MULTIPRODUCT COST MODELLING OF CANADIAN TRUST COMPANIES

MULTIPRODUCT COST MODELLING OF THE CANADIAN TRUST INDUSTRY

by

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TO MY PARENTS

ABSTRACT

The main objectives of this thesis are to develop and interpret an econometric model of the cost of operations of trust companies in Canada. A two-stage model of production is set up. In the first stage of production labour and capital are used to service deposits. Thus deposits are intermediate outputs. In the second stage of production, labour, capital and deposits are used to produce the services of the management of estates, trusts and agencies, and of the management of loans and securities. Using duality techniques, a cost function corresponding to the final-stage production transformation function is formulated.

Two sets of data are employed in the empirical analyses. The first data set consists of trust companies operating in Ontario in the period 1976-1981, omitting only a few whose circumstances were obviously quite unusual, as evidenced by inordinately low labour-shares. The second data set is a truncated form of the first data set -- it includes only observations with no zero outputs. The cost function was specified as a generalized translog function, but for purposes of comparison with earlier studies, an ordinary translog cost function was also specified and estimated. The cost function was in both cases estimated together with the labour-share equation as a system of seemingly unrelated simultaneous equations.

Parameters obtained from the estimated cost functions were used to evaluate measures of economies of scale, economies of scope, and product-specific economies of scale. Some of the results obtained are comparable to earlier studies. For example, this study found an inverse relation between asset size and scale-economies. Some new results also emerged, especially relating to economies of scope: it was found that there exists a positive relation between scope-economies and amalgamations of firms.

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TABLE OF CONTENTS

LIST OF TABLES, CHARTS, AND FIGURES

CHARTS AND FIGURES

CHAPTER 1

INTRODUCTION

Any production unit incurs some kind of cost. Such cost can be a one-time cost, which is not further related to the output produced by the unit; this is termed fixed cost. Another kind of cost -- variable cost -- relates to the amount of output the unit produces. Thus in the classic economic example of the mineral water producer, the cost of sinking of the well will constitute a fixed cost. If later on, the producer decides to bottle the mineral water, then the cost of the bottling will be a variable cost. Most production units incur both fixed and variable costs.

It is possible to formulate a relationship between costs and units of output produced. In this connection, textbook treatments tend to focus on the case of the single-product firm. In reality, though, a large proportion of producers must be regarded as multiproduct firms. Examples are, telecommunication firms, such as Bell Canada, railroad firms, such as Canadian Pacific, and financial institutions, such as Royal Trust. In the case of Bell Canada, for example, the outputs include local telephone services, message toll services and overseas services[1]. A railroad firm produces freight services and passenger services[2]. A financial firm may produce outputs such as the services of loans, securities and deposits[3].

There are certain economic hypotheses about cost-output relationships. Some are concerned with the variation of cost as the level of output changes. These hypotheses take on new meanings as one moves from the domain of the single-product firm to that of the multiproduct firm. For instance, one of the hypotheses concerns the fact that as a firm increases in size, its "unit cost" falls. This concept, as will be explained later, relates to "economies of scale". "Unit cost" or the cost of one unit of output is difficult to define when there are multiple outputs. In the case of a multi-product firm one should distinguish between what happens to costs when all the outputs are expanded together and when only some of the outputs are expanded. The multi-product analysis also brings into focus some new concepts. For instance, with multiproduct analysis, one can examine whether or not it is cheaper for a multiproduct firm to be producing its outputs jointly. Thus, one should be able to answer the question of whether or not a trust company can more cheaply perform the functions of 'fiduciaries and financial intermediation' jointly or separately.

Cost studies of the kind undertaken here may be of interest to three groups. These are: (a) regulators, (b) firm managers, and (c) economic theorists. Many multiproduct firms such as trust companies are regulated by governmental agencies. In Canada, for example, trust companies are subject to federal and provincial restrictions on their banking activities. For instance, the level of borrowing of a trust company is based on a certain multiple of the company's capital base, with the present normal maximum multiple allowed being 20 times this base. In 1967, the Canada Deposit Corporation was incorporated to provide insurance against loss of up to $20,000 for persons having a deposit with any member institution, provided that the term of the deposit is not in excess of five years.

Entry, mergers and expansions in the financial industry are subject to regulation. Estimates of the production and cost functions of such institutions can help the regulatory authorities in their decision making. As Fuss and Waverman (1981) pointed out, the following are some of the questions posed by a regulatory body:

i) What range of services is best supplied by a single firm?

ii) What are the production 'economies of scale'?

iii) What are the 'economies of scope'?

iv) What are the long-run marginal costs of producing one or more units of any one of the joint outputs?

Answers to these questions help to determine the appropriate size of the firm, the degree of competition to be allowed and the efficiency of any rate structure.

Cost studies can also benefit managers of an industry. From these studies they may be able to estimate the marginal costs of producing specific outputs. As Benston puts it, cost studies

> "should help managers evaluate the efficiency of their operations and estimate the costs to their institutions by expanding by de novo branching, merger, or growth at a single location, increasing or decreasing specific types of loans and other portfolio decisions"[4].

Cost studies are also important to the economic theorist. The definition of cost to the economist is different from that of the accountant. To the economist cost is defined in terms of the 'opportunity cost' of producing the outputs; that is the benefits foregone by the producer in producing one set of outputs instead of another. However, most data used for cost studies are recorded by accountants and therefore based on the accountant's definition of cost. The economic researcher, therefore, must make the necessary adjustments in order to be able to use such data. Interest, then, will focus on whether the theoretical cost properties known to the economist are satisfied when such data are used.

The Canadian trust industry is unique. In Canada, only trust companies are allowed to perform the fiduciary function. In other words, trust companies are the only corporate bodies allowed to manage real and financial assets on behalf of clients. This function is sometimes called estates, trusts and agency administration (or E.T.A.), and is the main thing which distinguishes a trust company from all other Canadian financial institutions. In addition, the trust company performs a financial intermediation function which involves its borrowing of funds (incurring of liabilities) and the lending of funds (acquiring of assets) on the Canadian capital market.

In spite of the uniqueness of the Canadian trust industry, and some of the advantages of cost studies mentioned above, there has been no cost study of the trust industry in Canada. In fact, there are few cost studies of financial institutions in Canada. Murray and White (1980, 1983), did some cost studies of Canadian deposit-taking financial institutions, as represented by credit unions in British Columbia. The present study draws on, and in a sense extends, the work of Murray and White. However, there are some significant differences of approach.

Apart from the data, there are definitional and econometric differences between the two studies. For example, Murray and White (1983) defined cost to include interest paid to depositors while the way in which cost is modelled in the present study suggests exclusion of interest costs from the definition of operating costs. Benston (1969) explained the exclusion of interest from the calculation of operating costs by saying that interest cost is "determined primarily by market conditions, rather than by the operations,"[5] of the institution. Also in defining the price of capital, Murray and White summed the major capital expenses such as rent, depreciation and utilities, and divided by the average dollar value of deposits. This author found very little rationale for the use of such a method in calculating the price of capital. Instead, we used what we refer to as the 'Jorgenson method' to calculate the price of capital. The rationale for this is given in chapter 5 of the thesis.

Econometrically, the two studies differ in the formulation of the functional form. Even though we recognize that the translog function, on which Murray and White based their study, is a useful 'flexible functional form', we have nevertheless used a more general 'flexible functional form' which can be used to give unambiguous meaning to a concept such as 'economies of scope'.

Thus, the motivation for this study is two-fold. The first motive is purely academic. It is to check whether the data on a service industry, with the necessary modifications, will satisfy some of the theoretical hypotheses relating to cost functions.

The second motive follows from the first: that is, to provide estimates of the key parameters describing the cost structure of the trust industry. Some of these parameters are those relating to economies of scale, economies of scope, and product-specific economies of scale.

ORGANIZATION

The thesis is organized into 7 chapters, including the present one. Below is a summary of the contents of the remaining chapters.

Chapter 2. This chapter discusses the Canadian trust industry. It considers how the industry came into being and its role in the Canadian economy. It also considers the growth and development of the trust industry. The main aim of this chapter is to bring out the multiple functions of the trust industry from which we derive the definitions of multiple outputs. In other words, this chapter prepares the ground for the application of multiproduct analysis to the trust industry.

Chapter 3. Some of the literature relating to cost studies is reviewed here. Emphasis is placed on multi-product cost studies and techniques of measuring economies of scale and scope. The chapter is divided into two sections. The first section discusses economies of scale/scope studies in general, giving examples from other industries such as telecommunications and railroads. The second section discusses economies of scale/scope studies of financial institutions.

Chapter 4. This chapter models the structure of cost of a trust company from the point of view of a basic multiproduct technology. Inputs and outputs of the industry are clearly identified here. The second section specifies the 'generalized translog' model as the appropriate functional form to be used for the analysis. Using the specified functional form, the last section derives some of the hypotheses to be tested empirically.

Chapter 5. This chapter discusses the sources of data and its treatment. Attention here is focussed on how the 'accountant's data' is transformed into the 'economist's data'.

Chapter 6. All empirical estimation and analyses are handled in this chapter. Estimation and results of the regressions are analyzed in section one. Section two is devoted to the calculation and interpretation of some statistics derived from the cost function.

Chapter 7. This concludes the thesis by highlighting some of the results and their implication for policy- and decision-making.

FOOTNOTES

Chapter 1

1. See, for example, Melvyn Fuss and Leonard Waverman (1981).

2. See, for example, Douglas W. Caves, Laurits R. Christensen and Michael W. Trethway (1980).

3. See, for example, C.W. Sealey, Jr., and James T. Lindley (1970).

4. George J. Benston (1972), p. 315.

5. See, for example, George J. Benston (1969).

CHAPTER 2

THE TRUST INDUSTRY OF CANADA

The trust industry of Canada includes corporations chartered under the Trust Companies Act and corresponding provincial legislation.

Trust companies have 'dual personalities' in the performance of their two functions -- the fiduciary function and the intermediary function. In their role as fiduciaries they act as agents working for a fee, while as financial intermediaries, they are the principals with their funds at risk. As an intermediary, a trust company takes in deposits from the public in the form of guaranteed investment certificates, savings deposits and time deposits. These funds are invested in first mortgates, securities and other loans. This aspect of their business is often referred to as "Guaranteed Funds" as opposed to "Company Funds" which refer to the management of the trust company's own capital.

The activities of trust companies have changed over the years. The industry has expanded both in number and by size.

Section 2.1 discusses the functions of the trust companies as laid down by the various types of legislations governing them. It is from these functions that later, we shall deduce the outputs and inputs to establish the production technology of the trust industry. Section 2.2 leads us through the development and growth of the trust industry. It highlights some of the structural changes within the trust industry.

2.1 STATUTORY POWERS AND FUNCTIONS OF TRUST COMPANIES

Trust companies in Canada are incorporated * by either federal or provincial legislation. Provincially incorporated trust companies proposing to do business in other provinces are required to qualify for deposit insurance under the Canada Deposit Insurance Corporation or the Quebec Deposit Insurance Board. The Superintendent of Insurance examines on behalf of the Deposit Insurance Corporation, the affairs of each federally incorporated trust company. Trust companies operating in Ontario are required to submit annual returns of their operations to the Registrar of Loan and Trust Companies; reports from this registrar formed the major source of our data.

While not uniform throughout Canada, trust company legislation is sufficiently similar to permit a general summary of trust company powers under the various acts. From the legislations it is immediately clear that trust companies perform two main functions: "fiduciary services function," and "financial intermediary function".

The fiduciary or trustee function is unique to trust companies since they are the only corporate entities in Canada having trustee powers. In the United States and the United Kingdom, for example, trustee functions are also performed by financial institutions specifically chartered for other functions (for example, commercial banks, life insurance companies and savings institutions). In their fiduciary function, trust companies serve as administrators of estates, trusts and agencies (E.T.A.). As administrators, trust companies do not obtain ownership of the assets under their administration; instead, they act as the trustee of a property. The trust deed defines the powers that the trust manager has in administering his client' assets and the client's rights to the income generated by the assets being so administered.

Benson (1962) summarized the fiduciary powers of the trust companies as follows:

"...The trust company is empowered
to receive property granted to it by persons, corporations, or courts, upon any trusts not contrary to law;
...to hold property in safe-keeping;
...to act as agents in management of property and collection of rents, interest, dividends, etc.;
...to act as corporation agents (transfer agencies, etc.);
...to act as executor, administrator, receiver, liquidator, custodian, trustee in bankruptcy, guardian of infant's property, committee for estates of mentally incompetent, etc.;
...to invest trust monies;
...to perform all acts necessary in dealing with property;
...to guarantee investments;

...to own real estate necessary to carry on its business;

...to charge renumeration for its services"[1].

The financial intermediary function of the trust company is best considered according to the sources (liabilities) and uses (assets) of funds involved in this aspect of a trust company's operation. Because of similarities in the various pieces of legislation only one (the Ontario Loan and Trust Companies Act) is used as a model for this presentation.

a) Sources (Liabilities)

Apart from capital, trust companies have two sources of funds available to them: deposits and borrowed funds. With respect to deposits the Ontario Act empowers trust companies to "recieve deposits of money repayable upon demand or after notice..." However, the law further specifies that any deposits received in the above manner are deemed to be trust monies in that they are held by the trust company as trustee for the depositors and the trust company guarantees repayment of these deposits. The trust company is allowed to pay interest on demand and notice deposits. The act also stipulates that the liquidity requirements on deposits in trust companies must be an aggregate of at least 20 per cent of the amount of deposits and of funds received for guaranteed investment coming due in less than 100 days.

With respect to borrowing, although the law prohibits trust companies from borrowing by issuing debentures, it sanctions trust companies borrowing on the security of "all or any of the real or personal property, present or future, of the company other than property deemed by this act to be held by the company as trustee or received for investment"[2]. The act also allows trust companies to borrow by issuing guaranteed investment certificates. There are restrictions on the amount of borrowing a trust company can obtain. In general, the restriction on the level of borrowings is based on a certain multiple of a company's capital base, with the present normal maximum multiple allowed being 20 times this base[3]. Currently the Trust and Loan Companies Act is under review to bring their intermediary activities in line with banks.

b) <u>Uses (Assets)</u>

The major items on the assets side of a trust company's balance sheet are loans and investments. A registered trust company may lend its own funds and monies received for guaranteed investment, or as deposits, on the security of mortgages and assignments of life insurance policies, government bonds, bonds secured by trust deed, conventional mortgages to 75% of value, N.H.A. mortgages, insured mortgages, the bonds, debentures, or other securities of various banks. The amount of investment in real estate is generally restricted to 10 per cent of the book value of the total assets of a trust company's funds. Restrictions on the type of investments made are usually intended to concentrate the use of funds on secure investments such as first mortgages and high-grade bonds.

Thus, the major difference between the financial intermediary function and the trustee function is that in the trustee business the trust company receives compensation or acts on a fee for service basis, while in the intermediary area it acts as a principal with its own funds and deposits at risk.

The financial intermediary function involves the companies receiving and borrowing deposits from the public in the form of demand deposits, notice deposits and guaranteed investment certificates. The proceeds are primarily invested in first mortgages, securities and other loans.

The above exposition of the legal powers of the trust companies brings into focus the peculiar characteristics distinguishing a trust company from any other financial institution. It demonstrates that the trust companies are unique in that they are the only financial institution operating as fiduciaries as well as financial intermediaries. Furthermore, in their intermediary capacity, the trust companies along with the loan companies have tended to the longer term maturities on both the asset and liability sides.

From the foregoing we realize that what constitutes the outputs of the trust industry can be deduced from the functions of a trust company as set out in the statutes. In the process of performing their fiduciary function, they produce the services of estates, trusts and agency administration. The outputs under their intermediary function can be deduced from their sources and uses of funds as the services of loans, securities and deposits.

Table 2.1 shows major assets and liabilities of Canadian trust companies as at December 1, 1980. Mortgages take the greatest percentage share of assets: 70.3% of total assets are in the form of mortgage loans. With the definition of loans broadened to include mortgages, personal and collateral loans, the proportion increases to more than 75%.

We define securities to include treasury bills and short term deposits, bonds and stocks. So defined, securities represent about 18% of total assets.

On the liabilities side, out of a total deposit from the public of $30,121 million, about $22,472 million -- being 67.2% of total liabilities -- are in the form of term deposits. Total deposits (demand plus term) represent more than 90% of total liabilities, with shareholders' equity taking only about 4%.

The above figures are consistent with the view that "the industry's banking operations are straightforward, with term deposits being used to make mortgage loans"[4]. Broadly speaking, deposits are inputs in the making of loans and securities, according to this view.

TABLE 2.1

TRUST COMPANIES MAJOR ASSETS AND LIABILITIES (31/12/80)
($ million)

MAJOR ASSETS	AMOUNT	PERCENTAGE
Cash	337	1.0
Treasury Bills and Short Term Deposits	2399	7.2
Bonds	2359	7.0
Stocks	1479	4.4
Mortgages	23558	70.3
Personal and Collateral Loans	1953	5.8
Other Assets	1414	4.3
TOTAL	33499	100.0

MAJOR LIABILITIES		
Demand Deposits	7649	22.9
Term Deposits	22472	67.2
Accrued Interest	1102	3.3
Other Liabilities	847	2.5
Shareholders Equity	1343	4.1
TOTAL	33433	100.0

SOURCE: Report of the Registrar of Loan and Trust Corporations, Ontario, 1980.

On the other hand, people derive benefits from the services of deposits and hence deposits can be considered as outputs too.

Thus, under the two functions of trust companies, the outputs of the industry are:

a) the services of loans;
b) the services of securities;
c) the services of estates, trusts, and agencies under administration.

Deposits are both inputs and outputs as will be further explained in chapter 4.

2.2 GROWTH AND DEVELOPMENT

Since the outputs of the industry are based on the functions of the trust companies as set out in the legislations establishing them, it follows that developments within the industry and legislative changes could affect our outputs. This section traces out the developments within the trust industry and the position of the trust industry in relation to other financial institutions.

The first trust company in Canada was incorporated in Ontario in 1872 and it started business in 1882. This was the Toronto General Trust Company. Later other companies were also given fiduciary powers by federal and provincial acts. By 1900 there were 14 trust companies across Canada; and this rapid growth, in the number of trust companies continued till the beginning of the First World War. By 1914 the number of trust companies in Canada had increased to 23. During the Second World War, there was a slow-down of trust activities and that slowed the rapid growth of the industry[5]. But business soon picked up after the war and by 1947 there were "60 companies of which 45 were provincially incorporated and accounted for 76% of total assets (company and guaranteed funds)"[6]. The number of companies declined again until by 1958 there were only 48 in total. After that there was a renewed expansion in the number of firms which continued till 1965 when there were 65 firms, 57 of which were provincially incorporated. Roughly a century after the advent of the first trust company in Canada, there are now over 80 companies in the industry employing over 31,000 people, including 9,000 real estate employees. Throughout Canada, these companies together operate over 900 deposit-taking branches and over 500 real estate offices[7]. Between 1976 and 1981, a period of six years, 20 firms were newly registered in Ontario to engage in trust business. (See Appendix 2.1(a).)

While the trust industry was expanding by means of increase in number of firms, existing firms were also expanding in size through mergers and increases in operations. Between 1976 and 1981, there was a total of 9 mergers. (See Appendix 2.1(b).) Such mergers and amalgamations resulted in the creation of a few big firms controlling a major proportion of the market. There are seven very big companies known in the industry circles as "the big seven." It should be pointed out that the "big seven" in Ontario are the "big seven" in Canada also. These are the first seven listed companies in Table 2.2. In 1981 these big companies alone controlled more than 70 per cent of the total assets (E.T.A., company and guaranteed funds) of the industry. In fact, two of these companies (Royal Corporation (Canada) and Royal Trust) were under the same directorship, so that essentially three companies alone control over half of the Canadian trust business and just six control over two-thirds of the business. Table 2.2 shows fifteen large companies controlling over 88 per cent of total assets and over 75 per cent of the guaranteed funds in 1981. This leaves less than a quarter of the business to be shared by the remaining trust companies (42 in 1981). Thus, there is a high degree of concentration among trust companies operating in Ontario. Since trust companies in Ontario control over 85 per cent of the total assets of the Canadian trust industry, the analysis of Table 2.2 is a good representation of the Canadian trust industry as well. Further discussion of the representativeness of the Ontario data is left till Chapter 5.

As the number of trust companies grew, so did the number and volume of their services to the public. Initially the strength of the trust industry came from

TABLE 2.2

ASSETS OF SELECTED TRUST COMPANIES OPERATING IN ONTARIO AS AT 31 DECEMBER, 1981

(MILLION DOLLARS)

TRUST COMPANY	GUARANTEED	COMPANY	E.T.A.	TOTAL	% OF ASSETS	CUMULATIVE %
Royal Trust Corporation (Canada)	5,157	261	12,982	18,400	17.8	17.8
Canada Trust	3,722	265	9,497	13,484	13.1	30.9
Royal Trust	1,853	116	9,950	11,889	11.5	42.4
National	2,471	106	8,543	11,120	10.8	53.2
Canada Permanent	3,348	236	3,974	7,558	7.3	60.5
Quebec Trust	610	34	4,669	5,313	5.1	65.3
Victoria and Grey	3,529	178	1,195	4,902	4.8	70.1
Guaranty	2,411	120	2,037	4,568	4.4	74.9
First City	1,579	100	1,165	2,844	2.8	77.6
Co-Operative	566	39	2,081	2,686	2.6	80.2
Morguard	152	16	2,053	2,221	2.2	82.4
International	159	16	1,535	1,710	1.7	84.1
Crown	632	31	1,038	1,701	1.6	85.7
Investors	106	9	1,531	1,646	1.6	87.3
Savings and Investment	215	9	1,078	1,302	1.3	88.6
Subtotal	26,480	1,536	63,328	91,344	88.6	
42 Others	8,619	576	2,601	11,979	11.4	100.0
TOTALS	35,099	2,112	65,929	103,141	100.0	

SOURCE: Report of the Registrar of Loan and Trust Companies, Ontario, 1981.

their trustee business. In fact, the law incorporating some of the early trust companies limited their scope by stating:

> "Nothing in this Act shall be construed to authorize the corporation to issue any note payable to the bearer thereof, or any promissory note intended to be circulated as money or as the note of a bank, or to engage in the business of banking or insurance."[8]

Thus the early trust companies concentrated more on the trustee business. This forced specialization continued throughout the late nineteenth century and early twentiety century. Table 2.3 shows the assets of trust companies registered in Ontario between 1900 and 1981. By 1910 the size of assets under trustee services was about 12 times the size of the other assets of the trust industry[9]. But this kind of specialization did not hold out for long. A look at columns (1) and (2) of Table 2.3 reveal the rapid acceleration of company and guaranteed funds. Over the years, the financial intermediary function has become almost as important as the fiduciary function of the trust companies. Between 1960 and 1975, total assets under financial intermediation increased thirteen-fold.

TABLE 2.3

ASSETS OF TRUST COMPANIES REGISTERED IN ONTARIO FROM 1900 to 1981
(SELECTED YEARS)

(THOUSAND DOLLARS)

Year	Company Funds	Guaranteed Funds	Total	Estates Trust and Agency Funds	(4)÷(3)
	(1)	(2)	(3)	(4)	(5)
1900	n.a.	n.a.	3,869	13,373	3.5
1910	n.a.	n.a.	10,812	132,416	12.3
1920	31,280	36,154	67,434	575,259	8.5
1930	60,849	147,472	208,321	1,867,622	9.0
1940	58,893	135,844	194,737	2,439,188	12.5
1950	72,730	319,719	392,449	3,262,472	8.3
1960	115,565	1,110,317	1,225,882	7,068,901	5.8
1970	450,529	5,511,943	5.962,472	21,986,464	3.7
1975	853,254	12,980,174	13,833,428	32,331,506	2.3
1976	1,123,144	16,457,313	17,580,457	37,018,700	2.1
1977	1,322,471	19,343,874	20,666,345	42,758,901	2.1
1978	1,443,528	23,193,945	24,637,473	50,080,197	2.0
1979	1,641,371	27,596,510	29,237,881	59,959,653	2.1
1980	1,859,176	31,691,512	33,550,688	60,732,001	1.8
1981	2,112,401	35,099,655	37,212,056	65,929,391	1.8

SOURCE: E.P. Neufeld, The Financial System of Canada, and Report of the Registrar of Loan and Trust Companies, Toronto, Annual.

n.a. indicated "not available".

