underline{\underline{I}}$ is the identity matrix and γ is a small positive number close to zero. The inverse of the response matrix, $\underline{\underline{A}}^{-1}$, is not explicitly evaluated at any stage of the calculation, instead the transformation matrix $\underline{\underline{H}}$ of Equation (5.5) is evaluated. Therefore, the matrix $\underline{\underline{\underline{H}}}$ is used instead of the matrix $\underline{\underline{\underline{A}}}^{-1}$ in Equation (5.7) to obtain;

$$\left| \left| \frac{H}{H_{k-1}} + \frac{\Lambda}{H_{k}} - \frac{I}{I} \right| \right| \leq \gamma$$
(5.8)

The above norm, in fact, represents the resolving error, and therefore is called the norm of the resolving error. The Euclidean norm of the resolving error is calculated using the relationship:

$$\left| \left| \begin{array}{c} H\\ =k-1 \end{array}\right|^{2} = \frac{1}{N^{2}} \\ N^{2} \\ i=1 \\ N^{2} \\ i=1 \\ k=1 \end{array}\right|^{N} \left(\begin{array}{c} \delta\\ j=1 \\ j=1 \\ j=1 \\ k=1 \\$$

where

$$if i = j$$

$$ij = \begin{cases}
 0 & if i \neq j$$

and A_{ij} and H_{ij} are elements of the matrices $\underline{\underline{A}}$ and $\underline{\underline{H}}$, respectively. The matrix $\underline{\underline{A}}$ is an MxN matrix, while the matrix $\underline{\underline{H}}$ is an NxM matrix.

The "global problem" is considered near-linear, and consquently the Bayesian procedure is used, when the Euclidean norm of the resolving error reaches an arbitrarily predetermined small fraction γ .

5.6 Adjustment of Local Solution

The solution of the "local problem" results in an approximation $\underline{\rho}_{\mathcal{V}}$ of the solution vector. The elements of the vector $\underline{\rho}_{\mathcal{V}}$ do . vector, not necessarily exist on the closed interval [0,1]. Instead some of these elements might have values greater than one and some of them might be negative. Since, the local water fraction cannot be greater than unity or negative, some adjustment of the elements outside the interval [0,1] is necessary. This adjustment is achieved by simply setting all elements of $\underline{\rho}_k$ that are greater than one to one, and those which are negative to zero. The logic behind this procedure is that, a cell for which local water fraction greater than unity is estimated is probably overfilled with water because an approximate response matrix is used in solving the response matrix. The closest physical situation to this overfilled cell is that of completely full of water; i.e. the water fraction in the cell is equal to one. A similar argument is used for the negative void fractions.

Containing the successive approximation of the solution within the interval [0,1] helps in guiding the "global problem" to convergence to the correct solution of the problem. That is because, as indicated before, the problem could have a solution outside the [0,1] interval, and the problem could converge to this solution if the successive

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approximations are not kept within [0,1]; (or the problem may not converge at all). Moreover, the local version of the Fixed Point Theorem, discussed earlier, requires for a unique solution within [0,1] that the problem maps itself to the interval [0,1]. The above adjustment procedure guides the mapping of the "global problem" to achieve self-mapping within the interval [0,1].

It should be mentioned that because of the error in the detector responses, the solution of the problem is in fact a quasi-solution. Therefore, some of the elements of the solution vector might lie slightly outside the interval [0,1] to allow for the error in detector responses. The deviation of the final solution is measured by the ||.>1|| and ||.<0|| norms defined by the relationships (5.3) and (5.4). If these norms lie within reasonable bounds the solution of the problem should be considered acceptable; as will be shown in the numerical examples given later.

5.7 Error Calculations

The resolving error associated with ρ_j , and jth element of the solution vector ρ_j is given by:

$$\operatorname{res}(\rho_{j}) = \sum_{\ell=1}^{N} | \mathbf{I}_{j\ell} - \sum_{i=1}^{M} | \mathbf{H}_{ji} - \sum_{i=1}^{M} | \mathbf{H}_{ji} - \rho_{j},$$
where H_{ji} is the (j,i) element of the transformation matrix, \underline{H} , used to construct ρ_{j} . The matrix \underline{H} is defined by Equation (5.5), if the direct inversion method is used for constructing the final solution. In the case of using the Bayesian technique for constructing the final solution, the response matrix of this solution is inverted to provide the matrix \underline{H} . Note that when the problem mapping is regularized the matrices \underline{H} and \underline{A}^{-1} may differ significantly resulting in a large resolving error. In this case a pseudo-solution of the problem is obtained, and the resolving error ceases to be meaningful.

The random error is evaluated using

std(
$$\rho_j$$
) = { $\sum_{i=1}^{M} H_{ji}\sigma_i^2$ }^b

where σ_1^2 is the variance associated with S_1 , the ith element of the detector responses vector, S_1 .

6.0 RESULTS AND DISCUSSION

In this section the inverse problem is numerically solved for three different kinds of problems. In the first kind, the detector responses are produced using the Single Scattering Approximation.

Therefore, from the view point of the inverse problem, the detector responses are error-free, except for round-off errors; since in the solution of the inverse problem the Single Scattering Approximation is implemented. The detector responses in the second kind of problems considered are obtained from a Monte Carlo simulation of the given problem. These responses differ from those calculated in the course of the inverse problem by including contributions of multiple scatterings, and by the inherent statistical error of the responses. Also, there is no homogenization in the Monte Carlo simulation; opposite to the homogenization of the two phases in each cell performed in the Single In third kind of problems, the detector Scattering Approximation. responses are measured in the laboratory by the method shown in Chapter 5. These detector responses are expected to have statistical errors less than those of the Monte Carlo experiment because of the relatively small number of source particles used in the latter. However the laboratory detector responses also include the contribution of second' and higher order scatterings, as well as the errors associated with the experimental process; namely, the errors of the unfolding the detector spectrum and the non-perfect collimation of the neutron source.

Different typical two-phase flow distributions are considered. Annular, inverted annular (core), and stratified two-phase flow regimes as well as, all water, single-phase, flow regimes are examined. In all the problems investigated, with the exception of the last one, the local void fraction in a 4x4 mesh are determined. In the last problem, an 8x8 mesh is considered, in order to show that the number of cells is not an obstacle in the phase reconstruction process.

As indicated in the previous Section, direct regularized or non-regularized inversion can be used in constructing the inverse problem. Also, one can use the Bayesian procedure, or a combination of direction inversion and Bayesian correction. These different solutionavenues are examined in detail for the first problem considered. For other problems, only the final result of the inversion process is presented.

The problems considered have, unless otherwise specified, the following specifications:

Test-section diameter = 100 mm (known)

Number of cells within

which the water (void) fraction is specified = 4x4 (preassigned) Number of given detector

responses

6.1

Problem 1 : Single Scattering Responses - Annular Flow

Nominal water distribution, as shown in Figure 11.A represents an annular flow (to be determined). Detector responses are calculated using the Single Scattering Approximation.

20

Error associated with detector responses is the round-off error associated with the calculations.

6.1.1 Solution Avenue 1:

Zeroth solution approximation of the identity vector (full of water).

Direct inversion (unregularized) is used until resolving error becomes close enough to zero, (equal to 0.005), the Bayesian procedure is implemented.

Direct inversion is used for the last approximation. 21 successive approximations are followed.



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FIG. 11.A. Nominal Water Distribution



- ||s-5"||=0.021V
- ||.-1 ||=0.0000
- ||.<0 ||=0.0003 -
- 22 Approximation

- 21 Approximations

||.>1 ||=0.0046

- FIGURE 11 Inversion of Annular Flow Regime

Details of the solution approximations and their norms are shown in the computer output listed in Appendix B.1.1 and the 22nd solution is shown in Figure 11.B.

6.1.1.1 Comments on Solution Strategy

Unregularized inversion is used since the problem is error-free (except for round-off error), consequently there is no need for regularization. When resolving error becomes close enough to zero the problem approaches linearity and the efficient Bayesian procedure can be used.

Direct inversion is used for the final approximation in order to show its consistency with the Bayesian procedure, and also to determine the errors associated with the solution, using the inverse mapping.

6.1.1.2 Comments on Results

The successive approximations show a contraction mapping, as indicated by the Euclidean norm between the iterations. The magnitude of mapping outside the closed interval [0,1], indicated by the ||.>1|| and ||.<0|| norms, decreases as the successive approximations approaches the final solution, or equivalently as the correctness

of the mapping (response matrix) is increased. Note that, since the inverse mapping is not produced during the Bayesian procedure, the inverse mapping of the last direct inversion is used to evaluate the resolving errors. This explains the increase in the norm of the resolution error through the Bayesian

.mapping.

Since a contraction mapping is achieved; the successive approximations, according to the fixed point theorem, are reaching a unique solution. This solution, within the given error norms, is the actual solution of the problem, shown in Figure 11.A. The statistical error associated with the final approximation indicates the propagation of the round-off error.

The norm of the resolving error indicates how the mapping of the final approximation differs from that of the previous approximation; in other word, how much nonlinearity still exists.

Solution Avenue 2

 Similar to Avenue 1, except that the zeroth
approximation is taken as the null vector (all void).
The results of the last few approximations are shown in Figure 12.A, while the details of the calculations are listed in Appendix B.1.2.

6.1.2.1 Comments

Comparing the final approximations that Avenue 1 and Avenue 2 led to, one can see that the choice of the zeroth approximation has no effect on the solution obtained.

Solution Avenues 1 and 2 show that the final solution of the problem does not depend on the zeroth approximation, (as long as the solution of the problem is unique within [0,1], i.e. as long as the mapping is contraction).

To further illustrate this fact, Figure 12.B shows the last few approximations of a mapping that started with a zeroth approximation of

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FIGURE 12.A Problem 1 with the Null Vector as the Zeroth Approximation (Ave.2)

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FIGURE 12.B Problem 1 with Half the Identity Vector as the the Zeroth Approximation (Ave.3)

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FIGURE 12.C Froblem 1 with the Identity Vector as the zeroth

POOR PRINT Epreuve illisible half identity vector (half full of water) and Figure 12.C shows the case for a zeroth approximation of the identity vector (used in Avenue 1).

6.1.3 Solution Avenue 3

Zeroth approximation of half identity vector (half full of water). Direct inversion (unregularized) is used throughout

the whole set of successive approximations.

The details of the successive approximations are

listed in Appendix B.1.3.

6.1.3.1 Comments

As indicated by the Euclidean norm between the approximations (iterations), the mapping is a contraction. Therefore, according to the fixed point theorem, the successive approximations are converging towards the unique solution of the problem. The negativity of the solution, indicated by the norm

of less than zero cut-off, does not tend to disappear. with increasing the number of the approximations. This is because the detector responses are not exact since they were rounded-off.

Comparing the results of this Avenue to those of other Avenues, in which the Bayesian procedure was used, one cannot see a clear advantage of the use of the direct inversion all the way throughout the mapping. In fact the Bayesian procedure is preferred since no matrix inversion is required and no negativity results in the solution. However, the Bayesian correction is only efficient when the problem is nearly linear, as indicated in the following Avenue.

6.1.4 Solution Avenue 4

Zeroth approximation of half identity vector The Bayesian procedure is used throughout the mapping Twenty approximations using the Bayesian correction, followed by an approximation using the direct inversion, are shown in Appendix B.1.4. Direct inversion was used, for sake of error calculations, as

well as for comparison.

6.1.4.1 Comments

Though the Bayesian procedure leads to a contraction mapping, consequently driving the problem to the right solution, the mapping is very slow as compared to .those of the previous avenues. When the direct inversion was used just once, the

resulting approximation came much closer to the solution, if compared to the approximations resulting

from the Bayesian procedure.

 This indicates that the Bayesian procedure is not as effective as the direct inversion in handling the non-linearity of the problem. Though, as shown before, when the problem bacomes near-linear the Bayesian procedure can be used efficiently.

6.1.5 General Comments on Problem 1

The four solution strategies used to solve this problem showed the following:

The selection of the zeroth approximation does not

affect the solution of the problem.

The use of the direct inversion for the first few approximations is desirable, since it brings the

problem quickly to near-linearity.

The use of the Bayesian procedure is recommended when the problem reaches near-linearity, after a few direct inversions. At this point the costly direct inversions is not necessary, since the Bayesian procedure can handle the problem efficiently.

Problem 2: Monte Carlo Responses - Annular Flow

Nominal water distribution, as shown in Figure 11.A, represents an annular flow (to be determined). Detector responses are obtained from a Monte Carlo experiment simulating the scattering problem. These responses opposite to those of the Single Scattering Approximation used in Problem 1, contain the contribution of higher order scatterings, as well as, the inherent randomness of the Monte Carlo experiment Errors associated with the detector responses are taken as the standard deviations associated with the Monte Carlo estimates of the responses.

6.2.1 Solution Avenue

As indicated in solving (the error-free) Problem 1, it is desirable to use direct inversion in the first few approximations until the problem becomes closer to linearity, then the Bayesian procedure can be used. However, in the present problem an error of about 5% is associated with the detector responses. This requires, as explained previously, the regularization of the inverse mapping; in order to accommodate this error. As shown in Section 4,4.4 the use of a regularization parameter, τ^2 , which is equal to the number of unknowns; in this case equal 16, is adequate. Further elaboration on the choice of τ^2 is given in Section 7.1.

The estimated water distribution after 21 approximations is shown in Figure 11.C, together with the associated errors in the estimated water fractions. The details of the problem are listed in Appendix B.2. The figure shows clearly that the unfolding process reproduces the nominal annular flow distribution.

6.2.2 Comments

The deviation of the estimated water fractions from the nominal values can be explained as follows. The water fractions of cells 5,8,9

and 12 are higher than their nominal value (0.6875) because of the homogenization of the water void mixture through the cell. The water layer in each of these cells has to spread to cover the whole cell but with lower density and consequently resulting in a lower probability of neutron scattering. In order to compensate for this loss of neutron scattering, the water density has to be increased to result in the same detector contributions as the original distribution. Homogenization of cells 2 and 14 does not seem to result in too much deviation of their estimated water fractions from the nominal values. This can be explained by the fact that source neutrons passing through the original woid have an average flight distance larger than those passing through the original void of cells 5,8,9 and 12. Therefore, while in homogenizing the latter cells more water is to be added in order to result in the same contribution to detector responses as the original cells, the homogenization of cells 2, 3, 14 and 15 does not require as much water addition since more source neutrons are scattered. The lower water fraction estimated in cells 3 and 15 can be explained by the fact that neutrons scattered in these cells have to pass through water rich cells on their way towards the detector. This increases the probability of multiple scattering and consequently results in more removal of contributing neutrons and in turn underestimation of the water fractions in the cells. The very low water fractions that appear in the core cells 6,7,10,11 are due to the multiscattered neutrons that reach the

detectors. Since detectors are supposed to record only single scatterings, multiscattered neutrons reaching the detectors result in false water positions.

The error associated with the estimated water fraction varies with the position and size of the cell. For instance, the cells that are exposed directly to the neutron beam tend to have higher error ds compared to their counterparts downstream of the beam. That is because neutron scattered by these cells have to cross many other cells before reaching the detectors. Therefore, errors in the detector responses

propagate to these cells more strongly than to cells that are not screened by other cells. Edge cells tend to have more errors in their estimated water fractions than other cells, because of their smaller size. That is, their contributions to the detector responses is not as large as other cells and consequently they tend to be affected more by errors in the responses.

6.3 Problem 3: Single Scattering Responses -

Inverted Annular Flow

Nominal water distribution, as shown in Figure 13.A, represents an inverted annular flow (to be determined).



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FIG. 13.A Nominal Water Fraction

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000	0'3314 +.004	0,3126 +.001	0,0000	0:0000	0,1158	0.2893 +.0056	0.0

(G. 13.)B	Water Distribution Resulting FIG. 13.C from Error-Free Detector Responses (Problem 3 & Ave.1)	Hater Distribution Result from Monte Carlo Detector + Responses (Problem 4)
	s-s ^C =0.005	s-s ^c =2.000
	.>1 =0.0003	_>1 =0.0330
	11. <0 1=0.0003	' .<0 =0.0130 -
	21 Approximations	20 Approximations

21 Approximations

FIGURE 13: Inversion of Inverted Annular Flow Regime

Detector responses are evaluated using the Single ' Scattering Approximation and the only error in the detector responses is the round-off error.

6.3.1.1 Solution Avenue

Zeroth approximation of identity vector (full of water).

Direct inversion (unregularized) is used till .

resolving error of 0.005 or less is reached, then the

Bayesian procedure is utilized.

21 successive approximations are followed. Results of the 21st approximation are shown in

Figure 13.B.

6.3.2 Comments

Mapping is contraction all over the 21 successive approximations. This indicates that the problem is converging towards the correct unique solution and the 21st approximation is a valid approximation of the actual solution within the given error bounds. The solution strategy adopted here leads to a successful solution; however, as indicated in Problem 1, this is not the only avenue that can be used.

Problem 4: Monte Carlo Responses - Inverted Annular Flow.

Nominal Water distribution, as shown in Figure 13.A, represents an inverted annular flow (to be determined). Detector responses are obtained from a Monte Carlo experiment simulating the scattering problem.

6.4.1 Solution Avenue

6.4

Regularized direct inversion of first five approximations, followed by Bayesian Correction, is 'the strategy of the successive approximations. Zeroth approximation of the identity vector is used. The estimated water fraction after 20 approximations

is shown in Figure 13.C.

6.4.2 Comments

The nominal water fraction of the problem is clearly

reconstructed. The errors associated with the inversion process, namely the homogenization of the two phases and the assumption of single

scatterings, affect the estimation of the water fraction in the circumferential cells. Homogenization in this water distribution,

opposite to the case of annular flow (Problem 2), tends to result in underestimation of the water content of the homogenized cells. That is because in the present case a small portion of water is distributed over a much larger space. This increases the probability of scattering of source neutrons in the cell, and consequently their contribution to the detector responses. Therefore, an underestimation of the cell water content is necessary to balance this increase in scattering.

The severe underestimation of water fractions of cells 2 and 14 is caused by the removal, by rescattering, of neutrons scattered at these cells and directed to the detectors on the far side of the cell. These cells are hidden from these detectors by the water in the core. Problem 5: Single Scattering Responses - Stratified Flow

Nominal water distribution, as shown in Figure 14.A, represents a stratified flow (to be reconstructed). Detector responses are evaluated using the Single Scattering Approximation and the only error in the detector responses in the round-off error.

6.5.1 Solution Avenue

. 5

Zeroth approximation of identity vector (full of water).

Direct inversion (unregularized) is used until resolving error of 0.005 or less is reached, then the

Bayesian procedure is applied.

21 successive approximations are followed. Results of the 21st approximation are shown in Figure 14.B.

•

6.5.2 Comments

The problem followed a contraction mapping reaching the correct

solution.

Problem 6: Monte Carlo Results - Stratified Flow

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- Nominal water distribution, as shown in Figure 14.A, represents a stratified flow (to be reconstructed).
- Detector responses are obtained from a Monte Carlo experiment simulating the scattering of neutrons.

6.6.1 Solution Avenue

6.6

Regularized direct inversion of first five approximations, then the Bayesian procedure is applied. Zeroth approximation of the identity vector is imposed. The estimated water fraction at the 20th approximation is shown in Figure 14.C.

6.6.2 Comments

The successive approximation process followed a contraction mapping, which means that the final approximation obtained is a true approximation of the nominal solution. However, as clear from Figure 14.C, this approximation deviates from the nominal solution especially at the edges of the water phase. This deviation is mainly caused by the rescattering of source neutrons. This rescattering









prevents neutrons scattered in cells that are screened from the detector. by other cells from reaching these detectors, and this consequently leads to an underestimation of the water fraction in these cells. Because of this underestimation, the water distribution in the affected cells drops below its nominal value of unity, and therefore has to be homogenized together with the void phase. This homogenization makes the approximated distribution deviate further from the nominal distribution. The large error associated with the estimation of the water fraction of cell l is because this cell does not contribute significantly to detector responses because it is screened by many other cells and because of its small size. This reduces the certainty of the estimation of the water fraction in the cell.

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6.7 Problem 7: Single Scattering Responses - All Water Flow.

Nominally the test section is full of water (single-phase), as shown in Figure 15.A. Detector responses are evaluated using the Single Scattering Approximation and the only error in the detector responses is the round-off error.





C



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Solution Avenue 1

Zeroth approximation of half the identity vector. Direct inversion till resolving error norm is less than 0.005, then the Bayesian procedure is applied.

6.7.1.1 Comments

.7.1

Under this solution strategy the problem is converging to a solution, as shown in Figure 16, outside the interval 10,11. However, this solution is not an acceptable solution of the problem, since the resulting detector responses differ significantly from the given responses, as indicated by the norm $||S-S^{C}||$ which indicates an error of 4.85% versus an error in the given detector responses of no more than 0.01% (round-off error). Also the greater than norm, ||.>1|| and the

333.

less than norm $||\cdot < 0||$ are large, as can one see by comparing the density before cut-off to that after adjustment. Therefore, all the three norms $||\underline{s}-\underline{s}^{c}||$, $||\cdot > 1||$, $||\cdot < 0||$ indicate that the problem has converged to a solution outside [0,1], that is the unique solution inside [0,1], which we are seeking, has not been reached.



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FIGURE 16 Problem 7 Converging outside [0,1]

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FIG. 16 (Continuation)

This behaviour of the successive approximation can be explained by the fact that the actual solution of the problem lies exactly on the upper edge of the interval [0,1] and the direct inversion tends always to

map the approximation outside, this interval; consequently violating the conditions of the Fixed Point Theorem for a unique solution within

[0,1]. The convergence of the problem to a solution outside [0,1] is not surprising because of the exponential nature of the response matrix; see the one-dimensional cases discussed earlier in this Chapter. Apparently the solution strategy adopted here failed in navigating the successive approximations within the interval [0,1]. Therefore, one has to approach the problem differently. This is a good example of introducing aesthetics in solving the inverse problem, that is, one has to examine carefully the given solution against the physics of the-problem.

6.7.2 Solution Avenue 2

Zeroth approximation of half the identity vector. Bayesian procedure is employed all the way through the problem.

The 20th approximation is shown in Figure 15.B, while the . last few approximations are shown in Figure 17.

18 DENSITY (BAYESIAN)

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FIGURE 17 Problem 7 Converging inside [0,1]

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DALCHLATED DETECTOR RESPONSES

	¥		PERPANSE
1.	1255E+02	=,7550E+01	- . 34715+64
2 .	1255F+02	=,7550E+01	5590E-04
3	1405E+05	-,5780E+01	4011F+04
4	1402F+02	-,5780F+01	35535+04
5	1405E+05	57X0E+01	5014E-04
6	1578E+02	- 4020E+01	*1-3555E-U4
7	1578E402	= 4020E+01	3614E-04
1 1 1	1573E+02	- 4050E+01	1296E-04
9 -	1755E+02	- 2550E+01	2020E-04
50	1755E+02	= 2550E+01	2804E-04
11 🦾	1755E+02	1255E+02	2050E+04
15	\$755F+02	+1255E+02	ZKOAE-04
13	1578E+02	1402E+05	32226=04.
4	1578E+02	14025+02	TALAE
15	15786+02	1402E+02	12951-04
16	1405E+05	15T8E+02	
17	14056+05	1578E+02	- 577 104114
18	1.405E+05	15785+02	74 31 C 04
19	1522+05	+1755F+72	5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
20 .	1755F+02	1755E+02 .	• • • • • • • • • • • • •

FIG.⁴17 (Continuation)

6.7.2.1 Comments

The Bayesian procedure, opposite to direct inversion, leads the problem to the nominal solution. The conditions of the Fixed Point Theorem, that is the successive approximations constitute a contraction mapping within the interval [0,1], are met; consequently leading to the unique solution within this interval. In the last approximation, direct inversion was used, feedlting in a solution very close to the actual solution. This indicates that, direct inversion is still capable of mapping the problem to its correct solution within [0,1], if the zeroth approximation is chosen close to the actual solution to prevent the mappings from going outside the [0,1] interval.

.8 Problem 8: Monte Carlo Résponses - All Water Flow

Nominal water distribution, as shown in Figure 15.A,
represents a one-phase flow of water (to be reconstructed).
Detector responses are obtained from a Monte Carlo
experiment simulating neutron scattering.

6.8.1 Solution Avenue

Regularized direct inversion is used for the first five approximations, followed by the Bayesian procedure. Zeroth approximation of half the identity vector

The estimated water fraction at the 20th approximation is shown in Figure 15.C.

6.8.2 Comments

The solution strategy of using direct inversion for the first few approximations succeeded in this case, though it failed for the error-free problem. This is because a regularized inversion is used. Regularization expands the mapping domain and tolerates the errors in the responses. In the case of error-free responses, regularization cannot be used, since the magnitude of uncertainty, upon which regularization is based, is zero (or near zero because of round-off error).

The estimated water fraction distribution (Figure 15.C) shows deviations from the nominal inventory at some of the edge cells. This is due to the removal of scoring neutrons from the detector responses by rescattering. The effect of neutron rescattering appears in these edge cells because the probability of rescattering is higher in these cells (larger rescattering path length). The errors associated with the estimated water fractions are higher in the centre of the object because the water fractions in these cells are constructed using greater number of detector responses, (more detectors observe this region), this causes an accumulation of errors.

Problem 9: Laboratory Measured Responses

In this problem the detector responses are taken from the laboratory measurements of the energy spectra of neutrons scattered by Test Séction A, as reported in Chapter 5. Test Section A, as shown in Figure 18.A, is a pipe of 50.9 mm outside diameter and 46.8 mm fiside, diameter filled with water. The pulse height spectra produced by the scattered neutrons in each of the four scintillation detectors positioned at one side of the test section were unfolded to obtain the energy spectrum in energy bins of 1 MeV covering the range . 2 to 14 MeV. Discarding energy bins below 3 MeV to reduce the effect of scattering and above 12 MeV to exclude neutron scattering by the oxygen in the water and the aluminum of the pipe wall, and excluding energy groups that do not correspond to angles of scattering from the test section, 18 effective detector responses are obtained. In contrast to the previous

theoretically simulated problems, the detectors used are not point detectors. A cylindrical detector, 125 mm in diaméter and 125 mm in length, was used in these laboratory experiments. Neutrons incident on


||.< 0 |=0.0000

10 Approximations

FIGURE 18 Inversion of Experimental Sesults (Test Section A)

||.< 0 ||,0.0000

10 Approximations

the detector from different angles do not have an equal chance of interaction with the detector material, since some have a higher path in the detector than others. This was taken care of in the calculations by adjusting the detector aperture for different angles of exposure.

Figure 18.B shows the water fraction distribution obtained from the inversion of the 18 detector responses. The details of the successive approximations are shown in Appendix B.3.2, (Appendix B.3.1 shows the phase reconstruction for an error-free problem). As shown in Figure 18.B the water fraction is underestimated for the cells that are closer to both the detectors and the incident neutron beam. This may be explained by the fact that these cells contribute to the detectors more than other cells since they are closer to the detectors and are exposed directly to the neutron source. Therefore, these cells tend to carry the effect of the deviation of the measured detector responses from those calculated by the Single Scattering Approximation used in the inversion process. The contribution of multiple scattering to this deviation is no more than 5%, as shown in Table 1 of Chapter 3. The deviation 1s not due to the degree of over-determinancy of the problem, since it also appears in the 3x3 mesh problem (Figure 18.C). This shows that the deviation is caused by errors inherent in the measurements; which can be due to the management of the experimental setup or the procedure used for unfolding

the neutron spectrum. These errors are discussed in Chapter 5.

In obtaining the water distribution in the present Problem, the Bayesian procedure was used for all the successive approximations. Attempts to use regularized or non-regularized direct inversion resulted in unacceptable solutions, (as shown at the end of Appendices B.3.2). This is because the problem is mapped into a region which is very close to the upper bound of the interval [0,1], using detector responses with relatively small statistical errors (about 2% errors). Therefore, a mapping that introduces very small changes is required in order to navigate the successive approximations inside the interval [0,1], consequently satisfying the Fixed Point Theorem. The Bayesian procedure provided the required mapping. Because of the use of the Bayesian progedure all the way through the successive approximations, no error.

estimation is given. The error estimation requires either the construction of the inverse mapping, or the propagation of the error through the successive approximations. Since the construction of the former was not possible and no error propagation algorithm for the Bayesian process is available in the present version of the water construction program, no error estimation is provided.

