FAULT ANALYSIS AND PARAMETER TUNING IN ANALOG CIRCUITS

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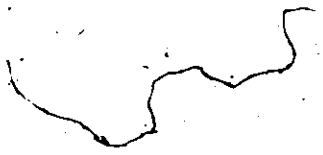

FAULT ANALYSIS AND PARAMETER TUNING IN ANALOG CIRCUITS

ABSTRACT

This thesis addresses itself to the two principal problems of computer-aided-testing in analog circuits, namely, fault analysis and postproduction tuning.

A unified approach to fault location in large analog circuits is introduced. The approach closely meets the goals of practical criteria for fault analysis. Network decomposition and logical analysis are incorporated to identify faulty subnetworks. Necessary and almost sufficient testing conditions for locating fault-free subnetworks are derived. These conditions are based on invoking KCL and topological relations between subnetworks. The application of the approach to practical linear and nonlinear networks is presented. Further fault analysis is carried out to identify faulty elements or regions inside the faulty subnetwork. Deterministic and approximate methods are introduced for that respect. The approximate method utilizes an estimation criterion, namely, the least-one objective function to predict the most likely faulty elements. The deterministic methods verify the existence of faults by examining the consistency of algebraic equations or by matching the subnetwork response using faulty models of the subnetwork elements. A number of network examples are considered to illustrate the application of the introduced methods.

The deviation in the response of a manufactured circuit can often be compensated by adjusting specified tunable elements. A number of

aspects of the postproduction tuning problem are studied. In particular, the relevant fundamental concepts and definitions are given, the tuning algorithms either functional or deterministic are reviewed and new techniques for choosing tunable parameters and critical response points are introduced. Two new functional tuning techniques are presented. The application of the new techniques in tuning a microwave network example is illustrated. A comparison and evaluation of four different tuning techniques are given by testing them in tuning an active filter example.

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1

INTRODUCTION

The increased complexity of analog circuits and the mass production of integrated circuits necessitate the use of computers in all aspects of the production and maintenance processes. Computer-aided-design (CAD) techniques that incorporate the multiple objectives of cost reduction, yield maximization, tolerance optimization and design centering are now well established and their application to actual design problems is steadily increasing. Similarly, the use of computer-aided-testing (CAT) techniques is in progress, especially for testing digital circuits.

For analog circuits, CAT techniques are primarily associated with the combined problems of fault location, parameter identification and postproduction tuning. The main objective is to adjust the manufactured circuit to meet the design specifications. The adjustment process could be carried out by two distinct approaches. In the first approach, the sources of faults in the circuit are identified, then the faulty elements and/or modules are replaced by nonfaulty ones. This is usually suitable for catastrophic faults, when the faulty element produces either a short circuit or an open circuit. In the second approach, the deviation in the network response is compensated by adjusting tunable parameters specified a priori. This is quite appropriate when the malfunction of the circuit is due to modest changes in the parameters of

the circuit, referred to as soft faults. These soft faults could result from manufacturing tolerances, aging or parasitic effects.

This thesis is addressed at the two main problems of computer-aided-testing in analog circuits, namely, fault analysis and postproduction tuning. In particular, a unified approach for locating faults inside large analog circuits is introduced and optimal postproduction tuning techniques and procedures are presented.

Chapter 2 presents a review of existing methods of fault analysis in analog circuits. The methods are categorized according to the time of simulation and number of excitations of the circuit under test. A comparison between the methods is given and a set of criteria which a practical fault analysis algorithm should achieve is discussed. This chapter provides an adequate state-of-the-art review of the fault analysis techniques.

In testing large analog circuits it is logical to locate the faults within smaller parts of the network (subnetworks), then to locate faults inside the identified faulty subnetworks. This usually permits a systematic, fast way of identifying faulty elements. In Chapter 3, the decomposition approach with logical analysis are utilized to locate faulty subnetworks. Necessary and almost sufficient testing conditions for fault-free subnetworks are derived. The effect of tolerances on the application of testing conditions is discussed and a probabilistic measure is introduced. The application of the technique to practical linear and nonlinear networks is presented.

Locating faults inside faulty subnetworks is addressed in

Chapters 4 and 5. Chapter 4 deals with the use of the deterministic techniques such as the fault verification method, internal-self-testing method, combinatorial method and matching method. The effect of tolerances on the application of the fault verification method is discussed and a linear programming formulation is introduced. The decomposition approach that is used for finding faulty subnetworks is utilized in the internal-self-testing method to locate faulty elements and/or regions inside the faulty subnetwork. The combinatorial approach is introduced to speed up the application of the fault verification method. The matching method utilizes the fault models of the faulty elements in matching the response obtained to that predicted using fault models. The latter technique is more applicable to nonlinear networks. The results of applying the techniques to different circuit examples are presented.

In Chapter 5, the ℓ_1 norm is utilized for estimating the most likely faulty elements. Two formulations are presented. The first considers the situation when the measurements are obtained using a single test vector. The second utilizes multiple test vectors to obtain the measurements. In both cases the number of measurements is less than the number of elements of the faulty subnetwork. The results of applying the techniques to linear subnetworks are presented.

Chapter 6 deals with some important aspects that are related to the postproducting tuning problem. The main aspects that are addressed are the formulation of error functions from design specifications, the tuning algorithm, either functional or deterministic, the choice of

tunable parameters and the choice of the critical samples of the response. A review of these aspects in the literature is given together with some original techniques for selecting tuning elements and critical frequencies.

In Chapter 7 two new functional tuning algorithms are presented. The techniques for selecting the tunable parameters and the frequencies to be monitored during the tuning process are applied in an integrated optimal tuning procedure for tuning a microwave circuit. A comparison between functional and deterministic tuning algorithms based upon testing four tuning techniques is then outlined.

The main contributions of this thesis are outlined in Chapter 8 together with some suggestions for future research work.

2

FAULT ANALYSIS IN ANALOG CIRCUITS— A REVIEW

2.1 INTRODUCTION

The problems which fall into the field of fault analysis can, in general, be classified into either fault location or parameter identification.

The first group corresponds to the situation where we want to locate an element or a number of elements which are faulty. By a fault we mean, in general, any large change in the value of an element w.r.t. its nominal value which can cause the failure of the whole circuit.

Parameter identification, in the context of electrical circuits, is the process of finding the actual values of circuit components. Dependent on a particular problem we may be interested in identification of selected parameters, assuming all the other parameters are known or in the identification of all parameters where only the circuit topology and model description are known. The fault location and parameter identification problems cannot be completely separated from each other.

Although many specific algorithms have been proposed for fault analysis, they may naturally be classified into three categories:

(a) simulation-before-test,

(b) simulation-after-test with a single test vector, and

(c) simulation-after-test with multiple test vectors.

2.2 SIMULATION-BEFORE-TEST APPROACH

The simulation-before-test approach is referred to as the dictionary approach for fault location. It is widely used for testing digital circuits. Although it suffers from a number of limitations in the testing of analog circuits it is still the most commonly used method.

2.2.1 General Description

The dictionary approach is a technique in which the unit under test (UUT) is simulated off-line using certain input signals for a number of hypothesized faults. The responses are stored as a dictionary. The UUT is excited by the specified input signals and the responses obtained in the field are compared with the simulated results in an attempt to determine the cause of the malfunction of the UUT. The fault candidate that produces the closest simulated responses w.r.t. a certain measure to that produced in the field is declared the actual fault.

It should be clear that the implementation of the dictionary approach consists of the following two stages. Before conducting the test, the dictionary is constructed to achieve the required degree of diagnosability. At the time of actual testing a search process is conducted using the stored data and the measurements to locate the fault or an ambiguity set that contains the possible fault.

The reported techniques of implementing the dictionary approach differ mainly in the following features:

(a) the kind of input/output measurements,

(b) the fault signature,

(c) the procedure for selecting optimal signatures,

(d) the fault location technique itself, and

(e) the degree of diagnosability and fault isolation.

2.2.2 Dictionary Construction

The construction of the dictionary is initiated by choosing the input signals to the circuit, the domain of analysis and the responses to be measured. In Fig. 2.1 the different techniques are summarized (Schreiber 1979).

One of the first techniques for constructing a fault dictionary was given by Seshu and Waxman (1966). In their proposed technique the linear, frequency-dependent circuit is excited at a number of frequencies close to the break points of its transfer function and the output voltage is measured at these frequencies. This is an example of a steady-state analysis method with sinusoidal input and voltage measurements.

DC testing of nonlinear circuits (video amplifier, power supply, etc.) has proved to be very effective for diagnosis of nonlinear circuits. The circuit is excited by a number of dc inputs such that the different states (off, on and linear) of the semiconductor devices are exercised. The purpose is to be able to observe the fault at any of the primary outputs (test nodes) of the circuit. Hochwald and Bastian (1979) utilized the voltages of the test nodes to construct their

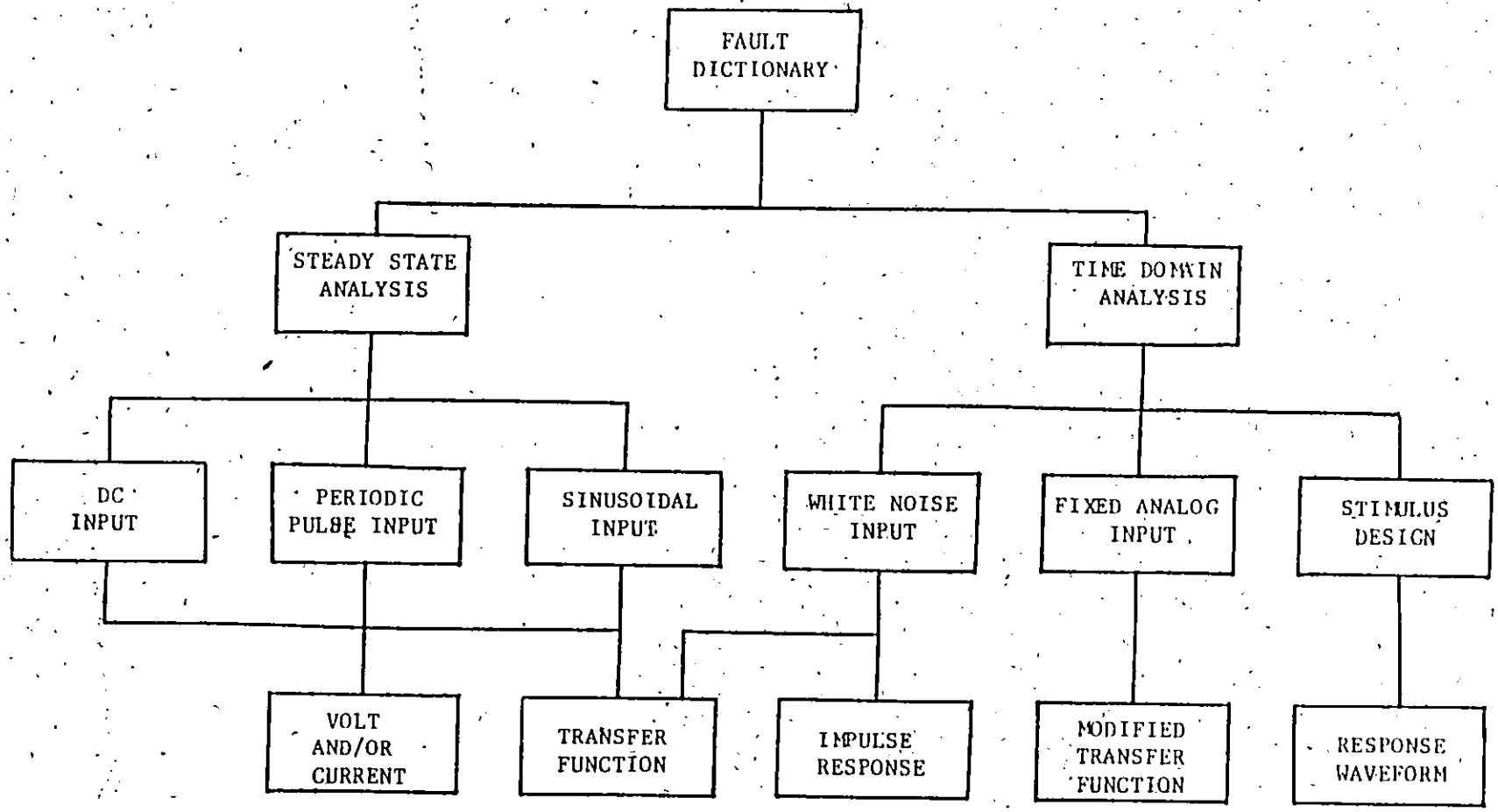

Fig. 2.1 Fault dictionary techniques categorized by analysis method, stimulus response and dictionary parameters.

dictionary. For the video amplifier example that was considered in their work two dc inputs were chosen to cover 95% of the assumed faults. Lin (1982) described the application of the complementary pivot theory in dc fault diagnosis of analog circuits. The approach has the advantage of faster dc analysis.

In the time domain analysis methods, the transient response of the system is monitored. The simplest input which can be applied to a circuit is a step input. Since the impulse response of a circuit is the derivative of the step response, the response to the step is utilized in determining the transfer function of the network under test. The transfer function, through its coefficients, may be utilized in constructing the dictionary.

Duhamel and Rault (1979) described a number of methods that utilize the time domain analysis in constructing the fault dictionary. Most of the methods are based on determining the impulse response of the circuit and/or the transfer function coefficients from the measured output using special types of inputs (e.g., white noise).

The complementary signal method by Schreiber (1979) requires the analytical determination of the shape of the test signals. The complementary signal consists of a sequence of piecewise constant impulses: the first impulse puts the system under test in a nontrivial state; the subsequent impulses bring back the system response to zero within a time interval that depends on both the number of poles of the transfer function and the passband of the system under test. The fault dictionary is constructed using the changes in the locations of the

poles in the complex s-plane and the changes in the locations of zero crossings of the response in the time domain.

It is clear that in constructing the dictionary there is a compromise between the degree of diagnosability and the stored information. The optimum choice of the measured outputs (number of test nodes, number of inputs, etc.) is required to store the minimum amount of test data without affecting the desired degree of diagnosability. For the specified hypothesized fault cases it is usually required, using the stored data, to uniquely identify every faulty case, or at least to identify it within very small ambiguous set. Two techniques are worth mentioning. The first is due to Varghese, Williams and Towill (1979), where the measurements are chosen according to the required degree of diagnosability and using a distance formula proposed to measure the discriminating information of the fault cases considered. The second is due to Hochwald and Bastian (1979). It uses logical manipulations in choosing the set of measurements to be stored.

Instead of storing the fault signatures as a set of voltages, Seshu and Waxman (1966) stored codes, which are much easier for inspection. Other types of fault signatures were also used, e.g., the loci of the changes in the poles and the zero crossings, as proposed by Schreiber (1979).

2.2.3 Fault Location

Different techniques are utilized to find the most probable fault using the stored data and measured response. Most techniques follow a

fault tree approach, as shown in Fig. 2.2. First the UUT is diagnosed to be either sick or healthy. If the UUT appears faulty the fault is first assigned to a certain ambiguity set Ω_i. Then, the fault is isolated to a component of this ambiguity set x_{ij}, if possible.

Let the fault dictionary corresponding to f faults and m measurements be represented by the entries h_{ij}, where i = 1, 2, ..., m and j = 1, 2, ..., f. Any unknown fault is represented by the pattern vector

$$\underset{\sim}{v} = [v_1 \quad v_2 \quad \ldots \quad v_m]^T, \tag{2.1}$$

where T denotes the transpose. Using the nearest neighbour rule the distance

$$d_j = [\sum_{i=1}^{m} (h_{ij} - v_i)^2]^{1/2}, \quad j = 1, 2, \ldots, f, \tag{2.2}$$

is computed for the f candidates and $\underset{\sim}{v}$ is associated with the candidate fault of minimum distance.

Following a completely probabilistic approach, Freeman (1979) considered the tolerance effects on the elements together with the errors in measurements.

If all faults are of equal probability the fault type is identified by finding the minimum over j of the quantity

$$\sum_{i=1}^{m} \sum_{\ell=1}^{m} (v_i - h_{ij})(v_\ell - h_{\ell j})\, w_{i\ell}, \tag{2.3}$$

where $w_{i\ell}$ is a weighting factor, which depends on measurement errors and

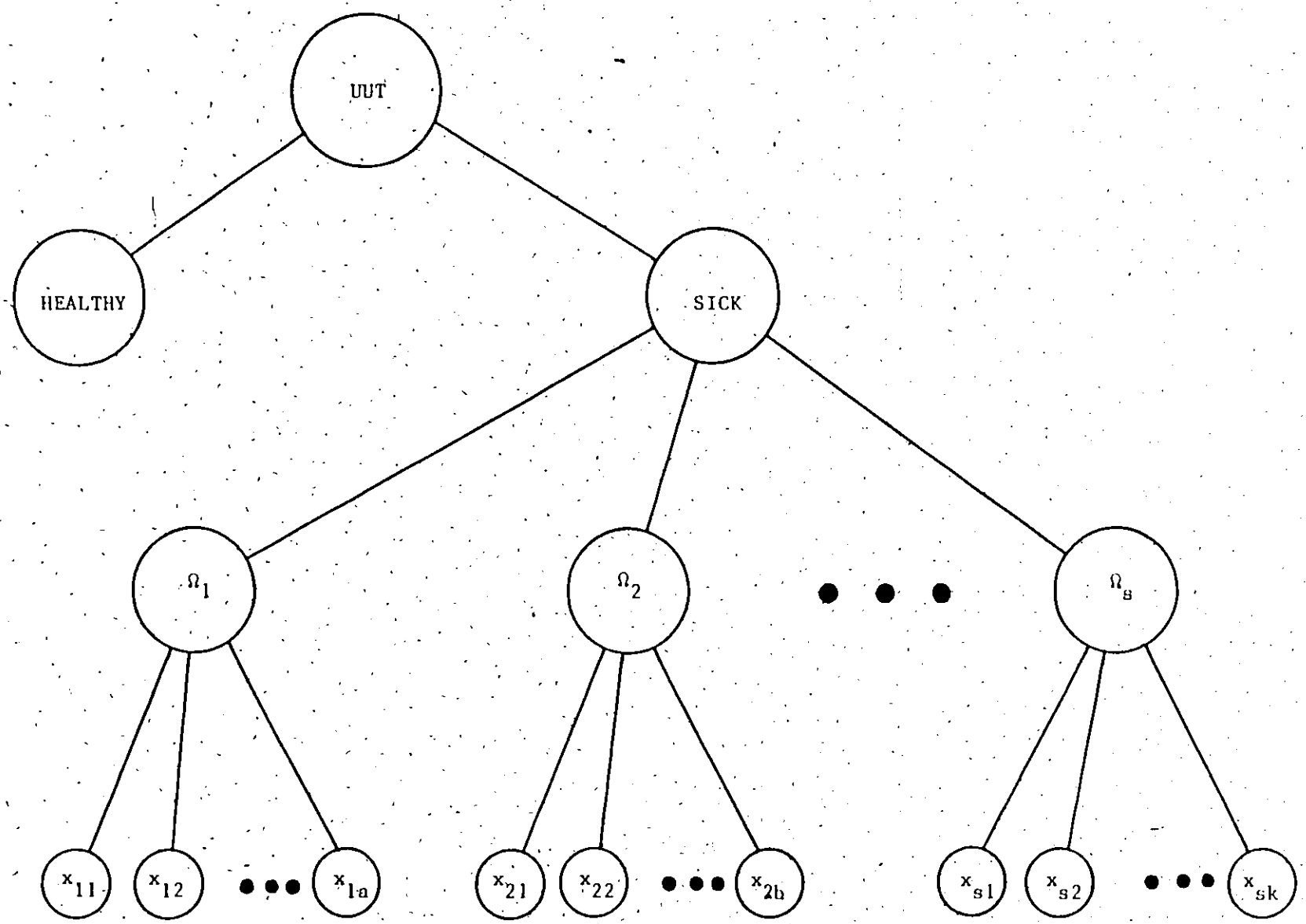

Fig. 2.2 Test and diagnosis scheme-fault tree approach.

the parameter tolerances. Freeman showed that (2.3) reduces to (2.2) under special circumstances.

If the fault signature is stored as a code, the response is transformed to this code then a complete matching to one of the stored codes is carried out. If the complete matching is not achieved the fault is not identified and the fault dictionary must be augmented to include this type of fault.

Most fault dictionaries are constructed for catastrophic single faults. As reported by Hochwald and Bastian (1979) 70 percent of single point field failures can be detected/isolated. This percentage can be further improved to 80 percent by including additional hard or soft failure modes that are unique to the specific electronic technology used in the system.

2.3 SIMULATION-AFTER-TEST USING A SINGLE TEST VECTOR

The responses of the network to a single input excitation are analyzed to determine the faulty elements of the network. Consequently, rather than simulating the network before the test as in the dictionary approach, most of the network analyses and simulations are performed after conducting the actual test. The techniques to be followed depend upon the number of available measurements. If the number of independent measurements is less than the number of the network elements, which is typical, two main approaches are followed:

(a) <u>estimation methods</u>, where an estimation criterion is used to identify the most probable fault, and

(b) fault verification techniques, where an upper bound is assumed on the number of simultaneous faults, usually less than the number of performed measurements.

If the number of independent measurements is sufficient an identification of all network elements can be carried out and the faulty elements are thereby isolated. This situation is rare and impractical, particularly if we are utilizing a single test vector.

2.3.1 Estimation Methods

Let the UUT have q input terminals, m output terminals and p parameters. The input-output model of the UUT is given by

$$\underset{\sim}{y} = \underset{\sim}{h}\,(\underset{\sim}{u},\ \underset{\sim}{\phi}), \tag{2.4}$$

where $\underset{\sim}{y} \in R^m$, $\underset{\sim}{u} \in R^q$ and $\underset{\sim}{\phi} \in R^p$. Using a single input vector $\underset{\sim}{u}^1$ we get a single output vector $\underset{\sim}{y}^1$

$$\underset{\sim}{y}^1 = \underset{\sim}{h}\,(\underset{\sim}{u}^1,\ \underset{\sim}{\phi}). \tag{2.5}$$

m is assumed to be less than p, so a complete identification of all network parameters is not possible. Different estimation criteria are utilized to estimate the values of the elements of the UUT.

Ransom and Saeks (1973) and Hankley and Merrill (1971) utilized the least-squares criterion in computing the deviations in the parameter values. The basic assumption is that small deviations in parameter values are more likely than large deviations. To a first-order approximation the change in the outputs $\underset{\sim}{y}^1$ from their nominal values $\underset{\sim}{y}^{10}$ is given by

$$\Delta \underset{\sim}{y} = \underset{\sim}{y}^1 - \underset{\sim}{y}^{10} \equiv \left(\frac{\partial \underset{\sim}{h}^T}{\partial \underset{\sim}{\phi}}\right)^T \Delta \underset{\sim}{\phi} = \underset{\sim}{B}\, \Delta \underset{\sim}{\phi}, \tag{2.6}$$

where

$$\underset{\sim}{B} \triangleq \left(\frac{\partial \underset{\sim}{h}^T}{\partial \underset{\sim}{\phi}}\right)^T, \tag{2.7}$$

is an m x p matrix of first derivatives evaluated at nominal and

$$\Delta \underset{\sim}{\phi} \triangleq \underset{\sim}{\phi} - \underset{\sim}{\phi}^0, \tag{2.8}$$

defines the change in the network parameters from their nominal $\underset{\sim}{\phi}^0$. Among all solutions of the linear system (2.6) the solution

$$\Delta \underset{\sim}{\phi} = \underset{\sim}{B}^+ \Delta \underset{\sim}{y}, \tag{2.9}$$

where

$$\underset{\sim}{B}^+ = \underset{\sim}{B}^T \left[\underset{\sim}{B}\, \underset{\sim}{B}^T\right]^{-1} \tag{2.10}$$

minimizes

$$\sum_{i=1}^{p} \Delta \phi_i^2. \tag{2.11}$$

A linear approximation is utilized in finding $\Delta \underset{\sim}{\phi}$. Consequently, the exact solution of the optimization problem that minimizes (2.11) subject to (2.5) is not realized exactly.

An iterative procedure could be carried out until $\| \Delta \underset{\sim}{\phi}^j \| \leq \text{EPS}$, where j refers to the jth iteration and EPS is a very small positive number.

Based on the hypotheses that the difference between actual and nominal values are much greater for the faulty parameters than for the remaining parameters and that the chosen solution is the one with the smallest number of faulty elements, Merrill (1973) devised the objective

function

$$\sum_{i=1}^{p} \sqrt{|\Delta\phi_i| + \delta} + \alpha\,(\Delta\underset{\sim}{y} - \underset{\sim}{B}\,\Delta\underset{\sim}{\phi})^T\,(\Delta\underset{\sim}{y} - \underset{\sim}{B}\,\Delta\underset{\sim}{\phi}). \qquad (2.12)$$

The first term penalizes the nonzero element values and the second term implies that the result should approximately satisfy (2.6). The choice of α is very crucial and usually a large value of α is used. δ is included in (2.12) to eliminate differential problems when $\Delta\phi_i = 0$. Merrill (1973) considered a sequence of quadratic approximations to (2.12). This led to solving a sequence of quadratic programming problems, which converge to the optimum of the original problem (2.12).

If the original linear approximation (2.6) is not adequate the procedure could be repeated until convergence is achieved. The reported results of this method were very encouraging and the hypotheses agree with the practical observation. This motivated Bandler, Biernacki and Salama (1981) and Bandler, Biernacki, Salama and Starzyk (1982) to consider a linear programming formulation of the problem.

For linear resistive networks, using just one single excitation, the change in measurements from nominal can be expressed as

$$\Delta\underset{\sim}{y} = \underset{\sim}{H}\,\underset{\sim}{s}, \qquad (2.13)$$

where $\underset{\sim}{s} = [s_1\ s_2\ \ldots\ s_p]^T$ are specially defined error parameters and $\underset{\sim}{H}$ is a constant matrix obtained from a sensitivity analysis. Relation (2.13) is exact. The error parameters $\underset{\sim}{s}$ are found by solving the linear program

$$\text{minimize} \sum_{i=1}^{p} |s_i| \qquad (2.14)$$

subject to (2.13). For frequency-dependent networks two error parameters are defined for every component such that all the optimization variables are real. The details of the approach are discussed in Section 5.2.

2.3.2 Fault Verification Techniques

If we assume that the number of simultaneous faults f is less than m, the number of measurements, a search procedure can be implemented to identify the faulty elements. Recalling (2.5) we may write

$$\underset{\sim}{y}^1 = \underset{\sim}{h}\,(\underset{\sim}{u}^1,\ \underset{\sim}{\phi}^0,\ \Delta\underset{\sim}{\phi}^F), \qquad (2.15)$$

where $\Delta\underset{\sim}{\phi}^F$ represents the vector of the changes in the f faulty elements. Under the assumption that $f < m$ (2.15) is an overdetermined system of equations. A necessary condition for $\Delta\underset{\sim}{\phi}^F$ to represent the changes in the exact faulty parameters is that (2.15) is consistent.

The results reported in the literature address two main issues. The first is the uniqueness of the diagnosable elements in both linear and nonlinear circuits. The second is the development of techniques to speed up the search for the faulty set.

Visvanathan and Sangiovanni-Vincentelli (1981) studied the problem of diagnosability of nonlinear circuits and systems. Following their results the f faulty elements $[\phi_1\ \phi_2\ \ldots\ \phi_f]^T$ are locally unique

if

$$\text{Rank}\left[\frac{\partial \underset{\sim}{h}}{\partial \phi_1}\ \frac{\partial \underset{\sim}{h}}{\partial \phi_2}\ \cdots\ \frac{\partial \underset{\sim}{h}}{\partial \phi_f}\ \frac{\partial \underset{\sim}{h}}{\partial \phi_x}\right] > \text{Rank}\left[\frac{\partial \underset{\sim}{h}}{\partial \phi_1}\ \frac{\partial \underset{\sim}{h}}{\partial \phi_2}\ \cdots\ \frac{\partial \underset{\sim}{h}}{\partial \phi_f}\right], \qquad (2.16)$$

for all elements ϕ_x of the network other than the faulty elements. The computation of $\Delta\underset{\sim}{\phi}^F$ is obtained by minimizing

$$\| \underset{\sim}{y}^1 - \underset{\sim}{h}(\underset{\sim}{u}^1, \underset{\sim}{\phi}^0, \Delta\underset{\sim}{\phi}^F) \|. \qquad (2.17)$$

Correct identification of the faulty parameters yields a zero value of (2.17) if the actual values of nonfaulty elements are nominal.

For linear networks global sufficient conditions for the uniqueness of the solution with graph theory implications have been presented by Starzyk and Bandler (1982) and Huang, Lin and Liu (1982).

Sakla, El-Masry and Trick (1980) have provided a necessary condition for locating single faults. They utilized Tellegen's theorem and the adjoint network approach in deriving this condition. Their technique could be extended to handle multiple fault location. Biernacki and Bandler (1980a, 1981) considered the problem of multiple fault location using a multiport approach. They developed a necessary condition for a set of elements F to be the exact faulty set. The condition is based on the consistency of a set of linear equations that corresponds to (2.15) in the linear case. They also pointed out a number of problems with the multiport approach.

Starzyk and Bandler (1981a) dealt in a rigorous way with the multiport approach. They also introduced the concept of faulty nodes (Starzyk and Bandler 1981b) and utilized the nodal admittance matrix

representation of the network for locating these nodes. They designated a node as faulty if a faulty element is incident with it. The location of faulty nodes rather than faulty elements is much easier and, as they reported, a graph theoretical concept (k-connection) (Starzyk 1980), could be utilized to check the uniqueness of the proposed faulty set. Huang et al. (1982) independently produced quite similar results. They stated a sufficient and almost necessary condition for a network to be k-fault testable. This condition could be utilized as a measure for designing for testability.

The problem of locating the exact faulty elements is combinatorial in nature. Different techniques have been devised to reduce the computational effort.

Wu et al. (1982) presented an exact algorithm for locating a single fault and a heuristic algorithm for the single and multiple fault cases. The latter algorithm is based on a heuristic that the effect of two faults will not cancel each other and that there is an upper bound on the number of faults.

Salama, Starzyk and Bandler (1983) very recently presented a new method for locating faults in very large analog networks. The details of this approach are given in Chapter 3. It is based on a proper choice of the measurement nodes such that a certain decomposition of the network into subnetworks results. Utilizing KCL and logical analysis the faulty subnetworks are isolated. Further fault analysis is carried out inside each relatively small subnetwork to isolate faulty elements or regions as is detailed in Chapters 4 and 5.

2.3.3 Parameter Identification Techniques

If the number of independent measured quantities using a single test vector is equal to the number of network parameters, full identification of the network parameters is possible. As reported by Saeks, Singh and Liu (1972) for linear systems, the resultant fault diagnosis equations are almost linear and may be solved using a single, sparse matrix inversion. If the number of measurements is less than the network parameters, selected elements could be identified (Biernacki and Bandler 1980b).

The main limitation of parameter identification is that the number of required measurements grows linearly with the complexity of the circuit.

2.4 SIMULATION-AFTER-TEST WITH MULTIPLE TEST VECTORS

One approach to reduce the test point requirements of the simulation-after-test algorithm is to use multiple test vectors to increase the number of equations derivable from a given set of test points.

Sen and Saeks (1979) described the utilization of several frequencies in diagnosing linear systems. A measure of testability and diagnosability is derived. Based on the value of this measure either full identification of all network elements or part of it is possible. The equations to be solved are nonlinear and the computational effort is quite excessive for the on-line implementation of the procedure.

Visvanathan and Sangiovanni-Vincentelli (1981) and Saeks et al.

(1981) utilized the multiple inputs to nonlinear systems to formulate the diagnosability conditions for dynamic and nondynamic nonlinear networks.

If all nodes of a linear network are accessible the problem of identifying all network elements reduces to that of solving a system of linear equations. Trick, Mayeda and Sakla (1979) and Sakla (1979) described techniques for isolating network elements that require accessibility to all network nodes. They utilized Tellegen's theorem with multiple test vectors to identify network elements. Trick (1980) further confirmed that the same results could be obtained by invoking Kirchhoff's Current Law.

If the numbers of measurements is still less than the number of network parameters the estimation criteria presented in Section 2.3.1 can be applied. Bandler et al. (1982) utilized the ℓ_1 norm with multiple test vectors. Details of their approach are presented in Section 5.3 of this thesis.

2.5 DISCUSSION AND COMPARISON

Saeks (1981) proposed criteria by which different fault location techniques can be judged. He examined the following points.

(a) Computational Requirements. The on-line and off-line computational requirements of the different techniques are rated. The optimum technique should have minimal on-line computational requirements and moderate off-line requirements.

(b) Test Points. An optimum technique will be based on the

utilization of a limited number of test points without complicating the fault analysis problem computationally.

(c) Robustness. The optimum approach should be robust against the changes in the nonfaulty elements inside their tolerance region.

(d) Models. The fault analysis technique should be able to utilize both the nominal models and faulty models of the network elements.

(e) Module vs. Parameter Testing. It has been noted that a module oriented CAT algorithm is preferred over a parameter oriented algorithm if it can be formulated without compromising other factors.

(f) In-Situ Testing. The ideal CAT algorithm should allow for in-situ testing. Such an algorithm should work with an arbitrary input signal rather than a fixed set of test vectors.

(g) Parallel Processing. The degree to which an algorithm can be implemented in a parallel processing mode becomes a significant factor in determining its viability with the increase in size of the networks.

In Table 2.1 the goals for an ideal algorithm are summarized and the degree to which the various techniques achieve these goals is indicated.

We believe that the simulation-after-test technique employing our decomposition approach for fault analysis satisfies quite closely the desired criteria, as we will attempt to show (in the following chapter). The technique requires moderate off-line computational requirements and

TABLE 2.1

PERFORMANCE OF ANALOG FAULT DIAGNOSIS ALGORITHMS
ACCORDING TO SAEKS (1981)

Criteria	Goal	Simulation-Before-Test	Simulation-After-Test With A Single Test Vector	Simulation-After-Test With Multiple Test Vectors
Off-line Comp.	Moderate	Very high	Moderate	Moderate
On-line Comp.	Minimal	Minimal	Minimal	High
Test Points	Less than linear growth	Less than linear growth	Linear growth	Less than linear growth
Robustness	Yes	No	Yes	Yes
Models	Nominal/Faulted	Faulted	Nominal	Nominal
Modules/Param.	Modules	Modules	Modules/Param.	Parameters
In-Situ Testing	Yes	No	Yes	No
Parallel Proc.	Yes	Yes	Yes	No

very reasonable on-line computational requirements. It needs only a limited number of test points and can handle the effect of tolerances on nonfaulty elements. Both nominal and faulty models are utilized. The technique is module oriented and very suitable for in-situ testing and parallel processing.

3

FAULT LOCATION IN ANALOG CIRCUITS A DECOMPOSITION APPROACH

3.1 INTRODUCTION

In this chapter, we present a new simulation-after-test method for fault location with the aim of keeping both the computations and measurements to acceptable bounds. A nodal decomposition, Happ (1974), of the network into smaller uncoupled subnetworks is carried out. The measurement nodes must include the nodes of decomposition. The voltage measurements are employed to isolate the faulty subnetworks. Utilizing the incidence relations between subnetworks and Kirchhoff's Current Law (KCL) we develop necessary and almost sufficient conditions for a subnetwork or a group of subnetworks to be fault free. Logical analysis of the results of these tests is carried out to identify faulty subnetworks.

In analog circuits the good network elements are usually not at their nominal values, but are randomly distributed within specified tolerance intervals. A probabilistic approach is used to check whether the testing conditions can be satisfied under these random changes.

The application of the method to both linear and nonlinear networks is discussed and illustrated using practical examples.

3.2 NETWORK DECOMPOSITION AND LOGICAL ANALYSIS

3.2.1 General Description

The topology of the network under test is known. In the pre-test stage we perform a nodal decomposition of the network. This results in subnetworks connected by the nodes of decomposition. There should be no mutual coupling between any two subnetworks and the nodes of decomposition should be chosen from the set where voltage measurements can be performed. The decomposition is either performed by inspection (for networks of relatively small size) or a special algorithm is used for that purpose, e.g., the heuristic algorithm proposed by Sangiovanni-Vincentelli et al. (1977).

Fig. 3.1 illustrates the situation. Subnetworks S_i and S_j are linked at nodes of decomposition C_{ij}. In actual testing we excite the network, usually with a current source, and perform voltage measurements at C_{ij}.

Testing conditions are applied to identify the nonfaulty subnetworks. The application of a testing condition is referred to as a test. The outcome of a test is classified simply as pass or fail. The test passes if and only if all subnetworks involved in the test are fault-free. The test fails if and only if at least one of those subnetworks is faulty. A subnetwork is faulty if it contains one or more faulty elements. A test that is applied to check whether subnetwork S_i is fault-free or not is described as a self-testing condition (STC). A test that is applied to check whether a group of k subnetworks $S_{j_1}, S_{j_2}, \ldots, S_{j_k}$ are fault-free or not is called a mutual-testing condition.

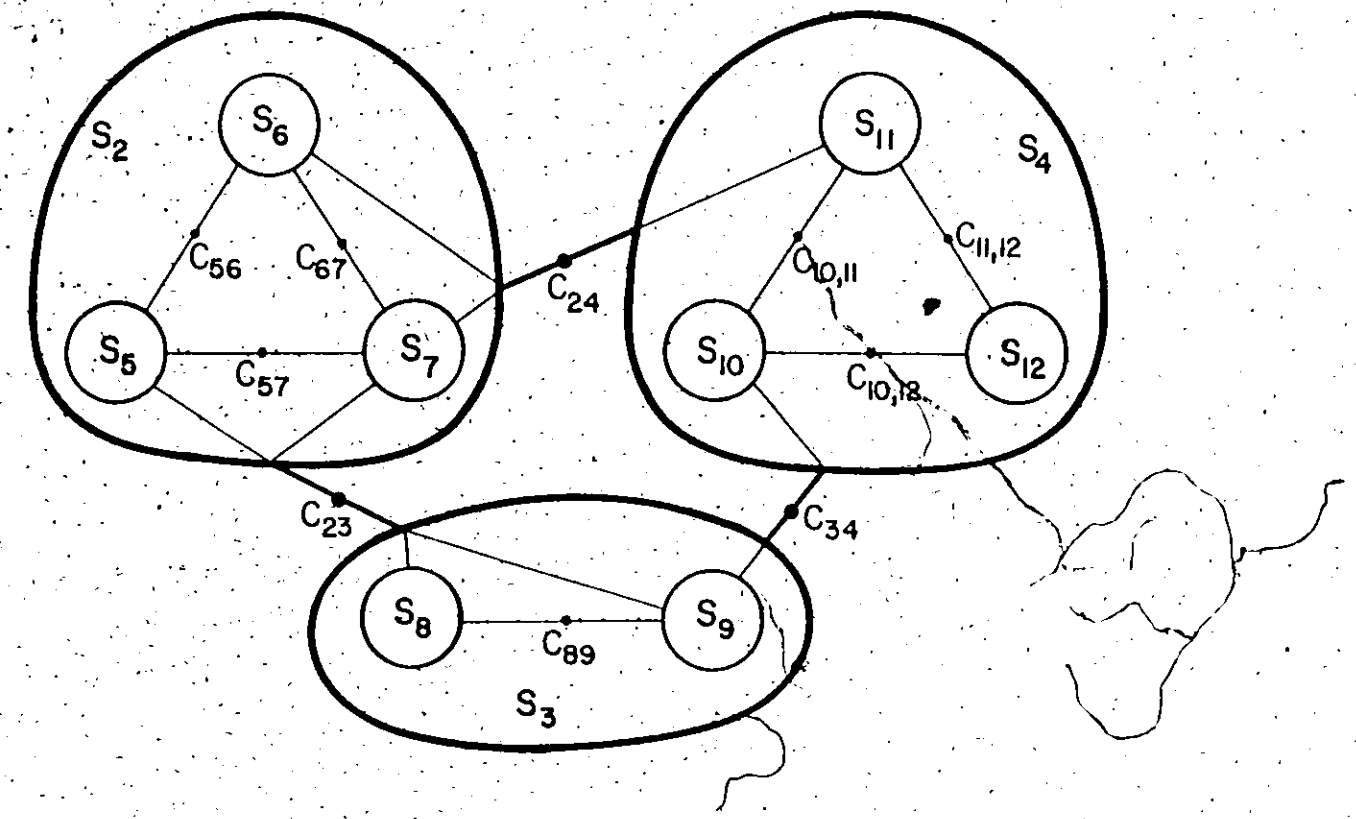

Fig. 3.1 Nodal decomposition of a network.

(MTC). In practice, we utilize the measurements together with the incidence relations between subnetworks to expedite these tests.

3.2.2 Hierarchical Decomposition

We follow a hierarchical decomposition approach (Gupta et al. 1982). This is illustrated in Fig. 3.2 and is represented by the so-called tree of decomposition. Vertices of this tree are assigned to subnetworks. Various levels of decomposition can be considered. The procedure can be continued until further division of subnetworks through the measurement nodes is impossible. The subnetworks at the final level are called blocks.

We begin by considering suitable STC and MTC situations for the subnetworks at the first level of decomposition. (We assume that a STC applied to S_1 has confirmed a fault). If a subnetwork is declared nonfaulty no further partitioning of it need be carried out. Faulty subnetworks and those which we are not sure about are decomposed further, if possible.

3.2.3 Logical Analysis

The results of different tests are analyzed to identify the faulty and nonfaulty subnetworks. Logical functions are utilized for this purpose. Every subnetwork has associated with it a logical variable σ, which takes the value 1 if the subnetwork is good and 0 if it is faulty. Every test is associated with a logical test function (LTF) which is equal to the complete product of variables σ_{j_i} if the

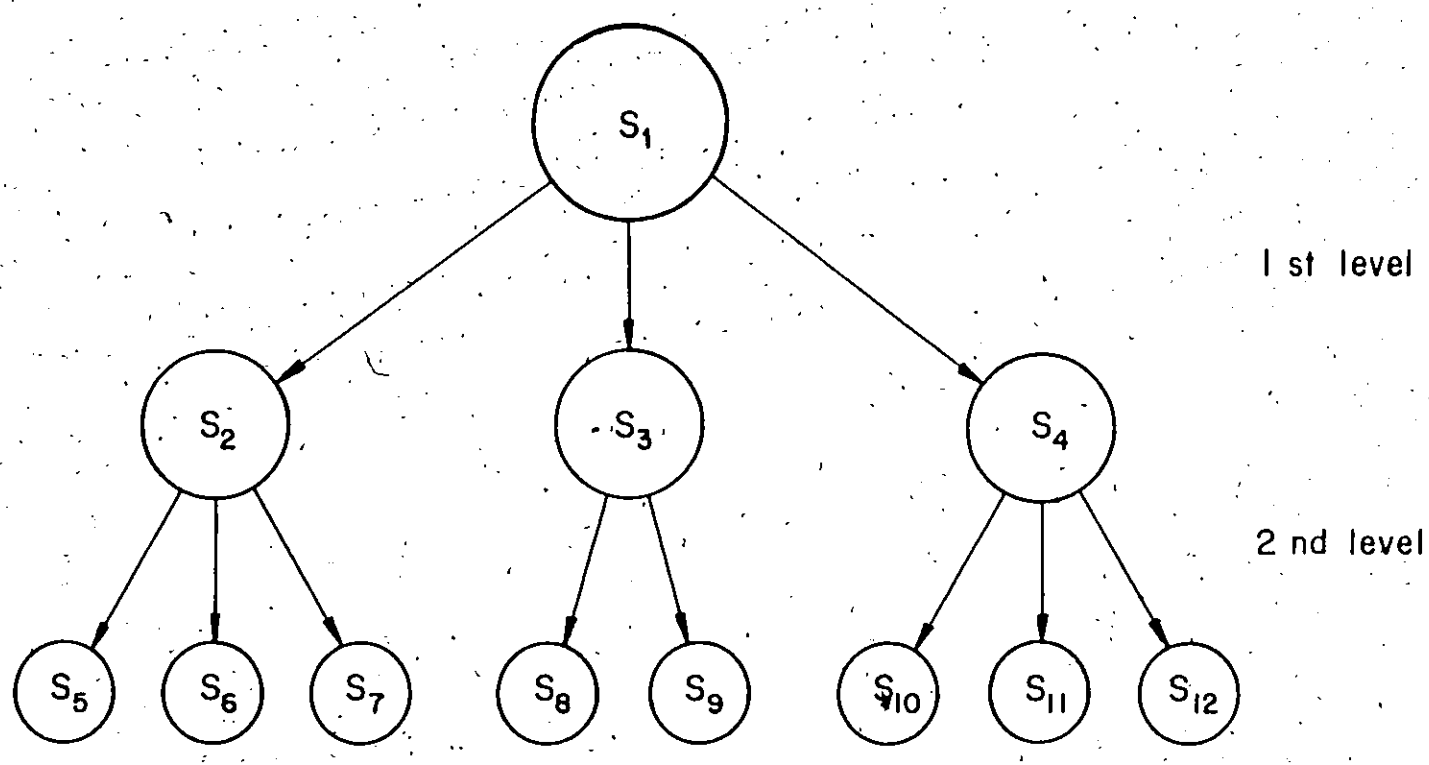

Fig. 3.2 Illustration of the hierarchical decomposition of the network of Fig. 3.1.

test is a pass

$$T_{J_t} \triangleq \sigma_{j_1} \cap \sigma_{j_2} \cap \ldots \cap \sigma_{j_k} , \tag{3.1}$$

where

$$J_t \triangleq \{j_1, j_2, \ldots, j_k\} . \tag{3.2}$$

j_i refers to subnetwork S_{j_i}, k is the number of subnetworks involved in the test, or the complete union of complemented variables $\bar{\sigma}_{j_i}$

$$T_{J_t} \triangleq \bar{\sigma}_{j_1} \cup \bar{\sigma}_{j_2} \cup \ldots \cup \bar{\sigma}_{j_k} \tag{3.3}$$

if the test is a fail.

