PRECISION SYNCHRONIZATION OF NAVIGATION SATELLITES IN INCLINED ORBIT FROM AN EARTH STATION

By

EHAB IBRAHIM SABRY, B.Eng.

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AUTHOR: EHAB IBRAHIM SABRY, B.Eng. (Alexandria University)

SUPERVISOR: Dr. C.R. Carter

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ABSTRACT

The possibility of establishing accurate timing on board a navigation satellite in inclined orbit using a timing reference from either an earth station or geostationary satellite has been an important task in the last fifteen years. More recently, there has been considerable effort placed in designing and developing the NAVSTAR system which uses atomic clocks on-board satellites in inclined orbits to establish accurate time.

In this thesis we discuss another possible mode of operation which is based on the transponding of timing information from an earth station to a navigation satellite in inclined orbit through a satellite in geostationary orbit. Assuming that the satellite in the geostationary orbit has constant space delay with an earth station, then the only change in the space delay between the earth station and the satellite in inclined orbit occurs between the two satellites.

The main advantages of our solution to this problem are:

1) The atomic clocks on board the navigation satellite are no longer required.

2) A communication link now exists between the earth station and all users of the system through the navigation satellite. This is due to the fact that the satellite in geostationary orbit has the capability of observing both the navigation satellite in inclined orbit and the earth station located inside its coverage area.

We assume the following:

1) The location of geostationary satellite is accurately known. This is usually true since its motion with respect to earth stations is small.

2) The space delay from earth station to geostationary satellite can be determined to within less than 1 ns using conventional timing methods in TDMA. Thus, accurate time at the geostationary satellite is established.

3) The distance between the geostationary satellite and the navigation satellite varies smoothly with time.

4) The location of the navigation satellite is known to lie within a sphere of certain radius centred at a known point.

If we calculate in advance the actual space delay between a geostationary satellite and a navigation satellite; and know the timing on-board the geostationary satellite, we can establish timing on-board the navigation satellite. The technique which we use in computing the uplink and downlink delays between the two satellites depends on the estimation of the navigation satellite location. The error in estimating the location of the navigation satellite does not affect the calculation of the space delay between the two satellites directly, but its effect will be reflected by the change in the space delay.

The computed results show that the estimated uplink and downlink space delay between the two satellites can be calculated to a high degree of accuracy (a fraction of a nanosecond or less). Thus it appears that this system could be practical especially for commercial use which may include communications links as well as navigation information.

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LIST OF SYMBOLS

a_{H_N} = the unit vector in the radial direction H_N.

a_Q = the unit vector in the angular direction Q.

a_i = the unit vector in the angular direction i.

$a_{\phi_{Lat}}$ = the unit vector in the latitude direction.

$a_{\phi_{Long}}$ = the unit vector in the longitude direction.

d = the distance between the geostationary satellite and the navigation satellite at any instant.

d_E = the distance measured between the earth station and the satellite at any instant.

h = the satellite altitude.

hsyn = the synchronous satellite altitude.

i = the orbital inclination angle of the navigation satellite.

r = the radius of the sphere of error.

$\left.\begin{matrix} r_M \\ \phi_{LatM} \\ \phi_{LongM} \end{matrix}\right\}$ = the coordinates of any point in the plane MM.

t = the reference time.

t_{n1}, t_{n2} = the time taken by the satellite to reach the node of the trace in two successive quarters.

t' = the time measured from the instant when the satellite crossed the equatorial plane.

$\left.\begin{matrix} x_M \\ y_M \\ z_M \end{matrix}\right\}$ = the Cartesian coordinates of any point in the plane MM.

$\left.\begin{matrix} x_N \\ y_N \\ z_N \end{matrix}\right\}$ = the cartesian coordinates of the navigation satellite subpoint N'.

B1 = $\sin \phi_{LatE}$.

B2 = $\cos \phi_{LatE}$.

C = the velocity of propagation.

D = $H_N H_G/d.C$.

Diff = the difference in longitude between the centers of the single antenna and the subantenna footprints.

EER1 = the error in estimating the change in space delay if the navigation satellite is fixed.

EER2 = the error in estimating the change in space delay if the geostationary satellite is fixed.

G = the geostationary satellite.

H_G = the distance measured from the center of the earth to the geostationary satellite.

H_N = the distance measured from the center of the earth to the navigation satellite.

ICA = the Instantaneous Coverage Area.

K = the nonsynchronous factor.

M_E = the mass of the earth.

M_S = the mass of the satellite.

N = the navigation satellite.

O = the center of the earth.

Q = the angle rotated by the navigation satellite measured from the center of the earth starting from the equatorial orbit.

Q′ = the angle rotated by the satellite in time TT.

R = the radius of the earth.

RR = the radius of the latitude circle ϕ_{Lat}.

SST = the Satellite Subpoint Trace.

T_{ij} = the space delay when the geostationary satellite located at G_i and the navigation satellite at N_j.

T_{12} = the true value for the uplink delay.

T_{23} = the true value for the downlink delay.

T_{34} = the new uplink delay.

T_{12E} = the estimated value of T_{12}.

T_{23E} = the estimated value of T_{23}.

T_{34E} = the estimated value of T_{34}.

T_{23th} = the theoretical value of T_{23}.

T_s = the time delay in passing through the navigation satellite receiving-transmission circuits.

T_t = the total delay for a round trip.

TT = the initial time phase of the navigation satellite.

U = the universal gravitational constant.

WEQ = the width of SST at the equator.

W_m = the maximum width of the SST.

α, β, γ = the directions of the normal to the plane MM.

δ = $\phi_{Long_N} - \omega_E t + \phi_{oN}$

η = $\phi_{Long_N} - \phi_{LongG}$

η_E = the difference in longitude between the satellite subpoint and the earth station.

λ = the angle between the geostationary satellite and the navigation satellite measured from the center of the earth.

λ_{max} = the maximum value of λ at the moment when the communications between the two satellites is blocked.

σ = the minimum angle of visibility.

ω_E = the angular velocity of the earth.

ω_S = the angular velocity of the navigation satellite.

Δ = half the coverage angle measured from the satellite.

ε = the angle between the earth station and the satellite subpoint measured from the center of the earth O.

θ_E = the angle rotated by the earth during time t'.

Λ = $\phi_{LongG} - \omega_E t + \phi_{oN}$

ϕ_g, ϕ_g' = the longitudes of two points on the boundary of the ICA with the same latitude of the satellite subpoint.

$\phi_{Latb}, \phi_{Longb}$ = the latitude and longitude of any point on the boundary of the ICA.

$\phi_{LatE}, \phi_{LongE}$ = the latitude and longitude of the earth station.

$\phi_{Lati}, \phi_{Longi}$ = the latitude and longitude of any point inside the boundary of the ICA.

$\phi_{Latic}, \phi_{Longic}$ = the latitude and longitude of the center of the ith subantenna footprint.

$\phi_{Latn}, \phi_{Longn}$ = the latitude and the longitude of the node of the SST

Φ_{oN} = the longitude of the point of intersection between the equator and the orbital projection of the navigation satellite on the surface of the earth at $t = 0$.

Φ_{LatN} = the latitude of the navigation satellite subpoint.

$\Phi_{LatNmin}$ = the minimum latitude reached by the navigation satellite subpoint before observing the whole geostationary orbit.

$\Phi_{LongI}, \Phi_{LongII}$ = the longitudes of two successive crossing points by the satellite through the equatorial plane.

Φ_{LongG} = the longitude of the geostationary satellite.

Φ_{LongN} = the longitude of the navigation satellite subpoint.

Φ'_{Latmax} = the maximum latitude reached by the satellite.

Φ'_{Long} = the projection of the angle Ω on the equatorial plane.

Φ'_{o} = the longitude at which the satellite crossed the equatorial plane.

$\hat{\Phi}_{Lat}, \hat{\Phi}_{Long}$ = the latitude and the longitude of any point on the surface of the earth located in the projection of the shadow zone on the earth.

$\hat{\Phi}_{Longmax}$ = the longitude of the two points on the edges of the blind segments.

ψ = the half arc observed angle.

ψ_1 = the angle between the satellite subpoint and any point inside the ICA.

$$\Omega = \frac{R}{R + hsyn}$$

$\frac{\partial T}{\partial t}$ = the change in the space delay

$$\frac{\partial^2 T}{\partial t^2} = \text{the change in } \frac{\partial T}{\partial t}$$

$$\frac{\partial^3 T}{\partial t^3} = \text{the change in } \frac{\partial^2 T}{\partial t^2}$$

$\left.\frac{\partial T}{\partial t}\right|_{G_i, N = N_{fix}}$ = the change in the space delay if G is moving starting from the location G_i and N is fixed at N_{fix}.

$\left.\frac{\partial T}{\partial t}\right|_{G = G_{fix}, N_j}$ = the change in the space delay if G is held fixed at G_{fix} and N is moving starting from the location N_j.

$$\left.\frac{\partial^2 T}{\partial t^2}\right|_{G_i, N = N_{fix}} = \text{the change in } \left.\frac{\partial T}{\partial t}\right|_{G_i, N = N_{fix}}$$

$$\left.\frac{\partial^2 T}{\partial t^2}\right|_{G = G_{fix}, N_j} = \text{the change in } \left.\frac{\partial T}{\partial t}\right|_{G = G_{fix}, N_j}$$

CHAPTER 1

INTRODUCTION

1.1 Historical Review

Almost exactly eleven years ago the idea of a plan that would revolutionize the art and science of navigation was born. The plan is named NAVSTAR Global Positioning System (GPS) and is being developed to fulfill the requirements of the Department of Defence in the United States [1, 2, 3].

The main reason behind this new program is the outstanding success of a simpler satellite network called the Navy Navigation Satellite System (NNSS), formerly known as TRANSIT. NNSS has been providing reliable service to the United States Navy submarines for nearly a decade [4, 5]. Also, NNSS has found ever- increasing use in the domestic and international communities.

NNSS consists of five satellites in circular polar orbit of approximately 1100- km altitude. Satellites broadcast their orbital parameters from an on-board memory system which is updated every 12 hours. The user computes his position based on data collected during a single satellite pass every 90 minutes. These data include the measured Doppler frequency shift, satellite orbital parameters and accurate time marks. The major problem with this system, although its coverage is global, is that position data are available only once every 90 minutes on the average. This waiting period is longer at the equator and shorter at the polar regions. The limited data rate of position determination makes the versatility of the NNSS quite limited [6, 7, 8].

In spite of such limitation, the NNSS has proved highly reliable through its satellite service. These satellites were originally designed to last up to two or three years in orbit. To insure the continued validity of the system the Navy put in stock several additional satellites, not knowing that most of the satellites would operate for 16 years. For this reason the NNSS has continued to provide service for almost twenty years, and there are several of those original satellites in stock which can still give life to the system at least through the 1980's. Such a success has encouraged both the Navy and the Air Force in the United States to investigate more advanced systems for a space-based navigation that would provide continuous position location information. The Air Force project was known simply as "Program 621B", while the Navy advanced program was called "TIMATION".

The TIMATION issue was basically a two-dimensional system and lacked the ability to provide continuous position updates in a high-dynamic aircraft environment. This was not surprising, since the Navy envisioned TIMATION as a system for use primarily by ships and submarines [1, 9, 10]. The Air Force concept would have provided the high-dynamic capacity, but the basic design of the system required at least four separate satellite constellations, each served by an independent ground-control station, to provide global coverage. This need for several ground-control stations (at least two of which would have to be located outside of the United States territory) was not acceptable from a survivability standpoint.

At first the complete NAVSTAR system was planned to be implemented with a total of 24 satellites equally distributed between three inclined orbit planes (with inclination angle equal to 63°) All the satellites are located at 20183 km (12 hour orbits) in order to provide whole globe coverage with at least four (and often, 6 to 9 satellites) in view with user position accuracies predicted to be in the ten meter range. Also it was planned that by 1984, 18 of 24 satellites were to be available, providing for the first time a global three-dimensional capability (ie., latitude, longitude and altitude) [11, 12].

Due to funding reduction amounting to nearly $500 million, there was a reduction in the number of satellites for the fully operational system from the 24 originally planned to 18. Not only that, but due to the complexity of the system, the funding constraints and the major technical advances required to develop the system equipment, it is not surprising that the time schedule has slipped four more years. Thus, the expected date for the project to be implemented is now 1988 [1, 13].

1.2 Satellite Navigation Technique

Usually four satellites are required for navigation purposes and those four satellites offering the best geometry can be selected by users using ephemeris information transmitted by the satellite [14]. By measuring the space delays from each one of the four satellites and multiplying by the propagation velocity (neglecting relativistic effects) the user can calculate the distance from his location to each of the four satellites [11, 12]. Also, the transmitted message contains ephemeris parameters that enable the user to calculate the position of each satellite at the time of transmission of the signal. Now the user solves four equations of four spheres with centers located at the satellite in order to determine four unknowns. Those unknowns are the three dimensional locations of the user and the correction of the user's clock. It is clear that if the user is equipped with a local accurate clock which is synchronized with the satellite clock and the GPS system time, the user needs only to be viewed by three satellites. Also by measuring the derivatives of the space delays the user can solve for his own velocity[11,12].

It is obvious that the operation of the system requires precise synchronization of the satellites clocks with the GPS system time. The suggested way to accomplish this is to use an atomic frequency standard in each satellite [11, 12, 15].

1.3 Precise Timing References

1.3.1 Atomic Clock

In order to keep accurate timing on-board the navigation satellite, it appears that the development of precise atomic clocks will open a new era for the achievement of such a purpose. Until now these precise frequency standards hold the key to the GPS concept where the satellites do not simply act as transponders for the ground-generated navigation and timing signals, but maintain and generate the navigation and timing signals on-board [1, 11, 12].

Although data on the exact performance of these advanced space-qualified clocks are not generally available, it can be expected that they will routinely achieve fractional long-term frequency stability in the range of a few parts in 10^{14} per day - about 1 s in 3,000,000 years [1]. This long-term stability is one of the keys to the operation of GPS, since it allows the autonomous, synchronized generation and transmission of the navigation and timing signals on-board each of the GPS satellites [15, 16, 17].

The main disadvantages with using atomic frequency standards on-board a satellite are that they are expensive and fragile. For example, seven satellites were launched in the period from 1978 to mid-1983. Of these, six are still operational although one of these operates on its quartz crystal oscillator rather than the more stable atomic frequency standard, ie., two atomic clocks were shut down on two satellites while another crystal clock on one of those two satellites is also out of order [1].

Although the satellites carry atomic standards on board, even these clocks are subject to drift and clock errors with time [12]. For this reason the clock timing errors are continually checked by receivers at ground monitor stations, and at least once per day a clock correction signal is uploaded to each satellite for relay down to each user as part of the satellite data stream, along with the satellite position information (ephemeris).

In addition to these slowly varying oscillator generated clock errors, there are also general and special relativistic clock shifts. The received clock frequency differs from the transmitted clock frequency by the expression given in [12].

1.3.2 Quartz Crystal Clock

Closely related to the GPS atomic frequency standard and to the user equipment that receives and processes the satellite navigation and timing signals, is the quartz crystal clock similar to those commonly used in modern digital watches. These clocks show excellent short-term stability[1, 12, 18].

Similar clocks are used in the user equipment, but for a much different reason. As described before the user equipment measures the range to three or more GPS satellites by measuring the time lapse from transmission to reception of the satellite signals. Clearly, to perform this measurement, the user equipment must have knowledge of common GPS time (ie., the master time reference to which all GPS satellite transmissions are synchronized). However, requiring each user to maintain a time reference as precise as an atomic standard would be prohibitively expensive. For this reason, the need for the user to be observed by four or more satellites is an essential need for the users clock to be in synchronization with the GPS standard time in order to provide the proper correction.

1.4 GPS Orbit Configuration and Multiple Access

Here, we discuss in more detail the description of the orbit configuration. The 18-satellite configuration will be equally distributed between six inclined orbital planes [1,7]. These planes will be equally spaced 60° apart in longitude and inclined to the equator at 55°. Three satellites will be located in each of the six orbital planes with equal interspacing of 120° between satellites in the same orbit satellite phasing from plane to plane will be 40° - that is, a satellite in one plane will have a satellite 40° "ahead" or North of it in the plane directly to the East, as depicted in Fig. 1-1.

Table 1.1 gives the orbit specification values for this configuration, including the three on-orbit spares which are located in every other orbit plane to provide quick reaction

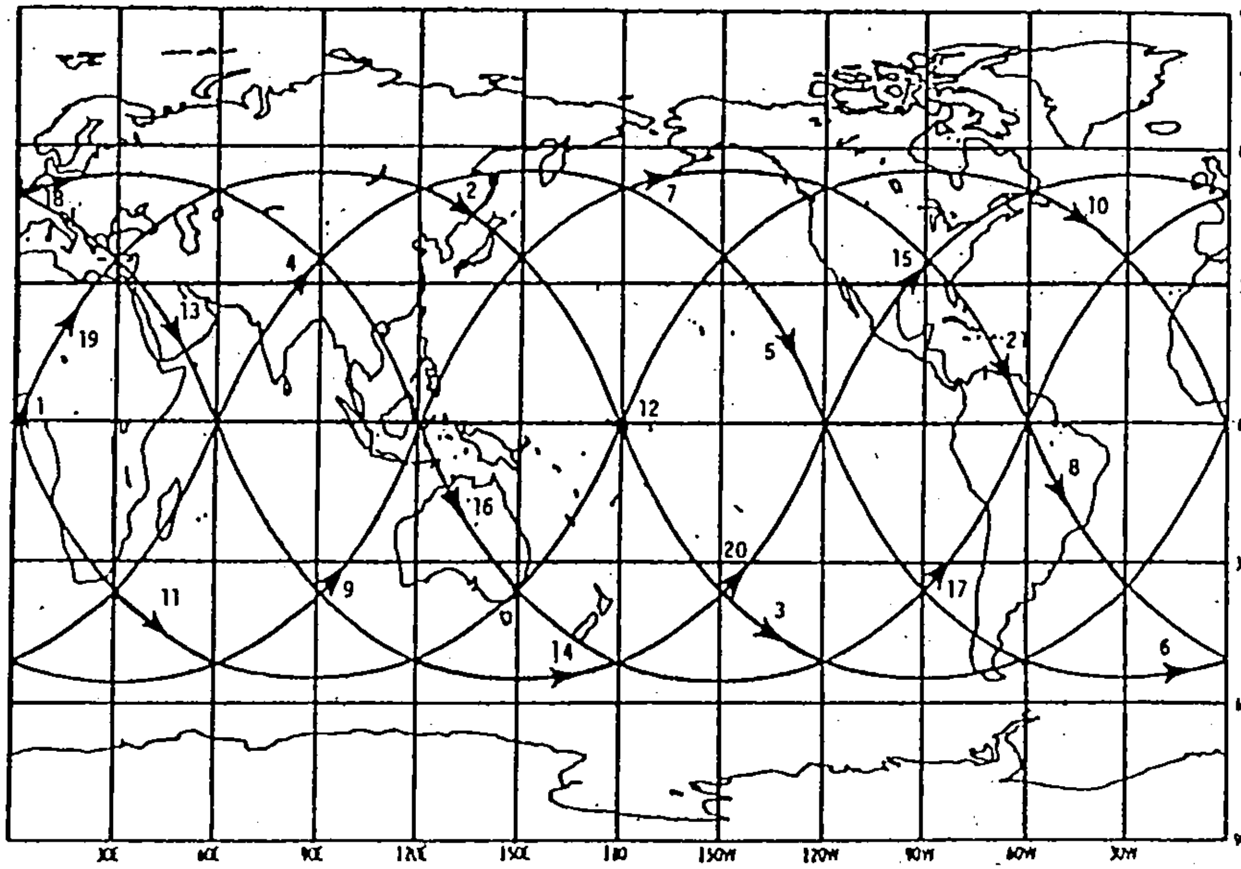

Fig. 1.1 Revised base line satellite development.

REFERENCE ORBIT PARAMETERS

BASELINE SATELLITE DEPLOYMENT

Satellite Number.	Orbit Plane	Longitude of Crossing the Equatorial Plane (Northerly)	Longitude of Orbital Intersection and Equatorial Plane.
1	1	0°, 180°	30° E
2	1	120° W, 60° E	30° E
3	1	60° W, 120° E	30° E
4	2	100° W, 80° E	90° E
5	2	40° W, 140° E	90° E
6	2	20° E, 160° W	90° E
7	3	20° W, 160° E	150° E
8	3	40° E, 140° W	150° E
9	3	100° E, 80° W	150° E
10	4	60° E, 120° W	150° W
11	4	120° E, 60° W	150° W
12	4	180°, 0°	150° W
13	5	140° E, 40° W	90° W
14	5	160° W, 20° E	90° W
15	5	80° E, 100° W	90° W
16	6	140° W, 40° E	30° W
17	6	80° W, 100° E	30° W
18	6	160° E, 20° W	30° W
Spares			
19	1	165° W, 15° E	30° E
20	5	145" W, 35° E	90° W
21	3	25° E, 155° W	150° E

TABLE 1.1

and recovery in the event of a primary satellite failure. The satellites are all in inclined circular orbits, at altitudes of approximately 20183 km (12 hour orbits).

This selection of parameters results in repeating daily the satellite subpoint trace, or in other words an orbital period that causes each satellite to pass over the same point on the earth every 23 h 55 min 56.6 s (a sidereal day), thus the orbital altitude was selected to provide a period exactly one half synchronous, ie. 11 h 57 min and 58.3 s. Another way to look at this is to imagine that each satellite completes two revolutions at the same time the earth completes one revolution around its axis which will produce the duplication of the satellite subpoint trace each sidereal day which will cause the satellite to pass over the same point on the earth almost four minutes earlier. The main purpose of this fact of the system is to allow each satellite to be viewed by a control earth station at least once a day.

1.5 Scope of Thesis

The main objective of this thesis is to find and develop a straight forward method for establishing accurate timing on board a navigation satellite in inclined orbit. Such a method should eliminate the use of the expensive and fragile atomic clocks on board the navigation satellite. The research plan includes the study and analysis of certain aspects of the satellite system, which were mainly dependent upon computer simulation.

In Chapter 2 a complete study of the satellite subpoint trace (SST) for any satellite in circular orbit has been analyzed and studied. Also the specification of SST has been demonstrated. In addition, the SST of a satellite constellation using a common reference time has been developed.

The basic concept of solving the problem of any satellite system coverage is established in Chapter 3, by deriving the equation of the instantaneous coverage area (ICA). This equation plays an important role in examining different kinds of coverage in both space and time.

In Chapter 4, the visibility and the space delay for two cases are discussed. In the first case, we consider a ground based system with different earth stations located at different locations on the surface of the earth. In this system, the study shows that the visibility is poor. In the second case, we consider an earth station-based satellite-to-satellite system using a geostationary satellite as an intermediate link. This last system shows a significant improvement in both the visibility and the space delay performance.

An accurate method has been developed in Chapter 5 for establishing accurate timing on board a navigation satellite in inclined orbit. This method is based on the transponding of time information from an earth station to a navigation satellite through a satellite in geostationary orbit. It is shown that timing can be established on board the navigation satellite with a high degree of accuracy (few nanoseconds or less) depending upon the accuracy of estimating the location of the navigation satellite.

Finally, conclusions and recommendations for further research are presented in Chapter 6.

CHAPTER 2

SATELLITE SUBPOINT TRACE (SST)

2.1 SUMMARY OF THE CHAPTER

This chapter deals with the problem of the satellite subpoint trace (SST), which is the set of points traced on the surface of the earth by the line connecting the center of the earth to the satellite. Assume that a reference clock is used at an earth station, and such a clock tracks a satellite constellation; thus, there will be more than one satellite under investigation, and all of them will use the same reference time. The SST is an important base for the design of navigation systems and the solution to the problems of whole globe coverage, partial coverage and earth based synchronization for any satellite constellation.

Section 2.2 of this chapter deals with deriving the equations of the SST in its general form, for any satellite rotating in any circular orbit. To implement these equations, we found that four initial conditions must be defined for each satellite and these initial conditions are: 1) the orbital inclination angle i; 2) the nonsynchronous factor K, which is directly related to the satellite angular velocity; 3) the longitude of the point of intersection between the equator and the projection of the orbit on the surface of the earth at the starting time measured by the reference clock; and, 4) initial time phase of the satellite in its orbit. After defining these four initial conditions, the location of satellite subpoint is only a function of the time measured by the reference clock.

In Section 2.3 the specifications of any satellite subpoint trace are discussed and defined in terms of the four initial conditions. These specifications are: 1) the maximum latitude-to-latitude variation, 2) the width of the trace at the equator, 3) the maximum width of the trace and the necessary condition for its existence and 4) the node of the trace and the required conditions to exist.

Section 2.4 deals mainly with the plots of the SST. A computer program is used for this purpose assuming a constellation of four satellites initially located symmetrically around the equatorial plane. The plots are drawn for the case of 48, 24, 16 and 12 hour orbits with an inclination angle equal to 60° and for successive values of the time measured by the reference clock. The aim is to show the effect of the initial conditions upon both the shape of the trace and the direction of motion of the satellite subpoint on its trace.

2.2 SATELLITE SUBPOINT LOCUS

First, we assume that the earth is stationary and find the locus of the satellite subpoint. This locus is simply the projection of the orbit on the earth's surface, which is the great circle of intersection between the earth and the orbital plane. Second, we take account of the rotation of the earth. The earth rotates with angular velocity ω_E, and, after time t' rotates through the angle θ_E. Note that due to the rotation of the earth, each point on the projection is delayed by the corresponding rotation angle θ_E and this delay is opposite in direction to the rotation of the earth.

Assume the earth is stationary and the satellite subpoint starts at point S on the equator and moves to point S', in time t', as shown in Fig. 2.1. In moving from S to S', the satellite subpoint moves through an angle of Q which corresponds to a change in longitude of ϕ'_{Long}. S' is the intersection between a latitude circle, with radius RR and latitude angle ϕ_{Lat}, and a longitude circle corresponding to a variation ϕ'_{Long} in longitude, measured from the longitude of S. Note that ϕ'_{Long} is the projection of the angle Q, measured in the inclined orbital plane, on the equatorial plane.

From the geometry illustrated in Fig. 2.2, we find

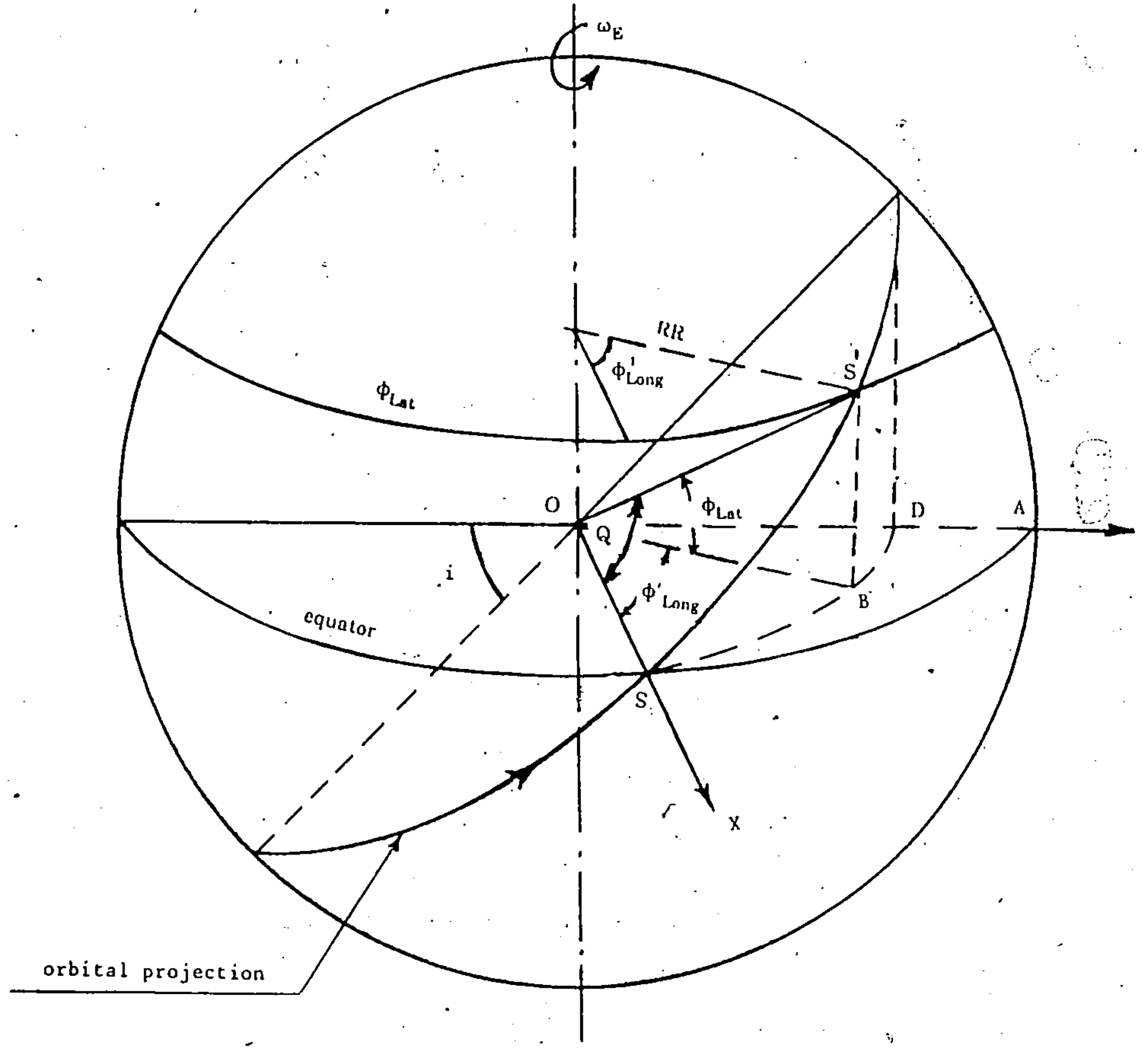

Fig. 2.1 Geometry of satellite sub-point assuming that the earth is stationary.

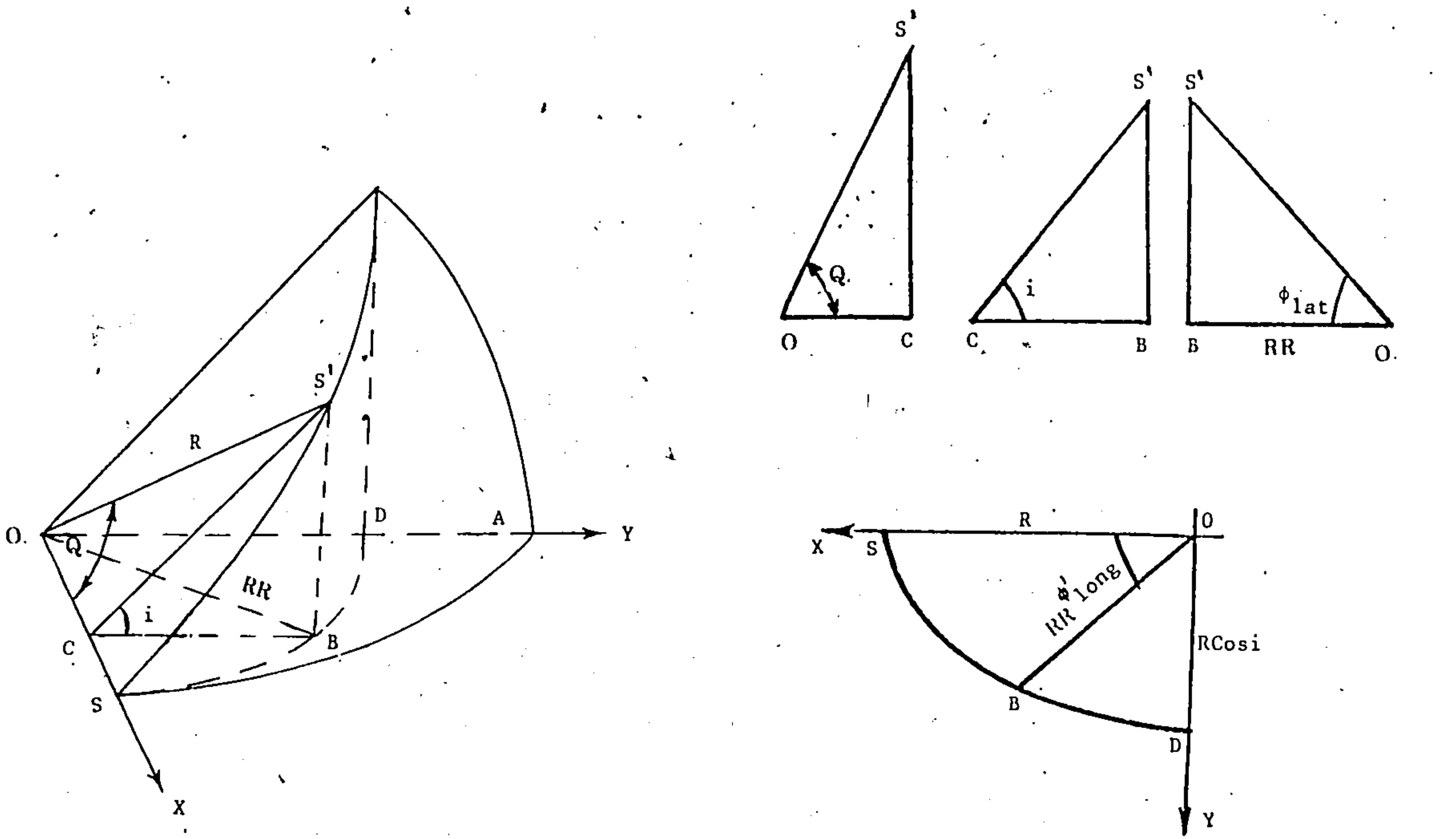

Fig. 2.2 Geometrical relations.

$$S'C = R \sin Q \text{ where R is the radius of the earth.}$$

Since the angle S'CB is equal to the inclination angle i, then the distance S'B, is given by

$$S'B = R \sin Q \sin i$$

and

$$\phi_{Lat} = \arc\sin (S'B/R)$$

or

$$\phi_{Lat} = \arc\sin [\sin Q \sin i] \tag{2.1}$$

Also,

$$RR = R\sqrt{1 - \sin^2 Q \sin^2 i} \tag{2.2}$$

The projection of the locus of the point S' on the equatorial plane is an ellipse with the following equation in polar coordinates

$$\frac{RR^2}{R^2}\left(\cos^2 \phi'_{Long} + \frac{\sin^2 \phi'_{Long}}{\cos^2 i}\right) = 1 \tag{2.3}$$

Substituting for RR in (2.3), from (2.2) and solving for ϕ'_{Long}, we get

$$\phi'_{Long} = \arc\sin\left[\frac{\cos i \sin Q}{\sqrt{1 - \sin^2 Q \sin^2 i}}\right] \tag{2.4}$$

So far, we have assumed that the earth is stationary. Now, in order to find the exact longitude variation ϕ''_{Long}, we subtract θ_E from ϕ'_{Long} (θ_E is the angle of rotation of the earth). Thus,

$$\phi''_{Long} = \arc\sin\left[\frac{\cos i \sin Q}{\sqrt{1 - \sin^2 Q \sin^2 i}}\right] - \theta_E \tag{2.5}$$

Note that the latitude of S' (ϕ_{Lat}) does not vary with the rotation of the earth. Substituting $Q = K\theta_E$ in Eq. (2.5) yields

$$\phi''_{Long} = \arc\sin\left[\frac{\cos i \sin K\theta_E}{(1 - \sin^2 i \sin^2 K\theta_E)^{1/2}}\right] - \theta_E \tag{2.6}$$

where K = nonsynchronous factor [19], which is the ratio between the satellite angular velocity ω_S and the earth angular velocity ω_E. Also K is related to the satellite altitude h by the formula

$$K = [(hsyn + R)/(h + R)]^{3/2} \tag{2.7}$$

where

hsyn = the altitude of the satellite in a 24 hour orbit

= 35784 km

R = the radius of the earth

= 6378 km.

Equations (2.1) and (2.6) give the locus of the satellite subpoint on the surface of the earth when the satellite subpoint crosses the equator at zero longitude and when t′ = 0. For the satellite at S′, it is clear that

$$\theta_E = \omega_E t'$$

and (2.8)

$$Q = \omega_S t'$$

Suppose that the satellite subpoint crosses the equator at a point with longitude ϕ_o'. We obtain the absolute value of the satellite subpoint longitude at time t′ by adding ϕ_o' to ϕ''_{Long} in Eq. (2.6) as follows:

$$\phi_{Long} = \phi''_{Long} + \phi_o'$$

or

$$\phi_{Long} = \arcsin \left| \frac{\cos i \sin K\theta_E}{(1 - \sin^2 i \sin^2 K\theta_E)^{1/2}} \right| - \theta_E + \phi_o' \tag{2.9}$$

where ϕ_{Long} = the value of the satellite subpoint longitude at time t′. In order to put Eq. (2.9) in the general form, let us assume that a reference clock is used, by an earth station, and such a clock tracks the satellite constellation.

At the instant $t = 0$, measured by the reference clock, it is clear that every satellite will be located at a specified point in its orbit. Now consider the case of a leading satellite, which passes through the equatorial plane at $t = -TT$ as shown in Fig. 2.3, where the time TT is simply equal to Q'/ω_S. By making use of Eq. (2.8), one can rewrite Eq. (2.1) as follows (noting $t' = t + TT$ and $Q = K\theta_E$):

$$\phi_{Lat} = \arcsin[\sin i \sin(K\omega_E (t + TT))] \tag{2.10}$$

Similarly, Eq. (2.9) will be

$$\phi_{Long} = \arcsin\left[\frac{\cos i \sin (K\omega_E (t + TT))}{[1 - \sin^2 i \sin^2 (K\omega_E (t + TT))]^{1/2}}\right] - \omega_E (t + TT) + \phi_o' \tag{2.11}$$

Note that if $TT > 0$, the satellite has a leading phase whereas if $TT < 0$, the satellite has a lagging phase. Also, TT is the time taken by any satellite from the point at which the satellite crosses the equatorial plane to its initial location at $t = 0$, where t is the time measured by the reference clock.

For every satellite in a given orbit, there is a specific value of phase angle ϕ'_o. It is more convenient, however, to find ϕ_o' in terms of TT.

Figure 2.4 shows the satellite subpoint S'_{TT} at the time $t = -TT$, and the satellite subpoint S' at $t = 0$. Let ϕ_o be the longitude at point P', which is the point of intersection between the equator and the orbital projection on the surface of the earth at $t = 0$. Similarly, when $t = -TT$, the point of intersection is at longitude ϕ'_o. During this interval of time, the earth rotates through an angle equal to $(-\omega_E TT)$ measured between OP' and OS'_{TT}, where O is the center of the earth.

Hence, we can get the following relation.

$$\phi'_o = \phi_o + \omega_E TT \tag{2.12}$$

Substituting Eq. (2.12) into Eq. (2.11) yields

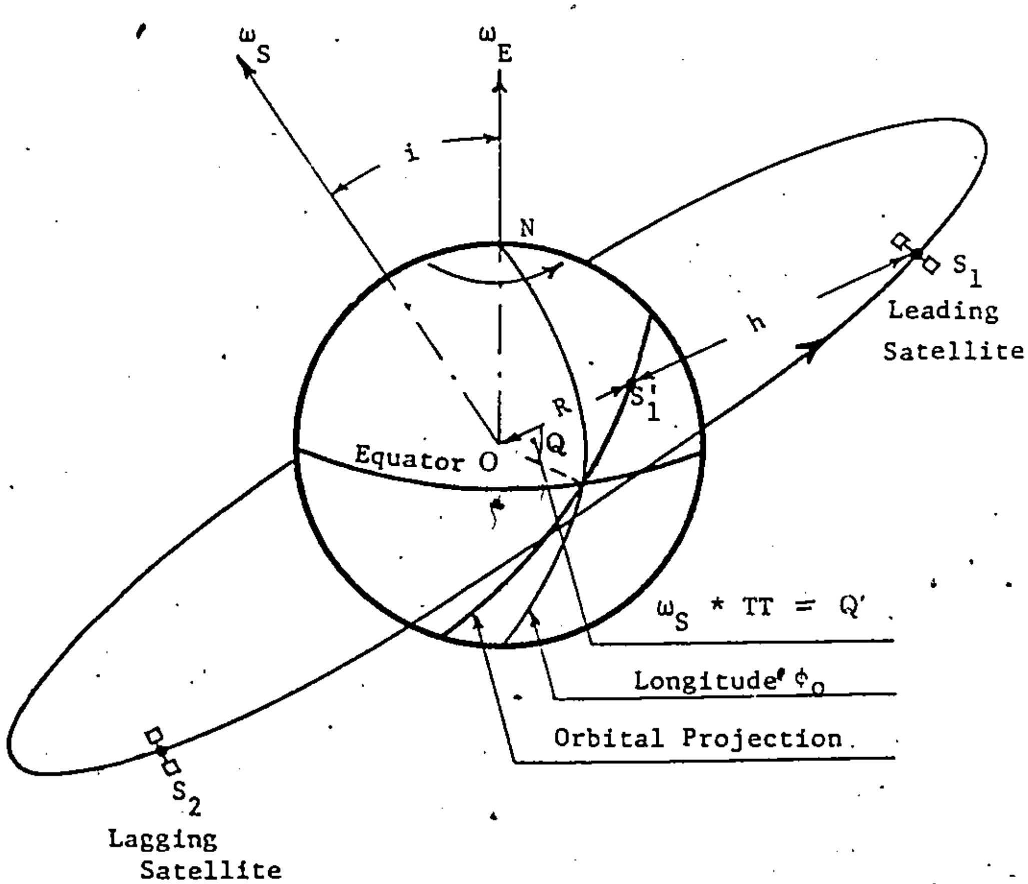

Fig. 2.3 Initial conditions required to define SST.

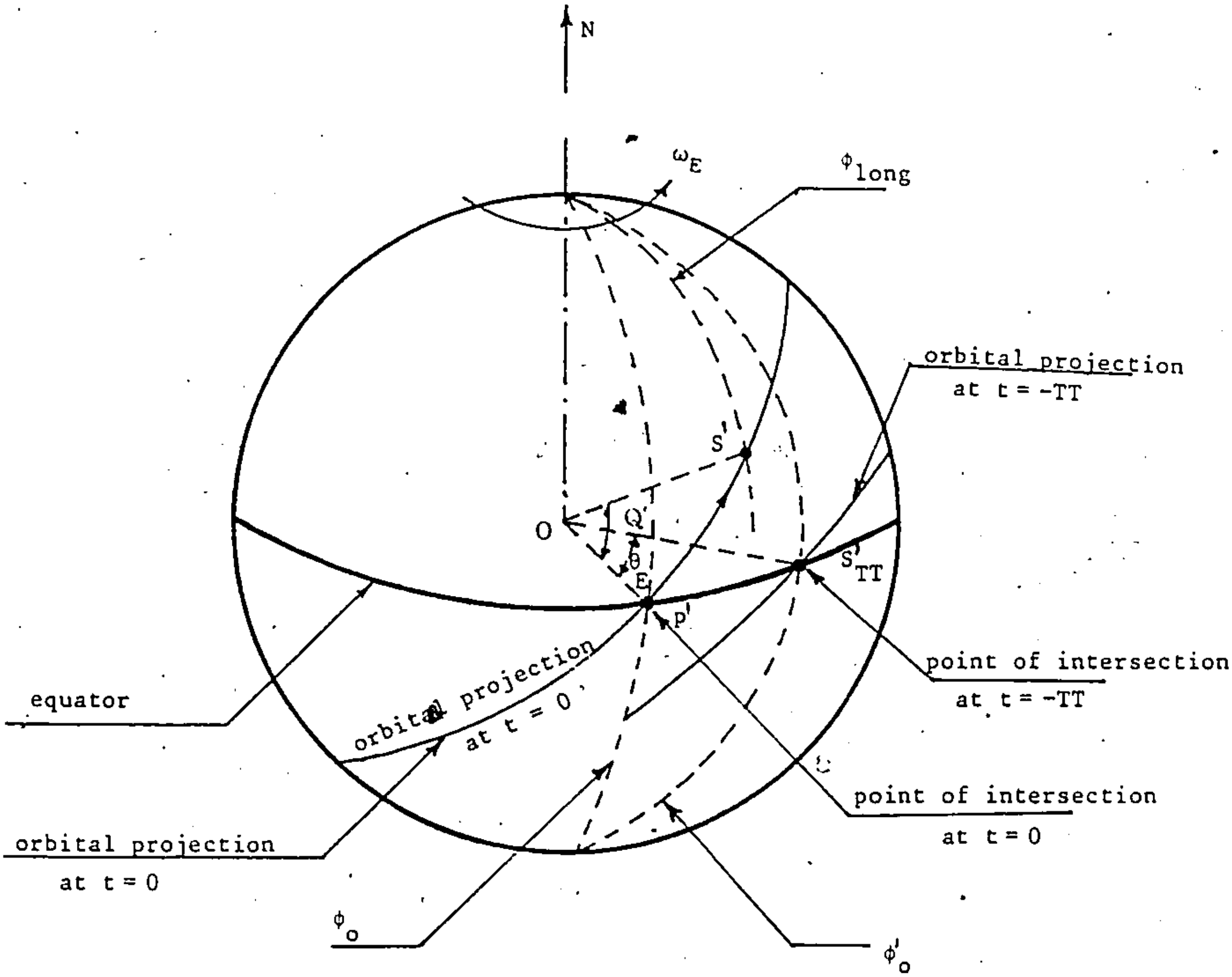

Fig. 2.4 Relation between the satellite subpoints at t = 0 and t = -TT.

$$\phi_{Long} = \arcsin\left[\frac{\cos i \sin (K\omega_E (t + TT))}{[1 - \sin^2 i \sin^2 (K\omega_E (t + TT))]^{1/2}}\right]$$
$$- \omega_E (t + TT) + (\phi_o + \omega_E TT)$$

After simplification, we have

$$\phi_{Long} = \arcsin\left[\frac{\cos i \sin (K\omega_E (t + TT))}{[1 - \sin^2 i \sin^2 (K\omega_E (t + TT))]^{1/2}}\right]$$
$$- \omega_E t + \phi_o \tag{2.13}$$

Equations (2.10) and (2.13) give the exact form for the satellite subpoint trace on the surface of the earth, at any altitude in any constellation.

To implement Eq. (2.10) and (2.13), two sets of initial conditions must be defined for each satellite, and they are:

1) The initial orbital location in space at time $t = 0$.

   This condition requires the following definitions:

   i) ϕ_o = the longitude of the point of intersection between the orbital projection on the surface of the earth and the equator at $t = 0$.

   ii) i = inclination angle.

   iii) K = nonsynchronous factor.

2) The initial time phase of each satellite in its orbit. This is the time taken for the satellite to move from its initial location at $t = 0$ to the point at which it crosses the equatorial plane. This time phase is given by TT. These initial conditions are shown in Fig. 2.3.

2.3 THE SPECIFICATION OF SST

In implementing Eq. (2.10) and (2.13), we use a rectangular map to represent the surface of the earth. All the easternly longitudes with respect to the prime meridian at Greenwich, are considered to be positive. Thus, the longitudes start with a value $-180°$ and end with a value $+180°$. In the same map we assume that all the northerly latitudes have positive signs, and the southerly latitudes will have negative signs.

As a general note for calculating the value of the satellite subpoint longitude ϕ_{Long}, great care must be taken when evaluating the angle

$$\arcsin\left[\frac{\cos i \sin(K\omega_E(t+TT))}{[1-\sin^2 i \sin^2(K\omega_E(t+TT))]^{1/2}}\right]$$

in the right hand side of Eq. (2.13). There are two possible angles with the same sine. For this reason Eq. (2.13) will be written with the following modifications:

1. If $\pi/2 \geq Q \geq -\pi/2$

$$\phi_{Long} = \arcsin\left[\frac{\cos i \sin(K\omega_E(t+TT))}{[1-\sin^2 i \sin^2(K\omega_E(t+TT))]^{1/2}}\right] - \omega_E t + \phi_o \tag{2.14}$$

2. If $3\pi/2 > Q > \pi/2$

$$\phi_{Long} = \pi - \arcsin\left[\frac{\cos i \sin(K\omega_E(t+TT))}{[1-\sin^2 i \sin^2(K\omega_E(t+TT))]^{1/2}}\right] - \omega_E t + \phi_o \tag{2.15}$$

Let us examine some specific cases.

I. The Peak-to-Peak Latitude Variation

This is height of the trace, recalling Eq. (2.10) which is given by

$$\phi_{Lat} = \arcsin[\sin i \sin[K\omega_E(t+TT)]]$$

Since the time t is the only independent variable after the initial conditions are defined, we conclude that ϕ_{Lat} is maximum when $\sin(K\omega_E(t+TT)) = \pm 1$. Therefore,

$$\phi_{Lat\,max} = \pm i \tag{2.16}$$

and the peak-to-peak latitude variation equals twice the inclination angle i.

II. The Width of the Trace at the Equator

This is the variation in longitude between two successive intersections between the satellite subpoint trace and the equator.

Since any point on the equator has $\phi_{Lat} = 0$, from Eq. (2.10) we get

$$0 = \sin i \sin(K\omega_E(t + TT))$$

Assuming that $\sin i \neq 0$, i.e., the satellite orbit is not located in the equatorial plane.

$$0 = \sin(K\omega_E(t + TT)), \text{ thus}$$

$$Q = K\omega_E(t + TT) = n\pi \qquad ; n = 0, 1, 2, \ldots \tag{2.17}$$

or

$$t = \frac{n\pi}{K\omega_E} - TT \qquad ; n = 0, 1, 2, \ldots \tag{2.18}$$

For the first intersection $n = 0$, hence, $t = -TT$ and the corresponding longitude, according to Eq. (2.14), will be

$$\phi_{Long_I} = 0 + \omega_E TT + \phi_o \tag{2.19}$$

For the second intersection $n = 1$, hence $t = \pi/K\omega_E - TT$. But we notice here that $Q = \pi$ which is greater than $\pi/2$. From Eq. (2.15) we get

$$\phi_{Long_{II}} = \pi - \omega_E \frac{\pi}{K\omega_E} + \omega_E TT + \phi_o \tag{2.20}$$

The variation in longitude between ϕ_{LongI} and ϕ_{LongII} is then equal to the width of the trace at the equator, W_{EQ}, where

$$W_{EQ} = \phi_{LongII} - \phi_{LongI}$$

i.e.,

$$W_{EQ} = \pi - \omega_E \frac{\pi}{K\omega_E}$$

or

$$W_{EQ} = \pi\left(1 - \frac{1}{K}\right) \tag{2.21}$$

From Eq. (2.21) we find that if $K < 1$ (i.e. the altitude of the satellite is higher than that of the synchronous orbit) the value of W_{EQ} in Eq. (2.21) will have negative sign; thus, the satellite subpoint is always moving in the direction of negative longitude, i.e., it moves towards the west. If $K > 1$, W_{EQ} is positive and the satellite subpoint moves towards the east. For the case of $K = 1$, i.e., for the case of a 24 hour orbit, the width of the pattern at the equator, according to Eq. (2.21), W_{EQ} is equal to zero, and that is because of the motion of the satellite subpoint from the equator to the peak of the pattern is always towards the west (negative longitude). On the other hand, the motion from the peak to the equator is always towards the east (positive longitude). Thus, the satellite subpoint crosses the equator every 12 hours at the same point, which is the node of the pattern. In order to show the effect of the non-synchronous factor K upon the width of the trace at the equator W_{EQ}, Fig. 2.5 is drawn to show the SST for the case of three satellites with different values of K. Note that, in Fig. 2.5, the three satellites have the same inclination angle 60° and they start their motion from the same point on the equatorial plane at $t = 0$.

III. The Maximum Width of the Trace

The width of the trace can be defined as the difference in two consecutive values of longitude at a given latitude for a specific satellite subpoint trace (note that the width can be positive or negative). Let us recall Eq. (2.11), which is

$$\phi_{Long} = \arcsin\left[\frac{\cos i \sin Q}{[1 - \sin^2 i \sin^2 Q]^{1/2}}\right] - \frac{Q}{K} + \phi_o \tag{2.22}$$

where

$$Q = K\omega_E(t + TT)$$

In order to find the maximum width of the trace, i.e., the maximum longitude change in two consecutive quarters of the trace, we differentiate ϕ_{Long} in Eq. (2.22) with respect to Q and equate the result with zero, yielding

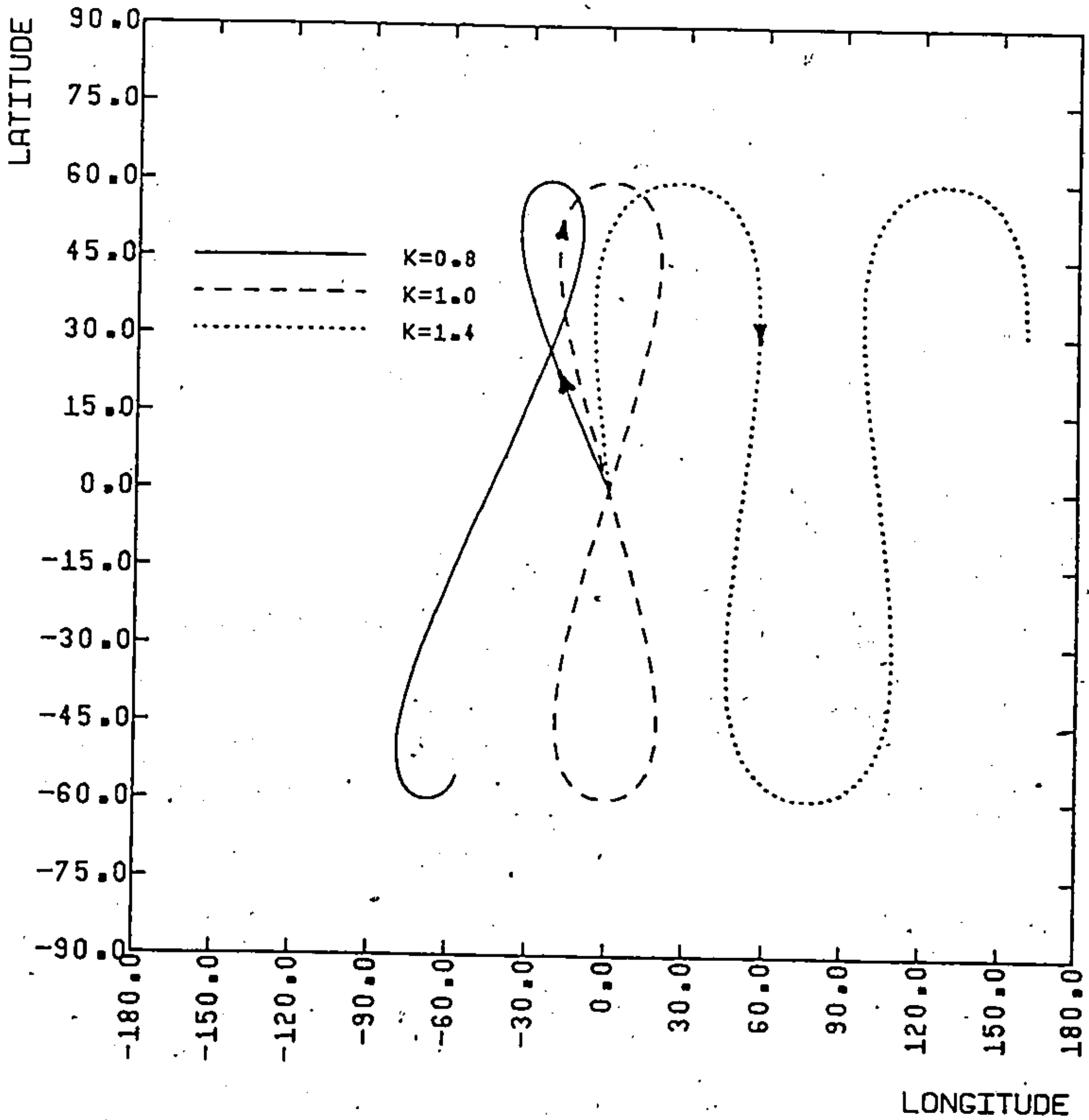

Fig. 2.5 SST for three satellites in inclined orbit with initial conditions $\Phi_o = 0°$, $i = 60°$, $TT = 0$ and $K = 0.8$, 1 and 1.4.

$$0 = \frac{1}{1 - \dfrac{\sin^2 Q \cos^2 i}{1 - \sin^2 Q \sin^2 i}} * \frac{\left[\left(-\dfrac{1}{2}\right)\cos i\,(-2)\,\dfrac{\cos Q}{\sin^3 Q}\right]}{\left(\dfrac{1}{\sin^2 Q} - \sin^2 i\right)^{3/2}} - \frac{1}{K} \tag{2.23}$$

Assuming that $\sin Q \neq 0$, Eq. (2.23) reduces to

$$\frac{1}{K} = \frac{1}{\left(\dfrac{\cos^2 Q}{1 - \sin^2 Q \sin^2 i}\right)^{1/2}} * \frac{\cos i \cos Q}{(1 - \sin^2 Q \sin^2 i)^{3/2}} \tag{2.24}$$

Assuming that $\cos Q \neq 0$, (i.e. the maximum width does not occur at the equator) implies

$$\frac{1}{K} = \frac{\cos i}{1 - \sin^2 i \sin^2 Q} \tag{2.25}$$

Therefore

$$\sin Q = \frac{\sqrt{1 - K\cos i}}{\sin i} \tag{2.26}$$

Substituting for $\sin Q$ in Eq. (2.22), we get the maximum value of the longitude $\phi_{\text{Long max}}$, at which the maximum width occurs

$$\phi_{\text{Long max}} = \arcsin\left(\sqrt{\frac{1 - K\cos i}{K\cos i}} * \frac{\cos i}{\sin i}\right) - \frac{1}{K}\arcsin\left(\frac{\sqrt{1 - K\cos i}}{\sin i}\right) + \phi_o \tag{2.27}$$

Equation (2.27) gives the value of the longitude at which the maximum occurs. Hence, to find the maximum width of the trace, W_m; we must find the difference between $\phi_{\text{Long max}}$ and the longitude of the peak of the trace. The peak of the trace occurs when $Q = \pi/2$, hence; the corresponding longitude will be $(\pi/2)(1 - 1/K) + \phi_o$.

Thus,

$$\begin{aligned} W_m &= 2 * (\text{the longitude of the peak} - \phi_{\text{Long max}}) \\ &= \pi\left(1 - \frac{1}{K}\right) - 2\arcsin\left(\sqrt{\frac{1 - K\cos i}{K\cos i}} * \frac{\cos i}{\sin i}\right) \\ &\quad + \frac{2}{K}\arcsin\left(\frac{\sqrt{1 - K\cos i}}{\sin i}\right) \end{aligned} \tag{2.28}$$

Equation (2.28) provides the maximum width of the trace as well as the upper and lower values for the non-synchronous factor K, to guarantee the presence of maximum width. This implies that the maximum should occur for $0 < Q < \pi/2$. The lower limit for K can be found as follows:

Note that $1 - K\cos i \geq 0$ in Eq. (2.28) for a solution to exist.

Thus,

$$\frac{1}{K} \geq \cos i \tag{2.29}$$

To find the upper limit for K, from Eq. (2.28), we consider the term

$$\arcsin\left[\sqrt{\frac{1-K\cos i}{K\cos i}} \cdot \frac{\cos i}{\sin i}\right]$$

Note that the maximum value for the sine is equal to 1. Consequently,

$$\left(\sqrt{\frac{1-K\cos i}{K\cos i}} \cdot \frac{\cos i}{\sin i}\right)^2 \leq 1$$

or

$$\frac{1}{K\cos i} \leq 1 + \tan^2 i$$

Therefore,

$$\sec i \geq \frac{1}{K} \tag{2.30}$$

Combining Eq. (2.29) and (2.30), we get

$$\sec i \geq K \geq \cos i \tag{2.31}$$

Equation (2.31) gives the upper and the lower limits for the nonsynchronous factor K (in terms of i) to guarantee the presence of the maximum width for the (SST). This width occurs for the values of $\pi/2 > Q > 0$.

If $K = \cos i$, i.e. K has the critical value of the lower limit in Eq. (2.31), then substituting for this value of K in Eq. (2.26) and (2.28) yields

$$Q = \pi/2 \tag{2.32}$$

and

$$W_m = 0 \tag{2.33}$$

Equations (2.32) and (2.33) indicate that the maximum width of the pattern is zero and it occurs at the peak of the trace due to the following arguments. Since $K = \cos i$, i.e. K is less than unity, then the angular velocity of the satellite is slower than that of the earth. Hence, the satellite subpoint moves towards the negative longitude (towards the West). Thus the width of the trace is always less than or equal to zero. At the peak of the pattern, the width of the trace will be zero.

If $K = \sec i$, i.e. when K is equal to the upper limit of Eq. (2.31), by substituting for K in Eq. (2.26) and (2.28) we get

$$W_m = \pi\left(1 - \frac{1}{K}\right) \tag{2.34}$$

and

$$Q = n\pi \qquad ; n = 0, 1, 2, \ldots \tag{2.35}$$

Equations (2.34) and (2.35) are exactly the same as equations (2.21) and (2.17) respectively; thus, the maximum width is equal to the width of the trace at equator (W_{EQ}).

Now, if K meets the conditions of Eq. (2.31), the presence of a maximum in width for the trace is ensured.

IV. <u>The Node of the Trace</u>

The node of the trace is defined as the point of intersection between two successive quarters of the trace, i.e. the latitude of the node must satisfy Eq. (2.10) for the two consecutive quarters of the trace and its longitude must satisfy both Eq. (2.14) and (2.15).

To check the existence of a node in the trace let us assume that Q1 and Q2 are the angles rotated by the satellite (measured from the equatorial plane) when the satellite subpoint passes through the node during the first and the second quarters of the trace, respectively. Substituting Q1 and Q2 in Eq. (2.10) yields

$$\sin \phi_{Latn} = \sin i \sin Q1 = \sin i \sin Q2$$

where ϕ_{Latn} is the latitude of the node, therefore,

$$Q1 = \pi - Q2 \tag{2.36}$$

By substituting Q1 and Q2 in Eq. (2.14) and (2.15) respectively, and solving by making use of Eq. (2.36), we get the longitude of any node by the formula

$$\phi_{Longn} = \frac{\pi}{2}\left(1 - \frac{1}{K}\right) + \phi_o + \omega_E TT \tag{2.37}$$

This is the longitude of the peak of the trace. To find the value Q1 at which the node occurs, we substitute ϕ_{Longn} from Eq. (2.37) in Eq. (2.14), yielding

$$\frac{\pi}{2}\left(1 - \frac{1}{K}\right) = \arcsin\left[\frac{\cos i \sin Q1}{[1 - \sin^2 i \sin^2 Q1]^{1/2}}\right] - \frac{Q1}{K} \tag{2.38}$$

Equation (2.38) can be reduced to

$$\cos\left[\frac{1}{K}\left(\frac{\pi}{2} - Q1\right)\right] = \frac{\sin Q1 \cos i}{(1 - \sin^2 i \sin^2 Q1)^{1/2}} \tag{2.39}$$

Also

$$\sin\left[\frac{1}{K}\left(\frac{\pi}{2} - Q1\right)\right] = \frac{\cos Q1}{(1 - \sin^2 i \sin^2 Q1)^{1/2}} \tag{2.40}$$

Dividing Eq. (2.40) by Eq. (2.39), we get

$$\tan\left[\frac{1}{K}\left(\frac{\pi}{2} - Q1\right)\right] = \cot Q1 \sec i$$

or

$$\tan Q1 \tan\left[\frac{1}{K}\left(\frac{\pi}{2} - Q1\right)\right] = \sec i \tag{2.41}$$

Note here that $0 < Q1 < \pi/2$ and $\sec i \geq 1$; consequently, to find a solution for Eq. (2.41) K must be less than unity. Let t_{n1} be the time measured by the reference clock, when the satellite subpoint reaches the node, in the first quarter of the trace.

Noting that $Q1 = K\omega_E t_{n1}$ and $\omega_E = \pi/12$ rad/hr, then t_{n1} can be calculated by the formula

$$t_{n1} = \frac{12\,Q1}{\pi K} - TT \tag{2.42}$$

and

$$t_{n2} = \frac{12}{K} - t_{n1} - 2\,TT \qquad (2.43)$$

where t_{n2} is the time measured by the reference clock, when the satellite subpoint reaches the node of the trace in the second quarter. Figure 2.6 shows the relation between the nonsynchronous factor K and the time t_{n1} (assuming that TT = 0) for different values of inclination angle i.

Figure 2.7 shows the SST for the case of three satellites in a 48 hour orbit (K = 0.5) with inclination angles i = 40°, 60° and 80°. Note here that when the inclination angle i = 80°, we have

$$\sec 80° > K > \cos 80° \text{ and } K < 1.$$

Thus the presence of a node is guaranteed, which agrees with plots of Fig. 2.7. By using Fig. 2.6, we note the satellite subpoint takes 6.8 hours to reach the node for the case of inclination = 80°. Substituting for t = 6.8 hours in Eq. (2.10) we find the latitude of the node is approximately 50°, which agrees with the plots of Fig. 2.7.

2.4 COMPUTER PLOTS

A computer program has been developed to plot the satellite subpoint for a case of four satellites with equal angular spacing in the same orbit. They are initially located symmetrically around the equatorial plane at the instant t = 0. Thus, we can calculate the initial time phase TT for each satellite. The initial orbital location is defined by $\phi_0 = 0$ and i = 60°. We consider four values for the nonsynchronous factor K which are 0.5, 1.0, 1.5 and 2. For each value of K, the four satellite subpoint traces are plotted on the same graph.

Referring to Fig. 2.8, we find TT for each satellite as follows:

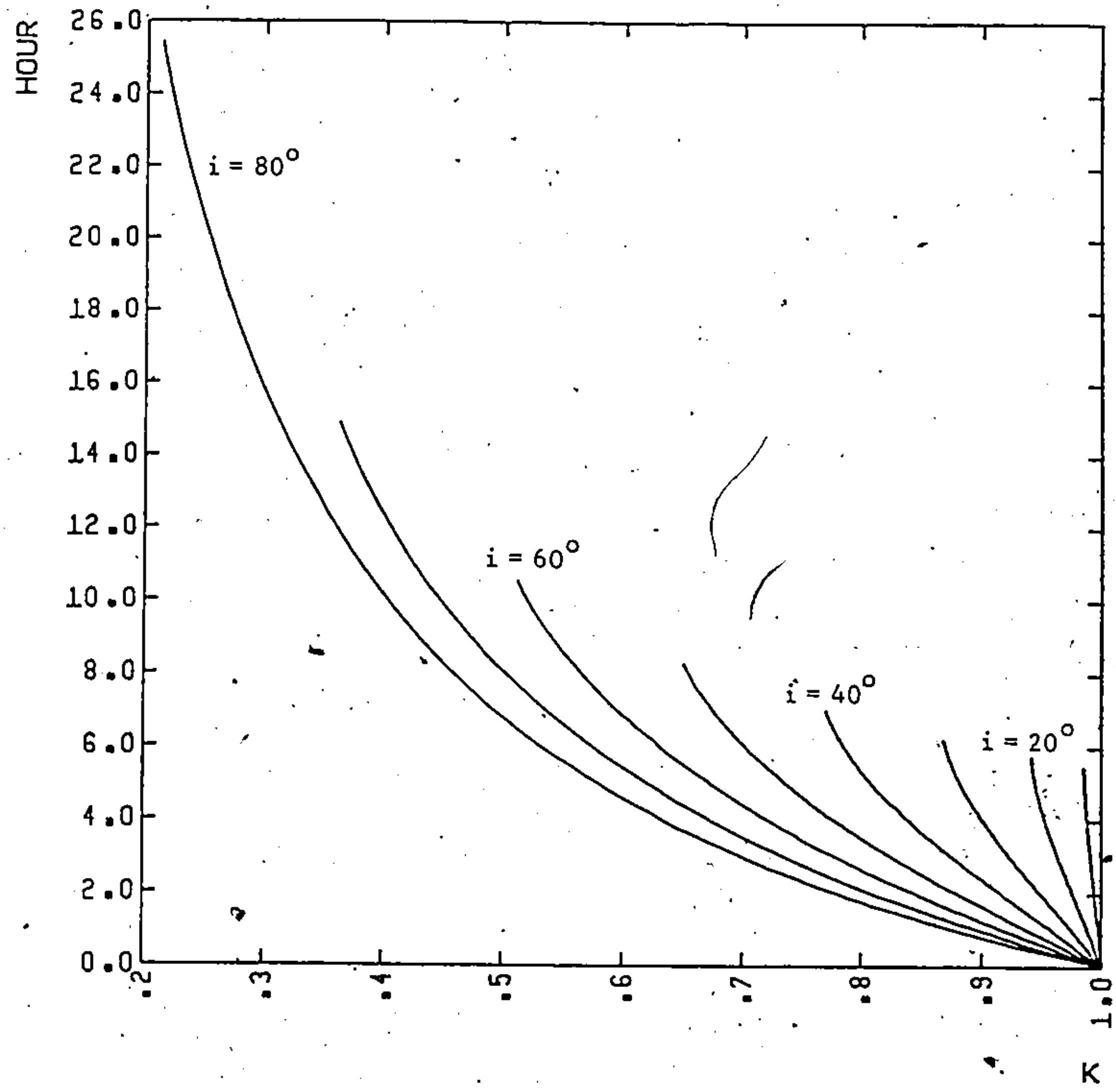

Fig. 2.6 Relation between K and t_{n_1} for different values of i.