6.10 Problem 10: 8x8 Mesh

In all the problems considered above, the water distribution was determined for 16 cells of a 4x4 mesh constructed inside the test section. However, there is no reason to restrict the problem to this

particular number of cells. The water distribution for 60 cells constructed in an Bx8 mesh is shown in Appendix B.4. The four cells of the mesh that lie completely outside the test section are excluded, since they contain no flow.

The success of the inverse algorithm in handling any number of cells was expected, since the algorithm is based on a general n-dimensional spaced with no restriction on n. Indeed, one does not have to restrict himself to the square cell structure use in the present study. One can easily implement any other kind of structure.

7.0 INVESTIGATION OF PARAMETERS AFFECTING THE SENT PROBLEM

In the SENT inverse problem different parameters that can influence the problem solution are encountered. Some of these parameters may affect the method of solution; such as the regularization parameter, τ^2 , the number of available measurements, the number and configuration of cells within which the phase fraction is to be determined, the arrangement with which the detectors are located. Other parameters affect the physics of the problem; such as the diameter of the test. section and its wall material and thickness, the neutron source geometry, energy and angular distribution, the detector geometry, as well as the distance between the source and the test section and that between the test section and the detectors. Most of these parameters are considered before and some are considered in the next Chapter. However, in view of the numerical examples given in the previous Section, the effect of these parameters on the SENT problem are examined in the following subsection.

Regularization Parameter

Based on a one-dimensional analysis (Section 4.4.4) a value of the regularization parameter, τ^2 , equal to the number of unknowns, N, was recommended. It was shown that this value expands the solution domain by a volume equivalent to one fractional standard deviation of the measurements. To further illustrate this fact, Problems 2,4,6-and 8 were solved using different values of τ^2 . The results are summarized in Table 1 to 4. In addition to the norms defined in Section 5.4, the norm $|\cdot| \rho^{S} - \rho^{a}$. $|\cdot|^{4+1}$ appears in the Tables. This norm is defined as:

$$|| \rho^{s} - \rho^{a} || = \frac{1}{N} \left\{ \sum_{i=1}^{N} (\rho_{i}^{s} - \rho_{i}^{a})^{2} \right\}^{l_{2}}$$

where ρ_1^s refers to the ith water fraction as estimated by the solution of the inverse problem, while ρ_1^a is the actual water fraction (already known in the problems considered). This norm provides a measure of the closeness of the estimated pseudo-solution to the actual solution of the problem. As one can see in Tables 1 to 4, the estimated solution closest to the actual solution is always obtained when a value of $\tau^2 = N = 16$ is used. This confirms the conclusion drawn in

Section 4.4.4 that it is appropriate to expand the solution domain by a value proportional, to the size of the errors. Since the error is estimated by one standard deviation of the measurement, the solution domain is expanded by a value equivalent to one fractional standard deviation by choosing τ^2 equal to N. In other words it allows the error in <u>S</u> to propagate to <u>p</u> with the same magnitude.

To the effect of the value of the regularization parameter can be examined from a different point of view. Consider Equation (5.4) (repeated here);

 $\varrho = \{\underline{\underline{A}}^{\mathrm{T}} \underline{\underline{W}} \underline{\underline{A}} + (\tau^{2}/N) \underline{\underline{Q}}^{-2}\}^{-1} \underline{\underline{A}}^{\mathrm{T}} \underline{\underline{W}} \underline{\underline{S}}$

S

Table 1: Regularization of Problem 2

Norm/ 7 ² 0	À 8	16	32	.64.	128
$ \rho^{s}-\rho^{a} $ 0.1056	0.0936 0.0595	0.0332	0.0481	0.0655	0.0789
$ \underline{s} - \underline{s}^{c} = 0.0277$	0.0165 0.0183	0.0183	0.0191	0.0351	0.0598
.>1 0.0618	0.0307 0.0	0.0461	0.0411	0.0761	0.0724
.<0 0.1885	0.0456 0.0183	0.0175	0.0094	0.0	0.0

Table 2: Regularization of Problem 4

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	Norm 72	0	4	. 8	16	32	64	128
.	$ \underline{\rho}^{s}-\underline{\rho}^{a} $	0.0298	0:0214	0.0206	0.0199	0.0335	0.0479	0.0464
	<u> s</u> -s ^c	0.0226	0.0216	0.0209	0.0201	0.0286	0.0286	0.0194
	.>1	0,0937	0.0668	0.0476	0.0333	0.0240	0.0179	0.0709
	. < 0	0.2601	0.1796	0.0797	0.0129	0.0031	0.0	0.0
								•

Table 3: Regularization of Problem 6

		•		Pr	• • • •
Norm $/\tau^2$ 0	8	16	32	64	128
$ \rho^{s}-\rho^{a} $ 0.0959 0.0664	0.0562	0.0205	0.0359	0.0368	0.0404
$ \underline{s} - \underline{s}^{c} $ 0.0608 0.1547	0.0144	0.0084	• 0.0189	0.0287	0.0476
.>1 0.1761 0.0254	0.0172	0.0056	0.00749	0.1141	0.1216
.<0 0.2883 0.0341	0.0021	0.0049	0.0008	0.0	0.0

. Table 4: Regularization of Problem 8

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·					<u> </u>		
Norm τ^2	0	4	8	16	32	64	128
$\left \left \underline{\rho}^{s}-\underline{\rho}^{a}\right \right $ 0	.1365	0.1275	0.0231	0.0121	0.0216	0.0112	0.0246
] <u>s</u> -s ^c] 0	.0516	0.049,6	0.0125	0.0117	0.0116	0.0122	0.0118
.>1 0	.2448	0.1780.	0.1357	0.1354	0.1367	0.0360	0.1135
.<0 0	.0692	0.0670	0.0	.0.0	0.0	0.0	0.0

- - -

Recalling that the matrix \underline{Q}^2 is a diagonal matrix whose elements are the squares of weak upper bounds of the solution, one can see from the above equation with τ^2 =N, the problem mapping is allowed to expand the solution domain up to one (weak) upper bound of the solution. It then seems reasonable not to allow the solution to exceed its upper bound.

Considering the different norms in Tables 1 to 4, one can see that for τ^2 =16, the norm $\lceil [\underline{S} - \underline{S}^C \rceil \rceil$ (S is the vector of the given detector responses and \underline{S}^C is that of the calculated response corresponding to $\underline{\rho}^S$), tends to be largest for both small and large values of τ . This is because the resulting solutions $\underline{\rho}^S$ for these values are far from the actual value $\underline{\rho}^a$ (upon which S depends). The cut-off norms $| \cdot \rangle 1 | \cdot |$ and $| \cdot \langle 0 | \cdot |$ tend to be largest for small values of τ^2 . One can explain that by the fact that for small values of τ^2 the solution is allowed to expand far beyond its upper bound; consequently resulting in solutions outside the interval [0,1].

One last note, in all the problems reported in Tables 1 to 4 the successive approximations represented contraction mappings with reasonable values of the convergence norm. This indicates that the value of τ^2 does not affect the Fixed Point Theorem, upon which the solution procedure is based. In conclusion, value of the regularization parameter equal to the number of unknowns seems to be the most appropriate choice.

7.2 Number of Measurements

A basic question one would tend to ask is: how many detector responses should be available in order to obtain an 'estimation of the phase volume fraction in N cells? A simple answer to this question is that at least N independent measurements must be provided. If more than N measurements are available the problem is overdetermined, and the reconstructed phase distribution is based on an average of the redundant information. The effect of this averaging is to reduce the effect of errors (as if N measurements with better statistics had been used). If fewer than N independent measurements are available the problem is underdetermined, and no unique solution of the problem is possible. However, if some "a priori" information regarding the phase distribution is available, such as circular symmetry, a solution of the problem could be obtained. No attempt has been made in the present study to solve an underdetermined problem. Further development should consider the study of reconstructing the phase distribution supported by "a priori" flow

regime conditions such as flow regime symmetry, stratification,, or homogenity. For flow regime known to be homogeneous, a smooth solution of minimum gradient of local phase fraction could be obtained from an underdetermined problem by applying the smoothness condition in a fashion similar to that of Phillips⁽¹²⁾, i.e., minimizing the second derivative of the phase fraction. The approach of Tewarson⁽¹³⁾ can also be used to implement "a priori" phase distribution into the solution of underdetermined problems using appropriate weighting matrices.

Supplementary "a priori" flow regime information can also be used in an overdetermined or fully determined problem to reduce the effect of measurement errors in the phase reconstruction process. Implementing "a priori" flow regime information reduces, the number of required measurements, and consequently reduces the cost of measurements.

Another note regarding the type of available information: In the present work only neutron scattered fluences were used as the measurement data. Neutron uncollided fluences were not considered since the detector used for measuring an uncollided fluence provides only a single piece of information, while a detector measuring the scattering fluence provides multiple pieces of information via the neutron. spectrum. Use of uncollided fluence requires modification of the

structure of the corresponding elements in the response matrix A.

Number and Configuration of Cells

7.3

The solution algorithm of the SENT problem as outlined in this Chapter can handle any number of cells, provided of course that an

353.

adequate number of measurements is a lable. For the sake of illustration, the solution of the inverse problem for an 8x8 mesh with 60

unknowns (discarding the edge cells lying Outside the test section) was shown in Problem 10. There is no reason to believe that the solution algorithm would fail if larger number of unknowns were considered. However, care should be taken in handling the cells lying outside the test section. These cells must be excluded, since the Single Scattering Algorithm does not trace neutron scattering from these cells; consequently resulting in singularity of the matrix A since the columns A corresponding to these cells are not assigned non-zero values.

One also should note that if all the detector responses are equal to zero, the inverse problem then becomes trivial since this means that the test section is completely empty. However, if all the detector responses are equal to zero except one, a solution of the inverse problem is still possible. This is because the matrix $\underline{A} (\underline{\rho})$, even in this case, is not necessarily singular. One can explain that by the fact that a zero detector response is only obtained from cells of water fraction zero. On the other hand, a cell i of zero water fraction does not necessarily result in a corresponding element A_{ji} in the matrix \underline{A}_{ji} unless it is not seen by the detector j. That is A_{ji} is zero if cell i is not seen by detector.j, but A_{ji} is not equal to zero if S_{j} , (the detector response j), is zero because the exponential nature of the matrix \underline{A} enables one to get around the singularity problem.

Regarding the cell configuration, square cells were considered " in the present analysis, (excluding parts of the cell that lies outside the test section). However, there is no reason to restrict the configuration to square cells. Any other cell geometry and cells sizes can be used. The Single Scattering Approximation algorithm has to be adjusted to accommodate any cell configuration different from the square configuration. The only restriction imposed on the cell size is that the cell width has to satisfy conditions (4.32) of contraction mapping. That is, for 14 MeV source neutrons and 3.5 MeV minimum detection energy, the cell width has to be less than 39.2 mm.

7.4 Detector Arrangement

The only condition imposed on arranging the neutron detectors is that they should be positioned such that they observe forward neutron scatterings. Other than that there are no other restrictions. In

principle one detector with a resolution good enough to provide the required number of energy-dependent fluences is required. Otherwise, more than one detector can be used.

Two restrictions are imposed on the energy of detection. The first is that the energy has to be less than (15/17) of the source energy, to avoid detection of scattering by oxygen. The second is that to avoid detecting multi-scattered neutrons, low energy neutrons should be discarded (one quarter of the source energy seems to be a reasonable detection energy cut-off as discussed in Chapter 3).

The considerations regarding detector size and geometry are discussed in Chapter 5.

7.5 Test Section Size

AB shown in Chapter 3, the diameter of the test section should not exceed 100 mm (equivalent to one-free-path of 14 MeV neutrons), to reduce contribution of multiple scattering. Of course, if a source of energy different from 14 MeV is used, one has to make sure that the test "section diameter does not exceed one mean-free-path of source neutrons.

If it is known that the global void fraction in the test section does not

exceed a specific limit, larger test sections can be used. For example, for a maximum global void fraction of 0.5, a test section of 200 mm diameter can be used.

7.6 Test Section Walls

The effect of test section walls can be éasily considered provided that its thickness does not exceed one-mean-free path of source neutrons in the wall material (about 44 mm for stainless steel and 96 mm for aluminum). For this thickness and for wall material of atomic number larger than that of oxygen(16), single scatterings in the wall would result in neutron scattering energies larger than (15/17) of the source energy. Since energies larger than this energy are discarded, as indicated earlier, the test section walls do not affect the detector responses used in the reconstruction process. The removal of scattered neutrons by the test section wall can be easily accounted for by introducing the neutron removal probability of the wall into thecalculation of the detector responses.

7.7 Neutron Source

A monoenergetic'l4 MeV neutron beam covering entirely the exposed surface of the test section is used in the present study. Using a source of different energy is possible, provided that the diameter of the test section does not exceed one-mean-free path of source neutrons, and a suitable detector is available to measure the spectrum of the scattered neutrons in the range corresponding to the source energy. In principle, multi-energetic sources can be also used provided that the conditions mentioned earlier are met for the lowest energy component. The use of multi-energetic sources would require significant modifications in the problem in order to accommodate the different energy components of the neutron source. This could be the subject of another study.

Although a neutron beam was used in the present study, it is possible to use isotropic sources or sources with known angular distributions. The problem has to be modified to take into account the source angular distribution. This is another, area for further development.

The width of the neutron beam in the present study is taken to be equal to the diameter of the test section. This beam width can be produced from an isotropic source using a collimator whose exit width is equal to the diameter of the test section, as shown in Chapter 5. If a neutron beam of smaller width is used, one would not be able to determine the phase fraction cells that are not fully exposed to the neutron beam; consequently resulting in zero corresponding elements in the response matrix that can cause singularity of the matrix. The beam height (in the direction perpendicular to the test) determines the height of the test section. Therefore the estimated local Phase fraction represents an average value over the volume defined by the cell area and the beam height. Again, the beam height can be controlled by the height of the collimator exit.

The beam profile used in the present study was assumed to be. uniform all over the exposed test section. Since producing a uniform beam profile is not an easy task, the fluence distribution of the beam can be measured and used as weighting function in the single scattering calculations. Use of a beam of width larger than the test section could be helpful in providing a reasonably uniform beam profile over the test secton; since the rapid fluence change at the edges of the beam profile can be avoided.

7.8 Relative Distances

The distance between the neutron beam (collimator exit) and the test section and that between the test section and the detector are generally governed by practical and radiation shielding considerations. However, one should try to minimize the distance between the neutron beam and the test section in order to fully utilize the source neutrons, (source bidence, decreases with distance). In determining the position of

a detector one should aim at minimizing the distance between the test section and the detector, in order to maximize the count rate. Also, one has to choose the detector position such that most of the recorded energies correspond to angles of scattering directed towards the test section, in order to make maximum use of the neutron spectrum in the reconstruction process. Moreover, one should position the detector at locations where the signal to background ratio is maximum.

8.0 CONCLUSIONS

This Chapter showed that the inverse problem of reconstructing the phase distribution from the scattered neutron fluences is solvable for different flow regimes and with different types of data: The abstract analysis given in Section 3 laid out the foundation for adressing the different aspects of the problem. In Section 4, these aspects were applied to our specific problem, consequently paving the way for the numerical algorithm (Section 5) used for solving the problem. The numerical examples discussed in Section 6 illustrated the problems encountered in reconstructing the phase distribution. The different parameters affecting the problem were examined, (in Section 7), showing the capabilities and limitations of the technique. The numerical examples showed that the water distribution always can be reconstructured form the error-free data. This may sound trivial however error-free problems can be considered as a test of the mathematical procedure used and as a check on the structure of the .

computer program. As shown in Problems 1,3,5 and 7, these objectives. have been achieved. Problems 2,4,6,8 and 9 showed that data with errors due to statistical fluctuations, can also provide a reasonable estimation of the phase distribution. The data in these problems were either provided by a Monte Carlo simulation or from actual laboratory measurements, and the number of available detector responses always exceeded the number of unknowns, i.e. the problem was overdetermined.

The parametric review indicated that there is no limitation on the number or structure of the cells within which the phase distribution can be detrmined. It also indicates that the test section walls can be accounted for, provided that the wall thickness does not exceed one-mean-free path of a source neutron in the wall material.

In conclusion, the inverse boblem of reconstructing the phase distribution from neutron scattering fluences is solvable as proved mathematically and numerically in this Chapter. The next step of development will be to automate the solution procedure such that the proper solution strategy can be selected with minimum external interference. An on-line solution is certainly a challenging task. Considering the effect of neutron multiple scattering is another area of development that can lead to allowing the use of larger sizes of test section. Applying the technique to different types of two-phase flows, other than the steam-water flow considered here, is another development task. The generalization of the technique to include multi-energetic and multi-directional neutron sources is also a challenging development

problem. Developing a reconstruction algorithm for underdetermined problems supported by "a priori" phase distribution information will be useful for routine work where the phase distribution does not

significantly change.

CHAPTER 5

MEASUREMENT OF SCATTERED NEUTRON.

ENERGY SPECTRUM

1.0 INTRODUCTION

As indicated in the previous chapter, the reconstruction of a local void fraction distribution requires the measurement of the neutron energy spectrum of the scattered (fast) neutrons. In this Chapter, a system for measuring the neutron spectrum is designed and testëd. A layout of the required measurements is first described, then the desired specifications of the detector to be used in such measurements are defined. A review of the existing fast neutron detectors and spectrometers is presented in Section 4. Based on this review, organic scintillation detectors are selected and the main characteristics of

these detectors are presented. ...

Since there are many types of organic scintillators , one has to choose from among them the type that best meets the specified

requirements. Also, one has to choose the appropriate dectector size and geometry. These design considerations are presented in Section 7 and the procedure used for the construction of the chosen detector is also described. The capability of the constructed detector to measure neutron fluences at different energies is demonstrated by calculating its efficiency as a function of the incident neutron energy. The method used for calculating the detection efficiency is outlined at the end of the Section.

The light produced by the scintillation process in the detector is collected and directed towards a photomultiplier tube, where its intensity is measured. There are different methods for light collection; these methods are reviewed and one of them is chosen for application in our detection system. The characteristics of the photomiltiplier tube used for the measurement of the light collected has to match those of the detector. Therefore, care has to be taken in choosing the appropriate tube; as demonstrated in Section 9.

Organic scintillators detect, in addition to neutrons, gamma-rays: Since gamma-rays usually accompany neutrons, discrimination , against gamma-rays is required to ensure that only neutrons are recorded. Discrimination techniques make use of the difference in shape of neutron and gamma-ray pulses produced by organic scintillation. The different available pulse shape discrimination methods are reviewed in Section 10. The chosen pulse shape discrimination method is also presented in this Section; together with the electronic circuit required to record the neutron spectrum. The procedures required for adjusting and calibrating the proposed detection system are outlined in Section 11: These procedures are required to ensure the adequate behaviour of the system.

The detection system provides the neutron energy spectrum in . terms of pulse height spectrum. Therefore, an unfolding procedure is required in order to obtain the neutron energy spectrum. The available unfolding methods are reviewed in Section 12, and a differential method is chosen and described in this Section.

Once the detection system was designed and tested, an experiment was set up to measure the energy spectrum of neutrons scattered by water samples exposed to a.14 MeV neutron beam. In Section 13, this experiment ℓ is described and the results obtained are presented. These results are then compared to those obtained from a Monte Carlo simulation. This comparison has a twofold purpose. Since the capability of the Monte Carlo method to simulate laboratory experiments has been demonstrated in Chapter 3, the comparison verifies the correctness of the experimental procedures; especially the calibration and unfolding of the neutron spectrum. On the other hand, hence the measurement method has proved to be capable of measuring the known intensity and energy of the beam neutrons, as shown in Subsection 13.3, the comparison of the Monte Carlo results to those obtained in the experiment verifies the accuracy of the Monte Carlo simulations.

The conclusions drawn from this experimental work are summarized in Section 14. However, the author would like to emphasize that no original work has been presented in this Chapter. All the measurement techniques presented here are widely used; as one can see from the extensive reference list provided at the end of the Chapter. These methods are reviewed in this Chapter, and the ones that meet our requirements are adopted, (sometimes with minor modifications). The fact that the measurement techniques used in the experiments are not new adds an'extra advantage to the local void fraction reconstruction technique presented in this work. That is, existing widely used measurement techniques are used; no new special measurement method is required.

2.0 REQUIRED MEASUREMENTS

As explained above, the neutron scattering technique for local void fraction measurement requires measurement of the energy spectrum of the scattered neutrons. If the energy fluence is defined as the neutron fluence in a specific energy range, then in order to determine the void fraction at N locations, at least N energy fluences of the scattered neutrons are required. By defining an energy fluence detector (efd) as the detector that measures the neutron fluence at a specific energy group at a specific position (spatial co-ordinates), then at least N efd's are required in order to estimate the void fraction at N locations. Note that the definition of an efd implies that a detector that is positioned at a specific location and measures M energy fluences is considered as M efd's. Also two detectors that measure the neutron fluence at the same energy but located at different positions are two different efd's Therefore, N efd's can be obtained either by using a detector located at a specific position and measures N different energy fluences, or by number of detectors located at different positions and provide in total N efd's (see Figure 1). Apparently in order to measure N energy fluences using one detector, the resolution of that detector has to be good enough to provide N different energy fluences: However, by using more than one

detector to provide N energy fluences, wider energy groups can be used



for each detector, allowing less restrictions on the detector resolution, as compared to the case when one detector is used to provide N energy fluences.

Since the energy and angle of scattering are related to each other, some of the efd's provided by some of the detectors are ineffective. These efd's are the ones that correspond to scattering from outside the scattering medium under considerations. Therefore, when N efd's are required, one has to make sure that these efd's correspond to scoring scattering angles, i.e. angles that correspond to scatterings inside the object. Also, as mentioned in Chapter 3, energies that correspond to oxegen scattering are to be excluded, since only single scattering by hydrogen is considered.

3.0 SPECIFICATIONS OF REQUIRED DETECTOR

The detector used to measure the required energy fluences should be small in size. That is because in solving the inverse problem a scattering ray that corresponds to the measured energy has to be drawn from the detector position towards the scattering target. If a large detector is used, the position inside the detector from which the scattering ray is to be drawn will be difficult to determine. The reason is that the angle of view of such a large detector is so wide that the

detected energy cor esponds to a wide angle of scattered neutrons. It is . then hard to determine which neutron in that angle has caused the measured energy fluence. This, in turn, increases the uncertainty and worsen the resolution of the estimated local void fraction. By using a small detector the measured energy fluence will correspond to a narrow scattered neutron angle, and consequently the uncertainty in the estimated local void fraction is decreased. In the limiting case of a point detector, the measured energy fluence corresponds to a specific scattered neutron as originated along the scattering ray.

From the above introduction one can specify the desired charcteristics of the neutron detectors required for the fast neutron scattering technique for local void fraction measurements. They are:

- (a) capability of measuring the energy fluence, i.e. the neutronfluence at a specific (preassigned) energy
- (c) insensitivity to radiation sources, other than fast neutrons

small size

(b)

The last requirement is added to eliminate from the measured neutron energy fluence the contribution of undesired radiation such as gamma rays.

REVIEW OF TECHNIQUES FOR MEASUREMENT OF FAST NEUTRON FLUENCES

Many techniques have been developed for measuring the energy and energy fluences of fast neutrons. In the following a brief review of the existing techniques is given. No attempt has been made to provide a comprehensive review. Excellent reviews can be found in References (1), (2), (3) and (4).

2), (3) and (4)

4.0

There are three fundamental techniques for measuring the

spectrum of neutrons: measuring the speed by time-of-flight over a known distance, measuring the energy of a secondary particle in a neutron reaction, and measuring the wave length by diffraction from a crystal. The latter method is limited to thermal neutrons.

In the time-of-flight techniques, the neutron velocity is

estimated by measuring the elapsed time for a neutron originated at a. source point "a" to reach a detector point "b". Therefore, the time of origin of a neutron is to be localized by producing the neutrons in burst of time duration, either by mechanically chopping the neutrons, by producing pulsed neutron source, or by recording a charged particle associated with the neutron production reaction. This technique requires a well-collimated beam, and long flight paths, (can reach 200 meters). These requirements limit the use of time-of-flight techniques in fast neutron spectroscopy.

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In techniques based on neutron-induced reactions, the fast neutron is made to induce a suitable nuclear reaction. The reaction products then have a total kinetic energy given by the sum of the incoming neutron kinetic energy and the Q-value of the reaction. Provided the neutron energy is not a hopelessly small fraction of the Q-valve, a measurement of the reaction product energies gives the neutron energy by simple subtraction of the Q-value. Excluding elastic scattering, which is considered as a recoil particle interaction, there, are only two reactions of major importance in fast neutron spectroscopy: 6 Li(n, α)³H and ³He(n,p)³H. A third reaction, neutron-induced fission, is not useful in spectroscopy because of the very high Q-value associated with this reaction.

Since there are no suitable gaseous compounds of lithium for a proportional counter, detectors using the 6 Li reaction (Q = 4.780 MeV) detect the ionization energy released in the reaction using either scintillation materials or semiconductor diodes. However, the use of scintillation materials is hampered by its nonlinear response to the tritons and alpha particles; since the scintillation light yield depends on the distribution of the reaction energy between the triton and the alpha particle. This nonlinearity results in a broad response function and consequently a poor resolution. The other way in which the lithium reaction has been used to measure fast neutron energy is using a sandwich detector in which lithium flouride or other lithium-containing material is incorporated as a thin film and placed between two semiconductor diode detectors. When the neutron energy is low, the two reaction products, the triton and the alpha particle, are emitted back to back and coincidence measurement should be observed from the two semiconductor detectors. The energy deposited in the two detectors is then equal to the incoming neutron energy plus the Q-value of the reaction, neglecting energy loss of the charged particles before they reach the active volume. If the neutron energy is significant compared to the Q-value, the reaction products must have some momentum in the direction of the incoming neutron and will not be exactly oppositely directed. Then, some fraction of all reactions will lead to two products, both of which strike the same detector and do not give rise to coincidences. The fraction of neutron events lost to this effect becomes more significant as the neutron energy increases. The main disadvantages of this semiconductor spectrometer are susceptibility to radiation damage at relatively low neutron fluences ($10^{12} n \text{ cm}^{-2}$), low efficiency, (about

 10^{-6} for 2MeV) and the necessity of shielding the spectrometer because \sim of the large thermal cross section of the ⁶Li reaction.

A neutron spectrometer based on the ${}^{3}\text{He}(n,p){}^{3}\text{H}$ reaction (Q = 764 keV) may be a proportional counter filled with ${}^{3}\text{He}$ gas at several atmospheres of pressure, or a coincidence - diode arrangement as for the

 6 Li spectrometer. This spectrometer is appropriate for investigating neutron spectra in the 200 keV to 2 MeV energy range. At higher neutron energies, the pulses arising from the ³He-recoil nuclei become noticeably perturbing: in an elastic collision between a neutron of energy E_n and a stationary helium nucleus the latter receives an energy between 0 and (3/4) E_n. For this reason, the neutron spectra at high neutron energy can be investigated only if the effect of the recoil nuclei can be eliminated, either by pulse rise time discrimination between the proton and the recoil ³He, or by making coincidence measurements between the two charged particles arising from the ' reaction. Another drawback of this spectrometer is the low Q-value of the reaction which makes discrimination against gamma rays difficult.

The detectors discussed above produce a prompt output pulse for each detected neutron. Neutron measurements can also be carried out indirectly through the radioactivity which is induced in some materials by neutron interactions. A sample of such material can be exposed to a flux of neutrons for a period of time, and then removed so that the induced radioactivity may be counted, using any of the available conventional methods. The measured radiation can then be used to deduce information about the number and/or energy distribution of the neutrons in the original field. Materials used in this way are often called activation detectors. Because neutron reaction cross sections are highest at low neutron energies, activation detectors are most commonly applied to the measurement of slow neutrons.

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The most common method of fast neutron detection is based on a detection scheme that differs from those discussed above. It is based on elastic scattering of neutrons by light nuclei. The scattering interaction transfers some portion of the neutron kinetic energy to the target nucleus, resulting in a recoil nucleus. The energy which can be transferred from a slow neutron is therefore very small, and the resulting recoil nuclei are too slow in energy to generate a usable detector signal. Once the neutron energy reaches the keV range, however, recoil nuclei can be detected directly, and assume a large importance for fast neutron detection. By far the most popular target nucleus is hydrogen. The cross section for neutron elastic scattering from hydrogen is quite large and its energy dependence is accurately known. The recoil nuclei which result from neutron elastic scattering are called recoil protons, and devices based on this neutron interaction are known as proton recoil fatectors.

