

THREE ESSAYS ON THE MACROECONOMIC IMPLICATIONS OF  
POPULATION AGING AND THE LABOR MARKET EFFECTS OF PAYROLL  
TAXATION

By  
MALICK SQUARE, Licence, M.A.

A Thesis  
Submitted to the School of Graduate Studies  
in Partial Fulfilment of the Requirements  
for the Degree  
Doctor of Philosophy

McMaster University

© Copyright by Malick Square, September 2003

ECONOMIC IMPLICATIONS OF AGING AND PAYROLL TAXATION

DOCTOR OF PHILOSOPHY (2003)  
(Economics)

McMaster University  
Hamilton, Ontario

TITLE: Three Essays on the Macroeconomic Implications of Population Aging  
and the Labor Market Effects of Payroll Taxation

AUTHOR: Malick Souare, Licence (Université Libre de Bruxelles),  
M. A. (Université Laval)

SUPERVISORY COMMITTEE: Professor William M. Scarth (Supervisor)  
Professor Lonnie Magee  
Professor Byron Spencer

NUMBER OF PAGES: ix, 140

## ABSTRACT

This dissertation is a collection of three essays on the macroeconomic implications of population aging and the labor market effects of payroll taxation.

In the first essay, we develop a calibrated overlapping generations model in a closed economy setting, which we apply to the North American economy to investigate the effect on living standards of the aging baby boom. The relative scarcity of labor when baby boomers retire raises the wage-rental ratio by an amount that is sufficient to ensure that the post-baby-boom generation can enjoy a modest increase in living standards – despite facing higher taxes. Nonetheless, the baby-boom cohort itself suffers a drop in consumption, and when the two generations are considered as a group, overall living standards fall by a modest amount. These results are robust to several changes in specification: the existence of liquidity constraints, alternative assumptions regarding individuals' expectations concerning future interest rates, and different fiscal policies concerning the tax treatment of private saving for retirement. Policy initiatives that bring significant hardship today to avoid a future "crisis" are not supported by the standard overlapping generations model.

In the second essay, we develop a general equilibrium model of a small open economy – calibrated to Canada – to examine some implications of population aging. First, we find that analysing the consequences of population aging in one country without taking into account the extent of aging throughout the world, one may misestimate the effect which aging may have on that country's living standards and its net foreign asset position. Second, the liquidity-constrained individuals are those whose living standards, both in the short run and in the long run, are most affected by the aging baby boom. On the whole, our analysis in this essay suggests that we should regard population aging as a serious upcoming threat to living standards. Finally, whether increases in payroll tax rates to finance the public pension system (as the elderly dependency ratio rises) are imposed on workers or firms has little effect on the impact of aging on average living standards; however, it does matter for the unemployment rate. More specifically, the firms' contribution rates to the social security programs are shown to have no influence on the natural unemployment rate, whereas increases in the employees' contribution rates raise it.

The third essay uses data on the Canadian economy to investigate the empirical evidence of the incidence and unemployment effects of the employer and employee payroll taxes. We estimate a disequilibrium aggregate labor market model that allows the separate effects of payroll tax rates imposed on employers and employees to be distinguished, and use the estimated model to simulate the impacts of payroll taxation. The results indicate that there is evidence of an asymmetric labor market effect according to whether payroll taxes levied on employers or employees are raised. Although there is no impact on unemployment in the long run in either case – which runs somewhat counter to the predictions in the second essay – the short-run effects differ. Moreover, we find that the elasticity of labor demand (with respect to the wage rate) is 10 times (in absolute value) as large as the labor supply elasticity. As a result, most of the burden of payroll taxes – whether levied on employers or employees – is found to be borne by labor in the long run. Finally, it appears that we should be reluctant to place too much weight on the time span over which payroll taxes may affect unemployment in the (most widely performed) deterministic simulations. In fact, the stochastic simulation results illustrate that this time span may be much shorter.

## PREFACE

I am sincerely grateful to my thesis supervisor, Professor William M. Scarth, for his tremendous assistance, patience and encouragement during the preparation of this thesis. My gratitude also goes to the other members of my committee, Professors Byron Spencer and Lonnie Magee, for their helpful comments and direction throughout the writing of this thesis. I would also like to thank Professors Herb Emery and John Burbidge for insightful suggestions and comments on the first chapter of this thesis

I also wish to thank the graduate secretary of the Economics Department, Mrs. Aloma Munian, for her kind support and encouragement during my years of study at McMaster University.

Without the love, support and understanding of my family and friends I would not have been able to complete my doctoral studies. Special thanks to my wife Mariame, her sister Adama and my daughter Fanta.

I gratefully acknowledge financial support from the Social Sciences and Humanities Research Council of Canada, the Ontario Graduate Scholarships, Tradeport International Corporation and the McMaster's SEDAP program.

Finally, I dedicate this work, with love, to my parents; I owe you all. This is for you.

The first chapter of this thesis was prepared with the intention of joint publication with my supervisor, Professor William M. Scarth. I had primary responsibility for solving the model and played an important role in the writing of the paper.

## CONTENTS

LIST OF TABLES	vii
LIST OF FIGURES	ix
INTRODUCTION	1
CHAPTERS	
1. Baby-Boom Aging and Average Living Standards	6
Endnotes	27
2. Population Aging, Public Pensions and the Macroeconomy: An Analysis in a Small-Open Economy Framework	28
Endnotes	72
Appendix A	84
3. The Effects of Payroll Taxes in a Labor Market with Unemployment and Unsatisfied Demand for Labor	89
Endnotes	125
Appendix A	134
CONCLUSION	137

## TABLES

### Chapter 1

Table 1: Percentage increase or decrease in living standards Perfect foresight – All households forward-looking planners	25
Table 2: Percentage increase or decrease in living standards Perfect foresight – 50% households forward-looking planners	25
Table 3: Percentage increase or decrease in living standards Static expectations – 50% households forward-looking planners	25
Table 4: Percentage increase or decrease in living standards Static expectations – 50% households forward-looking planners All savings sheltered from tax	26

### Chapter 2

Table 1: Baseline calibration	77
-------------------------------	----

#### Benchmark Simulation Experiments

Table 2: Effects of increased elderly dependency ratio on living standards and unemployment rate among low-income workers	78
Table 3: Effects of increased elderly dependency ratio on wages, capital stock, output, and foreign debt	79
Table 4: Effects of increased elderly dependency ratio (DR) on living standards and unemployment rate among low-income workers – DR increased by half of what is expected	80
Table 5: Effects of increased elderly dependency ratio on wages, capital stock, output, and foreign debt – DR increased by half of what is expected	80

#### Sensitivity Analysis Experiments

Table 6: Effects of increased elderly dependency ratio on living standards and unemployment rate among low-income workers – lower life expectancy and higher retirement age cases	81
Table 7: Effects of increased elderly dependency ratio on wages, capital stock, output, and foreign debt – lower life expectancy and higher retirement age cases	82



Table 8: Effects of increased elderly dependency ratio on living standards and unemployment rate among low-income workers – higher initial interest rate case	83
---	----

### **Chapter 3**

Table 1: Newspapers covered in the help-wanted index survey	127
Table 2: Linear and hyperbolic Beveridge relationships	127
Table 3: Tests for integration and cointegration	128
Table 4: GMM estimates of the parameters	129

## FIGURES

### Chapter 2

- Figure 1: Dynamic adjustment of domestic (forward-looking group) consumption and financial wealth to an increase in the elderly dependency ratio – *with no demographic change abroad* 75
- Figure 2: Dynamic adjustment of domestic (forward-looking group) consumption and financial wealth to an increase in the elderly dependency ratio – *with similar demographic change abroad* 76

### Chapter 3

- Figure 1: Employment and short-side principle 94
- Figure 2: The Beveridge curve 95
- Figure 3: Unemployment rate vs. HWI<sup>1</sup>: Canada 1981-2000 100
- Figure 4: Unemployment rate and HWI relationship, 1981-2000 101
- Figure 5: The simulated effect of a rise of one percentage point in employers' payroll tax rate – deviations from baseline 130
- Figure 6: The simulated effect of a rise of one percentage point in employees' payroll tax rate – deviations from baseline 131
- Figure 7: The simulated effect of a simultaneous rise of one percentage point in employers' and employees' payroll tax rates – deviations from baseline 132
- Figure 8: Stochastic simulations results – with 95% confidence interval (deviations from baseline) 133-34

---

<sup>1</sup> Help-Wanted Index.

## **Introduction**

This thesis comprises three essays on the economic implications of population aging and payroll taxation. More specifically, the first two essays examine some macroeconomic and distributional consequences of an aging population in the presence of a public pension system, within a closed and open economy framework, respectively. The third essay investigates the incidence and unemployment effects of payroll taxation.

Many countries, particularly those in the OECD, are facing major changes in the age structure of their populations. Because of the imminent retirement of the "baby boom" generation, combined with lower fertility and greater longevity, the ratio of elderly (people aged 65 and over) to the working-age population (people aged 15 to 64) will increase dramatically in most OECD countries over the coming decades. For example, in Canada, this ratio is expected to nearly double – from one fifth today – by 2030. Other major industrial countries such as Germany and Japan stand to experience even sharper increases in their retirement-age population.

Such a pronounced demographic change poses many challenges for a country's well-being, and concerns about these challenges have moved to the forefront of the public policy debate in many countries. One key concern is that the shift to an older population, by putting increased pressures on the cost of public pensions and health care, may have negative intergenerational equity implications. In industrial countries, public schemes for providing for the retired are predominantly of a pay-as-you-go (PAYG) type based on the notion of solidarity among generations. A standard PAYG system levies payroll taxes on the working population while paying benefits to the retired, but usually without the close person-based relationship between individual contributions and benefits that characterizes fully funded schemes. Thus, according to the conventional view, when the baby-boomers are retired, the much smaller number of workers in the next generation will face high tax rates (and consequently lower living standards) if the PAYG public pension and public health care systems are to be maintained at current benefit levels. However, some people have argued that the relative scarcity of workers resulting from aging populations is likely

to result in increases in real wages, which would provide (at least) some compensating effects to workers. With generous PAYG programs in place in most countries, policymakers, as well as the general public, are concerned about the demographic transition's impact on intergenerational distribution.

Thus, in the first essay of this dissertation, we develop an overlapping generations model in a closed economy setting, which we apply to the North American economy, to examine how the living standards of baby-boom and post-baby-boom generations are affected by the aging baby boom when modeled as a temporary event. The results indicate that the relative scarcity of labor when baby boomers are retired raises the wage-rental ratio by an amount that is sufficient to ensure that the post-baby-boom generation can enjoy a modest increase in living standards – despite facing higher taxes. Conversely, the baby-boom generation itself suffers a drop in living standards. Nonetheless, when the two generations are considered as a group, overall living standards fall by a modest amount. These results are robust across alternative specifications: no liquidity constrained consumers vs. 50 percent of the population living "hand to mouth;" perfect foresight vs. static expectations concerning future interest rates; and no tax-sheltered savings vs. a program that allows unlimited contributions. Overall, our analysis in this essay suggests that the impact of aging populations on average living standards cannot be dismissed as a trivial phenomenon, but nor should it be regarded as a crisis.

Applying the closed-economy model to examine the effect which aging may have on living standards can only be justified if the analysis refers at least to the entire North American economy. For the analysis to be exclusive to Canada, we have to consider a small open-economy model. In this case, the assumption of perfect capital mobility pegs the interest rate. When combined with a standard constant-returns-to-scale production technology, perfect capital mobility also pegs the wage rate. Thus, in a standard small open-economy model, there is no mechanism for the scarce labor (that works while the baby-boomers are retired) to receive higher pre-tax wages.

However, examining the economic implications of aging populations for a single economy – using a small open-economy framework – might also be problematic, because

assuming that the rate of interest is constant implies implicitly that no similar demographic changes take place abroad. In the real world, however, population aging appears to be a world-wide phenomenon. With a general aging process one would instead expect a fall in world real interest rates.

Furthermore, as mentioned earlier, the financial viability of the PAYG system can be maintained – despite an aging population – if payroll taxes are increased (as has been the ongoing plan in Canada). But some of these increases in payroll taxes may raise the natural unemployment rate, with the result that the transfers to retirees will be at least partly funded by reducing economic outcomes for younger individuals at the lower end of the income scale. This outcome is often overlooked, since in the literature pension systems and unemployment have traditionally been studied in separate classes of models. Also, it is not known whether it is better to have the contributions of workers or firms adjust to finance pension benefits as the elderly dependency ratio increases.

Thus, in the second essay of this thesis, we develop an overlapping generations small open-economy model – calibrated on Canadian data – to investigate some macroeconomic and distributional effects of an aging population when modeled as a permanent event. In this general equilibrium model, though the perfect capital mobility pegs the interest rate, it does not peg the wage rates, since there are two categories of labor (skilled and unskilled). Moreover, in the model, some households live hand-to-mouth, and therefore do not save. The study leads to three main results. First, it appears that by analyzing the aging problem in a small open-economy framework without taking into account its global dimensions, we may systematically misestimate the effect which aging may have on the country's living standards and its net foreign asset position. Second, the hand-to-mouth individuals are those whose living standards are most affected by aging. However, for both groups (liquidity and non-liquidity constrained individuals) the drop in average living standards is much *greater* than the fall in overall living standards reported in the first essay. Thus, our analysis in this essay suggests that we should regard population aging as a serious upcoming threat to living standards. This difference in the magnitude of findings is mainly due to three factors. The first is the fact

that, in the closed economy (in the first essay) the population aging leads to capital deepening and thus higher real wages; whereas in the open economy, aging results in capital shallowing – since part of the domestic capital stock is moved abroad – which leads to a reduction in the real wages. The second consideration is that variations in the nation's foreign indebtedness are important in the open economy environment. The final consideration is that the demographic shock is permanent in the second essay. Related to this, since agents plan much farther into the future in this specification (than they do in the first essay), a permanent shock has bigger short-run effects as well. Third, the actual incidence of increased contributions to the public pension system – that is, whether the workers' or firms' contribution rate should increase to maintain the financial viability of the system as the elderly dependency ratio rises – has little effect on the impact of aging on living standards. This conclusion emerges whether or not agents save for the future. However, the incidence of contributions matters for labor market distortions, and hence for the unemployment rate. More specifically, the firms' contribution rates to the social security programs are shown to have no influence on the natural unemployment rate, whereas increases in the employees' contribution rates raise it.

The third essay uses data on the Canadian economy to investigate the empirical evidence of the labor market effects of the employer and employee payroll taxes. Most of the existing macroeconomic-labor literature that has studied the incidence and employment (or unemployment) effects of payroll taxes has modeled taxation as a single total variable (employers' payroll taxes plus employees' social security and income taxes plus indirect taxes, as well as the term of trade). The use of a single combined variable rules out a differential analysis that distinguishes between taxes on labor that are imposed on the employer or employee, since both tax rates are constrained to have the same coefficient in estimation and/or simulation. Although this approach helps overcome the difficult problems of finding sufficient variation in payroll tax rates – problems which all empirical studies have to confront – it is not known whether the effects are symmetrical according to whether payroll taxes levied on employers or employees are raised.

Thus, in this third essay, we estimate a disequilibrium aggregate labor market model that allows the separate effects of payroll tax rates imposed on employers and employees to be distinguished, and use the estimated model to simulate the impacts of payroll taxation. Moreover, in contrast to the bulk of empirical studies of the labor market effects of payroll taxation, our approach recognizes that unfilled job vacancies and unemployed workers simultaneously coexist in the economy. The results of our study indicate that there is evidence of an asymmetric labor market effect according to whether payroll taxes levied on employers or employees are raised. Although there is no impact on unemployment in the long run in either case – which runs somewhat counter to the predictions in the second essay – the short-run effects differ. Moreover, we find that the elasticity of labor demand (with respect to the wage rate) is 10 times (in absolute value) as large as the labor supply elasticity. As a result, most of the burden of payroll taxes – whether levied on employers or employees – is found to be borne by labor in the long run. Finally, it appears that we should be reluctant to place too much weight on the time span over which payroll taxes may affect unemployment in the (most widely performed) deterministic simulations. In fact, the stochastic simulation results illustrate that this time span may be much shorter.

## Chapter 1

### Baby-Boom Aging and Average Living Standards

#### 1. Introduction

There is widespread concern about the living standards that will be available for the generation that follows the baby-boom cohort. According to conventional wisdom, when the baby-boomers are old, the much smaller number of workers in the next generation will face high tax rates (and consequently lower living standards) if the pay-as-we-go public pension and public health care programs are to be maintained at current benefit levels. For Lueth (2001), coping with these fiscal and distributional consequences of population aging is generally perceived as one of the central challenges of the decades to come. With generous pay-as-you-go schemes in place in most industrial countries, policymakers, as well as the general public, are particularly concerned about the demographic transition's impact on intergenerational distribution. The view that an increasing ratio of retirees to workers will impose a heavy fiscal burden on the young and future cohorts is underpinned by empirical evidence. Recent generational accounting studies find that 19 out of the 22 countries examined exhibit a fiscal imbalance to the detriment of future generations (see Kotlikoff and Raffelhuschen, 1999; Raffelhuschen, 1999). Similarly, in 1994, the World Bank referred to this challenge as a "crisis," and this view has been echoed by the OECD (1998), the Canadian Auditor General (1998), and a former Secretary of Commerce in the United States (Peterson, 1999) who emphasized that the aging crisis will bankrupt the world's wealthiest nations.<sup>1</sup>

The seriousness of the aging population has been challenged (see, for example, Denton and Spencer (2000), Emery and Rongve (1999) and Mérette (2001)). Denton and Spencer note that the *overall* dependency ratio<sup>2</sup> that will prevail when baby boomers are retired will not reach the level observed when the baby boom was young in the 1950s. Even though health-care costs for old dependents are higher than education costs for young dependents, western economies may not have to cope with any bigger challenge



than was faced earlier. Emery and Rongve argue that, with labor becoming relatively scarce when the baby-boomers are retired, factor price adjustments should provide a cushion for the living standards of the next generation. Indeed, if wages rise enough, it may be possible for that next generation to pay the higher payroll taxes necessary to finance the baby-boomers' pension and public health care, and still enjoy higher living standards themselves. To illustrate this possibility in a bold way, Emery and Rongve use a calibrated version of the simplest two-period overlapping generations model (very similar to Burbidge (1983)) to examine this possibility. Since they find that the post-baby-boom generation's living standard *is* higher than the steady-state outcome, they conclude that the concern expressed on behalf of this generation is "much ado about nothing." They go on to stress that imposing taxes on the lower-income members of the baby-boom cohort to avoid what they predict to be a non-existent crisis for the next generation is not recommended. In earlier work Auerbach *et al.* (1989), using a much more complex simulation study of demographic change, find results similar to that of Emery and Rongve (1999). They report that the aging of society could lead to capital deepening and higher real wages as the number of retirees with capital rises relative to the number of workers supplying labor. Higher real wages would raise the wage base and, thereby, limit the increase in payroll tax rates.

Mérette stresses two additional considerations. First, a rising wage-rental ratio should stimulate investment in human capital, with the result that the post-baby-boom generation can enjoy higher living standards. Second, Mérette draws attention to the existence of tax-sheltered private savings plans. Now, when baby boomers are working and contributing to these plans, they represent a drain on government revenues. In the future, when baby boomers are retired and withdrawing funds from these accounts, these programs will become net revenue-raisers for governments. This turnaround will limit the extent to which governments will have to raise taxes on the post-baby-boom generation while they are working. Mérette concludes that – far from there being a crisis for the post baby boomers – this generation should "welcome" the aging of the population.

It is an unsatisfactory state of affairs that economists have such disparate views on a central topic of public debate. Why has there not been a convergence of views on this broad topic? One reason is that most analyses involve quite complicated calibrated models – involving dozens of overlapping generations and an extensive disaggregation of government spending on many different age-specific categories. It is essentially impossible for one researcher to perform sensitivity tests with another's model. Since so many different assumptions are made – from one analysis to another – it is very difficult for researchers to ascertain the relative importance of each specification change in generating the different results.

Given this limited ability to discriminate between alternative studies, it seems appropriate to return to the basic analytical framework that underlies the overall approach to this policy question. As a result, in this chapter, we start with the textbook version of the overlapping generations model, and we apply it to the North American economy. This model involves bold assumptions – such as the absence of any liquidity constraints for all individuals, and perfect foresight over a time interval of an entire generation concerning future interest rates. The sensitivity of the results concerning the aging population to these very basic issues is virtually never assessed. We provide this assessment, in addition to examining the role of tax-sheltered savings plans. We are reassured to find that the results are robust across such boldly different alternative specifications: no liquidity constraints vs. 50% of the population living "hand to mouth;" perfect foresight vs. static expectations concerning future interest rates; and no tax-sheltered savings vs. a program that allows unlimited contributions.

A limitation of our analysis is that we do not explore the possibility that aging may stimulate investment in human capital. Such an investment can be expected since the pay-off from acquiring additional skills will be higher when the wage-rental ratio rises. Our defense for omitting this extension is based on our interpretation of the endogenous growth literature. Many studies, such as Devereux and Love (1994), reach the following two conclusions. First, changes in the wage-rental ratio have significant one-time *level* effects on living standards; and second, changes in the factor price ratio have extremely

modest effects on the growth *rate*. These results suggest that policy-oriented macroeconomists may lose little by focusing exclusively on the levels effects in this context. Moreover, Jones (1995) calls into question the prediction of many endogenous growth models that permanent changes in certain policy variables (such as investment in human capital) have permanent effects on the rate of economic growth. He argues that over the last 50 years, the U.S. economy has experienced substantial increases in both educational attainment and research intensity. Such changes, according to many endogenous growth models, should lead to permanent increases in the growth rate. However, the growth rate of U.S. per capita GDP has been surprisingly stable over the last century: the level of per capita GDP is well represented by a simple time trend. This raises the question of whether the prediction of endogenous growth models is misleading. In any case, our sorting out the importance of the several other issues in a simplified setting, that avoids being a "black box" for other participants in the debate, remains useful.

Our starting point is Emery and Rongve's focus on the core model that underlies this entire literature. Unfortunately, there are inconsistencies in the calibration of their model. After repairing these problems we report several sensitivity tests – how the overall conclusion on aging is affected by three central model-specification issues: Are some individuals liquidity constrained? Can individuals accurately forecast interest rates a generation into the future? and Does the government allow taxes on saving to be postponed until retirement? In addition to reporting the effects on the living standards of each generation, we focus on the average outcome for all those alive at each point in time.

## 2. The Model

In the standard two-period model, each cohort proceeds through a "young" period (when individuals work, consume, pay the taxes that support the public pension and save), and through an "old" period (when they consume their savings and public pension benefits). The length of a "period" is one generation – often taken to be 30 years (see Barro and Sala-i-Martin (1995) p. 131). One drawback of this framework – as noted by

Barro and Sala-i-Martin – is that it involves the assumption that there is a 30-year lag between the act of abstaining from consumption and the actual use of the newly produced output as capital. Nevertheless, there is an analytical convenience that accompanies this specification. The order of the dynamics can be reduced since it is a reasonable approximation to assume 100 percent depreciation of capital during one period of 30 years. It is unfortunate that some applications of this framework involve calibrations that are inconsistent with this 30-year time frame. For example, in Emery and Rongve, the length of the period is one generation for some parameters (such as the depreciation rate), while the length of the period is one year for others (such as the interest rate, the steady-state capital-output ratio, and the rate of time preference).

Concerning the rate of time preference, it is plausible to assume this parameter to be zero for a short time interval. But over a 30-year period, such a calibration is not appropriate. Indeed, we have found it to be impossible to calibrate the two-period overlapping-generations model – in a way that respects both the need to have all parameters "reasonable" for a 30-year period, and the need to be consistent with all the model's steady-state restrictions – without allowing a positive rate of time preference.

As just noted, in this overlapping generations framework, agents born at time  $t$ , live for only two periods:  $t$  and  $t+1$ . At any time  $t$ , the population consists of  $N_t$  young agents born at time  $t$ , and  $N_{t-1}$  old agents who were born in the previous period,  $t-1$ . One good is produced, and consumers derive utility from consuming this good in both their young period and their old period. Following Mankiw (2000), we consider two sorts of consumers. One group is forward looking; these individuals have a rate of time preference low enough to cause them to plan for the future, and they smooth consumption over their lifetimes. A second group has such a high rate of time preference that these individuals spend their entire after-tax income every period, and are, therefore, liquidity constrained. The two types of consumers represent proportions  $\pi$  and  $1 - \pi$  of both the young and old portions of the population. We examine two polar-case assumptions regarding the expectations of private agents. Initially, we follow convention and assume perfect foresight – that agents correctly anticipate future interest rates (since they correctly

forecast all demographic events). As a sensitivity test, we assume static expectations, and in this case agents are surprised by demographic developments and any change in the general level of interest rates from one generation to another.

All agents work in their first period, retire in the second period, and then die. Young consumers supply one unit of labor inelastically in period  $t$  for which they receive a wage of  $w_t$ . Since they pay a social security contribution,  $\theta_t w_t$  ( $0 < \theta_t < 1$ ), their after-tax income in time  $t$  equals  $(1 - \theta_t)w_t$ . At time  $t+1$ , they supply no labor. During this retirement period, the forward-looking consumers have both private savings and the public pension, but the public pension is all that the liquidity-constrained consumers have in old age.

The behavior of forward-looking consumers follows from standard intertemporal optimization. We define the consumption of each member of this group (born at time  $t$ ) in period  $t$  as  $C_{ft}(t)$ , and in period  $t+1$  as  $C_{ft}(t+1)$ , and their savings while young as  $S_t(t)$ . In the case without any tax-sheltered retirement savings plans, a young agent's budget constraint at time  $t$  is:

$$C_{ft}(t) + S_t(t) = (1 - \theta_t)w_t \quad (1)$$

Old consumers in this group have two sources of income for financing consumption in the second period of life. The first is their savings. In the second period, this capital earns a return equal to  $(r_{t+1} - 1)$ . Their second source of income is the social security payment denoted as  $\tau_{t+1}$ . Thus, without a tax-sheltered private savings plan, the second period budget constraint for a consumer of generation  $t$  is:

$$C_{ft}(t+1) = \tau_{t+1} + r_{t+1}S_t(t) \quad (2)$$

Taking  $w_t$  and  $r_{t+1}$  as given, each consumer chooses  $C_{ft}(t)$  and  $S_t(t)$  (and, hence,  $C_{ft}(t+1)$ ) to maximize lifetime utility given by:

$$U(C_{ft}(t), C_{ft}(t+1)) = \log(C_{ft}(t)) + \beta \log(C_{ft}(t+1)) \quad (3)$$

where  $\beta = \frac{1}{1+\rho}$  and  $\rho$  is the rate of time preference.

The first-order condition for this problem is:

$$\frac{C_{ft}(t+1)}{C_{ft}(t)} = \beta r_{t+1} \quad (4)$$

which can be solved to yield:

$$S_t(t) = \frac{\beta}{1+\beta} (1-\theta_t) w_t - \frac{\tau_{t+1}}{(1+\beta)r_{t+1}} \quad (5)$$

Consumption in either period can be determined by the budget constraints.

For the liquidity-constrained consumers, consumption in the two periods of life are given by:

$$C_{lt}(t) = (1-\theta_t) w_t \quad (6)$$

$$C_{lt}(t+1) = \tau_{t+1} \quad (7)$$

Considering both groups together, the average consumption (in period  $t$ ) of all individuals born at time  $t$ , and the average consumption (in period  $t+1$ ) of these same individuals are:

$$C_t(t) = \pi[(1-\theta_t)w_t - S_t(t)] + (1-\pi)[(1-\theta_t)w_t] \quad (8)$$

$$C_t(t+1) = \pi[\tau_{t+1} + r_{t+1}S_t(t)] + (1-\pi)[\tau_{t+1}] \quad (9)$$

On the supply side, firms operate with a standard production function:

$$y_t = \gamma_t^\alpha k_t^{1-\alpha} \quad (10)$$

where  $y_t$  is total production at time  $t$ ,  $l_t$  is the labor used at time  $t$ , and  $k_t$  is the total capital stock at time  $t$ . Since all factor markets clear, employment equals the total number of young people:

$$l_t = N_t \quad (11)$$

and since there is 100% depreciation of capital during one full generation of time, period  $t$ 's capital stock must equal period  $t-1$ 's total saving:

$$k_t = \pi N_{t-1} S_{t-1}(t-1) \quad (12)$$

(Recall that  $\pi$  is the proportion of young people that save.)

Capital and labor are paid their marginal products:

$$w_t = \gamma \alpha l_t^{\alpha-1} k_t^{1-\alpha} \quad (13)$$

and

$$r_t = \gamma(1-\alpha) l_t^\alpha k_t^{-\alpha} \quad (14)$$

Substituting the factor market equilibrium conditions into these two relationships, we have:

$$w_t = \gamma \alpha [(\pi N_{t-1} / N_t) S_{t-1}(t-1)]^{1-\alpha} \quad (15)$$

and

$$r_t = \gamma(1-\alpha) [(\pi N_{t-1} / N_t) S_{t-1}(t-1)]^{-\alpha} \quad (16)$$

Equations (15) and (16) describe how factor prices vary with both demographic changes and households' savings plans (as derived in equation (5)). As specified, equation (5) implies that young forward-looking consumers know the future return,  $r_{t+1}$ , when setting the amount to save in period  $t$ . To appreciate what must be known for this to be true, consider equation (16) – written one period forward in time. This relationship indicates that, to know the future interest rate, the pre-baby-boom generation must be aware that

the baby boom is coming – before it occurs. Since some may regard this property of the model as unappealing, we also derive simulation results that do not involve agents having such foresight. In particular, in the simulations referred to as the “static expectations” cases, we replace the  $N_t / N_{t+1}$  term in equation (16) – written one period ahead – by unity.

There is a pay-as-you-go social security system. Each young worker at time  $t$  pays taxes  $\theta_t w_t$ . These tax revenues are distributed to old consumers at time  $t$  as a lump sum pension benefit  $\tau_t$ . The balanced budget condition is:

$$N_t \theta_t w_t = N_{t-1} \tau_t \tag{17}$$

Since we assume that the pension payment is an exogenous constant, the tax rate  $\theta_t$  adjusts to satisfy equation (17) in each period.

The government’s budget constraint is more complicated if tax-sheltered private saving for retirement is allowed. In this case, the government receives less revenue while the baby-boom generation is young (since the majority of the population is taking advantage of the tax break). For the same reason, the government receives more revenue while baby-boomers are old (since, by that time, the majority of the population is withdrawing funds from these plans). To examine the importance of this variation in government revenue, we present results for two polar cases: one with no tax-sheltered private saving, and one with *no limit* on contributions to such a plan.

The detailed assumptions we make in this regard are as follows. Individuals are allowed to deduct their entire savings from their wage earnings, when calculating taxable income while young. Then when they are old, they pay tax on both that amount of saving and the interest earned thereon. This specification allows us to examine the implications of IRAs and 401(K) accounts (as in the United States) or RRSPs (as in Canada) – with the simplification that there is no limit on contributions. Also, for simplicity, we have assumed just one tax rate – that is involved in both the funding of the public pension and



this private savings plan. As a result, the variations in the household and government budget constraints that are appropriate in this setting are:

$$C_{ft}(t) + (1 - \theta_t)S_t(t) = (1 - \theta_t)w_t \quad (1a)$$

$$C_{ft}(t+1) = \tau_{t+1} + (1 - \theta_{t+1})r_{t+1}S_t(t) \quad (2a)$$

$$N_t\theta_t w_t + \pi N_{t-1}\theta_t r_t S_{t-1}(t-1) = N_{t-1}\tau_t + \pi N_t\theta_t S_t(t) \quad (17a)$$

When these revised constraints are involved, the first-order conditions for the forward-looking agents are affected. These changes have been incorporated in the simulations reported below.

### 3. Calibration and Results

We have assumed the following parameter values: labor's share,  $\alpha$ , 0.67; the constant term in the Cobb-Douglas production function,  $\gamma$ , 10; and steady-state values of labor and capital equal to 100. These values make the steady-state *annual* capital-output ratio equal to 3. Given that factors are paid their marginal products, these values make the steady-state wage equal to 6.67 and  $r$  (one plus the 30-year interest rate) equal to 3.33 (and this implies an annual interest rate of 4.1%). We assume that the public pension provides an amount equal to one-quarter of the steady-state wage (so  $\tau$  equals 1.67). The pension budget constraint then implies that the payroll tax rate,  $\theta$ , is one-quarter in the steady state. Finally, we consider two distributions of the population between the hand-to-mouth and the forward-looking planner groups:  $\pi = 1$  (all agents are consumption smoothers) and  $\pi = 0.5$  (only half of the agents are forward-looking)<sup>3</sup>. Concerning the forward-looking planners, the rate of time preference must be determined residually to ensure that the model's two expressions for saving generate exactly the same value in the steady state. One expression for saving is the market clearing condition – that one period's saving is the next period's capital stock (equation (12)). The second expression is the first-order condition that follows from (forward-looking) household utility maximization

(equation (5)). To have this consistency with the other parameter values already assumed (with  $\pi = 1$  and ignoring tax-sheltered private savings plans), we require  $\beta = 0.375$ . This value implies an *annual* rate of time preference equal to just under 3.3 %. Since this rate of impatience is less than the annual interest rate of 4.1 percent, consumption rises as agents age. A similar outcome occurs in the cases with  $\pi = 0.5$  and tax-sheltered private savings.