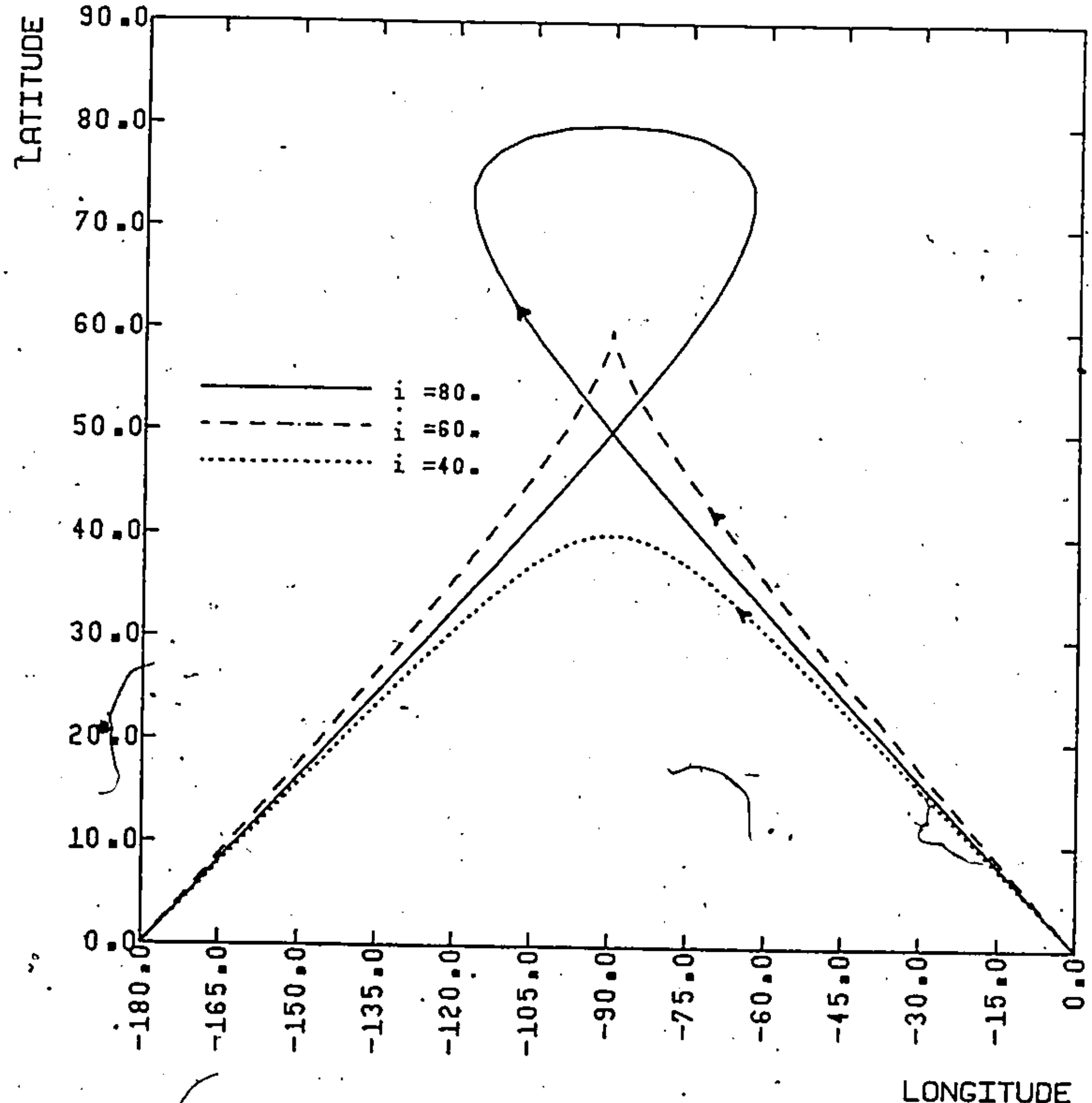

Fig. 2.7 SST for three satellites with initial conditions K = 0.5, TT = 0, $\Phi_o' = 0$ and i = 80°, 60° and 40°.

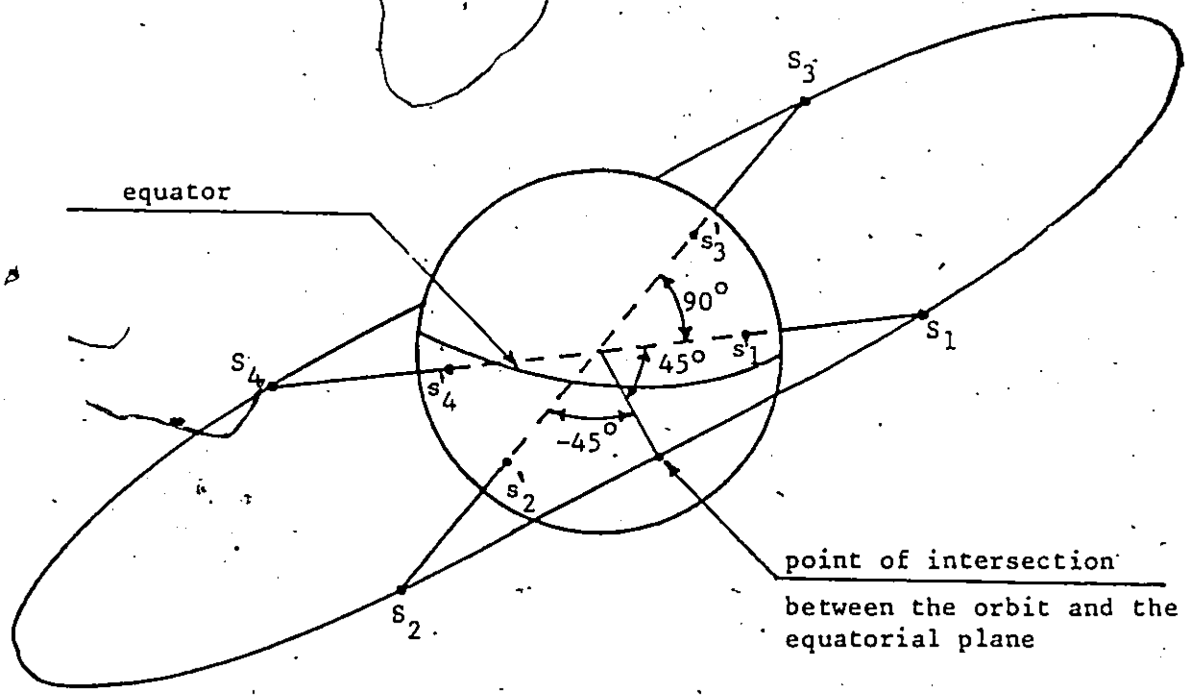

Fig. 2.8 Four satellites located symmetrically around the equatorial plane.

$$TT = \frac{Q'}{\omega_S} = \frac{Q'}{K\,\frac{2\pi}{24}}$$

$$= \frac{12}{\pi}\,\frac{Q'}{K}\ \text{hours} \tag{2.44}$$

In order to show the starting point and the direction of motion for each satellite subpoint, and the relative motion between the four satellite subpoints, the plot is drawn five times for progressively longer time intervals. Also, each satellite subpoint is calculated 72 times during each interval. Table 2.1 shows the values of these intervals and the period of time between two successive calculations.

Figures 2.9(a), through (e) show the satellite subpoint trace for the case of $K = 0.5$, which is the 48 hour orbit. S_1', S_2', S_3' and S_4' are the initial locations of the four satellite subpoints at the instant $t = 0$. The arrows indicate the direction of motion for each satellite subpoint. Using Eq. (2.44); the time phase TT of S_1 will be 6 hours, similarly, the corresponding values of TT for S_2, S_3 and S_4 are -6, 18, -18 hours, respectively. It is also clear, for all figures, that the maximum latitude reached by any satellite subpoint is 60°, which equals the inclination angle i. Also it is clear, from Fig. 2.9(e), that after 24 hours, each satellite completes one half of a revolution; hence, the satellite subpoint crosses the equator once. Note here that after 24 hours the earth completes one revolution and at the same time the satellite S_1 will reach the initial location of S_4. Similarly, S_4 will reach the initial location of S_1. The same situation holds for S_2 and S_3.

From Eq. (2.21), we can find the width of the pattern at the equator which is given by

$$W_{EQ} = \pi(1 - 1/K) = -\pi$$

This value of W_{EQ} agrees with the result of Fig. 2.9(a) to 2.9(e) which show the width W_{EQ} to be equal to $-\pi$.

The maximum width for the pattern W_m is given by Eq. (2.28) as follows:

TABLE 2.1

Interval			Period between Two Calculations
No.	From	To	
a	0.0	4.8 hours	4 min.
b	0.0	9.6 hours	8 min.
c	0.0	13.4 hours	12 min.
d	0.0	19.2 hours	16 min.
e	0.0	24.0 hours	20 min.

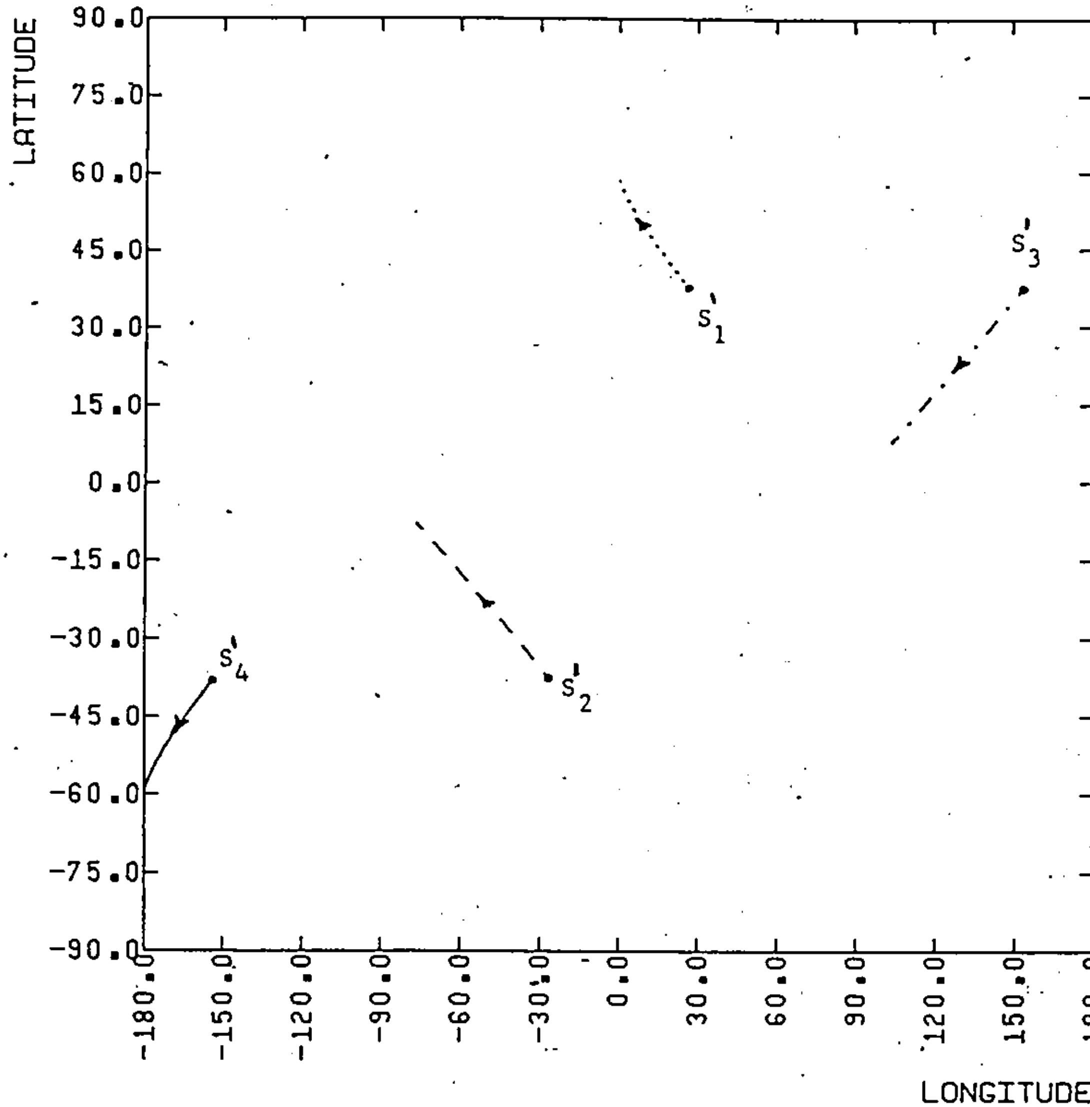

Fig. 2.9 (a) SST for 4 satellites, K = 0.5 and i = 60°, t from 0 to 4.8 hours,

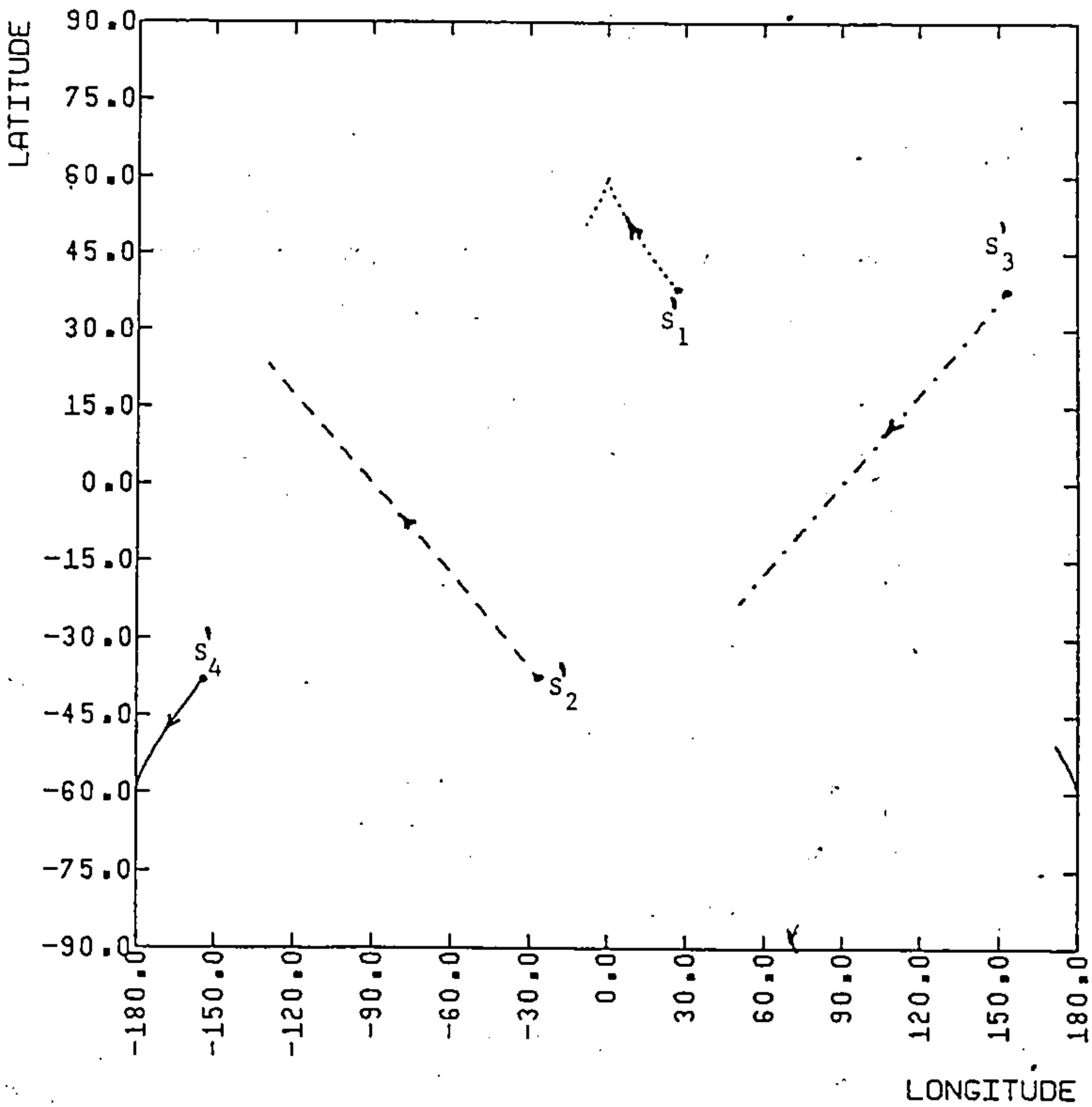

Fig. 2.9 (b) t from 0 to 9.6 hours

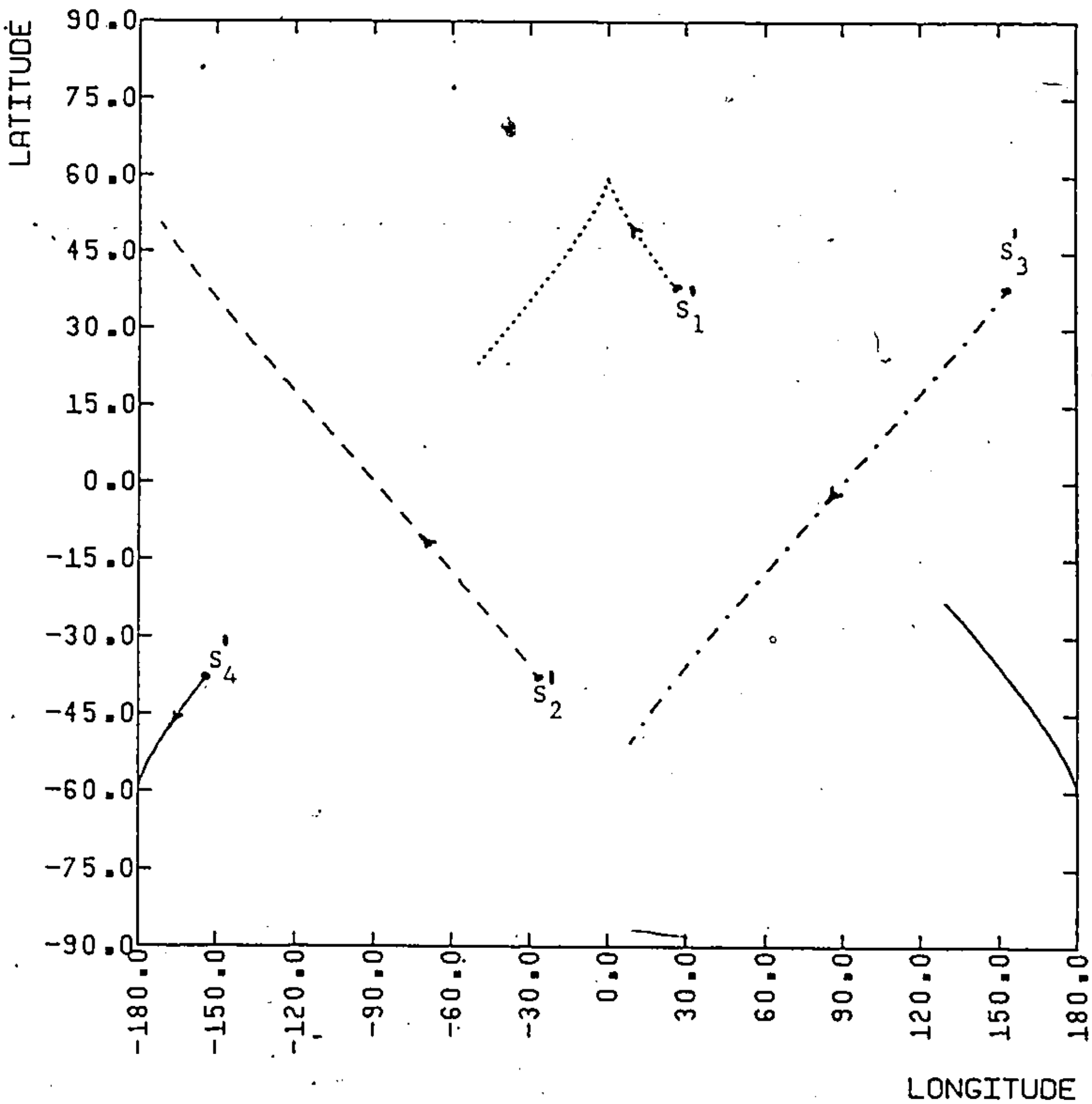

Fig. 2.9 (c) t from 0 to 14.4 hours

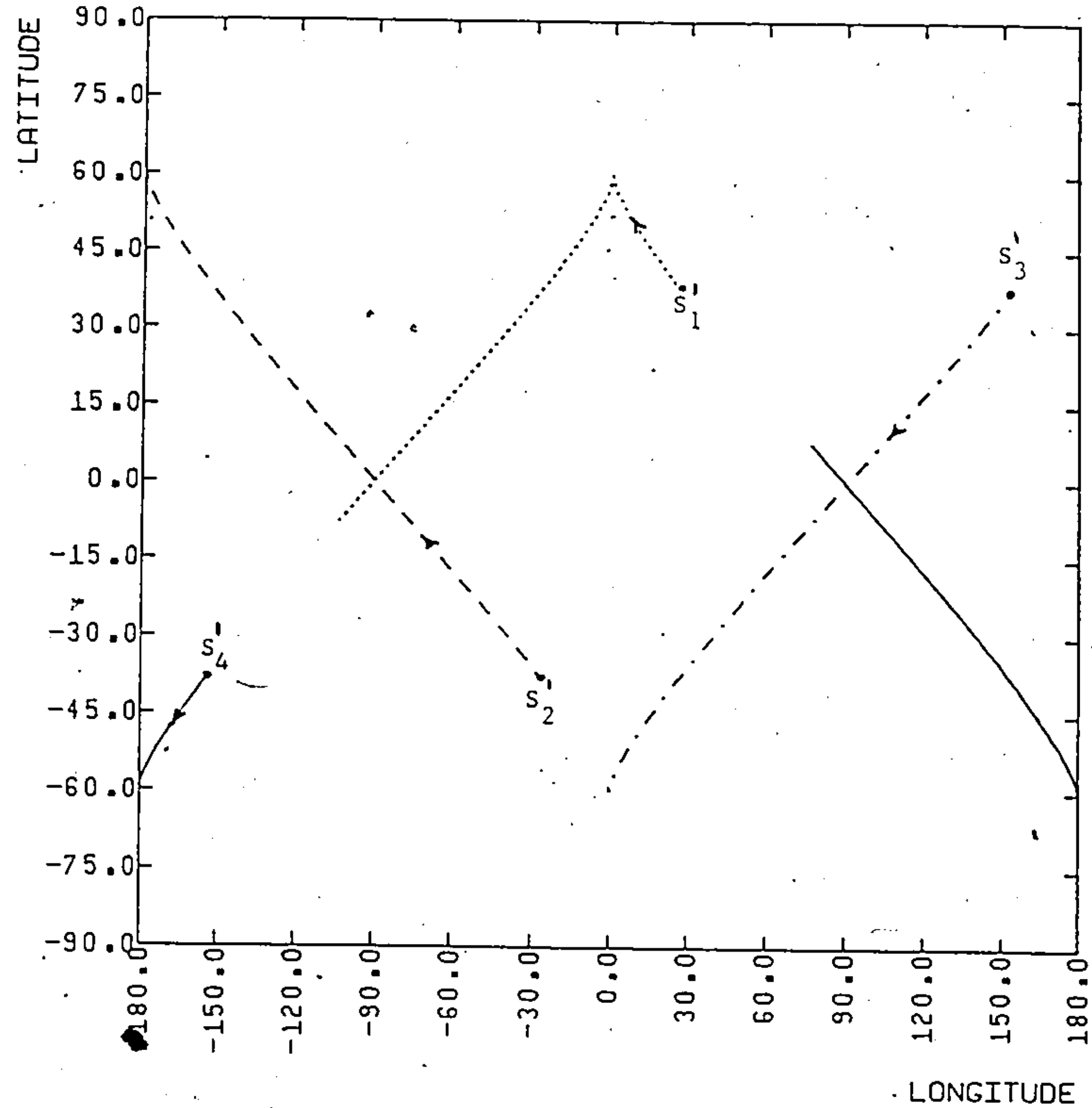

Fig. 2.9 (d) t from 0 to 19.2 hours

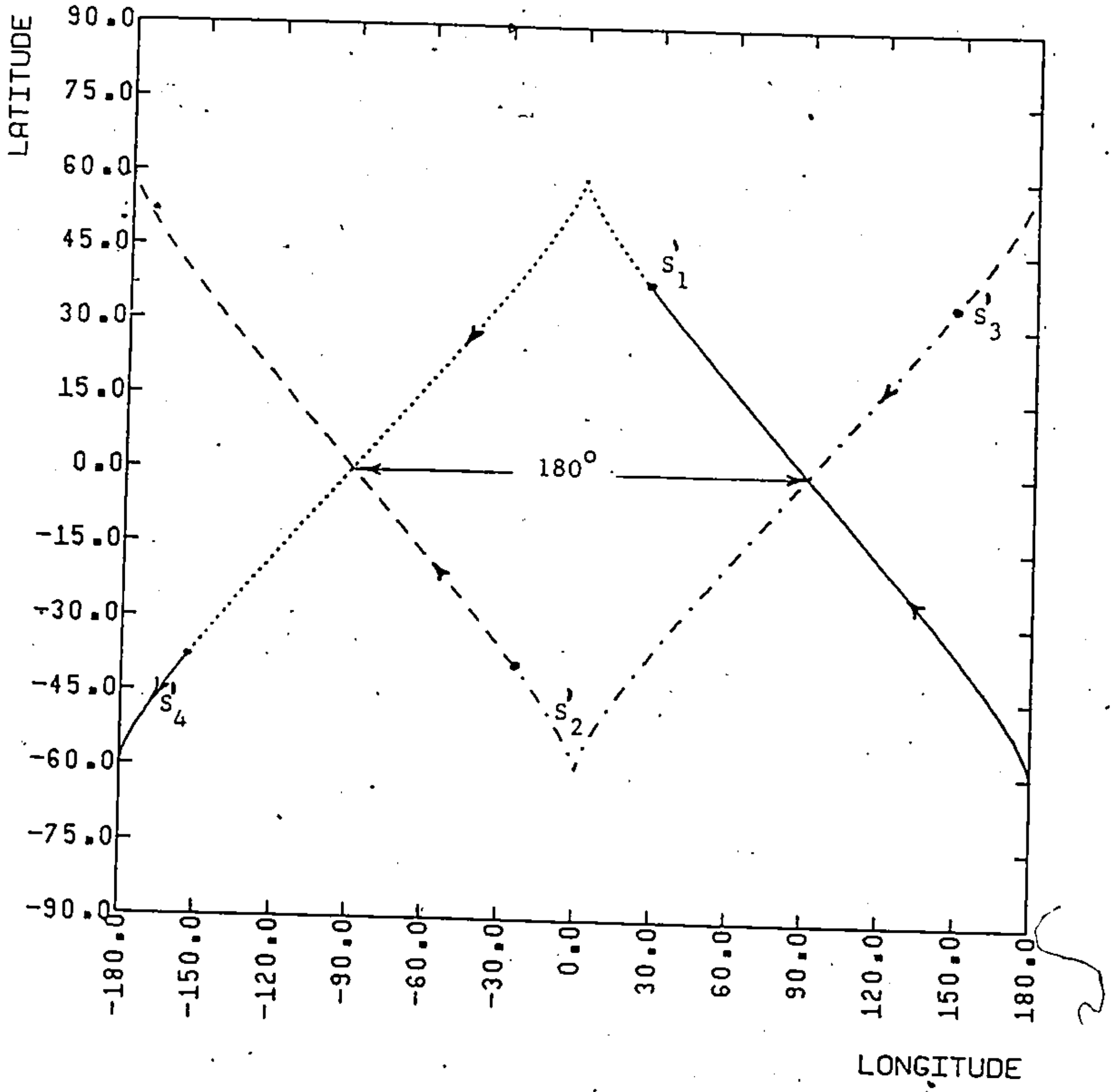

Fig. 2.9 (e) t from 0 to 24 hours

$$W_m = \pi\left(1 - \frac{1}{0.5}\right) - 2\arcsin\left(\frac{0.5}{\sqrt{3}/2} * \sqrt{3}\right) + \frac{2}{0.5}\arcsin(1.0) \qquad (2.45)$$
$$= 0$$

This arises due to the fact that K, which has a value of 0.5, meets the lower limit of Eq. (2.31), which is

$$2 \geq 0.5 = 0.5 \qquad (2.46)$$

The value of K is a critical value for the lower limit of Eq. (2.31), and the maximum width shrinks to zero at the peak of the pattern.

Figures 2.10(a), through (e) represent the satellite subpoint traces for the four satellites, when the nonsynchronous factor K is equal to unity (for example, the well known case of a 24 hour orbit). The satellite subpoint trace for each satellite will then have the shape of a figure 8. All the specifications for this case have been studied previously [19].

It is noted here, in Fig. 2.10(e), that the four nodes of the 8 patterns are separated by a 90° in longitude, since the four satellites are equally separated in their orbit.

Also, it is clear that the peak-to-peak latitude variation is equal to 120°, which equals twice the inclination angle i.

To check the width of each pattern at the equator, we substitute K = 1 in Eq. (2.21) to get

$$W_{EQ} = \pi(1 - 1/K) = 0° \qquad (2.47)$$

This is the width at the node, as shown in Figure 2.10(e).

Also, to check the presence of maximum width, we let K = 1 in Eq. (2.31)

$$\sec i \geq 1 \geq \cos i \qquad (2.48)$$

The last equation is valid for all values of i; thus, for the case of the 24 hour orbit, it is guaranteed to have a pattern with maximum width, and that maximum width occurs between the equator and the peak of the pattern. Substituting for K = 1, in Eq. (2.28) in order to find W_m yields

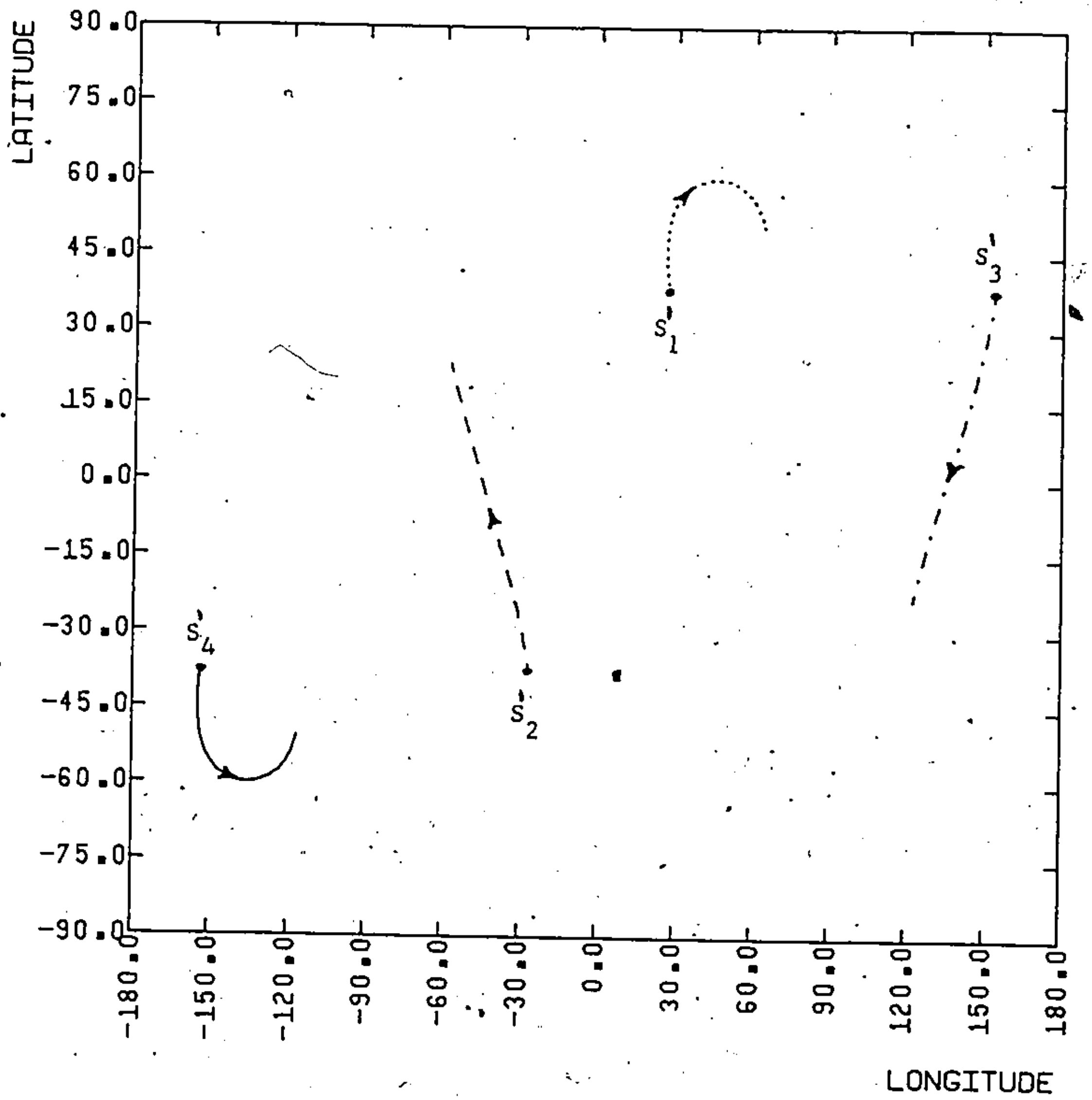

Fig. 2.10 (a)SST for 4 satellites, K = 1 and i = 60°, t from 0 to 4.8 hours

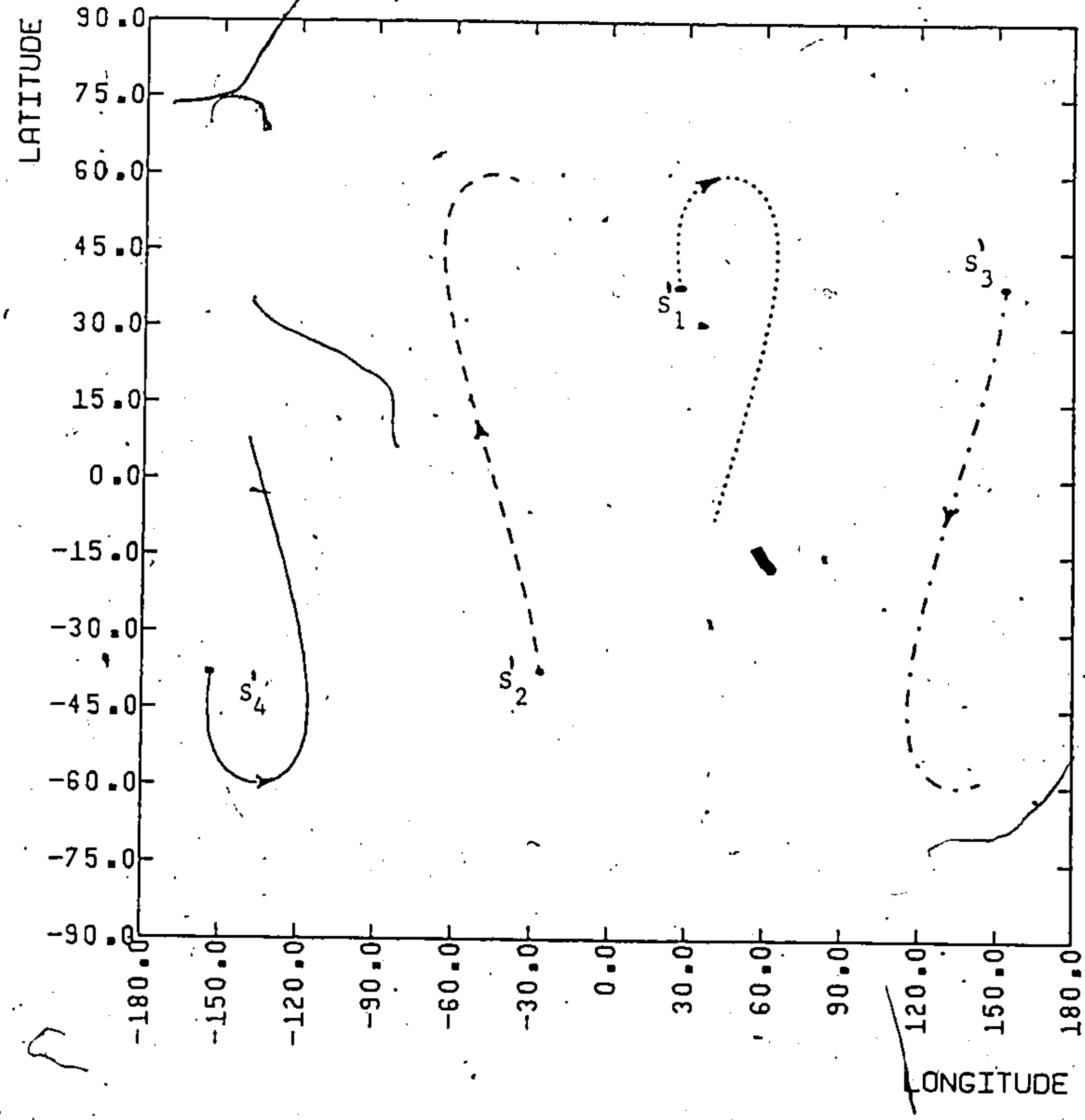

Fig. 2.10 (b) t from 0 to 9.6 hours

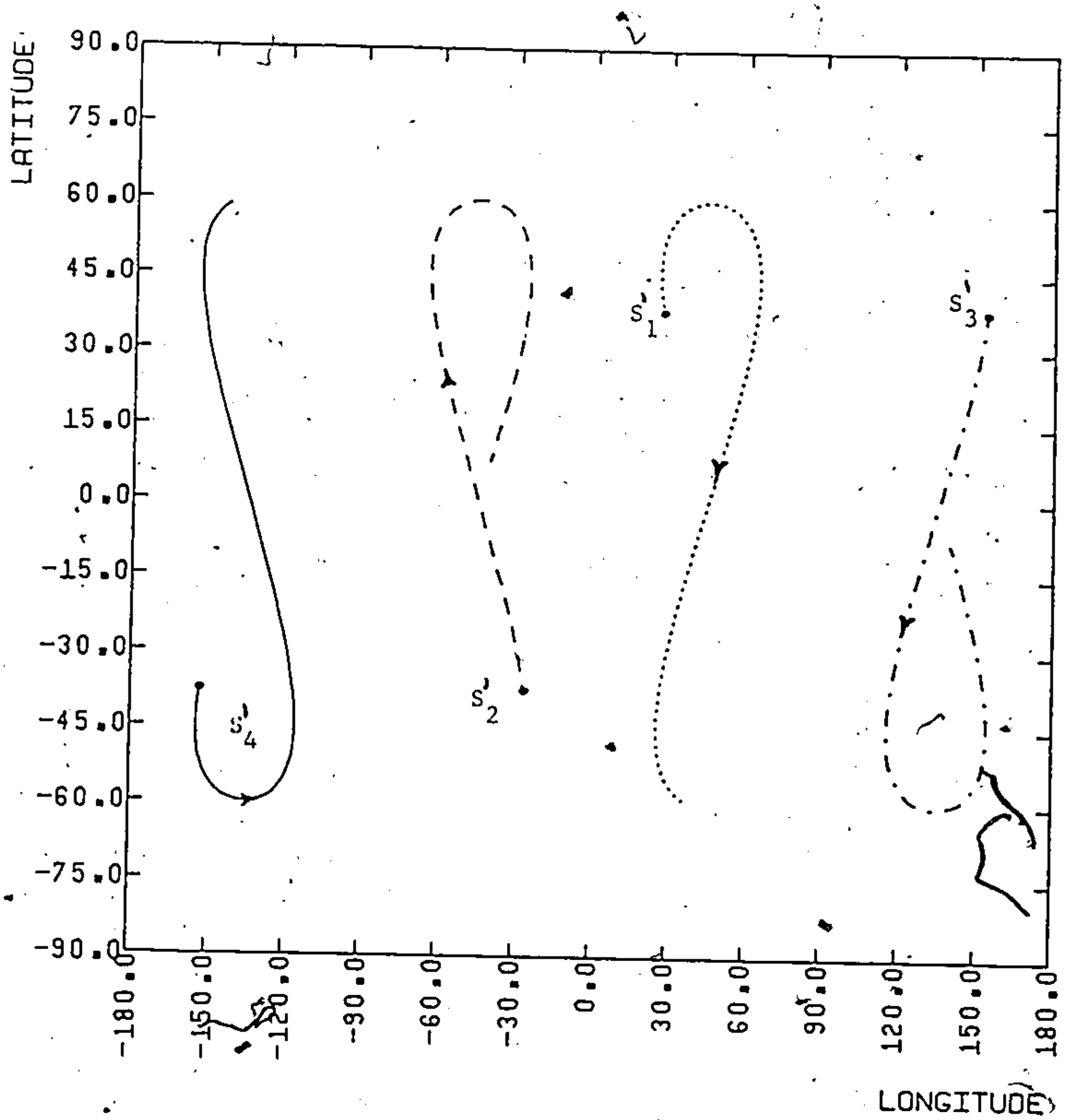

Fig. 2.10(c) t from 0 to 14.4 hours.

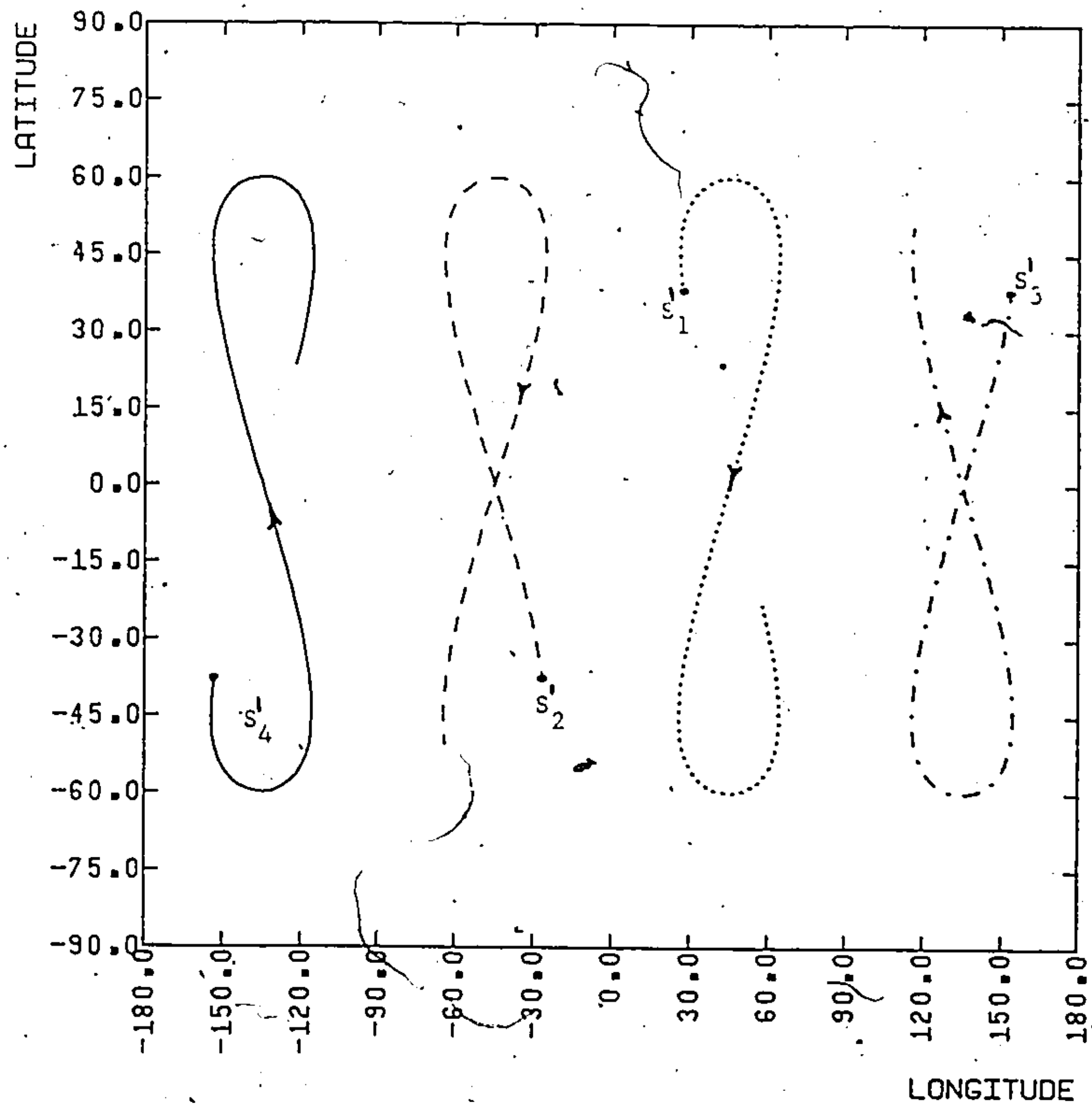

Fig.2.10 (d) t from 0 to 19.2 hours

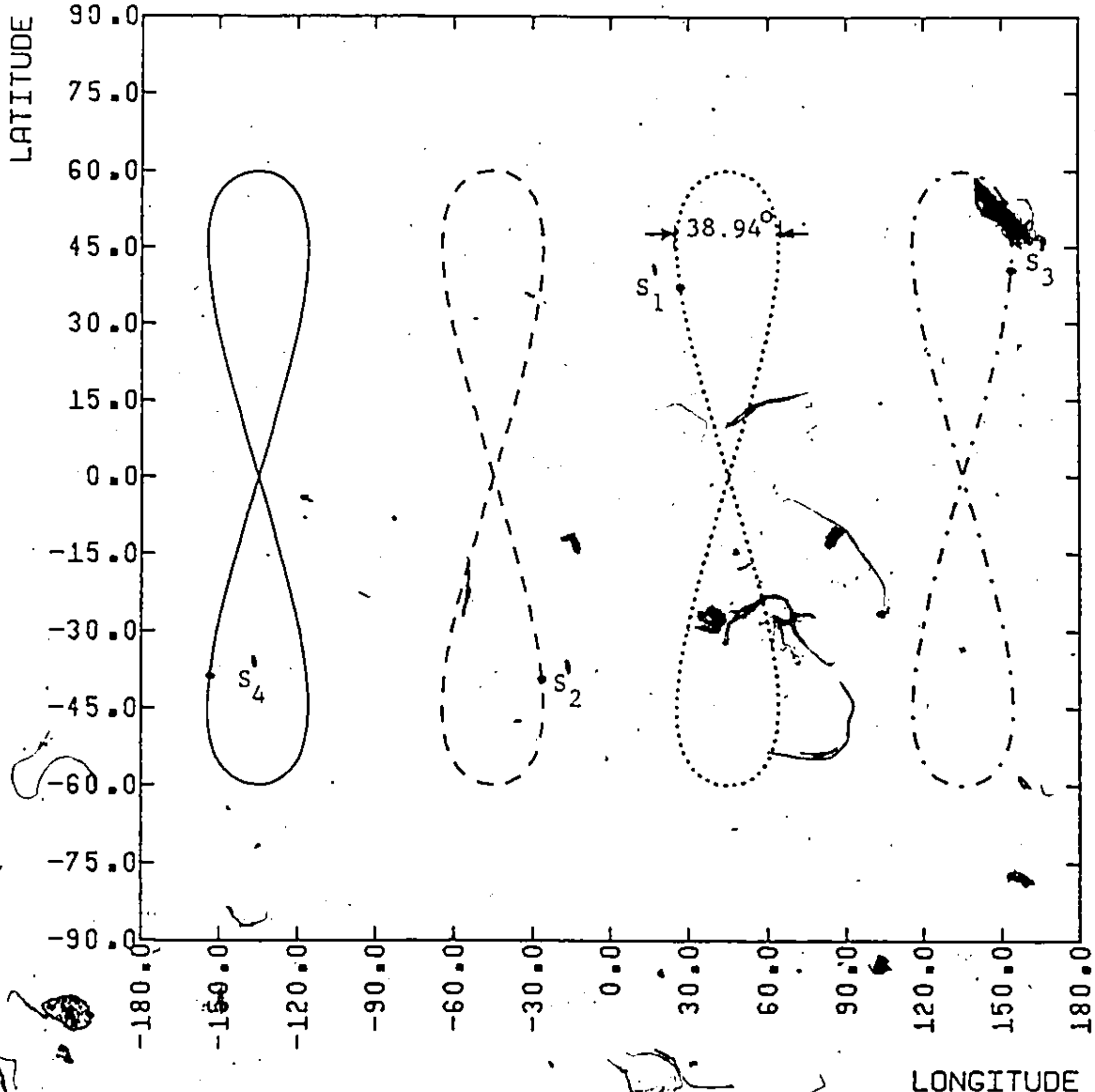

Fig. 2.10 (e) t from 0 to 24 hours

$$W_m = \pi(1-1) - 2\arcsin\left(\frac{\cos i}{\sin i} * \sqrt{\frac{1-\cos i}{\cos i}}\right)$$
$$+ 2\arcsin\left(\sqrt{\frac{1-\cos i}{\sin i}}\right)$$

or

$$W_m = 2\arcsin\left(\frac{1}{\sqrt{1+\cos i}}\right) - 2\arcsin\left(\sqrt{\frac{\cos i}{1+\cos i}}\right) \tag{2.49}$$

Similarly, from Eq. (2.26), we get

$$Q = \arcsin\left(\frac{1}{\sqrt{1+\cos i}}\right) \tag{2.50}$$

$$\frac{\pi}{2} - Q = \arcsin\left(\frac{\sqrt{\cos i}}{\sqrt{1+\cos i}}\right) \tag{2.51}$$

Substituting Eqs. (2.50) and (2.51) in Eq. (2.49) we get

$$W_m = 4Q - \pi \tag{2.52}$$

From Eq. (2.52), Q must be greater than $\pi/4$, i.e. the satellite takes more than 3 hours to move from the node of the Figure 8 to the point of maximum width, which agrees with previously obtained results [19,20].

Now, consider the case of inclination angle equal to 60° where the values for Q and W_m are Q = 54.7356° and W_m = 38.94°, respectively. These results agree with the values measured from Fig. 2.10(e).

Note here that the time phase, for the four satellites is given by Eq. (2.44) as follows:

$$S_1:\ TT1 = \frac{12}{\pi} * \frac{\pi/4}{K} = 3 \text{ hours}$$

$$S_2:\ TT2 = \frac{12}{\pi} * \frac{3\pi/4}{K} = 9 \text{ hours}$$

$$S_3:\ TT3 = \frac{12}{\pi} * \frac{-\pi/4}{K} = -3 \text{ hours}$$

$$S_4:\ TT4 = \frac{12}{\pi} * \frac{-3\pi/4}{K} = -9 \text{ hours}$$

Figure 2.11(a) through (e) represent the case of a nonsynchronous factor equal to 1.5 (the 16 hour orbit). According to Eq. (2.7), the altitude of the satellite will be 25798 km, which is less than that for the case of the synchronous orbit.

Since we have a 16 hour orbit, we expect that the satellite subpoint crosses the equator 3 times every 24 hours, and this is in agreement with Fig. 2.11(e).

Also, we notice here that the peak-to-peak latitude variation (as in the previous figures) equals 120°. The width of the pattern at the equator W_{EQ} can be found by substituting for K in Eq. (2.21).

$$W_{EQ} = (1 - 2/3) * \pi = 60° \tag{2.53}$$

which is exactly the same as shown in Fig. 2.11(e).

To check the presence of maximum width, we substitute for $K = 1.5$ in Eq. (2.31). This yields

$$2 > 1.5 > 0.5$$

Clearly, there is a maximum width between the equator and the peak. Substituting for K and i in Eq. (2.28), we get

$$\begin{aligned} W_m &= \pi\left(1 - \frac{2}{3}\right) - 2 * \arcsin\left(\frac{1}{3}\right) + \frac{4}{3} * \arcsin\left(\frac{1}{\sqrt{3}}\right) \\ &= 68.08° \end{aligned} \tag{2.54}$$

Note that W_m is greater than W_{EQ} as shown in Fig. 2.11(e).

Note here that the time phase TT for each satellite can be calculated using Eq. (2.44) as follows:

$$S_1: \ TT1 = \frac{12}{\pi} * \frac{Q'}{K} = \frac{12}{\pi} * \frac{0.25\pi}{1.5} = 2 \text{ hours}$$

$$S_2: \ TT2 = \frac{12}{\pi} * \frac{0.75\pi}{1.5} = 6 \text{ hours}$$

$$S_3: \ TT3 = \frac{12}{\pi} * \frac{-0.25\pi}{1.5} = -2 \text{ hours}$$

$$S_4: \ TT4 = \frac{12}{\pi} * \frac{-0.75\pi}{1.5} = -6 \text{ hours}$$

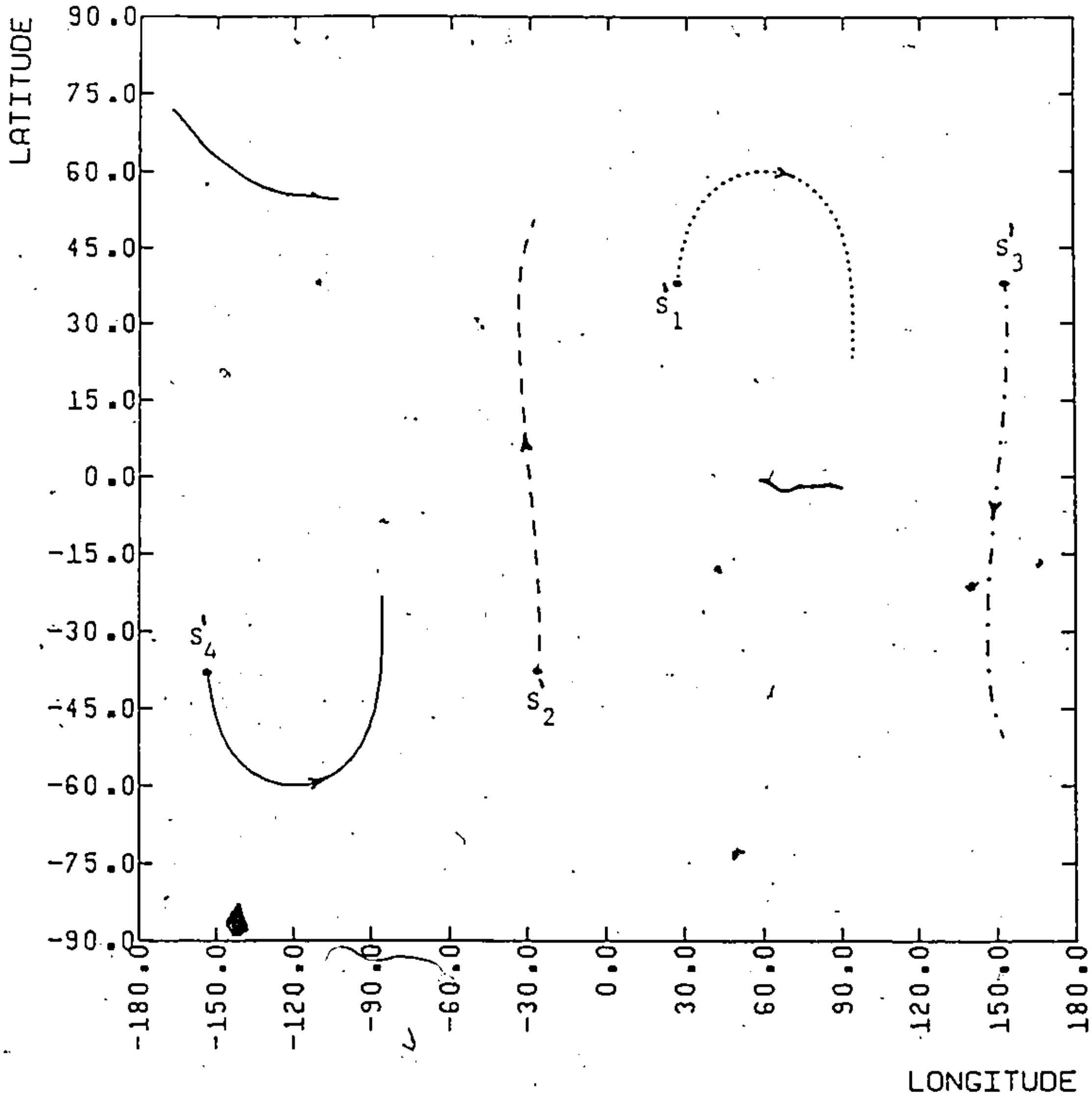

Fig. 2.11(a) SST for 4 satellites with K = 1.5 and i = 60°, t from 0 to 4.8 hours.

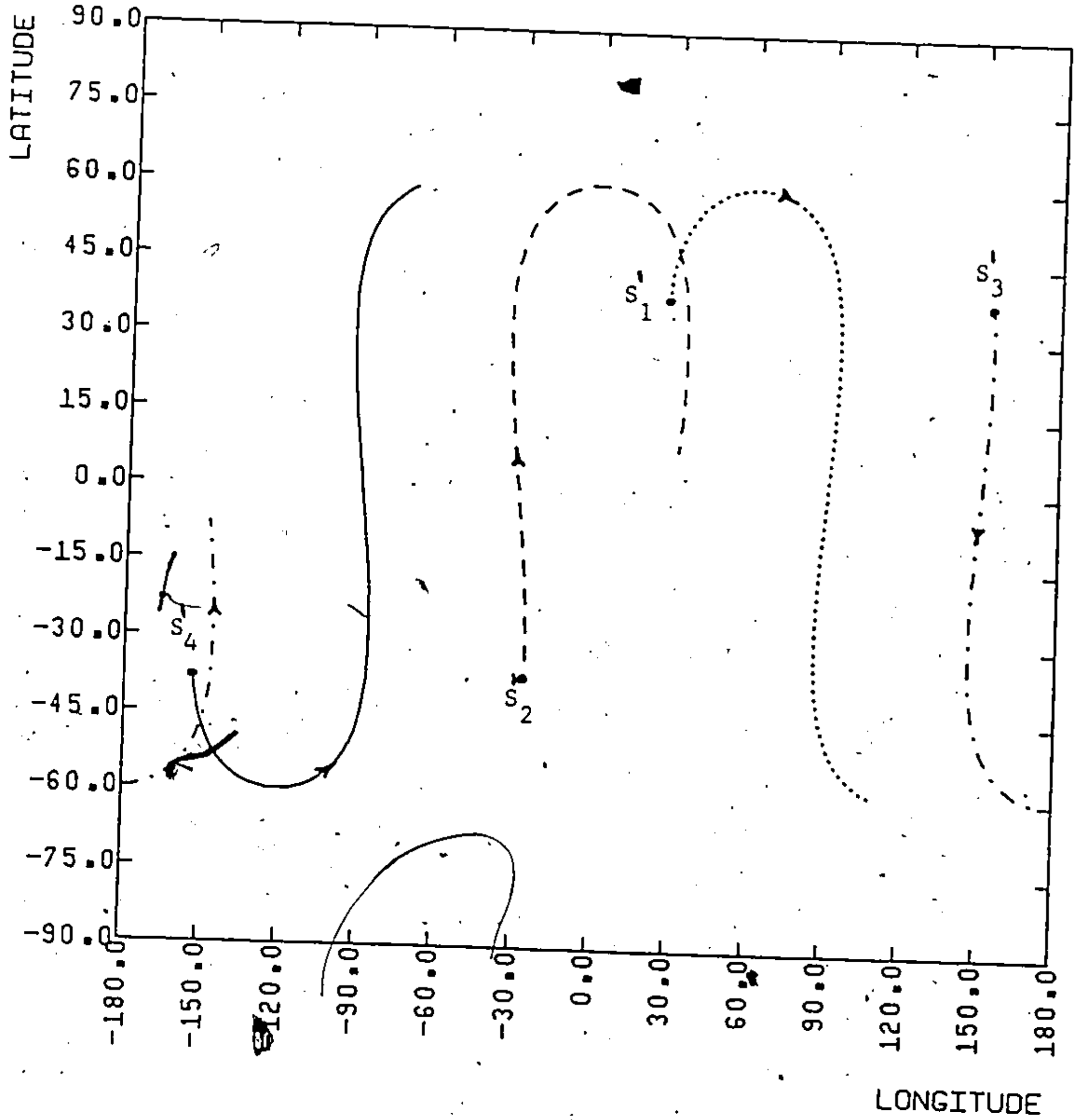

Fig. 2.11 (b) t from 0 to 9.6 hours

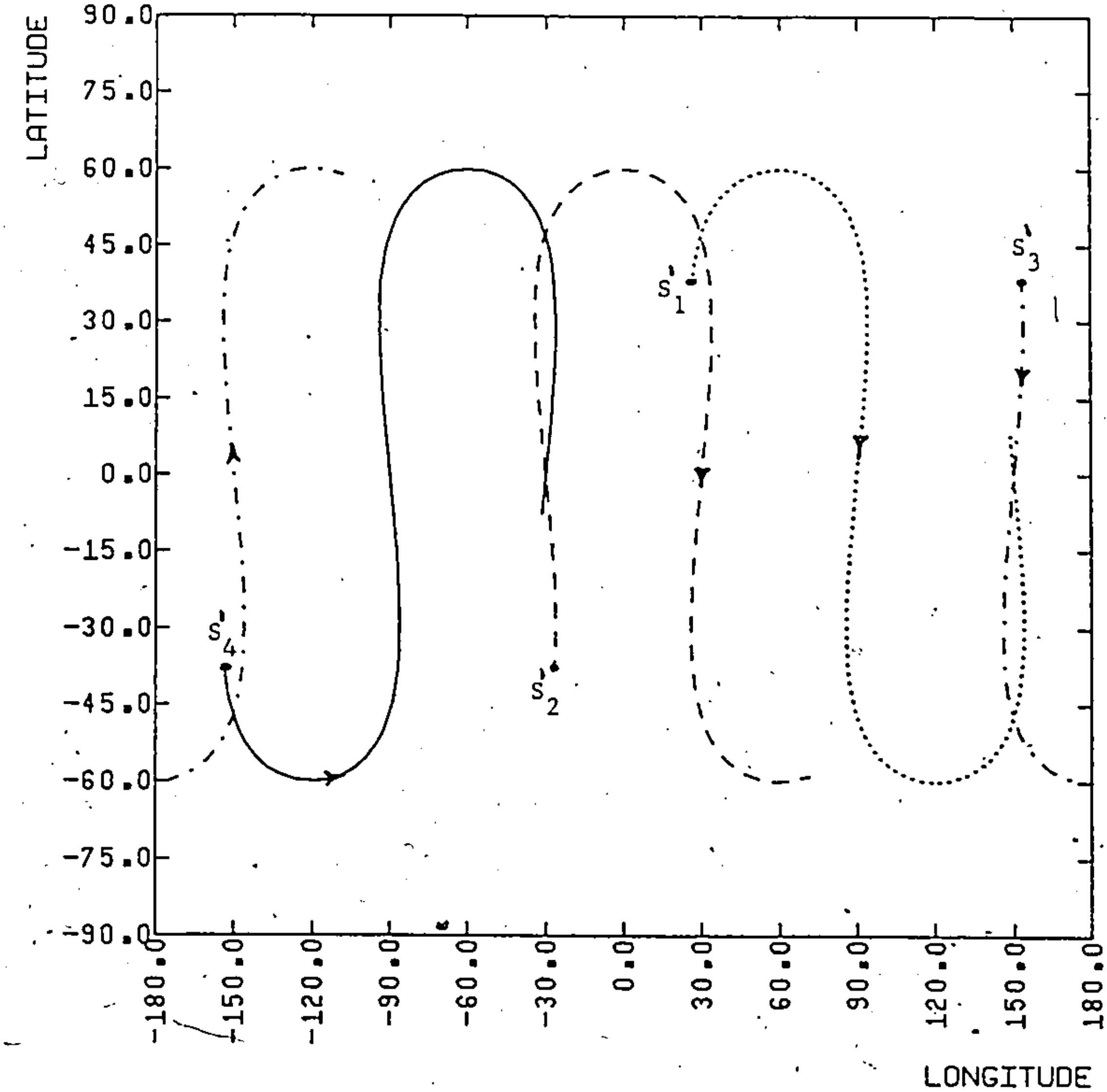

Fig.2.11 (c) t from 0 to 14.4 hours

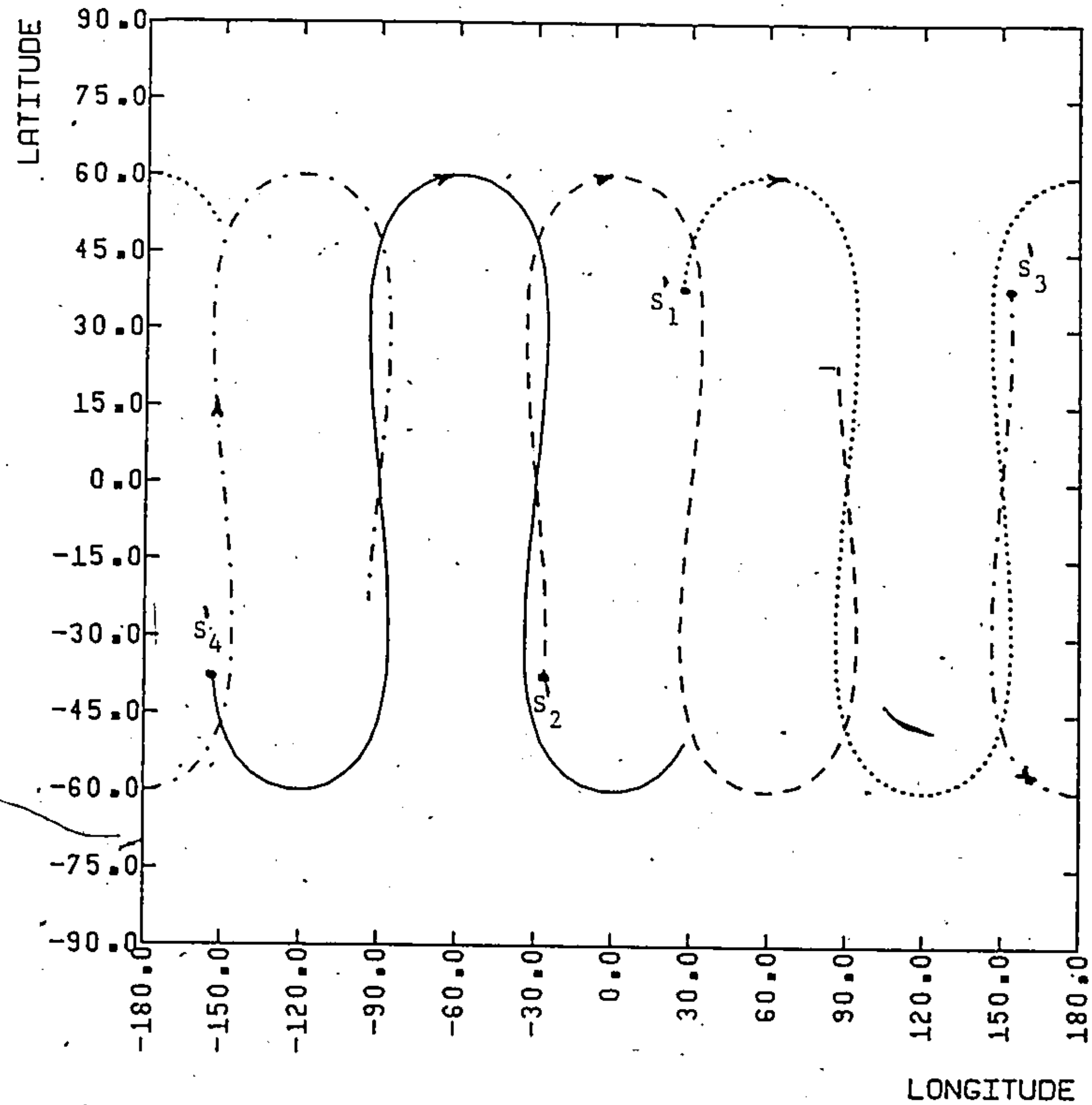

Fig. 2.11(d) t from 0 to 19.2 hours

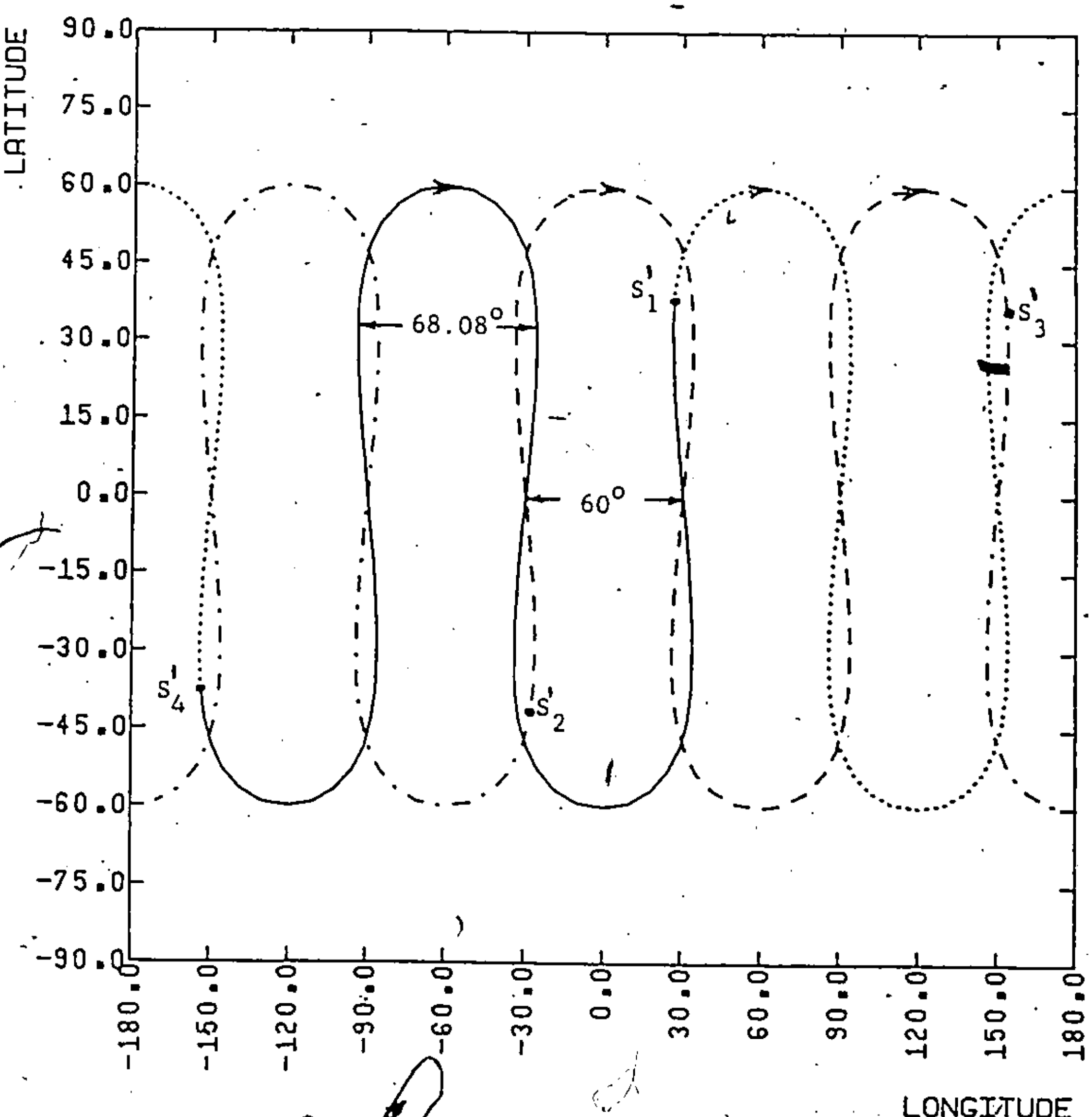

Fig. 2.11(e) t from 0 to 24 hours

Figures 2.12(a) through (e) show the satellite subpoint traces when K = 2. According to Eq. (2.7), the altitude of the satellite equals 20182 Km, which is the case of the NAVSTAR System (12 hour orbit), for this reason, we expect that the satellite subpoint will cross the equator 4 times every 24 hours. This agrees with Fig. 2.12(e). Also, the peak-to-peak latitude variation is the same as the previous figures, which is 120°.