Recoil protons can be detected using photographic plates, proportional counters, semiconductors or scintillators.' Neutron flux measurements are made with photographic plate detection through microscopic observation of recoil proton tracks left in the emulsion. The recoils may originate from scattering of neutrons by hydrogen in the emulsion, or may come from an external hydrogenous radiator. The advantages of the method are simplicity in neutron recording equipment, sensitivity which is continuous in time and over a wide neutron energy range, and creation of a permanent record of the experiment. The principal disadvantages are in the cost of the microscope and the time consumed in plate reading. The method has been used in neutron

Gas proportional counters can also be used to measure fast neutron through the proton recoil process. The fill gas is usually hydrogen or a hydrogen-containing gas such as methane. Because the detection medium is gas, recoil proportional counters inevitably have a low counting efficiency, (typical values in the MeV range are less than 1 percent).

Proton recoil telescopes are also used for fast neutron spectroscopy. These telescopes are based on a narrow selection of recoil directions. Only those proton recoils that occur at a fixed angle with respect to the neutron direction are-singled out, the recoil proton energy is then fixed for monoenergetic neutrons, since the energy of the proton and the energy of the neutron are uniquely related. Therefore, the response function approaches a simple narrow peak. The protons are produced in a thin hydrogeneous radiator film, for example, polyethylene. The proton energy may be measured using semiconductor diodes, gas proportional counters or scintillation detectors. The main advantage of these telescopes is their simple response function which eliminates the step of unfolding the measurements to obtain the neutron spectrum. The dominant disadvantage is the extremely low detection efficiency (typically one count per 10⁵ incident neutrons).

Probably the easiest way to use proton recoil in the detection of fast neutrons is through the application of hydrogen-containing scintillators. Fast neutrons incident on the scintillator give rise to recoil protons whose energy distribution is approximately rectangular, ranging from zero to the full neutron energy. Because the range of the recoil protons is usually small compared with the dimensions of the scintillator, their full energy is deposited in the scintillator and the expected pulse height distribution is also approximately rectangular. Organic scintillators are usually used because of their high-hydrogen content (1.1 atoms of hydrogen per atom of carbon and a density of $- lg/cm^3$). Since the hydrogen scattering cross-section is quite large (2.5 barns for 2.MeV neutrons), a relatively small detector (2cm thick) has quite a high efficiency (~10%) for 2.5 MeV neutrons. Organic scintillators are also relatively inexpensive, can be tailored to a wide

variety of sizes and shapes, and are totally nondirectional in their scintillation response. Scintillators, however, are sensitive to gamma rays, requiring the use of discrimination techniques against the gamma rays. The simple rectangular response function of the detector to monoenergetic neutrons is usually distributed by multiple neutron scattering, scattering by carbon and other effects. This requires special care in unfolding the measurements of proton recoil scintillators, in order to obtain the neutron energy spectrum.

SELECTION OF NEUTRON DETECTION SYSTEM

As mentioned in the introduction of this Chapter, the detector to be chosen for use in the fast neutron scattering technique has to possess the ability of measuring the energy as well as the fluence of the scattered neutrons and to be small in size and insensitive to gamma rays. All the detectors discussed above enjoy the capability of measuring the energy spectrum of fast neutrons. However, organic scintillation fast neutron detector were chosen. The main reason behind that choice is the relatively high efficiency that can be obtained even for small detector sizes. They are also inexpensive and can be constructed easily in the laboratory. The other types of detectors are large in size, (proportional counters) or susceptibile to radiation damage at relatively low neutron fluences (semiconductor detectors).
Other merits of organic scintillation detectors will be mentioned in the following Section which also covers in detail the properties of these detectors, considerations to be taken in their design, (since they are constructed in the laboratory), unfolding techniques used to obtain the neutron spectrum from the measurements, as well as other technical aspects.

ispects.

ORGANIC SCINTILLATION NEUTRON DETECTORS

Soon after the discovery that naphthalene and anthracene would scintillate when exposed to beta and gamma rays, Bell⁽⁵⁾ in 1948 showed that fast neutrons could be detected from the proton recoil produced in an anthracene scintillator. Later , attention was shifted to stilbene because of its superior gamma ray rejection characteristics⁽⁴⁾.

However, these organic crystals are expensive and subject to damage from thermal and mechanical shock. The rapid development of organic liquid. and plastic scintillators was a major breakthrough for fast neutron spectroscopy. For the past 15 years, the organic scintillation detector has been the detector used for almost all MeV neutron research, because it possesses many of the desired features for fast neutron spectrometers. In the following these features are reviewed. The five most important characteristics for neutron detectors are⁽⁵⁾ high efficiency, good timing resolution, good discrimination . against gamma ray background, good neutron energy discrimination and good pulse height resolution for neutron spectroscopy. Organic scintillation detectors meet all these characteristics.

Organic scintillators contain a large amount of hydrogen (1.1. atoms of hydrogen per atom of carbon and a density of 1.0). Since the hydrogen scattering cross section is quite large (~ 2.5b for 2.5 MeV neutrons) a relatively small detector (2 cm thick) has quite a high efficiency (~10%) for 2.5 MeV neutrons.

The fast response time of organic scintillators was noted from the very beginning (5). The timing resolution of organic scintillators was shown to be 1 ns. It is usually less than the time spread of the neutron source or that of the neutron passing through the detectors.

Although very small organic scintillators have low efficiency for gamma rays, larger scintillators have gamma ray efficiency comparable to their neutron efficiency. The observation that the shape of the light pulse for electrons was different than that for protons led to many pulse shape discrimination (PSD) systems which differentiate between gamma rays and neutrons. Special scintillators such as NE213 were developed which possess very good PSD properties, although pulse shape discrimination have been observed for most organic scintillators⁽⁵⁾.

It was also noted in the late 1950's⁽⁵⁾ that an organic scintillator was also useful as a neutron spectrometer, since the light output from that scintillator depends on the energies of the recoil protons produced by the initial neutron. Pulse height distributions have been computed using Monte carlo techniques for various sizes of detectors for energies from 23 keV to 75 MeV and compared to experimental distributions. By the use of unfolding codes, it was shown⁽⁶⁾ that incident neutron spectra could be obtained with an energy resolution of

The use of organic scintillators for neutron detection can most easily be discussed by considering two limiting cases. One limit is when the scintillator is large so that the probability that the initial neutron will interact with the hydrogen (or carbon) in the scintillator is nearly unity. At each scattering the neutron loses on the average one-half of its energy to the recoiling proton. Most of the initial energy is deposited before the neutron escapes from the scintillator or is captured. The light produced from all these recoiling protons for a single neutron produces a number of photoelectrons which are amplified in

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the photomultiplier to give a single pulse for each incident neutrom. The magnitude of the pulse is somewhat less than that produced by a proton whose energy equals that of the incident neutron. Since the light output versus proton energy for an organic scintillator is nonlinear, the total light output for a given neutron energy, depends on the distribution of the energies of the recoiling protons. For example, four 1 MeV protons produce less than half of the light of one 4 MeV proton (5)

The second limit is for a small or thin scintillator where the probability of an interaction between the initial neutron and the hydrogen in the scintillator is small, and the probability of the scattered neutron escaping from the scintillator after its initial scattering is near unity. For neutron energies up to - 10 MeV, the (n,p) scattering cross section is isotropic which results in a rectangular energy distribution of the fecoil protons up to a maximum proton energy equal to that of the incident neutron. However, the pulse height distribution from a thin scintillator may differ from this rectangular distribution for many reasons: (1) the escape of protons from the scintillator, (2) a second scattering of the initial neutron, (3) the nonlinearity of the light output vs proton energy (4) neutron interaction with carbon atoms in the scintillator, (5) variation in light collection

(to the photocathode) from different parts of the scintillator,

(6) variations in the number of photoelectrons produced by a given light intensity at the photocathode (Poisson distribution), (7) variations due to cathode nonuniformity, and (8) variations in gain of the photocathode. Some of these factors will be discussed in detail <u>later</u>, since the detector to be used for measuring the scattered neutron spectrum has to be small in size.

7.0 DETECTOR DESIGN AND CONSTRUCTION

7.1 Detector Material

Because scintillation materials that contain hydrogen are quite common, there is no shortage of candidates for use as fast neutron scintillators. Successful applications have been reported using organic crystals such as anthracene or stilbene, plastic scintillators in which an organic scintillant is incorporated in a bulk matrix of polymerized hydrocarbon, and liquid scintillators that combine an organic scintillant dissolved in a hydrogen containing organic solvent⁽⁴⁾. Organic crystals are expensive and subject to damage from thermal and mechanical shock⁽⁸⁾. A further disadvantage of these crystals stems from the directional variation of their light output, because of the channelling of recoil protons along different crystallographic directions. This effect greatly complicates the job of unfolding an observed pulse height spectrum to derive the incident fast neutron energy spectrum. Plastic scintillators are suggested and do not have to be encapsulated with provision of an expansion volume like the liquids, but can otherwise be used much like liquid scintillators.

The favoured scintillator has been NE-213 (made by Nuclear Enterprises, Ltd.), an xylene-based liquid, because it exhibits good pulse shape discrimination (PSD) properties. NE-213 has an enhanced emission of delayed light which gives it the good (PSD) properties. Since NE-213 is noncrystalline, it is isotropic in response to neutrons and is not sensitive to mechanical or thermal shock. Therefore NE-213 was chosen as the organic scintillator material to be used for constructing the detector used in the present work.

.2 Detector Size

As mentioned at the beginning of the present Chapter that a small detector is required for measuring the spectrum of scattered neutrons. Fortunately, a small size detector is a desirable requirement in the design of a proton recoil scintillator as explained in the following. By making the scintillator thick, the detector efficiency is obviously enhanced. As the neutron energy increases, the detection efficiency will decrease, and consequently there will be strong motivation to make the scintillator larger. However, it is more difficult to achieve uniform light collection from a large-volume scintillator, and the energy resolution will worsen. Another factor that favours a small size detector is the pulse rate due to gamma rays interacting within detector. This rate may exceed that from fast neutrons, and the scintillator must be kept sufficiently small so that the pile-up of gamma ray events is not a problem. Another factor that promotes the choice of a small size detector is that, in a small detector, a typical neutron is likely to scatter only once, and the energy spectrum of proton recoils will closely approximate the rectangular distribution discussed earlier. As long as the scintillator dimensions are larger than a few millimeters, escape of protons from the surface is unlikely, and the response function of the detector is simple and easily calculated . As the detector dimensions are increased, multiple scattering of the neutrons becomes more likely and the response function is more complicated and harder to predict. Because an accurate knowledge of the response function is critical for unfolding the energy spectrum, one would like to keep the scintillator small so that their complicating effects do not introduce large uncertainities. From the above discussion, one can see the requirements of a small size detector imposed by the neutron scattering technique for local void measurement is favoured by the physical aspects of neutron spectroscopy. Therefore, it is desirable to construct a miniature detector.

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A cylindrical detector cell, with outside dimensions of 12.5 mm high and 12.5 mm diameter, was made from pyrex glass, 0.8 mm thick, a section is shown in Figure 2. The design of the detector such that its diameter equals to its height reduces the sensitivity of the detector to the direction of incidence of neutrons. Therefore, with such a miniature size, the detector resembles to a great extent, a point detector.

7.3 Detector Construction

In constructing this detector, the procedure of Perkins and Scott⁽⁷⁾ was adopted. A small expansion chamber (100 mm³) connected to the main volume via a constricted neck of 0.1 mm diameter was used to allow for the difference in expansion of NE-213 and glass. The size of the constriction in the neck was chosen, such that, when the filled detector was inverted, the bubble in the expansion chamber did not pass into the main cell. The detector could therefore be used in any orientation without affecting its response. The cell was filled with NE-213 which had been de-oxygenäted by bubbling oxygen-free nitrogen through it for 15 minutes. When the filling was complete, with the scintillator filling the main cell and half the expansion reservoir, the cell was sealed off just above the reservoir, using a fine flame.



d=12.5 mm t= 0.8 mm

FIGURE 2 Cross Section of the Miniature NE-213 Scintillator Detector

7.4 Detector Efficiency

The detection efficiency of a hydrocarbon neutron detector based on recoil protons can be expressed, neglecting multiple scattering as:

$$\varepsilon(E) = (n_h \cdot \sigma_h / a) \int \exp(-az) dz \qquad (7.4.1)$$

where $\varepsilon(E)$ is the detection efficiency for an incident neutron energy E, $a=n_h \sigma_h + n_c \sigma_c$, with n_h and n_c being the number of hydrogen and carbon atoms per unit volume respectively and σ_h and σ_c being the scattering cross sections for hydrogen and carbon respectively, and L is the incident neutron path length through the detector.

Detection efficiency as expressed by eq. (7.4.1) ignores multiple neutron scattering with hydrogen and scattering with carbon. Many computer programs that take into account the effect of these scatterings have been developed. Some of these programs are deterministic (9,10) and others are probabilistic (11, 12). The following is a brief review of some of these programs. Kurz⁽⁹⁾ developed a computer program that includes first and second-scattering contributing from interactions with protons and carbon atoms in the calculations of plastic scintillator detection efficiency for neutron energies from 1 to 30 MeV. Thornton and Smith⁽¹⁰⁾ revised the Kurz code to vary hydrogen/carbon ratio, rather than having it always equal to unity, and to include light output information for various scintillators rather than only NE-102.

For large detectors, an incident neutron suffers more than just two scatterings and the above deterministic programs then tend to underestimate the detection efficiency. Programs based on the Monte Carlo method were developed to estimate detection efficiency for thick detectors. Stanton⁽¹¹⁾ developed a Monte-Carlo code to evaluate the neutron detection efficiencies for plastic scintillator of rectangular and cylindrical geometries. Many revisions of this code were developed later, however, the latest version is that of Cecil et al⁽¹²⁾. They improved Stanton's code by the adjustment of the inelastic cross-sections and kinematics for neutron-induced reaction on ¹²C, adopting of new (more accurate) light-response functions, use of relativistic kinematics for neutrons of energy above 100 MeV, and the proper determination of light deposited by escaping charged particles. Because of the miniature size of our detector, the probability of third order and higher order scattering is quite small. Therefore, the use of probabilistic programs for detection efficiency is not necessary. The deterministic programs of $\operatorname{Kurz}^{(9)}$ or Thornton and Smith⁽¹⁰⁾ are considered sufficient. In these codes C(E) is expressed

$$\varepsilon(E) = \varepsilon_h(E) + \varepsilon_c(E)$$

where c_h is the efficiency for an initial interaction with hydrogen and ε_c is the efficiency for an initial interaction with a carbon nucleus. ε_h can be further divided to:

$$h(E) = c \frac{1}{h}(E) + c \frac{2}{h}(E)$$
, (7.4.3)

(7.4.2)

where ϵ_h^1 is the hydrogen single-scattering efficiency and ϵ_h^2 is the efficiency due to rescattering of neutrons leaving the primary interaction. They are given by

$$c_{h}^{1}(E) = 2\pi W(E) \int_{-1}^{1} \frac{d\sigma(x)}{d\Omega} dx$$
(7.4.4)

and .

as:

$$\varepsilon_{h}^{2}(E) = 2\pi W(E) \int_{x}^{1} \frac{d\sigma(x)}{d\Omega} c^{1}(E^{1}) dx \qquad (7.4.5)$$

with .

$$W(E) = n_{b} \int exp(-az) dz$$
 (7.4.6)

where d_{σ}/d_{Ω} is the differential cross section, x is the cosine of the neutron angle of scattering in the centre of mass system, X^{1} is the value of x corresponding to neutron energy E^{1} , (note the unique relationship between the energy and angle of scattering for elastic scattering), and $\varepsilon^{1}(E^{1})$ is defined later.

If x^1 in Equations (7:4.4) and (7.4.5) is equal to unity, and considering isotropic scattering in the center of mass system, c_h^2 will be zero and ε_h will be equal to ε of Equation (7.4.1); that is the efficiency considering only single scattering. In order to include second scattering of the incident neutrons, x^1 is taken such that it corresponds to the average energy of a recoil proton produced by rescattering of the incident neutron. Since the average energy of an incident neutron after suffering a single scattering is half the incident energy; that is 1/2 E; then the average energy of a recoil proton resulting from a second scattering is 1/2x1/2 E, that is 1/4 E. This proton energy corresponds to $x^1 = 1/2$, in the center of mass system. Now the integrals (7.4.4) and (7.4.5) can be calculated provided that ε^1 (ε^1) is determined. ε^1 (ε^1) is the total single scattering efficiency from hydrogen and carbon for a thickness of scintillator equal the mean escape distance in a direction, in the laboratory system corresponding to x^1 , that is:

(E) =
$$\varepsilon_{\rm h}^{1}(E^{1}) + \varepsilon_{\rm c}^{1}(E^{1})$$
, (7.4.7)

With E^1 being the neutron energy corresponding to x^1 (3/4 E), and ϵ^1_h is given by Equation (7.4.4) with L replaced by d.

The efficiency corresponding to carbon interactions, (n, τ) , (n,n'3 α) and (n,p) required to evaluate ε_{c} for first and second scattering are not discussed here. The reader can refer to Reference (9) for details. However, it should be mentioned that the (n, α) reaction is treated as a two-body interaction, the (n,n'3 α) is tackled by assuming a four-body phase-space distribution of the final interaction stage energy, while the (n,p) reaction is considered, in the Kurz program, as a direct charge-exchange reaction. The Kurz code as modified by Thornton and Smith is proved to be accurate, when compared to experiment, to within 8% for a detector of 183 mm in depth⁽¹⁰⁾. This error was mainly caused by the uncertainty in the light output function used. It is to be noted that experimental results are determined in light output, while calculations produce recoil-proton distribution. Therefore a light output response function has to be imposed on the latter for comparison with experimental results. Since very few second order neutron scatterings are expected,... in the miniature detector used in the present work (125x125 mm), (as will be **sh**own later), Kurz code efficiency is expected to be as accurate as the hydrogen cross-sections, that is less that 1%.

Since the version of Kurz code developed by Thornton and Smith⁽¹⁰⁾ was not available, we modified the original code to include the alterations introduced in this version. Namely, varying the hydrogen/carbon ratio rather than having it always equal to unity, including the program to calculate the average escape probability as a subroutine in the main program and changing the program to FORTRAN IV.

The detector total efficiency as calculated by the Kurz code, for the miniature detector of 125 mm in length and 125 mm diameter is .shown in Figure 3. Table 1 shows the contribution of second neutron

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FIGURE 3 Detector Efficiency as a Function of Neutron Energy

· · · · · · · · · · · · · · · · · · ·			Ener Ener	rgy is in MeV	•••••••••••••••••••••••••••••••••••••••	•		
Neutron Energy	Total Efficiency	Ţ, u u	Carbo (n p)	n Efficiency (n, n'3α) ^I	escattering	Hydrogen (n, n)	Efficiency Rescattering	1
а. 	0.14670	0.0	0.0	d o	0.0	0.14599	17000.0	-
2	0.10828	0.0	0 0	0.0	0.0	0.10812	0.00016	1
m	1 0.08994	0 0	0.0	0.0	0 • 0	. 16680.0	0,00003	t i
4	0.07469	0.0	0 0	0.0	0.0	0.07468	10000 •0'	1
S	0.06550	0.0	0 0	0.0	0	0.06549	0.0001	• •
9	0.05805	0.0	0.0	D 0	.0 0	0.05804	10000.0	
7	0.05213	0, 00002	0.0	0.0	0.0	0.05211	0.0000	
α.	0.04855	0.00128	0 0	0.0	0•0	0.04727	0.0	
6	0.04908	0.00539	0.0	0.00045	0.0	0.04324	0.0	•
10	0.04868	0.00800	0.0	0.00086	0.0	0.03982	0.0	1
11	0.04774	0.00738	0.0	0,00354	0.0	0.03682	0.0	a.
12	0.04639	0.00624	0.0	0,00599	0*0	0.03416	0.0	1.
13,	0.04516	0.00519	0.0	1 0.00822	0.0	0.03175	0.0	
14	0.021395	0.00422	0.0	0.01023	0.0	0.02950	0.0	T
	•	-	•					
		-		•	•		-	

scattering and neutron-carbon interaction to the total efficiency. It is clear from this table how little interactions, other than the primary neutron interaction, affect the detection efficiency.

8.0 DETECTOR MOUNTING AND LIGHT COLLECTION

The performance of g scintillator detector depends on the efficiency with which the light is transmitted to the photocathode of the photomultiplier. This, in turn, depends on the method by which the scintillator is coupled onto the photomultiplier. The scintillator can be separated from the phototube, coupled by means of a light pipe, ordirectly coupled onto the photocathode window⁽⁵⁾. These different methods of mounting the scintillator are discussed briefly in the following and the method used for mounting our miniature detector is pointed out.

For some applications the scintillator is separated from the phototube. The scintillator is supported inside a container holding the phototube and the inside of the container may be polished or painted with a diffuse reflecting paint. In this configuration less light from the scintillator reaches the phototube than when a light guide (pipe) is used. When a light pipe is used to transmit the light from the scintillator to the photocathode, the light is propagated to the

phototube mainly by total internal reflection in the pipe. Therefore the walls of the light pipe should be polished or coated with a diffuse reflector. The disadvantage of this method is the use of coupling compounds at each touching interface. Light losses in each coupling interface can be as large as $20^{+(5)}$. Also substantial losses occur in the light pipe (quartz, lucite, arcyglass) due to their relatively short absorption lengths and due to reflection losses at polished surfaces (by light trapping).

The direct method of coupling is, therefore, preferred particularly when, working at low light levels. Because of the miniature size of the detector used in the present work, the light level is not very high and consequently the direct coupling method is to be used. The detector cell is coupled directly onto the photocathode window, using epoxy-coupling adhesives.

9.0. PHOTOMULTIPLIER -

In selecting a photomultiplier one should consider the following: the cathode diameter (to match the size of the scintillator), high photocathode sensitivity (for the given light wavelength), high overall sensitivity and low noise. The cathode area should be at least 5% of the scintillator area in order to collect most of the light because it is lost by repeated reflections⁽³⁾. The spectral sensitivity of the tube depends on the cathode material, and the optical transmission of the tube window. The sensitivity is usually designated by an "S" number assigned by the manufacturer. For example, the S-11 response, which matches most scintillators reasonably well, is achieved by an oxidized cerium - antimonide cathodes (CS₃Sb-Q) on lime glass⁽³⁾. The short wavelength cutoff is about 3500Å, (light output from NE-213 have a wavelength of 4250Å⁽³⁾).

The most useful measure of the overall sensitivity is the quantum efficiency, i.e., the number of photoelectrons emitted and collected by a perfect electron-optical system, per incident quantum. Noise in the tube can be reduced by selecting a photomultiplier of a low dark current, (current observed with no light illuminating the photocathodes). The magnitude of the dark current depends on the cathode area, the temperature and the work function of the cathode. The dark current can be reduced by cooling the tube.

Perkins and Scott⁽⁷⁾ recommended the RCA C31005C tube for their miniature NE-213 detector. This tube has a diameter of 1.9 cm and has a quantum efficiency higher than comparable tubes. It also exhibits a spectral response that reasonably matches the NE-213 fluorescence spectrum. However, this tube was not available in the market at the time the present work was done. Therefore, we replaced it by the EMF 9814KB tube, produced by EMI Industrial Electronics Limited. This photomultiplier tube has a diameter of 51 mm, (effective cathode diameter of 46 mm), a quantum efficiency of 26% and a bialkali cathode that has a spectral response that is bounded between 3300Å and 6200Å wavelengths, with the peak being around 4000Å. This spectral response matches well with that for NE-213 light output. Since the size of this tube is larger than the size of the detector, most of the light emitted is collected in the tube. For a better collection both the tube and the detector mounted onto it, were enclosed in a reflecting cage; as shown in Figure 4.

10.0 ELECTRONICS AND DISCRIMINATION AGAINST GAMMA-RAYS .

Since gamma radiation almost always accompanies neutron radiation, a very effective pulse-shape discrimination (PSD) electronic circuit is required to eliminate the gamma-ray pulses from the desired neutron pulse-height distribution. The relative intensity of fast and slow components of the light yield of the NE-213 organic scintillator and some other organic scintillator, depends on the specific ionization of the ionizing particle. Therefore, particles of different mass or change

will produce signal pulses with different time characteristics.

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FIGURE 4 Détector-Photomultiplier Tube Coupling .

Specifically, the scintillation response for gamma-ray induced Compton-electron ionization is faster than the response for recoil protons. This allows an individual scintillation process to be distinguished as a neutron event (proton recoil) or a gamma-ray event (electron recoil).

There is a wide variety of PSD electronic circuits, however, they can be classified into two major categories: methods based on charge comparison and methods that use zero crossover-timing. The following is a brief review of these techniques, succeeded by a detail lay out of the circuit used in the present work.

10.1 Charge Comparison PSD

This method is based on comparison of the light emitted during the early part of the pulse with the total light emitted. In principle, this system distinguishes between pulses induced by neutrons and gamma-rays in the following fashion. A photomultiplier (PM) converts the light from the scintillator into a current pulse, which is developed across the terminating resistor of a co-axial cable connected to the anode of the PM. The time constant of the anode circuit is small; therefore the voltage pulse developed across the resistor closely represents the profile of the current pulse from the photomultiplier. In the pulse shape discriminator, this voltage pulse is simultaneously admitted to two amplifiers: an integrating amplifier and a stretching amplifier. The voltage pulse is electronically integrated with a long time constant in the integrating amplifier to obtain an output pulse which is proportional to the total charge delivered during the current pulse. In the stretching amplifier, the voltage pulse is stretched to obtain a pulse whose amplitude is proportional only to the peak amplitude of the current pulse from the PM tube. The outputs from these two amplifiers are then compared in a difference amplifier. Because the ratio of the total charge to the peak current is greater for a pulse induced by a neutron than by a photon, it is possible to adjust the gain of the integrating amplifier to obtain outputs of opposite polarity for neutron and gamma-ray induced pulses. Circuits based on the charge comparison PSD method are described by Jones (13), Sabhah & Subami and Dance & Francois . A digital technique is also proposed by Morris et al⁽¹⁶⁾

10.2 Zero Crossover-Timing Technique

This technique takes advantage of the fact that the scintillation response for Compton-electron ionization is faster than the response for proton which allows an individual scintillation process to be distinguished as a neutron event (proton recoil) or a gamma-ray event (Compton-electron recoil). The main advantage of this method of PSD is its suitability for use over a large dynamic range (wide range of energies).

For this technique two signals are required. In addition to the pulse-height signal, a timing signal is needed to define the beginning of each pulse. This timing signal is taken from the anode of the PM tube. The pulse-height, also the energy, signal is taken from the highest dynode of the PM tube that gives a non-saturation signal for highly energetic neutrons. In order for the PSD circuit to accept recoil protons up to 20 MeV in energy, the fast component of the scintillation's light output must be highly saturated for these large pulses. These large non-linear (saturated) pulses produce severe pick-up problems in the lower dynodes where the linear non-saturation signal is obtained. For the 14 MeV neutrons, produced by D-T reaction, the pick-up problem is not expected to be very severe. However the use of the highest dynode that gives a non-saturation signal for 14 MeV neutrons, (if there is any saturation, can reduce this pick-up problem. Also, if the PM tube is selected for photocathode sensitivity greater than 80x10 A/lumer at about 3850A and an overall sensitivity of at least 12,000A/lumer, a. significant improvements in the spectrometer resolution and PSD circuit

operation can be achieved. The selected EMI 9814KB tube meets these requirements. All leads in the photomultiplier tube base are kept short and near the ground bus to reduce pick-up problems.