A logical diagnostic function (LDF) is given by

$$D_\ell \triangleq \left(\bigcap_{t=1}^{g} T_{J_t}\right) \cap \left(\bigcap_{t=g+1}^{\ell} \bar{T}_{J_t}\right) . \tag{3.4}$$

where the first g LTFs correspond to successful tests and ℓ is the total number of tests. In the LDF, the subnetworks which are represented by $\bar{\sigma}_i$ are faulty and those which are represented by σ_i are nonfaulty. If a subnetwork is not represented in the LDF we assume nothing about its status: more tests are necessary. We usually construct the LDF in a sequential manner by combining the results of the current test to previous tests. This usually reduces the number of tests needed since some of the tests could be redundant.

3.2.4 Example 3.1: Illustration of Logical Analysis

In the decomposed network of Fig. 3.3 let S_3 and S_5 be faulty blocks in an otherwise fault-free network. There is no subnetwork for

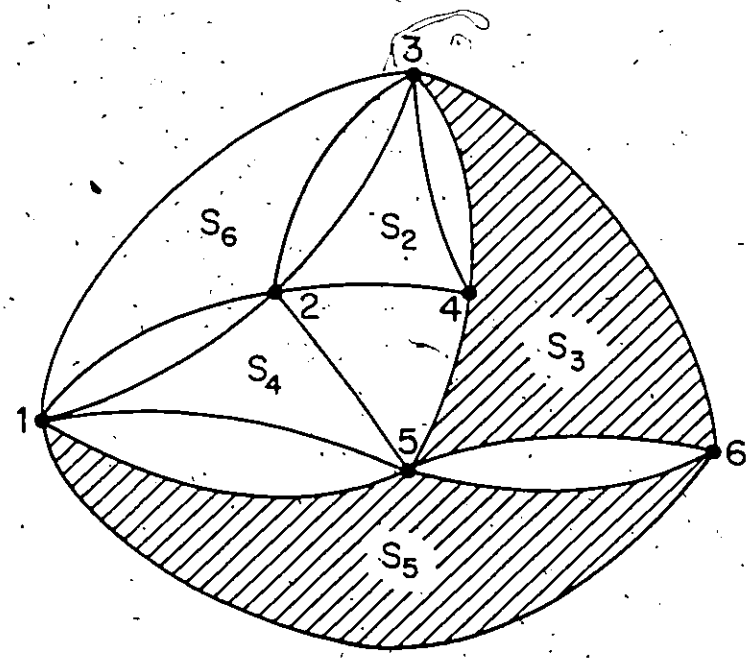

Fig. 3.3 Application of testing conditions and logical analysis to a decomposed faulty network.

which we can check the STC. We will apply MTCs to evaluate T_{35}, T_{23}, T_{236}, T_{246} and T_{456}. Only the test for T_{246} is a pass so we have

$$D_5 = (\bar{\sigma}_3 \cup \bar{\sigma}_5) \cap (\bar{\sigma}_2 \cup \bar{\sigma}_3) \cap (\bar{\sigma}_2 \cup \bar{\sigma}_3 \cup \bar{\sigma}_6) \cap (\sigma_2 \cap \sigma_4 \cap \sigma_6) \cap (\bar{\sigma}_4 \cup \bar{\sigma}_5 \cup \bar{\sigma}_6)$$

$$= \sigma_2 \cap \bar{\sigma}_3 \cap \sigma_4 \cap \bar{\sigma}_5 \cap \sigma_6,$$

from which it is evident that S_3 and S_5 are the only faulty subnetworks.

3.3 APPLICATION OF TESTING CONDITIONS TO SUBNETWORKS

In this section we give necessary and almost sufficient conditions for a subnetwork or group of subnetworks to be fault-free. The conditions are based on invoking KCL and topological relations.

For analog circuits the effect of two independent faults is highly unlikely to cancel at the measurement nodes. We adopt this reasonable heuristic (Wu et al. 1982).

3.3.1 Description of Subnetworks

The input-output relation for a subnetwork S_i, that is connected to the rest of the network by m_i+1 external nodes, as shown in Fig. 3.4, with one of the nodes taken as the reference, is given by

$$\underset{\sim}{i}^{M_i}(t) = \underset{\sim}{h}^{M_i}(\underset{\sim}{x}^{M_i}(t), \underset{\sim}{\phi}_i), \tag{3.5}$$

where $\underset{\sim}{\phi}_i$ is the vector of the subnetwork parameters and the cardinality of $\underset{\sim}{i}^{M_i}(t)$, $\underset{\sim}{h}^{M_i}$ and $\underset{\sim}{v}^{M_i}(t)$ is m_i. We assume that the subnetwork S_i is connected, i.e., there exists a path between any two nodes of subnetwork S_i and the m_i+1 external nodes do not decompose the subnetwork further,

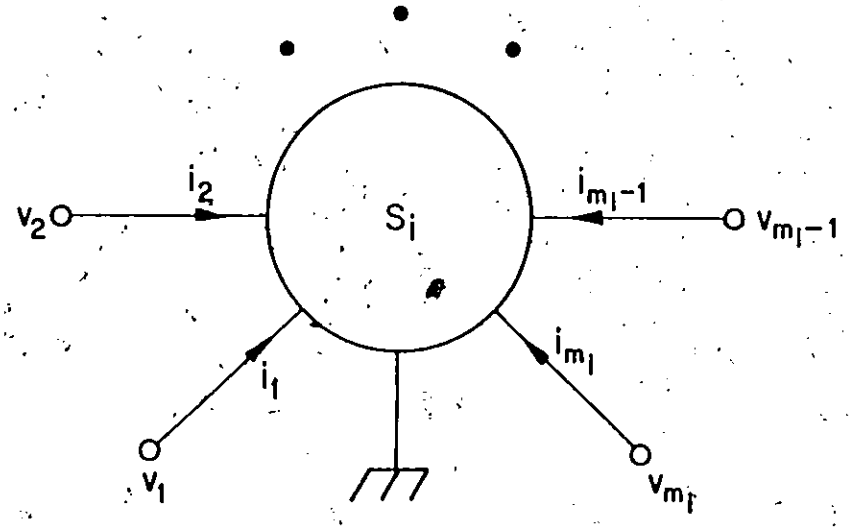

Fig. 3.4 Subnetwork S_i with m_i+1 external nodes.

i.e., we cannot partition S_i into smaller uncoupled subnetworks using only the set of m_i+1 external nodes. Let

$$M_i = M_{i\alpha} \cup M_{i\beta} \cup M_{i\gamma} \cup M_{i\delta}, \tag{3.6}$$

where $M_{i\alpha}$ is the set of nodes where both voltages and currents are known, $M_{i\beta}$ is the set of nodes where only voltages are known, $M_{i\gamma}$ is the set of nodes where only currents are known, $M_{i\delta}$ is the set of nodes where neither currents nor voltages are known, and M_i is the set of the m_i nodes. Accordingly, we can rewrite (3.5) as

$$\underset{\sim}{i}^{M_{i\alpha}}(t) = \underset{\sim}{h}^{M_{i\alpha}}(\underset{\sim}{v}^{M_{i\alpha}}(t), \underset{\sim}{v}^{M_{i\beta}}(t), \underset{\sim}{v}^{M_{i\gamma}}(t), \underset{\sim}{v}^{M_{i\delta}}(t), \underset{\sim}{\phi}_i), \tag{3.7a}$$

$$\underset{\sim}{i}^{M_{i\beta}}(t) = \underset{\sim}{h}^{M_{i\beta}}(\underset{\sim}{v}^{M_{i\alpha}}(t), \underset{\sim}{v}^{M_{i\beta}}(t), \underset{\sim}{v}^{M_{i\gamma}}(t), \underset{\sim}{v}^{M_{i\delta}}(t), \underset{\sim}{\phi}_i), \tag{3.7b}$$

$$\underset{\sim}{i}^{M_{i\gamma}}(t) = \underset{\sim}{h}^{M_{i\gamma}}(\underset{\sim}{v}^{M_{i\alpha}}(t), \underset{\sim}{v}^{M_{i\beta}}(t), \underset{\sim}{v}^{M_{i\gamma}}(t), \underset{\sim}{v}^{M_{i\delta}}(t), \underset{\sim}{\phi}_i), \tag{3.7c}$$

$$\underset{\sim}{i}^{M_{i\delta}}(t) = \underset{\sim}{h}^{M_{i\delta}}(\underset{\sim}{v}^{M_{i\alpha}}(t), \underset{\sim}{v}^{M_{i\beta}}(t), \underset{\sim}{v}^{M_{i\gamma}}(t), \underset{\sim}{v}^{M_{i\delta}}(t), \underset{\sim}{\phi}_i). \tag{3.7d}$$

If the cardinality of the set $M_{i\alpha}$ is greater than the cardinality of the set $M_{i\delta}$, i.e., $m_{i\alpha} > m_{i\delta}$, a necessary condition for the subnetwork S_i to be fault-free is that

$$\underset{\sim}{i}^{M_{i\alpha}}(t) = \underset{\sim}{h}^{M_{i\alpha}}(\underset{\sim}{v}^{M_{i\alpha}}(t), \underset{\sim}{v}^{M_{i\beta}}(t), \underset{\sim}{v}^{M_{i\gamma}}(t), \underset{\sim}{v}^{M_{i\delta}}(t), \underset{\sim}{\phi}_i^0), \tag{3.8a}$$

$$\underset{\sim}{i}^{M_{i\gamma}}(t) = \underset{\sim}{h}^{M_{i\gamma}}(\underset{\sim}{v}^{M_{i\alpha}}(t), \underset{\sim}{v}^{M_{i\beta}}(t), \underset{\sim}{v}^{M_{i\gamma}}(t), \underset{\sim}{v}^{M_{i\delta}}(t), \underset{\sim}{\phi}_i^0), \tag{3.8b}$$

is a consistent system of overdetermined equations at any instant of time, where $\underset{\sim}{\phi}_i^0$ is the vector of nominal parameter values of the subnetwork. We refer to this condition as the internal-self-testing

condition (ISTC). We utilize this condition in locating faulty regions inside faulty subnetworks.

When all the voltages of M_i are known and $m_{i\alpha}$ is greater than or equal to one, we can state the following stronger result.

3.3.2 Lemma 3.1: Self-Testing Condition (STC)

A necessary and almost sufficient condition for a connected subnetwork S_i with m_i+1 external nodes that do not decompose it further, $m_{i\alpha} > 1$ and $m_{i\gamma} = m_{i\delta} = 0$ to be fault-free is that

$$\underset{\sim}{i}^{M_{i\alpha}}(t) - \underset{\sim}{h}^{M_{i\alpha}}(\underset{\sim}{v}^{M_i}(t), \underset{\sim}{\phi}_i^0) = \underset{\sim}{0} \quad \forall\, t . \tag{3.9}$$

The necessity of (3.9) is obvious. For the sufficiency part of Lemma 3.1 the adjoint network concept (Director and Rohrer 1969) can be utilized to prove that any change in the subnetwork should be observable at the m_i nodes, thus changing $\underset{\sim}{i}^{M_i}$ if no change has occurred in $\underset{\sim}{i}^{M_{i\alpha}}$ from that computed using the given $\underset{\sim}{v}^{M_i}$ and the nominal parameters of the subnetwork, this implies that the subnetwork is fault-free. It is sufficient to check Lemma 3.1 using only one external current to the subnetwork. The proof is given in Appendix A.

Normally, the voltages of the m_i nodes are directly measured. The currents $\underset{\sim}{i}^{M_{i\alpha}}$ are not directly measured since it is difficult to do so practically except when they represent the input excitation to the whole network. The application of KCL and topological relations overcome this difficulty. The currents are not measured: they are computed using the nominal parameter values together with the measured

voltages, then KCL is invoked.

Let us assume we have a set of k subnetworks S_i, $i \in J_t$, which are incident on common node c as shown in Fig. 3.5. Each subnetwork is assumed to be connected and has m_i+1 external nodes that do not decompose the subnetwork further. The input-output relation for every subnetwork is similar to that given in (3.5). The voltages of the m_i external nodes are assumed to be measured. The current incident to the common node c from subnetwork S_i is given by

$$i_c^{M_i}(t) = h_c^{M_i}(\underset{\sim}{v}^{M_i}(t), \phi_i) . \qquad (3.10)$$

3.3.3 Lemma 3.2: Mutual-Testing Condition (MTC)

A necessary and almost sufficient condition for S_i, $i \in J_t$, to be fault-free is that

$$\sum_{i \in J_t} h_c^{M_i}(\underset{\sim}{v}^{M_i}(t), \phi_i^0) = 0 \quad \forall\, t , \qquad (3.11)$$

i.e., the currents incident to the common node c computed using the measured voltages and nominal parameter values should satisfy KCL.

Necessity is obvious. Sufficiency follows from the heuristic that no two faults will cancel each other at the measurement nodes. If (3.11) is satisfied, this implies that the current incident with the measurement node c from subnetwork S_i is given by

$$h_c^{M_i}(\underset{\sim}{v}^{M_i}(t), \phi_i^0) , \qquad (3.12)$$

but from Lemma 3.1 this implies that the subnetwork S_i is fault-free and thus all subnetworks S_i, $i \in J_t$ are nonfaulty.

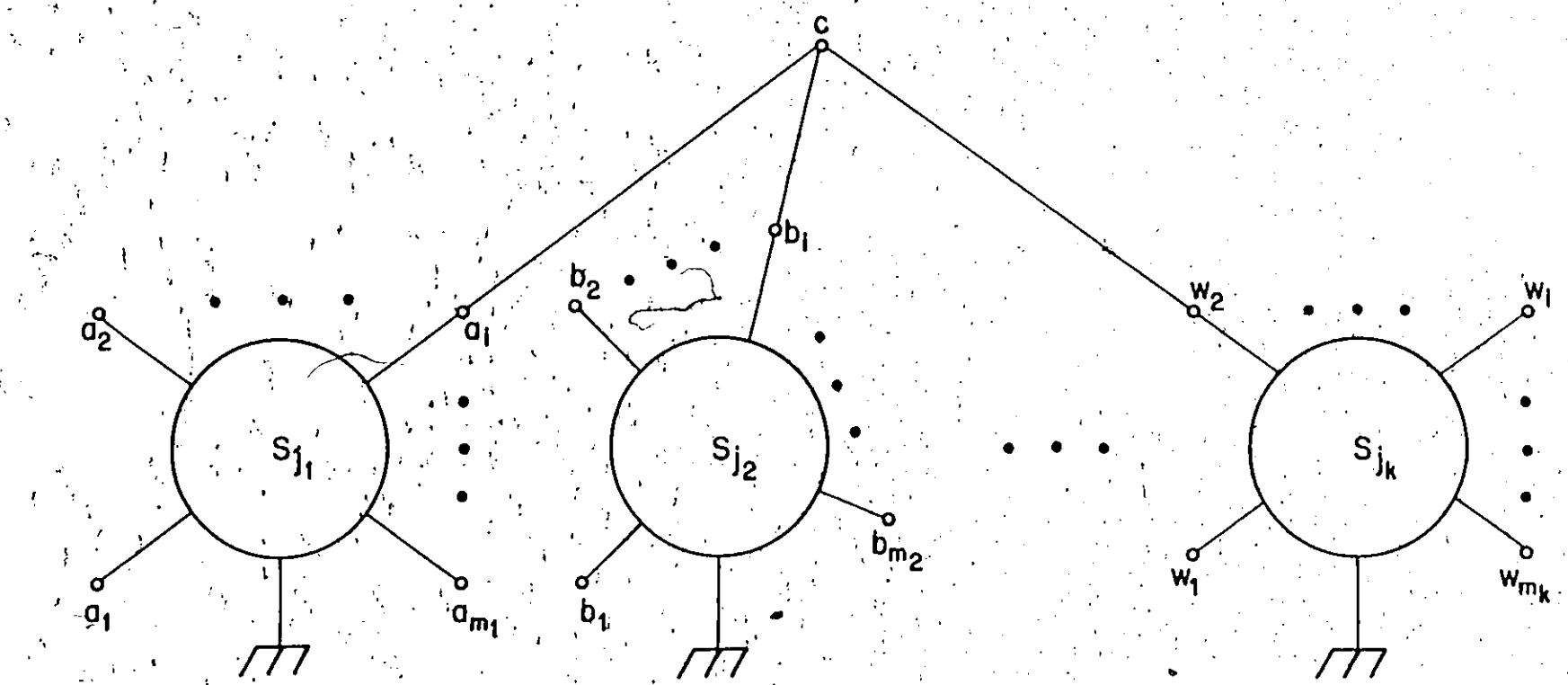

Fig. 3.5 k subnetworks incident at node c.

When the previous test is applied to two subnetworks which are incident at a common node c we refer to it as the bi-testing condition (BTC).

3.3.4 Lemma 3.3: Generalized-Mutual-Testing Condition (GMTC)

Let E_i, $i \in J_t$, denote some external nodes of the subnetwork S_i. Each subnetwork S_i is connected and has m_i+1 nodes that do not decompose it further, $E_i \subseteq M_i$. If the currents incident to E_i, $i \in J_t$, form a cut set, then a necessary and almost sufficient condition for these subnetworks to be fault-free is that

$$\sum_{i \in J_t} \sum_{j \in E_i} h_j^{M_i}(\underset{\sim}{v}^{M_i}(t), \underset{\sim}{\phi}_i^0) = 0 \quad \forall\, t. \tag{3.13}$$

3.3.5 Example 3.2: Illustration of Lemma 3.3

Consider the two subnetworks S_a and S_b that are incident with the subnetwork S_c, which is faulty, as shown in Fig. 3.6. The BTC fails for $S_a \cup S_c$ and $S_b \cup S_c$. But the output branches that connect S_a and S_b with S_c form a cut set. So, according to the GMTC S_a and S_b are fault free if and only if the computed currents using the measured external voltages and nominal design values of S_a and S_b through the cut set considered will sum to zero, i.e.,

$$i_1^a + i_2^a + i_3^a + i_4^b + i_5^b = 0 \quad .$$

3.4 TOLERANCE CONSIDERATIONS

The actual values of nonfaulty elements can deviate from their

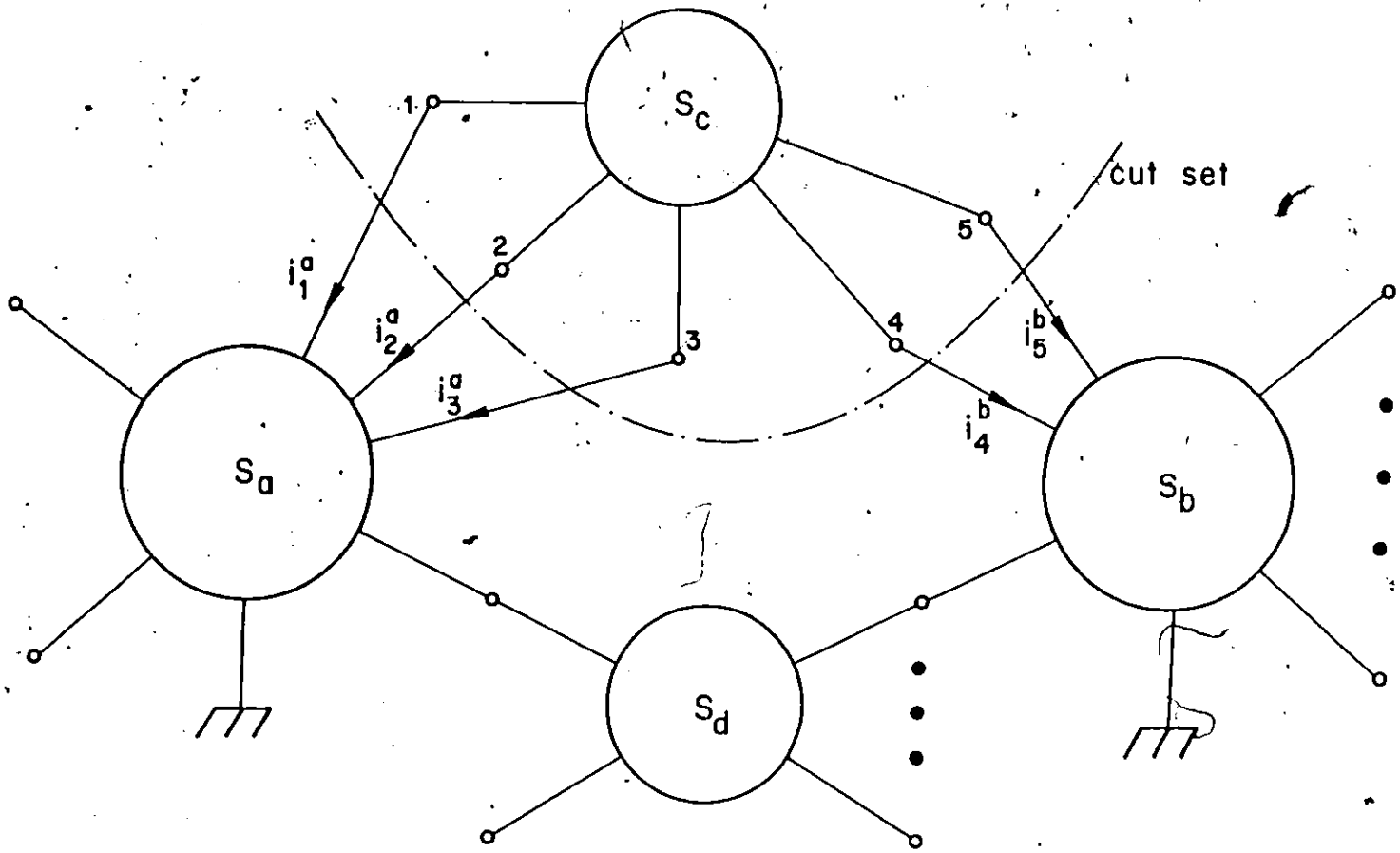

Fig. 3.6 Example of testing using cut sets.

nominal values within prescribed tolerance bounds. Thus, in practice, we face the situation that Lemmas 3.1-3.3 are not satisfied to the required degree of accuracy. Taking the tolerance changes in the subnetwork elements into consideration we may write condition (3.9) as

$$\underset{\sim}{i}^{M_{i\alpha}}(t) - \underset{\sim}{h}^{M_{i\alpha}}(\underset{\sim}{v}^{M_i}(t),\ \underset{\sim}{\phi}_i^0 + \Delta\underset{\sim}{\phi}_i) = \underset{\sim}{0}\ , \qquad (3.14)$$

where $\Delta\underset{\sim}{\phi}_i \triangleq [\Delta\phi_{i1}\ \Delta\phi_{i2}\ \cdots\ \Delta\phi_{ip}]^T$ defines the tolerance changes in the p elements of the subnetwork under consideration. For small tolerances the first-order approximation can be utilized to describe the changes in the network response. Accordingly, we may write (3.14) as

$$\underset{\sim}{i}^{M_{i\alpha}}(t) - \underset{\sim}{h}^{M_{i\alpha}}(\underset{\sim}{v}^{M_i}(t),\ \underset{\sim}{\phi}_i^0) = \sum_{j=1}^{p} \frac{\partial \underset{\sim}{h}^{M_{i\alpha}}}{\partial \phi_{ij}}\ \Delta\phi_{ij}\ . \qquad (3.15)$$

Let

$$\Delta\underset{\sim}{i}^{M_{i\alpha}}(t) \triangleq \underset{\sim}{i}^{M_{i\alpha}}(t) - \underset{\sim}{h}^{M_{i\alpha}}(\underset{\sim}{v}^{M_i}(t),\ \underset{\sim}{\phi}_i^0) = \underset{\sim}{B}_i\ \Delta\underset{\sim}{\phi}_i\ , \qquad (3.16)$$

where

$$\underset{\sim}{B}_i \triangleq \left[\frac{\partial \underset{\sim}{h}^{M_{i\alpha}}}{\partial \phi_{i1}}\ \frac{\partial \underset{\sim}{h}^{M_{i\alpha}}}{\partial \phi_{i2}}\ \cdots\ \frac{\partial \underset{\sim}{h}^{M_{i\alpha}}}{\partial \phi_{ip}}\right]\ . \qquad (3.17)$$

At a certain instant t_o of time equation (3.16) is an underdetermined system of linear equations in the variables $\Delta\underset{\sim}{\phi}_i$. The weighted-least-squares solution of (3.16) is given by

$$\Delta\underset{\sim}{\phi}_i = \underset{\sim}{B}_i^{+}\ \Delta\underset{\sim}{i}^{M_{i\alpha}}(t_o)\ , \qquad (3.18)$$

where

$$\underset{\sim}{B}_i^{+} \triangleq \underset{\sim}{C}_i \underset{\sim}{B}_i^T \left[\underset{\sim}{B}_i \underset{\sim}{C}_i \underset{\sim}{B}_i^T\right]^{-1} \tag{3.19}$$

and $\underset{\sim}{C}_i$ is a weighting matrix (Hankley and Merrill 1971, Merrill 1973). For $\Delta\underset{\sim}{\phi}_i$ normally distributed with mean $\underset{\sim}{0}$ and covariance matrix $\underset{\sim}{C}_i^{-1}$, the solution given in (3.18) is the conditional expected value of the parameters $\Delta\underset{\sim}{\phi}_i$ (Hankley et al. 1971) i.e.,

$$\Delta\underset{\sim}{\phi}_i = E\left[\Delta\underset{\sim}{\phi}_i \mid \Delta\underset{\sim}{i}^{M_i\alpha}(t_o)\right], \tag{3.20}$$

where E denotes the expectation. Moreover, the solution is a minimum in the weighted-least-squares sense. So $\Delta\underset{\sim}{\phi}_i$ is the solution of

$$\text{minimize } \Delta\underset{\sim}{\phi}_i^T \underset{\sim}{C}_i^{-1} \Delta\underset{\sim}{\phi}_i \tag{3.21}$$

subject to

$$\underset{\sim}{B}_i \Delta\underset{\sim}{\phi}_i = \Delta\underset{\sim}{i}^{M_i\alpha}(t_o) . \tag{3.22}$$

Using the probabilistic interpretation of the result, namely (3.20), we can have a measure of how far (3.9) is satisfied under the variations caused by the tolerances. If any component of the computed vector $\Delta\underset{\sim}{\phi}_i$ from (3.18) significantly exceeds its tolerance value we consider that the test is unsuccessful. The consideration of the matrix $\underset{\sim}{C}_i$ in (3.19) provides the possibility of considering the known correlation between the elements of the subnetworks.

The effect of tolerances on conditions (3.11) and (3.13) is treated in a similar way.

3.5 FAULT LOCATION IN LINEAR NETWORKS

For linear networks, the matrix description of the subnetworks greatly simplifies the computational effort needed for checking the testing conditions. Without loss of generality we assume sinusoidal excitations are applied. Whence, we represent the voltages and currents by their phasor variables.

3.5.1 General Description

Consider a subnetwork S_i which has m_i+1 external nodes, one of which is the reference node, and n_i internal nodes. The nodal equations are given by

$$\begin{bmatrix} \underset{\sim}{Y}_{M_i M_i} & \underset{\sim}{Y}_{M_i N_i} \\ \underset{\sim}{Y}_{N_i M_i} & \underset{\sim}{Y}_{N_i N_i} \end{bmatrix} \begin{bmatrix} \overset{M_i}{\underset{\sim}{V}} \\ \overset{N_i}{\underset{\sim}{V}} \end{bmatrix} = \begin{bmatrix} \overset{M_i}{\underset{\sim}{I}} + \overset{M_i}{\underset{\sim}{I}}_g \\ \overset{N_i}{\underset{\sim}{I}}_g \end{bmatrix}, \tag{3.23}$$

where

$$\underset{\sim}{I}_g^i \triangleq \begin{bmatrix} \overset{M_i}{\underset{\sim}{I}}_g \\ \overset{N_i}{\underset{\sim}{I}}_g \end{bmatrix}, \tag{3.24}$$

defines the current sources associated with the subnetwork, $\overset{M_i}{\underset{\sim}{V}}$ is the voltage vector of the external nodes, $\overset{N_i}{\underset{\sim}{V}}$ is the voltage vector of the internal nodes and $\overset{M_i}{\underset{\sim}{I}}$ is the current input vector to the subnetwork from outside through m_i external nodes. Eliminating the n_i internal nodes we get

$$\underset{\sim}{I}^{M_i} = -\left[\underset{\sim}{I}_g^{M_i} - \underset{\sim}{Y}_{M_i N_i} \underset{\sim}{Y}_{N_i N_i}^{-1} \underset{\sim}{I}_g^{N_i}\right] + \left[\underset{\sim}{Y}_{M_i M_i} - \underset{\sim}{Y}_{M_i N_i} \underset{\sim}{Y}_{N_i N_i}^{-1} \underset{\sim}{Y}_{N_i M_i}\right] \underset{\sim}{V}^{M_i}, \tag{3.25}$$

or more compactly

$$\underset{\sim}{I}^{M_i} = \underset{\sim}{H}_{M_i} \underset{\sim}{I}_g^{i} + \underset{\sim}{Y}_{M_i} \underset{\sim}{V}^{M_i}, \tag{3.26}$$

where

$$\underset{\sim}{H}_{M_i} \triangleq -\left[\underset{\sim}{1} - \underset{\sim}{Y}_{M_i N_i} \underset{\sim}{Y}_{N_i N_i}^{-1}\right], \tag{3.27}$$

$$\underset{\sim}{Y}_{M_i} \triangleq \left[\underset{\sim}{Y}_{M_i M_i} - \underset{\sim}{Y}_{M_i N_i} \underset{\sim}{Y}_{N_i N_i}^{-1} \underset{\sim}{Y}_{N_i M_i}\right] \tag{3.28}$$

and $\underset{\sim}{1}$ is a unit matrix of order m_i.

Equation (3.26) describes the input-output relation of the subnetwork. This relation is the one we are interested in to verify Lemmas 3.1-3.3.

Since the hierarchical decomposition is obtained prior to actual testing, it is fixed at the time of testing and the characteristics of the subnetworks, namely, $\underset{\sim}{Y}_{M_i}$ and $\underset{\sim}{H}_{M_i} \underset{\sim}{I}_g^{i}$ are computed off-line using nominal values and stored before conducting the actual testing. At the time of testing, the only on-line computation required is the matrix by vector multiplication, namely $\underset{\sim}{Y}_{M_i} \underset{\sim}{V}^{M_i}$. Let E define the set of faulty subnetworks or the ones which we are uncertain about. The procedure in carrying out the tests can be summarized as follows.

3.5.2 Procedure for Locating Faulty Subnetworks

Step 0 $j = 0$.

$E_j = \{S_1\}$. (S_1 is the network under test.)

Step 1 Partition using the least number of test nodes every $S_i \in E_j$, if possible, into smaller uncoupled subnetworks to constitute E_{j+1}. Otherwise go to 8.

Comment Only decomposed parts of every $S_i \in E_j$ will be contained in E_{j+1} or S_i itself if it is not decomposable.

Step 2 For every $S_i \in E_{j+1}$, find the sets $M_{i\alpha}$ and $M_{i\beta}$.

Step 3 Check the testing conditions of Lemmas 3.1-3.3.

Step 4 Identify faulty subnetworks using a logical analysis of the tests.

Step 5 Utilize the nonfaulty subnetworks to determine the external currents of the faulty subnetworks.

Step 6 Update the set E_{j+1} by removing nonfaulty subnetworks.

Step 7 $j = j+1$. Go to 1.

Step 8 Print out the components of the set $E = E_j$.

3.5.3 Computational Effort

The number of nodes where measurements are performed and the computational effort depend on the size of the blocks and the number of levels of decomposition. If we assume that we have L levels of decomposition and the resulting hierarchical decomposition is binary and symmetric, the number of subnetworks (blocks) at the final level of decomposition will be equal to 2^L. If each block has n nodes and b is the number of interconnection nodes between any two blocks, then the total number of network nodes is approximately given by

$$N_T \cong 2^L (n-b) + b \ . \qquad (3.29)$$

Assume that all the interconnection nodes are measurement nodes. Then their number N_m can be estimated from

$$N_m \leq 2^L b - b \ . \tag{3.30}$$

Accordingly, a measure of the needed degree of accessibility is given by the ratio

$$r = \frac{N_m}{N_T} \leq \frac{b}{n-b} \ . \tag{3.31}$$

For a smaller r, n should be much greater than b. On the other hand, we wish to have n as small as possible to obtain better diagnosis and decreased computational effort. There is clearly a compromise between the degree of accessibility and the size of the block.

If the faulty elements are in one block, following the hierarchical decomposition strategy and assuming binary partition, we check the testing conditions for just two subnetworks at each level. The total number of subnetworks to be considered is consequently 2L. In a number of steps proportional to log (N_T) we isolate the faulty subnetwork. Obviously, we do not need to measure all the voltages of the test nodes. Less than bL measurements are actually required.

3.5.4 Example 3.3: Linear Network Example

The network under test is composed of two identical low-pass filter sections in cascade. The low-pass filter section is shown in Fig. 3.7 and its nominal elements values are given in Table 3.1 (Semmelman et al. 1971). The operational amplifier is modeled by a controlled source and output resistance, as shown in Fig. 3.8. The

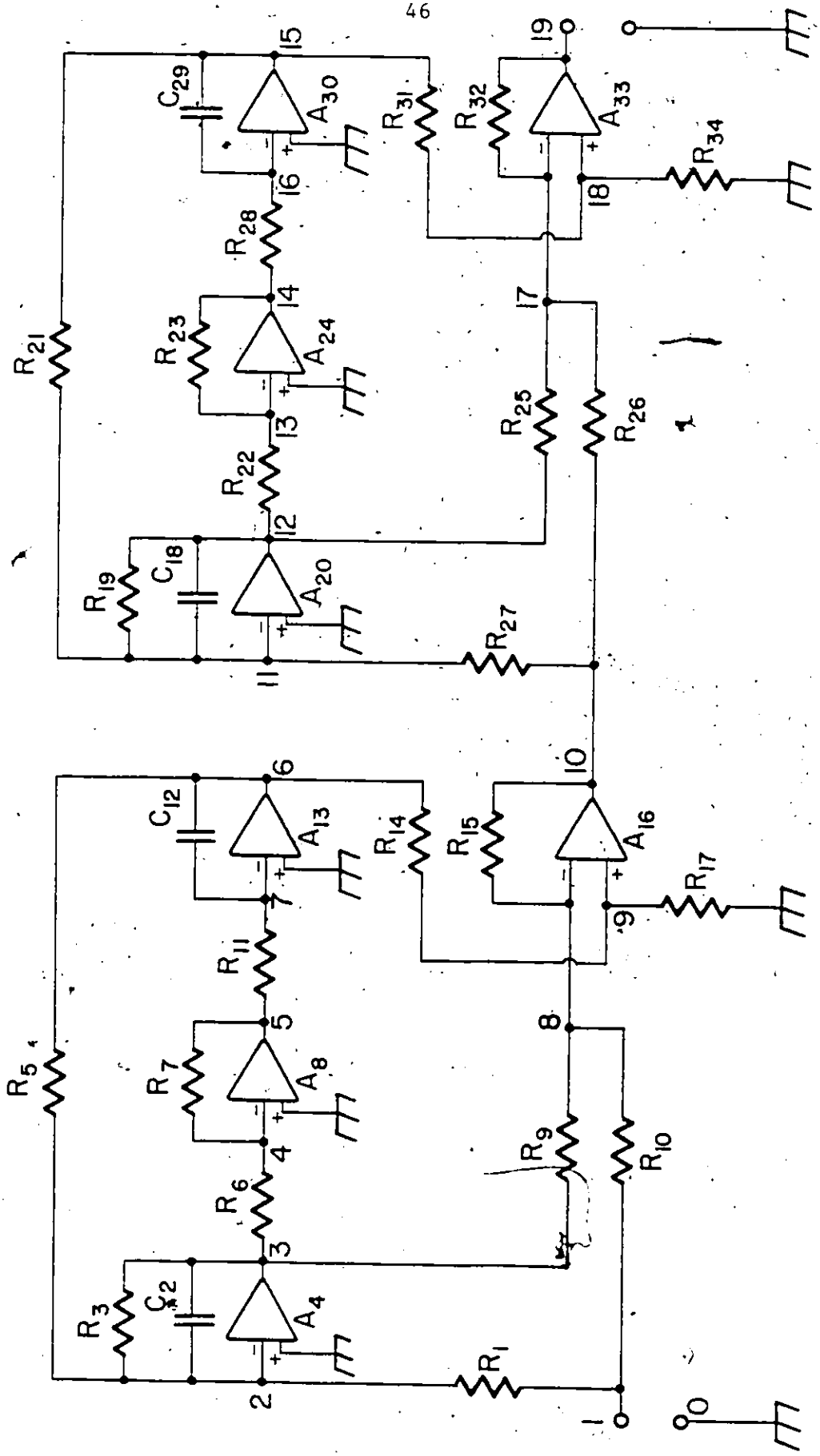

Fig. 3.7 First section of the active filter example.

TABLE 3.1

NOMINAL ELEMENT VALUES FOR THE LINEAR NETWORK EXAMPLE

Resistors (kΩ)		Capacitors (μF)	
R_1^*	0.182/0.1	C_2^*	0.01/0.02
R_3	1.57	C_{12}	0.01
R_5	2.64	C_{18}	0.01
R_6	10.00	C_{29}	0.01
R_7	10.00		
R_9	100.0		
R_{10}	11.1		
R_{11}	2.64		
R_{14}	5.41		
R_{15}	1.0		
R_{17}	1.0		
R_{19}	4.84		
R_{21}	2.32		
R_{22}	10.0		
R_{23}^*	10.0/6.0		
R_{25}^*	500.0/100.0		
R_{26}	11T.1		
R_{27}	1.14		
R_{28}	2.32		
R_{31}	72.4		
R_{32}	10.0		
R_{34}	10.0		

* refers to a faulty element: its faulty value follows the slash.

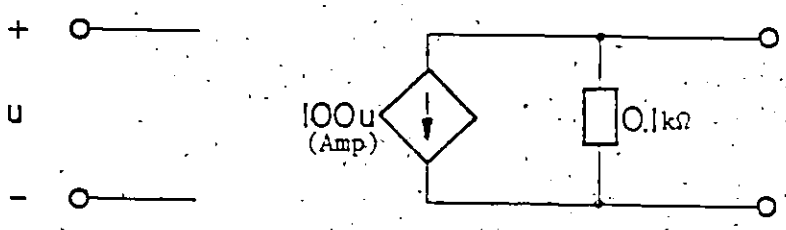

Fig. 3.8 Equivalent circuit for the Op-Amp.

network has 52 resistors and capacitors and 16 operational amplifiers. In the first section, nodes 1,3,5,6,8,10,12,14,15,17 and 19 are taken to be the measurement nodes. The corresponding nodes in the second section are chosen as measurement nodes. We simulated the network with a sinusoidal current source $i_g(t) = 0.01 \cos 2000t$ A applied at node 1. A number of faulty elements were randomly chosen in the first section and they are identified by an asterisk in Table 3.1. The procedure follows.

Stage 0 $E_0 = \{S_1\}$. (S_1 is the network under test).

Stage 1 S_1 is decomposed as shown in Fig. 3.9 into S_2 and S_3.

$E_1 = \{S_2, S_3\}$.

$M_{2\alpha} = \{1\}$, $M_{2\beta} = \{19\}$

$M_{3\alpha} = \{37\}$, $M_{3\beta} = \{19\}$

From the results of Table 3.2, S_2 is faulty and S_3 is nonfaulty.

$E_1 = \{S_2\}$.

Stage 2 S_2 is decomposed as shown in Fig. 3.9 into S_4 and S_5.

$E_2 = \{S_4, S_5\}$.

$M_{4\alpha} = \{1\}$, $M_{4\beta} = \{10\}$

$M_{5\alpha} = \{19\}$, $M_{5\beta} = \{10\}$

From the results of Table 3.3, S_4 and S_5 are both faulty.

$E_2 = \{S_4, S_5\}$.

Stage 3 S_4 and S_5 are decomposed as shown in Fig. 3.9. S_4 is decomposed into S_6, S_7 and S_8. S_5 is decomposed into S_9, S_{10} and S_{11}.

$E_3 = \{S_6, S_7, S_8, S_9, S_{10}, S_{11}\}$.

$M_{6\alpha} = \{\emptyset\}$, $M_{6\beta} = \{1, 3, 6\}$

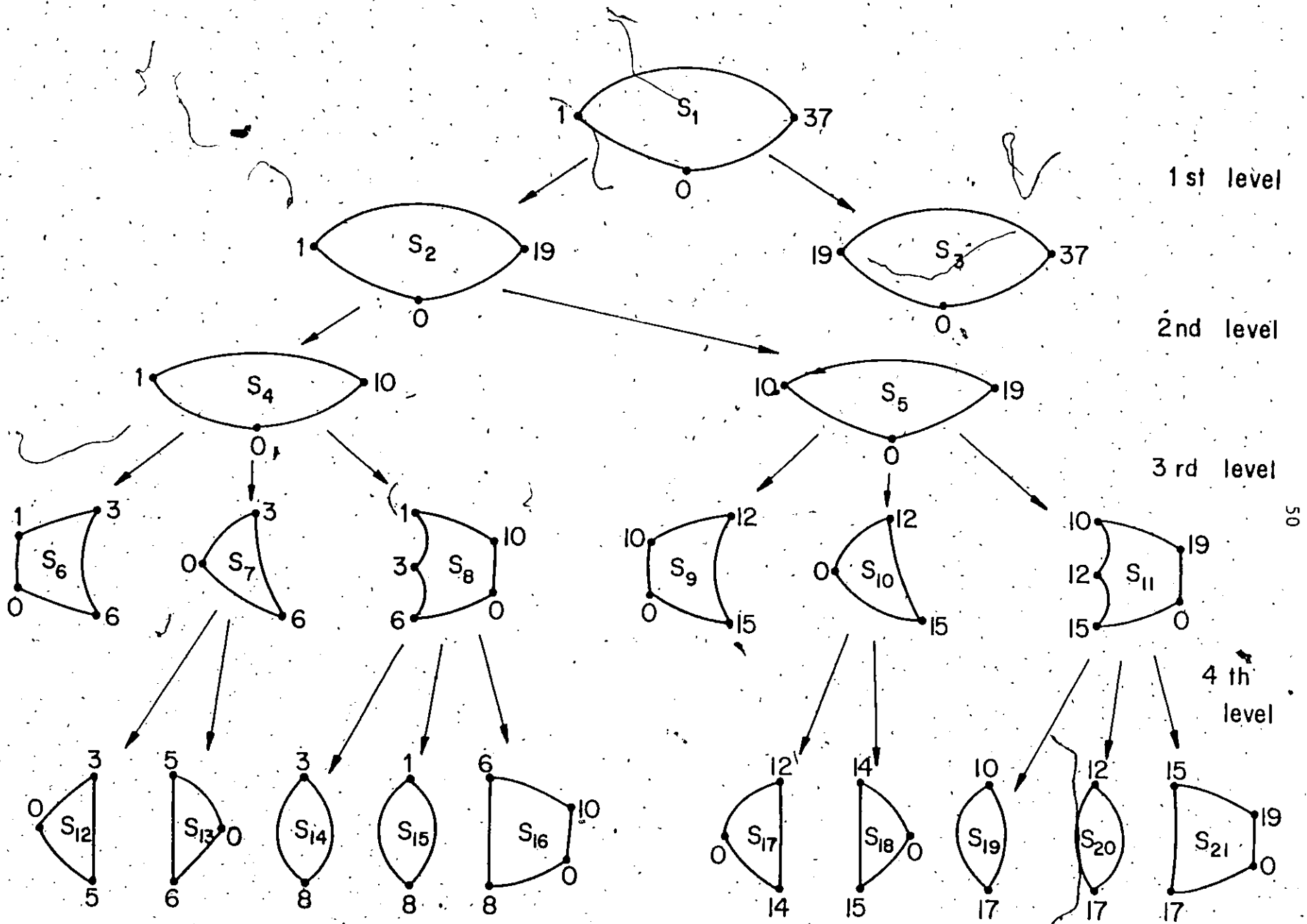

Fig. 3.9 Hierarchical decomposition of the linear network during testing.