To find the width at the equator, we substitute for K = 2 in Eq. (2.21) this yields

$$W_{EQ} = \pi(1 - 1/K) = 90° \tag{2.55}$$

which agrees with the value measured from Fig. 2.12(e).

To check the existence of a maximum width for the trace, we put K = 2 in equation (2.31)

$$\sec i \geq K \geq \cos i$$

i.e.,

$$2 = 2 \geq 0.5 \tag{2.56}$$

Thus it is a critical case of the upper limit of Eq. (2.31), hence, W_m will be equal to W_{EQ}.

Substituting for K = 2.0 in Eq. (2.28), we get

$$W_m = \frac{\pi}{2} - 2 \arcsin\left(\frac{0.5}{\sqrt{3}/2} * \sqrt{\frac{1-1}{2 * 0.5}}\right) + \arcsin\left(\frac{\sqrt{1 - 2 * 0.5}}{\sqrt{3}/2}\right) \tag{2.57}$$
$$= \pi/2 - 0 - 0 = \pi/2$$

which is equal to the width of the pattern at the equator W_{EQ} as shown in Fig. 2.12(e).

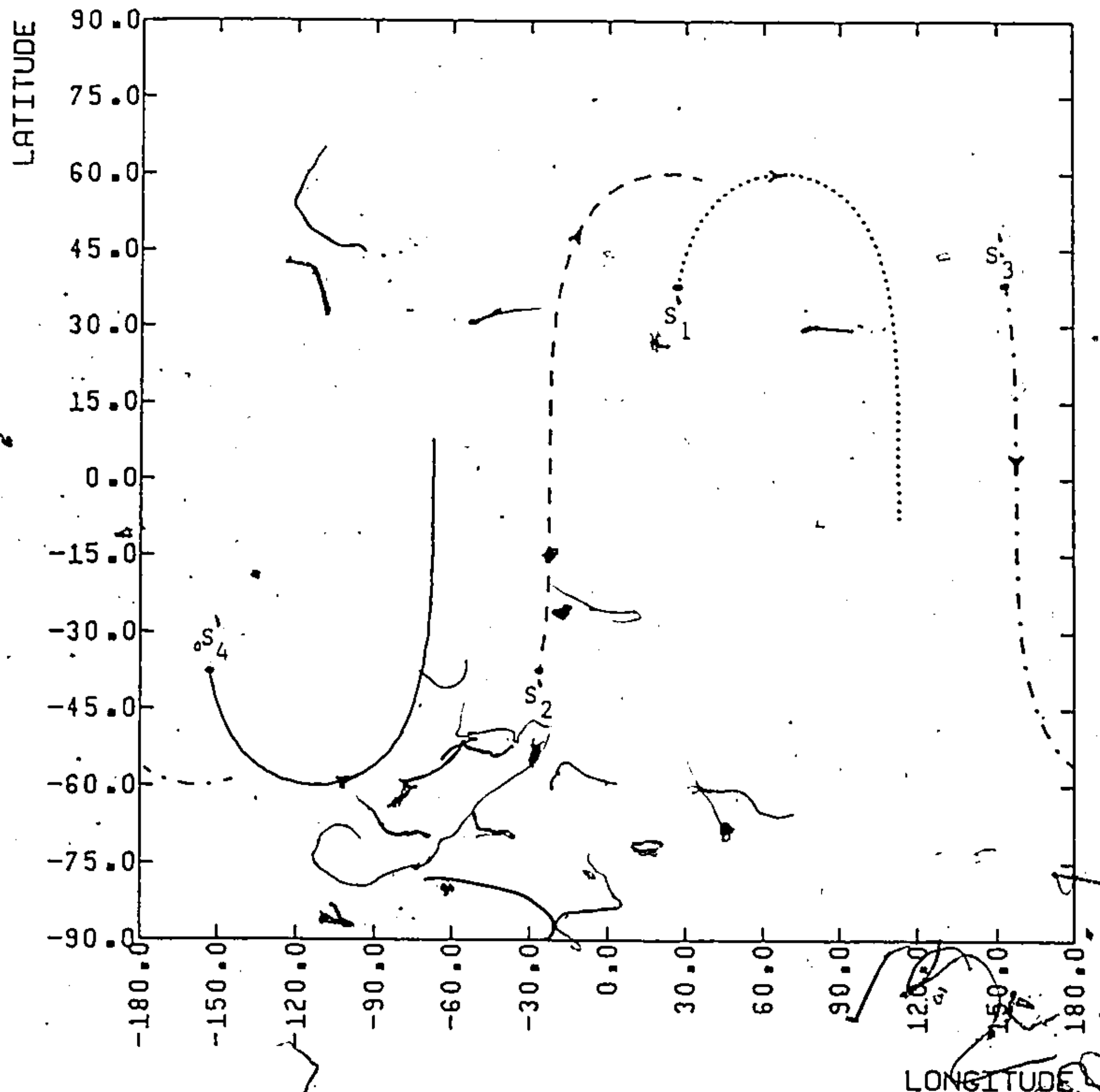

Fig. 2.12(a) SST for 4 satellites with K = 2 and i = 60°, t from 0 to 4.8 hours

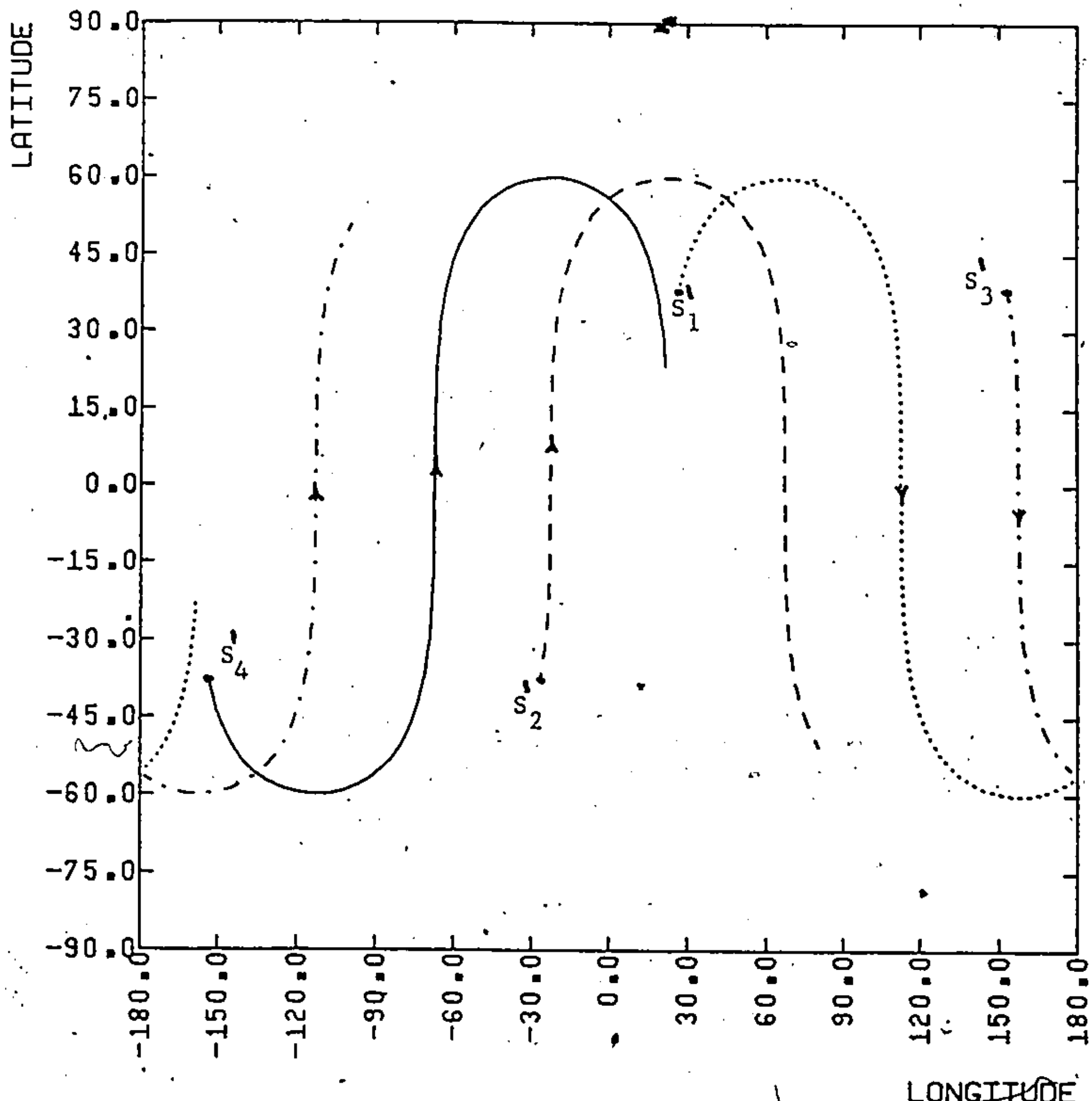

Fig.2.12 (b) t from 0 to 9.6 hours

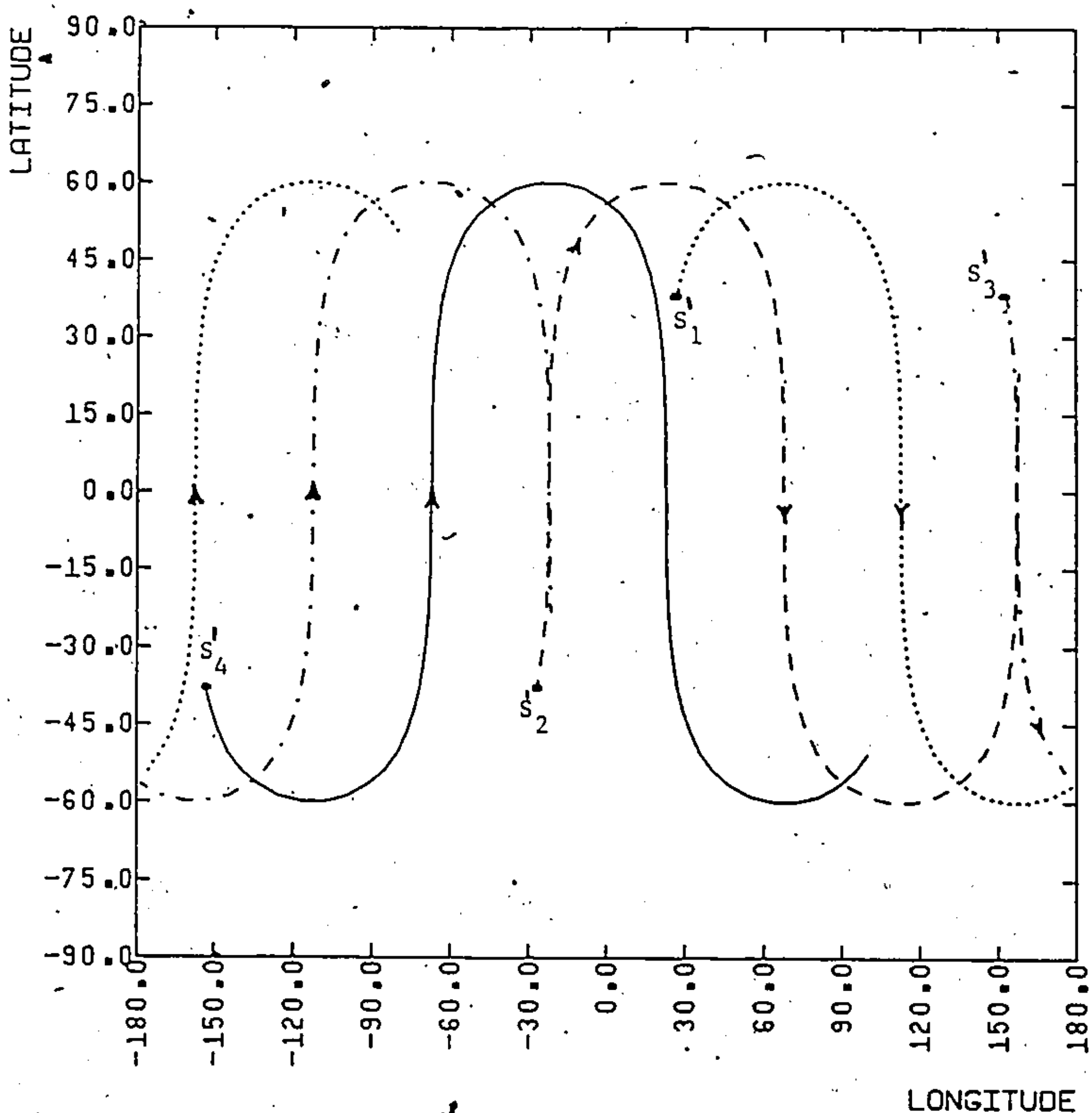

Fig. 2.12 (c)   t from 0 to 14.4 hours

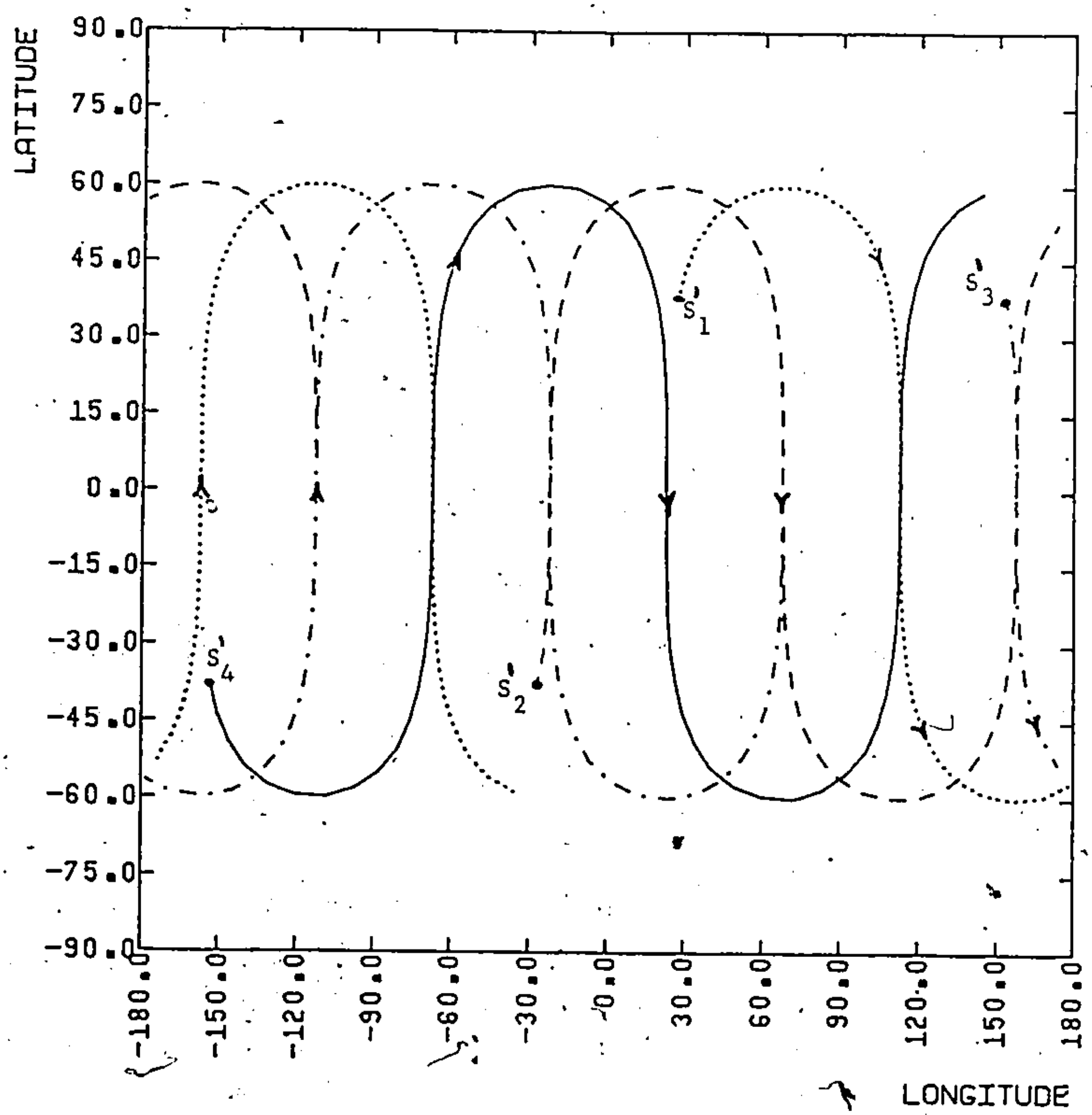

Fig. 2.12(d) t from 0 to 19.2 hours

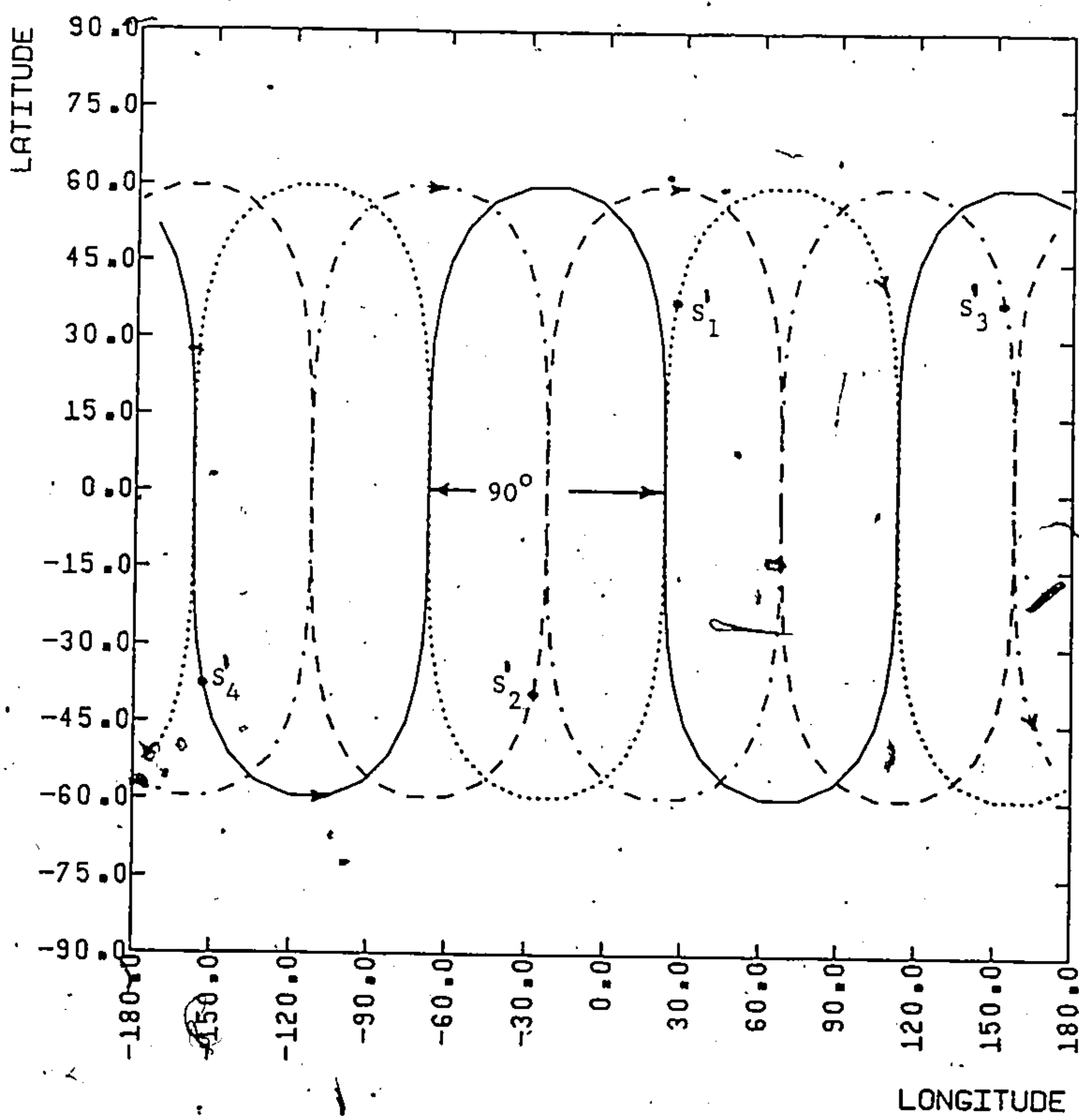

Fig. 2.12 (e) t from 0 to 24 hours

CHAPTER 3

THE INSTANTANEOUS COVERAGE AREA (ICA)

3.1 SUMMARY OF THE CHAPTER

The use of satellite based systems in navigation and positioning has received increasing consideration in the past few years. The need for improved performance and reliability in navigation and positioning systems has led to this interest in the use of satellites as a feasible alternative to ground-based and air-based system [22,23].

In order that the satellite can be used by different users, it must be visible to them. This implies that the users should be located inside what is called the instantaneous coverage area of the satellite.

Most of the research done in the area of satellite coverage is based upon computer modeling which may lead to an error. In fact, only the problem of partial coverage for a specific zone of the earth had been solved analytically using geostationary satellite covering a fixed area (around the equator) on the surface of the earth. On the other hand the analytic solution of the problem of partial coverage provided by a satellite in any nonstationary orbit does not appear in the literature. Also the problem of the whole globe coverage had been solved theoretically by assuming a network of satellites equally divided between a number of orbits. In each orbit the satellites are uniformly distributed, such that the same orbit satellites can provide a strip of coverage around the whole globe [24,25,26]. Similarly each orbit from the network will produce a replica of this coverage strip. Those coverage strips provided by the whole network are separated at the center of the equator by an angle equal to the interspacing angle between two adjacent orbits. Thus by arranging the orbits spacing, the number of satellites per orbit and the number of orbits, the network can provide the whole globe coverage. This method has a disadvantage of redundancy in the coverage and the excess in the number of satellites which leads to increased cost.

In this chapter we derive the equation of the instantaneous coverage area for any satellite in any circular orbit, and verify such an equation.

The equation of the instantaneous coverage area derived in this chapter represents the corner stone for solving any kind of coverage problem. Also in this chapter, we give some examples to show how to implement the equation of the ICA. These examples prove that the implementation of such an equation is rather easier and versatile, that is because such an equation depends only on the satellite subpoint which was studied comprehensively in Chapter 2.

3.2 DEFINITIONS

The Instantaneous Coverage Area (ICA) is that spherical segment of the earth (from which the satellite can be viewed) with half arc observed angle ψ measured by the great arc from the satellite subpoint to the small circle bounding the observed segment. In practice, an earth station can use the satellite effectively if it is above the horizon by some minimum angle σ at which the earth's atmosphere will not excessively impair transmission. This angle is called the minimum angle of visibility, and its value is usually equal to or greater than 5°.

For a spherical earth of radius R, Fig. 3.1 shows the geometric relations for determining the ICA of a satellite at distance $h+R$ from the center of the earth O, where h and R are the altitude of the satellite and the radius of the earth, respectively. The coverage boundary lies at an angle ψ with respect to the earth's center from the satellite subpoint S' and at a satellite angle Δ as shown.

Consider the triangle OSB. From the law of sines, we obtain

$$\frac{R}{\sin \Delta} = \frac{R+h}{\sin(90+\sigma)} \tag{3.1}$$

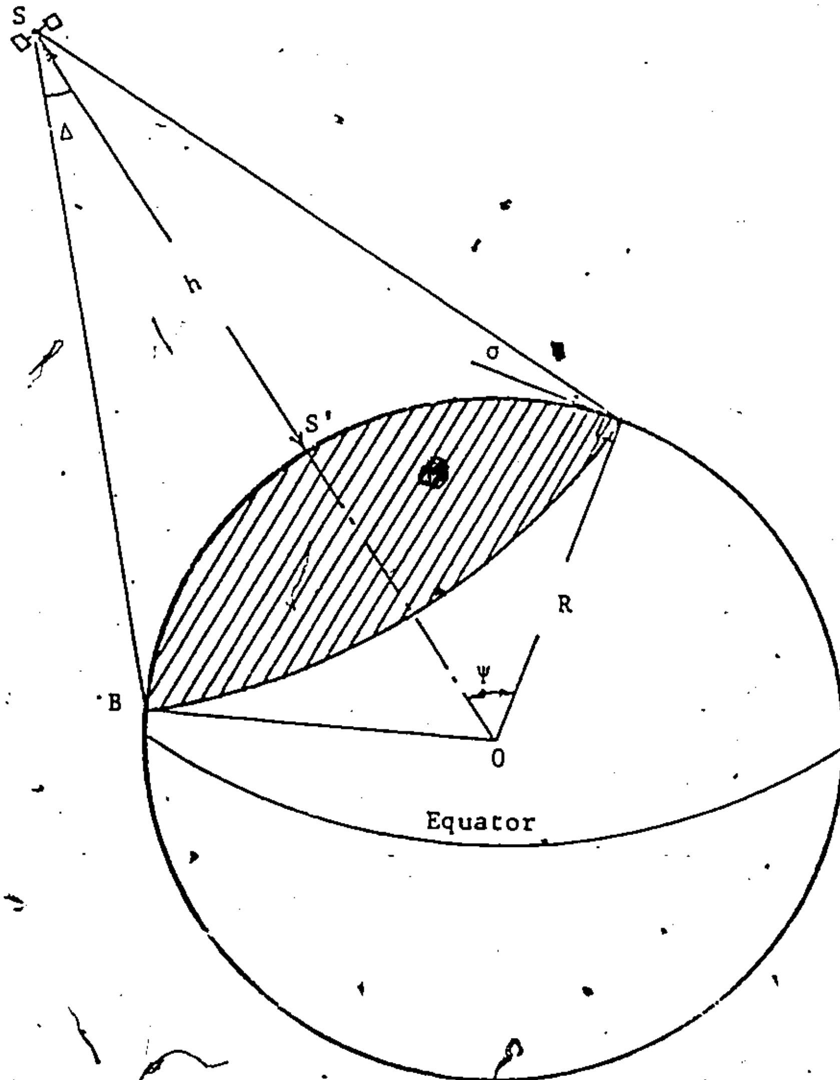

Fig. 3.1 The geometry of the ICA.

or

$$\frac{R}{\sin[90-(\psi+\sigma)]} = \frac{R+h}{\cos\sigma}$$

Equation (3.1) can be reduced to

$$\psi = -\sigma + \arccos\left[\left(\frac{R}{R+h}\right)\cos\sigma\right] \tag{3.2}$$

For constant angular velocities of the satellite ω_S and the earth ω_E we have the nonsynchronous factor K, which is defined by

$$K = \frac{\omega_S}{\omega_E} \tag{3.3}$$

From the law of Gravity [27], we have

$$U\frac{M_S M_E}{(R+h)^2} = M_S\,\omega_S^2\,(R+h) \tag{3.4}$$

where

U = Universal gravitational constant

M_E = Mass of the earth

M_S = Mass of the satellite

By applying Eq. (3.4) in order to compare the angular velocity of any satelite ω_S at altitude h and the angular velocity of any synchronous satellite ω_E at altitude hsyn = 35784 km, we get

$$K = \frac{\omega_S}{\omega_E} = \left(\frac{hsyn+R}{h+R}\right)^{3/2} \tag{3.5}$$

Substituting Eq. (3.5) in Eq. (3.2) and simplifying we obtain

$$\psi = -\sigma + \arccos[\Omega * K^{2/3} * \cos\sigma] \tag{3.6}$$

where

$$\Omega = \frac{R}{R+hsyn}$$

Fig. 3.2 shows the half arc observed angle ψ plotted against the nonsynchronous factor K for $\sigma = 0°, 5°, 10°$ and $15°$.

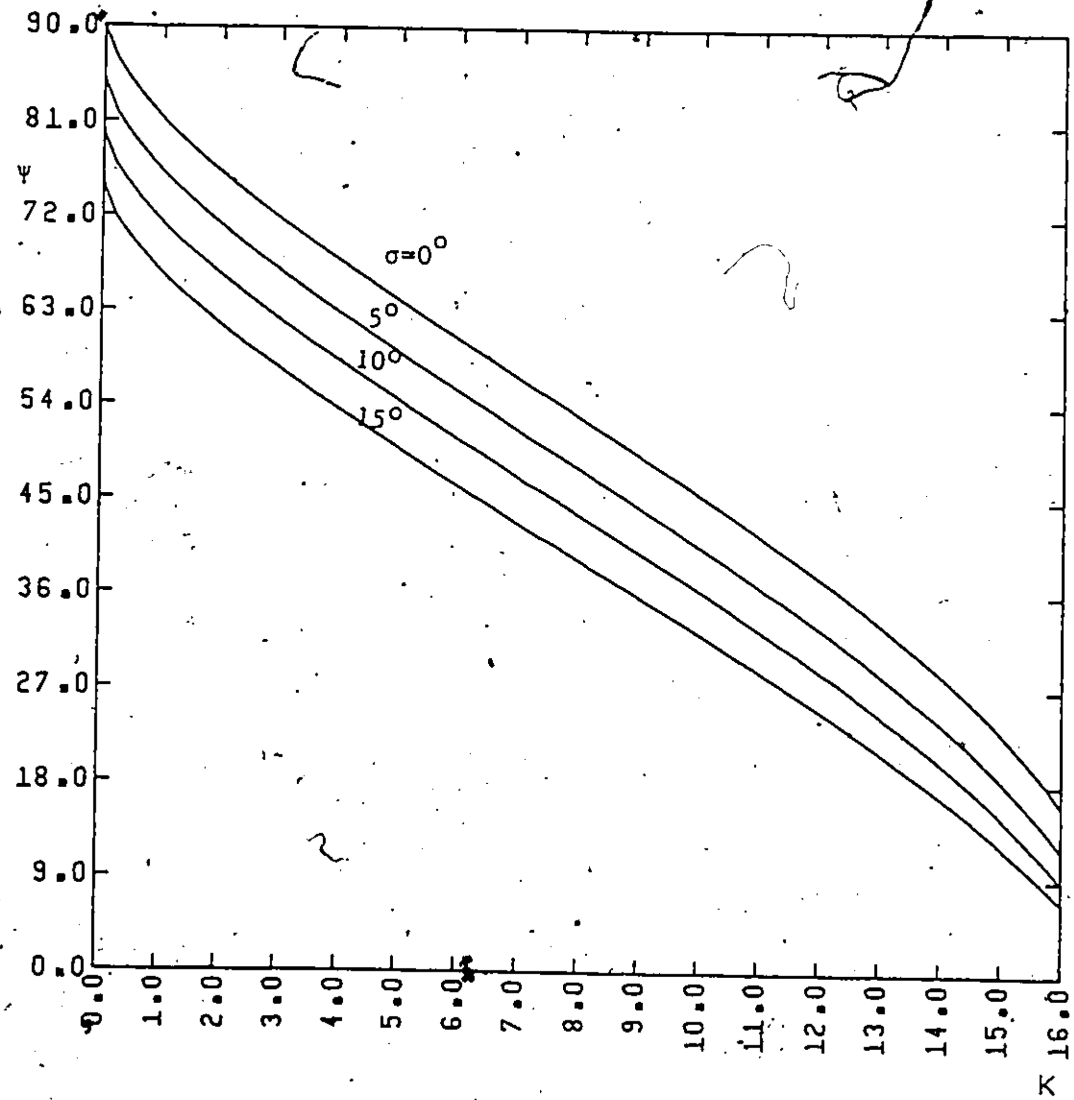

Fig. 3.2 The relation between the half arc observed agle and the non-synchronous factor for different values of σ.

The ICA as a percentage of the total area of the earth is plotted in Fig. 3.3 against K for $\sigma = 0°, 5°, 10°$ and $15°$, where the percentage ratio is given by

$$\text{Ratio} = \frac{1-\cos\psi}{2} \tag{3.7}$$

3.3 EQUATION OF THE ICA

Now, in order to derive the equation of the ICA, the satellite subpoint S′ must be a well defined point on the surface of the earth at any instant t. Let the latitude and longitude of the satellite subpoint S′ be ϕ_{Lat} and ϕ_{Long} respectively.

Referring to Fig. 3.4 which shows the satellite subpoint S′ at instant t, the equation of the ICA is simply the equation of the circle of intersection between the earth and the plane MM which passes through the point S″ and perpendicular to the line connecting the satelite subpoint S′ and the center of the earth O such that

$$OS'' = R\cos\psi \tag{3.8}$$

Referring to Fig. 3.5 the Cartesian equation of the plane MM in the normal form is given by

$$x_M \cos\alpha + y_M \cos\beta + z_M \cos\gamma = OS'' \tag{3.9}$$

where x_M, y_M, z_M are the Cartesian coordinates of any point in the plane MM, OS″ is the perpendicular distance from the origin (the center of the earth) to the plane MM and $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are the direction cosines of the normal OS″. For any point in the plane MM; with Cartesian coordinates x_M, y_M and z_M, we have

$$\begin{aligned} x_M &= r_M \cos\phi_{LatM} \cos\phi_{LongM} \\ y_M &= r_M \cos\phi_{LatM} \sin\phi_{LongM} \\ z_M &= r_M \sin\phi_{LatM} \end{aligned} \tag{3.10}$$

where r_M ϕ_{LatM} and ϕ_{LongM} are the polar coordinates for the same point in the plane MM. Substituting in Eq. (3.9) for OS″ from Eq. (3.8), x, y and z from Eq. (3.10) yields

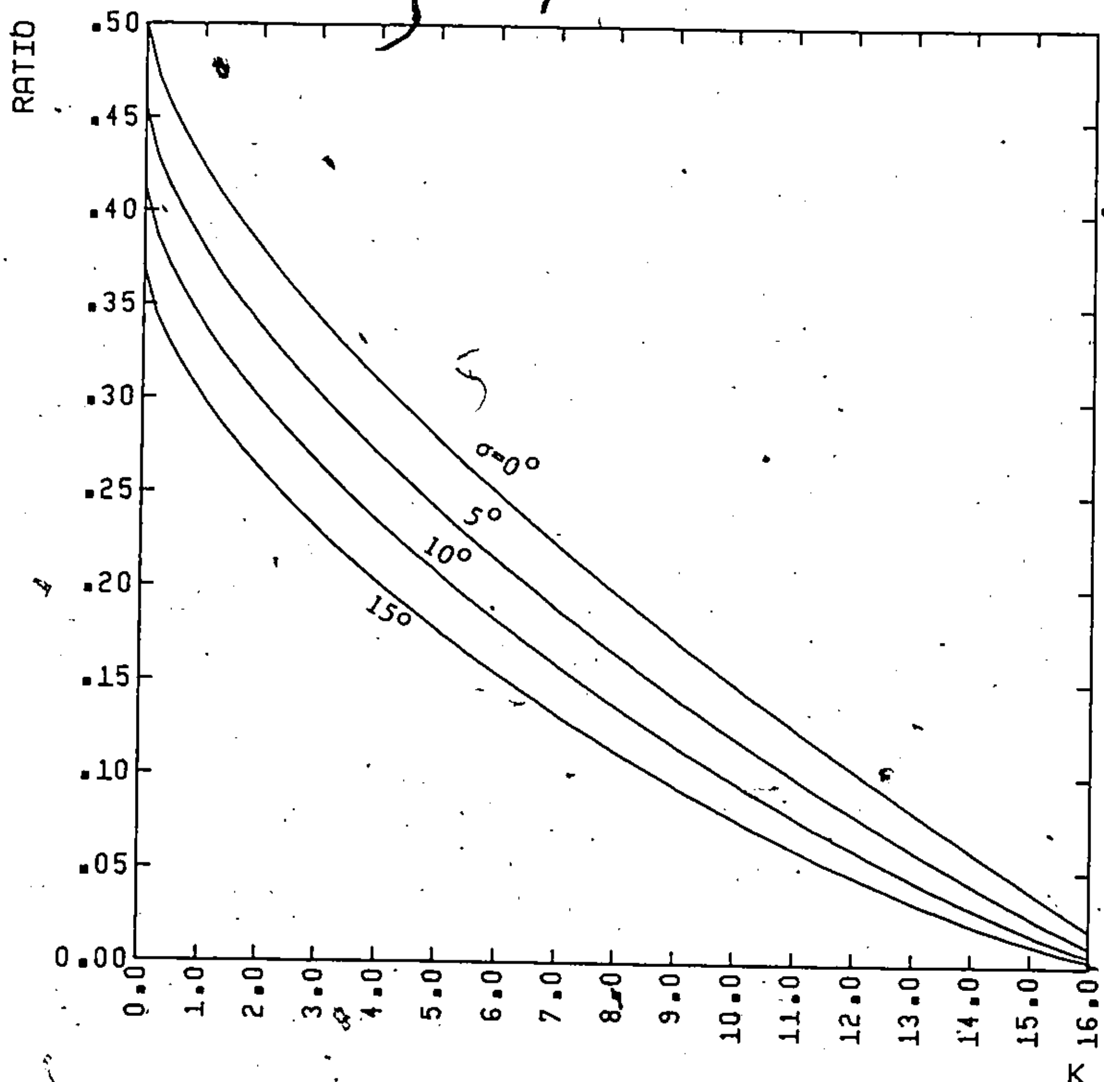

Fig. 3.3 The coverage area as a percentage of the total area of the earth versus the non-synchronous factor for different values of σ

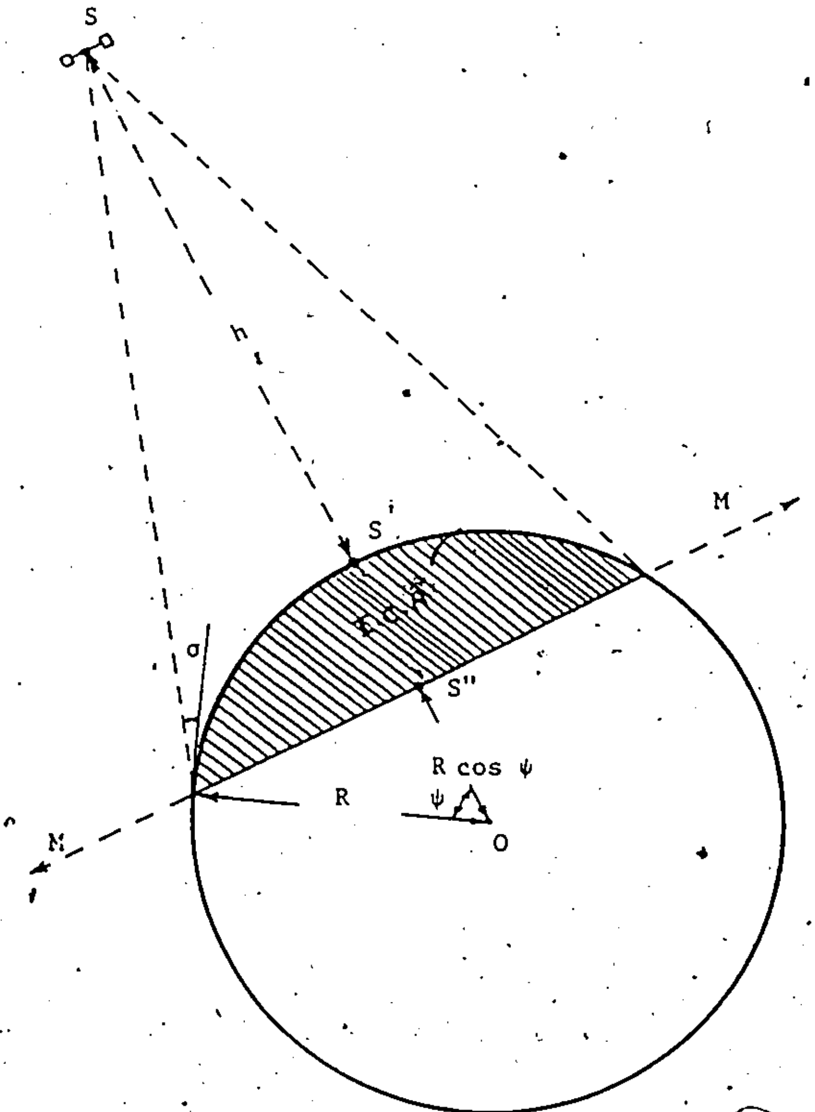

Fig. 3.4 Relationship between the parameters affecting the ICA.

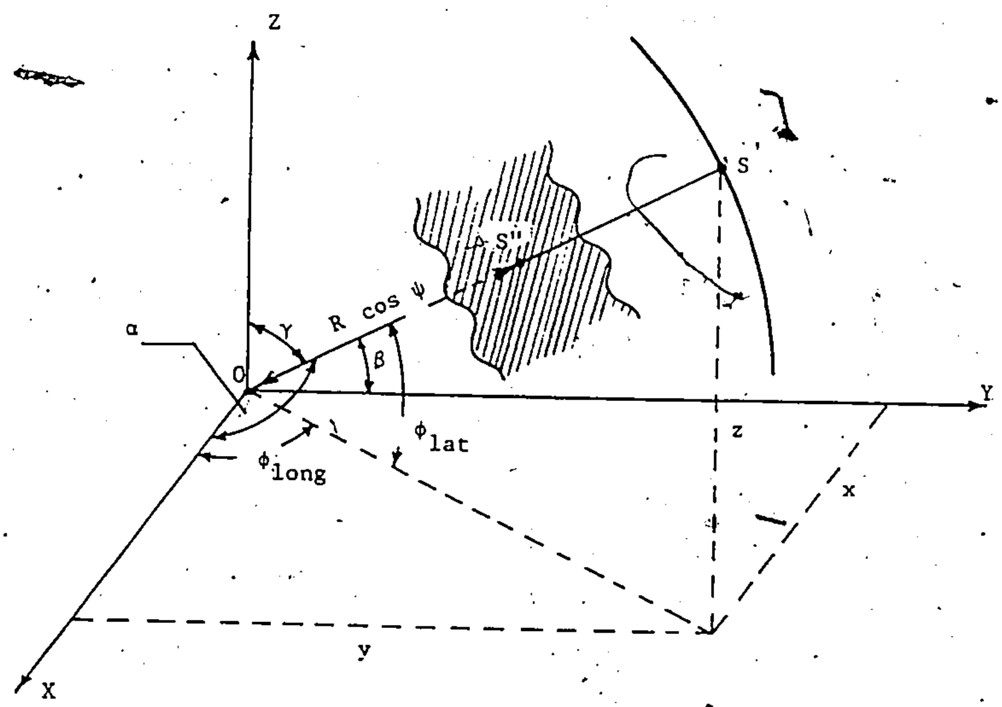

Fig. 3.5 Defining geometry for the plane MM

$$r_M \cos\phi_{LatM} \cos\phi_{LongM} \cos\alpha + r_M \cos\phi_{LatM} \sin\phi_{LongM} \cos\beta + r_M \sin\phi_{LatM} \cos\gamma = R \cos\psi \tag{3.11}$$

Now, from Fig. 3.5, the direction cosines of the normal OS″, can be defined by using the coordinates of the satellite subpoint R, ϕ_{Lat} and ϕ_{Long} as follows:

$$\begin{aligned} \cos\alpha &= \frac{x}{R} = \cos\phi_{Lat} \cos\phi_{Long} \\ \cos\beta &= \frac{y}{R} = \cos\phi_{Lat} \sin\phi_{Long} \\ \cos\gamma &= \frac{z}{R} = \sin\phi_{Lat} \end{aligned} \tag{3.12}$$

where x, y and z are the Cartesian coordinates of the satellite subpoint S′.

Substituting for cosα, cosβ, cosγ in Eq. (3.11) and simplifying, we get

$$r_M[\cos\phi_{Lat} \cos\phi_{LatM} \cos(\phi_{Long} - \phi_{LongM}) + \sin\phi_{Lat} \sin\phi_{LatM}] = R \cos\psi \tag{3.13}$$

Equation (3.13) gives the equation of the plane MM as a function of the satellite subpoint coordinates on the surface of the earth at any instant of the time.

By solving Eq. (3.13) with the equation of the earth which is the equation of a sphere defined by $r_M = R$ we obtain the equation of the intersection (which is the equation of the boundary of the ICA) given by

$$\cos\phi_{Lat} \cos\phi_{Latb} \cos(\phi_{Long} - \phi_{Longb}) + \sin\phi_{Lat} \sin\phi_{Latb} = \cos\psi \tag{3.14}$$

where, ϕ_{Latb} and ϕ_{Longb} are the latitude and longitude of any point on the boundary of the ICA respectively. For any point on the surface of earth located at latitude ϕ_{Lati} and longitude ϕ_{Longi} inside the ICA, the angle ψ_i between the satellite subpoint and this point, measured from the center of the earth, must be less than ψ thus

$$\cos\phi_{Lat} \cos\phi_{Lati} \cos(\phi_{Long} - \phi_{Longi}) + \sin\phi_{Lat} \sin\phi_{Lati} = \cos\psi_i \tag{3.15}$$

Now, the angle ψ_1 is less than ψ, hence, $\cos\psi_1$ is greater than $\cos\psi$. Therefore, comparing Eq. (3.14) and (3.15) we can say that any point on the surface of the earth, located inside the ICA must satisfy the following inequality:

$$\cos\phi_{Lat}\cos\phi_{Latb}\cos(\phi_{Long}-\phi_{Longb}) + \sin\phi_{Lat}\sin\phi_{Latb} - \cos\psi \geq 0 \qquad (3.16)$$

3.4 VALIDITY OF THE ICA EQUATION

Equation (3.14) provides the exact formula which represents the boundary of the ICA with size ψ for any satellite subpoint located at ϕ_{Lat} and ϕ_{Long} due to the following arguments:

1) The ICA boundary is a circle located o[illegible]e of the earth (sphere); therefore, the equation which represents the boundary of the ICA (after defining ψ, ϕ_{Lat} and ϕ_{Long}) must be a function of two variables, namely ϕ_{Latb} and ϕ_{Longb}, which is clear in Eq. (3.14).

2) Any circle, with any size, on the surface of the earth must be located symmetrically around the longitude plane which passes through its center. After defining the satellite subpoint (ϕ_{Lat}, ϕ_{Long}), and from Eq. (3.14) we find that there are two points on the boundary of the ICA with the same latitude but with two different values of longitude such that ϕ_{long} is the average of their longitude. Thus, we can say that the ICA is located symmetrically around the longitude plane which passes through the satellite subpoint.

3) Consider the case of a satellite rotating in equatorial orbit, where $\phi_{Lat} = 0$; thus, Eq. (3.14) reduces to

$$\cos\phi_{Latb}\cos(\phi_{Long}-\phi_{Longb}) = \cos\psi \qquad (3.17)$$

From the above equation, it is clear that the maximum latitude reached by any point on the boundary of the ICA is equal to $\pm\psi$ (when $\phi_{Longb} = \phi_{Long}$). Also, from Eq. (3.17) the maximum variation in longitude for the boundary of the ICA (measured from the longitude of the satellite subpoint on the equator ϕ_{Long}) is equal to $\pm\psi$, thus; the ICA for a satellite in equatorial orbit is located symmetrically around the satellite subpoint with a size ψ measured from the center of the earth.

4) Here, we are going to use the geometry of the earth in order to find the equation for the longitude of two points on the boundary of the ICA, such that both have the same latitude as the satellite subpoint S', namely ϕ_{Lat}. Note that a solution for this equation must exist for the case of a circle.

Refer to Fig. 3.6(a), which shows a section of the earth produced by a plane passing through the poles and the satellite subpoint S'. From the geometry we can say

$$RR = R\cos\phi_{Lat} \tag{3.18}$$

where

RR = the radius of the latitude circle ϕ_{Lat}

$$S'S'' = R(1 - \cos\psi) \tag{3.19}$$

and

$$S'E = \frac{R(1-\cos\psi)}{\cos\phi_{Lat}} \tag{3.20}$$

Referring to Fig. 3.6(b) which shows the latitude circle ϕ_{Lat} of the satellite subpoint S', it is obvious that the point g and g' are located on the boundary of the ICA. Also, from the same figure, the difference in longitude between S' and either g or g' equals $\pm(\phi_{Long} - \phi_g)$, as given by

$$\cos(\phi_{Long} - \phi_g) = \frac{RR - S'E}{RR} \tag{3.21}$$

Substituting Eq. (3.18) and Eq. (3.20) in Eq. (3.21) we get

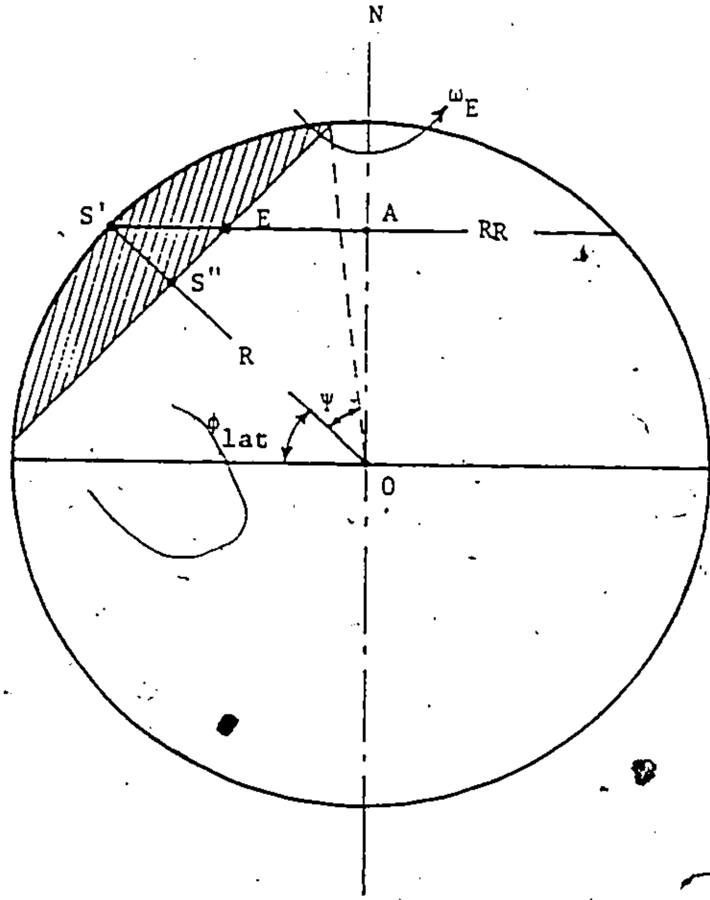

Fig. 3.6a Vertical section of the earth by longitude plane passes through the satellite subpoint S'.

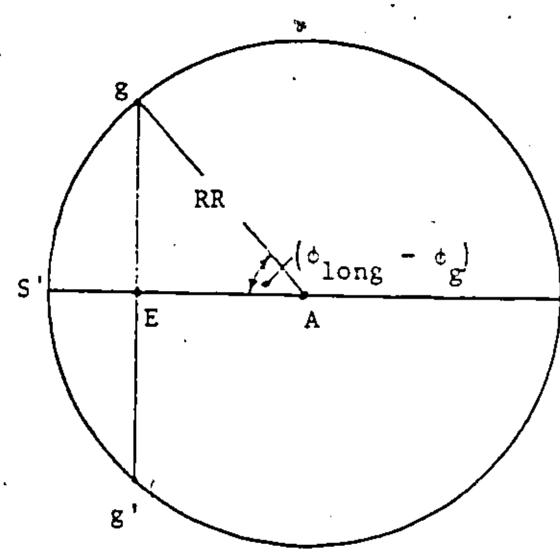

Fig. 3.6b Latitude circle of the satellite subpoint S'

$$\cos(\phi_{Long} - \phi_g) = \frac{R\cos\phi_{Lat} - [R(1-\cos\psi)/\cos\phi_{Lat}]}{R\cos\phi_{Lat}} \tag{3.22}$$

Eq. (3.22) reduces to

$$\cos(\phi_{Long} - \phi_g) = \frac{\cos^2\phi_{Lat} - 1 + \cos\psi}{\cos^2\phi_{Lat}} \tag{3.23}$$

or

$$\cos(\phi_{Long} - \phi_g) = \frac{\cos\psi - \sin^2\phi_{Lat}}{\cos^2(\phi_{Lat})} \tag{3.24}$$

Note here a solution for Eq. (3.24) always exists, because the R.H.S. is less than unity.

Substituting in Eq. (3.14) for $\phi_{Lat_b} = \phi_{Lat}$ and $\phi_{Long_b} = \phi_g$.

$$\cos^2\phi_{Lat}\cos(\phi_{Long} - \phi_g) + \sin^2\phi_{Lat} = \cos\psi \tag{3.25}$$

or

$$\cos(\phi_{Long} - \phi_g) = \frac{\cos\psi - \sin^2\phi_{Lat}}{\cos^2\phi_{Lat}} \tag{3.26}$$

Equation (3.24) is identical with Eq. (3.26) Thus, from arguments 1,2,3 and 4, we can say that Eq. (3.14) represents the ICA for any satellite.

The Instantaneous Coverage Area is plotted in Fig 3.7 for the case of a satellite rotating in a 12 hour orbit with inclination angle i=60° The satellite starts motion at t=0 (measured by the reference clock) when its subpoint on the surface of the earth is located at the equator at longitude 45° West. The ICA is also plotted when t = 3,6 and 9 hour, where the satellite subpoint S_2', S_3' and S_4' is located at (60° latitude N and 0° longitude), (0° latitude and 45° longitude E) and (60° latitude S and 90° longitude E) respectively.

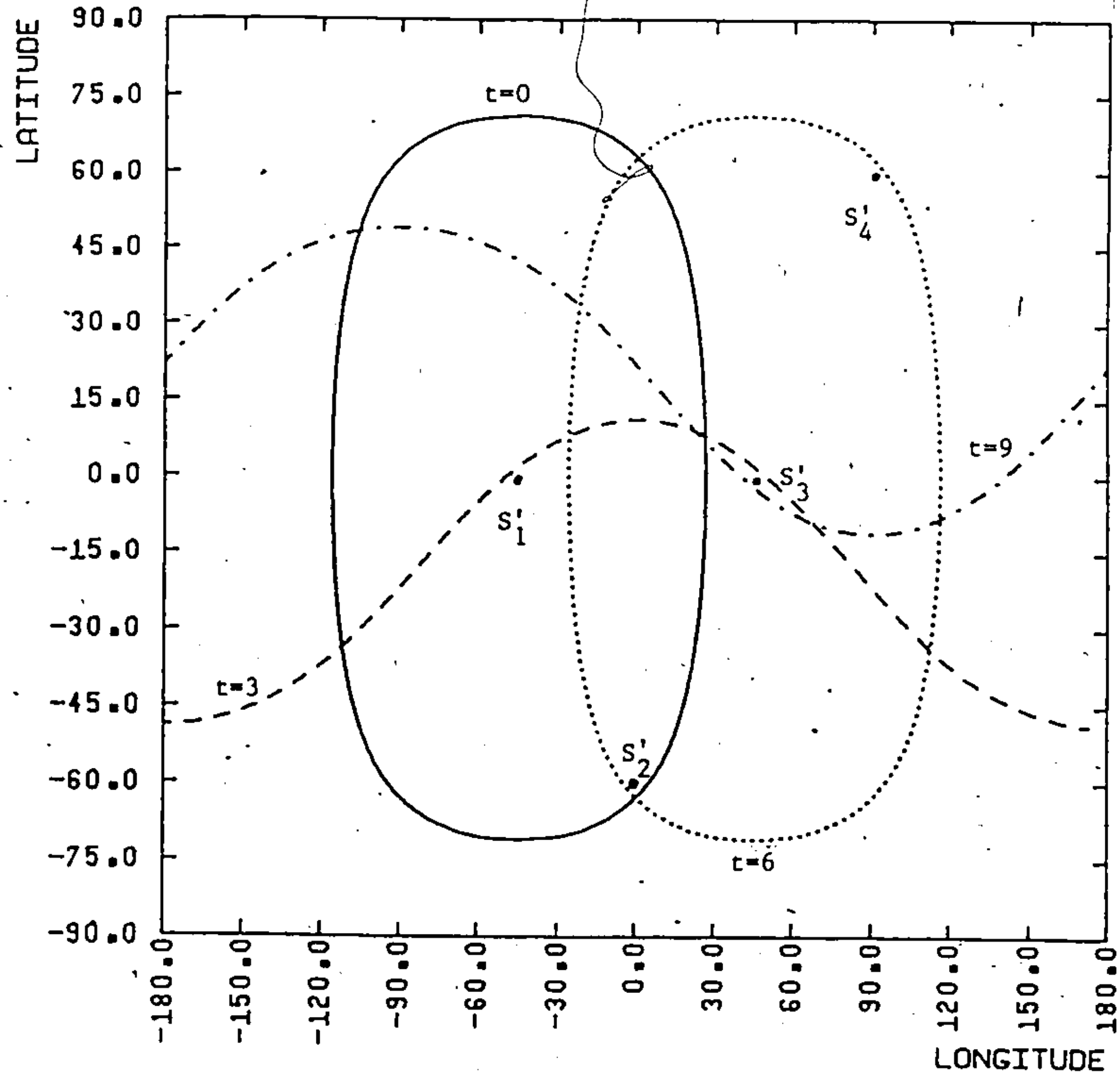

Fig. 3.7 ICA of a satellite with initial conditions $i = 60°$, $K = 2$, $TT = 0$ and $\phi_o = -45°$ at $t = 0, 3, 6$ and 9 hours.

3.5 IMPLEMENTATION OF THE ICA EQUATION

Now, in order to show how to implement Eq. (3.16), for designing satellite constellations to provide single fold or multifold coverage for either the whole globe, or specified zones on the earth (partial coverage), we use Eq. (3.16) and Monte Carlo analysis for this purpose.

Monte Carlo analysis is an optimization method which allows us to approximate the coverage area on the surface of the earth by a specified geometric shape; for example, square, rectangle, circle, ellipses, ... etc. Then, by calling one point from the perimeter of this specified shape and substituting the coordinates of this point in Eq. (3.14) for different values of time (to represent the migration of the ICA on the surface of the earth), we are able to determine if this point is always located either inside or outside the ICA. If it is located inside the ICA, we repeat the same procedure for another point on the perimeter of the specified geometric shape and so on. Thus we can determine if the whole shape is located inside the boundary of the ICA during its migration. If the specified shape is completely located inside the ICA for all values of time, we enlarge the dimensions of this shape by a certain amount, then repeat the same procedure.

In order to give an example, we consider two cases for single fold coverage for a satellite in 24 hour orbit. The first case we assume a satellite in geostationary orbit, and we approximate the coverage area of partial coverage (on the surface of the earth) by a square located symmetrically around the satellite subpoint on the equator, such that the sides of this square coincide with the latitudes and longitudes. The result from applying Eq. (3.16) and Monte Carlo analysis shows that a square with sides coincident with $\pm 60°$ longitude and latitude with respect to the satellite subpoint fits inside the ICA. Thus three satellites in geostationary orbit can provide the partial coverage of a strip around the earth and this strip is located symmetrically around the equator with width extending from 60° latitude N to 60° latitude S, which is a well known result.

The second case we assume is a satellite in 24 hour orbit with inclination angle equal to 15°. In this case, the maximum square (which receives single fold continuous coverage) fits inside the ICA during its migration on the earth and has a span of 96° located symmetrically around the node of the satellite subpoint trace.

In Appendix 1 we give an example which illustrates another possible application for the equation of the instantaneous coverage area. Assuming a satellite from a search and rescue by satellites system (SARSAT), we showed the effect on the coverage time provided by the satellite for (a specified point on the earth) if the single beam antenna on board the satellite is replaced by a four beam antenna. It is also shown in Appendix 1, how the rotation of the four beam antenna will effect the coverage time.

The method which we developed can provide the required information about the areas on the surface of the earth, which receive a coverage for specified interval of the time (not continuous coverage). In addition to that, we can design the optimal satellite constellation which consists of m satellites to provide n-fold coverage ($m \geq n$) of the whole globe or a specified zone of it, in such situation any point on the specified coverage area must satisfy at least n equations of the m equations of the instantaneous coverage areas at any instant of the time t, and we can adjust the initial conditions [28] of one or more satellites to reach the optimal constellation.

CHAPTER 4

THE SPACE DELAY

4.1 SUMMARY OF THE CHAPTER

The possibility of using geostationary satellites for communications was discussed in the popular literature as early as 1956 [29]. The first detailed proposal for a synchronous tracking satellite system for the purposes of orbit determination was provided by Vonbun in 1967 [30]. Since then a number of papers [31,32] have considered the use of satellite-to-satellite tracking for orbit determination and for gravity field model refinement. It was clear that with regard to coverage, a satellite-to-satellite tracking system has a significant advantage over a ground based tracking system. The future earth applications missions are likely to require satellite orbits with high inclination angle for global coverage, and low altitude for sensitivity.

For instance, with a single geostationary satellite, a satellite-to-satellite tracking system is capable of observing an earth orbiting satellite during almost half of every orbit. Equivalent coverage of a satellite in a high inclination angle orbit would be difficult to obtain with a ground based system.

In this Chapter the visibility and the space delay for two cases are discussed. In the first case we consider a ground based system with different earth stations located at different locations on the surface of the earth. In this system the study shows that the visibility is poor. In the second case we consider an earth station-based-satellite to satellite system using a geostationary satellite as intermediate link. This last system shows a significant improvement in both the visibility and the space delay performance.

4.2 THE SPACE DELAY BETWEEN AN EARTH STATION AND A SATELLITE

The space delay between an earth station ES, and a satellite S is defined as the distance between the earth station and the satellite divided by the propagation velocity.

Before evaluating the space delay T_E, we note that, both the earth station and the satellite are moving relative to each other in the space: thus, we are dealing with a dynamic case where the relative distance, velocity and acceleration are continually varying in magnitude and direction.

In order to find the space delay between an earth station and a satellite at any instant of the time t, we proceed as follows:

i) Define the satellite subpoint on the surface of the earth at any instant t. The equations which define the satellite subpoint given by Eq. (2.10) and (2.13) are

$$\Phi_{Lat} = \arcsin[\sin i \sin K\omega_E(t + TT)] \tag{4.1}$$

and

$$\Phi_{Long} = \arcsin\left|\frac{\cos i \sin K\omega_E(t+TT)}{\{1-\sin^2 i \sin^2 K\omega_E(t+TT)\}^{1/2}}\right| - \omega_E t + \Phi_o \tag{4.2}$$

where

t = the time measured by a reference clock at the earth station

Φ_{Lat} = the latitude of the satellite subpoint at any instant t

Φ_{Long} = the longitude of the satellite subpoint at any instant t

i = the orbital inclination angle

K = the nonsynchronous factor

ω_E = the angular velocity of the earth

TT = the initial time phase of the satellite in its orbit

Φ_o = the longitude of the point of intersection between the equator and the orbital projection on the surface of the earth at $t = 0$.

ii) Now, for any satellite, the subpoint is a well defined point on the surface of the earth at any instant t. By knowing the coordinates of the given earth station ES, we can form a plane passing through the earth station, the satellite subpoint and the center of the earth, intersecting the earth in a great circle as shown in Fig. 4.1. The size of the smallest arc ε between the earth station and the satellite subpoint can be found by forming a spherical segment on the surface of the earth located symmetrically around the satellite subpoint such that the earth station is located on its boundary, therefore from Eq. (3.14) we have:

$$\varepsilon = \arccos(\sin\phi_{Lat} \sin_{LatE} + \cos\phi_{Lat} \cos\phi_{LatE} \cos(\phi_{Long} - \phi_{LongE})) \tag{4.3}$$

where ϕ_{LongE} and ϕ_{LatE} are longitude and latitude of the earth station, respectively. Note that in Eq. (4.3) ε depends on the coordinates of both the satellite subpoint and the earth station. As well, we note that the location of the satellite subpoint depends upon time, while the location of the earth station is a fixed value, thus, the angle ε depends upon time through the coordinates of the satellite subpoint.

iii) By solving the plane triangle with sides being the lines connecting the center of the earth, the satellite and the earth station (noting that the radius of the earth and the satellite altitude are known in addition to the angle ε as shown in Fig. 4.1) the distance measured between the earth station and the satellite d_E is given by

$$d_E = [(h+R)^2 + R^2 - 2R(h+R)\cos\varepsilon]^{1/2} \tag{4.4}$$

or

$$d_E = [2R(h+R)(1-\cos\varepsilon) + h^2]^{1/2} \tag{4.5}$$

where

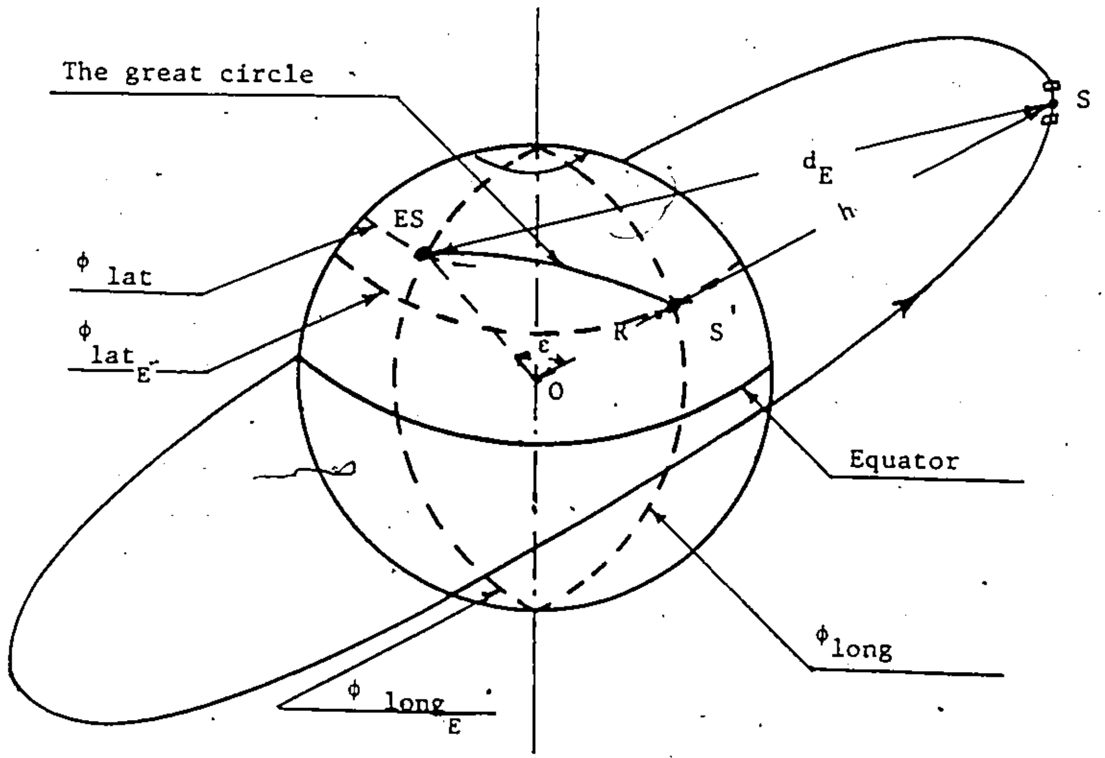

Fig. 4.1 Geometry of a satellite in inclined orbit and an earth station.

h = the satellite altitude

Recalling Eq.(2.7) we can write the satellite altitude h as follows:

$$h = (hsyn + R) K^{-2/3} - R \tag{4.6}$$

h_{syn} = altitude of a 24 hour orbit

R = radius of the earth.

iv) Now, the space delay T_E between the earth station and the satellite (at any instant t) is simply d_E/C, where C is the velocity of the propagation.

It is clear that, in order to guarantee the presence of a satellite-earth station link, the earth station must be located inside the boundary of the ICA. Hence, the angle ε between the earth station and the satellite must be less than or equal to the half arc angle ψ; otherwise the satellite is below horizon.