Since 1940 the proportion of trustee services to total assets has fallen dramatically. By 1981 the volume of the trustee business of the trust companies was less than twice the volume of their intermediary business. By 1981 the intermediary assets of trust companies stood at over 37 billion dollars.

A combination of legislative amendments and "practical market initiatives" has encouraged the trust companies to increase their financial intermediation business over the years. The Dominion Act of 1914 restricted the sum of a trust company's borrowing and its funds under guarantee, to an amount not exceeding five times the company's paid-up capital. In 1931 this proportion was increased to seven times, in 1947 to ten times, in 1958 to twelve-and-a-half times, in 1965 to fifteen times, "the excess ... of assets ... over liabilities" and in 1970 to twenty times that amount -- and it has been so since[10]. Column 2 of table 2.3 shows guaranteed funds increasing over the years. The only time there was a fall in the trust companies' intermediary activities was during World War II. Even with the proportion for funds under guarantee increased from five to seven times paid-up capital, still the

TABLE 2.4

ANNUAL PERCENTAGE DISTRIBUTION OF MAJOR ASSETS OF ONTARIO TRUST COMPANIES, 1965 AND 1976-1981

	1965	1976	1977	1978	1979	1980	1981
Mortgages	56.0	72.0	73.5	74.3	74.1	70.2	65.9
Bonds	33.1	8.8	8.5	6.9	6.9	7.0	8.5
Stocks	2.2	1.9	2.6	3.7	4.2	4.4	4.5
Collateral Loans	2.9	2.5	1.5	1.4	1.4	3.1	3.8
Cash and Short Term Deposits	2.7	8.6	8.1	8.2	7.1	8.2	9.0
Other Assets (1)	3.1	6.2	5.8	5.5	6.3	7.1	8.3
TOTAL ASSETS	100.0	100.0	100.0	100.0	100.0	100.0	100.0

(1) Accrued interest and dividends on mortgages, bonds, stocks, and collateral loans are included in other assets.

SOURCE: Ontario, Report of the Registrar of Trust and Loan Companies, Toronto, Annually.

funds under guarantee in 1940 were less than it was in 1930. However, this trend was reversed after the war.

Much of the post-war growth of the trust industry can be attributed to investment in mortgages. In a brief submitted to the Department of Finance of the Government of Canada for the Review of the Bank Act (1977), the Trust Companies Association of Canada concluded that investment in mortgages of funds obtained via guaranteed investment certificates and receipts was the major factor behind the trust companies' post-war growth[11]. Furthermore, the same factor -- mortgage investments -- underlies the fact that since the second World War the importance of the trust companies' financial intermediary role relative to their trustee role has been continually increasing[12].

Table 2.4 shows the annual percentage distribution of major assets of Ontario trust companies. We notice that a large percentage of guaranteed funds was invested in mortgages. In 1980 and 1981, high interest rates in Canada hurt the trust companies' investment in mortgages. Even though the shares of mortgages in total assets fell, still they were at levels above those of the sixties.

TABLE 2.5

MORTGAGE LOANS, PERSONAL LOANS AND PERSONAL SAVINGS BY SELECTED FINANCIAL INSTITUTIONS IN CANADA (1981)

	MORTGAGE LOANS		PERSONAL LOANS		PERSONAL SAVINGS	
	(a)	(b)	(a)	(b)	(a)	(b)
Chartered Banks	16,380	4.68	29,548	8.45	89,968	25.72
Trust Companies	27,887	63.90	1,799	4.12	38.930	89.21
Mortgage Companies	17,403	82.81	79	.38	13,678	65.51
Local Credit Unions	16,038	49.27	6,083	18.69	5,330	16.37
Life Insurers	16,428	29.13	2,644	4.69	29,938	70.83

Column (a) shows amounts in million dollars.

Columb (b) shows variable as percentage of institutions' total assets.

SOURCES: Statistics Canada: Financial Institutions - Financial Statistics, 61-006, 1981.

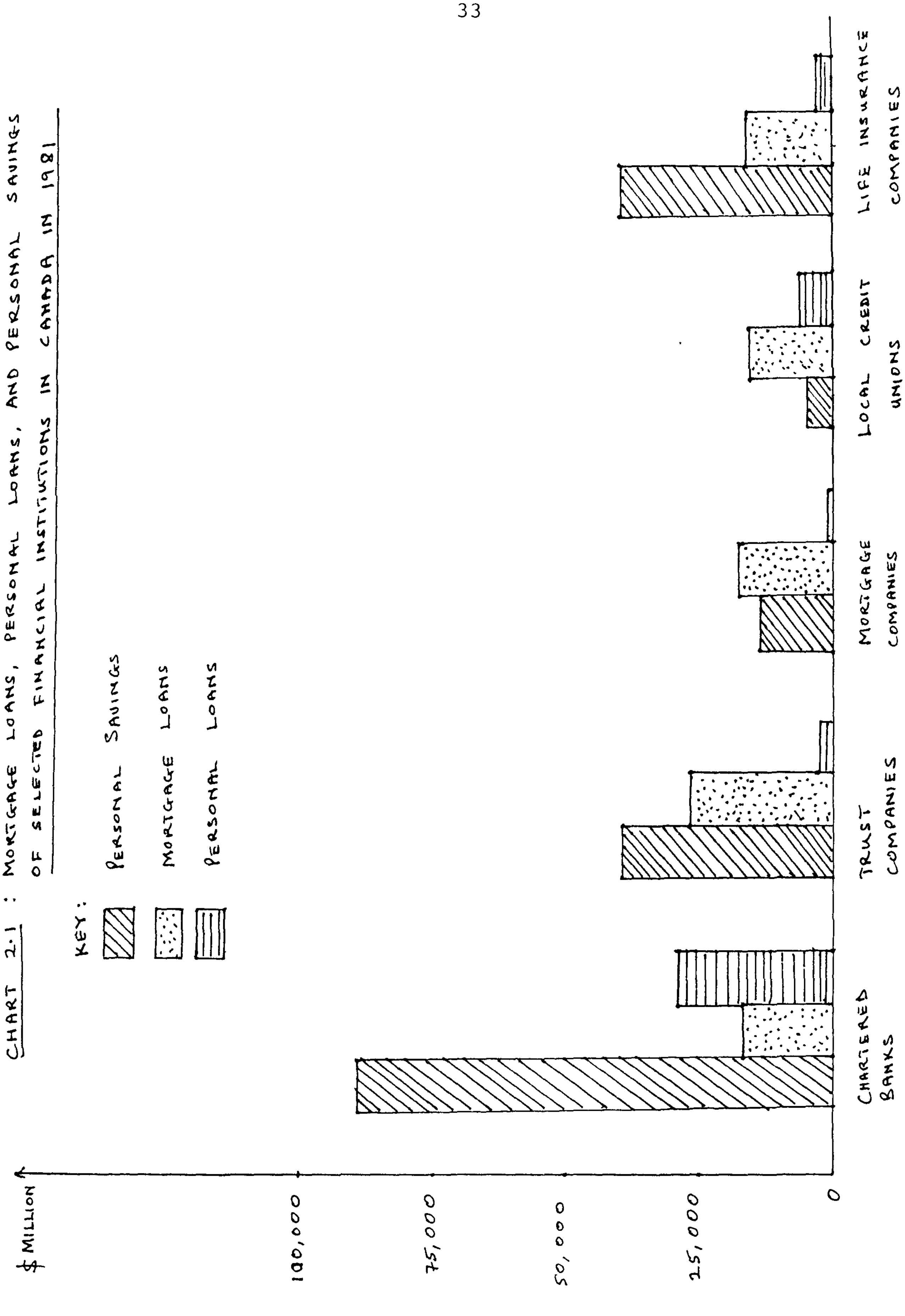

CHART 2.1 : MORTGAGE LOANS, PERSONAL LOANS, AND PERSONAL SAVINGS OF SELECTED FINANCIAL INSTITUTIONS IN CANADA IN 1981
KEY:
PERSONAL SAVINGS
MORTGAGE LOANS
PERSONAL LOANS
$ MILLION
100,000
75,000
50,000
25,000
0
CHARTERED BANKS
TRUST COMPANIES
MORTGAGE COMPANIES
LOCAL CREDIT UNIONS
LIFE INSURANCE COMPANIES

Some people think of a trust company as a bank, others as a "real estate company". However, only a small percentage of the trust companies' assets (about 3 percent in 1981) involves real estates held for sale.

The legislative changes mentioned above have favoured the trust companies, sometimes at the expense of other financial institutions. Whereas no other financial institution is allowed to compete with the trust industry in their fiduciary function, the trust industry competes well with the other financial institutions in their intermediary function. Table 2.5 shows mortgage loans, personal loans and personal savings of selected financial institutions. This is represented pictorially in chart 2.1. In absolute dollars, trust companies have more mortgage loans outstanding in 1981 than even mortgage loan companies.

Unlike banks, trust companies are not under strict liquidity controls. Banks have to maintain cash reserve ratio but trust companies are only required to keep a liquidity ratio of 20 per cent. While the Bank Act restricts the investment by the banks in mortgages, the Trust Companies Act also restricts the investment by trust companies in personal loans. Trust companies are allowed to give unsecured loans under what is termed the "basket clause". In 1981, the trust companies had one of the lowest shares of the personal loan business.

The trust companies also attract a good share of the personal savings market. In fact, in 1981, they had a greater proportion of their liabilities in personal savings than any other financial institution.

The federal Trust Companies Act and Loan Companies Act are now under review and will be replaced by the proposed Canada Savings Bank and Trust Companies Act. A major objective of this new act is "to put the regulation of the savings deposit activities of trust and loan companies on essentially the same basis as that of the chartered banks"[13].

Meanwhile in their fiduciary function the trust companies do not face any competition. Trust companies still continue to be the only fiduciaries in Canada while at the same time competing with other financial institutions in their intermediary function.

From the foregoing we have learned that legislation and market forces may affect the kinds of services the trust companies produce. In other words, the kinds of outputs produced by a trust company may be affected by legislative changes and developments in the Canadian capital market.

FOOTNOTES

Chapter 2

1. Winslow Benson (1982), p. 3.
2. Ontario Loan and Trust Corporation Act, 587(1).
3. Trust Companies Institute of Canada, (1980).
4. Trust Companies Association of Canada (1977).
5. E.P. Neufeld (1972).
6. E.P. Neufeld (1972), p. 301.
7. Trust Companies Institute of Canada, (1980).
8. Statutes of Canada (1895), 58-59 Vic., Cap. 84.
9. E.P. Neufeld (1972), p. 308.
10. E.P. Neufeld (1972), p. 299
11. Trust Companies Association of Canada (1977).
12. Trust Companies Association of Canada (1977).
13. Statistics Canada 61-006 (1984), p. xiii.

APPENDIX 2.1

ENTRIES, MERGERS AND EXITS OF COMPANIES BETWEEN 1976 AND 1981

a) ENTRIES[(i)]

1. Community Trust Company Limited, incorporated in Ontario in 1975 and registered 6 July, 1976.

2. Astra Trust Company, federally incorporated in 1976 and registered in Ontario, 1977.

3. Exchequer Trust Company, incorporated in Ontario and registered, 1977.

4. Financial Trust Company, incorporated in Ontario and registered in 1977.

5. Huronia Trust Company, was incorporated in Ontario and registered in 1977.

6. Security Trust Company was incorporated in Ontario and registered in 1977.

7. Effort Trust was incorporated and registered in Ontario, 1978.

8. McDonald-Cartier Trust Company was incorporated in Ontario in 1977 and registered in 1978.

9. Municipal Trust Company was incorporated in Ontario and registered in 1978.

10. Seaway Trust Company was incorporated and registered in Ontario in 1978.

11. Bayshore Trust Company was federally incorporated in 1977 and registered in Ontario in 1978.

12. The Merchant Trust Company, was federally incorporated and registered in Ontario in 1978.

13. Montreal Trust Company of Canada was federally incorporated and registered in Ontario in 1978.

14. Cabot Trust Company was incorporated in Ontario and registered in 1978.

15. Morgan Trust Company of Canada was federally incorporated and registered in Ontario in 1979.

16. Western Capital Trust Company was federally incorporated and registered in Ontario in 1979.

17. Executive Trust Company, Ontario incorporated, was registered in Ontario in 1981.

18. North Canadian Trust Company, federally incorporated, was registered in Ontario in 1981.

b) MERGERS

1. The Canada Trust Company, amalgamated with Ontario Trust Company and the Lincoln Trust and Savings Company as of December 10, 1976, to continue under the name of The Canada Trust Company.

2. The Eastern Canada Savings and Loan Company amalgamated with Central and Nova Scotia Trust Company both federally incorporated companies, as of 1 July, 1976, to form Central and Eastern Trust Company.

3. Canada Permanent Trust Company amalgamated with Hamilton Trust and Savings Company to continue under the name of Canada Permanent Trust Company in 1977.

4. Royal Trust Corporation of Canada amalgamated in 1977 with The Royal Trust Company (Ontario) to continue under the name of Royal Trust Corporation of Canada.

5. The Lambton Trust Company Limited was amalgamated with Victoria and Grey Trust Company to continue under the name Victoria and Grey Trust Company in 1978

6. Metropolitan Trust Company amalgamated with Victoria and Grey Trust Company under the name of Victoria and Grey Metro Trust Company in 1979. Victoria and Grey Metro Trust Company changed its name to Victoria and Grey Trust Company in 1980.

7. Fort Garry Trust Company, Manitoba, incorporated, merged into The Fidelity Trust Company in 1980.

8. Federal Trust Company, Ontario incorporated, merged with Central and Eastern Company, a federally incorporated company, which later changed its name to Central Trust Company, in 1981.

9. The Industrial Mortgage and Trust Company, Ontario incorporated, merged with Royal Trust Corporation of Canada, a federally incorporated company, in 1981.

c) EXITS

1. Effective June 13, 1980, The Clarkson Company Limited was appointed liquidator to wind up the affairs of Astra Trust Company, under the Winding-Up Act (Canada).

Source: Reports of the Registrar of Loans and Trust Companies. Ontario.

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[1] Entries exclude companies which have been incorporated and registered in other provinces before 1976 but were registered in Ontario between 1976 and 1981.

CHAPTER 3

A SURVEY OF THE LITERATURE

Considerable research has been conducted on cost functions in general. A number of these studies are concentrated on economies of scale in various kinds of industries, including both commodity output and service output industries. Until 1975, however, most researchers looked at economies of scale from the point of view of single output industry analysis.

As pointed out by Bailey and Friedlaender (1982), many industries in which the study of economies of scale may be of interest are multiproduct industries. Thus, for example, the railway industry has as outputs freight and passenger haulage; and banks have as outputs the services associated with loans, deposits and securities. A proper study of such industries requires multiproduct analysis, along the lines pioneered by Panzar and Willig (1977) and Baumol (1977). Multiproduct analysis allows a more detailed appraisal of the structure of an industry. In particular, it allows consideration of economies of scope and of product-specific economies of scale.

Financial institutions being multiproduct industries, one would expect cost analysis of banks and near banks to be done in a multiproduct fashion. However, except for some recent work by Murray and White (1983), it appears that none of the previous studies of economies of scale in financial institutions have taken a multiproduct approach.

This chapter attempts to establish that there is an important body of literature establishing methods for modelling multiproduct firms without collapsing output into a single dimension. There is also an indication that this multiproduct approach provides policy guidance not found in earlier studies.

In Section 1 of this chapter we review work on economies of scale in general, giving some particular attention to the question of functional form. Section 2 provides examples on economies of scale studies in the finance industry. Particular attention is paid to the problem of defining output variables.

3.1 MULTIPRODUCT COSTS AND ECONOMIES OF SCALE AND OF SCOPE

The historical development of the cost-function and its analytical properties are fully discussed in McFadden (1978). He starts from Hotelling's (1932) work on the properties of the price derivatives of the cost function, discusses Shephard's (1953) duality theory of cost and production functions and goes on to discuss the works of Uzawa (1964), McFadden (1962), Diewert (1974), Hanoch (1975), and Lau (1976), all of which contributed to the economic implications of the duality theory. This section, therefore, will omit discussion of the cost function itself and concentrate on concepts related to the cost function. Specifically, we shall discuss works related to the concepts of economies of scale, economies of scope, and product-specific economies of scale.

Economies of scale is a concept which has interested research economists since Adam Smith first extolled the virtues of 'division of labour' and 'specialization' in his book, "The Wealth of Nations." Economies of scale is useful in evaluating the efficiency of market structures; hence it is useful to both regulators and managers of the firm alike.

There are three principal ways in which people study economies of scale. The first, which may be called the "engineering technique," makes use of engineers' cost estimates. Haldi and Whitcomb (1967) have argued that these cost estimates are useful in the study of economies of scale, "because they embody assumptions consistent with those underlying the envelope curve"[1].

The second method is Stigler's "survivor technique"[2]. This technique involves classification of firms in an industry by size, and calculation of the share of industry output coming from each class over time. If the share of a given class falls, that class is relatively inefficient, and in general is more inefficient the more rapidly its share falls.

The third technique, which we shall call the "statistical cost analysis," involves the analysis of the average cost curve of the industry. This is the technique used in the present research and hence the literature surveyed is more or less limited to studies using this technique.

Traditional economic analysis generally involves single product firms. Until the mid-1970s researchers of cost functions were forced to use single product techniques to analyze economies of scale for multi-product firms. Such single product techniques failed to capture the structure of multiproduct firms. Johnston (1961) reviewed the statistical technique for analyzing cost functions. This technique usually involved the use of average cost and marginal cost functions. He applied his analysis to the railway and trucking industries. Analysts of economies of scale in single output firms normally check for scale economies by looking at the slope of the average cost curve within a particular range. The range within which the average cost curve falls indicates increasing returns to scale.

An example of work using single product approach is that of Christensen and Greene (1976). They estimated economies of scale for United States firms producing electric power. They used a translog cost function to analyze cross-section data for 1955 and 1970. Their model was a single output model, with economies of scale elasticity defined as

$$SCE = 1 - \frac{d \, \ln C}{d \, \ln Y} \qquad (3.1\text{-}1)$$

where C is operating cost and Y is output. There is economies of scale if SCE > 0 and diseconomies of scale if SCE < 0 and constant returns to scale if SCE = 0. They found that in 1955 there were significant scale economies available for all firms. In 1970, however, the bulk of U.S. electricity generation was by firms operating in the essentially flat area of the average cost curve. They concluded that a small number of extremely large firms are not required for efficient production and that policies designed to promote competition in electric power generation cannot be faulted in terms of sacrificing economies of scale.

There have been numerous formal discussions of the economies of multiproduct firms since Hicks (1939) early treatment. McFadden (1966), Jacobson (1968) and Shephard (1970) have used the principles of duality to demonstrate the existence of multiproduct cost functions corresponding to general production structures. Hall (1973) also showed the transformation of a separable production specification into a corresponding cost function. Most of the earlier analysts of multiproduct cost functions imposed theoretical restrictions on the cost structure though Brown, Caves and Christensen

(1979) have argued that, "the imposition of homogeneity and separability[3] can greatly distort estimates of marginal costs and scale economies"[4].

Panzar and Willig (1977) and Baumol (1977), apparently independently, introduced new ways of looking at economies of scale in multiproduct industries. These new ways brought in new concepts, such as 'economies of scope,' and 'product-specific economies of scale'. Economies of scope refer to the cost advantages of providing a large number of diversified products in one multiproduct firm instead of many single product firms. The existence of economies of scope provides a raison d'etre for multiproduct industries.

Bailey and Friedlaender (1982), in a survey article, discussed some cost concepts for multiproduct firms. They based their discussion mainly on the works of Baumol (1977) and Panzar and Willig (1977). Their discussion was centred mainly on the concepts of economies of scale, economies of scope, and product-specific economies of scale. Assuming $C(Y,r)$ is a multiproduct cost function, where Y is a vector of n outputs and r a vector of m input prices, they obtained a measure of multiproduct economies of scale by first computing

$$d \, \ell n \, C = \sum_{i=1}^{n} \frac{\partial \ell n C}{\partial \ell n Y_i} \frac{\partial Y_i}{Y_i} \qquad (3.1\text{-}2)$$

Then assuming all outputs are increased by a proportion $\lambda = dY_i/Y_i = d \, \ell n \, Y_i$, they defined the multiproduct economies of scale measure as

$$S = 1/\varepsilon \qquad (3.1\text{-}3)$$

where

$$\varepsilon = \frac{d \, \ell n \, C}{\lambda} = \sum_{i=1} \frac{\partial \, \ell n \, C}{\partial \, \ell n \, Y_i} \qquad (3.1\text{-}4)$$

There is multiproduct economies of scale if $S > 1$, diseconomies of scale if $S < 1$ and constant returns to scale of $S = 1$.

Some of the behaviour of cost with respect to changes in output which are neglected by the multi-product 'overall' measure of economies of scale are captured in a concept known as product-specific economies of scale. It is measured by calculating the cost elasticity of one output, holding all other outputs fixed.

Fuss and Waverman (1981) in a research on the Canadian Telecommunications Industry, claimed that "there is no unambiguous measure of output-specific returns to scale except in the case of non-joint production"[5]. This is because they defined incremental cost as marginal cost. Defining incremental cost of output 1 as

$$IC_1 = C(Y_1, Y_2, \ldots, Y_n) - C(0, Y_2, \ldots, Y_n), \qquad (3.1\text{-}5)$$

Panzar and Willig (1978) provided an unambiguous measure of product-specific returns to scale. However, this measure requires knowledge of the cost function in regions where one or more of the outputs are zero. Fuss and Waverman could not use the Willig and Panzar (1978) definition as their translog cost function does not allow for zero outputs.

Panzar and Willig (1978) defined average incremental cost (AIC) as the incremental cost (IC) of producing that output divided by the output. Where IC is the additional cost of producing an output where previously it was not produced at all. Therefore for output 1 in a two output case,

$$AIC_1(Y) = \frac{C(Y_1,Y_2) - C(0,Y_2)}{Y_1} \qquad (3.1\text{-}6)$$

Product-specific economies of scale for output 1 is measured by

$$S_1 = \frac{AIC_1(Y)}{MC_1} \qquad (3.1\text{-}7)$$

Where MC_1 is the marginal cost of producing Y_1.

If $S_1 > 1$ there are increasing returns to scale with respect to output 1; if $S_1 = 1$, constant returns and $S_1 < 1$, refers to decreasing returns to scale for product 1.

Another measure which captures the effect of changes in the composition of output on cost is the concept of economies of scope. Both global and local measures are suggested. According to Willig and Panzar (1975, 1981), economies of scope exist if the cost of producing a set of outputs jointly is less than the cost of producing them separately; that is, if

$$C(Y_1,Y_2) < C(Y_1,0) + C(0,Y_2) \qquad (3.1\text{-}8)$$

where Y_1 is the output level of product 1 and Y_2 is the output level of product 2.

Formally the degree of economies of scope is measured by

$$S_c = \frac{C(Y_1,0) + C(0,Y_2) - C(Y_1,Y_2)}{C(Y_1Y_2)} \qquad (3.1\text{-}9)$$

Thus, S_c is greater than zero if economies of scope exist and less than or equal to zero if no economies of scope exist.

Locally, economies of scope can be defined according to whether:

$$\frac{\partial^2 C}{\partial Y_1 \partial Y_2} \gtrless 0 \qquad (3.1\text{-}10)$$

If the left hand side of (3.1-10) is less than zero, then the marginal cost of one output is reduced by the increase in the output of the other, and local economies of scope are said to exist.

Algebraic manipulation of (3.1-3) shows the relation between the three concepts of multiproduct economies of scale (S), product-specific economies of scale (S_1, S_2) and global economies of scope (S_c) as

$$S = \frac{wS_1 + (1 - w)S_2}{1 - S_c} \qquad (3.1\text{-}11)$$

where

$$w = \frac{Y_1 \partial c/\partial Y_1}{Y_1 \partial C/\partial Y_1 + Y_2 \partial c/\partial Y_2} \qquad (3.1\text{-}12)$$

Bailey and Friedlaender (1982) concluded that an implication of (3.1-11) is that strong scope economies ($S_c > 0$) can confer scale economies ($S > 1$) on an entire product set. In fact, it is possible to have multiproduct economies of scale with product-specific diseconomies of scale in each output if S_c is sufficiently large. Also the sensitivity of the cost function to both the scale and composition of output imply that as the firm changes its level of output and product mix, there will be different reactions at different output levels.

Empirical works on multiproduct economies of scale span a wide range of industries, including railways and trucking, airlines, telecommunications, and banking and finance.