TABLE 3.2

DIAGNOSIS FOR THE FIRST LEVEL OF DECOMPOSITION

Voltage Measurements and Designed Currents	Computed Currents	Diagnosis	Test
Measured Voltages	$I_1^2 = 5.53-j0.0039$ mA	$I_1 - I_1^2 \neq 0$	T_2
$V_1 = 0.956+j0.0044$ V	$I_{19}^2 = 7.05-j1.52$ A		
$V_{19} = 1.67-j0.265$ V	$I_{19}^3 = 9.66-j1.57$ mA	$I_{19}^2 + I_{19}^3 \neq 0$	T_{23}
$V_{37} = 1.53-j0.421$ V	$I_{37}^3 = 0$	$I_{37} - I_{37}^3 = 0$	T_3
Designed Currents			
$I_1 = 10.0$ mA			
$I_{37} = 0$			

logical diagnostic function: $D_3 = \bar{\sigma}_2 n(\bar{\sigma}_2 \cup \bar{\sigma}_3) n \sigma_3 = \bar{\sigma}_2 n \sigma_3$.

result: S_3 is declared nonfaulty and I_{19}^3 is known in Table 3.3.

TABLE 3.3

DIAGNOSIS FOR THE SECOND LEVEL OF DECOMPOSITION

Voltage Measurements and Designed Currents	Computed Currents	Diagnosis	Test
Measured Voltages	$I_1^4 = 5.69\times10^{-3} - j1.05\times10^{-5}$ A	$I_1 - I_1^4 \neq 0$	T_4
V_1 known	$I_{10}^4 = -17.72 + j1.61$ A		
$V_{10} = -4.39 + j0.386$ V	$I_{10}^5 = -3.91\times10^{-3} + 3.44\times10^{-4}$ A	$I_{10}^4 + I_{10}^5 \neq 0$	T_{45}
V_{19} known	$I_{19}^5 = 0.67 - 0.81$ A	$I_{19}^3 + I_{19}^5 \neq 0$	T_5
Designed Currents			
$I_1 = 10.0$ mA			
$I_{19} = -I_{19}^3$			

logical diagnostic function $D_3 = \bar{\sigma}_4 \cap (\bar{\sigma}_4 \cup \bar{\sigma}_5) \cap \bar{\sigma}_5 = \bar{\sigma}_4 \cap \bar{\sigma}_5$.

result: no new currents are designable.

$M_{7\alpha} = \{\emptyset\}$, $M_{7\beta} = \{3, 6\}$

$M_{8\alpha} = \{\emptyset\}$, $M_{8\beta} = \{1, 3, 6, 10\}$

$M_{9\alpha} = \{\emptyset\}$, $M_{9\beta} = \{10, 12, 15\}$

$M_{10\alpha} = \{\emptyset\}$, $M_{10\beta} = \{12, 15\}$

$M_{11\alpha} = \{19\}$, $M_{11\beta} = \{10, 12, 15\}$

From the results of Table 3.4, subnetwork S_{11} is faulty and further tests are needed for all other subnetworks.

$E_3 = \{S_6, S_7, S_8, S_9, S_{10}, S_{11}\}$.

Stage 4 Subnetworks S_7, S_8, S_{10} and S_{11} are decomposed into S_{12}, S_{13}, S_{14}, S_{15}, S_{16}, S_{17}, S_{18}, S_{19}, S_{20} and S_{21} as shown in Fig. 3.9. No further decomposition of S_6 and S_9 is possible using only the measurement nodes.

$E_4 = \{S_6, S_{12}, S_{13}, S_{14}, S_{15}, S_{16}, S_9, S_{17}, S_{18}, S_{19}, S_{20}, S_{21}\}$

$M_{6\alpha} = \{\emptyset\}$, $M_{6\beta} = \{1,3,6\}$

$M_{12\alpha} = \{\emptyset\}$, $M_{12\beta} = \{3,5\}$

$M_{13\alpha} = \{\emptyset\}$, $M_{13\beta} = \{5,6\}$

$M_{14\alpha} = \{\emptyset\}$, $M_{14\beta} = \{3,8\}$

$M_{15\alpha} = \{\emptyset\}$, $M_{15\beta} = \{1,8\}$

$M_{16\alpha} = \{\emptyset\}$, $M_{16\beta} = \{6,8,10\}$

$M_{9\alpha} = \{\emptyset\}$, $M_{9\beta} = \{10,12,15\}$

$M_{17\alpha} = \{\emptyset\}$, $M_{17\beta} = \{12,14\}$

$M_{18\alpha} = \{\emptyset\}$, $M_{18\beta} = \{14,15\}$

$M_{19\alpha} = \{\emptyset\}$, $M_{19\beta} = \{10,17\}$

$M_{20\alpha} = \{\emptyset\}$, $M_{20\beta} = \{12,17\}$

$M_{21\alpha} = \{19\}$, $M_{21\beta} = \{15,17\}$

TABLE 3.4

DIAGNOSIS FOR THE THIRD LEVEL OF DECOMPOSITION

Voltage Measurements and Designed Currents	Computed Currents	Diagnosis	Test
Measured Voltages	$I_1^6 = 8.911+j0.0143$ mA	$I_1^6+I_1^8 \neq I_1$	T_{68}
V_1 known	$I_3^6 = -6657.25+j4.126$ mA	$I_3^6+I_3^7+I_3^8 \neq 0$	T_{678}
$V_3 = -0.142+j1.33$ V	$I_6^6 = -9.271+j0.823$ mA	$I_6^6+I_6^7+I_6^8 \neq 0$	T_{678}
$V_6 = -25.1+j2.17$ V	$I_3^7 = -0.0142-j0.1328$ mA	$I_{10}^8+I_{10}^9+I_{10}^{11} \tilde{=} 0$	$T_{8,9,11}$
V_{10} known	$I_6^7 = 13.446-j1.1625$ mA	$I_{12}^9+I_{12}^{10}+I_{12}^{11} \neq 0$	$T_{9,10,11}$
$V_{12} = 0.103+j0.69$ V	$I_1^8 = 0.439-j0.0301$ mA	$I_{15}^9+I_{15}^{10}+I_{15}^{11} \neq 0$	$T_{9,10,11}$
$V_{15} = 8.93-j1.12$ V	$I_3^8 = 0.0378-j0.0167$ mA	$I_{19}^{11} \neq -I_{19}^3$	T_{11}
V_{19} known	$I_6^8 = -3.9224+j0.3392$ mA		
Designed Currents	$I_{10}^8 = 3.9039-j0.3436$ mA		
$I_1 = 10.0$ mA	$I_{10}^9 = -3.8546+j0.3389$ mA		
$I_{19} = -I_{19}^3$	$I_{12}^9 = -0.0005-j0.0772$ mA		
	$I_{15}^9 = 3.8473-j0.4837$ mA		
	$I_{12}^{10} = 0.0103+j0.0689$ mA		
	$I_{15}^{10} = -540.09-j2728.18$ mA		
	$I_{10}^{11} = -0.0499+j0.0052$ mA		
	$I_{12}^{11} = -0.0021+j0.0018$ mA		
	$I_{15}^{11} = 0.1083-j0.0136$ mA		
	$I_{19}^{11} = 695.12-j593.77$ mA		

logical diagnostic function $D_6 = (\bar{\sigma}_6 \cup \bar{\sigma}_8) \cap \bar{\sigma}_{11}$.
although $T_{8,9,11}$ is almost 0 it contradicts T_{11}, hence we do not consider it in the LDF, otherwise the LDF = Ø.)
result: no new currents are designable.

From the results of Table 3.5 subnetworks S_6, S_{17} and S_{20} are faulty and all other subnetworks are nonfaulty.

$$E_4 = \{S_6, S_{17}, S_{20}\}.$$

No further decomposition is possible for subnetworks S_6, S_{17} and S_{20} using the measurement nodes. So we have $E = E_4 = \{S_6, S_{17}, S_{20}\}$. It is to be noted that in four steps (levels of decomposition) we were able to identify the faults to within very small subnetworks. Also since S_3 is fault-free after Stage 1, no further decomposition is carried out and, accordingly, we do not need to consider measurements at the accessible nodes inside S_3.

3.6 TESTING OF NONLINEAR NETWORKS

3.6.1 General Strategy

In typical nonlinear networks, the network is dominantly linear with a few nonlinear elements. The nodes of decomposition are chosen such that the part of the network that contains the nonlinear elements is decomposed into subnetworks, each of them having very few nonlinear elements or being completely linear. The part of the network that contains only linear elements is treated exactly as in the linear case. We decompose the network into blocks that contain the nonlinear elements and a number of subnetworks that contain only linear elements. The latter could be decomposed further. In applying Lemmas 3.1-3.3 for nonlinear networks we need a nonlinear network solver. Intuitively, by having very few nonlinear elements in each subnetwork the nonlinear network solver converges rapidly in just one or two iterations starting

TABLE 3.5

DIAGNOSIS FOR THE FOURTH LEVEL OF DECOMPOSITION

Voltage Measurements and Designed Currents	Computed Currents*	Diagnosis	Test
Measured Voltages	I_3^{12}=-0.0142-j0.1328 mA	$I_1^6+I_1^{15} \neq I_1$	$T_{6,15}$
V_1 known	I_5^{12}=-0.0435-j0.5034 mA	$I_3^6+I_3^{12}+I_3^{14} \neq 0$	$T_{6,12,14}$
V_3 known	I_5^{13}=0.0435+j0.5034 mA		
V_5=0.141+j1.33V	I_6^{13}=13.4462-j1.1625 mA	$I_5^{12} + I_5^{13} = 0$	$T_{12,13}$
V_6 known	I_3^{14}=-I_8^{14}=0.0378-j0.0167 mA	$I_6^6+I_6^{13}+I_6^{16} \neq 0$	$T_{6,13,16}$
V_8=-0.392+j0.339V	I_1^{15}=-I_8^{15}=0.4391-j0.0301 mA		
V_{10} known	I_6^{16}=-3.9224+j0.3392 mA	$I_8^{14}+I_8^{15}+I_8^{16} = 0$	$T_{14,15,16}$
V_{12} known	I_8^{16}=0.4768-j0.0468 mA	$I_{10}^9+I_{10}^{16}+I_{10}^{19} \neq 0$	$T_{9,16,19}$
V_{14}=-0.0615-j0.413V	I_{10}^{16}=3.9039-j0.3436 mA		
V_{15} known	I_{12}^{17}=0.0082+j0.0552 mA	$I_{12}^9+I_{12}^{17}+I_{12}^{20} \neq 0$	$T_{9,17,20}$
V_{17}=1.08-j0.136V	I_{14}^{17}=205.185+j1379.726 mA	$I_{14}^{17}+I_{14}^{18} \neq 0$	$T_{17,18}$
V_{19} known	I_{14}^{18}=-0.0225-j0.1787 mA		
Designed Currents	I_{15}^{18}=-3.9556+j0.4974 mA	$I_{15}^9+I_{15}^{18}+I_{15}^{21} = 0$	$T_{9,18,21}$
I_1=10.0 mA	I_{10}^{19}=-I_{17}^{19}=-0.0493+j0.0047 mA	$I_{17}^{19}+I_{17}^{20}+I_{17}^{21} \neq 0$	$T_{19,20,21}$
I_{19}=-I_{19}^3	I_{12}^{20}=-I_{17}^{20}=-0.0020+j0.0017 mA		
	I_{15}^{21}=0.1083-j0.0137 mA	$I_{19}^{21}+I_{19}^3 = 0$	T_{21}
	I_{17}^{21}=-0.0590+j0.0130 mA		
	I_{19}^{21}=-9.66+j1.57 mA		

logical diagnostic function $D_{11} = \bar{\sigma}_6 \cap \sigma_9 \cap \sigma_{12} \cap \sigma_{13} \cap \sigma_{14} \cap \sigma_{15} \cap \sigma_{16} \cap \bar{\sigma}_{17} \cap \sigma_{18} \cap \sigma_{19} \cap \bar{\sigma}_{20} \cap \sigma_{21}$.

result: S_6, S_{17} and S_{20} are faulty.

* for computed currents of S_6 and S_9 see Table 3.4.

from the nonfaulty state. Also, analyzing several subnetworks simultaneously is possible utilizing the parallel processing capabilities of modern Automatic Testing Equipment (ATE).

For locating faulty elements within faulty blocks that contain nonlinear elements we analyze the faulty block using assumed fault models of its elements, then we compare the different cases using the nearest neighbour rule. This technique is outlined in Section 4.7.

3.6.2 Example 3.4: Nonlinear Network Example

We considered the video amplifier circuit (Navid and Willson, Jr. 1979) shown in Fig. 3.10. The nodes of decomposition are chosen as nodes 1,2,5,7 and 10. The circuit is decomposed into eight uncoupled subnetworks as shown in Fig. 3.11 and in abstract form in Fig. 3.12. Every subnetwork contains at most one nonlinear element (transistor), which agrees with our requirements on the decomposition.

We considered dc testing of the circuit. All capacitors are therefore open circuits. To investigate faulty capacitors ac testing is needed. The nominal values of circuit elements are given in Table 3.6. We have considered the well known Ebers-Moll model of the transistor as shown in Fig. 3.13. The nominal operating conditions for the circuit are given in Table 3.7. All transistors are operating in their active regions.

Different faulty situations have been simulated. The results for four different cases with the nonfaulty parameters assumed at nominal values are summarized in Tables 3.8-3.11. In Case 1 (Table

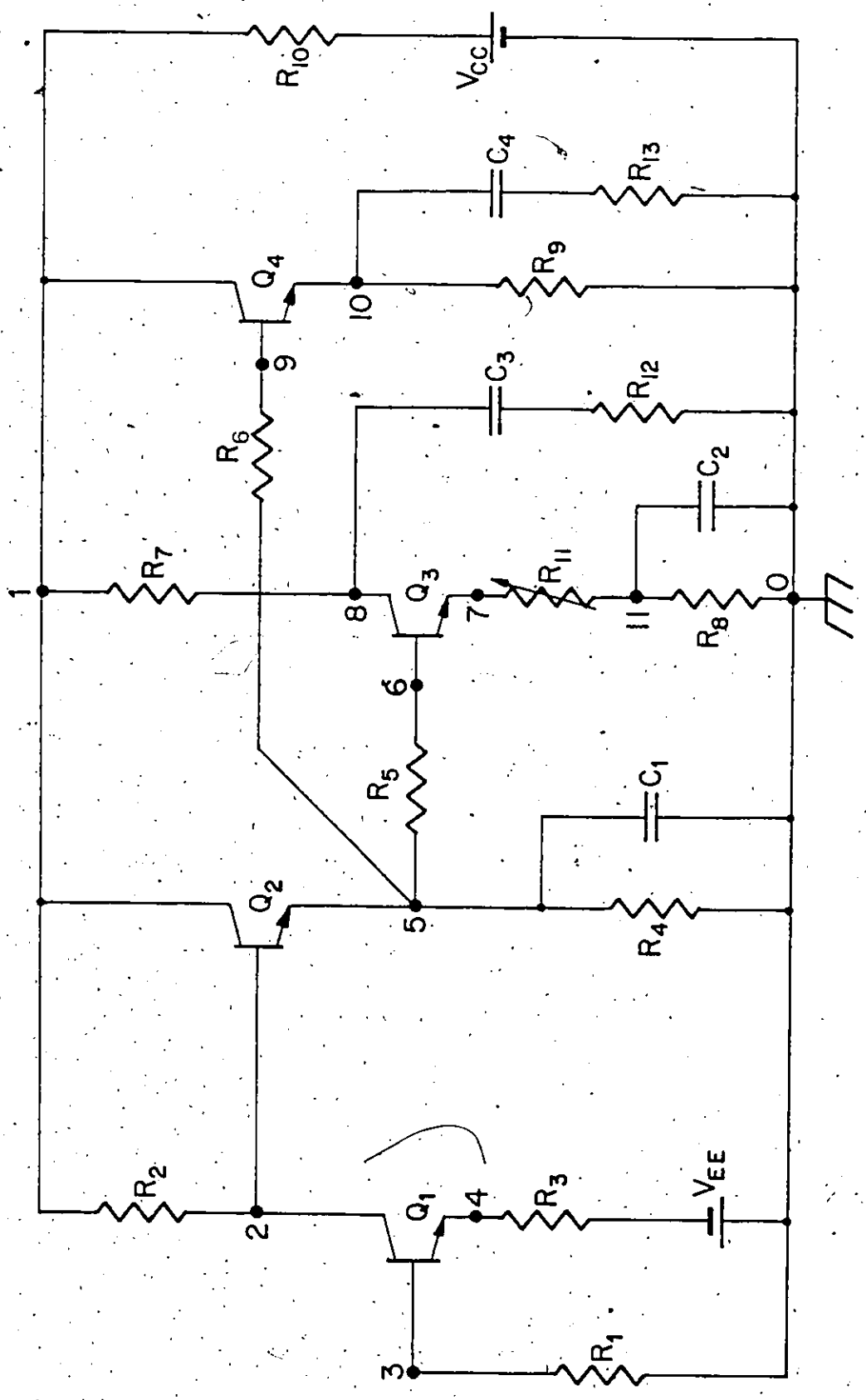

Fig. 3.10 Video amplifier circuit.

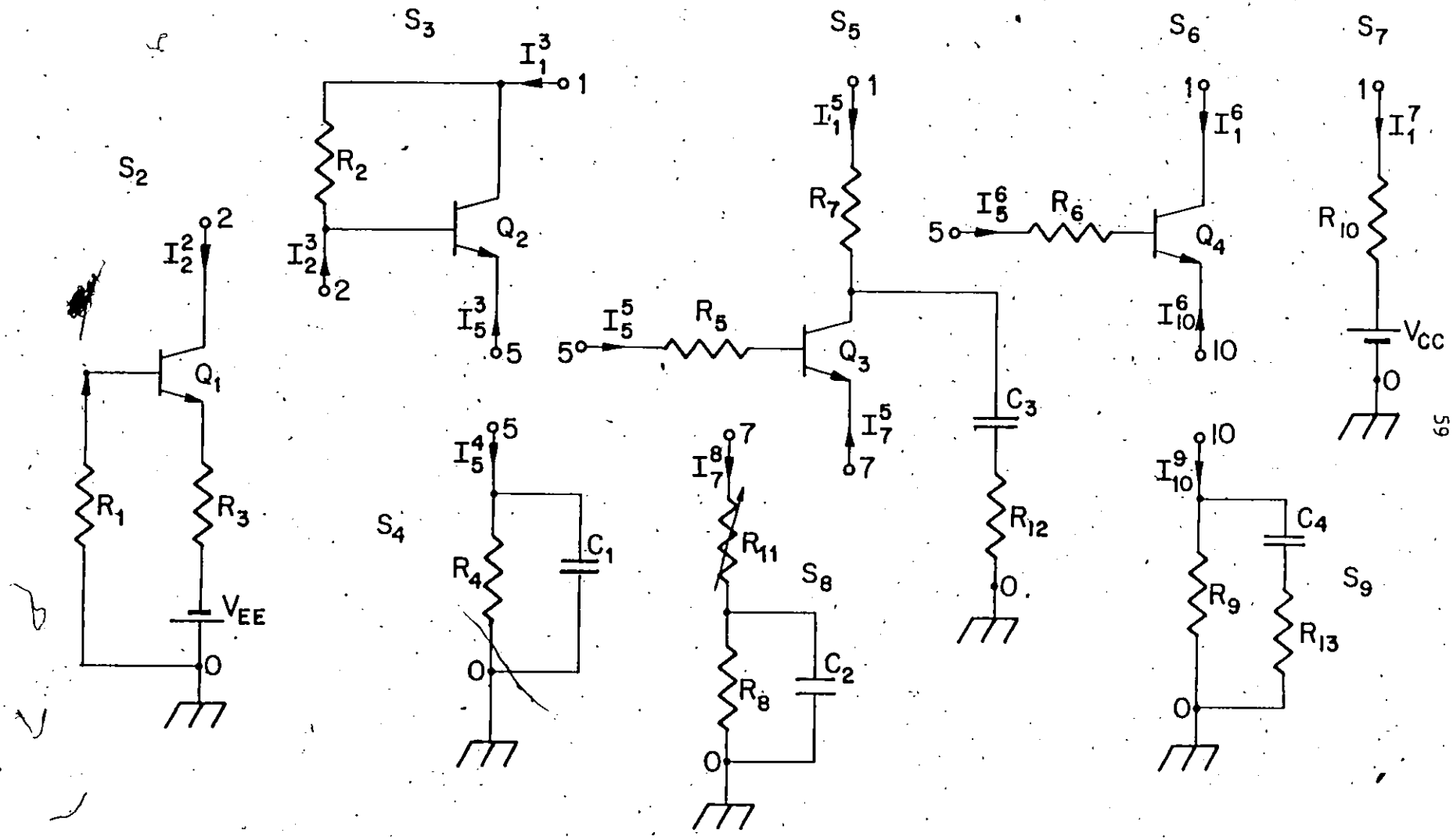

Fig. 3.11 Network decomposition of the video amplifier.

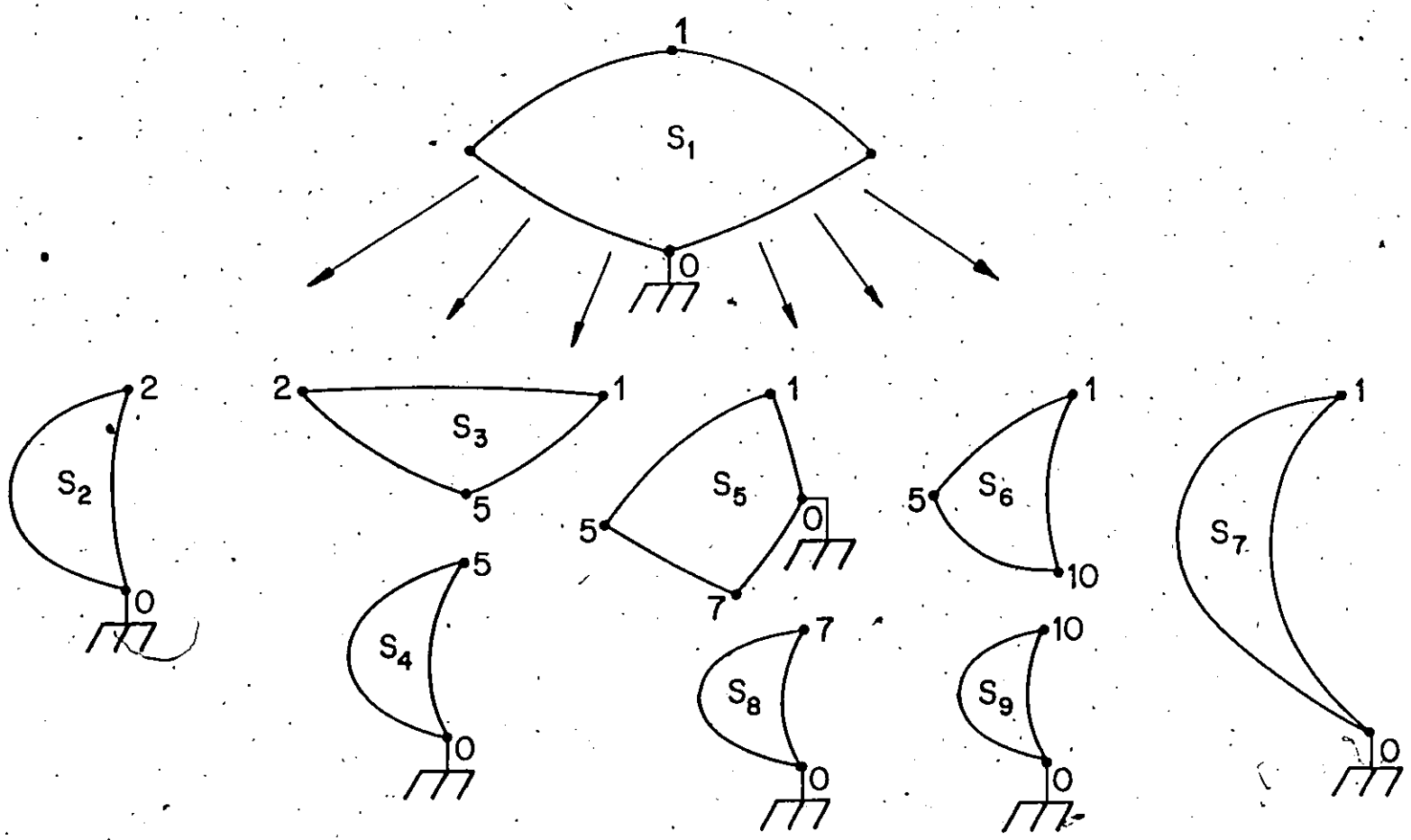

Fig. 3.12 The decomposition of the video amplifier in abstract form.

TABLE 3.6

NOMINAL VALUES OF THE ELEMENTS
OF THE VIDEO AMPLIFIER

Element	Value
R_1	1.2 kΩ
R_2	3.0 kΩ
R_3	5.672 kΩ
R_4	1.2 kΩ
R_5	0.33 kΩ
R_6	0.33 kΩ
R_7	1.0 kΩ
R_8	1.7 kΩ
R_9	3.3 kΩ
R_{10}	0.078 kΩ
R_{11}	0.5 kΩ
R_{12}	1.0 kΩ
R_{13}	1.0 kΩ
C_1	1.0 μF
C_2	3.3 μF
C_3	1.0 μF
C_4	1.0 μF
V_{CC}	28.0 V
V_{EE}	28.0 V

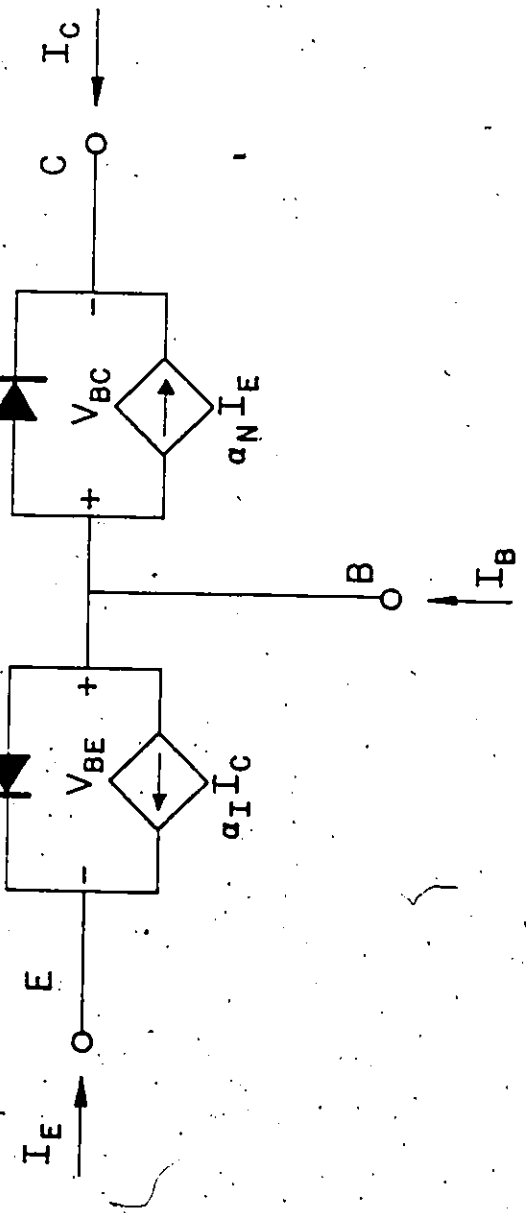

Fig. 3.13 Ebers-Moll model of the transistor.

TABLE 3.7

NOMINAL OPERATING POINT AND PARAMETERS OF THE TRANSISTORS

	Q_1	Q_2	Q_3	Q_4
I_C	4.744 mA	9.091 mA	5.891 mA	3.048 mA
I_E	-4.791 mA	-9.183 mA	-5.951 mA	-3.079 mA
V_{BE}	0.764 V	0.798 V	0.776 V	0.741 V
V_{BC}	-11.767 V	-14.506 V	-10.208 V	-15.315 V
α_N	0.99	0.99	0.99	0.99
α_I	0.5	0.5	0.5	0.5
I_{EO}	1.E-6 mA	1.E-6 mA	1.E-6 mA	1.E-6 mA

TABLE 3.8

CASE 1 - Q_1 FAULTY

Measurements	Computed Currents	Diagnosis	Test
V_1 = 27.1008 V	I_2^2 = 4.7437 mA	$I_2^2 + I_2^3 \neq 0$	T_{23}
V_2 = 3.1321 V	I_1^3 = 9.9952 mA	$I_1^3 + I_1^5 + I_1^6 + I_1^7 = 0$	T_{3567}
V_5 = 2.4126 V	I_2^3 = -7.9692 mA		
V_7 = 1.7256 V	I_5^3 = -1.9949 mA	$I_5^3 + I_5^4 + I_5^5 + I_5^6 = 0$	T_{3456}
V_{10} = 1.7606 V	I_5^4 = 2.0104 mA	$I_7^5 + I_7^8 = 0$	T_{58}
	I_1^5 = 1.0049 mA	$I_{10}^6 + I_{10}^9 = 0$	T_{69}
	I_5^5 = 0.0101 mA		
	I_7^5 = -1.050 mA		
	I_1^6 = 0.5282 mA		
	I_5^6 = 0.0053 mA		
	I_{10}^6 = -0.5335 mA		
	I_1^7 = -11.5283 mA		
	I_7^8 = 1.0150 mA		
	I_{10}^9 = 0.5335 mA		

logical diagnostic function $D_5 = (\bar{\sigma}_2 \cup \bar{\sigma}_3) \cap (\sigma_3 \cap \sigma_5 \cap \sigma_6 \cap \sigma_7) \cap (\sigma_3 \cap \sigma_6) \cap (\sigma_5 \cap \sigma_8) \cap (\sigma_6 \cap \sigma_9) = \bar{\sigma}_2 \cap \sigma_3 \cap \sigma_4 \cap \sigma_5 \cap \sigma_6 \cap \sigma_7 \cap \sigma_8 \cap \sigma_9$.

result: S_2 is the only faulty subnetwork.

TABLE 3.9

CASE 2 - Q_3 FAULTY

Measurements	Computed Currents	Diagnosis	Test
V_1 = 26.1850 V	I_2^2 = 4.7437 mA	$I_2^2 + I_2^3 = 0$	T_{23}
V_2 = 11.6790 V	I_1^3 = 13.9057 mA	$I_1^3 + I_1^5 + I_1^6 + I_1^7 \neq 0$	T_{3567}
V_5 = 10.8809 V	I_2^3 = -4.7437 mA		
V_7 = 10.8599 V	I_5^3 = -9.1620 mA	$I_5^3 + I_5^4 + I_5^5 + I_5^6 \neq 0$	T_{3456}
V_{10} = 10.1296 V	I_5^4 = 9.0675 mA	$I_7^5 + I_7^8 \neq 0$	T_{58}
	I_1^5 = 5.8736 mA	$I_{10}^6 + I_{10}^9 = 0$	T_{69}
	I_5^5 = 0.0593 mA	$I_2^2 + I_1^7 + I_7^8 + I_5^4$	
	I_7^5 = -5.9329 mA	$+ I_{10}^9 \equiv 0$	T_{24789}
	I_1^6 = 3.0389 mA		
	I_5^6 = 0.0307 mA		
	I_{10}^6 = -3.0696 mA		
	I_1^7 = -23.2685 mA		
	I_7^8 = 6.3882 mA		
	I_{10}^9 = 3.0696 mA		

logical diagnostic function $D_6 = (\sigma_2 \cap \sigma_3) \cap (\bar{\sigma}_3 \cup \bar{\sigma}_5 \cup \bar{\sigma}_6 \cup \bar{\sigma}_7) \cap (\bar{\sigma}_3 \cup \bar{\sigma}_4 \cup \bar{\sigma}_5 \cup \bar{\sigma}_6) \cap (\bar{\sigma}_5 \cup \bar{\sigma}_8) \cap (\sigma_6 \cap \sigma_9) \cap (\bar{\sigma}_2 \cap \sigma_7 \cap \sigma_8 \cap \sigma_4 \cap \sigma_9) = \sigma_2 \cap \sigma_3 \cap \sigma_4 \cap \bar{\sigma}_5 \cap \sigma_6 \cap \sigma_7 \cap \sigma_8 \cap \sigma_9$.

result: S_5 is the only faulty subnetwork.

TABLE 3.10

CASE 3 - R_{10} FAULTY

Measurements	Computed Currents	Diagnosis	Test
$V_1 = 5.8645$ V	$I_2^2 = 2.8379$ mA	$I_2^2 + I_2^3 = 0$	T_{23}
$V_2 = -2.6491$ V	$I_1^3 = 2.8379$ mA	$I_1^3 + I_1^5 + I_1^6 + I_1^7 \neq 0$	T_{3567}
$V_5 = 0$	$I_2^3 = -2.8379$ mA		
$V_7 = 0$	$I_5^3 = 0.0000$ mA	$I_5^3 + I_5^4 + I_5^5 + I_5^6 = 0$	T_{3456}
$V_{10} = 0$	$I_5^4 = 0.0000$ mA	$I_7^5 + I_7^8 = 0$	T_{58}
	$I_1^5 = 0.0000$ mA	$I_{10}^6 + I_{10}^9 = 0$	T_{69}
	$I_5^5 = 0.0000$ mA		
	$I_7^5 = 0.0000$ mA		
	$I_1^6 = 0.0000$ mA		
	$I_5^6 = 0.0000$ mA		
	$I_{10}^6 = 0.0000$ mA		
	$I_1^7 = -283.7879$ mA		
	$I_7^8 = 0.0000$ mA		
	$I_{10}^9 = 0.0000$ mA		

logical diagnostic function $D_5 = (\sigma_5 \cap \sigma_8) \cap (\sigma_6 \cap \sigma_9) \cap (\sigma_2 \cap \sigma_3) \cap (\bar{\sigma}_3 \cup \bar{\sigma}_5 \cup \bar{\sigma}_6 \cup \bar{\sigma}_7) \cap (\sigma_3 \cap \sigma_4 \cap \sigma_5 \cap \sigma_6) = \sigma_2 \cap \sigma_3 \cap \sigma_4 \cap \sigma_5 \cap \sigma_6 \cap \bar{\sigma}_7 \cap \sigma_8 \cap \sigma_9$.

result: S_7 is the only faulty subnetwork.

TABLE 3.11

CASE 4 - Q_2 FAULTY

Measurements	Computed Currents	Diagnosis	Test
V_1 = 26.1241 V	I_2^2 = 4.7437 mA	$I_2^2 + I_2^3 \neq 0$	T_{23}
V_2 = 11.6001 V	I_1^3 = 4.8414 mA	$I_1^3 + I_1^5 + I_1^6 + I_1^7 \neq 0$	T_{3567}
V_5 = 11.6001 V	I_2^3 = -4.8414 mA	$I_5^3 + I_5^4 + I_5^5 + I_5^6 \neq 0$	T_{3456}
V_7 = 10.8001 V	I_5^3 = 0.000 mA	$I_7^5 + I_7^8 = 0$	T_{58}
V_{10} = 10.8445 V	I_5^4 = 9.6667 mA	$I_{10}^6 + I_{10}^9 = 0$	T_{69}
	I_1^5 = 6.2895 mA	$I_2^2+I_1^7+I_7^8+I_5^4+I_{10}^9 \equiv 0$	T_{24789}
	I_5^5 = 0.0635 mA		
	I_7^5 = -6.3530 mA		
	I_1^6 = 3.2533 mA		
	I_5^6 = 0.0329 mA		
	I_{10}^6 = -3.2862 mA		
	I_1^7 = -24.0497 mA		
	I_7^8 = 6.3530 mA		
	I_{10}^9 = 3.2862 mA		

logical diagnostic function $D_6 = (\bar{\sigma}_2 \cup \bar{\sigma}_3) \cap (\bar{\sigma}_3 \cup \bar{\sigma}_5 \cup \bar{\sigma}_6 \cup \bar{\sigma}_7) \cap (\bar{\sigma}_3 \cup \bar{\sigma}_4 \cup \bar{\sigma}_5 \cup \bar{\sigma}_6) \cap (\sigma_5 \cap \sigma_8) \cap (\sigma_6 \cap \sigma_9) \cap (\sigma_2 \cap \sigma_4 \cap \sigma_7 \cap \sigma_8 \cap \sigma_9) = \sigma_2 \cap \bar{\sigma}_3 \cap \sigma_4 \cap \sigma_5 \cap \sigma_6 \cap \sigma_7 \cap \sigma_8 \cap \sigma_9$.

result: S_3 is the only faulty subnetwork.

3.8), we considered Q_1 faulty, namely its collector-base junction is shorted. A shorted junction is simulated by connecting a very small resistance across the junction. In Case 2 (Table 3.9), the base-emitter junction of Q_3 is shorted, and in Case 3 (Table 3.10), the resistor R_{10} is increased to 7.8 kΩ. In Case 4 (Table 3.11), transistor Q_2 has a shorted base emitter junction. We considered also Case 2 when all resistors are allowed to change within ± 10% of their nominal values and the transistor gain, $\beta = (\alpha_N/1-\alpha_N)$, is allowed to change within ± 10% of its nominal values or equivalently α_N to change within ± 0.1% of its nominal value. The predicted changes in the subnetworks elements using equation (3.18) for the different tests is summarized in Table 3.12 (Case 5). It is clear that the diagnosis of the different tests will be exactly as in the nontolerance case (Case 2). The matrix $\underset{\sim}{C}_i$ in (3.19) has been taken to be

$$\underset{\sim}{C}_i = \text{diag}\,\{w_1 \phi_{i1}^{0^2},\ w_2 \phi_{i2}^{0^2},\ \ldots,\ w_p \phi_{ip}^{0^2}\}\ ,$$

where p is the number of elements in the subnetworks considered in the test that are subjected to tolerance changes and w_i is an appropriate positive weighting function.

In all the cases considered we were quite successful in identifying the faulty subnetworks. In Case 3 further diagnosis may be needed after repairing the faulty element R_{10} since, due to abnormal operating conditions subnetworks S_4, S_8 and S_9 are short-circuited, and any fault in them will not show up until R_{10} is repaired. Also, knowing that S_7 is faulty we compute

TABLE 3.12

CASE 5 - Q_3 FAULTY WITH TOLERANCES ON THE NONFAULTY ELEMENTS

Percentage Deviation	Test 1 T_{23}	Test 2 T_{3567}	Test 3 T_{3456}	Test 4 T_{58}	Test 5 T_{69}	Test 6 T_{24789}
$\lvert\Delta R_1/R_1\rvert$	0.0					0.0
$\lvert\Delta R_2/R_2\rvert$	1.983	13.54	0.0			
$\lvert\Delta R_3/R_3\rvert$	1.774					2.11
$\lvert\Delta R_4/R_4\rvert$			26.99*			4.076
$\lvert\Delta R_5/R_5\rvert$		0.088	6.71	9.09		
$\lvert\Delta R_6/R_6\rvert$		0.012	0.0		0.136	
$\lvert\Delta R_7/R_7\rvert$		46.47*	0.31	70.82*		
$\lvert\Delta R_8/R_8\rvert$				27.74*		2.45
$\lvert\Delta R_9/R_9\rvert$					0.878	1.382
$\lvert\Delta R_{10}/R_{10}\rvert$		51.44*				8.96
$\lvert\Delta\alpha_{N1}/\alpha_{N1}\rvert$	0.0224					0.026
$\lvert\Delta\alpha_{N2}/\alpha_{N2}\rvert$	0.0513	0.71*	0.408*			
$\lvert\Delta\alpha_{N3}/\alpha_{N3}\rvert$		0.0	0.001	0.194*		
$\lvert\Delta\alpha_{N4}/\alpha_{N4}\rvert$		1.27*	0.072		0.14	
	Pass	Fail	Fail	Fail	Pass	Pass

logical diagnostic function $D_6 = T_{23} \cap T_{3567} \cap T_{3456} \cap T_{58} \cap T_{69} \cap T_{24789}$

$= \sigma_2 \cap \sigma_3 \cap \sigma_4 \cap \sigma_5 \cap \sigma_6 \cap \sigma_7 \cap \sigma_8 \cap \sigma_9$.

result: S_5 is the only faulty subnetwork (see Table 3.9).

* Deviation significantly exceeds tolerance.

$$I_1^7 = - I_1^3 - I_1^5 - I_1^6 = 2.8379 \text{ mA}$$

and

$$R_{10} = \frac{V_1 - V_{cc}}{I_1^7} = 7.8 \text{ k}\Omega ,$$

which is the actual fault value.

3.7 CONCLUSIONS

We have described a novel method for fault location in analog circuits. The method has the following characteristics:

(a) Due to the decomposition of the whole network into smaller uncoupled subnetworks, the method is directly applicable to large networks.

(b) The testing conditions are a result of network topology and KCL: they do not depend on network type, so the method is applicable to both linear and nonlinear networks. Also, depending on the type of circuits the network could be tested using different types of excitations.

(c) The measurement nodes are chosen as the nodes of decomposition. Their number can consequently be limited for practical implementation.

(d) For linear networks the on-line computational requirements are minimal (matrix by vector multiplications) and the off-line computation involves the analysis of the nominal network only. For nonlinear networks the on-line computation is reduced by performing the computation in a parallel processing mode.

(e) The decomposition of the network into subnetworks allowed us to deal with the tolerance problem at the subnetwork level, thus we have localized its effects.

(f) The method is initially modular, where nominal circuit models are used for the subnetworks. Subsequently, it is element oriented, at which time faulty elements are located inside subnetworks. Typical fault models may be utilized at this stage, as we will elaborate on in the next chapter.

A computer program realizing this method has been written and other practical examples (Hochwald and Bastian 1979) were tested yielding positive results.

4

FAULT LOCATION INSIDE FAULTY SUBNETWORKS— DETERMINISTIC METHODS

4.1 INTRODUCTION

After locating the faulty subnetwork, further fault analysis is carried out to identify the faulty elements in the subnetwork. Two main approaches are considered in this thesis. One of them utilizes an estimation criterion to predict the most likely faulty elements. This approach is referred to as the approximate method for fault location and is addressed in the next chapter. The deterministic approach methods are dealt with in this chapter.

In the fault verification method a set of subnetwork elements are designated as candidates for the faulty set. A set of algebraic relations that is invariant on the candidate set is constructed. We then test the consistency of the corresponding equations.

The effect of tolerances on the nonfaulty elements is considered and a linear programming formulation of the fault verification problem is devised. Since the search for the faulty set is combinatorial in nature different strategies are suggested to reduce the computational effort needed. The decomposition approach (Chapter 3) is utilized in locating faulty regions inside the faulty subnetworks. This is achieved by applying the internal-self-testing condition that is introduced in Section 3.3.1. A combinatorial algorithm, one which does not exploit topological interconnections within the subnetwork, is next presented.

In many subnetworks only a few elements are fault prone. In the directory approach, as explained in Chapter 2, these are the only elements that are considered in constructing the dictionary. A similar procedure can be followed in the present case by considering faulty models for the most probable faulty elements. The matching method utilizes these fault models together with the input excitation to the subnetwork to produce the subnetwork response. A matching between the actual response and simulated response is carried out to verify the existence of certain faults.

4.2 FAULT VERIFICATION METHOD

4.2.1 General Description

Let us assume that the subnetwork S has m+1 external nodes, one of which is considered as the reference node, as shown in Fig. 4.1. The m+1-pole subnetwork is considered as a subnetwork with m measurement ports. The voltages of the external nodes $\underline{V}^M$ are assumed to be measured. The currents $\underline{I}^M$ entering the subnetwork at these nodes are assumed to be either measured excitations or computed using the measured voltages and the nominal parameter values of the adjacent nonfaulty subnetworks.