The calibrations discussed in the previous paragraph involve two desirable features. They are consistent with the steady-state restrictions of the model, and they are "realistic" in the sense that each individual parameter value accords well with observations over a 30-year time interval. We now use these calibrations to examine how the initial steady state is disturbed by a demographic event that is intended to simulate the existence of the baby boom.

We introduce a cohort that is 10 percent larger than both the generations that precede and follow this group. This demographic development makes the overall dependency ratio rise by 10 percent as the baby-boom cohort moves from its working period to its retirement period, and this is what is expected by demographers in North America over the next 30 years.<sup>4</sup>

There is one aspect of the calibration that we find worrisome – but it is a problem for *any* application of this class of models. In the real world, it is the *overall* ratio of dependents to the population – not the ratio of the old-age component of dependents – that is approximately 50 percent. But in overlapping generations models of this sort (without population growth), the steady state requires that the ratio of old-age dependents to the population,  $(N_{t-1} / (N_t + N_{t-1}))$ , be equal to 0.5. This mismatch stems from the fact that this standard overlapping generations framework assumes that there is no truly "young" period when individuals do not work. Instead of assuming that agents live for some time without work while young, the model assumes that agents live just as long when they are retired (as they do when they are working). Relative to the real world, then, the model involves a retirement period that is "too long," and a pre-work youth period that is "too short." We find it helpful to think of the extra length of the retirement period

as a proxy for a pre-work youth period. This interpretation (which is equivalent to Barro's (1974) portrayal of an infinitely lived agent as a family dynasty) can be rationalized by thinking of retirees being the ones who take care of the children. The retirees plan their own life as if they will be retired for a long time. In fact, they die earlier, and it is their grandchildren that will take their place for the latter part of their "retirement" period. As long as the grandparents and the grandchildren have the same utility function, this interpretation can be defended. This application of the family dynasty concept implies that – to calibrate the "pension" – we should think of both social security and public health payments for the old, and public support for schooling the young, and (as noted above) we have assumed that this total is one-quarter of the working wage.

Tables 1 – 4 summarize the results; they show the percentage increase or decrease in consumption (compared to the steady state) that is available to several cohorts as the baby-boom phenomenon occurs. Table 1 involves the following assumptions – that no one is liquidity constrained, everyone correctly anticipates all changes in interest rates, and there are no tax-sheltered private savings plans. The first line in Table 1 refers to the period during which the baby-boom cohort is young. The larger labor supply pushes both the pre-tax wage rate and the per-person payroll-tax levy down. These competing effects are of approximately equal strength, so consumption rises by just over one-tenth of one percent relative to the consumption of young agents in the steady state. The pre-baby-boom generation (the old in period 1) enjoys a big rise in consumption – almost 5 percent increase compared to the consumption of retired agents in the steady state. Once again, the reason is the change in the wage-rental ratio. With capital now relatively scarce, its marginal product – and hence the interest rate that the retired earn on their savings – is high.

The second line in Table 1 refers to the period during which the baby-boomers are retired. Interest rates have come back down, so this group receives a level of consumption that is 4.6 percent below what retirees normally receive in the steady state. There is a very small reduction in the living standards of the post-baby-boom generation when these individuals are young. The reason is that there are competing effects that almost exactly

cancel off. With labor now more scarce, pre-tax wages are correspondingly higher. But this group is now paying higher payroll taxes to finance the pensions of the baby boomers, so *after-tax* wages are not increased.

The third line in Table 1 refers to the period when the baby-boomers have died and the post-baby-boom generation is retired. Again, during this later period of life, this group is affected very little by the baby-boom phenomenon. There is still some effect, nevertheless, on the generation that follows even this group. Since the post-baby-boom cohort has saved a little less, the generation that follows them has less capital to work with. (This is why there is a negative entry in the first column of the third row.)

Table 1 suggests that – contrary to much public discussion – the post-baby-boom generation is *not* affected significantly by this demographic event. Before concluding that all is well, however, we draw attention to an assumption which underlies the analysis. It is assumed that the level of benefits received by each pensioner is an exogenous constant. The model ignores the fact that baby-boomers may try to avoid the cut in living standards that is indicated for their old-age period by voting for an increase in the pension benefit. This group has the numbers to win such a vote, and if they exercised this political power, it would be the post-baby-boom generation that loses out after all.<sup>5</sup>

Rather than presenting an entire set of specifications concerning different political outcomes, we find it appealing to focus on the living standards of the average person who is alive in each period. The final column in Table 1 provides this information. We see that, no matter how the burden is shared at the time, the living standard for the average person alive when the baby-boomers are retired is estimated to be about 2.2 percentage points below the consumption enjoyed by individuals who populate the model's steady state. This estimate includes the increase in pre-tax wages enjoyed by those who will be young at that time.

Should a drop in average living standards of 2.2 percent for 30 years be regarded as significant? To answer this question, we must have some base for comparison. To provide this, we consider the debate in Canada surrounding the free-trade agreement with the United States, during the 1980s. The steady-state effect of free trade on the average

living standards (of Canadians) was estimated to be about 3 percent.<sup>6</sup> Assuming a discount rate of 5 percent and a GDP growth rate of 3 percent, a drop in living standards of 2.2 percent for 30 years is equivalent to a one-time loss of 51 percent of one year's GDP. Similarly, an annual drop of 3 percent forever is equivalent to a one-time loss of 158 percent of one year's GDP. So the model predicts that the aging economy affects living standards by about one-third of the amount that was the object of the fierce debates on free trade in the 1980s. This ratio rises to just over one-half if one uses the model's steady-state interest and growth rates, 4.1 percent and zero percent, respectively. Thus, this comparison suggests that we regard the effect of aging as important, but modest.<sup>7</sup>

How do the conclusions hold up in the face of sensitivity tests? Table 2 reports how things change when 50% of the households are liquidity constrained. Since there is less saving in this economy, the capital/labor ratio is smaller. This means that the bulge in the labor force causes a bigger deviation from the steady-state capital/labor ratio. As a result, the fall in the baby boomers' living standards while young (line 1 in Table 2) is bigger in this case. For a similar reason, wages rise more (than in the Table 1 scenario) for the post-baby-boom generation when labor is scarce in their youth period. As a result, as seen in line 2 of Table 2, this group's living standard goes up (by one-fifth of one percent), not down, as they support the aging baby boomers. Thus, the existence of liquidity constraints brings a more pronounced variation in the wage-rental ratio, and so it makes the model a little more supportive of those who view the aging population as a benign development. Nevertheless, our overall assessment is still appropriate: the average person living when the baby boomers are retired is worse off by a full 2 percentage points.

Tables 3 and 4 report how the results are affected by the assumption that the pre-baby-boom generation does not anticipate the changes in interest rates that are caused by the baby boom. We compare Table 3 to Table 2, since we continue to assume that 50% of the households live "hand to mouth." Because agents do not see the baby boom coming in the Table 3 simulations, the pre-baby-boom generation does not predict the same rise in the interest rate as before, and so these individuals do less saving. The result for them is

that – despite the “surprising” rise in the interest rate – they earn less in retirement. The result for the baby boomers is that they have less capital to work with when they are young, and their wages (and living standards) take a bigger hit in this case. A second reason for the baby boomers to suffer more during their work period follows from the fact that – like their predecessors – they make a forecast error concerning interest rates. With static expectations, they anticipate the same high interest rate that the previous generation had found surprising. Thus, compared to the Table 2 simulations, these baby boomers save more in their youth. This is the second reason why their consumption is lower. Once they retire, they find that the interest rate is lower than expected. Nevertheless, since they saved more when young, they are still better off during retirement (compared to the baby boomers who live in a perfect foresight world). The interest rate falls as baby boomers move into retirement – whether there is perfect foresight or static expectations. But since these individuals save more when they have static expectations, this mitigates the fall in living standards for their retirement period. What about the post-baby-boom generation? As just explained, the baby boomers save more in the static expectations case. This gives the next generation more capital to work with. Compared to the perfect foresight case, therefore, labour is more scarce. The resulting rise in wages makes the post baby boomers better off (than they were in the Table 2 simulations).

Our final simulations are reported in Table 4. We continue to assume that half the population lives hand to mouth and that all individuals have static expectations. The new feature is that there is a tax-sheltered private saving plan for retirement. This plan induces the forward-looking individuals to save more during their working stage of life. As a result, the pre-baby-boom generation has more interest income during retirement (and this is one reason why their living standard in old age is higher in Table 4 than it is in Table 3). The other reason follows from the fact that the large number of workers in the baby-boom generation allows the government to lower the economy’s tax rate. This benefits both the baby boomers, and the preceding generation – since the pre-baby-boomers pay less tax as they cash in their savings. For the baby boom generation, there are competing effects. On the one hand, the previous generation is now paying tax during their old age,

and this means that baby boomers face a lower tax rate during their youth. On the other hand, the existence of the tax shelter induces baby boomers to save more while working, and this lowers their consumption. By comparing the top left entries in Tables 3 and 4, we see that the second effect dominates. The possibility of tax-sheltered saving lowers the living standards of baby boomers in this first period of their lives. This generation suffers the same fate in old age. They save more (than in the Table 3 simulations), so they have higher pre-tax interest income. However, since the generation that is working when they themselves are old is smaller, the economy's tax rate has increased, and this lowers the after-tax income that baby boomers receive from their saving (while old). By comparing the middle entries in Tables 3 and 4, we can see that this second effect dominates. Finally, we consider the post-baby-boom generation. There are two reasons for this cohort to be doing better than in the simulations that do not involve tax-sheltered savings. First, those that are retired are now paying taxes, and this leaves a smaller burden for the working generation. Second, since the previous generation saved more, there is a bigger capital stock. This means that the increase in the wage-rental ratio is higher. Nevertheless, there is one reason for this group to be doing worse – the tax-sheltered savings opportunity induces these individuals to save more (which, other things equal, means lower consumption). Tables 3 and 4 indicate that this latter effect dominates, so that post-baby-boomers receive a smaller increase in living standards when the tax shelter opportunity exists.

#### **4. Conclusions**

There is widespread concern that the aging of the baby boom generation may put strain on our public finances, and that average living standards may suffer as a result. But there is significant disagreement about the possible magnitude of these effects, since studies involve different assumptions, and it is difficult for one researcher to perform sensitivity tests with another's complicated and disaggregated model. We have used the standard two-period overlapping generations framework to assess this concern – in a way that allows us to provide a series of sensitivity tests on some of the most basic

assumptions that underlie all analyses of this topic. Our conclusions are robust across several major changes in specification – concerning whether individuals plan ahead at all, whether they have accurate expectations over the time horizon of two generations, and whether the government allows a tax-free private saving plan for retirement (with no contribution limits) or not.

All versions of the model support the prediction that there will be a fall in the average living standards of all those alive when the baby boomers are retired – of about 2 percent. Compared to what has been at stake in other major policy debates, we regard this predicted reduction in living standards as significant, but certainly manageable. Thus, our analysis suggests that the aging population cannot be dismissed as a trivial phenomenon, but nor should it be regarded as a crisis.



## References

- Auerbach, A. J., L. J. Kotlikoff, R. P. Hagemann, and G. Nicoletti (1989) "The Economic Dynamics of an Aging Population: The Case of Four OECD Countries" *OECD Economic Review* 12, 97-130.
- Barro, R.J. (1974) "Are Government Bonds Net Wealth?" *Journal of Political Economy* 81, 1095-1117.
- Barro, R.J., and X. Sala-i-Martin (1995) *Economic Growth* (New York: McGraw-Hill).
- Bohn, H. (1999) "Will Social Security and Medicare Remain Viable as the U.S. Population is Aging?" *Carnegie-Rochester Conference Series on Public Policy* 50, 1-53.
- Burbidge, J.B. (1983) "Social Security and Savings Plans in Overlapping Generations Models", *Journal of Public Economics* 21, 79-92.
- Campbell, J. Y. and N. G. Mankiw (1989) "Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence", NBER Working Paper No. 2924.
- Campbell, J. Y. and N. G. Mankiw (1990) "Permanent Income, Current Income, and Consumption", *Journal of Business and Economic Statistics* 8, 265-279.
- Denton, F. T. and B. G. Spencer (2000) "Population Aging and Its Economic Costs: A Survey of the Issues and Evidence", *Canadian Journal on Aging* 19, supplement, 1-31.
- Devereux, M. and D. Love (1994) "The Effects of Factor Taxation in a Two-Sector Model of Endogenous Growth", *Canadian Journal of Economics* 27, 509-536.
- Emery, J. C. H. and I. Rongve (1999) "Much Ado About Nothing? Demographic Bulges, The Productivity Puzzle, and CPP Reform", *Contemporary Economic Policy* 17, 68- 78.
- Howitt, P. (1997) "Low Inflation and the Canadian Economy", in D. Laidler (ed.) *Where We Go From Here*, Policy Study 29 (Toronto: C.D. Howe Institute).
- Jones, C. I. (1995) "Time Series Tests of Endogenous Growth Models", *Quarterly Journal of Economics* 110, 495-525.
- Kotlikoff, L. J. and B. Raffelhuschen (1999) "Generational Accounting Around the Globe", *American Economic Review, Papers and Proceedings* 89, 161-166.

Lueth, E. (2001) "Can Inheritances Alleviate the Demographic Burden?", IMF Working Paper No. 01/97.

Mankiw, N. G. (2000) "The Savers-Spenders Theory of Fiscal Policy", *American Economic Review, Papers and Proceedings* 90, 120-125.

Mérette, M. (2001) *An Economic Welcome to the Population Ageing in Canada*, (Montreal: Institute for Research on Public Policy).

OECD (1998) *Maintaining Prosperity in an Ageing Society*, ISBN 92 64 163131.

Office of the Auditor General of Canada (1998) "Population Ageing and Information for Parliament: Understanding the Choices", *Report of the Auditor General to the House of Commons-1998*, vol. 1, chapter 6.

Peterson, P. (1999) *Gray Dawn* (Times Books).

Raffelhuschen, B. (1999) "Generational Accounting in Europe", *American Economic Review, Papers and Proceedings* 89, 167-170.

Scarth, W.M. (2000) *Economics: The Essentials*, Second Edition (Toronto: Harcourt Canada).

World Bank (1994) *Averting the Old Age Crisis: Policies to Protect the Old and Promote Growth*, (Oxford: Oxford University Press).

Young, A. (2001) "Demographic Fluctuations, Generational Welfare and Intergenerational Transfers", NBER Working Paper No. W8530.

**Table 1**

Percentage increase or decrease in living standards  
Perfect foresight – All households forward-looking planners

Time Period	Living Standards of Young Generation	Living Standards of Old Generation	Living Standards of Average person Alive
1	+0.12 %	+4.80 %	+2.07 %
2	-0.09 %	-4.59 %	-2.18 %
3	-0.05 %	-0.02 %	-0.03 %

**Table 2**

Percentage increase or decrease in living standards  
Perfect foresight – 50% households forward-looking planners

Time Period	Living Standards of Young Generation	Living Standards of Old Generation	Living Standards of Average person Alive
1	-0.59 %	+4.56 %	+1.61 %
2	+0.20 %	-4.45 %	-1.98 %
3	+0.09 %	+0.05 %	+0.07 %

**Table 3**

Percentage increase or decrease in living standards  
Static expectations – 50% households forward-looking planners

Time Period	Living Standards of Young Generation	Living Standards of Old Generation	Living Standards of Average person Alive
1	-1.30 %	+4.37 %	+1.78 %
2	+0.95 %	-4.20 %	-1.51 %
3	+0.01 %	+0.00 %	+0.00 %

**Table 4**

Percentage increase or decrease in living standards  
 Static expectations – 50% households forward-looking planners  
 All savings sheltered from tax

Time Period	Living Standards of Young Generation	Living Standards of Old Generation	Living Standards of Average person Alive
1	-1.94 %	+4.98 %	+1.39 %
2	+0.89 %	-5.21 %	-2.35 %
3	+0.66 %	+0.40 %	+0.54 %

## Endnotes

---

<sup>1</sup> See Mérette (2001) for additional related references.

<sup>2</sup> The overall dependency ratio refers to the ratio of the population who are not of working age (both young and old) to those who are.

<sup>3</sup> For the empirical evidence on about 50 percent of the population being liquidity constrained, see Campbell and Mankiw (1989; 1990).

<sup>4</sup> We view the analysis as referring to the entire North American economy. Only by making this assumption, are we justified in applying this closed-economy model. For the analysis to be exclusive to Canada, we would have to consider a small open-economy version. In this case, the assumption of perfectly mobile capital would preclude factor price changes, so there is no mechanism for the scarce labor (that works while the baby-boomers are retired) to receive higher pre-tax wages.

<sup>5</sup> Bohn (1999) and Young (2001) consider these issues. Bohn focuses on whether the baby-boom generation can dominate the political process enough to destroy the viability of existing government programs, while Young considers the intrinsic bias against large cohorts that exists when a social planner maximizes the discounted welfare of an endless stream of generations.

<sup>6</sup> See Scarth (2000, p123).

<sup>7</sup> Other bases of comparison are worth mentioning. Disinflation was also hotly debated in the 1980s, and there has been scant evidence of its favorable effect on steady-state living standards. (Howitt (1997) concludes that the evidence for any net benefits “is certainly not overwhelming.”) This comparison suggests that aging is important. On the other hand, it is believed that skill-biased technical change has caused the 20 percent drop in the wages of young, unskilled males in the last quarter century. This base for comparison suggests that aging causes only modest effects.

## **Chapter 2**

### **Population Aging, Public Pensions and the Macroeconomy: An Analysis in a Small-Open Economy Framework**

#### **1. Introduction**

Over the coming decades the populations of many countries, particularly the OECD countries, will experience aging. This demographic shock will be caused mainly by the post-war "baby boom" generation heading for retirement, although the lower fertility and the greater longevity are also contributing factors. For Canada, the so-called old-age dependency ratio, normally defined as the ratio of the number of elderly (aged 65 and over) to the population of working age (15-64), is expected to nearly double (from about 0.2 now) by 2035, and then remain at about that level (see Hviding and Mérette, 1998, p.7; Denton and Spencer, 2000, p.4). Among the economic implications of aging populations, the issue which is perhaps ultimately of most concern to policy makers is the effect which such aging may have on per capita living standards. The empirical magnitude of this effect is the object of much debate. For a single economy such as Canada, this question has been examined using two general approaches: closed and open-economy specifications. In the former case, real wages and real interest rates are sensitive to domestic demographic conditions. However, applying the closed-economy model can only be justified if the analysis refers at least to the entire North American economy. For the analysis to be exclusive to Canada, we have to consider a small open-economy model. In this case, the assumption of perfect capital mobility pegs the interest rate. When combined with a standard constant-returns-to-scale production technology, perfect capital mobility also pegs the wage rate. Thus, in a small open-economy model, factor prices are not affected by domestic demographic conditions.

Furthermore, examining the macroeconomic consequences of aging populations for a single economy – using a small open-economy framework – might also be problematic, because assuming that the rate of interest is constant implies implicitly that

no similar demographic changes take place abroad. However, in the real world, population aging appears to be a world-wide phenomenon – although not perfectly synchronized. With a general aging process one would instead expect a fall in the world interest rates. Therefore one objective of this present chapter, is to use an overlapping generations small open-economy model – calibrated to Canada – to highlight the concern that by ignoring the influence of population aging on the rate of interest, we may systematically misestimate the effect which aging may have on a country's living standards and its net foreign asset position. As a consequence, the scope of policy responses – those designed to ameliorate or offset these effects – may be very inaccurate.

Moreover, following Mankiw's (2000) advice that all applied public finance analyses involving household saving must allow for the fact that a significant number of households live "hand-to-mouth" (and so do not save), we evaluate how aging populations may affect people differently depending on whether or not they save for the future.

Another aspect of the aging is its implications for public pension funding. The aging population may undermine the sustainability of pay-as-you-go (PAYG) social insurance arrangements like the Canada/Quebec Pension Plan (C/QPP). However, the financial viability of the PAYG system can be maintained – despite an aging population – if payroll taxes are increased (as has been the ongoing plan in Canada). But some of these increases in payroll taxes may raise the natural unemployment rate, with the result that the transfers to retirees will be at least partly funded by reducing economic outcomes for younger individuals at the lower end of the income scale. This outcome is often overlooked. Thus, another purpose of this chapter is to illustrate the magnitude of this effect. We also examine whether it is better to have the contributions of workers or firms adjust to finance pension benefits as the elderly dependency ratio increases.

The remainder of the chapter is organized as follows. The next section briefly describes our methodology. Section 3 reviews the literature to motivate our modeling approach. In Section 4 we fully describe our model and in Section 5 we discuss its theoretical results regarding the effects of payroll taxes on unemployment, and compare

them to those derived in some other studies. Section 6 explains how we derived analytical expressions used in the computer simulation experiments to examine the broader consequences of an aging population. Section 7 presents and discusses the baseline calibration of the model. In Section 8 we report and analyze illustrative quantitative results (with sensitivity tests) from the computer simulations. Section 9 concludes the chapter.

## **2. Research Methodology**

The core of the analysis is a model of a small open economy that involves overlapping generations and life-cycle features. It builds on Blanchard's (1985) and Nielsen's (1994) extensions of the representative-agent model of household behavior.

In this chapter, the overlapping generations model is extended in two important ways. First, following Mankiw's (2000) suggestion, there is a subset of households that are liquidity constrained and cannot afford to save. Second, a model of asymmetric information in the labor market (Summers, 1988) is added, so that involuntary unemployment emerges for a subset of the liquidity constrained households. This model of efficiency wages (which is well-suited for numerical calibration) is inserted within the overlapping generations structure without exposing the analysis to Ambler's (1994) criticism of earlier work (e.g. Phelps, 1994) that shared these objectives. As in Nielsen, there is a PAYG public pension system. The forward-looking households who plan for the future have both private savings and the public pension available during their retirement years; the liquidity-constrained ones have only the public pension and transfer payments in old age. Some in the latter group rely on income transfers before retirement as well – since there is an employment insurance system. In the model, as in Canada, both the public pension and employment insurance programs are financed by earmarked proportional wage taxes levied on firms and their employees.

In the literature, pension systems and unemployment have traditionally been studied in separate classes of models. Pension systems are typically analyzed in intertemporal models that focus on savings incentives provided by the various systems.



The usual assumption is that the labor market is perfectly competitive, so that there is no unemployment. Models of equilibrium unemployment usually neglect the role of pension systems. In a recent paper, Corneo and Marquardt (2000) provide a step toward a more integrated analysis of the PAYG public pension and employment insurance programs. They develop a two-period overlapping generations model with endogenous growth that highlights the interaction between the two social security systems (SSS) in the presence of a labor market with union wage-setting. Although they do not examine the aging issues, they illustrate that valuable insights can be gained – particularly in terms of growth policy – by studying branches of the social security system in a unitary framework. For example, the contribution rates to the SSS are shown to exert growth effects, which would not arise if only one of these programs were present. As a result, some economic implications that arise from the externalities between programs may be overlooked if one program is analyzed in isolation. In line with these authors, this chapter provides a more integrated approach and analyzes the spillover effects between the public pension system and unemployment within the same macro model in the context of an aging population. Moreover, the behavior of other important macro variables, such as consumption and foreign debt accumulation, are addressed.

### **3. Literature Review**

In general there is no simple aggregate consumption function in an economy composed of finitely lived agents. This is because agents differ in two respects. Being of different ages, they have different levels and compositions of wealth. Having different horizons, they have different propensities to consume out of wealth. This problem makes exact or approximate aggregation impossible (Modigliani, 1966).

The solution adopted by Diamond (1965) was to choose a very simple population age structure, avoiding altogether the need for aggregation. The solution chosen by Blanchard (1985) is, instead, to make assumptions that allow aggregation. The central assumption is that agents face, throughout their lives, a constant instantaneous probability of death  $p$ . Thus their expected life is  $(1/p)$ . Agents are of different ages and have

different levels of wealth, but they have the same planning horizon and the same propensity to consume. This allows one to solve the aggregation problem.

The main advantage of this approach is its flexibility. If we think of  $(1/p)$  as the horizon index, we can choose it anywhere between zero and infinity. In particular, by letting  $p$  go to zero, agents have infinite horizons (Ramsey, 1928; Barro, 1974). The aggregate consumption function in this overlapping generations model with no bequests has a particularly simple and tractable form: it is proportional to the sum of aggregate human and nonhuman wealth. The marginal propensity to consume out of total wealth is  $(p+\delta)$ , where  $\delta$  is the rate of time preference. Introducing the government budget constraint, Blanchard demonstrates that government debt is a net contributor to wealth because future taxes are discounted by the probability of death plus the interest rate, implying that the present value of future primary balances is less than the stock of debt.

While this approach captures the finite horizon aspect of life, its main drawback is that it ignores the change in behavior over life, the "life-cycle" aspect. In that respect, Nielsen (1994) extended Blanchard's model by allowing retirement. He investigates the consequences of changes in a small open economy's unfunded social security system (PAYG public pension system) for its net foreign asset position. Assuming that the social security system is self-financing (in the sense that at any time  $t$  the taxes collected from the young correspond to the benefits paid to the old), Nielsen shows how an increase in the social security system's benefit rate is likely to lead to higher foreign indebtedness, even when "induced" retirement is allowed for. Feldstein (1974) identified two effects on private saving from the introduction of a social security (PAYG pension) system. On one hand, the value of promised social security benefits would reduce people's need to save: social security wealth would replace private wealth. On the other hand, expected social security benefits might alter the timing of retirement: individuals might retire earlier and accordingly wish to save more during their years of employment to prepare for a longer period of retirement. The question was whether the former "replacement" effect would dominate the latter "induced retirement" effect. If it did, social security would lead to a reduction in private savings and thereby national wealth.

In Blanchard's model, there is no population growth. Weil (1989) considers a variant of Blanchard's model where new households continuously enter the economy over time but existing households do not leave. He illustrates that the arrival of new households is sufficient to generate most of the main results of the Diamond and Blanchard models. Buiter (1988) demonstrates that the combination of a nonzero birth rate and the absence of operative intergenerational bequest motives yields debt nonneutrality in the Blanchard-Weil framework.

James (1994) seeks to determine the real effects of government deficits and debt in Canada. To perform the analysis, he simulates a dynamic, computable, general equilibrium model of an open economy, calibrated to Canadian data. His model uses the Blanchard-Weil-Buiter overlapping generations framework. The key result is that debt reduction, by increasing total saving today and by allowing future distorting taxes to be reduced, leads to increased output and consumption in the future. If, however, the initial debt reduction is achieved by raising highly distorting taxes, then the short-term costs thus induced exceed the long-term benefits.

Burbidge and Scarth (1995) use Blanchard's (1985) formulation of the overlapping generations model and the social welfare function advocated by Calvo and Obstfeld (1988) to analyze interest taxation and tariffs in a small open economy. They find that when the revenue lost from the elimination of these taxes is replaced by raising the tax on labor, as in the Canadian experience, a "low" social discount rate is required for the model to support lower interest-income taxes, while a "high" discount rate is needed to support the elimination of tariffs.

Furthermore, since consumption smoothing is far from perfect, many people have net worth near zero, and bequests are an important factor in wealth accumulation, Mankiw (2000) suggests an alternative model for analyzing fiscal policy. According to Mankiw, a better model to analyze the macroeconomic effects of fiscal policy should acknowledge the great heterogeneity in consumer behavior that is apparent in the data. It should include both low-wealth households who fail to smooth consumption over time and high-wealth households who smooth consumption not only from year to year, but

also from generation to generation. That is, there is a need for a model in which some consumers plan ahead for themselves and their descendants, while others are liquidity constrained and live paycheck-to-paycheck. All these features are shared by our model, except one: there is no bequest motive for the forward-looking households. There is a very good case against this assumption since the seminal work of Kotlikoff and Summers (1981). However, as noted by Rios-Rull (2001, p.2), two reasons can explain this assumption. First, assuming that all assets are held for life cycle motives is in part to be in line with the literature of population aging that has always used overlapping generations models with little role for bequests. Second, we do not have yet models that integrate a suitable theory of wealth inequality and a sophisticated demographic structure. In fact, this assumption simplifies the analysis (actually it makes it feasible). See Quadrini and Rios-Rull (1997) and Castaneda *et al.* (2000) for further arguments of what is a good theory of the wealth distribution.

In earlier work James (1994) considered an extension similar to that of Mankiw (2000) – with one group of households holding all the capital and receiving no labor income, the other holding all the government debt and earning all the labor income, and with neither being liquidity constrained.

Finally, although our model is much smaller, it has many broad characteristics in common with the Canadian Policy Analysis Model (CPAM) and the OECD's world model (Minilink).

CPAM is designed to provide a reasonably complete representation of the Canadian macroeconomy.<sup>1</sup> It represents a small open economy that produces a single domestic good. There are four groups of domestic agents in the model: firms, consumers, and the fiscal and monetary authorities. Profit-maximizing firms combine labor and capital in a Cobb-Douglas technology to produce the single good. Trend population growth and trend productivity growth are exogenous. Consumers come in two types. There are forward-looking consumers who make decisions with a view to picking the best path for current and future consumption, and "rule-of-thumb" consumers who spend all their income in each period. Thus, all assets are held by the forward-looking agents.

The behavior of the forward-looking consumers is characterized using the Blanchard (1985)-Weil (1989) model of overlapping generations, but in a discrete-time format. CPAM differs from our model in that our model has no monetary authority, no population growth, and our extension of Blanchard's model is in continuous time.

As for Minilink, it is a multi-region dynamic general equilibrium macroeconomic model of the world economy designed to examine policy issues over a time horizon of many decades.<sup>2</sup> It comprises five regional blocs – the United States, Japan, the European Union and the rest of the world, divided into fast and slow-aging regions – linked through international financial and goods market relationships. Each regional model involves three types of agents – households, firms and government. Some consumers are liquidity constrained and the behavior of the forward-looking consumers is based on an extended version of the Blanchard (1985) model – one in which labor income varies with age and the probability of death is a linear positive function of the old-age dependency ratio. The production technology used by the firms is a two-factor (capital and labor) constant-returns Cobb-Douglas function with exogenous labor-augmenting technical progress. Factors of production are assumed to be fully utilized. Each region is assumed to produce a single good, which is an imperfect substitute for the goods produced by other regions. Finally, the population growth rate is exogenous and the model operates in discrete time.

The OECD's economics department has used Minilink to analyze the macroeconomic implications of aging in global context. A "business-as-usual" case is examined in which, without improvements in labor market performance or specific policy adjustments to allow for the pressures of aging, economic growth is projected to slow significantly over the next 50 years in nearly all OECD countries; world real interest rates remain stable at current levels or even rise, because of the negative impact of aging on private savings, initially concentrated in the OECD where population aging occurs most rapidly. In the absence of specific policy adjustments, aging populations will also tend to reduce the growth of living standards (living standards measured as the level of output per capita) in the OECD as the output from any given number of workers is divided by a greater total population. Thus by 2050, the direct cumulative effects of the

rise in dependency ratios is to reduce per capita living standards by about 10, 18 and 23 percent for the United States, the European Union and Japan, respectively, relative to a situation where (other things being equal) dependency ratios remain at current levels. Further analysis using model simulations, illustrate that only a combination of policies (such as measures which promote improvement in total factor productivity and increase public saving), carried out on a timely and coordinated international basis is likely to be successful in avoiding the slowdown in living standards that will be caused by aging.<sup>3</sup>

However, empirical evidence on the impact of population aging on private savings is very mixed. Meredith (1995), reviewing the empirical literature, shows that estimates of the sensitivity of the savings ratio to the dependency ratio vary considerably depending on whether they are based on microeconomic or macroeconomic, time series or cross-section data. For example, on the basis of pooled time-series estimation across the major industrial countries, Masson and Tryon (1990) obtain an estimate suggesting that an increase of 1 percentage point in the dependency ratio causes a corresponding fall of 1 percentage point in the savings ratio. At the other extreme, household survey evidence typically suggests only a weak (or even positive) effect of the dependency ratio on private savings rates (see for example Auerbach *et al.*, 1991, Canari, 1994, and Borsch-Supan, 1996). A recent empirical study using pooled time-series evidence for 21 industrial countries by Masson *et al.* (1995) provides further empirical support for a weak effect of dependency ratios on private savings rates (according to which a 1 percentage point rise in the dependency ratio reduces the private savings rate by 0.14 percent).

In the version of the Minilink model used to generate the long-term reference scenario, it is assumed that a 1 percentage point rise in the dependency ratio reduces the private savings rate by 0.3 percent. This effect operates through a “probability of death” parameter which has a direct effect on the propensity to consume out of wealth.<sup>4</sup> In this case, world real interest rates rise by up to 0.7 of a percentage point by 2035. However, in an alternative simulation where there is no demographic effect on private savings behavior, world interest rates fall by 1.5 percentage points by 2035.<sup>5</sup> This effect stems from the world-wide slowdown in output growth, caused by a decline in the labor force,

which reduces the marginal productivity of capital and so reduces interest rates. At the slower growth rate, less investment is needed to maintain the required capital stock.

Thus, from the perspective of the world economy, there is likely to be an *ex ante* reduction in both investment and savings, and the uncertainties follow from which is likely to fall by more.

One of the main contributions in this chapter is to insert *appropriately* a model of asymmetric information in the labor market within the overlapping generations framework, so that involuntary unemployment emerges. Phelps' (1994) earlier attempt has been criticized by Ambler (1994). As mentioned, the assumption of a constant probability of death facilitates the aggregation of individuals' demands. Since there is labor market rationing in Phelps' model, Ambler stresses that individual's demand and asset accumulation equations depend both on previously accumulated wealth and on employment status. This introduces an additional degree of heterogeneity compared to the original Blanchard model, and the associated aggregation problem is ignored by Phelps.