The principle of operation of the zero crossover-timing (ZCT) technique can be explained as follows. Bipolar voltage pulses are produced by integrating and subsequent differentiation of the dynode current pulses. These bipolar pulses cross the base line at a time uniquely determined by the shape of the dynode current pulse, (independent of the amplitude), i.e., at a time uniquely determined by the nature of the exciting particle. Measurement of the zero crossing time of the bipolar pulse then determines the particle type. Figure 5 shows the waveforms in a simple ZCT circuit. The bipolar pulse, A, is fed into the non-inverting input of a differential comparator which compares the input signal with ground level. This causes the output, B, to change state whenever the input pulse crosses the zero level. Signal C placed in coincidence with the comparator output is derived from the fast current pulse of the anode of the PM tube and defines the start of the pulse. The pulse length of the logic signal D is transformed into amplitude signal with a time-to-amplitude converter (TAC). A single channel analyzer (SCA) is adjusted such that a proton recoil pulse causes the output to fall in the window of the SCA. A pulse from this SCA is used to gate the multi-channel analyzer (MCA) for analysis of energy

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FIGURE 5 A Simplified Diagram of a Pulse Shape Discrimination Circuit and Waveforms

signal (dynode current signal). The proper window setting is determined by analyzing the TAC output pulses for a neutron and gamma-ray source using the MCA gated by a second SCA (not shown) and analyzing the energy signal. The rise time spectrum has a gamma-ray peak and a neutron peak. The lower level of discrimination of the SCA can, be set at the centre of the valley between the gamma-ray and neutron event peaks, while the higher level of discrimination can be set on the neutron event peak of the rise-time spectrum.

A number of circuits have been devised for PSD, based on the zero-crossover timing method. The difference between these circuits lies in the kind of electronic devices used to achieve the purpose. Johnson et al⁽¹⁷⁾ used a cross-over pick-off which provides the stop signal when the bipolar pulse crosses zero. Heistek and Van Der Zwan⁽¹⁸⁾ employed an integrated circuit differential comparator to measure the zero crossover time of the bipolar pulse relative to the start of the pulse. In the circuit suggested by McBeth et al⁽¹⁹⁾, an anticoincidence voting of zero-crossing signals with the undesired time relation with respect to the anode signal is substituted for time to amplitude conversion. The constant fraction pulse-height triggering

(CFPHT) was employed by Glasgow et al. $\binom{20}{}$, Plischke et al $\binom{21}{}$ and in the Munich PSD system $\binom{22}{}$. The use of a CFPHT to provide either the start or the stop signal on both enhances the discrimination over a wide

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dynamic range by reducing the excessive time walking. (Time walking is an effect derived from the variable amplitudes of input pulses, namely, pulses with identical shape and time of occurrence but different amplitude produce time triggering level at different times).

10.3 Description of PSD Electronic Circuit

A PSD circuit based on the zero crossover timing method was chosen for measuring the scattered neutron spectrum required for constructing the local void-fraction distribution. This is because the zero crossing technique gives, the best discrimination in practice when PSD is required over a large range of neutron energies (large dynamic range of pulse amplitudes)⁽¹⁹⁾. Also, the performance of this type of circuit can be determined rapidly and easily because the pulse shape signal is independent of the pulse amplitude⁽¹⁸⁾. On the other hand, methods based on charge comparison are proved to be superior to the zero-cross timing methods only for low energies⁽¹⁴⁾. Since our problem is a scattering one where neutrons scatter over a wide range of energies, and since we are not interested in very low energies, a method based on the zero cross-over timing was chosen.



1.

FIGURE 6 Block Diagram of Pulse Shape Discrimination Circuit

Used in the Experiments

As mentioned before a number of designs were reported for constructing a zero cross-over timing PSD circuit. After testing some of these designs, we finally adopted a circuit similar to the one reported by Perkins and Scott⁽⁷⁾ because of its simplicity and consequent ease of adjustment. The only modification introduced was the replacement of the two-parameter multichannel analyzer by an ordinary (one-parameter) multichannel analyzer. The two-parameter analyzer was used to accumulate pulse height and zero-crossing time data and display them as a two-parameter response surface (pulse height versus zero-crossing time, for an elaborate discrimination between electrons, protons, alpha particles and "C nuclei): Two-parameter analyzers are available but are expensive because of the extra analog-to-digital converter (ADC), control and readout circuitry, and enlarged memory. Furthermore, we are interested in discriminating between only two particles, electrons and Therefore, the use of a two-parameter analyzer was not protons. necessary.

A block diagram of the PSD discrimination circuit used measuring the scattered neutrons is shown in Figure 6. The timing signal taken from the anode of the PM tube defines the beginning of each signal. The START pulse is based on a leading edge triggering of the timing pulse. The START (logic) pulse is produced when the leading edge reaches a constant fraction (0.1) of the peak signal amplitude. Therefore the START pulse is independent of timing signal amplitude for all signals of constant shape. (as it is the case for PM anode pulses). This constant fraction timing method is very effective in reducing amplitude walk (time jitter).

The pulse height of the energy signal taken from the 10th dynode of the PM tube defines the light output of an event. The preamplifier.

and RC-shaping amplifier shape and amplify the energy signal which is then fed directly to the multichannel analyzer (MCA). The dynode signal

is also passed through an CR-RC-CR network to produce a bipolar shape. The time at which this bipolar pulse crosses zero is picked-off using a

timing single channel analyzer (SCA) which produces a logic pulse (STOP) when the bipolar pulse passes through zero. This zero-crossing time does not depend on pulse amplitude, but instead is a function of the pulse shape and rise time.

The timing interval between the START pulse and the STOP pulse is an indication of the differences in pulse shape prior to the shaping network. This time difference is converted by a TAC (time-to-amplitude converter) onto a pulse amplitude. The TAC output is fed to a single channel analyzer (SCA) to select only those intervals that correspond to neutrons. The SCA output is then used to gate the MCA only when

acceptance criteria are met.

The proper setting of the SCA window, for cross-over times corresponding to neutron events, is determined by analyzing the TAC

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butput for a neutron and gamma-ray source using the MCA gated by a second SCA (not shown in Figure 6) analyzing the energy signal. The rise-time spectrum of the neutron and gamma-ray fields associated with the T(d,n) He reaction is shown in Figure 7. The rise time spectrum accumulated by the MCA has a gamma-ray event peak and a neutron event peak. The lower level of the SCA was set at the center of the valley between the gamma-ray and neutron event peaks and the upper level at the far edge of the neutron event spectrum. A pure gamma-ray source (22 Na) was then used to adjust the window setting such that a minimum passage of gamma-ray events is allowed.

11.0 ADJUSTMENT AND CALIBRATION

11.1 Linearity and Correct Zero-Channel Adjustment

The linearity of the different electronic components in the PSD has to be insured before taking any measurements. This is done by replacing the PM tube and its associated bridge by a precision pulser. The pulser is adjusted such that the shape of the pulses out of, the amplifier is the same as those of recoil-proton pulses. The attenuated output of the pulser is considered as the energy signal, while the direct

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output is used as a timing signal. The attenuated signal is used to adjust the multi-channel analyzer so that zero energy falls in the middle of the channel zero. Also, the variation of the attenuation of the energy signal is used to optimize the circuit such that the analyzer and the linear amplifier are adjusted for linearity.

11.2 Timing Walk Minimization

The walk of the electronic system is measured by using the direct output of the pulser as the timing signal. Timing walk as a function of pulse height is then measured by varying the attenuation of the energy signal. The timing walk is reduced by adjusting the gain setting of the bipolar amplifier and also the setting of the timing of the SCA.

11.3 • Calibration (in terms of light units)

The pulse-height spectrum must be calibrated in the units used in the unfolding process. For a NE-213 scintillator, the light unit is defined with a ²²Na gamma-ray source. The standard light unit

corresponds to 1/0.89 times the half height point at the upper shoulder (Compton edge) of the 1.28 MeV Na gamma-ray calibration

spectrum⁽¹⁷⁾. A ⁶⁰Co gamma-ray source can be also used for

calibration. Although, it has two energy groups of gamma-rays, (1.17 and 1.33 MeV), these two groups average approximately an energy equivalent to. that of 22 Na. Figure 8 shows the pulse height spectrum for 22 Na gamma-ray source. This spectrum was recorded on a scale of 2048 channels to increase the accuracy of the calibration. The spectrum, though, appears mainly in the first 256 channels.

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11.4 Gain Standarization

Each spectrum must be accompanied by an explicit statement of the gain in units of (light units/channel). This gain must be standarized. In other words, the ²²Na pulse-height spectrum, measured at the time of a neutron spectroscopy experiment, must have the same position on the pulse-height scale as a 'specified calibration. This calibration is obtained by adjusting the fine-gain control such that the half height of the 1.28 MeV Compton edge always lies on the same channel.

12.0_ Unfolding of Neutron Spectrum

Proton-recoil spectroscopy methods are based on measurement of the energy of the recoil proton generated in elastic scattering of the neutron on hydrogen. If the angle of scattering is restricted, there is a unique relationship between proton energy and neutron energy, namely

(12.1)


where E_n and E_p are the energies of the incident neutron and the recoil proton, respectively, and ψ_p is the angle of scattering. Since no information regarding the scattering angle is obtained from a scintillation detector, the proton spectrum is still related quantitatively to the neutron spectrum, but an unfolding process is required. For the special case of monoenergetic neutrons, at energies below 10 MeV where isotropic center of mass scattering is assumed, the probability of observing a proton energy interval E_p to $E_p + \Lambda E_p$ is

$$(E_{p}) = 1 / E_{n}, \quad 0 \le E_{p} \le E_{n}$$

$$= 0 \quad , \qquad E_{p} \ge E_{n}$$

$$(12)$$

or in other words, a rectangular distribution between 0 and the incident neutron energy, with unit area. The neutron spectrum can then be unfolded by simple differentiation,

$$n(E_n) = dN(E_n)/dE_n$$
(12.3)

where $n(E_n)$ is the number of neutrons per eV at energy E_n , ε is the efficiency (protons generated per incident neutron), and $N(E_p)$ is the number of protons per eV at E_p . Nevertheless, one does not measure the proton distribution, but the number of counts as a function of pulse-height. Therefore, conversion of pulse-height distribution to proton energy distribution is necessary.

In thick detectors, multiple scattering will modify $N(E_p)$. Therefore one must calculate or measure the response function (counts per incident neutron of known energy, versus pulse-height), then construct a response matrix (three-dimensional table of counts per incident neutron versus pulse-height, versus neutron energy, arranged in selected contiguous pulse-height intervals or bins, and contiguous energy groups). Response functions for NE-213 thick detectors have been reported by many authors and a Monte Carlo computer program is available for response function calculations⁽²³⁾. Verbinski and Burrus^(6,24) at Oak Ridge National Laboratory and Wilensky⁽²⁵⁾ at MIT, and others, have reported measured response functions and developed unfolding programs for fast-neutron spectroscopy using organic scintillators. Nevertheless, the FORIST computer code⁽³⁵⁾ is the most widely used program for neutron spectrum unfolding.

The detector used in the present work is small in size (≈ 1.5 cm³). This allows a differential method⁽⁷⁾, rather than response matrix inversion method, to be used for unfolding neutron spectrum measurements, avoiding the production of the detector response matrix. The NEUTSP⁽²⁶⁾ differential unfolding code is used for this purpose. The unfolding procedure used in this code is presented in the following.

12.1 A Differential Method for Spectrum Unfolding

When factors that give rise to distortions of the proton recoil response function are minimized or allowed for, a differential method can be used to unfold the neutron spectrum from the recoil proton response. These factors include multiple neutron scattering in the scintillation volume, spatially dependent proton transport effects and interactions with carbon. Because of the miniature size of the detector used in the present work the effect of these distortions is not significant, as can be seen from the detector efficiency analysis reported earlier. Differential unfolding methods are much simpler than response matrix inversion methods, since the latter requires the production of the detector response function, either experimentally or by computation. Response metrix inversion methods also require more computation time and computer memory storage.

In the following a modified version of the NEUTSP⁽²⁶⁾ code for neutron spectrum unfolding using the differential mothod is presented. This code includes corrections for second scattering of neutrons and for effects due to the shape of the scintillator. The calculation algorithm used in this code can be illustrated in two steps. First, conversion of the pulse-height distribution to energy distribution of the recoil protons. The second step is to convert the recoil-proton energy spectrum to a neutron spectrum. These two steps are explained in the following.

In converting the pulse-height distribution to energy

distribution, the following relationship of Birks⁽²⁷⁾, is used for describing the non-linear response of scintillators to protons, (with respect to their energy):

(12.1.1)

$dL/dr = (S dE_p/dr) / (1+kB.dE_p/dr)$

where L is the scintillation response, r is the proton range, E_p is the proton energy, S is the absolute scintillation efficiency, B is a constant when multiplied by dE_p/dr gives the specific density of ionized and excited molecules along the particle track and k is a quenching parameter. For electrons dE_e/dr , where E_e is the electron energy, is sufficiently small that $dL/dr = S.dE_e/dr$, which agrees with the observed linearity of the scintillation response of electrons. The response to other particles can be given, then in terms of electron equivalence. A unit called MeVee (MeV electron equivalence) is used to express this equivalence, where one MeVee is the kinetic energy of a stopping electron which produces the same amount of light as the stopping heavy particle in question. The light output (pulse-height) for

units. Using the Compton scattering relationship it can be shown that 1.28 MeV protons give rise to a maximum electron energy of 1.067 MeV.

Since the NE-213 response is linear for electrons, then one light unit corresponds to electron energy of 1.199 MeV (=1.067/0.89) that is 1.199

MeVee.

)(2)

Given the pulse height distribution as counts versus channel number, the procedure to obtain the recoil-proton energy distribution, i then as follows:

(1) convert channel numbers to light units, using the ²²Na Compton edge calibration,

> relate light units to electron equivalent energy, MeVee, using the fact that the scintillator response to electrons is linear. That is

L (light units) = K.P(MeVee), (12.1.2)

where L is the pulse-height in light units, P is the electron equivalent energy and K is the constant of proportionality. (The constant K is 1/1.199 light units/MeVee). obtain proton energy from the electron equivalence, MeVee, using the proton electron equivalence relationship. The NEUTSP⁻ program reported by Toms⁽²⁶⁾, uses a relationship similar to that of Brock and Anderson⁽²⁷⁾:

$P(MeVee) = 0.19 E_{p}^{1.42}$

(3)

(12.1.3)

However, these relationships were based on measurements in a stilbene scintillation crystal. We adopt the relationship measured (and partly calculated using the Monte Carlo method) by Verbinski et al⁽²⁴⁾ for an NE-213 scintillator. This relationship is given in a table form and assumes that the protons have fully deposited their energies into the scintillator. Loss of recoil protons through the edges of the detector is accounted for by the method discussed later. In fact this table relates directly the pulse-height of protons in light units to the proton energy, consequently step (2) can be bypassed.

The second part in the neutron spectrum unfolding process is the conversion of the recoil-proton energy distribution to the neutron spectrum. For this conversion the following relationship is used:

 $\phi(E) = E/[A_{E}|(E) B] (-q/\Delta E)$ (12.1.4).

where $\varphi\left(E\right)$ is the number of neutrons per cm 2 per unit energy at energy

E, A is the cross secctional area of the detector, exposed to neutrons, ϵ (E) is the efficiency for neutron detected at energy E, by means of

recoil-protons, B is a shape factor, and $(-q/\Delta E)$ is the slope of the proton distribution at the center of the energy interval E. A slope - averaged over six energy bins surrounding E is employed.

For the efficiency, $\mathcal{T}(E)$ Toms⁽²⁶⁾ used the simple formula:

$c(E) = n_h \sigma_h [1-exp(-aL)]/aL$

(12.1.5)

with L the detector length and $a = n_0 \sigma + n_0 \sigma$ where n_n and n_c are the number of hydrogen and carbon atoms per cm³, respectively, while σ_h and σ_c are the hydrogen and carbon cross-section, respectively, evaluated at the neutron energy E. In this formula only single scattering in hydrogen is considered and scattering by hydrogen following scattering by carbon is neglected. The second scattering of neutrons by hydrogen and loss of recoil-protons through the ends of the detector are accounted for by introducing the correction factor B given by the relationship

 $B = 1 - 0.780 (R_m/L) + 0.09 n_h \sigma_h + 0.077 n_h \sigma'_h r_h^{-1}$

where σ_{h}^{\prime} is the hydrogen cross-section at 0.068E, r is the detector radius and R is the range of a proton that receives the full neutron energy. In NE-213 R (in mg/cm²) is given by the expressions⁽²⁶⁾:

$R_{m} = 1.7382 (E+0.1504)^{1.8194}$

(12.1.7)

However, a more elaborate detector efficiency calculation that includes contribution of second-scattering and detector shape effects, is available through the modified version of the program of Kurz⁽⁹⁾, as reported earlier. The results of this program are then used for $\varepsilon(E)$ in Equation (12.1.4) and the shape factor is set equal to unity, since its effect is implied in the calculation of $\varepsilon(E)$ by the program. The area A in Equation (12.1.4) is taken as the area of the detector surface exposed to neutrons. Results of this unfolding program are shown in the next Section where the experimental setup and results are presented.

13.0 EXPERIMENTAL SETUP AND RESULTS

The experimental measurement of the spectra of fast neutrons scattered by water is discussed in this Section. A few preliminary experiments were performed using McMaster University tandem Wan de Graaff accelerator to produce neutrons by bombarding a ⁹ Be target with a 2 MéV deuteron beam. The main purpose of these preliminary experiments was to familiarize ourselves with the used electronic system. The experience gained from these experiments was utilized in setting up the final experiment using a small low-voltage accelerator at Chalk River Nuclear Laboratories (CRNL) to produce 14 MeV neutrons using the T(d,n). He reaction. This reaction needs a low energy deuteron beam, #keV range) which cannot be obtained from a large accelerator like the tandem accelerator of McMaster University. Therefore, the experiment was performed at CRNL.

Neutrons produced by the T(d,n) ⁴He reaction have a nominal energy of 14.1 MeV. The actual neutron energy is a function of the incident ion energy and observation angle with respect to the ion beam direction. However, because of the high Q-value of the reaction (17.6 MeV), the neutron energy is relatively insensitive to the angle of emission for low deuteron bombarding energy. For a deuteron energy of a few hundred keV the neutron energy varies around 14.1 MeV by only about 7%.

A fresh thin zirconium tritiated target of 5 Ci per in tritium loading was used. This target was bombarded with a 0.4 mA deuteron beam at energy of 300 keV. At this beam current and bombarding energy, a nearly isotopic neutron emission of about 4×10^{10}

neutrons per second is expected

13.1 Design and Description of Experimental Setup

The experimental setup for measuring the energy fluence of neutrons scattered by water is shown in Figure 9. Two 40x4x4 cm³ iron blocks each backed by a 12 cm thick layer of wax were used to act as a neutron collimator and as a shadow shield to reduce the background neutron field. This collimator design was based on the studies on fast neutron collimators for neutron radiotherapy, (see for examples, References 28 to 33). A good collimator, in these studies, has to meet the following two requirements:

(a) The spectrum of source neutrons should not be softened by neutrons of low or intermediate energy.

(b) A reasonably uniform beam profile should be obtained.

and,

These requirements are essentially those demanded for our neutron scattering experiment. The studies mentioned above indicated that in order to achieve the above requirements, the best form of a

collimator is given by considerations according to geometrical optics, and the best collimator materials are iron and tungsten, since they have

forward peaking of elastic scattering at 14 MeV. Iron or tungsten serve.



also as neutron attenuators. The combination of iron or tungsten and a hydrogenous material reduces the background at the detection site. Tungsten is used, instead of iron, in medical applications, in order to overcome the induced activity associated with the activation of iron by the reaction 56 Fe(n,p) 56 Mn. However, reducing the radiation dose was not a major concern, since the experiment was run in a well shielded room and the data was remotely collected. Therefore, iron was used for neutron collimation. In a future design of a system for routine use in the laboratory, tungster is probably recommended. Finally, one should mention that the dimensions of the collimator used here are comparable to those reported in Reference 28, where a fairly uniform beam profile was obtained.

13.2 Measurement of Source Energy Spectrum

The measurement of the neutron source strength and energy is useful for two reasons. First, since the source is monoenergetic of a nominal energy of 14.1 MeV and expected strength of 4×10^{10} neutrons per second (as indicated earlier), the experimental procedure can be verified if the measurements reproduce the nominal energy and strength. The second benefit of measuring the source spectrum is to use the measured. strength to normalize the spectra of the scattered neutrons and consequently enable comparison with results obtained using Monte Carlo simulations. (The latter produces neutron fluences per source neutron).

A direct measurement of source neutrons was taken by positioning the detector such that its exposed surface. faced the beam at a distance of 39 cm from the tritium target (see Figure 9). The

pulse-height distributions, collected over a period of 30 seconds, is shown in Figure 10; collapsed to 256 channels instead of the 1024 used for measurement. The background associated with this distribution was measured in the absence of the neutron beam and the detector. However, the pulse height of the background was found to be so small that it was neglected. The pulse-height distribution of the neutron source was recorded after the adjustment and calibration procedures discussed in Section 11 were implemented; resulting in the relationship between channel number and energy shown in Table 2.

The differential method of spectrum unfolding, discussed in. detail in Section 12.1, was applied to obtain the energy spectrum of the 1024 channels pulse-height spectrum for a 100 keV resolution. Figure 11 shows the resulting energy spectrum. The spectrum depicts clearly the 14 MeV expected peak, however, it shows also an oscillation below the peak and a small peak at 1.2 MeV.



Table 2

Energy - Pulse Height - Channel Number Conversion Table (Only selected channels are given)

Energy	Pulse Height	Channel No.	Channel No:	Channel No.
Mev	light units	(200 Channets)	(1024 Chamers)	
0	. 0	~ 0	0	0 *
1.78	.0.3845	13	52	
2.81	0.798	27	108	
3.05	0.890	30.125	120.5	241 *
3.30	1.000	33.85	135.4	270.8
4.50	1.566	ົ 53	212	
6.05	2.334	· 79 ^	316	
7.45	3.102	105	420	
8.85	3.870	131	524	
10.15	4.638	157	628	
11.37	5.377	182	728	
12.70	6.175	209	836	
13.95	6.943	235	940	
13,98	7.563	256	1024	· · · · · · · · ·

Calibration points obtained from Figure 8, which shows the ²²Na gamma-rays spectrum as it appears on the first 256 channels of a 2048 channels scale.



The oscillation below the 14 MeV peak and the negative flux component associated with it can be explained by taking a second look into the pulse-height spectrum of Figure 10.. One can see in this Figure that the pulse height distribution increases before it sharply drops at the right edge of the spectrum. Since a differential method is used for unfolding this pulse height distribution, the increase in the pulse height results in a positive slope, which consequently produces negative neutron flux, (see Equation 12.1.4). Nevertheless, one cannot hold the differential method of unfolding responsible for this negative flux, since this negative flux was also observed when the spectrum of a pulsed. accelerator (deuterium-tritium) neutron source was unfolded by Profio et $\widehat{al}^{(34)}$ using the FORIST Code (35), which employs a response matrix inversion method. Profio attributed this negative flux to an error in the response matrix, however, he did not explain the nature of the error. In our opinion, this error is due to assuming neutron scattering by hydrogen in the detector to be isotropic. Anisotropic hydrogen scattering, which occurs above 10 MeV, tends to take recoil protons from the middle of the distribution and place them at the ends. The redistribution occurs because the probability of forward or backward scattering is enhanced relative to medium angle interactions (36). This explains the deviation of the pulse-height distribution, shown in Figure 10, from the ideal rectangular distribution expected for the monoenergetic 14 MeV Reutron source.

The small neutron peak that appears at 1.2 MeV is due to assigning a recoil-proton energy threshold of 1.1 MeV. Below this proton energy, which corresponds to channel number 90 of 1024 channels scale, the pulse shape discrimination circuit ceases to be effective. This channel number was determined by testing the circuit against a pure gamma-ray source (²²Na). Therefore, the 1.1 MeV energy corresponding to this channel number was assigned as an energy threshold for the unfolding program. But, since the program utilizes six energy bins to calculate the pulse height slope at a given energy bin, and since three of these bins are below the considered energy, some of the energy bins that are contaminated with gamma-rays are used in estimating the flux at the energy bin corresponding to 1.2 MeV. This resulted in a relatively large slope and consequently the flux peak at 1.2 MeV.

Despite the two defects in the source energy spectrum discussed above, the measurement of the energy spectrum is considered successful. That is because it resulted in a very distinguishable peak at the expected neutron energy of 14.1 MeV. The distribution of neutrons around this peak is due to the fact that the resolution of the detector is about 10% of full width at half maximum (FWHM) at this neutron energy (36). The success of the measurement verifies the calibration procedure, as well as, the unfolding algorithm: The negative flux encountered near the

neutron peak is not expected to be encountered in the neutron scattering .

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measurements, since the amount of neutrons of energy above 10 MeV that reach the detectors is limited. Also, the small neutron peak encountered at the low energy edge of the spectrum, is not expected to be observed in the scattering measurements, since a background measurement is available.

13.3 Estimation of Neutron Yield

The source strength is estimated using the source energy spectrum shown in Figure 11. Integrating the spectrum under the 14 MeV neutron peak results in a flux of $1.8 \times 10^6 + 3.1 \times 10^4$

neutrons/cm²/sec. Assuming a point isotropic source, the source \mathcal{U} strength is then estimated to be 3.44x10¹⁰ neutrons/sec. According to the Bulletin "H" of the High Voltage Engineering Corporation ; Burlington, quoted in Reference 2, a neutron yield of about 2×10^{11} neutrons/mA is obtained from a thick ³H-Zr target bombarded with 300 keV deuterons. Since a current of 0.4 amps was used in the experiment, then a neutron yield of 8×10^{10} neutron/sec is expected according to this Bulletin. The discrepancy between the value obtained in our experiment and that reported in the Bulletin is probably due to

the difference in the tritium loadings of the two targets. (The loading of the target reported in the Bulletin is not specified). However, the two values of neutron yield have the same order of magnitude, which further verifies the experimental procedure.

13.4 Scattered Neutron Energy Spectrum

Neutron scattering was investigated for three cylindrical test sections of different diameters (see Figure 12). Each test section, (of length 14 cm), was filled with water to a vertical level that coincided with the level of the collimator, then a (60 seconds) background measurement for the scattered neutrons was taken. The Tevel of water was increased by 25 mm to the top of the collimator and a (60 seconds) foreground measurement was recorded. Figure 13 shows a typical foreground and background measured pulse-height distribution. Only one detector was employed and was moved into the positions 1 to 4 marked on Figure 12, and the experiment was repeated for each detector site. In order to take into account variations in the neutron output from run to run, the deuteron beam current was recorded, since it is directly proportional to the neutron output.

The pulse height distribution per light unit, after subtracting the background, measured at detector at different detector positions for the three test sections are shown in Figures 14 to 17. The oscillating behaviour of the distributions, caused by statistical fluctuations as well as by scaling to light units, necessitated the smoothing of these distributions. Without smoothing, the differential unfolding method would result in negative neutron fluences, while in an unfolding method









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based on the inversion of the detector response matrix, large error propagation would occur. Smoothing also reduces the effect of neutrons removed by the discrimination against gamma ray, that appear at the lower edge of the distribution.

The simplest method of smoothing is to collapse the data into wider intervals. This method results smoother data, but this is at the The main interest in these experiments was expense of resolution. to compare the experimental results with those obtained from Monte Carlo simulations. Since Monte Carlo results were obtained within energy bins wider than 1 MeV, there was no need to unfold the experimental results within a resolution better than 1 MeV. Also, as can be seen from Figures 14-16, the counting statistics were poor. Therefore, it was decided to collapse the pulse-height distribution into wider channels. such that every collapsed channel corresponding to a 1 MeV energy bin. Table 3 shows the channels between which the polse-height distributions are collapsed from the data given on 1024 channels. In applying the differential unfolding code NEUTSP, discussed in Section 12.1, two energy bins surround the considered energy interval were used for obtaining an average value for the slope of the proton distribution at this interval. Note that the original version of NEUTSP uses six bins for the slope calculations, but this is not necessary in the present case since the

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hilse	Height	Distribution	Collapsing	Tabl

Table 3

•	New Channel		Old Channel (1024 Channels)	Old Channel (256 Channels)**	Energy MeV
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2			63.884 -	13.9/1	· • • • • • • • • • • • • • • • • • • •
з			117.133	29.283	3.0
4			178.570	44.643	4.0
.5	· · ·	7	245.786	61.447	5.0
. 6			312.541	- 78.135	6.0
7		•	384.263	96.066	: . 7.0 ·
 		•. •	4597597	114.899	8.0
a			537.042	134.261	9.0
10		••	615.666	153.917	10.0
11	, 1		696.859	174.215	1i 0
13			799,407	199.852	12.0
12		•	860,602	215.151	13.0
22			944.502	236.126	14.0
14			1024.6	256.0	15.0
· · ·		·] .	- t		

Fractional channel implies that a fraction of the data in the channel is used in the collapsing procedure.