A change of value of a component with respect to nominal can be represented by either a current or a voltage source in parallel or in series, respectively, with the component, as shown in Fig. 4.2. Let us assume that we have f faulty elements in the subnetworks, f_1 of which are represented by voltage sources $\underline{V}^{F1}$ and f_2 of which are represented

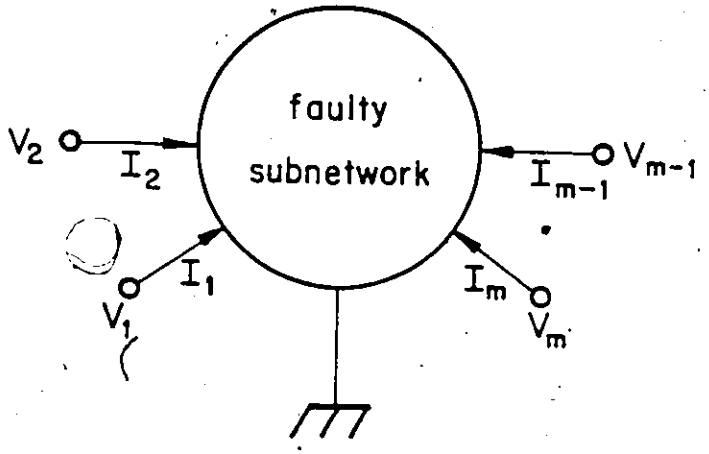

Fig. 4.1 Faulty subnetwork with m+1 external nodes.

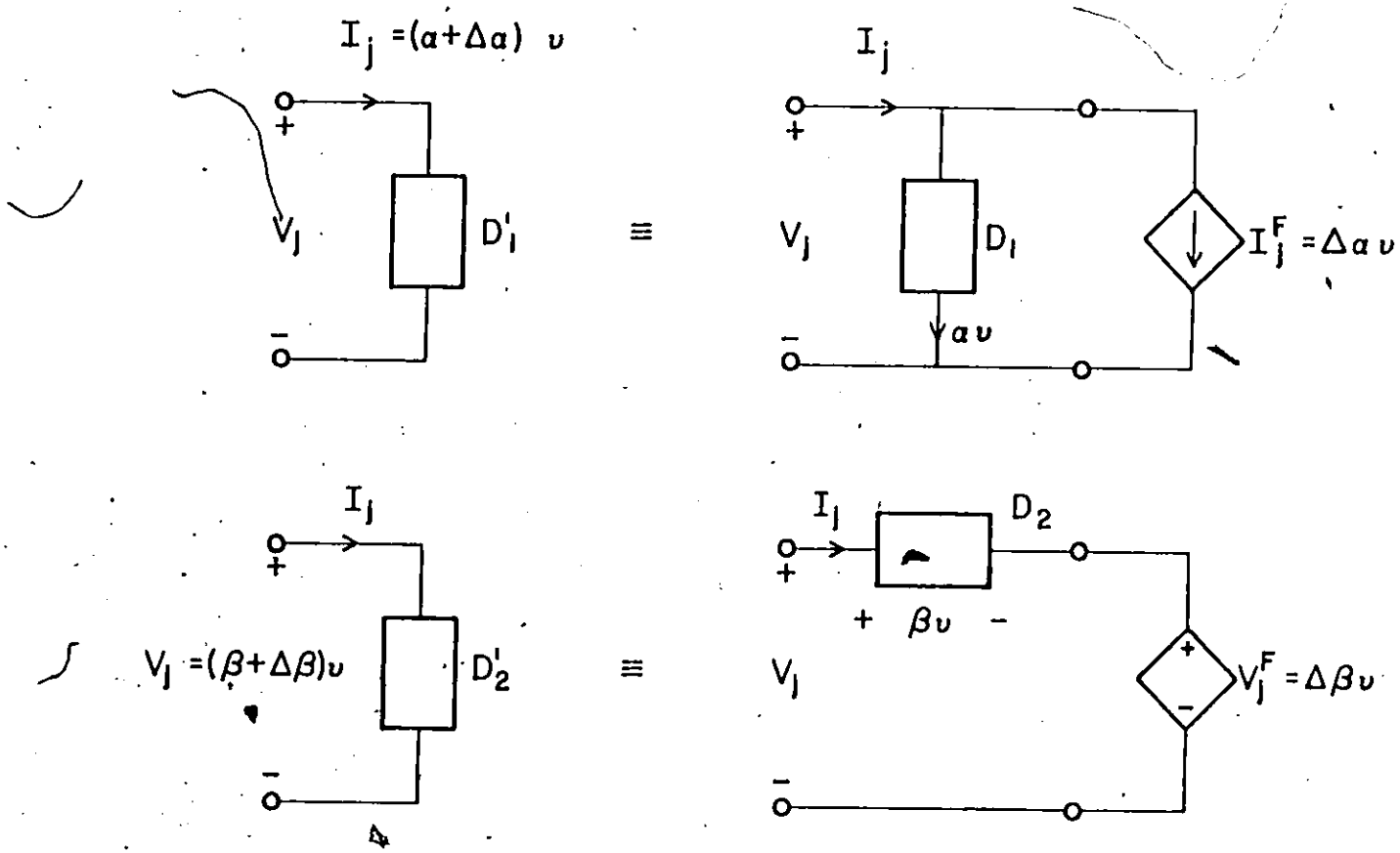

Fig. 4.2 Representation for changes in element values.

by current sources, $\underset{\sim}{I}^{F2}$ so that $f \triangleq f_1 + f_2$. If we treat these additional sources as external to the subnetwork, as shown in Fig. 4.3, then the interior of the subnetwork is known, since it consists of elements with nominal component values. Assuming that the hybrid matrix $\underset{\sim}{H}$ of the (m+f) - port network exists we obtain the following relation:

$$\begin{bmatrix} \underset{\sim}{I}^{M} \\ \underset{\sim}{I}^{F_1} \\ \underset{\sim}{V}^{F_2} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{H}_{MM} & \underset{\sim}{H}_{MF_1} & \underset{\sim}{H}_{MF_2} \\ \underset{\sim}{H}_{F_1M} & \underset{\sim}{H}_{F_1F_1} & \underset{\sim}{H}_{F_1F_2} \\ \underset{\sim}{H}_{F_2M} & \underset{\sim}{H}_{F_2F_1} & \underset{\sim}{H}_{F_2F_2} \end{bmatrix} \begin{bmatrix} \underset{\sim}{V}^{M} \\ \underset{\sim}{V}^{F_1} \\ \underset{\sim}{I}^{F_2} \end{bmatrix}, \tag{4.1}$$

where

$$\underset{\sim}{V}^{M} \triangleq [V_1^M \; V_2^M \; \dots \; V_m^M]^T, \quad \underset{\sim}{I}^{M} \triangleq [I_1^M \; I_2^M \; \dots \; I_m^M]^T \tag{4.2}$$

are the measured voltages and assumed currents,

$$\underset{\sim}{V}^{F_1} \triangleq [V_1^F \; V_2^F \; \dots \; V_{f_1}^F]^T, \quad \underset{\sim}{V}^{F_2} \triangleq [V_{f_1+1}^F \; V_{f_1+2}^F \; \dots \; V_f^F]^T \tag{4.3}$$

are the voltages at the fault ports and

$$\underset{\sim}{I}^{F_1} \triangleq [I_1^F \; I_2^F \; \dots \; I_{f_1}^F]^T, \quad \underset{\sim}{I}^{F_2} \triangleq [I_{f_1+1}^F \; I_{f_2+2}^F \; \dots \; I_f^F]^T \tag{4.4}$$

are the currents associated with the fault ports.

From (4.1) the currents of the external nodes are given by

$$\underset{\sim}{I}^{M} = \underset{\sim}{H}_{MM} \underset{\sim}{V}^{M} + [\underset{\sim}{H}_{MF_1} \; \underset{\sim}{H}_{MF_2}] \begin{bmatrix} \underset{\sim}{V}^{F_1} \\ \underset{\sim}{I}^{F_2} \end{bmatrix} \tag{4.5}$$

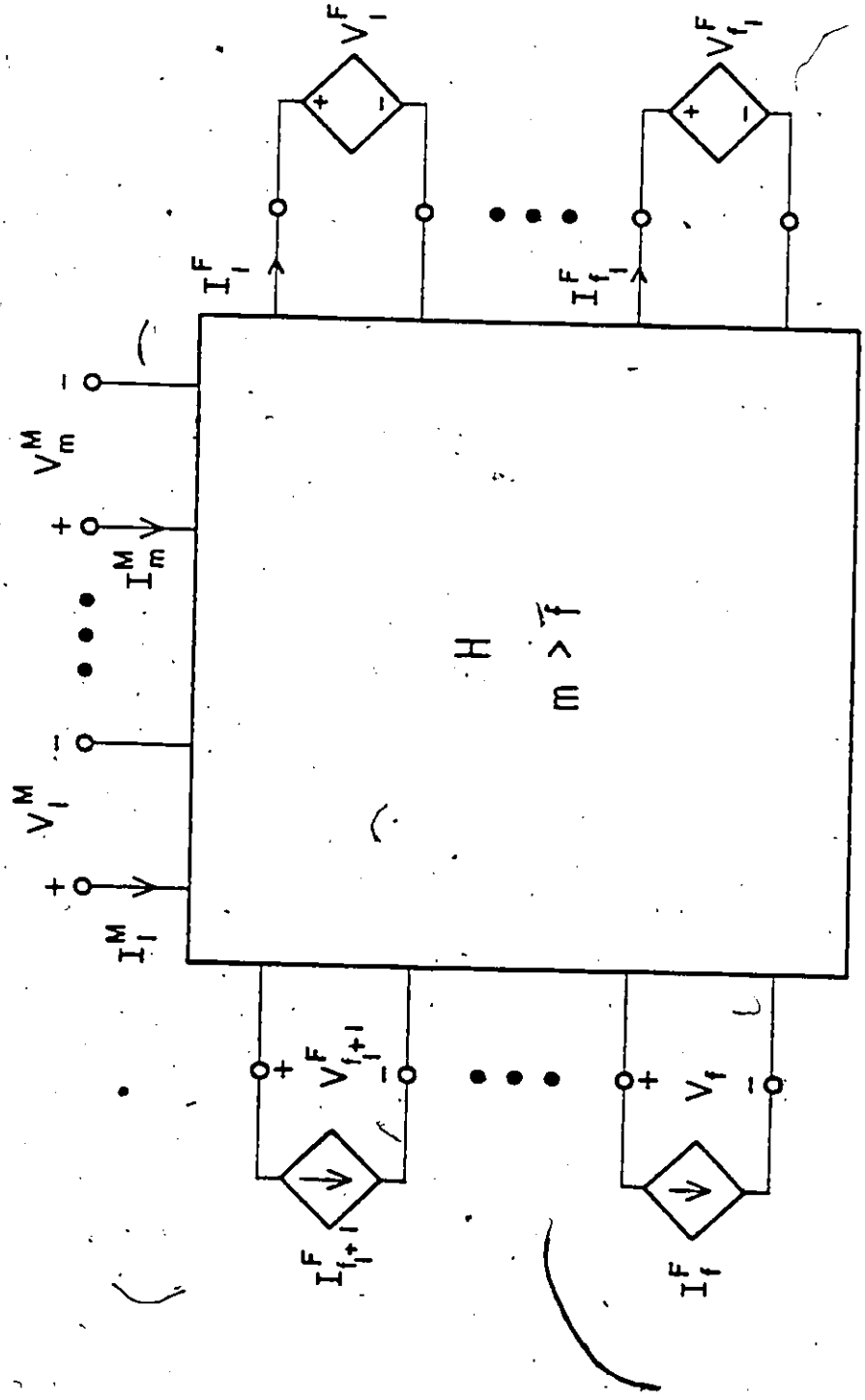

Fig. 4.3 Subnetwork S after extracting f fault ports and m measurement ports.

and we may define

$$\Delta \underset{\sim}{I}^M \triangleq \underset{\sim}{I}^M - \underset{\sim}{H}_{MM}\, \underset{\sim}{V}^M = [\underset{\sim}{H}_{MF_1}\ \underset{\sim}{H}_{MF_2}] \begin{bmatrix} \underset{\sim}{V}^{F_1} \\ \underset{\sim}{I}^{F_2} \end{bmatrix} = \underset{\sim}{H}_{MF} \begin{bmatrix} \underset{\sim}{V}^{F_1} \\ \underset{\sim}{I}^{F_2} \end{bmatrix}, \tag{4.6}$$

which we attribute to the faulty elements. $\Delta \underset{\sim}{I}^M$ is known since $\underset{\sim}{I}^M$ and $\underset{\sim}{V}^M$ are both assumed known and $\underset{\sim}{H}_{MM}$ is computed using nominal parameter values of the subnetwork. If $m > f$, i.e., the cardinality of the measured voltages and currents is larger than the cardinality of the faulty set F, then (4.6) is an overdetermined system of equations. A necessary condition for F_1 and F_2 to be candidates for the exact faulty set is that (4.6) is a consistent overdetermined system of equations (Biernacki and Bandler 1981). If $\underset{\sim}{H}_{MF}$ is of full column rank this condition reduces to

$$(\underset{\sim}{\Pi}_{MF} - \underset{\sim}{1})\, \Delta \underset{\sim}{I}^M = \underset{\sim}{0}, \tag{4.7}$$

where

$$\underset{\sim}{\Pi}_{MF} \triangleq \underset{\sim}{H}_{MF} (\underset{\sim}{H}_{MF}^T\, \underset{\sim}{H}_{MF})^{-1}\, \underset{\sim}{H}_{MF}^T, \tag{4.8}$$

and $\underset{\sim}{1}$ is an identity matrix of appropriate dimension. For the set F to be unique the following condition should be satisfied.

4.2.2 Uniqueness of the Faulty Elements

Lemma 4.1

The set of faulty elements F is unique if and only if

$$\text{Rank}\, [\underset{\sim}{H}_{MF_1}\ \underset{\sim}{H}_{MF_2}\ \underset{\sim}{H}_{Mx}] > \text{Rank}\, [\underset{\sim}{H}_{MF_1}\ \underset{\sim}{H}_{MF_2}] = f \quad \forall\, x \in A-F\,, \tag{4.9}$$

where A is the set of all elements of the subnetwork. (Note that

element x is not in the set F.)

Proof

If (4.9) is not true then

$$\text{Rank}[\underset{\sim}{H}_{MF_1}\ \underset{\sim}{H}_{MF_2}\ \underset{\sim}{H}_{Mx}] = \text{Rank}\ [\underset{\sim}{H}_{MF_1}\ \underset{\sim}{H}_{MF_2}]$$

so the column vector $\underset{\sim}{H}_{Mx}$ is of a nonzero linear combination of $[\underset{\sim}{H}_{MF_1}\ \underset{\sim}{H}_{MF_2}]$, i.e.,

$$\underset{\sim}{H}_{Mx} = [\underset{\sim}{H}_{MF_1}\ \underset{\sim}{H}_{MF_2}]\ \underset{\sim}{\lambda}$$

and we can replace a column of $[\underset{\sim}{H}_{MF_1}\ \underset{\sim}{H}_{MF_2}]$ that corresponds to nonzero component value of $\underset{\sim}{\lambda}$ with $\underset{\sim}{H}_{Mx}$. The resultant matrix will span the same subspace spanned by $[\underset{\sim}{H}_{MF_1}\ \underset{\sim}{H}_{MF_2}]$. Another consistent solution of (4.6) exists, hence the solution is not unique.

If (4.9) is true, but there exist two sets of f elements which satisfy (4.6), then the columns corresponding to each set span the same subspace. This contradicts (4.9). Therefore, if (4.9) is satisfied, the solution is unique.

4.2.3 Computation of the Coefficients

The adjoint network concept is utilized to compute the required coefficients to construct (4.6). With $\underset{\sim}{\hat{V}}^{F_1} = \underset{\sim}{0}$ and $\underset{\sim}{\hat{I}}^{F_2} = \underset{\sim}{0}$ the equations adjoint to (4.1) are given by

$$\begin{bmatrix} \hat{\underset{\sim}{I}}^{M} \\ \hat{\underset{\sim}{I}}^{F_1} \\ -\hat{\underset{\sim}{V}}^{F_2} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{H}_{MM}^{T} \\ \underset{\sim}{H}_{MF_1}^{T} \\ \underset{\sim}{H}_{MF_2}^{T} \end{bmatrix} \cdot \hat{\underset{\sim}{V}}^{M} \cdot \qquad (4.10)$$

The currents $\hat{\underset{\sim}{I}}^{M}$ and $\hat{\underset{\sim}{I}}^{F_1}$ as well as the voltages $\hat{\underset{\sim}{V}}^{F_2}$ are computed with a unit voltage source connected to the ith measurement port and the remaining measurement ports are short-circuited. This gives the elements of the ith column of the matrices $\underset{\sim}{H}_{MM}^{T}$, $\underset{\sim}{H}_{MF_1}^{T}$ and $\underset{\sim}{H}_{MF_2}^{T}$. Repeating this procedure for all measurement ports all the coefficients of (4.5) are directly obtained as

$$\begin{bmatrix} \hat{\underset{\sim}{I}}^{M1} & \hat{\underset{\sim}{I}}^{M2} & \cdots & \hat{\underset{\sim}{I}}^{Mm} \\ \hat{\underset{\sim}{I}}^{F_1 1} & \hat{\underset{\sim}{I}}^{F_1 2} & \cdots & \hat{\underset{\sim}{I}}^{F_1 m} \\ -\hat{\underset{\sim}{V}}^{F_2 1} & -\hat{\underset{\sim}{V}}^{F_2 2} & \cdots & -\hat{\underset{\sim}{V}}^{F_2 m} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{H}_{MM}^{T} \\ \underset{\sim}{H}_{MF_1}^{T} \\ \underset{\sim}{H}_{MF_2}^{T} \end{bmatrix} \cdot \qquad (4.11)$$

Having calculated the voltages and currents in all the adjoint network elements with different voltage excitations, $\hat{\underset{\sim}{V}}^{Mi}$, i = 1, 2, ..., m, we can obtain the matrix $\underset{\sim}{H}_{MF}$ corresponding to any set F of faulty elements. Thus, the adjoint network analysis does not have to be repeated if the set of predicted faulty elements is changed.

The manner in which the matrix $\underset{\sim}{H}_{MF}$ is computed yields insight into faulty situations when $\underset{\sim}{H}_{MF}$ is not of full column rank. Result 4.1 follows immediately.

4.2.4 Result 4.1

If the set of faulty elements contains a subset consisting of either

(a) a circuit formed by one-port elements, controlled current sources or currents that control faulty voltage or current sources;

(b) a cut set formed by one-port elements, controlled voltage sources or voltages that control faulty voltage or current sources;

then the test matrix $\underset{\sim}{H}_{MF}$ contains linearly dependent columns.

Proof

The proof follows Starzyk and Bandler (1981a). Result 4.1 can be used in constructing the appropriate combinations of the faulty elements that produce a full column $\underset{\sim}{H}_{MF}$ matrix for test purposes. In general, the sets of Result 4.1 define regions that contain the faulty set. Identification of the elements within the faulty region is possible using multitest vectors, as reported in Biernacki and Starzyk (1980) and Starzyk, Biernacki and Bandler (1981).

4.3 TOLERANCE CONSIDERATIONS

The exact consistency of (4.6) can appear only if all nonfaulty elements assume exact nominal values. In this section we consider the effect of deviations in the nonfaulty elements. A linear programming formulation for checking the consistency of (4.6), which has been proposed by Bandler, Biernacki and Salama (1981), will be elaborated on.

Let us assume that $\underset{\sim}{H}_{MF}$ of (4.6) is of full column rank. We can

then find f rows such that the submatrix $\underset{\sim}{H}_{FF}$ that is constructed using these rows is nonsingular. Without loss of generality, we can assume that they are the first f rows of $\underset{\sim}{H}_{MF}$. Considering the first f equations of (4.6) we have

$$\Delta \underset{\sim}{I}_F^M = \underset{\sim}{H}_{FF} \underset{\sim}{s} \ , \tag{4.12}$$

where

$$\Delta \underset{\sim}{I}_F^M \triangleq [\Delta I_1^M \ \Delta I_2^M \ \ldots \ \Delta I_f^M]^T \tag{4.13}$$

and

$$\underset{\sim}{s} \triangleq \begin{bmatrix} \underset{\sim}{V}^{F_1} \\ \\ \underset{\sim}{I}^{F_2} \end{bmatrix} . \tag{4.14}$$

Solving for the vector $\underset{\sim}{s}$ we obtain

$$s_j = \sum_{i=1}^{f} (\Delta I_i^M \ \Delta_{ij}/\Delta) = \sum_{i=1}^{f} (\Delta I_i^M \ \bar{\Delta}_{ij}) \ , \ j = 1, 2, \ldots, f \ , \tag{4.15a}$$

where

$$\bar{\Delta}_{ij} \triangleq \Delta_{ij}/\Delta \ . \tag{4.15b}$$

Δ_{ij} is the cofactor of the element (i,j) of $\underset{\sim}{H}_{FF}$ and Δ denotes its determinant. For any equation other than the first f equations of (4.6) we have

$$\Delta I_\ell^M = \sum_{j=1}^{f} H_{\ell j} \ s_j \ , \ \ell = f+1, f+2, \ldots, m \ . \tag{4.16}$$

Subsituting (4.15) into (4.16) we obtain

$$\Delta I_{\ell}^{M} = \sum_{j=1}^{f} H_{\ell j} \left(\sum_{i=1}^{f} \Delta I_{i}^{M} \bar{\Delta}_{ij} \right), \quad \ell = f+1, f+2, \ldots, m . \tag{4.17}$$

Due to the deviation in the nonfaulty elements from their nominal values, the values of $H_{\ell j}$ and $\bar{\Delta}_{ij}$ will deviate from their nominal and (4.17) becomes

$$\Delta I_{\ell}^{M} = \sum_{j=1}^{f} (H_{\ell j} + \delta H_{\ell j}) \left(\sum_{i=1}^{f} \Delta I_{i}^{M} (\bar{\Delta}_{ij} + \delta \bar{\Delta}_{ij}) \right) ,$$

$$\ell = f+1, f+2, \ldots, m . \tag{4.18}$$

The tolerance region of the elements supposed to be nonfaulty is defined by

$$-\varepsilon_i \leq \Delta\phi_i \leq \varepsilon_i, \quad i \in I_{\varepsilon} , \tag{4.19}$$

where $\Delta\phi_i \triangleq \phi_i - \phi_i^0$ is the change of the ith element value w.r.t. its nominal, ε_i is its associated tolerance and I_{ε} denotes the set of indices of the p-f nonfaulty elements.

Now the consistency of (4.18) requires the existence of $\delta H_{\ell j}$ and $\delta \bar{\Delta}_{ij}$ satisfying (4.19). It should be noted that although (4.18) gives an exact relationship it is a nonlinear function in the unknown variables. However, for a small tolerance region we can use the first-order approximation and substantially reduce the computational effort required. With $\delta H_{\ell j}$ and $\delta \bar{\Delta}_{ij}$ given by the first-order changes (4.18) becomes

$$\Delta I_{\ell}^{M} \equiv \sum_{j=1}^{f} \left(\left(H_{\ell j} + \sum_{k \in I_{\varepsilon}} \frac{\partial H_{\ell j}}{\partial \phi_k} \Delta \phi_k \right) \left(\sum_{i=1}^{f} \Delta I_i^{M} \left(\bar{\Delta}_{ij} + \sum_{k \in I_{\varepsilon}} \frac{\partial \bar{\Delta}_{ij}}{\partial \phi_k} \Delta \phi_k \right) \right) \right)$$

$$, \ell = f+1, f+2, \ldots, m . \qquad (4.20)$$

After some manipulations and neglecting second-order terms we get

$$b_{\ell} = \sum_{k \in I_{\varepsilon}} a_{\ell k} \Delta \phi_k, \ \ell = f+1, f+2, \ldots, m , \qquad (4.21)$$

where

$$b_{\ell} = \Delta I_{\ell}^{M} - \sum_{j=1}^{f} H_{\ell j} \left(\sum_{i=1}^{f} \Delta I_i^{M} \bar{\Delta}_{ij} \right), \ \ell = f+1, f+2, \ldots, m \qquad (4.22a)$$

and

$$a_{\ell k} = \sum_{j=1}^{f} H_{\ell j} \left(\sum_{i=1}^{f} \Delta I_i^{M} \frac{\partial \bar{\Delta}_{ij}}{\partial \phi_k} \right) + \sum_{j=1}^{f} \frac{\partial H_{\ell j}}{\partial \phi_k} \left(\sum_{i=1}^{f} \Delta I_i^{M} \bar{\Delta}_{ij} \right)$$

$$, \ell = f+1, f+2, \ldots, m . \qquad (4.22b)$$

In general, (4.21) is a complex system of linear equations. After separating out the real and imaginary parts we have

$$b_{\ell 1} = \underset{\sim}{c}_{\ell 1}^{T} \Delta \underset{\sim}{\phi}, \ \ell = f+1, f+2, \ldots, m, \qquad (4.23a)$$

$$b_{\ell 2} = \underset{\sim}{c}_{\ell 2}^{T} \Delta \underset{\sim}{\phi}, \ \ell = f+1, f+2, \ldots, m, \qquad (4.23b)$$

where

$$b_{\ell 1} = \mathrm{Re}\,(b_{\ell}), \qquad (4.24a)$$

$$b_{\ell 2} = \mathrm{Im}\,(b_{\ell}), \qquad (4.24b)$$

$$\underset{\sim}{c}_{\ell 1} = \mathrm{Re}\,[a_{\ell k_1} \ a_{\ell k_2} \ \ldots \ a_{\ell k_{p-f}}]^{T}, \qquad (4.24c)$$

$$\underset{\sim}{c}_{\ell 2} = \mathrm{Im}\,[a_{\ell k_1} \ a_{\ell k_2} \ \ldots \ a_{\ell k_{p-f}}]^{T}. \qquad (4.24d)$$

Equations (4.23) together with (4.19) determine the consistency in question. In other words, if a feasible solution exists we consider (4.6) is consistent and, simultaneously, we verify the f selected elements as a possible faulty set. If no feasible solution exists we can be sure that there exist one or more faulty elements besides those selected ones as long as the first-order approximation is justifiable.

In order to find a meaningful feasible solution, we propose a linear programming formulation of the form

$$\text{minimize } f(\Delta\underline{\phi}) \tag{4.25a}$$

subject to

$$b_{\ell 1} = \underline{c}_{\ell 1}^T \Delta\underline{\phi} \quad , \qquad \ell = f+1, f+2, \ldots, m, \tag{4.25b}$$

$$b_{\ell 2} = \underline{c}_{\ell 2}^T \Delta\underline{\phi} \quad , \qquad \ell = f+1, f+2, \ldots, m, \tag{4.25c}$$

$$-\epsilon_i \leq \Delta\phi_i \leq \epsilon_i \quad , \qquad i \in I_\epsilon \, , \tag{4.25d}$$

where $f(\Delta\underline{\phi})$ is a suitable linear function of $\Delta\underline{\phi}$.

It is to be noted that instead of the linear programming formulation a weighted-least-squares-solution similar to that considered in Section 3.4 could be used. In this case $f(\Delta\underline{\phi})$ is a weighted-least-squares objective function in the parameters $\Delta\underline{\phi}$ and constraints (4.25d) are not considered in the optimization problem. The solution obtained is then compared to the tolerance deviations described by (4.25d). A solution with any component of the computed $\Delta\underline{\phi}$ which significantly exceeds its tolerance value indicates that equation (4.6) is inconsistent under the tolerance variations in the nonfaulty elements of the subnetwork.

4.3.1 Coefficients Computation

The coefficients and sensitivities which appear in (4.20), (4.21) and (4.22) are calculated using the adjoint network approach. Recalling (4.10) we have

$$\begin{bmatrix} \hat{\underset{\sim}{I}}^{F_1} \\ -\hat{\underset{\sim}{V}}^{F_2} \end{bmatrix} = \underset{\sim}{H}_{MF}^{T} \hat{\underset{\sim}{V}}^{M} . \tag{4.26}$$

Considering only the first f columns of $\underset{\sim}{H}_{MF}^{T}$, (4.26) becomes

$$\begin{bmatrix} \hat{\underset{\sim}{I}}^{F_1} \\ -\hat{\underset{\sim}{V}}^{F_2} \end{bmatrix} = \underset{\sim}{H}_{FF}^{T} \hat{\underset{\sim}{V}}_{F}^{M} , \tag{4.27}$$

where $\hat{\underset{\sim}{V}}_{F}^{M}$ corresponds to the first f components of $\hat{\underset{\sim}{V}}^{M}$. In general, $\underset{\sim}{H}_{FF}^{T}$ is a nonsingular square matrix. Solving (4.27) for the ith measurement voltage $\hat{V}_i^M$ we obtain

$$\hat{V}_i^M = \sum_{j=1}^{f_1} \bar{\Delta}_{ij} \hat{I}_j^{F_1} - \sum_{j=f_1+1}^{f} \bar{\Delta}_{ij} \hat{V}_{j-f_1}^{F_2} . \tag{4.28}$$

Applying f independent excitations to the measurement ports of the adjoint network we get

$$\begin{bmatrix} \hat{V}_i^{M1} \\ \hat{V}_i^{M2} \\ \vdots \\ \hat{V}_i^{Mf} \end{bmatrix} = \begin{bmatrix} [\underset{\sim}{I}^{F1^1}]^T & [-\underset{\sim}{V}^{F2^1}]^T \\ [\underset{\sim}{I}^{F1^2}]^T & [-\underset{\sim}{V}^{F2^2}]^T \\ \vdots & \vdots \\ [\underset{\sim}{I}^{F1^f}]^T & [-\underset{\sim}{V}^{F2^f}]^T \end{bmatrix} \begin{bmatrix} \bar{\Delta}_{i1} \\ \bar{\Delta}_{i2} \\ \vdots \\ \bar{\Delta}_{if} \end{bmatrix} \tag{4.29}$$

or, more compactly,

$$\underset{\sim}{\hat{V}}_i^M = \underset{\sim}{R}\ \underset{\sim}{\bar{\Delta}}_i \tag{4.30}$$

where the superscripts 1 to f stand for the f independent excitations. The matrix $\underset{\sim}{R}$ is independent of i. Using the LHS of (4.29) for different i all the $\bar{\Delta}_{ij}$ coefficients can be computed.

The sensitivites of $\bar{\Delta}_{ij}$ relative to the nonfaulty elements are evaluated by computing the sensitivities of the elements of the matrix $\underset{\sim}{R}$ w.r.t. the nonfaulty elements. These sensitivities as well as the sensitivities of the $H_{\ell j}$ coefficients are directly obtained using the adjoint network concept.

4.3.2 Calculation of Faulty Element Values

The following procedure could be followed to compute the faulty element values.

(a) Perturb the nonfaulty elements of the subnetwork under test from their nominal values by the feasible tolerance vector $\Delta\phi$ obtained from the solution of (4.25).

(b) Evaluate the matrix $\underset{\sim}{H}_{MF}$ of the perturbed subnetwork.

(c) Solve equation (4.6) for $\underset{\sim}{V}^{F1}$ and $\underset{\sim}{I}^{F2}$. Note that (4.6) is an overdetermined system of equations. A direct way for solving it is to choose f independent equations from (4.6).

(d) Simulate the perturbed subnetwork with the input excitations $\underset{\sim}{V}^{M}$, $\underset{\sim}{V}^{F1}$ and $\underset{\sim}{I}^{F2}$ and calculate the responses at the fault ports and also the controlling currents or voltages if one or more of the f faulty elements are controlled sources.

(e) From (c), (d) and Fig. 4.2 calculate the changes in the faulty parameters.

4.4 EXAMPLES

The examples considered in this section were treated as networks rather than as subnetworks by Bandler, Biernacki and Salama (1981). In general, any network can be considered as a subnetwork as long as the appropriate nodes are identified as external nodes.

We utilized the IMSL Library routine ZX3LP for solving the linear programming problem. The Library is available as a part of McMaster University Computer Centre Library.

4.4.1 Ladder Subnetwork Example

Consider the ladder subnetwork shown in Fig. 4.4 with nominal values of elements $G_i = 1$ and tolerances $\varepsilon_i = 0.05$, $i = 1, 2, \ldots, 5$. The subnetwork is connected to the rest of the network through nodes 1, 2, 3 and the reference node. The voltages of these nodes and the currents incident to these nodes are assumed to be known. In this

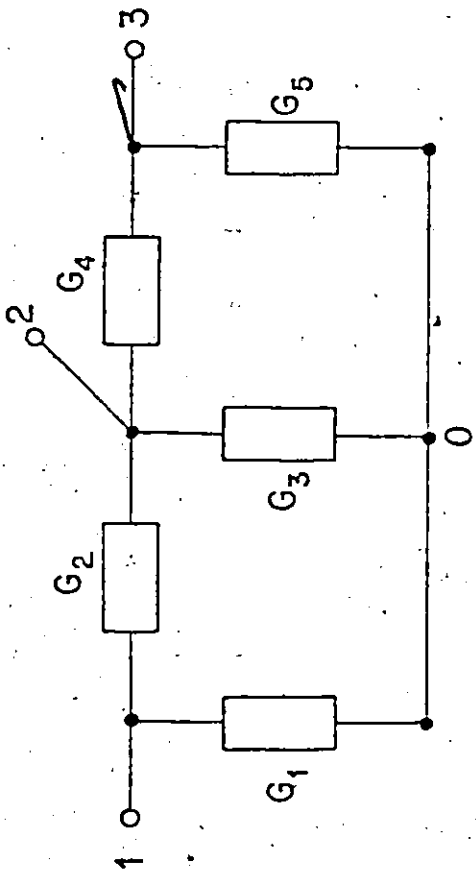

Fig. 4.4 Ladder subnetwork example.

example we have taken the subnetwork elements to be $G_1 = 1.02$, $G_2 = 0.5$, $G_3 = 0.98$, $G_4 = 0.98$ and $G_5 = 0.95$. It is clear that element G_2 is faulty and all other elements are within tolerances. The voltages of the external currents are given by $V_1 = 0.718$ V, $V_2 = 0.183$ V and $V_3 = 0.093$ V. The currents incident to these nodes from outside of the subnetwork are given by $I_1 = 1$A, $I_2 = 0$ and $I_3 = 0$. We have applied the verification algorithm with a single fault hypothesis.

From the available measurements, two equality constraints are constructed of the form of (4.25b). Using the optimization problem (4.25) and taking $f(\Delta\phi) = \sum_{j \in I_\varepsilon} |\Delta\phi_j|$, we check the single fault hypothesis. The linear program gives a feasible solution for the case when G_2 is considered faulty. No feasible solution was found for the case when any of the other remaining faulty elements is considered faulty.

Carrying out the procedure of Section 4.3.2 to calculate the faulty element value, we find that $G_2 = 0.532$, which is very close to the actual value. The value differs because the feasible tolerance vector found by the linear program is not the actual one.

4.4.2 Active Filter Subnetwork Example

Consider the active filter shown in Fig. 4.5 with the nominal element values $G_1 = G_2 = 1$, $C_1 = C_2 = 1$ and $K = 1$. Also, we assume that the amplifier has an output conductance $G_{out} = 1$. All elements are assumed to have design tolerances of ±5%. The subnetwork is assumed to be connected to the rest of the network through nodes 1, 2, 3 and the

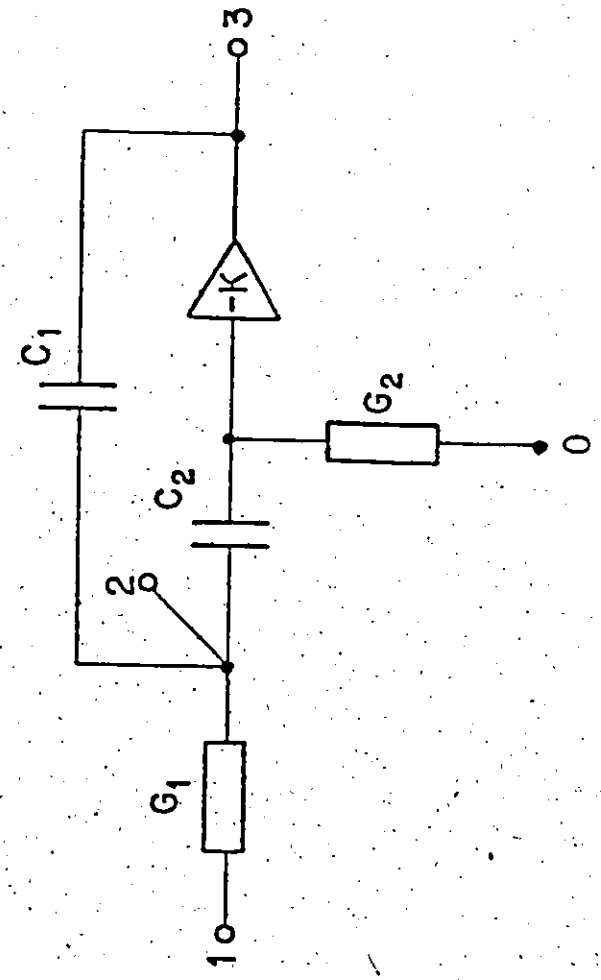

Fig. 4.5 Active filter subnetwork example.

reference node. The voltages of these nodes together with the currents input to the subnetwork though these nodes are assumed known.

In this example we considered $G_1 = 0.5$, $G_2 = 1.02$, $C_1 = 0.98$, $C_2 = 0.5$ and for the amplifier $K = 1.02$ and $G_{out} = 0.98$. The changes in C_2 and G_1 substantially violate the design tolerances and they are assumed to be faulty.

We have applied the verification method to check whether or not G_1 and G_2 are faulty. We have three measurements (complex), therefore we have four equality constraints (4.25b, 4.25c). Using the same approach as in Example 4.4.1 the program gives a feasible solution corresponding to the combination of G_1 and C_2. We have checked the remaining 9 combinations for the double fault hypothesis and no feasible solution was detected. Following the procedure for computing the faulty elements we found $G_1 = 0.5$ and $C_2 = 0.488$ which are close to the exact values.

4.5 INTERNAL-SELF-TESTING METHOD

The main drawback of the technique presented in Section 4.2 is guessing the set F which contains all faulty elements and has a number of elements $f<m$. Utilizing the internal-self-testing condition (ISTC) presented in Section 3.3.1 we can identify fault-free regions and regions that contain the faulty elements.

4.5.1 General Description

Let us decompose the subnetwork S into subnetworks S_1, S_2, ...,

S_k. Let c_i denote the cardinality of the set of S_i nodes incident with the other subnetworks, namely,

$$c_i \triangleq \text{card} \left(\bigcup_{\substack{j=1 \\ j \neq i}}^{k} N_j \cap N_i \right), \tag{4.31}$$

where N_i and N_j represent, respectively, sets of nodes of subnetworks S_i and S_j. Some of the C_i nodes could be measurement nodes. For the subnetwork S_i the four types of external nodes $M_{i\alpha}$, $M_{i\beta}$, $M_{i\gamma}$ and $M_{i\delta}$ defined in Section 3.3.1 may exist and

$$C_i \subseteq M_{i\beta} \cup M_{i\gamma} \cup M_{i\delta} . \tag{4.32}$$

The input-output equations for S_i are given by :

$$\begin{bmatrix} \underset{\sim}{I}^{M_\alpha} \\ \underset{\sim}{I}^{M_\beta} \\ \underset{\sim}{I}^{M_\gamma} \\ \underset{\sim}{I}^{M_\delta} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{H}_{M_\alpha M_\alpha} & \underset{\sim}{H}_{M_\alpha M_\beta} & \underset{\sim}{H}_{M_\alpha M_\gamma} & \underset{\sim}{H}_{M_\alpha M_\delta} \\ \underset{\sim}{H}_{M_\beta M_\alpha} & \underset{\sim}{H}_{M_\beta M_\beta} & \underset{\sim}{H}_{M_\beta M_\gamma} & \underset{\sim}{H}_{M_\beta M_\delta} \\ \underset{\sim}{H}_{M_\gamma M_\alpha} & \underset{\sim}{H}_{M_\gamma M_\beta} & \underset{\sim}{H}_{M_\gamma M_\gamma} & \underset{\sim}{H}_{M_\gamma M_\delta} \\ \underset{\sim}{H}_{M_\delta M_\alpha} & \underset{\sim}{H}_{M_\delta M_\beta} & \underset{\sim}{H}_{M_\delta M_\gamma} & \underset{\sim}{H}_{M_\delta M_\delta} \end{bmatrix} \cdot \begin{bmatrix} \underset{\sim}{V}^{M_\alpha} \\ \underset{\sim}{V}^{M_\beta} \\ \underset{\sim}{V}^{M_\gamma} \\ \underset{\sim}{V}^{M_\delta} \end{bmatrix}, \tag{4.33}$$

where the subscript i is dropped for the interest of simpler notations. From (4.33) we have

$$\begin{bmatrix} \Delta\underset{\sim}{I}^{M_\alpha} \\ \Delta\underset{\sim}{I}^{M_\gamma} \end{bmatrix} \triangleq \begin{bmatrix} \underset{\sim}{I}^{M_\alpha} \\ \underset{\sim}{I}^{M_\gamma} \end{bmatrix} - \begin{bmatrix} \underset{\sim}{H}_{M_\alpha M_\alpha} & \underset{\sim}{H}_{M_\alpha M_\beta} \\ \underset{\sim}{H}_{M_\gamma M_\alpha} & \underset{\sim}{H}_{M_\gamma M_\beta} \end{bmatrix} \begin{bmatrix} \underset{\sim}{V}^{M_\alpha} \\ \underset{\sim}{V}^{M_\beta} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{H}_{M_\alpha M_\gamma} & \underset{\sim}{H}_{M_\alpha M_\delta} \\ \underset{\sim}{H}_{M_\gamma M_\gamma} & \underset{\sim}{H}_{M_\gamma M_\delta} \end{bmatrix} \begin{bmatrix} \underset{\sim}{V}^{M_\gamma} \\ \underset{\sim}{V}^{M_\delta} \end{bmatrix}, \tag{4.34}$$

which is quite similar to (4.6), with $\underset{\sim}{V}^{M_\gamma}$ and $\underset{\sim}{V}^{M_\delta}$ representing the unknown faulty sources. If $m_{i\alpha} > m_{i\delta}$ (4.34) is an overdetermined system of equations. A necessary condition for S_i to be fault free is that (4.34) is consistent, which is the internal-self-testing condition of Section 3.3.1 for linear network.

Similar to Lemma 4.1 a sufficient condition for the subnetwork could be obtained.

4.5.2 Result 4.2 (fault-free subnetwork)

If the system of equations (4.34) is consistent and

$$\text{Rank}\begin{bmatrix} \underset{\sim}{H}_{M_\alpha M_\gamma} & \underset{\sim}{H}_{M_\alpha M_\delta} & \underset{\sim}{H}_{M_\alpha x} \\ \underset{\sim}{H}_{M_\gamma M_\gamma} & \underset{\sim}{H}_{M_\gamma M_\delta} & \underset{\sim}{H}_{M_\gamma x} \end{bmatrix} > \text{Rank}\begin{bmatrix} \underset{\sim}{H}_{M_\alpha M_\gamma} & \underset{\sim}{H}_{M_\alpha M_\delta} \\ \underset{\sim}{H}_{M_\gamma M_\gamma} & \underset{\sim}{H}_{M_\gamma M_\delta} \end{bmatrix} \tag{4.35}$$

for all x, where x is any internal node of subnetwork S_i, then there are no faulty elements incident with nodes x. $\underset{\sim}{H}_{M_\alpha x}$ and $\underset{\sim}{H}_{M_\gamma x}$ represent transfer functions from a current source connected to node x to the nodes M_α and M_γ.