In their work on the telecommunications industry in Canada, Fuss and Waverman (1981) used a translog cost function. This kind of functional form limited their use of some of the definitions of some of the cost concepts listed above. Since their functional form does not allow for zero outputs, they had to limit themselves to only local definitions when it came to checking for economies of scope. In checking for economies of scope, they used a definition of local cost complementarities:

$$\frac{\partial^2 C}{\partial Y_i \partial Y_j} < 0 \quad (i \neq j, \quad i,j = 1,\ldots,n)$$

But as shown by Panzar and Willig (1979) the existence of cost complementarities is only a sufficient condition for a twice differentiable multiproduct cost function to exhibit economies of scope. On multiproduct economies of scale they concluded that "estimates of the overall economies of scale elasticity are not sufficiently precise to enable one to reject the hypotheses of increasing, constant, or decreasing returns to scale"[6].

An example of multiproduct cost study of the trucking industry can be found in Wang-Chiang (1981) reported in Bailey and Friedlaender (1982). In an effort to distinguish between the economies of scale and economies of scope in the trucking industry, Wang Chiang (1981) analyzed trucking costs by estimating a "translog cost function that incorporates a disaggregate output vector and variables that reflect the configuration and utilization of the network over which the firm operates"[7]. She found out that returns to distribution networks appear to be sufficiently strong to generate fairly marked product-specific economies of scale and economies of scope in the intermediate-haul trucking markets. It is concluded that these economies of network utilization and of network configuration suggest that there are strong economies of joint production associated with short-haul and intermediate-haul trucking shipments, particularly of small and medium sized firms -- thus this explains the observed merger movement in the U.S. trucking industry. This suggests that in the absence of regulations, firms would attempt to merge and to grow to obtain the full range of economies of joint production afforded by efficient network utilization. Wang Chiang (1981) also found that there was no evidence of global economies of scale and that product-specific economies disappear for large firms, suggesting there may be a limit to the efficient growth of trucking firms.

An important work which provides evidence of the output disaggregation is Jara-Diaz and Winston (1981). They estimated a quadratic cost function at the totally disaggregate level of the point-to-point of shipment and compared it with a model with aggregate output measures. They found substantial biases resulted concerning measures of economies of scale and elasticities of substitution. They also found out that economies of scale existed with respect to shipments throughout the network.

An important part in the analyses of multi-product cost functions is the formulation of the cost function. Baumol, Panzar and Willig (1982) explained some of the implications of the definitions of economies of scale/scope as far as the functional form is concerned. They argued that one should not let the functional form determine the results of the analysis but that rather should allow the data to do so. They warned that

> "any function that, like the Cobb-Douglas or the translog, takes the value zero whenever the output of any product set is zero automatically precludes the possibility of economies of scope or of subadditivity, if C(Y) > 0 for other relevant values of Y. For then the costs of an industry can always be driven (ostensively) to zero by dividing its outputs among specialized firms, none of which produces every one of the industry's products"[8] .

They gave an example using a Cobb-Douglas cost function and showed that such a function "preimposes the conclusion that weak cost complimentarities are always absent and that, in fact, diseconomies of scope prevail in the absence of fixed costs"[9] The Cobb-Douglas cost function, in fact, has nothing to recommend it except tractability. The translog, on the other hand, is a very useful flexible functional form, except that it cannot handle zero outputs. Caves, Christensen and Tretheway (1980) examined different functional forms which were flexible in nature. They proposed a new functional form which they called the Generalized Translog Multiproduct Cost Function (GTMCF). This functional form applies a Box-Cox transformation to the output variables. This kind of transformation allows for zero outputs. We shall learn more about GTMCF in Chapter 4.

In conclusion, it seems clear from existing econometric studies of multiproduct industries that explicit disaggregation of the output vector to take the heterogeneity of output into account provides information and policy guidance that cannot be gained from single-product analysis. Such multiproduct studies have taken place in the telecommunication, railroad and trucking industries, to mention a few. Discussion of studies on cost functions in the finance industries, including multiproduct studies are deferred till the next section.

3.2 SCALE STUDIES IN FINANCIAL INSTITUTIONS

Even though the concept of economies of scale has long been recognized, it was not until 1954[10] that it was tested for in the finance industry. The delay might be due to conceptual and definitional problems which continue to plague analysts of the industry. Being a service industry, it has not been easy to find unambiguous definitions of outputs. In some cases, there is limitation in the data on individual banks and near banks. This is particularly so with Canadian financial institutions. In the United States, data is readily available on banks belonging to the Federal Reserve's Functional Cost Analysis Program. Banks join the program to get the analyses of their operations and comparisons with other banks provided them by the Bank Relations Division of the district Federal Reserve Banks. Benston (1965, 1970, 1972) and Bell and Murphy (1968) utilized data from the Functional Cost Analysis Program.

As mentioned above, the definition of output is one of the problems researchers have faced. Benston (1965, 1970) and Bell and Murphy (1968) have defined output "in terms of what banks or savings and loan associations do to cause them to incur operating costs"[11]. They claim that

> "the operating costs are related primarily to the number of documents handled and customers served rather than to the dollars deposited or loaned"[12].

Thus for Benston, and Bell and Murphy, outputs are represented by the numbers of loan and deposit accounts serviced. They also claimed that computing these numbers as averages per year allows the outputs to be interpreted as flows. In addition to using the numbers of accounts as outputs, they included in their estimation average dollar balances as "cost homogeneity variables".

Greenbaum (1967) and Powers (1969) used an estimate of "real value" of output in their analysis of commercial banking. Greenbaum argued that bank output "must be related to community well-being"[13]. He and Powers measured the amount of "community well-being" produced by a bank with a variant of the bank's gross income. But these authors differ in other definitions; Powers' definition accounted for interbank differences in expected yields whereas Greenbaum's did not. Powers therefore used gross operating income to measure output.

Alhadeff (1954), Horwitz (1963), Schweiger and McGee (1961), Brigham and Grebler (1963), Gramley (1962), and Brigham and Pettit (1970) all used real-valued unweighted indexes of bank output. Alhadeff and Horwitz used loans plus investments; Schweiger and McGee used total deposits and Gramley, Brigham and Grebler, and Brigham and Pettit used total assets. All these authors used dollar balances of accounts to represent output. Murray and White (1980, 1982) also used the dollar amounts of investments, loans and deposits as outputs.

Sealey and Lindley (1977), in a theoretical paper, concluded that the appropriate concept of output is "the services provided to the debtors of financial institutions". They also measured their output volumes in dollar units. Their model was a two-stage production model in which, in the first stage, loanable funds borrowed from depositors are serviced by the firm, and in the second stage the deposits are serviced by capital, labour and other inputs to produce earning assets.

Different researchers have used different techniques in analyzing economies of scale in financial institutions. Alhadeff (1954) and Horwitz (1963) analyzed economies of scale by relating total operating costs per thousand dollars of loan and securities to banks classified in different size groups. They both used tabular analysis. Alhadeff used California bank data for the years 1938-1950 while Horwitz, who replicated Alhadeff's work, used data for all commercial banks for the period 1949-1960. Like Alhadeff, Horwitz concluded that

> "once a bank reaches the relatively small size of $5 million of deposits, additional size does not result in reduced costs to any great extent until a bank reaches the giant size of over $500 million"[14]

and that branch banks have uniformly higher unit costs than unit banks.

Schweiger and McGee (1961), Gramley (1962), Grebler and Brigham (1963), Brigham and Pettit (1970), Benston (1965, 1970), Bell and Murphy (1968) and Murray and White (1980, 1982) all used multiple regression analysis.

The following equation was estimated by Schweiger and McGee and Gramley for banks in 1959 (Schweiger and McGee used data for banks in Chicago and surrounding areas and Gramley used data from a sample of Tenth District member banks):

$$C_i/A_i = b_0 + b_1S_i + b_2D_i + \sum b_jE_{ji}$$

$$+ b_mG_i + \sum b_KO_{Ki} \qquad (3.2\text{-}1)$$

where

C_i/A_i = total operating cost/total assets of the i^{th} bank.

S_i = size variable (for Schweiger and McGee 1,2,...,9 deposit size groups; for Gramley, log total assets).

D_i = time deposits/total deposits.

E_j = earning asset structure (for Schweiger and McGee business loans/assets, consumer loans/assets, farm loans/assets; for Gramley, total loans/assets, nongovernment securities/assets, consumer loans/total loans).

G = percentage growth of assets.

O_K = other structure variables (for Schweiger and McGee, branch-asset dummy, state population increase, size of community).

For both studies the coefficient of their size variable is negatively biased because it appears as the denominator of the dependent variable[15]. These scale coefficients, b_1, are also negative and of magnitudes that indicate large economies of scale.

Brigham and Grebler (1963) and Brigham and Pettit (1970) used similar equations to analyze the operating costs of savings and loan associations. Brigham and Pettit used 1961 data from 221 California loan associations (which held 96 percent of the industry's asset in the state). They ran separate regressions for subsamples of associations grouped according to location, type of charter and type of ownership. They found that the sign of the coefficient of the size variable was negative in all cases, and for the most part the coefficients were not statistically significant. Furthermore, their partial correlation coefficients were generally neither statistically significant nor large enough to be considered important in an economic sense. This means, in effect, that no important economies of scale appeared in the 1961 data.

Brigham and Grebler analyzed a five-year data (1962-1966) from all associations in three markets: Los Angeles, Chicago, and Detroit-Cleveland. They found consistent economies of scale for all subsamples in all the five years.

Greenbaum (1962) and Powers (1969) related their measure of output to unit cost as follows:

$$C_i/A_i = b_0 + b_1 R_i/A_i + b_2(R_i/A_i)^2 + b_3(R_i/A_i)^3 \quad (3.2\text{-}2)$$

where R_i is total operating revenue (or variant thereof) of the i^{th} bank and C_i and A_i are as defined above. Both divided their samples into unit and branch banks and Greenbaum also added a code of 1-9 for the number of branches. Powers divided his sample into three asset size groups, each of which was subdivided into two groups according to the ratio of current operating revenue to total revenue. Powers also concluded that he found no "definitive conclusion as to the existence of either diseconomies of branching or economies of size in the entire banking industry"[16].

Benston (1965) and Bell and Murphy (1968) used multiple regression analysis to analyze the direct and indirect operating costs of individual banking services. They used data gathered by the Federal Reserve in its Functional Cost Analysis Program. Direct costs were assigned to the deposits and loans departments and indirect costs were allocated between administration and business development and occupancy.

Bell and Murphy (1968) showed that if the underlying production function is Cobb-Douglas and banks are assumed to minimize costs, then the cost function is also Cobb-Douglas. However, there is no reason for the production function to be assumed to be Cobb-Douglas in the first place. The equation used by Bell and Murphy and Benston, with all variables in common logarithms, is as follows:

$$DC_i = b_0 + b_1N_i + b_2S_i + b_3A_i + b_4M_i + b_5R_i + b_iC_i + b_7O_i + b_8W + \sum_{j=1}^{5} b_jB_{ij} \qquad (3.2\text{-}3)$$

where DC_i = direct cost of each type of banking service of the i^{th} bank.

N_i = number of accounts serviced per year.

S_i = size of accounts.

A_i = activity per account.

M_i = mixed types of accounts

R_i = riskiness of loans (average interest rate charged)

C_i = concentration in types of business (the ratio of time deposits to demand deposits for the deposit services, and business loans to total loans for the business loan services).

O_i = other cost homogeneity factors.

W_i = relative wage index as a proxy for differences in factor prices.

B_{ij} = structure variables for branching, where B_{i1} = 1(log 10) for banks with one branch and zero otherwise, B_2 = 1 (log 10) for banks with 2 branches and zero otherwise, and so on.

The same formulation was used for indirect costs, with only definitional changes.

For savings and loan associations, Benston (1970) used the following equation:

$$C_i = a + bQP_i + \sum C_j QD_{ji} + \sum d_K QH_{Ki} + \sum g_m M_{mi} + h_n P_{ni} \qquad (3.2\text{-}4)$$

where C_i = operating costs (salaries, occupancy, miscellaneous or total expenses) of the i^{th} association.

QP_i = primary output variable (number of loans made, number of loans serviced, or number of savings accounts serviced).

QD_{ji} = two variables that account for different proportions of output (ratio of number of loans made to the number serviced and percentage of borrowing to mortgates).

QH_{ki} = six output homogeneity variables.

M_{mi} = ten managerial and structural variables.

P_{ni} = six input price and other cost homogeneity variables.

Benston (1965) used data for 80 to 83 banks for the years 1959, 1960, and 1961. Bell and Murphy (1968) used data for 210 to 283 banks in Boston, New York and Philadelphia for the years 1963, 1964, and 1965. In both studies, consistent and significant economies of scale were found for the demand deposits and real estate loan functions. Time deposits and instalment loans have estimated scale elasticities that are significantly less than unity for Benston's sample and less than unity but generally not significantly so for Bell's and Murphy's sample. Business loans also showed no significant economies of scale. Administration did not show consistently significant economies of scale.

Benston (1970), in analyzing operating costs for savings and loans associations, used a six-year data (1962-1966) to study 3159 of the 4332 insured associations (as of December 31, 1962). He excluded 205 firms because they merged during the period, 889 firms because some of their reports were missing and 79 firms because their data "failed to pass edit checks". He found relatively consistent economies of scale for savings and loan associations. Economies of scale do not appear to be proportionately greater for larger institutions than for smaller associations. For regulation purposes, Benston concluded that the existence of consistent and significant economies of scale indicate that larger firms are preferable, ceteris paribus. However, he advised that for specific decisions, the amount of expected savings in operating costs should be compared to an estimate of the disadvantages (like loss of effective competition).

Baltensperger (1973) used a much different formulation from all of the ones discussed above. He feels that "operating cost analysis" neglects one important function of banks -- consolidation of risks. He said that

> "to the extent that different debtors and creditors are independent, an increase in the number of customers reduces the bank's uncertainty about changes in cash reserves and capital account, and the inventory and adjustment costs associated with uncertainty. A large bank has thus an advantage over a small bank, as far as these costs are concerned."[17]

Unfortunately, Baltensperger did not do any empirical work for us to compare the results with the operating cost analyses.

All the literature surveyed so far utilized U.S. data. It appears that only Murray and White (1980, 1983) have done work using Canadian data. Murray and White analyzed economies of scale for deposit-taking financial institutions in Canada, using data for 152 credit unions in British Columbia, over the period 1972-1975. In their 1980 paper, they formulated a Cobb-Douglas cost function. They found no significant economies of scale in credit unions. Like all the other papers cited above in this section, Murray and White (1980) did not really tackle economies of scale from a multiproduct angle. Their 1983 paper was full of new ideas on multiproduct analysis. They used a translog functional form and defined the outputs as mortgage loans, other loans and investments. They defined costs to include all labour and real capital expenses, as well as the interest and dividends paid to depositors and shareholders. They defined the price of capital somewhat crudely as the sum of the major capital expenses (such as rent, depreciation and utilities) divided by the average dollar deposits in 1977. There is no theoretical reason given for defining the price of capital this way. Murray and White also incorporated other control variables, such as 'branch', 'risk', and 'growth'. They found that there exist economies of scale in all credit unions. Like Fuss and Waverman, Murray and White (1983) used the existence of cost complementarity as a sufficient criterion for the existence of economies of scope. Weak evidence of cost complementarity was found between investments and loans, but the results were not statistically significant. They also detected significant economies of scope between mortgage lending and other lending activity.

In summary, numerous studies on economies of scale in financial institutions, have either failed to approach the problem from a multiproduct angle or have used a functional form which has limited the full interpretation of multiproduct concepts. None of the works I have surveyed so far have done anything on the Canadian trust industry. This will be our task in this paper.

FOOTNOTES

Chapter 3

1. John Haldi and David Whitcomb (1967), p. 374.

2. George Stigler (1968), p. 71.

3. If $f(Y_1, Y_2, \ldots, Y_m, X_1, X_2, \ldots X_n) = 0$ is a general multiproduct transformation function, separability implies:

$$F(h(Y_1, Y_2, \ldots, Y_m), X_1, X_2, \ldots, X_n) = 0$$

and homogeneity implies

$$f(\lambda^r Y_1, \ldots, \lambda^r Y_m; \lambda X_, \ldots, X_n) = f(Y_1, \ldots, Y_m; X_1, \ldots, X_n) = 0$$

where

Y_i is output $\quad i = 1 \ldots m$

X_j is input $\quad j = 1 \ldots n$

λ is proportion in which inputs are increased.

r is degree of homogeneity.

4. Brown, Caves and Christensen (1979), p. 256.

5. Fuss and Waverman (1981) used the term 'joint production' to refer to either joint or common costs. Common costs are defined as the costs of inputs utilized by two or more outputs. Joint costs occur when two or more outputs are produced in fixed proportions. (p. 273-283.)

6. Fuss and Waverman (1981), p. 309.

7. E.E. Bailey and Ann F. Friedlaender (1982), p. 1036.

8. W. Baumol, J. Panzar and R. Willig (1982), p. 449

9. W. Baumol, J. Panzar and R. Willig (1982), p. 449.

10. D.A. Alhadeff (1954) is, according to Horwitz (1963), first economist to have examined empirical cost data in banking.

11. G. Benston (1972), p. 320.

12. G. Benston (1972), p. 320.

13. S. Greenbaum (1967), p. 466

14. P.M. Horwitz (1963), p. 37

15. G. Benston (1972), p. 321

16. J.A. Powers (1969), p. 164.

17. E. Baltensperger (1973), p. 601.

CHAPTER 4

THE MODEL

In chapter 2, we noted that the outputs for the trust industry are the services of loans, securities and the administration of estates, trusts and agencies. We also noted that deposits are both inputs as well as outputs.

In this chapter, we shall attempt to model the cost determination of the trust industry, by starting from a basic production structure using the said outputs of the trust industry and a set of inputs. We shall demonstrate that our cost function satisfies the homogeneity requirements and Shepherd's lemma.

Section 1 will be concerned with the structure of the model. Section 2 will be devoted to model specification. We shall talk about two different functional forms here: translog multiproduct cost function and generalized translog multiproduct cost function. In Section 3, we shall use the two functional forms to derive some specific measures of multiproduct economies of scale, economies of scope and product specific economies of scale.

4.1 THE STRUCTURE OF THE MODEL

The trust industry has a two-stage production structure. In the first stage, labour and capital are used as inputs to produce the services of deposits. The production technology at this stage can be represented by

$$Y_4 = f(X_1, X_2) \qquad (4.1\text{-}1)$$

where

Y_4 = the services relating to deposits, measured in "dollar-years".

X_1 = labour, measured in "man-years".

X_2 = capital, measured in "machine-years".

In the second stage, the services of deposits (Y_4) joins labour (X_1) and capital (X_2) to produce the other set of outputs of the trust industry. These outputs are

Y_1 - services relating to the administration of estates, trusts and agencies.

Y_2 - services relating to loans (mortgages, personal and colateral).

Y_3 - services relating to securities held (bonds, stocks, and treasury bills).

Assume there is a direct proportional relationship between stocks and flows. Thus, we can write the following stock balance equation in flow form as

$$Y_2 + Y_3 = Y_4 + S \qquad (4.1\text{-}2)$$

where S is shareholders' equity and the other variables are as defined above.

Assume that at any point in time shareholders' equity, S, is given and that the firm, being a cost minimizer, considers output as exogenously determined. As Benston has observed, the assumption of "exogenously determined rates of output appears valid for most output of regulated financial institutions. Banks and savings and loan associations are limited, on the whole, to their local market areas"[1]. Given the above structure, Y_4 can be determined from (4.1-2).

The final stage production technology of the trust industry can, therefore, be represented as

$$g(Y_1, Y_2, Y_3; Y_4, X_1, X_2, S) = 0 \qquad (4.1\text{-}3)$$

We assume that $g(\cdot)$ has continuous first and second order derivatives for positive values of the arguments, that it has positive first derivatives for the outputs and negative ones for the inputs, and that it is quasi-concave.

Total cost for a firm in the industry will be the sum of deposit costs, labour costs, and capital costs. That is, if

C_1 = operating costs

r_1 = unit price of labour

r_2 = unit price of capital services

and

r_d = unit price of deposits

then

$$C_1 = r_d Y_4 + r_1 X_1 + r_2 X_2 \qquad (4.1\text{-}4)$$

Under the assumption of cost-minimization, we can minimize C_1 subject to (4.1-3). Note that our assumptions imply that Y_4 is already determined. With λ as a Lagrange multiplier, we can set up the problem as follows:

$$\underset{(X_1, X_2, \lambda)}{\text{Min}}\ Z = r_d Y_4 + r_1 X_1 + r_2 X_2$$

$$- \lambda g(Y_1, Y_2, Y_3;\ Y_4, X_1, X_2, S) \qquad (4.1\text{-}5)$$

The first-order conditions for (4.1-5) are

$$\frac{\partial Z}{\partial X_1} = r_1 - \lambda \frac{\partial g}{\partial X_1} = 0 \qquad \ldots(i)$$

$$\frac{\partial Z}{\partial X_2} = r_2 - \lambda \frac{\partial g}{\partial X_2} = 0 \qquad \ldots(ii)$$

$$\frac{\partial Z}{\partial \lambda} = -g(Y_1, Y_2, Y_3;\ Y_4, X_1, X_2, S) = 0 \qquad \ldots(iii)$$

We then solve (i), (ii) and (iii) for the input demand functions, as follows:

$$X_1^* = X_1(Y_1, Y_2, Y_3, Y_4, S, r_1, r_2)$$

$$X_2^* = X_2(Y_1, Y_2, Y_3, Y_4, S, r_1, r_2)$$

We can then write the minimized cost function as follows:

$$C_1^* = r_d Y_4 + r_1 X_1^* + r_2 X_2^* \qquad (4.1\text{-}6)$$

or

$$C_1^* = C_1(Y_1, Y_2, Y_3, Y_4, S, r_1, r_2)$$

$$+ r_d Y_4 \qquad (4.1\text{-}7)$$

or

$$C_1^* - r_d Y_4 = C_1(Y_1, Y_2, Y_3, Y_4, S, r_1, r_2) \qquad (4.1\text{-}8)$$

from (4.1-2), $S = Y_2 + Y_3 - Y_4$ so (4.1-8) can be written as

$$C = C(Y_1, Y_2, Y_3, Y_4, r_1, r_2) \qquad (4.1\text{-}9)$$

where

$$C = C_1^* - r_d Y_4 \qquad (4.1\text{-}10)$$

C is, therefore, operating cost minus deposit costs.

Since X_1^* and X_2^* are homogeneous of degree zero in r_1 and r_2, the cost function C_1^* is homogeneous of degree one in all input prices. If we represent cost as operating cost less deposit cost, then C is homogeneous of degree one in r_1 and r_2.

We can demonstrate Shepherd's Lemma as follows:

Write

$$Z^* = r_d Y_4 + r_1 X_1^* + r_2 X_2^*$$

$$- \lambda g(Y_1, Y_2, Y_3; Y_4, X_1^*, X_2^*, S), \text{ where}$$

Z^* is the minimized Lagrangian. Differentiating Z^* with respect to r_1, gives

$$\frac{\partial Z^*}{\partial r_1} = X_1^* + r_1 \frac{\partial X_1^*}{\partial r_1} + r_2 \frac{\partial X_2^*}{\partial r_1} - \lambda \frac{\partial g}{\partial X_1^*} \cdot \frac{\partial X_1^*}{\partial r_1} - \lambda \frac{\partial g}{\partial X_2^*} \cdot \frac{\partial X_2^*}{\partial r_1}$$

$$= X_1^* + \left(r_1 - \lambda \frac{\partial g}{\partial X_1^*}\right) \frac{\partial X_1^*}{\partial r_1} + \left(r_2 - \lambda \frac{\partial g}{\partial X_2^*}\right) \frac{\partial X_2^*}{\partial r_1}$$

$= X_1^*$ (where the terms in parentheses equal zero by the first order conditions).

$$\therefore \frac{\partial C_1^*}{\partial r_1} = \frac{\partial Z^*}{\partial r_1} = X_1^* \text{ (Shepherd's Lemma).}$$

We can demonstrate a similar result for X_2^*.

4.2 THE SPECIFICATION OF THE MODEL

The choice of functional form is very important in the analysis of an industry. As noted in chapter 3, the function should not "prejudice the presence or absence of any cost properties" important to the analysis. A flexible functional form such as the translog is generally used to avoid the imposition of undue restrictions on the function.