Actual collapsing is performed on the 1024 channels scale; the 256 channels scale is given here since the pulse-height spectrum was plotted using this scale.

distribution is smoothed and the number of energy intervals is limited. Because of this use of two neighbouring intervals for every energy big, no estimation of the slope of the proton distribution is calculated for the edge intervals (0-1) and (14-15) MeV, consequently, no estimation of the neutron fluence in these intervals is calculated.

Results of unfolding the pulse-height distributions are shown in Figures 18-21, for the different detection positions and test sections illustrated in Figure 12. In these Figures the resulting neutron fluences are shown connected by a continuous line only to aid the eye. Each fluence line is surrounded by two lines indicating the error range, within a 68% confidence level. The validity of these experimentally measured neutron fluences is confirmed by comparing them to those obtained from a Monte Carlo Simulation, as shown in the following paragraphs.

13.5 Comparison with Monte Carlo Results

The laboratory experiment was simulated using the MORSE code in the same fashion presented in Chapter 3. This Monte Carló simulation results in neutron fluences normalized to one source neutron. Therefore, in order to compare Monte Carlo results to the experimental results, it is necessary to calculate the source strength. The neutron source used









in the Monte Carlo simulation was a simple line source of 5.09 cm length that emits a beam of neutrons, while in the experiment the source is essentially a point isotropic source, which has been Collimated. In order to compare the strength of these source, the number of neutrons incident on the test section A, shown in Figure 12, is calculated. The flux at the surface of this test section, as shown in Section 13.3, is estimated to be 1.8×10^6 neutrons/cm²/sec. Multiplying this flux by the area of the test section exposed to the neutron source, and the exposure time, results in

$S = 1.8 \times 10^6 \times \pi \times D \times L \times T$

where D is the diameter of test Section A (5.09 cm)., L is the effective length of the test section (2.5 cm) (see Section 13.4), and T is the neutron exposure time, (60 S). Therefore, test Section A is exposed to 4.3×10^9 neutrons. The error associated with this number is \pm 7.4×10⁷ neutrons. The neutron fluence obtained by the Monte Carlo method are then multiplied by S; resulting in the values shown in Figures 18 to 21.

As one can see from these Figures, the measured results generally agree with those obtained using the Monte Carlo method. The discrepancies between the measured and calculated results can be due to many factors. The neutron beam used in the experiment though collimated is not a perfect beam compared to the monodirectional beam used in the Monte Carlo analysis. Also, one cannot guarantee that the neutron flux at the surface of the test section is uniformly distributed, as it is assumed in the Monte Carlo simulation. Discrepancies could be also due to the fact that a point detector is used in the Monte Carlo experiment, while a volume detector, (though small) is used in the laboratory experiment. The energy spectrum unfolding procedure can also be held responsible for these discrepancies. These factors are examined briefly in the following.

Deviation of the source neutrons from a perfectly collimated beam is estimated from the geometrical configuration of the setup of the experiment (Figure 9) to be no more than 3.2°. However, because of the energy width of the detector bin (1-MeV), neutrons deviated by less than 2° would score in the same energy bin. Moreover, because of the detector aperture, neutrons which are deviated by up to 1.6° from the perfect beam configuration reach to the same detector. Therefore, one cannot consider the non-perfect collimation of the neutron source as a major contributor to the discrepancy of the measurements from the Monte Carlo calculations.

As shown in Figure 9, the test sections are not placed near the edges of the neutron beam where the beam profile is expected to exhibit a severe gradient in flux. Therefore, the neutron flux incident on the
test section is expected to be reasonably uniform, and the effect of the non-uniformity of the beam profile on the measurements could not be considered to be a dominant factor in the discrepancy of the measurements from the calculations.

The fact that volume detectors are used in the experiment, while point detectors are used in the calculations, might have contributed to the discrepancies at the edge detectors, where the flux of incident neutrons could suffer a severe gradient. However, in the middle detectors, the response of a point detector is not expected to be different from that of a volume detector, because of the expected uniformity of the neutron flux incident on the latter.

Many factors in the procedure of unfolding the neutron energy spectrum could be blamed for the discrepancies of the measurements from the calculations. The adequacy of relationship (21.1.1) that describes the non-linear response of the scintillator is questionable because of the many empirical parameters involved. Also, the relationships used to relate the proton energy to the calibrated electron energy could cause significant errors since it was estimated for a larger detector. Moreover, there are no error bounds associated with the parameters in these relationships, so that one can associate a corresponding error with the unfolded spectrum. Finally, the smoothing of the measured pulse height distributions, required for the adequate use of the differential

unfolding method, could produce errors in unfolding the energy spectrum because of the information lost by smoothing the data.

. In order to overcome the error introduced in the unfolding procedure, an unfolding method based on the inversion of the detector responstmatrix, such as that of the FORIST code (35), should have been used. In these methods, one relies on a measured (or adequately calculated) detector response matrix, and consequently avoids many of the empirical relationships used in a differential unfolding method. Also, since matrix inversion methods do not depend on direct differentiation of the pulse height distribution, they are less subject to error propagation than are differential methods. Though matrix inversion methods are thought to be sufficiently accurate these are more complicated to use, and require generation of the detector response matrix which relates the light pulse-height to the incident neutron energy. Since the generation of the response matrix is an expensive process that requires many experiments and calculations, matrix inversion methods were not used in the present work.

It is interesting to note that both the experimental and computed (Monte Carlo) fluences reflect strongly the existence of the water core by tending to peak at energies corresponding to the angles at which neutrons are scattered by the hydrogen in the water. The fluences recorded at the high energy tail reflect the scattering caused by the oxygen in water and the aluminum walls of the test sections, while fluences at the low energy tail are caused by multiscattering in the test

sections.

6.0 <u>CONCLUSIONS</u>

The laboratory experiments reported in this chapter indicate clearly that the energy spectrum can be measured in the laboratory. They also indicate that there is no need to collimate the scattered neutrons as long as the detector used is small enough to resemble a point detector. The energy angle relationship provides an automatic scattering collimation. The detectors used are guite simple, cheap and are widely used in routine measurements of fast neutron spectra.

In future work, to obtain more accurate neutron spectra, it is recommended to measure (or adequately calculate) the detector response matrix, and to unfold the spectrum using a matrix inversion method.

Finally, this Chapter has shown that the techniques required to measure the scattered neutron spectrum are well established. Consequently, the method of reconstructing the void fraction distribution utilizing the scattered neutron spectrum, proposed in this work, is applicable and is not just a theoretical idea.

APPENDIX A

LISTING OF COMPUTER PROGRAM FOR

FORWARD AND INVERSE CALCULATIONS

Persons who wish to obtain a copy of the program should request the most updated version from the office of the Engineering Physics Department, McMaster University, Hamilton, Ontario, Canada L8S 4M1. Please refer to the program SENT (Single Exposure Neutron Tomography).

APPENDIX B

DETAILS OF SOME INVERSE PROBLEMS

This Appendix contains the computer outputs of some of the inverse problems considered in Chapter 4. The parameters used in these outputs have the following meanings :

FOLD nn TIMES : nn folds are used in the calculations of the detector responses

SWITCH TO THE ITERATIVE TECHNIQUE AFTER nn ITERATIONS * switch to

the Bayesian procedure after nn successive approximations performed using the direct inversion procedure, (the Bayesian procedure is referred to here as the iterative technique since the solution is not obtained by the direct inversion of the response matrix, rather it is obtained by iterations using the Bayesian probabilistic concept).

KEY= nn

A key defining the amount of output sequired •

ISM= nn

If nn is not equal zero flow regime radial symmetry is imposed.

MAXIMUM NUMBER OF ITERATIONS ALLOWED= nn : maximum number of successive approximations performed.

ERROR ALLOWED IN UNFOLDING= pp : The successive aprroximation procedure is performed until the value $||\underline{s}-\underline{s}^{c}|| / \Sigma s_{i} \times 100$ is less than pp, where \underline{S} and \underline{S}^{C} are the given and the calculated detector responses, respectively, and S_i is the ith_element of the vector S. If pp is zero the procedure is followed till the above vale is less than or equal $\Sigma \sigma_i / \Sigma S_i \times 100$, where σ_i is the standard deviation of the ith detector response.

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TAW= pp

 τ^2 (the regularization parameter) is equal to pp

GAMMA= pp

If the quantity used to measure the linearity of the response matrix is less than or equal to pp the system is near-linear, then switch to the Bayesian procedure.

IPÓINT= nn

If nn is not equal to zero, the detector used are ponint detector and APERTURE refers to the angular width corresponding to the detector response considerd. If nn is zero, then volume detector are considered and APERTURE refers to the physical petector witch.

: refers to the water fractions in the pells

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ERROR = qq PER CENT : the value $||\underline{s}-\underline{s}^{c}|| / \sum_{i} x 100$ is calculated and is equal to qq at the Kth approximation . considered.

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ENRUR = . ENCLIDEAN N= 10 DES .98769 .69193 .69193 .98769 EUCLIDEAN DIS EUCLIDEAN EUCLIDEAN	C327L-01 RE NUK (F RESU 3 177 (BATES .66620 .09000 .66620 TA-CE 0ET. L NUR: UF .GT. NGR: UF .LT.	<pre>K CEFT LVING ERMOR = IAN) .665747 .05155 .00155 .00155 .66797 A5T 2 ITERAT I CUT-UFF = 0 CUT-UFF =</pre>	.94274 .50867 .66867 .94279 .66867 .94279	
ENRUR . ENCLIDEA NE 10 DE 5 .98769 .69193 .69193 .98769 EUCLIDEAN DIS EUCLIDEAN EUCLIDEAN	CS27L-01 RE NUK (F RESU J 177 (BATES .66620 .09000 .66620 TA-CE 8ET. L NUR: UF .67. NGR: UF .LT.	<pre># CE: T Lv1NG ERMOR = IAN) .66747 .05155 .05155 .66747 A5T 2 ITERAT: .CUT-UFF = 0 CUT-UFF = </pre>	.94279 .94279 .60867 .44279 .66867 .44279	
ENRIR # ENCLIDEAN NE 10 DE 5 .98769 .69193 .69193 .69193 .98769 EUCLIDEAN DIS EUCLIDEAN EUCLIDEAN EHRUR =	C327L-01 RE NUK (F RESU 117 (BATES .66620 .00000 .66620 TA-CE BET. L NUR. UF .GT. NUR. UF .LT. 1496E-01 PE	<pre># CEFT LVING ERMAR # IAN)</pre>	. 3027E+02 . 3027E+02 . 0 0007 . 60067 . 94279 . 60067 . 94279 . 1621F-03 0. 0.	
EHRLR = EHRLR = EHRLR = 10 DEAS .98769 .69193 .69193 .69193 .69193 .98769 EUCLIDEAN DIS EUCLIDEAN EHRUR = EUCLIDEAN	C327L-01 RE NUK (F RESU 117 (BATES .66620 .00000 .66620 TA-CE 8ET. L NUR. UF .6T. NUR. UF .6T. 1496E-01 PE NOR: UF RESO	<pre># CEFT LVING ERMOR == IAN) .665747 .00155 .00155 .00155 .66797 AST 2 ITERAT I CUT-UFF == 0 CUT-UFF == 0 CUT-UFF == R CENT SLVING ERMOR == </pre>	.94274 .50867 .60867 .44279 .50867 .44279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .94279 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .50867 .5	
ENRUR = ENCLIDEAN NE 10 DES 98769 69193 69193 98769 EUCLIDEAN DIS EUCLIDEAN EUCLIDEAN EHRUR = EUCLIDEAN	C327L-01 RE NUK (F RESU 117 (BATES .66620 .00000 .66620 TA-CE 0ET. L NUR. UF .67. NGR. UF .67. 14966-01 PE NORS UF RESO	<pre># CEFT LVING ERROR = IAN) .665747 .07155 .07155 .07155 .07155 .65797 A5T 2 ITERAT I CUT-UFF = U CUT-UFF = U CUT-UFF = IK CENT ILVING ERROR = </pre>	.94274 .50007 .94274 .50007 .66067 .94279 .66067 .94279 .000 .000 .000 .000 .000 .000 .000 .0	
ENRUR = ENCLIDEAN NE 10 DE 5 .98769 .69193 .69193 .98769 EUCLIDEAN DIS EUCLIDEAN EUCLIDEAN EHRUR = EUCLIDEAN	C327L-01 RE NUK UF RESU 117 (BATES .66620 .00000 .66620 TA-CE 0ET. L NUR. UF .67. NUR. UF .67. 14966-01 PE NUR. UF RESU	<pre># CEFT LVING ERMOR = IAN) .665747 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .051747 .A51 2 ITERAT: I CUT-UFF = 0 CUT-UFF = 0</pre>	.94274 .50867 .60867 .94279 .60867 .94279 .000 0. 0. 0.	
EHRLR = EHRLR = EHRLR = 98769 98769 69193 98769 EUCLIDEAN DIS EUCLIDEAN EHRUR = EUCLIDEAN EHRUR = EUCLIDEAN EHRUR = EUCLIDEAN	C 527L-01 RE NUK (F RESU ITY (BATES .65620 .09000 .66020 TA-CE 0ET. L NUR: UF .6T. NUR: UF .6T. 1496E-01 PE NOR: UF NESO 114 (BATES	<pre># CE: T LVING ERMOR = 1A+) .66747 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05175 .05175 .05175 .05175 .05175 .05175 .05175 .05175 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05177 .05155 .05155 .05155 .05177 .05155 .05155 .05177 .05155 .05177 .0515 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0517.055 .0557.055 .0557.0557.0557.0557.055</pre>	.3527E-52 .94279 .55867 .66867 .44279 .66867 .44279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .56867 .94279 .576667 .94279 .57667 .94279 .57667 .94279 .58867 .94279 .94279 .58867 .94279 .58867 .94279 .59867 .94279 .59867 .94279 .59867 .94279 .59867 .94279 .59867 .94279 .59867 .94279 .59867 .94279 .59867 .94279 .59867 .94279 .59867 .94279 .59867 .94279 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .59867 .5986	
ENRUR = ENCLIDEA NE 10 DEAS .98769 .99193 .99193 .98769 EUCLIDEAN DIS EUCLIDEAN EUCLIDEAN EHRUR = EUCLIDEAN EHRUR = .92764	C327L-01 RE NUK UF RESU 117 (BATES .66620 .00000 .66620 TA-CE 0ET. L NUR: UF .67. NUR: UF .67. 14966-01 PE NOR: UF RESO 117 (BATES	<pre># CEIT LVING ERMOR = 1AH) .66747 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05155 .05175 .05175 .051747 </pre>	.94279 .94279 .60867 .44279 .66867 .44279 .94279 .66867 .94279 	
ENRUR = . ENCLIDEA NE 10 DE-5 .98769 .99193 .99193 .98769 EUCLIDEAN DIS EUCLIDEAN EUCLIDEAN EHRUR = . EUCLIDEAN KE 11 DENS .93756	C 527L-01 RE NUK (F RESU ITT (BATES .66620 .00000 .66000 .56620 TA-CE SET. L NUR: UF .6T. NUR: UF .6T. 1496E-01 PE NOR: UF NESU 117 (BATES .68615 .0000	<pre># CEFT # CEFT LVING ERMORE IAN) .66747 .05155 .05155 .05155 .05155 .05155 .05155 .051797 AST 2 ITERAT: I CUT-UFF = 0 CUT-</pre>	.3562E=02 .94279 .60867 .44279 .66867 .44279 .1621F=03 0. 0. 0.	
ENRLR = ENCLIDEA N= 10 DE 5 .98769 .69193 .69193 .98769 EUCLIDEAN DIS EUCLIDEAN EUCLIDEAN EHRUR = EUCLIDEAN EHRUR = .98756 .99756 .69157 .04157	C327L-J1 RE NUK (F RESU 117 (BATES .66620 .00000 .66620 TA-CE 0ET. L NUR: UF .67. NUR: UF .67. NUR: UF .67. 14966-01 PE NOR: UF RESO 177 (BATES .65515 .0000 .0000	<pre># CEFT # CEFT LVING ERMOR 00155 00155 00155 00155 00155 AST 2 ITERAT 1 CUT-UFF = 0 CUT-UFF = N CENT EVING ERROR 00155 00155 00155 00155</pre>	.3562E=02 .94279 .60867 .44279 .66867 .44279 .1621F=03 0. 0. 0. 0.	
EHRLR = EHRLR = EHRLR = POCLIDEAN N= 10 DE S .98769 .69193 .69193 .69193 .98769 EUCLIDEAN DIS EUCLIDEAN EHRUR = EUCLIDEAN EHRUR = EUCLIDEAN K= 11 DENS .98756 .69157 .04157 .04157 .94736	C327L-J1 RE NUK (F RESU 117 (BATES .66620 .00000 .66620 TA-CE 0ET. L NUR: UF .67. NUR: UF .67. NUR: UF .67. 14966-01 PE NOR: UF RESO 177 (BATES .65515 .0000 .0000 .0000 .0000	<pre># CEFT LVING ERMOR = 1AH) .66747 .05155 .05155 .05155 .66747 AST 2 ITERAT! 1 CUT-UFF = 0 CUT-UFF = 0 CUT-UFF = N CENT SEVING ERROR SEVING SEVING S</pre>	.3562E=02 .94279 .60867 .44279 .60867 .44279 .94279 .60867 .94279 .60867 .94279 .60867 .94279 .94279 .66857 .94216 .66859 .94216	
EHRLR = EHRLR = EHRLR = POCLIDEAN N= 10 DE 5 .98769 .69193 .69193 .69193 .98769 EUCLIDEAN DIS EUCLIDEAN EHRUR = EUCLIDEAN EHRUR = EUCLIDEAN K= 11 DENS .98756 .69157 .04157 .04157 .94736	C327L=J1 RE NUK (F RESU 117 (BATES .66620 .00000 .66620 TA-CE 0ET. L NUR: UF .67. NUR: UF .67. 14966=01 PE NOR: UF RESO 14966=01 PE NOR: UF RESO 177 (BATES .65515 .0000 .0000 .0000 .0000	<pre># CEFT LVING ERMOR = 1A%) .66747 .00155 .00155 .00155 .66747 AST 2 ITERAT? 1 CUT-UFF = 0 CUT-UFF = 0 CUT-UFF = .8 CENT .9LVING ERMOR .00155 .00155 .00155 .00155 .00155 .00155 .00155 .00155 .00155 .00155</pre>	.3562E=02 .94274 .50867 .60867 .44279 .60867 .44279 .1621F=03 0. 0. 0. 0. 0. 0. 0. 0. 0.	

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CONTROLLER COMPOSITION (CONTROLP) A CONTROL CONTROL CONTROL CONTROL CONTROL FOR THE CONTROL FRONT CONTROL FRONT RE ELIGENT OF OF PESCENTIE ENPOREN ,4.05E= c e 8 - - -10 161511Y (BAYES Daws)-. 46725 .00025. .64155 . 971/6. . n. 15-• 14 Sec. 14 .e. 54141 .07141 .00154 ູປະຊະບ . oc 05/ .99176 .94726 .00720 .65625 EUCLIDEAN NURS OF .GT. 1 CUT-OFF = C. EUCLIDEAN NURS OF .LT. 0 CUT-OFF = C. ENNOR BO LIGHT-UI PERSOREST EQUIDENCE THE OF RESULVING ERRIE = .45428- . F, 2 10411514-) .00037 ...703 . 46725 -9+150 .09135 0.154 ▲GV 0000 +50020 والمتا المنبع041:35 1 .00037 . 48725 +00/1-3 .44150 EUCLIDEAN DISTANCE BET, LAST 2 ITERATIONS = . SHIYE-04 -EUCLIDEAN WORK OF .LT. O CUTHUFF # 0. _ERRUR # . . 9607E+U2 _PER CENT EUCLIDEAN NURH OF RESOLVING ERROR = . . 4576E-02 қ э 14 DENSITY (BAYESIAN) .68642 ,90727 .66651 ,99133 .00154 .00000 .68859 .69134 .69154 .00000 .00154-.68859 .98727 .68651 .68692 -,99133 EUCLIDEAN DISTANCE BET. LAST 2 ITERATIONS # .21784-04 EUCLIDEAN WERE UF .GT. 1 CUTHOFF = 0. EUCLIDEAN NORE UF .LT. 0 CUTHOFF = 0. ELELIBEAN NERM OF RESILVING EARIP = - 4/40E-UP *** 15 0E2/5114 (BAYESIAN) 0 164135 100144 49460

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***** + CO C P B -00159 -**EBSW** +00005 CONCLICEN SISTANCE BET, 1487 2 ITERNTICAS = .16417-04 RECLIPEN SERVICE LET I DETERME UL A Pasta REAL REPORT EVENIE FR. U CAN IN APSILATAGE EXERCIC E CANADA - 2 16 DE 5117 - (64185145) K 8 ,99113 . . bhoill , 107.51 .59137 1.0020 .+ 15+ .00701 . 14154 .oddal .64137 •0000a .99115 .66551 . 45731 .00075 素化乙酸盐化化物 的复数形式 不可 化色形合 (山谷) PEA 6月11000 ● 1 含于PA1++---LOUETOFAL SEAR OF LATE I DETACHE = 0. F LETTERS SPACE PRAINT, A COLLEGE # 0. ERNOR & BAULDFUZ PER CENT E IDELDER - WAY OF ALMERIAN FRANK - . 44052 - 10 17 0E-SITY (HATESIAN) R = 991uh .40731 +60677° 01000 00055 .64135 ູປຸງ154 **.**6915a .00154 .68955 .65677 .99198 .96731 66684 EUCLIDEAN NURM UF .GT. 1 CUT-OFF .= 0. 4 EUCLIDEAN NORH OF .LT. D CUT-OFF # 0. # ERRUR . . BEERE-US PER CENT -EUCLIDEAN NORM OF RESULVING ERROR = .5034E+02 1.1 18 DENSITY (BAYESIAN) K₽ .99104 .98729 :68675 .56464 00000 .00154 .69140. .68863 .68863 .00000 .01154 .6914ù F .08642 . .98729 .99104 .66075 ENCLIDENT NUMP OF .GT. 1 CUT-OFF = 0. EUCLIDENT NUMP OF .LT. 0 CUT-OFF = 0. ERACE = .8111E-02 PER CENT - .6094E+02

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x. .

14 DENSITY (HAVESIAN) . 40727 60098 . 66673 .99101 .00000 .09141 .00154 .60803 . 69141 .0.15 * 000 U .60863 . 40727 - .60696 \$ 441 01 .ete75

EUCLIDEAN NURH OF .DT. 1 CUT-OFF = 0; EUCLIDEAN NORM OF .LT. 0 CUT-OFF = 0.

EUCLIDEAN NORM OF RESOLVING ERROR = .5122E-02

K= 20 DENSITY" (BAYESIAN) .

K'#

0			
.70/23	.00/05 -	66612	- 44100
.64)42	° 00000	00154	. 68664
• 6 9] 4 5	• 06000 j. 🔍	. 00154	(
.987-23	.6d705	.6dn72	44160

EUCLIDEAN NURH UF .GT. 1 CUT-DFF # 0.

K= 21 DEWSITY (BAYESIAN)

98719	66708	.65571	99048
69143	. 00000	00154	66864
69143	200000	00154	66864
98719	.68738	+68671	99098

EUCLIDEAN DISTANCE BET, LAST 2 ITERATIONS = .5719E-05-

EUCLIDEAN NURH OF .GT. 1 CUT-OFF = 0. EUCLIDEAN NORH OF .LT. 0 CUT-OFF = 0.

ERRUR . . 7942E-02 PER CENT

EUCLIDEAN NORM OF RESOLVING ERROR # ,5171E-02

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EBBUB	81.	_2078E-01 PER	CEN
	•	and the second	

ANGLE

.1255E+02 1 -.7550E+01 .27,95E=04 -_4500E+02 .1255E+U2 -.7550E+01 2 . 10001-06 -. 55008+02 3 -.5780E+01 .1402E+02 ·5454F-04 -,4500F+02 4 .1402E+02 -. 3500F+02 .2775E-04 -.5180E+01 5 .14C2É+U2 -. 5500F+02 .1615E-04 -.40206+01. .1578E+U2 Þ .25F2E-04 -.45006+02 .1578E+02 7 -.4020E+01 .2175E-04 +.3500E+02 8 .1578E+02 -. 4020E+01 -1640E-04 -.5500E+02 Q ,1755E+Ú2 -.25\$UE+U1 -. 4500F+02-.2327E-04 -10 .1755E+02 -.255GE+01 -.3500F+02 11 .1755E+02 .1255E#02-.2327E-04 .4500F+02 12 ,1755E+02 .3500F+02 .12552+02 .2163E+04 13 .1578E+02 -1402E+02 .4500E+02 .2582E-04 .1578E+02 .1402E+02 14 .3500E+02 .2175E-04 15 .1578E+02 14026+02 \$500E+02 .1648E-04 1578E+02 .2424E-04 16 .1402E+02 .4500E+02 17 ·,1402E+02 +1578E+U2 3500E+02 .2775E-04 .1578E+02 \$500E+02 .18 .14D2E+02 .1615E-04 .1255E+02 19. .1755E+02 .279HE-04 .4500E+02 20 ,1255E+02 -1755E+02 . 5500E+02 .1444E-C4

CALCULATED DETECTOR RESPONSES

x

.02413	.00240	10.511
	· · · · ·	

.05240

STATISIJC=L	ENKOR IN DER	5171	•
.02413	.00240	.01211	. Ú-45n
1.0947	.00240	.01375	. U5054
.01427	.00362	.01373	. 05054

EUCLIDEAN 015	14*CE BET. L	AST 2 ITERAT	1005 =	U71-63
• • •			. ·	•
.40420	.0573	.05050	.991/6	•
.09371	10000	.00179 -	03630	
.04371	.00000	-00174	00000	
• 10 - EV	· • • • • • • • •	*P6053	.99175	

. 3-12.11

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. 30458

RESPONSE

DE4SITA

EUCLIDEAN NORH OF .GT. 1 CUT-OFF. = 0.

T**≜**-<u></u>€9.

NEL DE JIERATIONS = 55

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B.1.2 Problem 1 - Avenue 2

INVERSE PROBLES 1 (ANNULAR FLOW) AVENUE 2

FOLD 21 TIMES SHITCH IN THE ITEMATIVE TECHNISCH AFTER 11 ITERATIONS KEYE HI ISME O HAVING MUNEER OF ITERATION ALLOWEDE 21 EKROR ALLOWED IN OVERLOINGED. TAMED. GAMMAE .5000E=02

(INVERSE PROPLEM) CANNING FLOWS AVELUE P

DIA FTERE ,1400E+02 CM SCHARE VELTERN TRIAL CRUSS-SECTIONE ,1011E+00 SOURCE VEUTERN KONABBORFTING PROBABILITYE ,9395E+00

STREAT ATTAC FAT ATTACS FOR TO BE AV 4 GRT AND

20 DETECTORS

. . SIVER DETECTOR RESPONSES

	¥	v	RESPONSE	ERROR
1	.12558+02	-,7550E+01	.2797E-04	.5000E-08
2	-1255E+02	- - 755rE+01	.1441E=04	-5000E+0A
- 3-	. *1405E+05 .	5780F+01	5050E-04	-5000E-08
<u> </u>	- 145.2E+C2 -	57ANE+31	2774E-04	.5000E-08
5	* (465L+15	- 57875+71	1615E-04	- 5000E+08
b	.1578E+1.2	=_402:F+01	25826-04	5000E-08
7	.157AF+02	- 4120F+01	21726-04	5000E-08
	4157AE+52	-,402(F+0).	16496-04	.5000E-0A
` G	.17556+02	- 255:5+01	23286-04	5000E-08
10	1755E+12	25505+01	21625-04	- 5000E-08
11	17555412	1255E+02	2328E+04	.5000E-08
1.2	17556+02	.1255E+02	21625-04	5000E-08
13	.157*E+02	1402E+02	2582E-04	50008-05
14	157 AF+02	1402F+02	2172E+04	5000E+08
15	1578F+02	1402E+02	1649E-04	5000E+0E
. 16	.14022+02	1578E+02	2420f=04	5000E=08
17	.1402E+02	.1578E+02	27745-04	5000E-08
18 .	,1402E÷02	.1578E+02	1615F=C4	5000E+08
19.	1255E+02	1755E+02	27971+04	.5000E-08
20 .	1255E+02	.1755F+02	1441F-00	SOODE-DE

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ERROR = .2285E+02 PER CENT.