Involuntary unemployment appears to be a persistent feature of many modern labor markets (Shapiro and Stiglitz, 1984, 1985). The presence of such unemployment raises the question of why wages do not fall to clear labor markets. One explanation is that, in an asymmetric information and moral hazard situation, firms may not lower their wages, even in the face of unemployment, if net productivity depends on wages. Profits may fall when wages are reduced, if reducing wages influences productivity by affecting workers' effort. As a result, wage rigidities and involuntary unemployment arise.<sup>6</sup>

Theories in which there is a cost as well as a benefit to the firm of paying lower wages are known as *efficiency-wage* theories. (The name comes from the idea that higher wages may raise the productivity, or efficiency, of labor.) The central idea is that if firms cannot monitor their workers' effort perfectly, they may pay more than market-clearing wages to induce workers not to shirk. If it pays one firm to raise its wage, it will pay all firms to raise their wages. When they all raise their wages, the demand for labor decreases, and unemployment results.

However, different kinds of models come under the heading of "efficiency-wage theory". Three of the most important are: moral hazard or shirking, adverse selection, and sociological models (summaries of efficiency wage models appear in Johnson and Layard, 1986; Weiss, 1991; and Phelps, 1994). Here, we focus on shirking models. Nonetheless, the ideas behind the latter two are the following: paying a higher wage can improve workers' ability along dimensions the firm cannot observe. Specifically, if higher-ability workers have higher reservation wages, offering a higher wage raises the average quality of the applicant pool, and thus raises the average ability of the workers the firm hires. In the last case, a high wage can build loyalty among workers, and hence induce high effort; conversely, a low wage can cause anger and desire for revenge, and thereby lead to shirking or sabotage. Thus, all efficiency wage models have one common feature, namely that the wage enters the production function in a labor-augmenting way.

The efficiency-wage theory of structural unemployment, of which Summers' (1988) model is an example, combines the rigor demanded by classical economists (given its grounding in formal optimization) with a well-defined source of market failure (which is at the core of Keynesian economics). As a result, many influential economists regard it as the most promising framework for understanding unemployment. (See, for example, Blanchard and Fischer, 1989, p.463).

The final prerequisite for model selection is that it can be readily calibrated, so that illustrative and empirically relevant *quantitative* results can be derived. We have chosen Summers' (1988) model of efficiency wages for this reason.<sup>7</sup> Romer (1996) has drawn attention to this model as a particularly compact version of efficiency wage theory, and we have extended it to allow for various taxes and transfer payments, a variable capital stock, and two types of labor, high and low-skilled workers or high and low-paid workers. It is worth mentioning that both the Summers model and our extension of it have a property which is consistent with economic history. It relates to the vast increase in productivity over the past century, associated with no long-term trend in the unemployment rate. In our model, productivity has no effect on the unemployment rate;



some other formulations (for example, Shapiro and Stiglitz, 1984) do not involve this appealing property.

Finally, regarding the integrated approach, our model on the whole is very close in spirit to a recent paper by Corneo and Marquardt (2000). They develop a two-period overlapping generations model with endogenous growth that highlights the interaction between PAYG public pension and employment insurance programs in the presence of a labor market with union wage-setting. Although they do not examine the aging issues, they illustrate that valuable insights can be gained – particularly in terms of growth policy – by studying branches of the social security system in a unitary framework. As a result, some economic implications that arise from the externalities between programs may be overlooked if one program is analyzed in isolation.

However, in the two-period overlapping generations model, the length of a "period" is one generation – often taken to be 30 years (see Barro and Sala-i-Martin, 1995, p.131). One drawback of this framework – as noted by Barro and Sala-i-Martin – is that it involves the assumption that there is a 30-year lag between the act of abstaining from consumption and the actual use of the newly produced output as capital. Therefore, we opt to extend the Blanchard (1985)-Nielsen (1994) specification of overlapping generations. Moreover, while the presence of unions in the labor market is an important institutional feature of European labor markets – where over three-quarters of the workforce have wages that are covered by collective bargaining – the rate of unionization is very small in North America. In Canada, the rate is about 30 percent and mostly concentrated in the public sector. As a result, we instead assume a model of asymmetric information in the labor market where firms set wages. This model of efficiency wages as well as the union model generates involuntary unemployment.

#### **4. The Model**

The core of the analysis is a model of a small open economy that involves overlapping generations and life-cycle features. There are three groups of domestic agents: firms, consumers, and the government. Profit-maximizing firms combine labor

and capital in a Cobb-Douglas technology to produce a single good. Trend productivity growth is exogenous and there is no population growth. At any time, the economy comprises two age groups: the working age people (workers and unemployed) and the old (pensioners). Consumers come in two types. There are forward-looking consumers (high-paid workers) who make decisions with a view to picking the best path for current and future consumption, and "rule-of-thumb" consumers (low-paid workers) who spend all their income in each period. Thus, all assets are held by the forward-looking agents. These two types of consumers are in the same proportion in both working age people and pensioners,  $\pi$  and  $(1 - \pi)$ . The behavior of the forward-looking consumers is characterized using the Blanchard (1985)-Nielsen (1994) model of disjoint overlapping generations with retirement. In the model, as in Canada, both the PAYG pension and employment insurance programs are financed by earmarked proportional wage taxes levied on firms and their employees.

We now give the details of each of the "building blocks" of the model.

#### **4.1 Firms**

There is a large number of identical competitive firms. The production function for the representative firm is of the form:

$$Y = K^\alpha (QN_h)^\beta (Q\mu N_l)^{1-\alpha-\beta} \quad (1)$$

where  $N_h$  and  $N_l$  denote, respectively, the employment of high and low-paid workers,  $K$  is the capital stock,  $Q$  the labor-augmenting technical progress that grows exogenously at rate  $n$ , and  $\mu$  is an index of effort per low-paid worker. Notice that there is no work effort index for high-wage workers because we make the assumption that these workers do not shirk, since they have "good" jobs which they enjoy. Assuming that firms cannot perfectly monitor the effort supplied by low-paid workers, they may pay them more than market-clearing wages to induce these workers not to shirk. As a result, only the low-wage workers can be unemployed in the economy. Though this assumption may be a little restrictive, in practice this group of workers represents the largest proportion of

the unemployed (see Layard *et al*, 1991, p.22; OECD, 1994, p.243). In addition, as Shapiro and Stiglitz (1984, p.443) state in their conclusion, involuntary unemployment is a significant factor in the observed level of unemployment, especially in low-paid, lower-skilled, blue-collar occupations.

Compared to the Cobb-Douglas production technology with only one type of labor input, an appealing feature of having two types of labor is that the assumption of perfect capital mobility – in the small open economy model – does not peg the wage rates.

Define  $y = Y/Q$ ,  $k = K/Q$  and  $w = W/Q$ , where  $W$  is the wage level. The problem facing the representative firm is to maximize its profits, which are given by

$$\underset{k, N_h, N_l, w_l}{Max} \quad Pr = k^\alpha N_h^\beta (\mu N_l)^{1-\alpha-\beta} - (r + \phi)k - w_h(1 + \tau_f + \theta_f)N_h - w_l(1 + \tau_f + \theta_f)N_l$$

subject to

$$\mu = \{w_l(1 - t_w - \tau_w - \theta_w) - x\}^\varepsilon, \quad (2)^8$$

where  $w_h$  and  $w_l$  denote, respectively, the wage paid to high and low-skilled workers (without including the effect of exogenous ongoing productivity growth on wages);  $\tau_f$  and  $\theta_f$  ( $\tau_w$  and  $\theta_w$ ) are the firm's (employees') contribution rates to the pension and employment insurance programs, respectively;  $r$  is the interest rate (which is assumed to be fixed by perfect capital mobility internationally);  $\phi$  the capital depreciation rate;  $t_w$  the personal tax rate applied to labor income; and  $x$  represents the alternative option available to low-wage workers, which is considered as given to the firm. Since  $\varepsilon$  is a positive fraction, equation (2) shows that a higher wage in the current job raises the low-paid worker's return relative to her alternative and thereby induces higher productivity. That gives rise to the interpretation of  $\varepsilon$  as the propensity to shirk.

Unlike financial and physical capital, both kinds of labor are assumed to be completely immobile across borders.

Setting the derivatives of profits with respect to  $k$ ,  $N_h$ ,  $N_l$ , and  $w_l$ <sup>9</sup> to zero, we have the following first-order conditions:

$$k: \quad \alpha \frac{y}{k} = r + \phi \quad (3)$$

$$N_h: \quad \beta \frac{y}{N_h} = w_h(1 + \tau_f + \theta_f) \quad (4)$$

$$N_l: \quad (1 - \alpha - \beta) \frac{y}{N_l} = w_l(1 + \tau_f + \theta_f) \quad (5)$$

$$w_l: \quad (1 - \alpha - \beta) \frac{y}{N_l} = \frac{[w_l(1 - t_w - \tau_w - \theta_w) - x]}{\varepsilon(1 - t_w - \tau_w - \theta_w)}(1 + \tau_f + \theta_f) \quad (6)$$

Combining equations (5) and (6) yields the wage setting rule for the low-paid workers

$$w_l = \frac{x}{(1 - \varepsilon)(1 - t_w - \tau_w - \theta_w)} \quad (7)$$

Without the asymmetric information that leads to shirking (that is, with  $\varepsilon = 0$ ), firms simply set the wage such that the after-tax wage is equal to the workers' outside option. With shirking, however, the wage is higher, and with all firms behaving in this fashion, employment is lower when the threat of shirking exists. The resulting unemployment is involuntary since firms do not accept offers from the unemployed to work for less than the profit-maximizing wage.

The low-paid workers' alternative is a weighted average of the after-tax wage received at the other firms and what is received if the individual cannot find work (which Summers assumes is a fraction  $f$  times  $w$ ). The weights in the average are the employment rate,  $(1-u)$ , and the unemployment rate,  $u$ , respectively. Hence, the alternative income is  $x = (1 - u)w_l^o(1 - t_w - \tau_w - \theta_w) + uf w_l$ , where  $w_l^o$  is the pre-tax wage offered at other firms.<sup>10</sup> When this definition is substituted into the wage-setting rule (equation (7)) with  $w_l^o = w_l$  in equilibrium, we get the following unemployment rate

$$u = \frac{\varepsilon(1 - t_w - \tau_w - \theta_w)}{(1 - t_w - \tau_w - \theta_w) - f} \quad (8)$$

Thus, unemployment rises with the propensity to shirk ( $\varepsilon$ ), the personal tax rate applied to labor income ( $t_w$ ), employees' contribution rates to the social security systems ( $\tau_w$  and  $\theta_w$ ), and with the generosity of employment insurance ( $f$ ). Again, note that a rise in labor productivity has no effect on unemployment; instead, it simply raises real wages. This property of the model is consistent with economic history. There was a vast increase in productivity in the course of the twentieth century, with a corresponding increase in wages, but with no long-term trend in the unemployment rate. As noted earlier, many formulations of the efficiency-wage hypothesis do not have this appealing property.

#### 4.2 The public pension system

The population of the economy is set equal to unity (at any time  $t$ ). As in Blanchard's model, every individual has, regardless of the time of her birth, the same instantaneous probability of death,  $p$ , and the birth rate at any time is also equal to  $p$ . Thus, a cohort born at time zero has a size, as of time  $t$ , of  $pe^{-pt}$ . Since the population is split into those of working age and pensioners, the social security (pension) scheme determines – by way of a pension age,  $\lambda$  – the relative size of these two groups. Given the death (birth) rate  $p$  and the pension age  $\lambda$ , the former and latter groups constitute the shares  $(1 - e^{-p\lambda})$  and  $e^{-p\lambda}$  of the population, respectively.<sup>11</sup> Because the pension system is constrained to be self-financing in the sense that all pension benefits in any given time are financed by the total amount of contributions paid in that time by the firms and their employees, we must have

$$(\tau_f + \tau_w)(N_h w_h + N_l w_l) = e^{-p\lambda} m \quad (9)$$

where  $m$  is the pension benefit adjusted for the productivity growth.

Furthermore, since high-paid workers (forward-looking consumers) and low-paid workers (liquidity-constrained consumers) are in the same proportion in both working age people and pensioners,  $\pi$  and  $(1 - \pi)$ , and since only the low-paid workers can be unemployed, we have

$$N_h = \pi(1 - e^{-p\lambda}) \quad (10)$$

and

$$N_l = (1 - u)(1 - \pi)(1 - e^{-p\lambda}) \quad (11)$$

Inserting equations (10) and (11) into the pension plan budget constraint (equation (9)) yields

$$(\tau_f + \tau_w)[\pi w_h + (1 - u)(1 - \pi)w_l] = \frac{e^{-p\lambda}}{1 - e^{-p\lambda}} m \quad (12)$$

where  $\frac{e^{-p\lambda}}{1 - e^{-p\lambda}}$  is the old-age dependency ratio – the ratio of retirees to those of working age.

Given that equation (12) is balanced in different demographic environments, it is important for the analysis that follows to know which variable – whether  $\tau_w$ ,  $\tau_f$  (or both) or  $m$  – is set residually to adjust in response to unforeseen demographic changes (here, an increase in the elderly dependency ratio). This suggests a potential trade-off between efficiency and equity in the adjustment of the pension system to demographic shocks. On one hand, by holding the benefit payment fixed a higher dependency ratio would constitute a squeeze on the pension budget and the required rise in contribution rates could well have distortionary effects in e.g. the labor market. On the other hand, by targeting the contribution rates – and letting pension benefits fall endogenously – the associated intergenerational redistribution of income against pensioners would hardly be acceptable from a viewpoint of social equity. Since in the Canadian experience the contribution rates have been increased and since we are interested in estimating the magnitude of the distortionary effects on unemployment, we will fix (or target) the benefit payment at its previous steady state level and let the contribution rates adjust endogenously.

Note also that the projected increase in the old-age dependency ratio can be simulated by lowering either the retirement age,  $\lambda$ , or the probability of death,  $p$  (which is the birth rate as well; and  $1/p$  is the life expectancy). In this chapter, we will proceed

by lowering the retirement age. While the share of elderly people will continue to increase with both lower fertility and greater longevity, demographers predict that the increase in the next three decades will be caused mainly by the baby-boom generation heading for retirement, and the only way that can be simulated in the current framework is through an assumed reduction in  $\lambda$ .

### 4.3 Consumers

#### 4.3.1 Forward-looking consumers

In this subsection, we derive the aggregate consumption function for the forward-looking consumers. These individuals do not face any liquidity constraints. Their behavior is characterized using the Blanchard/Nielsen framework of overlapping generations with retirement. Since Blanchard's generalization of the representative agent formulation is familiar, we focus more on Nielsen's extension of this specification. The main drawback of Blanchard's approach is that while it captures the finite horizon aspect of life it ignores change in behavior over life, the "life-cycle" aspect. In that respect, Nielsen extended Blanchard's model by allowing for retirement. Here, we extend Nielsen's model by introducing exogenous productivity growth and various taxes and transfer payments, and apply it only to the subset of households that are forward looking.

To simplify the exposition, we ignore for the moment the tax on interest income and the fact that only a portion of the population has a rate of time preference low enough to cause them to operate with a forward-looking plan. As in Blanchard (1985), each decision maker has an instantaneous utility function equal to the log of consumption, and she discounts the future at rate  $(p + \delta)$ , where  $\delta$  and  $p$  are the rate of time preference and (constant) probability of death, respectively.

It is well-known under Blanchard's formulation (1985, p. 229, equation (5)) that aggregate consumption is a linear function of aggregate human wealth,  $H(t)$ , and financial wealth,  $V(t)$ ,

$$C(t) = (p + \delta)[H(t) + V(t)]$$

Taking the time derivative of this equation gives:

$$\dot{C}(t) = (p + \delta)[\dot{H}(t) + \dot{V}(t)]$$

Now what are  $\dot{V}(t)$  and  $\dot{H}(t)$ ? The accumulation identity for aggregate financial wealth,  $V(t)$ , is standard:

$$\dot{V}(t) = rV(t) + (1 - e^{-p\lambda})[1 - t_w(t) - \tau_w(t) - \theta_w(t)]W(t) + e^{-p\lambda}M(t) - C(t)$$

where  $M$  is the pension benefit. Three assets are available to citizens in the economy: physical capital stock,  $K(t)$ , public debt,  $B(t)$ , and net foreign debt,  $A(t)$ . In symbols,

$$V(t) = K(t) + B(t) - A(t)$$

As for human wealth we have

$$\begin{aligned} H(t) = & \int_{-\infty}^{t-\lambda} \int_t^{\infty} p e^{-p(t-s)} M(i) R(t, i) di ds \\ & + \int_{t-\lambda}^t \int_t^{s+\lambda} p e^{-p(t-s)} [1 - t_w(i) - \tau_w(i) - \theta_w(i)] W(i) R(t, i) di ds \\ & + \int_{t-\lambda}^t \int_{s+\lambda}^{\infty} p e^{-p(t-s)} M(i) R(t, i) di ds \end{aligned}$$

where  $R(t, i)$  denotes the death-rate-inclusive discount factor,  $R(t, i) = e^{-(r+p)(i-t)}$ .

This expression consists of three integrals. The first is the discounted pension benefits to be received by that part of the population that is already old (pensioners) at time  $t$  - i.e. those who were born prior to time  $t - \lambda$ . The second integral corresponds to the after-tax wages to be received in the future by the young (workers) from time  $t$  to their retirement at  $s + \lambda$ , after which they will receive pension benefits. The latter benefits are captured by the third integral.

Reversing the order of integration and differentiating by making use of Leibniz's formula, we have

$$\begin{aligned} \dot{H}(t) = & (r + p)H(t) - (1 - e^{-p\lambda})[1 - t_w(t) - \tau_w(t) - \theta_w(t)]W(t) \\ & + p e^{-p\lambda} \int_t^{t+\lambda} [(1 - t_w(i) - \tau_w(i) - \theta_w(i))W(i) - M(i)] e^{-r(i-t)} di \end{aligned}$$

Substituting  $\dot{V}(t)$  and  $\dot{H}(t)$  in the  $\dot{C}(t)$  equation, we obtain



$$\dot{C}(t) = (r - \delta)C(t) - p(p + \delta)V(t) + (p + \delta)e^{-p\lambda}M(t) + p(p + \delta)e^{-p\lambda} \int_t^{t+\lambda} [(1 - t_w(i) - \tau_w(i) - \theta_w(i))W(i) - M(i)] e^{-r(i-t)} di$$

To achieve a model which is stationary in equilibrium, this identity and the ones that follow are expressed in terms of variables that are ratios of each aggregate to labor-augmenting technical progress,  $Q$ , which grows at rate  $n$ .<sup>12</sup> Moreover, since tax rates are constant – except once-and-for-all changes – we have

$$\begin{aligned} \dot{c} = & (r - \delta - n)c - p(p + \delta)v + (p + \delta)e^{-p\lambda}m \\ & + \frac{p(p + \delta)}{r - n} e^{-p\lambda} (1 - e^{-(r-n)\lambda}) [(1 - t_w - \tau_w - \theta_w)w - m] \end{aligned} \quad (13)$$

This equation reduces to Blanchard's specification when  $\lambda = \infty$ , and it further reduces to the standard infinitely lived representative agent analysis if  $p = 0$ . Given this nesting, the model provides an appealing overlapping generations structure. Note that this equation differs from Nielsen's in three respects. First, since we split labor into two groups, equation (13) is derived without inserting the self-financing pension plan condition (equation (12)), and this explains the third element on the right-hand side of the equation (13). Second, tax payments are proportional to wages. And third, we have productivity growth.

Next, when we add the tax rate on interest income and use the fact that this theory applies only to the proportion  $\pi$  of the population (forward-looking consumers or high-paid workers) who hold all assets ( $k + b - a$ ) in the economy, we get

$$\begin{aligned} \dot{c}_h = & (r(1 - t_i) - \delta - n)c_h - p(p + \delta)(k + b - a) + \pi(p + \delta)e^{-p\lambda}m \\ & + \pi \frac{p(p + \delta)}{(r(1 - t_i) - n)} e^{-p\lambda} [1 - e^{-(r(1-t_i)-n)\lambda}] [(1 - t_w - \tau_w - \theta_w)w_h - m] \end{aligned} \quad (14)$$

where  $t_i$  is the tax rate applied to interest income.

### 4.3.2 Liquidity-constrained consumers

These individuals live hand-to-mouth and therefore fail to smooth consumption over time. One can associate hand-to-mouth behavior with individuals who have high discount rates and often face binding borrowing constraints. Thus, the workers, unemployed, and pensioners of this group spend all of their incomes (which are after-tax wages, employment insurance benefits, and pension benefits, respectively, plus their transfer payments). By use of equation (11), we have the following aggregate spending equation

$$c_l = (1 - u)(1 - \pi)(1 - e^{-\rho\lambda})w_l(1 - t_w - \tau_w - \theta_w) + fw_lu(1 - \pi)(1 - e^{-\rho\lambda}) + (1 - \pi)e^{-\rho\lambda}m + tp \quad (15)$$

where  $c_l$  is consumption of the low income group,  $f$  and  $tp$  denote, respectively, the generosity of employment insurance and transfer payments. On the right-hand side of equation (15), the first term represents the after-tax labor income of workers, the second term the employment insurance paid to those unemployed, the third element the pension payments, and the fourth – transfer payments that go to low income individuals whether or not they are working, and whether or not they are retired.<sup>13</sup>

### 4.4 Government and foreign debts

For simplicity we assume that the public debt is owned entirely by domestic residents and that the domestic physical capital stock and net foreign assets are perfectly substitutable. Then, one might think of foreign debt as the share of the physical capital stock employed domestically that is owned by foreigners. Foreign debt,  $a$ , increases whenever net interest payments on pre-existing foreign debt exceed the country's net exports<sup>14</sup>,

$$\dot{a} = (r(1 - t_i) - n)a - [y - ((n + \phi)k + \dot{k}) - g - c_h - c_l] \quad (16)$$

where  $g$  is government direct spending. Since there are no adjustment costs involved with installing physical capital, investment is given by  $(n + \phi)k + \dot{k}$ . Thus, only the amount

$z = k - a$  of the physical capital that is employed within the country is domestically owned.<sup>15</sup> By use of this definition, equation (16) can be written as:

$$\dot{z} = (r(1 - t_i) - n)z + y - (r(1 - t_i) + \phi)k - g - c_h - c_i \quad (17)$$

As for the evolution of government debt,  $b$ , we have

$$\dot{b} = d - nb \quad (18)$$

which states that public debt increases whenever the deficit,  $d$ , exceeds the economy's growth rate times the pre-existing debt. The government budget deficit is the excess of government spending over tax revenue. Using equations (10) and (11), we have

$$d = g + r(1 - t_i)b + fw_i u(1 - \pi)(1 - e^{-p\lambda}) + tp - t_i r k - (t_w + \theta_w + \theta_f)[\pi w_h + (1 - u)(1 - \pi)w_i](1 - e^{-p\lambda}) \quad (19)$$

The government spends on direct purchases, net interest payments on the outstanding debt, employment insurance benefits, and transfer payments. Revenue is derived from taxes on interest income and wages, and from the contributions to the employment insurance program made by firms and their employees.

We have assumed that the government collects tax on all the interest income that is generated within the country. In effect, the government levies a withholding tax on foreigners that is equal to the tax that is levied on domestic residents. Also, as in Canada, we considered employment insurance program surpluses (or deficits) as an element of the overall government budget constraint.

## 5. Theoretical results and comparisons with the existing literature

Some relevant theoretical results stem from the present model. In contrast to the employees', the firms' contribution rates to the pension and employment insurance programs are shown to have no influence on the unemployment rate; this follows from:

$$u = \frac{\varepsilon(1 - t_w - \tau_w - \theta_w)}{(1 - t_w - \tau_w - \theta_w) - f} \quad \text{or} \quad u = \frac{\varepsilon}{1 - \frac{f}{1 - t_w - \tau_w - \theta_w}}$$

Hence, attempts to fight unemployment by means of reductions in the payroll tax on firms are useless in this model. Nevertheless, a decrease in tax rates on workers results in a lower unemployment rate. To understand the intuition behind these results, consider each category in turn.

#### *Tax rates on firms*

An increase in firms' contribution rates decreases their willingness to pay for labor. Workers recognize this fact and so generally reduce their wage claims in line with the firms' decreased ability to pay. The fact that all firms have a similarly decreased willingness to pay is what leaves workers no option but to accept the wage cut. Thus, an increase in the employer payroll tax will reduce the net return to working; and since employment insurance benefit is a fixed proportion of the gross wage, it will also reduce (proportionately) the net returns from being unemployed. As a result, raising employer taxes will not influence the comparative attractions of working and being jobless with benefits, so unemployment will be unaffected by firms' tax rates changes. In fact, most empirical studies find that these payroll taxes are, for all intents and purposes, fully passed on to employees in the form of lower wages<sup>16</sup>, so firms' contribution rates are actually a "wage killer", not a "job killer". To put the point differently, in our model a cut in these payroll taxes does not raise employment, it only raises the wages of those who already have jobs.

#### *Tax rates on employees*

An increase in taxes levied on employees makes the work less attractive for them and therefore raises their propensity to shirk. More specifically, if an employee payroll tax rate (or income tax rate) is increased, this will reduce the return to working but not reduce the prospective benefit if jobless, since that benefit is computed as a fixed proportion of the gross wage and assumed to be tax free. Hence, a hike in the employee tax rates will make work relatively less attractive and induce workers to shirk. Firms react

by paying higher wages. As a result, the demand for labor decreases, and higher unemployment results. Thus, these taxes are a job killer. Clearly, cutting them decreases the propensity of employees to shirk. As the wage employers must pay decreases, firms hire more workers.

Regarding the effects of labor taxation in efficiency wage models, the existing literature leads to surprisingly different conclusions. Pissaro (1991) illustrates that the features of the effort function,  $\mu$ , are the source of the difference in the outcomes. In particular, three elements turn out to be important:

- (i) whether  $\mu(w, \cdot)$ <sup>17</sup> is a continuous and concave function;
- (ii) whether  $\mu(w, \cdot)$  depends on the expected alternative income earned outside the firm, say  $x$ ; and
- (iii) whether  $\mu(w, x)$  is homogeneous of degree zero in  $w$  and  $x$ .

The effort function in our model is consistent with (i) and (ii).

We can now compare our results with what other efficiency wage models predict. In the "rudimentary" efficiency wage model proposed for expository purposes by Yellen (1984)<sup>18</sup>,  $\mu$  is continuous but does not depend on  $x$ ; that is  $\mu = \mu(w)$ . In this case, an ad valorem tax on labor (a wage tax) has no effect on the after-tax wage (i.e. the tax is borne entirely by firms) and there is a negative effect on employment. On the other hand, a specific tax (an employment tax) leads to an increase in the after-tax wage.

In Johnson and Layard (1986),  $\mu$  is continuous and homogeneous of degree zero in  $w$  and  $x$ , that is  $\mu = \mu(w/x)$ . In this framework, an ad valorem tax will be borne entirely by workers, leading to a reduction in the after-tax wage by the full amount of the tax and leaving employment unaffected (the same results as in our model, but the prediction is completely reversed when compared to the "rudimentary" model). And a specific tax may either lower or raise the after-tax wage but it will definitely cause a decrease in employment.

In Shapiro and Stiglitz (1984),  $\mu$  depends also on  $x$ , but is not homogeneous of degree zero, and can assume only two values: 0 and  $\mu^* > 0$ . In this setting, the effect of

an increase in the two taxes is exactly the same: a decrease in the after-tax wage, but by less than the full amount of the tax, and a decrease in employment.

Finally, in Pisauro (1991),  $\mu$  is continuous between 0 and 1, concave, and is a positive function of the wage, unemployment rate, and detection rate, and a negative function of unemployment benefits and the (constant) marginal disutility of effort. (Recall that in our model,  $x$  also depends on  $w, u, f$ ). In Pisauro's model, ad valorem and specific taxes on labor do not have the same impact on the real wage. The imposition of a specific tax causes an increase in the net real wage. However, both taxes have a negative effect on employment. This difference with respect to what our model predicts comes from the fact that in Pisauro's model, the effort function involves the restriction  $\mu_{wu} = 0$ .

Regarding these various assumptions about the effort function, Pisauro states that the alternative income,  $x$ , seems an essential ingredient of a moral hazard model with some micro foundation. But there are no reasons, apart from simplifying the problem (with some costs), to suppose either that there is only one possible positive level of effort (as should be obvious on the ground of realism), or that  $\mu(w, x)$  should be homogenous of degree zero in  $w$  and  $x$ . On a priori grounds, a desirable effort function should therefore be continuous and have the general form  $\mu(w, x)$ . In that respect, readers can notice that the effort function in our model is consistent with these recommendations.

## 6. Population aging effects

In order to examine the broader effects of an aging population (as represented by an increase in the old-age dependency ratio) we have constructed a calibrated version of the model presented above. Collecting some equations together the model reduces to the following system:

$$y = \psi k^\alpha (1 - e^{-p\lambda})^{1-\alpha} (1 - u)^{1-\alpha-\beta} [w_l(1 - t_w - \tau_w - \theta_w)]^{\alpha(1-\alpha-\beta)} \quad (1')$$

where  $\psi$  embodies all constant terms.

$$\alpha \frac{y}{k} = r + \phi \quad (3)$$

$$\beta \frac{y}{\pi(1 - e^{-\rho\lambda})} = w_h(1 + \tau_f + \theta_f) \quad (4')$$

$$(1 - \alpha - \beta) \frac{y}{(1 - u)(1 - \pi)(1 - e^{-\rho\lambda})} = w_l(1 + \tau_f + \theta_f) \quad (5')$$

$$u = \frac{\varepsilon(1 - t_w - \tau_w - \theta_w)}{(1 - t_w - \tau_w - \theta_w) - f} \quad (8)$$

$$(\tau_f + \tau_w)[\pi w_h + (1 - u)(1 - \pi)w_l] = \frac{e^{-\rho\lambda}}{1 - e^{-\rho\lambda}} m \quad (12)$$

$$\begin{aligned} \dot{c}_h &= (r(1 - t_i) - \delta - n)c_h - p(p + \delta)(z + b) + \pi(p + \delta)e^{-\rho\lambda}m \\ &+ \pi \frac{p(p + \delta)}{(r(1 - t_i) - n)} e^{-\rho\lambda} (1 - e^{-(r(1 - t_i) - n)\lambda}) [(1 - t_w - \tau_w - \theta_w)w_h - m] \end{aligned} \quad (14')$$

$$\begin{aligned} c_l &= (1 - u)(1 - \pi)(1 - e^{-\rho\lambda})(1 - \tau_w + \theta_f)w_l + (1 - \pi)e^{-\rho\lambda}m \\ &+ d - g - r(1 - t_i)b + t_i r k + (t_w + \theta_w + \theta_f)\pi(1 - e^{-\rho\lambda})w_h \end{aligned} \quad (15')$$

$$\dot{z} = (r(1 - t_i) - n)z + y - (r(1 - t_i) + \phi)k - g - c_h - c_l \quad (17)$$

These equations determine  $y$ ,  $k$ ,  $w_h$ ,  $w_l$ ,  $u$ ,  $\tau$ <sup>19</sup>,  $c_h$ ,  $c_l$ ,  $z$ .

To get equation (1') we divided (1) by  $Q$ , then substituted in equations (2) (using the  $x$  definition), (10) and (11). Equations (4') and (5') result from using equations (10) and (11), respectively. The use of the definition,  $z = k - a$ , implies (14'). Equation (15') follows from inserting (19) (substituting out transfer payments,  $tp$ ) into (15). Finally, note that equation (18) is ignored since the government debt is kept constant in all experiments reported below. In fact, the level of government transfer payments adjusts residually to keep the budget deficit ratio constant.

Next, we take the total differential of the system and eliminate the changes in  $y$  and  $k$  by substitution. Then, we focus on a set of seven equations that determine changes

in  $z$ ,  $c_h$ ,  $c_l$ ,  $w_h$ ,  $w_l$ ,  $u$  and  $\tau$ . We denote absolute changes by  $d$  and percentage changes by  $\Delta$ .

Thus, the model can be summarized as follows:

$$\Omega [dz \ dc_h \ dc_l \ \Delta w_h \ \Delta w_l \ du \ d\tau]' = \Theta [dz \ dc_h]' + \Gamma [d\lambda \ dr]' \quad (20)$$

where  $\Omega$ ,  $\Theta$ , and  $\Gamma$  are  $(7 \times 7)$ ,  $(7 \times 2)$  and  $(7 \times 2)$  matrices, respectively.<sup>20</sup>

## 7. Model calibration

As a starting point for the analysis, the model is solved for an initial steady-state equilibrium in which all key ratios (most stock and flow variables being expressed as proportion of output)<sup>21</sup> are assumed to be constant. This numerical steady state is obtained by choosing values for the fundamental parameters of the model and all the stock and flow variables, to reflect as far as possible the economic situation in Canada in the years 1999-2000. This is achieved by selecting ratios on the basis of values observed in data and then using the model's equations to determine the remaining variables residually to insure internal consistency. The values for the key ratios and parameters are reported in Table 1.

Since we exclude childhood and the period of life when people receive their education (people aged 1-20, say), our model does not distinguish between start of life and start of *working* life. Hence the period from age zero to  $\lambda$  covers the working life period which we set to 40. These assumptions imply that people retire on average at about age 60. Also, the assumed probability of death, 0.02, implies a life expectancy of 50 years<sup>22</sup> (or 70 years if we think of life "beginning" at age 20). These values appear to be a fair approximation for Canada and most OECD countries.