The space delay T_E between four earth stations at different locations on the surface of the earth, and a satellite located in an inclined orbit, has been evaluated and plotted versus the time of navigation measured by the reference clock. The earth stations are located at latitude and longitude of [0°, 0°], [30°, 30°], [60°, 60°] and [90°, 90°].

In our calculations we assume that the radius of the earth R = 6378 km, the altitude of the 24 hour orbit Hsyn = 35784 km, the velocity of propagation C = 300,000 km/s and a minimum angle of visibility σ = 5°.

Figures (4.2), (4.3) and (4.4) show the space delay T_E in ms between those earth stations and a satellite in 24 hour orbit with inclination angles 30°, 60° and 90° respectively. As a general note, for all the coming Figures, the discontinuity in all the curves represents the interval in which the communications is blocked, ie., when $\varepsilon > \psi$.

Figures 4.5, 4.6 and 4.7 show the space delay T_E between the same four earth stations and a satellite in a 12 hour orbit with inclination angle 30°, 60° and 90° respectively.

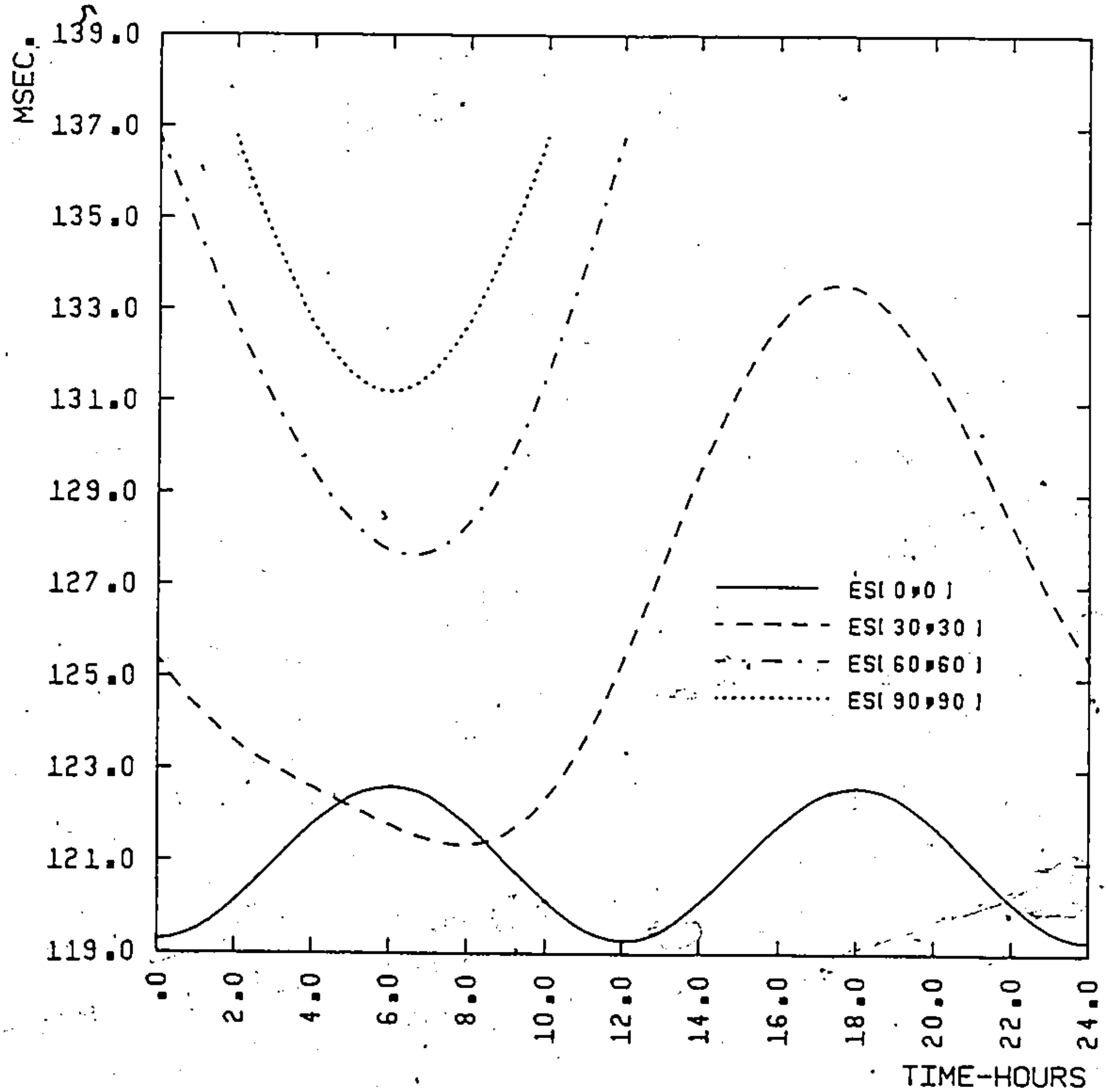

Fig. 4.2 Space delay between a satellite in 24 hour orbit with inclination 30° and each of four earth stations.

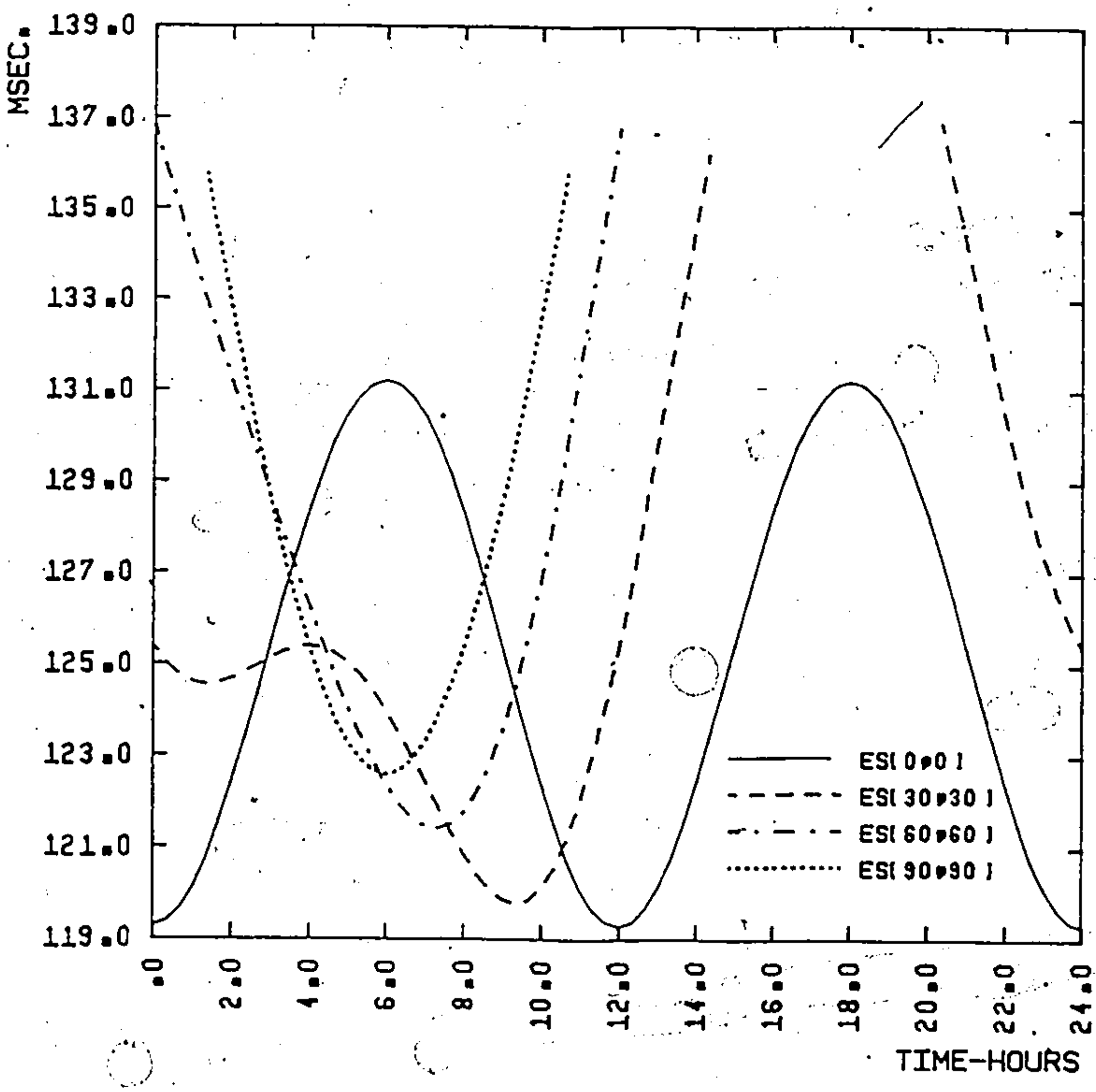

Fig. 4.3 Space delay between a satellite in a 24 hour orbit with inclination 60° and each of four earth stations.

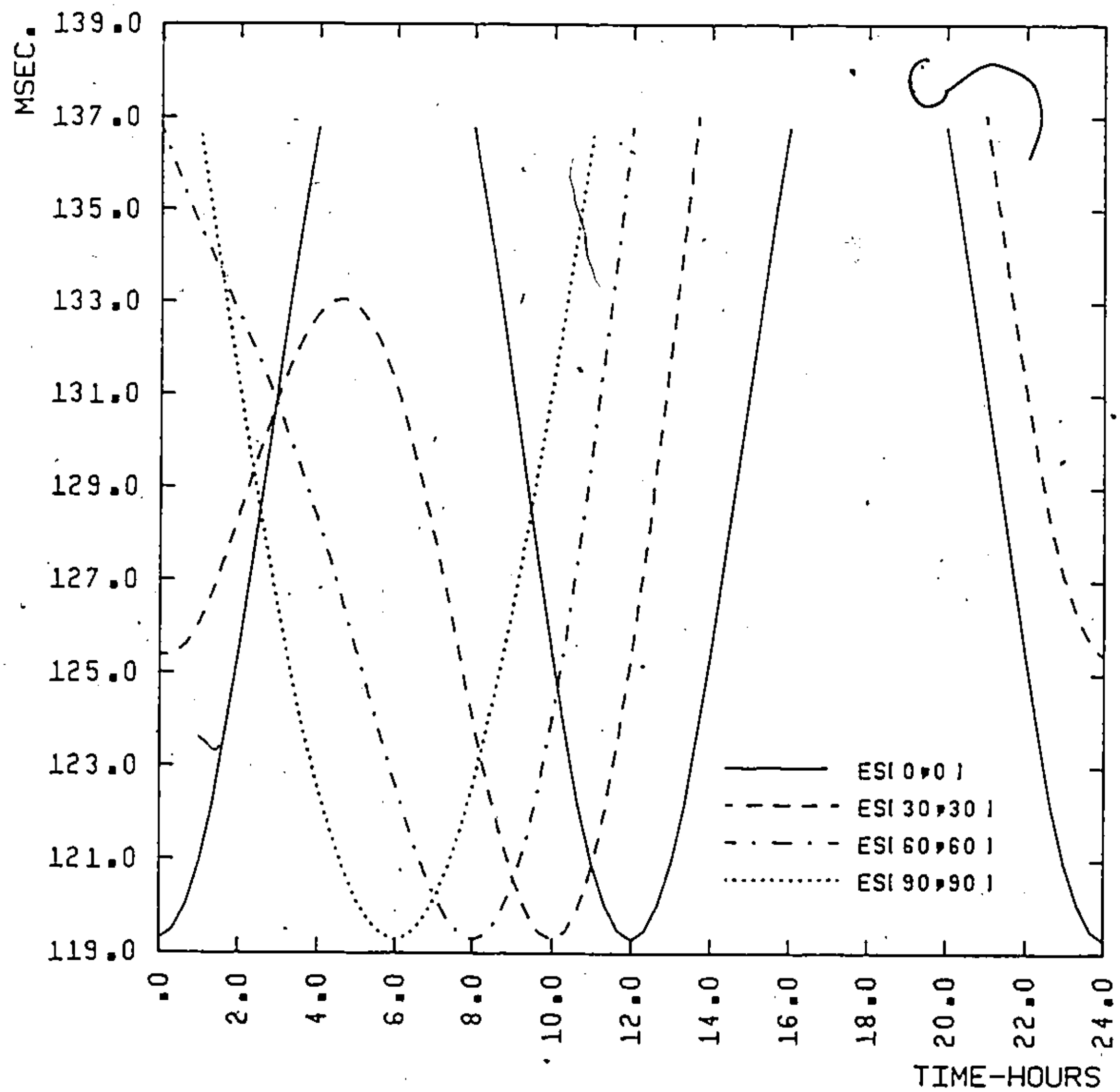

Fig. 4.4 Space delay between a satellite in a 24 hour polar orbit and each of four earth stations.

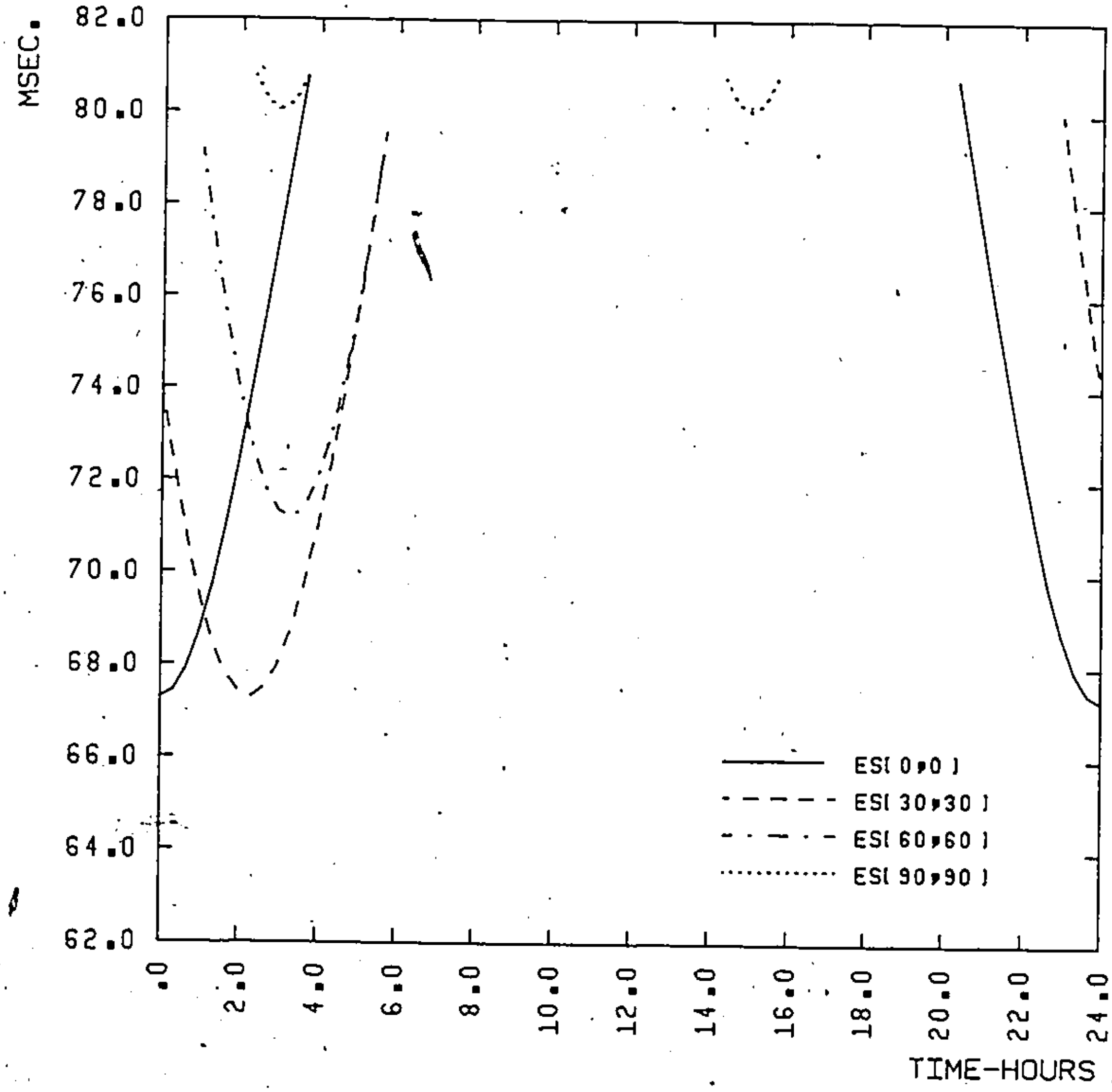

Fig. 4.5 Space delay between a satellite in 12 hour orbit with inclination 30° and each of four earth stations.

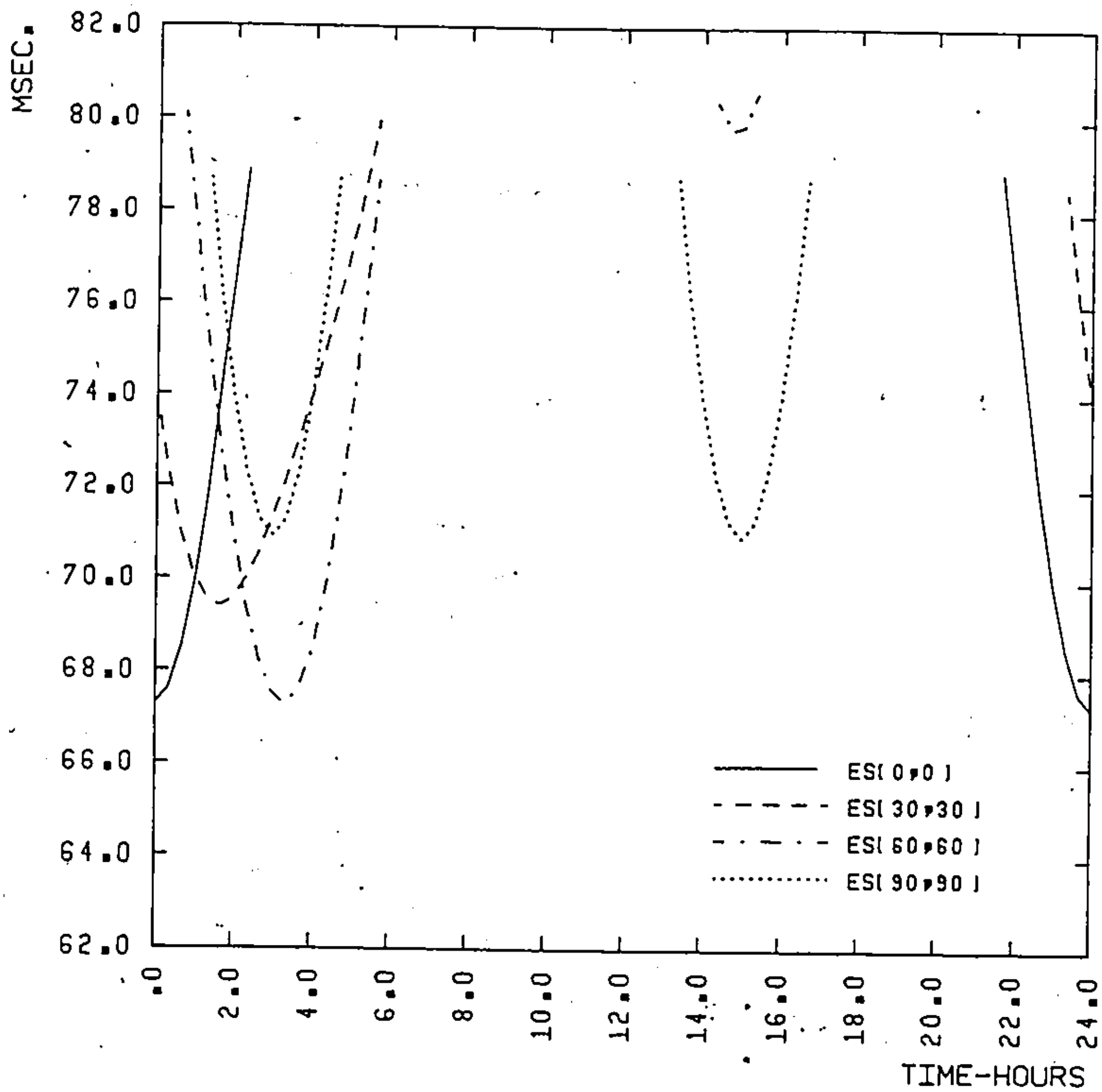

Fig. 4.6 Space delay between a satellite in 12 hour orbit with inclination 60° and each of four earth stations

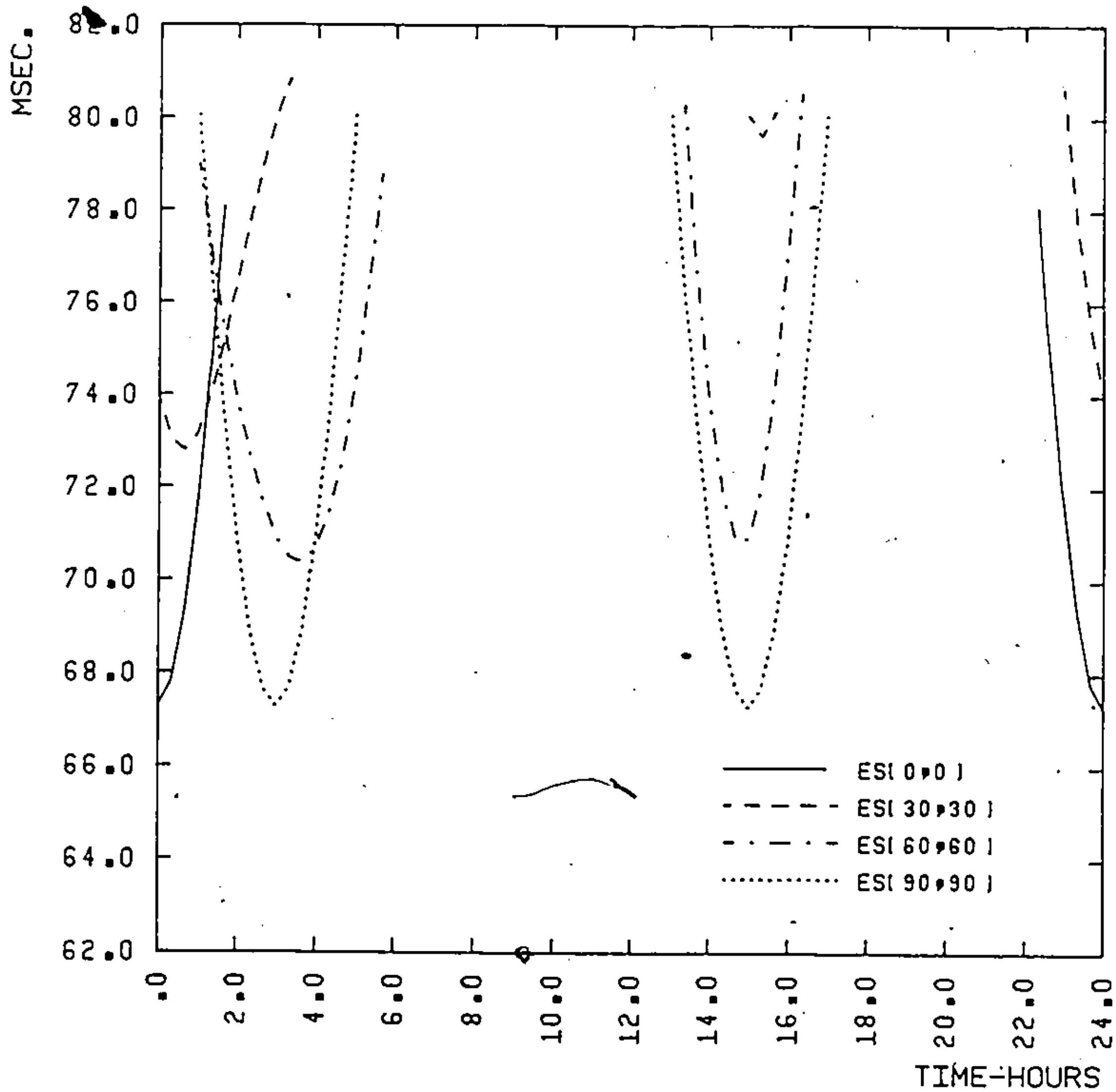

Fig. 4 7 Space delay between a satellite in a 12 hour polar orbit and each of four earth stations.

4.3 CHANGE IN SPACE DELAY WITH TIME

In order to find the change in the space delay with respect to the time measured by the reference clock t, we differentiate Eq. (4.4) and divide by the velocity of propagation C. Noting that ε is the only time-dependent variable, we find:

$$\frac{\partial T_E}{\partial t} = \frac{-R(h+R)}{d_E \cdot C} \frac{\partial \cos\varepsilon}{\partial t} \tag{4.7}$$

To find $\partial\cos\varepsilon/\partial t$, we rewrite Eq. (4.3) as follows

$$\cos\varepsilon = \cos\phi_{Lat}\cos\phi_{LatE}\cos(\phi_{Long}-\phi_{LongE}) + \sin\phi_{Lat}\sin_{LatE} \tag{4.8}$$

Differentiating Eq. (4.8) with respect to t, noting that ϕ_{LatE} and ϕ_{LongE} are independent of t, yields

$$\frac{\partial\cos\varepsilon}{\partial t} = -\cos\phi_{LatE}\left[\sin\phi_{Lat}\cos(\phi_{Long}-\phi_{LongE})\frac{\partial\phi_{Lat}}{\partial t} + \cos\phi_{Lat}\sin(\phi_{Long}-\phi_{LongE})\frac{\partial\phi_{Long}}{dt}\right] + \sin\phi_{LatE}\cos\phi_{Lat}\frac{\partial\phi_{Lat}}{\partial t} \tag{4.9}$$

or

$$\frac{\partial\cos\varepsilon}{\partial t} = \frac{\partial\phi_{Lat}}{\partial t}\left[\sin_{LatE}\cos\phi_{Lat} - \cos\phi_{LatE}\sin\phi_{Lat}\cos(\phi_{Long}-\phi_{LongE})\right] - \cos\phi_{Lat}\cos\phi_{LatE}\sin(\phi_{Long}-\phi_{LongE})\frac{\partial\phi_{Long}}{\partial t} \tag{4.10}$$

Let $K\omega_E(t+TT) = Q$ and $\phi_{Long} + \omega_E t - \phi_o = \delta$ in both of Eq. (4.1) and (4.2). From these equations we can get the following relations:

$$\cos\phi_{Lat} = (1-\sin^2 i\,\sin^2 Q)^{1/2} \tag{4.11}$$

$$\sin\delta = \frac{\sin Q\,\cos i}{[1-\sin^2 i\,\sin^2 Q]^{1/2}} \tag{4.12}$$

and

$$\cos^2 \delta = 1 - \frac{\sin^2 Q \cos^2 i}{1 - \sin^2 i \sin^2 Q} \tag{4.13}$$

therefore

$$\cos^2 \delta = \frac{1 - \sin^2 Q}{1 - \sin^2 i \sin^2 Q} \tag{4.14}$$

or

$$\cos \delta = \frac{\cos Q}{[1 - \sin^2 i \sin^2 Q]^{1/2}} \tag{4.15}$$

Differentiating Eq. (4.1) with respect to t, we get

$$\cos\phi_{Lat} \frac{\partial \phi_{Lat}}{\partial t} = \sin i \cos Q \frac{\partial Q}{\partial t} \tag{4.16}$$

Noting that $\partial Q/\partial t = K\omega_E$, the last Equation will be

$$\cos\phi_{Lat} \frac{\partial \phi_{Lat}}{\partial t} = K \omega_E \sin i \cos Q \tag{4.17}$$

Substituting in Eq. (4.17) from Eq. (4.11) gives

$$\frac{\partial \phi_{Lat}}{\partial t} = \frac{K \omega_E \sin i \cos Q}{[1 - \sin^2 i \sin^2 Q]^{1/2}} \tag{4.18}$$

Now, substituting Eq. (4.15) in Eq. (4.18), we obtain

$$\frac{\partial \phi_{Lat}}{\partial t} = K \omega_E \sin i \cos\delta \tag{4.19}$$

Also, by differentiating Eq. (4.12) with respect to t, noting that ϕ_o is constant and $\phi_{Long} + \omega_E t - \phi_o$ we get

$$\cos\delta \left(\frac{\partial \phi_{Long}}{\partial t} + \omega_E \right) = K \omega_E \frac{\cos i \cos Q}{(1 - \sin^2 i \sin^2 Q)^{3/2}} \tag{4.20}$$

Substituting for $\cos\delta$ from Eq. (4.15), in Eq. (4.20) (after algebraic manipulation), we arrive at

$$\frac{\partial \phi_{Long}}{\partial t} = K \omega_E \frac{\cos i}{(1 - \sin^2 i \sin^2 Q)} - \omega_E \tag{4.21}$$

By making use of Eq. (4.11); Eq. (4.21) reduces to

$$\frac{\partial\phi_{Long}}{\partial t} = \frac{K\,\omega_E \cos i}{\cos^2 \phi_{Lat}} - \omega_E \tag{4.22}$$

Substituting both Eq. (4.19) and (4.22) in Eq. (4.10) reduces to (after simplifications)

$$\frac{\partial T_E}{\partial t} = A\,\omega_E \left[K \cos\delta \sin i \,\{B_2 \cos \eta_E \sin\phi_{Lat} - B_1 \cos\phi_{Lat}\} + B_2 \sin \eta_E \left\{ \frac{K \cos i}{\cos\phi_{Lat}} - \cos\phi_{Lat} \right\} \right]. \tag{4.23}$$

where

$A = R(R+h)/(d_E \cdot C)$

$\delta = \phi_{Long} + \omega_E t - \phi_o$

$\eta_E = \phi_{Long} - \phi_{LongE}$

$B_1 = \sin \phi_{LatE}$

$B_2 = \cos \phi_{LatE}$

Equation (4.23) gives the exact formula for the change of the space delay

$$\frac{\partial T_E}{\partial t}$$

for a satellite-earth station link.

Note that for the special case of polar orbit, where $i = 90°$, Eq. (4.1) and (4.2) reduces to

$$\phi_{Lat} = Q \tag{4.24}$$

$$\phi_{Long} = -\omega_E t + \phi_o \tag{4.25}$$

Hence, the relation between the latitude and the longitude of the satellite subpoint is a linear relation. Thus, we have

$$\frac{\partial\phi_{Lat}}{\partial t} = K\,\omega_E \tag{4.26}$$

and

$$\frac{\partial \phi_{Long}}{\partial t} = -\omega_E \tag{4.27}$$

Therefore, substituting Eq. (4.26) and Eq. (4.27) in Eq. (4.10) and inserting it in Eq. (4.7), we obtain

$$\frac{\partial T_E}{\partial t} = A\,\omega_E \left[K\,\{B_2 \cos \eta_E \sin Q - B_1 \cos Q\} + B_2 \sin \eta_E \cos Q \right] \tag{4.28}$$

Now let the inclination angle $i = 0$, i.e., the satellite subpoint is always located on the equator, therefore

$$\phi_{Lat} = 0 \tag{4.29}$$

$$\phi_{Long} = Q - \omega_E t + \phi_0 \tag{4.30}$$

Substituting in Eq. (4.23) this yields to

$$\frac{\partial T_E}{\partial t} = B_2 \sin \eta_E (K - 1) \tag{4.31}$$

If $K = 1$, i.e., the satellite is a geostationary Eq. (4.31) will be

$$\frac{\partial T_E}{\partial t} = 0$$

which is true, since the space delay from any earth station to any geostationary satellite is assumed to be fixed.

Figures (4.8), (4.9) and (4.10) show the change in the space delay

$$\frac{\partial T_E}{\partial t}$$

versus the time measured by the reference clock for the curves shown by Figures (4.2), (4.3) and (4.4) respectively

Figures 4.11, 4.12 and 4.13 are plotted for the case of the 12 hour orbit satellite as in Figures 4.5, 4.6 and 4.7. The main difficulty demonstrated by the Figures from (4.2) to (4.7) is that the satellite is below the horizon for a long period.

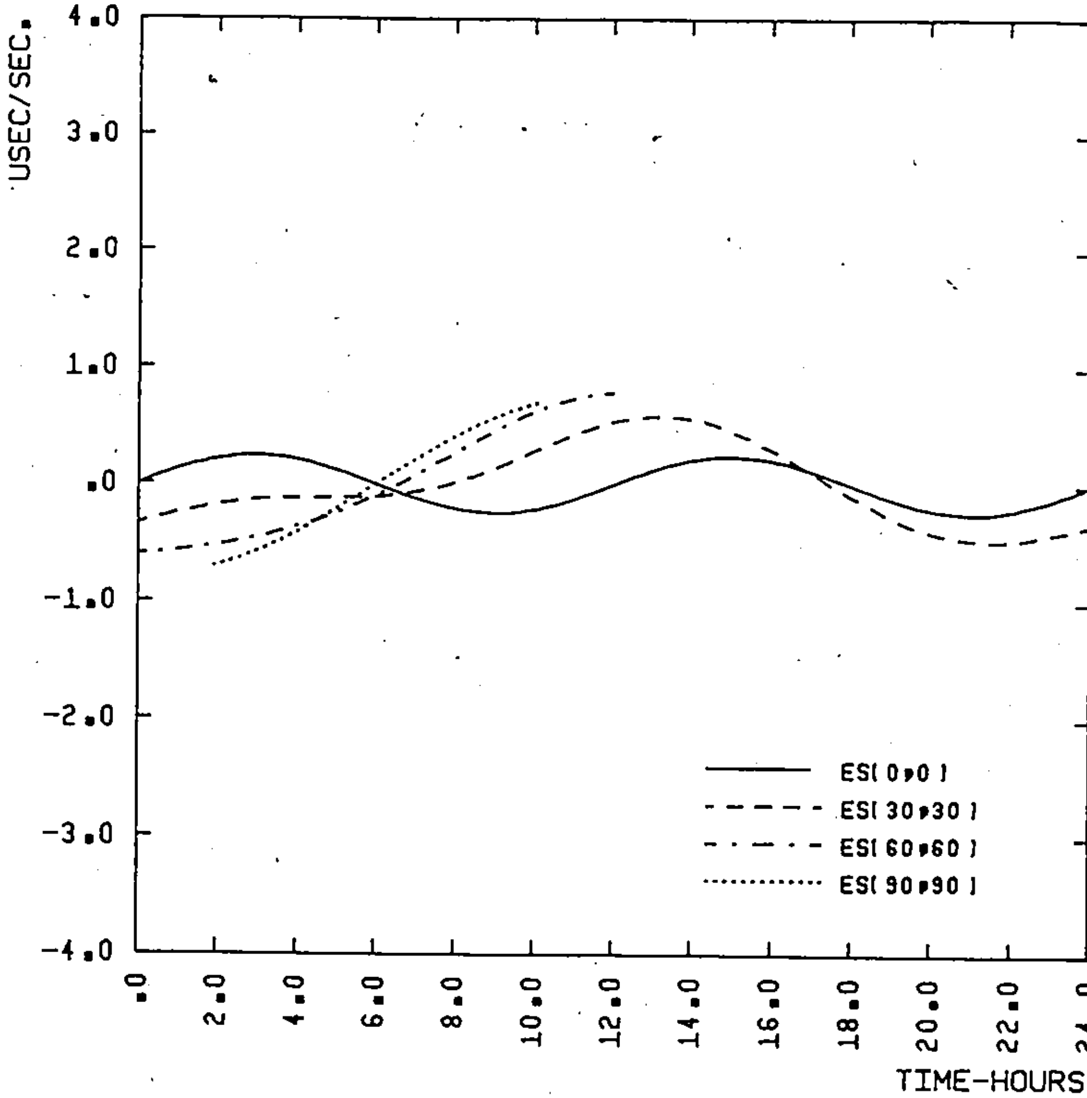

Fig. 4.8 Change in the space delays plotted in Fig. 4.2 for the same four earth stations.

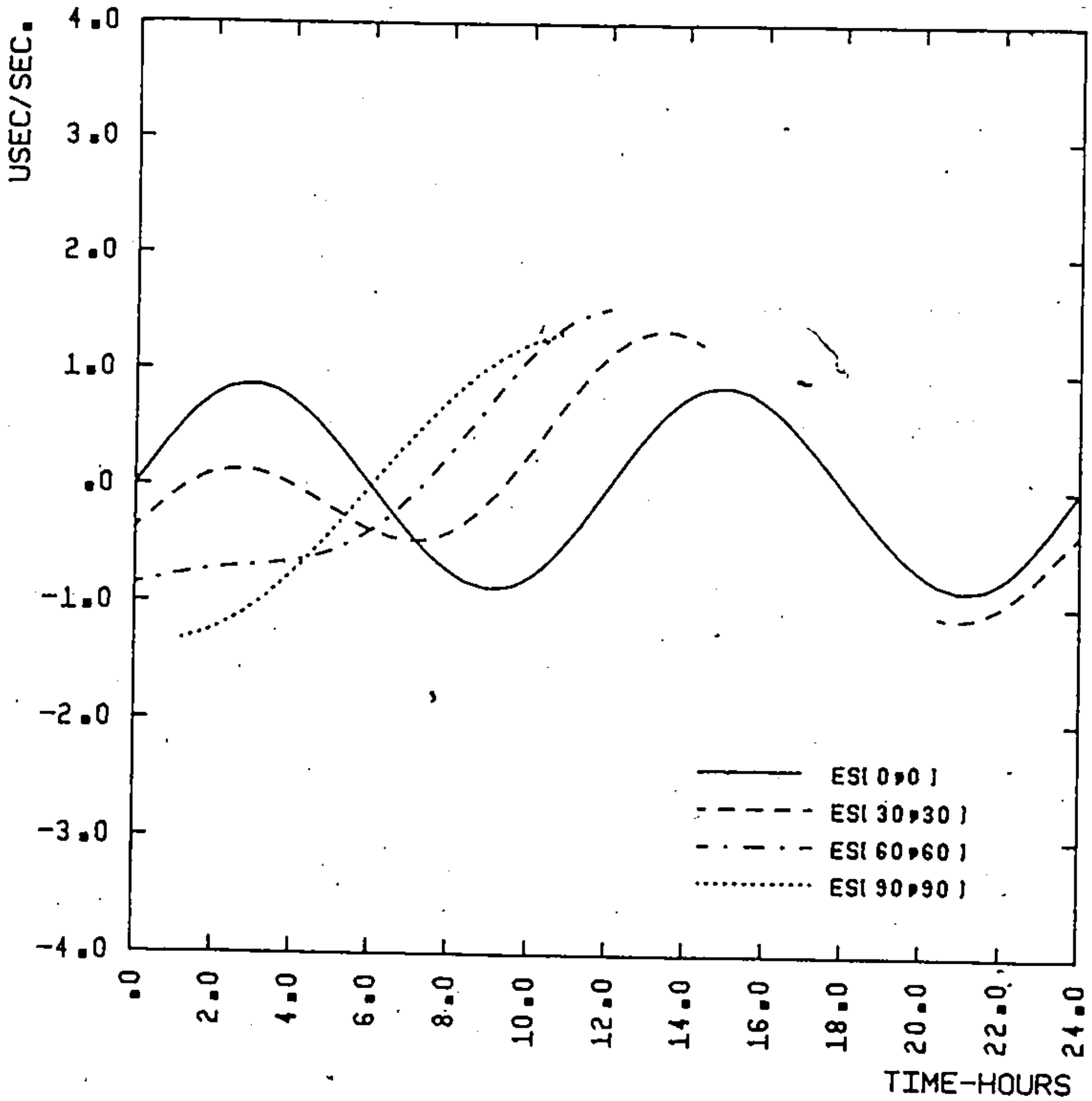

Fig. 4.9 Change in the space delays plotted in Fig. 4.3 for the same four earth stations.

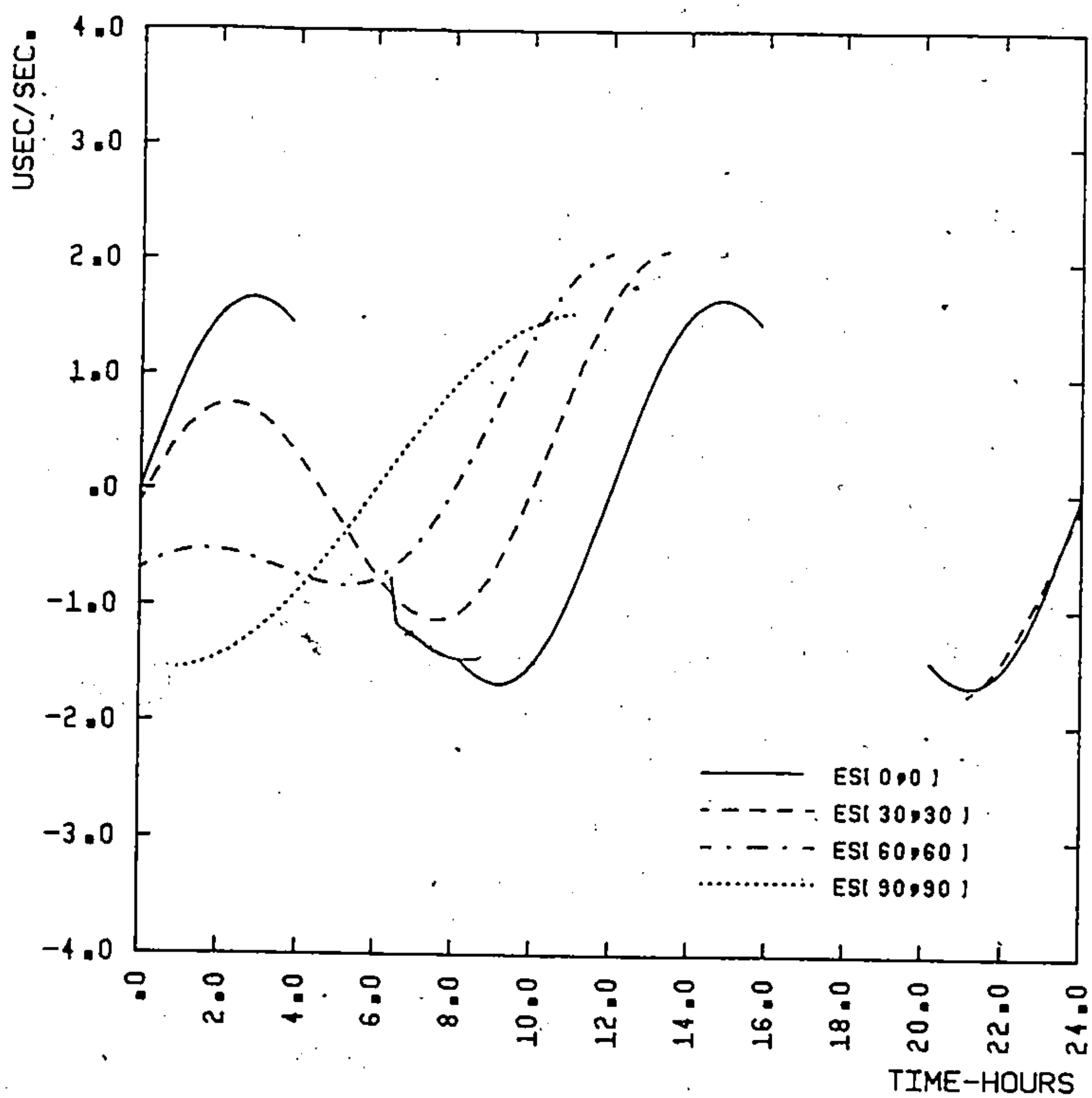

Fig. 4.10 Change in the space delays plotted in Fig. 4.4 for the same four earth stations.

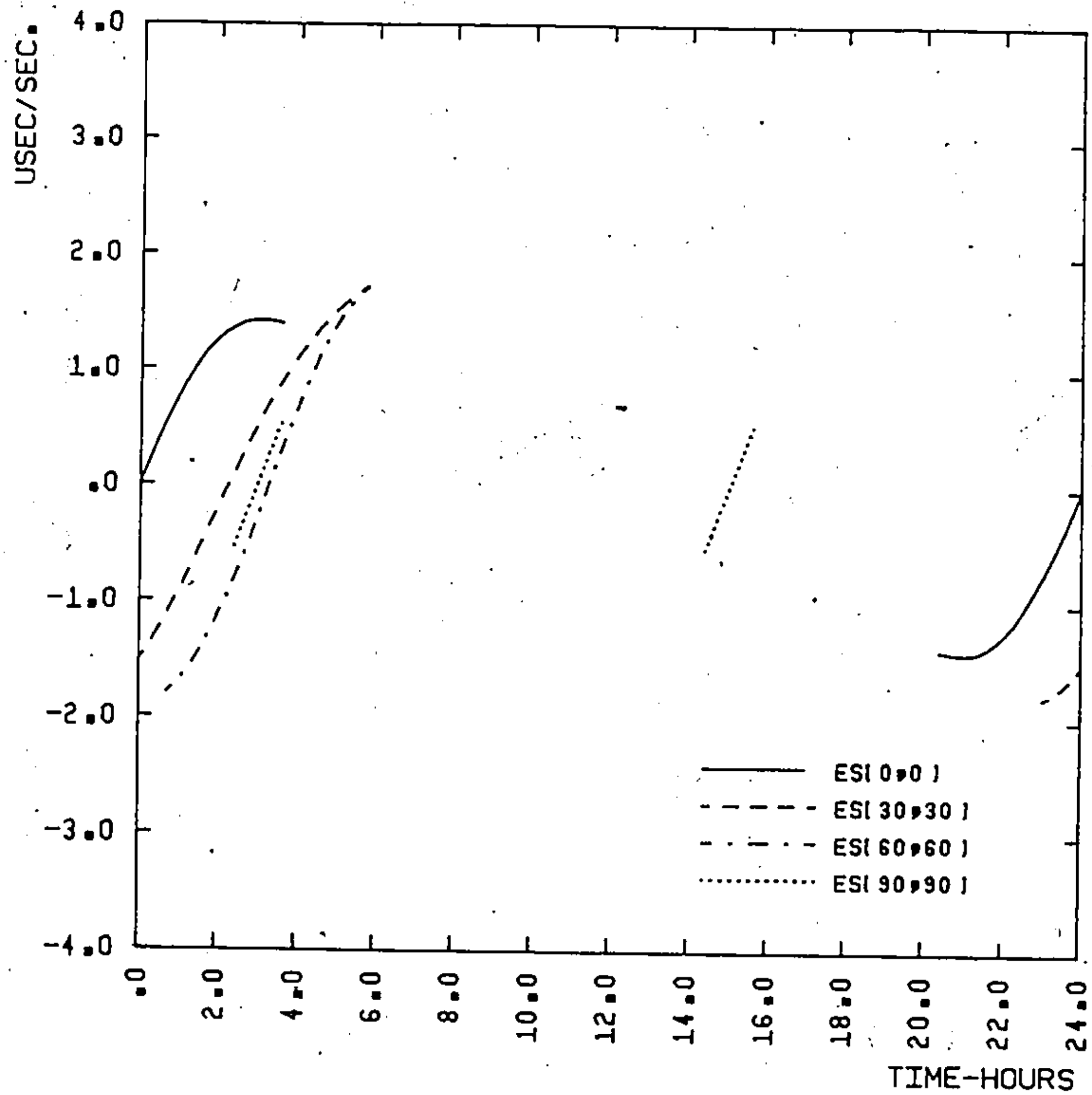

Fig. 4.11 Change in the space delays plotted in Fig. 4.5 for the same four earth stations.

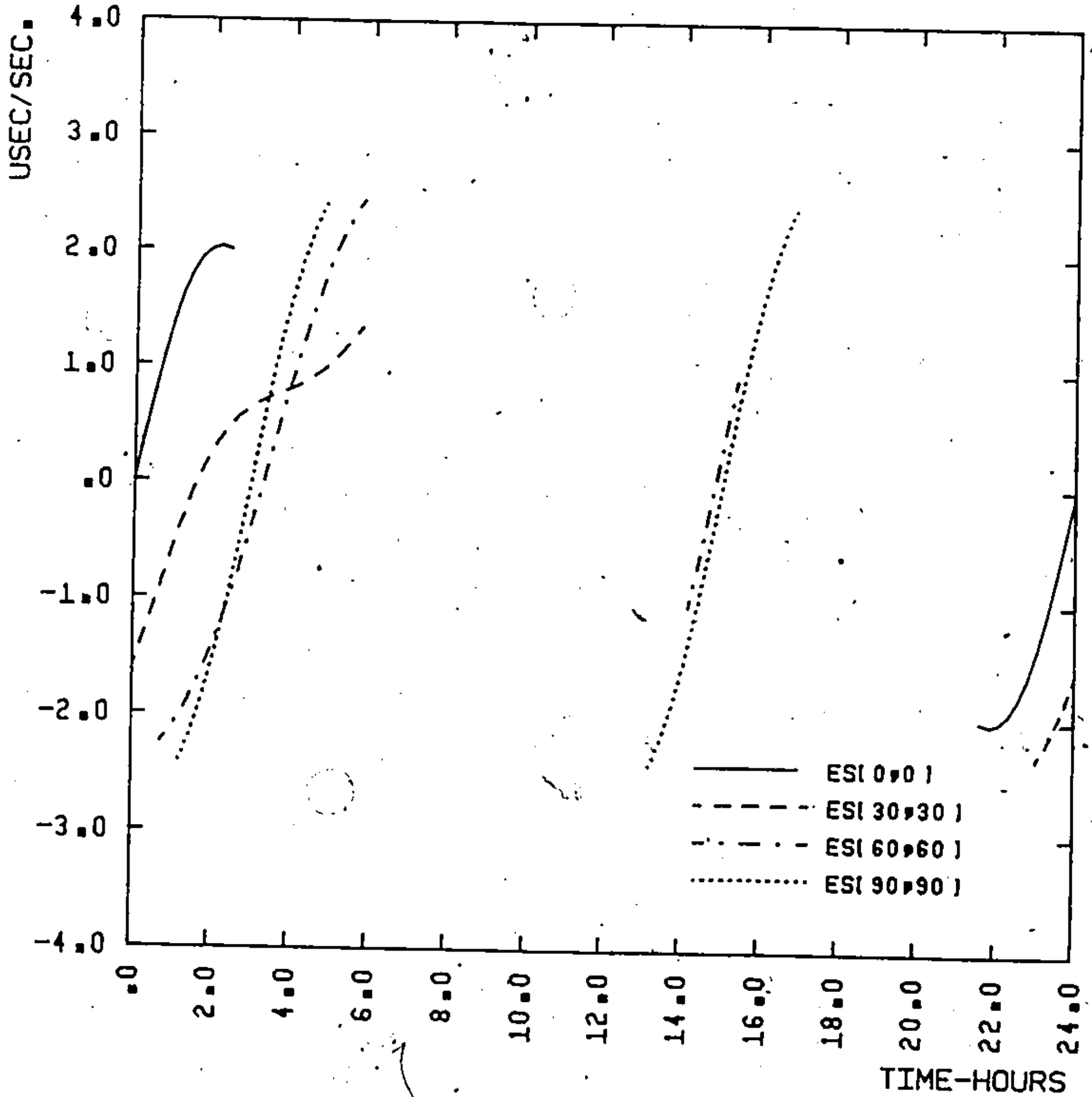

Fig. 4.12 Change in the space delays plotted in Fig. 4.6 for the same four earth stations.

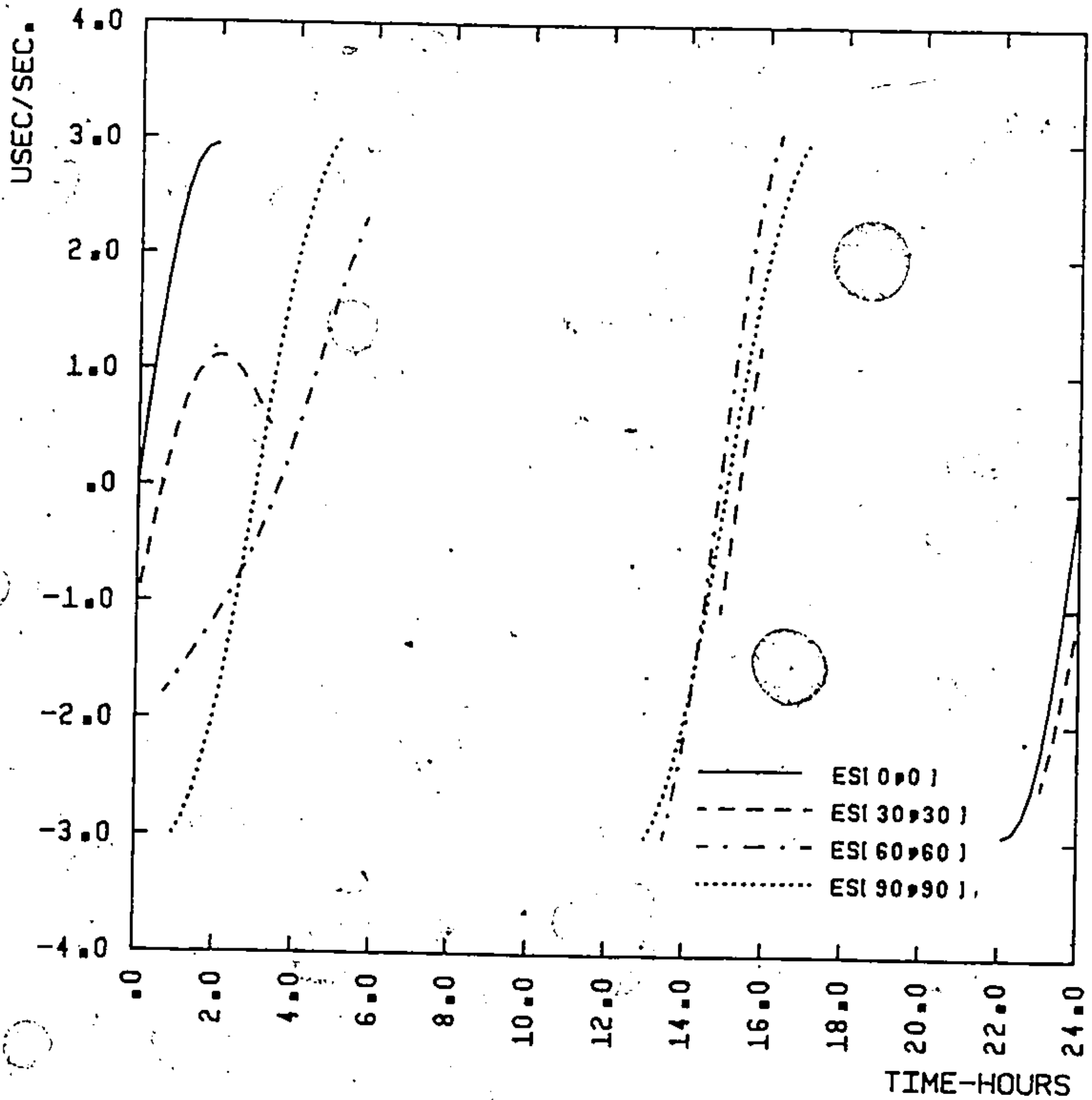

Fig. 4.13 Change in the space delays plotted in Fig. 4.7 for the same four earth stations.

4.4 THE SPACE DELAY BETWEEN TWO SATELLITES

At this point, let us define the space delay between two satellites at an instant of time as the distance between the two divided by the velocity of propagation. Note that this is not the same as the uplink or downlink space delay when relative motion exists between the two satellites; as will be investigated in Chapter 5. Refer to Fig. 4.14 which shows two satellites S_1 and S_2 in two different orbits. The satellite S_1 rotates at an altitude h_1 while the corresponding altitude for S_2 is h_2. At any instant of time t the subpoints for S_1 and S_2 on the surface of the earth are S_1' and S_2', respectively. From Chapter 2, the locations of S_1' and S_2' are well defined at any instant, by forming a plane passing through the center of the earth O and the two satellite subpoints, this plane intersects the surface of the earth in a great circle. The size of the smallest arc λ between the two satellite subpoints, measured from the center of the earth, as shown in Fig. 4.14, is given by

$$\cos\lambda = \cos\phi_{Lat1}\cos\phi_{Lat2}\cos(\phi_{Long1} - \phi_{Long2}) + \sin\phi_{Lat1}\sin_{Lat2} \tag{4.32}$$

where ϕ_{Lat_1}, ϕ_{Long_1} are the latitude and longitude of the satellite subpoint S_1'; and, ϕ_{Lat_2}, ϕ_{Long_2} are the corresponding values for the satellite subpoint S_2'. Solving for the plane triangle S_1S_2O we get

$$d = \sqrt{(R+h_1)^2 + (R+h_2)^2 - 2(R+h_1)(R+h_2)\cos\lambda} \tag{4.33}$$

where R is the radius of the earth and d is the distance between the two satellites.

Assume that the satellite S_1 is a navigation satellite in inclined orbit (S_1' has latitude ϕ_{LatN} and longitude ϕ_{LongN}). Further, we consider the satellite S_2 is a geostationary satellite with latitude $\phi_{LatG} = 0$ and longitude ϕ_{LongG}. Substituting these values in Eq. (4.32) and (4.33), we get

$$\cos\lambda = \cos\phi_{LatN}\cos(\phi_{LongN} - \phi_{LongG}) \tag{4.34}$$

and

$$d = \sqrt{H_N^2 + H_G^2 - 2H_N H_G \cos\lambda} \tag{4.35}$$

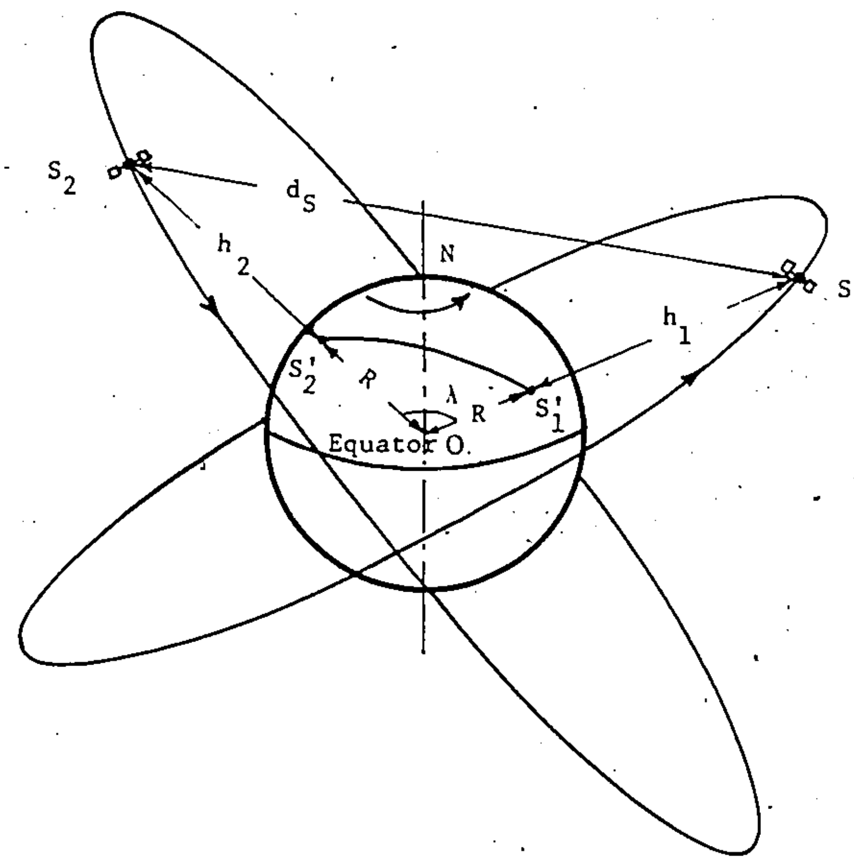

Fig. 4.14 Geometry of two satellites in different orbits.

where H_N and H_G are the distance measured from the center of the earth to the navigation satellite and the geostationary satellite respectively. Dividing d by the velocity of propagation C, we can evaluate the space delay T between two satellites at any instant t.

$$T = d/C \tag{4.36}$$

Consider the case of a navigation satellite S_1 in a 12 hour inclined orbit with initial conditions $TT = 0$, $\phi_{oN} = 0°$ and $i = 60°$. The space delay T between this satellite and each of six satellites S_2 in geostationary orbit with initial condition $TT = 0$ and $\phi_{LongG} = 0°, 30°, 60°, 90°, 120°$ and $150°$ is plotted, as a function of time, in Fig. 4.15. Figure 4.16 represents the space delay T for the same set of six geostationary satellite locations and a navigation satellite in 16 hour orbit.

It is clear that the communication between any two satellites depends upon the relative locations of both satellites and the earth, i.e., the communications between the two satellites is guaranteed except for the case when the earth is located between the two satellites which produces the shadow zone of the earth. For two satellites, the shadow zone is defined by the condition when line connecting them is tangent to or intersecting the surface of the earth [28,33]. Thus, the communication link is blocked if

$$d \geq \sqrt{H_N^2 - R^2} + \sqrt{H_G^2 - R^2} \tag{4.37}$$

It is clear, from Fig. 4.15 that the satellite in inclined orbit always sees the geostationary satellites located at longitudes 30°, 60°, 120° and 150°. However, if the geostationary satellite is located at longitudes 0° and 90° the shadow zone exists for a period of almost 1.76 hours every 24 hours.

4.5 CHANGE IN SPACE DELAY WITH TIME

To find the change in the space delay for the curves of Fig. 4.15 and 4.16, we differentiate Eq. (4.36) with respect to the time t measured by the reference clock. Note that

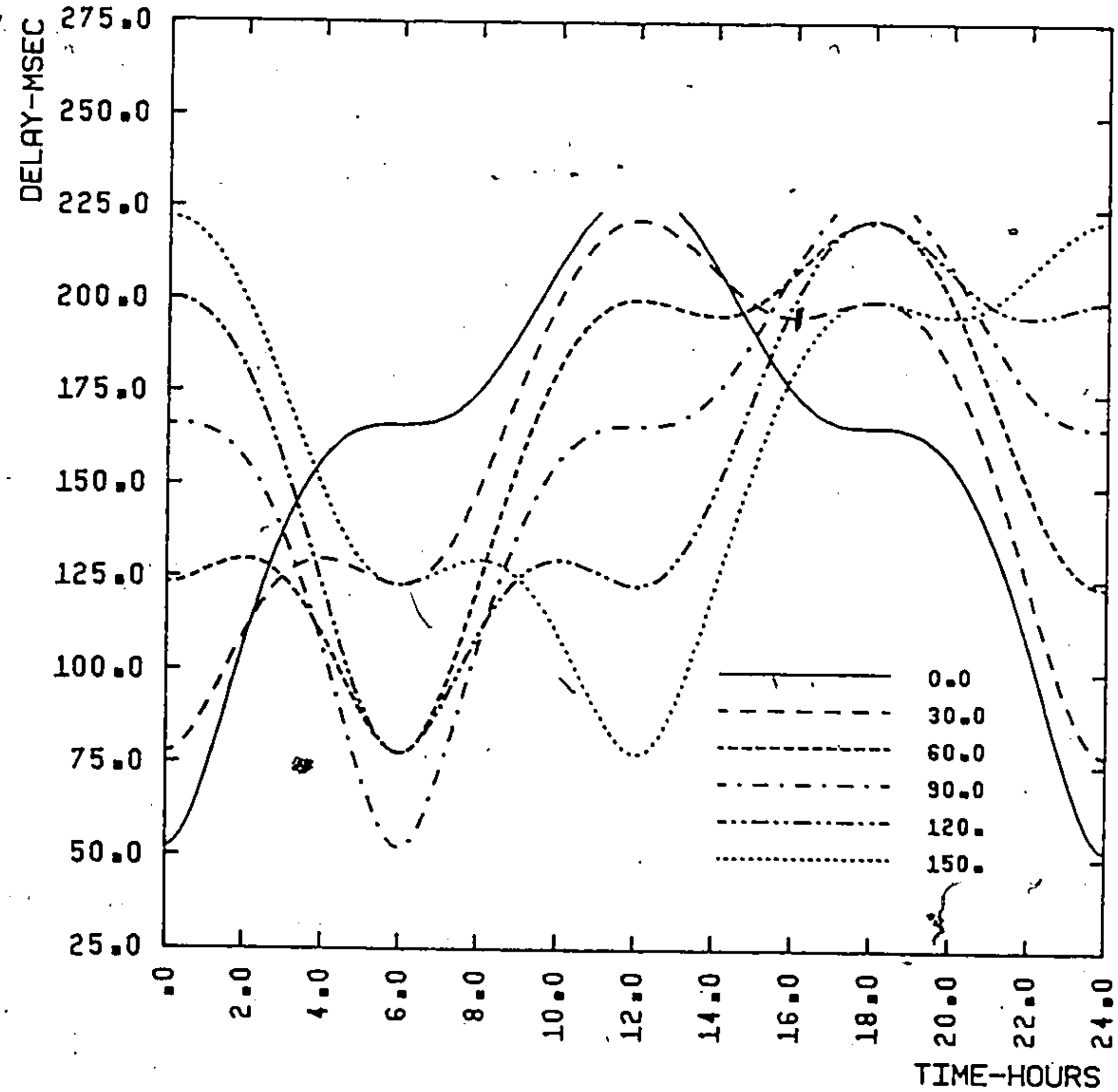

Fig. 4.15 Space delay between a satellite in 12 four orbit with i = 60° and six geostationary satellites located at different longitudes.

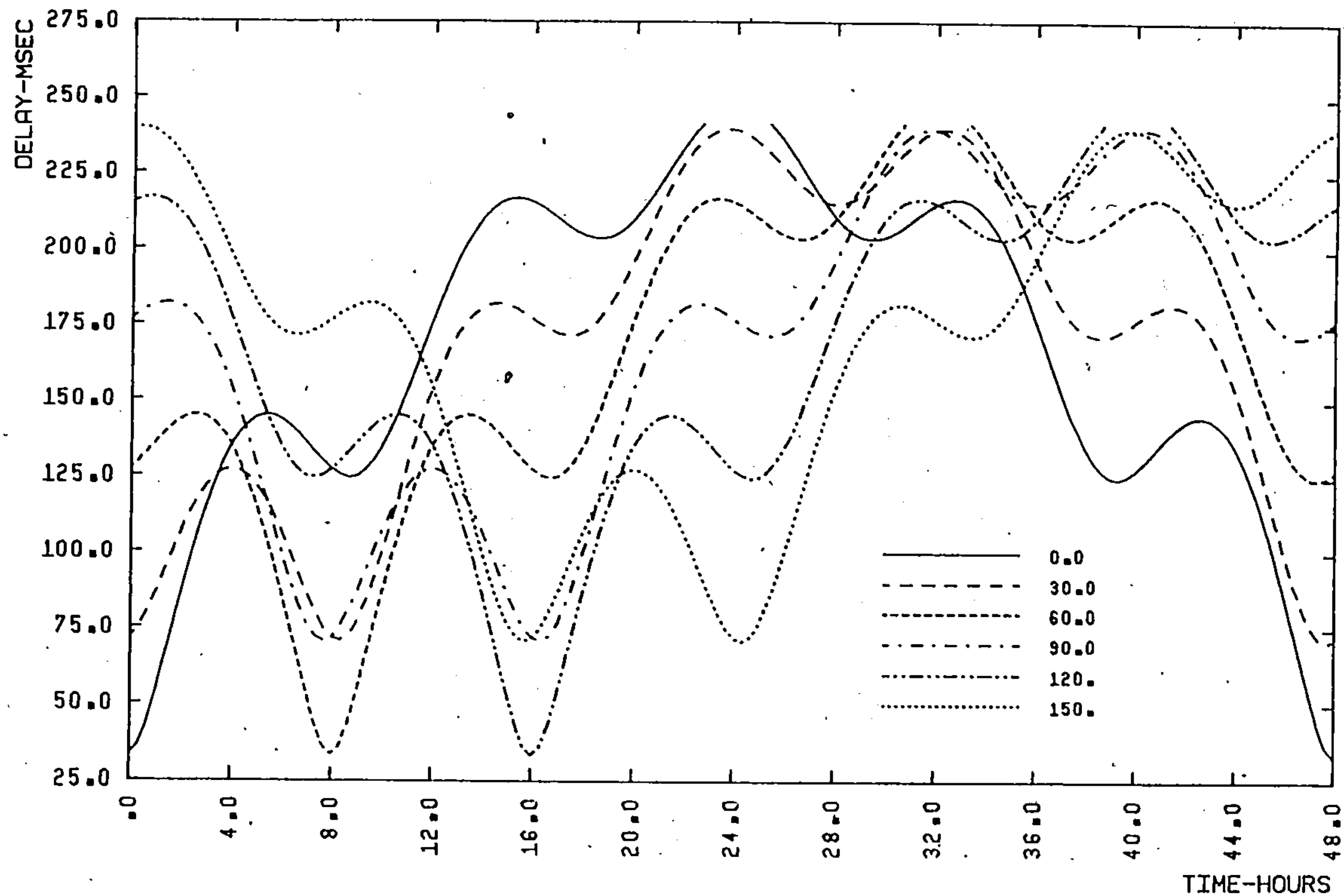

Fig. 4.16 Space delay between a satellite in 16 hour orbit with $i = 60°$ and six geostationary satellies located at different longitudes.