Using the variables defined in Section 1, we write the translog multiproduct cost function (TMCF) for the cost equation in(4.1-9) as

$$\ell n\ C = \alpha_0 + \sum_{i=1}^{4} \alpha_i\ \ell n\ Y_i + \sum_{h=1}^{2} \beta_h\ \ell n\ r_h$$

$$+ 1/2 \sum_{i=1}^{4} \sum_{j=1}^{4} \delta_{ij}\ \ell n\ Y_i\ \ell n\ Y_j$$

$$+ 1/2 \sum_{h=1}^{2} \sum_{k=1}^{2} \gamma_{hk}\ \ell n\ r_h\ \ell n\ r_k$$

$$+ \sum_{i=1}^{4} \sum_{h=1}^{2} \delta_{ih}\ \ell n\ Y_i\ \ell n\ r_h \qquad (4.2\text{-}1)$$

For symmetry, we impose the restrictions

$$\gamma_{hk} = \gamma_{kh} \qquad (4.2\text{-}2a)$$

and

$$\delta_{ij} = \delta_{ji} \qquad (4.2\text{-}2b)$$

For the cost function to be linearly homogeneous, we impose the further restrictions

$$\sum_{h=1}^{2} \beta_h = 1 \qquad (4.2\text{-}3a)$$

$$\sum_{h=1}^{2} \gamma_{hk} = 0 \qquad k = 1,2 \qquad (4.2\text{-}3b)$$

$$\sum_{h=1}^{2} \delta_{ih} = 0 \qquad i = 1,2,3,4 \qquad (4.2\text{-}3c)$$

The translog cost function is a possible choice so long as there are no zero output quantities in our data set. Not all trust companies produce all specified outputs. There are some trust companies which produce only the services of loans and deposits; some produce only the services of E.T.A. and loans, and so on. The translog is undefined for such data observations. In such a situation a general functional form which can take on zero outputs as well, is preferred. In choosing such a function, one must keep in mind some of the properties set down by Caves, Christensen and Trethway:

> "To be attractive for empirical applications a flexible form for the MCF (Multiproduct Cost Function) should be linearly homogeneous in prices for all possible price and output levels; be parsimonious in parameters; and contain the value zero on permissible domain of output quantities."[2]

All the conditions are satisfied by the TMCF except that which requires it to allow for zero output values. A Box-Cox transformation is therefore applied to the output values to circumvent this difficulty. This transformation defines a function

$$f_i(\theta) = \frac{Y_i^{\theta} - 1}{\theta}, \qquad \theta \neq 0 \qquad (4.2\text{-}4)$$

such that

$$\lim_{\theta \to 0} \left[\frac{Y_i^{\theta} - 1}{\theta} \right] = \ln Y_i \qquad (4.2\text{-}5)$$

Applying the Box-Cox transformation to all output variables in the TMCF in (4.2-1), we obtain

$$\ln C = \alpha_0 + \sum_{i=1}^{4} \alpha_i \left(\frac{Y_i^{\theta} - 1}{\theta}\right) + \sum_{h=1}^{2} \beta_h \ln r_h$$

$$+ 1/2 \sum_{i=1}^{4} \sum_{j=1}^{4} \delta_{ij} \left(\frac{Y_i^{\theta}-1}{\theta}\right)\left(\frac{Y_j^{\theta}-1}{\theta}\right)$$

$$+ 1/2 \sum_{h=1}^{2} \sum_{k=1}^{2} \gamma_{hk} \ln r_h \ln r_k$$

$$+ \sum_{i=1} \sum_{h=1} \rho_{ih} \left(\frac{Y_i^{\theta}-1}{\theta}\right) \ln r_h \qquad (4.2\text{-}6)$$

This functional form, which is a hybrid of the translog multiproduct cost function and the Box-Cox transformation, was introduced, apparently independently, by Caves, Christensen and Trethway (1980) and by Fuss and Waverman (1981). It is generally referred to as the Generalized Translog Multiproduct Cost Function (GTMCF). Symmetry and homogeneity restrictions are the same as for the TMCF. We note that the function in (4.2-6) is defined for zero outputs.

If we define cost as in (4.1-10), that is, total cost less deposit costs, then using Shepherd's Lemma, we can derive the input-share equations as follows:

For labour, we have

$$S_1 = \frac{r_1 X_1}{C} = \frac{\partial C}{\partial r_1} \cdot \frac{r_1}{C}$$

$$= \frac{\partial \ \ell n \ C}{\partial \ \ell n \ r_1}$$

$$= \beta_1 + \sum_{k=1}^{2} \gamma_{1k} \ell n \ r_k + \sum_{i=1}^{4} \rho_{i1} \left(\frac{Y_i^{\theta}-1}{\theta}\right)$$

(4.2-7)

Similarly, for capital, the share equation is

$$S_2 = \frac{r_2 X_2}{C}$$

$$= \beta_2 + \sum_{h=1}^{4} \gamma_{h2} \ \ell n \ r_h + \sum_{i=1} \rho_{i2} \left(\frac{Y_i^{\theta}-1}{\theta}\right)$$

(4.2-8)

If we impose the homogeneity and symmetry restrictions on the cost function and the share equations, our relevant system will look as follows:

$$
\begin{aligned}
\ell n\ C = {} & \alpha_0 + \alpha_1\left(\frac{Y_1^\theta - 1}{\theta}\right) + \alpha_2\left(\frac{Y_2^\theta - 1}{\theta}\right)^2 + \alpha_3\left(\frac{Y_3^\theta - 1}{\theta}\right) \\
& + \alpha_4\left(\frac{Y_4^\theta - 1}{\theta}\right) + 1/2\ \delta_{11}\left(\frac{Y_1^\theta - 1}{\theta}\right) + 1/2\ \delta_{22}\left(\frac{Y_2^\theta - 1}{\theta}\right)^2 \\
& + 1/2\ \delta_{33}\left(\frac{Y_3^\theta - 1}{\theta}\right)^2 + 1/2\ \delta_{44}\left(\frac{Y_4^\theta - 1}{\theta}\right)^2 \\
& + \delta_{12}\left(\frac{Y_1^\theta - 1}{\theta}\right)\left(\frac{Y_2^\theta - 1}{\theta}\right) + \delta_{13}\left(\frac{Y_1^\theta - 1}{\theta}\right)\left(\frac{Y_3^\theta - 1}{\theta}\right) \\
& + \delta_{14}\left(\frac{Y_1^\theta - 1}{\theta}\right)\left(\frac{Y_4^\theta - 1}{\theta}\right) + \delta_{23}\left(\frac{Y_2^\theta - 1}{\theta}\right)\left(\frac{Y_3^\theta - 1}{\theta}\right) \\
& + \delta_{24}\left(\frac{Y_2^\theta - 1}{\theta}\right)\left(\frac{Y_4^\theta - 1}{\theta}\right) + \delta_{34}\left(\frac{Y_3^\theta - 1}{\theta}\right)\left(\frac{Y_4^\theta - 1}{\theta}\right) \\
& + \ell n\ r_2 + \beta_1(\ell n\ r_1 - \ell n\ r_2) \\
& + 1/2\ \gamma_{11}[(\ell n\, r_1)^2 + (\ell n\ r_2)^2 \\
& - 2(\ell n\ r_1)(\ell n\ r_2)]
\end{aligned}
$$

...continued

$$+ \rho_{11}\left(\frac{Y_1^\theta - 1}{\theta}\right)[\ell n\ r_1 - \ell n\ r_2]$$

$$+ \rho_{21}\left(\frac{Y_2^\theta - 1}{\theta}\right)[\ell n\ r_1 - \ell_n\ r_2]$$

$$+ \rho_{31}\left(\frac{Y_3^\theta - 1}{\theta}\right)[\ell n\ r_1 - \ell n\ r_2]$$

$$+ \rho_{41}\left(\frac{Y_4^\theta - 1}{\theta}\right)[\ell n\ r_1 - \ell n\ r_2] \qquad (4.2\text{-}9)$$

The labour share equation is as follows:

$$S_1 = \beta_1 + \gamma_{11}[\ell n\ r_1 - \ell n\ r_2]$$

$$+ \rho_{11}\left(\frac{Y_1^\theta - 1}{\theta}\right) + \rho_{21}\left(\frac{Y_2^\theta - 1}{\theta}\right)$$

$$+ \rho_{31}\left(\frac{Y_3^\theta - 1}{\theta}\right) + \rho_{41}\left(\frac{Y_4^\theta - 1}{\theta}\right) \qquad (4.2\text{-}10)$$

The capital share equation is correspondingly defined. However, for the purposes of estimation we shall need the cost equation in (4.2-9) and only one share equation[3]. We shall use the labour share equation in (4.2-10).

4.3 SPECIFIC FORMULATION OF MEASURES OF SOME COST CONCEPTS

In chapter 3, we defined some measures of economies of scale, economies of scope, and product-specific economies of scale. In this section we shall use the generalized translog multiproduct cost function in (4.2-9) to give specific definition to these measures.

Multiproduct economies of scale measure has been defined in (3.1- 3) as

$$S = 1/\varepsilon \qquad (4.3\text{-}1)$$

where

$$\varepsilon = \sum_{i=1} \frac{\partial \ \ell n \ C}{\partial \ \ell n \ Y_1} \qquad (4.3\text{-}2)$$

Returning to the GTMCF in equation (4.2-9) we notice that we can define the elasticity of cost with respect to output 1 (estates, trusts and agency administration) as follows:

$$\frac{\partial \ \ell n \ C}{\partial \ \ell n \ Y_1} = [\alpha_1 + \delta_{11}(Y_1^\theta - 1)/\theta + \delta_{12}(Y_2^\theta - 1)/\theta + \delta_{13}(Y_3^\theta - 1)/\theta + \delta_{14}(Y_4^\theta - 1)/\theta + \rho_{11}(\ell n \ r_1 - \ell n \ r_2)]Y_1^\theta \qquad (4.3\text{-}3)$$

We can similarly define the elasticity of cost with respect to loans, securities, and deposits. It follows that, for the GTMCF,

$$\varepsilon = \sum_{i=1}^{4}\left[\alpha_i + \sum_{j=1}^{4} \delta_{ij}(Y_j^{\theta}-1)/\theta + \rho_{i1}(\ell n\ r_1 - \ell n\ r_2)\right]Y_i^{\theta}$$

(4.3-4)

For the translog multiproduct cost function (TMCF) in (4.1-1),

$$\varepsilon = \sum_{i=1}^{4}\left[\alpha_i + \sum_{j=1}^{4} \delta_{ij}\ \ell n\ Y_j + \rho_{i1}(\ell n\ r_1 - \ell n\ r_2)\right]$$

(4.3-5)

There are two ways we can measure global economies of scope for the trust industry. The first measure which we shall call Scope-a, considers the cost of producing all outputs separately relative to the cost of producing them jointly. Thus Scope-a is given by

$$S_{ca} = \{C(Y_1,0,0,0) + C(0,Y_2,0,0) + C(0,0,Y_3,0) + C(0,0,0,Y_4) - C(Y_1,Y_2,Y_3,Y_4)\}/C(Y_1,Y_2,Y_3,Y_4)$$

(4.3-6)

S_{ca} is termed weak economies of scope[4].

The second global measure of economies of scope in the trust industry, separates the outputs into the two main functions. This measure we shall call Scope-b. The output under the trustee function is the services relating to the administration of estates, trusts, and agencies (Y_1). Under financial intermediation the outputs are the services of loans (Y_2), securities (Y_3) and deposits (Y_4). Thus Scope-b is given by

$$S_{cb} = \frac{C(Y_1, 0,0,0) + C(0,Y_2,Y_3,Y_4) - C(Y_1,Y_2,Y_3,Y_4)}{C(Y_1,Y_2,Y_3,Y_4)} \qquad (4.3\text{-}7)$$

S_{cb} is referred to by Mintz (1981) as incremental economies of scope[5].

S_{ca} checks whether there are cost advantages to a trust company for producing all the four outputs, while S_{cb} checks whether there is a cost advantage to a trust company for performing its two major functions. Because they involve zero outputs, both S_{ca} and S_{cb} can only be calculated using the generalized translog multiproduct cost function.

The sufficient condition for local economies of scope between Y_1 and Y_2 is given by

$$\frac{\partial^2 C}{\partial Y_1 \partial Y_2} < 0 \qquad (4.3\text{-}8)$$

Using the translog function in (4.2-1) and differentiating it with respect to Y_i, we obtain

$$\frac{\partial C}{\partial Y_i} = C\, Y_i^{-1}[\alpha_i + \sum_{j=1}^{4} \delta_{ij}\ \ell n\ Y_j + \rho_{i1}(\ell n\ r_1 - \ell n\ r_2)], \quad i \ldots 4 \qquad (4.3\text{-}9)$$

Second differentiation with respect to Y_j yields

$$\frac{\partial^2 C}{\partial Y_i \partial Y_j} = C\, Y_i^{-1} Y_j^{-1}\Big[\delta_{ij} + (\alpha_i + \sum_{j=1}^{4} \delta_{ij}\ \ell n\ Y_j + \rho_{i1}(\ell n\ r_1 - \ell n\ r_2))(\alpha_j + \sum_{i=1}^{4} \delta_{ij}\ \ell n\ Y_i + \rho_{ji}(\ell n\ r_1 - \ell n\ r_2))\Big] \quad i \neq j,\ j = 1 \ldots 4$$

$$(4.3\text{-}10)$$

Since for local economies of scope we are only interested in the sign, the real test for scope between say Y_i and Y_j, is whether

$$\text{L-Scope}(i:j) = \left[\delta_{ij} + (\alpha_i + \sum_{j=1}^{4} \delta_{ij} \ln Y_j) (\alpha_j + \sum_{i=1}^{4} \delta_{ij} \ln Y_i)\right] \gtrless 0$$

(4.3-11)

where L-Scope (i:j) is to be evaluated at the means of the prices. Prices have been normalized to their means — see chapter 5.

For the generalized translog function (GTMCF), local economies of scope is tested for by examining whether

$$\text{L-Scope}(i:j) = \left[\delta_{ij} + (\alpha_i + \sum_{j=1}^{4} \delta_{ij} (\frac{Y_j^{\theta}-1}{\theta})) (\alpha_j + \sum_{i=1}^{4} \delta_{ij} (\frac{Y_i^{\theta}-1}{\theta}))\right] \gtrless 0$$

$$i \neq j, \; i,j = 1 \ldots 4$$

(4.3-12)

To check for local economies of scope between outputs under the intermediation and the fiduciary functions, assume that

$$\frac{dY_2}{Y_2} = \frac{dY_3}{Y_3} = \frac{dY_4}{Y_4} \qquad (4.3\text{-}13)$$

Let

$$C_i = \frac{\partial C}{\partial Y_i} \quad \text{and} \quad C_{ij} = \frac{\partial^2 C}{\partial Y_i \partial Y_j}$$

Thus to check for economies of scope between outputs under the two distinct functions, we examine whether

$$\text{Scope }(1{:}2,3,4) = \left.\frac{\partial^2 C}{\partial Y_1 \partial Y_2}\right|_{\frac{dY_2}{Y_2} = \frac{dY_3}{Y_3} = \frac{dY_4}{Y_4}}$$

$$= \left[C_{12} + C_{13}\frac{Y_3}{Y_2} + C_{14}\frac{Y_4}{Y_2}\right] \gtrless 0 \qquad (4.3\text{-}14)$$

The condition for product-specific economies of scale is given by

$$S_i = AIC_i / \frac{\partial C}{\partial Y_i} > 1 \qquad (4.3\text{-}15)$$

where $$AIC_i = \frac{C(Y_1,Y_2,Y_3,Y_4) - C(Y_1,0,Y_3,Y_4)}{Y_i}, \text{ if } i=2, \qquad (4.3\text{-}16)$$

FOOTNOTES

Chapter 4

1. George J. Benston (1972), p. 317.

2. D.W. Caves, L.R. Christensen, and M.W. Tretheway (1980), p. 478.

3. See L.R. Christensen and W. Greene (1976), and also Caves, D.W., L.R. Christensen, and M.W. Tretheway (1980).

4. J.M. Mintz (1981), p. 30.

5. J.M. Mintz (1981), p. 30.

CHAPTER 5

DATA AND DEFINITION OF VARIABLES

DATA

There were almost 90 firms in Canada by 1981 doing general trust business. Of this number, 57 were registered and doing business in Ontario. These 57 firms together control total guaranteed and company assets valued at 37 billion dollars, which is about 85 per cent of the guaranteed and company assets of all trust companies operating in Canada. Firms registered in Ontario also control about 88 per cent of the total assets under estates, trusts and agency administration controlled by all trust companies in Canada. In addition, all the major trust companies in Canada are also registered in Ontario. Thus, the data on trust companies operating in Ontario is representative of the Canadian trust industry.

The data used for analysis in this thesis has been obtained from reports submitted by the individual trust companies to the Ontario Registrar of Loan and Trust Companies. These reports are published annually by the Ontario Ministry of Consumer Affairs. Other sources of data include Statistics Canada reports (catalogue numbers 13-211, 13-568, 61-006, and 72-002), and Bank of Canada reviews. The population consists of trust companies registered and doing business in Ontario between 1976 and 1981, inclusive. The number of firms is not the same from year to year because of entries, exits and mergers taking place over the years. (See Appendix 2.1 for details on these.)

Some firms were found to have unusually low labour shares in the order of 1% as opposed to the average of 46%. Such outliers distorted the initial regression runs and so were later excluded from the observations. It was not always the case that a firm with an unusually low share in one year continued to have low labour shares in subsequent years. Out of the initial 326 observations for the six years, 27 observations were dropped because they have low labour shares. The remaining data set, containing 299 observations, is referred to here as Data-1. Data-1 contains observations with zero output values for some firms for some of the years. This implies that Data-1 cannot use any functional form which takes logarithms of outputs or other transformations that would be undefined at zero outputs. A second data-set, Data-2, was therefore prepared from Data-1. Data-2 excludes all observations with zero outputs. 39 observations were further lost in this way. Analyses of Data-2 allows for easy comparisons with previous studies which have used the translog functional form.

The data is shown in Appendix A. It contains all observations for the six years.

COST

Cost is defined in this thesis to reflect the 'economic' or 'opportunity cost' of operating a firm during a time period at a given rate of output. It refers in this case to cost of all inputs. That is, it is the cost of labour (wages, salaries and staff benefits, and real estate commissions) plus cost of deposits (interest incurred) plus cost of capital (rental cost and an imputed cost of capital). Capital in this case is defined to include all other inputs apart from labour and deposits.

'Total expenses' of the trust companies include wages and salaries and commissions plus interest incurred plus the cost of rented capital. It does not, however, include the cost of capital the firm owns. Thus, to get the opportunity cost of capital, an imputed cost of the capital the firm owns is added on to the cost of rented capital. Assume that the value of the firm's equity is an approximation of the replacement cost of capital.

If we define the opportunity cost of capital in Jorgenson's sense[1] to be

$$r_{2t} = r_t + \delta_t - g_t \qquad (4.1\text{-}1)$$

then the imputed cost of capital the firm owns shall be

$$r_{2t} \times \text{equity}$$

where

r_t = the rate of interest on 3-5 year government bonds in period t.

δ_t = depreciation rate = 5.5 [2]

$g_t = (P_{t+1}/P_{t-1})^{.5} - 1$ = capital gain/loss

To obtain r_t, we calculated the geometric average for the monthly rates from July to June of each year of the yields on 3-5 year Government of Canada Bonds. P_t is the price index for capital expenditure on plant and equipment[3] by the finance, insurance and real estate industry, with 1971 as base year.

P_t increases at the rate g_t calculated as

$$P_{t+1} = P_{t-1}(g_t + 1)^2$$

or

$$g_t + 1 = \left(\frac{P_{t+1}}{P_{t-1}}\right)^{.5}$$

Table 5.1 shows the calculated values of P_t, g_t and r_t for the period 1975 to 1981.

TABLE 5.1

Calculated Values of P_t, g_t, r_t

Year	P_t	g_t	r_t
1975	143.4		
1976	155.7	.042	.08
1977	167.3	.037	.08
1978	182.8	.045	.10
1979	196.7	.037	.12
1980	214.8	.045	.13
1981	237.5	.052	.16

SOURCES: Computed from data obtained from Statistics Canada, Fixed Capital Flows and Stocks, Catalogue Nos. 13-568 and 13-211; Bank of Canada Reviews, various years.

Depreciation is already accounted for in the total expenses figure. So that the imputed cost of capital added on to 'total expenses' should exclude $\delta_t \times$ equity. Hence, operating costs is calculated as follows:

$$\text{Operating Costs} = \text{Total Expenses} + (r_t - g_t)\ \text{Equity} - \text{Interest Incurred} \qquad (5.1\text{-}2)$$

OUTPUTS

In chapter 4, outputs in the trust industry have been defined as the services of estates, trusts, and agencies (Y_1), loans (Y_2), and securities (Y_3). To obtain loans we summed mortgages and sale agreements, collateral loans and consumer loans. Securities are calculated as the sum of bonds, stocks and treasury bills. Deposits (Y_4), which originally had been assumed to be both an input and an output turned out to be an argument in the cost function.

We have assumed in chapter 4 that there is a direct and perfect relation between flows of outputs and their stock values. Thus, following Sealey and Lindley (1977) and Murray and White (1980, 1983), each of these services mentioned above is represented by the dollar amounts outstanding on the accounts.

INPUT PRICES

Let us assume that all firms in the finance industry buy their inputs from one competitive market. This implies that at any time the price of an input is the same for all financial institutions. The inputs for the trust industry are deposits, labour and capital; but the model specification in chapter 4 implies that only the prices of labour and capital will be needed. Capital in this case is defined to be all other inputs apart from labour and deposits. The price of capital is r_2, already defined above. Since r_2 enters the cost equation in logarithmic form, evaluations at the mean are simplified by dividing each r_2 observation by the mean of r_2. The normalized r_2 is called r_2^* and is shown in Table 5.2

Average weekly earnings of labour for the

TABLE 5.2

Price of Capital

	1976	1977	1978	1979	1980	1981
r_2	0.093	0.098	0.110	0.138	.140	.163
r_2^*	0.744	0.784	0.880	1.080	1.120	1.304

SOURCE: Calculated from Table 5.1

finance, insurance and real estate industry is taken as the wage index (r_1) for the trust industry. This is shown in Table 5.3. Again, because of the way r_1 appears in the model, it is normalized by dividing each observation by the mean of r_1 to obtain r_1^*.

TABLE 5.3

PRICE OF LABOUR

	1976	1977	1978	1979	1980	1981
r_1	213.71	229.57	248.43	272.10	304.37	353.71
r_1^*	.78	.83	.90	.99	1.11	1.29

SOURCE: Statistics Canada: Employment, Earnings and Hours. Catalogue #72-002.

INPUT SHARES

By defining variable cost as the cost of labour and capital, we can derive only two share equations. To obtain values for the share of labour (S1), we divided the sum of wages, salaries and real estate commissions by the operating net cost. To obtain values for S2, the share of capital, we subtracted S1 from unity.

FOOTNOTES

Chapter 5

1. See, for example, Frank Brechling (1975) p. 12.

2. Because of the complexity of the definition of capital in this study and for simplicity we just assumed the rate of depreciation of capital to be 5.5%. If we had a unique definition of capital we would have used one of the conventional methods of calculating depreciation like

   $$d_i = 1 - \frac{1}{L} T \qquad (T = 0,1,\ldots,L-1)$$

   where L is the assumed life-span of the capital. (See for example, Statistics Canada, Catalogue #13-568.)

3. For the finance, insurance and real estate industry, the components of plant and equipment are building construction, and machinery and equipment.

CHAPTER 6

EMPIRICAL ANALYSIS

Studies on cost functions have used various functional forms in their econometric analyses of different industries. Murray and White (1980, 1983) for example, analyzed cost in deposit-taking financial institutions by using both Cobb-Douglas and translog functional forms. Fuss and Waverman (1981) analyzed the cost structure in the telecommunication industry by using the translog and other restricted forms of the translog. As we have pointed out in previous chapters, the use of the translog in a multiproduct context, limits the usefulness of such concepts as economies of scope and product specific economies. In Chapter 4 we suggested the generalized translog multiproduct cost function (GTMCF) as a more general flexible functional form which overcomes some of the shortcomings of the translog multiproduct cost function (TMCF).

In chapter 5 we divided the data into two sets -- data-1 and data-2. The truncated data (data-2) which contains no zero outputs and hence can accommodate the translog are analyzed to make our results comparable to earlier studies. Using data-2, the translog and the generalized translog are both estimated and tested for the best production structure that fits the data. We also estimated and analyzed data-1, which comprises all firms, except those with very low labour shares. Only the generalized translog is used in this case.