However, as just indicated, given the assumptions regarding the steady-state of certain stock variables, the values of many flow variables are automatically determined by equilibrium conditions and, hence, some differ significantly from recently-observed values. For example, since the government debt and economy's growth rate are positive,



one of the government budget identities (equation (18)) implies a budget deficit-to-GDP ratio of 1.06 percent in the baseline "steady state".<sup>23</sup> The initial ratios of total private consumption, investment and net exports to GDP (0.614, 0.24 and 0.006 respectively) are implied by the variable and parameter values in Table 1.

As noted, the selected parameter values must satisfy all steady-state restrictions of the model. To accomplish this, some parameters have to be chosen residually, and the rate of time preference for forward-looking consumers and capital's share of output were selected in this regard. Since this implied rate of impatience (0.0332) is less than the interest rate (0.05), the consumption of these individuals rises as they age. (No specific rate of time preference is involved for the hand-to-mouth group; it is assumed that these individuals, in addition to earning lower labor income, are sufficiently impatient that they never save.)

Beach and Slotsve's (1996) examination of data indicates that the lowest two quintiles of the earnings distribution in Canada get about 20 percent of earnings. On the other hand, Wolff (1998) reports that the lowest two quintiles of the financial wealth distribution hold only 0.2 percent of household wealth. Based on these facts, we have chosen appropriate values for  $\beta$  and  $\pi$ .

The reported values for the employment insurance (EI) contribution and payout rates are lower than the actual legislated values. The reason for this discrepancy is that the actual program involves no payroll taxes beyond certain income levels, a qualifying period involving no EI benefits, and a maximum number of weeks of benefits. Since we have followed Summers and modeled an EI system without such limits, we have had to reduce the replacement ratio and the payroll tax rates accordingly (as OECD analysts do when making international comparisons; see Martin, 1996). Nonetheless, the ratio of the employer and the employee contribution rates reflects the 1.4 factor in the Canadian system.<sup>24</sup>

Next, we justify the calibrated values for the employee and employer pension contribution rates (the last two parameters in Table 1) which are higher than the actual values. The reason for this discrepancy is that in the model, the old-age dependency ratio

is given by  $DR = \frac{e^{-p\lambda}}{1 - e^{-p\lambda}}$  (the ratio of the share of retirees to the share of working-age people in the population), where  $p$  and  $\lambda$  are the death probability and retirement age, respectively. Hence, it is obvious that we cannot choose independent values for  $p$ ,  $\lambda$  and  $DR$ ; one parameter has to be determined by the equation. As a consequence, although our selected values for the probability of death and retirement age are individually reasonable, together they imply a dependency ratio that corresponds more closely to the “overall” dependency ratio<sup>25</sup> than to the elderly dependency ratio in Canada. With this implied (higher) dependency ratio, the actual contribution rates to the public pensions (CPP/QPP) lead to a relatively small individual pension payment (as a proportion of the average wage) of 9 percent. Therefore, in the baseline steady state, we have raised the contribution rates such that the pension payment as a proportion of the average wage is about 21 percent – which is a sensible number.

Finally, the third column in Table 1 shows our "estimate" of the shock to the retirement age that leads to the projected increase in the dependency ratio as the baby-boom generation retires, and the change in the world real interest rates that might be expected to accompany this demographic event. Over the next 35 years, the ratio of the Canadian labor force to the Canadian population is expected to fall from 0.525 to 0.475 – a 10 percent reduction.<sup>26</sup> The main reason for this development is that the baby-boom generation is growing older (see Mankiw and Scarth (2001, p.104)). The perturbation to the retirement age in Table 1 is consistent with this fact.<sup>27</sup> Furthermore, according to the estimates reported by the OECD's economics department, world-wide population aging would lead to changes in world real interest rates that vary between (-1.5 to + 0.7) percentage points by 2035, depending on the assumed effect of population aging on private savings (see Figure 7 in Turner *et al.* (1998, p. 69)).<sup>28</sup> We have chosen a number that lies in the middle of this range – a drop of 40 basis points.

Furthermore, although we focus on the small open economy, the model could be solved in steady state as a closed economy to generate an endogenous interest rate change following an increase in the dependency ratio. Obviously, this procedure may be

appealing if we assume that the calibration to the Canadian economy is representative of the entire Western world, and that the aging population phenomenon will proceed roughly the same way throughout all these countries. Thus, applying this procedure generates a drop in the interest rate that is just below 0.4 percentage point.

## 8. Quantitative results

### 8.1 Long and short-run effects of increased elderly dependency ratio

We consider first the cases in which domestic population aging (or an increase in the domestic old-age dependency ratio) is not accompanied by aging in the rest of the world, and so does not involve any change in the rate of interest.<sup>29</sup> At the start, we assume that both the employee and employer contribution rates adjust (equally) to balance the pension plan budget constraint as the dependency ratio increases. Then, using Gauss to solve the model yields the following two differential equations

$$\begin{bmatrix} \dot{dz} \\ \dot{dc}_h \end{bmatrix} = \begin{bmatrix} 0.020000 & -1.000000 \\ -0.001064 & -0.013209 \end{bmatrix} \begin{bmatrix} dz \\ dc_h \end{bmatrix} + \begin{bmatrix} 0.007102 \\ -0.000022 \end{bmatrix} d\lambda \quad (21)$$

Denote the 2x2 matrix above by  $\Psi$ , henceforth. Since  $c_h$  is a "jump" variable, while  $z$  is predetermined at each point in time, a unique convergent solution requires that there be just one negative eigenvalue for matrix  $\Psi$ . Given that the determinant of  $\Psi$  is the product of the two eigenvalues, saddlepath stability requires  $\det(\Psi) < 0$ ; that condition is met.

Following an unexpected but permanent reduction in the retirement age, which raises the dependency ratio, the dynamics of evolution in (domestic) forward-looking aggregate consumption and financial wealth (composed of ownership of domestic capital stock and net financial claims on foreigners) is illustrated in the phase diagram in Figure

1.<sup>30</sup> From matrix  $\Psi$  it is clear that the  $\dot{dc}_h = 0$  locus is negatively sloped and the  $\dot{dz} = 0$

locus is positively sloped. Further, since equation system (21) is linear, so is the saddlepath, SS, which equation is given by

$$c_h^o = c_h^* + \varphi(z^o - z^*) \quad (22)$$

where  $c_h^o$  and  $z^o$  are the initial values (after the jump onto the saddlepath in the case of consumption),  $c_h^*$  and  $z^*$  are the steady-state values, and  $\varphi$  is the slope of the saddlepath. Both this slope and the negative eigenvalue – which defines the system’s speed of adjustment once on the saddlepath – can be readily calculated from the matrix  $\Psi$ . (See appendix A for a brief sketch of that procedure.)

In Figure 1, the initial steady state is represented by the point 1. After a once-and-for-all reduction in the retirement age, consumption drops immediately to a lower level corresponding to point 2 on the saddlepath SS, and then the system moves over time (along SS) toward point 3 (the new steady state, where consumption is still below its initial steady-state level).<sup>31</sup>

On impact, consumption falls by 12.88 percent of its initial full-equilibrium value (see Table 2, Bm.1, Col.1)<sup>32</sup>. This effect is calculated according to equation (22), and the outcome is simply a weighted average of the steady-state effects. (Note that since  $z$  is a predetermined variable,  $dz^o/d\lambda$  is zero.) During the transition period, consumption increases gradually until the new full equilibrium is reached; at that stage the drop is 6.91 percent of the initial steady-state value (see Table 2, Bm.1, Col.1).<sup>33</sup> What are the mechanisms that lead to these results?

The initial fall in consumption is the net effect of forces working in opposite directions. On the one hand, a lowering of the retirement age means that the labor force shrinks. This (other things being equal) reduces the marginal productivity of capital – which leads to a downward pressure on the interest rate – and increases the marginal product of labor – which pushes pre-tax wages up. On the other hand, since perfect capital mobility pegs the rate of interest (and since we assume initially that the increased dependency ratio has no effect on the interest rate), the physical capital stock has to decrease. This reduction in the capital stock (that ensures the constant interest rate) lowers

the marginal product of labor – and that reduces pre-tax wages. In addition, firms are now willing to pay less for labor – since their contribution rate to the public pension system increases with demographic changes. According to our results (see Table 3, Bm.1, Col.1), these downward pressures on pre-tax wages dominate. Moreover, after-tax wages also decrease – since the employees’ pension contribution rate has increased as well. Thus, these negative effects on pre and after-tax wages combined with the fact that the number of workers has shrunk explain why the average consumption of this group has declined on impact.

Furthermore, since  $z$  is a predetermined variable,  $z = k - a$  implies that on impact, the variations in domestic physical capital stock correspond to the variations in foreign indebtedness. Therefore, as indicated in Table 3 (Bm.1, Col.1), the physical capital stock ratio falls on impact by an amount that is just equal to the initial foreign debt ratio. Hence, following the “aging” shock, the remaining physical capital is entirely owned by domestic (forward-looking) residents. After these initial adjustments, the dynamics of  $z$  is determined by that of  $a$  – since the change in the physical capital stock is once-and-for-all. Thus, as shown in Figure 1, during the transition to the new full equilibrium, the forward-looking group’s consumption rises gradually as their claims on foreign agents increase. However, in the new steady state their living standards remain below the initial full-equilibrium level – despite the resulting higher level of non-human wealth.

As for the hand-to-mouth group, the decline in their average consumption is *greater* (18.54 percent of the initial steady-state value – both initially and in new full equilibrium) (see Table 2, Bm.1, Col.1). The causes of that large fall are the following. First, for the same reasons as before, the labor force has shrunk and also the low-paid workers in this group suffer a reduction in their pre- and after-tax wages. However, since firms induce these workers not to shirk, the fall in their pre-tax wages is lower than that of the high-wage workers (see Table 3, Bm.1, Col.1). As a result, the unemployment rate among lower-skilled workers increases by 0.7 of one percentage point (see Table 2, Bm.1, Col.1). Second, the government transfer payments – which go only to this group of

low-income individuals – have decreased. The reduction in pre-tax wages, physical capital stock employed within the country, and the lower number of workers result in a loss of revenue for the government. Therefore, transfer payments have to adjust accordingly to maintain the government budget deficit. Although, the domestic (forward-looking) residents have built up a positive stock of net foreign assets (see Table 3, Bm.1, Col.1), that does not have any revenue implications for the domestic government – since by assumption it collects tax on the interest income that is generated within the country irrespective of who owns that capital.

The first column of benchmark 2 in Tables 2 and 3 show how the results change when *only* the employees' pension contribution rate adjusts residually as the dependency ratio rises. Relative to the previous case, there are fewer downward pressures on the pre-tax wages – since the firms' contribution rate is unaffected. Besides, the wage paid to lower-skilled workers increases (see Table 3, Bm.2, Col.1). The intuition runs as follows. A relatively higher increase in taxes levied on employees makes the work less attractive for low-paid workers and therefore raises their propensity to shirk. Knowing that, firms react by paying higher wages. As a result, the labor demand for these workers decreases more, and a consequent higher unemployment emerges. In this case, the rate of unemployment among these workers increases by 1.3 percentage points (see Table 2, Bm.2, Col.1) – compared to 0.7 percentage point above.

Finally, the first column of benchmark 3 in Tables 2 and 3 report how the results are affected by the assumption that *only* the firms' contribution rate varies endogenously following an increase in the dependency ratio. In this case, the effort supplied by lower-skilled workers (and hence their productivity) is unaffected. This is because an increase in the firms' pension contribution rate – which reduces market wage rates – lowers the net return both to working and to drawing employment insurance benefits, since raising this tax affects the net returns to work and non-work proportionately. Therefore, the low-paid workers' comparative attractions of working and being jobless with benefits are unaffected. Then, since there is no threat of shirking, both groups' pre-tax wages decline

by an equivalent amount (see Table 3, Bm.3, Col.1). As a result, the unemployment rate is unchanged (see Table 2, Bm.3, Col.1).

In summary, comparisons of the first column of the three benchmarks in Table 2 suggest that the choice of which contribution rate becomes the dependent variable (as the dependency ratio increases) has little effect on the impact of aging on average living standards – this is true whether or not agents are forward-looking. However, the choice matters for labor market distortions, and therefore, for the unemployment rate. More specifically, in contrast to the employees', the firms' contribution rates to both the public pension and employment insurance programs are shown to have no influence on the unemployment rate. It also appears that the hand-to-mouth individuals are those who take a bigger hit in their average living standards – both in the short run and in the long run. Overall, however, we should regard population aging as a serious upcoming threat to living standards.

Furthermore, notice the substantial swings in net foreign asset positions (see the last row in Table 3, Col.1 for each benchmark). Although, the magnitude of these swings is considerably larger than has been experienced, it is reassuring to note that it is similar to ones reported by the OECD's world model, Minilink (see Figures 6, 9 on pages 68 and 71, respectively, in Turner *et al.* (1998)).

So far, we have assumed that there are no demographic changes in the rest of the world and more explicitly that the rate of interest is constant. However, in the real world, the aging population seems to be a world-wide phenomenon – although not perfectly synchronized. With a general aging process one would instead expect a fall in the world interest rates. Therefore, we now consider the possibility that an increase in the elderly dependency ratio will have a negative effect on the world rate of interest – which we assume decreases by four tenths of one percentage point. To examine how this eventuality modifies the results, we initially revert to the assumption that both the employee and employer pension contribution rates adjust (in equal proportion) as the dependency ratio increases. Then using Gauss to solve the model yields the following two differential equations

$$\begin{bmatrix} dz \\ dc_h \end{bmatrix} = \begin{bmatrix} 0.020000 & -1.000000 \\ -0.001064 & -0.013209 \end{bmatrix} \begin{bmatrix} dz \\ dc_h \end{bmatrix} + \begin{bmatrix} 0.007102 & 0.691234 \\ -0.000022 & 0.239747 \end{bmatrix} \begin{bmatrix} d\lambda \\ dr \end{bmatrix} \quad (23)$$

Following an increase in the dependency ratio – which is accompanied by a reduction in the rate of interest – the full dynamic response of (domestic) forward-looking aggregate consumption and financial wealth is shown in Figure 2; and the numerical results are reported on the second column of benchmark 1 in Tables 2 and 3.

As illustrated in Figure 2, after the “aging” shock (and the accompanying interest rate effect), the system moves from its initial full equilibrium, point 1, to point 2 immediately; then as time passes, the system moves to point 3. The initial fall in consumption is less, 8.52 percent – compared to 12.88 percent in the reference scenario case (see the two columns of benchmark 1 in Table 2). This result stems from the fact that there are fewer downward pressures on both pre- and after-tax wages. First, since the rate of interest decreases, the required reduction in the physical capital stock is smaller (see Table 3, Bm.1, Col.2). As a result, the negative effect on pre-tax wages of a decline in capital stock is mitigated. Second, with the lower reduction in pre-tax wages, the rise in the two contribution rates that is needed to balance the pension plan is lower – for the same increase in the elderly dependency ratio. This implies a smaller decrease in the after-tax wage. Consequently, by ignoring the world-wide aging – and hence a reduction in world real interest rates, we overestimate the initial impact of population aging on domestic savers’ living standards.

Furthermore, during the transition to the new full equilibrium, the forward-looking group’s living standards decrease gradually as their level of foreign indebtedness increases; that is seen in Figure 2. Indeed, the lower interest rate means a reduced incentive for private saving and therefore a lower level of non-human wealth. At the new steady state, the average consumption of this group is reduced by 10.48 percent – compared to 6.91 percent in the reference scenario case (see the two columns of benchmark 1 in Table 2). Therefore, by ignoring demographic changes in the rest of the



world, we underestimate the final impact of population aging on domestic forward-looking individuals' living standards.

As for the hand-to-mouth group, things are worsened less than in the reference scenario (see the two columns of benchmark 1 in Table 2). The smaller reduction in physical capital stock and its effect on pre-tax wages have another implication for this group; the required reduction in government transfer payments is lower. Here also, the reference scenario case overestimates the effect of aging on this group's living standard.

Finally, we also derive the results for the cases in which either the employee or employer pension contribution rate is the only rate that adjusts when the demographic structure changes. These results are reported on the second column of benchmarks 2 and 3, respectively, in both Tables 2 and 3.

To sum up, comparisons of the two columns of each benchmark in Table 2 suggest that by ignoring the impact of the general aging process on the world rate of interest, we may systematically overestimate or underestimate the effect which aging populations may have on per-capita living standards. As a consequence, policy responses – that are designed to offset this effect – may be inappropriate. Therefore, any conclusions we make about the implications of population aging on living standards should be based on what we believe regarding the effect of aging throughout the world on interest rates. Moreover, when domestic population aging is accompanied by aging in the rest of the world, and hence a reduction in world real interest rates, changes in the country's net external liabilities are very small (see the last row in Table 3, Col.2 for each benchmark). The intuition runs as follows. Broadly speaking, to the extent that imbalances in savings and investment – which arise from aging – occur at the global level they are likely to be reflected in movements in real interest rates, whereas to the extent that they occur in particular regions they will be reflected in changes in net foreign asset positions (and exchange rates).<sup>34</sup> In addition, when we compare the second column of the three benchmarks in Table 2, it appears again that the results are not much affected by whose contribution rate changes, except for the unemployment rate. Also, the hand-to-mouth individuals are those who are most affected by aging.

Furthermore, it is conceivable that the pressure on the public pension system due to population aging may induce people to postpone their retirement age. But since the retirement age is exogenous in this model, the only way we can examine this eventuality is to assume a lower increase in the elderly dependency ratio. For example, one may assume the limit case where the dependency ratio increases by half of what is expected. The results concerning this possibility are presented in Tables 4 and 5. Although the aging consequences are mitigated in this case, it appears that our basic results are not much affected.

## 8.2 Sensitivity analysis

To evaluate the relative influence of selected parameters for model results, we have conducted a number of sensitivity tests in the case where both the employee and employer pension contribution rates adjust in equal measure as the dependency ratio increases.

As explained earlier, one unappealing aspect of this model is that we cannot simultaneously choose appropriate values for the death probability ( $p$ ), retirement age ( $\lambda$ ), and the old-age dependency ratio ( $DR$ ). In our baseline calibration, we have chosen sensible values for the first two parameters, and the model generates a dependency ratio that corresponds more closely to the overall dependency ratio than to the elderly dependency ratio in Canada. Therefore, we have had to raise the pension contribution rates such that the individual pension payment (as a proportion of the average wage) equals about 21 percent. Since the old-age dependency ratio may give a more accurate description of the economic burden arising from population aging, we now consider two sets of values for  $p$  and  $\lambda$  that generate a sensible elderly dependency ratio of roughly 25 percent. As a result, we set both pension contribution rates at the actual values of 4.5 percent.

*Parameter Set I* involves:  $p = 0.04$  (which implies a life expectancy of 45 years at age 20),  $\lambda = 40$  (as in the baseline calibration), and  $\delta = 0.0165$  (a corresponding adjustment in the forward-looking rate of time preference that keeps all steady state

restrictions satisfied). Furthermore, according to Hviding and Mérette (1998, p.7) or Mérette (2001, p.56), the elderly dependency ratio in Canada is expected to nearly double by 2035. The “shock” to the retirement age that is consistent with this fact is  $d\lambda = -10$ . The simulation results corresponding to this set are reported in Tables 6 and 7 – lower life expectancy case.

*Parameter Set II* implies:  $\lambda = 80$ ,  $p = 0.02$  (as in the baseline calibration),  $\delta = 0.0215$ , and  $d\lambda = -20$ . These results are reported in Tables 6 and 7 – higher retirement age case.

When we compare the lower life expectancy and higher retirement age results in Table 6, and in turn compare each to the benchmark 1 results in Table 2, the sensitivity analyses suggest that our initial findings are fairly robust.

Furthermore, calibrations for all aggregative models of this sort are always a little strained by the requirement that one interest rate must represent both the net marginal product of capital and the rate of return on risk-free government debt. For a risky investment, our baseline calibration of the interest rate is relatively small. Therefore, we now set the interest and capital depreciation rates (at 8 percent and 3 percent, respectively) such that the capital-output ratio and capital’s share of output remain unaffected (see equation (3)). The results of these simulations are reported in Table 8. Again, it appears that our initial findings are truly robust.

## **9. Conclusions**

This chapter has examined some fiscal, macroeconomic and distributional consequences of population aging for a single economy. Our vehicle for addressing these issues was a general equilibrium model that involves overlapping generations with a public pension system, asymmetric information in the labor market, and a subset of households that are liquidity constrained. The analysis leads to three main conclusions.

First, it shows that by analyzing the aging problem in a small open-economy framework without taking into account its global dimensions, we may systematically misestimate the effect which aging may have on the country’s living standards and its net

foreign asset position. As a result, the scope of policy responses – that are designed to ameliorate or offset these effects – may be very inaccurate.

Second, the magnitude of the effect of an aging population on people's average living standards, in the short run as well as in the long run, significantly depends on whether or not they are liquidity constrained. On the whole, however, we should regard population aging as a serious upcoming threat to living standards.

Third, the actual incidence of increased contributions to the public pension system – that is, whether the workers' or firms' contribution rate should increase to maintain the financial viability of the system as the elderly dependency ratio rises – has little effect on the impact of aging on living standards. This conclusion emerges whether or not agents save for the future. However, the incidence of contributions (to both the public pension and employment insurance programs) matters for labor market distortions, and hence for the unemployment rate. More specifically, in contrast to the employees', the firms' contribution rates to both programs are shown to have no influence on the unemployment rate.

For future research, it could be relevant to make a distinction between start of life and start of working life. Given the fall in birth rates it could be envisaged that the pressure on the public pension system could be eased by transferring resources from the group of young (people aged 1-20, say) to the old. Furthermore, it would be interesting to examine alternative adjustment schemes for the pension contribution and benefit rates. In this chapter, as well as in the chapter 1, we have assumed the benefit payment constant and let the contribution rates adjust endogenously. However, in order to achieve 'relative' fairness in the burden of adjustment following demographic shocks, one could reform the public pension system by fixing (or targeting) the net replacement ratio, defined as the ratio of the average pension benefit to the average post-tax wage income. Recently, Germany has implemented a pension system along these lines.

## References

- Ambler, S. (1994) "Book Review", *Journal of Monetary Economics* 34, 567-79.
- Auerbach, A. J., J. Cai, and L. J. Kotlikoff (1991) "U.S. Demographics and Saving: Predictions of Three Saving Models", *Carnegie-Rochester Conference Series on Public Policy*, 34, 135-156.
- Barro, R. J. (1974) "Are Government Bonds Nets Wealth?", *Journal of Political Economy* 82, 1095-1117.
- Barro, R. J. and X. Sala-i-Martin (1995) *Economic Growth* (New York: McGraw-Hill).
- Beach, C. M. and G. A. Slotsve (1996) *Are We Becoming Two Societies: Income Polarization and the Myth of the Declining Middle Class in Canada*, in *The Social Policy Challenge* 12, (Toronto: C.D. Howe Institute).
- Black, R. and D. Rose (1997) "The Canadian Policy Analysis Model: CPAM", Working Paper No. 97-16, Bank of Canada, Ottawa.
- Blanchard, O. J. (1985) "Debts, Deficits, and Finite Horizons", *Journal of Political Economy* 93, 223-247.
- Blanchard, O. J. and S. Fischer (1989) *Lectures on Macroeconomics* (Cambridge, Mass.: M.I.T. Press) p. 463.
- Borsch-Supan, A. (1996) "The Impact of Population Aging on Savings, Investment and Growth in the OECD Area", in *Future Global Capital Shortages: Real Threat or Pure Fiction?*, OECD, Paris.
- Buiter, W. H. (1988) "Death, Birth, Productivity Growth and Debt Neutrality", *The Economic Journal* 391, 279-293.
- Burbidge, J. B. and W. Scarth (1995) "Eliminating Interest Taxation and Tariffs: the Underpinnings for Recent Canadian Policy", *Canadian Journal of Economics* 28, 437-49.
- Calvo, G. and M. Obstfeld (1988) "Optimal Time-Consistent Fiscal Policy with Finite Lifetimes: Analysis and Extensions", in *Economic Effects of the Government Budget*, ed. E. Helpman, A. Razin, and E. Sadka (Cambridge, MA: MIT Press).

- Canari, L. (1994) "Do Demographic Changes Explain the Decline in the Saving Rate of Italian Households?", in Ando A., L. Guiso and I. Visco, eds., *Saving and the Accumulation of Wealth*, Cambridge University Press.
- Castaneda, A., J. Diaz-Giménez, and J. V. Rios-Rull (2000) "Accounting for Earnings and Wealth Inequality", Mimeo, University of Pennsylvania.
- Corneo, G. and M. Marquardt (2000) "Public Pensions, Unemployment Insurance, and Growth", *Journal of Public Economics* 75, 293-311.
- Dahlby, B. (1992) "Taxation and Social Insurance", in R.M. Bird and J.M. Mintz, eds., *Taxation to 2000 and beyond* (Toronto: Canadian Tax Foundation).
- Dahlby, B. (1993) "Payroll Taxes", in A.M. Maslove, eds., *Business Taxation in Ontario* (Toronto: University of Toronto Press).
- Denton, F. T. and B. G. Spencer (2000) "Population Aging and Its Economic Costs: A Survey of the Issues and Evidence", *Canadian Journal on Aging* 19 suppl. 1, 1-31.
- Diamond, P. A. (1965) "National Debt in a Neoclassical Growth Model", *American Economic Review* 55, 1126-50.
- Feldstein, M. J. (1974) "Social Security, Induced Retirement, and Aggregate Capital Accumulation", *Journal of Political Economy* 82, 905-926.
- Hviding, K. and M. Mérette (1998) "Macroeconomic Effects of Pension Reforms in the Context of Aging Populations: Overlapping Generations Model Simulations for Seven OECD Countries", Working Paper AWP 1.3 Eng, OECD, Economics Department, Paris.
- James, S. (1994) "Debt Reduction with Distorting Taxes and Incomplete Ricardianism: A Computable General Equilibrium Analysis", in W.B.P. Robson and W. Scarth, eds., *Deficit Reduction: What Pain, What Gain ?* (Toronto: C.D. Howe Institute).
- Johnson, G. E. and P. R. G. Layard (1986) "The Natural Rate of Unemployment: Explanation and Policy", in O. Ashenfelter and R. Layard, eds., *Handbook of labor economics* vol. II (Elsevier Science Publishers, Amsterdam), 921-999.
- Kesselman, J. R (1997) *General Payroll Taxes: Economics, Politics, and Design*. Canadian Tax Paper No. 101 (Toronto: Canadian Tax Foundation).
- Kotlikoff, L. J. and L. H. Summers (1981) "The Role of Intergenerational Transfers in Aggregate Capital Accumulation", *Journal of Political Economy* 89, 706-732.

Layard, R., S. Nickell, and R. Jackman (1991) *Unemployment: Macroeconomic Performance and the Labor Market*. Oxford: Oxford University Press.

Lee, R. (2001) "The Fiscal Impact of Population Aging", testimony prepared for the U.S. Senate Budget Meeting, February 7, 2001 in Washington D.C.

Lee, R. and R. Edwards (2001) "The Fiscal Impact of Population Aging in the US: Assessing the Uncertainties", paper presented at the National Bureau of Economic Research Tax Policy and the Economy meeting, organized by James Poterba, October 30, 2001, in Washington D.C.

Lucas, R. E., Jr. (1976) "Econometric Policy Evaluations: A Critique," in K. Brunner and A. H. Meltzer, eds., *The Phillips Curve and the Labor Market*, Amsterdam, New York, and Oxford: North Holland.

Mankiw, N. G. (2000) "The Savers-Spenders Theory of Fiscal Policy", *American Economic Review, Papers and Proceedings* 90, 120-125.

Mankiw, N. G. and W. Scarth (2001) *Macroeconomics*, Canadian edition, second edition (New York: Worth Publishers Press).

Martin, J. (1996) "Unemployment and Related Welfare Benefits in OECD Countries and Their Impact on the Labor Market", paper presented at the CILN conference, McMaster University (September 1996).

Masson, P. R. and R. Tryon (1990) "Macroeconomic Effects of Projected Population Aging in Industrial Countries", *IMF Working Paper*, January.

Masson, P. R., T. Bayoumi and H. Samiei (1995) "International Evidence on the Determinants of Private Savings" *IMF Working Paper*, May.

Meredith, G. (1995) "Demographic Change and Household Saving in Japan", in *Saving Behavior and the Asset Price "Bubble" in Japan*, U. Baumgartner and G. Meredith, eds, *IMF Occasional Paper* No. 124.

Mérette, M. (2001) "An Economic Welcome to the Population Aging in Canada", Paper Prepared for the Institute for Research on Public Policy, Montréal.

Modigliani, F. (1966) "The Life Cycle Hypothesis of Saving, the Demand for Wealth and the Supply of Capital", *Soc. Res.* 33, 160-217.

Nielsen, S. B. (1994) "Social Security and Foreign Indebtedness in a Small Open Economy", *Open Economies Review* 5, 47-63.

OECD (1994) *The OECD Jobs Study: Evidence and Explanations, Part II, The Adjustment Potential of the Labor Market*. Paris.

Phelps, E. S. (1994) *Structural Slumps: The Modern Equilibrium Theory of Unemployment, Interest, and Assets* (Harvard University Press, Cambridge, MA.)

Pisauro, J. (1991) "The Effect of Taxes on Labor in Efficiency Wage Models", *Journal of Public Economics* 46, 329-345.

Pissarides, C. A. (1998) "The Impact of Employment Tax Cuts on Unemployment and Wages; The Role of Unemployment Benefits and Tax Structure", *European Economic Review* 42, 155-183.

Quadrini, V. and J. V. Rios-Rull (1997) "Understanding the U.S. Distribution of Wealth", *Federal Reserve Bank of Minneapolis Quarterly Review* 21, 22-36.

Ramsey, F. P. (1928) "A Mathematical Theory of Saving", *Economic Journal* 38, 543-9.

Rios-Rull, J. V. (2001) "Population Changes and Capital Accumulation: The Aging of the Baby Boom", *Advances in Macroeconomics* 1, No.1, Article 7.

Romer, D. (1996) *Advanced Macroeconomic Theory* ( New York: McGraw-Hill)

Scarth, W. (1997) *A Job-Creation Strategy for Governments with No Money*, Commentary No 92 (Toronto: C.D. Howe Institute).

Scarth, W. and T. Moutos (2000) "Work-Sharing: An Efficiency-Wage Analysis", *CESifo Working Paper Series* 386, Institute for Economic Research, Munich.

Shapiro, C. and J. Stiglitz (1984) "Equilibrium Unemployment as a Worker Discipline Device", *American Economic Review* 74, 433-444.

Shapiro, C. and J. Stiglitz (1985) "Can Unemployment Be Involuntary? Reply", *American Economic Review* 75, 1215-1217.

Summers, L. (1988) "Relative Wages, Efficiency Wages, and Keynesian Unemployment", *American Economic Review* 78, 383-388.

Turner D., C. Giorno, A. De Serres, A. Vourc'h and P. Richardson (1998) "The Macroeconomic Implications of Aging in a Global Context", Working Paper AWP 1.2, OECD, Macroeconomic Analysis and Systems Management Division, Paris.

Yellen, J. L. (1984) "Efficiency Wage Models of Unemployment", *American Economic Review* 74, 200-205.



Weil, P. (1989) "Overlapping Families of Infinitely-Lived Agents", *Journal of Public Economics* 38, 183-198.

Weiss, A. (1991) *Efficiency Wages*. Princeton University Press. Princeton, NJ.

Wolff, E. N. (1998) "Recent Trends in the Size Distribution of Household Wealth", *Journal of Economic Perspectives* 12, 131-150.

## Endnotes

<sup>1</sup> For more details about CPAM see Black and Rose (1997).

<sup>2</sup> See the technical annex in Turner *et al.* (1998) for a complete and detailed description of Minilink and its properties.

<sup>3</sup> Nonetheless, Lee (2001) and Lee and Edwards (2001) examine the fiscal impact of population aging in a probabilistic setting. They find that even in the experimental projections in which the rate of productivity growth is fixed at a higher level, the fiscal problems arising from aging are highly unlikely to go away. Moreover, they indicate that the fiscal consequences of population aging will indeed be severe for the US, but in many other industrial nations they will be simply staggering.

<sup>4</sup> Note that in Minilink, the death probability is endogenised as a function of the old-age dependency ratio.

<sup>5</sup> See Figure 7 in Turner *et al.* (1998, p. 69).

<sup>6</sup> By involuntary unemployment we mean a situation where an unemployed worker is willing to work for less than the wage received by an equally skilled employed worker, yet no job offers are forthcoming.

<sup>7</sup> Pissarides (1998) notes that Shapiro and Stiglitz's (1984) model has a serious limitation for numerical simulations, since it cannot be readily calibrated.

<sup>8</sup> Although Summers (1988) and Romer (1996) did not provide micro foundations for this work effort function, Scarth and Moutos (2000) showed that it derives support from household optimization behavior.

<sup>9</sup> If there are unemployed workers, the firm can choose the wage freely. If unemployment is zero, on the other hand, the firm must pay at least the wage paid by other firms.