Eq. (4.36) depends on the time t through the location of the subpoint of the navigation satellite on the surface of the earth only, because the latitude of any geostationary satellite equals zero and its longitude is a fixed value. By substituting Eq. (4.34) and (4.35) in Eq. (4.36), and making use of Eq. (3.19) and (3.22) we have after simplification

$$\frac{\partial T}{\partial t} = D\,\omega_E \left| K\cos\delta \sin i \cos\eta \sin\phi_{Lat_N} + \sin\eta\left(\frac{K\cos i}{\cos\phi_{Lat_N}} - \cos\phi_{Lat_N}\right)\right| \tag{4.38}$$

where

$$D = \frac{H_N H_G}{d \cdot C}$$

$$\delta = \phi_{LongN} + \omega_E t - \phi_{oN}$$

ϕ_{LatN} = the latitude of the subpoint of the navigation satellite

ϕ_{LongN} = the longitude of the subpoint of the navigation satellite

ϕ_{oN} = the longitude of the point of intersection between the equator and the orbital projection of the navigation satellite orbit (on the surface of the earth) at t = 0

η = $\phi_{LongN} - \phi_{LongG}$

ϕ_{LongG} = the longitude of the subpoint of the geostationary satellite.

From Eq. (4.38), we see that by choosing the inclination angle i, we can force the change in the space delay ($\partial T/\partial t$) to be equal to zero when the navigation satellite passes through the equatorial plane, ie., $(\partial T/\partial t) = 0$ when $\phi_{Lat_N} = 0$. Such a value of the inclination angle i is given by

$$i = \arccos(1/K) \tag{4.39}$$

which is the lower critical limit for the nonsynchronous factor K given by Eq. (2.31). For example if K = 2, then the proper inclination angle i must be 60°. Fig. 4.17 shows the change

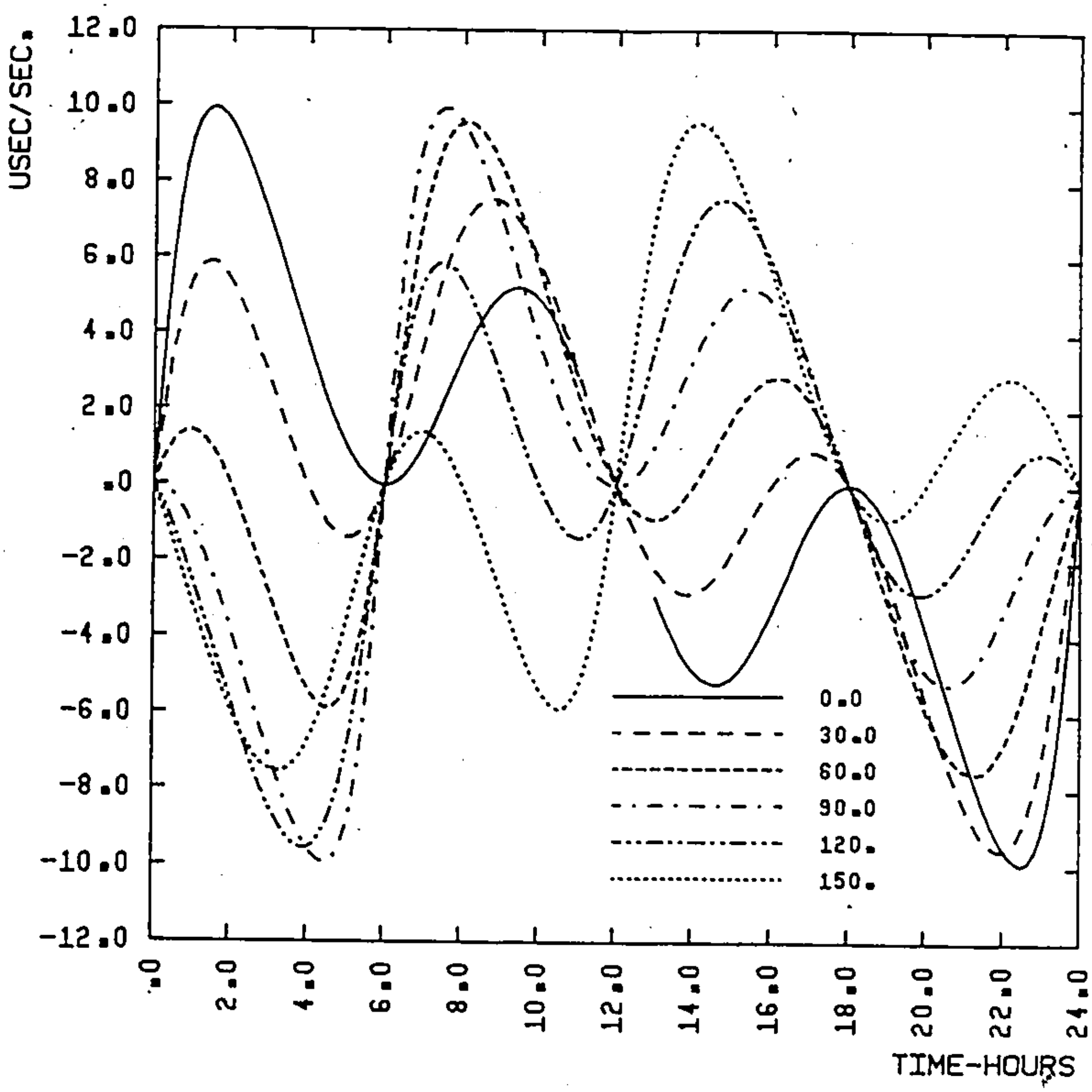

Fig. 4.17 Change in the space delays plotted in Fig. 4.15 for the same six geostationary satellites.

in the space delay ($\partial T/\partial t$) versus the navigation time t in hours, for the case plotted in Fig. 4.15. From Fig. 4.17, it is clear that the change in the space delay equals zero when the satellite crosses the equatorial plane, i.e. every six hours. Also it is clear that the maximum change in the space delay for the case of 12 hour orbit is equal to $\pm$ 10 μ s/s. Fig. 4.18 represents the case of a 16 hour orbit.

In order to find the higher order derivatives of the space delay T, it is more convenient to reformulate Eq. (4.33) in a format suitable for differentiation.

Let us consider the difference in longitude $\phi_{LongN} - \phi_{LongG}$ given in Eq. (4.33). This angle can be written in the form

$$\phi_{LongN} - \phi_{LongG} = (\phi_{LongN} + \omega_E t - \phi_{oN}) - (\phi_{LongG} + \omega_E t - \phi_{oN}) \tag{4.40}$$

or

$$\phi_{LongN} - \phi_{LongG} = \delta - \Lambda \tag{4.41}$$

where δ is defined in Eq. (4.38) and

$$\Lambda = \phi_{LongG} + \omega_E t - \phi_{oN} \tag{4.42}$$

Substituting from Eq. (4.41) in Eq. (4.33) yields

$$\cos\lambda = \cos\phi_{LatN} (\cos\delta \cos\Lambda + \sin\delta \sin\Lambda) \tag{4.43}$$

Substituting in Eq. (4.43) for $\cos\phi_{LatN}$, $\sin\delta$, $\cos\delta$ from Eq. (4.11), (4.12) and (4.15), respectively, we get after simplification

$$\cos\lambda = \cos Q \cos\Lambda + \cos i \sin Q \sin\Lambda \tag{4.44}$$

We notice here, that Q and Λ are the only time dependent variables in Eq. (4.44). Differentiating Eq. (4.44) with respect to time noting that

$$\frac{\partial Q}{\partial t} = K\omega_E \text{ and } \frac{\partial \Lambda}{\partial t} = \omega_E ,$$

yields

$$\frac{\partial \cos\lambda}{\partial t} = -\omega_E (K \sin Q \cos\Lambda + \cos Q \sin\Lambda) + \omega_E \cos i (K \cos Q \sin\Lambda + \sin Q \cos\Lambda) \tag{4.45}$$

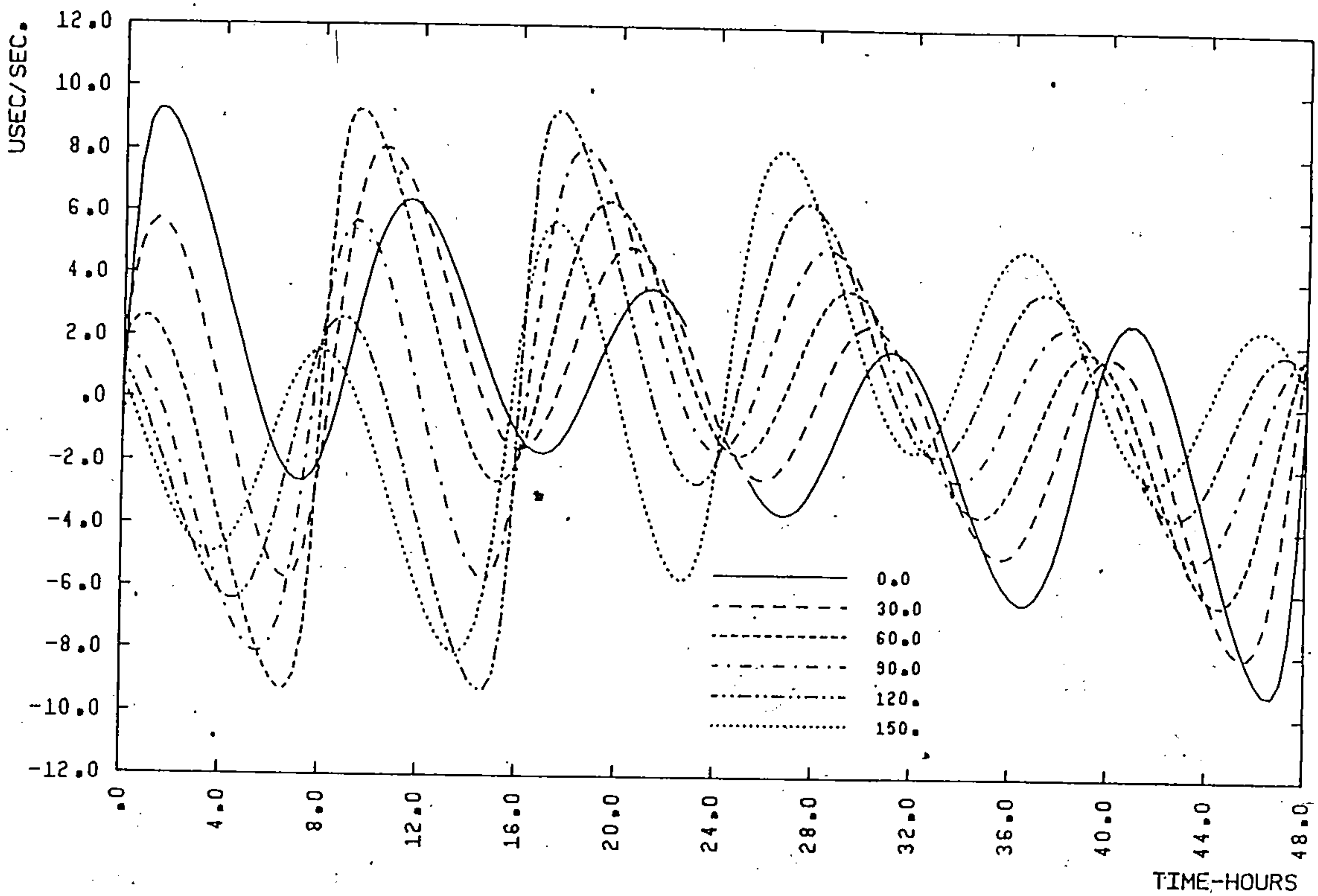

Fig. 4.18 Change in the space delays plotted in Fig. 4.16 for the same six geostationary satellites.

or

$$\frac{\partial \cos\lambda}{\partial t} = (\cos i - K)\sin Q \cos \Lambda + (K\cos i - 1)\cos Q \sin \Lambda \tag{4.46}$$

Differentiating Eq. (4.36) with respect to time, making use of Eq. (4.34) and (4.35); and noting that Λ is the only time dependent variable, we get

$$\frac{\partial T}{\partial t} = \frac{-H_N H_G}{d \cdot C} \frac{\partial \cos\lambda}{\partial t} \tag{4.47}$$

Substituting in Eq. (4.47) from Eq. (4.45), yields

$$\frac{\partial T}{\partial t} = D\omega_E [(K - \cos i)\sin Q \cos \Lambda + (1 - K\cos i)\cos Q \sin \Lambda] \tag{4.48}$$

where

$$D = \frac{H_N H_G}{d \cdot C} \tag{4.49}$$

Equation (4.38) and (4.48) are identical but Eq. (4.48) is more suitable for further differentiation.

To prove that the change in the space delay can be represented by either Eq. (4.38) or (4.48), we will show that the two equations are identical.

Let us put Eq. (4.38) in the form

$$\frac{\partial T}{\partial t} = D\omega_E (A + B)$$

where,

$$A = K\cos\delta \sin i \cos\eta \sin\phi_{LatN} \tag{4.50}$$

and

$$B = \sin\eta \left[\frac{K\cos i}{\cos\phi_{LatN}} - \cos\phi_{LatN} \right] \tag{4.51}$$

Substituting Eq. (4.1), (4.15) and (4.41) in Eq. (4.50) we get

$$A = K \frac{\cos Q}{\cos\phi_{LatN}} \sin i \cos(\delta - \Lambda) \sin i \sin Q \tag{4.52}$$

By expanding the term cos (δ–Λ) in Eq. (4.52) and using Eq. (4.12) and (4.15) we get

$$A = K \frac{\sin^2 i \cos Q \sin Q}{\cos \phi_{LatN}} \left[\cos \Lambda \frac{\cos Q}{\cos \phi_{LatN}} + \sin \Lambda \frac{\sin Q \cos i}{\cos \phi_{LatN}} \right] \tag{4.53}$$

The last equation is reduced to

$$A = \frac{K \sin^2 i \cos Q \sin Q}{\cos^2 \phi_{LatN}} [\cos \Lambda \cos Q + \sin \Lambda \sin Q \cos i] \tag{4.54}$$

Now, consider the remaining terms given by B in Eq. (4.51).

Substituting from Eq. (A1.12) in Eq. (A1.21) we have:

$$B = [\sin \delta \cos \Lambda - \cos \delta \sin \Lambda] \left[\frac{K \cos i}{\cos \phi_{LatN}} - \cos \phi_{LatN} \right] \tag{4.55}$$

Substituting from Eq. (4.11) in Eq. (4.12) and (4.15) and then, in Eq. (4.55), this yields:

$$B = [\sin Q \cos i \cos \Lambda - \cos Q \sin \Lambda] \left[\frac{K \cos i}{\cos^2 \phi_{LatN}} - 1 \right] \tag{4.56}$$

By adding Eq. (4.53) and (4.56) and examining the terms with $\sin \Lambda$ as a common factor we get:

$$E = \sin \Lambda \left[\frac{K \sin^2 i \cos Q \sin^2 Q \cos i - K \cos i \cos Q}{\cos^2 \phi_{LatN}} + \cos Q \right] \tag{4.57}$$

The last equation can be reduced to

$$E = \sin \Lambda \cos Q \left[\frac{K \cos i (\sin^2 Q \sin^2 i - 1)}{\cos^2 \phi_{LatN}} + 1 \right] \tag{4.58}$$

Noting that $\cos \phi_{LatN} = [1 - \sin^2 Q \sin^2 i]^{1/2}$, Eq. (4.58) will be:

$$E = \sin \Lambda \cos Q [1 - K \cos i] \tag{4.59}$$

Now consider the remaining term from A+B, we find that $\cos \Lambda$ is a common factor, these remaining terms are given by:

$$F = \cos \Lambda \left[\left\{ \frac{K \sin^2 i \cos^2 Q \sin Q + K \cos^2 i \sin Q}{\cos^2 \phi_{LatN}} \right\} - \sin Q \cos i \right] \tag{4.60}$$

or

$$F = \cos \Lambda \left[\left\{ \frac{K \sin Q (\cos^2 Q \sin^2 i + \cos^2 i)}{\cos^2 \phi_{LatN}} \right\} - \sin Q \cos i \right] \tag{4.61}$$

Substituting in Eq. (4.61) for $\cos^2 i = 1 - \sin^2 i$ and $\cos \phi_{LatN}$ from Eq. (4.11) and after simplification we have:

$$F = \cos\Lambda \left[\left\{ \frac{K \sin Q \,(1 - \sin^2 i \sin^2 Q)}{(1 - \sin^2 i \sin Q)} \right\} - \sin Q \cos i \right] \tag{4.62}$$

or

$$F = \cos\Lambda \sin Q \,(K - \cos i) \tag{4.63}$$

Now, by adding E+F and multiplying by $D\omega_E$, we can get the change in the space delay $\partial T/\partial t$, this is given by

$$\frac{\partial T}{\partial t} = D\omega_E \left[\cos\Lambda \sin Q\,(K - \cos i) + \sin\Lambda \cos Q\,(1 - K\cos i)\right] \tag{4.64}$$

Equation (4.64) is the same as Eq. (4.48), therefore, both Eq. (4.38) and (4.48) are identical.

Now, to find the change in the change in the space delay (second order derivative) we differentiate Eq. (4.48) with respect to t taking in consideration that d, Λ and Q are time dependent variables such that,

$$\frac{\partial \Lambda}{\partial t} = \omega_E, \quad \frac{\partial Q}{\partial t} = K\omega_E \quad \text{and} \quad \frac{\partial d}{\partial t} = C\,\frac{\partial T}{\partial t}$$

Therefore

$$\begin{aligned} \frac{\partial^2 T}{\partial t^2} = {} & D\omega_E \left[(K - \cos i)(\omega_S \cos\Lambda \cos Q - \omega_E \sin\Lambda \sin Q)\right. \\ & \left. + (1 - K\cos i)(\omega_E \cos\Lambda \cos Q - \omega_S \sin\Lambda \sin Q)\right] \\ & + \left(\frac{\partial T}{\partial t}\cdot d\right)\left(-\frac{1}{d^2}\right)\frac{\partial d}{\partial t} \end{aligned} \tag{4.65}$$

Noting that $d/C = T$ and $\omega_S = K\,\omega_E$ and after simplifications we get

$$\frac{\partial^2 T}{\partial t^2} = D\omega_E^2 \left[(K^2 - 2K\cos i + 1)\cos\Lambda\cos Q \right.$$

$$\left. + (K^2\cos i - 2K + \cos i)\sin\Lambda\sin Q \right] \tag{4.66}$$

$$- \frac{1}{T}\left(\frac{\partial T}{\partial t}\right)^2 .$$

Figures 4.19 show the change in the change in the space delay for the case plotted in Fig. 4.15 and 4.17. Figure 4.19 shows that the maximum change in the change in the space delay is $\underline{+}4$ ns/s^2. Figure 4.20 is plotted for the case of the 16 hour orbit satellite illustrated in Fig. 4.16 and 4.18.

Now to find the third order derivative we differentiate Eq. (4.66) with respect to time noting that $D = H_N H_G/d.C$ this gives

$$\frac{\partial^3 T}{\partial t^3} = D\omega_E^2 \left[AA\left(-\omega_E \sin\Lambda\cos Q - \omega_S\cos\Lambda\sin Q\right) \right.$$

$$\left. + BB\left(\omega_E\cos\Lambda\sin Q + \omega_S\sin\Lambda\cos Q\right)\right] \tag{4.67}$$

$$- \left(\frac{\partial^2 T}{\partial t^2} + \frac{1}{T}\left(\frac{\partial T}{\partial t}\right)^2\right) d\left(-\frac{1}{d^2}\right)\frac{\partial d}{\partial t}$$

$$- \left(\frac{1}{T}\cdot 2\,\frac{\partial T}{\partial t}\cdot\frac{\partial^2 T}{\partial t^2}\right) - \left(\frac{-1}{T^2}\right)\frac{\partial T}{\partial t}\cdot\left(\frac{\partial T}{\partial t}\right)^2$$

where

$AA = K^2 - 2K\cos i + 1$

and

$BB = K^2\cos i - 2K + \cos i$

Equation (4.67) can be reduced to

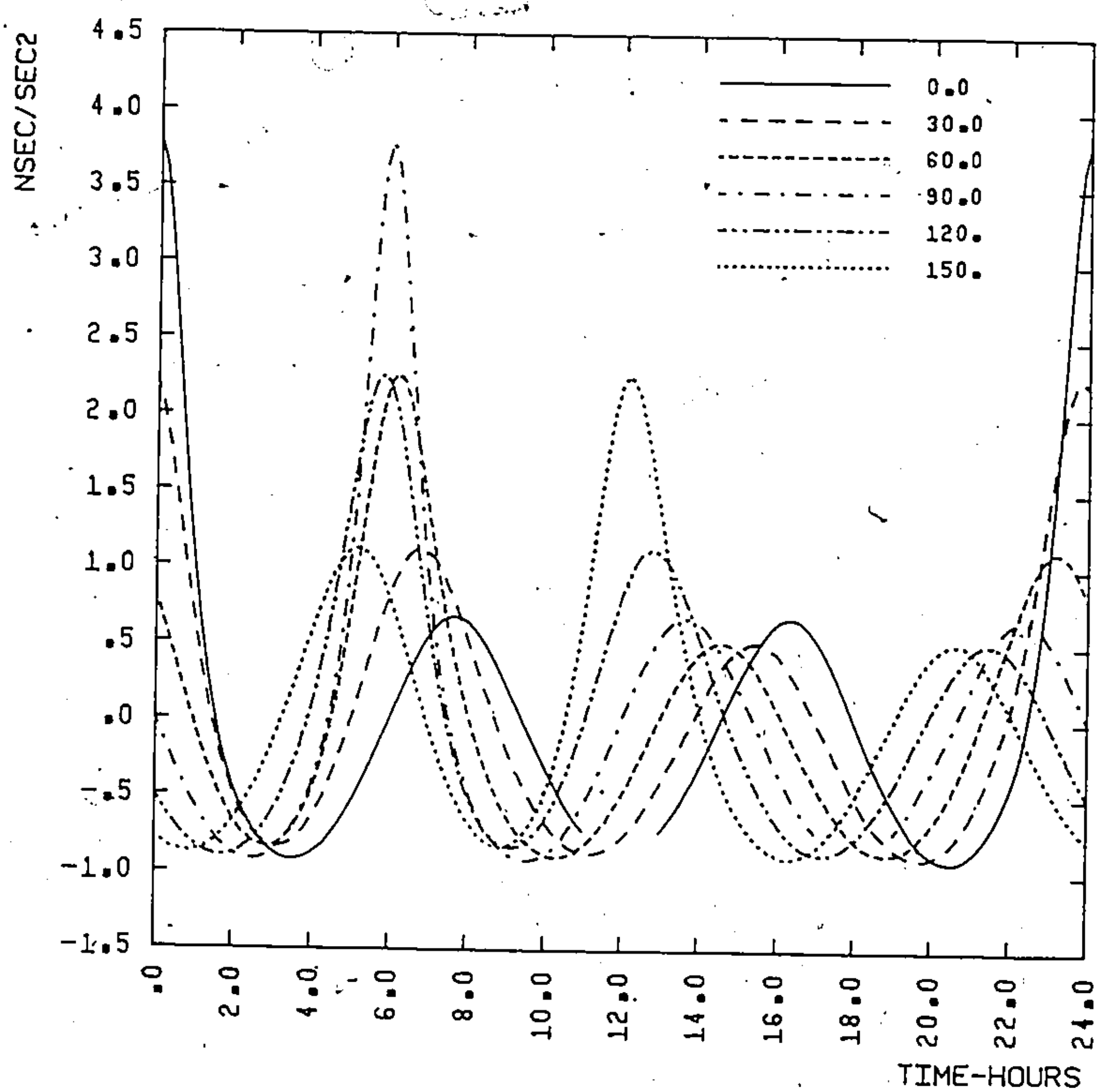

Fig. 4.19 Change in the change in space delays plotted in Fig. 4.17 for the same six geostationary satellites.

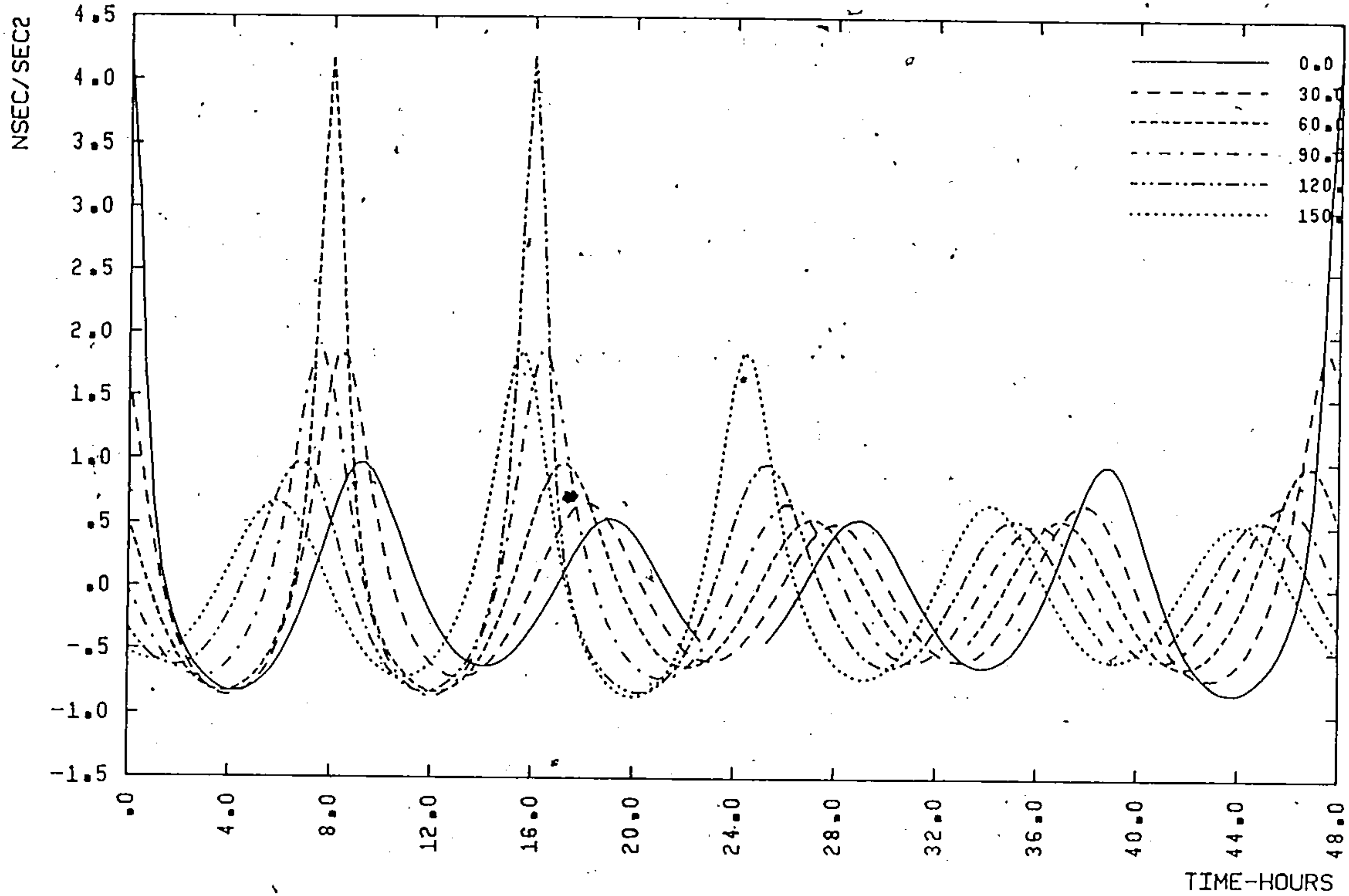

Fig. 4 20 Change in the change in space delays plotted in Fig. 4.18 for the same six geostationary satellites.

$$\frac{\partial^3 T}{\partial t^3} = D\omega_E^3 [(K^3 \cos i - 3K + 3K\cos i - 1) \sin\Lambda \cos Q$$

$$- (K^3 - 3K^2 \cos i + 3K - \cos i) \cos\Lambda \sin Q] \tag{4.68}$$

$$- \frac{3}{T} \frac{\partial T}{\partial t} \frac{\partial^2 T}{\partial t^2}$$

The higher order derivatives can be obtained following the same procedure.

4.6 BLIND SEGMENTS ON GEOSTATIONARY ORBIT

In order to find the segments on the geostationary orbit, from which the navigation satellite in any inclined orbit can be seen 24 hours a day, we form a circle on the surface of the earth, located symmetrically around the subpoint of the navigation satellite, such that the size of this circle (measured from the center of the earth) corresponds to the equality in Eq. (4.37). Let this angle be λ_{max}. It is clear that λ_{max} is the maximum angle between the navigation satellite and any point on the geostationary orbit maintaining visibility [28]. This value of λ_{max} is given by

$$\lambda_{max} = \arccos\left(\frac{R}{H_N}\right) + \arccos\left(\frac{R}{H_G}\right) \tag{4.69}$$

Substituting for $R = 6378$ km, $H_G = R + hsyn = 42162$ and by making use of Eq. (3.5); Eq. (4.69) can be reduced to

$$\lambda_{max} = 81.3° + \arccos(0.15127 * K^{2/3}) \tag{4.70}$$

Note here that if the angle between the navigation satellite and any point on the geostationary orbit (measured from the center of the earth) is greater than λ_{max} the projection of such a point on the surface of the earth is located in the projection of the shadow zone. Thus for any point on the surface of the earth to be located in the protection of the shadow zone, it must satisfy the following equation:

$$\cos\phi_{LatN}\cos\phi_{Lat}\cos(\phi_{LongN}-\phi_{Long}) + \sin\phi_{LatN}\sin_{Lat} \leq \cos\lambda_{max} \tag{4.71}$$

where ϕ_{Lat} and ϕ_{Long} are the latitude and the longitude of any point in the projection of the shadow zone, respectively, ϕLat_N and $\phi Long_N$ are the corresponding values for the subpoint of the navigation satellite. Now, the projection of the shadow zone on the surface of the earth moves with the motion of the navigation satellite; hence, may or may not intersect with the equator (which is the projection of the geostationary orbit). If it intersects with the equator, the existence of a blind segment on the geostationary orbit (from which the communications is blocked) is ensured. But we know that any point on the geostationary orbit has a latitude equal to zero. Substituting for $\phi_{Lat} = 0$ in Eq. (4.71) we get the longitude of the point of intersection between the equator and the boundary of the shadow zone and this is given by

$$\phi_{Long} = \phi_{LongN} \pm \arccos\left(\frac{\cos\lambda_{max}}{\cos\phi_{LatN}}\right) \tag{4.72}$$

It is clear that the maximum size for the blind segment occurs when $\phi Lat_N = 0$, i.e., when the navigation satellite passes through the equatorial plane and the maximum value for longitudes of the edge points on the boundary of the shadow zone are given by

$$\phi_{Longmax} = \phi_o' \pm \lambda_{max} \tag{4.73}$$

where ϕ'_o is the longitude of the subpoint for the navigation satellite when it crosses the equator. From Eq. (2.21), we know that the satellite subpoint crosses the equator every $\pi(1-1/K)$ radians in longitude; thus, Eq. (4.73) is rewritten as

$$\phi_{Longmax} = \phi_o' + n\pi\left(1-\frac{1}{K}\right) \pm \lambda_{max}; \qquad n = 0, 1, 2, 3, \ldots \tag{4.74}$$

By applying Eq. (4.70) for the case of a 12 hour orbit, where $K = 2$, we get $\lambda_{max} = 157.4°$. Substituting for λ_{max} and K in Eq. (4.74) we find that four blind segments with width 45.2° exist on the geostationary orbit, centered at longitudes 0°, 90°, 180° and 270° with respect to the point at which the navigation satellite crosses the equatorial plane.

In order to find the maximum time for the geostationary satellite to remain in the shadow zone hidden from the navigation satellite, we consider the worst location on the geostationary orbit with respect to the navigation satellite. This worst location is opposite to the point at which the navigation satellite crosses the equatorial plane; i.e., $\phi_{Long_N} - \phi_{Long} = \pi$. Substituting in Eq. (4.72) we can evaluate the minimum latitude reached by the navigation satellite subpoint before observing the whole geostationary orbit; this latitude is given by

$$\phi_{Lat_{N\,min}} = \pm 22.6° \tag{4.75}$$

By assuming an inclination angle $i = 60°$, we substitute for both $\phi_{Lat_{N\,min}}$ and i in Eq. (4.1) we can calculate the maximum time for the communications to be blocked. This time equals 1.756 hours every 24 hours of navigation.

Thus, we conclude that there exist four segments on the geostationary orbit with angle = 44.8° located symmetrically at longitudes 45°, 135°, 225° and 315° with respect to the point at which the 12 hour satellite crosses the equatorial plane. If any geostationary satellite is located at any point on those segments, such a satellite can provide 24 hour visibility for this 12 hour satellite.

CHAPTER 5

ESTABLISHMENT OF ACCURATE TIMING ON-BOARD A NAVIGATION SATELLITE IN INCLINED ORBIT

5.1 SUMMARY OF CHAPTER

The establishment of accurate timing on-board a navigation satellite in inclined orbit using a timing reference from either an earth station or geostationary satellite is important [1,2]. More recently, there has been considerable effort placed in designing and developing the NAVSTAR system which uses atomic clocks on-board satellites in inclined orbits to establish accurate time [1,11,12].

In this chapter, we are going to discuss another possible mode of operation which is based on the transponding of timing information from an earth station to a navigation satellite in inclined orbits through a satellite in geostationary orbit. Assuming that the satellite in the geostationary orbit has constant space delay with an earth station, then the only change in the space delay between the earth station and the satellite in inclined orbit occurs between the two satellites.

The main advantages of our solution to this problem are:

1) The atomic clocks on-board the navigation satellite are no longer required.

2) A communication link now exists between the earth station and all users of the system through the navigation satellite. This is due to the fact that the satellite in geostationary orbit has the capability of observing both the navigation satellite in inclined orbit and the earth station located inside its coverage area.

We assume the following:

1) The location of the geostationary satellite is accurately known. This is usually true since its motion with respect to earth station is small.

2) The space delay from earth station to geostationary satellite can be determined to within less than 1 ns using conventional timing methods in TDMA. Thus, accurate time at the geostationary satellite is established [34].

3) The distance between the geostationary satellite and the navigation satellite varies smoothly with time [28,33].

4) The location of the navigation satellite is known to lie within a sphere of certain radius centred at a known point.

If we calculate in advance the actual uplink delay between a geostationary satellite and a navigation satellite; and know the timing on-board the geostationary satellite, we can establish timing on-board the navigation satellite. The technique which we use in computing the uplink and downlink delays between the two satellites depends on the estimation of the navigation satellite location. The error in estimating the location of the navigation satellite does not strongly affect the calculation of the uplink delays between the two satellites.

The computed results show that the estimated uplink and downlink space delay between the two satellites can be calculated to a high degree of accuracy (a fraction of a nanosecond or less).

5.2 DESIGN CONFIGURATION

In our procedure to solve the problem of estimating the uplink delay between a geostationary satellite and a navigation satellite in inclined orbit, we assume that the location of the geostationary satellite is known at any instant of time with high degree of accuracy, which is in fact true.

Before going into further details, we want to emphasize that the expression space delay is used to define the distance between the two satellites at any instant of time divided by the propagation velocity; on the other hand the expression uplink delay or downlink delay is used to define the actual space delay taken by a message to propagate between the two satellites. The term 'fixed' is used here to denote the location of a satellite at the instant when a signal is received. The satellite continues to move, but the location when the signal was received is kept fixed.

Refer to Fig. 5.1 which shows the location of the geostationary satellite G_1 and the navigation satellite N_1 at time $t = t_1$. Let the geostationary satellite G_1 start its transmission at time t_1. The transmitted message takes time T_{12} to reach the navigation satellite at N_2 where N_1N_2 is the distance moved by the navigation satellite during the uplink delay T_{12}. Now, the navigation satellite at N_2 loops this message back such that it is received by the geostationary satellite at location G_3 after a downlink delay T_{23}. Upon receiving the message at G_3; the first piece of information we can calculate is the sum of the uplink delay and the downlink delays ($T_{12} + T_{23}$) which is then available at the geostationary satellite at G_3.

Note that the total delays measured by the geostationary satellite at G_3 will include the time delay in passing through the navigation satellite receiving-transmission circuits T_S, (which is of the order of nanoseconds). Such a delay is known in advance and a proper compensation for its value can be done at G_3.

Assuming that the geostationary satellite has clock information from the earth station, therefore, the round trip delay T_t can be measured, where:

$$T_t = T_{12} + T_{23} \tag{5.1}$$

From Fig. 5.1 it is clear that T_{12} and T_{23} are related as space delays if the navigation satellite is held fixed at N_2 and the geostationary satellite moves from G_1 to G_3

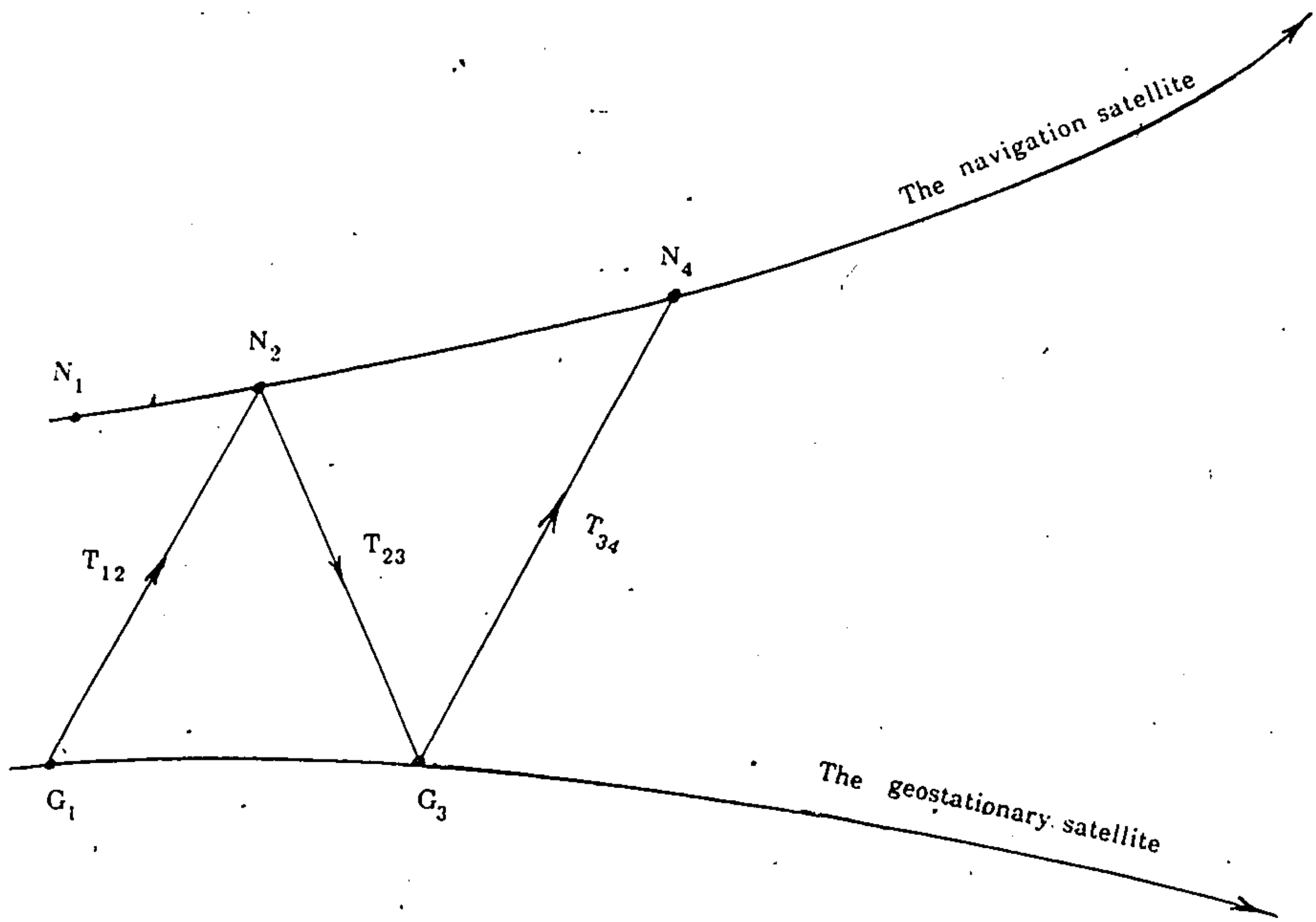

Fig. 5.1 The relation between the space delay, uplink and downlink delays

during the time $\Delta t_t = T_t$. It was shown before [5,6] that the change in the space delay between the two satellites is a smooth, slowly varying function of time when both satellites are in motion. Now, if N is held fixed at N_2, we expect that the change in the space delay will still have this same property. Therefore, the space delays T_{23} and T_{12} can be related by Taylor Series given by

$$T_{23} = T_{12} + \Delta t_1 \left.\frac{\partial T}{\partial t}\right|_{G_1, N=N_2} + \frac{\Delta t_1^2}{2!} \left.\frac{\partial^2 T}{\partial t^2}\right|_{G_1, N=N_2} + \text{higher order terms} \tag{5.2}$$

Substituting for $\Delta t_1 = T_{12} + T_{23}$ in the last equation, and ignoring the higher order terms we get

$$T_{23} = T_{12} + (T_{12} + T_{23}) \left.\frac{\partial T}{\partial t}\right|_{G_1, N=N_2} + \frac{(T_{12} + T_{23})^2}{2!} \left.\frac{\partial^2 T}{\partial t^2}\right|_{G_1, N=N_2} \tag{5.3}$$

where

$\left.\frac{\partial T}{\partial t}\right|_{G_1, N=N_2}$ = the change in the space delay when G is at G_1 (moving) and N is at N_2 (fixed).

$\left.\frac{\partial^2 T}{\partial t^2}\right|_{G_1, N=N_2}$ = the change in the change in the space delay when G is at G_1 (moving) and N is at N_2 (fixed).

Therefore, if the values of

$$\left.\frac{\partial T}{\partial t}\right|_{G_1, N=N_2} \quad \text{and} \quad \left.\frac{\partial^2 T}{\partial t^2}\right|_{G_1, N=N_2}$$

are known, one can solve both of Eq. (5.1) and Eq. (5.3) for the values of T_{12} and T_{23}. These values are given by:

$$T_{23} = 0.5\, T_t \left| 1 + \left.\frac{\partial T}{\partial t}\right|_{G_1, N=N_2} + 0.5\, T_t \cdot \left.\frac{\partial^2 T}{\partial t^2}\right|_{G_1, N=N_2} \right| \tag{5.4}$$

and

$$T_{12} = 0.5\, T_t \left| 1 - \left.\frac{\partial T}{\partial t}\right|_{G_1, N=N_2} - 0.5\, T_t \cdot \left.\frac{\partial^2 T}{\partial t^2}\right|_{G_1, N=N_2} \right| \tag{5.5}$$

If,

$$\left.\frac{\partial T}{\partial t}\right|_{G_1,N=N_2} >> 0.5\,T_t \cdot \left.\frac{\partial^2 T}{\partial t^2}\right|_{G_1,N=N_2},$$

the downlink delay T_{23} given by Eq. (5.4) can be rewritten as:

$$T_{23} = 0.5\,T_t\left(1 + \left.\frac{\partial T}{\partial t}\right|_{G_1,N=N_2}\right) \tag{5.6}$$

In order to solve either Eq. (5.4) or Eq. (5.5), we need an expression for the change in the space delay and the change in the change of the space delay under the condition that the navigation satellite is held fixed while the geostationary satellite is moving.

Until now, we did not show how to predict the new value of the uplink delay which establishes timing on-board the navigation satellite. Referring to Fig. 5.1, let the geostationary satellite upon receiving the routed back message at G_3 retransmit a new message which reaches the navigation satellite at N_4. Now, the new uplink delay which we want to calculate is T_{34}, which is the time taken by the message to propagate from G_3 to N_4.

From Fig. 5.1 it is clear that T_{23} and T_{34} are related to each other as space delays if G is held fixed at G_3 and N moves from N_2 to N_4 during a short period of time $\Delta t_2 = T_{23} + T_{34}$. Since the space delay varies slowly with time, T_{34} and T_{23} are related by the Taylor Series:

$$T_{34} = T_{23} + (T_{23} + T_{34}) \cdot \left.\frac{\partial T}{\partial t}\right|_{G=G_3,N_2} + \frac{(T_{23} + T_{34})^2}{2!} \cdot \left.\frac{\partial^2 T}{\partial t^2}\right|_{G=G_3,N_2} + \ldots \tag{5.7}$$

where

$\left.\frac{\partial T}{\partial t}\right|_{G=G_3,N_2}$ = the change in the space delay if the geostationary satellite is fixed at G_3 and the navigation satellilte moves from N_2.

$\left.\frac{\partial^2 T}{\partial t^2}\right|_{G=G_3,N_2}$ = the change in $\left.\frac{\partial T}{\partial t}\right|_{G=G_3,N_2}$

From Eq. (5.7), T_{34} can be calculated directly if the derivatives of T are known when N is at N_2 and moving and G is at G_3 and fixed in addition to the calculated downlink delay T_{23} from either Eq. (5.4) or Eq. (5.6). Therefore upon receiving the looped back message by the geostationary satellite at G_3 (before transmitting the new message at G_3), we can solve Eq. (5.7) and the value of the new uplink delay T_{34} can be calculated. Thus, an expression for the derivatives of the space delay under the previous condition should be available. As it will be shown later, the derivatives of the space delay required to solve Eq. (5.4) and Eq. (5.6) are dependent on the location of the geostationary satellite at G_1 and G_3 which are known accurately, also they depend upon the location of the navigation satellite at N_2 which is the second piece of information needed to predict the new uplink delay, T_{34}.

Solving Eq. (5.7) for the value T_{34}, we obtain

$$T_{34} = \left[\left(1 - \left.\frac{\partial T}{\partial t}\right|_{G=G_3,N_2} - T_{23} \cdot \left.\frac{\partial^2 T}{\partial t^2}\right|_{G=G_3,N_2} \right) - \sqrt{\left(1 - \left.\frac{\partial T}{\partial t}\right|_{G=G_3,N_2} \right)^2 - \left(4T_{23} \cdot \left.\frac{\partial^2 T}{\partial t^2}\right|_{G=G_3,N_2} \right)} \right] \Bigg/ \left.\frac{\partial^2 T}{\partial t^2}\right|_{G=G_3,N_2} \tag{5.8}$$

If

$$\left(1 - \left.\frac{\partial T}{\partial t}\right|_{G=G_3,N_2} \right)^2 >> \left(4T_{23} \cdot \left.\frac{\partial^2 T}{\partial t^2}\right|_{G=G_3,N_2} \right),$$

which is always true since the LHS is of the order of unity and the RHS is approximately 10^{-9}, then Eq. (5.8) reduces to:

$$T_{34} = T_{23} \cdot \frac{\left| 1 + \left.\frac{\partial T}{\partial t}\right|_{G=G_3,N_2} \right|}{\left| 1 - \left.\frac{\partial T}{\partial t}\right|_{G=G_3,N_2} \right|} \tag{5.9}$$

5.3 THE CHANGE IN THE SPACE DELAY IF THE NAVIGATION SATELLITE IS FIXED

At any instant of time t, the angle λ between the geostationary satellite subpoint and the navigation satellite subpoint measured from the center of the earth is given by Eq. (4.44)

$$\cos\lambda = \cos Q \cos\Lambda + \cos i \sin Q \sin\Lambda \tag{5.10}$$

where

Q	=	the angle rotated by the navigation satellite in its orbit measured from the equatorial plane.
Q	=	$\omega_S(t + TT)$
TT	=	the initial time phase of the navigation satellite
i	=	the inclination angle of the navigation satellite orbit
Λ	=	$\phi_{LongG} + \omega_E t - \phi_{oN}$
ϕ_{LongG}	=	the longitude of the geostationary satellite subpoint.
ϕ_{oN}	=	the longitude of the intersection point between the orbital projection of the navigation satellite and the equator at $t = 0$.
ω_E	=	the angular velocity of the earth.
ω_S	=	the angular velocity of the navigation satellite.
t	=	the reference time.

The space delay T at time t is given by:

$$T = d/C \tag{5.11}$$

or

$$T = [H_N^2 + H_G^2 - 2 H_N H_G \cos\lambda]^{1/2}/C \tag{5.12}$$

where

d	=	the distance between the two satellites at any instant t.

H_N, H_G = the distance measured from the center of the earth to the navigation satellite and the geostationary satellite respectively.

C = the propagation velocity.

Assuming that the navigation satellite is fixed, Q does not change with time. Thus differentiating T in Eq. (5.12) with respect to time, noting that λ is the only time dependent variable, yields:

$$\left.\frac{\partial T}{\partial t}\right|_{G,N=N_{fix}} = \frac{(1/2)}{(H_N^2 + H_G^2 - 2\,H_N\,H_G \cos\lambda)^{1/2}\cdot C}\left(-2\,H_N\,H_G \left.\frac{\partial \cos\lambda}{\partial t}\right|_{G,N=N_{fix}}\right) \quad (5.13)$$

Equation (5.13) can be rewritten as

$$\left.\frac{\partial T}{\partial t}\right|_{G,N=N_{fix}} = \frac{-H_N\,H_G}{d\cdot C}\,\frac{\partial \cos\lambda}{\partial t} \quad (5.14)$$

By differentiating Eq. (5.10) with respect to time keeping Q fixed, (noting that $(\partial\Lambda)/\partial t) = \omega_E$), we get

$$\left.\frac{\partial \cos\lambda}{\partial t}\right|_{G,N=N_{fix}} = [\cos Q\,(-\omega_E \sin\Lambda) + \cos i \sin Q\,(\omega_E \cos\Lambda)] \quad (5.15)$$

or

$$\left.\frac{\partial \cos\lambda}{\partial t}\right|_{G,N=N_{fix}} = \omega_E[-\cos Q\ \sin\Lambda + \cos i \sin Q\ \cos\Lambda] \quad (5.16)$$

Substituting Eq. (5.16) into Eq. (5.14) gives:

$$\left.\frac{\partial T}{\partial t}\right|_{G,N=N_{fix}} = \frac{-H_N\,H_G\,\omega_E}{d\cdot C}\,[-\cos Q\ \sin\Lambda + \cos i \sin Q\ \cos\Lambda] \quad (5.17)$$

The last Equation provides the change in the space delay with respect to time assuming that the navigation satellite is held fixed starting from this specified instant of time.

Equation (5.17) can be expressed in terms of the latitude and longitude of the satellite subpoint given in Appendix 2 by Eq. (A.2.13) as follows:

$$\left.\frac{\partial T}{\partial t}\right|_{G,N=N_{fix}} = \frac{-H_N H_G \omega_E}{d \cdot C} \cos\phi_{LatN} \sin\eta \tag{5.18}$$

where

ϕ_{LatN} = the latitude of the navigation satellite subpoint at the instant of time t.

η = $\phi_{LongN} - \phi_{LongG}$

= the difference in longitude between the two satellite subpoints at any instant of time t.

Figures 5.2 and 5.3 show the change in the space delay

$$\left.\frac{\partial T}{\partial t}\right|_{G,N=N_{fix}}$$

between a geostationary satellite and a navigation satellite located in a 12 hour orbit ($K=2$). The inclination angle varies from zero degrees to 60° in 15° steps, such that $TT=0$, and, the difference in longitude $\phi_{LongG} - \phi_{oN}$ is 15° as in Fig. 5.2 and 45° as in Fig. 5.3. Note here that the discontinuity in all the plotted curves represents the situation when the communications is blocked between the two satellites due to the location of the earth between them.

Figures 5.4 and 5.5 are the same as the Figs. 5.2 and 5.3 except that the navigation satellite is located in 16 hour orbit, i.e., $K=1.5$; where K is the nonsynchronous factor. Figures 5.4 and 5.5 are plotted for an interval of 48 hours in order to cover two revolutions by the geostationary satellite and three revolutions by the navigation satellite.

Let us now calculate the change in the change of the space delay with respect to time when the navigation satellite is fixed

Recalling Eq. (4.48) which gives the formula for the change in the space delay (when both satellites are moving), we have

$$\frac{\partial T}{\partial t} = \frac{H_N H_G}{d \cdot C} \omega_E [(K - \cos i)\sin Q \cos \Lambda + (1 - K\cos i)\cos Q \sin \Lambda] \tag{5.19}$$

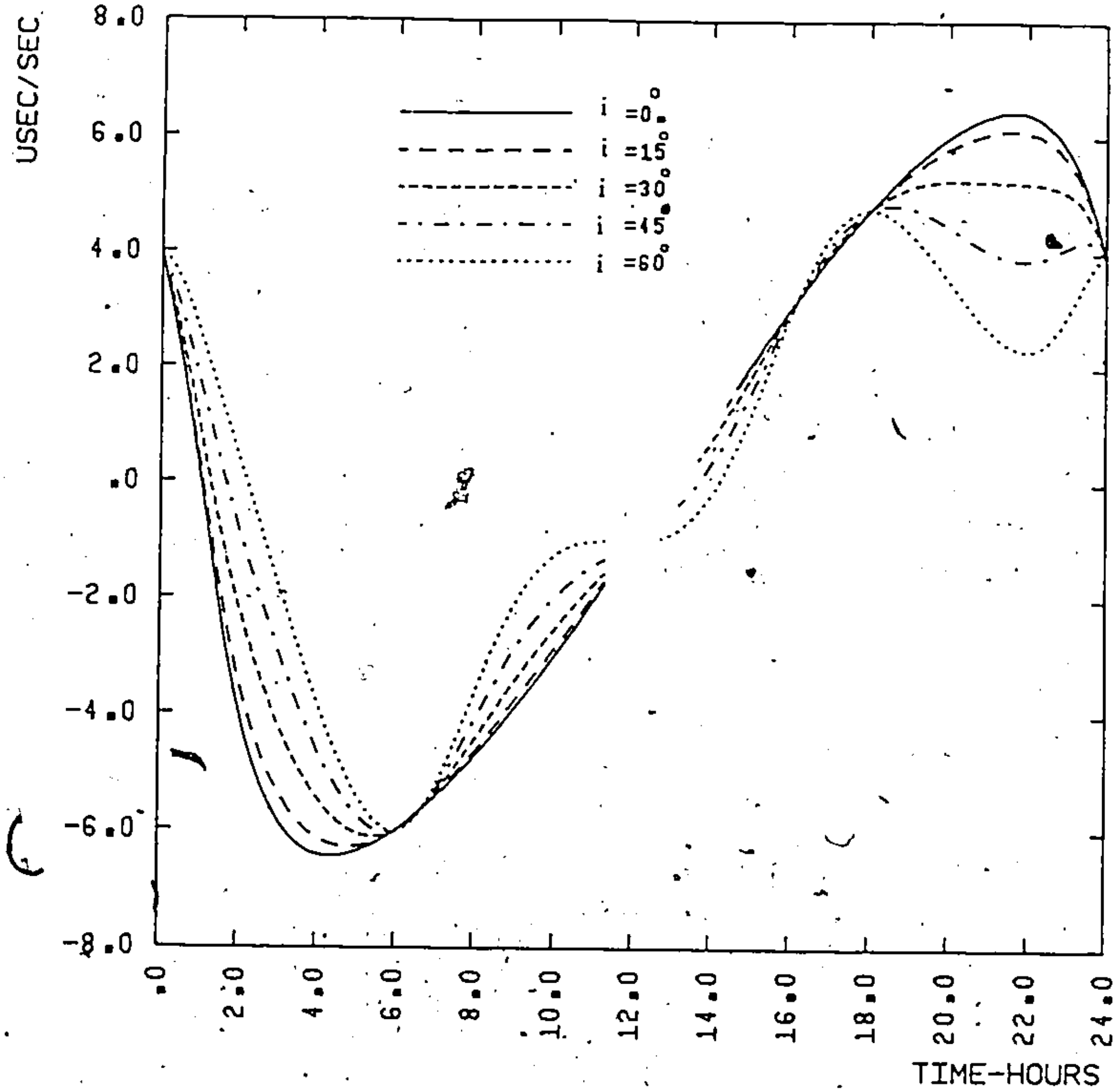

Fig. 5.2 The change in the space delay

$$\frac{\partial \Gamma}{\partial t}\bigg|_{G,N=N_{fix}}$$

between a geostationary satellite and 12 hour navigation satellite with

$\Phi_{LongG} - \Phi_{oN} = 15°$, $i = 0°, 15°, 30°, 45°$ and $60°$

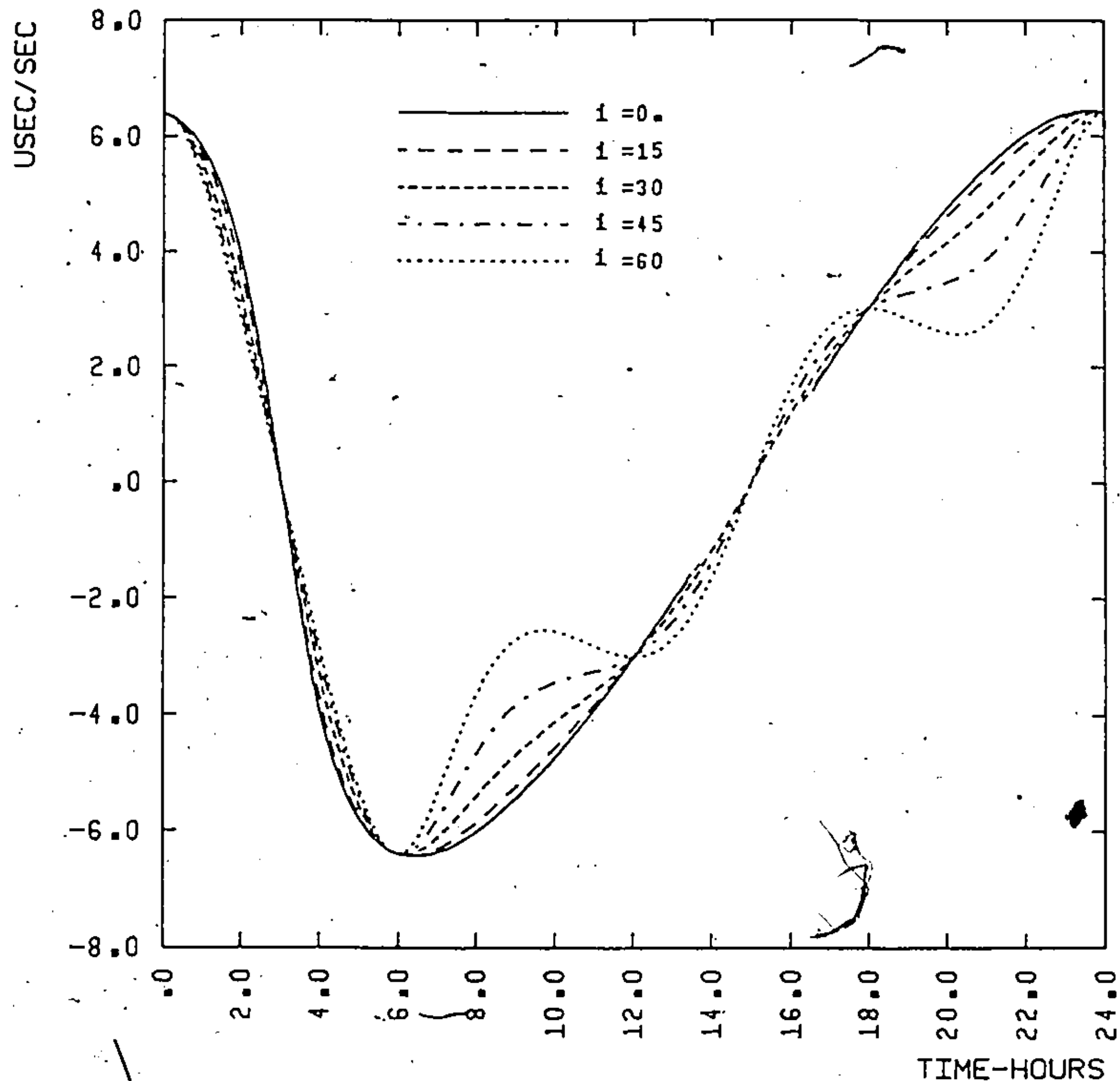

Fig 5.3 The change in the space delay

$$\left.\frac{\partial T}{\partial t}\right|_{G,N=N_{fix}}$$

between a geostationary satellite and 12 hour navigation satellite with

$\Phi_{LongG} - \Phi_{oN} = 45°$, $i = 0°, 15°, 30°, 45°$ and $60°$

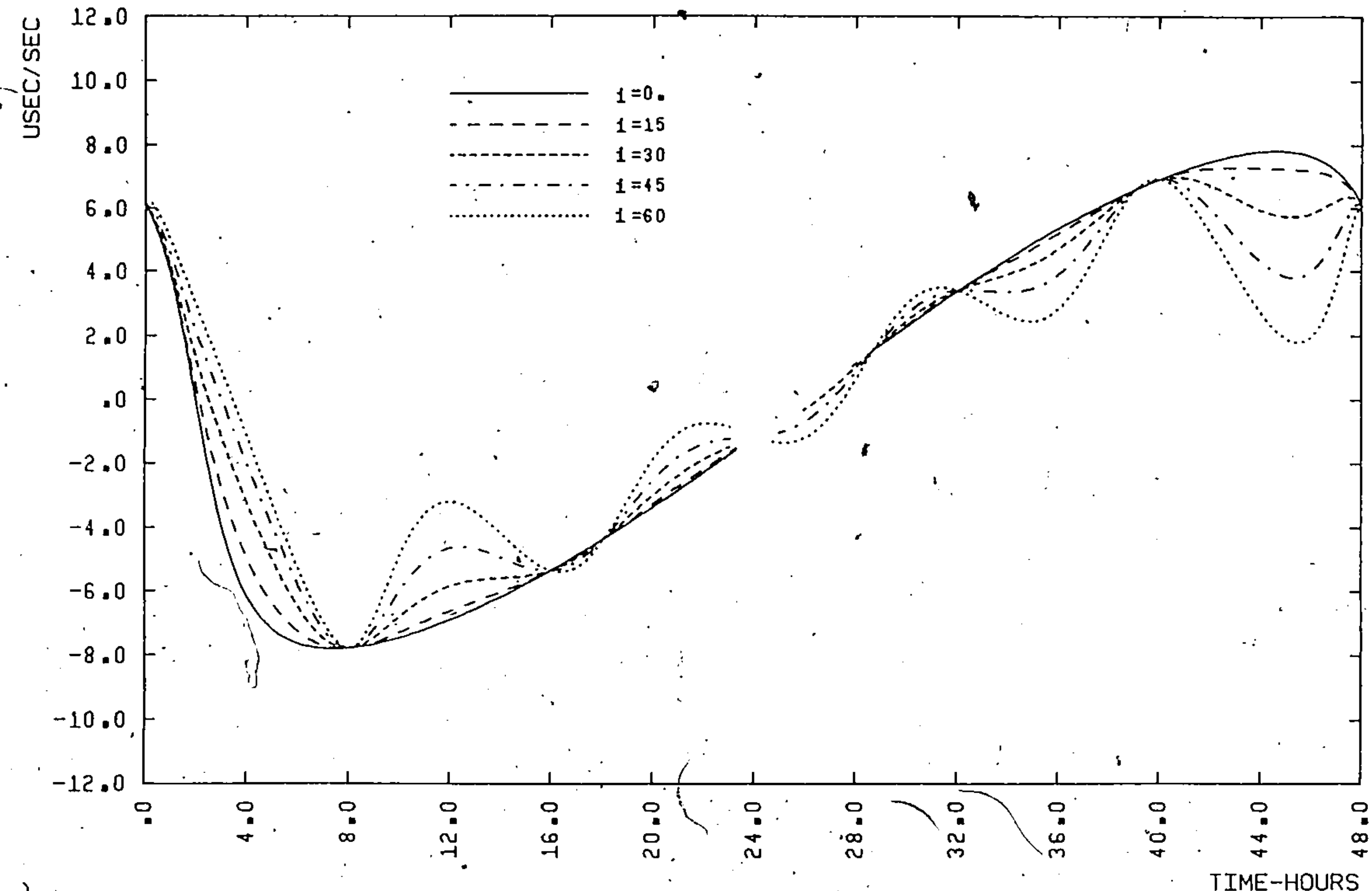

Fig. 5.4 The change in the space delay

$$\left.\frac{\partial T}{\partial t}\right|_{G,N=N_{fix}}$$

between a geostationary satellite and 16 hour navigation satellite with

$\phi_{G} - \phi_{N} = 15°$, $i = 0°, 15°, 30°, 45°$ and $60°$

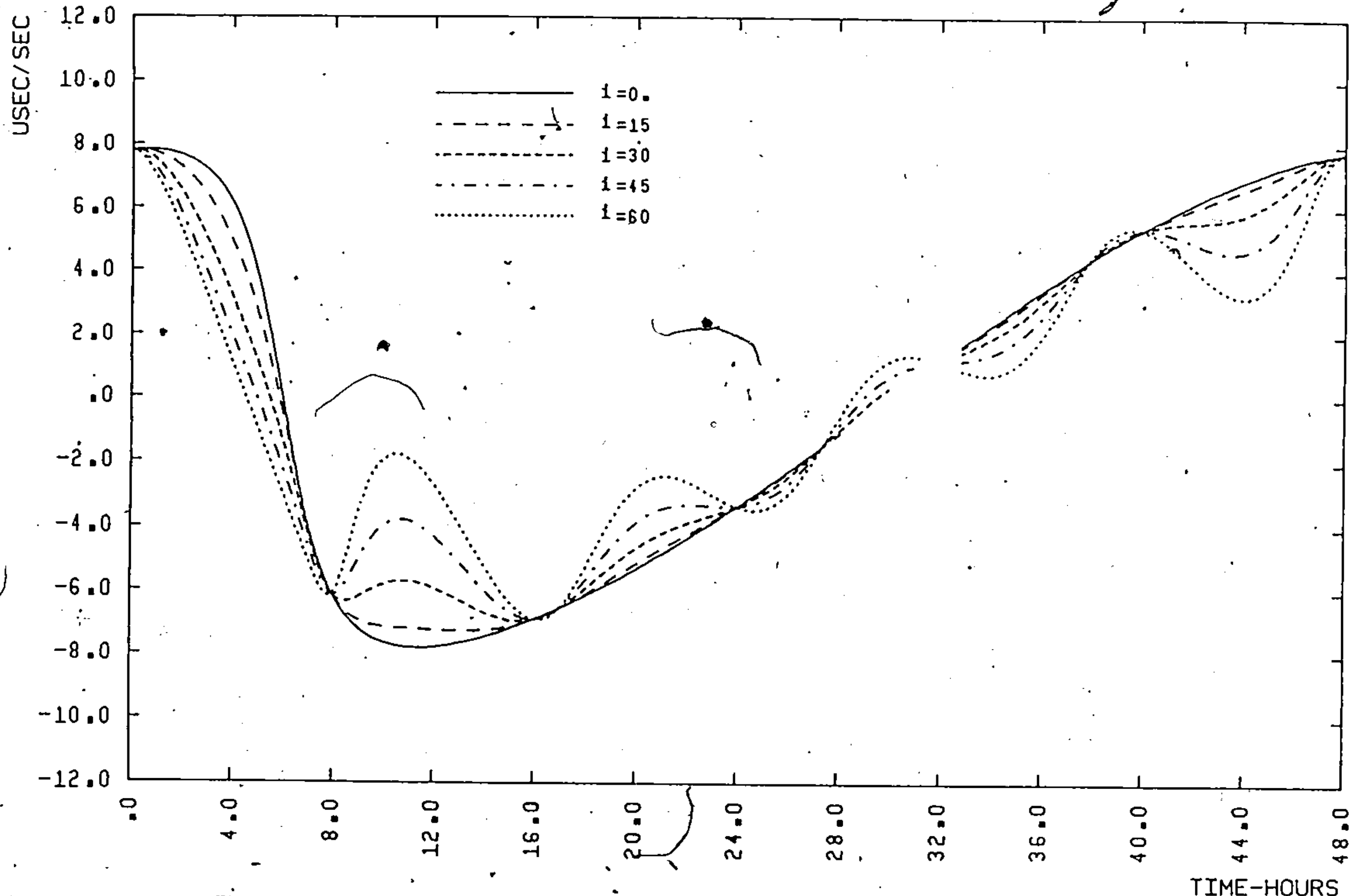

Fig. 5.5 The change in the space delay

$$\left.\frac{\partial T}{\partial t}\right|_{G,N=N_{fix}}$$

between a geostationary satellite and 16 hour navigation satellite with

$\Phi_{LongG} - \Phi_{oN} = 45°$, $i = 0°, 15°, 30°, 45°$ and $60°$

Differentiating Eq. (5.19) with respect to t while holding Q constant, noting that both Λ and d are functions of time, gives:

$$\left.\frac{\partial^2 T}{\partial t^2}\right|_{G,N=N_{fix}} = \frac{H_N H_G}{d \cdot C}\omega_E \left|(K-\cos i)\sin Q(-\omega_E \sin\Lambda) + (1-K\cos i)\cos Q(\omega_E \cos\Lambda)\right| +$$

$$\frac{H_N H_G}{C}\omega_E \left|[(K-\cos i)\sin Q\cos\Lambda + (1-K\cos i)\cos Q\sin\Lambda]\cdot\left(\frac{-1}{d^2}\right)\left.\frac{\partial d}{\partial t}\right|_{G,N=N_{fix}}\right| \tag{5.20}$$

Using Eq. (5.19) the last Equation can be reduced to:

$$\left.\frac{\partial^2 T}{\partial t^2}\right|_{G,N=N_{fix}} = \frac{H_N H_G}{d \cdot C}\omega_E^2[(\cos i - K)\sin Q\sin\Lambda + (1-K\cos i)\cos Q\cos\Lambda]$$

$$-\left.\frac{\partial T}{\partial t}\cdot\frac{1}{d}\cdot\frac{\partial d}{\partial t}\right|_{G,N=N_{fix}} \tag{5.21}$$

where:

$\partial T/\partial t$ = the change in the space delay.