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EUCLIFEAN DISTANCE RET. LAST P ITERATIONS . .12798+00

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FRAME SCAREFON PER CENT

DENSITY -۰. 63287 92817 1,00000 .64665 . . 00120 •00000 75063 ,68633 .68633 00150 .00000 .75065 1,00000 . . 64665 , 92817 .63287

"ENCLIDEAN DISTANCE RET, LAST 2 TTERATIONS # ... 2460E-01

EUCLIDEAN NORM OF RESOLVING ERPOR . . . 2016E+00 4 (INVERSION TANED. Кæ

.R4359 • • • • • • • • • • • .07193 .73625 .93249 56850 .55468 .75286 ENCLIDEAN DISTANCE BET, LAST 2 ITERATIONS . . 2668E-01 EUCLIDEAN NORM OF .GT. 1 CUT-OFF = D. EUCLIDEAN NORM OF .LT. O CUT-OFF = .6429F-02.

.93249-.73286 .50850 4 .6546A .84359 .00000 .07193 ,73625

H 3 3 [] · \ F & S + C Т∆-жЗ,

EUCLIDEAN NORM OF LT. D DOT-DEF # 0. FUCLIDEAN NORM OF LT. D DOT-DEF #5 .1688E=01

. 7 52 Su .1-268 . 12345 . 52035 .93250 16268 43471 .51910 .98435 .53632 EURLINESS DISTANCE HET, LAST & TIFRATIONS = ... 3451E-01

DE -5174

. 53632

FUELTHEAR HERE OF RESPLOYED FRAME = - PSSAE. NE

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C EPRIM = _174-F+01 PFH OFHT

ERRUR = _5070E+00 PER CENT

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8, B 5 (]:VEESI_ TALER. DENSITY .6-675 1. 6.699 1,1 65414 ۰. . 640.0 129nb . . esari k 71 07 .64030 .00966 . 101.ip 1.00000 1 03000 . .45413 EUCLIPEAN DISTNICE ALT: LAST 2 TIFRATIONS . . 67165-02. EUCLIDEAN NORM OF RESILVING ERPIR + 1031F+00 6. TEVERSTON TA- 40. X E ↓ 51751 1 Manual J 6109 /1058 .67129 1.0000 66090 ...69502 .nnoon <u>, 61958</u> .56747 , nañañ .64602 . . 68009 1 nonan 67129 1,00000 · •

EUCLIDEAN NORN UF .GT. 1 CHT-OFF . FUCLIDEAN HORH UF .LT. O CUT-DF64733E-02 -1017E-05

TERROR . 2104E+00 PER CENT

7 CINVERSING TAWED. ,K.#., **)** \sim

DENSITY

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1.00000	4 68337	68307	99572 -

FUCLIDEAN DISTANCE BET. LAST 2 ITERATIONS . . 2662F-02

EUCLIDEAN NORH OF .GT. 1 CUT-DFF = .4058E-03 EUCLIDEAN NORH OF .LT. 0 CUT-OFF = .1592E-03

ERROR ... 15434F-01 PER CENT

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E-OLIDEAN NORY OF RESOLVING FRADE = . 3207E-01

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ENCLIDEAD HORH OF RESOLVING ENROR 2651E-02

ERRIR . . 1645F+01 PER CENT

ENCLIDEAN NORM OF .GT. 1 CUTHOFF = 0. EUCLIDEAN NORM OF .LT. 0. CUTHOFF = .1612E-03

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.96191

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.99257 .99141 68544 .687.90

68544 00000 .5879n .9.9141" .00112 .66983 .00112 .68983 .69264 00000 -.69264

DENSITY .99257

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EUCLIDEAN NORH OF RESOLVING ERROR . . . 5374E-02

ERRUR #. , 2880F+01 - PFH CENT

EUCLIDEEN DISTANCE HET, LAST P ITENITIONS # 28465-03

. 94250 ··· . 99005 1-5495

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A. ULARBADH - LEMENT KŻ HELSTTY

ERROR = _ SPASE-01 PER CENT " E ICENTIFEN NER OF RESELVING EXPERIE . RISTEND

EUCLIDEAN NONN UP .GT. 1 COTHOFF # 0. FOCLIDEAN NON OF .LT. D CUTHOFF # 10079E-04

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OF 18114

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EARIR # .115 FFO1 PER DENT

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HE 15 DENSITY (HAVESIAN)

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EUCLIDEAN DISTANCE BET. LAST 2 ITERATIONS # .2412F-04

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ENCLIDEAN NORY OF .GT. 1 CHT-DEF . 0. FOCLIDEAN NORY OF .LT. 0. CUT-DEF . 0.

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KE 18 DENSITY (BAYESTAN)

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ENCLIDEAN NORH OF RESOLVING FRROR #14 .3415E-02

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EUCLIDEAN NORM OF .GF. 1 CUIT-OFF . 0. FUCLIDEAN NORM OF .LT. D.CUIT-OFF. 0.

.56591 .68669 99073 ,9913A .6890a ູດວວວດັ .00112 .6920A .68905 .00115 00000 .99073 .68591 .68669. 99134

RE 17 DENSITY (BAYESTAN)

- ENCLIDEAN NORH OF RESOLVING ERROR . . . 3356E-02 .

ERROR = . . 7929F=02 PER CENT

FUELTDEAN NOR OF LT. D CHT-OFF = 0.

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15 DEWSTTY (BAYESTAN) K 18

ENELTHELN HERE HERELYING ENROR = 3274E-12

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15 DEFATTY (844151ax)

FLOID # 7877F-C2 PER CENT E CLIDERT TORN OF RESULVING ERROR # 1.3462E-32 19 / FE1513+ K \$ INTERSTAN) .68516 .99145 .69208 .ncoba .00112 .6#906 .+9294 . Autor .0011299068 .65605 **.**58564 ,991/15 EUCLIDEAN DISTANCE BET. LAST P ITFHATIONS = .7941F-05 EUCLIDEAN NORM OF .GT. 1 CUT-OFF # 0. EUCLIDEAN NORM OF .LT. D CUT-OFF # 0. EANDE E JAYAE-US DES CENT 20 DEHSITY K 8 (HAVESTAN) .49165. .6#513 .00000 .00000 .68662 .99149 64205 ne111 .68905 5926A +02111 .99065 .65613 56866 .99149 SENCLIDEAN DISTANCE BET. LAST 2 TTERATIONS # .7553E-05 EUCLIDEAN NORH OF .GT. 1 CUT-OFF # 0. EUCLIDEAN NORH OF .LT. D CUT-OFF # 0. ERROR = .7375E-02 PER CENT 3 6 KE 21 DENSITY (BAYESIAN) .99060 .68560 ,99153 .69207 00000 .00111 .68904 .69207-. :00000 _001F1 .68904 6 .99060 .68618 .99153 .68660 - 19 EUCLIDEAN DISTANCE RET. LAST 2 ITERATIONS . .73110 EUCLIDEAN NORH OF .GT. 1 CUT-OFF = 0. EUCLIDEAN NORH OF .LT. 0 CUT-DFF = 0. ERROR # . . 7302E-02 PER CENT

ENCLIDEAN NORM OF PESOLVING ERROR = .3559E+02

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Epreuve illisible

INVERSE PROBLEM 1 (ANNOLAR FLOW) AVENNE 2 -HU. DE ITERATIONS = 22 KΞ 55 DENZITA (BAYESIAN) .99157 . 99.756 .55653 .00000 .00111 .68903 .69207 .59267 .00111 .66903 .00000 .68623 .99056 . 99157. .58658 FUCLIDEAN DISTANCE BET, LAST & ITERATIONS = .7146E-05 EUCLIDEAN NORM OF .GT. 1 CUTHDEF = 0. EUCLIDEAN NOBH OF .LT. 0 CUTHDEF = 0. STATISTICLE ERROR IN DENSITY 00245 .n2¤1n .05458 • UL S 1 1 .00059 ,00374 00363 .00945 .00374 .00948 .00353 . 11-059 .02410 ູກຄວັມຄ 100511 CALCULATED DETECTOR RESPONSES RESPONSE ANGLE. 1255E+02 -,7550E+01 27978-04 1 1462F-04 1255E+02. -,7550E+01 З + 5500E+02 1402E+02 _2420E=04 - 4500F+02 - 57A0E+01 3 = 57 BOE+01 1402E+02 - 3500E+02 4 27735-04 1405E+05 -,5780E+01 1615E-04 5 - \$500E+02 -. 4020E+015 . 1576E+02 -2582E=04 -.4500F+02 6 .1578E+02 -,4020E+01 -2172E-04 -.3500E+02 7 -. 4020E+01 .16496-04 _1578E+02 8. -.4500E+02 .1755E+02 ...23276-04 +.2550E+01 9 1755E+02 -,3500F+02 -.2550E+01 -2163E-04 10 .1755E+02 12556+02 -2327E-04 4500E+02 11 -2163E+04 1755E+02 .12556+02 -3500E+02 12. .1578E+02 .4500E+02 -1402E+02 -5285E-04 13 .3500E+02 1578E+02 1402E+02 .2172E-0ª 14 1649E-04 \$500E+02 -1578E+02 1405E+05 15 4500E+02 -1402E+02 .1578E+02 . 2420F+04 16 1402E+02 .1578E+02 .2773E-04 3500E+02 17 -5500E+02 1402E+02 . .1578E+02 18 -1615E-04 4500E+02 2797E-04 -1255E+02 ~.1755E+02 19 11755E+02 ' -5500E+02 1442E-04 ,1255E+02 20--٦ ERRUR # 7239E+02 PER CENT

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B.1.3 Problem 1 - Avenue 3

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INVERSE PRUBLED 1 (A WULLAR FLOW) AVENUE 3

FULD 21 TIMES SHITCH TO THE ITERATIVE TECH-19UF AFTER 22 IFERATIONS REVE ==1 ISC= 0 HAXING SUMDER OF ITERATION ALLO DOT 200

475

ERROR ALLUMED IN UNFOLDINGEN. TANED. GAM MAED. I IPDINTE I

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- DIA ETER# (10000+02)CM SUUREE NEUTRO, NUTAL CHUSS+SECTIOU# (1011E+0) SUUREE NEUTRON (DNABSORFTION PROMABILITY# (0395E+00
 - FOLOING CALCOLATIONS FOR 37 a 641 a.u 2: FIELIERS

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•		x _	Ť Y	RESPONSE	. EBBAB
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	2	1255E+02	- 7550E+01	_1441E+04	5000E-08
	3	14026+02	+ 5783E+01	2420E-04	-500000-08
-	4	1402E+02	5780E+01	. 2774E-04	-5600E=68
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	· 6	1578E+02	-,4020E+01	.25826-04	SUCOF-CA
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	r	-15/er+u2	44216421	.15498+04	15000E-06
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•	10	.17556+-2	-,255.E+ul	.2162E-34	5000E-08
•	- 1 -1	.17556+-2	•1522F+<5	.2328E-04	5000E-CH
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	17	14026+05	+1570E+32	.2774E-C4	
	18	1402E+35	.1576E+02	.1015E-64	- 5000E-CB
. .	19	15225E+05	.1755E+02 *	.21975-04	.5464E-06
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DENSITY .95771 .58541 .60754 . 59075 00000 .75993 . 69353. .12010 :69353 00000 15010 .76993 95771 56541 .60754 64075

EUCLIDENT DISTANCE BET. LAST 2 ITERATIONS = .17532-01

EUCLIDEAN NORM OF .GT. 1 CUT-DFF = 0. EUCLIDEAN NORM-OF .LT. 0 CUT-OFF = .5873E-03

ERHIR = .1294E+U1 ... PER CENT

16F HESHENT & EKHAR #1 _1785E+11 Ę

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2 (INVENSION TARED. K. . DENSITY 1. 2.000 +46457 .nisubu . 45-64 . 1:51 .05.25 1-5-2 . · · · · · · · · .65025 .01851 -**,** 0000-,74042 .02957 .63550 1.00000 ,93846 EUCLIDEAN DISTANCE BET. LAST 2 TTERATIONS . .2551E-01 EUCLIDEAN NUMB OF .GT. 1 CUT-OFF # EUCLIDEAN NUMB OF .LT. U CUT-OFF # .1951t=01v .coolt=u2 ERROR # .7842E+00 PER CENT EUCLIDEAN ABRM OF RESULVING ERROR = .2309L+00 S JINENSIU TANES. 56-5111 . 05410 1.05060 1.00000 .60115 1.1051 • • • • • • .1.014 61881 .70014 .01900a 1.000000 ,66115 10041 J 1.00000 EUCLIDEAN DISTANCE BET, LAST & ITERATIONS = 7165E-02 EUCLIDEAN NURH OF .GT. 1 LUT-OFF = .1244E-01 EUCLIDEAN NURH OF .LT. 0 CUT-OFF = .2432E-02 / ERRUR = _4410E+00 PER CENT EUCLIDEAN NORM OF RESULVING ENRICH = .1224E+OF

K= 4 (INVERSID: TAW=0.) DENSITY 1.00000 .07512 .07566 .9979

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EUCLIDEAN NORM OF .GT. 1 CUIT-OFF . .3449E-02 ENCLIDEAN NORM OF .UT. D CUIT-OFF. . .4157E-03

ENKUR = .1766E+UU PER CENT

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EICLIPEA- NURY OF REGULVING ERROR = .20111-01 K= 6 (TAVERSID) TANED.)

EUCLIDEAN DISLANCE DET. LAST 2 ITERATIONS = ,1439E-02 EUCLIDEAN NURS OF .GT. 1 CUT-UFF = 0. EUCLIDEAN NORM OF .LT. 0 CUT-UFF = .4091E+03

ERRUR . . 4261E-31 PER CENT

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EUCLIDEAN DISTANCE BET. LAST & ITERATIONS = .9739E-03

EUCLIGEAN NURH LF .GT. 'I CUT-OFF = 0. EUCLIDEAN NORH UF .LT. O CUT-OFF = .2125E-05

ERRUR # .1919E-U1 PER CENT

EUCLIDEAN WORK OF RESULVING ERROR A. SSATEMER.

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EUCLIDEAN NORH OF RESULTING ERROR = .31816-02

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ERROR = .1452E-01 PER CENT.

EUCLIDEAN NORM OF RESULVING ERROR = _2108E+02-

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EUCLIDEAN DISTANCE BET. LAST 2 ITERATIONS * .1195E-03

ERRUR . . 1815E-01 PER CENT

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EUCLIDEAN NORM OF RESOLVING ERHUR = . . 1447E-03

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EUCLIDEAN WISTANCE HET. LAST 2 ITEANTT WS 5 ... 2205-14

EUCLIDER NUMBER OF ALT. DELOT-OFFICE . 1/005-05

ERHOR = VIB77E-01 PER CENT

ENELIDEAN NURS OF RESIDENTS FRANK # 1225521- 5

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EUELINEAR RISTARCE DET. LAST & ITEANTING F AND YER H

HUGLIGEAN ANRY OF GOTA ALCOLANTE DU LA TALEN COLANTA DE LA TALENCE DE LA T ERRIR - . 1631E-01 / PER LENTS

EUCLIDEAS NURS OF RESILVING FRANK . .1445E-03

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EUCLIDEAN NURH OF .GT. 1 CUT-UFF = 0, EUCLIDEAN NURF OF .ET. 0 CUT-UFF = .1720E-03

ERROR . . IHOBE-UI- PER CENT

EUCLIDEAN NORM OF RESOLVING ERROR . . . 4602E-04

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r RAUH = .18116-01 HER CE .1

ny na **Laβûd** Estan Angla Frank Estan Statistika (* angla βαβαλαγία). Na Statistika

К# 15 (1+хЕ∀SI/ Такад.)

NC 25113 48954 ---**-**66955 .06631 .97309 المعادية الجو .04246 . mies .6-908 .04240 _____U_1_1_1_1_ Sec. 125 .96964 .02131 .44344. .00055

C. EVELLORA CONTRACTOR OF CASE & TERMITING TO APPRISH S

Endury = .18142-01 Prof CL-1

K= 16 (157£651,0) 14-#µ,

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EUCLIDEAN DISTANCE HET, 1457 2 ITERATIONS - .56418-00.

*ERROR ** ** .1814E=01 _ PER CENT

KA 17 CINVERSION TANEU.

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60622	.00331	. 99555
	,00522 ,13000 ,03000 ,03000 ,60522	.00022 .00125 .0000 .00125 .00100 .00125

🖣 ាមិនជាមនុស្ស ខណ្ឌាស្នាស់ ដែល មេត្ត ដែលសារ ២ ដោយសារសេន សារាសន៍សៅទីសារី

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E CULCEAR MARCH OF STATE 1 CHIMPLEF & OF

.63522 .0-831 94,559 ,98454 FUCCIDEAN MISTANCE SET. LAST 2. TTEMATIONS - 44151-07

.6,6908-ຸຍະບົດຢີ.... . 69246

.94389. - 08d31 ,98464 .04055 ູບບົວຈີນ . . . 125 . 60.908' .04540 .0.125

R. (135 MAY 10 A ... HOLLT, D.C.) - AP. = (17475-03

20 CINVERSIUN TANED. .K # UENSITY

EUCLIDEAN NOR OF SET. 1 CUTHOFF # 0, EUCLIDEAN NUR SF .LT. 0 CUTHOFF # 0,1729E=03 ERROR . 1818E-01 PER CENT EUCLIDEAN NORE OF RESOLVING ERROR = 6395E-06

69246 .50908 55434 . . 64831 .94389 95454

.00125

.60908

P0055 . 568-1 . 97389

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39 (INERSIA JA =0. N # - UENS117.

ENROR = . ININE VI - MER DE T

ENGLISEAN NERVICE, REACENT OF EVEN AN AN AND AND A

EUCLIDEAN LISTANCE ULT. LAST 2 ITEMATIONS = .19176-05

98964 . 97364 .0co22 . •n#831 .69246 ូមរបន់ព្ .00125 .00400 .09246 .00000 16c430 .95954 *Pan55 ence31 . 44304

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- I-E - SIT'

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ENELTHEAN NORD OF ALTA D. COTADER . ATCHENS ENHIS A ... IDINE-UI PLA-DE-1:

ENCLIPEAN AUR OF NESHLYI & EVROR = 151044-5

TOVERSE PROBLEM 1 LAGRANDE FLU D AVEGUE 3 TOVE OF ITEMATIONS = 21

•	K≭ 21	6111EK510-	1 north	,
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	. 54246	.00000	,00125	66408	
	69246	aua	- 0.0125	60908	

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EUCLIDEAN DISTANCE HET. LAST 2 ITERATIONS * .13518-07.

.68831

99389

EUCLIDEAN NURN OF .GT. 1 CUI-UFF # 0. EUCLIDEAN NURN OF .LT. D CUI-UFF = .17292-03

STATISTICAL ERRORS IN DENSITY

96984

			•	
•		: <u>نا ہے ہ</u> ی	. 3 1211	.00450
		10352	.00573	.0.059
	94A	0.362	0.1373	0.0059
	. 1. 2 4 1 5	100040	.0.211	0.458

CALCULATED DETECTOR RESPONSES

	X	۲	- ESPINSE	ANGLE
3	.1255E+02	- ,7550E+01	.279HE-74	4500E+02
2	1255E+02	-,7550E+01		-,5500E+02
ંક	.1402E+U2	5780E+01	<u>'58542- R</u>	-,45J0E+02
4	1405E+05	5780E+01	2775E- 4	-,3500E+02
. 5	14026+02	5780E+01	.16168	550DE+02
b	1578E+U2	4020E+01	.2533L+.14	-,45002+02
7	1578E+U2	4020E+01	.2175E-04	-,3500E+02
8	1578E+02	-,4020E+01	.10492-04	-,5500E+02
- 9	17556+42	-,2550E+01	.23286-04	-,4500E+02
10	1755E+U2	2550E+01	.21,546-04	3500E+02
11	1755E+U2	.1255E+02	* "5354E-('A	.4500E+02
12	1755E+UZ	.1255E+02	.2164E-04	.3500E+02
13	1578E+02	1402E+02	.2563E=04	.4500E+02
14	1576E+02 -	1402E+05	21.755-04	.3500E+02
15 -	1578E+02	.1402£+0Ż	<u>,1449</u> E=04	\$500E+02
16 -	1402E+02	.1578£+02 ·	u24Eu	4500E+02
17	1402E+U2	.1578E+02	.27/52-04	.3500E+02
18	1402E+02	.1578E+02	.10165-04	• . 5500E+02
. 19.1	1295E+02	1755E+02	. 27938-14	_4500E+02
20	1255E+02	.1755E+02	. 1 4 6 9 E 4	\$500E+02
	-			•

.1815E-01 PER CENT ERROR =

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B.1.4 Problem 1 - Avenue 4

INVERSE PROBLEM 1 (ANNULLE FLOL) AVENUE 4 FOLD 21 TIMES SAITCH TO THE ITERATIVE TECHNIQUE AFTER OLITERATIONS SET = -1 18M= Q MAXIMUM NUMBER OF ITERATION ALLOWED: 20 ERROR ALLOWED IN UNFOLDINGEO. TAMEO. GAMMARC.

484

TANEO. TPUTNTE STORES

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P OFISTTY CALLBARRY ...

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ERROR . . . 3838E+01 PER CENT

ENCLIDEAN NORM OF .LT. O CHT-OFF = 0.

EUCLIDEAN NORN OF RESILVING ERROR . . . 1246E+00

EUCLIDEAN DISTANCE BET. LAST 2 ITERATIONS = .1146E-01 EUCLIDEAN NORM OF .GT. 1 CUT-OFF . 0.

.56284 .53178 . 47722 ,46282 44796 .54874 50693 43051 \$0693 . 447,96 .54874 53178 56284 .47722 46282

1 DENSITY (BAYESTAN)

.50000 .50000. 50000 .50000 50000. 50000 .50000 50000 .50000 .50000 .50000 ERROR = .4626E+01 PER CENT

0 DENSITY Kar. .50000 .50000 ...

. 50000

* **K #** :

	10396400		14415-04	5000E-08
	1402E+02		*5450E=04 ···	5000E-04
	1405E+05	- ,5780E+01.	2774E-04	5000E-08
	*1405E+05		1615E-04	SOODEWOR
	-157 <u>8E+02</u>	4020E+01, "	25825-04	5000E=08
· .	- 1578E+02	-,4020E÷01	21725-04	5000E-00
	<u>-15788+02</u>	40205+01	16495-04	50005-04
	1755E+02	255hF+01	2328F=04	
	17555+02	=.255hE+h1	21625-04	5000E-08
	1755E+02	12555+02	23266-04	500000000
•	17556+02	1255E+02	21625-04	- 3000E - 08
• .	1578F+62	1402E+02	25826_00	
•	157AE+92	14725+02	21125-00	20101-04
	1578F+42-	14025+02	14005-04	2000F-09
	14025-02-	14745-03	10045-00	-2000E-08
	14021-02	15785-07	.24206-04	.5000E⇒08
	14626-02	15705402	2//4E=04	.5000E≠0A
	12556.12		.1615E=04	S0nnE=na
	12555-02	\$1/33E+02	27976-04	5000E-0A
· ·	********	- +1(35E+02	1441E-04	5000E-08

.50000

GIVEN DETECTOR RESPONSES Χ.

2

.1255E+02

-1522+05

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HEFOLDING CALCULATIONS FOR 4 - 9Y - 4 GRTD AND 20 DETECTORS

RESPINSE

.27978-04

.50000

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.14418-04

ERROR

.5000E-0a

SOURCE "EUTEON LONABSURPTIO" PRUBASILITYS - 9395E+00

-.7550E+01

#.7550E+01

INVERSE PROPLEM & CANNILLAR FLOUD AVENUE &
• •	. 55571		44177	45454
6 . T	. 51245	50165	41476	58711
	.51243	311165	1 4447h	55711
	- 55571		. 46177	45454
-				

FORTIFAN FORM OF ST. 1 COTHORE # 0. EURETORAN NORM OR ST. D. CUTHORE # 0.

FARER = . 325 F+01 PEP CE .1

ENCETDEAN NORM DE RESELVING ERROR = .1253E+00

3 OFNETTY PRAVERTEES

	* # *		
57290	60729	45244	41375
51767	34455	36403	61595
51767	34455	2-975	61595
4724		15,24,4	41 175
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FUELTOERS WERE OF LET. 3 CHTHUEF & 0. FUELTOERS NORMORE LET. DEUT-GEFOR 0.

E CLEDEAN WERE DE RESOLATING ENROR = .1266E+00

F. 4 DENSITY (HAYESTAN)

.58453	.70757	44815	.39908
52329	31805	33948	63701
\$52320	31808		63701
,58453 ···	76757		39908

ENCLIDEAN NORM OF .GT. 1 CUT-OFF = 0. ENCLIDEAN NORM OF .LT. 0 CUT-OFF = 0.

ERROR # 2531E+01. PER CENT

5 DENSITY (HAYESIAN)

,59166	74017			. 38927
.52917	29833	1.11	31486	65219
52917	,29833	· .	.31486	65219
59166	74017	· .	44764	38927

EUCLIDEAN DISTANCE HET, LAST P ITERATIONS = .43954-02

FORTERATION OF STATEMENTED FOR

POOR COPY COPIE DE QUALITEE INFERIEURE

E. MITOPAN MOR'S FERENENTING FRRIP & *1540£+00' " -7 . (34455145) -DE STTY 6 . 1432-.59520 8 LOG 21.325 , 29415 1-1515 53517 66513 .53547 29415 \$9359 .59520 .70585 . 04995 . 44526 ENCLIDEAN NORM OF ... OF . 1 CUT-NEE . D. EUCLIDEAN WONN UF C CHI-DEF = 0. ERHUR = _2170F+01 PER CHNT ENCLIDEAN MORE OF RESULVING ERROR = .1306F+00 : (BAYESTAR) / DF1 S1TY * 8 - 36014 . 545 55 - 74551 .05424 ٠. .67111 .27147 .27557 .54194 21650 .67111 **,**541.9a 27147 36018 45428 .595×h .705.11 EUCLIDEAN DISTANCE BET. LAST 2 TIEPATIONS . . . P755H-02 LEUCLIDEAN NORM DE .GT. 1 CUT-DEF # 0. EUCLIDEAN NORH OF .LT. O CUT-OFF . 0. ERROR = 120565+01 PER CENT -EUCLIDEAN NORY OF RESULVING ERROR = .1316E+CO 8 DENSITY (RAYESIAN) .37932 59427 45995 .80012 ,67705 .54840 26196 .26124 .26154-.67705 .20196 .54540 .37932 .45995 59427 10012 FUCLIDEAN DISTANCE BET. LAST 2 ITERATIONS . . 22528-02 EUCLIDEAN NORM DE .GT. 4 CUT-OFF. . 0. ENGLIDEAN NORH OF .LT. O CUTHOFF # 0. ERROR . . 1964F+01 PER CENT-EUCLIDEAN NORH OF RESOLVING ERROR . 1324E+00 . - ^В (р. . 9. DENSITY (BAYESTAN) ,46646 .59693 38016 _81060⁽_______) ,25405 24789 .68158 .55471

25405

_A1050____

\$5471

,54093,

.24789

.46545

,6815A

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EUCLIDEAN NORM OF GT. 1 CHT-OFF # 0. EUCLIDEAN NORM OF .LT. 0 CHT-OFF # 0. FRRDR . . . 1702F+01 PER CENT E GUIDEAN SHEE OF RESOLVING ERROP' + 1342F+00

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57.429 69041 .57.211 23593 .21586 23593 . 59041 .21586 .57211 .48779 .82504 .38923 57 / 2.9 EUCLIDEAN DISTANCE BET, LAST P ITERATIONS . . 1453E-02

KE SZ DEHSTTY (BAYESTAN) . 4A7.79 .82504 .38923

FUELIDEAN NORH-OF RESPLATING FRROR #1 .1339E+00

EUCLIDEAN DISTANCE BET, LAST 2 ITERATIONS = .1542E+02 EUCLIDEAN NORM OF .GT. 1 CUT-OFF = 0. EUCLIDEAN NORM OF .LT. 0 CUT-OFF = 0.