<sup>10</sup> Notice that employment insurance benefit,  $fw_t$  – which is paid to those unemployed among low-income individuals – is tax free. Two arguments could be mentioned to justify this assumption. First, there exists a basic exemption for personal income tax. And second, only the employed individuals contribute to the public pension and employment insurance programs.

$$^{11} \int_{t-\lambda}^t pe^{-p(t-s)} ds = (1 - e^{-p\lambda}) \text{ and } \int_{-\infty}^{t-\lambda} pe^{-p(t-s)} ds = e^{-p\lambda}, \text{ where } s \text{ is the birth date.}$$

<sup>12</sup> As before, we will use lower case letters to denote variables adjusted for the productivity growth. Moreover, if we choose unity as the initial value for  $y$ , some lower case letters can be interpreted as the ratio of those variables to GDP.

<sup>13</sup> Assuming that transfer payments go only to the “poor” can be justified by the fact that income taxes are proportional in our model, whereas they are actually progressive.

<sup>14</sup> Although there is only one good, foreigners can buy domestic output, and domestic residents can buy foreign output. Thus, the only function of international trade in this model is to allow domestic

production to diverge from domestic aggregate demand. In other words, we consider the intertemporal aspects of international trade but neglect the implications for patterns of specialization in production.

<sup>15</sup> Indeed, if  $a = k - z$ , is positive, it corresponds to net claims by foreigners on the domestic economy. Conversely, if  $a$  is negative, it represents net claims by domestic residents on foreign economies.

<sup>16</sup> There is an important empirical literature that reports that in the long-run, employer payroll taxes are fully shifted to labor in the form of lower wages. See, for example, Dahlby (1992, 1993); Kesselman (1997) for a survey.

<sup>17</sup> The notation indicates that the effort or efficiency function depends on the real wage and other variables.

<sup>18</sup> Note that in Yellen's model, and in the subsequent (moral hazard) ones, tax refers to the employers' tax.

<sup>19</sup> We have removed the subscript, since following an increase in the elderly dependency ratio the government might choose  $\tau_w$  or  $\tau_f$  (or both) to balance the pension plan budget constraint. We derive quantitative results for all three cases.

<sup>20</sup> The elements of these matrices are fully reported in Appendix A.

<sup>21</sup> As noted above, we have chosen unity as the initial value for  $y$ , so that most of the variables in lower case letters can be interpreted as relative to GDP.

<sup>22</sup> Recall that, in the model, expected life is given by  $1/p$ .

<sup>23</sup> It is worth noting that such a discrepancy also appears in the OECD's world model, Minilink. Besides, in Minilink, since the United States has net foreign liabilities, it is necessary to assume that it is running a trade surplus (which is not what we observe) in the baseline steady state to cover interest payments, so that net liabilities remain a stable share of GDP.

<sup>24</sup> Since 1971 the employer contribution rate for EI is 1.4 times the rate paid by employees. Pre-1971, employers and employees paid matching contributions which varied with earnings.

<sup>25</sup> The overall dependency ratio refers to the ratio of the population who are not of working age (both young and old) to those who are.

<sup>26</sup> If we normalize the population at 1, these numbers imply that the Canadian overall dependency ratio will increase by 22.16 percent in three decades.

<sup>27</sup> Since our baseline calibration generates an overall dependency ratio, we found it consistent to use the projection of this ratio in estimating the shock to the retirement age – despite the fact that the projection of the old-age dependency ratio may give a more accurate description of the economic burden arising from population aging.

<sup>28</sup> The first number follows by assuming that demographic change has no effect on private savings behavior; whereas, for the last one, it is assumed that a 1 percentage point rise in the dependency ratio reduces the private savings rate by 0.3 percent. However, as mentioned earlier, the empirical evidence on the impact of population aging on savings is very mixed.

<sup>29</sup> Hereafter, we refer to these cases as the reference scenario cases.

<sup>30</sup> Although the public debt belongs in this group's financial wealth, it is irrelevant in the dynamic analyses, since it is kept constant through time. Furthermore, given the complexity of this model, relative to the one in the first essay, it would be very difficult – if not impossible – to examine the case of the perfect foresight assumption regarding the aging event in this framework. Thus, this sensitivity analysis will not be performed here.

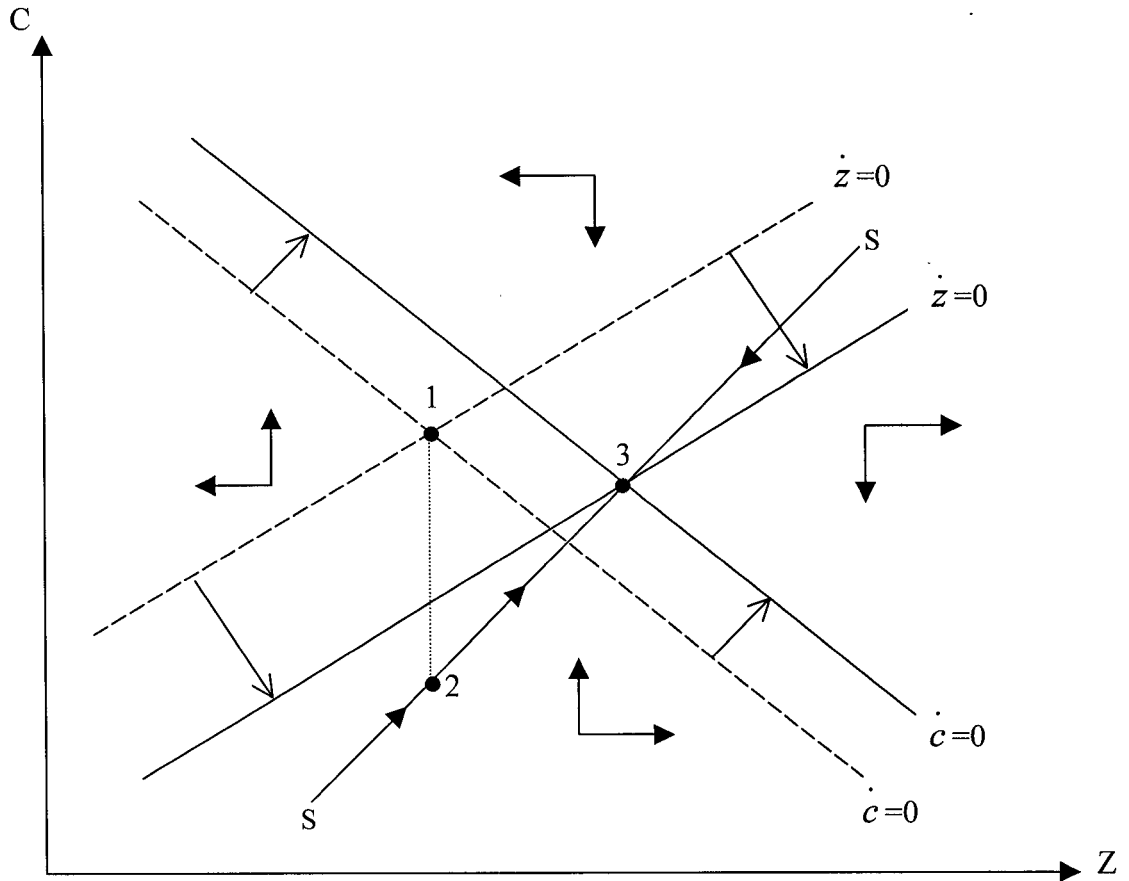
<sup>31</sup> Since the share of forward-looking individuals in the population is constant through time, the average consumption of this group also displays a similar pattern.

<sup>32</sup> Bm. and Col. denote Benchmark and Column, respectively.

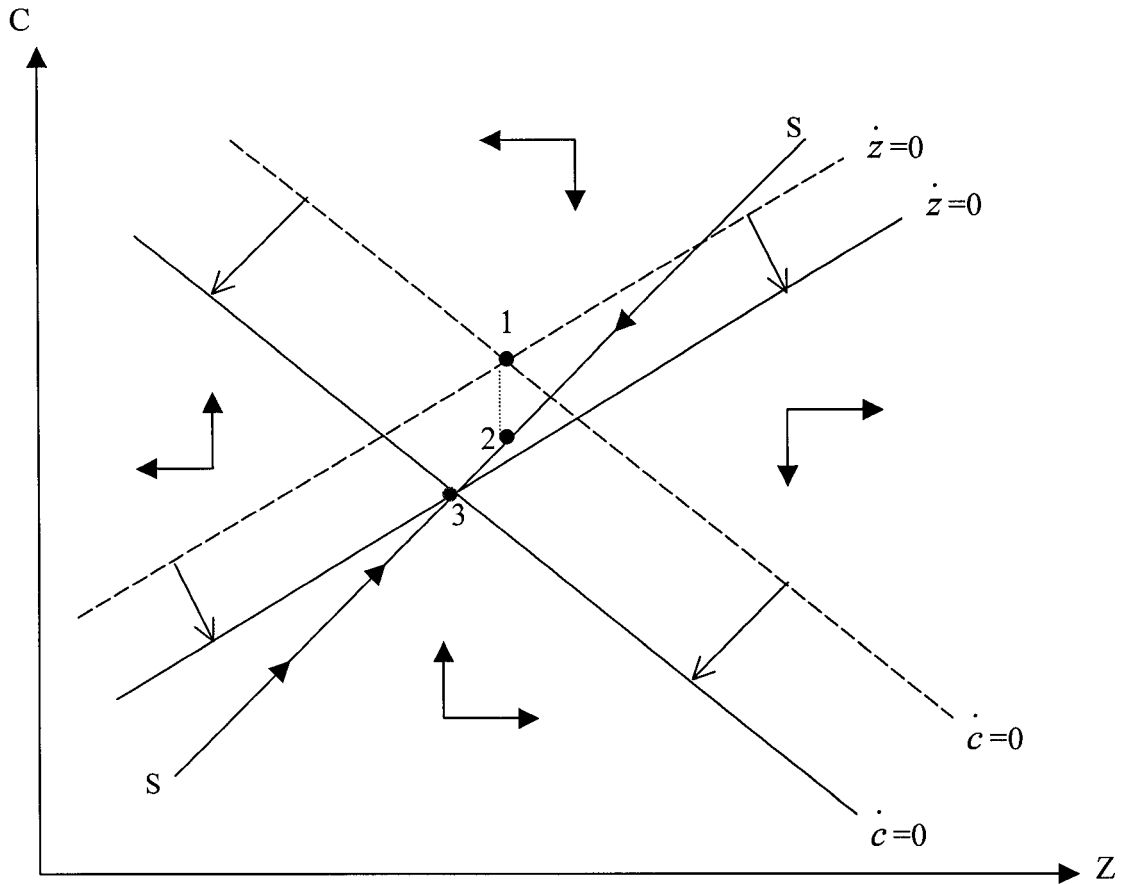
<sup>33</sup> For the same reason as in the footnote 31, these numbers also correspond to the variations in average consumption among forward-looking agents.

<sup>34</sup> Regional imbalances need not lead to global imbalances if the weight of the region in the world economy is small and/or if there are offsetting imbalances in other regions.

**Figure 1:** Dynamic adjustment of domestic (forward-looking group) consumption and financial wealth to an increase in the elderly dependency ratio – *with no demographic change abroad.*



**Figure 2:** Dynamic adjustment of domestic (forward-looking group) consumption and financial wealth to an increase in the elderly dependency ratio – *with* similar demographic change abroad.



**Table 1:** Baseline calibration

Variables and parameters		benchmark	perturbation
$r$	interest rate	0.05	0.046
$n$	productivity growth rate	0.02	
$\phi$	capital depreciation rate	0.06	
$\lambda$	retirement age	40	34
$p$	probability of death	0.02	
$\pi$	proportion of population not liquidity constrained	0.6	
$\delta$	rate of time preference, forward-looking consumers	0.0332	
$\alpha$	capital's share of output	0.33	
$\beta$	high-skilled labor's share of output	0.536	
$k$	capital-output ratio	3	
$a$	initial foreign debt ratio	0.3	
$b$	government debt ratio	0.53	
$g$	government spending ratio	0.14	
$t_i$	tax rate on interest income	0.2	
$t_w$	tax rate on labor income	0.2	
$u$	proportion of unemployed among low-paid workers	0.175	
$f$	unemployment benefits as a proportion of wage	0.4	
$\theta_w$	employee contribution rate to employment insurance	0.015	
$\theta_f$	employer contribution rate to employment insurance	0.021	
$\tau_w$	employee contribution rate to the PAYG pension plan	0.085	
$\tau_f$	employer contribution rate to the PAYG pension plan	0.085	

## Long and Short-Run Effects of Increased Elderly Dependency Ratio

### I – Benchmark Simulation Experiments

**Table 2:** Effects of increased elderly dependency ratio on living standards and unemployment rate among low-income workers.

Variable	Benchmark 1		Benchmark 2		Benchmark 3	
	$\lambda$ shock	$\lambda$ and $r$ shocks	$\lambda$ shock	$\lambda$ and $r$ shocks	$\lambda$ shock	$\lambda$ and $r$ shocks
$C_h$ , impact effect	-12.88%	-8.52%	-13.31%	-8.91%	-12.39%	-8.07%
$C_h$ , long-run effect	-6.91%	-10.48%	-7.58%	-11.10%	-6.12%	-9.76%
$C_l$ , I and LR effects	-18.54%	-14.72%	-17.40%	-13.67%	-19.88%	-15.94%
$u$	+0.007	+0.006	+0.013	+0.011	0.000	0.000

Notes:

I: Denotes Impact

LR: Denotes Long Run

Benchmark 1: The employee and employer contribution rates adjust (equally) to balance the pension plan budget constraint as the dependency ratio increases.

Benchmark 2: The employee contribution rate alone adjusts to balance the pension plan budget constraint as the dependency ratio increases.

Benchmark 3: The employer contribution rate alone adjusts to balance the pension plan budget constraint as the dependency ratio increases.



**Table 3:** Effects of increased elderly dependency ratio on wages, capital stock, output, and foreign debt.

Variable	Benchmark 1		Benchmark 2		Benchmark 3	
	$\lambda$ shock	$\lambda$ and $r$ shocks	$\lambda$ shock	$\lambda$ and $r$ shocks	$\lambda$ shock	$\lambda$ and $r$ shocks
$\Delta w_h$	-2.06%	-0.07%	-0.37%	+1.48%	-4.03%	-1.87%
$\Delta w_l$	-1.24%	+0.70%	+1.15%	+2.88%	-4.03%	-1.87%
$dk$	-0.300	-0.136	-0.305	-0.140	-0.295	-0.132
$\Delta y$	-10.00%	-8.18%	-10.16%	-8.31%	-9.85%	-8.03%
$da^*$	-0.823	+0.035	-0.806	+0.051	-0.843	+0.017

Notes:

$\Delta$  : Denotes percentage changes

$d$  : Denotes absolute changes

**Benchmark 1:** The employee and employer contribution rates adjust (equally) to balance the pension plan budget constraint as the dependency ratio increases.

**Benchmark 2:** The employee contribution rate alone adjusts to balance the pension plan budget constraint as the dependency ratio increases.

**Benchmark 3:** The employer contribution rate alone adjusts to balance the pension plan budget constraint as the dependency ratio increases.

**Table 4:** Effects of increased elderly dependency ratio on living standards and unemployment rate among low-income workers – DR increased by half of what is expected.

Variable	Benchmark 1		Benchmark 2		Benchmark 3	
	$\lambda$ shock	$\lambda$ and $r$ shocks	$\lambda$ shock	$\lambda$ and $r$ shocks	$\lambda$ shock	$\lambda$ and $r$ shocks
$C_h$ , impact effect	-6.63%	-2.27%	-6.85%	-2.46%	-6.38%	-2.06%
$C_h$ , long-run effect	-3.56%	-7.13%	-3.90%	-7.42%	-3.15%	-6.80%
$C_l$ , I and LR effects	-9.55%	-5.72%	-8.96%	-5.23%	-10.24%	-6.29%
$u$	+0.003	+0.003	+0.006	+0.005	0.000	0.000

Note: DR denotes Dependency Ratio

**Table 5:** Effects of increased elderly dependency ratio on wages, capital stock, output, and foreign debt – DR increased by half of what is expected.

Variable	Benchmark 1		Benchmark 2		Benchmark 3	
	$\lambda$ shock	$\lambda$ and $r$ shocks	$\lambda$ shock	$\lambda$ and $r$ shocks	$\lambda$ shock	$\lambda$ and $r$ shocks
$\Delta w_h$	-1.06%	+0.93%	-0.19%	+1.66%	-2.08%	+0.08%
$\Delta w_l$	-0.64%	+1.28%	+0.59%	+2.32%	-2.08%	+0.08%
$dk$	-0.155	+0.009	-0.157	+0.008	-0.152	+0.012
$\Delta y$	-5.16%	-3.32%	-5.23%	-3.38%	-5.07%	-3.25%
$da^*$	-0.424	+0.435	-0.415	+0.442	-0.435	+0.426

Note: DR denotes Dependency Ratio

## II – Sensitivity Analysis Experiments

Unless otherwise indicated, the sensitivity analyses are conducted in the case where both the employee and employer contribution rates adjust (equally) to balance the pension plan budget constraint as the dependency ratio increases.

**Table 6:** Effects of increased elderly dependency ratio on living standards and unemployment rate among low-income workers – lower life expectancy and higher retirement age cases.

Variable	Lower life expectancy		Higher retirement age	
	$\lambda$ shock	$\lambda$ and $r$ shocks	$\lambda$ shock	$\lambda$ and $r$ shocks
$C_h$ , impact effect	-13.72%	-10.10%	-15.18%	-9.14%
$C_h$ , long-run effect	-7.32%	-10.22%	-4.39%	-11.15%
$C_l$ , I and LR effects	-19.10%	-15.16%	-19.10%	-15.16%
$u$	+0.007	+0.006	+0.007	+0.006

**Table 7:** Effects of increased elderly dependency ratio on wages, capital stock, output, and foreign debt – lower life expectancy and higher retirement age cases.

Variable	Lower life expectancy		Higher retirement age	
	$\lambda$ shock	$\lambda$ and $r$ shocks	$\lambda$ shock	$\lambda$ and $r$ shocks
$\Delta w_h$	-2.46%	-0.55%	-2.46%	-0.55%
$\Delta w_l$	-1.66%	+0.22%	-1.66%	+0.22%
$dk$	-0.311	-0.147	-0.311	-0.147
$\Delta y$	-10.36%	-8.53%	-10.36%	-8.53%
$da^*$	-0.843	-0.136	-1.533	+0.081

**Table 8:** Effects of increased elderly dependency ratio on living standards and unemployment rate among low-income workers – higher initial interest rate case.

Variable	S1		S2		S3	
	$\lambda$ shock	$\lambda$ and $r$ shocks	$\lambda$ shock	$\lambda$ and $r$ shocks	$\lambda$ shock	$\lambda$ and $r$ shocks
$C_h$ , impact effect	-10.78%	-7.68%	-11.18%	-8.05%	-10.31%	-7.25%
$C_h$ , long-run effect	-3.64%	-9.32%	-4.28%	-9.90%	-3.00%	-8.65%
$C_l$ , I and LR effects	-19.08%	-14.74%	-18.00%	-13.74%	-20.35%	-15.90%
$u$	+0.007	+0.006	+0.013	+0.011	0.000	0.000

Notes:

- S1: The employee and employer contribution rates adjust (equally) to balance the pension plan budget constraint as the dependency ratio increases.
- S2: The employee contribution rate alone adjusts to balance the pension plan budget constraint as the dependency ratio increases.
- S3: The employer contribution rate alone adjusts to balance the pension plan budget constraint as the dependency ratio increases.

## Appendix A:

### Complete derivation of the simulation model

Taking the total differential of the equations (1'), (3) and (17) (with  $z = k - a$ ) gives

$$\Delta y = \alpha \Delta k + \frac{(1-\alpha)pe^{-p\lambda}}{(1-e^{-p\lambda})} d\lambda - \frac{(1-\alpha-\beta)}{(1-u)} du + \varepsilon(1-\alpha-\beta)\Delta w_l - \frac{\varepsilon(1-\alpha-\beta)}{(1-t_w-\tau_w-\theta_w)} d\tau_w \quad (\text{A1})$$

$$\Delta k = \Delta y - \frac{1}{r+\phi} dr \quad (\text{A2})$$

and

$$d\dot{z} = (r(1-t_i) - n)dz - dc_h - dc_l + y\Delta y - k(r(1-t_i) + \phi)\Delta k - a(1-t_i)dr \quad (\text{A3})$$

respectively.

Using (A2) and  $k = \frac{\alpha}{r+\phi}y$  (from equation (3)), (A3) can be written as:

$$d\dot{z} = (r(1-t_i) - n)dz - dc_h - dc_l + A_1\Delta y + \left[ \frac{(r(1-t_i) + \phi)k}{r+\phi} - (1-t_i)a \right] dr \quad (\text{A4})$$

where  $A_1 = \left( (1-\alpha) + \frac{\alpha r t_i}{r+\phi} \right) y$ .

Combining (A1) and (A2), then substituting the resulting equation into (A4) yields

$$d\dot{z} + dc_l - \frac{A_1\varepsilon(1-\alpha-\beta)}{(1-\alpha)}\Delta w_l + \frac{A_1(1-\alpha-\beta)}{(1-\alpha)(1-u)}du + \frac{A_1\varepsilon(1-\alpha-\beta)}{(1-\alpha)(1-t_w-\tau_w-\theta_w)}d\tau_w = \quad (\text{A5})$$

$$(r(1-t_i) - n)dz - dc_h + \frac{A_1pe^{-p\lambda}}{(1-e^{-p\lambda})}d\lambda + \left[ \frac{(r(1-t_i) + \phi)k}{r+\phi} - (1-t_i)a - \frac{\alpha A_1}{(1-\alpha)(r+\phi)} \right] dr$$

This equation represents the first line in the matrix system.

After setting the total differential of the equations (14') and (15') (in this case, eliminating the change in  $k$ ), we have respectively

$$\begin{aligned}
 d\dot{c}_h - A_5(1-t_w - \tau_w - \theta_w)w_h\Delta w_h + A_5w_h d\tau_w = -p(p+\delta)dz + (r(1-t_i) - \delta - n)dc_h \\
 + \left[ A_6 \left( (r(1-t_i) - n) \frac{e^{-(r(1-t_i)-n)\lambda}}{(1-e^{-(r(1-t_i)-n)\lambda})} - p \right) - m\pi p(p+\delta)e^{-p\lambda} \right] d\lambda \\
 + \left[ A_6 \left( \frac{(1-t_i)\lambda e^{-(r(1-t_i)-n)\lambda}}{(1-e^{-(r(1-t_i)-n)\lambda})} - \frac{(1-t_i)}{(r(1-t_i)-n)} \right) + (1-t_i)c_h \right] dr
 \end{aligned} \tag{A6}$$

where

$$A_5 = \frac{\pi p(p+\delta)}{(r(1-t_i)-n)} e^{-p\lambda} (1-e^{-(r(1-t_i)-n)\lambda}) \quad \text{and} \quad A_6 = A_5((1-t_w - \tau_w - \theta_w)w_h - m)$$

and

$$\begin{aligned}
 dc_l - A_3\Delta w_h - \left[ \frac{A_2(1-\alpha) + A_4\varepsilon(1-\alpha-\beta)}{(1-\alpha)} \right] \Delta w_l + \left[ \frac{A_2(1-\alpha) + A_4(1-\alpha-\beta)}{(1-\alpha)(1-u)} \right] du \\
 + \left[ \frac{A_2}{(1-\tau_w + \theta_f)} + \frac{A_4\varepsilon(1-\alpha-\beta)}{(1-\alpha)(1-t_w - \tau_w - \theta_w)} \right] d\tau_w \\
 = \left[ (A_2 + A_3 + A_4) \frac{pe^{-p\lambda}}{(1-e^{-p\lambda})} - m(1-\pi)pe^{-p\lambda} \right] d\lambda + \left[ t_i k - (1-t_i)b - \frac{A_4}{(1-\alpha)(r+\phi)} \right] dr
 \end{aligned} \tag{A7}$$

where

$$A_2 = (1-u)(1-\pi)(1-e^{-p\lambda})w_l(1-\tau_w + \theta_f)$$

$$A_3 = (t_w + \theta_w + \theta_f)\pi(1-e^{-p\lambda})w_h \quad \text{and} \quad A_4 = \frac{rt_i\alpha y}{r+\phi}$$

These equations correspond respectively to the second and third rows in the matrix system.

By taking the total differential of the equations (4' ) and (5'), we get respectively

$$\Delta y = \Delta w_h + \frac{1}{(1 + \tau_f + \theta_f)} d\tau_f + \frac{pe^{-p\lambda}}{(1 - e^{-p\lambda})} d\lambda \quad (A8)$$

and

$$\Delta y = \Delta w_l + \frac{1}{(1 + \tau_f + \theta_f)} d\tau_f - \frac{1}{(1 - u)} du + \frac{pe^{-p\lambda}}{(1 - e^{-p\lambda})} d\lambda \quad (A9)$$

Combining (A1) and (A2), then substituting the resulting equation into (A8) and (A9) gives

$$- \Delta w_h + \frac{\varepsilon(1 - \alpha - \beta)}{(1 - \alpha)} \Delta w_l - \frac{(1 - \alpha - \beta)}{(1 - \alpha)(1 - u)} du - \frac{\varepsilon(1 - \alpha - \beta)}{(1 - \alpha)(1 - t_w - \tau_w - \theta_w)} d\tau_w \quad (A8')$$

$$- \frac{1}{(1 + \tau_f + \theta_f)} d\tau_f = \frac{\alpha}{(1 - \alpha)(r + \phi)} dr$$

and

$$\frac{\varepsilon(1 - \alpha - \beta) - (1 - \alpha)}{(1 - \alpha)} \Delta w_l + \frac{\beta}{(1 - \alpha)(1 - u)} du - \frac{\varepsilon(1 - \alpha - \beta)}{(1 - \alpha)(1 - t_w - \tau_w - \theta_w)} d\tau_w \quad (A9')$$

$$- \frac{1}{(1 + \tau_f + \theta_f)} d\tau_f = \frac{\alpha}{(1 - \alpha)(r + \phi)} dr$$

These equations stand for the fourth and fifth rows in the matrix system, respectively.

The total differential of the equation (8) is



$$(1 - t_w - \tau_w - \theta_w - f)du - (u - \varepsilon)d\tau_w = 0 \quad (\text{A10})$$

and it represents the sixth row in the matrix.

Finally, the seventh row in the system is the total differential of the equation (12), which is given by

$$\begin{aligned} & (\tau_f + \tau_w)\pi w_h \Delta w_h + (\tau_f + \tau_w)(1 - u)(1 - \pi)w_l \Delta w_l - (\tau_f + \tau_w)(1 - \pi)w_l du \\ & + [\pi w_h + (1 - u)(1 - \pi)w_l](d\tau_w + d\tau_f) = -m \frac{pe^{-p\lambda}}{(1 - e^{-p\lambda})^2} d\lambda \end{aligned} \quad (\text{A11})$$

## Determination of the saddlepath slope and the negative eigenvalue

The solutions of the dynamic system defined by (21) are

$$z_t = z^* + \kappa_1 e^{\sigma_1 t} + \kappa_2 e^{\sigma_2 t}$$

$$c_t = c^* + \kappa_3 e^{\sigma_1 t} + \kappa_4 e^{\sigma_2 t}$$

But since  $\sigma_1 \cdot \sigma_2 = \det(\Psi) < 0$ , we can take  $\sigma_1 < 0$  and  $\sigma_2 > 0$ . The initial jump to the saddlepath means that the unstable root,  $\sigma_2$ , does not operate, so  $\kappa_2 = \kappa_4 = 0$ . Further, since equation system (21) is linear, so is the saddlepath, equation (22). That equation and the two above are consistent if and only if  $\kappa_3 = \varphi \kappa_1$ . Thus, once on the saddlepath, the solutions simplify to

$$z_t = z^* + \kappa_1 e^{\sigma_1 t}$$

$$c_t = c^* + \varphi \kappa_1 e^{\sigma_1 t}$$

If these two equations and their time derivatives are substituted into system (21) and  $d\lambda$  is set to zero, the result simplifies to

$$\begin{bmatrix} \sigma_1 & \varphi \sigma_1 \end{bmatrix}' = \Psi \begin{bmatrix} 1 & \varphi \end{bmatrix}'$$

which can be used to solve for  $\varphi$  and  $\sigma_1$ .

## **Chapter 3**

### **The Effects of Payroll Taxes in a Labor Market with Unemployment and Unsatisfied Demand for Labor**

#### **1. Introduction**

Since the beginning of the 1980s, chronic unemployment problems in most industrialized countries, including Canada, have prompted a heightened interest in empirical research on the causes of persistent unemployment. Among the areas examined has been payroll taxation and its effects on employment (or unemployment) and wages. To date, one unappealing feature of the large literature that has modeled the labor market effects of payroll taxation is that in estimating the empirical impacts of payroll taxes, no distinction is made between the firms' notional demand for labor – that is, the amount of labor that firms *desire* to employ at a given wage – and the observed quantity of labor that they end up hiring. In other words, the existing literature ignores the unsatisfied demand for labor as measured by the number of unfilled vacancies in the economy. One possible explanation is that many countries (including Canada) do not collect such data or else do not have continuous time series on the number of job vacancies.

Between 1971 and 1978, Statistics Canada conducted the Job Vacancy Survey (JVS) among employers representing approximately 90 percent of employment in Canada. This survey was designed to allow estimates of the number of vacancies for detailed occupations and thus to provide useful information for manpower programs aimed at reducing the amount of mismatch in the labor market. The survey was discontinued after 1978. As a result, before the advent of the 1999 Workplace and Employee Survey, there was no nation-wide measure of job vacancies in Canada between 1978 and 1999. The job vacancy rate – the proportion of unmet demand for labor in relation to the total demand – is an important economic indicator that summarizes firms' hiring intentions and indicates the future direction of labor demand. A hope of labor market analysts has been that vacancies and unemployment could be combined to

produce a measure of excess demand (or supply) for labor – a measure which plays a vital role in theories of labor market behavior. The absence of such data has long been regarded as a basic gap in Canadian labor market statistics, previous efforts being heavily concentrated on the supply-side data. In this regard, Gera *et al.* (1991, p. 45) conclude "a job vacancy survey would assist policy makers in analyzing the growing mismatches between unfilled job vacancies and the attributes of the unemployed. ...The cost of providing such data would possibly not be more than the cost incurred by misguided policies pursued without the insights the job vacancy data might provide."

However, Statistics Canada has been producing a help-wanted index since 1962, which has frequently been relied on for evidence of job vacancies (e.g., Reid and Meltz, 1979; Gera *et al.*, 1991; Archambault and Fortin, 1997). Two statistical series have been published, the first from January 1962 to December 1988 and the second from January 1981 until the present day. While trends in the help-wanted index may be a good indicator of trends in available jobs, the index itself does not provide a measure of the number of job vacancies (Morissette and Zhang, 2001). For Galarneau *et al.* (2001), the method used to provide help-wanted index data is simply not exhaustive.<sup>1</sup> Moreover, we find that the correlation between direct measures of vacancies produced by the JVS and the help-wanted series is only 0.46 for the period they overlap. This indicates that the help-wanted index is not such a good proxy for unfilled job vacancies. In this regard, Dziechciarz (1990) stresses that 'bad' proxy variables reflect only partially the signal of the variable they seek to account for and consequently lead to instability of the coefficients on the proxy variables.

The absence of direct measures or adequate indicators of job vacancies does not imply, however, that there is no satisfactory procedure to take into account the existence of unfilled vacancies in the economy.<sup>2</sup> There is, on the contrary, an appealing approach that would appear to merit serious consideration: the use of the so-called Beveridge curve – the inverse relationship between the rate of unemployment and the vacancy rate. According to Blanchard and Diamond (1989), macroeconomists thinking about aggregate labor market dynamics have organized their thoughts around two relations, the Phillips

curve and the Beveridge curve. Blanchard and Diamond (1989) observe further that the Beveridge curve has very much played second fiddle, and consider that this emphasis is wrong. They also stress that the Beveridge relation comes conceptually first and contains essential information about the functioning of the labor market and the shocks that affect it. This point of view is shared by Janet Yellen and George Akerlof (see Hall and Yellen, 1989, p.65 and footnote no. 1), who consider the Beveridge curve as the neglected stepsister of macroeconomics.

Thus, the purpose of this chapter is to examine empirically the labor market effects of payroll taxation while recognizing the existence of unfilled job vacancies, and use the Beveridge relation to overcome the lack of appropriate data on job vacancies. This work is done within the context of a disequilibrium model, which allows for the possibility that the real wage may not move quickly enough to clear the labor market, and that some agents may be rationed. The choice of this framework is not arbitrary. Typically in disequilibrium models<sup>3</sup> it is assumed that the observed quantity of labor in any given period is the minimum of the quantity demanded and the quantity supplied. As will be shown in the next section, this has the unappealing implication that unfilled job vacancies and unemployed workers do not coexist in the economy. This chapter represents an improvement over the existing literature that models the labor market effects of payroll taxation, in that it recognizes the coexistence of unemployment and unsatisfied demand for labor when estimating the effects of payroll taxes.

All empirical studies of payroll tax incidence and employment (or unemployment) effects have to confront difficult problems in finding data sets that contain sufficient variation in tax rates relative to the other factors affecting labor markets. For example, in the macroeconomic-labor literature – which has focused more on the employment and unemployment effects of payroll (and other) taxes rather than their incidence – this problem is masked by the use of a single combined variable, the so-called “wedge”. The wedge is the gap (in logarithm) between the real labor costs of the firm, on the one hand, and the real, post-tax consumption wage of the worker, on the other. Thus, the wedge consists of the tax rates applying to both employers and employees and the price of

consumer goods relative to value-added price.<sup>4</sup> However, with this aggregative approach, a differential analysis according to whether taxes on labor are imposed on the employer or employee is not possible, since both tax rates are constrained to have the same coefficient in estimation and/or simulation.