Section 1 of this chapter deals with the estimation and econometric analyses of the results. This section is subdivided into two parts: the first part deals with estimation and results using data-1, while the second part deals with analyses using data-2. The second section is devoted to the interpretation of results. Economic meanings are given to the theoretical hypotheses of cost functions enumerated in chapter 4.

6.1 ESTIMATION AND EMPIRICAL RESULTS

The cost equation can be estimated more efficiently as a simultaneous system along with the share equations than as a single equation[1]. In this instance there are two share equations representing the two inputs: labour and capital. To avoid singularity, only one of the share equations is estimated, together with the cost equation. It is generally immaterial which of the two share equations is deleted. Here, the cost equation and the labour-share equation are treated as a simultaneous equation system. This system is treated as reduced form except for the error terms (u_1, u_2) which are assumed to be contemporaneously related. This suggests that the procedure best suited for estimation would be Zellner's seemingly unrelated iterative technique. This technique yields estimates asymptotically equivalent to maximum likelihood estimates[2]. The system to be estimated is the following:

$$\ell n\ C - \ell n\ r_2 = \alpha_0 + \alpha_1\left(\frac{Y_1^\theta - 1}{\theta}\right) + \alpha_2\left(\frac{Y_2^\theta - 1}{\theta}\right) + \alpha_3\left(\frac{Y_3^\theta - 1}{\theta}\right)$$

$$+ \alpha_4\left(\frac{Y_4^\theta - 1}{\theta}\right) + 1/2\delta_{11}\left(\frac{Y_1^\theta - 1}{\theta}\right)^2 + 1/2\delta_{22}\left(\frac{Y_2^\theta - 1}{\theta}\right)^2$$

...continued

$$+ 1/2\delta_{33}\left(\frac{Y_3^{\theta}-1}{\theta}\right)^2 + 1/2\delta_{44}\left(\frac{Y_4^{\theta}-1}{\theta}\right)^2$$

$$+ \delta_{12}\left(\frac{Y_1^{\theta}-1}{\theta}\right)\left(\frac{Y_2^{\theta}-1}{\theta}\right) + \delta_{13}\left(\frac{Y_1^{\theta}-1}{\theta}\right)\left(\frac{Y_3^{\theta}-1}{\theta}\right)$$

$$+ \delta_{14}\left(\frac{Y_1^{\theta}-1}{\theta}\right)\left(\frac{Y_4^{\theta}-1}{\theta}\right) + \delta_{23}\left(\frac{Y_2^{\theta}-1}{\theta}\right)\left(\frac{Y_3^{\theta}-1}{\theta}\right)$$

$$+ \delta_{24}\left(\frac{Y_2^{\theta}-1}{\theta}\right)\left(\frac{Y_4^{\theta}-1}{\theta}\right) + \delta_{34}\left(\frac{Y_3^{\theta}-1}{\theta}\right)\left(\frac{Y_4^{\theta}-1}{\theta}\right)$$

$$+ \beta_1(\ell n\ r_1 - \ell n\ r_2) + 1/2\gamma_{11}[(\ell n\ r_1)^2$$

$$+ (\ell n\ r_2)^2 - 2(\ell n\ r_1)(\ell n\ r_2)]$$

$$+ \rho_{11}\left(\frac{Y_1^{\theta}-1}{\theta}\right)(\ell n\ r_1 - \ell n\ r_2) + \rho_{21}\left(\frac{Y_2^{\theta}-1}{\theta}\right)(\ell n\ r_1 - \ell n\ r_2)$$

$$+ \rho_{31}\left(\frac{Y_3^{\theta}-1}{\theta}\right)(\ell n\ r_1 - \ell n\ r_2) + \rho_{41}\left(\frac{Y_4^{\theta}-1}{\theta}\right)(\ell n\ r_1 - \ell n\ r_2)$$

$$+ \psi_1 T_1 + \psi_2 T_2 + \psi_3 T_3 + \psi_4 T_4 + \psi_5 T_5 + u_1$$

(6.1-1)

$$S_1 = \beta_1 + \gamma_{11}(\ell n\ r_1 - \ell n\ r_2)$$

...continued

$$+ \rho_{11}\left(\frac{Y_1^{\theta}-1}{\theta}\right) + \rho_{21}\left(\frac{Y_2^{\theta}-1}{\theta}\right) + \rho_{31}\left(\frac{Y_3^{\theta}-1}{\theta}\right)$$

$$+ \rho_{41}\left(\frac{Y_4^{\theta}-1}{\theta}\right) + u_2 \qquad (6.1\text{-}2)$$

All variables are as defined previously. T_i $(i = 1...5)$ are time dummies. T_1 takes the value 1 in 1976 while all others are zero; T_2 takes the value 1 in 1977 while all others are zero, and so on. 1981 has no dummies at all. u_1 and u_2 are random disturbance terms (error terms). It is assumed that there is no autocorrelation within equations, but there exists cross-equation correlation since the share equation (6.1-2) is derived from the cost equation (4.2-9).

Zellner's seemingly unrelated iterative technique is applied to the two sets of data separately. In all cases the initial parameter values for the iterative process were changed several times to check for global convergence. For all regressions, the data are pooled over the six years to increase the efficiency of the parameter estimates. Since it has been assumed that the input prices are the same across firms in a given year, it implies that were we to estimate cross-sectional equations, we would have to modify the model specification to exclude all price terms. If this is not done, cross-sectional estimations will encounter a perfect multicollinearity problem.

The problem of multicollinearity is avoided in the pooled data case when the dummies are introduced since only intercept dummies are allowed. Slope dummies attached to the input price terms, would again lead to perfect multicollinearity.

Using data-2, a third production structure is tested by estimating the translog cost function with the following constant returns to scale restrictions:

$$\sum_i \alpha_i = 1$$

$$\sum_i \delta_{ij} = 0 \qquad j = 1,\ldots,4 \qquad (4.1\text{-}3)$$

and

$$\sum_i \rho_{i1} = 0$$

The results of the estimation are presented below in two parts. The first part shows the results of estimation using data-1 and the second part shows that of data-2.

DATA-1 RESULTS

Data-1 contains all firms in the Canadian trust industry doing business in Ontario, except those with very low labour shares. Since this data-set contains some zero outputs, the translog production structure could not be estimated here. Instead only the generalized translog is examined with this data. Because of the presence of zero output values we could not allow Zellner's iterative process to iterate freely over all real values. When that is attempted, it iterated over zero theta (θ) values and that caused the zero output terms to be undefined. The best alternate way to estimate this nonlinear system is by first linearizing it by fixing values for theta (θ). Theta (θ) is searched over the range 0.01 to 3.0. The result of the search is presented in Table 6.1. Theta (θ) = .15 maximizes the likelihood function and is hence, selected as the best value of θ to linearize the system.

Two kinds of models are estimated (see Table 6.1). In the first model no time-dummies are included while the second included five time-dummies for the first five years. To distinguish between the two models we formulate the hypotheses

TABLE 6.1

SEARCH FOR θ FOR THE GTMCF -- DATA-1

Values of θ	Log of Likelihood Functions: No dummies	Log of Likelihood Functions: Intercept dummies
3.0	-241.595	-238.033
2.0	-205.372	-202.734
1.0	-91.1184	-89.4381
.30	80.7229	82.2886
.25	86.5111	88.1231
.20	92.2528	93.9812
.15*	94.2805*	96.2076*
.10	90.3993	92.6987
.05	76.3492	79.2172
.015	50.6070	53.8548
.01	44.7309	47.9717

* logs of the likelihood functions are at maximum where θ = .15

$$H_0 \text{ : GTMCF with no dummies}$$
$$H_1 \text{ : GTMCF with dummies} \tag{6.1-4}$$

If the likelihood function under the null hypothesis is L_0 and that under the alternate hypothesis is L_1, then by the log-likelihood ratio test

$$-2\log(L_0/L_1) = -2[\log(L_0) - \log(L_1)] \sim \chi^2(r)$$

where r is the degrees of freedom.

The test statistic for the hypothesis in (6.1-4) is

$$-2[\log(L_0) - \log(L_1)] = -2[94.2805 - 96.2076]$$

$$= 3.8542$$

The critical value for $\chi^2(5)$ at 5% significance level is 11.07 and at 10% is 9.24. At either significance level, we do not reject the null-hypothesis (H_0). Moreover, all the dummy variables are not significantly different from zero. Hence, we choose the generalized translog model without time dummies as a better fit than the one with the time dummies. Our analysis will be based on the former only.

Ignoring cross-interactions between the outputs and the input prices in equation (6.1-1), Table 6.2 indicates that the marginal costs for E.T.A., loans, and securities are all positive while that for deposits is negative. The latter result may be due to the fact that deposits are inputs as well as outputs as noted earlier.

Table 6.3 provides the descriptive statistics for both the cost equation and the labour share equation.

To examine the goodness of fit of the two equations in the model, R^2 is calculated as follows:

$$R^2 = 1 - \frac{S^2}{Var(Q)} \left(\frac{N-K}{N-1}\right) \qquad (6.1\text{-}5)$$

where

R^2 is the coefficient of determination,

S is the standard error of the regression,

Var(Q) is the variance of the dependent variable,

N is the number of observations, and

K is the number of parameters in the equation.

Thus, for the cost equation,

$$R^2 = 1 - \frac{.5485^2}{1.8166^2} \left(\frac{278}{298}\right) = .915$$

TABLE 6.2

PARAMETER ESTIMATES FOR THE GTMCF WITH AND WITHOUT ANNUAL DUMMIES -- DATA-1

Coefficients	GTMCF(1) With Dummies	GTMCF(1) Without Dummies
α	1.8843 (.1811)	1.9203 (.1687)
α_1	.0535 (.0069)	.0540 (.0070)
α_2	.0914 (.0344)	.0902 (.0344)
α_3	.1176 (.0173)	.1181 (.0173)
α_4	-.0354 (.0352)	-.0354 (.0352)
δ_{11}	.0016 (.0005)	.0016 (.0005)
δ_{22}	.0077 (.0023)	.0076 (.0023)
δ_{33}	.0116 (.0024)	.0118 (.0023)
δ_{44}	-.0074 (.0049)	-.0068 (.0048)
δ_{12}	-.0021 (.0010)	-.0023 (.0010)
δ_{13}	-.0031 (.0009)	-.0033 (.0009)
δ_{14}	.0022 (.0011)	.0024 (.0010)
δ_{23}	-.0080 (.0048)	-.0075 (.0048)

...continued

Coefficients	GTMCF(1) With Dummies	GTMCF(1) Without Dummies
δ_{24}	.0046 (.0022)	.0043 (.0022)
δ_{34}	-.0006 (.0050)	-.0011 (.0049)
β_1	.3419 (.0140)	.3424 (.0140)
γ_{11}	.3457 (.1252)	.2488 (.0954)
ρ_{11}	.0044 (.0007)	.0044 (.0007)
ρ_{21}	-.0020 (.0021)	-.0020 (.0021)
ρ_{31}	-.0031 (.0018)	-.0030 (.0018)
ρ_{41}	.0058 (.0022)	.0057 (.0023)
Ψ_1	.0215 (.0918)	
Ψ_2	.0994 (.0911)	
Ψ_3	.1132 (.0841)	
Ψ_4	-.0242 (.0886)	
Ψ_5	-.0129 (.0812)	
θ[a]	.15	.15

(a) θ was estimated by searching over the range .01 to 3.0 for the best fit. See Table 6.1

NOTE: Asymptotic standard errors are reported in parentheses.

TABLE 6.3

Descriptive Statistics for the GTMCF Without Annual Dummies -- Data-1

Description	Cost Equation	Labour Share Equation
Dependent Variable	$\ell n(C) - \ell n(r_2)$	S_1
Sum of Squared Residuals	89.9523	3.1473
Standard Error of the Regression	.5485	.1026
Mean of Dependent Variable	5.9205	.4625
Standard Deviation	1.8166	.1266
Number of Observations	299	299
Sum of Residuals	-.6474E-01	-.1853E-01

Hence, 85% of the variation in the dependent variable is explained by variations in the independent variables. Adjusted for degrees of freedom, we find the adjusted coefficient of determination to be

$$\bar{R}^2 = 1-(1 - R^2)(\frac{N-1}{N-k}) = .909 \qquad (6.1\text{-}6)$$

Thus, the specification of the cost equation as GTMCF is a good fit for the data. Similarly, for the share equation, $R^2 = .354$ and $\bar{R}^2 = .343$. Thus the share equation is not a good fit.

DATA-2 RESULTS

Data-2, which includes only firms with non-zero outputs, is used to estimate three different functional forms: (i) generalized translog (GTMCF), (ii) translog (TMCF), and (iii) translog with constant returns to scale(TMCFCRS). Results of these regressions are reported in Table 6.4. Standard errors of the estimates appear under them in parentheses.

Comparing the results of the GTMCF and the TMCF models reported in Table 6.4, one notices that the signs on the parameters are the same for both models. Most of the estimates in both models are statistically significant. It is interesting to note that the estimate for theta (θ) is very close to zero and this indicates that the best fit equation could be the limiting case of θ approaching zero (the translog model). To be more precise on this matter, a comparison is made of the logs of the likelihood functions of the GTMCF and TMCF models. The result is tabulated in Table 6.5. To test which of the two models best describes the production structure that fits data-2, we formulated the null hypothesis (H_0) and compared it with the alternate hypothesis (H_1).

TABLE 6.4

PARAMETER ESTIMATES FOR GTMCF AND TMCF (WITHOUT ANNUAL DUMMIES) USING DATA-2

Parameters	GTMCF	TMCF	TMCFCRS
θ	.0116 (.0391)	$\theta \to 0$	$\theta \to 0$
α	2.3328 (1.0644)	2.6437 (.6956)	-1.3588 (.2382)
α_1	.2030 (.0775)	.2119 (.0828)	
α_2	1.9344 (.6323)	1.9894 (.6353)	3.2794 (.6569)
α_3	.3838 (.2380)	.3910 (.2719)	.7304 (.2412)
α_4	-2.4308 (.9081)	-2.6171 (.6762)	-3.2455 (.7205)
δ_{11}	.0369 (.0297)	.0466 (.0107)	
δ_{22}	1.0242 (.6362)	1.2333 (.3151)	1.0099 (.3416)
δ_{33}	-.0448 (.0679)	-.0459 (.0774)	.0778 (.0823)
δ_{44}	.5467 (.4659)	.6270 (.4094)	.0663 (.4361)
δ_{12}	-.0041 (.0758)	-.0005 (.0923)	-.0752 (.0847)
δ_{13}	-.0529 (.0365)	-.0627 (.0298)	-.0603 (.0306)
δ_{14}	.0093 (.0919)	.0060 (.1122)	.0720 (.1071)
δ_{23}	-.4311 (.2580)	-.5176 (.1963)	-.7481 (.1879)

...continued

Parameters	GTMCF	TMCF	TMCFCRS
δ_{24}	-.7634 (.5094)	-.8995 (.3311)	-.5005 (.3524)
δ_{34}	.4895 (.2871)	.5856 9.2016)	.6965 (.2082)
β_1	.3220 (.0377)	.3164 (.0357)	.5203 (.0120)
γ_{11}	.1592 (.0795)	.1580 (.0796)	.1491 (.0857)
ρ_{11}	.0193 (.0070)	.0214 (.0032)	
ρ_{21}	.0338 (.0274)	.0396 (.0273)	-.0016 (.0288)
ρ_{31}	.0224 (.0106)	.0238 (.0088)	.0212 (.0095)
ρ_{41}	-.0549 (.0341)	-.0625 (.0309)	-.0405 (.0333)

NOTE: Asymptotic standard errors are reported in parentheses.

Number of observations = 260.

TABLE 6.5

TEST OF THE PRODUCTION STRUCTURE -- DATA-2

Structure	Likelihood	Test Statistic ($-2\log(L_1/L_0)$)	Degrees of Freedom	Critical Value (5%)
GTMCF	117.327			
TMCF	117.375	-.096	1	3.841
Constant returns to scale - TMCF	73.9457	86.8586	4	9.488

H_0 : Translog production structure ($\theta = 0$)

H_1 : Generalized translog ($\theta \neq 0$)

(6.1-7)

The test statistic for the hypotheses in (6.1-6) is

$$-2[\log(L_0) - \log(L_1)] = -2[117.375 - 117.327]$$
$$= -.096$$

The critical value of $\chi^2(1)$ is 3.841 at the 5% significance level. Thus, since the calculated test statistic is less than the critical value, we do not reject the null-hypothesis at the 5% significance level. Notice that the estimated value for θ in the GTMCF is .0116 and is not significantly different from zero. This implies that if we assume that all trust companies produce all the specified outputs then the production structure can be represented by the translog.

As part of the search for the production structure which best fits data-2, we estimated a restricted form of the translog cost function -- constant returns to scale was imposed on the structure. The hypothesis tested here is as follows:

$$H_0 : \text{Translog with Constant Returns to Scale}$$

$$H_1 : \text{Unrestricted Translog} \tag{6.1-8}$$

First, we treated the translog as an exact form and then imposed the restrictions in (6.1-3). The likelihood functions obtained from the two regressions are then used to test the hypotheses in (6.1-8) as follows:

$$-2[\log(L_0) - \log(L_1)] = -2[73.9457 - 117.357]$$
$$= 86.8586$$

The critical value of $\chi^2(4) = 9.488$ at 5% significance level. Hence we reject the null hypothesis at 5% significance level and in fact, even at .5% significance level. Thus even if we assume the translog to best describe the production structure, we must reject the hypothesis of constant returns to scale.

In summary, this sub-section has demonstrated that if we consider only firms that produce all the specified output set, then the translog model as opposed to the generalized translog, may best describe the production structure. This agrees with Murray and White (1983) and Fuss and Waverman (1981). Nevertheless, within the Canadian trust industry, there are some firms which do not produce all the specified outputs of loans, E.T.A. and securities at some particular times. Sometimes for lack of necessary staff or for legislative reasons, some companies produce only a few of the specified outputs. In 1976, for example, Central and Eastern Trust Company was registered in Ontario for limited purposes. (See Appendix 2.1(a)).

6.2 INTERPRETATION OF THE RESULTS

In this section, we evaluate and interpret in economic terms some of the measures of the cost concepts we have considered in chapter 4. Where possible we shall use results from estimations using the two data sets. With data-2, since we did not reject the translog cost function (TMCF) as a better fit than the generalized translog, results will mostly be based on TMCF. Nevertheless, we shall sometimes refer to the GTMCF results for data-2, for purposes of comparison. Data-1 results are solely based on GTMCF estimates.

Tables 6.6 and 6.7 show own- and cross-price elasticities for labour and capital evaluated at the means of the outputs and input prices. The cost-minimizing own - price elasticities are negative in the case of the TMCF; this is consistent with the theoretical hypothesis on the cost function. In the case of the GTMCF (Data-1) the signs on the elasticities are quite the opposite of what are expected. However, in all cases, the elasticities are not statistically significant.

TABLE 6.6

OWN-AND CROSS-PRICE ELASTICITIES FOR LABOUR AND CAPITAL EVALUATED AT THE MEANS USING TMCF (DATA-2)

	Labour	Capital
Labour	-.007 (.157)	.007 (.157)
Capital	.008 (.179)	-.008 (.179)

NOTE: The first row refers to percentage changes in factor demands (dX_i/X_i), and first column refers to percentage changes in factor prices (dr_j/r_j). Standard errors are in parentheses.

TABLE 6.7

OWN-AND CROSS-PRICE ELASTICITIES FOR LABOUR AND CAPITAL EVALUATED AT THE MEANS USING GTMCF (DATA-1)

	Labour	Capital
Labour	.0005 (.2069)	-.0005 (.2069)
Capital	-.0004 (.1780)	.0004 (.1780)

NOTE: The first row refers to percentage changes in factor demands (dX_i/X_i), and first column refers to percentage changes in factor prices (dr_j/r_j). Standard errors are in parentheses.

The own-price elasticities are calculated as

$$\varepsilon_{ii} = \gamma_{ii}/S_i + S_i - 1$$

Most studies on economies of scale in financial institutions reported the existence of unexploited economies of scale for such institutions. Murray and White (1983) 'decisively' rejected constant returns to scale production structure for credit unions of British Columbia. In section 6.1, we also 'decisively' rejected constant returns to scale production structure for the trust industry, using data-2.

Multiproduct-economies-of-scale estimates are provided in Tables 6.8, 6.9 and Appendix 6.3.

We notice from Table 6.8 that the multiproduct scale measure at the mean for the translog is 1.009 and for the GTMCF is 1.011. These figures indicate an almost constant returns to scale production structure for the trust industry at the mean value of outputs.

Considering the ranges of the outputs (see Appendix B) and also the high concentration in the industry (70% of total assets of trust companies in 1981 is controlled by only 7 firms), calculation of scale and scope at the means do not give us the true picture. We therefore calculated economies of scale at each observation point. This is presented in Appendix 6.2. Most firms in the industry have unex-

TABLE 6.8

ESTIMATES OF MULTIPRODUCT ECONOMIES OF SCALE, PRODUCT-SPECIFIC ECONOMIES OF SCALE AND SOME GLOBAL MEASURES OF ECONOMIES OF SCOPE EVALUATED AT THE MEANS

	GTMCF (Data-1)	TMCF (Data-2)
1. Multiproduct-Economies-of-Scale, S	1.011	1.021
2. Product-specific-Economies-of-Scale (E.T.A.), S_1	3.761	n.a.
3. Product-specific-economies-of-scale (loans), S_2	.270	n.a.
4. Product-specific-economies-of-scale (securities), S_3	-4.865	n.a.
5. Product-specific-economies-of-scale (deposits), S_4	-3.639	n.a.
6. Weak-economies-of-scope, S_{ca}	1.236	n.a.
7. Incremental-economies-of-scope, S_{cb}	-.570	n.a.

NOTE: n.a. = not available. Because the measures involve zero outputs, estimate with the translog are not obtainable.

TABLE 6.9

ESTIMATES OF MULTIPRODUCT-ECONOMIES-OF-SCALE ELASTICITIES FOR SMALL AND LARGE COMPANIES IN 1981 USING GTMCF (DATA-1), GTMCF (DATA-2), TMCF (DATA-2)

7 Largest Companies	GTMCF (DATA-1)	GTMCF (DATA-2)	TMCF (DATA-2)
1. Royal Trust (Canada)	.809	.837	.852
2. Canada Trust	.825	.902	.907
3. Royal Trust	.831	.931	.938
4. National Trust	.889	.883	.894
5. Canada Permanent	.781	.868	.878
6. Quebec Trust	.939	1.030	1.031
7. Victoria and Grey	.745	.860	.870
7 Smallest Companies			
1. Cabot Trust	1.673	1.323	1.325
2. Regional Trust	1.648	1.471	1.462
3. Family Trust	1.548	1.680	1.718
4. Effort Trust	1.807	1.680	1.685
5. Counsel Trust	1.962	n.a.	n.a.
6. Dominion Trust	2.066	n.a.	n.a.
7. Merchant Trust	1.717	n.a.	n.a.

NOTE: Counsel, Dominion and Merchant Trusts were not included in Data-2 sample because they have some zero outputs in 1981.