If (4.34) is not a consistent system of equations then S_i is a faulty subnetwork. The fault verification technique can be applied to locate these faults. Let S_i contain f faults and $m_{i\alpha} > m_{i\delta} + f$. Instead of (4.34) we consider

$$\begin{bmatrix} \Delta \underset{\sim}{I}^{M_\alpha} \\ \Delta \underset{\sim}{I}^{M_\gamma} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{H}_{M_\alpha M_\gamma} & \underset{\sim}{H}_{M_\alpha M_\delta} & \underset{\sim}{H}_{M_\alpha F} \\ \underset{\sim}{H}_{M_\gamma M_\gamma} & \underset{\sim}{H}_{M_\gamma M_\delta} & \underset{\sim}{H}_{M_\gamma F} \end{bmatrix} \begin{bmatrix} \underset{\sim}{V}^{M_\gamma} \\ \underset{\sim}{V}^{M_\delta} \\ \underset{\sim}{V}^{F} \end{bmatrix}, \qquad (4.36)$$

where the added terms account for the faults inside the subnetwork. Since $m_{i_\alpha} > m_{i_\delta}+f$ (4.36) is overdetermined. A necessary condition for F to contain the faulty set is that (4.36) is consistent. The set F is unique if

$$\text{Rank} \begin{bmatrix} \underset{\sim}{H}_{M_\alpha M_\gamma} & \underset{\sim}{H}_{M_\alpha M_\delta} & \underset{\sim}{H}_{M_\alpha F} & \underset{\sim}{H}_{M_\alpha x} \\ \underset{\sim}{H}_{M_\gamma M_\gamma} & \underset{\sim}{H}_{M_\gamma M_\delta} & \underset{\sim}{H}_{M_\gamma F} & \underset{\sim}{H}_{M_\gamma x} \end{bmatrix} > \text{Rank} \begin{bmatrix} \underset{\sim}{H}_{M_\alpha M_\gamma} & \underset{\sim}{H}_{M_\alpha M_\delta} & \underset{\sim}{H}_{M_\alpha F} \\ \underset{\sim}{H}_{M_\gamma M_\gamma} & \underset{\sim}{H}_{M_\gamma M_\delta} & \underset{\sim}{H}_{M_\gamma F} \end{bmatrix} \qquad (4.37)$$

for all x, where x is any internal node not incident with the faulty set F.

All matrices used in (4.33-4.37) are computed using nominal element values and can be stored before performing the test. The on-line computational effort will be only that of verifying the consistency of (4.34) or (4.36), which requires elementary operations on the matrices considered.

By combining the fault verification technique and internal-self-testing condition method we reduce the number of different combinations to be considered, since the detected faulty region inside the subnetwork is usually much smaller than the subnetwork (contains fewer elements). The procedure for carrying out the internal-self-testing-method is summarized as follows.

4.5.3 Procedure for Locating Faulty Regions and/or Identifying Faulty Elements

The application of ISTC starts by partitioning the faulty subnetwork S_i into two smaller subnetworks S_j, S_k such that $S_i = S_j \cup S_k$, as shown in Fig. 4.6. For at least one of these subnetworks and preferably for both of them, $m_{\ell\alpha} > m_{\ell\delta}$, where ℓ = j or k, as appropriate.

Utilizing (4.34) we can identify whether S_j or S_k are fault-free or not. We utilize the nonfaulty subnetwork to determine the currents and voltages of the common nodes that are incident to it.

We continue the binary partitioning process in the identified faulty region until we cannot find a partition that satisfies the cardinality condition, namely $m_{\ell\alpha} > m_{\ell\delta}$. At this stage we utilize (4.36) if possible to identify the faulty elements inside the detected faulty region.

4.5.4 Example of Applying ISTC

Consider the resistive subnetwork shown in Fig. 4.7 with the nominal values of elements $G_i = 1$, $i = 1, 2, \ldots, 20$. All outside nodes {1, 2, 3, 6, 7, 10, 11, 12} constitute our measurement nodes where both voltages and currents are known and are given in Table 4.1. Node 12 is taken as the reference ground node. Nodes 4, 5, 8 and 9 are internal nodes where no measurements can be performed. Two faults are assumed in the subnetwork in elements G_2 and G_{18}. The process of identifying the faulty elements is summarized in the following steps, where E defines the set of faulty regions.

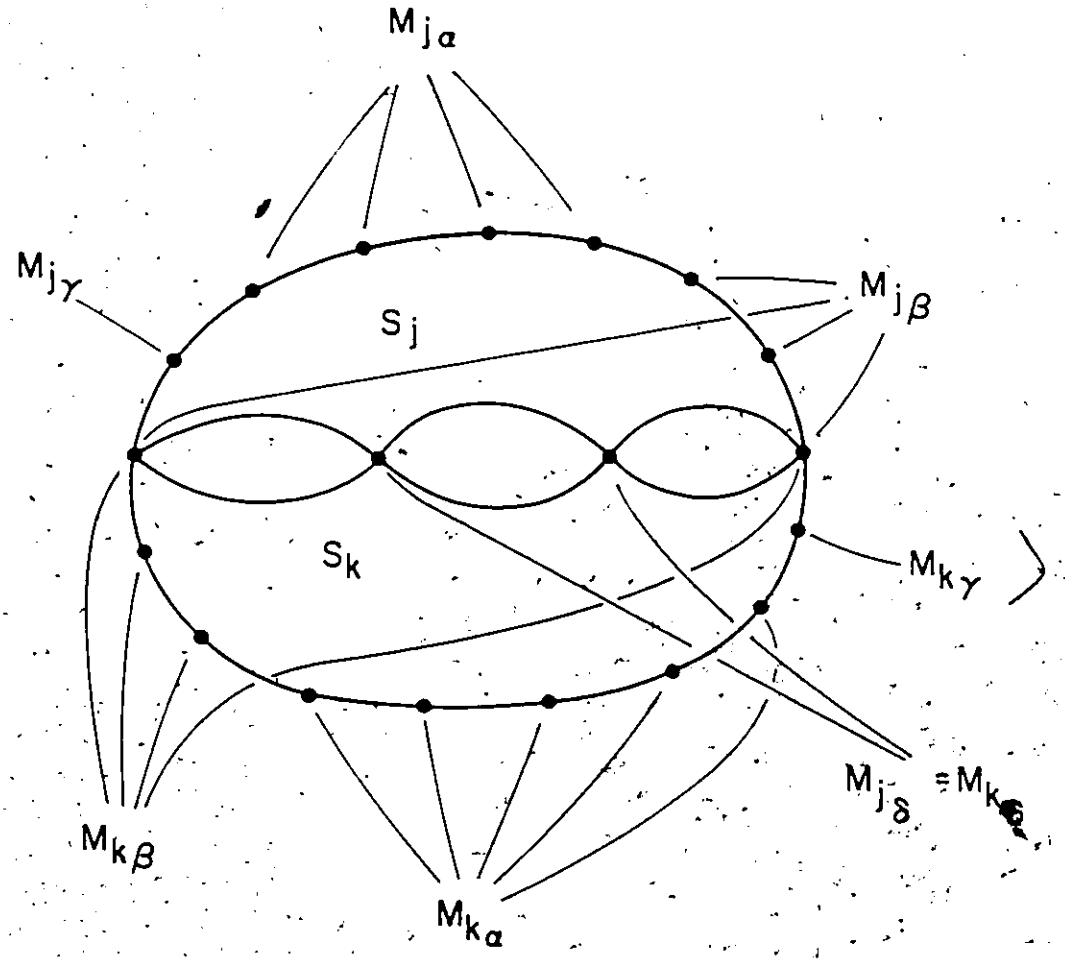

Fig. 4.6 Decomposition of S into two subnetworks.

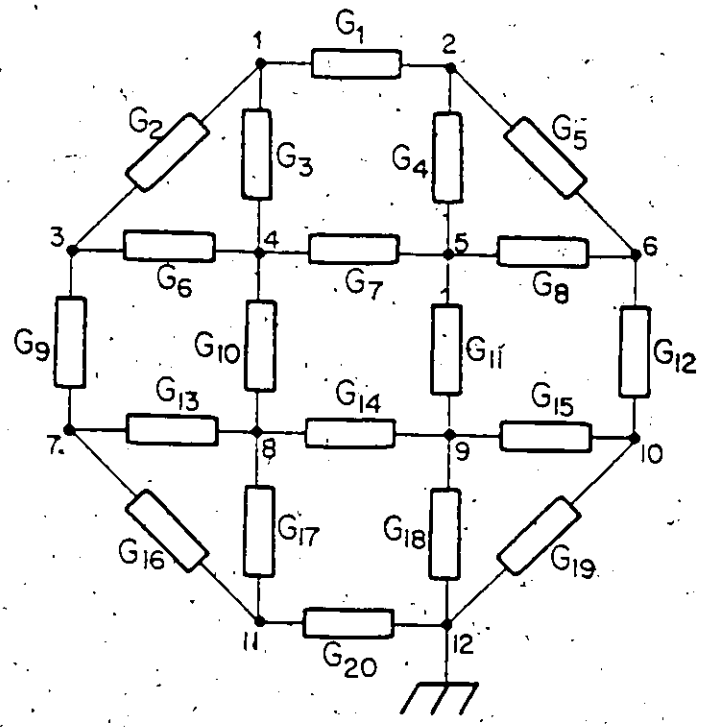

Fig. 4.7 The resistive subnetwork S.

TABLE 4.1

VOLTAGES AND CURRENTS OF MEASUREMENT NODES OF THE MESH SUBNETWORK

Node No.	Voltage (Volt)	Current (Amp)
1	1.246	1.0
2	0.8675	0.0
3	0.8222	0.0
6	0.6494	0.0
7	0.5952	0.0
10	0.3740	0.0
11	0.3896	0.0

Stage 1

S is decomposed as shown in Figs. 4.8 and 4.9 into S_1 and S_2.

$M_{1\alpha} = \{7, 11\}$, $M_{1\beta} = \{3, 12\}$, $M_{1\gamma} = \{\emptyset\}$, $M_{1\delta} = \{8\}$

$M_{2\alpha} = \{1, 2, 6, 10\}$, $M_{2\beta} = \{3, 12\}$, $M_{2\gamma} = \{\emptyset\}$, $M_{2\delta} = \{8\}$.

Applying condition (4.34), S_1 was found to be fault free and S_2 was found to be faulty and the following currents and voltages were computed

$$I_3^1 = -I_3^2 = 0.227 \text{ A},$$
$$I_8^1 = -I_8^2 = 0.162 \text{ A},$$
$$I_{12}^1 = -I_{12}^2 = -0.389 \text{ A},$$
$$V_8 = 0.5736 \text{ V}.$$

Stage 2

S_2 is decomposed as shown in Figs. 4.8 and 4.9 into S_3 and S_4.

$M_{3\alpha} = \{3, 8, 12\}$, $M_{3\beta} = \{1, 10\}$, $M_{3\gamma} = \{\emptyset\}$, $M_{3\delta} = \{5\}$,

$M_{4\alpha} = \{2, 6\}$, $M_{4\beta} = \{1, 10\}$, $M_{4\gamma} = \{\emptyset\}$, $M_{4\delta} = \{5\}$.

Applying (4.34) S_3 was found to be faulty and S_4 to be nonfaulty and the following currents and voltages were computed

$$I_1^4 = -I_1^3 = 0.3785 \text{ A},$$
$$I_5^4 = -I_5^3 = -0.1017 \text{ A},$$
$$I_{10}^4 = -I_{10}^3 = -0.2754 \text{ A},$$
$$V_5 = 0.7071 \text{ V}.$$

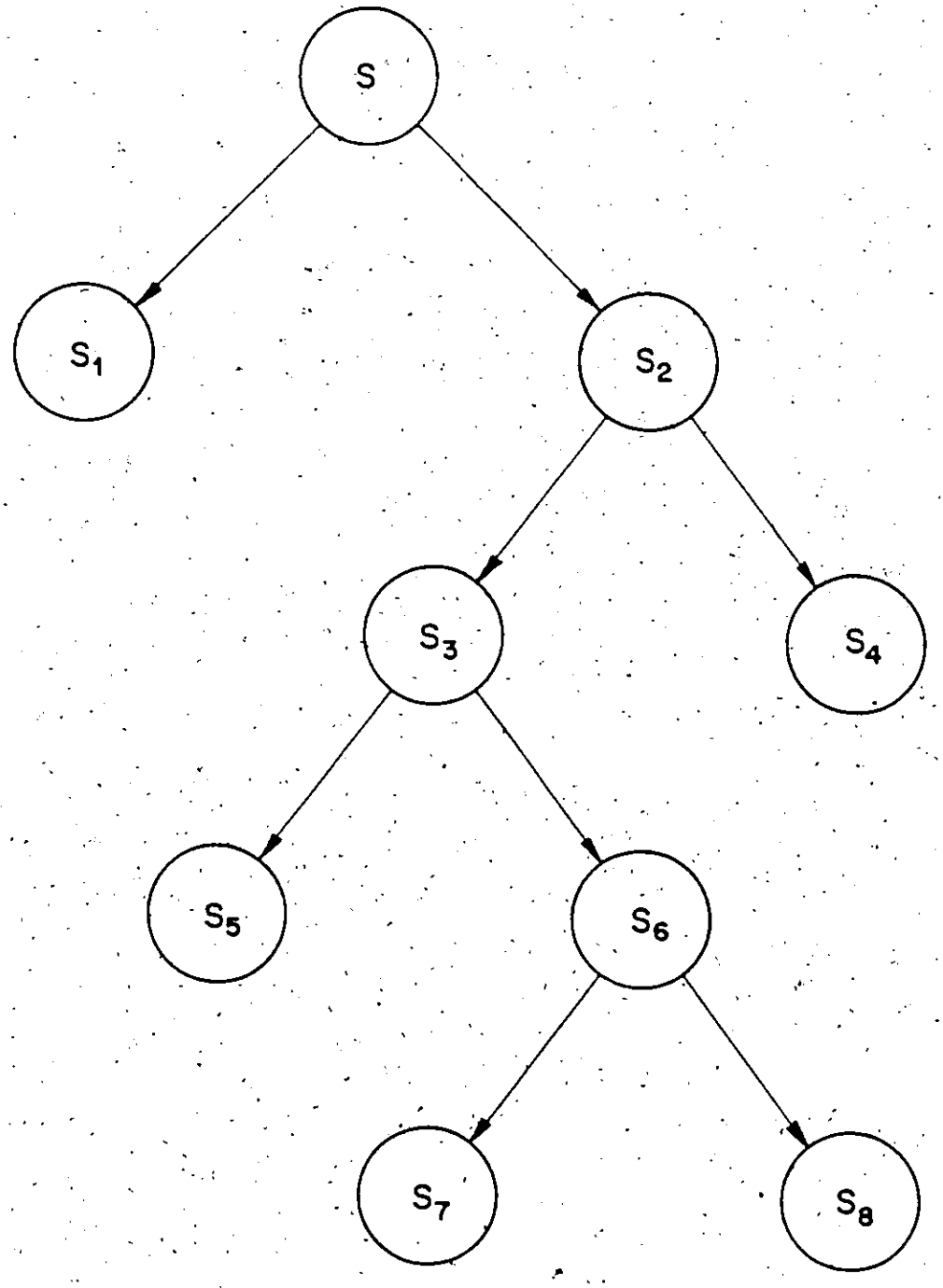

Fig. 4.8 The tree of decomposition for testing the resistive subnetwork.

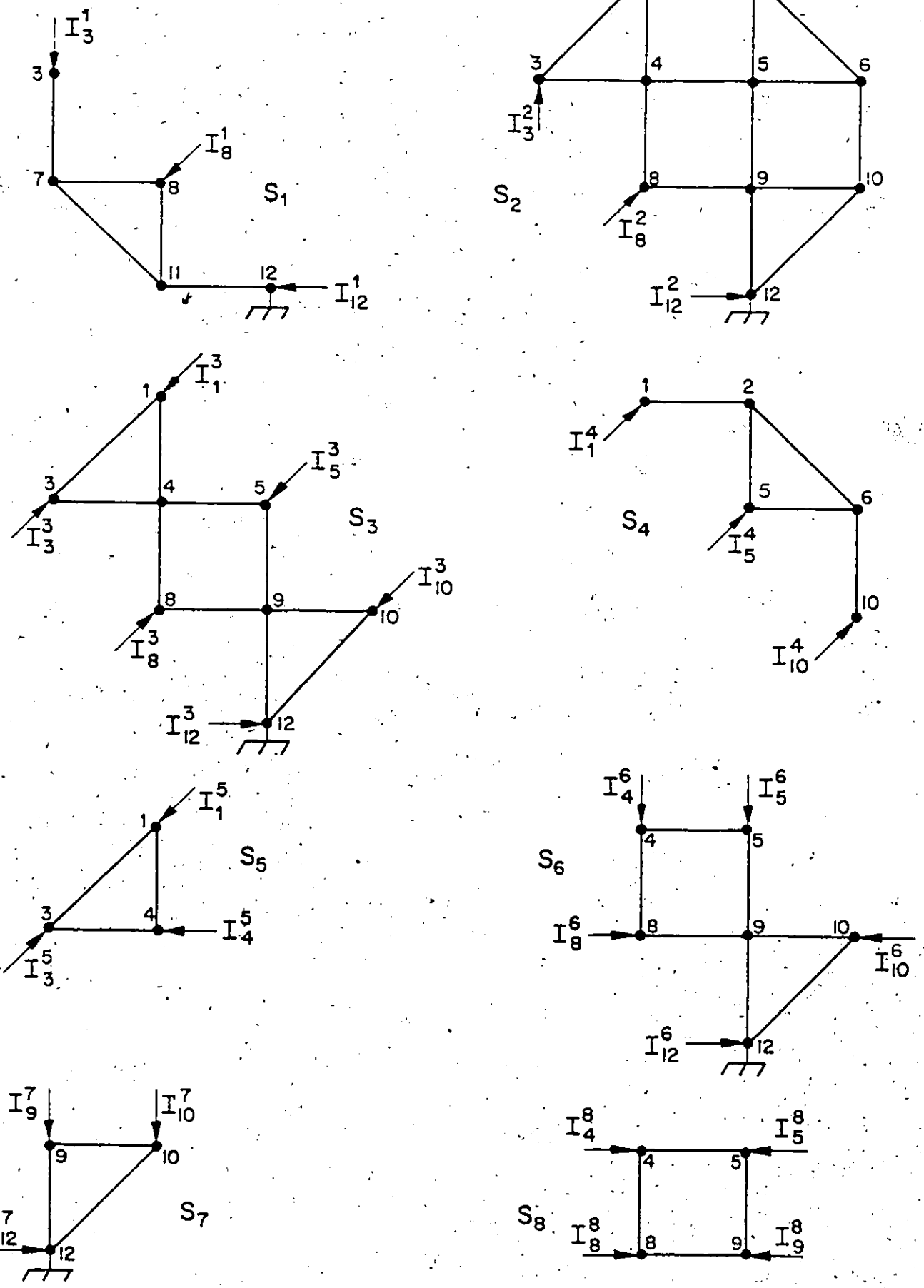

Fig. 4.9 The subnetworks resulting during testing.

Stage 3

S_3 is decomposed as shown in Figs. 4.8 and 4.9 into S_5 and S_6.

$M_{5\alpha} = \{1,3\}$, $M_{5\beta} = \{\emptyset\}$, $M_{5\gamma} = \{\emptyset\}$, $M_{5\delta} = \{4\}$.

$M_{6\alpha} = \{5, 8, 10, 12\}$, $M_{6\beta} = \{\emptyset\}$, $M_{6\gamma} = \{\emptyset\}$, $M_{6\delta} = \{4\}$.

Applying (4.34), S_5 and S_6 were found faulty.

Stage 4

S_6 is decomposed into S_7 and S_8 as shown in Figs. 4.8 and 4.9. S_5 is not decomposable according to our condition.

$M_{5\alpha} = \{1, 3\}$, $M_{5\beta} = \{\emptyset\}$, $M_{5\gamma} = \{\emptyset\}$, $M_{5\delta} = \{4\}$,

$M_{7\alpha} = \{10, 12\}$, $M_{7\beta} = \{\emptyset\}$, $M_{7\gamma} = \{\emptyset\}$, $M_{7\delta} = \{9\}$,

$M_{8\alpha} = \{5, 8\}$; $M_{8\beta} = \{\emptyset\}$, $M_{8\gamma} = \{\emptyset\}$, $M_{8\delta} = \{4, 9\}$.

Using KCL we compute the following currents.

$$I_4^5 = -I_4^8 = -0.3945 \text{ A},$$

$$I_9^8 = -I_9^7 = 0.335 \text{ A},$$

and thus $M_{5\gamma} = \{4\}$, $M_{7\gamma} = \{9\}$ and $M_{8\gamma} = \{4,9\}$.

Applying (4.34) S_8 was found to be nonfaulty and S_7 to be faulty and the following voltages were computed

$$V_4 = 0.8579 \text{ V},$$

$$V_9 = 0.4731 \text{ V}.$$

Stage 5

For both S_5 and S_7 all external voltages and currents are known. Searching for a single fault in both of them we found G_2 in S_5 to be faulty and equal 0.5 and G_{18} in S_7 to be faulty and equal 0.5. Hence,

we were able to locate the faults in 5 steps.

4.6 COMBINATORIAL ALGORITHM

For a subnetwork with p elements if f of them are faulty, the number of different combinations that is considered by the fault verification technique will be equal to $\binom{p}{f}$. For large subnetworks (p large) this number is enormous and the required computations will be prohibitive. The combinatorial algorithm (Starzyk and Bandler 1982) utilizes the available measurements to reduce the number of combinations. As the number of measurements increases the number of combinations considered will decrease.

Similar to the internal-self-testing technique for fault location, the combinatorial algorithm searches for a faulty set of w elements that contains the faulty set F.

It is evident from the fault verification technique that if we have m independent measurements then the maximum number of faulty elements we can identify is equal to m-1. The algorithm begins by identifying the set R_w of cardinality w that contains the faulty set F of cardinality $f << m-1$. The elements of this set could be identified using a technique similar to that presented in Section 4.3.2, and consequently the faulty elements are immediately identified. Alternatively, instead of identifying the faulty elements immediately the algorithm could be carried further to reduce the cardinality of the set R_w until no further reduction is possible (in this case the cardinality of the set equals f if the faults are unique). The

algorithm is summarized in the following steps.

4.6.1 Algorithm

Step 1 Divide arbitrarily the set of all subnetwork elements on k distinct subsets $\Omega_1, \ldots, \Omega_k$, each of them of cardinality equal to $E((m-1)/f)$, where $E(x)$ denotes the largest integer number smaller than or equal to x.

Comment If $p/E((m-1)/f)$ is not integer the last subset will have less than $E((m-1)/f)$ elements. So $k \geq p/E((m-1)/f)$.

Step 2 Examine all combinations of f subsets out of k subsets using the fault verification technique.

Comments (i) By dividing the subnetwork elements into k subsets, every f subsets will constitute a set of cardinality less than or equal to m-1 which can be checked by the fault verification technique.

(ii) The number of combinations considered in this step is equal to $\binom{k}{f}$ which is smaller than $\binom{p}{f}$. As the number of measurements increases the number of subsets k decreases and we have fewer combinations.

Step 3 Construct a set R_w that corresponds to the f subsets which contain the faulty elements. This set constitutes the first identified fault region

$$R_w = \bigcup_{i \in I} \Omega_i ,$$

where the set R_w of cardinality $w = f \cdot E((m-1)/f)$ and $I = \{i_1, i_2, \ldots, i_f\}$ designates the f subsets.

Comment Since $w \leq m-1$ a complete identification of the w elements is possible using the given m measurements and the fault verification technique could be ended at this point.

Step 4 Divide the set R_w arbitrarily on f+1 subsets and then check which combination of f subsets contain all faults. Update the set R_w by including only those subsets corresponding to verified faults. If there is no such combination stop.

Comment At least $E(w/(f+1))$ elements are eliminated from the set R_w and the cardinality of the updated set R_w is equal $w = w-E(w/(f+1))$.

Step 5 If w=f stop. Otherwise go to 4.

4.6.2 Example of Applying the Combinatorial Algorithm

For the mesh circuit example considered in the previous section we have p = 20, m = 7 and f = 2. The algorithm is realized as follows:

1. $k \geq \frac{p}{E(\frac{m-1}{f})} \geq \frac{20}{3}$, we take k = 7.

2. We check $\binom{k}{f} = \binom{7}{2} = 21$ different combinations to find the first fault region R_w with w = 6.

3. In every step of this stage (Step 4 of the algorithm) we check 3 combinations obtaining successively the following fault regions: R_4, R_3, and R_2.

In this example, therefore, we have to check not more than 30 combinations instead of $\binom{20}{2} = 190$ if we apply the fault verification technique directly.

As we stated in the algorithm we divide the elements arbitrarily on k subsets. This constitutes the main difference between this approach and the internal-self-testing approach, where we divide the elements according to the topology of the subnetwork.

4.6.3 Computational Aspects

In applying (4.7), $\underset{\sim}{H}_{MF}$ should be of full column rank. Since the elements are arbitrarily chosen there is a good probability that $\underset{\sim}{H}_{MF}$ will not be of full column rank. In this case we cannot apply (4.7) directly.

A system of equations

$$\underset{\sim}{A}\underset{\sim}{y} = \underset{\sim}{b}, \qquad (4.38)$$

where $\underset{\sim}{A}$ is m x f matrix (not necessary of full column rank) and m>f has a solution if $\underset{\sim}{b}$ belongs to the column space of $\underset{\sim}{A}$. The solution will be unique if rank $\underset{\sim}{A}$ equals f. To check the existance of a solution (consistency of the system of equations) we transform the system of equations by elementary row operations into the row-echelon form

$$\begin{bmatrix} 1 & x & 0 & \dots & x & x & x \\ 0 & 0 & 1 & \dots & x & x & x \\ \vdots & & & & & & \vdots \\ 0 & 0 & 0 & \dots & 1 & x & \dots x \\ \hline & & & \underset{\sim}{0} & & & \end{bmatrix} \underset{\sim}{y} = \begin{bmatrix} \underset{\sim}{b}_1 \\ \hline \underset{\sim}{b}_2 \end{bmatrix}, \qquad (4.39)$$

where x indicate a non-zero value. If $\underset{\sim}{b}_2 = \underset{\sim}{0}$ the system of equations (4.38) is consistent. Even if matrix $\underset{\sim}{A}$ is of full column rank the

number of mathematical operations (multiplications and divisions) in checking (4.7) is much higher than by transforming the matrix $\underset{\sim}{A}$ to the row-echelon form (see Starzyk and Bandler 1982). Also, (4.39) provides us immediately with a solution $\underset{\sim}{y}$ to (4.38). For ill-conditioned systems the method of Householder for orthogonal transformations can be used to reduce to zero the subdiagonal elements of $\underset{\sim}{A}$.

4.7 MATCHING TECHNIQUE FOR FAULT VERIFICATION

Practically, in any subnetwork there are some elements that are fault prone. Fault models of these elements are usually known and in the directory approach they are used to construct the dictionary. We exploit this to our advantage by computing using the measured voltages and the fault models

$$\underset{\sim}{I}^{M}_{F_j} = \left(\underset{\sim}{H}^{F_j}_{MM}\right) \underset{\sim}{V}^{M}, \quad j = 1, 2, \ldots, k, \tag{4.40}$$

where $\underset{\sim}{H}^{F_j}_{MM}$ is constructed using the jth fault model and k different faulty cases are considered. Utilizing the nearest neighbour rule (Varghase et al. 1979), the exact faulty case is assumed to be the one that has the minimum distance d_j from the actual measured $\underset{\sim}{I}^{M}$, where

$$d_j = \| \underset{\sim}{I}^{M}_{F_j} - \underset{\sim}{I}^{M} \| . \tag{4.41}$$

The on-line implementation of the method needs matrix by vector multiplication to compute $\underset{\sim}{I}^{M}_{F_j}$ since $\underset{\sim}{H}^{F_j}_{MM}$ could be computed off-line using the fault model and stored for the on-line use. If the number of possible faulty cases, k, is small, e.g., by considering catastrophic

faults only, the method could be very attractive for practical implementation (Salama, Starzyk and Bandler 1983).

4.8 CONCLUDING REMARKS

In this chapter we presented a number of techniques for locating faults inside subnetworks. Except for the matching technique all other techniques check the consistency of certain algebraic relations that are invariant on the faulty set. We investigated the effect of tolerances on the nonfaulty elements on the consistency of the algebraic equations. A linear programming formulation for checking the consistency of the equations was introduced.

The fault verification technique as presented by Biernacki and Bandler (1981) considers a very large number of combinations. The internal-self-testing condition is utilized in cutting down the number of combinations. The technique locates faulty regions. It takes advantage of the topology of the subnetwork to perform the required partition.

The combinatorial algorithm searches for the faulty elements in a very efficient way. The algorithm exploits all measurements, which should be larger than the number of faults. The reduction of the system of equations under consideration to the row-echelon form permits arbitrary choice of partitions as devised in the algorithm. The algorithm is not topology oriented and may be useful when the implementation of the ISTC technique cannot be continued.

All the techniques could be applied to nonlinear networks, but

the matching technique is the most suitable. A nonlinear network will require a nonlinear equation solver. In the matching technique the number of faulty cases is usually low and the utilization of on-line parallel processing speeds up the required computations.

5

FAULT LOCATION INSIDE FAULTY SUBNETWORKS— APPROXIMATE METHOD

5.1 INTRODUCTION

In this chapter we utilize an estimation criterion to infer the most likely faulty elements inside a faulty subnetwork. Practically, the faulty components are very few and the relative change in their values is significantly larger than in the nonfaulty ones (Merrill 1973).

Here, we use the least-one-objective function in estimating the faulty elements in any of the detected faulty subnetworks. The number of independent measurements is less than the total number of the components of the subnetwork. The ℓ_1 norm identifies the most likely faulty elements according to the aforementioned practical observations.

Two cases are considered in this chapter. The first considers the situation when the measurements are obtained using a single test vector (Bandler, Biernacki and Salama 1981). The second utilizes multiple test vectors to obtain the measurements (Bandler, Biernacki, Salama and Starzyk 1982).

5.2 FAULT ISOLATION USING THE ℓ_1 NORM WITH A SINGLE TEST VECTOR

Let us assume that for the subnetwork S (Fig. 4.1) the voltages $\underset{\sim}{V}^M$ and the currents $\underset{\sim}{I}^M$ are both known. The change in any subnetwork component is represented by either a current source or a voltage source,

as explained in Section 4.2.1.

We consider all the subnetwork elements as different from their nominal values. Most of them have changes within their tolerances and a few of them have changes far from assigned tolerances. Recalling equation (4.6) we have

$$\Delta \underset{\sim}{I}^M \triangleq \underset{\sim}{I}^M - \underset{\sim}{H}_{MM}\, \underset{\sim}{V}^M = \underset{\sim}{H}_{MF}\, \underset{\sim}{s}, \tag{5.1}$$

where $\underset{\sim}{s}$ represents the changes in the subnetwork elements and the set F is of cardinality equal to the total number of subnetwork elements p. The matrices $\underset{\sim}{H}_{MM}$ and $\underset{\sim}{H}_{MF}$ are computed using the nominal component values of the subnetwork.

Every component of $\underset{\sim}{s}$ has the form

$$s_i = (j\omega)^{\alpha i}\, \nu_i\, \Delta\phi_i = s_{i1} + j\, s_{i2},\ i = 1, 2, \ldots, p, \tag{5.2}$$

where $\alpha i = 0$, 1 or -1 depending on the component type, ω is the angular frequency, $\Delta\phi_i$ is the change in component value, ν_i is the actual controlling voltage or current for the component as shown in Fig. 4.2 and j is the complex square root of -1. The symbol s_i represents a voltage or a current source, s_{i1} and s_{i2} are its real and imaginary parts, respectively. Normally, ν_i is unknown unless a direct measurement of its value has been carried out.

Equation 5.1 is an underdetermined system of linear equations in the parameters $\underset{\sim}{s}$. Taking advantage of the dependency of $\underset{\sim}{s}$ on the changes in the parameters $\Delta\underset{\sim}{\phi}$ we propose to assume that $\underset{\sim}{s}$ defines the error parameters. To estimate the value of the components of $\underset{\sim}{s}$ we construct the following linear optimization problem for linear resistive

subnetworks.

$$\text{Minimize} \sum_{i=1}^{p} |s_i| \tag{5.3a}$$

subject to

$$\Delta \underset{\sim}{I}^{M} = \underset{\sim}{H}_{MF}\, \underset{\sim}{s}. \tag{5.3b}$$

For linear frequency dependent networks, the error parameters $\underset{\sim}{s}$ are, in general, complex and formulation (5.3) should be modified by redefining our error parameters. Consider the 2p-dimensional real vector

$$\underset{\sim}{e} \triangleq \begin{bmatrix} \underset{\sim}{s}_1 \\ \underset{\sim}{s}_2 \end{bmatrix}, \tag{5.4a}$$

where

$$\underset{\sim}{s}_1 \triangleq \text{Re}\,[\underset{\sim}{s}] \quad \text{and} \quad \underset{\sim}{s}_2 \triangleq \text{Im}\,[\underset{\sim}{s}]. \tag{5.4b}$$

Hence, the optimization problem can be stated as follows.

$$\text{Minimize} \sum_{i=1}^{2p} |e_i| \tag{5.5a}$$

subject to the real linear equality constraints

$$\begin{bmatrix} \text{Re}[\Delta \underset{\sim}{I}^{M}] \\ \text{Im}[\Delta \underset{\sim}{I}^{M}] \end{bmatrix} = \begin{bmatrix} \text{Re}[\underset{\sim}{H}_{MF}] & -\text{Im}\,[\underset{\sim}{H}_{MF}] \\ \text{Im}[\underset{\sim}{H}_{MF}] & \text{Re}[\underset{\sim}{H}_{MF}] \end{bmatrix} \underset{\sim}{e} \tag{5.5b}$$

The optimization problems (5.3) and (5.5) can be easily converted to the regular linear programming form by an appropriate transformation of the variables.

The least-one objective function tends to satisfy the equality

constraints (5.1) with the minimum number of error parameters different from zero. This is consistent with the assumption that a few elements are faulty inside the subnetwork.

Although the proposed optimization problem does not account for chain faults, fault correlation or a knowledge of the most likely areas of fault , this can be done by using appropriate weighting factors, based on experience and knowledge of the particular UUT.

The result of the optimization problem provides us with $\underset{\sim}{e}$. The subnetwork is then simulated using the known input currents to the subnetwork $\underset{\sim}{I}^M$ and $\underset{\sim}{e}$ to find ν_i, i = 1, 2, ..., p. The change in every subnetwork component can be easily computed using (5.2). Comparing the change in every component with its allowed tolerance the most likely faulty components could be isolated.

It should be noted that the change in the element value is obtained by the complex division

$$s_i/((j\omega)^{\alpha i}\ \nu_i) \qquad (5.6)$$

and since both s_i and ν_i are approximations to the actual values, a complex quantity may be obtained.

To overcome this difficulty we may either take $\Delta\phi_i$ as the absolute value or the real value of the quantity obtained in (5.6). In both cases an approximate value of the true $\Delta\phi_i$ is obtained. This value is then used to identify the faulty elements.

The formulations (5.3) and (5.5) do not destroy the linear relation in (5.1) and they do not create more unknowns, since the

unknown network response v_i is included in the error parameter (5.2). But, at the same time, for nonresistive networks, two error parameters are defined for every component, which cannot be easily correlated and this leads to obtaining a complex quantity in (5.6). These formulations are only valid for a single excitation at any time since any new excitation will add a new set of error parameters.

5.3 FAULT ISOLATION USING THE ℓ_1 NORM WITH MULTIPLE TEST VECTORS

Practically, we would like to have (5.1) less underdetermined. This is achieved if we increase the number of measurements by exciting the network more than once using different excitations, e.g., a different frequency or different input signals to the network. We then consider $\Delta\phi_i$ as the error parameter.

Equation (5.1) can be written as

$$\Delta \underset{\sim}{I}^M = \underset{\sim}{H}_{MF}\, \underset{\sim}{\Omega}_M\, \underset{\sim}{\nu}_M\, \Delta\underset{\sim}{\phi}, \tag{5.7}$$

where

$$\underset{\sim}{\nu}_M \triangleq \text{diag}\,\{\nu_1, \nu_2, \ldots, \nu_p\}, \tag{5.8}$$

$$\underset{\sim}{\Omega}_M \triangleq \text{diag}\,\{(j\omega)^{\alpha 1}, (j\omega)^{\alpha 2}, \ldots, (j\omega)^{\alpha p}\}, \tag{5.9}$$

$$\Delta\underset{\sim}{\phi} \triangleq [\Delta\phi_1\ \Delta\phi_2 \ldots \Delta\phi_p]^T. \tag{5.10}$$

To preserve the linear system (5.1) the subnetwork responses v_i, $i = 1, 2, \ldots, p$ have to be known. Hence, they are initially assumed. Conceptually, this is a linearization of the nonlinear functions that relate $\underset{\sim}{I}^M$ to $\underset{\sim}{\phi}$, as we discuss in Section 5.3.2. An iterative procedure updates the values v_i, $i = 1, 2, \ldots, p$ and at the same time computes the changes in the component values.

For k different excitations applied to the faulty network, we consider, instead of (5.3) the following optimization problem.

$$\underset{\Delta\underset{\sim}{\phi}}{\text{Minimize}} \sum_{i=1}^{p} |\Delta\phi_i/\phi_i^0| \tag{5.11a}$$

subject to

$$\begin{bmatrix} \Delta\underset{\sim}{I}^{M1} \\ \Delta\underset{\sim}{I}^{M2} \\ \vdots \\ \Delta\underset{\sim}{I}^{Mk} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{H}_{MF}^{1} & \underset{\sim}{\Omega}_{M}^{1} & \underset{\sim}{\nu}_{M}^{1} \\ \underset{\sim}{H}_{MF}^{2} & \underset{\sim}{\Omega}_{M}^{2} & \underset{\sim}{\nu}_{M}^{2} \\ & \vdots & \\ \underset{\sim}{H}_{MF}^{k} & \underset{\sim}{\Omega}_{M}^{k} & \underset{\sim}{\nu}_{M}^{k} \end{bmatrix} \Delta\underset{\sim}{\phi}, \tag{5.11b}$$

where the superscripts 1 to k refer to the different k excitations. The normalization w.r.t. ϕ_i^0 in (5.11a) is needed when the nominal values of the components are varied over a wide range. Formulation (5.11) can be modified by adding the inequality constraints

$$\Delta\phi_i > -\phi_i^x, \; i = 1, 2, \ldots, p, \tag{5.11c}$$

where ϕ_i^x is the value of the network element after being updated. Initially, ϕ_i^x is equal to the nominal design value ϕ_i^0. These constraints prevent nonphysical solutions.

We utilize the measurements together with an updated model of the subnetwork to compute $\Delta\underset{\sim}{I}^{M\ell}$, $\underset{\sim}{H}_{MF}^{\ell}$ and $\underset{\sim}{\nu}_{M}^{\ell}$, $\ell = 1, 2, \ldots, k$. The responses ν_i^{ℓ}, $\ell = 1, 2, \ldots, k$, $i = 1, 2, \ldots, p$ are functions of the nodal voltages. We update only the values of the internal node voltages since the external node voltages are known from measurements.

The number of equations in (5.11) $r = km$ is less than the number

of unknowns p. In every iteration, the linear program will find a solution that lies in a subspace spanned by at most r components of $\Delta\underset{\sim}{\phi}$. All other components are set to zero.

The iterative process for computing the changes in element values can be summarized in the following steps.

5.3.1 Algorithm

Step 1 Using the nominal component values, construct the matrices required by (5.11).

Step 2 Solve the linear optimization problem (5.11) and update the subnetwork component values.

Step 3 If there is no appreciable change in the component values, stop.

Step 4 Recompute the matrices required by (5.11) using the updated subnetwork. Return to step 2.

5.3.2 Convergence Properties of the Method

Recalling (3.5), the input-output relation of a subnetwork is given by

$$\underset{\sim}{I}^M = \underset{\sim}{h}^M(\underset{\sim}{V}^M, \underset{\sim}{\phi}). \tag{5.12}$$

The changes $\Delta\underset{\sim}{\phi}$ from nominal values are not known. Utilizing a first-order approximation we have, using the Taylor series expansion,

$$\Delta\underset{\sim}{I}^M \triangleq \underset{\sim}{I}^M - \underset{\sim}{I}^{MO} = \underset{\sim}{B}\,\Delta\underset{\sim}{\phi}\,, \tag{5.13}$$

where

$$\underset{\sim}{B} = \left[\frac{\partial \underset{\sim}{h}^M}{\partial \phi_1}\ \frac{\partial \underset{\sim}{h}^M}{\partial \phi_2}\ \cdots\ \frac{\partial \underset{\sim}{h}^M}{\partial \phi_p}\right]. \tag{5.14}$$

System (5.13) is underdetermined. For k different excitations (5.13) can be written as

$$\begin{bmatrix} \Delta \underset{\sim}{I}^{M1} \\ \Delta \underset{\sim}{I}^{M2} \\ \vdots \\ \Delta \underset{\sim}{I}^{Mk} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{B}^{1} \\ \underset{\sim}{B}^{2} \\ \vdots \\ \underset{\sim}{B}^{k} \end{bmatrix} \cdot \Delta \underset{\sim}{\phi} . \qquad (5.15)$$

Equation (5.15) is equivalent to (5.11b). Since (5.15) is still an underdetermined system of equations it has many solutions. Since only r components of $\Delta \underset{\sim}{\phi}$ are allowed to vary from zero in every solution of (5.11), we are implementing a restricted Newton iteration solving the nonlinear equations (5.12). For a starting point that is close enough to the final solution Newton's method is guaranteed to converge quadratically. The nominal component values define a good starting point, since just a few elements change significantly from nominal.

From (5.12) it is clear that the technique presented in this section is equally applicable to nonlinear networks. This is mainly due to the iterative nature of the procedure used in finding the changes in the subnetwork elements.

5.4 EXAMPLES

Similar to the examples of Section 4.4, the examples considered in this section were treated as networks rather than as subnetworks by Bandler, Biernacki, Salama and Starzyk (1982). We consider them here as subnetworks with the appropriate currents and voltages of external nodes

are assumed known.

5.4.1 Mesh Subnetwork Example

Consider the resistive subnetwork shown in Fig. 4.7 with the nominal values of elements $G_i = 1$ and tolerances $\epsilon_i = \pm 0.05$, $i = 1, 2, \ldots, 20$. This subnetwork has been considered earlier in Section 4.5.3. All outside nodes are assumed to be accessible with node 12 taken as the reference node. Nodes 4, 5, 8 and 9 are assumed internal, where no measurements can be performed.

Two faults are assumed in the subnetworks in elements G_2 and G_{18}. For Case 1, we applied the fault isolation method with a single excitation. We solved the optimization problem (5.3) for the error parameters $\underset{\sim}{s}$. Then, the changes in the component values were computed. For Case 2, we considered two excitations applied sequentially. We constructed optimization problem (5.11) and applied 3 iterations of our iterative procedure. See Table 5.1 for the results.

Both methods have identified the actual faulty elements, but in Case 2 the estimated changes in the faulty elements approach their true values. Some of the changes in the nonfaulty components have slightly exceeded their allowed tolerances, but these changes are small so we can still consider them nonfaulty.

Subroutine ZX3LP of the IMSL Library was utilized in solving the linear programming problems.

TABLE 5.1

RESULTS FOR THE MESH SUBNETWORK EXAMPLE

Element	Nominal Value	Actual Value	Percentage Deviation		
			Actual	Case 1	Case 2
G_1	1.0	0.98	-2.0	-4.15	-2.20
G_2	1.0	0.50	-50.0	-50.42	-47.86
G_3	1.0	1.04	4.0	0.0	8.60
G_4	1.0	0.97	-3.0	0.0	-2.46
G_5	1.0	0.95	-5.0	0.0	-2.36
G_6	1.0	0.99	-1.0	0.0	-0.14
G_7	1.0	1.02	2.0	0.0	-2.49
G_8	1.0	1.05	5.0	0.0	0.39
G_9	1.0	1.02	2.0	0.0	0.84
G_{10}	1.0	0.98	-2.0	0.0	-1.28
G_{11}	1.0	1.04	4.0	0.0	0.0
G_{12}	1.0	1.01	1.0	-3.06	1.55
G_{13}	1.0	0.99	-1.0	0.0	0.0
G_{14}	1.0	0.98	-2.0	0.0	2.47
G_{15}	1.0	1.02	2.0	0.0	1.51
G_{16}	1.0	0.96	-4.0	-7.07	-6.69
G_{17}	1.0	1.02	2.0	0.0	1.51
G_{18}	1.0	0.50	-50.0	-40.1	-46.90
G_{19}	1.0	0.98	-2.0	-9.21	-3.38
G_{20}	1.0	0.96	-4.0	-6.81	-6.14

5.4.2 Amplifier Subnetwork Example

Consider the one stage transistor amplifier of Fig. 5.1 with its equivalent circuit in Fig. 5.2. This example was originally considered by Chen and Saeks (1979). The nominal component values together with the actual values are listed in Table 5.2. Three faults are assumed in the subnetwork, namely, C_1, r_π and g_m, with all other elements within their relative tolerances. We first considered optimization problem (5.5). A single current excitation is applied at node 1 of angular frequency 0.01 rad/sec. Voltage measurements are simulated at the assumed accessible nodes 1, 2, 4, 5 and 6. For optimization problem (5.11) we excited the network twice at node 1 using two different frequencies, namely, 0.01 rad/sec and 0.0075 rad/sec, and voltage measurements are obtained from the same accessible nodes. The results of both cases are given in Table 5.2. Convergence in Case 2 occurred after 5 iterations. It is clear that the results of Case 2 are much sharper and they identify exactly the faulty elements. The solutions of both optimization problems are obtained using the linear programming routine ZX3LP.