In contrast, our approach in this chapter consists of estimating a (basic) disequilibrium aggregate labor market model that does not impose such a restriction, and then use the estimated model to simulate the impacts of payroll taxation, even though we do not have data on payroll tax rates, or we observe very little variation in the available data.<sup>5</sup>

As will be seen in the next sections, our approach involves a minimal use of theory. Regarding the role of theory, there are in general three approaches. First, one can study a model based on a precise theory that imposes many over-identifying restrictions (as done for example in the other two essays). Second, one can do empirical work with essentially no reference to theory (like an unrestricted Vector Autoregression (VAR)). Third, one can do empirical work that relies on just a bit (what is intended to be the least controversial aspects of) theory – as do those who perform just-identified VARs. Often, the "least controversial" bit of theory is assumed to be the proposition that agents know enough to separate real from nominal items. This chapter is in the tradition of the third approach. Given the lack of (variation in the) data on the tax rates, we use (a bit of) theory to generate estimating equations and identify the parameters needed to do the simulations. But the theory we rely on is truly basic. It is limited to five simple ideas: demand curves slope down, supply curves slope up, prices rise when a market is in excess demand, labour market matches are best when there is extreme excess demand or supply, and the price deflator that is relevant for firms is different than the one that is relevant for households.

The remainder of the chapter is organized as follows. In the following section, we briefly review the concept of the Beveridge curve. Section 3 discusses the measures of job vacancies and help-wanted index in Canada. Section 4 presents evidence on job vacancies, help-wanted index and unemployment rate relationships in Canada. In Section

5 we review the empirical literature on the labor market effects of payroll taxes. Section 6 presents the model. The estimation and simulation procedures and results are discussed in Section 7. Section 8 concludes the chapter.

## 2. The concept of the Beveridge curve

A stable, declining relation between the rate of unemployment and the vacancy rate was postulated as early as the 1960s when the Phillips curve was developed. This postulate was necessary in order to express the excess demand for labor, as well as the change in the wage rate, as a function solely of the unemployment rate (e.g. Lipsey, 1960; Quandt and Rosen, 1986). The relation was heuristically deduced from the short-side principle and is depicted in Figure 1 (see Dow and Dicks-Mireaux, 1958, Holt and David, 1966, and in particular Hansen, 1970).

**Figure 1:** Employment and short-side principle

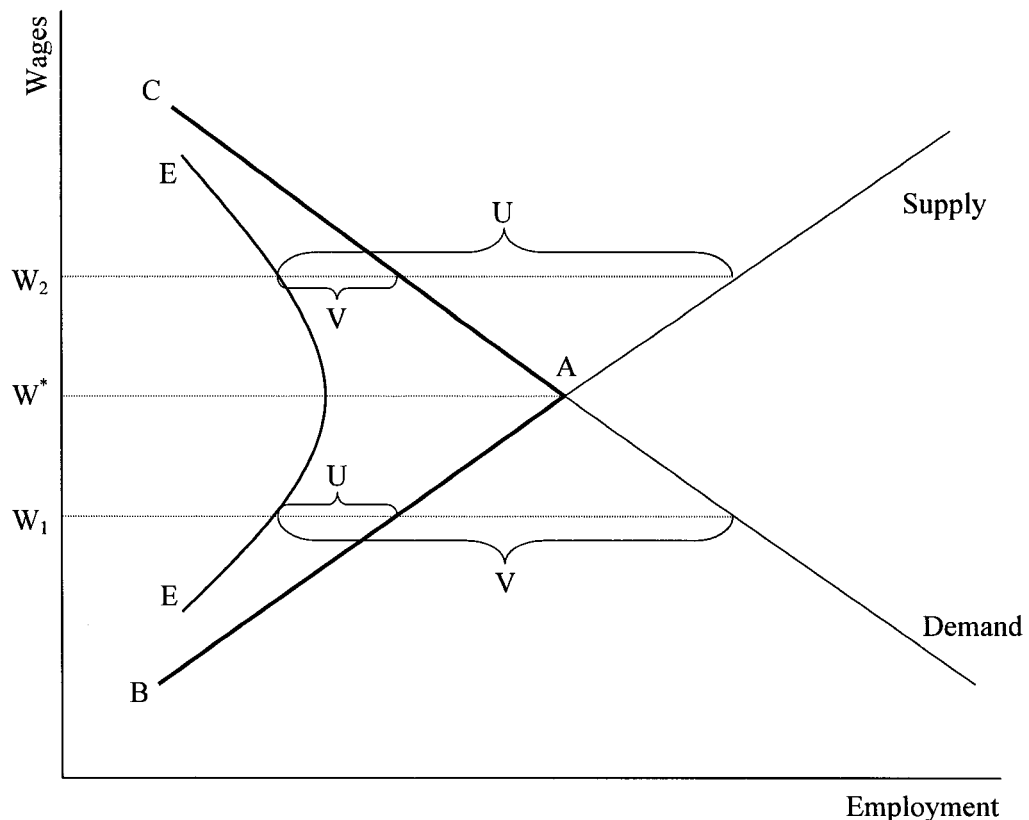
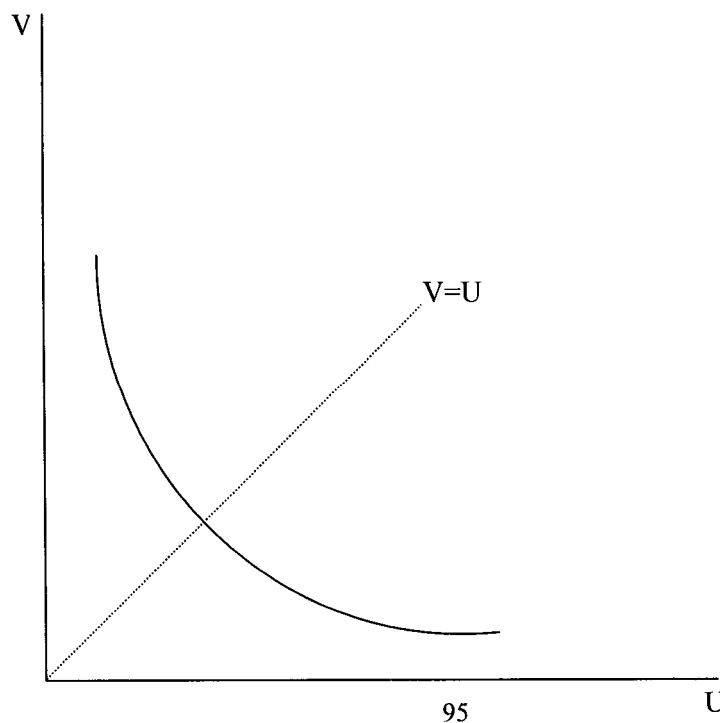


Figure 1 presents a simple demand and supply model of the labor market which depicts the simultaneous coexistence of unfilled jobs and unemployed workers. If vacancies and unemployed workers can perfectly and instantaneously be matched, the



branch AB on the supply curve is realized as long as labor demand is in excess of labor supply ( $W_1 < W^*$ ); otherwise the segment AC on the demand curve is realized. In the first case, the distance between the demand and supply curves represents the number of vacancies. In the second case ( $W_2 > W^*$ ), the distance between the supply and demand curves indicates the number of workers who are unemployed. If the matching of vacancies and unemployed workers is not perfect, as is the case in reality, only the amount of labor  $EE$  is realized. The shape of the  $EE$  curve results from the assumption that the matching becomes better when the pressure of excess demand or excess supply increases. The distance between the  $EE$  curve and the labor supply curve represents the number of unemployed, and the distance between the  $EE$  curve and the labor demand curve represents the number of vacancies. In labor market equilibrium ( $W = W^*$ ), the number of vacancies coincides with the number of unemployed, both numbers being positive. By plotting the number of unemployed  $U$  against the number of vacancies  $V$ , one generates a relation that has been called the Beveridge curve (Figure 2), in which the  $45^\circ$  line represents labor market equilibrium.

**Figure 2:** The Beveridge curve



It has frequently been assumed that the Beveridge curve can be approximated by either a hyperbolic or a linear relationship between  $V$  and  $U$  (see, e.g., Pencavel, 1974; Quandt and Rosen, 1986; Borsch-Supan, 1991; Stegman and Stegman, 1999; Osberg and Lin, 2000).

In an equilibrium model of the labor market, the observed quantity of labor is determined by the intersection of the supply and demand curves. This implies that the number of vacancies coincides with the number of unemployed, both numbers being equal to zero. Therefore, the Beveridge curve is a point, the origin of the axes in Figure 2.

In contrast, the bulk of the disequilibrium models assume that the observed quantity of labor during any given period is the minimum of the quantities supplied and demanded at the current wage. If more labor is demanded by the firms than is supplied by the population, unfilled vacancies will exist in the labor market; similarly, if more labor is supplied than is demanded, unemployment will exist. Thus, the min condition has the unappealing implication that unfilled jobs and unemployed workers do not simultaneously coexist in the economy. Therefore, the Beveridge curve is the horizontal and vertical axes in Figure 2.

### **3. Measures of job vacancies and help-wanted index in Canada**

The number of job vacancies is usually measured in three ways: by surveys, by job postings in employment centers and by newspaper ads. According to Galarneau *et al.* (2001), methods based on job postings in employment centers and newspapers ads are not exhaustive, since some job vacancies will not be advertised if the firm believes it can fill them easily. Therefore, job postings in employment centers will often be for positions that are more difficult to fill. Positions advertised in newspapers may appear more than once in different papers, and therefore the same job vacancy will be counted more than once. Also, with the advent of the Internet, the number of openings posted in newspapers represents no more than a portion of all job vacancies. The Statistics Canada help-wanted index provides the number of ads of vacant jobs in large metropolitan area newspapers since 1962. Two statistical series have been published, the first – old help-wanted index –

from January 1962 to December 1988 and the second – new help-wanted index – from January 1981 until the present day.

The old help-wanted index was constructed by measuring the column space of job advertisements published under the classified section of 18 major metropolitan area newspapers for 17 metropolitan areas (see Table 1). The measured column space for any given month was then compared to the respective average column space in the base year and adjusted by the appropriate population weights. Data were collected one Saturday a month – specifically, the Saturday corresponding to the reference week for the Labor Force Survey, usually the week which contains the 15<sup>th</sup> day of the month. When the ad space was measured, care was taken to exclude special headings and materials not pertaining to help-wanted advertisements. Ads which did not appear in the classified section (sections titled "careers" or "opportunities", ads for "position wanted", or ads for newspaper carriers) were excluded.

Since 1981, Statistics Canada has published another series, the new help-wanted index. The new index is constructed using the number of help-wanted advertisements in the classified section of the newspaper instead of measuring the column space. This eliminates the distortions caused by varying sizes of help-wanted ads across different newspapers at any given time or the changing format and layout of the classified section of newspapers over time. The number of metropolitan areas covered also increased from 17 to 20, and the number of newspapers covered rose from 18 to 22 (see Table 1). This improves the population representation both nationally and regionally.<sup>6</sup>

However, in spite of the differing methodology, Haggard-Guenette (1988) and Osberg and Lin (2000) show that the two series are nearly perfectly correlated. Shifting the two series from a lag of up to five months to a lead of up to eight months, Haggard-Guenette finds the cross-correlation coefficient between the two series ranges from 0.86 to 0.99; whereas Osberg and Lin report a (simple) correlation coefficient of 0.98 for the period the two series overlap (from January 1981 to December 1988).

It is generally accepted that surveys of job vacancies are one of the most reliable methods to collect information on vacancies. Between 1971 and 1978, Statistics Canada

conducted the Job Vacancy Survey (JVS). This survey was conducted twice a month by both mail and interviews among employers representing approximately 90 percent of employment in Canada. The excluded sectors were agriculture, domestic services, the military and fishing and trapping. Because of concern that no single respondent in a large establishment would be able to give an accurate report on all current job openings, large establishments were broken into subunits on the basis of where hiring points were located and these subunits became the sampled entities. As mentioned above, the survey was conducted in two phases: a mail phase and a follow-up interview phase. The results of the interview phase were used to correct for systematic over- or under-reporting on mail forms and for nonresponse bias. Job vacancies were defined in the same spirit as unemployed workers are defined, namely as jobs: 1) which were vacant for the entire survey day, 2) which employers tried to fill within four weeks prior to survey day (by advertising, contacting Canada Manpower Centers, interviewing walk-ins, etc.), 3) which were available immediately, and 4) which were available to persons outside the firm. By definition, the following job openings were excluded: 1) those that had a future starting date and thus were not immediately available, 2) those for which no recruiting action was undertaken or recruiting action stopped four weeks prior to the reference day, 3) those that could be filled immediately from employers' or unions' waiting lists and thus were not vacant for the entire reference day, 4) those that were open only to employees of the firm (either working or on temporary layoff).

Since the JVS program was discontinued after 1978, before the advent of the new Workplace and Employee Survey (WES), no nation-wide survey was available to analysts to measure directly the number of job vacancies between 1978 and 1999 in Canada. As a result, some authors (e.g., Gera *et al.*, 1991; Archambault and Fortin, 1997) used the help-wanted data as evidence of job vacancies. However, we find that the correlation between direct measures of vacancies available from the JVS and the help-wanted series is only 0.46 for the period they overlap. This indicates that the help-wanted index is not a very good proxy for unfilled job vacancies.

Finally, the new WES conducted in 1999 by Statistics Canada with the support of Human Resources Development Canada is a linked employer-employee file: it consists of both employer and employee components. Usable information was collected from 6,351 business locations and 24,597 employees, representing response rates of 94 percent and 83 percent, respectively.<sup>7</sup>

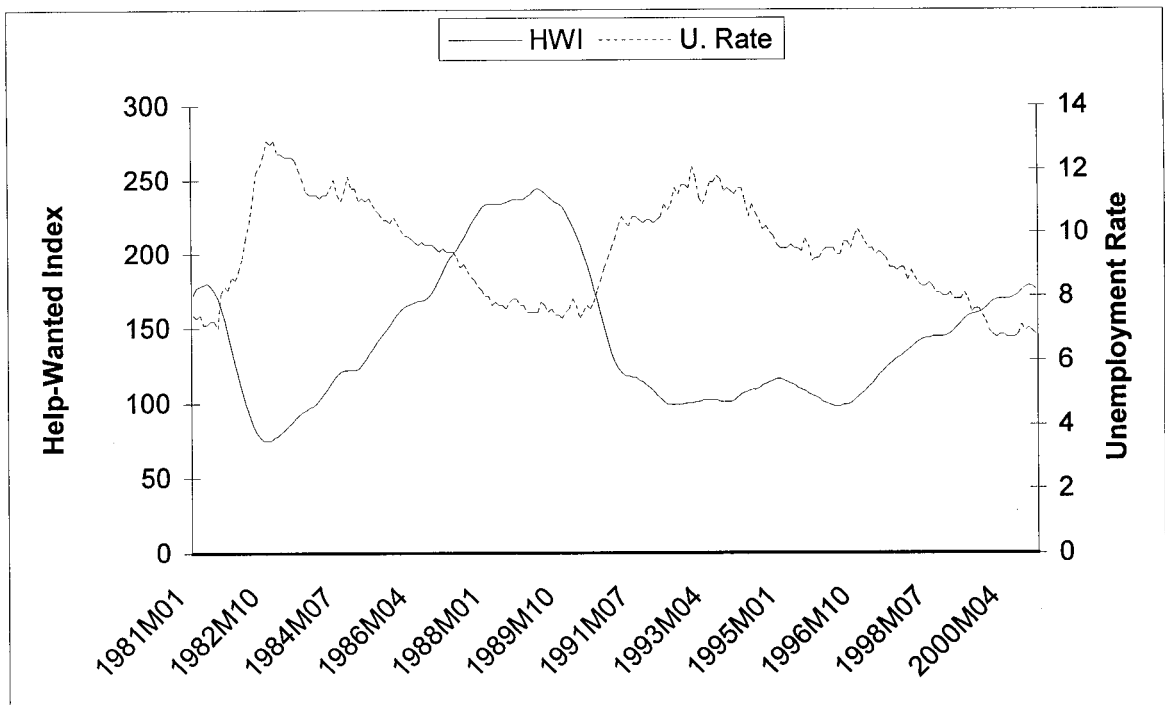
#### **4. Evidence on job vacancies, help-wanted index and unemployment rate relationships in Canada**

Abraham (1983) associated the vacancy rates produced by the Job Vacancy Survey with different unemployment rate ranges and reported a very strong negative relationship between the vacancy rate and the unemployment rate; that is, when the unemployment rate is high, the vacancy rate tends to be low, and vice versa.

Using data from the 1999 Workplace and Employee Survey (WES), Galarneau *et al.* (2001) found that Alberta, Manitoba and Ontario had the lowest unemployment rates and the highest vacancy rates. Conversely, the Atlantic provinces, British Columbia and Quebec had the largest unemployment rates and the lowest vacancy rates (see Figure 1 in Galarneau *et al.*, 2001, p. 12). By Census Metropolitan Areas, Toronto and Calgary showed relatively low unemployment rates and high vacancy rates, while the largest unemployment rates were seen in Quebec and Montreal, which had relatively low vacancy rates (see Figure 2 in Galarneau *et al.*, 2001, p. 12). Moreover, contrary to the common believe that most of the job vacancies are in the high tech sector, Morissette and Zhang (2001) reported from the 1999 WES data that in profit-oriented establishments, more than 40 percent of all job vacancies and 50 percent of long-term vacancies (i.e. vacancies unfilled for at least four months) originated from retail trade and consumer services industries, a sector that pays relatively low wages and has a high rate of labor turnover. This sector accounted for 30 percent of jobs in the private sector. This finding suggests that even in periods of strong growth in the high-technology industries – such as occurred in the late 1990s – a substantial share of job vacancies is found outside those industries.

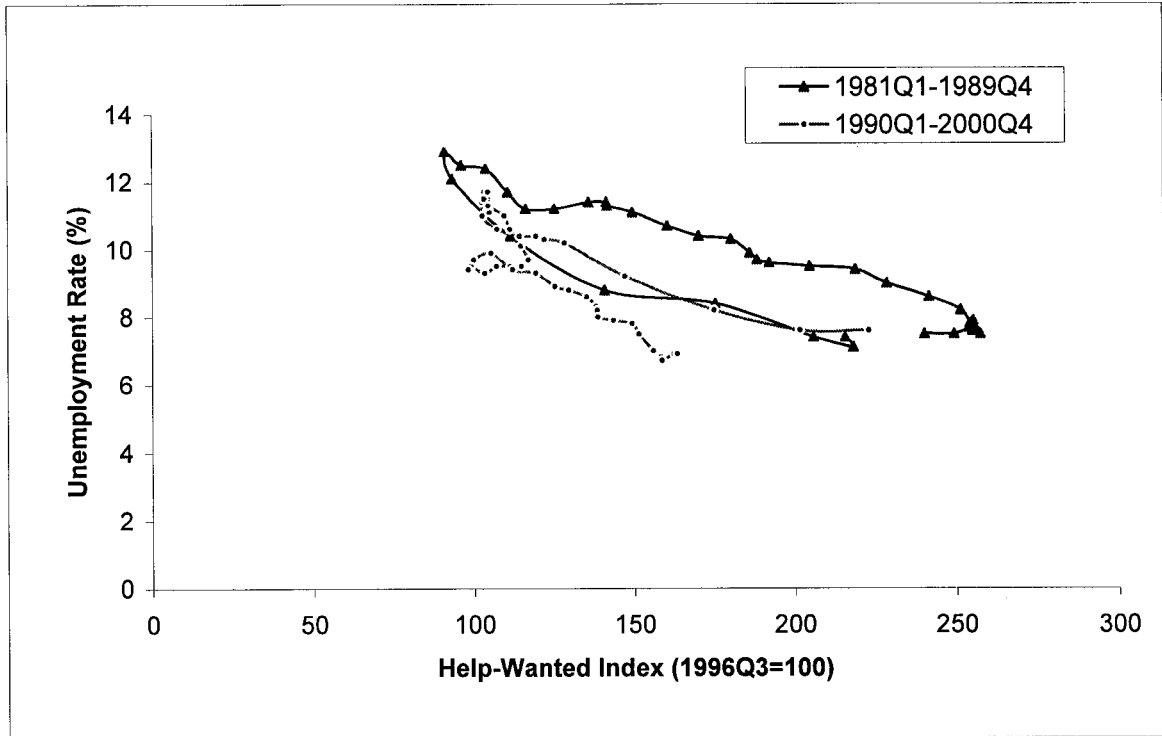
As mentioned earlier, the lack of direct measures of job vacancies in Canada between 1978 and 1999 has led some authors to use the help-wanted index as evidence of vacancy trends. As Figure 3 illustrates, over time the help-wanted index moves in almost perfect opposition to the unemployment rate.

**Figure 3:** Unemployment rate vs. HWI: Canada 1981-2000



Furthermore, many authors (such as Abraham, 1987; Jackman *et al.*, 1989; Blanchard and Diamond, 1989) have interpreted the shift to the right (outward shifts) of the Beveridge curve – which represents an increase in the unemployment rate for a given number of vacancies – as an increase in structural unemployment. As Figure 4 indicates, there appears to have been a significant outward shift in the help-wanted index/unemployment rate relationship during the 1980s, which Gera *et al.* (1991, p.44) saw as indicative of growing structural imbalances.

**Figure 4:** Unemployment rate and HWI<sup>8</sup> relationship, 1981-2000



Gera *et al.* (1991) argued that growing interregional disparities and the scarring effect of increasingly prevalent long-term unemployment were responsible for this increase in structural imbalances. More interestingly, they reject explanations based on generosity of the unemployment insurance system because "the evidence suggests that the UI system became less generous during the 1980s compared with the mid-1970s and that, as a result, ...the unemployment/vacancy relationship should therefore have shifted inward, not outward as it did in fact" (Gera *et al.*, 1991, p.22). This contrasts with Reid and Meltz (1979) who argued that the 1971 revisions to UI were largely responsible for a shift outward in the Beveridge curve from the mid-1960s to the mid-1970s. If the Reid and Meltz position is correct, the 1990s revisions to UI/EI (which have put the system back to 1950s levels of generosity) should have substantially shifted the Beveridge curve in. Similarly, Gera *et al.* argue that employment protection laws, minimum wages and unionization rates have the "wrong" trend to explain rising structural imbalances in the 1980s.

However, whatever happened to the help-wanted index/unemployment rate relationship in the 1980s, the 1990s have been very different. It is clear from Figure 4 that this relationship has shifted inwards during the 1990s, to such an extent that all of the outward shift of the 1980s has been more than reversed<sup>9</sup> – which appears (see Gera *et al.*, 1991, p.6) to be essentially identical to the late 1970s.

## **5. Survey of empirical literature on the labor market effects of payroll taxation**

Much of the earlier work on the labor market effects of payroll taxation was surveyed by Dahlby (1992,1993), Hamermesh (1993) and the OECD (1995) Jobs Study publication. In this section, we will briefly review the conclusions drawn in these surveys, and then examine more recent empirical studies that have focused on Canada.

Considerable debate exists over whether the effect of payroll taxes is to reduce wages or lower employment levels (or both) over the long term. In surveying the literature, Dahlby concluded that in the long run over 80 percent of the employer payroll tax burden is shifted backward to workers in the form of lower wages. Therefore, payroll taxes do not directly inhibit job creation, at least in the long run, but they may have an indirect effect on employment in the long run, by distorting labor market decisions. While the direct effect on wages of a payroll tax increase has only a short-run effect on employment, the short-run increase in unemployment reduces the real wage rate and decreases the size of the labor force. This short-term reduction in the labor force can lead to the ‘discouraged worker effect’ which can have long-term implications on employment.

Hamermesh (1993) surveyed a number of empirical studies of the incidence of payroll taxes and calculated the implied fraction of the payroll tax that is borne by labor through reduced wages. The labor demand models which use cross-sectional data imply complete backward shifting of payroll tax increases. However, the time-series analyses show widely divergent results, none of which imply complete shifting. Consequently, Hamermesh’s survey of the empirical literature led him to deduce that the lack of



consensus regarding the incidence of payroll taxes and its long-run impact on employment makes it impossible to draw any firm conclusions. Therefore, Hamermesh reached the following verdict: the lack of convincing direct estimates of payroll tax shifting and the well-established values of the labor supply and demand elasticities suggest that there is little scope in the long run for a payroll tax to reduce employment, or for a payroll tax subsidy to increase it. Barring substantial improvement in empirical studies of tax incidence, we must tentatively infer that most of the burden of payroll taxes is on wages.

The survey of literature conducted by the OECD (1995) Jobs Study concentrated more on studies that have focused on the employment and unemployment effects of payroll taxes rather than their incidence. These studies typically are set in macroeconomic models and often explore the dynamic path of employment responses to payroll (and other) tax rate changes. Based on the empirical studies surveyed, the Jobs Study concluded that there exists a general consensus that high taxes will raise the labor costs, hence reducing employment in the short run, although it is unclear whether they do so in the long run. There is less agreement on the extent to which taxes feed through to higher wages and higher unemployment, on the speed with which any effect takes place, and on whether there are symmetrical according to whether taxes levied on employers, employees, or consumers are raised.

While the Jobs Study literature survey found that the short-run effects on unemployment were generally less than one year, it drew attention, as Dahlby did, to the notion that short-term unemployment effects can become long-term effects. The Jobs Study referred to the notion of ‘hysteresis’, the loss of skills due to lack of use. This, in turn, can lead to long-term unemployment, even if the initial causes of unemployment are reversed. Hysteresis is not generally addressed in empirical studies which attempt to assess the employment impacts of payroll taxes. Thus, the OECD Jobs Study stressed, as did Hamermesh (1993), that it is important to recognise that there is no empirical evidence to suggest any simple linkage between the tax burden on labor and unemployment.

Several more recent empirical studies of the labor market effects of payroll taxes are worth reviewing. First is Tyrvainen (1995), a study commissioned by the OECD that examined the paths and long-run effects of various tax changes on wages, employment, and unemployment for 10 countries. It found that labor costs react differently to tax changes depending on the country and that those impacts can persist over long periods – more than 10 years in some countries. Simulations indicated that, even for economies with the most flexible wage adjustment – the United States and Sweden – it takes 10 years for the employment and unemployment effects of an employer payroll tax hike to vanish; and significant effects remain even after 5 years. For countries with greater wage rigidity, such as Canada and France, a portion of the labor market impacts was simulated to persist indefinitely. However, these results differ from other studies (surveyed by the OECD Jobs Study, 1995) for that do not suggest a long-run persistence of the impacts. Thus, different studies produced different results, even for the same country, regarding the impact of taxes on wages and employment.

The conflicting findings on payroll taxes even for the same economy are well illustrated by a number of ‘recent’ studies for Canada. Wilton and Prescott (1993), using micro-data on 2,529 private sector wage contracts (signed during the period from 1979 to 1992), estimated reduced-form equations to explain the effects of taxes on wage changes. Using several alternative specifications, they found that an increase in employer payroll taxes results in larger wage increases. In other words, the results suggest that firms are unable to shift any of the payroll tax increase onto labor and, in fact, may even end up paying higher wages when payroll taxes increase.

Beach *et al.* (1995) estimated a labor demand equation derived from a CES production function using pooled provincial employment and wage data for 1966-1993. Payroll taxes were analysed at a provincial level to take advantage of the variation of such levies across provinces. They found that the employer payroll tax is associated with a highly significant downward shift in the labor demand curve that is more consistent with full shifting of the tax back onto labor than with no or only partial shifting. Another Canadian study, Abbott and Beach (1997), also used cross-provincial data to estimate the

employment and wage effects of changes in employer payroll tax rates. Their main findings were that tax rate increases have strong and statistically significant impacts on both long-run employment and real wages. A one percentage point rise in the employer tax rate was estimated to reduce employment by 0.9 to 3.3 percent and annual wages per worker by 1.7 to 3.5 percent. These results were robust for estimates involving year- and province-specific fixed effects and cyclical macro controls, and they included both reduced-form and structural estimates.

Di Matteo and Shannon (1995) examined the impact of marginal and average payroll tax rates on levels of wages and employment. They conceded that there exists a wide range of empirical estimates as to the elasticities of labor demand and supply and, consequently, to the magnitude of job losses associated with payroll tax increases. However, their evidence is consistent with the view that payroll tax increases reduce employment.

Finally, based on a rudimentary model of the empirical relationship between the producer real wage and the level of productivity, as outlined in the simple theory of the firm, Parker (1995) estimated that the increase in supplementary labor income paid by employers between 1988 and 1993 reduced the level of employment by about one percent in 1993, *ceteris paribus*.

## **6. The Model**

In this section we construct a basic model of the labor market which consists of three equations: one each for the demand of labor, the supply of labor and the real wage adjustment. The deterministic version of each equation is discussed in turn.

### **Demand for labor**

A necessary condition for profit maximization by the representative firm is that the marginal product of labor equals the real labor costs of the firm:

$$WP_t(1 + \theta_t^f) = F_L(L_t, K_t, t) \quad (1)$$

where  $WP_t$  is the pretax real product wage (the nominal wage,  $W_t$ , divided by the value added price,  $P_t^o$ ),  $\theta_t^f$  is the payroll tax rate on firms, and  $F_L$  is the partial derivative of the production function,

$$Y_t = F(L_t, K_t, t) \quad (2)$$

and where  $Y_t$  is real output,  $L_t$  labor input,  $K_t$  the capital stock, and  $t$  is a time trend representing the state of technical progress in period  $t$ . Substituting out  $K_t$  from (2) to (1) and solving the resulting equation for  $L_t$ :

$$L_t^D = H(WP_t(1 + \theta_t^f), Y_t, t) \quad (3)$$

The superscript  $D$  indicates that this is the quantity of labor demanded given the level of output.  $L_t^D$  is the notional demand in the sense that it is the amount of labor that firms wish to employ – not necessarily the quantity they in fact hire.

Taking the log-linear approximation (except for  $t$ ) of equation (3), the labor demand is

$$\ln L_t^D = \alpha_0 + \alpha_1 \ln WP_t + \alpha_1 \theta_t^f + \alpha_2 \ln Y_t + \alpha_3 t \quad (4)$$

Although equation (4) is a proper structural relationship, it should be noted that since output is assumed to be exogenous the labor market is treated here in isolation from the rest of the economy.<sup>10</sup> However, equation (4) (or a minor variant) is a common starting point for studying labor markets (see, for example, Hamermesh, 1980; Quandt and Rosen, 1986; Rudebusch, 1986; Beach *et al.*, 1995; Tyrvaenen, 1995). Thus, for tractability it will be assumed that output is exogenous.

If the underlying production function is assumed to be CES, the coefficients in (4) can be used to solve the following set of interdependent equations for the CES parameters:  $\alpha_1 = -\sigma$ ,  $\alpha_2 = (\sigma h + 1 - \sigma) / h$ , and  $\alpha_3 = -\lambda \sigma (1 - \sigma) / h$ , where  $\sigma$  is the elasticity of substitution between capital and labor,  $h$  measures returns to scale, and  $\lambda$  is

the rate of Hicks-neutral technological change. Although this is an interesting interpretation, the usefulness of (4) does not rest upon the CES specification.

Finally, the labor demand in (4) is based on the assumption that labor is costlessly variable. Many authors have noted, however, the existence of adjustment costs in changing the amount of labor demanded; in consequence past levels of labor affect current desired levels. These costs lead firms to adjust gradually to the level indicated by the static labor demand equation, rather than maintaining it continually through time. To capture the effect of such partial adjustment, a lagged value of labor demand will be included as an explanatory variable (as in, for example, Hamermesh, 1980; Rudebusch, 1986, 1989; Keil and Symons, 1994), so labor demand is

$$\ln L_t^D = \alpha_0 + \alpha_1 \ln WP_t + \alpha_1 \theta_t^f + \alpha_2 \ln Y_t + \alpha_3 t + l_1 \ln L_{t-1}^D \quad (5)$$

### Supply of labor

The labor supply function is based upon the theory of labor-leisure choice. Consider a static version of the household optimization problem. The household maximizes utility over consumption ( $C_t$ ) and labor supplied ( $L_t$ ), subject to the budget constraint, that is,

$$\max U(C_t, L_t) \quad (6)$$

subject to

$$WC_t(1 - \theta_t^e)L_t + A_t - C_t = 0 \quad (7)$$

where  $WC_t$  is the pretax real consumption wage (the nominal wage,  $W_t$ , divided by the consumer price index,  $P_t^c$ ),  $\theta_t^e$  is the payroll tax rate on workers, and  $A_t$  is nonlabor real income – which is the nominal nonlabor income deflated by the consumer price index. The resulting labor supply is

$$L_t^S = Z(WC_t(1 - \theta_t^e), A_t) \quad (8)$$

which identifies consumer real wage net of taxes and nonlabor income as important influences on the decision to seek employment.