Consider the last term in Eq. (5.21) which is given by

$$\left.\frac{\partial T}{\partial t}\cdot\frac{1}{d}\cdot\frac{\partial d}{\partial t}\right|_{G,N=N_{fix}} = \left.\frac{\partial T}{\partial t}\frac{1}{d/C}\frac{\partial d/C}{\partial t}\right|_{G,N=N_{fix}} \tag{5.22}$$

This reduces to:

$$\left.\frac{\partial T}{\partial t}\cdot\frac{1}{d}\cdot\frac{\partial d}{\partial t}\right|_{G,N=N_{fix}} = \left.\frac{\partial T}{\partial t}\frac{1}{T}\frac{\partial T}{\partial t}\right|_{G,N=N_{fix}} \tag{5.23}$$

Substituting from Eq. (5.23) into Eq. (5.21) yields

$$\left.\frac{\partial^2 T}{\partial t^2}\right|_{G,N=N_{fix}} = \frac{H_N H_G \omega_E^2}{d \cdot C}[(\cos i - K))\sin Q\sin\Lambda + (1-K\cos i)\cos Q\cos\Lambda]$$

$$-\left.\frac{\partial T}{\partial t}\frac{1}{T}\frac{\partial T}{\partial t}\right|_{G,N=N_{fix}} \tag{5.24}$$

The change of the change in the space delay has been plotted in Figs. 5.6, 5.7, 5.8 and 5.9 for the same cases plotted in Figs. 5.2, 5.3, 5.4 and 5.5, respectively. From Fig. 5.6 to

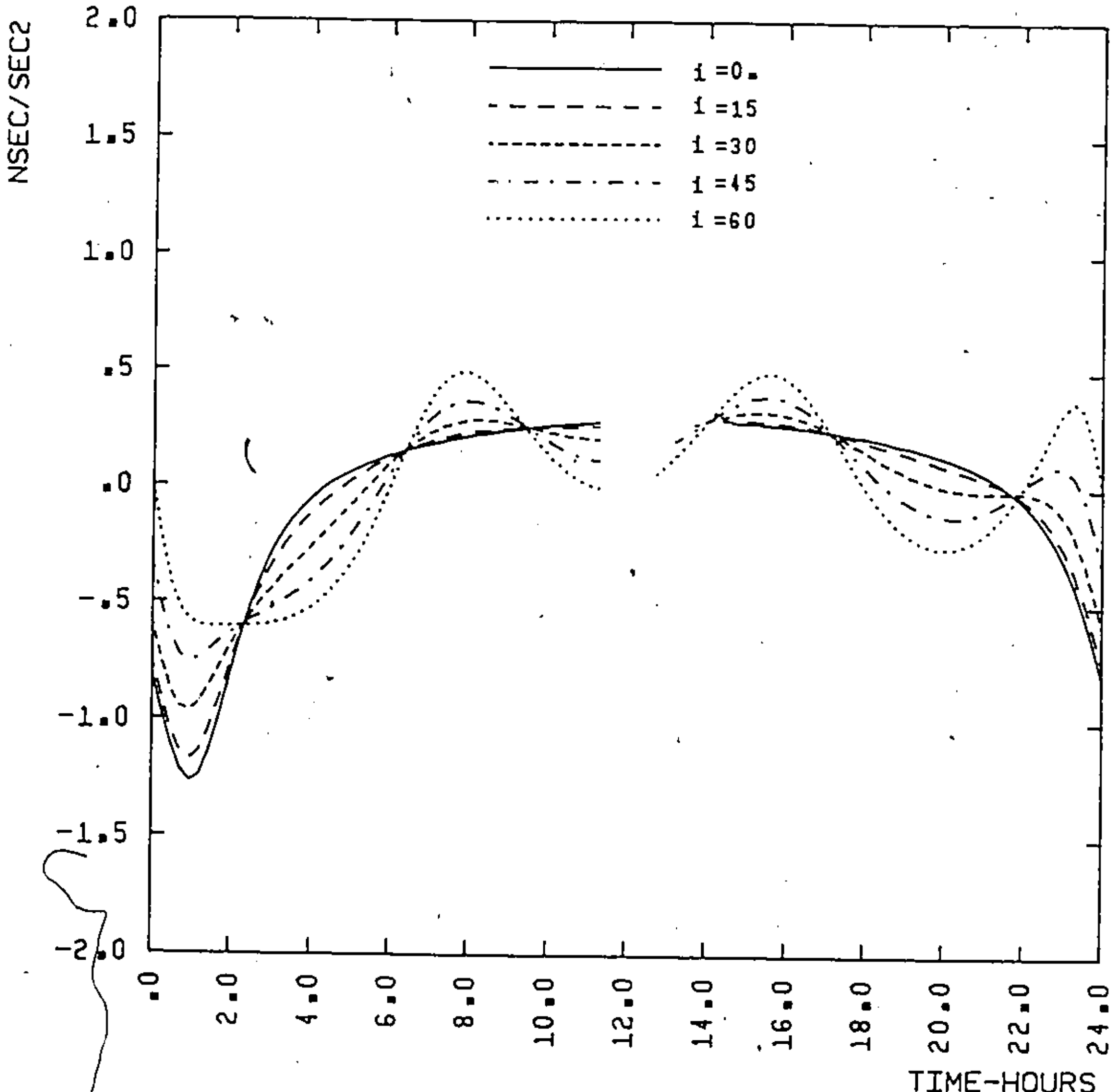

Fig. 5.6 The change in the change in the space delay

$$\left.\frac{\partial^2 T}{\partial t^2}\right|_{G.N = N_{fix}}$$

between a geostationary satellite and 12 hour navigation satellite with

$\Phi_{LongG} - \Phi_{oN} = 15°$, $i = 0°, 15°, 30°, 45°$ and $60°$

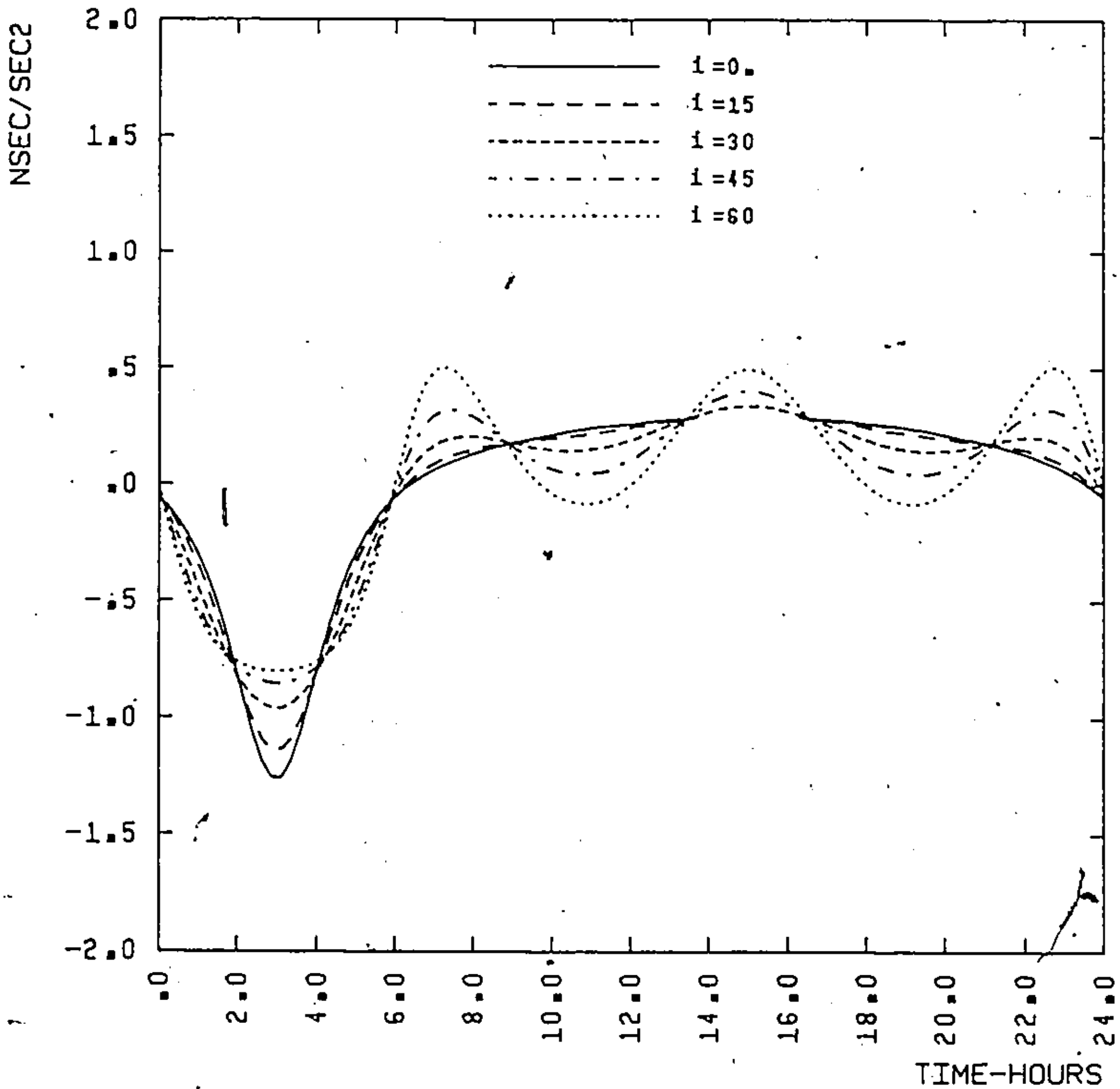

Fig. 5.7 The change in the change in the space delay

$$\left.\frac{\partial^2 T}{\partial t^2}\right|_{G,N=N_{fix}}$$

between a geostationary satellite and 12 hour navigation satellite with

$\Phi_{LongG} - \Phi_{oN} = 45°$, $i = 0°, 15°, 30°, 45°$ and $60°$

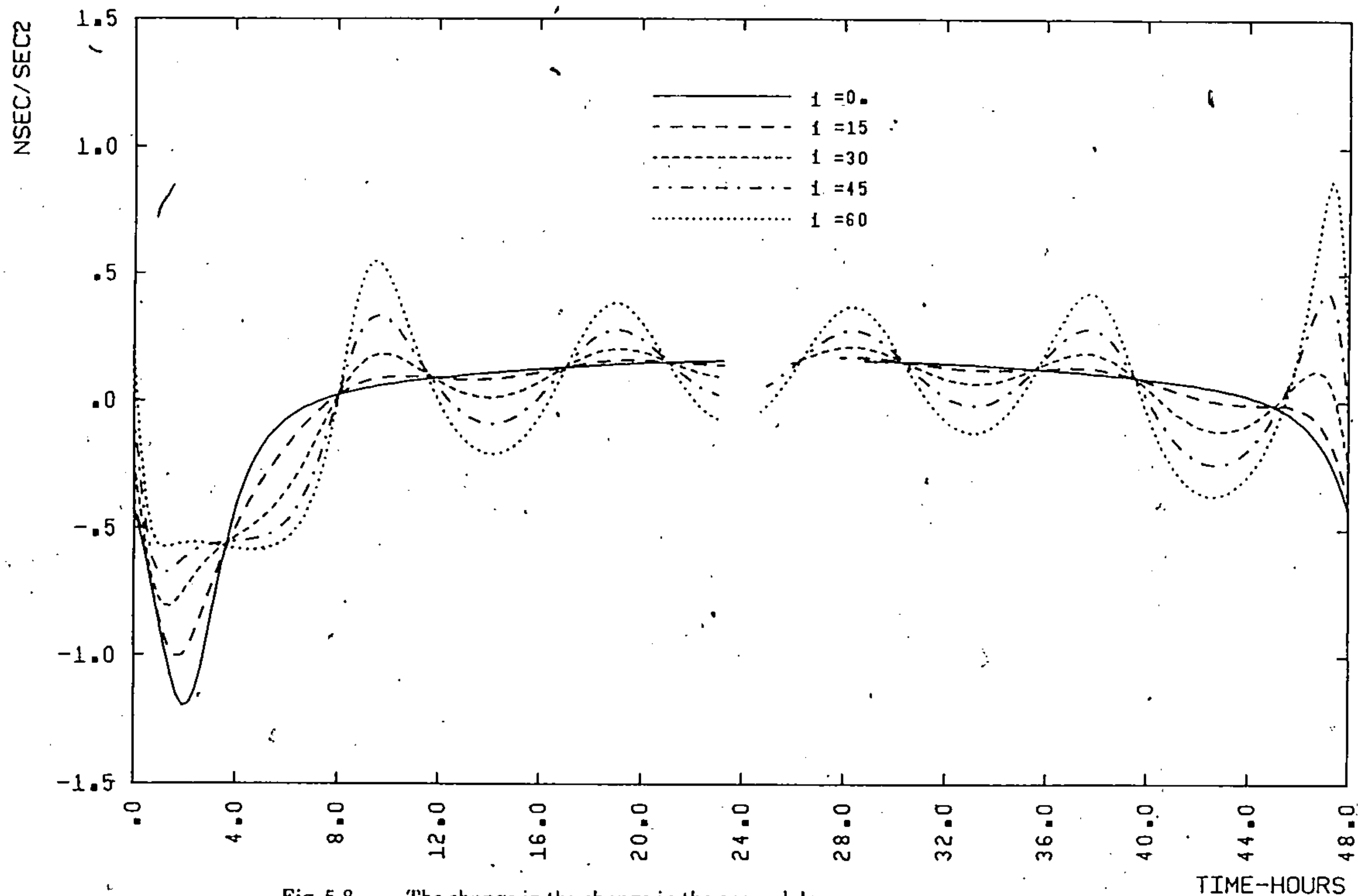

Fig. 5.8 The change in the change in the space delay

$$\left.\frac{\partial^2 T}{\partial t^2}\right|_{G,N=N_{fix}}$$

between a geostationary satellite and 16 hour navigation satellite with

$\Phi_{LongG} - \Phi_{oN} = 15°$, $i = 0°, 15°, 30°, 45°$ and $60°$

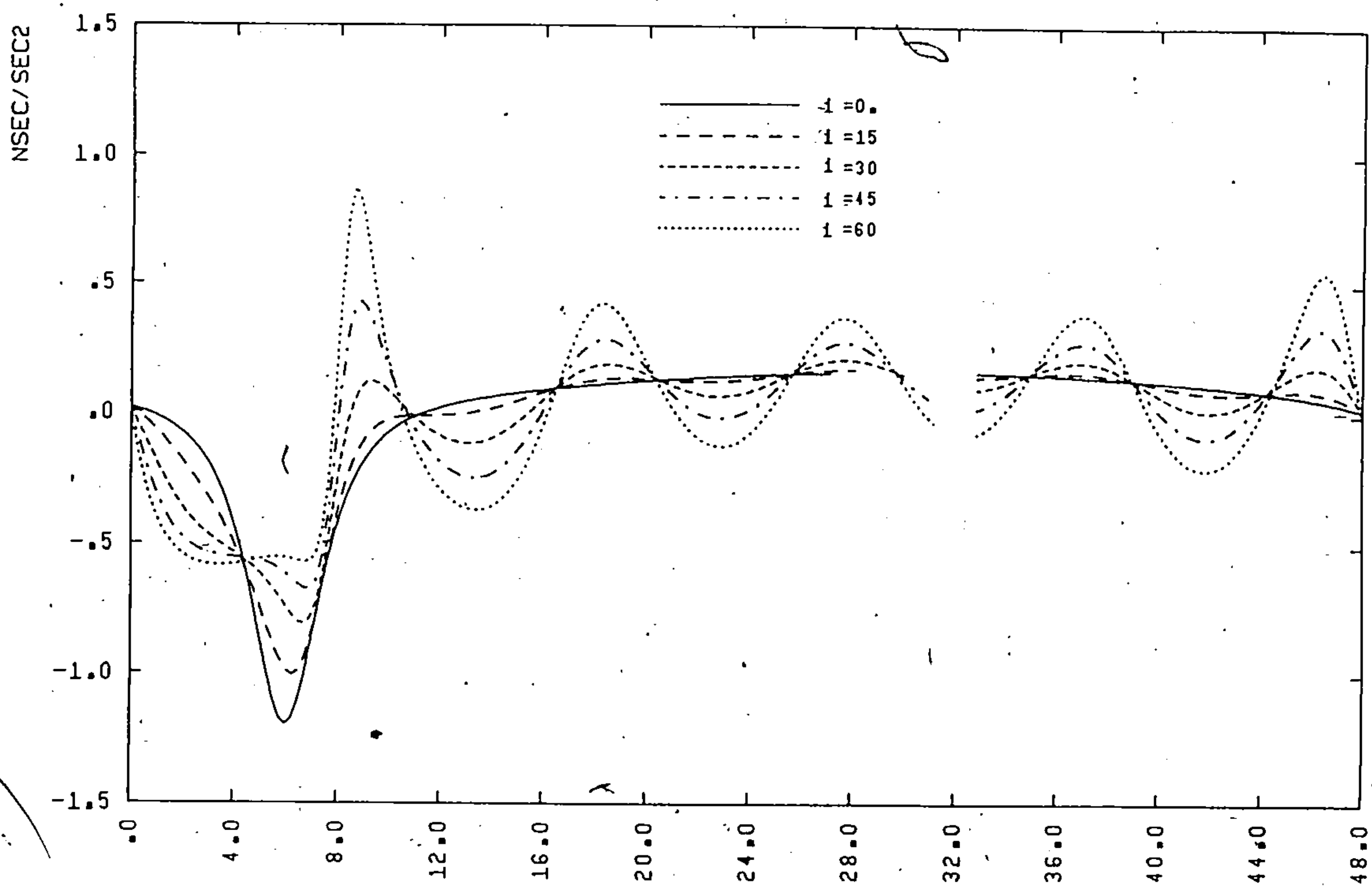

Fig. 5.9 The change in the change in the space delay

$$\frac{\partial^2 T}{\partial t^2}\Big|_{G,N=N_{fix}}$$

between a geostationary satellite and 16 hour navigation satellite with

Fig. 5.9, it is clear that the change of the change in the space delay varies very slowly with time (maximum value is slightly greater than 1 ns/s^2) which shows that Eq. (5.6) will perform to a high degree of accuracy by (comparing the values of

$$\frac{\partial T}{\partial t}\bigg|_{G,N=N_{fix}} \quad \text{to the value of } 0.5\, T \cdot \frac{\partial^2 T}{\partial t^2}\bigg|_{G,N=N_{fix}}$$

see Eq. (5.5)).

5.4 THE CHANGE IN THE SPACE DELAY IF THE GEOSTATIONARY SATELLITE IS FIXED

Here, we want to find an expression for the change in the space delay between the navigation satellite and the geostationary satellite when the latter is held fixed over a short interval of time starting from any instant t.

As in Section (5.3) we are going to differentiate the expression of the space delay while holding the geostationary satellite fixed, i.e., the angle Λ does not change with time starting from this instant. Recalling Eq. (5.12) and differentiating under the previous condition noting that Λ is only dependent on time, yields

$$\frac{\partial T}{\partial t}\bigg|_{G=G_{fix},N} = \frac{1/2}{d \cdot C_-}\left(-2\, H_N\, H_G\, \frac{\partial \cos\Lambda}{\partial t}\bigg|_{G=G_{fix},N} \right) \tag{5.25}$$

Differentiating Eq. (5.10) with respect to t while holding the geostationary satellite fixed, noting that Q is the only time dependent variable, gives:

$$\frac{\partial \cos\Lambda}{\partial t}\bigg|_{G=G_{fix},N} = [(-\omega_S \sin(Q)\cos\Lambda + \cos i\,(\omega_S \cos Q)\sin\Lambda] \tag{5.26}$$

or

$$\frac{\partial \cos\Lambda}{\partial t}\bigg|_{G=G_{fix},N} = \omega_S\,[-\sin Q\cos\Lambda + \cos i\,\cos Q\sin\Lambda] \tag{5.27}$$

Substituting Eq. (5.27) in Eq. (5.25) and simplifying produces:

$$\left.\frac{\partial T}{\partial t}\right|_{G=G_{fix},N} = \frac{H_N H_G}{d \cdot C}\,\omega_S\,(\sin Q \cos \Lambda - \cos i \,\cos Q \sin \Lambda) \tag{5.28}$$

The last equation can be expressed in terms of latitude and the longitude of the two satellites. They are given in Appendix 2 by Eq. (A2.24) as follows:

$$\left.\frac{\partial T}{\partial t}\right|_{G=G_{fix},N} = \frac{H_N H_G}{d \cdot C}\,\omega_S\,[\sin i \cos \delta \cos \eta \sin \phi_{LatN} + (\sin \eta \,\cos i / \cos \phi_{LatN})] \tag{5.29}$$

The change in the space delay

$$\left.\frac{\partial T}{\partial t}\right|_{G=G_{fix},N}$$

from Eq. (5.28) or (5.29) has been plotted versus time in Figures 5.10 and 5.11 for a navigation satellite located in a 12 hour orbit. The inclination angle varies from 0° to 60° in 15° steps while the difference in longitude $\phi_{LongG} - \phi_{oN} = 15°$ in Fig. 5.10 and $\phi_{LongG} - \phi_{oN} = 45°$ in Fig. 5.11.

Figures 5.12 and 5.13 give the change in the space delay as in Figs. 5.10 and 5.11 respectively but for the case of 16 hour orbit.

Figures from 5.10 to 5.13 show that the maximum value for the change in the space delay is ±12 μs/s which is higher than that shown in Figures from 5.2 to 5.5 when the navigation satellite is held fixed. The reason is simply because the navigation satellite in a 12 or 16 hour orbit is much faster than the geostationary satellite in a 24 hour orbit.

To find the change in the change of the space delay when the geostationary satellite is fixed, we follow the same steps which we followed in Section (5.3). Here, however, with G fixed we keep the angle Λ constant, thus by differentiating Eq. (5.19) under the previous condition and after simplification, we have:

$$\left.\frac{\partial^2 T}{\partial t^2}\right|_{G=G_{fix},N} = \frac{H_N H_G}{d \cdot C}\,\omega_S^2\,[(K - \cos i) \cos Q \cos \Lambda - (1 - K\cos i)\, \sin Q \sin \Lambda]$$

$$- \frac{\partial T}{\partial t} \cdot \frac{1}{T} \cdot \left.\frac{\partial T}{\partial t}\right|_{G=G_{fix},N} \tag{5.30}$$

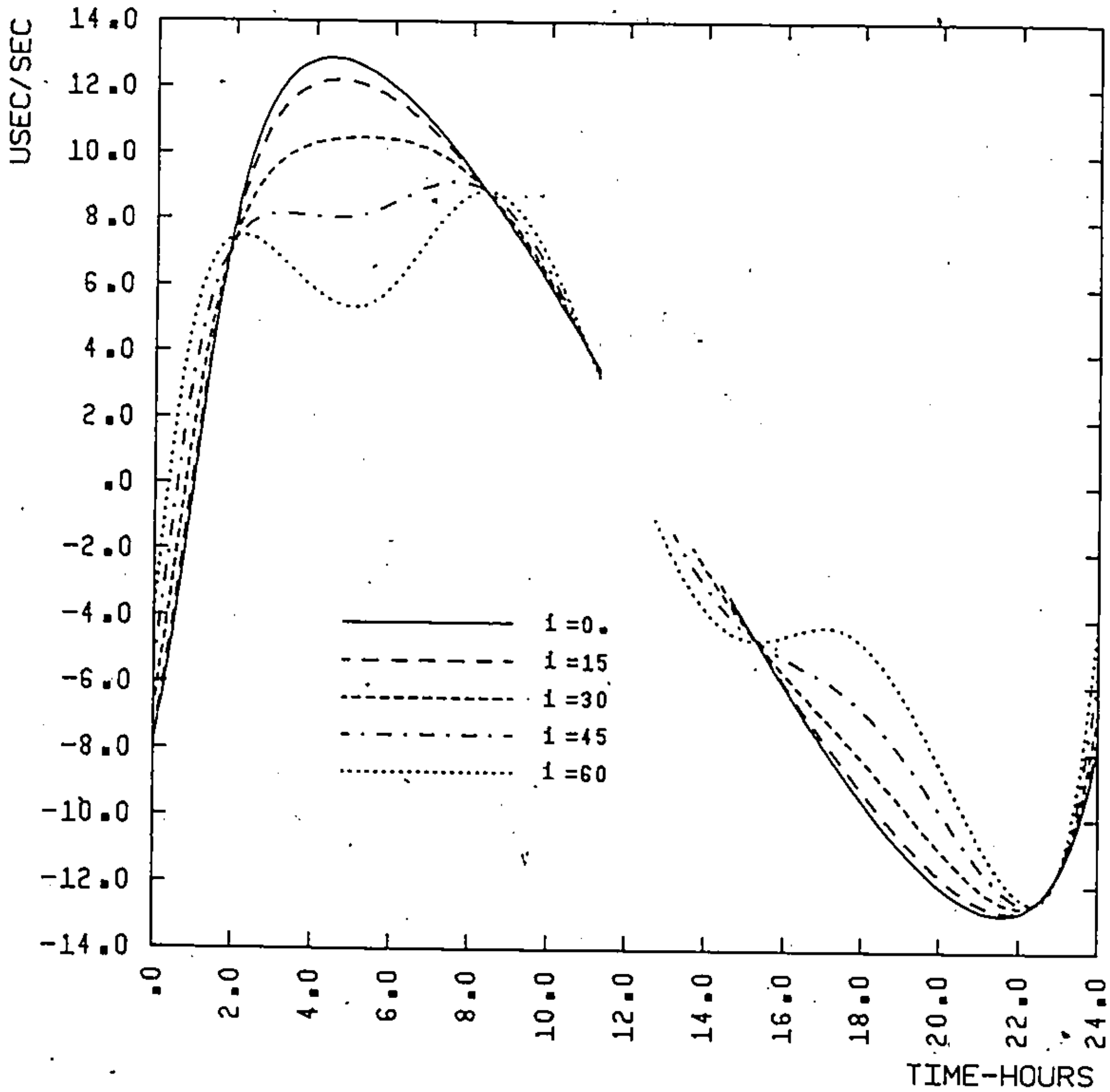

Fig. 5.10 The change in the space delay

$$\left.\frac{\partial T}{\partial t}\right|_{G=G_{fix},N}$$

between a geostationary satellite and 12 hour navigation satellite with

$\Phi_{LongG} - \Phi_{oN} = 15°$, $i = 0°, 15°, 30°, 45°$ and $60°$

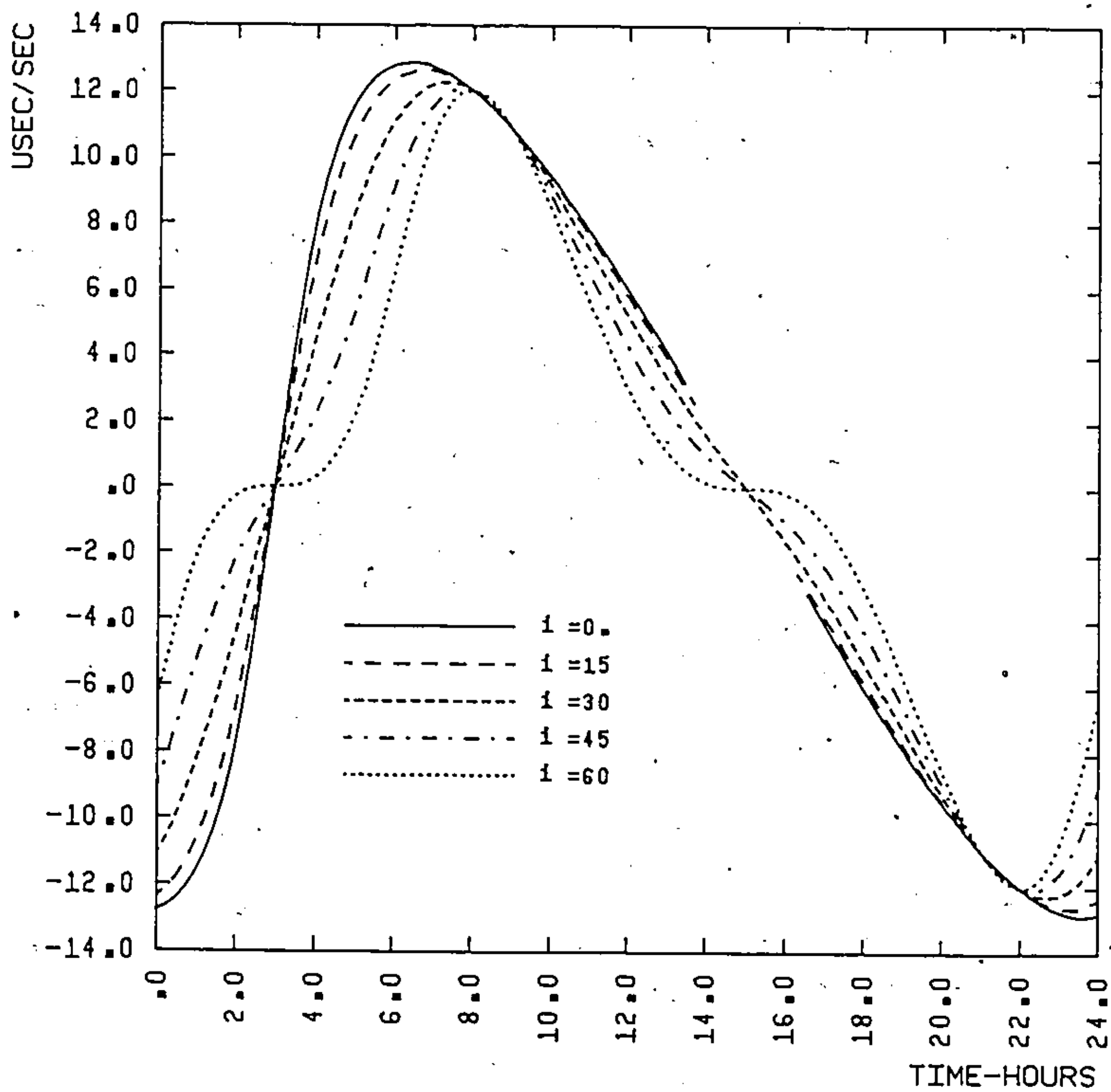

Fig. 5.11 The change in the space delay

$$\left.\frac{\partial T}{\partial t}\right|_{G=G_{fix},N}$$

between a geostationary satellite and 12 hour navigation satellite with

$\Phi_{LongG} - \Phi_{oN} = 45°$, $i = 0°, 15°, 30°, 45°$ and $60°$

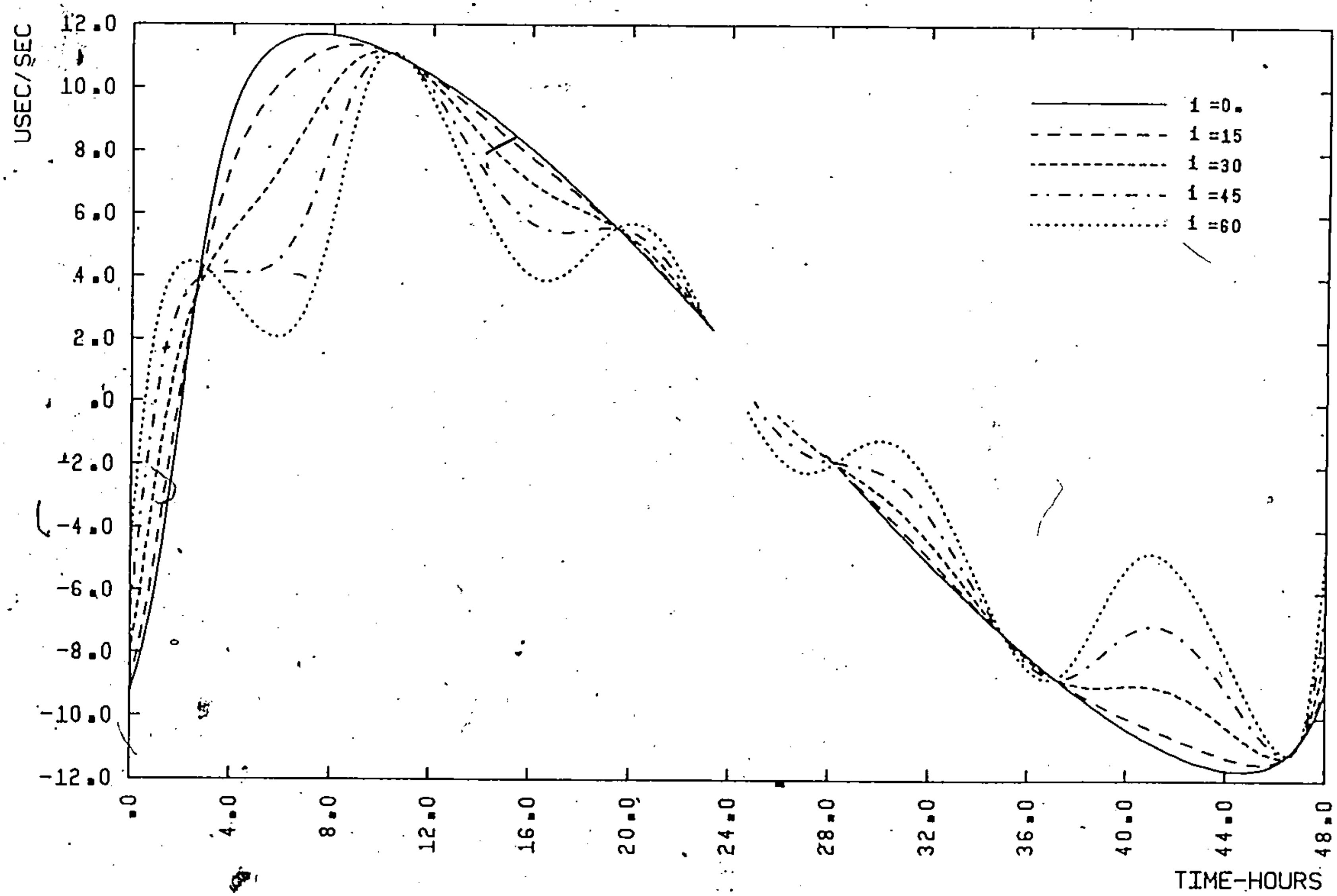

Fig. 5.12 The change in the space delay

$$\frac{dT}{dt}\bigg|_{G=G_{fix},N}$$

between a geostationary satellite and 16 hour navigation satellite with

$\phi_{LongG} - \phi_{oN} = 15°$, $i = 0°, 15°, 30°, 45°$ and $60°$

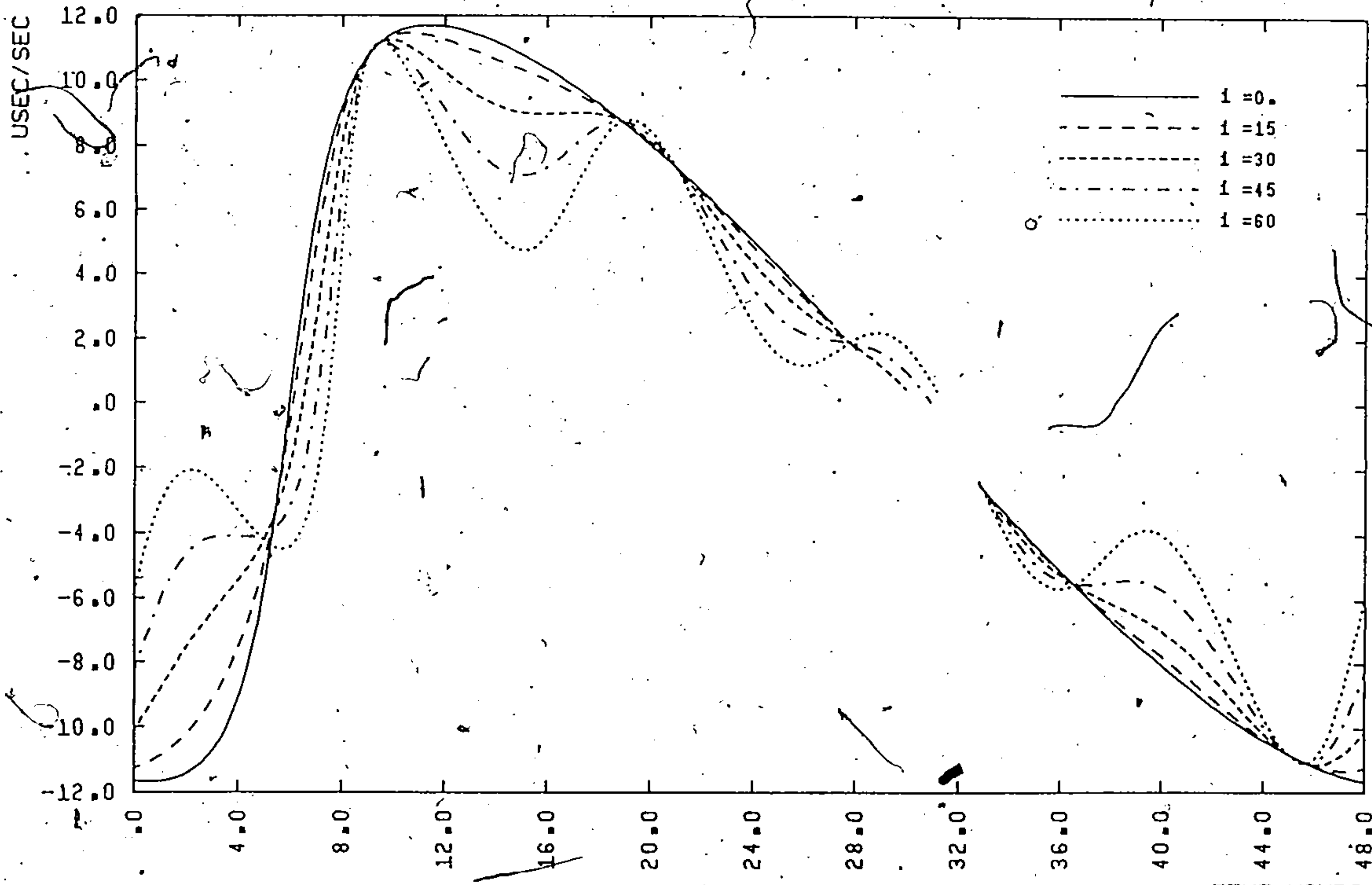

Fig. 5.13 The change in the space delay

$$\left.\frac{\partial T}{\partial t}\right|_{G=G_{fix},N}$$

between a geostationary satellite and 16 hour navigation satellite with

$\Phi_{LongG} - \Phi_{oN} = 45°$, $i = 0°, 15°, 30°, 45°$ and $60°$

The change in the change in space delay given by Eq. (5.30) has been plotted, in Figures 5.14, 5.15, 5.16 and 5.17 for the same cases plotted in Figures 5.10, 5.11, 5.12 and 5.13, respectively.

Figures 5.14 to 5.17 show that the maximum change if the change in the space delay does not exceed the value of 3 n s/s^2, therefore, Eq. (5.9) is a suitable formula to calculate T_{34} with high degree of accuracy.

5.5 THE CALCULATION OF THE UPLINK AND DOWNLINK DELAYS

It is now clear that if the location of the navigation satellite is known accurately, the space delay derivatives given by Eq. (5.17), Eq. (5.24), Eq. (5.28) and Eq. (5.30) can be calculated. Hence, substituting these derivatives in Eq. (5.4) and Eq. (5.8) the new uplink delay can be determined.

Although the location of the navigation satellite is not known precisely, we assume that its location in space does lie within a sphere which has a known center and radius. We designate this the sphere of error and find the maximum effect of this error upon the derivatives of the space delay given by Eq. (5.17) and (5.28) if the navigation satellite located at any point inside such a sphere. The reason why we consider the effect of the sphere of error upon the change in the space delays given by Eq. (5.17) and (5.28) only is that the computed results using Eq. (5.6) and Eq. (5.9) give a high accuracy in calculating the new uplink delay.

Now to find the effect of the change in navigation satellite location upon the change in the space delays, we find the gradient of the change in the space delay given by the Eqs. (5.17) and (5.28) in three orthogonal directions in space at the center of the sphere of error. By calculating the magnitude of the gradient at the center of the sphere of error, the maximum change in the space delay per unit length is obtained in the direction of the gradient. As assumed before, the navigation satellite may be located at any point inside the sphere of

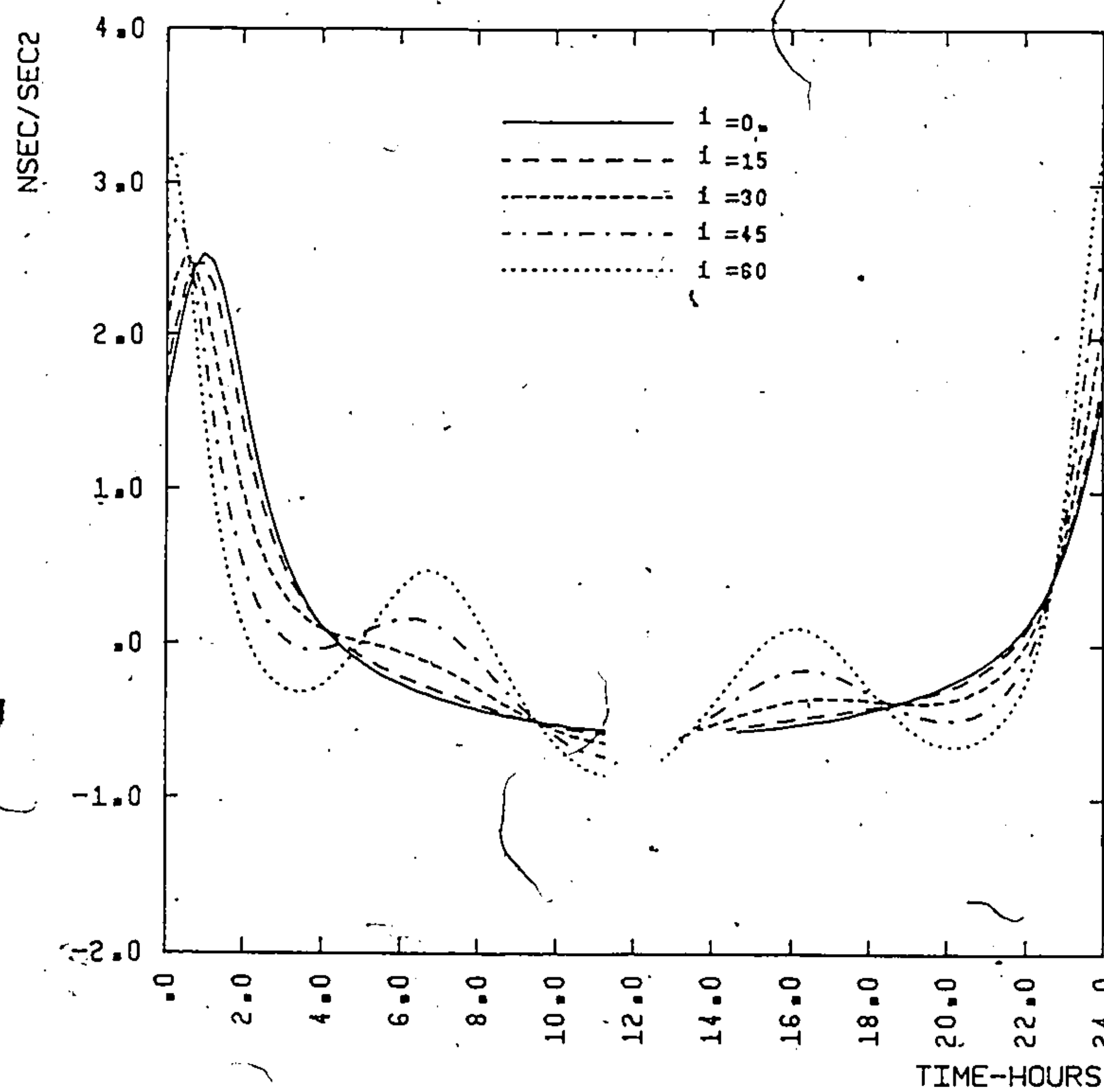

Fig. 5.14 The change in the change in the space delay

$$\left.\frac{\partial^2 T}{\partial t^2}\right|_{G=G_{fix},N}$$

between a geostationary satellite and 12 hour navigation satellite with

$\Phi_{LongG} - \Phi_{oN} = 15°$, $i = 0°, 15°, 30°, 45°$ and $60°$

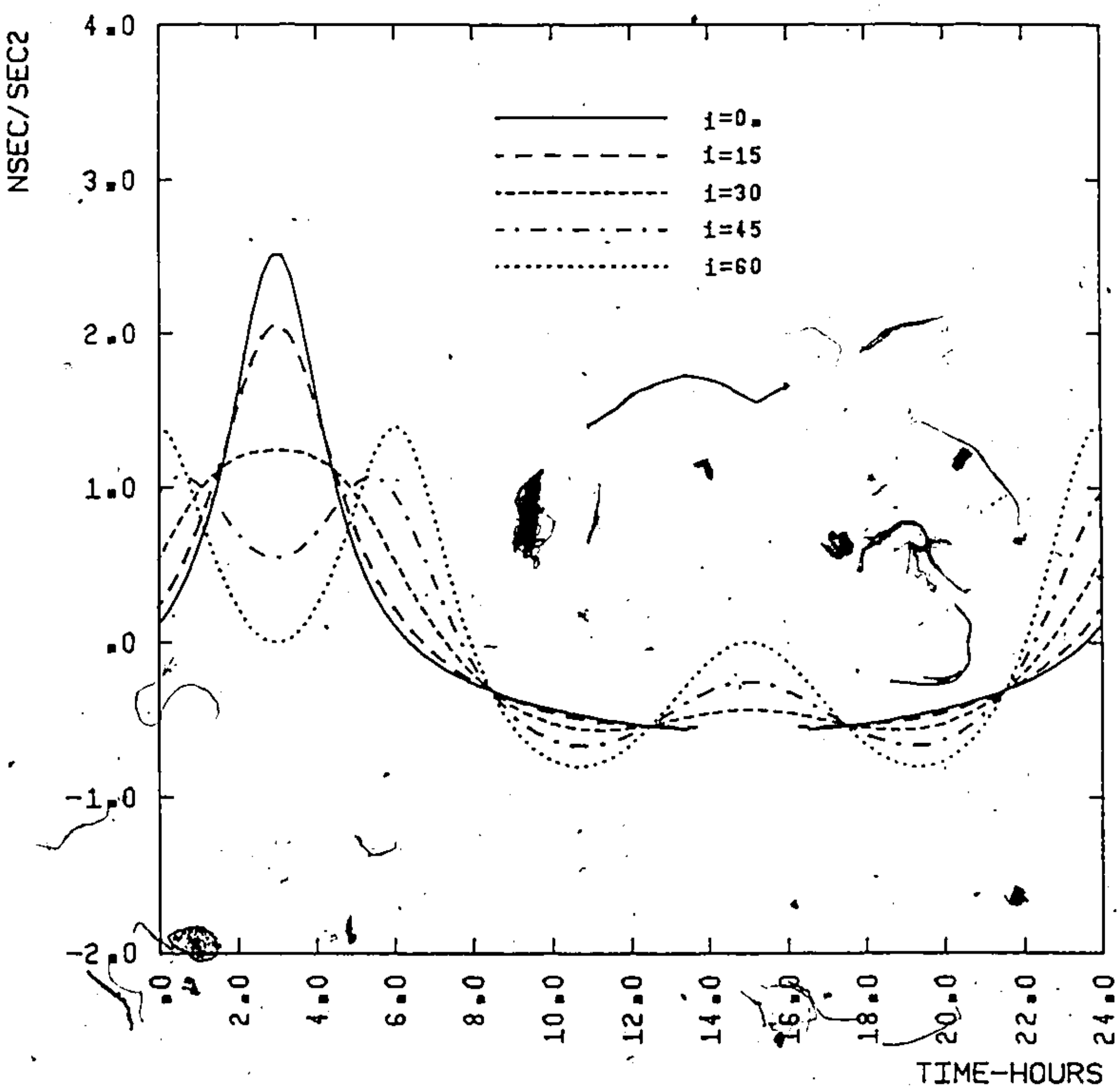

Fig. 5.15 The change in the change in the space delay

$$\left.\frac{\partial^2 T}{\partial t^2}\right|_{G=G_{fix},N}$$

between a geostationary satellite and 12 hour navigation satellite with $\Phi_{LongG} - \Phi_{oN} = 45°$, $i = 0°, 15°, 30°, 45°$ and $60°$

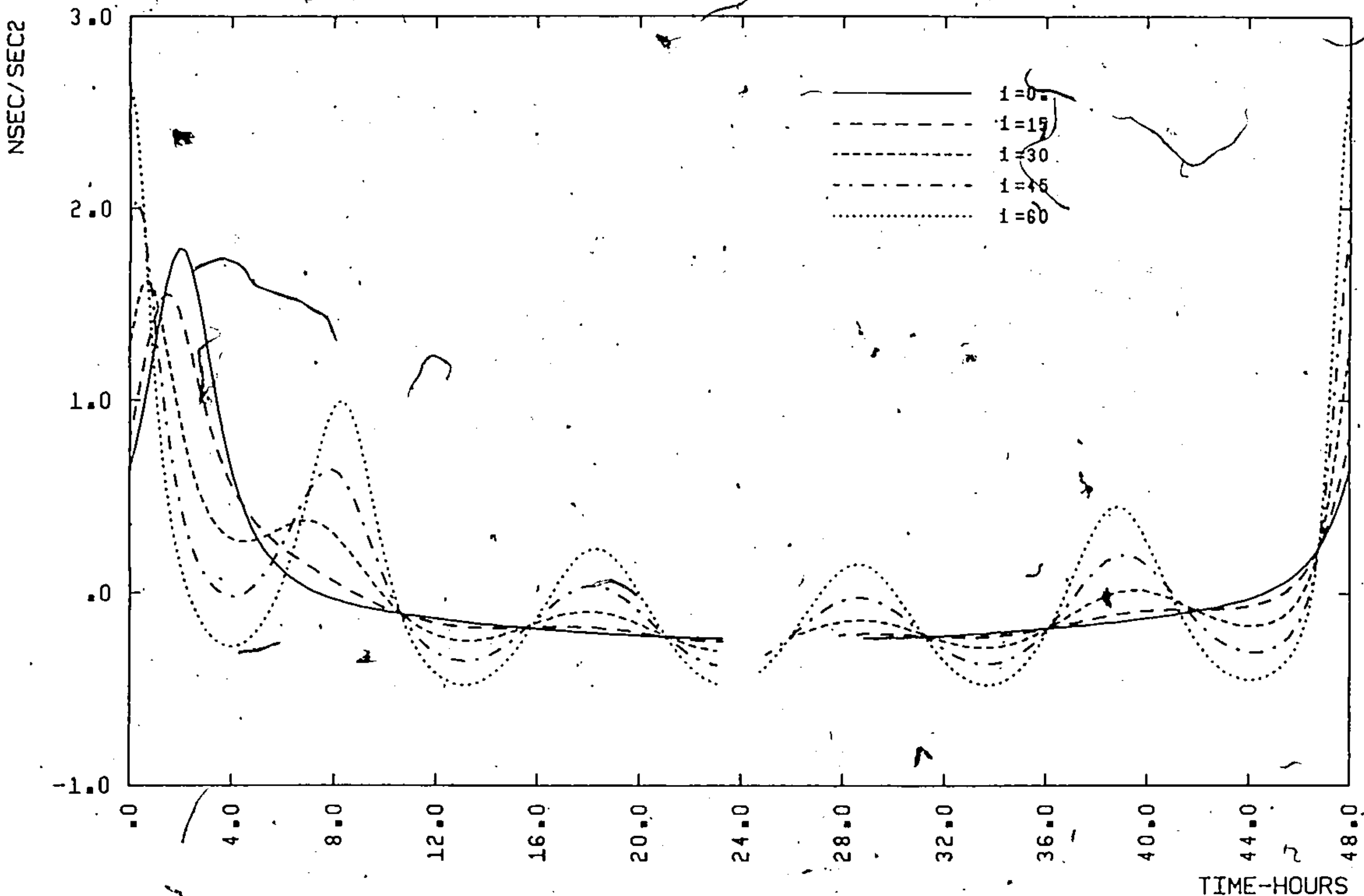

Fig. 5.16 The change in the change in the space delay

$$\left.\frac{\partial^2 T}{\partial t^2}\right|_{G=G_{fix},N}$$

between a geostationary satellite and 16 hour navigation satellite with

$\Phi_{LongG} - \Phi_{oN} = 15°$, $i = 0°, 15°, 30°, 45°$ and $60°$

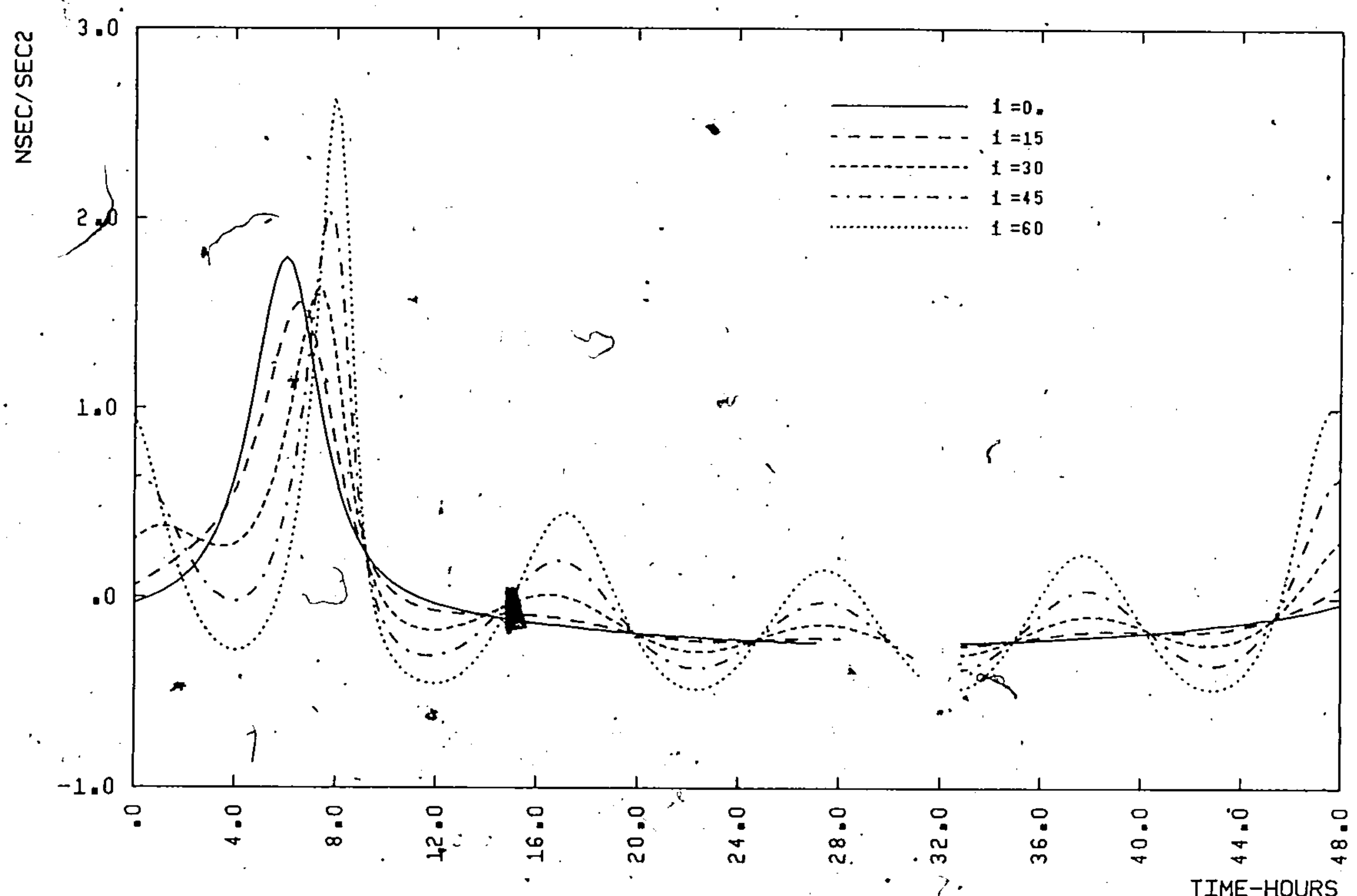

Fig. 5.17 The change in the change in the space delay

$$\left.\frac{\partial^2 T}{\partial t^2}\right|_{G=G_{fix},N}$$

between a geostationary satellite and 16 hour navigation satellite with

$\Phi_{LongG} - \Phi_{oN} = 45°$, $i = 0°, 15°, 30°, 45°$ and $60°$

error, therefore; for certain; one diameter of such a sphere will coincide with the direction of the gradient. Thus the maximum error in the change in the space delay due to the location of the navigation satellite inside the sphere of error will be equal to the magnitude of the gradient multiplied by the sphere of error radius. Such an error should be added to or subtracted from the change in the space delay (as we will explain later) in order to calculate the estimated value of the change in the space delay with maximum error. Now, the estimated changes in space delays are available; therefore, substituting in Eq. (5.6) and then, Eq. (5.9), we can find the new estimated uplink delay.

5.5.1 ESTABLISHING THE PROPER COORDINATES

In order to find the gradient of Eq. (5.17) and Eq. (5.28), we should select the proper coordinates suitable for using such equations. Refer to Fig. 5.18 which shows the orbital projection of both the equatorial orbit and the navigation satellite at any instant of time t on the surface of the earth.

It is clear that A is the point of intersection between the two orbital projections at time t, such a point will have a longitude equal to $\phi_{oN} - \omega_E t$, where ϕ_{oN} is the longitude of the point of intersection at $t = 0$.

Let the navigation satellite subpoint N' have a latitude and longitude ϕ_{LatN} and ϕ_{LongN} respectively.

The Cartesian coordinates x, y, z of N' is given by:

$$x_{N'} = R\cos\phi_{LatN}\cos\phi_{LongN} \tag{5.31}$$

$$y_{N'} = R\cos\phi_{LatN}\sin\phi_{LongN} \tag{5.32}$$

$$Z_{N'} = R\sin\phi_{LatN} \tag{5.33}$$

where R is the radius of the earth.

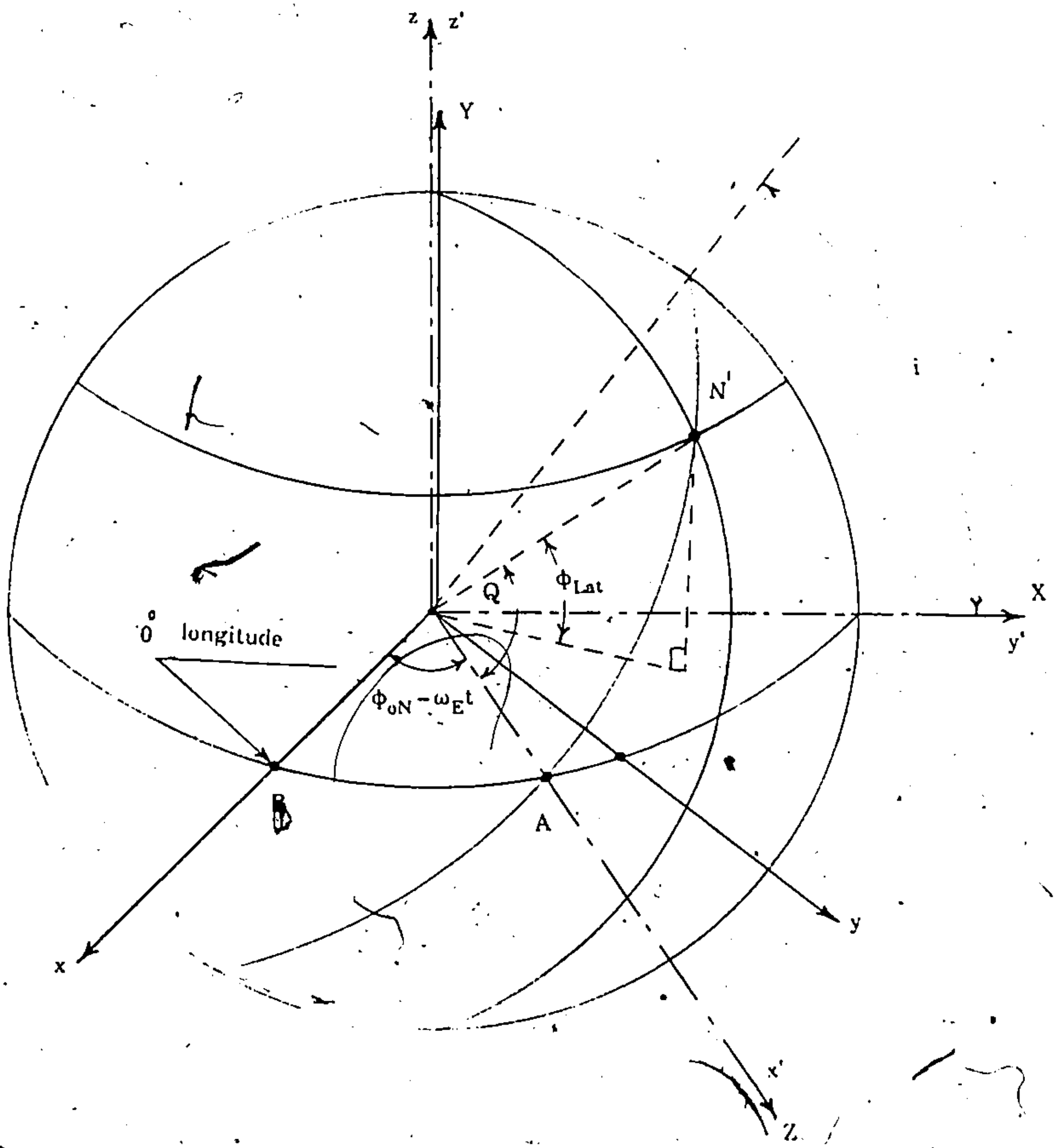

Fig. 5.18 Geometric relation for selecting the proper axis

By rotating the x axis and the y axis through an angle $\phi_{oN} - \omega_E t$ in the equatorial plane, the x axis becomes x' (coinciding with the line OA) and the y axis becomes y'. The new values for the navigation satellite subpoint coordinates $x'_{N'}$ and $y'_{N'}$ in the Cartesian coordinate system x', y' is given by:

$$x'_{N'} = x_{N'} \cos(\phi_{oN} - \omega_E t) + y_{N'} \sin(\phi_{oN} - \omega_E t) \tag{5.34}$$

$$y'_{N'} = -x_{N'} \sin(\phi_{oN} - \omega_E t) + y_{N'} \cos(\phi_{oN} - \omega_E t) \tag{5.35}$$

Substituting from Eq. (5.31) and Eq. (5.32) in Eq. (5.34) yields

$$x'_{N'} = R \cos\phi_{LatN} [\cos(\phi_{oN} - \omega_E t) \cos\phi_{LongN} + \sin(\phi_{oN} - \omega_E t) \sin\phi_{LongN} \tag{5.36}$$

this reduced to

$$x'_{N'} = R \cos\phi_{LatN} \cos(\phi_{LongN} + \omega_E t - \phi_{oN}) \tag{5.37}$$

From Eq. (4.11) and (4.15), we have

$$\cos(\phi_{LongN} + \omega_E t - \phi_{oN}) = \frac{\cos Q}{\cos\phi_{LatN}} \tag{5.38}$$

Substituting from Eq. (5.38) into Eq. (5.37) gives

$$x'_{N'} = R \cos Q \tag{5.39}$$

Similarly, substituting for $x_{N'}$ and $y_{N'}$ in Eq. (5.35) yields

$$y'_{N'} = R \cos\phi_{LatN} \sin(\phi_{LongN} + \omega_E t - \phi_{oN}) \tag{5.40}$$

Recalling Eq. (4.2) which is given by

$$\sin(\phi_{LongN} + \omega_E t - \phi_{oN}) = \frac{\cos i \sin Q}{\cos\phi_{LatN}} \tag{5.41}$$

Substituting into Eq. (5.40) yields

$$y'_{N'} = R \cos i \sin Q \tag{5.42}$$

Substituting for $\sin\phi_{LatN}$ in Eq. (5.33) from Eq. (4.1) gives:

$$z'_{N'} = R \sin i \sin Q \tag{5.43}$$

Now if we replace the directions x', y', z' by Z, X, Y respectively we can rewrite Eq. (5.39), Eq. (5.42) and Eq. (5.43) as follows:

$$X_{N'} = R \sin Q \cos i \tag{5.44}$$

$$Y_{N'} = R \sin Q \sin i \tag{5.45}$$

$$Z_{N'} = R \cos Q \tag{5.46}$$

Equations (5.44), (5.45) and (5.46) give us a new set of Cartesian coordinates. Comparing these equations with the relation between the Cartesian and polar coordinates, we can say that the equivalent polar coordinates for the navigation satellite will be H_N, Q and i. Thus the gradient can be calculated using the formula:

$$\nabla = \frac{\partial}{\partial H_N} a_{H_N} + \frac{1}{H_N} \frac{\partial}{\partial Q} a_Q + \frac{1}{H_N \sin Q} \frac{\partial}{\partial i} a_i \tag{5.47}$$

where a_{H_N}, a_Q and a_i are the unit vectors in the directions H_N, Q and i, respectively.

5.5.2 THE GRADIENT OF THE CHANGE IN THE SPACE DELAYS

Now to find the gradient of the change in the space delays with either of the two satellites being fixed, we apply Eq. (5.47) directly to either Eq. (5.17) or Eq. (5.28). In order to make the derivations shorter we are going to use the following formula:

$$\nabla \frac{\partial T}{\partial t} = \frac{\partial}{\partial t} (\nabla T) \tag{5.48}$$

Substituting for the gradient from Eq. (5.47) this gives:

$$\nabla \frac{\partial T}{\partial t} = \frac{\partial}{\partial t} \left(\frac{\partial T}{\partial H_N} a_{H_N} + \frac{1}{H_N} \frac{\partial T}{\partial Q} a_Q + \frac{1}{H_N \sin Q} \frac{\partial T}{\partial i} a_i \right) \tag{5.49}$$

where T is given by Eq. (5.12), noting that Λ in Eq. (5.12) is given by Eq. (5.10).

Consider the component of the gradient in the H direction. This is given by:

$$\frac{\partial T}{\partial H_N} = \frac{\partial}{\partial H_N} \frac{\sqrt{H_N^2 + H_G^2 - 2 H_N H_G \cos\Lambda}}{C} \tag{5.50}$$

$$\frac{\partial T}{\partial H_N} = \frac{2 H_N - 2 H_G \cos\Lambda}{2 C \sqrt{H_N^2 + H_G^2 - 2 H_N H_G \cos\Lambda}} \tag{5.51}$$

The last equation may be put in the form

$$\frac{\partial T}{\partial H_N} = \frac{(H_N - H_G \cos \lambda)}{d \cdot C} \tag{5.52}$$

Now, consider the component of the gradient in the Q direction noting that λ is the only variable in Q. This gives:

$$\frac{1}{H_N}\frac{\partial T}{\partial Q} = \frac{1}{H_N}\frac{\partial}{\partial Q}\frac{\sqrt{H_N^2 + H_G^2 - 2\,H_N\,H_G\cos\lambda}}{C} \tag{5.53}$$

$$\frac{1}{H_N}\frac{\partial T}{\partial Q} = \frac{1}{H_N}\frac{(1/2)(-2\,H_N\,H_G)}{\sqrt{(H_N^2 + H_G^2 - 2\,H_N\,H_G\cos\lambda)}\cdot C}\cdot\frac{\partial\cos\lambda}{\partial Q} \tag{5.54}$$

Substituting for $\cos\lambda$ from Eq. (5.10) and after simplification we get:

$$\frac{1}{H_N}\frac{\partial T}{\partial Q} = \frac{H_G}{d \cdot C}\,[\sin Q\cos\Lambda - \cos i\cos Q\sin\Lambda] \tag{5.55}$$

Consider the component of the gradient in the i direction:

$$\frac{1}{H_N\sin Q}\frac{\partial T}{\partial i} = \frac{1}{H_N\sin Q}\frac{\partial}{\partial i}\frac{\sqrt{H_N^2 + H_G^2 - 2\,H_N\,H_G\cos\lambda}}{C} \tag{5.56}$$

Substituting for $\cos\lambda$ and d yields:

$$\frac{1}{H_N\sin Q}\frac{\partial T}{\partial i} = \frac{1}{H_N\sin Q}\frac{-2\,H_N\,H_G}{2\,d\cdot C}\frac{\partial}{\partial i}[\cos Q\cos\Lambda + \cos i\sin Q\sin\Lambda] \tag{5.57}$$

After simplification we get:

$$\frac{1}{H_N\sin Q}\frac{\partial T}{\partial i} = \frac{H_G}{d\cdot C}\sin i\,\sin\Lambda \tag{5.58}$$

Now the gradient component of the change in the space delay if one satellite is fixed can be calculated simply by differentiating Eq. (5.52), Eq. (5.55) and Eq. (5.58) with respect to time for two cases.

Case 1 - The Gradient of the Change in the Space Delay if N is Fixed:

Consider the navigation satellite is held fixed, therefore, Q does not change when we differentiate the components of the gradient of the space delay with respect to time.