5.5 CONCLUSIONS

We have presented a method for approximate fault isolation. The method utilizes the properties of the ℓ_1 norm in isolating the most likely subnetwork components which have exhibited large changes in their values.

For a single test vector the solution is obtained by solving a

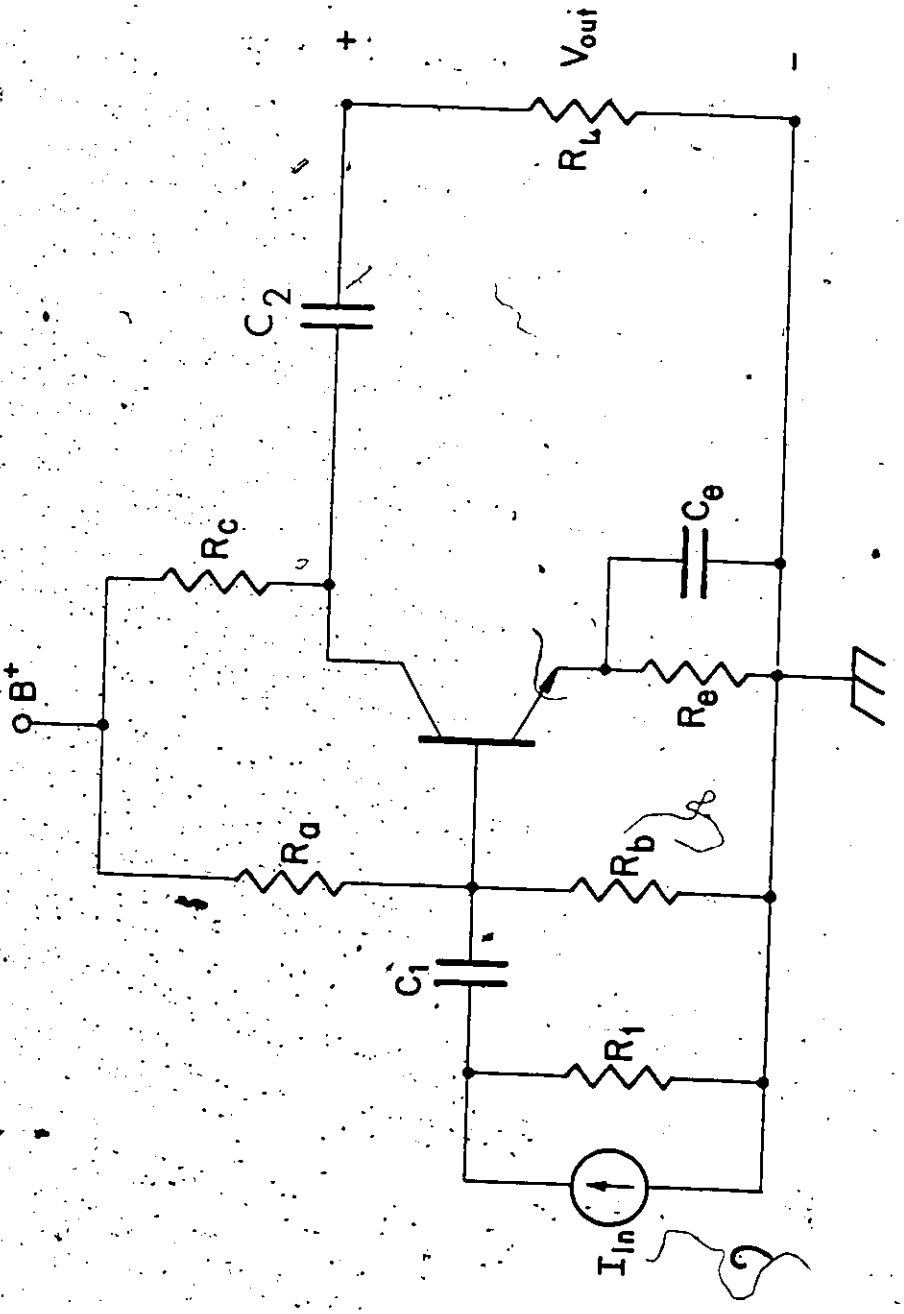

Fig. 5.1 The transistor amplifier circuit.

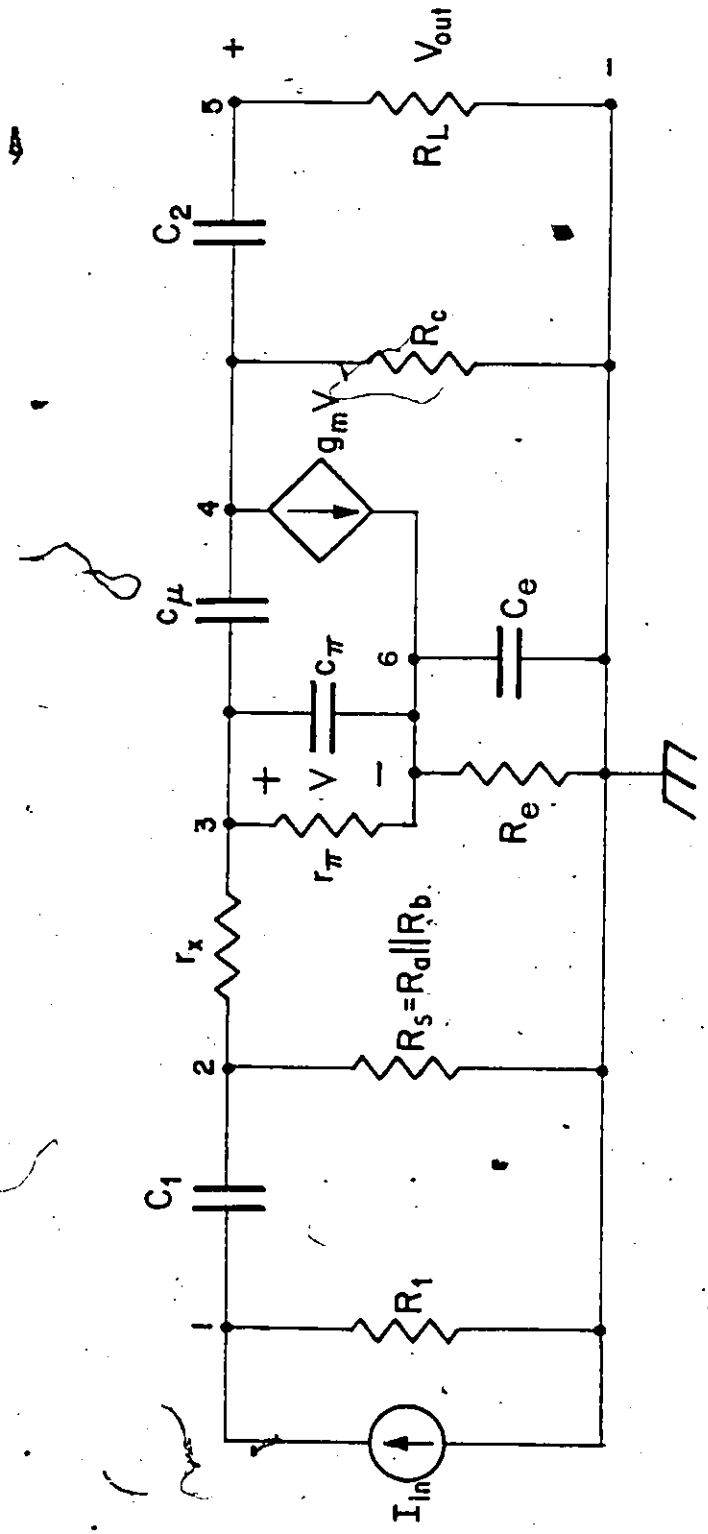

Fig. 5.2 The amplifier equivalent circuit.

TABLE 5.2

RESULTS FOR THE AMPLIFIER SUBNETWORK EXAMPLE

Element	Nominal Value	Actual Value	Percentage Deviation		
			Actual	Case 1	Case 2
C_1	20.0	10.0	-50.0	-48.68	-50.0
R_s	75.0	76.92	2.56	0.0	-3.04
r_x	10.0	10.2	2.00	0.0	-2.06
r_π	40.0	66.67	66.66	-12.93	53.63
c_π	15.0	14.0	-6.66	0.0	6.62
c_μ	25.0	24.0	-4.00	-0.32	-4.12
g_m	10.0	5.0	-50.00	0.0*	-46.21
R_c	10.0	9.8	-2.00	-1.36	2.55
C_2	20.0	19.0	-5.00	-0.65	-5.59
R_L	20.0	20.6	3.0	1.43	-3.44
R_e	30.0	29.4	-1.96	4.97	1.37
C_e	10.0	9.5	-5.00	0.0	-5.01
R_1	10.0	10.05	0.5	-1.43	-0.50

* A faulty element has not been detected.

linear programming problem and it does not destroy the linear relationship between the deviations in measurements and the defined error parameters.

For multiple test vectors, the formulation of the problem necessitates the application of an iterative procedure for its solution. A linear programming problem is solved in every iteration to provide us with the most likely changes in the subnetwork components. Linear programming is very efficient and, from our experience with the method, the iterative procedure converges rapidly.

With multiple test vectors the approximate method provided sharper results. This is easily seen from the results of the examples considered in this chapter.

The approximate method could be used by itself for fault isolation, or we could verify the results obtained by applying the deterministic methods of Chapter 4.

6

FORMULATION AND TECHNIQUES FOR THE POSTPRODUCTION TUNING PROBLEM

6.1 INTRODUCTION

Postproduction tuning is often essential in the manufacturing of electrical circuits. Tolerances on the circuit components, parasitic effects, and uncertainties in the circuit model cause deviations in the manufactured circuit performance and violation of the design specifications may result. Therefore, postproduction tuning is included in the final stages of the production process to readjust the network performance in an effort to meet the specifications (Bandler and Biernacki 1980).

In specifying a procedure for the computer-aided tuning of a circuit we are faced with a number of problems to be solved. Among these problems are the choice of the tunable parameters, the selection of samples of the responses (e.g. frequency points) and the tuning algorithm itself.

This chapter deals with some important aspects that are related to the aforementioned problems. First, we consider the relevant fundamental definitions and concepts. We next define error functions and their derivation from the design specifications. In practice, one of two classes of methods for tuning is usually employed. We devote Section 6.4 to the functional tuning approach, where a number of algorithms are reviewed. In Section 6.5 we discuss deterministic tuning

algorithms. We then discuss some of the techniques for the selection of both the tunable parameters and the tuning frequencies.

6.2 FUNDAMENTAL CONCEPTS AND DEFINITIONS

The manufactured circuit is characterized by the actual values of the p design parameters

$$\underset{\sim}{\phi}^a \triangleq [\phi_1^a \ \phi_2^a \ \cdots \ \phi_p^a]^T \tag{6.1}$$

and the parasitic effects associated with the produced circuit, which can be represented by the additional variables (Bandler and Salama 1983)

$$\underset{\sim}{\phi}_d \triangleq [\phi_{p+1} \ \phi_{p+2} \ \cdots \ \phi_{p+d}]^T \tag{6.2}$$

For convenience, we assume that nominal or ideal values for the parasitic variables are zeros.

The parameters $\underset{\sim}{\phi}^a$ for zero tuning can be expressed as

$$\underset{\sim}{\phi}^a = \underset{\sim}{\phi}^0 + \underset{\sim}{E}\,\underset{\sim}{\mu}_\varepsilon^a , \tag{6.3}$$

where $\underset{\sim}{\phi}^0$ is the nominal design parameter values,

$$\underset{\sim}{E} \triangleq \text{diag}\,\{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_p\}, \tag{6.4}$$

where ε_i is the tolerance associated with ϕ_i and $\underset{\sim}{\mu}_\varepsilon^a \in R_{\mu_\varepsilon}$. R_{μ_ε} defines a set of multipliers determined from realistic situations of the tolerance spread (Bandler, Liu and Tromp 1976a). For example,

$$R_{\mu_\varepsilon} \triangleq \{\underset{\sim}{\mu}_\varepsilon \mid -1 \leq \mu_{\varepsilon i} \leq 1, \ i \in I_\phi\}, \tag{6.5}$$

where

$$I_\phi = \{1, 2, \ldots, p\}. \tag{6.6}$$

Similarly, we may express $\underline{\phi}_d$ as

$$\underline{\phi}_d \triangleq \underline{\Delta}\, \underline{\mu}_\delta , \tag{6.7}$$

where

$$\underline{\Delta} \triangleq \text{diag}\, \{\delta_1, \delta_2, \ldots, \delta_d\}, \tag{6.8}$$

where δ_j is the expected deviation of ϕ_{p+j} from nominal, $j \in I_d$, where

$$I_d \triangleq \{1, 2, \ldots, d\} \tag{6.9}$$

and $\underline{\mu}_\delta \in R_{\mu_\delta}$ (Bandler, Liu and Tromp 1976b). R_{μ_δ} may be of the form

$$R_{\mu_\delta} \triangleq \{\underline{\mu}_\delta \mid 0 \leq \mu_{\delta i} \leq 1, \; i \in I_d\}. \tag{6.10}$$

The region of uncertainty $R_{\varepsilon\delta}$ is a set of points defined by

$$R_{\varepsilon\delta} \triangleq \{\underline{\phi} \mid \phi_i = \phi_i^0 + \varepsilon_i \mu_{\varepsilon i}, \; -1 \leq \mu_{\varepsilon i} \leq 1, \text{ for } i \in I_\phi,$$
$$\phi_{p+j} = \delta_j \mu_{\delta j}, \; 0 \leq \mu_{\delta j} \leq 1, \text{ for } j \in I_d\}, \tag{6.11}$$

which is a convex regular polytope of p+d dimensions with sides of length $2\varepsilon_i$, $i \in I_\phi$, and δ_j, $j \in I_d$.

The extreme points of $R_{\varepsilon\delta}$ are obtained by setting $\mu_{\varepsilon i} = \pm 1$ and $\mu_{\delta j} \in \{0, 1\}$. Thus, the set of vertices may be defined as

$$R_v \triangleq \{\underline{\phi} \mid \phi_i = \phi_i^0 + \varepsilon_i \mu_{\varepsilon i}, \; \mu_{\varepsilon i} \in \{-1, 1\}, \; i \in I_\phi, \text{ and}$$
$$\phi_{p+j} = \delta_j \mu_{\delta j}, \; \mu_{\delta j} \in \{0, 1\}, \; j \in I_d\}. \tag{6.12}$$

The number of points in R_v is 2^{p+d}. Let each of these points be indexed by $i \in I_v$, where

$$I_v \triangleq \{1, 2, \ldots, 2^{p+d}\}. \tag{6.13}$$

A subset of the design parameters is often used for tuning. Let the k tunable parameters be indexed by I_t^* and be given by

$$\phi_t \triangleq [\phi_{i_1} \; \phi_{i_2} \; \cdots \; \phi_{i_k}]^T \tag{6.14}$$

The remaining nontunable parameters are given by

$$\phi_r \triangleq [\phi_{i_{k+1}} \; \phi_{i_{k+2}} \; \cdots \; \phi_{i_p}]^T. \tag{6.15}$$

The objective of the tuning assignment problem is to find the required changes in the tunable parameters, namely ϕ_t, such that the manufactured circuit satisfies the design specifications.

6.3 DESIGN SPECIFICATIONS AND ERROR FUNCTIONS

In electrical circuit design more than one response function might have to meet given specifications. As an example, a circuit can be designed to meet desired specifications in both the frequency and the time domains (Bandler and Rizk 1979). In this case we have more than one independent variable ψ, namely, ψ^1, ψ^2, ..., ψ^λ, where λ is the number of these independent variables. Accordingly, we have λ response functions $F^1(\phi, \psi^1)$, $F^2(\phi, \psi^2)$, ..., $F^\lambda(\phi, \psi^\lambda)$. In general, we can have λ upper specifications $S_u^1(\psi^1)$, $S_u^2(\psi^2)$, ..., $S_u^\lambda(\psi^\lambda)$ and λ lower specifications $S_\ell^1(\psi^1)$, $S_\ell^2(\psi^2)$, ... $S_\ell^\lambda(\psi^\lambda)$. The error functions will be of the form

$$e_u^j(\underset{\sim}{x}, \psi^j) = w_u^j(\psi^j)\,(F^j(\underset{\sim}{x}, \psi^j) - S_u^j(\psi^j)),$$
$$j = 1, 2, \ldots, \lambda, \tag{6.16a}$$

$$e_\ell^j(\underset{\sim}{x}, \psi^j) = w_\ell^j(\psi^j)\,(F^j(\underset{\sim}{x}, \psi^j) - S_\ell^j(\psi^j)),$$
$$j = 1, 2, \ldots, \lambda, \tag{6.16b}$$

where $w_u^j(\psi^j)$ and $w_\ell^j(\psi^j)$ are positive weighting functions and the subscripts u and ℓ refer to upper and lower specifications, respectively.

In a typical tuning assignment problem the independent variable is the frequency and we are interested in the output response of the circuit at a discrete set of frequency points. Without loss of generality, we consider the following error functions

$$e_{ui}(\underset{\sim}{x}) \triangleq e_u(\underset{\sim}{x}, \psi_i) = w_{ui}\,(F_i(\underset{\sim}{x}) - S_{ui}),\ i \in I_u, \tag{6.17a}$$

$$e_{\ell i}(\underset{\sim}{x}) \triangleq e_\ell(\underset{\sim}{x}, \psi_i) = w_{\ell i}\,(F_i(\underset{\sim}{x}) - S_{\ell i}),\ i \in I_\ell, \tag{6.17b}$$

where

$$F_i(\underset{\sim}{x}) \triangleq F(\underset{\sim}{x}, \psi_i). \tag{6.17c}$$

I_u and I_ℓ are index sets, not necessarily disjoint. Let

$$f_i \triangleq \begin{cases} e_{uj}, & j \in I_u \\ -e_{\ell k}, & k \in I_\ell, \end{cases} \quad i \in I_c, \tag{6.18}$$

where

$$I_u \triangleq \{1, 2, \ldots, n_u\}, \tag{6.19a}$$

$$I_\ell \triangleq \{1, 2, \ldots, n_\ell\}, \tag{6.19b}$$

$$I_c \triangleq \{1, 2, \ldots, m\}, \tag{6.19c}$$

and $m = n_u + n_\ell$. The m functions

$$\underset{\sim}{f} = [f_1\ f_2 \ \ldots\ f_m]^T, \tag{6.20}$$

characterize the circuit performance, which is monitored during the tuning process. If we let

$$M_f(\underset{\sim}{\phi}) \triangleq \max_{i \in I_c} f_i(\underset{\sim}{\phi}), \tag{6.21}$$

then the sign of M_f indicates whether the specifications are satisfied or violated. The objective of a zero tolerance, ideal design is usually to minimize $M_f(\underset{\sim}{\phi})$ w.r.t. $\underset{\sim}{\phi}^0$, whereas the postproduction tuning problem is to minimize $M_f(\underset{\sim}{\phi})$ w.r.t. $\underset{\sim}{\phi}_t$ for a given production outcome whose actual parameter values and parasitic effects may not be known precisely.

Since the changes in the tunable parameters are predicted to be small, the first-order approximation is utilized in most tuning algorithms to estimate the changes in the functions $\underset{\sim}{f}$ and to provide the required tuning amounts $\Delta\underset{\sim}{\phi}_t$. The first-order change of the functions $\underset{\sim}{f}$ is given by

$$\Delta\underset{\sim}{f} = \sum_{j \in I_t^*} \phi_j^x \frac{\partial \underset{\sim}{f}}{\partial \phi_j} \frac{\Delta\phi_j}{\phi_j^x} = \underset{\sim}{S}\, \underset{\sim}{x}, \tag{6.22}$$

where $\underset{\sim}{S}$ defines the sensitivity matrix which is computed for a suitable network model $(\underset{\sim}{\phi}_t^x, \underset{\sim}{\phi}_r^x, \underset{\sim}{\phi}_d^x)$. $\underset{\sim}{x}$ is the vector of the relative changes in the tunable parameters.

Sensitivity analysis and optimization play an important role in constructing appropriate postproduction tuning algorithms. In the next two sections we consider functional and deterministic tuning methods.

6.4 FUNCTIONAL TUNING APPROACH

Functional tuning is the traditional way of tuning electrical circuits. After manufacturing and assembling, the circuit performance specifications are checked. If tuning is necessary a sequence of tunable parameter adjustments is carried out until the specifications are met. In functional tuning methods the actual network element values are generally assumed unknown, for example it may be difficult to measure or identify the actual circuit element values.

6.4.1 Traditional Functional Tuning Methods

Functional tuning is obviously suitable when the sensitivity matrix $\underset{\sim}{S}$ is diagonal, as has been pointed out by Moschytz (1975, 1978). For every function f_i there is a separate tunable parameter for its adjustment. The tuning process is noniterative, very fast and does not need a skillful operator. Normally, $\underset{\sim}{S}$ is not diagonal, but if the tunable parameters are chosen appropriately it can be arranged to be triangular or diagonally dominant. In this case the tuning process is carried out in a certain specified sequence to reduce the iterative adjustment of the tuning elements.

The reported success of this method is confined to small and simple circuits, particularly when the circuit design is chosen with tuning in mind.

6.4.2 Matrix Inversion Method

Shockley and Morris (1973) proposed the evaluation of the tuning

amounts by the direct inversion of the sensitivity matrix $\underset{\sim}{S}$. In this case the number of tunable parameters is equal to the number of functions f_i, i.e., m=k. From (6.22) the tunable amounts are given by

$$\underset{\sim}{x} = \underset{\sim}{S}^{-1} \Delta\underset{\sim}{f}, \tag{6.23}$$

where

$$\Delta\underset{\sim}{f} \triangleq \underset{\sim}{f}^0 - \underset{\sim}{f}^a. \tag{6.24}$$

$\underset{\sim}{f}^0$ represents the nominal values of the functions $\underset{\sim}{f}$ and $\underset{\sim}{f}^a$ the actual values as obtained from response measurements.

The nominal sensitivities are used in constructing $\underset{\sim}{S}$. The method is simple and its on-line implementation requires the pre-storage of $\underset{\sim}{S}^{-1}$, e.g., as LU factors.

When the changes in the manufactured elements are not small enough to permit the use of the differential sensitivities the method may not converge and is not reliable (Shockley et al. 1973).

6.4.3 Least-Squares Method

A number of authors (Antreich et al., 1975, Adams et al. 1975 and Müller 1976) suggested the use of the least-squares solution of (6.22) to find the values of the tunable parameters. The number of tunable parameters is assumed less than m. The tunable amounts are given by

$$\underset{\sim}{x} = [\underset{\sim}{S}^T \underset{\sim}{S}]^{-1} \underset{\sim}{S}^T \Delta\underset{\sim}{f} \tag{6.25}$$

and $\Delta\underset{\sim}{f}$ is as defined in (6.24). The sensitivity matrix $\underset{\sim}{S}$ is assumed of full column rank. The method is based on the assumption that functions

f_i, i = 1, 2, ..., m are almost linear in the tunable parameters. A prior investigation of the linearity of the functions f_i is usually required to guarantee the success of the method.

The method is usually applied iteratively (Adams et al. 1975) and is reported to have fast convergence. The matrix $\underline{S}$ is computed using the nominal parameter values and stored before the actual implementation of the tuning process. The method is very simple and suitable for on-line implementation.

Garrison et al. (1974), pointed out that the solution of (6.25) often requires an adjustment which is infeasible either in magnitude or direction. The least-squares solution should then be constrained and gradient search techniques employed to determine the best constrained solution.

6.4.4 Linear Minimax Optimization Method

Another approach which utilizes the response measurements as well as the simulated network sensitivities was developed by Bandler, Rizk and Salama (1981). The method is based on formulating the tuning process as a linear minimax optimization problem.

The network sensitivities are evaluated using a good approximate model of the circuit. They are usually later updated during the tuning process using the Broyden (1965) rank one updating formula.

The on-line implementation of the method requires more computing facilities than those needed by the aforementioned methods. The method is discussed in more detail in Section 7.2.

6.4.5 Modelling Technique for Functional Tuning

The actual network response is assumed to be given by

$$F^a(\underset{\sim}{\phi}_t^a, \underset{\sim}{\phi}_r^a, \underset{\sim}{\phi}_d^a) = F^0(\underset{\sim}{\phi}_t^0, \underset{\sim}{\phi}_r^0, \underset{\sim}{0}) + F^d(\Delta\underset{\sim}{\phi}_t^a, \Delta\underset{\sim}{\phi}_r^a, \underset{\sim}{\phi}_d^a), \qquad (6.26)$$

where a refers to the actual values, 0 to the nominal and d to the deviational effect due to any changes in parameters of the network.

F^d is modeled by a transfer function. The coefficients of the T.F. are obtained by utilizing the nominal response and the measured actual response. The model is later used for finding the tunable amounts by optimization. The approach is detailed in Section 7.3.

6.4.6 Quadratic Approximation Technique

Tromp (1982b), proposed a method that depends on successive quadratic approximation of an objective function $U(\underset{\sim}{\phi})$, which could be a function of f_i, $i = 1, 2, \ldots, m$. The quadratic approximation is obtained by interpolation (Bandler and Abdel-Malek 1978).

The tuning problem is formulated as

$$\underset{\underset{\sim}{\phi}_t}{\text{minimize}} \quad Q^j(\underset{\sim}{\phi}_t) \qquad (6.27a)$$

subject to

$$\underset{\sim}{\ell} \leq \underset{\sim}{\phi}_t \leq \underset{\sim}{u}, \qquad (6.27b)$$

where $Q^j(\underset{\sim}{\phi})$ is the jth quadratic approximation of $U(\underset{\sim}{\phi})$. Problem (6.27) is solved by a special algorithm developed by Tromp (1982a) that is noniterative. The interpolation region and the tuning parameters are updated and the process is repeated until convergence has occurred.

The on-line implementation of the method requires the availability of the quadratic optimization routine and performing enough measurements to construct the quadratic approximations. For k tunable parameters (k+1)(k+2)/2 settings of the tunable parameters and the corresponding response measurements are needed.

6.4.7 Predistortion Technique

The method proposed by Schaumann and Farrell (1981) is a combination of two procedures. First, the circuit is designed assuming ideal conditions and using well established design processes. This circuit serves as an initial condition for the following steps.

Next, based on a more accurate model of the circuit that may take first-order and second-order parasitic effects into account the measurable circuit parameters (e.g., gain, quality factor, pole-frequency) are altered to minimize a suitable error criterion (e.g., function of f_i, i = 1, 2, ..., m).

Finally, the circuit is functionally tuned under load to the parameter values that are generated by the predistortion stage.

The inclusion of parasitic effects, if known, in the model of the circuit to be tuned facilitates the prediction of the actual response and, consequently, the tuning process. The on-line implementation of the approach requires the clarification of the functional tuning procedure to be used. Schaumann and Farrell considered the possibility of replacing the functional tuning stage with a deterministic tuning algorithm.

In general, functional tuning methods are not fast. Deterministic tuning methods are considerably faster and much more efficient. In the next section we examine the deterministic methods of tuning.

6.5 DETERMINISTIC TUNING APPROACH

Deterministic tuning is primarily proposed to simplify the tuning assignment problem and eliminate the iterative process needed in the functional tuning approach.

The tuning process is usually carried out by measuring all the parameters of the manufactured network and the possible parasitic effects. Then, a matching procedure is carried out, where it is required to match the performance functions by varying the tunable parameter values.

6.5.1 Basic Deterministic Methods

Lüeder and Kaiser (1976) and Lüeder and Malek (1976) considered deterministic tuning methods for second-order active filters. The problem can be expressed as finding $\Delta\phi_t$ such that

$$f_i(\phi_t^a+\Delta\phi_t, \phi_r^a, \phi_d^a) = f_i^0(\phi^0),\ i = 1, 2, \ldots, k, \qquad (6.28)$$

where k is the number of tunable parameters and the f_i are assumed real functions.

A closed form solution of the nonlinear system of equations (6.28) can be obtained for simple problems such as those considered by

Lüeder and Malek (1976). The on-line implementation of the method is quite fast and usually results in a successful tuning procedure.

The network model used should quite closely simulate the actual performance of the manufactured circuit and this necessitates the consideration of all possible parasitic effects, which is impossible. It is, therefore, customary to functionally tune the network after performing the deterministic tuning procedure to improve the final circuit performance (Friedenson et al. 1975).

In general, a closed form solution is hard to obtain and it needs a formidable algebraic manipulation task (Lüeder and Kaiser 1976). As such, other methods have been devised that utilize network sensitivities and first-order approximation.

6.5.2 Sequential Tuning Method

As a matching problem, the tuning assignment problem is not well posed, i.e., a solution may or may not exist using the set of the tunable parameters chosen and if it exists it may not be unique. The problem of uniqueness is usually less important than the problem of existency.

Lopresti (1977) and Lopresti and Laker (1980) formulated the problem using discrete optimal control theory. Recalling (6.22) the deviation of the functions $\underset{\sim}{f}$ after tuning from their nominal values is given by

$$\delta\underset{\sim}{f} = \underset{\sim}{f} - \underset{\sim}{f}^0 = \underset{\sim}{f}^a - \underset{\sim}{f}^0 + \sum_{j\varepsilon I_t^*} \phi_j^x \frac{\partial \underset{\sim}{f}}{\partial \phi_j} \frac{\Delta\phi_j}{\phi_j^x}, \qquad (6.29)$$

where $\underset{\sim}{f}^0$ and $\underset{\sim}{f}^a$ are as defined before: The above differential function can be rewritten as

$$\underset{\sim}{x}_{j+1} = \underset{\sim}{x}_j + \underset{\sim}{S}_j \chi_j, \quad j = 1, 2, \ldots, k, \tag{6.30a}$$

$$\underset{\sim}{x}_1 = \underset{\sim}{f}^a - \underset{\sim}{f}^0, \tag{6.30b}$$

where $\underset{\sim}{S}_j$ is a column vector and is given by

$$\underset{\sim}{S}_j = \phi^x_{i_j} \frac{\partial \underset{\sim}{f}}{\partial \phi_{i_j}}, i_j \epsilon I_t^*. \tag{6.30c}$$

The partial derivatives are evaluated for the nominal circuit and χ_j is defined as

$$\chi_j \triangleq \Delta\phi_{i_j} / \phi^x_{i_j}, i_j \epsilon I_t^*. \tag{6.30d}$$

The tuning problem can be stated as a quadratic optimal control problem as follows. Minimize

$$\underset{\sim}{x}^T_{k+1} \underset{\sim}{Q} \underset{\sim}{x}_{k+1} + \sum_{i=1}^{k} \gamma_i \chi_i^2 \tag{6.31}$$

subject to (6.30). The second part of the objective function penalizes the excessive tuning amounts and guarantees the uniqueness of the solution.

A closed form solution is obtained using the Riccati equation (Lopresti 1977). The process is usually performed sequentially by measuring the tunable elements after adjustment and recalculating the remaining tuning amounts accordingly. This will partially compensate for inaccurate element adjustments and imprecise circuit model. The matrix $\underset{\sim}{Q}$ plays an important role in this process and an intelligent

choice of its elements usually affects the success of the method.

6.5.3 Large-Change Sensitivity Method

Another quite successful deterministic tuning algorithm was proposed by Alajajian (1979) and Alajajian et al. (1980).

The method is based on the observation that the first-order approximation which utilizes differential sensitivities is not always reliable. It utilizes Tellegen's theorem to derive a large change sensitivity expression, relating large changes in the tuning elements to the desired changes in the functions $\underset{\sim}{f}$ in the manufactured circuit. The response of the filter is matched to within a multiplicative constant c. Both the multiplicative constant and the tuning amounts are obtained through solving a system of linear equations.

Recalling (6.22) this system of equations could be in the form

$$\underset{\sim}{\bar{S}}\,\underset{\sim}{\chi} = c\,\underset{\sim}{f}^{0} - \underset{\sim}{f}^{a}\,, \tag{6.32}$$

which can be rearranged as

$$[\underset{\sim}{\bar{S}} \quad -\underset{\sim}{f}^{0}] \begin{bmatrix} \underset{\sim}{\chi} \\ c \end{bmatrix} = -\underset{\sim}{f}^{a}. \tag{6.33}$$

The matrix $[\underset{\sim}{\bar{S}} \quad -\underset{\sim}{f}^{0}]$ should be nonsingular and this will restrict the number of tunable parameters for certain independent functions $\underset{\sim}{f}$.

The matrix $\underset{\sim}{\bar{S}}$ is constructed such that its elements approximate the large change sensitivities. Since the initial state of the circuit is known from the direct measurement of the circuit parameters and the final desired state is almost known (approximately it could be considered the nominal state), an approximate expression of the

incremental sensitivities could be derived (Alajajian 1979).

In general, the method seems efficient and simple at the same time. Hocevar and Trick (1982), have compared both the sequential tuning method and the large change sensitivity method. Both methods performed quite well for the considered examples. For the on-line implementation the sequential method requires less computational requirements and usually produces better results.

6.6 SELECTION OF TUNING FREQUENCIES

6.6.1 Heuristic Method

Hocevar and Trick (1982) provided some heuristic guidelines for frequency selection. The effect of the choice of a particular frequency point is to greatly reduce the error f_i at that frequency. Since the response gradients for two closely spaced frequencies will be almost collinear, the frequencies should be reasonably spaced and placed in areas where tight control over the response is desired. The poles and zeros determine the response. The tuning frequencies located near the poles and zeros have strong control over their locations and, consequently, will have strong control on the response.

They also showed that the rank of $\underset{\sim}{S}$ will not be defective for logical frequency choices.

6.6.2 Matrix Inversion Method

Gleissner (1976) utilized the sensitivity matrix $\underset{\sim}{S}$ in finding the appropriate sampling frequencies. For ℓ frequencies the matrix $\underset{\sim}{S}$ is an

$\ell \times p_t$ matrix, where ℓ is greater than p_t, and where p_t is the number of elements that could be tunable. Let the rank of $\underset{\sim}{S}$ be ρ. The matrix

$$\underset{\sim}{H}_{\omega} \triangleq \underset{\sim}{S}\ \underset{\sim}{S}^T \tag{6.34}$$

is utilized in finding the appropriate frequency samples. Since matrix $\underset{\sim}{S}$ is of rank ρ, for the ℓ sampling frequencies only ρ are linearly independent of each other. For tuning, sampling points which optimally combine high sensitivity and minimum interdependency are selected.

This is achieved by a step by step inversion of matrix $\underset{\sim}{H}_{\omega}$ using as a pivot element always the maximum main diagonal element of the residual matrix (matrix results after pivoting). After ρ steps the $(\ell-\rho)$ main diagonal elements of the remaining partial matrix are equal to zero. Usually, the matrix inversion process is stopped when the remaining elements on the main diagonal are below a certain value. This will result in a number of frequencies less than ρ, say m.

6.6.3 Minimax Approach

In Section 6.3 we indicated that the network design problem can be formulated as a minimax optimization problem as follows.

$$\underset{\underset{\sim}{\phi}, z}{\text{Minimize}}\ z \tag{6.35a}$$

subject to

$$f_i(\underset{\sim}{\phi}) \leq z, \quad i \in I_c, \tag{6.35b}$$

where $\underset{\sim}{\phi}$ is the p-vector of design components. The solution of (6.35) provides us with theoretically justifiable critical (or active

functions) $f_j(\underset{\sim}{\phi}^0)$, $j \in I_c^*$, where $I_c^* \subset I_c$ is the index set of active functions. The critical functions are those approximately equal to z at the solution.

Normally, each critical function corresponds to a sample frequency, consequently we determine using (6.35) the frequencies to be monitored during tuning. As such, the information obtained during the design process that implements a minimax criterion is utilized directly in specifying the required set of frequencies I_c^* (Bandler and Salama 1983).

6.7 TUNABLE ELEMENT SELECTION

6.7.1 Matrix Inversion Method

Gleissner (1976) proposed a similar procedure to that of choosing the tuning frequencies to find the tunable elements. Let $\underset{\sim}{S}$ be an $m \times p_t$ matrix where $m < p_t$ and rank $\underset{\sim}{S}$ is less than or equal to m. It is required to find k maximum-linear independent columns of $\underset{\sim}{S}$. We construct the matrix

$$\underset{\sim}{H}_t = \underset{\sim}{S}^T \underset{\sim}{S}, \tag{6.36}$$

whose rank is less than or equal to m. A step by step inversion of matrix $\underset{\sim}{H}_t$ using as pivot element the maximum main diagonal element will lead to finding the tunable parameters.

Other aspects of tuning elements have to be considered, e.g., the linearity of functions f_i w.r.t. the chosen tunable elements.

6.7.2 QR Factorization Method (Hocevar and Trick 1982)

Let $\underset{\sim}{S}$ be an m x p_t sensitivity matrix, where $m < p_t$. It is required to choose k tunable parameters, where $k < p_t$. The matrix $\underset{\sim}{S}$ is factored as

$$\underset{\sim}{S} = \underset{\sim}{Q}\underset{\sim}{R}, \tag{6.37a}$$

$$[\underset{\sim}{S}_1 \; \underset{\sim}{S}_2] = [\underset{\sim}{Q}_1 \; \underset{\sim}{Q}_2] \begin{bmatrix} \underset{\sim}{R}_{11} & \underset{\sim}{R}_{12} \\ \underset{\sim}{0} & \underset{\sim}{R}_{22} \end{bmatrix}, \tag{6.37b}$$

$$\underset{\sim}{S}_1, \; \underset{\sim}{Q}_1 \in R^{mxk}, \quad \underset{\sim}{R}_{11} \in R^{kxk}, \tag{6.37c}$$

where $\underset{\sim}{Q}$ is an orthogonal matrix and $\underset{\sim}{R}$ is an upper triangular matrix.

The goal is to find a set of strong linearly independent columns of $\underset{\sim}{S}$ and these can be found in $\underset{\sim}{S}_1$ for the proper choice of k.

Column pivoting is usually used in QR factorization. Then, the factorization would be for a matrix

$$\hat{\underset{\sim}{S}} = \underset{\sim}{S}\,\underset{\sim}{P}, \tag{6.38}$$

where $\underset{\sim}{P}$ is a square permutation matrix. Consequently, the diagonal elements of matrix $\underset{\sim}{R}$ satisfy

$$|r_{11}| \geq |r_{22}| \geq \ldots \geq |r_{p_t p_t}|. \tag{6.39}$$

The singular values of $\underset{\sim}{S}_1$ can be approximated by the diagonal elements $|r_{ii}|$. The choice of k element values depends on the value of $|r_{kk}|$ since there is a strong relation between the number of relatively large singular values and the number of strong independent columns of the

matrix $\underset{\sim}{S}$ (Hocevar and Trick 1982).

6.7.3 Optimal Tuning Method

Bandler, Liu and Tromp (1976a, 1976b) and Liu (1975) considered tuning as an integral part of the optimal design problem. The centering, tolerancing and tuning problem was formulated as a nonlinear programming problem. They considered an objective function of the form

$$\sum_{i=1}^{p} c_i \frac{\phi_i^0}{\varepsilon_i} + \sum_{i=1}^{p} c_i' \frac{t_i}{\phi_i^0}, \tag{6.40}$$

where t_i defines the tunable amount associated with the ϕ_i parameter, c_i and c_i' are positive constants. The worst-case solution must satisfy

$$R_t(\underset{\sim}{\mu}_\varepsilon) \cap R_c \neq \emptyset, \tag{6.41}$$

for all $\underset{\sim}{\mu}_\varepsilon \in R_{\mu_\varepsilon}$, where $\emptyset$ denotes the null set. $R_t(\underset{\sim}{\mu}_\varepsilon)$ is given by

$$R_t(\underset{\sim}{\mu}_\varepsilon) \triangleq \{\underset{\sim}{\phi} \mid \underset{\sim}{\phi} = \underset{\sim}{\phi}^0 + \underset{\sim}{E}\,\underset{\sim}{\mu}_\varepsilon + \underset{\sim}{T}\,\underset{\sim}{\rho},\ \underset{\sim}{\rho} \in R_\rho\}, \tag{6.42a}$$

where

$$\underset{\sim}{T} = \text{diag}\,\{t_1, t_2, \ldots, t_p\} \tag{6.42b}$$

and

$$R_\rho \triangleq \{\underset{\sim}{\rho} \mid -1 \leq \rho_i \leq 1,\ i \in I_\phi\} \tag{6.42c}$$

for the two-way tuning case. The feasible region is defined by

$$R_c \triangleq \{\underset{\sim}{\phi} \mid f_i(\underset{\sim}{\phi}) \leq 0,\ i \in I_c\}. \tag{6.43}$$

For the worst-case problem they considered only worst-case vertices (6.12). This is usually adequate under the condition that R_c

is one-dimensionally convex (Bandler 1972). The results of investigating optimal design centering, tolerancing and tuning utilizing objective function (6.40) resulted in a few parameters having nonzero values of t_i and the rest having exactly zero tuning amounts.

At that point the authors did not realize that this could provide the basis of a general algorithm for identifying tunable elements based on the worst-case vertices, namely, the elements which have nonzero tunable amounts t_i. We have exploited the aforementioned observation in devising a procedure for finding the tunable elements (Bandler and Salama 1983).

A manufactured outcome of the circuit would be a point of the region $R_{\varepsilon\delta}$ (6.11). Worst-case analysis is carried out to identify the critical points of this region. A worst-case point is assumed to occur at a vertex of the region $R_{\varepsilon\delta}$ (6.12). $R_{\varepsilon\delta}$ has a 2^{p+d} vertices and they are indexed by the set I_v of (6.13). A worst-case algorithm for vertex selection is employed (Brayton et al. 1979). For every critical function $f_i(\underline{\phi})$, $i \in I_c^*$, (obtained from 6.35) one or more vertices are selected. Let $I_{vi} \subset I_v$ be the index set of worst-case vertices corresponding to the function $f_i(\underline{\phi})$, $i \in I_c^*$, and let

$$I_v^* \triangleq \bigcup_i I_{vi}, \quad i \in I_c^* \tag{6.44}$$

define the index set of critical vertices, $I_v^* \subset I_v$.

To compute the tunable parameters, we solve the following optimization problem.

$$\text{Minimize} \sum_{i \in I_t} t_i \tag{6.45a}$$

w.r.t. t_i, ρ_i^r, $i \in I_t$, $r \in I_v^*$, where

$$t_i \geq 0, \quad i \in I_t, \tag{6.45b}$$

$$-1 \leq \rho_i^r \leq 1, \quad i \in I_t, \quad r \in I_v^*, \tag{6.45c}$$

such that

$$\underline{\phi} \in R_c^*, \tag{6.45d}$$

for all $r \in I_v^*$, where

$$\phi_i = \begin{cases} \phi_i^r, & i \notin I_t, \quad (6.45e) \\ \phi_i^r + t_i \rho_i^r, & i \in I_t, \quad (6.45f) \end{cases}$$

and

$$R_c^* \triangleq \{\underline{\phi} \mid f_i(\underline{\phi}) \leq 0, \ i \in I_c^*\}. \tag{6.45g}$$

It can be shown that this problem is a least pth optimization problem with p = 1. It finds the minimum number of tunable parameters k required to tune all worst-case vertices. At the solution we obtain $I_t^* \subseteq I_t$, where

$$I_t^* \triangleq \{i \mid t_i \neq 0, \ i \in I_t\} \tag{6.46}$$

and I_t is the index set for the tunable parameter candidates.

It is readily recognized that the same objective function has been utilized in conjunction with the approximate method for fault analysis to find the minimum number of faulty elements (Sections 5.2 and 5.3). In selecting the tunable elements the objective here is quite similar, namely, to find the minimum number of tunable parameters to tune all worst-case vertices.

6.8 CONCLUDING REMARKS

In this chapter we presented a number of aspects that are related to the postproduction tuning assignment problem.

As was first pointed out by Pinel (1971), network sensitivities provide a valuable tool for network tuning. As we examined in this chapter they are utilized in many functional and deterministic tuning algorithms to provide the required tuning amounts. They are also utilized in selecting the tunable parameters and the tuning frequencies.

Minimax optimization and least pth optimization with $p = 1$, subject to suitable constraints provide an effective technique for determining both the frequencies at which tuning should be monitored and the tunable parameters. In the next chapter the application of these techniques to tune a microwave example is presented.