Equation (8) must be augmented by a scale variable that captures changes in the size of the potential labor force,  $TP$ . Also suppliers of labor may adjust slowly to attain the optimal levels because of habit or adjustment costs, such that previous labor supply may be a factor in current labor supply. Using a log-linear form, aggregate labor supply is then specified as

$$\ln L_t^S = \beta_0 + \beta_1 \ln WC_t - \beta_1 \theta_t^e + \beta_2 \ln A_t + \beta_3 \ln TP_t + l_2 \ln L_{t-1}^S \quad (9)$$

By definition,  $WC_t = W_t / P_t^c$ , which can be written as  $WC_t = (W_t / P_t^o) * (P_t^o / P_t^c)$ , where  $W_t / P_t^o = WP_t$ . Using these expressions in (9), labor supply becomes

$$\ln L_t^S = \beta_0 + \beta_1 \ln WP_t - \beta_1 \ln(P_t^c / P_t^o) - \beta_1 \theta_t^e + \beta_2 \ln A_t + \beta_3 \ln TP_t + l_2 \ln L_{t-1}^S \quad (10)$$

### Real wage adjustment

Equilibrium models of the labor market assume that real wage rates adjust instantaneously so as to bring the supply of and demand for labor into equality. However, according to some authors (see, for example, Boschen and Grossman, 1982; McCallum, 1986), although the assumption that markets are continuously cleared by prices aids in the formulation of rigorous theoretical macroeconomic models, it is still in doubt as to whether such equilibrium models can be reconciled with the short-run behavior of the economy.

In contrast, disequilibrium models of the labor market allow for the possibility that the real wage may not adjust quickly enough to clear the labor market. However, the ability to articulate a compelling choice-theoretic explanation for real wage stickiness has been a problem for disequilibrium advocates. Proponents are apt to point out that, despite difficulties in explaining why the labor market does not clear at every moment in time,

the real world does seem to be like that, and this fact should be reflected in economic analysis. As Rees (1970, p.234) observes,

Although we know very little about the exact nature of the costs of making wage changes, we can infer that they exist. Wages are, next to house rents, the stickiest general class of prices in the economy, seldom adjusted more frequently than once a year. This stickiness may be reinforced by unionism and collective bargaining, but it was present long before unions arrived.

The common assumption (see, for example, McCallum, 1974; Rosen and Quandt, 1978; Hamermesh, 1980, 1986; Rudebusch, 1986; Lee, 1988; Marrewijk and Verbeek, 1993, among others) has been that relative changes in the (real product) wage are proportional to (relative) levels of excess demand, that is,

$$\Delta \ln WP_t = \phi E_t, \tag{11}$$

where  $\Delta \ln WP_t = \ln WP_t - \ln WP_{t-1}$ ,  $E_t = (\ln L_t^D - \ln L_t^S)$  is the excess demand (or supply), and  $\phi > 0$ .<sup>11</sup>

Since we will be utilizing quarterly data, and because of the possibility that different economic units react with different speeds, we adopt a distributed lag formulation of equation (11), that is,

$$\Delta \ln WP_t = \phi(1 - \lambda)[E_t + \lambda E_{t-1} + \lambda^2 E_{t-2} + \dots] \tag{12}$$

where  $0 < \lambda < 1$ .

The Koyck transformation of equation (12) is

$$\Delta \ln WP_t = (1 - \lambda)\phi E_t + \lambda \Delta \ln WP_{t-1} \tag{13}$$

Now if excess demand for labor were directly observable there would be no need to go any further. But since this is not the case, it is necessary (at least for estimation purposes) to relate excess demand to something that is directly observable. In this regard, two approaches can be followed.

### “Phillips-Lipsev” Approach

The difference between unfilled vacancies and unemployed workers might provide a reasonable direct measure of excess demand (see the graphical exposition in Figure 1). To see this formally, recall that

$$E_t = (\ln L_t^D - \ln L_t^S)$$

As a result of the approximation formula  $(x - z) / z \approx \ln x - \ln z$ , we have

$$E_t = (L_t^D - L_t^S) / L_t^S$$

Let  $L_t$  be the observed quantity of labor in period  $t$  (which is represented along curve EE in Figure I). Then we have

$$E_t = [(L_t^D - L_t) / L_t^S] - [(L_t^S - L_t) / L_t^S] = V_t - U_t \quad (14)$$

where  $V_t$  and  $U_t$  denote the vacancy and unemployment rates, respectively.<sup>12</sup>

Combining (13) and (14), we get

$$\Delta \ln WP_t = (1 - \lambda)\phi[V_t - U_t] + \lambda \Delta \ln WP_{t-1} \quad (15)$$

This form is consistent with some earlier work that indicates that vacancies are an important determinant of wages, at least as important as unemployment (see, for example, Brownlie and Hampton, 1967; Schultze, 1971; Baily and Tobin, 1977; Abraham and Medoff, 1984; Jackman *et al.*, 1989). Unfortunately, Canadian data for the vacancy rate do not exist for our sample period. However, various authors (such as Pencavel, 1974; Quandt and Rosen, 1986; Borsch-Supan, 1991; Stegman and Stegman, 1999) suggest that the relationship between the vacancy and unemployment rates (a version of the Beveridge curve) can be approximated by either a hyperbolic or a linear relationship. That is,

$$V_t = \omega_0 + \omega_1 / U_t, \text{ where } \omega_1 > 0. \quad (16)$$

or

$$V_t = \eta_0 + \eta_1 U_t, \text{ where } \eta_1 < 0. \quad (17)$$



Using direct (quarterly) measures of vacancy rates produced by the JVS, we estimated equations (16) and (17) – augmented with quarterly dummy variables – and found a very strong negative relationship between the vacancy and unemployment rates (see the results in Table 2). Since the  $R^2$  's of the two estimations are almost equal, for simplicity, we will be using the linear relationship, equation (17).

Because the Beveridge curve may shift over time, some authors (such as Jackman *et al.*, 1989; Borsch-Supan, 1991; Stegman and Stegman, 1999) have augmented the vacancy-unemployment relationship with variables such as the youth and women shares of the labor force, the employment insurance benefits, and the share of long-term unemployed, that may account for these shifts.<sup>13</sup> Recall that the shift to the right (outward shifts) of the Beveridge curve has been interpreted as an increase in structural unemployment.

Using equation (17), augmented with shift variables, to substitute out  $V_t$  in (15), and then solving for  $U_t$ , we get

$$U_t = \delta_0 + \delta_2 \Delta \ln WP_t + \delta_3 \Delta \ln WP_{t-1} + \delta_4 Youth + \delta_5 Women + \delta_6 EIB \quad (18)$$

where *Youth* and *Women* represent the shares in the labor force, and *EIB* is the (real) employment insurance benefits.

An equation similar to (18) (that is, one in which the relative change in the wage rate is the dependent variable and there are no shift variables) was first estimated by Lipsey in his attempt to construct a theoretical model capable of explaining the Phillips curve.

### “McCallum” Approach

This approach consists of substituting expressions (5) and (10) into equation (13) (using the definition of  $E_t$ ) in order to express wage changes in terms of labor demand and supply determinants which are directly observable. This approach was initiated by McCallum (1974). The resulting equation is

$$\begin{aligned}\Delta \ln WP_t &= \pi_o + \pi_1 \ln WP_t + \pi_2 \ln Y_t + \pi_3 t + \pi_4 \ln A_t + \pi_5 \ln TP_t + \pi_6 \Delta \ln WP_{t-1} \\ &\quad + \pi_7 \ln(P_t^c / P_t^o) + \pi_7 \theta_t^e + \pi_8 \theta_t^f + \pi_9 (l_1 \ln L_{t-1}^D - l_2 \ln L_{t-1}^S)\end{aligned}\quad (19)$$

where  $\pi_7 = (1 - \lambda)\phi\beta_1$ ,  $\pi_8 = (1 - \lambda)\phi\alpha_1$  and  $\pi_1 = (1 - \lambda)\phi(\alpha_1 - \beta_1)$  or  $\pi_1 = \pi_8 - \pi_7$ .

For estimation, equation (19) must be transformed to eliminate the difference between the unobservable lagged demand and supply,  $(l_1 \ln L_{t-1}^D - l_2 \ln L_{t-1}^S)$ . Assuming for simplicity that  $l_1 = l_2$ , and substituting out the difference  $(\ln L_{t-1}^D - \ln L_{t-1}^S) = E_{t-1}$  in (19) using a lagged version of (13), we have

$$\begin{aligned}\Delta \ln WP_t &= \pi_o + \pi_1 \ln WP_t + \pi_2 \ln Y_t + \pi_3 t + \pi_4 \ln A_t + \pi_5 \ln TP_t + \pi_6^* \Delta \ln WP_{t-1} \\ &\quad + \pi_7 \ln(P_t^c / P_t^o) + \pi_7 \theta_t^e + \pi_8 \theta_t^f + \pi_9^* \Delta \ln WP_{t-2}\end{aligned}\quad (20)$$

Because of the simultaneity problem between the regressor  $\ln WP_t$  and the dependent variable  $\Delta \ln WP_t$ , we transform equation (20) as follows. We add and subtract  $\pi_1 \ln WP_{t-1}$  on the right-hand side of the equation, and after some manipulations we get

$$\begin{aligned}\Delta \ln WP_t &= \gamma_o + \gamma_1 \ln WP_{t-1} + \gamma_2 \ln Y_t + \gamma_3 t + \gamma_4 \ln A_t + \gamma_5 \ln TP_t + \gamma_6 \Delta \ln WP_{t-1} \\ &\quad + \gamma_7 \ln(P_t^c / P_t^o) + \gamma_7 \theta_t^e + \gamma_8 \theta_t^f + \gamma_9 \Delta \ln WP_{t-2}\end{aligned}\quad (21)$$

where  $\gamma_1 = \gamma_8 - \gamma_7$ . Compared to equation (20), note that  $\gamma_i = (1 / (1 - \pi_1))\pi_i$ .

Thus, taking the ratio of  $\gamma_1$  to  $\gamma_7$  (and substituting out  $\pi_1$  and  $\pi_7$ ), the relationship between the labor demand and supply elasticities (with respect to the wage rate),  $\alpha_1$  and  $\beta_1$  respectively, is as follows:

$$\alpha_1 = (1 + (\gamma_1 / \gamma_7))\beta_1 \quad (22)$$

Recall that in the macroeconomic-labor literature, the composite term,  $\ln(P_t^c / P_t^o) + \theta_t^e + \theta_t^f$ , is the so-called wedge. As mentioned earlier, the use of this single combined variable rules out differential responses depending on whether taxes on labor

are imposed on the employer or employee, since both tax rates are constrained to have the same coefficient in the ‘wage’ equation. Thus, it will not be known whether the effects are symmetrical according to whether payroll taxes are levied on employers or employees. In contrast, in equation (21), the ‘price’ wedge and the employee payroll tax rate enter with the same coefficient while the coefficient on the employer tax rate may differ.

Furthermore, since we observe very little variation in payroll tax rates over time, they will be dropped from the estimation equation – with minor or negligible resulting bias – in the next section.<sup>14</sup> But from equation (21), one can still recover the marginal effect of employer and employee payroll tax rates, and thereby avoid an aggregative analysis of the impacts of payroll taxes. Recall that in equation (21)  $\gamma_1 = \gamma_8 - \gamma_7$ . Thus the estimates for  $\gamma_1$  (coefficient on  $\ln WP_{t-1}$ ) and  $\gamma_7$  (coefficient on  $\ln(P_t^c / P_t^o)$ ) can be used to solve for  $\gamma_8$ . The resulting estimates of  $\gamma_7$  and  $\gamma_8$  allow us to simulate the impacts of an increase in both payroll taxes.

## 7. Estimation and simulation procedures and results

In this section, we combine the two approaches outlined above and estimate jointly the following two-equation system:

$$U_t = \delta_0 + \delta_1 U_{t-1} + \delta_2 \Delta \ln WP_t + \delta_3 \Delta \ln WP_{t-1} + \delta_4 Youth + \delta_5 Women + \delta_6 EIB + \varepsilon_{1t} \quad (18')$$

$$\Delta \ln WP_t = \gamma_0 + \gamma_1 \ln WP_{t-1} + \gamma_2 \ln Y_t + \gamma_3 t + \gamma_4 \ln A_t + \gamma_5 \ln TP_t + \gamma_6 \Delta \ln WP_{t-1} + \gamma_7 \ln(P_t^c / P_t^o) + \gamma_9 \Delta \ln WP_{t-2} + \varepsilon_{2t} \quad (21')$$

Equation (18') differs from (18) by the addition of both the lagged dependent variable and the error term  $\varepsilon_{1t}$ , and equation (21') differs from (21) both by the suppression of the payroll tax rate variables and the addition of the error term  $\varepsilon_{2t}$ .

We estimate this system using quarterly data relating to the Canadian economy for the period 1976-2000. Because of a profound difference between wage behavior in the

public and private sectors (see, for example, OECD, 1995; Tyrvaïnen, 1995), we consider only the private sector. A complete listing of the definitions of variables is given in the Appendix A. Before discussing results, we first characterize the unit root property of the data included in the analysis. Most macroeconomic time series are non-stationary in levels and the use of conventional regression techniques with such data tends to produce spurious results. However, non-stationary time series data may be *cointegrated* if some linear combinations of the series become stationary. That is, the series may wander around, but in the long run there may be economic forces that will tend to push them to an equilibrium. If so, cointegrated series will not move far away from each other since they are linked in the long run. In other words, the spurious regression problem does not arise if the non-stationary variables are cointegrated.

Thus, we test each of the variables included in the estimated system for unit roots using the Augmented Dickey-Fuller (ADF) test (1979), the Phillips-Perron (PP) test (1988) and the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test (1992). The results of these tests are presented in Table 3 (top panel). The test statistics suggest that some of the variables in equations (18') and (21') exhibit non-stationary behavior over the estimation period and are integrated of order one  $I(1)$ <sup>15</sup>. In order to test for cointegration between non-stationary variables in each equation, we perform the unit root tests for the estimated residuals,  $\varepsilon_{1t}$  and  $\varepsilon_{2t}$ , of the two-equation system. These tests are reported in Table 3 (bottom panel). The ADF and PP statistics indicate that the data reject the null hypothesis of no cointegration between non-stationary variables in each equation at the 5% significance level.

We turn to the estimation results. The technique of estimation is the Generalized Method of Moments (GMM). In particular, we estimate the system using the GMM-time series (HAC) procedure, which is robust to heteroskedasticity, contemporaneous correlation and autocorrelation of unknown form. The results are reported in Table 4.<sup>16</sup> Consider first the unemployment equation (18'). As suggested by the wage Phillips curve, our parameter estimates indicate that there exists a negative relationship between the (current) rate of change in real wages and the unemployment rate. Most of the existing

macroeconometric evidence lends support to this negative relation – labeled the Phillips curve – in Canada and in the United States (see, for example, Farès, 2002)<sup>17</sup>. The estimated slope of our ‘augmented’ Phillips curve ( $\delta_2$ ) is negative and statistically significant at the 5% level. The coefficient of the lagged unemployment rate ( $\delta_1$ ) also has the expected positive sign, and is strongly significant. This coefficient estimate indicates that there is a fair amount of persistence in the unemployment process. Regarding the ‘structural variables’ – the youth and women shares of the labor force and the employment insurance benefit payments – that are intended to capture the shifts in the Beveridge curve, their coefficient estimates ( $\delta_4$ ,  $\delta_5$ , and  $\delta_6$ , respectively) are negative (even if two of them are insignificant), which is counterintuitive. However, similar results have been found in other Canadian studies. For example, in examining the trend unemployment rate in Canada, Côté and Hostland (1996) also found that these three variables have negative effects on the unemployment rate.

Next, consider the wage equation (21'). The estimated coefficients on the lagged real wage ( $\gamma_1$ ), real output ( $\gamma_2$ ) and the price wedge ( $\gamma_7$ ) are all of the expected sign, and statistically significant at the 5% level. An apparently counterintuitive result is the sign of the coefficient on nonlabor income ( $\gamma_4$ ), which suggests a positive elasticity of labor supply with respect to real nonlabor income. This phenomenon has been noted in a number of previous studies (for example, Rudebusch, 1989). The sign of the estimated coefficient on the potential labor force ( $\delta_5$ ) accords with prior expectation, but it is not statistically significant. Following equation (22), we find that the elasticity of labor demand (with respect to the wage rate) is 10 times (in absolute value) as large as the labor supply elasticity. Furthermore, as pointed out earlier the parameter estimates of the wage equation (21') allow us to recover the marginal effect of employer and employee payroll tax rates. Recall from equation (21) that  $\gamma_1 = \gamma_8 - \gamma_7$ . The estimation provided estimates for  $\gamma_1$  (coefficient on  $\ln WP_{t-1}$ ) and  $\gamma_7$  (coefficient on  $\ln(P_t^c / P_t^o)$ ), which can be used to solve for  $\gamma_8$ . Then, one can use estimates of  $\gamma_7$  and  $\gamma_8$  to simulate the dynamic impacts of an increase in the payroll tax rates.<sup>18</sup>

The first step in producing such a simulation is to decide on values for our exogenous variables during the simulation period. For that, we employed the widely used Hannan and Rissanen's (1982) procedure to identify an ARIMA model for all exogenous variables whose coefficients are of the expected sign and statistically significant. Essentially, for a given I(1) exogenous variable this amounts to selecting an ARMA model for its first-difference on the basis of the minimization of the Schwarz Information Criterion.<sup>19</sup> Thereafter, the selected model is used to obtain values for that exogenous variable over the simulation period.

Next, we simulated the model under various scenarios and reported the results in terms of deviations from the baseline simulation. The first scenario we considered is a one time (permanent) increase of 1 percentage point in the employer payroll tax rate. As Figure 5 indicates, this rise in the employer tax rate causes a reduction in the pretax real product wage and an immediate increase of 0.32 percentage point in the unemployment rate. In the third quarter after the shock, the unemployment rate increases by 0.46 percentage point. Although unemployment declines thereafter – along with the reduction in the pretax real product wage – it takes about 5-6 years for the effect to die out almost fully. Eventually, around 91 percent of the employer payroll tax burden is shifted back onto labor through lower wages.

Figure 6 presents the simulation results of our second scenario, which consists of a one time rise of 1 percentage point in the employee payroll tax rate. As noted earlier, this leads to adjustment paths of the real wage and unemployment rate that differ from those for an increase in the employer tax rate. In this case, the pretax real wage rises until 9 percent of the increase in the employee payroll tax burden is shifted to employers in the form of higher wages. The impact on the unemployment rate is nil in the long run (as it was in the first scenario) *but* the short-run effect, although very small, is favorable. Though surprising, a similar short-run effect would have arisen in Hamermesh (1980) (whose model structure is similar to ours) if a distinction between the employer and employee payroll tax rates had been made. Since in his model (and subsequent

simulation) such a distinction was not made, his payroll tax rate increase simulation is equivalent to our next scenario; and let's say here that the results are very similar.

Finally, Figure 7 reports simulations in which both employer and employee payroll tax rates increase by 1 percentage point. In this case, the shapes of the simulated time paths are similar to those in the first scenario although the magnitude is slightly smaller here.

Thus, these results indicate that there is evidence of an asymmetric labor market effect according to whether payroll tax increases are levied on employers or employees. Moreover, it appears that most of the burden of payroll taxes – whether levied on employers or employees – is borne by labor in the long run.

Up until now, we have worked – as far as the simulations are concerned – in a deterministic environment. That is, we have been working under the assumption that our (stochastic) equations hold exactly over the simulation period. In reality, we would expect to see the same sort of errors occurring as we have seen in history. More importantly, we have also been ignoring the fact that the coefficients in our equations are estimated, rather than fixed at known values – that is, their point estimates. It would be interesting to reflect these uncertainties in our simulations results. This is a novelty in the sense that, to the best of our knowledge, none of the previous work that estimated empirically the labor market effects of payroll taxes has attempted to do it.

To incorporate these stochastic features into our simulations, we use a Monte Carlo approach, where the model is solved many times – here 3000 times – with both the estimated coefficients allowed to vary randomly during the simulation period and pseudo-random numbers substituted for the unknown errors at each repetition. More specifically, the coefficients are redrawn randomly once at the beginning of each repetition, and a set of independent random numbers are drawn from the standard normal distribution, then these numbers are scaled to match the variance-covariance matrix estimated from the model residuals.

Figure 8 presents our stochastic simulations results with 95% confidence intervals. When the model is simulated stochastically, it appears that we should be reluctant to

place too much weight on the time span over which payroll taxes affected unemployment in the deterministic simulations. The error bounds in the Figure 8 show that the short-run effects of payroll tax increases may last for a much shorter period.

## **8. Conclusions**

Much of the existing literature that models the labor market effects of payroll taxation does not distinguish between the firms' notional demand for labor and the observed quantity of labor when estimating empirically the effects of payroll taxes. In fact, it ignores the unsatisfied demand for labor as measured by the number of unfilled vacancies available in the economy. One possible explanation is that many countries (including Canada) do not collect such data or else do not have continuous time series on the number of job vacancies.

Furthermore, all empirical studies of payroll tax incidence and employment (or unemployment) effects have to confront difficult problems in finding data sets that contain sufficient variation in tax rates relative to other factors that affect labor markets. Because of the variation among provinces, some studies have overcome the lack of sufficient variations in payroll tax rates by analyzing payroll taxes at a provincial level. Others have addressed these problems by modeling taxation as a single total variable (employers' payroll taxes plus employees' social security and income taxes plus indirect taxes, as well as the term of trade). The use of a single combined variable rules out a differential analysis that distinguishes between taxes on labor that are imposed on the employer or employee, since both tax rates are constrained to have the same coefficient in estimation and/or simulation.

The purpose of this chapter is to examine empirically the labor market effects of payroll taxation while recognizing that unfilled job vacancies and unemployed workers simultaneously coexist in the economy, and then use the Beveridge relation to overcome the lack of (appropriate) data on job vacancies. This work is done within the context of a disequilibrium model, which allows for the possibility that the real wage may not move quickly enough to clear the labor market, and that some agents may be rationed. More



specifically, our approach consists of estimating a (basic) disequilibrium aggregate labor market model that allows the separate effects of payroll tax rates imposed on employers and employees to be distinguished, and using the estimated model to simulate the impacts of payroll taxation. This empirical work is done with the minimal use of theory just-identified approach.

The results indicate that there is evidence of an asymmetric labor market effect according to whether payroll taxes levied on employers or employees are raised. Although there is no impact on unemployment in the long run in either case, the short-run effects differ. Moreover, we find that the elasticity of labor demand (with respect to the wage rate) is 10 times (in absolute value) as large as the labor supply elasticity. As a result, most of the burden of payroll taxes – whether levied on employers or employees – is found to be borne by labor in the long run. Finally, it appears that we should be reluctant to place too much weight on the time span over which payroll taxes may affect unemployment in the (most widely performed) deterministic simulations. Indeed, the stochastic simulation results show that this time span may be much shorter.

Needless to say, the finding of a positive – even extremely small – short-run unemployment effect of an increase in the employee payroll tax rate is an issue on which more research is needed. It will be very interesting to find a convincing economic intuition behind the short-run asymmetric unemployment effect of payroll taxes. It is hoped that the current chapter's illustration will provide an impetus for the required research.

## References

- Abbott, M. and C. Beach (1997) "The Impact of Employer Payroll Taxes on Employment and Wages: Evidence for Canada, 1970-1993", in M. Abbott, C. Beach, and R. Chaykowsi, eds., *Transition and Structural Change in the North American Labor Market* (Kingston, Ont.: Queen's University, IRC Press), 154-234.
- Abraham, K. G. (1983) "Structural/Frictional vs. Deficient Demand Unemployment: Some New Evidence", *American Economic Review* 73, 708-724.
- Abraham, K. G. (1987) "Help-Wanted Advertising, Job Vacancies, and Unemployment", *Brookings Papers on Economic Activity* 1, 207-248.
- Abraham, K. G. and J. L. Medoff (1984) "Unemployment, Unsatisfied Demand for Labor, and Compensation Growth, 1956-1980", in *Workers, Jobs, and Inflation*, edited by Martin N. Baily. Washington: Brookings.
- Archambault, R. and M. Fortin (1997) "The Beveridge Curve and Unemployment Fluctuations in Canada", Working Paper No. W-97-4E, Ottawa: Applied Research Branch, Human Resources Development Canada.
- Baily, M. N. and J. Tobin (1977) "Macroeconomic Effects of Selective Public Employment and Wage Subsidies", *Brookings Papers on Economic Activity* 2, 511-541.
- Beach, C., Z. Lin and G. Picot (1995) "The Employer Payroll Tax in Canada and Its Effects on the Demand for Labor", (mimeograph, Queen's University, Department of Economics, Kingston, Ontario), 38.
- Blanchard, O. J. and P. Diamond (1989) "The Beveridge Curve", *Brookings Papers on Economic Activity* 1, 1-60.
- Borsch-Supan, A. H. (1991) "Panel Data Analysis of the Beveridge Curve: Is There a Macroeconomic Relation between the Rate of Unemployment and the Vacancy Rate?", *Economica*, New Series, 58, 279-297.
- Boschen, J. F. and H. Grossman (1982) "Tests of Equilibrium Macroeconomics Using Contemporaneous Monetary Data", *Journal of Monetary Economics* 10, 309-333.
- Brownlie, A. D. and P. Hampton (1967) "An Econometric Study of Wage Determination in New Zealand", *International Economic Review* 8, 327-334.

Côté, D. and D. Hostland (1996) "An Econometric Examination of the Trend Unemployment Rate in Canada", Bank of Canada Working Paper No. 96-7.

Dahlby, B. (1992) "Taxation and Social Insurance", in R.M. Bird and J.M. Mintz, eds., *Taxation to 2000 and beyond* (Toronto: Canadian Tax Foundation).

Dahlby, B. (1993) "Payroll Taxes", in A.M. Maslove, eds., *Business Taxation in Ontario* (Toronto: University of Toronto Press).

Di Matteo, L. and M. Shannon (1995) "Payroll Taxation in Canada: An Overview", *Canadian Business Economics* 3, 5-22.

Dickey, D.A. and W.A. Fuller (1979) "Distribution of the Estimators for Autoregressive Time Series with a Unit Root", *Journal of the American Statistical Association* 74, 427-431.

Dow, J. C. R. and L. A. Dicks-Mireaux (1958) "The Excess Demand for Labor: A Study of Conditions in Great Britain, 1946-56", *Oxford Economic Papers* 10, 1-33.

Dziechciarz, J. (1990) "Changing and Random Coefficient Models. A Survey", in Peter Hackl (ed.), *Statistical Analysis and Forecasting of Economic Structural Change*, Berlin: Springer-verlag, 217-251.

Farès, J. (2002) "Does Micro Evidence Support the Wage Phillips Curve in Canada?", Bank of Canada Working Paper No. 2002-4.

Galarneau, D., R. Morissette, H. Krebs and X. Zhang (2001) *The Quest for Workers: A New Portrait of Job Vacancies in Canada*, Research Paper No. 71-584-XIE No. 2, Ottawa: Statistics Canada, Human Resources Development Canada.

Gera, S., S. Rahman and J.L. Arcand (1991) "Structural Imbalances in Canadian Labor Markets", Working Paper No. 18, Ottawa: Economic Council of Canada.

Haggag-Guenette, C. (1988) "The 'Old' and the 'Revised' Help-Wanted Index: A Comparison", *Help-Wanted Index* 1988, Cat. No. 71-204. Ottawa: Statistics Canada, 33-35.

Hall, R. E. and J. Yellen (1989) "The Beveridge Curve. Comments and Discussion", *Brookings Papers on Economic Activity* 1, 61-76.

Hamermesh, D. S. (1980) "Factor Market Dynamics and the Incidence of Taxes and Subsidies", *Quarterly Journal of Economics* 95, 751-764.

Hamermesh, D. S. (1986) "Inflation and Labor Market Adjustment", *Economica* 53, 63-73.

Hamermesh, D. S. (1993) *Labor Demand*, Princeton, NJ: Princeton University Press.

Hannan, E. J. and J. Rissanen (1982) "Recursive Estimation of Mixed Autoregressive-Moving Average Order", *Biometrika* 69, 81-94. Correction (1983), 70, P. 303.

Hansen, B. (1970) "Excess Demand, Unemployment, Vacancies and Wages", *Quarterly Journal of Economics* 84, 1-23.

Holt, C. C. and M. H. David (1966) *The Concept of Job Vacancies in a Dynamic Theory of the Labor Market*, in NBER, *The Measurement and Interpretation of Job Vacancies*. New York: Columbia University Press.

Jackman, R., R. Layard and C. Pissarides (1989) "On Vacancies", *Oxford Bulletin of Economics and Statistics* 51, 377-394.

Keil, M. W. and J. S. Symons (1994) "An Aggregate Model of the Canadian Labor Market", *Journal of Macroeconomics* 16, 589-612.

Kwiatkowski, D., Phillips, P., Schmidt, P., and Y. Shin (1992) "Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root", *Journal of Econometrics* 54, 159-178.

Lee, K. (1988) "Inflation and Labor Market Adjustment: The UK Experience", *Economica*, New Series, 55, 409-416.

Lipsey, R. G. (1960) "The Relation between Unemployment and the Rate of Change in Money Wage Rates in the United Kingdom, 1862-1957: A Further Analysis", *Economica* 27, 1-31.

Marrewijk, C. van and J. Verbeek (1993) "Disequilibrium Growth Theory in an International Perspective", *Oxford Economic Papers* 45, 311-331.

McCallum, B. T. (1974) "Wage Rate Changes and the Excess Demand for Labor: An Alternative Formulation", *Economica*, New Series, 41, 269-277.

McCallum, B. T. (1986) "On 'Real' and 'Sticky-Price' Theories of the Business Cycle", *Journal of Money, Credit, and Banking* 18, 397-414.

Morissette, R. and X. Zhang (2001) *Which Firms Have High Job Vacancy Rates in Canada?* Research Paper 11F0019MIE No. 176. Analytical Studies, Ottawa: Statistics Canada.

OECD (1995) "The OECD Jobs Study: Taxation, Employment and Unemployment", OECD, Paris.

Osberg, L. and Z. Lin (2000) "How Much of Canada's Unemployment is Structural", *Canadian Public Policy*, Special Supplement on Structural Aspects of Unemployment in Canada, Vol. 26, No. 1, S141-S157.

Pancavel, J. H. (1974) "A Note on the Use of the Unemployment and Vacancy Statistics to Measure some Effects of Government Legislation", mimeo, Stanford University.

Parker, R. (1995) "Aspects of Economic Restructuring in Canada, 1989-1994", *Bank of Canada Review*, Summer 1995.

Phillips, P. and P. Perron (1988) "Testing for a Unit Root in Time Series Regression", *Biometrika* 75, 335-346.

Quandt, R. E. and H. S. Rosen (1986) "Unemployment, Disequilibrium and the Short Run Phillips Curve: An Econometric Approach", *Journal of Applied Econometrics* 1, 235-253.

Rees, A. (1970) "The Phillips Curve as a Menu for Policy Choice", *Economica* 37, 227-238.

Reid, F. and N. Meltz (1979) "Causes of Shifts in the Unemployment-Vacancy Relationship: An Empirical Analysis for Canada", *Review of Economics and Statistics* 61, 470-475.

Rosen, H. S. and R. E. Quandt (1978) "Estimation of a Disequilibrium Aggregate Labor Market", *Review of Economics and Statistics* 60, 371-379.

Rudebusch, G. D. (1986) "Testing for Labor Market Equilibrium with an Exact Excess Demand Disequilibrium Model", *The Review of Economics and Statistics* 68, 468-476.

Rudebusch, G. D. (1989) "An Empirical Disequilibrium Model of Labor, Consumption, and Investment", *International Economic Review* 30, 633-654.

Schultze, C. L. (1971) "Has the Phillips Curve Shifted? Some Additional Evidence", *Brookings Papers on Economic Activity* 1, 452-467.

Statistics Canada (1988) *Help-Wanted Index 1988*, Cat. No. 71-204. Ottawa: Supply and Services Canada.

Stegman, A. and T. Stegman (1999) "Labor Market Flexibility, the Beveridge Curve, and the Output-Employment Ratio in Australia", Discussion Paper 99/16, School of Economics, University of New South Wales, Sydney.

Tyrvainen, T. (1995) "Real Wage Resistance and Unemployment: Multivariate Analysis of Cointegrating Relations in 10 OECD Countries", The OECD Jobs Study Working Papers Series, No. 10. OECD, Paris.

Wilton, D. and D. Prescott (1993) "The Effects of Tax Increases on Wage and Labor costs", Discussion Paper no. 93-29 (Kingston, Ont.: Queen's University, School of Policy Studies), 26.

## Endnotes

---

<sup>1</sup> See Abraham (1987) for more details about the strengths and weaknesses of help-wanted index as a measure of the vacancy rate.

<sup>2</sup> For example, using data from the 1999 Workplace and Employee Survey, Galarneau *et al.* (2001) reported a job vacancy rate of 2.6 percent. In the same year, the unemployment rate was 7.6 percent. Note that the job vacancy rate was calculated as follows: (number of vacant positions/(number of employees + number of vacant positions))\*100.

<sup>3</sup> See, for example, Rosen and Quandt (1978); Hamermesh (1980); Quandt and Rosen (1986); Rudebusch (1986, 1989); Marrewijk and Verbeek (1993) among others.

<sup>4</sup> Formally, the wedge is given by  $\theta^f + \theta^e + p^c - p^o$ , where  $\theta^f$  and  $\theta^e$  are the tax rates on employers and employees, respectively,  $p^c$  is the consumer price index, and  $p^o$  the value added price.

<sup>5</sup> In other studies (such as Beach *et al.*, 1995), the lack of sufficient variations in payroll tax rates has been addressed by using cross-provincial payroll tax rates in Canada.

<sup>6</sup> For more details on the construction of the new series, see Statistics Canada, *Help-Wanted Index*, Catalogue 71-204.

<sup>7</sup> For more details on the 1999 WES and its differences from the JVS of 1971-1978, see Galarneau *et al.* (2001) or Morissette and Zhang (2001).

<sup>8</sup> The help-wanted index (HWI) is adjusted by the working-age (15+) population.