Differentiating the radial component of the gradient given by Eq. (5.52) with respect to time gives:

$$\frac{\partial}{\partial t}\frac{\partial T}{\partial H_N}\bigg|_{G,N=N_{fix}} = \frac{\partial}{\partial t}\left(\frac{1}{d\cdot C}\left(H_N - H_G\cos\lambda\right)\right)\bigg|_{G,N=N_{fix}} \tag{5.59}$$

Note here that $\cos\lambda$ and d are the only variables dependent on time, therefore Eq. (5.59) can be written as

$$\frac{\partial}{\partial t}\frac{\partial T}{\partial H_N}\bigg|_{G,N=N_{fix}} = \frac{(H_N - H_G\cos\lambda)}{d^2\cdot C}\left(-\frac{\partial d}{\partial t}\bigg|_{G,N=N_{fix}}\right) - \frac{1}{d\cdot C}H_G\frac{\partial\cos\lambda}{\partial t}\bigg|_{G,N=N_{fix}} \tag{5.60}$$

Substituting for

$$\frac{-H_G}{d\cdot C}\frac{\partial\cos\lambda}{\partial t}\bigg|_{G,N=N_{fix}}$$

from Eq. (5.14) and noting that

$$\frac{1}{C}\frac{\partial d}{\partial t}\bigg|_{G,N=N_{fix}} = \frac{\partial T}{\partial t}\bigg|_{G,N=N_{fix}}$$

we can write Eq. (5.60) as follows:

$$\frac{\partial}{\partial H_N}\frac{\partial T}{\partial t}\bigg|_{G,N=N_{fix}} = \frac{H_N - H_G\cos\lambda}{d^2}\left(-\frac{\partial T}{\partial t}\bigg|_{G,N=N_{fix}}\right) + \frac{1}{H_N}\frac{\partial T}{\partial t}\bigg|_{G,N=N_{fix}} \tag{5.61}$$

The last Equation can be reduced to:

$$\frac{\partial}{\partial H_N}\frac{\partial T}{\partial t}\bigg|_{G,N=N_{fix}} = \left(\frac{H_G\cos\lambda - H_N}{d^2} + \frac{1}{H_N}\right)\frac{\partial T}{\partial t}\bigg|_{G,N=N_{fix}} \tag{5.62}$$

Let us consider the change in the gradient of the space delay in Q direction; that is:

$$\frac{\partial}{\partial t}\,\frac{1}{H_N}\,\frac{\partial \Gamma}{\partial Q}\Bigg|_{G,N=N_{fix}} = \frac{\partial}{\partial t}\left(\frac{H_G}{d\cdot C}\,(\sin Q\cos\Lambda - \cos i\cos Q\sin\Lambda)\right)\Bigg|_{G,N=N_{fix}} \tag{5.63}$$

or

$$\frac{1}{H_N}\frac{\partial}{\partial Q}\frac{\partial \Gamma}{\partial t}\Bigg|_{G,N=N_{fix}} = \frac{H_G}{d^2\cdot C}\left|(\sin Q\cos\Lambda - \cos i\cos Q\sin\Lambda)\left(-\frac{\partial d}{\partial t}\Bigg|_{G,N=N_{fix}}\right)\right|$$

$$+\frac{H_G\,\omega_E}{d\cdot C}\,(-\sin Q\sin\Lambda - \cos i\cos Q\cos\Lambda) \tag{5.64}$$

Noting that

$$\frac{1}{C}\,\frac{\partial d}{\partial t}\Bigg|_{G,N=N_{fix}} = \frac{\partial \Gamma}{\partial t}\Bigg|_{G,N=N_{fix}}$$

The last equation can be written in the form:

$$\frac{1}{H_N}\,\frac{\partial}{\partial Q}\frac{\partial \Gamma}{\partial t}\Bigg|_{G,N=N_{fix}} = \frac{H_G}{d}\left|\frac{(\cos i\cos Q\sin\Lambda - \sin Q\sin\Lambda)}{d}\,\frac{\partial \Gamma}{\partial t}\Bigg|_{G,N=N_{fix}}\right.$$

$$\left. - \frac{\omega_E}{C}\,(\cos i\cos Q\cos\Lambda + \sin Q\sin\Lambda)\right| \tag{5.65}$$

The last equation gives the gradient component in Q direction of the change in the space delay. The gradient component of the change in the space delay in the i direction can be calculated by differentiating Eq. (5.58) with respect to time: noting that Λ and d are dependent on time. This gives:

$$\frac{\partial}{\partial t}\,\frac{1}{H_N\sin Q}\,\frac{\partial \Gamma}{\partial i}\Bigg|_{G,N=N_{fix}} = \frac{\partial}{\partial t}\,\frac{H_G}{d\cdot C}\sin i\sin\Lambda\Bigg|_{G,N=N_{fix}} \tag{5.66}$$

Thus, we have

$$\frac{1}{H_N\sin Q}\,\frac{\partial}{\partial i}\,\frac{\partial \Gamma}{\partial t}\Bigg|_{G,N=N_{fix}} = \frac{H_G}{C}\sin i\left|\frac{\sin\Lambda}{d^2}\left(\frac{-\partial d}{\partial t}\right)\Bigg|_{G,N=N_{fix}} + \frac{\omega_E\cos\Lambda}{d}\right| \tag{5.67}$$

The last equation can be reduced to

$$\frac{1}{H_N\sin Q}\frac{\partial}{\partial i}\,\frac{\partial \Gamma}{\partial t}\Bigg|_{G,N=N_{fix}} = \frac{H_G}{d}\sin i\left|\frac{\omega_E\cos\Lambda}{C} - \frac{\sin\Lambda}{d}\,\frac{\partial \Gamma}{\partial t}\Bigg|_{G,N=N_{fix}}\right| \tag{5.68}$$

The magnitude of the gradient of the change in the space delay and its three perpendicular components, given by Eq. (5.62), (5.65) and (5.68) have been plotted in Figs. 5.19, 5.20, 5.21 and 5.22, where the magnitude of the gradient is equal to the vector addition of its components.

Figure 5.19 represents the case of a navigation satellite in a 12-hour orbit with inclination angle $i = 15°$, TT = 0 and the difference in longitude, $(\phi_{LongG} - \phi_{oN}) = 0$. Figure 5.20 is plotted for inclination angle = 45°, TT = 0, and $(\phi_{LongG} - \phi_{oN}) = 30°$.

Figures 5.21 and 5.22 are plotted when the inclination angle $i = 60°$ with $(\phi_{LongG} - \phi_{oN}) = 15°$, and $(\phi_{LongG} - \phi_{oN}) = 45°$, respectively.

The corresponding gradient components of the change in the space delay in the radial direction H_N, the latitude direction ϕ_{LatN} and the longitude direction ϕ_{LongN} for the case when the navigation satellite is fixed have been derived in Appendix 3, and are given by:

$$\frac{\partial}{\partial H_N} \left. \frac{\partial T}{\partial t} \right|_{G,N=N_{fix}} = \left(\frac{1}{H_N} + \frac{H_G \cos\lambda - H_N}{d^2} \right) \left. \frac{\partial T}{\partial t} \right|_{G,N=N_{fix}} \tag{5.69}$$

$$\frac{-1}{H_N} \frac{\partial}{\partial \phi_{LatN}} \left. \frac{\partial}{\partial t} \right|_{G,N=N_{fix}} = \frac{H_G \sin\phi_{LatN}}{d} \left| \frac{\cos\eta \left. \frac{\partial T}{\partial t} \right|_{G,N=N_{fix}}}{d} - \frac{\omega_E \sin\eta}{C} \right| \tag{5.70}$$

$$\frac{1}{H_N \cos\phi_{LatN}} \frac{\partial}{\partial \phi_{LongN}} \left. \frac{\partial T}{\partial t} \right|_{G,N=N_{fix}} = \frac{-H_G}{d} \left| \frac{\sin\eta}{d} \left. \frac{\partial T}{\partial t} \right|_{G,N=N_{fix}} + \frac{\omega_E \sin\eta}{C} \right| \tag{5.71}$$

where

$$\eta = \phi_{LongN} - \phi_{LongG}$$

Case 2 - The Gradient of the Change in the Space Delay if the Geostationary Satellite is Fixed

Now, to find the gradient of the change in the space delay while keeping the geostationary satellite fixed momentarily, we differentiate Eqs (5.52), (5.55) and (5.58) with

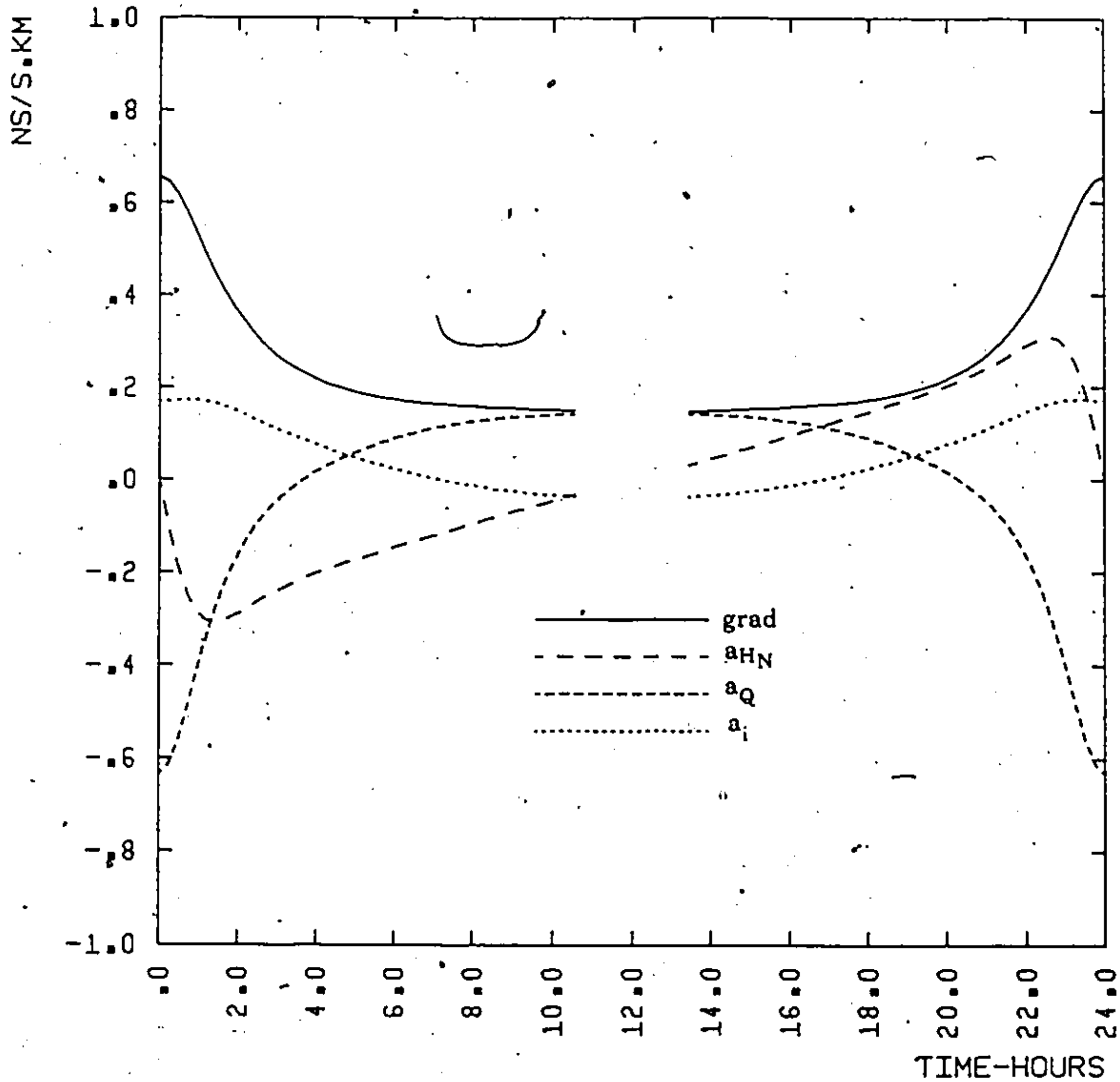

Fig. 5.19. The gradient and its components of

$$\left.\frac{\partial \Gamma}{\partial t}\right|_{G,N=N_{fix}}$$

between a geostationary satellite and a navigation satellite in 12 hour orbit with $i = 15°$ and $\phi_{LongG} - \phi_{oN} = 0°$

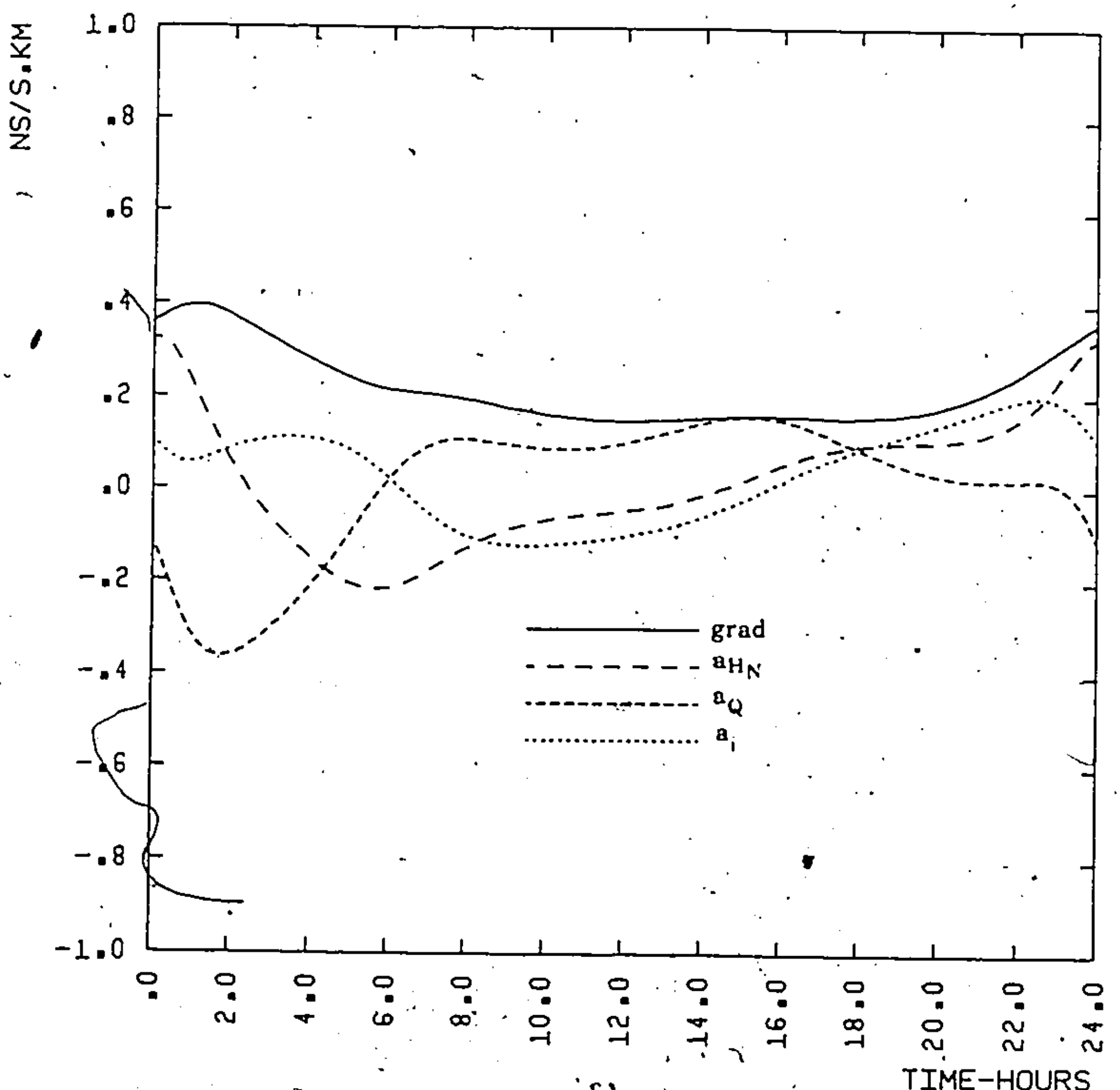

Fig. 5.20 The gradient and its components of

$$\left.\frac{\partial T}{\partial t}\right|_{G,N=N_{fix}}$$

between a geostationary satellite and a navigation satellite in 12 hour orbit with $i = 45°$ and $\phi_{LongG} - \phi_{oN} = 30°$.

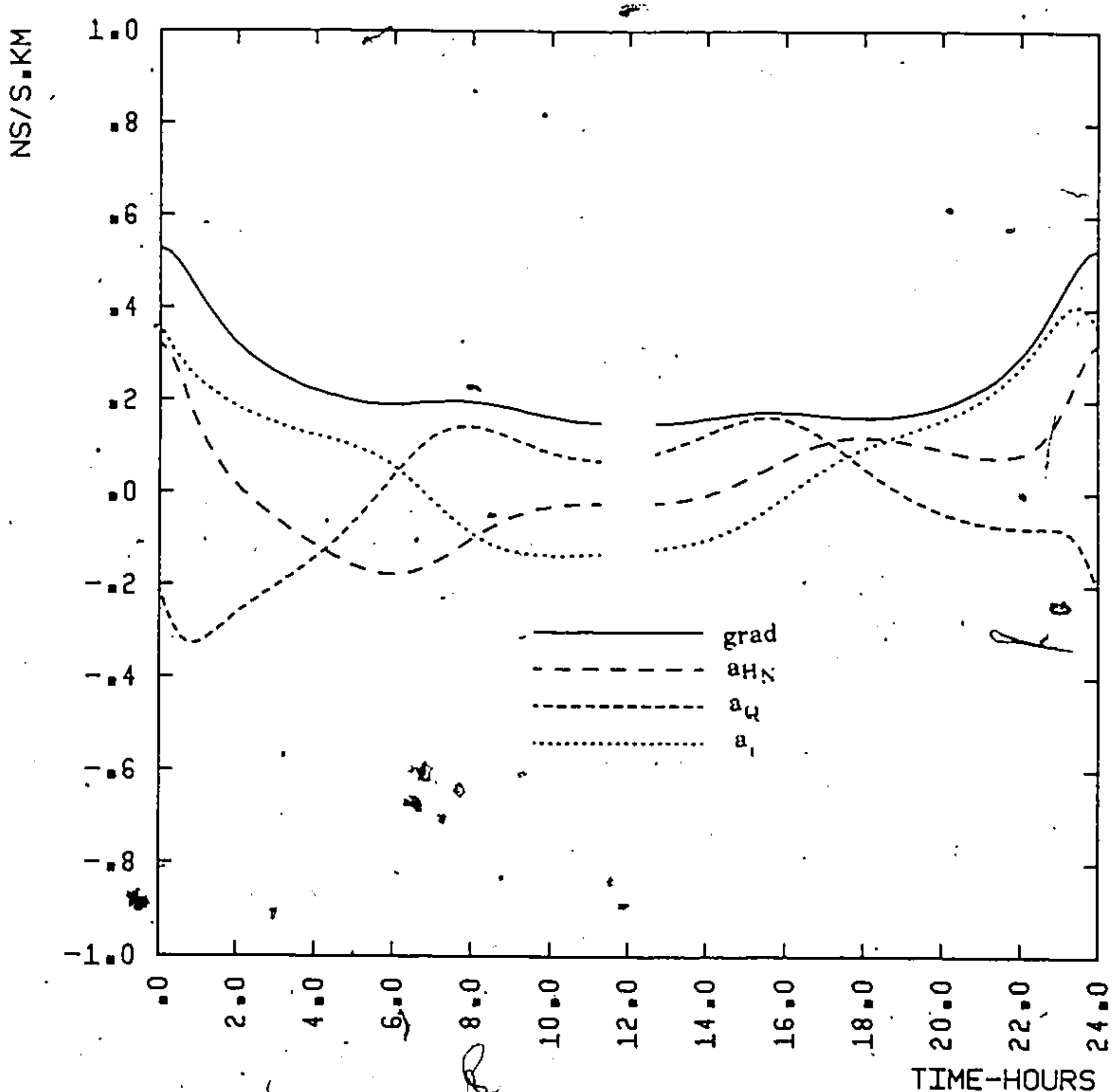

Fig. 5.21. The gradient and its components of

$$\left.\frac{\partial T}{\partial t}\right|_{G,N=N_{fix}}$$

between a geostationary satellite and a navigation satellite in 12 hour orbit with $i = 60°$ and $\phi_{LongG} - \phi_{oN} = 15°$.

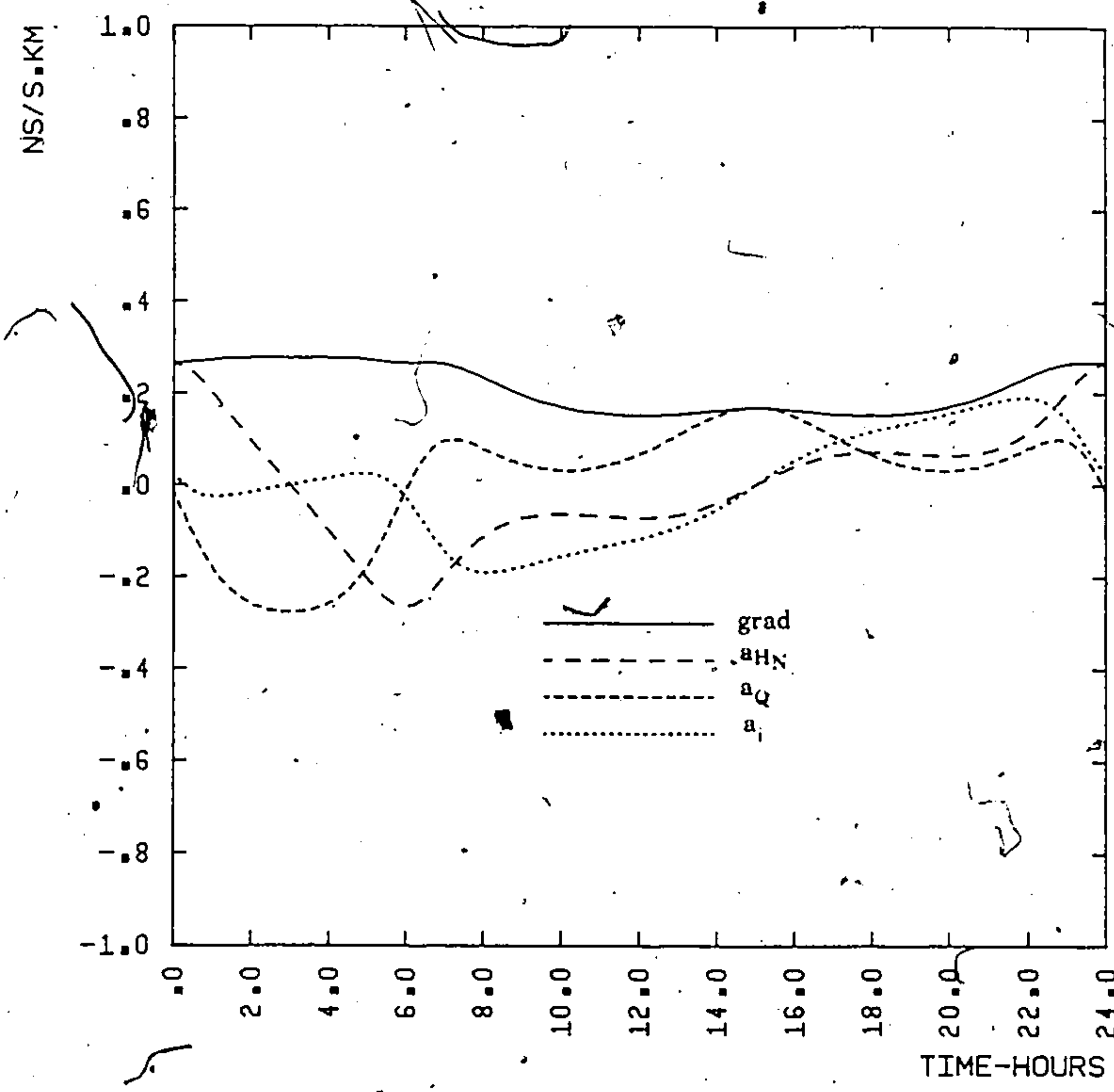

Fig. 5.22. The gradient and its components of

$$\left.\frac{\partial T}{\partial t}\right|_{G,N=N_{fix}}$$

between a geostationary satellite and a navigation satellite in 12 hour orbit with $i = 60°$ and $\phi_{LongG} - \phi_{oN} = 45°$.

respect to time while holding Λ fixed. The gradient component of the change in the space delay in the radial direction a_{H_N} is given by:

$$\frac{\partial}{\partial t}\frac{\partial T}{\partial H_N}\bigg|_{G=G_{fix},N} = \frac{\partial}{\partial t}\frac{1}{d\cdot C}(H_N - H_G\cos\lambda)\bigg|_{G=G_{fix},N} \qquad (5.72)$$

Noting that d and λ are the only variables dependent on time, the last equation becomes:

$$\frac{\partial}{\partial t}\frac{\partial T}{\partial H_N}\bigg|_{G=G_{fix},N} = -\frac{1}{d^2}(H_N - H_G\cos\lambda)\frac{\partial d/c}{\partial t}\bigg|_{G=G_{fix},N}$$

$$+\frac{1}{d\cdot C}\left(-H_G\frac{\partial\cos\lambda}{\partial t}\bigg|_{G=G_{fix},N}\right) \qquad (5.73)$$

Now, recalling Eq. (5.10) which gives cos λ and differentiating it with respect to t while keeping Λ = constant this gives

$$\frac{\partial\cos\lambda}{\partial t}\bigg|_{G=G_{fix},N} = \omega_S(-\sin Q\cos\Lambda + \cos i\cos Q\sin\Lambda) \qquad (5.74)$$

Substituting Eq. (5.74) in Eq. (5.73) and after simplifying we get

$$\frac{\partial}{\partial H_N}\frac{\partial T}{\partial t}\bigg|_{G,G_{fix}=N} = -\frac{1}{d^2}(H_N - H_G\cos\lambda)\frac{\partial T}{\partial t}\bigg|_{G=G_{fix},N}$$

$$+\frac{\omega_S H_G}{d\cdot C}(\sin Q\cos\Lambda - \cos i\cos Q\sin\Lambda) \qquad (5.75)$$

By making use of Eq. (5.28) the last equation can be rewritten in the form:

$$\frac{\partial}{\partial H_N}\frac{\partial T}{\partial t}\bigg|_{G=G_{fix},N} = \left(\frac{H_G\cos\lambda - H_N}{d^2} + \frac{1}{H_N}\right)\frac{\partial T}{\partial t}\bigg|_{G=G_{fix},N} \qquad (5.76)$$

The last equation gives the gradient of the change in the space delay in the H_N direction if G is fixed.

To find the gradient of the change in the space delay in Q direction if G is fixed, we differentiate Eq. (5.55) with respect to time under the condition that Λ is constant. This yields:

$$\frac{1}{H_N}\frac{\partial}{\partial Q}\frac{\partial T}{\partial t}\bigg|_{G=G_{fix},N} = \frac{\partial}{\partial t}\frac{H_G}{d\cdot C}\,[\sin Q\cos\Lambda - \cos i\cos Q\sin\Lambda]\bigg|_{G=G_{fix},N} \tag{5.77}$$

Noting that Q and d are the only variables dependent on time, we get

$$\frac{1}{H_N}\frac{\partial}{\partial Q}\frac{\partial T}{\partial t}\bigg|_{G=G_{fix},N} = \frac{-H_G}{d^2}\,[\sin Q\cos\Lambda - \cos i\cos Q\sin\Lambda]\,\frac{\partial d/C}{\partial t}\bigg|_{G=G_{fix},N}$$

$$+\frac{H_G\,\omega_S}{d\cdot C}\,[\cos Q\cos\Lambda + \cos i\sin Q\sin\Lambda] \tag{5.78}$$

By making use of Eq. (5.10) the gradient of the change in the space delay in the a_Q direction will be:

$$\frac{1}{H_N}\frac{\partial}{\partial Q}\frac{\partial T}{\partial t}\bigg|_{G=G_{fix},N} = \frac{-H_G}{d^2}\,[\sin Q\cos\Lambda - \cos i\cos Q\sin\Lambda]\,\frac{\partial T}{\partial t}\bigg|_{G=G_{fix},N}$$

$$+\frac{H_G\,\omega_S}{d\cdot C}\cos\lambda \tag{5.79}$$

The last component of the gradient which is in the a_i direction can be found by differentiating Eq. (5.58) with respect to time yielding:

$$\frac{1}{H_N\sin Q}\frac{\partial}{\partial i}\frac{\partial T}{\partial t}\bigg|_{G=G_{fix},N} = \frac{\partial}{\partial t}\left(\frac{H_G}{d\cdot C}\sin i\sin\Lambda\right)\bigg|_{G=G_{fix},N} \tag{5.80}$$

Keeping Λ fixed, the last equation becomes

$$\frac{1}{H_N\sin Q}\frac{\partial}{\partial i}\frac{\partial T}{\partial t}\bigg|_{G=G_{fix},N} = \frac{H_G}{d^2}\sin i\sin\Lambda\cdot\frac{-\partial d/C}{dt}\bigg|_{G=G_{fix},N} \tag{5.81}$$

Thus, the gradient of the change in the space delay in a_i direction if G is fixed will be given by:

$$\frac{1}{H_N\sin Q}\frac{\partial}{\partial i}\frac{\partial T}{\partial t}\bigg|_{G=G_{fix},N} = -\frac{H_G}{d^2}\sin i\sin\Lambda\,\frac{\partial T}{\partial t}\bigg|_{G=G_{fix},N} \tag{5.82}$$

The total gradient of the change in the space delay if the geostationary satellite is fixed can be calculated by the vector addition of its components given by Eq. (5.76), (5.79) and (5.82).

The gradient and its components has been plotted in Figs. 5.23, 5.24, 5.25 and 5.26, where the vertical axis represents the gradient or its components in ns/(s·Km) and the horizontal axis represents the time measured by the reference clock.

Figures 5.23 and 5.24 are plotted for the case when the navigation satellite is located in 12 hour orbit with an inclination angle equal to 30° with TT = 0, $\phi_{LongG} - \phi_{oN} = 0$ and $\phi_{LongG} - \phi_{oN} = 45°$, respectively.

Figures 5.25 and 5.26 are the same as the previous Figures but the inclination angle equals 60° with $\phi_{LongG} - \phi_{oN} = 30°$ in Fig. 5.24 and $\phi_{LongG} - \phi_{oN} = 45°$ in Fig. 5.25.

5.6 THE COMPUTED RESULTS AND THE ERROR

From the previous analysis it is clear that the new uplink delay can be estimated by knowing the approximate location of the navigation satellite. A computer program has been developed to calculate the new estimated uplink delay and to find the error in its calculation. The program can be divided into two parts. The first part assumes that the location of the geostationary satellite at G_1 and G_3 is known precisely; the location of the navigation satellite at N_1 (and consequently at N_2) is precisely known; and the round trip time T_t is accurately measured. Then, T_{12}, T_{23} and T_{34} can all be calculated precisely.

The second part assumes the same conditions as the first part except now the navigation satellite location at N_2 is not precisely known. The values for T_{12}, T_{23} and T_{34} are recalculated and compared with the previously computed accurate values. The difference is then the error.

Note that all the calculation of the second part should take place on board the geostationary satellite before transmitting the new messages.

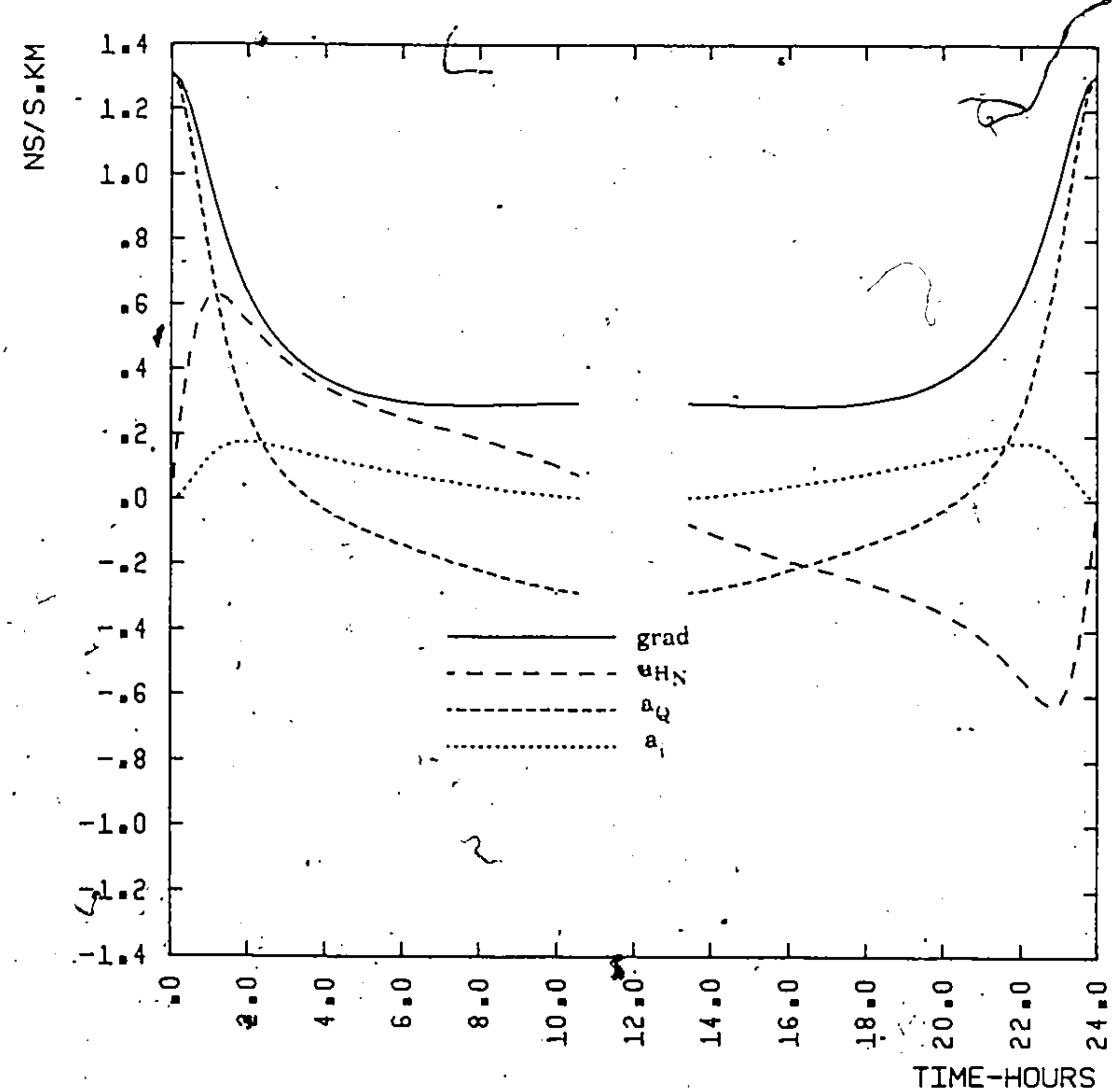

Fig. 5.23. The gradient and its components of

$$\left.\frac{\partial T}{\partial t}\right|_{G=G_{fix},N}$$

between a geostationary satellite and a navigation satellite in 12 hour orbit with $i = 30°$ and $\phi_{LongG} - \phi_{oN} = 0°$.

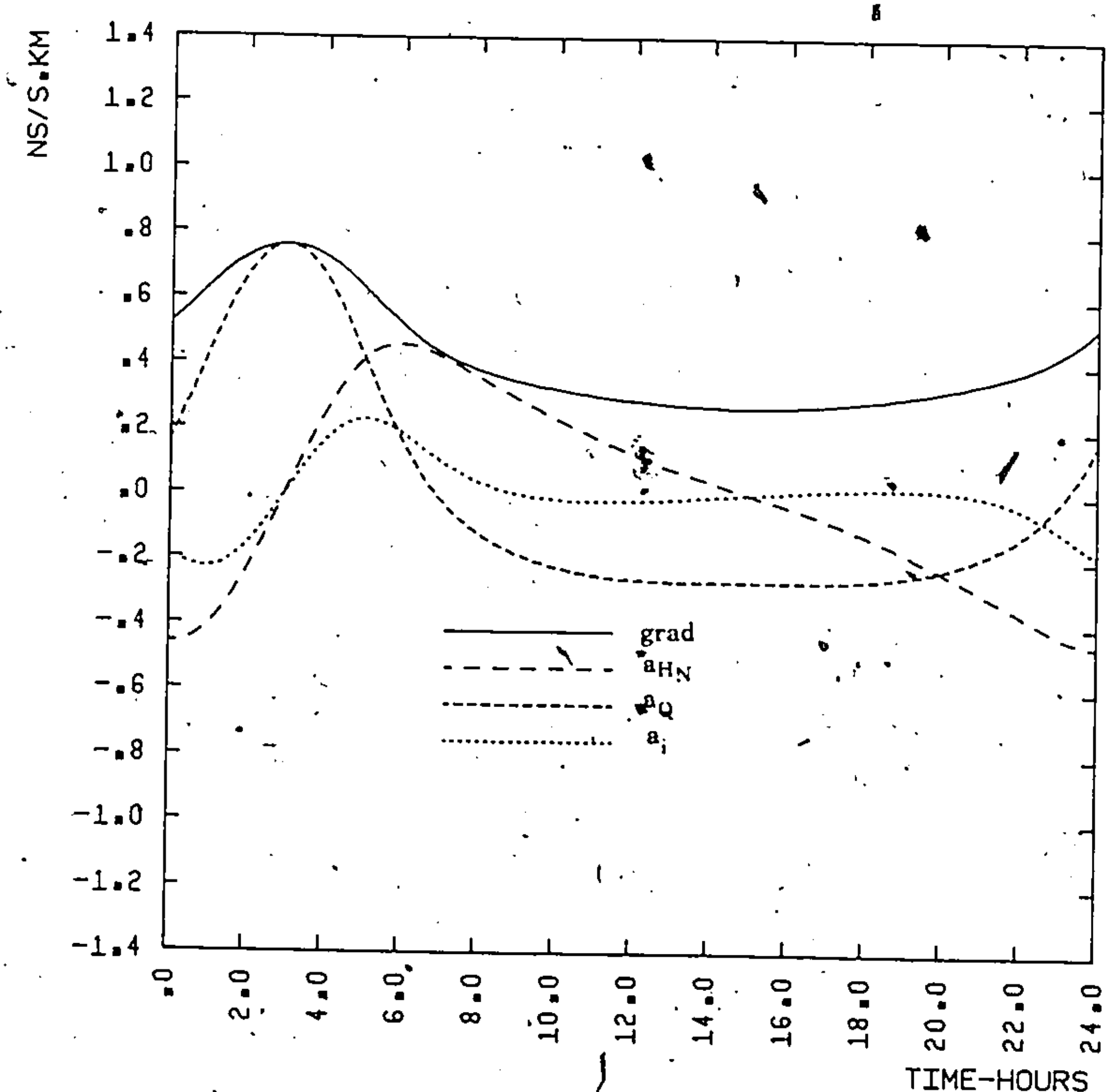

Fig. 5.24. The gradient and its components of

$$\left.\frac{\partial T}{\partial t}\right|_{G=G_{fix}N}$$

between a geostationary satellite and a navigation satellite in 12 hour orbit with $i = 30°$ and $\phi_{LongG} - \phi_{oN} = 45°$

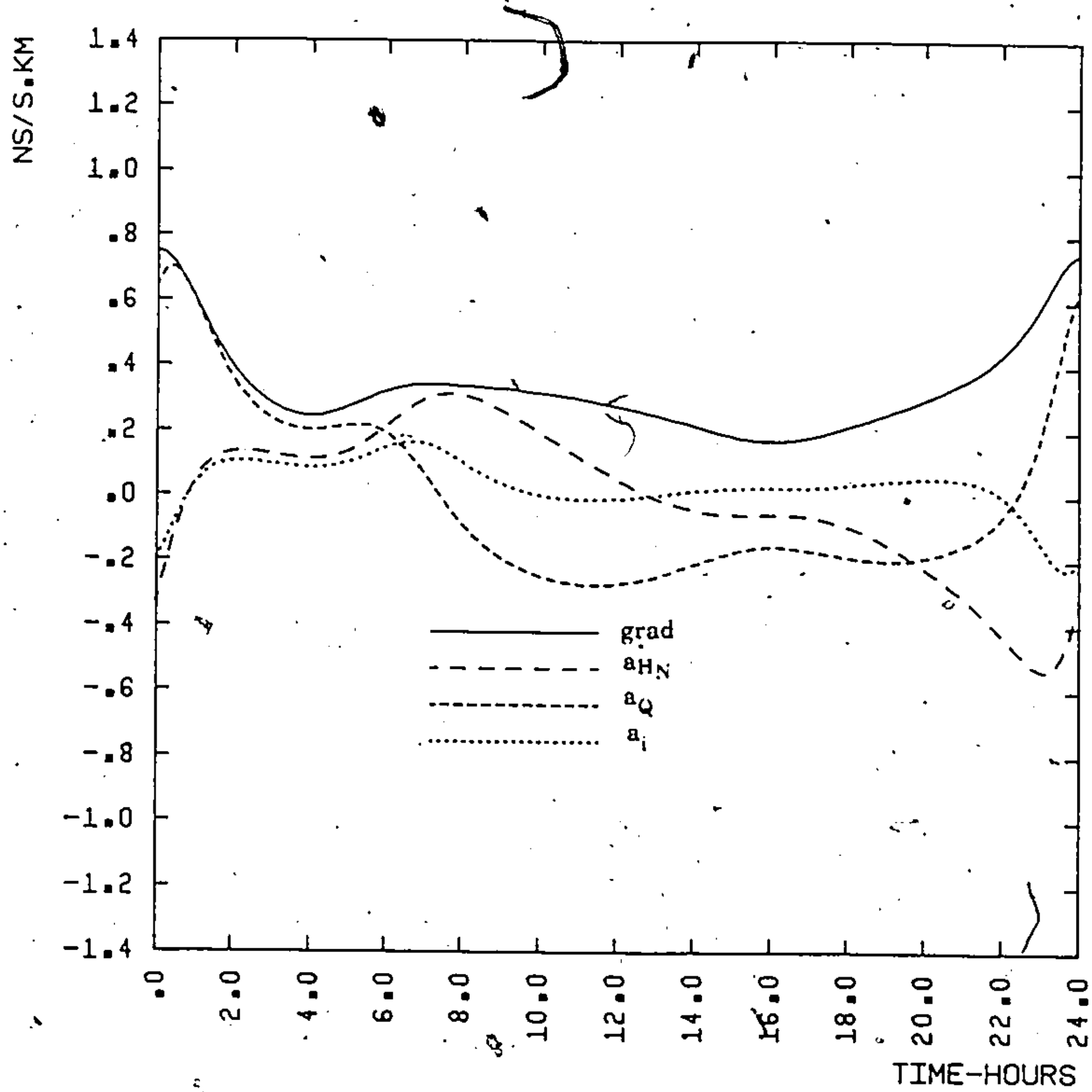

Fig. 5.25. The gradient and its components of

$$\left.\frac{\partial T}{\partial t}\right|_{G=G_{lin},N}$$

between a geostationary satellite and a navigation satellite in 12 hour orbit with $i = 60°$ and $\Phi_{LongG} - \Phi_{oN} = 30°$.

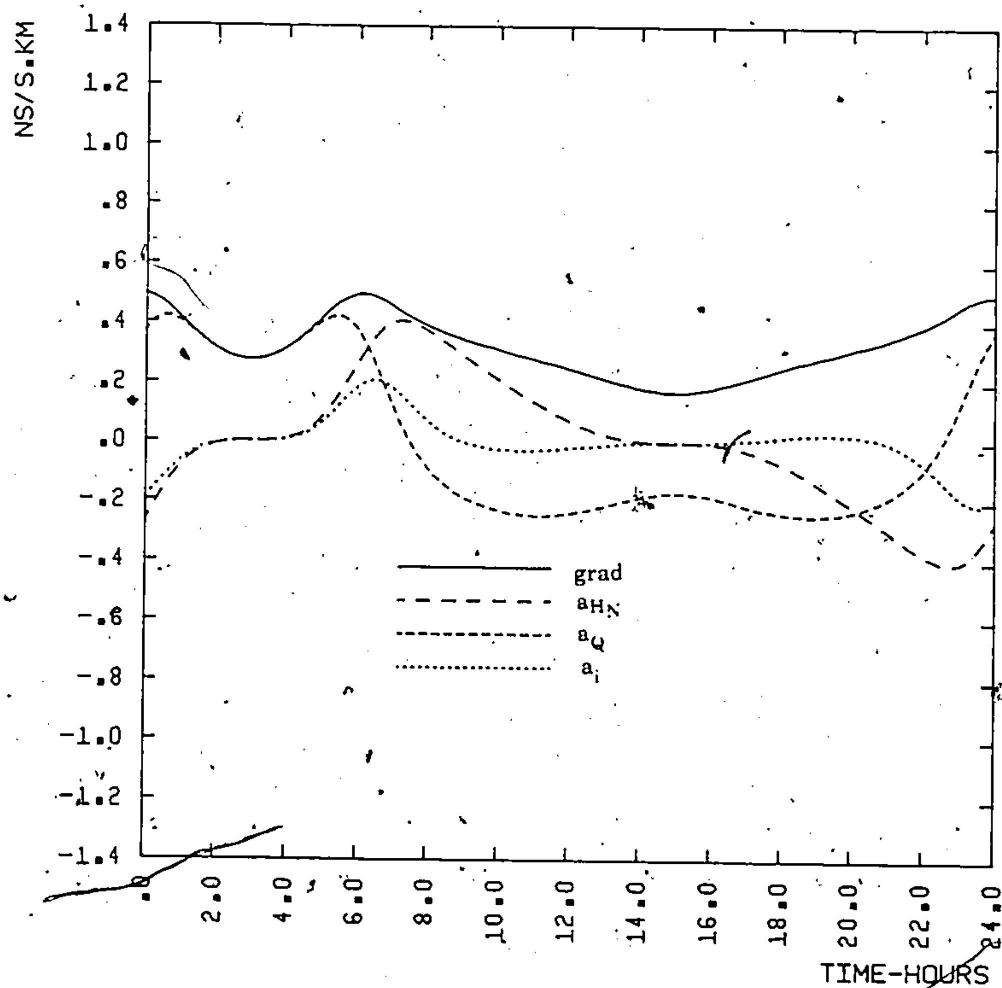

Fig. 5.26. The gradient and its components of

$$\left.\frac{\partial T}{\partial t}\right|_{G=G_{fix},N}$$

between a geostationary satellite and a navigation satellite in 12 hour orbit with $i = 60^\circ$ and $\phi_{LongG} - \phi_{oN} = 45^\circ$

Part 1

With the aid of Fig. 5.27, we can summarize the steps as follows:

1) By knowing the location of G_1 and N_1 at any instant of time t, we can calculate the space delay T_{11} using Eq. (5.10) and Eq. (5.12).

2) Holding the geostationary satellite fixed at G_1 we calculate:

a) The change in the space delay and the change in the change in the space delay if the navigation satellite is at N_1 and moving from Eq. (5.28) and (5.30) respectively.

b) Using Taylor Series we can calculate the theoretical value of the uplink delay which is given by:

$$T_{12th} = T_{11} + T_{11} \left.\frac{\partial T}{\partial t}\right|_{G=G_1,N_1} + \frac{T_{11}^2}{2!} \left.\frac{\partial^2 T}{\partial t^2}\right|_{G=G_1,N_1} \tag{5.83}$$

c) Moving the navigation satellite from its location at N_1 an angular distance corresponding to the theoretical delay T_{12th}; the true space delay T_{12} can be calculated using Eqs. (5.10) and (5.12).

The difference between the true value of space delay T_{12} and the theoretical value T_{12th} in ns is plotted in Fig. 5.28 for the case of a navigation satellite in 12 hour orbit with inclination angle i having values 0°, 15°, 30°, 45° and 60° while the difference in longitude $\phi_{LongG} - \phi_{oN} =$ 45.0°.

d) Moving the navigation satellite a distance corresponding to T_{12} measured from its initial location at N_1 we can find the exact location of the navigation satellite N_2 upon receiving the message transmitted by the geostationary satellite at G_1.

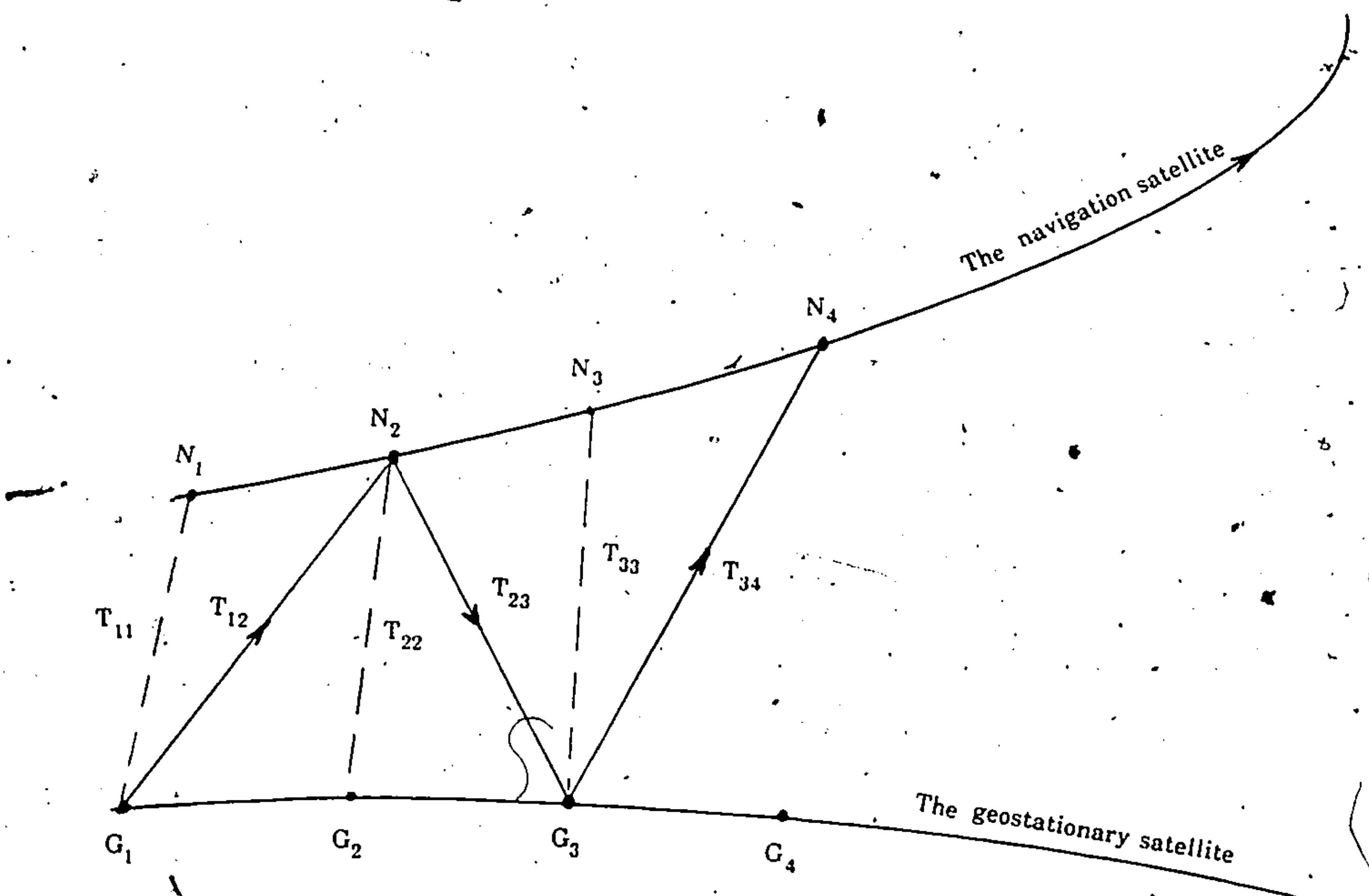

Fig. 5.27 Relation between the space delays, uplink delay, downlink delay and new uplink delay.

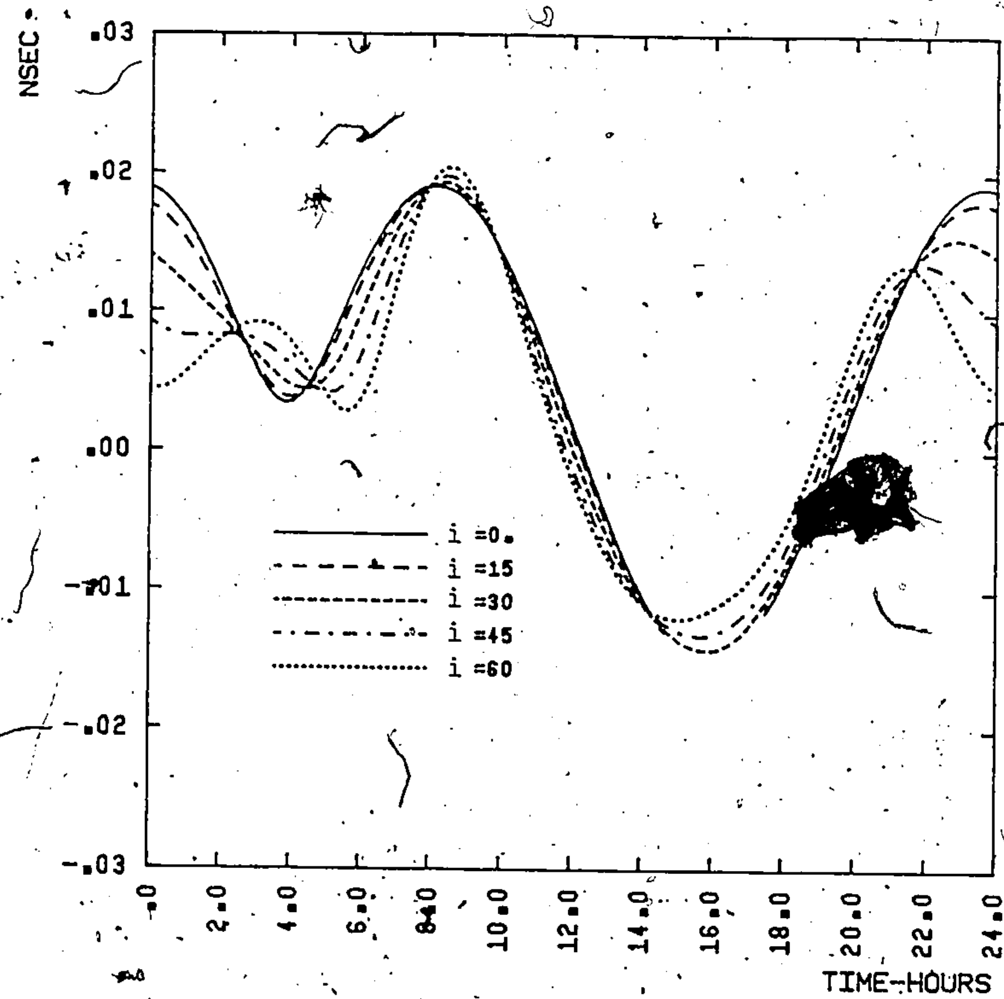

Fig. 5.28 Theoretical error in calculating the uplink delay between the geostationary satellite and a navigation satellite in 12 hour orbit with $\phi_{LongG} - \phi_{oN} = 45°$ and i = 0°, 15°, 30°, 45° and 60°.

3) Moving the geostationary satellite an angular distance corresponding to the true up-link space delay T_{12} we can find its location G_2 when the message was received by the navigation satellite at N_2. Thus, from Eq. (5.10) and (5.12) we can calculate the space delay T_{22} between G_2 and N_2.

4) Holding the navigation satellite fixed at N_2 we calculate:

a) The change in the space delay and the change in the change of the space delay from Eq. (5.17) and Eq. (5.24) respectively.

b) Using Taylor Series we can find the theoretical value for the downlink space delay T_{23th} from the relation:

$$T_{23th} = T_{22} + T_{22} \left. \frac{\partial T}{\partial t} \right|_{G_2, N = N_2} + \frac{T_{22}^2}{2!} \left. \frac{\partial^2 T}{\partial t^2} \right|_{G_2, N = N_2} \tag{5.84}$$

c) Moving the geostationary satellite from position G_2 a distance corresponding to T_{23th} and recalling Eq. (5.10) and Eq. (5.12) we can find the true space delay T_{23}.

d) Moving the geostationary satellite from G_2 a distance corresponding to T_{23}, we find the location of the geostationary satellite at G_3.

Now the information available on-board the geostationary satellite after the complete transmission loop $G_1 N_2 G_3$ is given by

1. The total space delay $T_t = T_{12} + T_{23}$ from the steps 2) c and 4) c.

2. The location of the navigation satellite at N_2 (which is the center of the sphere of error) from the step 2) d and the location of the geostationary satellite at G_1 and G_3 from steps 1) and 4) d using the same procedures, we find the precise value for the new uplink delay T_{34}.

Part 2

Now we start the second part of the program which is developed to estimate the new uplink delay T_{34E}. By knowing the location of G_1, the center of the sphere of error N_2 and its radius r we can calculate the estimate values for the uplink delay T_{12E} and the estimate downlink delay T_{23E} as follows:

1) The total space delay for round trip which is

$$T_t = T_{12E} + T_{23E} \tag{5.85}$$

2) Substituting the location N_2 and G_1 in Eq. (5.16) we can calculate true value of the change in the space delay if the navigation satellite is fixed at N_2 and G_1 is moving.

3) Substituting the locations of the geostationary satellite at G_1 and the navigation satellite at N_2 in Eqs. (5.62), (5.65) and (5.68) we can calculate the gradient components of the change in the space-delay if the navigation satellite is fixed at N_2 and the geostationary is at G_1 and moving.

4) From step 3 we can calculate the total gradient, and by knowing the radius of the sphere of error r, the maximum error ERR1 in calculating the change in the space delay can be found simply by multiplying the total gradient by the radius r.

5) By adding the maximum error ERR1 calculated from step 4 to the change in the space delay calculated from step 2 we can find the estimate value for the change in the space delay.

Substituting this value in Eq. (5.6) we can find the estimated downlink T_{23E}

$$T_{23E} = 0.5\, T_t \left| 1 + \frac{\partial \Gamma}{\partial t} \right|_{G_1, N = N_2} + \mathrm{EER1} \Big| \tag{5.86}$$

6) Repeat step 5 but this time subtract ERR1 from the change in the space delay. Find another value for T_{23E} and comparing the two values with the true space delay T_{23} calculated from step 2.c in the first part. We then select the value of T_{23E} which has the higher error.

7) Now, we start to calculate the estimated value of the new uplink delay T_{34E}. By substituting the location N_2 and G_3 in Eqs. (5.28), (5.76), (5.79) and (5.82), the change in the space delay and the components of the gradient can be calculated respectively if the geostationary satellite is fixed at G_3 and the navigation satellite is at N_2 moving.

8) The total gradient of step 7 can be calculated and the maximum error of the change in the space delay ERR2 equals the total gradient multiplied by the radius of the sphere of error r.

9) By adding ERR2 to the change in the space delay calculated from step 7 we can calculate the estimated value of the change in the space delay. Substituting this value and the estimated downlink delay T_{23E} calculated from step 5 in Eq. (5.9) we can calculate the estimated value of the new uplink delay T_{34E} which is given by:

$$T_{34E} = T_{23E} \frac{\left(1 + \left.\frac{\partial T}{\partial t}\right|_{G=G_3,N_2} + \text{ERR2}\right)}{\left(1 - \left.\frac{\partial T}{\partial t}\right|_{G=G_3,N_2} + \text{ERR2}\right)} \tag{5.88}$$

10) Repeating step 9 but now subtract ERR2 from the change in the space delay, we get another value for the estimated uplink delay T_{34E}. Comparing the two values of T_{34E} with the precise value (part 1), we choose the value of T_{34E} which has the higher error.

The highest, the lowest and the average values of the maximum error (in ns) in the estimated value of the uplink delay T_{34E} occurring over a period of 24 hours are tabulated in Tables 5.1 to 5.6. We assume a navigation satellite located in a 12 hour orbit with TT = 0, and five different values of the inclination angle i. Also we assume five different values of the angle $\phi_{LongG} - \phi_{oN}$ and six different values of the radius r for the sphere of error (1 km, 2 km, 5 km, 10 km, 50 km and 100 km).

Tables 5.7 to 5.12 document the same information for a navigation satellite in 16 hour orbit. Here the errors are calculated over a period of 48 hours to cover two complete revolutions by the geostationary satellite and three complete revolutions by the navigation satellite.

Figure 5.29 is plotted for the maximum values of the maximum error versus the radius r of the sphere of error. It is clear that Fig. 5.29 presents the design chart for the maximum possible error (occurring in practical) in estimating the new uplink delay T_{34E}

5.7 CONCLUSIONS

This report examines the possibility of establishing accurate timing on board a navigation satellite in inclined orbit using a timing reference from an earth station. By using a geostationary satellite as an intermediate link between the earth station and the navigation satellite, an effective method for predicting the uplink and downlink delays between the two satellites has been established. This method depends upon estimating the location of the navigation satellite in the space within a sphere of different radii. Such a method has the advantages of easy implementation and high degree of accuracy.

Based on the results presented in the preceding sections, we can conclude that:

1. For navigation satellites at lower altitude than the synchronous satellites, a geostationary satellite is capable of observing the navigation satellite throughout its entire orbit [28,33,35].
2. An accuracy on the order of few nanoseconds or much less, can be achieved for predicting the uplink and downlink delays by using this method.
3. The accuracy of the predicted uplink downlink delays can be improved by estimating the location of the navigation satellite within a smaller sphere, which is available in practice [36,37,38].

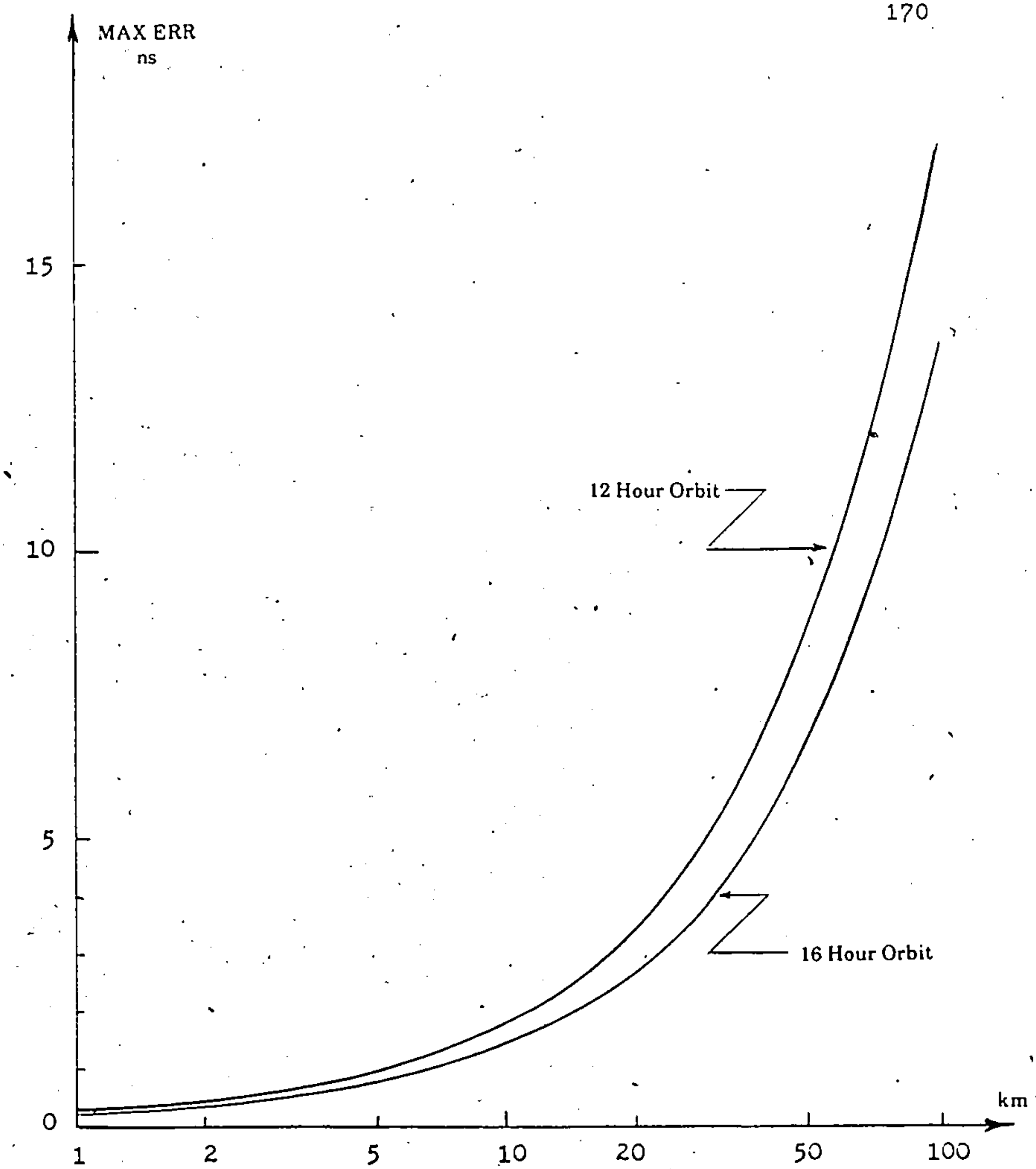

Fig. 5.29 The maximum values of the maximum error in calculating the new uplink delay versus the sphere of error radius r in Km

4. Now, since accurate timing can be established on board the navigation satellite, the need for the fragile atomic clock on board the satellites can be eliminated.