11 DEVELTA (HAVESTAN) _A2240 38539 _480h1 +58062 24131 22545 .68802 .56660 24131 +68802 -56560 .22545 . 38539 82240 .48061 .58062

E HOLTOFAN WARA OF ALTA & CHITHIPPE # 14 FURDA = _1H14F+D1 PER CENT FUELTDEAM NOR OF RESILVING ERROR - 1336E+004 .

FUELTDEAM KORN OF SITE TENTEDER # 01 -

. 47 544 .19559 .58626 .41778

. 24728 .23605 .68514 .56080

.47304

.23605

49556

.68514

REPORTER HERE OF RESILVE G FRAME . . 1341E+05

(RAVESTAR)

.81778

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ENCLIDENT WERE OF ST. 1 CHT-OFF = 0. FICE TOFARE ST.

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К.#	15° DE					14 g		16
	15° DE		F 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3					
K.	15 DE	.82	364	.50821		40360		
K =	15 DE .55322	.82	364 198	.50821	•	40360		•
K a	15 DE 55322 56693	28. 28.	364 195	.50821 .19173		40360 69573		•
K.	15 DE .55322 .56693 .56693	28 22 22	195 195 198	.50821 .19173 .19173		40360 69573 69573		
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K #	15 DE .55322 .68693 .58693 .55322	28. 22. 28. 82.	564 564	.50821 .19173 .19173 .50821		40360 69573 69573 40360		
K #	15 DE .55322 .55693 .56693 .55322 CLIDEAN D	.82 .22 .22 .82 .82	564 195 198 564 Ret. Lai	,50821 ,19173 ,19173 ,50821 ST 2 ITERI	TIONS	40366 69573 69573 40360	\$11E-D2	•
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K # E UC	15 DE .55322 .68693 .58693 .55322 CLIDEAN D FUCLIDEA	.82 .22 .82 .82	964 198 198 564 BET. LA	.50821 .19173 .19173 .50821 ST 2 ITER	ATIONS = 0.	40360 69573 69573 40360	111E-D2	•
K #	15 DE .55322 .56693 .58693 .55322 CLIDEAN D EUCLIDEA EUCLIDEA	.82 .22 .82 15TAMCE N NORH D	564 198 564 BET. LAS F .GT. S	50821 .19173 .19173 .50821 ST 2 ITER/ I CUT-OFF CUT-OFF	ATIONS = 0.	40368 69573 69573 40360	311F- <u>D</u> 2	
K# E110	15 DE .55322 .58693 .56693 .55322 CLIDEAN D EUCLIDEA EUCLIDEA	.82 .22 .82 .82 .82 N NORH 0	564 198 564 BET. LAS F .GT. S F .LT. S	.50821 .19173 .19173 .50821 ST 2 ITER I CUT-OFF D CUT-OFF	TICNS = 0.	40368 69573 69573 40360	\$11E-02	•
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K #	15 DE .55322 .56693 .55322 CLIDEAN D EUCLIDEA EUCLIDEA ERRUR = EUCLIDEA	.82 .22 .82 TSTAMCE N NORM D .1557E+ N NORM D	564 198 564 BET. LA F. GT. F F. LT. C D1 PFR F. RESOLV	.50821 .19173 .19173 .50821 ST 2 ITER I CUT-OFF CUT-OFF CENT- VING ERRUE	TICINS . 0.	40368 69573 69573 40360	511E-02	
K.	15 DE .55322 .56693 .56693 .55322 CLIDEAN D EUCLIDEA EUCLIDEA ENRUR = EUCLIDEA	.82 .22 .82 1514MCE N NORH D .1552E+ N NORH D	564 198 564 F .GT. LAS F .LT. (01 PER F RESOLD	50821 .19173 .19173 .50821 ST 2 ITER I CUT-OFF CCUT-OFF CENT- VING ERRUE	■ 0.	40368 69573 69573 40368 3 • : • 1	311F-D2	
K = E 10 	15 DE .55322 .5693 .5693 .55322 CLIDEAN D EUCLIDEA EUCLIDEA ENROR = EUCLIDEA	.82 .22 .82 .82 .82 .82 .82 .82 .82 .82	564 198 564 BET. LAS F .GT. F F .LT. G O1 PFR F RESOLV	.50821 .19173 .19173 .50821 ST 2 ITER I CUT-OFF CUT-OFF CENT- VING ERRUE	■ 0.	40368 69573 69573 40360 3 • 1	511F-02	
K =	15 DE .55322 .56693 .55322 CLIDEAN D EUCLIDEA EUCLIDEA ERRUR = EUCLIDEA 16 DE	.82 .22 .82 TSTAMCE N NORH D .1557E+ N NORH D NSITY	564 198 564 BET. LAS F .GT. S F .LT. S O1 PFR F RESOLS (BAYEST)	,50821 ,19173 ,19173 ,50821 ST 2 ITER I CUT-OFF C CUT-OFF CENT- VING ERRUE	■ 0.	40368 69573 69573 40360 3 • . 1	511E-02	
κ	15 DE .55322 .56693 .5693 .55322 CLIDEAN D EUCLIDEA EUCLIDEA ENCUIDEA ENCLIDEA ENCLIDEA	.82 .22 .82 TSTANCE N NORH D .1552E+ N NORH D NSITY	564 198 564 F .GT. LAS F .LT. (01 PER F RESOLD	.50821 .19173 .19173 .50821 ST 2 ITERA I CUT-OFF CCUT-OFF CENT- VING ERRUE	■ 0.	40366 69573 69573 40360 3 • : .1	\$11 F-D2	
K=	15 DE .55322 .56693 .56693 .55322 CLIDEAN D EUCLIDEA EUCLIDEA ENCLIDEA ENCLIDEA 16 DE .54596	.82 .22 .82 ISTANCE N NORM D .1552E+ N NORM D NSITY .82	564 198 564 8ET. LAS F .GT. F .LT. 01 PFR F RESOLV (BAYEST/ 443	.50821 .19173 .19173 .50821 ST 2 ITER I CUT-OFF CCUT-OFF CENT- VING ERRUE	атј(INS # 0. # 0.	40360 69573 69573 40360 3 • 1 1347E+(311F-02	
K =	15 DE .55322 .56693 .56693 .55322 CLIDEAN D EUCLIDEA EUCLIDEA ERRUR = EUCLIDEA 16 DE .54596 .59138	.82 .22 .82 N NORH D N NORH D .1557E+ N NORH D NSITY	564 198 564 BET. LAS F .GT. S F .LT. S O1 PFR F RESOLD (BAYEST) 443 781.	.50821 .19173 .19173 .50821 ST 2 ITER I CUT-OFF CUT-OFF CENT- VING ERRIE LN) .51444 .18488	E 0.	40360 69573 69573 40360 3 • 1 1347E+0 40899 69710	511E-D2	
κ	15 DE .55322 .56693 .55322 CLIDEAN D EUCLIDEA EUCLIDEA ERRUR = EUCLIDEA 16 DE .54596 .59138	.82 .22 .82 TSTAMCE N NORH DI .15572+ N NORH DI .15572+ N NORH DI 	564 198 564 8ET. LA F. GT. F F. LT. (01 PER F. RESOLI (BAYEST) 443 781	.50821 .19173 .19173 .50821 ST 2 ITER I CUIT-OFF CENT- VING ERRIE IN) .51444 .18488	TIGNS = 0. -	40360 69573 69573 40360 3 * . 1 1347E+(40899 69710	411E-02	
κ Ευ κ	15 DE .55322 .56693 .56693 .55322 CLIDEAN D EUCLIDEA EUCLIDEA EUCLIDEA EUCLIDEA EUCLIDEA 16 DE .54596 .59138 .59138	82. 22. 82 1574MCE N NORH D 1552E+ N NORH D NSITY 82 21 .21	564 198 564 8ET. LAS F .GT. S F .LT. S 01 PFR F RESOLD (BAYEST) 443 781 781	.50821 .19173 .19173 .50821 ST 2 ITER I CUT-OFF CENT- VING ERRUE .18488 .18488	■ 0 . ■ 0 .	40360 69573 69573 40360 3 • 1 1347E+(40899 69710 69710	311F-02	
K =	15 DE .55322 .58693 .58693 .55322 CLIDEAN D FUCLIDEA FUCLIDEA ENCLIDEA ENCLIDEA 16 DE .54596 .59138 .59138 .59138	.82 .22 .82 .82 .82 .82 .82 .82 .82 .82	564 198 564 8ET. LAS F .GT. S F .LT. S O1 PFR F RESOLS (BAYEST) 443 781 781 443	.50821 .19173 .19173 .50821 ST 2 ITER I CUT-OFF CUT-OFF CENT- VING ERRUS .18488 .18488 .51444	■ 0.	40360 69573 69573 40360 3 • 1 3 • 1 40899 69710 69710 40899	511 F-02	
κ	15 DE .55322 .56693 .55322 CLIDEAN D EUCLIDEA EUCLIDEA ENCUIDEA ERRUR = EUCLIDEA 16 DE .54596 .59138 .59138 .59138	.82 .22 .82 TSTAMCE N NORM D .1557E+ N NORM D .1557E+ N NORM D .21 .21 .82	564 198 564 BET. LAS F .GT. S F .LT. S O1 PFR F RESOLS (84YEST) 443 781 781 443	.50821 .19173 .19173 .50821 ST 2 ITER I CUIT-OFF C CUIT-OFF CENT- VING ERRIE .18488 .18488 .18488 .51444	E 0.	40360 69573 69573 40360 3 • . 1 3 • . 1 40899 69710 69710 40899	411 F-D 2	

ET DI ICEAN NORM OF .GT. 1 CHT-OFF . 0. F FLIDER HORADOF .CT. G CHT-OFF ...G.

FORETHEAN MORE DE RESOLVING ERROR . 1548F+00

490

17 DE STIY (PAYESIA-)

53475	HUDAR	- 5pn (5	41457
54553	23.589	17452	69832
54563	213an	17852	69432
55875	тыланы	\$52035	.41457

· · · ·

FUELIDEAN NORM OF .GT. 1 CUT-DEF # 0. FUELIDEAN NORM OF .LT. 0 CUT-DEF # 0.

ERROP # _1465F+01 PER CENT

EUCLIDEAN NORS OF RESILVING FRROR # 1349F+00

18 DEI STTY (HAVES144) N 2

<u>_54164</u> .82111 42028 59970 59970 211993 69943 .17258. 2:1943 .17254 69943 .53164 A2111 52591 42028

ENCLIDEAN NORM OF .GT. 1 CHITHOFF & 0. ENCLIDEAN NORM OF .LT. 0 CHITHOFF . 0.

ERROR . _ 1425E+01 PER CENT

EUCLIDEAN NORM OF RESOLVING ERROP = .1350E+00

19 "DENSITY (BAYESIAN) K 🕿

.52466	- 81925	.53115	.42610
.60361	20619	16702	70044
60361	20619	16702	70044
52466	81922	53115	42610
• - ·			÷

FUCLIDEAN DISTANCE BET, LAST 2 ITERATIONS =' .1168E-02

EUCLIDEAN NORM OF .GT. 1 CUT-OFF # 0. EUCLIDEAN NORM OF .LT. 0 CUT-DFF # 0.

ERRAR = 1386E+01 YPER CENT

20 DENSITY (BAYESIAN) K E

		•	
-51784	81728	53607	43197
61738	20256	16180	.70137
67758	20256	16180	70137
51744	A1729	53697	43197
• •			the first second

POOR COPY COPIE DE QUALITEE INFERIEURE



INVERSE PRUBLER 1. (ANNULAR FLOW) AVENUE 2

22 DENSITY (BAYESTAN) KΞ

.99056	68623	.68658	99157
69207	00000	00111	68903
69207	00000	.00111	68903
99056	68653)	68658	99157

EUCLIDEAN DISTANCE BET. LAST ? ITERATIONS = .7146E-05

-EUCLIDEAN NORH OF .GT. 1 CUT-OFF . 0. EUCLIDEAN NORH OF .LT. O CHI-DEF + 0.

STATISTICAL ERROR IN DENSITY

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1 O

.12

13

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18

20.

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02010 00240		.00211		00456	
	•	.00374	· ·	00059	•
200948	• •	00374		.00059	
02410 00240		.00211		_0045B	
				· .	· ·

CALCULATED DETECTOR RESPONSED

		Y	PESPONSE	ANGLE
.	12556+02	7550E+01	.2792E+04	. 4500F+02
•	1255E#02	7550E+01 (1412E-04	5500E+02
<u>,</u> .	14056+05	57.80E+01	2341E-04	- 4500E+02
•	1402E+02		2752F-04	4,3500E+02
	1402E+02	+,5780E+01	1541F=04	5500E+02
• : ~	1578E+02	-,4020E+01	_2648E=04	
	1578E+02	+ 4020E+01	· 2057E+04	3500E+02
r	1578E+02	+,4020E+01	1670F-04	= 550DE+02
:	1755E+U2	2550E+01	23801-04	
	1755E+VC		23075-04	=.3500E+02
•	17556445	+123324VE	22595-04	-4500E+02
	15785402	10025+02	.2648F=04	4500E+02
	15785+02	1902E+02	2057E=04	-3500F+02
	1578E+02	1402E+02.	1670E-00	5500E+02
•	1402E+02	- ,1578E+02	.2341E-Ca	4500E+02
•	1402E+02	.1578E+02	.2752E=04	3500E+02
ن ې	1405E+05	- 1578E+02	.1581E+04	.5500E+02
•	.1255E+02 .	.1755E+02	.2792F-04	45005+02
	. 1522E+05	.1755E+02	,14 <u>12E-04</u>	\$500E+02

7239E+02 PER CENT ERRIR #



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B.2 Problem 2 /

ANNULAR FLOW (015) MENTE CARLO RESULTS -

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Ċ.

FOLD - 21 TIMES SWITCH TO THE TERATIVE TECHNIQUE AFTER 5 ITERATIONS KEY = -1 ISM = 0

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MAXIMUM SHAREA OF ITERATION ALLOUED: 20 ERROR ALLOWED IN UNFOLDING:0. TAW: .1600E+02 TPOINT: 1

POOR COPY COPIE DE QUALITEE INFERIEURE

POOR COPY COPIE DE QUALITEE INFERIEURE

EUGLIDEAN NORM DE .GT. 1 CHT-DEE . EUGLIDEAN NORM DE .LT. O CHT-DEE . .1913E-01

HERE BERRENTER BRUDE STOREFTS

.87815 1.00000 1.00000 7214A 94390 16288 1,00000 ្នំពល់ព័ល្ 18111 1.00000 95473. ຸດດາດກໍ .87361 1,00000 1.00000 . 71634

DENSITY

ERATE - ASSAFTAL PER CENT

1 (INVERSION TAWE . 1600F+02)

FRADE = . 9228E+01 PFR CENT

, bonon. 🦾 😗	_00000	1.00000	1	00000
ຸດດາວດ - 🤅 🤅	00000-	1.00000	- 1	00000%
	ຸດບົດກໍດີ	1 00000	1	.00000
ounda 1	10000	1,00000	1	00000

. 0 DENSITY

2.				
	-1255E+02	.17558+02	1405E=04	94155-06
•	.1255E+02	1755F+02	-502 at =04	1438E#05
· · ·	* 1/402E+05	. 1978F+92	1521E+N4	1019F+05
	.1402E+02	1578F+02	,2907F=04	
	1402E+02	- 1578E+02	2331E-04	. "1492E=05
	157#F+F2		.1801E=04	
	.15785+02	. 14n2F+n2.	-5139E=04	1391F=05
	157AF+62	. 140pt+12	.2495F-04	1621F=05
1 L	.17556+12	125hF+02	\$155E=04	1422E+05
	.17557+1211	12555+12	2662E=04	.1411E=05
	.1755F+02	+ 2550A+03	\$125E=04	1422F-05
	.1755#+22	- 255"F+11	2423E=04	1390F=05
-	157,46+2	0821 F+11	1401E=04	,1171E=05
·.	157AF+02	-,4020F+01	.2139E=04	1301E+n5
	.1578E+02 ···	=_4020F+01	_2495E+04	16216-05
•	_1402F+02	- 5780F+01	15218-04	. 1019E=05

GIVEN DETECTOR RESPONSES

.1.255E+02

1255F+02

14026+05

1405E+05

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50%

SOURCE, NEUTRON NOVARSORPTION PROBABILITYE HUPPH DING CALCULATIONS FOR 4 GRIE AND - **4** Y - 4

93956700

20 DETECTINES

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.14308-05

.94155-04

14925-05

1686E=05

RESPINSE

2978E=04

14056-04

2331F=04

.2907E=04

DIAHETERS INDUE+02 [4 STURTI FUTPON TOTAL CRUSS-SECTIONS 1011E+05

=17550E+01

-,755 1F+01

-.578(E+1)

-.57,80F+01

ASTONIAN FURN (0,5) HONTE CARLO RESOLTS

K# 2 11-9F#ST99 TA+# 1600F+021

DEVSITY

	-				. 1
11	. 45a5u	7 / 4 3 4	.73413	1,00000	``
2	.87727	1.5745	00000	94025	
	.88773	กลุ่มักส	00000	1. 1. 10000	
	97254	711944	73401	1,0000	
			-		

495

EUCLINEAN DISTANCE BET. LAST & TIFKATIONS # .171PE-01

EUCLIDEAN NORM OF .GT. 1 COT-DEE . 1525E-01 EUCLIDEAN NORM OF .LT. 0 COT-DEE . 2038E-01

ERROR # "2807F+01 PER CENT

ENGLIDEAN NORM OF RESOLVING FRADE . 5394E+00

DEUST 71941 95839 - 68044 1.00000 0477A94911 45264 .57214 85845 .00000 ,9751A 1,00000 ********* 1.00000 .74452

BUCLIDEAN DISTANCE HET. LAST 2 ITERATIONS . .6993E-02

ENCLIDEAN NORM OF LC. 1 CHI-DEF = .1248E-01 ENCLIDEAN NORM OF LT. D. CHI-DEF = .2038E-01

ERROR # "P469F+01 PER CENT

4 (I***FRSTN: TAHE .1600E+021-

DENSITY

97227	7193A	66100	1,00000
84547	04614	0000	92576
85165	1 16853	10000	95472
1.00000	70400	.65810	1.00000

ENCLIDEAN DISTANCE BET. LAST 2 TTERATIONS . . PAIGE-02

EUCLIDEAN NORM UF .GT. 1 CUITHDEF # .1119E-01 EUCLIDEAN NORM UF .LT. 0 CUITHDEF # .1968E-01 ERROR # .2305F+01 PFR CENT

POOR PRINT Epreuve illisible

92169	49571	62930	91 490	
AINAN	14212	0.00.10	84925	
51717	10264	ักกกษ์ก 🦢	A7452	
95112	64262	627.50	91211	
-		a de la composición d	a set a s	

FUCLIDEAN NOP OF LT. O CUT-DEF . 0.

EPHOR = 1759E+01 PER JOLAT

EPERTORNAL OF RESILVING FRANK . . . 5523E4.05

6 DENSITY (BAYESIAN) . K #

90143 64336 61804 A	6755
40100 03070 00000	
	1244
17786 05924 00000 B	3564
	HURA

 $\sim t$

FURDIDEA DISTANCE HET, LAST P TTERATIONS # - 5751<u>E</u>-02

ERRIF + 13570F+114 PER DENT

FUCUTTEAN FORM OF RESOLVING ERROPIE ST. 5418FAND

7 DEMSTTY (HAYESIAN) ĸ

89261	69882	61573	A4068
50125	03820	00000	79538
80795	05694	00000	A1741
92252	68794	.61550	A3752

EUCLIDEAN NORH OF .GT. 1 CHT-DEF = 0. EUCLIDEAN NORH OF .LT. O CHT-DEF = 0.

ERROR . . . ISAUE+01 - PER CENT

8 DE: SITY (HAYFSJAN)

.88697	79613	.51687	A2338
60413	03695	00000 5	78786
A1179	5512	ູ້ຄຸດຄວາມ	AC916
91729	6 A9581	61742	A1997

FILLTHEAS SERVICE OF TO POST OF THE SO.

POOR PRUT Epreuve illisible

FUCLIDEAN NORM OF RESOLVING ERROR . .5334F+00 9 DE STTY (HAYESTAN) ,71302 .41923 _A6139 -AC724 . 1506 nanan. . 18480 .05153 .80565 41381 91193 .70309 . . 2046 .80742 EUCLIDEAN NURH OF .GT. 1 CHI-OFE = 0. EUCLIDEAN WORK DE .LT. O DUT-NEE . O. ERROR = _1449F+01 PER CENT EURLIDERS NURS OF RESOLVING FRRDR # 15301E+00 10 DE-STIV (HAYESTAN) A7492 . 62193 .46128 . **,**71881 . 8 3992 . 26020 . 28390 .×0432 .61626 .05205 **,** hisoual . .70917 .90553 .42374 .79760 EUCLIDEAN NORM OF .GT. 1 CHILDEE # G. EUCLIDEAN NURH DE .LT. D CHT-DEE = D. , FRROR ... 1036F+01 PER CENT 11 DEKSTTY KΞ (HAYESTAN) .86745 .72347 ,79319 .62463 .81196 . 03387 .00000 78367 .A0397 +81808 .05064 ,00000 . 49804 78941 .71404 .62695 EUCLIDEAN DISTANCE BET, LAST 2 ITERATIONS # .1114E-02 EUCLIDEAN NORH OF .GT. 1. DUT-OFF = 0. ENCLIDEAN NORM OF LT. O CHT-OFF # 0. DEHROR # ,1426F+01 PER CENT Κ∎ - (HAYESIAN) .85913 .72717 ...78619 .62724 .41348 .03244 .00000 78421 AGUAN .4194.00 .nonn _04928

.71745

R 964

76251

. 42444

POOR COPY COPIE DE QUALITEE INFERIEURE ENCLIDEAN DISTANCE HET, LAST 2 TIERATIONS = 1.10650-02

TO FUELTOFAN WORN OF GT. 1 DUT-OFF # 0.1. EURLTOFAN WORN OF LIT. O CUT-OFF # 0.1.

FRANK # 141PF+01 PF9 CF07

FUELTREAM MORE MESNERTHE FRANK # . STARTAGE

- PE 13 DESTTY (RAVESIAN)

	-	•		
ABASO	•	72198	A 3 7 1	77500
.82045	- <u>-</u>	04797	.00000	<u>80419</u>
A1465	•	03205		76467
85-20		73010	.62972	.77997
· •	· .	•	· _ ·	

[™]F<ΩLTDFAR HOR'S DF ".DT. 1 DHT-"FF = 0. SIMUTERT STRUCK F.T. P. DHT-"FF = 0.

1 FAR 10 = (14)1F+11 PFR FF.1

FURLTHEAS OTHER PRESENTER FRENCH = . . \$137F+11

KE AU OF STTY FAAVESTAN

A		• 44	
840A4	73247	63204	77435
.51557	63118	, annon,	78515
82107	r 4670.	. <u>, nonco</u> :	R0439
87109	78346	63545	77027

EUCLIDEAN MORM OF .LT. 9 CHT-DEF . 0.

FRROR # 1405F+01 PER CENT

HE 15 DENSITY (BAYESTAN)

	and the second		
83120	73441	63435	76923
A16.54	03035		78561
82165	04548	nnnnn 🔆 📑	80458 ···
86131	72550	63746	76505

ENCLIDEAN DISTANCE BET. LAST 2 ITERATIONS = .10176-02

ENCLIDENT HOPE OF .GT. 3. CUT-OFF . 0. ENCLIDENT NOT OF .CT. D.C.T-OFF . D.

F44-9 . 1400F+01 PFR CENT

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STATES AND A THREE OF RESIDENT & FRANCE A STREET, AND

CHARGE TO

POOR COPY. COPIE DE QUALITEE INFERIEURE

,7:56:03 . 661 (1) +53651 274452 78604 ~2953 (0442481703 . = = = 1'a acono .90474 .55136 ,76722 . . 64135 ,76025 ENCLIFENS WORK DE .GT. 1 CHTWDEF . D. ENCLIDEAN MORE UP .LT. D DUT-DEF. . D. ERROR = (180% + 0) DER DE T EUCLIDEAN MARY OF RESOLVING FRADR. . . 49936+00 A KE 1117 DEMSTTY (RAYESTAN) 73742 +3957 .81154- * .76014 .81765 .02875 .78644 04313 .A2261 <u>, honon</u> • , A0468 7-570 ****** ,75583 .5-251 ç. FORLTOFAN MORE OF ST. 1 DUA-OFF & G. EUCLIDFAN MORE OF ST. 0 CUITOFF * C. K CRYCH = .1389F+01 PER CENT KE 18 DENSTTY (BAYESTAN) .80165 . 64154 . 75607 73864 .02799 .81A31 -> 786A1 ...8230.4 .00000 . 40499 04201 .831,30 .73000 .75166 EUCLIDEAN NORM OF .GT. 1 CUT-OFF # 0. EUCLIDEAN NORM OF .LT. 0 CUT-OFF #-0. ERROR = .1384F+01 PER CENT EUCLIDEAN MORM OF RESOLVING FRROR . . 4896E+00 19 DENSITY (BAYESTAN) K B

73972 64242 .75225 .79179 02725 787.57 A\$894 _`00000 _______ A0509 .82352 ______ 6/1679 7477A 73117 <u>, 82129</u>,

ENCLIDEAN DISTANCE BET, LAST 2 TTERATIONS # . . 94806-03

E FETDEAN WORN OF .GT. 1. CHT-DEF. . 0. F DITOTAN MORY OF .I.T. C FOT-DEF. . D.



POOR COPY COPIE DE QUALITEE INFERIEURE

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F DITOFAN WORK OF RESPLIT D FRENC . 48476+00

)					. 3	
	18.8	24	DE-51TY	14445	51711	
)		. 7~1	4.4	7		 7

	1-166	7 , 7 2	. 6 9 9 7 1	_74495	
	41449	12655		78751	
•	82400 ·	0.39AA	ู้กกอออ	80518	
	41133	73225	- 64512	74413	

EUCLIDEAN UTSTANCE BET, LAST P TTEMATTONS + _ 9541E-03.

ENCLIDEAN NUR- IF .GT.) DUT-UFF & G. FUCLIDEAN NURM OF .LT. D CUTHUFF & O.

EPROR = .1374F+01 PER CENT

POOR PRINT Epreuve illisible

ERRIH = 1625F+11 PER CENT,

				• •
:	. 1255E+02)		29265-00	- 4500F+02
2	+1522+05	7556E+01	14444-04	÷.5500E+02
- 3	1402F+72	- 57A0E+01	25100-04	- 4500F+02
4	-1402F+(2)	=_57A0E+01	2852E-04	- 3500E+02
- 5	.1402E+∂200	-,5730F+01	17105-04	+ 5500F+02
5 6	1578F+02	35-4020E+01.	29304-04	+.4500E+02
7	.157AE+02		23205-04	+ 1500F+02
- H	157AE402	-,4020E+01	1732E-04	+ 1500F+02
່ ວໍ.	- 1755E+02	2552E+01	2448E=04	- 5005+12
11 - Carl	1755F+02(****	2550F+01	2415F-04	- 3500F+02
11	.1755E+72	12555+02	2459F=04	4-00F+02
12	1755E+02	.1235F+02	24046-04	3500F+02
13	- 1578F+02	1402E+12	24125-04	4500F+02
10	.1578E+02	. 1402F#02	.2307F=0#	35005+02
15	1578E+02	.1402F+02	1734F=na	5500F+02
16	(,)402E+02:)	.1578F+02	. 249AF -na	4500F+02
17	,1407E+97.	+157AF+02	.2859E+04	3500F402
14	-1402E+02	.157HE+12	17025-04	.5500E+02
.19	:.1255E+^?	1755E+02	2934F-04	4500++12
્ ટ ્ર	-1255E+32	.1755F+02	.1458F.=04	5500F+02
		and the second		

.12045

.15486 , 15552 .15772

RESPOSE

ANGLE

CALCOLATEN DETECTOR RESPONSES

X.