<sup>9</sup> In regressing the unemployment rate on the HWI and squared HWI, Osberg and Lin (2000) test for this inward shift by using a dummy variable indicating the 1990s. They report that in comparison to the 1980s, in the 1990s a 1.6 percentage point lower unemployment rate is observed for any given level of the HWI.

<sup>10</sup> Ideally, one would want to study a multi-market model in which output was treated econometrically as an endogenous variable. This task is beyond the scope of the current chapter.

<sup>11</sup> If  $\phi \rightarrow \infty$  (infinitely fast wage adjustment),  $L_t^D = L_t^S$ , and the disequilibrium model reduces exactly to the equilibrium model. Thus, a proper test of disequilibrium is the significance of  $1/\phi$ . If it is significantly different from zero, equilibrium is rejected. Rosen and Quandt use a real wage adjustment equation similar to (11) and apply this parameter test to reject the hypothesis of equilibrium.

<sup>12</sup> Note that  $V_t$  and  $U_t$  represent the rates (instead of numbers) henceforth.

<sup>13</sup> In previous studies, only the proportion of long-term unemployed appeared to be relevant. Unfortunately, there is no continuous time series data on such variable in Canada.

<sup>14</sup> Beach *et al.* (1995) overcame the lack of sufficient variations in payroll tax rates by generating data on the cross-provincial tax rates in Canada. Interestingly, the Canada-wide payroll tax rate follows almost a time trend in Beach *et al.* (see their Figure 3, p. 17).

<sup>15</sup> Subsequent testing on the first differences of non-stationary variables suggested none of them were I(2).

<sup>16</sup> Since we use quarterly data, notice that the implementation of the estimation included a set of three quarterly dummies (with fourth quarter as the default) to sweep out seasonality effects.

<sup>17</sup> Note that in the standard Phillips curve specification, the rate of change of real wages is the dependent variable.

<sup>18</sup> At this point, it is interesting to note the difference between the estimates of  $\gamma_7$  and  $\gamma_8$ , respectively the marginal effect of employee and employer payroll tax rates in the wage equation. This difference indicates (at least on impact) a differential response according to whether taxes on labor are imposed on the employer or employee.

<sup>19</sup> For more details about Hannan and Rissanen's (1982) procedure, see Appendix A.



**Table 1:** Newspapers covered in the help-wanted index survey

Region	Newspaper
Atlantic:	St. John's Evening Telegram, Charlottetown Guardian, Halifax Chronicle Herald, Saint John Telegraph Journal, Moncton Times Transcript*
Quebec:	Quebec Le Soleil, Sherbrooke la Tribune, Montreal Gazette, Montreal La Presse, Ottawa-Hull Le Droit, Ottawa-Hull Citizen*
Ontario:	Ottawa-Hull Le Droit, Ottawa-Hull Citizen*, Toronto Star, Hamilton Spectator, London Free Press, Sudbury Star*
Prairie:	Winnipeg Free Press, Regina Leader Post, Saskatoon Star Phoenix*, Edmonton Journal, Calgary Herald
BC:	Vancouver Sun, Victoria Times Colonist

\* Added to the new series.

**Table 2:** Linear and hyperbolic Beveridge relationships

Dependent Variable:  $V$

Independent Variable	Linear Relation		Hyperbolic Relation	
	Coeff.	t-ratio	Coeff.	t-ratio
Constant	21.772	10.38	-9.360	-5.35
$U$	-2.279	-7.64		
$1/U$			103.529	9.53
Quarter 1	2.283	2.34	2.237	2.43
Quarter 2	1.511	1.61	1.485	1.74
Quarter 3	1.389	1.31	1.227	1.23
$R^2$	0.686		0.690	

Note: Estimation by least squares with White heteroskedasticity-consistent standard errors & covariance.

**Table 3:** Tests for integration and cointegration

<b>I- Unit Root and Stationarity Tests</b>			
Variable	ADF	PP	KPSS <sup>a</sup>
<i>U</i>	-3.38	-2.73	0.13
<i>U</i> <sub>-1</sub>	-3.41	-2.80	0.14
$\Delta \ln WP$	-3.77	-19.41	0.33
$\Delta \ln WP_{-1}$	-3.75	-19.36	0.29
<i>Youth</i>	-2.15	-0.94	1.18
<i>Women</i>	-2.47	-1.16	1.19
<i>EIB</i>	-2.81	-2.64	0.26
$\ln WP_{-1}$	1.29	-0.85	1.23
$\ln Y$	-0.26	-0.80	1.21
$\ln A$	-2.50	-6.48	0.97
$\ln TP$	-1.51	-3.75	1.22
$\ln(P^c / P^o)$	-2.40	-1.27	1.12
$\Delta \ln WP_{-2}$	-3.77	-18.90	0.34
5% Critical Value	-2.89	-2.89	0.463
<b>II- Cointegration Tests (using residual-based versions of ADF and PP)</b>			
	ADF	PP	5% Critical V <sup>b</sup>
Residual1, $\varepsilon_{1t}$	-9.13	-9.12	-4.12
Residual2, $\varepsilon_{2t}$	-7.00	-6.95	-4.72

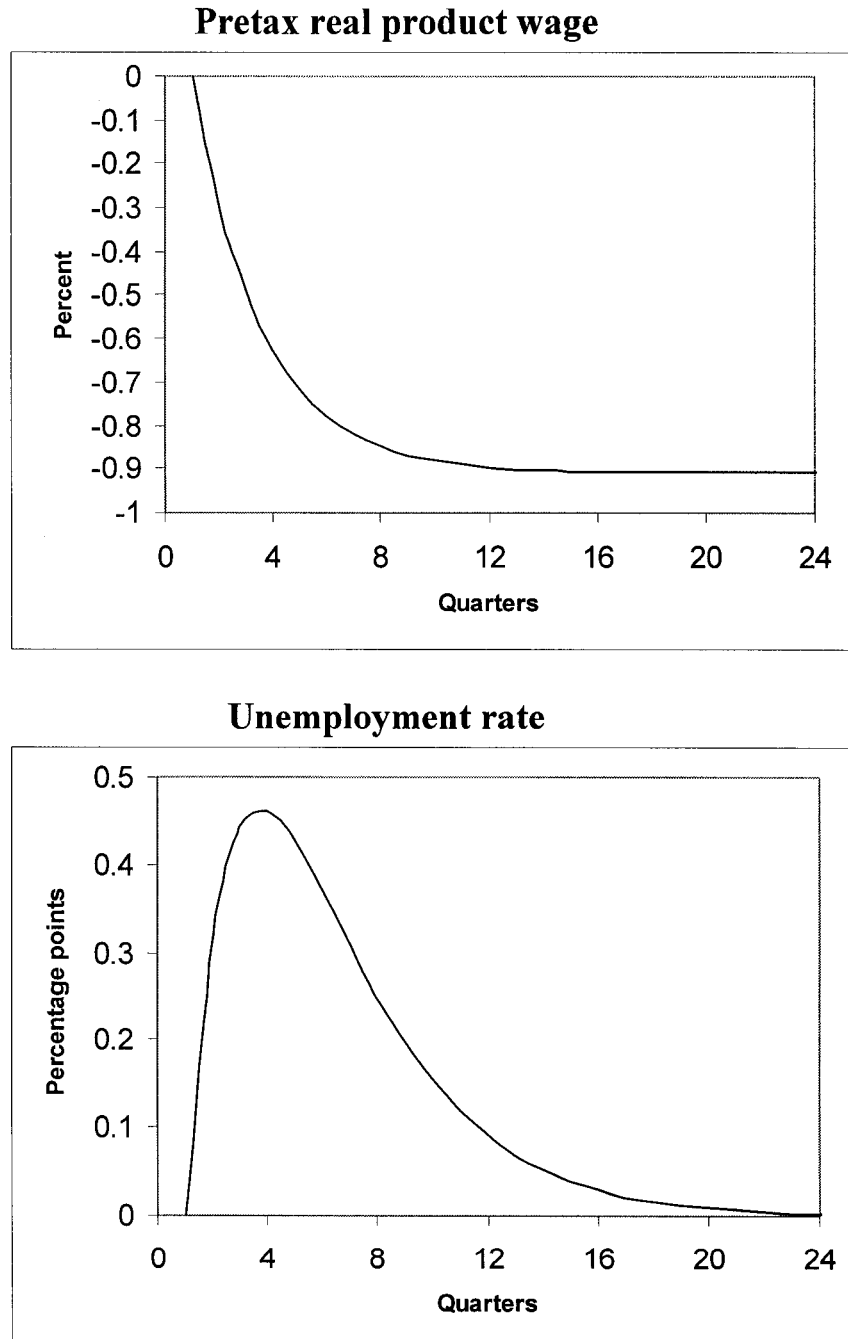
Notes: <sup>a</sup> The KPSS Lagrange Multiplier test differs from the other two unit root tests in that the series is assumed to be stationary under the null hypothesis. Moreover, one rejects the null if the LM test statistic is greater than the critical value.

<sup>b</sup> Note that since the unit root tests are applied upon residuals obtained as a by-product of estimation rather than on raw data, the critical values differ from those mentioned above. Moreover, they depend also upon the number of I(1) variables in the regression equation.

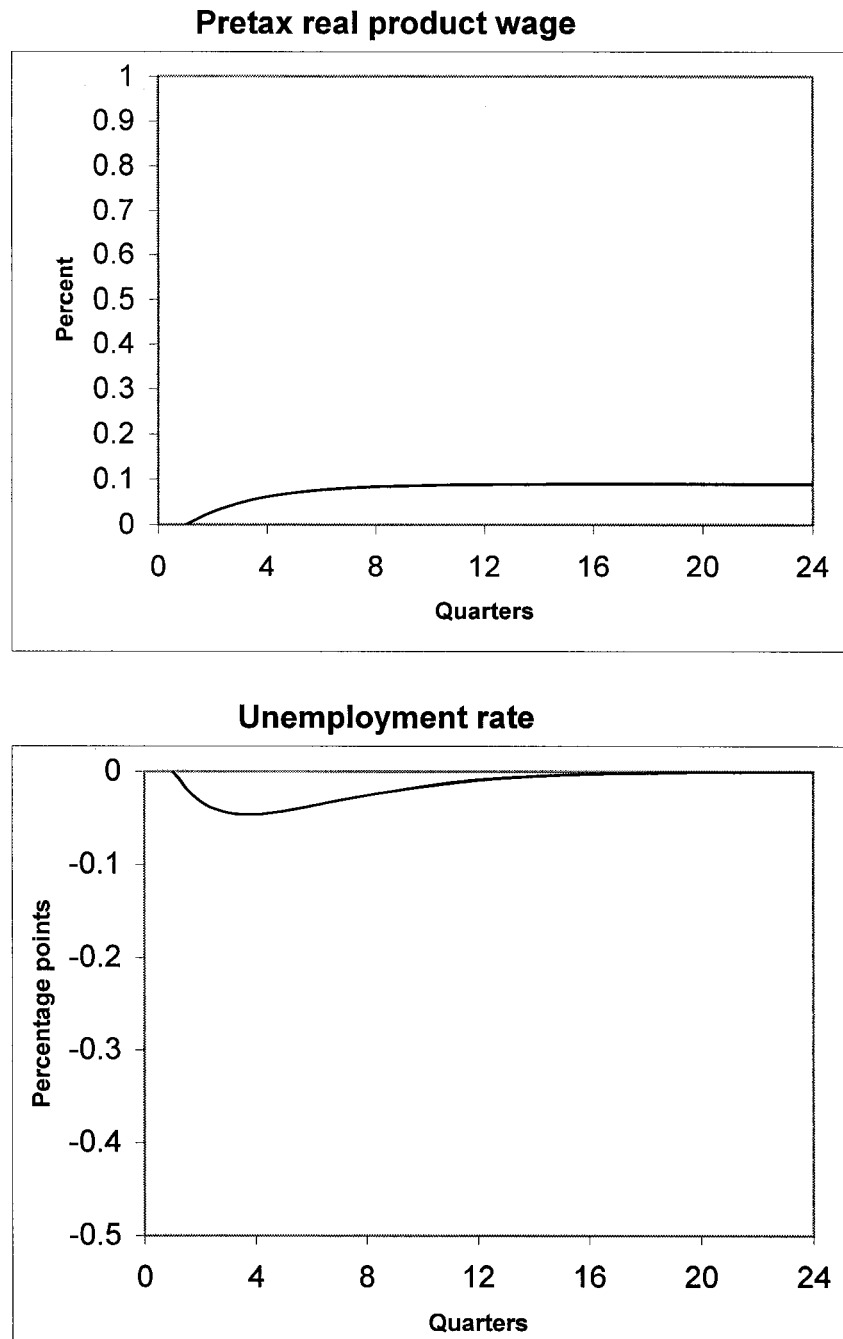
**Table 4:** GMM estimates of the parameters

Equation (18')			Equation (21')		
Dependent Variable: $U_t$			Dependent Variable: $\Delta \ln WP_t$		
Parameter	Estimate	t-Statistics	Parameter	Estimate	t-Statistics
$\delta_0$	1.048	2.04	$\gamma_0$	0.443	0.17
$\delta_1$	0.709	4.76	$\gamma_1$	-0.322	-3.60
$\delta_2$	-1.090	-2.00	$\gamma_2$	0.157	3.31
$\delta_3$	-0.290	-1.57	$\gamma_3$	0.001	0.40
$\delta_4$	-1.947	-2.11	$\gamma_4$	-0.089	-6.31
$\delta_5$	-0.996	-1.73	$\gamma_5$	-0.219	-0.48
$\delta_6$	-0.083	-1.81	$\gamma_6$	-0.114	-1.19
Quarter 1	0.006	0.92	$\gamma_7$	0.029	2.51
Quarter 2	-0.017	-1.75	$\gamma_9$	0.053	0.70
Quarter 3	-0.028	-1.64	Quarter 1	-0.013	-3.04
			Quarter 2	-0.024	-5.84
			Quarter 3	-0.029	-7.55

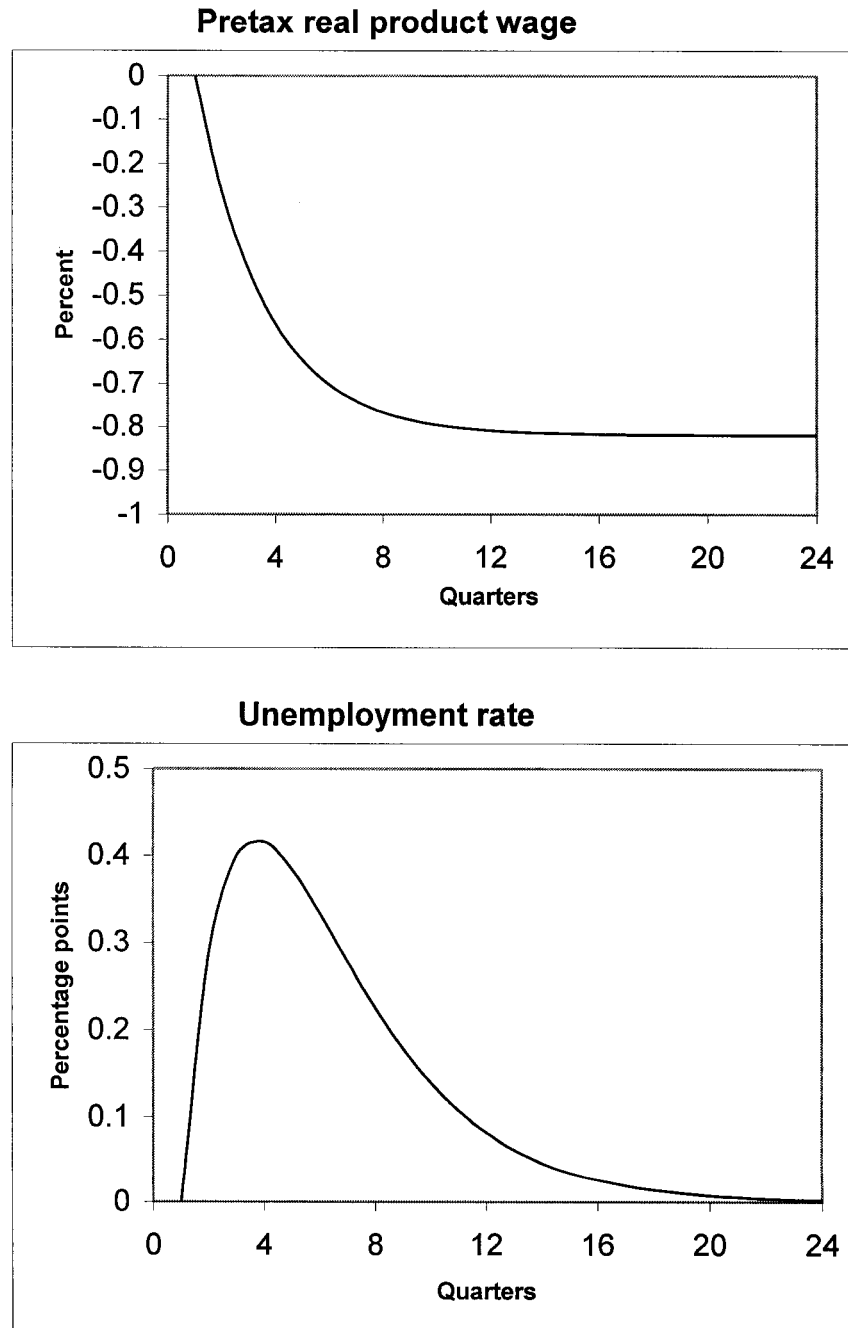
**Figure 5:** The simulated effect of a rise of one percentage point in employers' payroll tax rate – deviations from baseline.



**Figure 6:** The simulated effect of a rise of one percentage point in employees' payroll tax rate – deviations from baseline.

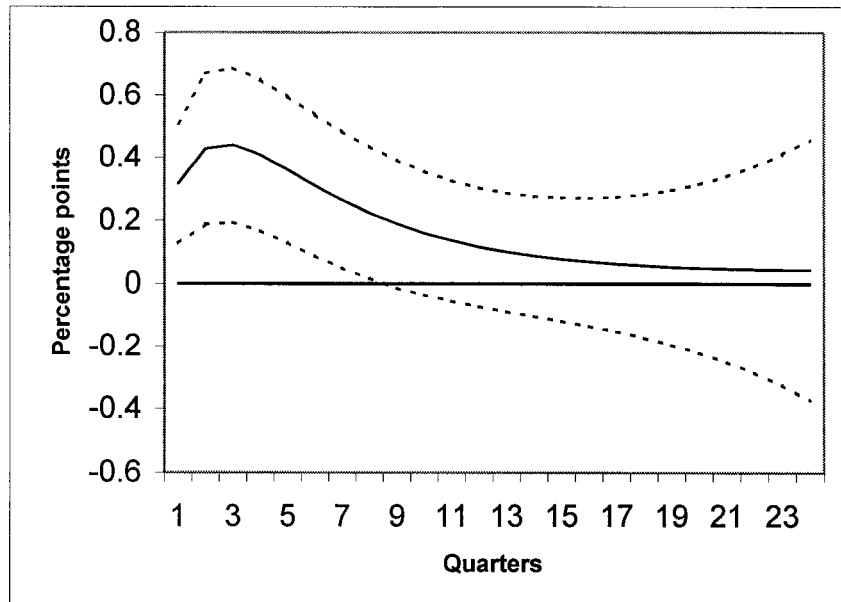


**Figure 7:** The simulated effect of a simultaneous rise of one percentage point in employers' and employees' payroll tax rates – deviations from baseline.

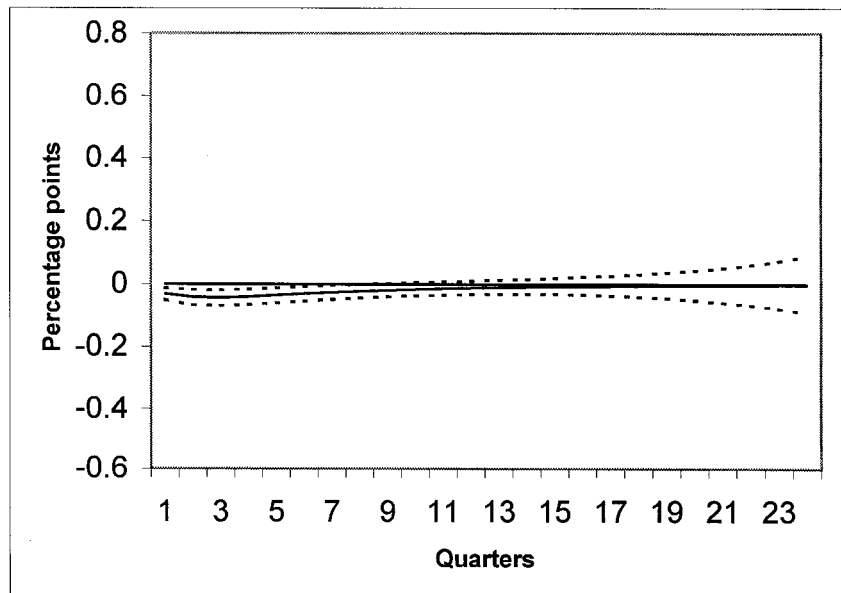


**Figure 8:** Stochastic simulations results – deviations from baseline. The simulated effect of a rise of one percentage point in i) employers', ii) employees' and iii) both payroll tax rates on the unemployment rate – *with 95% confidence interval*.

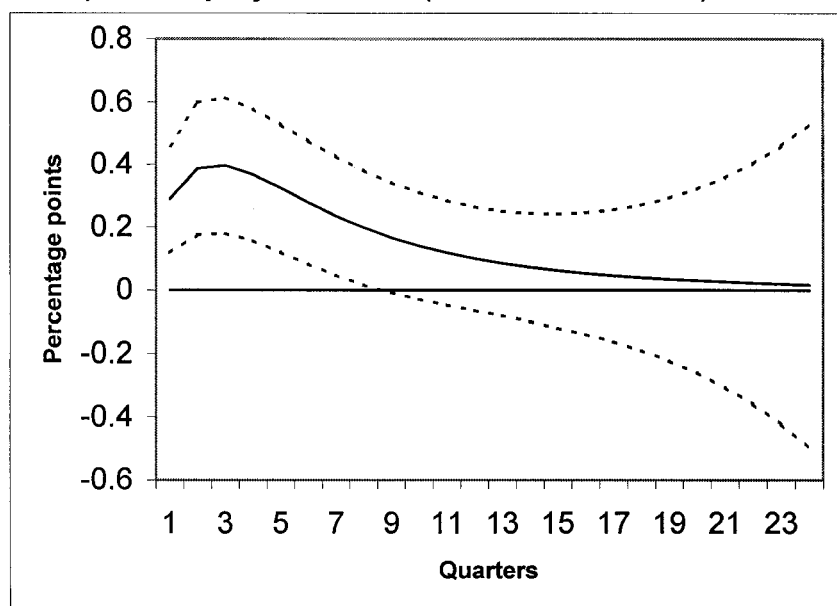
**i)- Unemployment rate (Mean +/- 1.96 S.E.)**



**ii)- Unemployment rate (Mean +/- 1.96 S.E.)**



iii)- Unemployment rate (Mean +/- 1.96 S.E.)



## Appendix A:

### Definitions and Methods of Construction of the Variables

This section gives the definitions of the variables and describes the construction methods of some of them. The data are taken from Statistics Canada's CANSIM databank.

- |                      |   |
|----------------------|---|
| <i>U</i>             | Unemployment rate of the working-age population.  |
| <i>Y</i>             | Real private output, the aggregate output of the private business sector, a component of aggregate gross domestic product (GDP), at 1992 prices.                              |
| <i>W</i>             | Nominal wage paid per employee, the ratio of total wages and salaries to number of employed persons in the private sector (because data on working hours were not available). |
| <i>P<sup>o</sup></i> | Producer price, the value added deflator for business GDP, calculated as nominal GDP divided by real GDP of the private sector; $P^o = 100$ in 1992Q3.                        |



<i>WP</i>	real product wage (the nominal wage, $W$ , divided by the producer price, $P^o$ )
$P^c$	Consumer price index; $P^c=100$ in 1992Q3.
<i>Youth</i>	Youth share of labor force, the proportion of the labor force accounted for by men and women aged 15 to 24.
<i>Women</i>	Women share of labor force, the proportion of the labor force accounted for by women 25 and over.
<i>EIB</i>	Employment insurance benefit, the (nominal) average weekly employment insurance benefit deflated by the consumer price index.
$t$	Time trend.
$A$	Net nonlabor real income, the nominal nonlabor income (net of taxes) deflated by the consumer price index. The nonlabor income is the sum of unincorporated business net income (including rent), interest, dividends, and miscellaneous investment receipts.
<i>TP</i>	Potential labor force, the population over 15 years of age.

## Hannan and Rissanen's methodology for the identification of univariate stochastic processes

Hannan and Rissanen (1982) have proposed a procedure which has been widely used to identify the nature of a stochastic process. Let us consider a sample of  $T$  observations of a stationary series  $y_t$  whose true underlying stochastic process is an ARMA ( $p^*$ ,  $q^*$ ).<sup>1</sup>

1. First, approximate the process by some AR( $k$ ) process, for different values of  $k$ , from 1 to 10. For each value of  $k$ , estimate the following equation:

$$y_t = \delta_k + \phi_{k1}y_{t-1} + \phi_{k2}y_{t-2} + \dots + \phi_{kk}y_{t-k} + \varepsilon_{kt}$$

and get  $\sigma_k^2$ , i.e. the estimate of the residual variance.

2. For each value of  $k$ , compute the Akaike Information Criterion (AIC):

---

<sup>1</sup> If  $y_t$  were I( $d$ ), we could difference the original series  $d$  times to obtain a stationary variable, i.e.  $[(1-L)^d y_t]$ .

$\left[ \log \sigma_k^2 + \frac{2k}{T} \right]$ , and choose  $k^*$  that minimizes this criterion.<sup>2</sup>

3. Compute the residuals associated with the  $AR(k^*)$  approximation:

$$\varepsilon_{k^*t} = y_t - \phi_{k^*1}y_{t-1} - \dots - \phi_{k^*k^*}y_{t-k^*}.$$

4. Compute the *OLS* estimator of the model below, for different combinations of  $p$  and  $q$ :

$$y_t = \delta + \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} + \theta_1 \varepsilon_{k^*t-1} + \dots + \theta_q \varepsilon_{k^*t-q} + e_t$$

and get  $\sigma_{p,q}^2 = T^{-1} \sum_{t=1}^T e_t^2$ . In practice, you choose  $p \leq k^*$ , and you can restrict the

alternative values of  $p$  and  $q$  such that  $p + q \leq k^*$ .

5. For each considered pair  $(p, q)$ , compute the following criterion:

$\left[ \log \sigma_{p,q}^2 + \frac{(p+q) \log T}{T} \right]$ , and choose the model that minimizes this criterion.

This procedure has been widely used in macroeconometrics to restrict the class of alternative models to be considered.

---

<sup>2</sup> Notice that this stage of the procedure is not consistent and  $k^*$  tends asymptotically to overestimate the true order of the model. The second term of the AIC is a penalty for models that contain many regressors.

## **Conclusion**

This dissertation is composed of three essays on the macroeconomic implications of population aging and the labor market effects of payroll taxation.

Over the coming decades the demographic profile of many countries, particularly those in the OECD, will change dramatically as a consequence of population aging. The percentage of people who are 65 and over (retirees) is projected to increase rapidly while the percentage of people in their working years (those aged 15 to 64) is expected to decline. In Canada, there are currently five Canadians of working age for every Canadian over 65. In three decades, there will be half that many. The shift to an older population is not unique to Canada, however: this phenomenon is projected to be global in nature. Indeed, in other major industrial countries such as Germany and Japan, the ratio of working-age people to retirees will drop to still lower levels by 2030. This demographic shock will be caused mainly by the imminent retirement of the post-war baby boom generation, although lower birth rates and longer life spans are also contributing factors.

The coming demographic transformation of Canada and other industrial countries poses significant challenges, and concerns about these challenges have moved to the forefront of the public policy debate in many countries. The first two essays of this dissertation examine some macroeconomic and distributional consequences of population aging – in the presence of a pay-as-you-go public pension system – within a closed and open economy framework, respectively.

In the first essay, we develop an overlapping generations closed-economy model and apply it to the North American economy to examine how the living standards of baby-boom and post-baby-boom generations are affected by the aging baby boom when modeled as a temporary event. The results indicate that the relative scarcity of labor when baby boomers retire raises the wage-rental ratio by an amount that is sufficient to ensure that the post-baby-boom generation can enjoy a modest increase in living standards – despite facing higher taxes. Conversely, the baby-boom generation itself suffers a drop in consumption. When the two generations are considered as a group, overall living

standards fall by a modest amount. These results are robust to several changes in specification: the existence of liquidity constraints, alternative assumptions regarding individuals' expectations concerning future interest rates, and different fiscal policies concerning the tax treatment of private saving for retirement. Compared to what has been at stake in other major policy debates, one may regard the model's predicted reduction in average living standards as significant, but manageable.

In the second essay, we develop a general equilibrium model of a small open economy to allow the analysis to be exclusive to Canada. The model features overlapping generations with a public pension system, asymmetric information in the labor market, and includes some individuals that are liquidity constrained. In this case, the increase in the elderly dependency ratio is treated as a permanent event, to reflect the projections of demographers – the Canadian elderly dependency ratio is expected to nearly double (from about 0.2 now) by 2030, and then remain at about that level until at least 2050. The analysis leads to three main results. First, it shows that by analyzing the aging problem in a small open-economy framework without taking into account its global dimensions, we may systematically misestimate the effect which aging may have on the country's living standards and its net foreign asset position. Second, the liquidity-constrained individuals are those whose living standards, both in the short run and in the long run, are most affected by the aging baby boom. However, for both groups (liquidity and non-liquidity constrained individuals) the drop in average living standards is much *greater* than the fall in overall living standards reported in the first essay. Thus, our analysis in this essay suggests that we should regard population aging as a serious upcoming threat to living standards. This difference in the magnitude of findings is mainly due to three factors. The first is the fact that, in the closed economy (in the first essay) the population aging leads to capital deepening and thus higher real wages; whereas in the open economy, aging results in capital shallowing – since part of the domestic capital stock is moved abroad – which leads to a reduction in the real wages. The second consideration is that variations in the nation's foreign indebtedness are important in the open economy environment. The final consideration is that the demographic shock is permanent in the second essay.

Related to this, since agents plan much farther into the future in this specification (than they do in the first essay), a permanent shock has bigger short-run effects as well. Third, whether increases in payroll tax rates to finance the public pension system (as the elderly dependency ratio rises) are imposed on workers or firms has little effect on the impact of aging on average living standards; however, it does matter for the unemployment rate. More specifically, the firms' contribution rates to the social security programs are shown to have no influence on the natural unemployment rate, whereas increases in the employees' contribution rates raise it.

It is difficult to know how to assess the relative importance of the first and second essays. Both underlying methodologies, the two-period overlapping generations formulation in the first essay and the infinite-period perpetual-youth specification of overlapping generations in the second essay, are mainstream approaches within the profession. The strategy of this thesis has been to use both approaches in separate chapters. If the analyses had led to similar conclusions, we could have advised policy makers that they need not be overly concerned about the lack of a single approach to studying this broad issue within the economics profession. Unfortunately, our results indicate that the answer is sensitive to changes across these specification issues, so we must conclude that more definitive conclusions must await further research that can help us discriminate between alternative modelling strategies. In the meantime, however, we feel that, overall, the assumptions that underlie the second essay are more applicable to Canada. Hence, our conclusion is that aging population has the potential to substantially lower living standards, and therefore there is a need for policy actions that may ameliorate or reverse its effects.

The differential effect of the employer and employee payroll taxes on the natural unemployment rate in the second essay has partly motivated the empirical work in the third essay.

The third essay of this thesis uses data on the Canadian economy to investigate the empirical evidence of the incidence and unemployment effects of the employer and employee payroll taxes. We estimate a disequilibrium aggregate labor market model that

allows the separate effects of payroll tax rates imposed on employers and employees to be distinguished, and use the estimated model to simulate the impacts of payroll taxation. The results indicate that there is evidence of an asymmetric labor market effect according to whether payroll taxes levied on employers or employees are raised. Although there is no impact on unemployment in the long run in either case – which runs somewhat counter to the predictions in the second essay – the short-run effects differ. Moreover, we find that the elasticity of labor demand (with respect to the wage rate) is 10 times (in absolute value) as large as the labor supply elasticity. As a result, most of the burden of payroll taxes – whether levied on employers or employees – is found to be borne by labor in the long run. Finally, it appears that we should be reluctant to place too much weight on the time span over which payroll taxes may affect unemployment in the (most widely performed) deterministic simulations. In fact, the stochastic simulation results illustrate that this time span may be much shorter.

The results in the second and third essays regarding the unemployment effects of payroll taxes can only be compared in the long run, since the second essay, by ignoring temporary ‘real’ rigidities, rules out the analyses of the short-run effects. The empirical finding that increases in the employer payroll taxes have no influence on the long-run unemployment rate is consistent with the efficiency wage model’s finding in the second essay. In contrast, the empirical work does not lend support to the efficiency wage model’s result that increases in the employees’ social security contribution rates raise the natural unemployment rate. Since no specific theory underlies this empirical work, the only conclusion one can make at this point is that there is a need for further research to reconcile the efficiency wage theory’s prediction with the data. It is hoped that, by identifying this need for further work, our extension of the disequilibrium labor market econometrics literature has made a contribution.