7

NEW FUNCTIONAL TUNING ALGORITHMS AND A COMPARISON WITH DETERMINISTIC TUNING METHODS

7.1 INTRODUCTION

In this chapter, two functional tuning algorithms are presented. Both algorithms are based on measuring the response of the circuit at a number of critical frequencies and formulating the postproduction tuning problem as an optimization problem.

The first algorithm (Bandler, Rizk and Salama 1981), as is pointed out in Section 7.2, utilizes linear programming iteratively for estimating necessary tuning amounts. The technique exploits the availability of a good approximation model simulating the actual network under consideration. Each step of the iterative tuning procedure requires one set of response measurements. A linear approximation of the minimax optimization problem is solved, providing the amounts of tuning to be implemented. The tunable parameters are adjusted to the extent possible and the process is repeated until an optimum is obtained.

The second algorithm, which we present in Section 7.3, models the deviation in the network response from nominal by a rational transfer function in the complex frequency. The tuning problem is next formulated as a linearly constrained minimax optimization problem. The tunable amounts are implemented and the procedure is repeated until

convergence is achieved.

Both algorithms were tested for tuning a microwave network amplifier and highpass notch active filter example. For the latter example, the two deterministic tuning algorithms of Sections 6.5.2 and 6.5.3 were also implemented. The results of the four techniques are then compared.

The techniques introduced in Sections 6.6.3 and 6.7.3 for the selection of the critical frequency points and the tunable parameters are utilized with the tuning algorithm of Section 7.3 to tune the microwave network amplifier.

7.2 A LINEAR APPROXIMATION TECHNIQUE FOR FUNCTIONAL TUNING

7.2.1 Mathematical Formulation

The tuning assignment problem is similar to the design problem but with the tunable elements taken as variables.

Similarly to (6.35), the tuning problem can be formulated as a minimax problem as follows.

$$\underset{\Delta\underline{\phi}_t,\ z}{\text{Minimize } z} \tag{7.1a}$$

subject to

$$f_i(\underline{\phi}^a + \Delta\underline{\phi}_t,\ \underline{\phi}_r^a,\ \underline{\phi}_d^a) \leq z,\quad i \in I_c, \tag{7.1b}$$

$$\underset{\sim}{\ell} \leq \Delta\underline{\phi}_t \leq \underset{\sim}{u}. \tag{7.1c}$$

The minimization is carried out by varying $\Delta\underline{\phi}_t$. The definition of functions f_i and parameters $\underline{\phi}_t^a$, $\underline{\phi}_r^a$ and $\underline{\phi}_d^a$ is exactly as considered in Sections 6.2 and 6.3. $\underset{\sim}{\ell}$ and $\underset{\sim}{u}$ represent limits on the tuning amounts.

In the case of irreversible tuning where, for example, the elements are only permitted to increase, the limits are non-negative.

Since we restrict the tuning amounts by (7.1c), a differentiable approximation can be used to estimate the change in the functions and the minimax optimization problem, namely (7.1), can be approximated as follows:

$$\underset{\chi,\, z}{\text{Minimize}}\ z \tag{7.2a}$$

subject to

$$f_i(\ell_t^a, \ell_r^a, \ell_d^a) + \sum_{j\in I_t^*} \phi_j^x \frac{\partial f_i}{\partial \phi_j} \chi_j \le z, \quad i \in I_c, \tag{7.2b}$$

$$\bar{\ell}_j \le \chi_j \le \bar{u}_j, \quad j\in I_t^* \tag{7.2c}$$

where

$$\chi_j \triangleq \frac{\Delta\phi_j}{\phi_j^x} \tag{7.2d}$$

and z is an additional independent variable.

The sensitivities should be evaluated at the actual manufactured values (ℓ^a, ℓ_d^a). In our implementation, we utilize a suitable approximate network model ℓ^x for simulating these sensitivities since the actual manufactured values are usually unknown, and the functions f_i are evaluated by directly measuring the response.

The above mathematical formulation is of the form considered by Madson et al. (1975) and Hachtel et al. (1980). Their reported success in solving different circuit design problems motivated us to employ

similar concepts in the tuning problem.

7.2.2 Tuning Algorithm

The tuning procedure can be summarized by the following steps.

Step 1 Measure the network response. Check whether the design specifications are satisfied. If they are satisfied, stop.

Comment This is carried out by constructing f_i, i=1,2,...,m, and by checking that $\max_i f_i \leq EPS$, $i \in I_c$, where EPS is designer specified value.

Step 2 Construct the linear program defined in (7.2). Use sensitivities which are derived from a suitable network model with the design parameters assumed at certain reasonable values $\underset{\sim}{\phi}^x$ (e.g., their nominal values). The upper and lower limits $\bar{\ell}_i$, $\bar{u}_i$, i=1,2,...,k, are defined to ensure the validity of linear approximation and the type of tuning (reversible or irreversible).

Step 3 Check the output of the linear program. If the absolute value of the tunable amount of any tunable element is less than the minimum amount of tuning which can be carried out in practice, we assume that it is zero. If all the absolute values of the tunable amounts are less than their corresponding minimum allowable values stop.

Step 4 Adjust the parameters to the extent possible by the amounts obtained from the linear program. If the maximum number of measurement iterations specified has not been exceeded return

to Step 1.

The network sensitivities could be updated using the Broyden (1965) rank one updating formula.

$$\underset{\sim}{S}^{i+1} = \underset{\sim}{S}^{i} + \frac{(\underset{\sim}{f}(\underset{\sim}{\phi}^{i} + \Delta\underset{\sim}{\phi}_t^{i}) - \underset{\sim}{f}(\underset{\sim}{\phi}^{i}) - \underset{\sim}{S}^{i}\underset{\sim}{x}^{i})(\underset{\sim}{x}^{i})^{T}}{(\underset{\sim}{x}^{i})^{T}\underset{\sim}{x}^{i}}, \tag{7.3}$$

where i refers to the ith iteration of the algorithm. Initially, $\underset{\sim}{S}^{0}$ is computed at $\underset{\sim}{\phi}^{x}$.

The use of the Broyden formula exploits the measurements in improving the initial network model $\underset{\sim}{\phi}^{x}$. A better approximation is obtained after each iteration.

7.2.3 Tuning of a Microwave Matching Amplifier

As an example, we consider a broadband amplifier with a complex antenna load as shown in Fig. 7.1. The object is to match the antenna load over the frequency band 150 MHz to 300 MHz. We considered the design given by Sanchez-Sinencio (1973) as the nominal design for simulation purposes. The power gain at a certain frequency is given by

$$4 R_S G_L \frac{|V_L|^2}{|V_S|^2}, \tag{7.4}$$

where R_S is the source resistance, G_L is the real part of the admittance of the load, $|V_L|$ is the absolute value of the voltage across the load, and $|V_S|$ is the absolute value of the input voltage, which we assumed to be unity. The response was assumed to be measured at sixteen uniformly

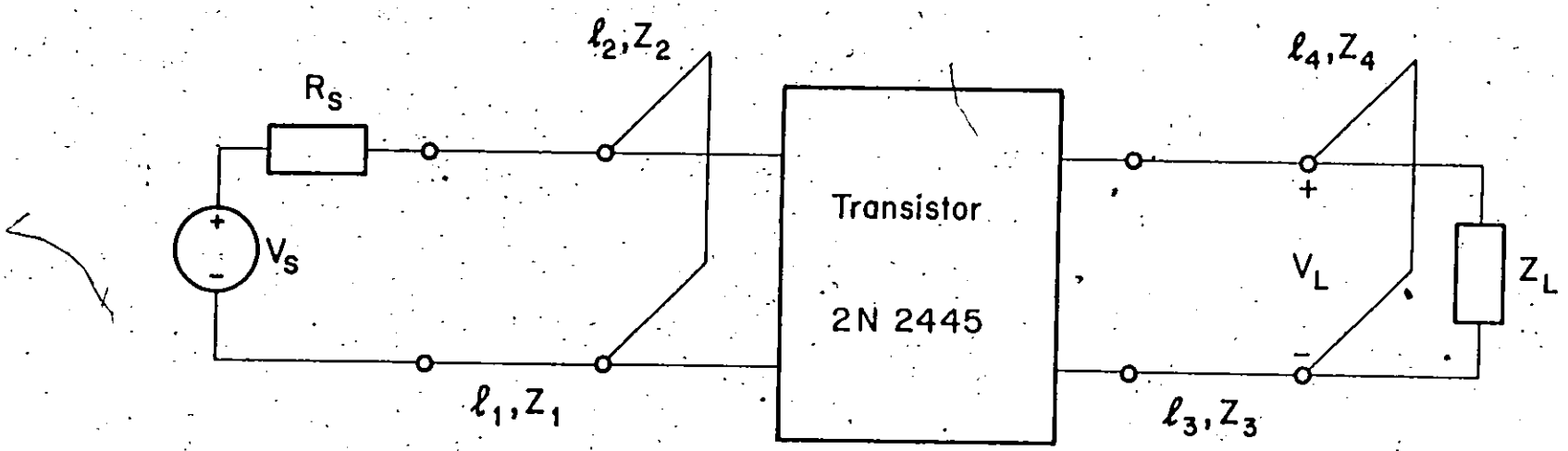

Fig. 7.1 The broadband microwave amplifier.

distributed frequencies over the given frequency band. At each frequency, an error function was defined as the absolute difference between the measured response and the 10 dB specified power gain value. The source resistance was assumed to be 50 ohms. The transistor scattering parameter values and the antenna impedances at the sixteen frequencies were obtained from Sanchez-Sinencio (1973).

A number of different cases are considered here. In all of them, the tuning process is stopped when the response is within $\pm$ 0.8 dB of the specification. Also, we have assumed that the minimum tuning amount to be implemented is $\pm$ 0.1 percent of the nominal element value for all tunable elements. Any amount less than that is neglected.

For Case 1 and Case 2, the four characteristic impedances are considered as the tunable elements. In Case 1, we have taken $\bar{u}_i = -\bar{\ell}_i = 0.1$, $i=1,2,\ldots,4$. In Case 2, we have taken $\bar{u}_i = -\bar{\ell}_i = 0.05$, $i=1,2,\ldots,4$. The four transmission-line lengths are considered to be the tunable elements in Case 3, with $\bar{u}_i = -\bar{\ell}_i = 0.1$, $i=1,2,\ldots,4$.

The network sensitivities have been calculated using the network model with the components assumed at their nominal values. Tables 7.1 and 7.2 summarize the results for the three cases. A plot for the response before and after tuning for the three cases is given in Figs. 7.2, 7.3 and 7.4, respectively. It should be noted that the elements used in Case 3 are closer to nominal than in the first two cases, which manifests itself by tuning converging in one step.

TABLE 7.1

ELEMENT VALUES FOR THE MICROWAVE AMPLIFIER EXAMPLE

Element	Nominal Value	Actual Value		Percentage Deviation	
		Case 1&2	Case 3	Case 1&2	Case 3
ℓ_1	2.012	2.25	1.9	11.82	-5.56
Z_1	86.76	74.07	93.4	-14.62	7.65
ℓ_2	0.976	0.85	0.982	-12.9	0.60
Z_2	97.57	83.33	93.45	-14.59	-4.25
ℓ_3	0.833	0.72	0.85	-13.56	2.04
Z_3	125.	111.11	129.87	-11.11	3.89
ℓ_4	0.927	1.07	0.91	15.42	-1.83
Z_4	132.	113.63	128.2	-13.91	-2.87

ℓ is the normalized length. The actual length equals $\ell \lambda_n/2\pi$, where λ_n is the wavelength at 230 MHz.

Z is the characteristic impedance in ohms.

TABLE 7.2

RESULTS OF TUNING
FOR THE LINEAR APPROXIMATION TECHNIQUE

	Case 1	Case 2	Case 3
Maximum Relative Tunable Amount ($\bar{u}_i = -\bar{\ell}_i$)	0.1	0.05	0.1
No. of Iterations	6	8	1
Tuned Element Values	$Z_1 = 112.35\Omega$	$Z_1 = 109.89\Omega$	$\ell_1 = 2.090$
	$Z_2 = 102.04\Omega$	$Z_2 = 106.38\Omega$	$\ell_2 = 0.976$
	$Z_3 = 121.21\Omega$	$Z_3 = 117.64\Omega$	$\ell_3 = 0.788$
	$Z_4 = 169.49\Omega$	$Z_4 = 147.70\Omega$	$\ell_4 = 1.001$

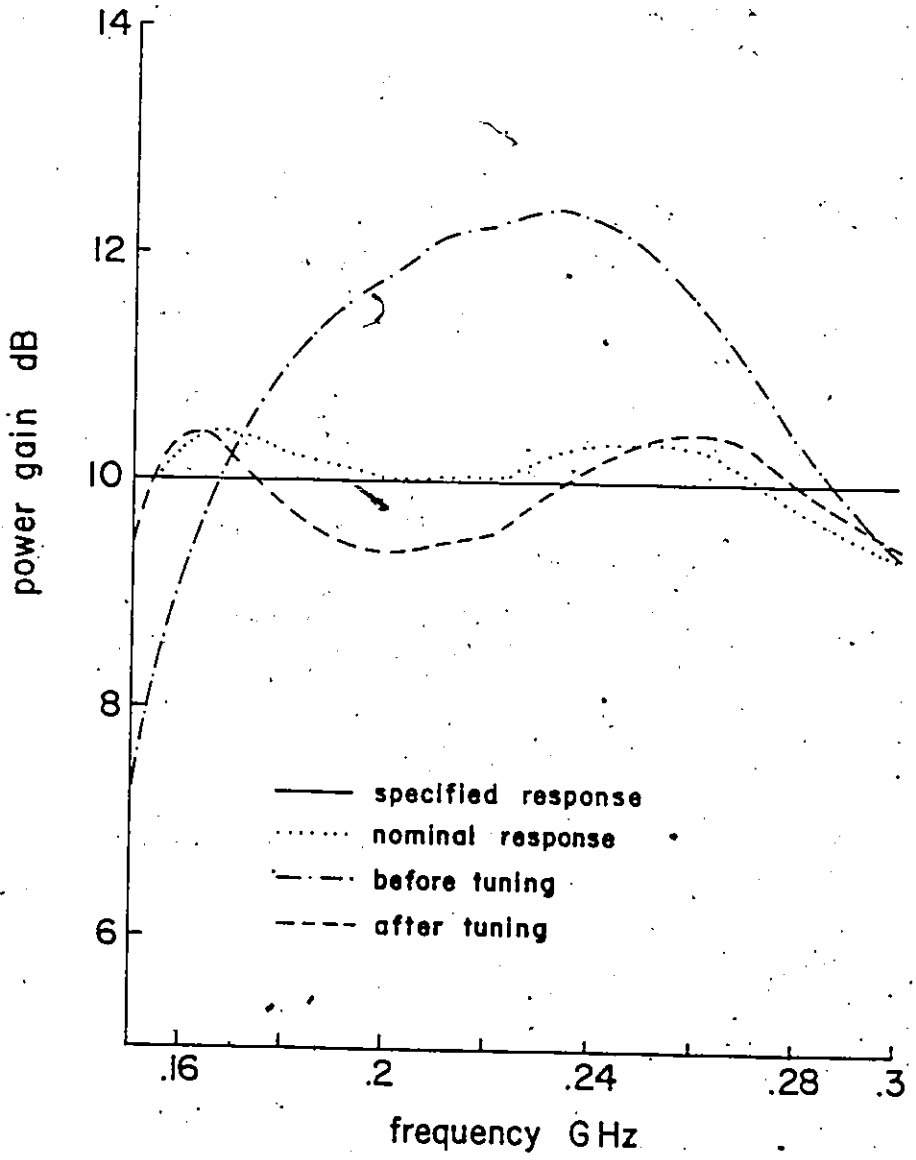

Fig. 7.2 The results of tuning using the linear approximation technique for Case 1 of Table 7.2.

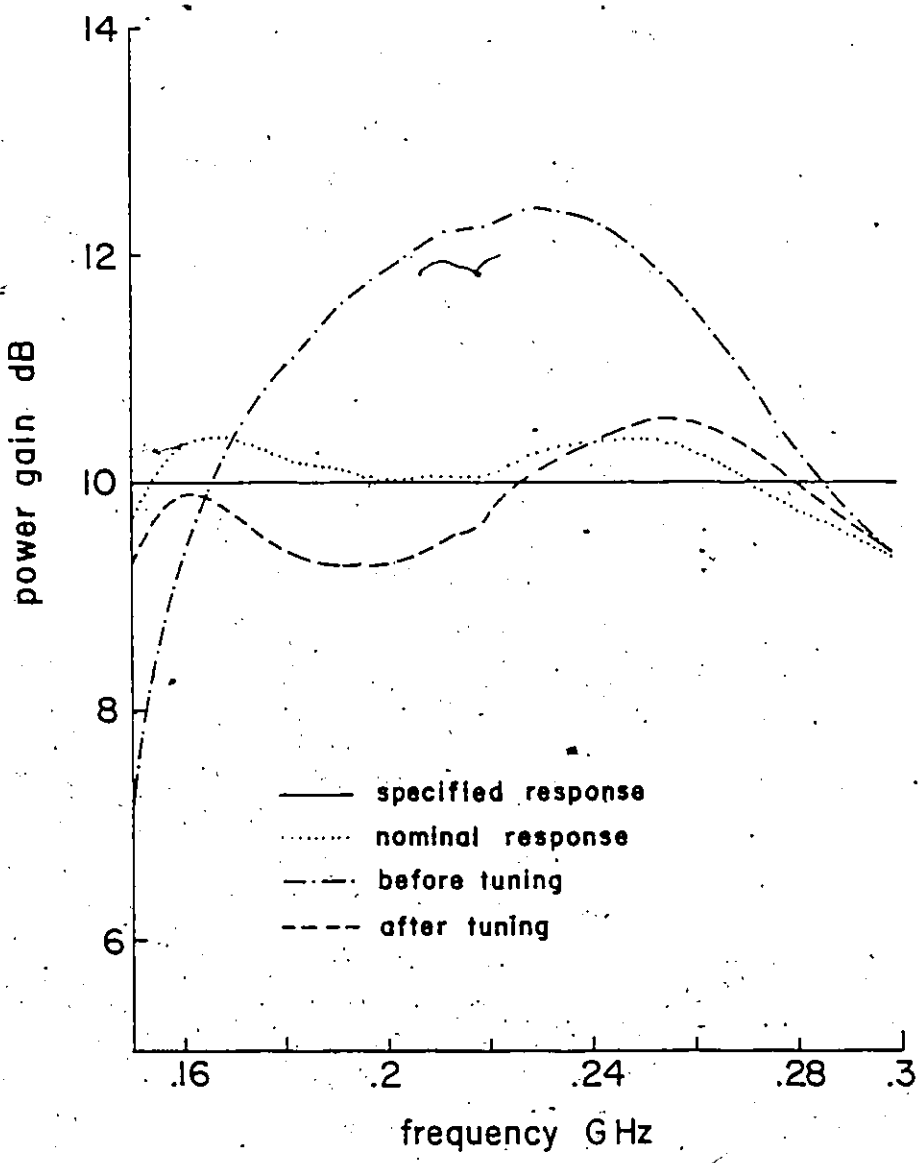

Fig. 7.3 The results of tuning using the linear approximation technique for Case 2 of Table 7.2.

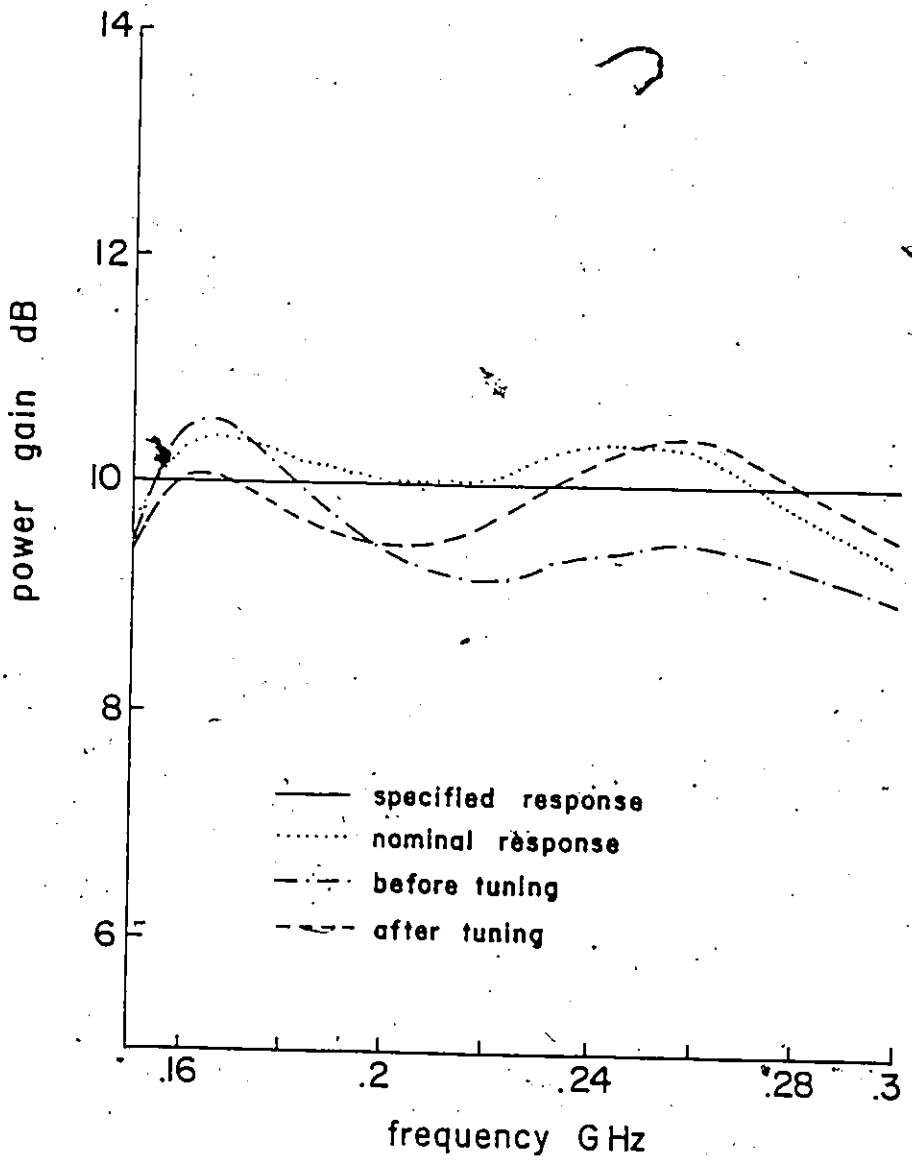

Fig. 7.4 The results of tuning using the linear approximation technique for Case 3 of Table 7.2.

7.3 MODELLING TECHNIQUE FOR FUNCTIONAL TUNING

7.3.1 Problem Formulation

Let $F(\underline{\ell}_t, \underline{\ell}_r, \underline{\ell}_d, s)$ represents the response function that is monitored during the tuning process. We assume that the actual network response is given by

$$F^a(\underline{\ell}_t^a, \underline{\ell}_r^a, \underline{\ell}_d^a, s) \triangleq F^0(\underline{\ell}_t^0, \underline{\ell}_r^0, \underline{0}, s) + F^d(\Delta\underline{\ell}_t^a, \Delta\underline{\ell}_r^a, \underline{\ell}_d^a, s), \qquad (7.5)$$

where the superscript a refers to the actual values, superscript 0 refers to the nominal values, and F^d gives the deviational effect due to the changes in the circuit parameters from nominal and due to the parasitic effects, namely, $\Delta\underline{\ell}_t^a$, $\Delta\underline{\ell}_r^a$, and $\underline{\ell}_d^a$.

We model the deviational effect by a rational transfer function in the complex frequency s. Let

$$F^d(s) \triangleq \frac{a_N s^N + a_{N-1} s^{N-1} + \ldots + a_0}{s^D + b_{D-1} s^{D-1} + \ldots + b_0}, \qquad (7.6)$$

where the degree of the numerator and that of the denominator, namely N and D, are determined according to the order of the transfer function of the nominal circuit, together with the different known parasitic effects that affect the performance of the network. Some of the coefficients of (7.6) are set to zero, as appropriate, if $F^d(s)$ is a pure real or a pure imaginary function.

The coefficients of (7.6) are obtained from (7.5), since the nominal responses are known and the actual response is measured directly. Since $F^d(s_i) = F^a(s_i) - F^0(s_i)$, we may write (7.6) as

$$(s_i^D + b_{D-1}s_i^{D-1} + \ldots + b_0)(F^a(s_i) - F^0(s_i)) = a_N s_i^N + a_{N-1} s_i^{N-1} + \ldots + a_0. \quad (7.7)$$

Taking ℓ different frequencies such that $2\ell > N + D + 1$, we get an overdetermined system of equations in the coefficients a_k, $k=0,1,\ldots,N$, and b_j, $j=0,1,\ldots,D-1$. The least-squares method is used to solve the resulting linear system of equations. It provides an estimate of the coefficients of (7.6).

Recalling (6.17) the error functions are defined by

$$\bar{e}_{ui}(\Delta\phi_t) \triangleq w_{ui}(\bar{F}_i^0(\Delta\phi_t) + F^d(s_i) - S_{ui}), \; i \in I_u, \quad (7.8a)$$

$$\bar{e}_{\ell i}(\Delta\phi_t) \triangleq w_{\ell i}(\bar{F}_i^0(\Delta\phi_t) + F^d(s_i) - S_{\ell i}), \; i \in I_\ell, \quad (7.8b)$$

where

$$\bar{F}_i^0(\Delta\phi_t) \triangleq F(\phi_t^0 + \Delta\phi_t, \phi_r^0, \underset{\sim}{0}, s_i). \quad (7.8c)$$

Correspondingly, we let

$$\bar{f}_i \triangleq \begin{cases} \bar{e}_{uj}, & j \in I_u \\ -\bar{e}_{\ell k}, & k \in I_\ell \end{cases} \quad i \in I_c, \quad (7.9)$$

where I_u, I_ℓ and I_c are as defined by (6.19).

Similarly to the tuning assignment problem (7.1), the following linearly constrained minimax problem is formulated.

$$\underset{\Delta\phi_t,\, z}{\text{Minimize}} \; z \quad (7.10a)$$

subject to

$$z \geq \bar{f}_i, \; i \in I_c, \quad (7.10b)$$

$$\bar{\ell}_i \leq \Delta\phi_i^*/\phi_i^0 \leq \bar{u}_i \quad , \quad i \in I_t^* \qquad (7.10c)$$

The solution of the optimization problem provides us with $\Delta\underset{\sim}{\phi}_t$. The network parameters are adjusted by amounts indicated by $\Delta\underset{\sim}{\phi}_t$. The process is repeated until the circuit meets its design specifications.

7.3.2 Tuning Algorithm

The tuning algorithm is quite similar to that of Section 7.2.2. The procedure can be summarized as follows.

Step 1 Measure the network response. Check whether the design specifications are satisified. If they are satisfied, stop.

Step 2 Utilize the performed measurements and the nominal network responses in constructing the transfer function (7.6) which represents the deviational effect.

Step 3 Solve the optimization problem (7.10) for the changes in the tunable parameters $\Delta\underset{\sim}{\phi}_t$. The upper and lower limits of (7.10c) $\underset{\sim}{\ell}$ and $\underset{\sim}{u}$ are defined to ensure the validity of the approximating deviational model and the type of tuning (reversible or irreversible).

Step 4 If the absolute value of the tunable element is less than the minimum amount of tuning which can be carried out in practice, we assume that it is zero. If all the absolute values of the tunable amounts are less than their corresponding minimum allowable values, stop.

Step 5 Adjust the parameters to the extent possible by the amounts obtained from the optimization program. If the maximum number of iterations has not been exceeded, return to 1.

The optimization problem (7.10) is solved using a recent algorithm by Hald and Madsen (1981). The optimization algorithm has two stages and always starts in Stage 1. In Stage 1, the error functions $\bar{f}_j$, j=1,2,...m, are approximated by linear functions using the first-order derivatives. The Stage 2 iteration is introduced in order to speed up the final rate of convergence for problems which are singular at the solution. The Stage 2 algorithm is a modified quasi-Newton method, i.e., approximate second-order information is utilized.

7.3.3 Applying the Modelling Technique for Tuning the Microwave Amplifier

The modelling technique for functional tuning is applied on the example of 7.2.3. We assumed that the actual power gain is given by

$$F^a(s) = F^0(s) + F^d(s), \qquad (7.11a)$$

where

$$F^d(s) = \frac{a_4 s^4 + a_2 s^2 + a_0}{s^4 + b_2 s^2 + b_0} . \qquad (7.11b)$$

Cases 1 and 3 of Table 7.1 were considered. In Case 1, we have taken $\bar{u}_i = -\bar{\ell}_i = 0.4$. Z_1, Z_2, Z_3 and Z_4 are considered as the tunable parameters. The algorithm converged in 4 iterations. In Case 3 we have taken $\bar{u}_i = -\bar{\ell}_i = 0.1$ and the four lengths ℓ_1, ℓ_2, ℓ_3 and ℓ_4 are assumed

to be the tunable parameters. The results are summarized in Table 7.3. The tuned responses are given in Figs. 7.5 and 7.6, respectively.

7.3.4 Integrated Tuning Procedure

The techniques which are detailed in Sections 6.6.3 and 6.7.3 for the selection of the critical frequency points and the tunable parameters are utilized with the modelling tuning algorithm to tune the microwave network amplifier with no parasitics. The integration of these techniques provides an efficient tuning procedure (Bandler and Salama 1983).

Using optimization problem (6.35), the nominal parameters of the circuit were reoptimized to a 10 dB power gain over the frequency range 150 MHz to 300 MHz. We utilized the optimization package MMLC by Bandler and Zuberek (1982a) for linearly constrained minimax optimization, as described by Hald and Madsen (1981). The MMLC package is an adaptation of the MMLA1Q package (Hald 1981). The characteristic impedances are assumed to be less than 200 ohms. The response achieved is superior to that obtained earlier (Sanchez-Sinencio 1973). This is partly because we relaxed the bounds on the design parameters. The new and previous nominal design parameters are given in Table 7.4. From the response obtained, the frequencies 150, 160, 170, 220, 250, 280 and 300 MHz are candidates for the critical frequencies. The response at 16 uniformly distributed frequencies in the 150-300 MHz band is given in Table 7.5.

TABLE 7.3

RESULTS OF TUNING USING THE MODELLING TECHNIQUE

	Case 1	Case 3
Maximum Relative Tunable Amount ($\bar{u}_i = -\bar{\ell}_i$)	0.4	0.1
No. of Iterations	4	1
Tuned Element Values	$Z_1 = 111.42\ \Omega$	$\ell_1 = 2.042$
	$Z_2 = 89.77\ \Omega$	$\ell_2 = 0.8838$
	$Z_3 = 124.97\ \Omega$	$\ell_3 = 0.7726$
	$Z_4 = 144.26\ \Omega$	$\ell_4 = 0.9446$

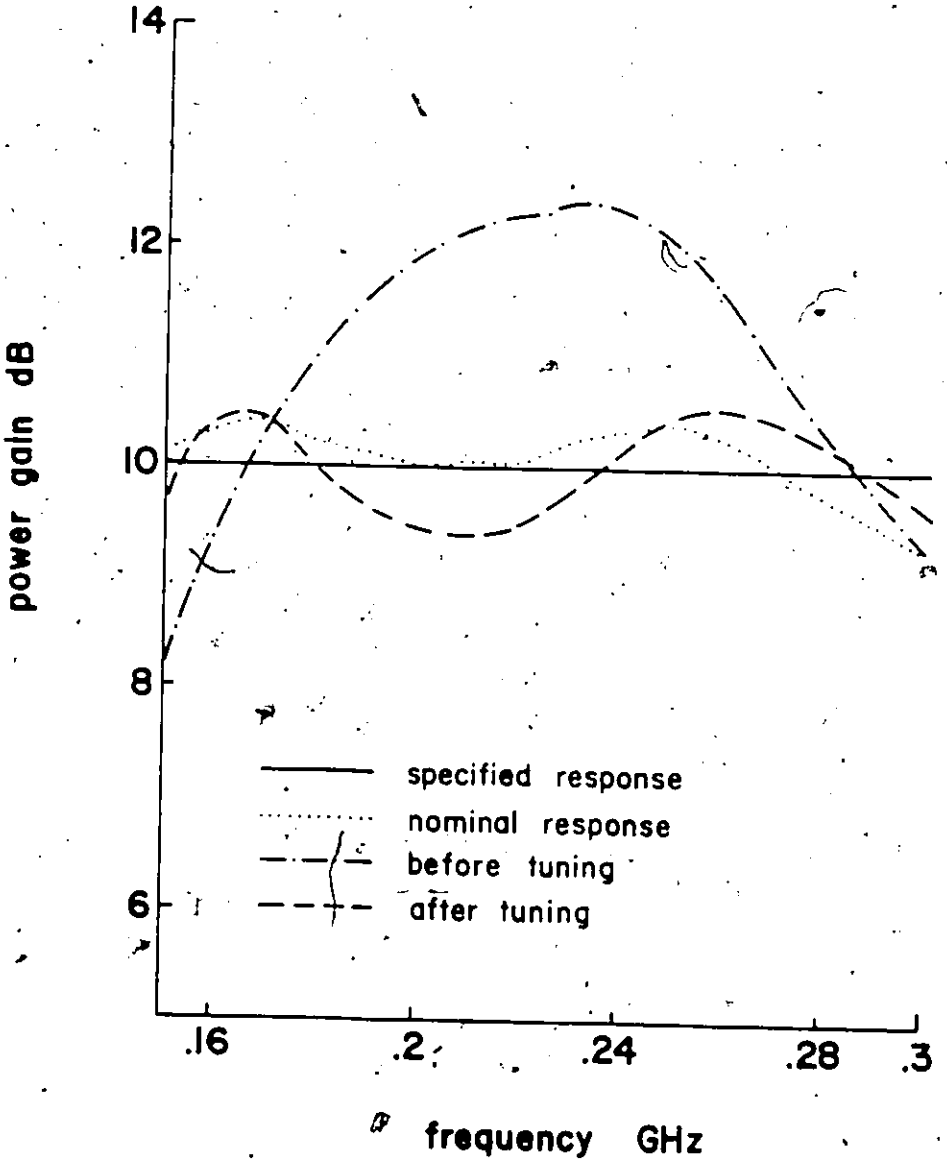

Fig. 7.5. The results of tuning using the modelling technique for Case 1 of Table 7.1.

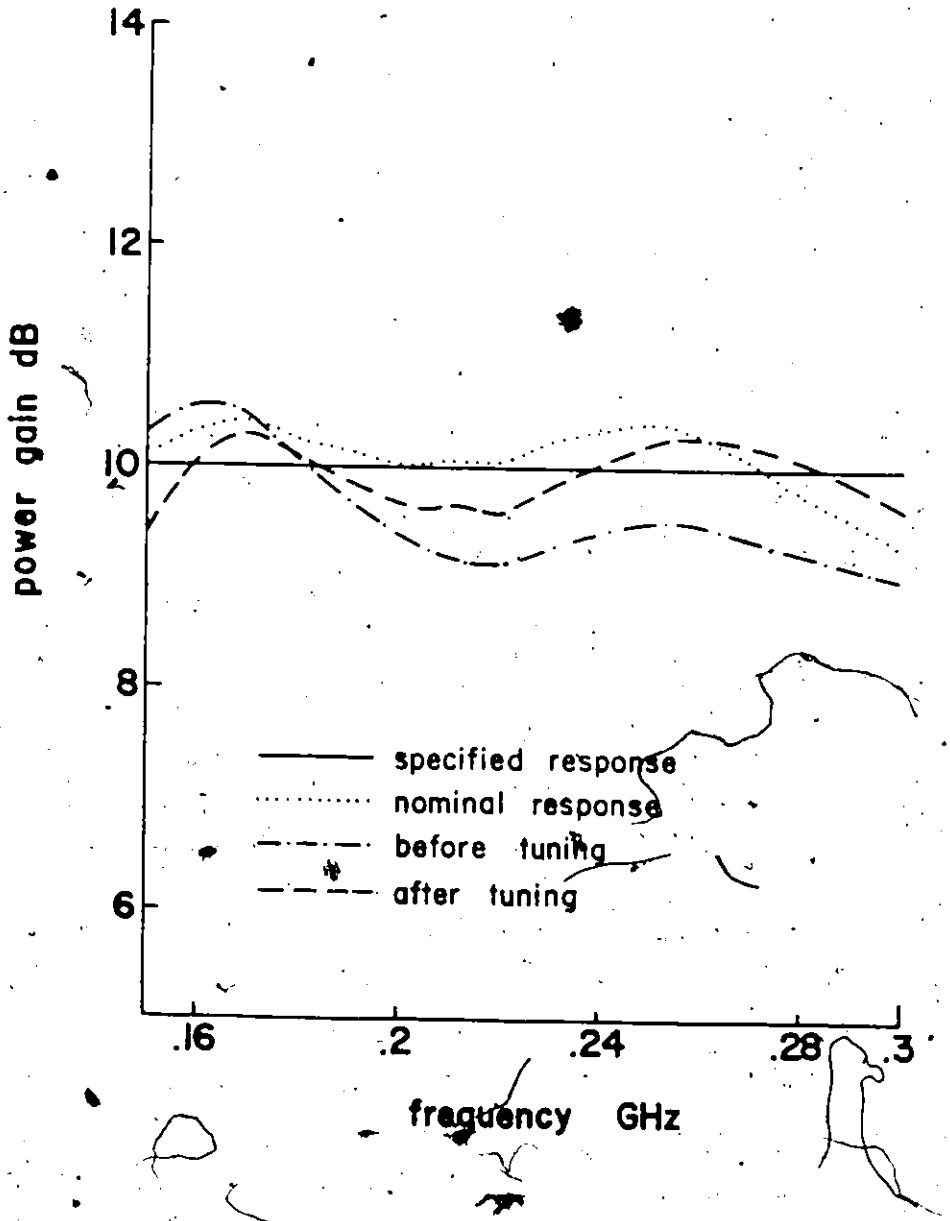

Fig. 7.6 The results of tuning using the modelling technique for Case 3 of Table 7.1.

TABLE 7.4

NOMINAL ELEMENT VALUES OF THE MICROWAVE AMPLIFIER

Element	Nominal Value	
	Original	New
ℓ_1	2.012	1.741
$Z_1(\Omega)$	86.76	68.778
ℓ_2	0.976	1.534
$Z_2(\Omega)$	97.57	200.0
ℓ_3	0.833	1.140
$Z_3(\Omega)$	125.	181.252
ℓ_4	0.927	1.280
$Z_4(\Omega)$	132.	105.105

TABLE 7.5

THE OPTIMUM NOMINAL RESPONSE
OF THE MICROWAVE AMPLIFIER

Frequency (MHz)	Power Gain (dB)	
150	10.058	*
160	9.926	*
170	10.072	*
180	10.043	
190	10.053	
200	10.006	
210	10.028	
220	9.926	*
230	10.031	
240	10.028	
250	10.072	*
260	10.31	
270	9.965	
280	9.926	*
290	9.983	
300	10.072	*

* identifies critical frequencies

We assume that the design specifications tolerate ± 1 dB deviation from a specified value of 10 dB. Worst-case responses for the candidate set of critical frequencies are computed with $\pm$ 5% tolerances on the design parameters. The first four worst-case responses violate the design specifications, as is shown in Table 7.6. We performed the optimization problem (6.45), using the four critical violating responses to determine the tunable parameters. The results of this optimization problem are given in Table 7.7. It is clear that Z_1 and ℓ_4 are the tunable parameters. The optimization package MFNC by Bandler and Zuberek(1982b), which implements the Han-Powell algorithm described by Powell (1977), is utilized in solving this problem. The MFNC package is an adaptation of the VF02AD (1978) subroutine of the Harwell Subroutine Library.

The functional tuning algorithm of Section 7.3.2 is used to tune the previous four critical cases (Case 1 and Case 3 consider the same vertex, so they are identical). The results of tuning for these cases are given in Table 7.8. The responses before and after tuning are shown in Figs. 7.7, 7.8 and 7.9. The solution of (7.10) is obtained by the optimization package MMLC.

7.4 COMPARISON BETWEEN FOUR TUNING ALGORITHMS

The two new functional tuning methods and the two deterministic tuning methods of Sections 6.5.2 and 6.5.3 were tested by applying them to test the highpass notch circuit shown in Fig. 7.10. This example has originally appeared in Alajajian's thesis (1979). The nominal circuit component values and the actual circuit values are given in Table 7.9.

TABLE 7.6

WORST CASE RESPONSE FOR $\pm 5\%$ TOLERANCES

Frequency (MHz)	Vertex Number	Power Gain (dB)
150	123	11.318
160	134	8.559
170	123	11.274
220	153	8.794
250	80	10.726
280	189	9.302
300	212	10.657

The vertex number is given by the formula

$$r = 1 + \sum_{j=1}^{p} \left\lfloor \frac{\mu_{j+1}^{r}}{2} \right\rfloor 2^{j-1}, \quad \mu_j^r \in \{-1, 1\}.$$

TABLE 7.7

DETERMINATION OF THE RELATIVE TUNABLE AMOUNTS

Element	t_i/ϕ_i^0
ℓ_1	0.0
Z_1	0.0088
ℓ_2	0.0
Z_2	0.0
ℓ_3	0.0
Z_3	0.0
ℓ_4	0.079
Z_4	0.0

TABLE 7.8

RESULTS OF TUNING USING THE INTEGRATED TUNING PROCEDURE

	Cases 1 & 3	Case 2	Case 4
Vertex No.	123	134	153
No. of Iterations of Functional Tuning Algorithm	1	1	1
Tunable Element Values	$Z_1 = 66.66\Omega$	$Z_1 = 70.616\Omega$	$Z_1 = 66.66\Omega$
	$\ell_4 = 1.209$	$\ell_4 = 1.331$	$\ell_4 = 1.154$

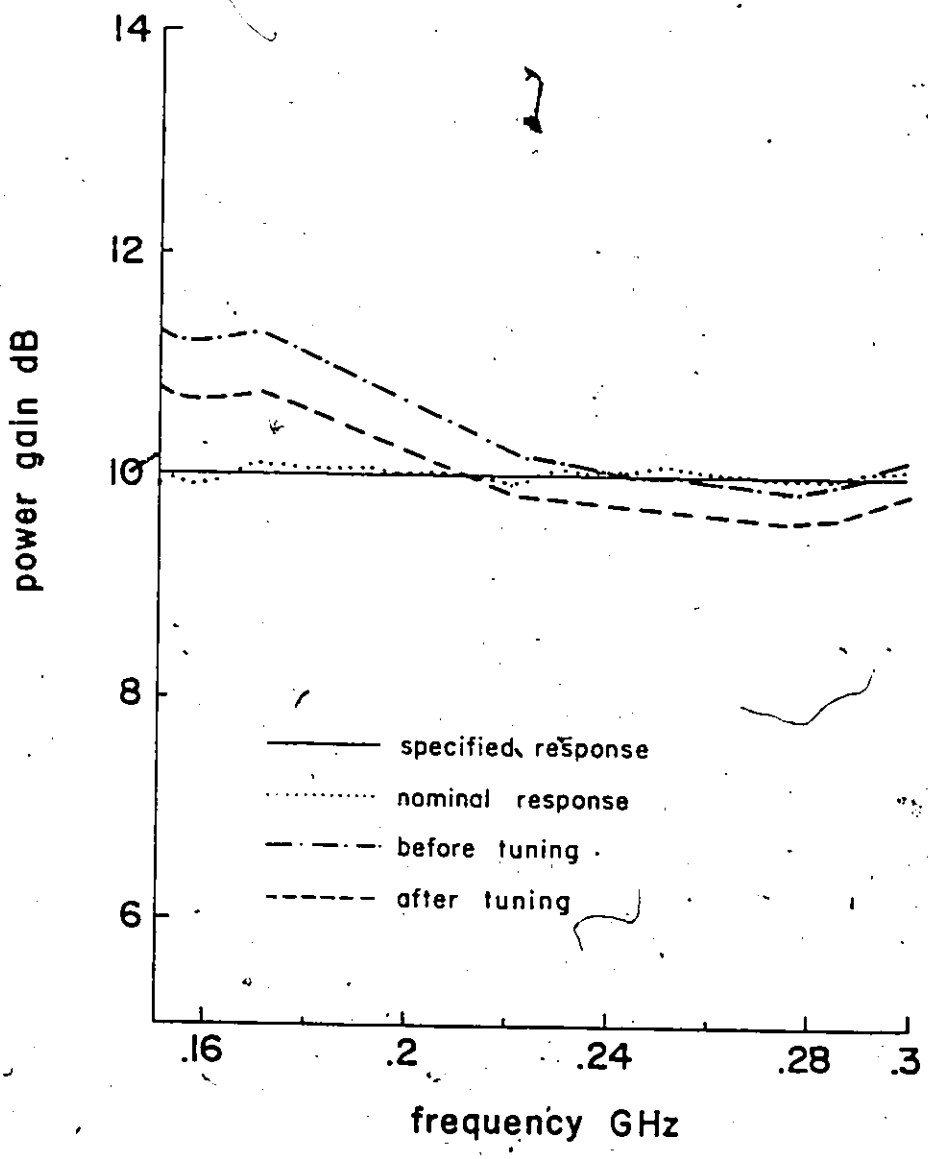

Fig. 7.7 The results of tuning an outcome corresponding to vertex number 123.