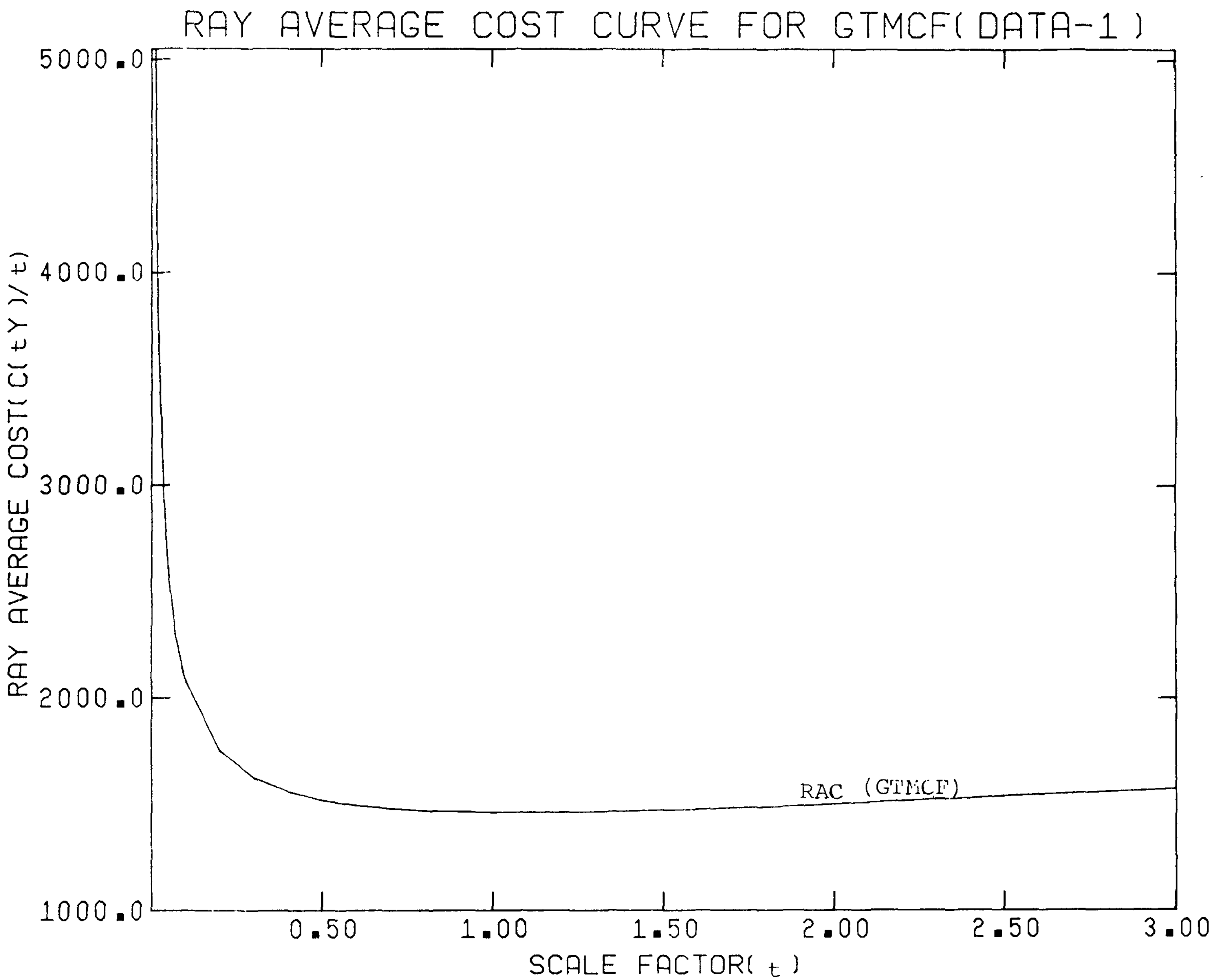
ploited economies of scale throughout the six years under study. The scale factor remained almost the same throughout the six years, for most firms.

Out of a total of 37 firms which were in operation between 1976 and 1981, only 8 firms consistently showed decreasing returns to scale ($S < 1$). All these 8 firms were very large firms. The picture is much clearer when we look at Table 6.9. For data-1, all the 7 largest companies show economies of scale elasticity of less than unity (diseconomies of scale). All the 7 smallest firms in Table 6.9 have scale elasticities of greater than unity (economies of scale). The data-2 results with either the TMCF or the GTMCF showed estimates very similar to that obtained with the data-1 (GTMCF). Note, however, that with data-2 the TMCF was a better fit than the GTMCF. These results are also evident in Figures 6.1 and 6.2, showing the ray average cost curves for GTMCF (data-1), TMCF (data-2), and GTMCF (data-2). The ray average cost is calculated as

$$RAC = \frac{C(tY^{o})}{t}$$

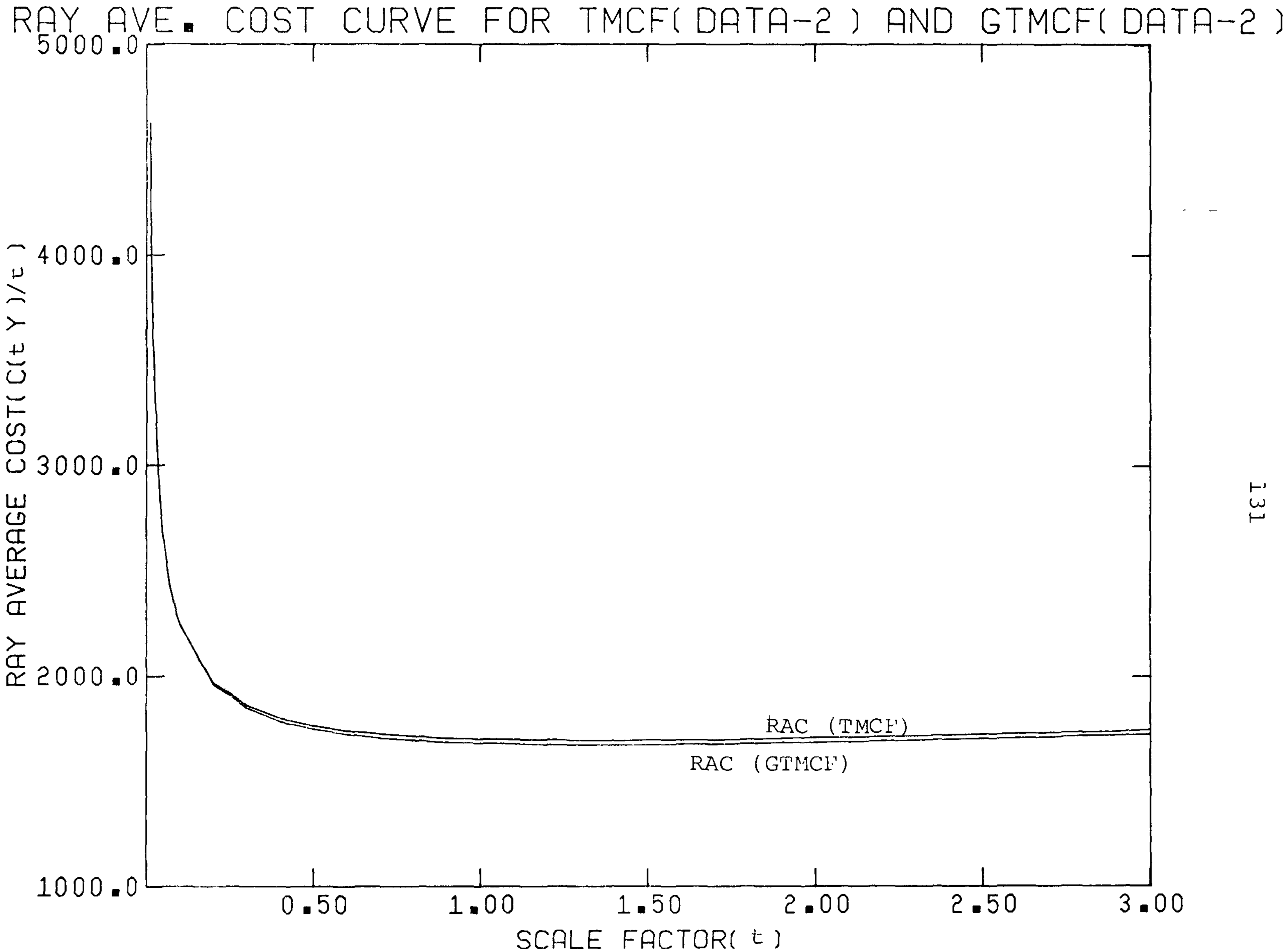
where Y^{o} is composite output comprising the mean value of the outputs (E.T.A., loans, securities and deposits), and t is the scale factor.

FIGURE 6.1



130

FIGURE 6.2


RAY AVE. COST CURVE FOR TMCF(DATA-2) AND GTMCF(DATA-2)
RAY AVERAGE COST(C(t Y)/t)
5000.0
4000.0
3000.0
2000.0
1000.0
0.50
1.00
1.50
2.00
2.50
3.00
SCALE FACTOR(t)
RAC (TMCF)
RAC (GTMCF)

To plot the ray average cost curves, different values of t ($.01 \leq t \leq 3$) were matched against RAC. Using the GTMCF in data-1 and TMCF in data-2, we realize that not only do the shapes of the RAC look similar in both cases but also the minimum efficient scale is just about the average output level. The minimum efficient scale factor was 1.1 in the case of data 1 and 1.3 in the case of data-2. In the case of data-2, the RAC curves for both the GTMCF and the TMCF are almost close together.

Table 6.10 arranges scale elasticities by asset size of firms. The first 10 firms all seem to have exhausted all economies of scale. It seems the cut-off size is about that of Morguard Trust Company. It seems the distribution of the assets affect the size of the scale elasticity. The last 4 firms in Table 6.10 all have small guaranteed and company assets compared to E.T.A.; and all these firms still have unexploited economies of scale. On the other hand, a company like the Central Trust Company (number 11 in Appendix 6.2) which has a large financial intermediary asset but a small fiduciary asset, has exhausted all multiproduct economies of scale. Almost all the small companies have multiproduct economies of scale elasticities greater than unity.

TABLE 6.10

ASSET SIZE AND SCALE ELASTICITIES FOR 15 LARGE COMPANIES IN 1981 (DATA-1)

		ASSETS IN MILLION DOLLARS				Scale
		Guaranteed	Company	E.T.A.	Total	Elasticity
1.	Royal Trust (Canada)	5157	261	12982	18400	.809
2.	Canada Trust	3722	265	9497	13484	.825
3.	Royal Trust	1853	116	9950	11889	.831
4.	National	2471	106	8543	11120	.889
5.	Canada Permanent	3348	236	3974	7558	.781
6.	Quebec	610	34	4669	5313	.939
7.	Victoria and Grey	3529	178	1195	4902	.745
8.	Guaranty	2411	120	2037	4568	.827
9.	First City	1579	100	1165	2844	.895
10.	Co-Operative	566	39	2081	2686	.947
11.	Morguard	152	16	2053	2221	1.003
12.	International	159	16	1535	1710	1.178
13.	Crown	632	31	1038	1701	.944
14.	Investors	106	9	1531	1646	1.235
15.	Savings and Investment	215	9	1078	1302	1.178

Murray and White (1983) found all credit unions in their sample to have unexploited economies of scale. On the basis of either the translog estimation of data-2 or the GTMCF estimation of data-1, this study concludes that, very large trust companies have exhausted all multiproduct economies of scale while medium and small size firms continue to enjoy multiproduct economies of scale.

The above analysis suggests that whereas there are economies of scale in the production of some outputs, with others there are diseconomies. Estimates of product-specific economies of scale evaluated at the means are provided in Table 6.8. Some of the elasticities are negative in sign -- indicating that it costs less to produce the whole output set than to leave out some particular outputs. For example, deposits have negative sign on their product-specific scale elasticity. This implies that on the average it is generally cheaper for firms to service deposits than not to service deposits. Again, the negative sign could also be due to the fact that deposits are inputs as well. There is unexploited product-specific economies in the production of E.T.A. while firms in the industry have exhausted all product-specific economies with respect to loans.

The cost concept which may help us to understand the multiproduct nature of trust companies is the economies of scope measure. As explained in chapter 4, there are two measures of economies of scope -- local and global. Estimates of global economies of scope at the means are provided in Table 6.8 for data-1. Weak economies of scope are calculated based on the expression in (4.3-6) while the incremental economies of scope are calculated based on the expression in (4.3-7). The weak scope measure of 1.236 at the means indicate that trust companies benefit in their costs by producing all the specified output sets -- loans, E.T.A., securities and deposits -- jointly, rather than by producing them separately. That is, the weak economies of scope measure gives support to the fact that trust companies are multiproduct firms. The incremental economies of scope measure on the other hand has an estimated value of -.570 at the means. Thus, trust companies have diseconomies of scope in producing E.T.A. and financial intermediary services. That is, it costs more for a trust company to produce E.T.A. and financial intermediary services jointly than separately. Thus, on the basis of the incremental economies of scope, trust companies will be better off in their costs if they function only as trustees or simply as banks.

Individual weak economies of scope estimates are provided in Appendix 6.3. It is apparent that mergers increase the scope of a firm. Firms which merged with others had their scope elasticities increased substantially. For example, Victoria and Grey Trust Company (firm number 8 in Appendix 63.) had its scope elasticity increased from 3.606 to 6.125 after merging with Lambton Trust Company in 1978. Also Canada Trust (firm number 10 in Appendix 6.3) increased its scope economies in 1977 after merging with Lincoln Trust and Savings Company in December 1976. Similar results were observed for Central Trust (firm number 11 in Appendix 6.3) after merging in July, 1976; Fidelity Trust (firm number 17 in Appendix 6.3) after merging in 1980; and Royal Trust Corporation of Canada (firm number 34 in Appendix 6.3) after merging in 1977 with Royal Trust Company (Ontario) and in 1981 with Industrial Mortgage and Trust Company.

Table 6.11 shows estimates of weak economies of scope measures for some large and small firms. We notice that all the seven large firms have substantial economies of scope in the production of loans, E.T.A., securities and deposits. On the other hand most small firms exhibit diseconomies of scope. In other words it will be cheaper for a small company to specialize in the production of one of the outputs than attempting to produce all the outputs jointly.

TABLE 6.11

ESTIMATES OF WEAK ECONOMIES OF SCOPE MEASURES FOR SOME SMALL AND LARGE TRUST COMPANIES, 1967-1981
(USING GTMCF (DATA-1))

7 LARGE COMPANIES	YEARS 1976	1977	1978	1979	1980	1981
1. Royal Trust Canada	4.438	7.328	11.144	19.221	76.553	156.538
2. Canada Trust	7.341	17.318	15.016	23.512	23.975	46.719
3. Royal Trust	14.880	4.299	4.306	4.829	3.002	3.465
4. National Trust	11.931	.147	10.601	9.703	16.842	39.391
5. Canada Permanent	3.476	9.674	12.318	12.882	13.901	11.519
6. Quebec Trust	.736	.499	.295	.666	.547	.623
7. Victoria and Grey	2.871	3.606	6.125	7.086	9.108	11.961
7 SMALL COMPANIES						
1. Cabot Trust	n.a.	n.a.	n.a.	.754	.299	.647
2. Regional Trust	.272	.460	.306	.472	-.117	.970
3. Family Trust	.049	-.695	-.435	-.418	-.453	-.414
4. Effort Trust	n.a.	n.a.	.080	-.502	-.134	.074
5. Counsel Trust	n.a.	n.a.	n.a.	.330	.425	.423
6. Dominion Trust	1.458	1.395	1.446	1.633	1.754	1.929
7. Merchant Trust	n.a.	n.a.	-.062	-.186	.190	.212

NOTES: 1. Companies were ranked by their 1981 asset size.

2. n.a. implies the company was not registered in Ontario in that year.

Another interesting result on weak economies of scope is in connection with new entrants into the trust industry. Established firms generally showed economies or diseconomies of scope consistently over the six years. New entrants, on the other hand, generally have economies of scope in their year of entry before adjusting to either consistent economies or diseconomies of scope (see Table 6.12).

Local estimates of economies of scope are shown in Table 6.13. Calculations were based on expressions in (4.3-11), (4.3-12) and (4.3-14), evaluated at the means. Using Data-1, we find local economies of scope between E.T.A. (Y_1) and deposits (Y_4); loans (Y_2) and deposits (Y_4); and securities (Y_3) and deposits (Y_4). Weak evidence of cost complementarities were found between E.T.A. and loans; E.T.A. and securities; loans and securities; and between E.T.A. on the one hand, and loans, securities and deposits on the other.

Using Data-2 we find economies of scope between E.T.A. and securities; loans and securities; and loans and deposits. Weak evidence of cost complementarities were found between E.T.A. and loans, E.T.A. and deposits, securities and deposits and between E.T.A. on the one hand, and loans, deposits and securities on the other.

TABLE 6.12

ESTIMATES OF WEAK-ECONOMIES-OF-SCOPE ELASTICITIES FOR SOME NEW ENTRANTS

Trust Company	1976	1977	1978	1979	1980	1981
Community Trust	.369	.184	.030	-.023	.122	.029
Exchequer Trust		.215	-.018	-.011	-.126	1.291
Huronia Trust		.519	.134	-.027	.007	-.025
Security Trust		.115	1.293	.537	.478	.132
Effort Trust			.080	-.502	-.134	.074
McDonald-Cartier			.069	-.234	-.271	-.137
Seaway Trust			.167	.159	.084	.133
Merchant Trust			-.062	-.186	.190	.212
Cabot Trust				.754	.299	.647
Western Capital				.135	.382	-.065
North Canadian						.032

TABLE 6.13

LOCAL MEASURES OF ECONOMIES OF SCOPE EVALUATED AT THE MEANS

	GTMCF (DATA-1)	TMCF (DATA-2)
L-Scope $(Y_1:Y_2)$	.232	.142
L-Scope $(Y_1:Y_3)$	.020	-.051
L-Scope $(Y_1:Y_4)$	-.068	.025
L-Scope $(Y_2:Y_3)$	.077	-.470
L-Scope $(Y_2:Y_4)$	-.247	-.866
L-Scope $(Y_3:Y_4)$	-.026	.854
L-Scope $(Y_1:Y_2,Y_3,Y_4)$	.157	.130

NOTE: Y_1 is E.T.A.

Y_2 is loans

Y_3 is securities, and

Y_4 is deposits.

In summary, the economies of scope estimates indicate that most firms in the trust industry enjoy economies of scope in the production of E.T.A., loans, securities and deposits. After merging, firms increase their scope measures. Locally, there is economies of scope in the production of E.T.A. and deposits, loans and deposits, and securities and deposits.

To conclude, we have shown that if the full data set on trust companies is considered, then the GTMCF seems the appropriate functional form to use. On the other hand, with a truncated data set which contains no zero outputs, the translog function seems the most appropriate one. Scale parameters derived from GTMCF (data-1) and TMCF (data-2), however, indicate that similar conclusions can be reached with either data set. It is observed that at the means, the trust industry tends to exhibit an almost constant returns to scale, with the minimum efficient scale being 1.1 and 1.3 times the mean output values for the Data-1 and Data-2, respectively. To calculate economies of scope and product-specific economies, however, we have to rely solely on the GTMCF. A lot of interesting results emerged concerning economies of scope. Most firms enjoy economies of scope -- giving credence to the fact that the trust industry is indeed a multiproduct industry. Most large firms, while enjoying strong economies of scope, have exhausted all economies of scale. There is a strong correlation between mergers among firms and economies of scope. Most of the Data-2 results agree in a general way with results obtained by Murray and White (1983) for the credit union industry of British Columbia.

FOOTNOTES

Chapter 6

1. See, for example, Christensen and Greene (1976), Caves, Christensen and Tretheway (1980) and Murray and White (1983).

2. In TSP, all of the statistical techniques involve minimization of a criterion function, Q, over the parameters. For the Zellner's seemingly unrelated regression, the criterion, Q, is the negative of the log-likelihood function. Minimization of Q gives maximum likelihood estimates.

   In multivariate regression, if λ is the log-likelihood function, then

   $$-\lambda = \alpha + \log |\Sigma|$$

   where α is a constant and

   $|\Sigma|$ is the determinant of the covariance matrix of the regression disturbances.

   (See, for example, the TSP User's Manual, University of Western Ontario, London, Canada).

APPENDIX 6.1

TRUST COMPANIES REGISTERED AND CONDUCTING BUSINESS IN ONTARIO THROUGHOUT ALL THE SIX YEARS (1976-1981)*

A. Ontario Incorporated Companies

1. Community Trust Company Limited.
2. Crown Trust Company.
3. District Trust Company.
4. The Dominion Trust Company.
5. Family Trust Company.
6. National Trust Company.
7. Vanguard Trust of Canada Limited.
8. Victoria and Grey Trust Company.

B. Federal and Other Provinces Incorporated Companies

9. Canada Permanent Trust Company (Federal).
10. The Canada Trust Company (Federal).
11. Central Trust Company (Federal).
12. Continental Trust Company (Federal).
13. Co-Operative Trust Company of Canada (Federal).
14. Eaton Bay Trust Company (Alberta).
15. Eaton/Bay Trust Company (Federal).

16. The Equitable Trust Company (Federal).

17. The Fidelity Trust Company (Federal).

18. First City Trust Company (Alberta).

19. General Trust of Canada (Quebec).

20. Guaranty Trust Company of Canada (Federal).

21. Guardian Trust Company (Quebec).

22. Income Trust Company (Federal).

23. Citicorp Trust Company (Federal).

24. Investors Group Trust Company Limited (Manitoba).

25. Montreal Trust Company (Quebec).

26. The Morgan Trust Company (Federal).

27. Morguard Trust Company (Federal).

28. North America Trust (Quebec).

29. The International Trust Company (Federal).

30. The Premier Trust Company (Federal).

31. Quebec Trust (Quebec).

32. The Regional Trust Company (Federal).

33. The Royal Trust Company (Quebec).

34. Royal Trust Corporation of Canada (Federal).

35. Savings and Investment Trust (Quebec).

36. Standard Trust Company (Federal).

37. Sterling Trust Corporation (Federal).

* Bankers Trust and Credit Foncier operated through all the six years but were excluded because they have extraordinarily low labour shares.

APPENDIX 6.2

ESTIMATES OF MULTIPRODUCT-ECONOMIES-OF-SCALE[b] ELASTICITIES FOR SELECTED TRUST COMPANIES USING GTMCF (DATA-1)

Firm[a] Number	YEARS 1976	1977	1978	1979	1980	1981
1	2.254	1.965	1.774	1.690	1.559	1.535
2	1.010	1.008	.995	.998	.978	.944
3	1.289	1.247	1.200	1.155	1.126	1.093
4	2.054	2.004	2.021	2.036	2.054	2.066
5	3.047	1.144	1.641	1.602	1.541	1.548
6	.951	.741	.896	.873	.875	.889
7	2.014	1.624	1.561	1.298	1.273	1.273
8	.806	.786	.713	.719	.727	.745
9	.814	.823	.805	.779	.761	.781
10	.821	.827	.808	.796	.780	.825
11	.876	.872	.874	.869	.863	.829
12	1.630	1.697	1.548	1.428	1.322	1.261
13	1.090	1.049	.998	.977	.985	.947
14	1.133	1.104	1.054	1.055	1.008	.988
15	1.405	1.333	1.264	1.212	1.125	1.105
16	1.537	1.449	1.411	1.306	1.433	1.543
17	1.229	1.133	1.051	1.001	.951	.850
18	1.083	1.046	1.000	.965	.927	.895
19	1.085	1.077	.991	1.062	1.042	.933
20	.913	.915	.889	.837	.818	.828
21	1.625	1.642	1.616	1.509	1.403	1.368
22	1.412	1.297	1.254	1.256	1.229	1.129
23	1.285	1.197	1.163	1.099	1.053	1.046
24	1.529	1.493	1.443	1.410	1.316	1.235
25	.932	.928	.913	.936	.958	.983
26	1.723	1.583	1.424	1.305	1.237	1.229
27	1.275	1.188	1.128	1.081	1.043	1.003
28	1.637	1.527	1.443	1.407	1.317	1.276
29	1.587	1.414	.999	1.297	1.183	1.178
30	1.367	1.358	1.358	1.359	1.309	1.281
31	1.089	1.050	1.023	.998	.961	.939
32	1.549	1.157	1.648	1.577	1.629	1.648
33	.809	.769	.786	.795	.810	.831
34	1.322	.934	.844	.806	.806	.809
35	1.337	1.310	1.284	1.259	1.183	1.178
36	1.290	1.259	1.187	1.145	1.102	1.052
37	1.150	1.128	1.081	1.073	1.043	1.031

(a) Firm number corresponds to listings in Appendix 6.1

(b) Calculations based on equations (4.3-1) and (4.3-4).