ъ. 3

• .	5(34)	13585	
	Ph/25	10414	16797
N	. P4718	17007	1.1171
-	.5 47 2	1.5977	102,501
•			1

STATISTICAL FARDA IN DENSITY

FUCLIDEAN NER OF .GT. 1 ("THEF . 4607E-02" E-CLIDEAN NER DE .LT. (CHI-SEE . . 1747F-01)

EHOLISCAN DISTANCE BET. LAST P. TIERATIONS = 2967E-01

	DF451	T.Y		
. :	1. 1. 175	7 450	- 3004	1.0.01
	.84909	03789	กว่าเรื่อง 🍐	
	.85375	05833	or him .	.A124
-	1,00000	59074	.h3516	949n
· .		•		

P) (11) (+#51)) (TA +# 13) (5) (+ 12)

AN HEASE FORM CHEST - HAMTE DARLH RESULTS DE OF TTERATED SEE - AT JA

B.3.1 Problem 9 (Error-Free Results)

EXPERIMENTAL TEST SECTION A (ERROR-FREE)

FOLD 21 TIMES SWITCH TO THE ITERATIVE TECHNIQUE AFTER O ITERATIONS KEY= -1 ISM= 0 MAXIMUH NUMBER DE ITERATION ALLOWED= 10 ERROR ALLOWED IN UNEDLDING=0. TAM=0. IPDINT= 0

502

POOR PRINT Epreuve illisible

POOR COPY COPIE DE QUALITEE INFERIEURE

K. 2 DEMSTTY CHAVESTAND

EUCLIDEAN HORH OF RESULVING ERROR . . . 2008E+00

ERROR = .3073E+G1 PER CENT

EUCLIDEAN HURH UF .GT. 1 CUT-OFF = 0. EUCLIDEAN NURH OF .LT. 0 CUT-OFF = 0.

·***.

- >

.78737	76734	,75576	.74934
. 82229	78850	76544	75354 .
. 86803	82482.	78848	7.6283
.89777	87811	SS122	78774

KE WALL DENSITY CUAYESIAND

ERRUR = .9218F+01 PER CENT

				· ·
•	.50000	50000	.50000	Sanon
۰.	50000	50000	50000	50000
	50000	50003	50000	Sunnn
	.5000h	50000	50101	50000

KE O DEFSTTY

	X	Υ,	RESPONSE	ERROR
1	.1167E+07	4590E+11	.244AE+04	5000E+08
2	*1195E+0S	-,9590E+01-	2771E-04	5000E-08
3	*1195E+05	-,9590E+01	3369E+04	-5000F+04
4	1165E+05	÷.9593E+01	3125E+04	5000F-04
5	.1250E+02	8710F+01	14118-04	5000E=0A
5	~1250E+42	8710E+01	29055-14	5000F=08
7	1250E+02	871.9E+01	3378E-04	5000F=04
5	1250E+02	871.5+01	31005-04	50005-04
9	121250E+02	- 8710F+01	23085-04	5000F-04
٥.	1339E+CS	- 7826F+01	1748F=04	SOOVEROA
1	13396+05	- 78205+01	21568-64	SCORE-04
2	13305+05	- 7427F+01	- 31418-04	50005-0
3	13396+02	- 78225+01	13350E-04	50005-04
	13396+12	- 7820E+01	3326F=04	50000-08
5 ·	14276+02	- 6940F+01	2420F-04	50005-04
5	14276+02	- 59/06+01	32156-04	5000E-08
7	1427F+02	4.6900F+01	43331-14	STACE-na
5	1427E+02	= 6940E+01	15706-16	SCOOF-ON

GIVEN DETECTOR RESPONSES

Ø

UNFOLDING CALCULATIONS FOR 4 BY 4 GRID AND 18 DETECTORS

DIAMETER# "4679E+01 CM Source Neutron Total Cruss-Section# .1011E+00 Source Neutron Novabsorptio: Prubability# .9395E+00

EXPERIMENTAL TEST SECTION A (ERROR-FREE)

	• •	1. S.	•				••
	••				•	÷.,	
	POOR	COPY		••••••••••		- <u>-</u>	
· .	COPI	E DE (QUAL	ITE	E IN	FERI	EURE

EUCLIDEAN DISTANCE BET, LAST 2 ITERATIONS # 2563E-02

.04949

.98926.

EUCLIDEAN NORH DE .GT. 1 CUT-OFF # .1732E-03 -EUCLIDEAN NORH DE .LT. TO CUT-OFF # 0.

5 DENSITY (BAYESIAN) . 99189 498940-.99861 .98175 99049 93682 9953A 99879 1.00000 .98750 .94169 . . .

EUCLIDEAN HORM OF RESULVING ERROR = ...2463E+00

1, 10000. 1,00000

ERROR # 22362E+00 PER CENT

EUCLIDEAN DISTANCE BET. LAST P. ITERATIONS = \$701E+02

≺ ≊	n nEXS1	ITY (BAMES	TAN)	<u>.</u>
	96851	9789/	97414	96575
	99443	98566	97733	9712A
	1_00000	99350	98244	97407
	1.00000	s nonon 🤅	49669	.97990

ERROR = _5395E+00 PER CENT

EUCLIDEAN NOPH OF RESULVING FARDA # _2413F+00

EVELIDEAN DISTANCE BET. LAST P ITERATIO S. . . 1250E-01

3 RENSITY K≢, (BAYESTAN) .95039 .93374 _9641R .94262 ,97883 94555 93956 .94853 97874 91964 1,00000 94505

ERROR # .1240F+01 PER CENT

EUCLIDEAN NORM OF RESOLVING FRROM # .2299E+00

1.000.00

EUCLIDEAN MORH DE .GT. 1 CHT-OFE = .1249E-03 ENCLIDEAN NORH DE .LT. D CHT-OFE = 0.

EUCLIDEAN DISTANCE BET, LAST 2 TTERATIONS = __2924E=DI

\$9022

,90837

93763

.98439

190878

.93645

97585

1,00000

1,00000

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.87954 0

.88815

,90695

,94292

.44154

×#7105

.876h1

SA8477

90526

_9573Å

FUCLIDEAN NORM HE RESOLVING ERROR = 2485E+00

N= 6 DENSTTY (BAYESIAN)

	1.00000	99764	.99660	. 98964
	1.00000	99927	9963P	99431
	1.00000	1 01000	- 995u7 - 45 -	99362
•	1.00000	1 00000	1.0000	99301

- - EUCLIDEAN HURE OF .GT. 1 CHT-OFF . P139E-03, EUCLIDEAN NORM DF. LT. 0 CUT-OFF . 0.
 - ERRUR # .4550E-01 PER CENT
 - EUCL THEAM NORM OF RESOLVING ERROR # 228493E+00
- KE 7 DESSTTY (BAYFSIAS)

1_00000	1_06000	acque	99335
1 10000	ງ ດະວຽງ	749.9	99771
1,00000	1 00000 1	04731	09620
94994	1 01.000	1. orecon	99464

EUCLIDEAN DISTANCE BET, LAST POITEPATINGS - 4487 2003

 $(x_{i},y_{i}) \in \mathcal{A}$

- EUCLIDEAN HURE OF .GT. 1 CHI-UFF = .14505-03 EUCLIDEAN 1004 CF .LT. C CHI-UFF = 6.
- FRROR . . . 2103E-01 PEN CENT
- EURLIDEAN NORH OF RESILVING FRADE #1 2497E+06
- K. B. DEMSITY (HAVESIAL)

	1.00000	1,00000	1, nundin
·	1.00000	1 . nuoun .	09495
	1 00000	1.00000	UGR16 99745
	99989	1,06000	1 00000 99548

- EUCLIDEAN DISTANCE BET. LAST & ITERATIONS # .14768-03:
 - EUCLIDEAN NORM DE .GT. 1. CUTHUEF . 1361E-03 EUCLIDEAN NORM DE .LT. C CUTHUEF . 0.
 - ENROR 1308E-01 PER CENT
 - EUCLIDEAN NORH OF RESOLVING ERROR 2498E+00

9 DENSITY (BAXESTAN)

1.00000	1:00000	5.0000	99603
1 00000	1.00000	1 00000 - 1	00000
1 10000	1,00000	99873	99811
99985	1,00000	1,00000	99605

POOR PRINT Epreuve illisible EUCLIDEAN DISTANCE BET. LAST 2 ITERATIONS = .1024E-03 EUCLIDEAN NORM DF .GT. 1 CUT-OFF = .9336E-04 EUCLIDEAN NORM UF .LT. 0 CUT-OFF = 0 EUCLIDEAN NORM UF .LT. 0 CUT-OFF = 0 EUCLIDEAN NORM OF .LT. 0 CUT-OFF = 0

KA 10 DENSITY (BAYESIAN)

1.00000	1.00000	1.00000	99659
1,00000 2	1 00000	1,00000	1,00000
1.00000	He00000	99916	. 99855
.99981	1-00009	1.00000	99647

FUELTDEWN MARY OF .GT. 1 DHT-OFF = .7220E-04 EVELTDEAN MARY UF .LT. 0 DHT-OFF = 0;

ENROR . TERSENCE PER CENT

EXPERIMENTAL TEST SECTION & (ERFORMEREE) NO. OF ITERATIONS = 11

= 11 FINVERSIDA TANED

•	DENSITY			
	.94137	1,00000	1,00000	No Aphon
	1,0000000000000000000000000000000000000	00000	95996	90537
7	1.00000	97 47	96423	1 00000
	1_00000	99524	000001.0000	1 00000
			• • • • •	

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ENCLIDEAN DISTINCE BET. LAST & ITERATIONS - 5390F-02

EUCLIDEAN MENN OF .GT. 1 CHT-OFF . 4554E-02 EUCLIDEAN MENN OF .HF .LT. 0. CHT-OFF . 0.

STATISTICAL EPHON IN DENSITY

338.54	-1971	annuo ta	.00785
•01485	1717B	.02310	-07596
	. 11735	. 5557r	03974
_U07Gn	104 561	a 1/5 - 1	.04152

CALCHLATED DETECTOR RESIMANSES

× ×	\mathbf{N}	Y A.	RESPONSE 🗠 🗽	, AUGLE :
<u> </u>	\mathbf{X}	in the second second		
1-1	E+119 4	5456+01	.2046E=04	6000F+02
2 .1167	E+07 -49	5906+01	.2755E-04	5500r+n2
5	E+n5 3/0	590E+01	3334F=04	- 51005+02
	E+05 4.4	590F+01	31125-04	4700F+02
5 1250	E+02	711-5-011	19195-04 5 5	- 5500F+02.
6 6 5,1250	E+059	710E+01	28905-04	5100F+02
7 7 1 1 2 50	E+02	710E+01 -	.3343E=04	470DE+02
8 1250	E+02	710E+01	.30A5E-04	-14300F+02
9	E+02 - 8	710E+01 :: :	2304E+04	3900F+02
10 🌋 🖓 33	E+07	20F+01	1747E=04	1.5100F+02
11-1339	E+02 - 7	10+305	27445-04	4700F+02
12 .1339	E+02 - 7	10+3058	3108E-04	43005402
13 1339	E+02 = 7	820E+01	332°E-04	39005+02
1.4 1339	E+02 - 7	• 10+30'53	3319E-04	.35005+02
421	E+02 = 6	9405+01	2410E-04	43005+02
16 1427	E+026	9405+01	.3183E-04	39006+02
1027	E+02 - 6	9406+01	-4351F=04	3900F+02
18 .1427	E+02	946F+01	.1567E-04	3000E+02

ERROR . . 1643F+00 PEH CENT

B.3.2 Problem 9 (Experimental Results)

EXPERIMENTAL TEST SECTION A

FOLD 21 TIMES SWITCH TO THE ITERATIVE TECHNIQUE AFTER O ITERATIONS KEY= -1 ISM= 0 MAXIMUM NUMBER OF ITERATION ALLOWED= 10 ERROR ALLOWED JW UNFOLDING=0. TAW=0. IPDINT= 0

ERROR . . . 5880F+01 PER CENT EUCLIDEAN NORH OF RESOLVING ERROR . .2104E+00 2 DESSTY

(BAYESIAN)

EUCLIDEAN DISTANCE BET. LAST 2 IJERATIONS = .0762E-01 EUCLIDEAN NORM OF .GT. 1 CUT-OFF # . EUCLIDEAN NORM OF .LT. 0 CUT-OFF = 0. 9304E-02

.62981 1:00000 87812 1.00000. .76239* .A2481 \$ 89629 -i,00000 75636 .A1944 91397 .71564 -74965 .70034 .70666 1.81784+1

K 🖿

1 DEPSITY (HAVESIAD)

ERROR . .1094E+02 PER CEUT

50000 *50001 50000 . 4

.50000 .50000 .50260 .50000 500n0 ··· .50000 50000 ,50000. sanco **.**50000

DUDEWSITY

.50000

;50001

.50000

+1 · · ·		• • • •	RESPUNSE	2-40-
i	1162E+C2	95906+01	2512E=0/	73718-06.
. 5.	1167E+02	-,9590F+01	3136E-04	9261E=06
. 3	1162E+02	- 9590E+01	37186-04	9685E-06
4	11626+02	- 9570E+N)	2347F=04	95365-06
5	1250E+02	- 8710F+01	318AF=04	. 9471E+06
- .6	12506+02	+ A7-10E+01	3434F=04	10126-05
7	12505-72	=_8710E+01	2406E=04	1043F-05
5	12506+02	[™] =,8710F+01	2406F=04	1043F+05
9	1520E+05	871GE+01	1473E=00	9499F=06
10	1330E+05	+,7820E+01	33345-00	1168E-05
11	.1339E+02	- 7,620E+0)	2979F=04	1072E-05
12	.1339E+02	- 7820F+01	2979E-04	30746-05
_13	1339E+07 -	- 7620E301	2980E=04	12556-05
14	1339E+U2		2988E-04	12428-05
15 ∖	.14272+12	6940E+01	3085F=0/	19946-05
16.	.1427E+02		4654F=04	19555-05
17	.1427F+02	6940F+01	46576-04	23745-05
18	1427F+02	- 6940E+31	1317E-0/L	1447F-05
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·4 4Y

4 GRID AND

18 DETECTORS

EDBAD

GIVEN DETECTOR RESPONSES

UNFOLDING CALCULATIONS FOR

.4679F+01 CH SOURCE NEUTRON TOTAL CROSS-SECTION= .1011E+no SOURCE NEUTRON NONABSORPTION PROBABILITY= ,9395E+00

EXPERIMENTAL TEST SECTION A

DIAMETER#

	1 00000	1.00000	
* *00au	1 1 100001	1,00000	1,00000
.83382	95007	1,00000	1,00000
74674	A2389-	.94060	1,00000
.71877	72998	, B1159	93728
		•	

EUCLIDEAN DISTANCE BET. LAST 2 ITENATIONS = .207PE-01

EUCLIDEAN NORM OF GT. 1 CUT-OFF = .4305E-01 EUCLIDEAN MORM OF LT. O CUT-OFF = 0.

EUCLIDEAN NORM OF RESOLVING FRROP = .2273E+00

KE 3 DENSITY (BAYESIAN)

1.00000	1 00000	t_nnnnn	1,00000
.85362	1 00000	1.00000	1,00000
.74315	84142	1,00000	0.0000.1
,71203	72764	"H26n5.	1,01000
	and the first state of the second		

EUCLIDEAN NERH OF .GT. 1 COTHOFF = .4668E-01 - . CENCLIDEAN NORH OF .LT. D CUTHOFF = 0.

ERROR # .5516E+01 PER CENT

EUCLIDEAN YORH OF RESOLVING ERROR = . 2313E+00

4 DEWSITY (HAYESTAM)

<u>.</u>				
1.	0000	S. P. W. W. B. S. P.	J . UGUCC	1.ncoog
	62855	<u>` < 1,00000</u> -	_	1.00000
	73553	84452		1. 20000
	70753		P0658-	1,00000
	•	•	1. S.	
È				

EUCLIDEAN NORM OF RESPLATING ERROR . .2312E+00

5 DENSITY (BAYESTAN)

•

توالمدتوم فأتك

1.00000	1, 00000	1.00000	1,00000
86246	1 00000	1,00000	1,00000
73046	84742	1.00000	1,00000
,70761	70851	.83282	1.00,000

EUCLIDEAN DISTANCE BET, LAST 2 ITERATIONS = .5997F-03 EUCLIDEAN NORM OF .GT. 1 CUT-DFF = .4755E-01 EUCLIDEAN NORM DF .LT. 0 CUT-DFF = 0.

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EUCLIDEAN DISTANCE BET, LAST 2 ITERATIONS = .4668E+03

- EUCLIDEAN NURH UF .GT. 1 CUT-OFF # .4759E-01 FUCLIDEAN NORH UF .LT. 0 CUT-OFF # C.
- ERROR . . 5516E+D1 PER CENT

KE 10 DENSITY (BAYESIAN)

1.00000	1.00000	1.0000	1.00000
.87541	1_00000	1,00000	1,00000
7136B	85430	1_00000	1,00000
.73101	_6948B	_84600	1.00000

ENCLIDEAN DISTANCE BET, LAST 2 TTERATIONS = _______

۰.

EUCLIDEAN NORM OF .GT 1 CUT-OFF = .4759E-C1 EUCLIDEAN NORM OF .LT. 0 CUT-OFF = C.

ERROP . LASISE+01 PEN CENT

EUCLIDEAN NORM OF RESOLVING ERPOP = 22317E+00

EXPERIMENTAL TEST SECTION A ND. DF ITERATIONS . 11 (INVERSION TAHEO, K 🗉 11 DENSITY 00000 00000 ;;00000 .00000 .00000 1,00000 1.00000 1.00000 1.00000 .00000 · .0000n. .00000 1_10000 .00000 1.00000 -00000 EUCLIDEAN NORM OF .GT. 1 CUT-OFF = .2167E+02 EUCLIDEAN NORM OF .LT. 0 CUT-OFF = .4104E+02 STATISTICAL FRENH IN DENSITY 2.64363. 1.91498 90,73323 5.1717A 3,91534 45:32743 5,92091 2.27652 2 246414 18,00770 1.88679 4_03028 1,75335 1.56489 1_55693 1945407 CALCULATED DETECTOR RESPONSES RESPE-SE ANGLE -11655+65 .10255+64 14,9590E+01; - . 6000F+-2 - 9590E+71 2 +11656+95 .1494F≓n4 -.5500F+02 3 *1195E+05 - 9590E+01 .1879E=04 -.51005+02 *1195E+05 4 -,9590E+01: .16596-04 +,4700F+02 5 1520E+05 -. 5500F+02 -,8710F+01 1012E-04 ,1250E+02 ,1491E=04 6 +_8710E+01 #.5100E+02 -1520E+05 7 -,8710E+01 .1869E-04" -.4700E+D2 -1520E+05 -1550E=04 B -.8710E+01 1171E-04 9 *1520E+05 +_8710E+01 -. 3900F+02 10 *1336±05 -.7820E+01 94738-05 -,5100E+02 1408E-04 1339E+05 11 -,7820E+01 -.47000+02 -_4300F+02 .13396+02 -,7820E+01 1729E=04 12 41334E+05 - 7850E+01 13 - 39006+02 .1626E-04 14 1339E+02 .1705E-04 - 3500E+02 15 .1427E+02 -- 6940E+01 212818=04 -,4300E+02 1749E-04 -,3900E+02 16 1427F+02 -.6940E+01 1427E+02 17 -,6940E+01 2192E-04 #,3500F+02 18 -1427E+02 -.6940E+01. - 3000E+n2 .8181E=05 ERROR . .1301F+02 - PER CENT

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513.

B.4 Problem 10 : 8 x 8 Mesh

8 X 8 MESH

1.,

FOLD 1 TIMES SWITCH TO THE ITERATIVE TECHNIQUE AFTER 9 ITERATIONS KEY= -1 ISM= 0 MAXIMUM NUMBER OF ITERATION ALLOWED= 20 ERRUR ALLOWED IN UNFOLDING= .1000E=02 TAW=0. GAMMA= .5000E=02 IPOINT= 1

S X 5 HESH

DIAMETER: ,1000E+02 CM SHURCE NEUTROM TOTAL CRUSS-SECTION: ,1011E+00 SHURCE NEUTROM NONARSORPTIDE PROBABILITY: ,5345E+00

	ÚNFOL	DING CALCHLATIC	DHS,FDR B B	Y . B GRID AND	70 DETECTORS
	GIVEN	DETECTOR RESPI	INSR5		
	•				
• .		X	Υ .	RESPANSE	ERROR
	5	12555-02	- 75505-01		
·	2	32556+02		24/52-04	_ 3007E=08
	3	14025-02	- \$780E+01	11/2-04 31/27-04	5000E-00
. •	4	1402E+02	5701.E+01	24237-04	SDDDE=10
	5	.1402E+02	- 5760E+01	16526-04	5000E-08
	6	1578E+02	414020E+01	28166-04	SCORE-OB
	7 1	1576E+02	:4020E+01	26365-04	SQNFE+DA
• :	5	.1578E+02	+ (6+31594+	1. 0 10-30PS:	SONDE-NA
	9	- 1122E-55		1866E-04	SUDCE+05
	10	.17558+22		\$399F+0F	SUDCE-DB
	11	1755F+92	12556+02	. LEESE-DE	-SODDE-DA
	12		1255E+02	.73595-84	SCOPE-DB
· .	121	- 1 3/01 +VC	14022+025	PAINE	5-07E-0A
	15		14026+02	.28365+74	-SD05E-78
•		14025-02	15785-02	24449E-44	
	17	14025402	15786+02	31-22-34	50005-0A
	16	14026+02	157%1402	126725-14	500/5-05
	19 .	12556+02	17551402		SCOCE-0-
	20 .	12556+02	1755-02	19075-14	5000F=06
- ¹	21	1129E 12	- 6676E+01	20575-14	50075-04
	22	14906+92	(+ 4905E+01	1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 10000 - 10000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 - 1000 -	SONCE-OF
	23	1667E+02	- 3296E+01	28105-06	STODE-DE
	2	167E+62	13246+02	241 JF-04	SCOTE-D*
.*	25	.1490E-12	16906+32	35045-14	50025-05
	26	13256+02	1667E+32	29572+06	50-001-08
	21	.1329E+02	6678E+01	10241-04	5052E+68 ***
	25	14901+J2	490%E+01	16478-64	SCORE-OF
	29	1667.E+02	329#E+01	5545F-75	SCODE+OK
· •	3.0	.1667E+02	13246-02	55655-05	50CPE-06
	31	.1497E+02	1241E+05	16472404	50005+05
1 . Î	32 .	1354E+DS	16676+02	10505-04	5000E-0F
	33	1356E+05	- 6670E+01	11165-04	ScotE-ns
· `*	54	1490E+U2	- 49008-01	27976-04	500CE-08
	35	.16675402	-,3290E+01	.2663E-C-C	- SSOCE-DA
	36	*1667E+02	13296+02	.2663E-G4	STOCE+68
1.1.1.1	37 (° .	14406+02	1490L+02	2797E-04	SCORE-05
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	4 1	.130004402	- 530/6+01	,30046-04	SODOE-DA
	42	15305-02		50575-04	5000
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	44		- 2920F+01	20176-04-20 4	SDODEPDA
	45	.)7110+02	1292F+02	21655-04	SUDAE-DE
	46	16230+02	1364E+02	2617F=04	5000E-AR
• • • •	47	1534F+02	1446E+02	>9575-04	SPOCE-DE
• • •	48	14461+12	1534E+02	31255-04	SDODE-OF
• •	49	13665402	-1p23E+02	3046F-94	50005-08

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•	55	1623E+U2 .		.1018E-04	SCOPE+08
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	57	1534E+02	1465E+U2	14405-04	SCODE-OR
• .	58	14465+65	. 153-E+02	1770E-04	-500CE-06
•	\$9	.1366E+U2-	1623E+02	1900E=04	SOCOE HOR
-	60	50+35951.	-171 E+02	19275-04	SCOSE+CE.
· · ·	61	1364E+02	E+01	19621-04	50075-08
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· · .	P 64	,1653E+05	=,365 E+01	-C138E=DE	50005-08
::: :● ,	65	,1711E+02	- 29251+01		SOCCE-OF
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APPENDIX C

CONTRACTION MAPPING OF A SIMPLE PROBLEM

The Contraction Mapping proof of subsection 4.1.1 of Chapter 4 is examined here for a simple problem. Consider the problem of finding the water fraction distribution in a test section divided into two cells from the response of two detectors. Let the water fractions in cells 1 and 2 be ρ_1 and ρ_2 , respectively. Also, let detector 1 observes the neutron scattering from cell 1 and detector 2 observes that from cell 2, giving the detector responses S_1 and S_2 , as shown in the following schematic representation :

DETECTOR SOURCE ្នុំរូវ Cell 1 RAY

SCATTERED * RAY + DETECTOR 2

The response matrix A can be then written as

 $\begin{bmatrix} K_1 \exp(-a\rho_1) & 0 \\ 0 & K_2 \exp(-b\rho_1 - c\rho_2) \end{bmatrix}$

where an element A_{ij} of \underline{A} represents the contribution of cell j to detector i, and a, b and c are non-negative constants related to the neutron path length in the cells, and K_1 and K_2 are non-negative constants proportional to the scattering probability and the inverse of the square of the distance between the scattering cell and the detector position. Let

$$= \frac{1}{N} \underline{\lambda}^{-1} \underline{S} ,$$

where N is the number of cells, $(=2)_{n}$, and

$$\underline{s} = [s_1 \ s_2]^{\mathrm{T}}$$
(3)

(2)

From (1),

$$\mathbf{A}_{=}^{-1} = \begin{bmatrix} \mathbf{K}_{1}^{-1} \exp(a\rho_{1}) & & \\ & \mathbf{K}_{2}^{-1} \exp(b\rho_{1} + c\rho_{2}) \end{bmatrix} , \quad (4)$$

Therefore

then

$$P_{1}^{\prime} = \frac{\partial \phi}{\partial \rho_{1}} = \frac{1}{N} \begin{bmatrix} s_{1} \kappa_{1}^{-1} a \exp(a\rho_{1}) \\ s_{2} \kappa_{2}^{-1} b \exp(b\rho_{1} + c\rho_{2}) \end{bmatrix}$$

... ≤ β₁

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 $\beta_1 := \max(a,b)$, i.e. $a \leq \beta_1$ and $b \leq \beta$ (7)

Similarly,

where,

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(8) where = max(c), i.e. $c = \beta_2$ β_2/ (9)

Let o be a matrix that consists of N (=2) rows, each is equivalent to the vector ϕ , i.e.

$$\underline{\Phi} = [\Phi : \Phi]$$
, (40)

then using (6) and (8)

$$\underline{\Phi}' = \begin{bmatrix} \underline{\Phi}_1' : \underline{\Phi}_2' \end{bmatrix} \leq \begin{bmatrix} \beta_1 \underline{\Phi} : \beta_2 \underline{\Phi} \end{bmatrix}$$
(11)

•

Let

that is

$$\beta = \max (\beta_1, \beta_2)$$

$$\beta_1 \leq \beta$$
 and $\beta_2 \leq \beta$ (12)

Now, as shown in subsection (4.1.1 of Chapter 4, relationship (2) represents a contraction mapping if

β [

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(13)

(14)

which requires according to (13) that

<u>||∳</u>'|| <u><</u>

<u>φ' ≤</u> β <u>φ</u>

 $\beta ||\underline{\phi}|| \leq 1$ (15)

Let us define $||\underline{\phi}||$ as

then,

or,

 $|| \Phi || = \max_{i} |\Sigma \Phi_{ij}|, \qquad (16)$

where ϕ_{ij} is the ij^{th} element of the matrix Φ_{ij} , which according to (10) is equal to ϕ_{ij} , then

 $||\underline{\phi}|| = \max |\Sigma \phi_{i}| = \max |N\phi_{i}|$

 $= N \max |\phi_i|$ (17)

If $\phi_i \in [0, \frac{1}{N}]$, then

and condition (15) would be satisfied if

$$\beta \leq 1$$
, (18)

and then relationship (2) would be a contraction mapping. From relationships (7), (9), and (12), to satisfy (18), the maximum value of a, b, and c has to be less than unity. This requires that the maximum neutron path in cell be less than one mean-free-path of the neutron in the medium of the cell; in order for the successive approximations represented by relationship (2) to be a contraction mapping.



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