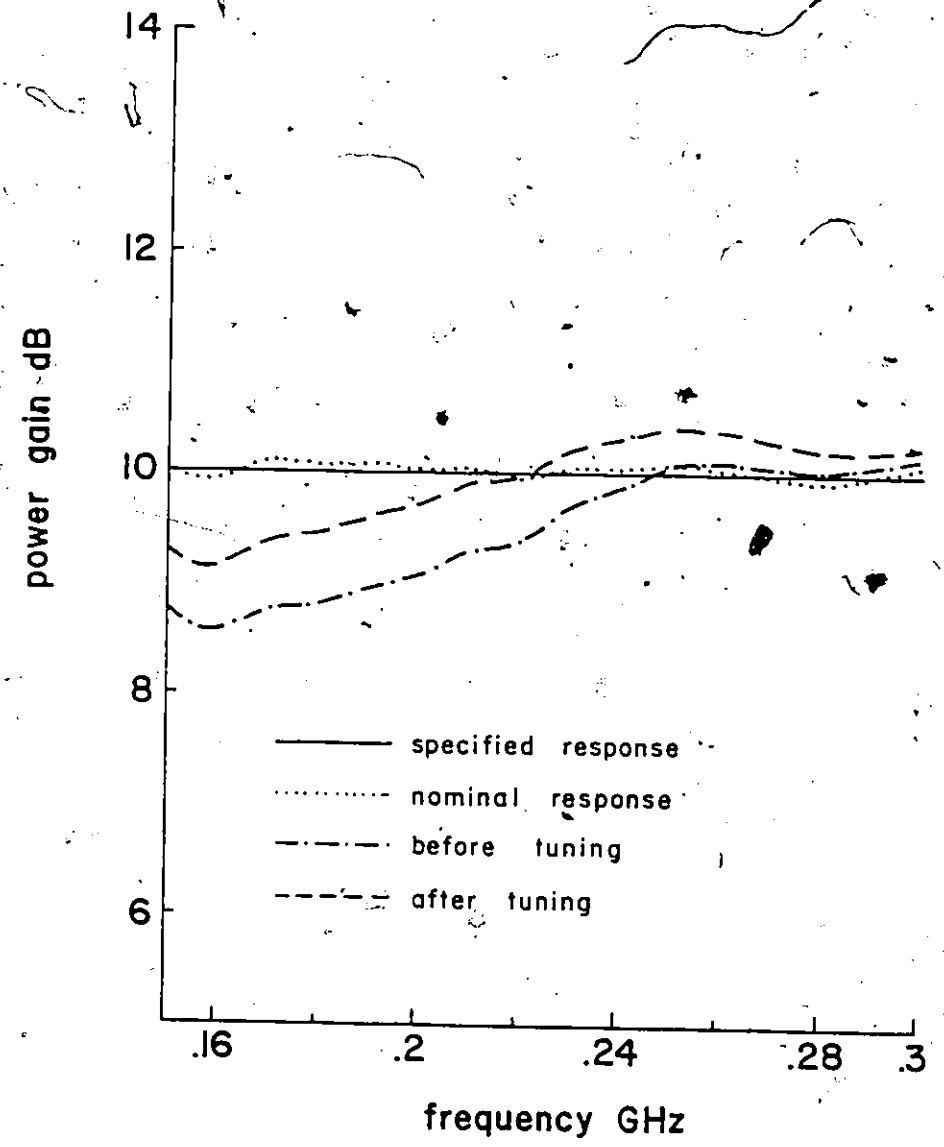

Fig. 7.8 The results of tuning an outcome corresponding to vertex number 134.

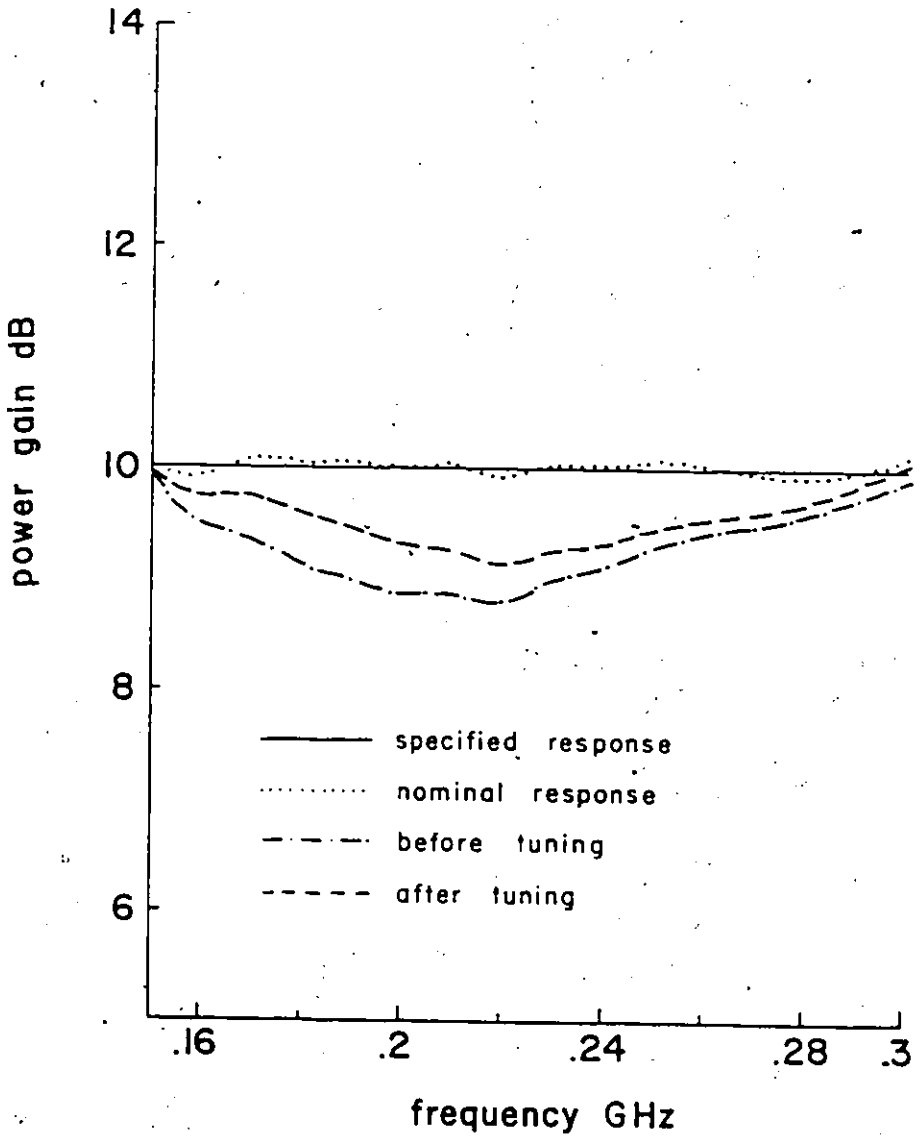

Fig. 7.9 The results of tuning an outcome corresponding to vertex number 153.

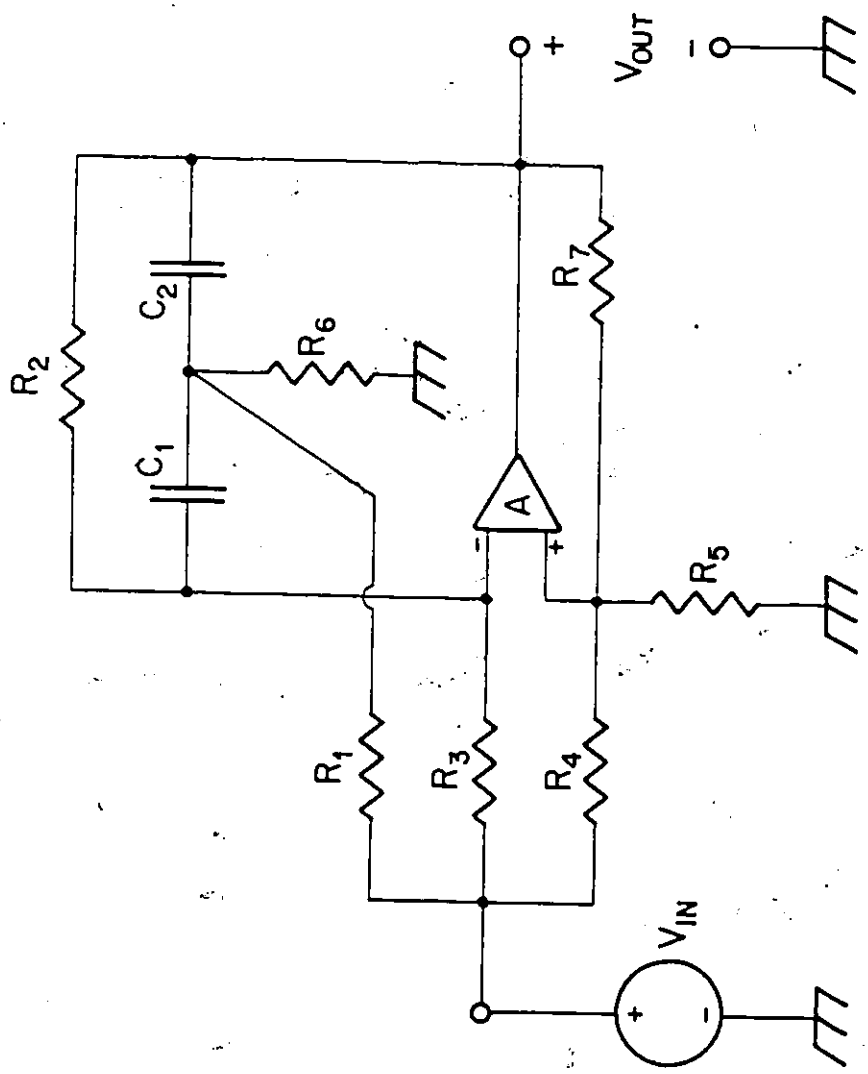

Fig. 7.10 The highpass notch filter circuit.

TABLE 7.9

ELEMENT VALUES OF THE HIGHPASS NOTCH FILTER CIRCUIT

Element	Nominal Value	Actual Value	Percentage Deviation
R_1 (kΩ)	13.260	13.260	0.0
R_2 (kΩ)	93.0	93.0	0.0
R_3 (kΩ)	214.0	192.6	-10.0
R_4 (kΩ)	2.0	2.0	0.0
R_5 (kΩ)	2.0	1.8	-10.0
R_6 (kΩ)	12.467	11.221	-10.0
R_7 (kΩ)	10.00	9.00	-10.0
C_1 (μF)	0.01	0.00973	-2.07
C_2 (μF)	0.01	0.00965	-3.35
A	10000.0	10000.0	0.0

R_3, R_5, R_6 and R_7 are assumed to be the tunable parameters.

7.4.1 Linear Approximation Technique (Method 1)

We first tested the functional tuning algorithm of Section 7.2.2. To construct the optimization problem of (7.2), we defined the functions f_i as the absolute deviation of the output voltage from its nominal value at 20 frequencies in the interval 410-505 Hz, where the notch lies. The limits given in (7.2c) are $\bar{u}_j = 0.02$ and $\bar{\ell}_j = 0$. After 12 iterations, the tuned responses closely approached the nominal response, as shown in Fig. 7.11.

The sensitivities are updated after every iteration using the Broyden formula and the initial sensitivities are computed with the components at the nominal starting values. A tunable amount less than 0.1 percent is assumed to be infeasible and no adjustment is carried out for the corresponding tunable parameter.

The final values of the tunable parameters are given in Table 7.10.

7.4.2 Modelling Technique (Method 2)

The modelling technique for functional tuning is next tested. The response of the network is considered at the same 20 frequencies. We assumed that the actual output is given by

$$V^a_{out}(s) = V^0_{out}(s) + V^d_{out}(s) , \qquad (7.12a)$$

where

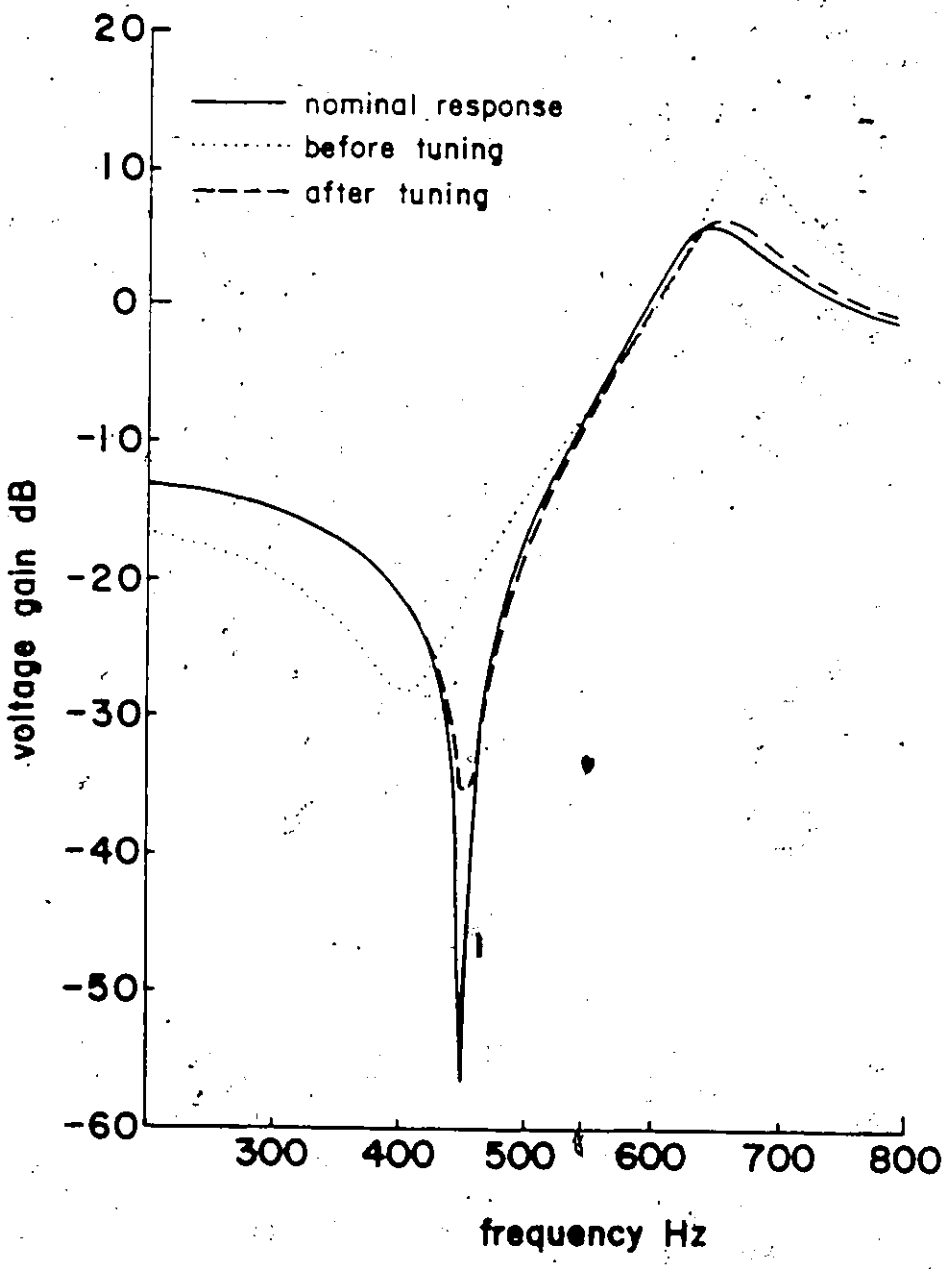

Fig. 7.11 The results of tuning the highpass notch filter examle using the algorithm of Section 7.2.

TABLE 7.10

RESULTS OF TUNING USING THE FOUR TECHNIQUES

Tuned Element	Linear Approximation Technique Method 1	Modelling Technique Method 2	Sequential Tuning Technique Method 3	Large Change Sensitivity Technique Method 4
R_3 (kΩ)	208.769	204.53	184.487	192.94
R_5 (kΩ)	2.194	2.065	2.241	2.2459
R_6 (kΩ)	13.711	12.475	13.747	13.892
R_7 (kΩ)	9.741	9.049	9.993	10.06
Type of Method	Functional	Functional	Deterministic	Deterministic
No. of Iterations	12	2	1	3

$$V^{d}_{out}(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0} \quad (7.12b)$$

represents the deviational effect.

The functions f_i considered for the optimization problem (7.10) are exactly as those considered for the optimization problem (7.2). In (7.10c), $\underline{\ell}_i = 0$ and no upper limit is considered.

After two iterations, the algorithm converged. The tuned response is shown in Fig. 7.12 together with untuned and nominal responses. The final values for the four tunable resistors, namely R_3, R_5, R_6 and R_7, are given in Table 7.10.

7.4.3 Sequential Tuning Technique (Method 3)

We next considered the deterministic tuning algorithm proposed by Lopresti (1977). The transfer function of the filter considered is given by

$$T(s) = \frac{\beta_2 s^2 + \beta_1 s + \beta_0}{s^2 + \alpha_1 s + \alpha_0} \quad (7.13)$$

where $\beta_2 = 0.5$, $\beta_1 = 0.0$, $\beta_0 = 4\text{x}10^6$, $\alpha_1 = 500.0$ and $\alpha_0 = 16\text{x}10^6$. Due to the wide variation in the tunable parameters frequency scaling was necessary. With $\Im = 1000$ s, the coefficients become $\hat{\beta}_2 = 0.5$, $\hat{\beta}_1 = 0.0$, $\hat{\beta}_0 = 4$, $\hat{\alpha}_1 = 0.5$ and $\hat{\alpha}_0 = 16.0$ A complete matching of the coefficients is required. We take $\underline{F} \triangleq [\hat{\alpha}_1\ \hat{\alpha}_0\ \beta_2\ \beta_1\ \beta_0]^T$ and the error functions are defined as the absolute deviations of the coefficients from their nominal values.

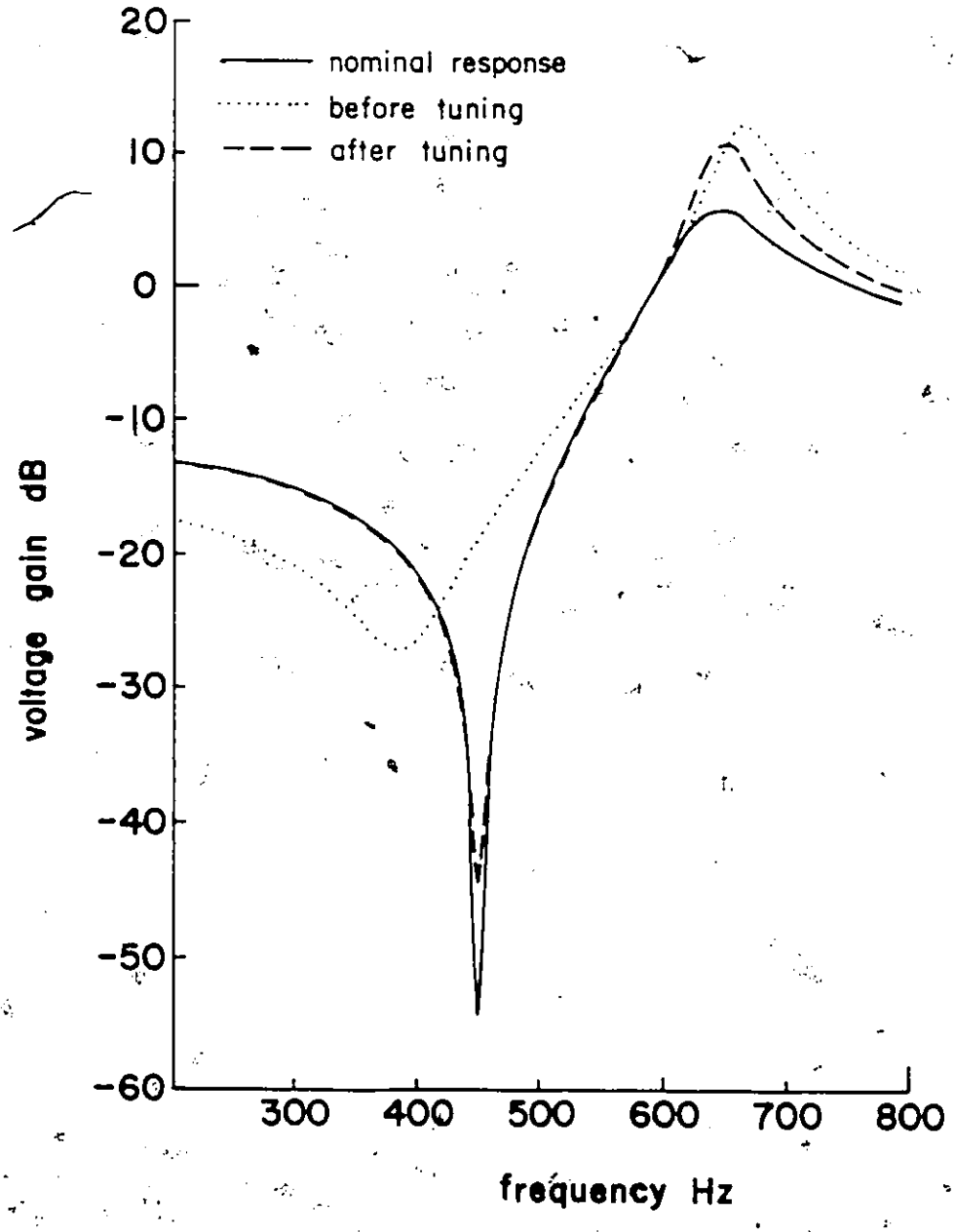

Fig. 7.12 The results of tuning the highpass notch filter examle using the algorithm of Section 7.3.

The sensitivities of the coefficients were evaluated using the actual parameter values. This improved the performance of the algorithm (Bandler and Salama 1981).

The tuned reponse is shown in Fig. 7.13. The final values of the tuned parameters are given in Table 7.10.

7.4.4 Large-Change Sensitivity Technique (Method 4)

To construct (6.33), the output voltage of the filter is considered at three critical frequencies, namely, 450.38, 636.94 and 676.75 Hz.

Using Tellegen's theorem, we have

$$\sum_{\ell=1}^{4} (V_\ell^i + \Delta V_\ell^i)\, \hat{V}_\ell^i\, \Delta G_\ell - c\, \bar{V}_{out} = -V_{out}^i \,, \qquad (7.14)$$

where i = 1, 2 and 3 corresponds to the three frequencies, V_ℓ and $\hat{V}_\ell$ are the branch voltage of the tunable parameters in the original circuit and its adjoint before tuning, respectively. $V_\ell + \Delta V_\ell$ is the tunable branch voltage after tuning, which can be approximated by the nominal branch voltage. $\bar{V}_{out}$ is the specified output voltage. Solving (7.14) we get the required changes in the tunable parameters and the multiplication factor c.

After three iterations, the tuned response approached the nominal response very closely. The responses are given in Fig. 7.14 and the values of tunable parameters are listed in Table 7.10.

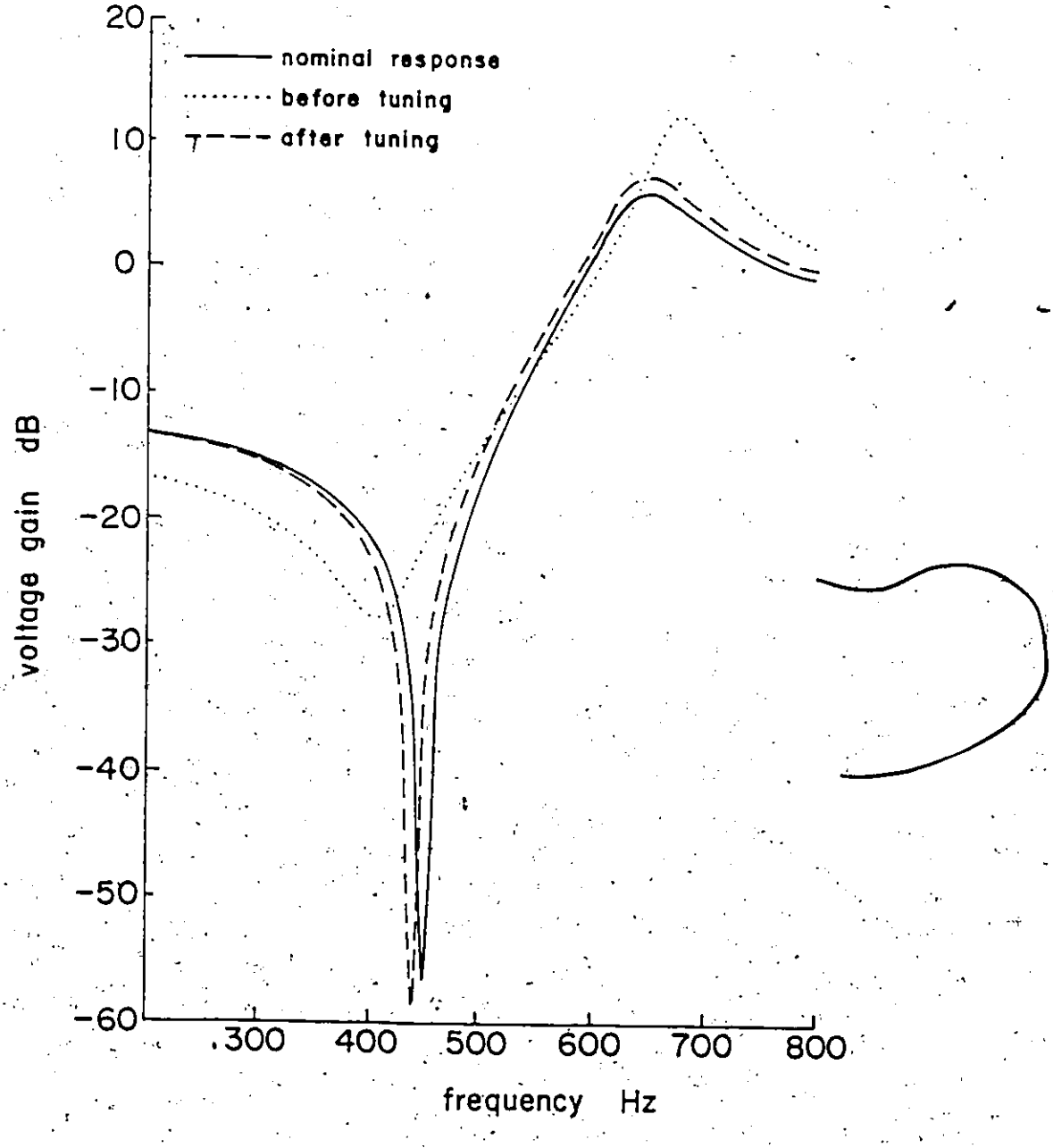

Fig. 7.13 The results of tuning the highpass notch filter example using the sequential tuning technique.

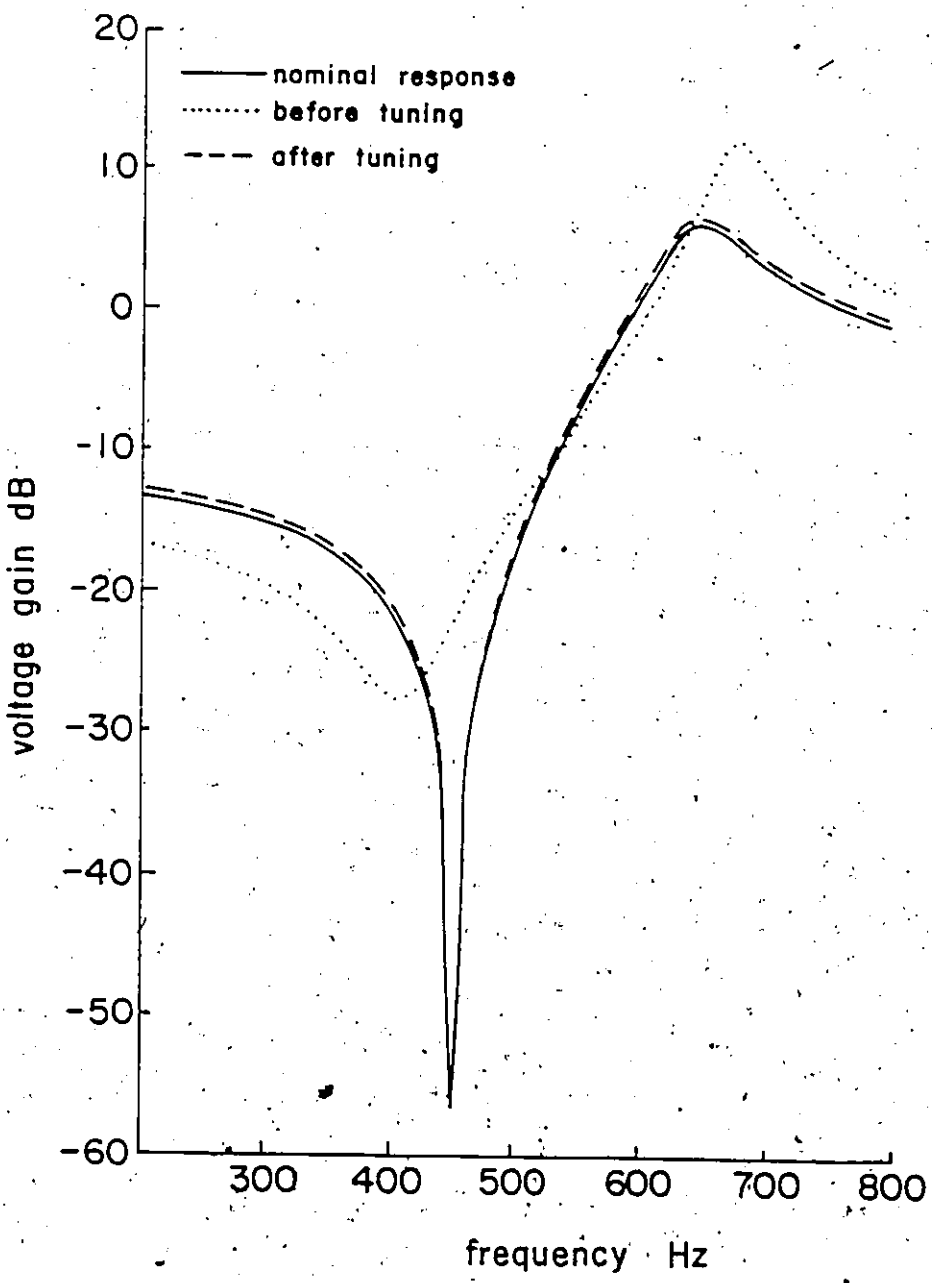

Fig. 7.14 The results of tuning the highpass notch filter example using the large-change sensitivity technique.

7.5 CONCLUDING REMARKS

In this chapter, we have presented two postproduction functional tuning techniques. The techniques optimally use available response measurements and eliminate completely the experimental trial-and-error and one-at-a-time approaches. They are quite general and can be applied to any network for both reversible and irreversible tuning.

The formulation of the postproduction tuning problem as an optimization problem facilitates the inclusion of many physical constraints such as the direction of tuning, the tuning amounts, and the constraints on other functions obtained by simulation.

The linear approximation technique will perform quite reasonably as long as the approximation is valid. Carrying out the tuning procedure in stages and updating the sensitivity matrix by the Broyden formula will ensure the validity of the linear approximation.

The modelling technique usually needs fewer response measurements than the linear approximation technique, but requires much more on-line computational capabilities. For reasonably small deviations of the network elements from nominal the technique converges in one or two iterations.

Deterministic tuning methods performed better than the functional tuning methods. This is expected, since the deterministic technique assumes that more information about the network is known (all network components are assumed known).

Large-change sensitivities seem more promising than the differential sensitivities for deterministic tuning methods. This appears

very clearly in the results obtained by Alajajian (1979) and our implementation of his algorithm.

From our experience with both functional tuning and deterministic tuning algorithms, the latter are superior. A comparison based on our results is given in Table 7.11.

The integration of a number of concepts and techniques produced an efficient postproduction tuning procedure. This is manifested by the results we have obtained in Section 7.3.4. We believe that this integrated approach utilizes effectively the information that could be obtained at the design stage in specifying an optimal tuning procedure.

TABLE 7.1

COMPARISON OF THE TUNING APPROACHES

Feature	Functional Tuning	Deterministic Tuning
Actual Component Values Identified	No	Yes
Parasitics Measured	No	Yes
Exact Network Model Needed	No	Yes
Post-tuning Adjustment Needed	No	Yes
Tuning Speed	Slow	Fast
Tuning Assigrment	Difficult	Simpler
Hardware Requirements	Moderate	Sophisticated
Software Support	Sophisticated	Moderate
Computational Effort	Method dependent	Method dependent
One Way Tuning	Difficult	Possible
Applicability	General	Quasi-general

8

CONCLUSIONS

This thesis has considered two important problems associated with computer-aided-testing of large analog circuits, namely, fault analysis methods and postproduction tuning algorithms and procedures. A review of the fault analysis methods revealed the need for the development of an efficient on-line implementable technique for both soft and hard fault location in large analog networks. The simulation-after-test approach that is developed in this thesis quite closely meets these requirements.

Accessible test nodes, where voltage measurements are made, are limited in number. They are chosen according to a nodal decomposition of the whole network and such that the network is decomposed into small mutually uncoupled subnetworks. Necessary and almost sufficient conditions are derived to identify faulty and nonfaulty subnetworks. These testing conditions are based on invoking KCL and topological relations between subnetworks. Logical analysis provides an efficient way for processing the results of the application of different testing conditions. During real testing, a hierarchical decomposition procedure is strongly recommended to be followed, particularly if the network is dominantly linear. Hierarchical decomposition provides a systematic and fast way for testing very large faulty networks. The effect of tolerances on the nonfaulty elements is handled by utilizing the

weighted-least-squares criterion for solving underdetermined systems of equations. The criterion has a significant probabilistic implication.

After localizing the faults to the subnetwork level, the identification of faulty elements inside the faulty subnetwork is then carried out by either an exact fault location technique or an approximate fault estimation method.

Most of the deterministic methods that have been described in Chapter 4 verify the existence of faults by checking the consistency of certain overdetermined systems of equations. The effect of tolerances on the nonfaulty elements on the consistency of these equations is handled by utilizing a linear programming formulation of the problem. Linear programming is used to check the existence of a feasible solution for the assumed consistent system of equations. The internal-self-testing technique and the combinatorial algorithm are introduced to speed up the location of faulty elements inside the assumed faulty subnetwork. The internal-self-testing technique utilizes the subnetwork topology to perform an internal partition of the subnetwork. Necessary and sufficient conditions for a subnetwork region to be fault free are derived. The combinatorial algorithm is usually applied when the ISTC technique cannot be continued. It permits arbitrary choices of partitions.

The matching technique utilizes the fault models of the fault prone elements of the subnetwork. Under the condition that the number of possible faulty cases is small and these faulty cases are assumed known, the method is quite fast and efficient.

Under the physically realistic assumption that a few subnetwork elements are faulty, the ℓ_1 norm is used to estimate the most likely faulty elements, as outlined in Chapter 5. A linear programming formulation is proposed for both the single test and multiple test techniques. The method is simple, efficient and readily accounts for the tolerances on the nonfaulty elements. The method is applicable to both linear and nonlinear subnetworks.

Postproduction tuning provides a valuable means of correcting the faulty network and improving its performance. In Chapter 6 a number of functional and deterministic tuning algorithms are reviewed. The information gained during the design process provides theoretically justifiable new techniques for choosing the critical samples of the response and the tunable parameters. Minimax optimization and least pth optimization with p=1, subject to suitable constraints, are utilized in these techniques, as outlined in Chapter 6. They provide the necessary ingredients for a general, successful tuning procedure.

Two new functional tuning techniques are presented in Chapter 7. The techniques optimally use the available response measurements. A tuning procedure based upon the techniques introduced in Chapter 6 for choosing tunable parameters and critical frequencies and the modelling tuning technique provide an optimal integrated tuning procedure. This is illustrated by the results obtained.

A comparison between deterministic and functional tuning algorithms based upon testing four different techniques verify that the deterministic techniques are faster and more reliable.

A number of points are worth further research and development.

(a) The test nodes, and consequently the nodes of decomposition, should be chosen such that the network is k-fault testable, i.e., we can locate exactly the faults in at most k subnetworks without any ambiguities. A similar problem has been addressed in testing digital circuits.

(b) From our experience with the method presented in Chapter 3 the application to nonlinear networks could produce some uncertainties due to the blocking effect of some nonlinear elements (e.g., transistors) under certain biasing conditions. This could be eliminated by the suitable choice of the testing signals and an investigation of the subnetwork elements. This problem should be addressed such that a general procedure independent of the network considered will always produce appropriate satisfactory results.

(c) The application of the ISTC technique to very large networks needs further investigation. It appears that hierarchical decomposition could be the most appropriate procedure to conduct the ISTC. Further application of the technique to nonelectrical networks, but for which analogs are available, e.g., in human body testing could prove to be very successful in locating faulty tissues (Wexler 1982).

(d) The use of the ℓ_1 norm is very appropriate for locating a few faulty elements. As is well known, an appropriately weighted-least-squares criterion could produce the same result as the ℓ_1 norm. The choice of these weights needs further research. The significance of using the ℓ_2 norm is in the predicted reduction of the needed computational effect.

(e) The use of the functional tuning algorithms with more measurements at some of the internal network nodes could increase the efficiency and reliability of the methods. It is quite appropriate to assume the voltages across the tunable elements to be known. An algorithm taking this into consideration is worth further research.

(f) The integrated tuning procedure that utilizes the proposed techniques for choosing tunable elements and critical frequencies needs to be tested with a number of functional and deterministic tuning algorithms. The results could establish the reliability and efficiency of the procedure.

APPENDIX A

ON THE PROOF OF LEMMA 3.1

Necessity is obvious, since if (3.9) is not satisfied for $\underset{\sim}{\phi}_i = \underset{\sim}{\phi}_i^0$, this implies that S_i is faulty. For the sufficiency part of the proof we utilize the adjoint network concept. Assuming that the subnetwork S_i has b internal branches, Tellegen's sum can be written as

$$\sum_{p=1}^{m_i} (v_p(t)\ \hat{i}_p(\tau) - i_p(t)\ \hat{v}_p(\tau)) = \sum_{j=1}^{b} (v_j(t)\ \hat{i}_j(\tau) - i_j(t)\ \hat{v}_j(\tau)), \quad (A.1)$$

where t and τ are arbitrary. The summation on the R.H.S. is over the internal branches and on the L.H.S. is over the measurement ports. The adjoint network has the same topology as the original network and its currents and voltages are identified by the hat symbol ($\hat{}$).

If some of the subnetwork element values change from nominal, the currents and voltages in the subnetwork change accordingly. The change in the element value can be represented by a current source or a voltage source (Bandler, Biernacki and Salama 1981). Without loss of generality we assume that the change in element value is represented by a current source which is in parallel with the nominal element. These sources constitute additional branches in the network. For the nominal network the values of these current sources are equal to zero and we may write (A.1) as

$$\sum_{p=1}^{m_i} (v_p(t)\ \hat{i}_p(\tau) - i_p(t)\ \hat{v}_p(\tau)) = \sum_{j=1}^{b} (v_j(t)\ \hat{i}_j(\tau) - i_j(t)\ \hat{v}_j(\tau))$$

$$+ \sum_{\ell=1}^{f} v_\ell(t)\ \hat{i}_\ell(\tau)\ , \qquad (A.2)$$

where f denotes the number of elements which have changed from nominal. For the perturbed subnetwork equation (A.2) becomes

$$\sum_{p=1}^{m_i} \left((v_p(t) + \Delta v_p(t))\ \hat{i}_p(\tau) - (i_p(t) + \Delta i_p(t))\ \hat{v}_p(\tau)\right) =$$

$$\sum_{j=1}^{b} \left((v_j(t) + \Delta v_j(t))\ \hat{i}_j(\tau) - (i_j(t) + \Delta i_j(t))\ \hat{v}_j(\tau)\right)$$

$$+ \sum_{\ell=1}^{f} \left((v_\ell(t) + \Delta v_\ell(t))\ \hat{i}_\ell(\tau) - \Delta i_\ell(t)\ \hat{v}_\ell(\tau)\right)\ , \qquad (A.3)$$

where $\Delta i_\ell(t)$ is the current source that represents the variation in the subnetwork element.

Subtracting (A.2) from (A.3),

$$\sum_{p=1}^{m_i} (\Delta v_p(t)\ \hat{i}_p(\tau) - \Delta i_p(t)\ \hat{v}_p(\tau)) = \sum_{j=1}^{b} (\Delta v_j(t)\ \hat{i}_j(\tau) - \Delta i_j(t)\ \hat{v}_j(\tau))$$

$$+ \sum_{\ell=1}^{f} (\Delta v_\ell(t)\ \hat{i}_\ell(\tau) - \Delta i_\ell(t)\ \hat{v}_\ell(\tau))\ . \qquad (A.4)$$

The internal branches of adjoint network are defined such that, for incremental changes in the f branch elements,

$$\Delta v_j(t)\ \hat{i}_j(\tau) - \Delta i_j(t)\ \hat{v}_j(\tau) = 0\ , \qquad j=1,2,\ldots,b \tag{A.5}$$

and the branches corresponding to the assumed current sources are open circuit. That is

$$\hat{i}_\ell\ (\tau) = 0\ , \qquad \ell=1,2,\ldots,f\ . \tag{A.6}$$

We may then write (A.4) as

$$\sum_{p=1}^{m_i} (\Delta v_p(t)\ \hat{i}_p(\tau) - \Delta i_p(t)\ \hat{v}_p(\tau)) = -\sum_{\ell=1}^{f} \Delta i_\ell(t)\ \hat{v}_\ell(\tau)\ . \tag{A.7}$$

We are interested in the variation in the measured currents $\Delta i_k(t)$, $k \in M_{i\alpha}$. We apply an impulse voltage source $\delta_k(\tau)$ across its corresponding port in the adjoint network and all other ports are short-circuited, i.e., $\hat{v}_p(\tau) = 0$, $p=1,2,\ldots,m_i$, $p \neq k$. The cardinalities of the sets $M_{i\gamma}$ and $M_{i\delta}$ are assumed to be zero as the voltages of all m_i nodes are known and the subnetwork is considered to be voltage excited; i.e., $\Delta v_p(t) = 0$, $p=1,2,\ldots,m_i$. Hence, (A.7) becomes

$$\delta_k(\tau)\ \Delta i_k(t) = \sum_{\ell=1}^{f} \hat{v}_{\ell k}(\tau)\ \Delta i_\ell(t)\ , \qquad k=1,2,\ldots,m_{i\alpha}\ . \tag{A.8}$$

Taking $\tau = t_f - t$ and integrating over the interval $0-t_f$, we get

$$\Delta i_k(t_f) = \int_0^{t_f} \sum_{\ell=1}^{f} \hat{v}_{\ell k}\ (t-t_f)\ \Delta i_\ell(t)\ dt,\ k=1,2,\ldots,m_{i\alpha}. \tag{A.9}$$

If (3.9) is satisfied, this implies that $\Delta i_k(t_f) = 0\ \forall\ t_f$. Since we adopt the assumption that the effect of faults will not cancel at the measurement ports, every term of the summation on the R.H.S. of (A.9)

should be equal 0, namely,

$$\int_0^{t_f} \hat{v}_{\ell k}(t-t_f)\,\Delta i_\ell(t)\,dt = 0 \quad \forall\, t_f \ , \quad k=1,2,\ldots,m_{i\alpha} \qquad (A.10)$$

For $\Delta i_\ell(t) \neq 0$ $\hat{v}_{\ell k}(\tau)$ should be 0, $\forall\, \tau$, which contradicts the assumption that the subnetwork is connected and the decomposition nodes m_i do not decompose it further. So (A.10) is equal to 0 only if $\Delta i_\ell(t) = 0$ and this implies that the subnetwork is nonfaulty.

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