APPENDIX 6.3

ESTIMATES OF WEAK ECONOMIES OF SCOPE[b] FOR SELECTED TRUST COMPANIES USING GTMCF (DATA-1)

Firm Number[a]	YEARS 1976	1977	1978	1979	1980	1981
1	.369	.839	.030	-.023	.122	.029
2	-.124	.121	.228	-.095	.886	.842
3	.276	.335	.443	.578	.706	.308
4	1.458	1.395	1.446	1.633	1.754	1.929
5	.049	-.695	-.435	-.418	-.453	-.414
6	11.931	.147	10.601	9.703	16.842	39.391
7	-.127	-.245	.150	-.301	-.308	-.273
8	2.871	3.606	6.125+	7.086	9.108+	11.961
9	3.476	9.674+	12.318	12.882	13.901	11.519
10	7.341	17.318+	15.016	23.512	23.975	46.719
11	1.921	2.327+	3.121	4.686	6.025	5.662
12	-.314	-.388	-.254	-.206	-.168	.169
13	1.052	1.291	1.064	.966	1.104	.407
14	1.394	1.940	1.432	2.401	1.188	1.375
15	-.307	-.226	-.232	.005	-.160	.971
16	-.261	-.330	-.331	-.452	-.187	-.123
17	.065	-.010	-.007	-.035	.331	.418
18	1.621	1.654	2.354	2.927	4.343	3.803
19	2.092	3.113	8.310	5.607	6.554	12.341
20	4.807	12.328	9.530	6.184	6.992	10.802
21	-.371	-.207	-.108	-.042	0.142	1.207
22	-.211	-.340	-.284	-.025	-.208	-.201
23	-.260	-.253	.442	.626	1.135	.928
24	-.395	-.378	-.354	-.384	-.082	.590
25	1.729	2.449	2.457	1.575	6.228	4.324
26	.061	-.145	-.082	-.077	-.213	-.208
27	-.407	-.435	-.421	-.504	-.522	-.513
28	.076	.726	-.076	.017	-.158	.353
29	.298	-.129	-.105	-.089	-.019	.125
30	.459	.506	.598	.636	.465	.602
31	.736	.499	.295	.666	.547	.623
32	.329	-.512	-.212	-.270	-.173	-.066
33	4.880	4.299	4.306	4.829	3.002	3.465
34	4.438	7.328+	11.221	76.553	76.553	156.538+
35	.272	.460	.306	.472	-.117	.970
36	.224	.355	.582	1.279	1.506	2.444
37	.369	.595	.532	.932	.886	1.382

(a) Calculation based on expression in (4.3-6).

(b) Firm number corresponds to listing in Appendix 6.1.

\+ Post-merger scope measure.

CHAPTER 7

CONCLUSION

Trust companies in Canada, like most other financial institutions, are multiproduct firms. They have two main functions: (i) a fiduciary function, and (ii) a financial intermediation function. Under these two functions the outputs produced by a trust company are the services of the management of estates, trusts and agencies (E.T.A.) and of the management of loans and of securities. The services of deposits are both inputs and outputs. The services of deposits are produced by labour and capital, and are best viewed as an intermediate output. When labour and capital combine with the services of deposits to produce the services of loans and securities, then deposits join with labour and capital as inputs.

Using the set of inputs and outputs we were able to formulate a production technology for Canadian trust companies. Using duality techniques, we derived a multiproduct cost function from the basic production transformation function.

Even though we consider the average trust company as producing E.T.A., loans, deposits and securities, not all firms produce all these outputs at a particular time; sometimes for lack of manpower a trust company may produce only a subset of the specified output set. A trust company may also abstain from the production of a particular output based on market forces: for instance, if the company finds it less 'profitable' to invest in securities then it will not do so. A trust company may also be restricted by the law in its operations. For example, in 1976, Quebec Trust and Central Trust were registered in Ontario for 'limited purposes'. Because of these reasons we specified a cost function which could accommodate zero output values. This function is the generalized translog multiproduct cost function (GTMCF).

Aside from the fact that the GTMCF can accommodate zero output values it has the following advantage: one can meaningfully define economies of scope and product-specific economies of scale. It is more general than the translog but like the translog it is parsimonious in parameters.

Data on trust companies was obtained from Reports of the Registrar of Loan and Trust Corporations for Ontario. Based on their number, asset size and the fact that all the large trust companies are registered in Ontario, the Ontario data is considered as representative of the Canadian trust industry data. After the initial editing of the data, two data sets were defined. Data-1 comprises the whole population except for a small number of firms dropped in consequence of the initial editing rule. Data-2 is a truncated form of Data-1, consisting only of observations with non-zero output points.

A generalized translog multiproduct cost function was fitted to Data-1 and both the generalized and an ordinary translog was fitted to Data-2. A hypothesis test revealed that we could not reject the translog as the best fit of the truncated data against the alternate hypothesis of the generalized translog. This suggests that Murray and White (1983) may be right in fitting a translog function to their data, although they did not provide tests against the more general function, the generalized translog.

A translog cost function restricted to be linearly homogeneous in output was also fitted to Data-2 but this was also rejected in favour of the translog cost function. Again, this result agrees with what Murray and White (1983) found for the credit unions in British Columbia.

Having satisfied ourselves that the translog cost function (TMCF) best represents Data-2 and that the generalized translog (GTMCF) best represents Data-1, we then evaluated measures of multiproduct economies of scale, product-specific economies of scale, and economies of scope. Unless otherwise stated, the summaries of the findings below apply to both Data-1 and Data-2.

a) The minimum efficient scale for the trust industry is about the average size of all outputs.

b) All small firms (firms of less than average size) have unexploited multiproduct economies of scale, while most firms of above average size have exhausted all multiproduct economies. This agrees with the Murray and White (1983) finding of the existence of an inverse relation between returns to scale and asset size. We should add that since the average size of a trust company is larger than the average size of a credit union, our model yielded a minimum efficient scale while that of Murray and White (1983) did not.

c) It appears that firms with a large proportion of trustee assets have increasing returns to scale while those with a larger proportion of financial intermediary assets have decreasing returns to scale.

d) For most companies, multiproduct economies of scale elasticities remained the same over the six years.

e) Using Data-1, we found that most firms have unexploited product-specific economies in the production of E.T.A., securities and deposits, and product-specific diseconomies in the production of loans.

f) Most large firms appear to enjoy economies of scope in the production of loans, securities, deposits and E.T.A. while most small firms do not have economies of scope in the production of these outputs.

g) Most firms entering the trust industry enjoy economies of scope in their years of entry but as they stay on most of them (especially the small ones) start having weak scope economies.

h) After merging, most firms increase their scope economies.

i) Local results of economies of scope differ between Data-1 and Data-2. With Data-1, scope economies were detected between deposits and E.T.A., between deposits and loans, and between deposits and securities. This is not a strange result since deposits were also considered a necessary input in producing loans and securities and E.T.A. Data-1, however, showed weak evidence of cost complementarity between E.T.A. and loans, E.T.A. and securities, and loans and securities. With Data-2, scope economies were detected between E.T.A. and securities, between loans and securities, and between loans and deposits.

These results have important policy implications. Even though the larger trust companies are more cost efficient than small ones, care must be taken in granting permission for expansion. Firms which expand past the 'average firm size' start experiencing diseconomies of scale. It should be noted, however, that increases in say, average cost after the minimum efficient scale is very gradual; thus, such diseconomies arising from further expansion should be matched against other cost advantages of large firms. For instance, although large firms might have some diseconomies arising from expansion, these diseconomies may be offset because of a reduction in risk due to exposure to a wider geographical area. Also, if the aim of regulators is for trust companies to produce the diversified outputs specified in our model, then our results show that large firms are preferable. It should, however, be noted that most of our conclusions have not been subject to rigorous statistical tests. Care should be taken in using the product-specific economies of scale results since they do not seem precise enough -- probably an indication that the GTMCF does not behave well around zero values.

TRUST COMPANIES OPERATING IN ONTARIO

APPENDIX A.

DATA-1 (POOLED DATA FOR 1976 TO 1981)

FIRM NO.	EXPENSES	E.T.A	LOANS	SECURITY	EQUITY	SALARY	INTEREST	DEPOSIT
1	4.0781	0	38	79	120	2.0819	.0102	7.7
2	3175	72599	25566	2123	1101	667	2175	27771
3	959	1250	8805	1167	1337	122	739	9295
4	58	0	124	465	92	12	38	517
5	6.7921	3019	167	15	113	4.2903	1.1738	0
6	1359	3695	12097	1491	865	184	1016	13264
7	1532	11441	16177	1905	923	148	1265	17386
10	3326	77371	29146	6584	2299	531	2481	32963
11	12895	384053	84220	41830	6295	2499	8973	122262
12	4803	5229	19621	894	1281	1891	1957	13672
13	42	740	271	101	104	24	8.8495	260
14	11497	18793	128821	9928	5407	636	10363	128078
16	20056	227756	146242	28948	9058	4779	12871	16461
17	21568	318886	181216	38860	11851	3303	16524	205750
18	9102	26157	87815	8990	4085	800	7743	89298
19	3316	9423	28293	7045	1549	350	2665	32779
20	2828	15638	21916	6364	1332	235	2412	28038
21	89.6894	2655	0	132	123	50	0	0
22	3338	39459	25861	6712	1547	479	2579	30414
23	6.9273	173	0	418	408	2.5000	0	0
24	331	3126	3307	129	196	55	213	3164
25	186	2647	2065	178	139	18	157	1973
26	1138	32563	9258	2289	589	177	809	10327
27	726	5622	6483	735	343	94	552	6572
28	4512	155410	26165	12346	1639	919	3165	37708
29	11228	82244	94184	22965	5128	1482	8836	113419
30	232	6862	931	112	125	83	91	1042
31	336	7901	3517	542	241	21	283	3596
32	673	80275	3983	1433	557	107	435	4959
33	96	67379	210	282	221	60	0	302
34	10763	578334	61649	18178	4075	2944	6337	78221
35	114	3302	594	583	594	57	21	783
36	762	86195	3432	895	373	244	304	3813
37	194	2170	1417	617	152	18	160	1862
38	45	129	0	403	362	6.0721	30	13
39	637	733	5976	1440	858	68	541	6602
40	3415	114129	22586	7010	1544	675	2459	28216
41	47	0	1003	0	129	8.6070	31	807
42	36877.	1207995	231041	67222	14273	7583	25859	305594
43	441	142	4994	6590	494	121	262	11615
44	744	28924	4657	2498	397	157	513	5854
45	916	1334	8706	1030	406	69	782	9220
46	1734	5892	17178	2168	729	167	1434	18050
47	31	35	511	190	143	7.2056	17	569
48	3788	83830	30924	3633	1746	675	2705	35364

...continued

49	1251	1803	10954	1442	1341	130	1001	11922
50	56	0	143	482	100	12	36	515
51	12	0	0	125	122	4.2795	0	0
52	54	4270	415	0	122	19	15	296
53	1659	7241	13262	2206	728	231	1236	15058
55	11	0	79	70	107	1.8899	.2648	68
56	14	495	581	95	679	8.8393	0	0
58	3868	90045	30978	9799	3582	569	2906	39301
59	15797	436923	104992	4260	6699	3226	10659	143043
60	11	0	80	68	148	3.5474	0	0
61	106	1778	1481	136	171	28	62	1416
62	14317	23266	153018	12869	6520	806	12844	152003
63	59	426	447	326	130	14	24	836
65	24394	301890	190898	44858	11036	5416	15682	222361
66	24678	345978	203876	60939	15329	3808	18831	239316
67	10102	27001	95252	10647	4552	937	8532	101546
68	3950	21281	35496	8368	2356	389	3222	40863
69	3337	15865	26173	8266	160	306	2787	33253
70	84	14003	361	125	145	34	9.6689	314
71	4429	68085	33102	8829	1989	698	3254	38511
73	527	4097	5217	307	310	70	394	5163
74	249	3415	2618	122	152	27	203	2384
75	1532	37583	15245	2247	1191	262	1033	15872
76	937	11429	9012	664	458	129	677	9027
77	5206	180476	28536	15747	1757	1102	3575	42662
78	12441	92090	106137	39204	5899	1611	9711	139930
79	284	2930	1302	239	100	93	144	1445
80	501	8885	5074	279	273	31	420	4849
81	782	1856	7570	623	298	48	665	7665
82	647	91759	7471	1806	735	107	435	8638
83	138	79809	243	384	253	65	26	408
84	11998	620340	68001	21888	4389	3341	6999	87521
85	253	3592	1624	421	572	71	119	1601
86	1147	113994	5705	1082	500	351	494	6035
87	21	84	0	362	311	7.30	0	0
88	295	2358	2282	1792	206	28	239	3873
89	687	697	6146	1554	937	75	584	6770
90	4095	147783	26737	6728	1603	878	2798	31788
91	126	140	1204	0	143	20	92	1031
92	36464.	1288400	214403	38916	14211	6625	25641	257735
93	11636	92791	89741	28930	4838	5116	5017	117215
94	957	30924	5728	3172	417	209	655	7570
95	1093	1103	10288	1301	473	86	928	10933
96	2183	7610	20228	3206	966	223	1776	21812
97	95	174	1131	196	155	8.6760	77	1161
98	4247	93008	34578	4460	1845	744	3065	36382
99	1523	2282	13586	1927	1446	149	1225	13996

...continued

APPENDIX A. (continued)

100	55	0	138	485	111	13	34	535
101	3.2131	7707	65	62	121	1.1251	0	0
102	77	98	260	122	261	31	5.6232	103
103	657	5437	674	30	136	454	40	524
104	1754	11591	14562	2428	931	215	1360	16681
106	58	53	693	153	100	9.8696	35	861
108	43	3	546	32	348	18	6.8851	336
109	4743	91255	42315	9040	4107	636	3583	50278
110	22	46	533	79	116	3.6152	11	483
112	17588	486497	122520	44738	7299	3481	11975	163091
113	4.8922	0	95	15	110	2.7297	0	0
114	44	0	259	529	146	7.750	19	692
115	241	16	2918	140	284	22	194	2708
116	18612	890	199422	12935	7471	1062	16597	196378
117	176	980	1227	868	167	25	113	1852
119	11	0	35	62	105	6.0556	.3244	.38250
120	30782	319763	223775	51807	12272	6515	20891	261724
121	28665	509781	230224	60626	11301	4046	22269	276799
122	11188	30845	102789	14400	4700	998	9496	113022
123	1053	2042	16522	1699	879	84	874	17589
124	4106	21167	34154	7368	1498	398	3344	40156
125	178	14091	1295	461	210	40	84	1611
126	5611	102028	42759	8982	2396	839	4114	47462
128	719	20675	6750	889	375	53	554	7020
129	306	6118	3079	217	163	32	253	2857
130	2297	42531	22916	2187	1271	306	1725	23669
131	5258	29934	49615	11483	4050	398	4427	59078
132	1398	20360	12994	1854	680	195	1005	14226
133	5956	201	33142	18525	1890	1212	4162	49708
134	14761	109235	123115	36167	7641	1831	11631	153761
135	365	3013	1540	397	150	120	134	1832
136	649	11149	6789	467	304	44	544	6452
137	941	2596	9489	558	371	49	816	9352
138	189	108505	0	2441	444	60	46	2031
139	173	99548	371	569	301	78	45	632
140	32	5168	0	100	104	15	6.1514	111
141	13416	741147	73514	23033	4817	3790	7779	93941
143	400	5111	3680	728	343	76	238	3577
144	1581	133451	8692	1340	508	409	745	9400
145	407	4754	3604	651	165	35	337	4198
146	718	795	6081	1775	1014	82	601	6849
147	4501	242077	27534	6727	1654	1044	2975	32273
148	145	346	1428	27	156	20	110	1209
149	31369.	1286472	193613	38795	6141	5469	22675	228705
150	22111	335246	174532	47260	9502	6146	13479	215565

...continued

APPENDIX A. (continued)

151	1125	53533	6296	3118	434	240	780	9665
152	1471	1454	15024	2078	635	93	1269	16179
153	2641	8775	25086	2933	1249	272	2155	26295
154	39	0	310	287	209	11	12	409
155	156	263	1589	172	176	10	134	1549
156	45	0	316	5	248	26	13	117
157	4868	146408	28040	2891	1876	793	3604	39656
158	1895	2831	17477	2359	1428	155	1579	17910
159	69.6912	0	118	531	123	14	46	566
160	123	673	250	0	159	86	11	168
161	889	6492	911	59	139	647	60	712
162	2100	13983	15877	2403	879	230	1675	17881
163	184	1482	1792	428	195	24	131	1952
164	139	128	1470	88	116	14	110	1458
166	208	603	2284	32	339	49	119	1971
167	99	40	1155	68	130	6.2696	86	1057
169	21591	607086	141268	45100	7648	3831	15448	181837
170	46	5	417	67	109	18	17	377
171	97	0	944	9	113	15	61	944
172	422	3272	4861	103	321	33	354	4490
173	28304	100857	276159	24014	12760	1835	24968	282024
175	11	0	83	31	107	5.6887	.9129	9.3250
176	37411	350566	264937	53839	18101	7047	26635	294885
177	36818	625878	275392	76243	11318	4867	29384	329059
178	13690	32214	115401	19958	4796	1104	11784	129268
179	2238	2021	22963	1974	821	138	1965	24162
180	5143	23693	36498	10379	1994	445	4312	45361
181	366	15701	2869	694	223	46	247	3214
182	6766	147322	47619	9066	2327	931	5135	54796
184	1060	22838	10566	1730	632	52	849	11262
185	395	9557	3431	85	169	50	307	3082
186	3500	42207	27415	1606	1087	334	2851	29174
187	8137	34550	64831	13444	4333	499	7031	79421
188	2097	24318	16406	1498	555	222	1635	16752
189	7022	228397	35352	22091	2466	1276	5069	56862
190	19058	139285	157676	29555	8261	2305	15284	179598
191	556	2992	2757	557	236	176	251	3034
192	952	13081	8584	1278	357	54	794	8828
193	1185	3162	11461	832	398	54	1045	11483
194	470	122049	3124	2126	494	88	275	4818
195	217	120542	486	549	356	93	63	709
196	57	5946	0	122	105	28	11	43
197	13590	817379	54883	18158	4843	4192	7157	71687
198	2813	12890	28769	13177	2209	325	2483	37347
199	868	8633	6687	913	380	122	634	6721

...continued

APPENDIX A. (continued)

201	1889	149586	9627	945	554	485	971	9868
202	555	2701	4739	684	252	53	454	5156
203	765	827	6137	1814	1075	86	639	6892
204	5432	280711	36430	9063	1693	1225	3612	43751
205	171	645	1564	19	169	25	130	1341
206	31003.	1001818	191331	37727	3962	5632	22136	219954
207	36730	780287	252161	70192	11811	8131	25120	315689
208	1371	68817	7376	3893	440	264	977	11171
209	2178	1260	19270	4080	985	138	1869	22066
210	3241	11130	28463	4656	1319	320	2662	30212
211	51	1222	102	90	214	25	0	0
212	108	48	796	345	240	15	69	972
213	233	394	2567	486	237	15	205	2636
214	57	0	377	119	248	31	21	271
215	6405	123398	46558	8433	2418	966	4801	53074
216	2527	2939	20692	2804	1486	171	2184	22149
217	84	0	110	559	132	13	60	619
218	149	835	454	136	213	95	27	378
219	1150	7869	1060	51	148	774	79	896
220	2326	15404	15710	3811	799	196	1943	18812
221	346	1268	2834	246	244	39	259	2900
222	213	223	1901	200	141	18	179	1934
224	442	1141	4147	33	317	49	332	3882
225	140	56	1357	224	142	6.8595	128	1396
226	193	67	1849	105	119	3.3415	175	1918
227	25840	722669	152506	59576	8118	4429	18797	206075
228	130	71	1753	192	150	26	85	1722
229	156	0	1508	81	166	20	117	1428
230	597	3428	5327	108	411	48	498	4831
231	34274	112334	294287	34328	12461	2080	30677	306241
234	45550	338290	296514	57098	18380	8353	33193	327463
235	45629	773126	302787	81690	13935	5084	38019	348897
236	16626	41829	126831	25209	4638	1216	14481	147267
237	3167	4420	31923	4531	1978	131	2934	34976
238	624	19330	5209	1014	297	46	493	5611
239	7366	184780	45637	10446	2647	841	5831	55299
241	1848	22838	14163	1117	848	171	1583	13730
242	5981	23893	41934	6687	2076	443	5118	45564
243	424	10728	3037	657	158	52	339	3223
244	6803	64293	44283	3682	1383	479	5784	49179
245	12165	51885	88292	19239	6828	698	10709	114796
246	8535	266532	41331	25413	4592	1330	6505	68174
247	24593	172140	181363	33696	8487	2915	19664	206672
248	1005	9432	3828	737	373	308	438	4218
249	1091	16389	8393	821	407	66	960	8669
250	1441	4026	12393	991	486	52	1301	12526

...continued

APPENDIX A. (continued)

251	1351	130991	8750	3144	1013	212	1000	11402
252	452	148734	1829	2102	430	102	271	3478
253	28	40	0	115	114	8.6385	6.1637	1.4694
254	14264	757	50951	17510	5027	4638	6896	67938
255	5693	267	41846	13224	2828	656	5018	50209
256	994	10394	8019	621	407	130	732	7543
257	199	8289	1244	827	409	33	121	1787
258	2085	162718	11541	967	603	545	1114	11598
259	785	1969	5863	217	253	89	609	5834
260	911	689	8018	1477	1139	96	762	8462
261	7154	338338	43658	9374	1799	1289	5164	51499
262	193	665	1710	106	176	27	148	1575
263	27948.	1044042	151702	30332	5379	5721	18893	173884
264	58930.	1102405	321819	127115	12976	12141	42315	436121
265	1789	88261	9621	2320	475	321	1315	13007
266	3079	1440	24239	5041	1321	207	2659	27918
267	3976	14895	33182	4834	1423	380	3303	35523
268	234	13705	398	1292	229	103	18	1428
269	278	115	1326	886	243	24	213	2029
270	419	775	2884	412	254	21	382	3176
271	106	0	872	31	250	36	63	716
272	8782	103833	55565	7639	2549	891	7220	61280
273	3157	3003	20788	1076	404	181	2789	21821
274	105	0	102	605	149	14	82	685
275	256	1012	602	386	273	137	85	750
277	1873	6937	1241	89	169	1401	92	972
278	935	1902	5623	3127	599	103	732	9103
279	875	1815	5872	222	683	93	657	6124
280	312	307	2186	258	153	25	267	2325
281	294	49	1800	593	173	13	274	2119
283	32849	854340	160405	83562	8470	4076	25952	241811
284	815	887	9210	543	609	46	715	9536
285	309	78	2290	182	161	30	250	2297
286	743	3184	5715	130	448	69	605	5446
287	41739	119588	295581	44255	13557	2334	37786	328934
290	55299	397461	261898	51120	17087	9441	40385	314579
291	51168	949779	248498	101649	12771	5946	42290	304448
292	23849	36050	150393	23166	6639	1587	21055	171602
293	3672	4614	32100	3660	2079	113	3419	34620
294	1395	19378	8554	2183	464	105	1159	9586
295	7396	208146	47193	7139	2656	613	6295	52153
296	4621	1524	31184	12374	1883	204	4222	39767
297	1865	0	13139	1381	865	100	1692	13053
298	7077	13060	46563	6634	2137	437	6147	48694
299	357	1259	2443	268	171	19	328	2348

...continued

APPENDIX A. (continued)

300	9220	55724	64334	2397	1196	509	8087	69719
301	19512	116593	108532	19493	3461	1135	17091	150190
302	11056	320	51607	26340	4747	1197	9103	79808
303	33299	203708	191156	44073	10705	3480	27583	234070
304	1808	6382	4993	3614	542	349	1158	7457
305	1187	21678	8314	1012	306	70	1051	8496
306	1752	4159	13531	1155	442	54	1597	13617
307	2887	153578	9545	3861	1102	187	2477	15613
308	1250	153196	5808	5148	765	128	969	9708
309	2.9	0	0	128	123	2.5	.2	0
310	15175	635	44138	12931	4828	5228	7374	56446
311	9580	460	55784	16001	2750	809	8212	67682
312	1179	11173	8359	681	411	130	955	7551
313	491	7379	3304	712	399	63	361	3258
314	2747	205306	14846	1225	858	613	1677	14764
315	728	0	5645	101	256	19	696	2786
316	516	1245	2974	531	250	44	403	3202
317	1311	377	9205	1652	1139	121	1126	9893
318	8293	466981	49183	11029	2045	858	6862	58229
319	236	643	1685	237	187	33	184	1670
320	31870	995080	139149	31666	5514	5734	22488	168033
321	86538.	1298211	352092	163152	17013	14680	65663	501679
322	2988	107833	12998	6657	767	392	2375	20586
323	5142	3007	33012	8589	1432	295	4445	39045
324	5593	15515	38175	6808	1294	450	4779	43064
325	796	20817	3612	1237	446	200	422	4389

SOURCE: Reports of the Registrar of Loan and Trust Companies (1976-1981), Ontario.

NOTES TO APPENDIX A.

A1. This appendix refers to data on trust companies used in the estimation of GTMCF. This data which consists of 299 observations is referred to as Data-1 in the text.

A2. Except for column 1, all other columns are in units of $10,000.

A3. "Firm No." refers to the 'pooled' position of a firm in the Report of the Registrar of Loan and Trust Companies, 1976 to 1981. There are gaps in the numbering of firms showing which firms were edited out by editing principles in the text.

A4. "Expenses," include Interest Incurred, Salaries and Staff Benefits, Real Estate Commissions, Other Operating Expenses, and Depreciation.

A5. "E.T.A." -- Estates, Trusts and Agencies under Administration.

A6. "Loans," include Collateral Loans, Consumer Loans, Mortgages and Sale Agreements, and Commercial Loans.

A7. "Security," include Bonds, Stocks and Treasury Bills and Short Term Deposits.

A8. "Salary," include Salaries and Staff Benefits plus Real Estate Commissions.

A9. "Interest" -- Interest Incurred.

A10. "Deposit" -- Demand and Term Deposits.

APPENDIX B

RANGES OF VARIABLES IN THE COST FUNCTIONS

Variable	DATA-1				DATA-2			
	Mean	S.D.	Minimum	Maximum	Mean	S.D.	Miminum	Maximum
Operating Cost (C)	1655	3204	8.63	22707	1896	3371	16	22707
E.T.A. (Y_1)	105565	236875	0.	1298210	120861	250427	3	1298210
Loans (Y_2)	40882	71087	0.	352092	46906	74393	210	352092
Securities (Y_3)	9737	19905	0.	163152	11157	20984	19	163152
Deposits (Y_4)	48156	85330	0.	501679	55270	89373	103	501679
Price of labour (r_1)	1.0	.174	.780	1.290	1	.175	.78	1.29
Price of capital (r_2)	1.0	.198	.744	1.304	1	.200	.73	1.29

NOTE: Mean, Minimum and Maximun values are in units of $10,000.

APPENDIX C.

RAY AVERAGE COST

t	RAC (GTMCF - DATA-1)	RAC (TMCF - DATA-2)	RAC (GTMCF - DATA-2)
.01	5049.7	4617.9	4625.6
.07	3679.6	3547.3	3559.9
.03	3113.1	3097.3	3107.5
.04	2788.7	2836.6	3107.5
.05	2573.6	2662.3	2667.4
.06	2418.5	2535.7	2538.7
.07	2300.3	2438.7	2439.8
.08	2216.7	2361.4	2361.0
.09	2140.1	2298.2	2296.3
.10	2076.3	2245.2	2242.1
.20	1750.8	1971.1	1960.4
.30	1623.8	1860.9	1846.4
.40	1557.2	1801.4	1784.6
.50	1517.9	1765.0	1746.6
.60	1493.4	1741.2	1721.6
.70	1477.8	1725.0	1704.7
.80	1468.1	1713.9	1692.9
.90	1462.5	1706.3	1684.8
1.0	1459.7	1701.2	1679.3
1.1	*1459.2	1697.9	1675.8
1.2	1460.2	1696.0	1673.8
1.3	1462.5	*1695.3	*1672.9
1.4	1465.9	1695.4	1672.9
1.5	1470.0	1696.2	1673.7
1.6	1474.8	1697.6	1675.1
1.7	1480.1	1699.4	1686.9
1.8	1485.9	1701.6	1679.2
1.9	1492.0	1704.1	1681.8
2.0	1498.4	1706.9	1684.7
2.1	1505.1	170918	1687.8
2.2	1512.0	1713.0	1691.1
2.3	1519.1	1716.4	1694.6
2.4	1526.3	1719.8	1698.2
2.5	1533.7	1723.4	1702.0
2.6	1541.1	1727.0	1705.8
2.7	1548.7	1730.8	1709.8
2.8	1556.3	1734.6	1713.8
2.9	1564.0	1738.5	1717.9
3.0	1571.8	1742.4	1722.0

* Minimum RAC

APPENDIX D

FORMAL DERIVATION OF THE MULTIPRODUCT ECONOMIES OF SCALE MEASURE

Define a multiproduct transformation technology.

$$\phi(y_1 \ldots y_m, x_1, \ldots x_n) = 0$$

where the y_j are outputs and x_i are inputs.

Let cost, $C = \sum r_i x_i$ and minimize it subject to $\phi(y,x) \geq 0$.

Setting the Lagrangian, we have

$$\underset{x}{\text{Min}}\ L = \sum r_i x_i - \gamma(\phi(y,x))$$

The Kuhn-Tucker conditions are

a) $$r_i - \gamma \frac{\partial \phi}{\partial x_i} \geq 0$$

b) $$x_i (r_i - \gamma \frac{\partial \phi}{\partial x_i}) = 0$$

c) $$\phi(y,x) \geq 0, \quad \gamma \geq 0$$

d) $$\gamma \phi(y,x) = 0$$

Now suppose outputs increase by λ^S when inputs are increased by a factor λ. Then

$$H(\lambda) = \phi(\lambda^S y_1 \ldots \lambda^S y_m, \lambda x_1 \ldots \lambda x_n) = 0$$

To select the λ which maximizes $H(\lambda)$, differentiate $H(\lambda)$ with respect to . Thus,

$$\frac{dH(\lambda)}{d\lambda} = S\ \lambda^{S-1} \sum_j \frac{\partial \phi}{\partial y_j} y_j + \sum_i \frac{\partial \phi}{\partial x_i} x_i = 0$$

There is no loss of generality by setting = 1. Solving for S, we have

$$S = \frac{- \sum_i \frac{\partial \phi}{\partial x_i} x_i}{\sum_j \frac{\partial \phi}{\partial y_i} y_i}$$

S is a measure for multiproduct economies of scale.

Using the 'Envelope Theorem' and the Kuhn-Tucker conditions, it can be deduced that

$$\frac{\partial C}{\partial y_i} = -\gamma \frac{\partial \phi}{\partial y_i}$$

Also from the Kuhn-Tucker condition (b),

$$\sum w_i x_i = - \gamma \sum \frac{\partial \phi}{\partial x_i} x_i$$

Therefore,

$$S = \frac{\sum_i r_i x_i}{\sum_j \frac{\partial C}{\partial y_j} y_j}$$

$$= \frac{C(y,r)}{\sum_j \frac{\partial C}{\partial y_j} y_j}$$

$$= \sum_{j=1} \frac{\partial \ \ln C}{\partial \ \ln y_j}$$

(This corresponds to S in (3.1-4).

BIBLIOGRAPHY

Alhadeff, D.A. (1954), Monopoly and Competition in Banking, Berkeley: University of California.

Alhadeff, D.A. (1968), Competition and Controls in Banking, Berkeley: University of California.

Allen, R.G.D. (1959), Mathematical Economics, Second Edition, London: McMilland and Co.

Amemiya, T. (1982), "Correction to a Lemma (The Maximum Likelihood and Non-Linear Three Stage Least Squares Estimator in the General Non-Linear Simultaneous Equations Model)", Econometrica, 50 (5), p. 1009-29.

Arrow, Kenneth J. (1971), Essays in The Theory of Risk Bearing, Chicago, Illinois: Markham.

Bailey, Elizabeth E., and Ann F. Friedlaender (1982), "Market Structure and Multiproduct Industries," Journal of Economic Literature, Vol. XX, p. 1024-48.

Bain, J.S. (1956), Barriers to New Competition, Cambridge: Harvard University.

Baltensperger, E. (1970), "Costs of Banking Activities - Interactions Between Risk and Operating Costs," Journal of Money, Credit and Banking.

Baumol, William J. (1976), "Scale Economies, Average Cost and the Profitability of Marginal Cost Pricing," Public and Urban Economics: Essays in Honour of William S. Vickery: Ed. Ronald E. Grieson, Lexington, MA: Lexington Books.

Baumol, William J. (1977), "On the Proper Cost Tests for Natural Monopoly in a Multiproduct Industry," American Economic Review, 67 (5), p. 809-822.

Baumol, William J., Panzar, J.C., and Willig, R.D. (1982), Contestable Markets and the Theory of Industry Structure, San Diego, CA: Harcourt Brace Jovanovich.

Baumol, William J., Bailey, E., and Willig, R.D. (1977), "Weak Invisible Hand Theorems on the Sustainability of Multiproduct Natural Monopoly," American Economic Review, 67 (3), p. 350-365.

Baumol, William J., and Fischer, Dietrich (1978), "Cost-Minimizing Number of Firms and Determination of Industry Structure," Quarterly Journal of Economics, 92 (3), p. 439-467.

Baumol, William J. and Willig, R.D. (1981), "Fixed Costs, Sunk Costs, Entry Barriers and Sustainability of Monopoly," Quarterly Journal of Economics, 96 (3), p. 405-431.

Beazer, William F., (1975), Optimization of Bank Portfolios, Lexington: D.C. Heath & Co.

Bell, F.W. and N.B. Murphy, (1976), "Economies of Scale and the Division of Labour in Commercial Banking," The Southern Economic Journal, XXXV, p. 131-139.

Bell, F.W., and N.B. Murphy, (1968), "Costs in Commercial Banking: A Quantitative Analysis of Bank Behaviour and its Relation to Bank Regulators," Research Report No. 41, Boston: Federal Reserve Bank of Boston.

Benson, Winslow, (1962), Business Methods of Canadian Trust Companies, Toronto: The Ryerson Press.

Benston, George J. (1964), "Commercial Bank Price Discrimination Against Small Loans: An Empirical Study," Journal of Finance, XIX, p. 631-643.

Benston, George J. (1965a), "Economies of Scale and Marginal Costs in Banking Operations," The National Banking Review, 2, p. 507-49.

Benston, George J. (1965), "Branch Banking and Economies of Scale," The Journal of Finance, XX, p. 312-331.

Benston, George J. (1969), "Cost of Operations and Economies of Scale in Savings and Loan Associations," in Study of the Savings and Loan Industry, Vol. 2, Washington D.C.: U.S. Government Printing Office.

Benston, George J. (1972), "Economies of Scale of Financial Institutions," Journal of Money, Credit and Banking, 4, p. 312-341.

Berndt, E.R., and L.R. Christensen (1973), "The Translog Function and the Substitution of Equipment, Structures and Labour in U.S. Manufacturing, 1929-68," Journal of Econometrics, 1, p. 81-113.

Binhammer, H.H. (1977), Money, Banking and the Canadian Financial System. (3rd edition), Toronto: Methuen.

Bogert, G.C. (1962), "Trusts," in Encyclopedia Britannica, Vol. 22, p. 516-519.

Bond, D.E., and R.A. Shearer (1972), The Economics of the Canadian Financial System: Theory, Policy and Institutions. Scarborough, Ontario: Prentice-Hall.

Box, G.E.P., and D.R. Cox (1964), "An Analysis of Transformations," Journal of the Royal Statistical Society, Series B, 26, p. 211-243.

Braeutigan, R.R., Daughety, A.F., and Turnquist, M.A. (1982), "The Estimation of a Hybrid Cost Function for a Railroad Firm," Review of Economics and Statistics, Vol. 64, No. 3.

Brechling, Frank (1975), Investment and Employment Decisions. Manchester (England): Manchester University Press; Totowa, N.J.: Rowman and Littlefield.

Brigham, Eugene F. and R.R. Pettit (1970), "Effects of Structure on Performance in the Savings and Loan Industry," in Study of the Savings and Loan Industry, Federal Home Loan Bank Board, Washington: U.S. Government Printing Office, p. 971-1209.

Brown, R.S., D.W. Caves, and L.R. Christensen (1979), "Modelling the Structure of Cost and Production for Multiproduct Firms," Southern Economic Journal, 46, p. 256-273.

Brozen, Yale (1982), Concentration, Mergers and Public Policy, New York: MacMillan.

Canada (1964), Report of the Royal Commission on Banking and Finance, Ottawa: Government of Canada.

Carter, H. and I. Partington (1979), Applied Economics in Banking and Finance, New York: Oxford University Press.

Caves, D.W., L.R. Christensen and M.W. Tretheway (1980), "Flexible Cost Functions for Multiproduct Firms," Review of Economics and Statistics, 62 (3), p. 477-481.

Cowes, R.E., J. Khalilzadeh-Shirazi and M.E. Porter (1975), "Scale Economies in Statistical Analyses of Market Power," Review of Economics and Statistics, p. 133-40.

Christensen, L.R., and W.H. Greene (1976), "Economies of Scale in U.S. Electric Power Generation," Journal of Political Economy, 84 (4), p. 655-676.

Christensen, L.R., D.W. Jorgenson and L.J. Lau (1973), "Transcendental Logarithmic Production Frontiers," Review of Economics and Statistics, 55, p. 28-45.

Christensen, L.R., and M.E. Manser (1977), "Estimating U.S. Consumer Preferences for Meat with a Flexible Utility Function," Journal of Econometrics, 5, p. 37-53.

Chu, F.S., D.J. Aigner and M. Frankel (1970), "On the Log-Quadratic Law of Production," Southern Economic Journal, 37, p. 32-39.

Cobb, C.W., and P.H. Douglas (1928), "A Theory of Production," American Economic Review, 18, Supplement, p. 139-165.

Dhrymes, P.J., and B.M. Mitchell (1969), "Estimation of Joint Production Functions," Econometrica, 37, p. 732-736.

Dhrymes, P.J. (1967), "On a Class of Utility and Production Functions Yielding Everywhere Differentiable Demand Functions," Review of Economic Studies, 34, p. 399-408.

Diewert, W.E., (1974), "Applications of Duality Theory," in Frontiers of Quantitative Economics, Vol. 2, ed. M.D. Intriligator and D.A. Kendrick, Amsterdam: North-Holland.

Diewert, W.E. (1971), "An Application of the Shepherd Duality Theorem: A Generalized Leontief Production Function," Journal of Political Economy, 79, p. 481-507.

Diewert, W.E. (1973), "Functional Forms for Profit and Transformation Functions," Journal of Economic Theory, 6, p. 284-316.

Eads, G., M. Nerlove, and W. Raduchel (1969), "A Long-Run Cost Function for the local Service Airline Industry: An Experiment in Non-Linear Estimation," Review of Economics and Statistics, 51, p. 258-270.

Frankel, Dave (1972), The Law of Trusts, Second Edition, Vanvouver: Coast Legal Publications.

Fuchs, V.R. (ed.), (1969), Production and Productivity in the Service Industries, New York: NBER and Columbia University Press.

Fuss, Melvyn and Daniel McFadden (ed.), (1978), Production Economics: A Dual Approach to Theory and Applications, Vol. 1, Amsterdam: North-Holland.

Fuss, Melvyn and Waverman, L. (1981), "Regulation of the Multiproduct Firm: The Case of Telecommunications in Canada," in Studies in Public Regulation, ed. Gary Fromm. Cambridge, MA: MIT Press.

Gold, Bela (1981), "Changing Perspectives on Size, Scale, and Returns: An Interpretative Survey," Journal of Economic Literature, Vol. XIX, p. 5-33.

Gramley, L.E. (1962), A Study of Economies of Scale in Banking, Kansas City, MO: Federal Reserve Bank of Kansas City.

Grebler, Leo and E.F. Brigham (1963), Savings and Mortgage Markets in California, Pasadena: California Savings and Loan League.

Greenbaum, S.I. (1967), "Competition and Efficiency in the Banking System - Empirical Research and its Policy Implications," The Journal of Political Economy, 75, Supplement, p. 461-479.

Guilkey, D.K., and C.A.K. Lovel (1980), "On the Flexibility of the Translog Approximation," International Economic Review, 21 (1), p. 137-147.

Haldi, John and David Whitcomb, (1967), "Economies of Scale in Industrial Plants," Journal of Political Economy, 75, p. 373-385.

Hall, R.E. (1973), "The Specification of Technologies with Several Kinds of Outputs," Journal of Political Economy, 81 (4), p. 878-892.

Hanoch, G. (1970), "Homotheticity in Joint Production," Journal of Economic Theory, 2, p. 423-426.

Hasenkamp, G. (1976), "A Study of Multiple-Output Production Functions: Klien's Railroad Study Revisited," Journal of Econometrics, 4 (3), p. 253-262.

Hasenkamp, G. (1976), Specification and Estimation of Multiple Output Production Functions, Berlin: Springer-Verlag.

Heaton, Ernest (1904), The Trust Company Idea and Its Development, Toronto: Hunter Rose Co.

Henderson, J.M. and R.E. Quandt (1972), Microeconomic Theory, (second edition), New York: McGraw-Hill.

Horwitz, Paul M. (1963), "Economies of Scale in Banking," in Private Financial Institutions, Englewood Cliffs, NJ: Prentice-Hall.

Hotelling, H. (1932), "Edgeworth's Taxation Paradox and the Nature of Demand and Supply Functions," Journal of Political Economy, 40, p. 577-616.

Intriligator, Michael D. (1978), Econometric Models, Techniques, and Applications. Englewood Cliffs, NJ: Prentice-Hall.

Jara-Diaz, S. and Winston, C. (1981), "Multiproduct Transportation Cost Functions: Scale and Scope in Railway Operations," Eighth European Association for Research in Industrial Economics, Vol. 1. Eds. Nicklaus Blattner, et. al., U. of Basle.

Johnston, J. (1972), Econometric Methods, New York: McGraw-Hill.

Johnston, J. (1960), Statistical Cost Analysis, New York: McGraw-Hill.

Keeler, J.E. (1974), "Railroad Costs, Returns to Scale, and Excess Capacity," Review of Economic Statistics, 56 (2), p. 201-208.

Khalilzadeh-Shirazi, J. (1974), "Market Structure and Price-Cost Margins in U.K. Manufacturing Industries," The Review of Economics and Statistics, 56, p. 67-76.

Klein, L.R. (1953), A Textbook of Econometrics, Evanston, Illinois: Row Paterson and Co.

Kmenta, J. (1971), Elements of Econometrics, New York: MacMillan.

Knudsen, N.C. (1973), Production and Cost Models of a Multiproduct Firm - A Mathematical Programming Approach. Odense University: Studies in History and Social Sciences. Vol. 13. Odense University Press.

Koenker, Roger (1977), "Optimal Scale and the Size Distribution of American Trucking Firms," Journal of Transport Economics and Policy, 11 (1), p. 54-67.

Koutsoyiannis, A. (1979), Theory of Econometrics, Toronto: MacMillan.

Kuhn, H.W. and A. Tucker (1951), "Non-Linear Programming in Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability, ed. J. Neyman, Berkeley and Los Angeles: University of California Press.

Laitinen, Kenneth (1980), A Theory of the Multiproduct Firm, Amsterdam: North-Holland.

Lau, L.J. (1972), "Profit Functions of Technologies With Multiple Inputs and Outputs," Review of Economics and Statistics, 54 (3), p. 281-289.

Lau, L.J. (1976), "A Characterization of the Normalized Restricted Profit Function," Journal of Economic Theory, 12, p. 131-163.

Layard, P.R.G. and Walters, A.A. (1978), Microeconomic Theory, London: McGraw-Hill.

Magnen, S.D. (1971), The Cost of Funds to Commercial Banks - An Econometric Analysis. New York: Dunellen.

Matthews, John O. (1981), "Multiproduct Economies of Scale in the Securities Industry: An Application of Developing Theory," in Report of Securities and Exchange Commission.

McFadden, Daniel (1978), "Cost, Revenue and Profit Functions," in Production Economics: A Dual Approach to Theory and Applications, Vol. 1, Melvyn Fuss and Daniel McFadden (eds.), Amsterdam: North-Holland.

McFadden, Daniel (1963), "Constant Elasticity of Substitution Production Functions," Review of Economic Studies 30, p. 73-83.

McIvor, R.C. (1973), "Postwar Trends in the Financing of Canadian Economic Activity," Transactions of the Royal Society of Canada, Series IV. Vol. XI.

Mintz, J.M. (1981), "A Note on Multiproduct Economies of Scale and Economies of Scope," Economic Letters, 8 29-33.

Murray, John D. and Robert W. White (1980), "Economies of Scale and Deposit-Taking Financial Institutions in Canada - A Study of British Columbia Credit Unions," Journal of Money, Credit and Banking, 12 (1), p. 58-70.

Murray, John D. and Robert W. White (1983), "Economies of Scale and Economies of Scope in Multi-Product Financial Institutions," Journal of Finance, XXXVIII (3), p. 887-902.

Neufeld, E.P. (1972), The Financial System of Canada - Its Growth and Development. New York: St. Martin's Press.

Panzar, J.C. and R.D. Willig (1977), "Economies of Scale in Multi-Output Production," Quarterly Journal of Economics, 91, p. 481-493.

Panzar, J.C. and R.D. Willig (1977b), "Free Entry and the Sustainability of Natural Monopoly." Bell Journal of Economics, 8, p. 1-22.

Panzar, J.C. and R.D. Willig (1981), "Economies of Scope," American Economic Review, 71 (2), p. 268-72.

Pegrum, Dudley F. (1965), Public Regulation of Business (second edition). Illinois: Richard D. Irwin, Inc.

Pfouts, R.W. (1961), "The Theory of Cost and Production in the Multiproduct Firm," Econometrica, 29, p. 650-58.

Phillips, P.C.B. (1982), "On the Consistency of Non-Linear FIML," Econometrica, 50 (5).

Pindyck, R.S. and D.L. Rubinfeld (1976), Econometric Models and Economic Forecasts. New York: McGraw-Hill.

Poapst, J.V. (1975), Studies for Decision Making - Liquidity in the Credit Union System. Study #1. National Association of Canadian Credit Unions.

Pontecorve, Giulio, R.P. Shay and A.G. Hart (1967), Issues in Banking and Monetary Analyses. New York Holt, Rinehart and Winston, Inc.

Powell, A.A. and F.H.G. Gruen (1968), "The Constant Elasticity of Transformation Production Frontier and Linear Supply System." International Economic Review, 9, p. 315-328.

Powers, J.A. (1969), "Branch versus Unit Banking: Bank Output and Cost Economies," The Southern Economic Journal, XXXVI, p. 153-164.

Rao, P. (1969), "A Note on Econometrics of Joint Production," Econometrica, 37, p. 737-738.

Rousakis, E.N. (1977), Managing Commercial Bank Funds. New York: Praeger Publishers.

Sakai, Y. (1974), "Substitution and Expansion Effects in Production: The Case of Joint Production," Journal of Economic Theory, 9, p. 255-274.

Scherer, F.M. (1980), Industrial Market Structure and Economic Performance (second edition). Chicago, Il: Rand McNally College Publishing Co.

Schweiger, I. and J.S. McGee (1961), "Chicago Banking: The Structure and Performance at Banks and Related Financial Institutions in Chicago and Other Areas," Journal of Business, XXXIV, p. 201-366.

Sharkey, William W. (1982), The Theory of Natural Monopoly. Cambridge: Cambridge University Press.

Shephard, R.W. (1970), Theory of Cost and Production Functions. Princeton: Princeton University Press.

Smith, Adam (1974). The Wealth of Nations. Andrew Skinner (editor). New York: Penguin.

Spady, Richard H. (1979), Econometric Estimation of Cost Functions for the Regulated Transportation Industry. New York: Garland Press.

Starrett, David A. (1977), "Measuring Returns to Scale in the Aggregate and the Scale Effect of Public Goods," Econometrica, 45 (6), p. 1439-55.

Statutes of Canada. (Elizabeth II) Vol. 1. 1969-70 (23), 63A1.

Stigler, George J. (1968), The Organization of Industry, Homewood, IL: R.D. Irwin.

Teece, David J. (1980), "Economies of Scope and the Scope of Enterprise," Journal of Economic Behaviour and Organization. 1(3), p. 223-247.

Theil, H. (1980), The System-Wide Approach to Microeconomics. Chicago: University of Chicago Press.

Trust Companies Institute of Canada (1980), The Canadian Trust Industry. Toronto: McGraw-Hill.

Uzawa, H. (1964), Duality Principles in the Theory of Cost and Production." International Economic Review.

Vinod, H.D. (1968), "Econometrics of Joint Production." Econometrica, 36, p. 322-336.

Von Weizsacker, C.C. (1980), Barriers to Entry: A Theoretical Treatment. New York: Springer-Verlag.

Walters, A.A. (1963), "Production and Cost Functions: An Econometric Survey," Econometrica, XXXI, p. 1-66.

Wang Chiang, J. Shaw-er (1981), Economies of Scale and Scope in Multiproduct Industries: A Case Study of The Regulated U.S. Trucking Industry. Ph.D. Dissertation, Department of Civil Engineering, M.I.T.

Willig, Robert D. (1979), "Multiproduct Technology and Market Structure," American Economic Review, 69 (2), p. 346-351.

Wolken, John D. a-d Frank J. Navratil (1980), "Economies of Scale in Credit Unions: Further Evidence." Journal of Finance, Vol. XXXV No. 3.

Yip, George S. (1982), Barriers to Entry (A Corporate-Strategy Perspective). Lexington: Lexington Books.

Zarembka, P. (1974), "Transformation of Variables in Econometrics," in P. Zarembka, edited, Frontiers in Econometrics. New York: Academic.