12 Hour Orbit Navigation Satellite and Sphere of Error Radius = 1 km

$\phi_{LongG}-\phi_{oN}$	Inc. Angle i	MAX ERR ns	MIN ERR ns	AVER ERR ns
0.0°	0.0°	0.2658	0.1139	0.1662
	15.0°	0.2670	0.1105	0.1647
	30.0°	0.2627	0.1021	0.1559
	45.0°	0.2620	0.0904	0.1437
	60.0°	0.2633	0.0815	0.1314
15.0°	0.0°	0.2663	0.1143	0.1652
	15.0°	0.2630	0.1117	0.1642
	30.0°	0.2589	0.1065	0.1602
	45.0°	0.2419	0.1013	0.1496
	60.0°	0.1993	0.0996	0.1337
30.0°	0.0°	0.2667	0.1146	0.1657
	15.0°	0.2604	0.1127	0.1655
	30.0°	0.2456	0.1114	0.1652
	45.0°	0.2071	0.1051	0.1539
	60.0°	0.1859	0.0838	0.1403
45.0°	0.0°	0.2672	0.1139	0.1659
	15.0°	0.2621	0.11354	0.1660
	30.0°	0.2405	0.11352	0.1657
	45.0°	0.2232	0.0979	0.1548
	60.0°	0.2210	0.0739	0.1419
60.0°	0.0°	0.2676	0.1132	0.1662
	15.0°	0.2650	0.1130	0.1643
	30.0°	0.2575	0.1112	0.1652
	45.0°	0.2513	0.0925	0.1540
	60.0°	0.2502	0.0703	0.1404

Table 5.1

12 Hour Orbit Navigation Satellite and Sphere of Error Radius = 2 km

$\phi_{LongG}-\phi_{oN}$	Inc. Angle i	MAX ERR ns	MIN ERR ns	AVER ERR ns
0.0°	0.0°	0.4337	0.2488	0.3041
	15.0°	0.4348	0.2439	0.3013
	30.0°	0.4287	0.2312	0.2888
	45.0°	0.4274	0.2054	0.2701
	60.0°	0.4290	0.1680	0.2487
15.0°	0.0°	0.4343	0.2486	0.3031
	15.0°	0.4288	0.2472	0.3013
	30.0°	0.4213	0.2395	0.2943
	45.0°	0.3961	0.2204	0.2754
	60.0°	0.3219	0.1866	0.2471
30.0°	0.0°	0.4349	0.2487	0.3032
	15.0°	0.4246	0.2489	0.3022
	30.0°	0.4006	0.2495	0.2962
	45.0°	0.3425	0.2130	0.2767
	60.0°	0.3253	0.1734	0.2534
45.0°	0.0°	0.4354	0.2489	0.3034
	15.0°	0.4269	0.2479	0.3023
	30.0°	0.3934	0.2453	0.2969
	45.0°	0.3739	0.1939	0.2781
	60.0°	0.3720	0.1584	0.2573
60.0°	0.0°	0.4360	0.2490	0.3039
	15.0°	0.4318	0.2440	0.3003
	30.0°	0.4200	0.2288	0.2973
	45.0°	0.4125	0.1935	0.2803
	60.0°	0.4113	0.1590	0.2617

Table 5.2

12 Hour Orbit Navigation Satellite and Sphere of Error Radius = 5 km

$\phi_{LongG}-\phi_{oN}$	Inc. Angle i	MAX ERR ns	MIN ERR ns	AVER ERR ns
0.0°	0.0°	0.9375	0.6594	0.7481
	15.0°	0.9385	0.6535	0.7412
	30.0°	0.9267	0.6161	0.7134
	45.0°	0.9235	0.5370	0.6698
	60.0°	0.9262	0.4275	0.6172
15.0°	0.0°	0.9384	0.6594	0.7467
	15.0°	0.9262	0.6495	0.7409
	30.0°	0.9085	0.6116	0.7208
	45.0°	0.8587	0.5446	0.6769
	60.0°	0.7840	0.4471	0.6174
30.0°	0.0°	0.9393	0.6594	0.7463
	15.0°	0.9173	0.6517	0.7424
	30.0°	0.8657	0.6287	0.7253
	45.0°	0.7485	0.5929	0.6839
	60.0°	0.7500	0.5036	0.6326
45.0°	0.0°	0.9401	0.6595	0.7464
	15.0°	0.9213	0.6547	0.7432
	30.0°	0.8563	0.6400	0.7287
	45.0°	0.8298	0.5831	0.6902
	60.0°	0.8289	0.4653	0.6413
60.0°	0.0°	0.9401	0.6595	0.7470
	15.0°	0.9321	0.6494	0.7406
	30.0°	0.9091	0.6145	0.7304
	45.0°	0.8978	0.5540	0.6930
	60.0°	0.8969	0.4534	0.6444

Table 5.3

12 Hour Orbit Navigation Satellite and Sphere of Error Radius = 10 km

$\phi_{LongG}-\phi_{oN}$	Inc. Angle i	MAX ERR ns	MIN ERR ns	AVER ERR ns
0.0°	0.0°	1.7770	1.3220	1.4956
	15.0°	1.7779	1.3136	1.4818
	30.0°	1.7567	1.2482	1.4262
	45.0°	1.7504	1.0875	1.3401
	60.0°	1.7549	0.8598	1.2349
15.0°	0.0°	1.7785	1.3220	1.4935
	15.0°	1.7551	1.3031	1.4807
	30.0°	1.7206	1.2255	1.4377
	45.0°	1.6298	1.0835	1.3531
	60.0°	1.5956	0.8811	1.2426
30.0°	0.0°	1.7799	1.3220	1.4931
	15.0°	1.7384	1.3052	1.4842
	30.0°	1.6409	1.2532	1.4537
	45.0°	1.5372	1.1680	1.3732
	60.0°	1.5390	0.9820	1.2722
45.0°	0.0°	1.7813	1.3221	1.4929
	15.0°	1.7453	1.3129	1.4865
	30.0°	1.6310	1.2890	1.4575
	45.0°	1.5924	1.2354	1.3807
	60.0°	1.5935	0.9768	1.2828
60.0°	0.0°	1.7827	1.3221	1.4935
	15.0°	1.7661	1.3030	1.4818
	30.0°	1.7251	1.2383	1.4579
	45.0°	1.7077	1.1288	1.3810
	60.0°	1.7069	0.9318	1.2822

Table 5.4

12 Hour Orbit Navigation Satellite and Sphere of Error Radius = .50 km

$\Phi_{LongG}-\Phi_{oN}$	Inc. Angle i	MAX ERR ns	MIN ERR ns	AVER ERR ns
0.0°	0.0°	8.5409	6.6142	7.4757
	15.0°	8.5409	6.5836	7.4069
	30.0°	8.5409	6.2937	7.1290
	45.0°	8.4930	5.4898	6.7028
	60.0°	8.4930	4.3189	6.1764
15.0°	0.0°	8.5073	6.6144	7.4677
	15.0°	8.4355	6.5230	7.3984
	30.0°	8.2969	6.1300	7.1731
	45.0°	8.3026	5.3908	6.7622
	60.0°	8.3099	4.3534	6.2440
30.0°	0.0°	8.5193	6.6151	7.4682
	15.0°	8.3355	6.5259	7.4186
	30.0°	8.0753	6.2407	7.2806
	45.0°	8.0230	5.7663	6.8876
	60.0°	8.0278	4.8099	6.3897
45.0°	0.0°	8.5287	6.6148	7.4647
	15.0°	8.3374	6.5772	7.4327
	30.0°	7.8401	6.4790	7.2878
	45.0°	7.7042	6.3661	6.9041
	60.0°	7.7162	5.0687	6.4151
60.0°	0.0°	8.5355	6.6145	7.4655
	15.0°	8.4376	6.5224	7.4121
	30.0°	8.2580	6.2276	7.2784
	45.0°	8.1938	5.7253	6.8847
	60.0°	8.1966	4.7596	6.3845

Table 5.5

12 Hour Orbit Navigation Satellite and Sphere of Error Radius = 100 km

$\Phi_{LongG}-\Phi_{oN}$	Inc. Angle i	MAX ERR ns	MIN ERR ns	AVER ERR ns
0.0°	0.0°	17.0578	13.2287	14.9509
	15.0°	17.0578	13.1686	14.8133
	30.0°	17.0578	12.5986	14.2574
	45.0°	17.0099	10.9928	13.4060
	60.0°	17.0099	8.6427	12.3533
15.0°	0.0°	17.0073	13.2287	14.9355
	15.0°	16.8957	13.0479	14.7956
	30.0°	16.6918	12.2607	14.3424
	45.0°	16.7037	10.7743	13.5237
	60.0°	16.7186	8.6938	12.4958
30.0°	0.0°	17.0148	13.2297	14.9369
	15.0°	16.6643	13.0501	14.8367
	30.0°	16.2419	12.4749	14.5643
	45.0°	16.1314	11.5122	13.7805
	60.0°	16.1401	9.5946	12.7866
45.0°	0.0°	17.0336	13.2305	14.9294
	15.0°	16.5793	13.1553	14.8655
	30.0°	15.6034	12.9637	14.5756
	45.0°	15.3441	12.7489	13.8084
	60.0°	15.3719	10.1835	12.8305
60.0°	0.0°	17.0471	13.2294	14.9305
	15.0°	16.7770	13.0463	14.8250
	30.0°	16.4246	12.4616	14.5539
	45.0°	16.3022	11.4710	13.7643
	60.0°	16.3088	9.5443	12.7623

Table 5.6

16 Hour Orbit Navigation Satellite and Sphere of Error Radius = 1 km

$\phi_{LongG}-\phi_{oN}$	Inc. Angle i	MAX ERR ns	MIN ERR ns	AVER ERR ns
0.0°	0.0°	0.1999	0.0838	0.1175
	15.0°	0.1992	0.0837	0.1180
	30.0°	0.1968	0.0835	0.1153
	45.0°	0.1964	0.0693	0.1089
	60.0°	0.1957	0.0562	0.1009
15.0°	0.0°	0.2003	0.0839	0.1177
	15.0°	0.1973	0.0827	0.1190
	30.0°	0.1885	0.0768	0.1173
	45.0°	0.1728	0.0707	0.1098
	60.0°	0.1749	0.0574	0.1005
30.0°	0.0°	0.2005	0.0837	0.1177
	15.0°	0.1976	0.0824	0.1180
	30.0°	0.1911	0.0796	0.1189
	45.0°	0.1902	0.0760	0.1123
	60.0°	0.1911	0.0552	0.1039
45.0°	0.0°	0.1999	0.0838	0.1174
	15.0°	0.2001	0.0834	0.1171
	30.0°	0.2003	0.0829	0.1152
	45.0°	0.2007	0.0766	0.1108
	60.0°	0.2010	0.0561	0.1034
60.0°	0.0°	0.2003	0.08385	0.1176
	15.0°	0.1996	0.08381	0.1180
	30.0°	0.1974	0.0835	0.1153
	45.0°	0.1973	0.0692	0.1088
	60.0°	0.1980	0.0562	0.1009

Table 5.7

16 Hour Orbit Navigation Satellite and Sphere of Error Radius = 2 km

$\phi_{LongG}-\phi_{oN}$	Inc. Angle i	MAX ERR ns	MIN ERR ns	AVER ERR ns
0.0°	0.0°	0.3344	0.1755	0.2213
	15.0°	0.3330	0.1786	0.2222
	30.0°	0.3290	0.1723	0.2193
	45.0°	0.3283	0.1548	0.2103
	60.0°	0.3273	0.1288	0.1966
15.0°	0.0°	0.3348	0.1755	0.2212
	15.0°	0.3296	0.1771	0.2237
	30.0°	0.3157	0.1799	0.2216
	45.0°	0.2966	0.1578	0.2097
	60.0°	0.2995	0.1242	0.1947
30.0°	0.0°	0.3352	0.1755	0.2213
	15.0°	0.3304	0.1767	0.2217
	30.0°	0.3216	0.1786	0.2216
	45.0°	0.3207	0.1606	0.2119
	60.0°	0.3219	0.1347	0.1984
45.0°	0.0°	0.3344	0.1755	0.2210
	15.0°	0.3346	0.1765	0.2201
	30.0°	0.3349	0.1745	0.2173
	45.0°	0.3354	0.1638	0.2112
	60.0°	0.3359	0.1397	0.1990
60.0°	0.0°	0.3348	0.1755	0.2211
	15.0°	0.3335	0.1786	0.2221
	30.0°	0.3300	0.1724	0.2190
	45.0°	0.3297	0.1548	0.2100
	60.0°	0.3308	0.1288	0.1963

Table 5.8

16 Hour Orbit Navigation Satellite and Sphere of Error Radius = 5 km

$\Phi_{LongG}-\Phi_{oN}$	Inc. Angle i	MAX ERR ns	MIN ERR ns	AVER ERR ns
0.0°	0.0°	0.7378	0.4401	0.5535
	15.0°	0.7343	0.4490	0.5558
	30.0°	0.7258	0.4378	0.5483
	45.0°	0.7241	0.4051	0.5259
	60.0°	0.7222	0.3458	0.4917
15.0°	0.0°	0.7385	0.4401	0.5533
	15.0°	0.7266	0.4444	0.5570
	30.0°	0.6972	0.4500	0.5509
	45.0°	0.6686	0.3991	0.5255
	60.0°	0.6742	0.3243	0.4901
30.0°	0.0°	0.7391	0.4401	0.5531
	15.0°	0.7289	0.4433	0.5546
	30.0°	0.7135	0.4524	0.5538
	45.0°	0.7125	0.4143	0.5296
	60.0°	0.7151	0.3376	0.4961
45.0°	0.0°	0.7378	0.04401	0.5523
	15.0°	0.7381	0.4435	0.5528
	30.0°	0.7387	0.4442	0.5462
	45.0°	0.7395	0.4050	0.5266
	60.0°	0.7405	0.3413	0.4940
60.0°	0.0°	0.7385	0.4401	0.5530
	15.0°	0.7353	0.4490	0.5549
	30.0°	0.7277	0.4379	0.5476
	45.0°	0.7269	0.4050	0.5248
	60.0°	0.7291	0.3456	0.4911

Table 5.9

16 Hour Orbit Navigation Satellite and Sphere of Error Radius = 10 km

$\Phi_{LongG}-\Phi_{oN}$	Inc. Angle i	MAX ERR ns	MIN ERR ns	AVER ERR ns
0.0°	0.0°	1.4102	0.8805	1.1071
	15.0°	1.4032	0.8988	1.1116
	30.0°	1.3872	0.8797	1.0967
	45.0°	1.3838	0.8222	1.0518
	60.0°	1.3804	0.7064	0.9835
15.0°	0.0°	1.4114	0.8805	1.1068
	15.0°	1.3884	0.8889	1.1126
	30.0°	1.3331	0.8984	1.0996
	45.0°	1.2889	0.8012	1.0517
	60.0°	1.2989	0.6571	0.9825
30.0°	0.0°	1.4125	0.8806	1.1061
	15.0°	1.3930	0.8871	1.1095
	30.0°	1.3671	0.9058	1.1075
	45.0°	1.3657	0.8370	1.0591
	60.0°	1.3704	0.6758	0.9922
45.0°	0.0°	1.4102	0.8805	1.1046
	15.0°	1.4105	0.8882	1.1072
	30.0°	1.4116	0.8924	1.0943
	45.0°	1.4131	0.8070	1.0522
	60.0°	1.4148	0.6743	0.9858
60.0°	0.0°	1.4114	0.8805	1.1060
	15.0°	1.4050	0.8987	1.1095
	30.0°	1.3906	0.8796	1.0954
	45.0°	1.3888	0.8221	1.0494
	60.0°	1.3929	0.7063	0.9823

Table 5.10

16 Hour Orbit Navigation Satellite and Sphere of Error Radius = 50 km .

$\phi_{LongG}-\phi_{oN}$	Inc. Angle i	MAX ERR ns	MIN ERR ns	AVER ERR ns
0.0°	0.0°	6.8043	4.4033	5.5358
	15.0°	6.8042	4.4948	5.5585
	30.0°	6.8042	4.4135	5.4839
	45.0°	6.8042	4.1580	5.2595
	60.0°	6.8042	3.5909	4.9180
15.0°	0.0°	6.8074	4.4030	5.5348
	15.0°	6.7145	4.4440	5.5574
	30.0°	6.6592	4.4813	5.4893
	45.0°	6.6654	4.0182	5.2620
	60.0°	6.6718	3.3199	4.9219
30.0°	0.0°	6.8190	4.4032	5.5300
	15.0°	6.7058	4.4373	5.5485
	30.0°	6.5955	4.5316	5.5373
	45.0°	6.5920	4.2185	5.2954
	60.0°	6.6130	3.3814	4.9612
45.0°	0.0°	6.8229	4.4033	5.5226
	15.0°	6.7905	4.4425	5.5422
	30.0°	6.7952	4.4760	5.4793
	45.0°	6.8019	4.0229	5.2573
	60.0°	6.8092	3.3387	4.9196
60.0°	0.0°	6.8074	4.4030	5.5305
	15.0°	6.8063	4.4952	5.5467
	30.0°	6.8019	4.4137	5.4774
	45.0°	6.7964	4.1589	5.2462
	60.0°	6.8102	3.5919	4.9122

Table 5.11

16 Hour Orbit Navigation Satellite and Sphere of Error Radius = 100 km

$\phi_{LongG}-\phi_{oN}$	Inc. Angle i	MAX ERR ns	MIN ERR ns	AVER ERR ns
0.0°	0.0°	13.6179	8.8067	11.0718
	15.0°	13.6178	8.9895	11.1170
	30.0°	13.6178	8.8304	10.9679
	45.0°	13.6178	8.3278	10.5192
	60.0°	13.6179	7.1966	9.8360
15.0°	0.0°	13.6101	8.8061	11.0698
	15.0°	13.4400	8.8873	11.1133
	30.0°	13.3838	8.9597	10.9764
	45.0°	13.3964	8.0395	10.5248
	60.0°	13.4094	6.6483	9.8461
30.0°	0.0°	13.6287	8.8065	11.0598
	15.0°	13.3468	8.8749	11.0973
	30.0°	13.1313	9.0638	11.0745
	45.0°	13.1253	8.4455	10.5907
	60.0°	13.1661	6.7634	9.9224
45.0°	0.0°	13.6364	8.8067	11.0450
	15.0°	13.5155	8.8853	11.0860
	30.0°	13.5247	8.9846	10.9607
	45.0°	13.5378	8.0429	10.5137
	60.0°	13.5522	6.6692	9.8369
60.0°	0.0°	13.6101	8.8061	11.0610
	15.0°	13.6095	8.9902	11.0931
	30.0°	13.6073	8.8313	10.9550
	45.0°	13.6022	8.3296	10.4922
	60.0°	13.6112	7.1990	9.8245

Table 5.12

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

6.1 Conclusions

This thesis presents a theoretical basis for establishing accurate timing on board a navigation satellite in a dynamic satellite constellation in which the navigation satellite is located in any inclined circular orbit. The theoretical basis not only eliminates the need for computer modelling but also corrects some previously published mathematical and computer results. In addition, all the implemented equations are easy to handle, widely applicable and in terms of one reference time and initial location conditions of the satellites.

Specifically, the thesis provides the following contributions:

1) A detailed study of the satellite subpoint trace (S.S.T.) for any satellite located in any circular orbit has been presented in a new analytical form. All the specifications of S.S.T. are studied and expressed mathematically in terms of the satellite initial conditions. Computer plots are used to verify the mathematical forms.

2) A new analytical expression of the instantaneous coverage area (I.C.A.) for any satellite located in any circular orbit. We show how such an expression can be widely implemented for solving the problems of either single-fold coverage or multi-fold coverage. Also, by using a simple optimization technique it is demonstrated that the coverage pattern can be sized and measured to determine whether any holes in the coverage pattern exist.

3) A new theoretical method for calculating the space delays for two cases. In the first case, the space delay between a navigation satellite and an earth station is calculated while in the second case the earth station is replaced by a geostationary

satellite. The method is supported by computer plots in order to show the advantages and disadvantages in each case.

4) A new theoretical expression for the derivatives of the change in the space delay, formulated in a suitable format for higher order differentiation.

5) A new theoretical definition for the blind segments on the geostationary orbit. Outside these blind segments are other segments which guarantee the existence of a continuous communications link with any navigation satellite in inclined orbit. Also a theoretical expression is developed for the time period of which the communications is blocked if the geostationary satellite is located in a blind segment.

6) A new theoretical analysis for establishing accurate timing on board a navigation satellite using timing reference from either an earth station or geostationary satellite. This method depends only upon estimating the location of the navigation satellite within spheres of different radii and shows a high degree of accuracy in establishing timing at the navigation satellite. Such a method has the advantage of eliminating the fragile atomic clocks on board the navigation satellite.

6.2 Recommendations for Future Research

It is recommended that:

1) The design of a geostationary satellite network which can view all navigation satellites simultaneously be investigated.

2) The coverage provided by the geostationary satellites and the navigation satellites be evaluated.

3) A method for accurately locating navigation satellites, with respect to geostationary satellites, by using the precisely determined space delays be developed.

4) The problem of passing control of navigation satellites to different geostationary satellites be studied.

5) The cost of this system as compared to the NAVSTAR system be determined.

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APPENDIX 1

COVERAGE TIME PROVIDED BY A SINGLE AND MULTIPLE BEAM ANTENNA ASSUMING A SATELLITE FROM SARSAT SYSTEM

Here we are going to give another example of how to apply the equation of the instantaneous coverage area derived in Section 3.3; for the case of implementing multiple beam antenna on board a satellite instead of using single beam antenna and we will show what will be the effect upon the coverage time provided by the satellite for a specific point on the earth if the multiple beam antenna is rotated around its axis.

In this example we assume a satellite from the Search And Rescue by Satellite Aided Tracking system (SARSAT). Basically the system operates with a number of satellites in low polar orbits (altitude 860 km).

If a distressed aircraft is equipped with an Emergency Locator Transmitter (ELT), a transmitted signal will be picked up by the approaching satellite.

Substituting the altitude of the SARSAT satellite in Eq. (3.5) the nonsynchronous factor K can be calculated, which will be equal to 12.522. This value for the nonsynchronous factor K corresponds to an orbital period of almost 115 min. Substituting the value of K in Eq. (3.6) and assuming $\sigma = 0$, the half arc observed angle ψ will be equal to 35.34° which is the angle covered by the satellite single beam antenna.

By calling Eq. (2.10), Eq. (2.14) and Eq. (2.15) which describe the SST; and by substituting for the inclination angle $i = 90°$ assuming that at $t = 0$ satellite subpoint is located on the equator at the point with longitude equals zero, therefore, $TT = 0$ and $\phi_0 = 0$. Thus the equation representing the SST will be given by

$$\phi_{Lat} = K\,\omega_E t \tag{A1.1}$$

$$\phi_{Long} = -\omega_E t \qquad -\frac{\pi}{2} > K\,\omega_E t > \frac{\pi}{2} \tag{A1.2}$$

$$\phi_{Long} = \pi - \omega_E t \qquad \frac{3\pi}{2} > K\,\omega_E t > \frac{\pi}{2} \tag{A1.3}$$

Figure A1.1 shows the SST of this specific case (which is a linear relation between ϕ_{Lat} and ϕ_{Long}) for one revolution by the satellite around the earth.

It is clear that during the motion of the satellite in its orbit; the satellite subpoint on the surface of the earth will be the center of the antenna footprint, therefore, the satellite subpoint coordinates should be defined at any instant of time which is given by Eqs. (A1.1), (A1.2) and (A1.3).

Assuming that we are going to replace the single beam antenna by four subantennas. Their footprints can be represented by the shape shown in Fig. A1.2 which shows that each subantenna footprint will have half arc observed angle equals $\psi/2$. While the satellite moves in its orbit, the footprints of its subantennas are moving on the surface of the earth. The shape and the size of the footprint should always be kept fixed during its motion. Assuming the satellite subpoint is S' (which is the center of the footprint of a single antenna) we should find the location of S_1', S_2', S_3' and S_4' which are the centers of the subantennas footprints at any instant of time as a function of the satellite subpoint S'. This can be done as follows.

Suppose that we want to find the location of the center S_1', i.e., to find its latitude and longitude ϕ_{Lat1e} and ϕ_{Long1e} respectively. First, we should establish the location of the point S'. Second, knowing that the subantenna footprints are located symmetrically around S' with half arc angle $\psi/2$ and for the specific shape given by Fig. A1.2 the angle between the tangent drawn at S' to the arc $S'S_1'$ and the north direction will be equal to 45° as shown in Fig. A1.3. Third, by solving the spherical triangle $S'NS_1'$ in Fig. A1.3 whose sides are arcs from great circle we get:

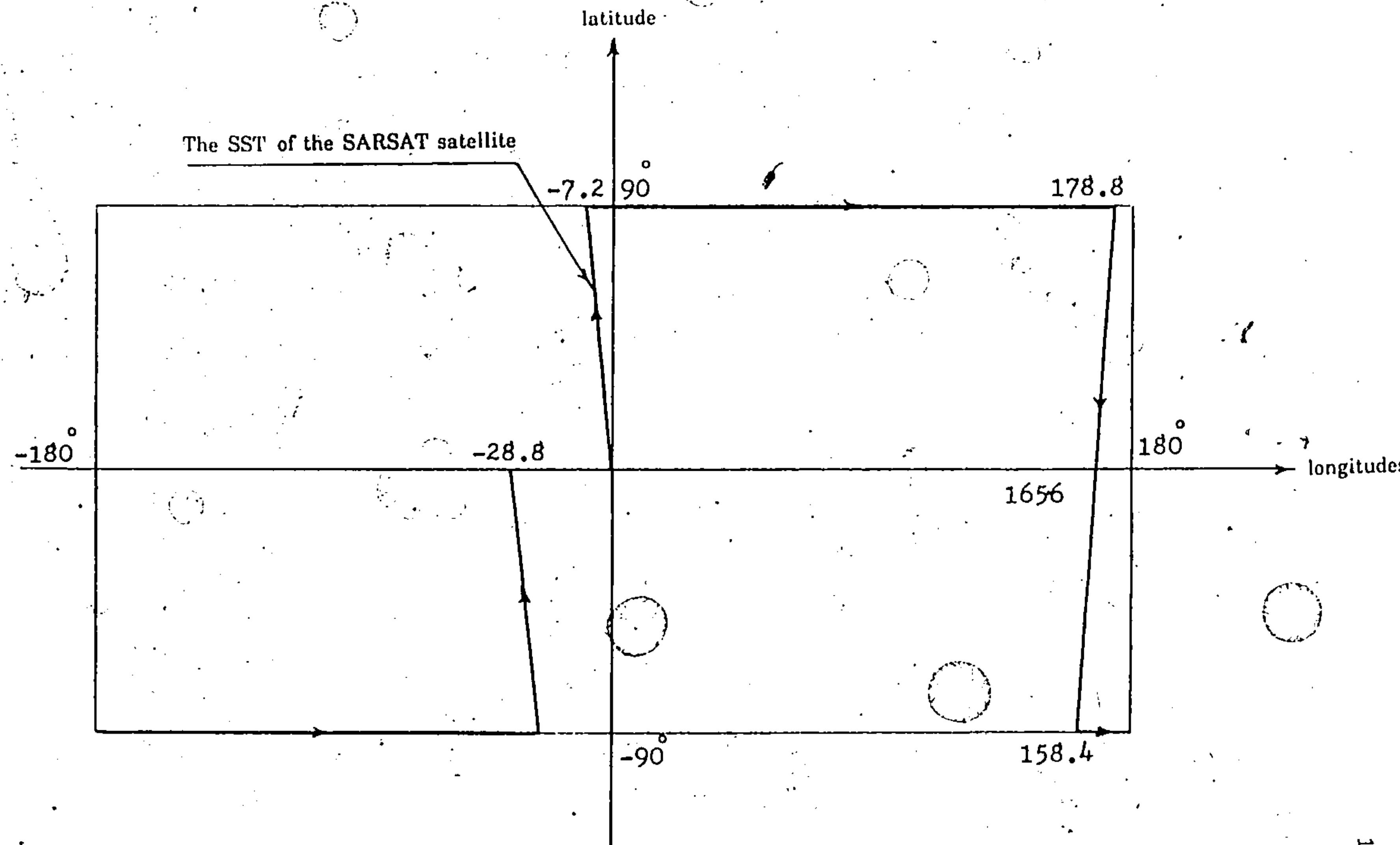

Fig. A1.1 The SST (one revolution) by a SARSAT satellite.

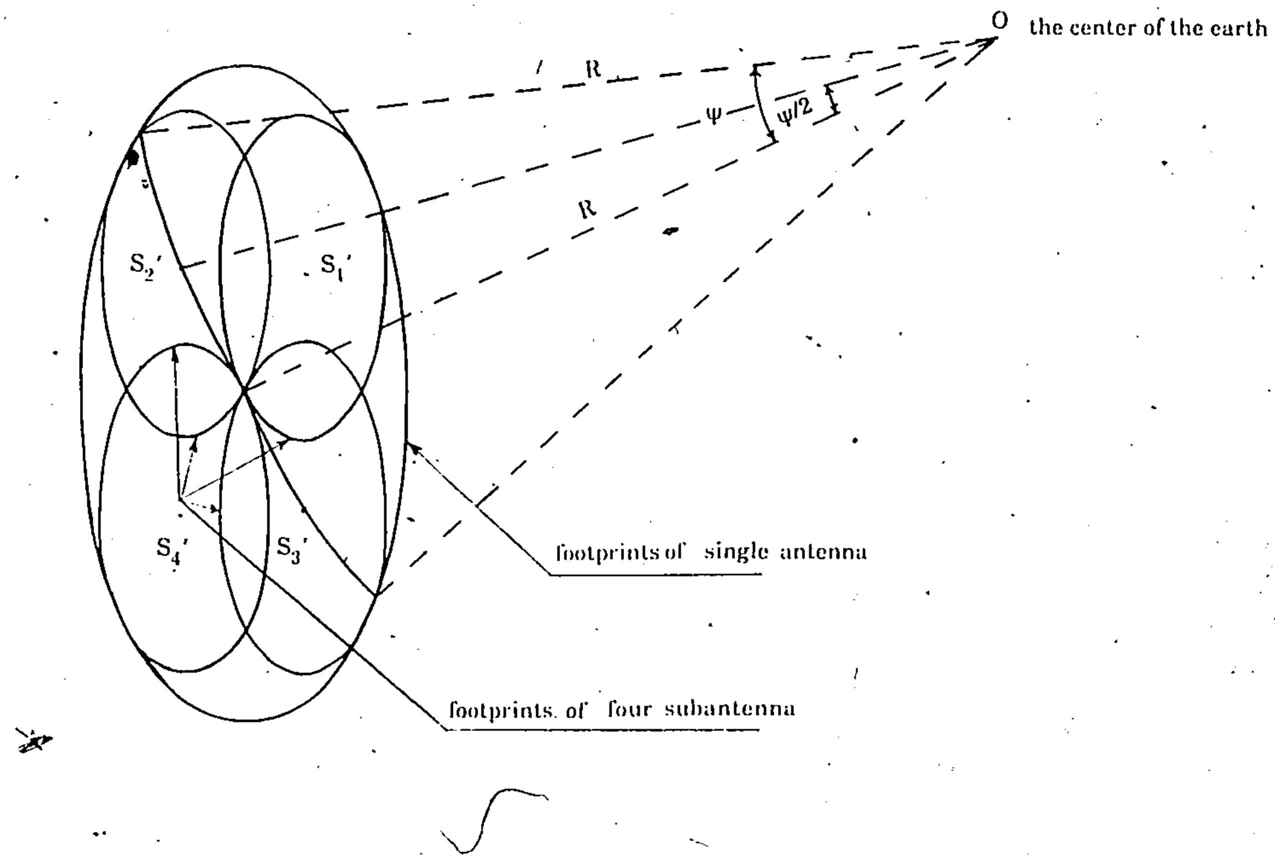

Fig. A1.2 Geometric relation between the single antenna and four subantenna footprints.

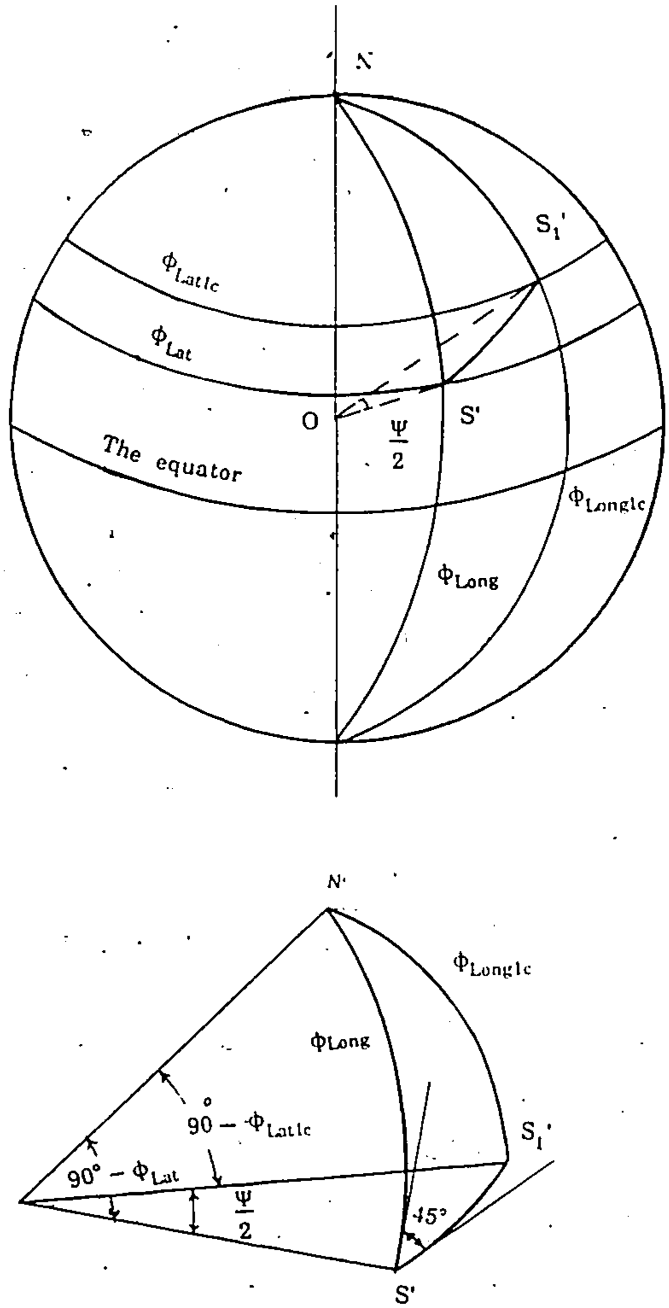

Fig. A1.3 Geometric relation between the centers of the single antenna and the subantenna footprints.

$$\sin \phi_{Lat1c} = \sin \phi_{Lat} \cos \frac{\psi}{2} + \cos \phi_{Lat} \sin \frac{\psi}{2} \cos 45 \qquad (A1.4)$$

From the last equation, the latitude ϕ_{Lat1c} can be calculated. Fourth, in order to find ϕ_{Long1c} we form a semisphere located symmetrically around S' and its half arc measured from the center of the earth equals $\psi/2$, thus the points S_1', S_2', S_3' and S_4' must be located on the boundary of this semisphere. By substituting in Eq. (3.14) by the value $\psi/2$ instead of ψ and ϕ_{Lat1c}, this yields to

$$\cos(\frac{\psi}{2}) = \cos \phi_{Lat} \cos \phi_{Lat1c} \cos(\phi_{Long} - \phi_{Long1c}) + \sin \phi_{Lat} \sin \phi_{Lat1c} \qquad (A1.5)$$

From the last Equation we can calculate the difference in longitude $\phi_{Long} - \phi_{Long1c} = \pm$ Diff, therefore,

$$\phi_{Long1c} = \phi_{Long} + \text{Diff} \qquad (A1.6)$$

We choose the positive sign because the direction of the tangent is positive.

From the symmetric shape of the subantenna footprint we can say that the center coordinates of the second subantenna footprint S_2' will be:

$$\phi_{Lat2c} = \phi_{Lat1c} \qquad (A1.7)$$

and

$$\phi_{Long2c} = \phi_{Long} + \text{Diff} \qquad (A1.8)$$

Repeating the same procedure in order to evaluate ϕ_{Lat3c} but in such a case the angle between the tangent and the north direction will be 135°, thus, replacing the 45° tangent angle in Eq. (A1.4) by 135° we can calculate ϕ_{Lat3c} which equals ϕ_{Lat4c}, then substituting ϕ_{Lat3c} in Eq. (A1.5) instead of ϕ_{Lat1c} we can find ϕ_{Long3c} and ϕ_{Long4c}.

Figures A1.4.a, A1.4.b and A1.4.c show the footprints for both a single antenna and the four subantennas at time equals 0, 4 and 8 minutes after the satellite crossed the equatorial orbit at $\phi_o' = 0$.

Now S_1', S_2', S_3' and S_4' are well defined points on the surface of the earth at any instant t; hence; the Equation which represents the boundary of the footprint for any

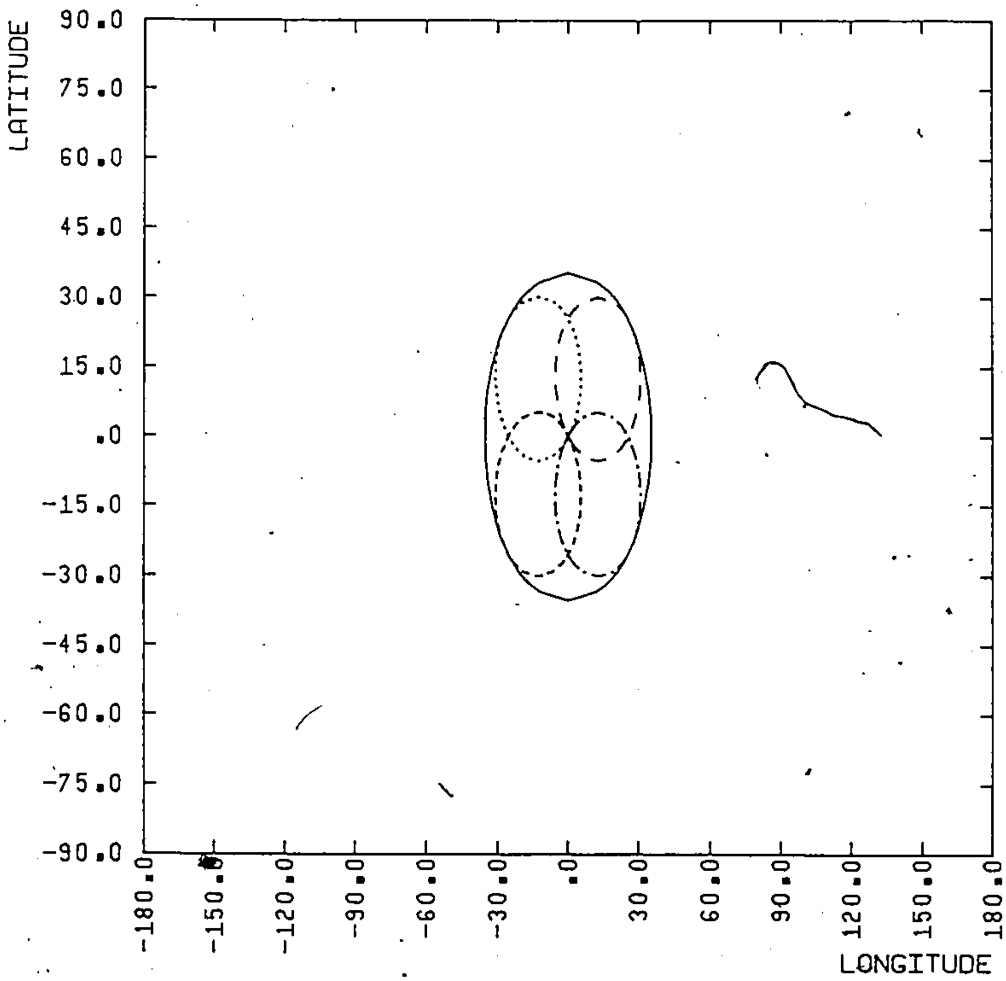

Fig A1.4. (a) The footprints for both a single antenna and the four subantennas at $t = 0$

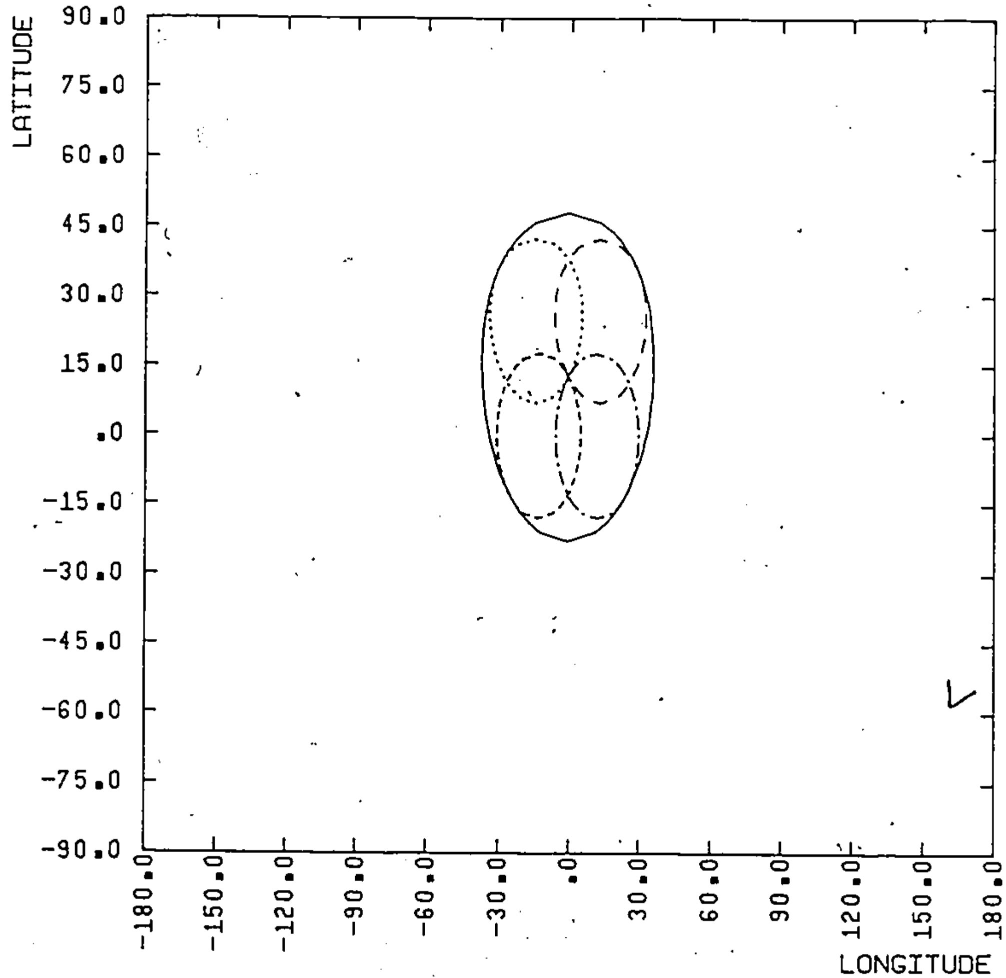

Fig. A1.4 (b) t = 4 min

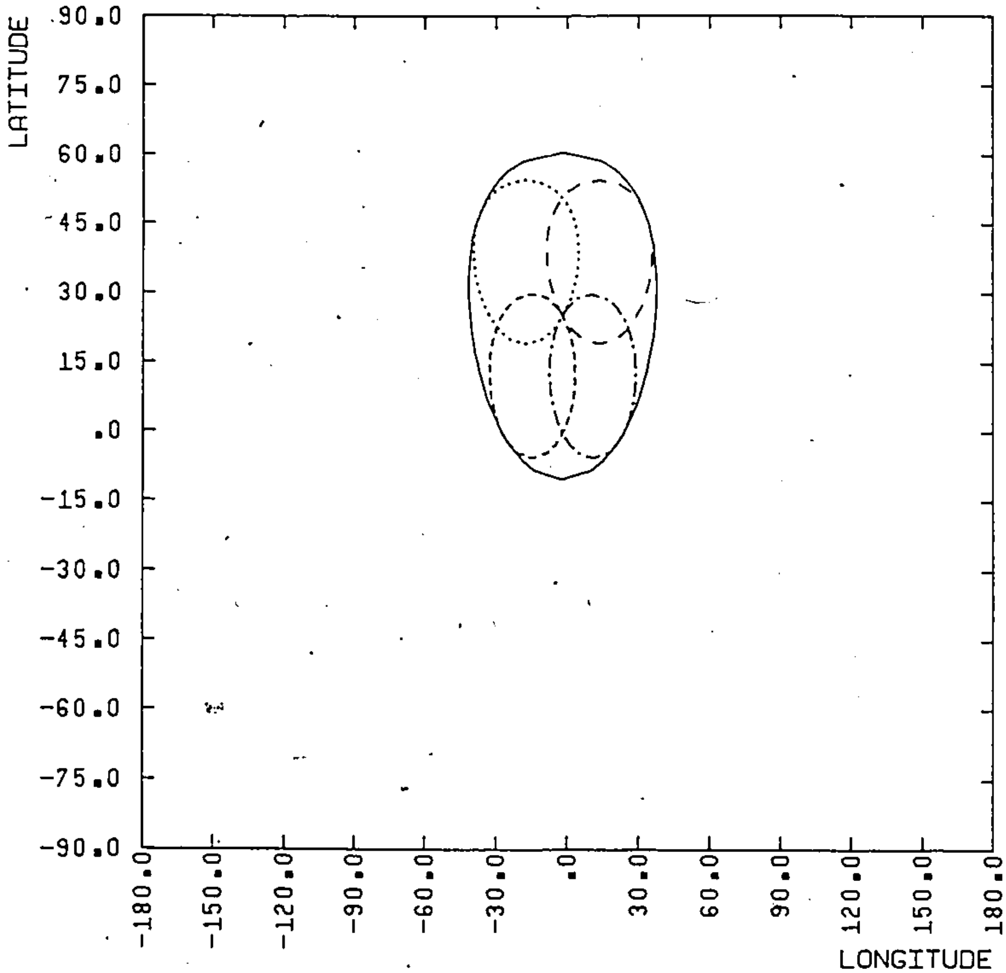

Fig. A1.4 (c) t = 8 min

subantenna will be given by Eq. (A1.5) as follows:

$$\cos(\frac{\Psi}{2}) = \cos\phi_{Latjc}\cdot\cos\phi_{Latb}\cos(\phi_{Longjc} - \phi_{Longb}) + \sin\phi_{Latjc}\sin\phi_{Latb}\,; \qquad j = 1, 2, 3, 4. \tag{A1.9}$$

where ϕ_{Latb} and ϕ_{Longb} are the latitude and longitude of any point on the jth footprint boundary respectively.

For any point located inside the jth footprint (the latitude and longitude of this point, namely ϕ_{Lati} and ϕ_{Longi}) must satisfy Eq. (3.16) which is

$$\cos\phi_{Latjc}\cos\phi_{Lati}\cos(\phi_{Longjc} - \phi_{Longi}) + \sin\phi_{Latjc}\sin\phi_{Lati}$$

$$- \cos(\frac{\Psi}{2}) \geq 0\,; \qquad j = 1, 2, 3, 4. \tag{A1.10}$$

Thus, the footprints of both the single antenna and the four pattern antenna, are well defined zones upon the surface of the earth at any instant t. By assuming a distressed aircraft located near Toronto airport, i.e. at latitude = 44° and lognitude = -80°, substituting these coordinates in both of ~~Eq. (3.16)~~ and (A1.10), we can determine the coverage time provided by both of the single beam antenna and the four beam antenna.

A computer program has been used for this purpose, and we checked the location of the aircraft to be either inside or outside the used pattern every 12 seconds and for a total interval of 10 hours, to cover the time period of five complete revolutions by the satellite. The results are drawn in Fig. A1.5 which shows for the first revolution by the satellite, there is no chance to pick a signal from the distressed aircraft but in the second revolution there is a possibility of picking the ELT signal for an interval of 10.2 minutes if a single antenna is used, for the three next revolutions Fig. A1.5 shows the coverage time received by each subantenna and the holes which would not occur if we used a single antenna.

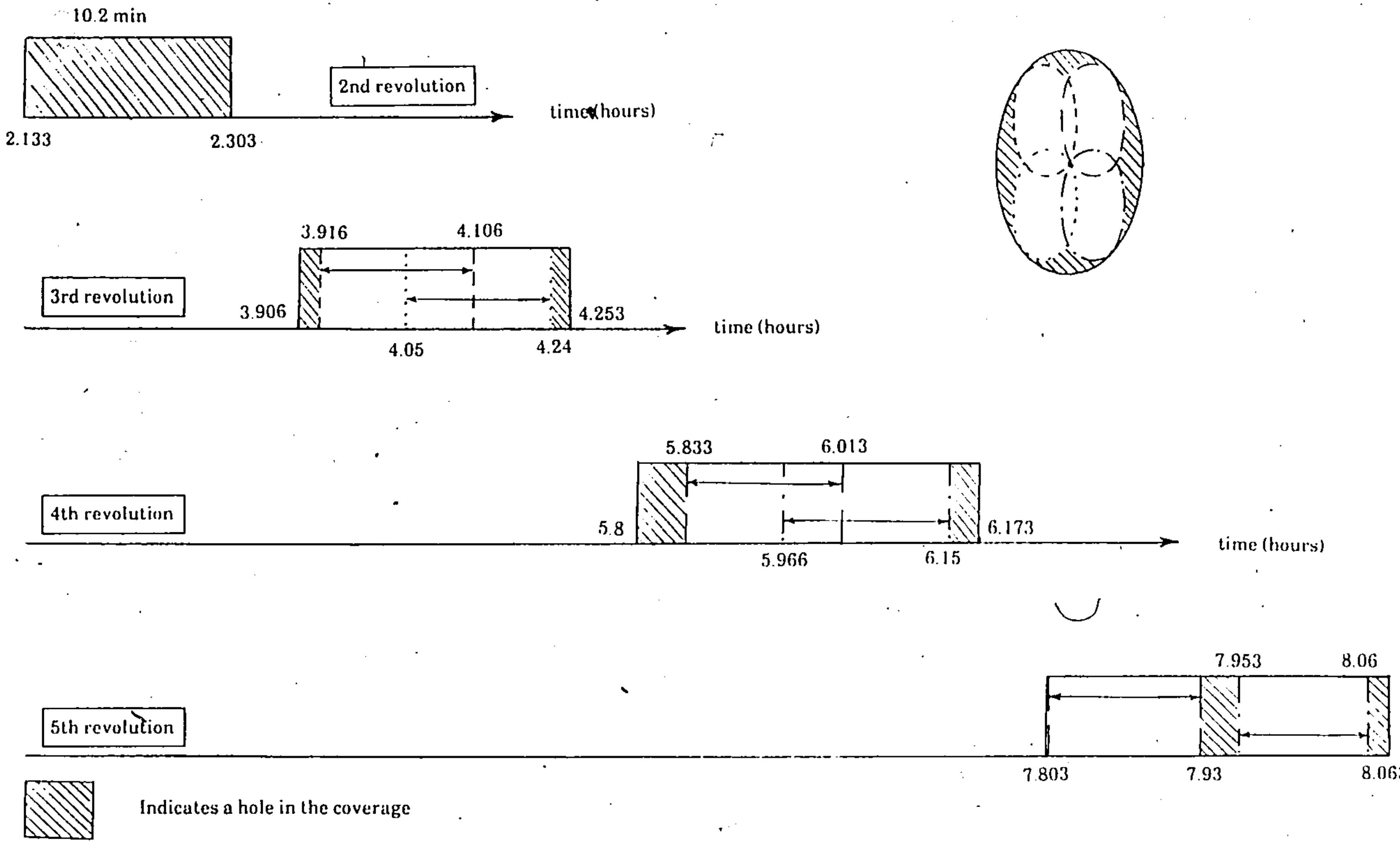

Fig. A1.5 The coverage time provided by the single antenna and four subantennas.

By rotating the antenna of the satellite, around its axis by an angle of 45°, the shape of the footprints of the four pattern antenna will be as shown in Fig. A1.6.a, A1.6.b and A1.6.c. These Figures are obtained at $t = 0, 4$, and 8 minutes respectively.

Using the same procedure which we used before, the coverage time will be as shown in Fig. A1.7 during five complete revolutions by the satellite which is different from the coverage time demonstrated in Fig. A1.5.

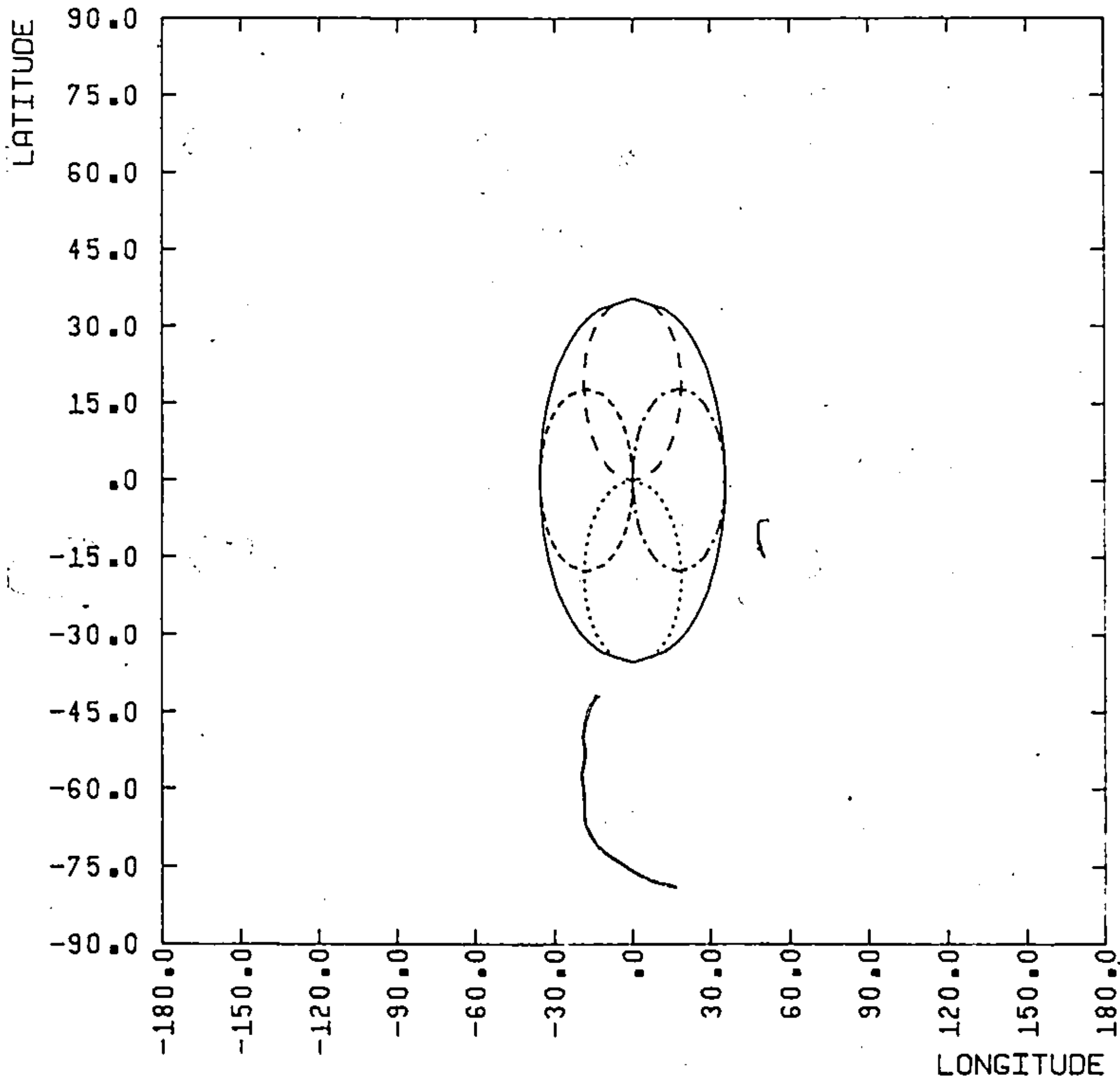

Fig. A1.6 (a) The footprints of the single antenna and the four subantennas after a rotation of 90° at t = 0

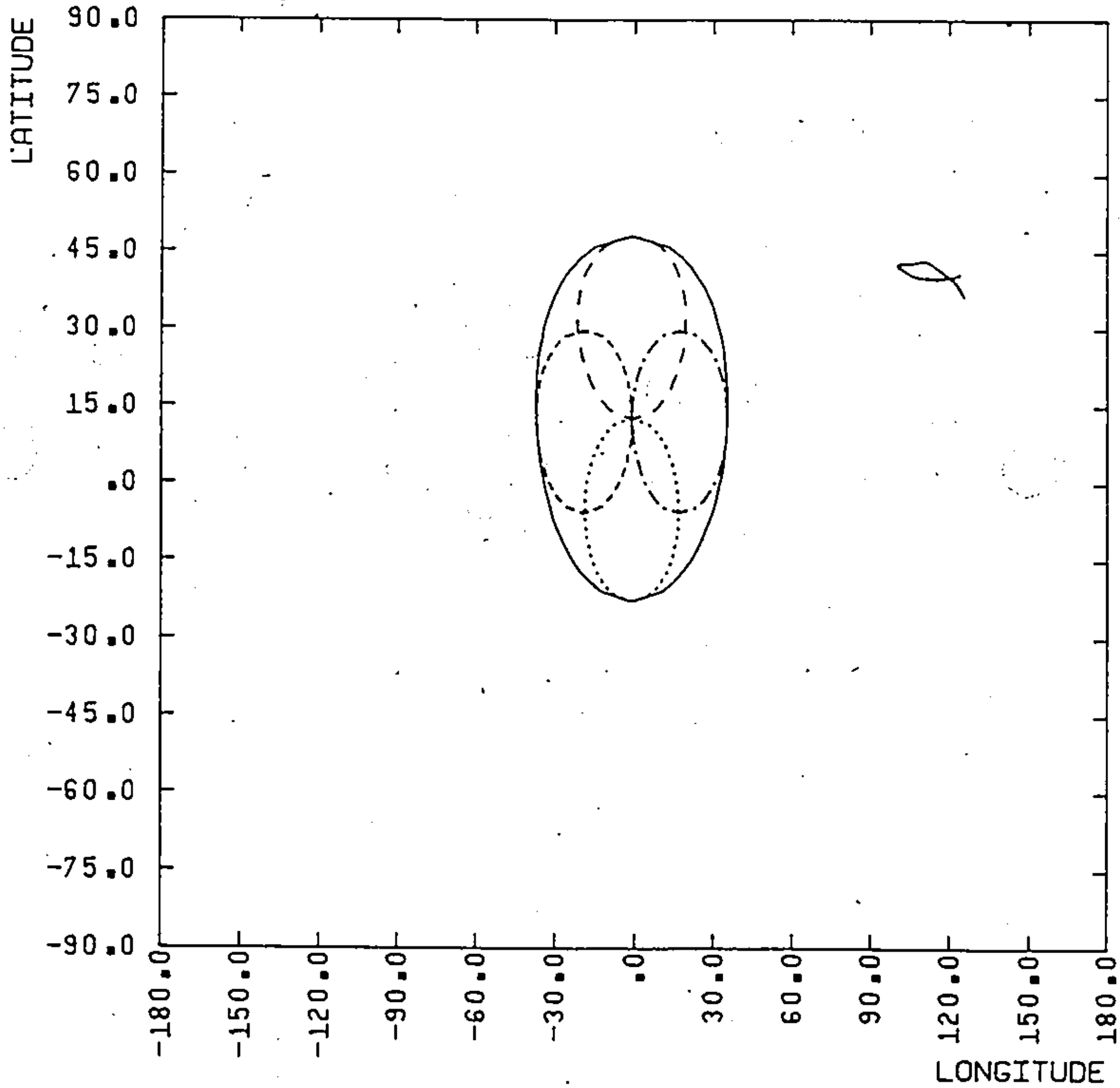

Fig. A1.6 (b) t = 4 min

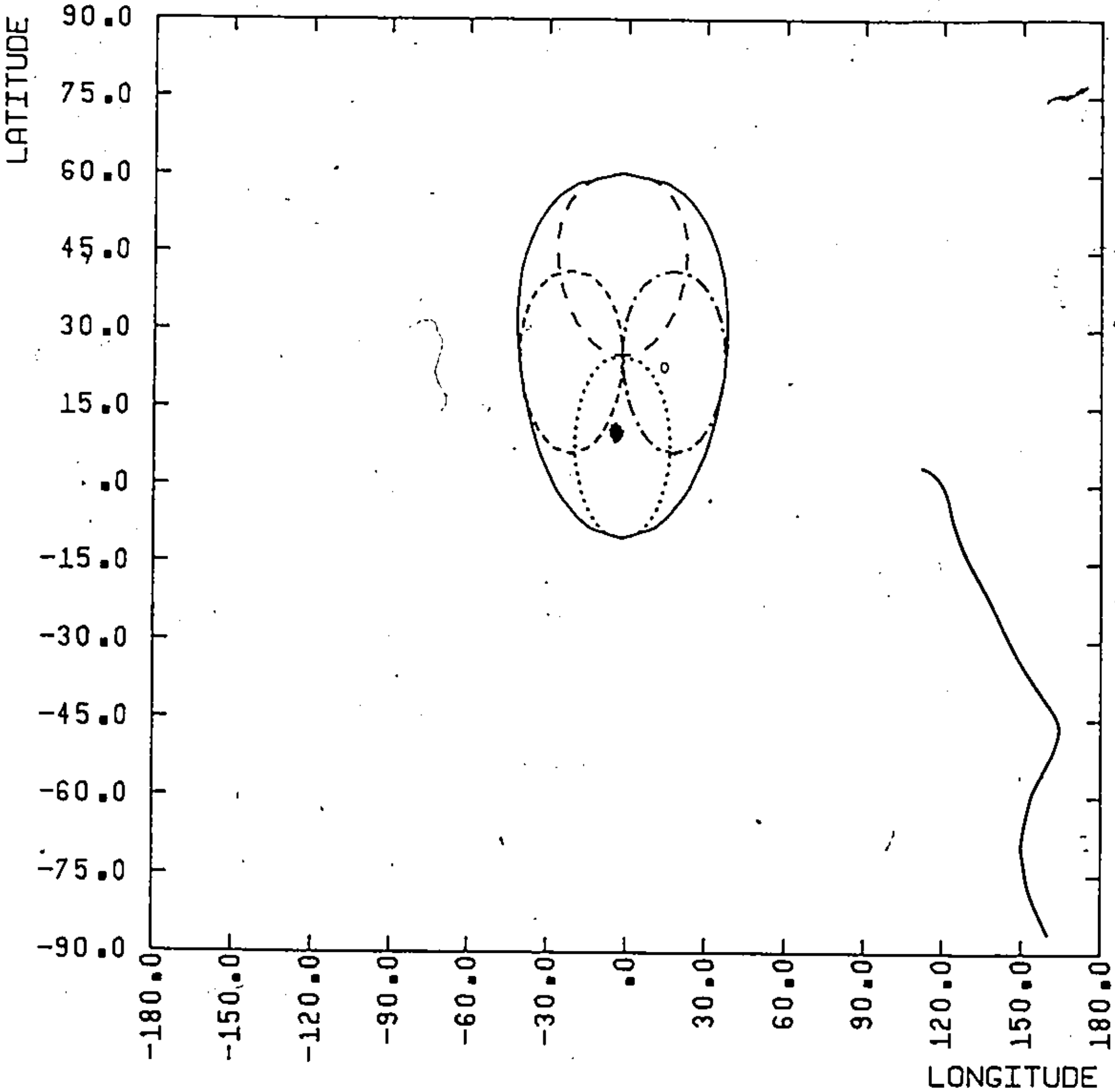

Fig. A1.6 (c) t=8 min.

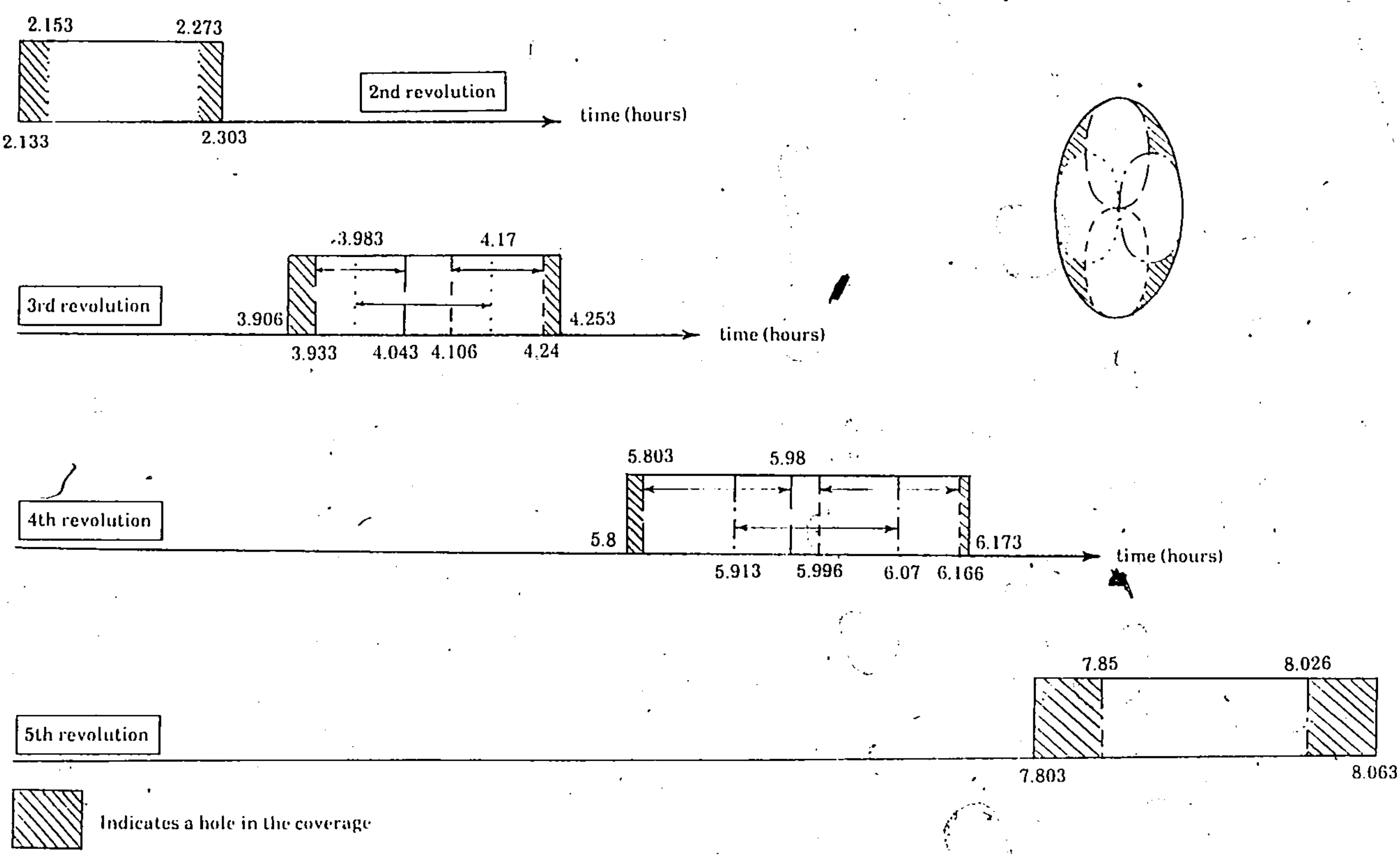

Fig. A1.7 The coverage time provided by the single antenna and the four subantenna after a rotation of 90°.

APPENDIX 2

THE CHANGE IN SPACE DELAY BETWEEN A GEOSTATIONARY SATELLITE AND A NAVIGATION SATELLITE IF EITHER ONE IS FIXED

Here, we are going to find an expression for the change in the space delay if either the navigation satellite or the geostationary satellite is fixed in terms of the latitude and longitude of the navigation satellite subpoint.

The angle λ between the navigation satellite subpoint and the geostationary satellite subpoint measured from the center of the earth at any instant of time t is given by Eq. (4.34) which is:

$$\cos\lambda = \cos\phi_{LatN} \cos(\phi_{LongN} - \phi_{LongG}) \tag{A2.1}$$

and the space delay between two satellites at this specified instant of time is given by Eq. (4.36)

$$T = d/C \tag{A2.2}$$

or

$$T = [H_N^2 + H_G^2 - 2H_N H_G \cos\lambda]^{1/2}/C \tag{A2.3}$$

where:

d = the distance between the two satellites.

H_N = the distance measured from the center of the earth to the navigation satellite.

H_G = the distance measured from the center of the earth to the geostationary satellite.

C = the propagation velocity

Also, we have from Eq. (4.1) and Eq. (4.2) the following relations:

$$\sin\phi_{LatN} = \sin i \sin Q \tag{A2.4}$$

$$\sin(\phi_{LongN} + \omega_E t - \phi_{oN}) = \frac{\cos i \sin Q}{(1 - \sin^2 i \sin^2 Q)^{1/2}} \tag{A2.5}$$

where

i	=	the orbital inclination angle of the navigation satellite.
Q	=	the angle rotated by the navigation satellite in its orbit measured from the center of the earth starting from the equatorial orbit.
Q	=	$\omega_S(t + TT)$
ω_S	=	the angular velocity of the navigation satellite.
TT	=	initial time phase of the navigation satellite.
ω_E	=	the angular velocity of the earth.
ϕ_{oN}	=	longitude of the intersection point between the navigation satellite orbital projection on the surface of the earth and the equator at $t=0$.

To find the change in the space delay when the navigation satellite is fixed, we differentiate Eq. (A2.3) with respect to time, while keeping Q constant, noting that $\cos\lambda$ is the only time dependent variable in Eq. (A2.3). This yields

$$\left.\frac{\partial T}{\partial t}\right|_{G,N=N_{fix}} = \frac{(-2\,H_N H_G/C)}{2[H_N^2 + H_G^2 - 2\,H_N H_G \cos\lambda]^{1/2}}\,\left.\frac{\partial \cos\lambda}{\partial t}\right|_{G,N=N_{fix}} \tag{A2.6}$$

Now by differentiating $\cos\lambda$ in Eq. (A2.1) with respect to t under the previous condition this gives:

$$\left.\frac{\partial \cos\lambda}{\partial t}\right|_{G,N=N_{fix}} = \cos(\phi_{LongN} - \phi_{LongG})\cdot\left[(-\sin\phi_{LatN})\cdot\left.\frac{\partial\phi_{LatN}}{\partial t}\right|_{G,N=N_{fix}}\right]$$

$$+ \cos\phi_{LatN}\left[\left(\sin(\phi_{LongN} - \phi_{LongG})\right)\cdot\left.\frac{\partial\phi_{LongN}}{\partial t}\right|_{G,N=N_{fix}}\right] \tag{A2.7}$$

Now to find

$$\left.\frac{\partial\phi_{LatN}}{\partial t}\right|_{G,N=N_{fix}},$$

we differentiate Eq. (A2.4) with respect to t while holding Q constant. Noting that i is independent of time, this gives:

$$\cos\phi_{LatN} \left.\frac{\partial\phi_{LatN}}{\partial t}\right|_{G,N=N_{fix}} = 0 \tag{A2.8}$$

Therefore:

$$\left.\frac{\partial\phi_{LatN}}{\partial t}\right|_{G,N=N_{fix}} = 0 \tag{A2.9}$$

In order to find

$$\left.\frac{\partial\phi_{LongN}}{\partial t}\right|_{G,N=N_{fix}},$$

we differentiate Eq. (A2.5) with respect to t under the mentioned condition this gives:

$$\cos(\phi_{LongN} + \omega_E t - \phi_{oN})\left(\left.\frac{\partial\phi_{LongN}}{\partial t}\right|_{G,N=N_{fix}} + \omega_E\right) = 0 \tag{A2.10}$$

$$\therefore \left.\frac{\partial\phi_{LongN}}{\partial t}\right|_{G,N=N_{fix}} = -\omega_E \tag{A2.11}$$

Substituting Eq. (A2.9) and Eq. (A2.11) in Eq. (A2.7) this yields:

$$\left.\frac{\partial\cos\lambda}{\partial t}\right|_{G,N=N_{fix}} = \omega_E \cos\phi_{LatN} \sin(\phi_{LongN} - \phi_{LongG}) \tag{A2.12}$$

Substituting Eq. (A2.12) in Eq. (A2.6) and after simplification we get:

$$\left.\frac{\partial\Gamma}{\partial t}\right|_{G,N=N_{fix}} = -\frac{H_N H_G}{d.C}\,\omega_E \cos\phi_{LatN} \sin\eta \tag{A2.13}$$

where

$$\eta = \phi_{LongN} - \phi_{LongG}$$

Now, to find the change in the space delay if the geostationary satellite is fixed, ie, $\omega_E t$ = constant, we follow the same steps followed before but now Q is time dependent. Equations (A2.6) and Eq. (A2.7) can be written as follows:

$$\left.\frac{\partial \Gamma}{\partial t}\right|_{G=G_{fix},N} = \frac{-H_N H_G}{d.C} \left.\frac{\partial \cos\lambda}{\partial t}\right|_{G=G_{fix},N} \tag{A2.14}$$

$$\left.\frac{\partial \cos\lambda}{\partial t}\right|_{G=G_{fix},N} = \cos\eta\left(-\sin\phi_{LatN}\left.\frac{\partial \phi_{LatN}}{\partial t}\right|_{G=G_{fix},N}\right)$$

$$+\cos\phi_{LatN}\left(-\sin\eta\left.\frac{\partial \phi_{LongN}}{\partial t}\right|_{G=G_{fix},N}\right) \tag{A2.15}$$

To find

$$\left.\frac{\partial \phi_{LatN}}{\partial t}\right|_{G=G_{fix},N},$$

we differentiate Eq. (A2.4) with respect to t, this gives:

$$\cos\phi_{LatN}\left.\frac{\partial \phi_{LatN}}{\partial t}\right|_{G=G_{fix},N} = \sin i\ \ (\omega_S \cos Q) \tag{A2.16}$$

Therefore:

$$\left.\frac{\partial \phi_{LatN}}{\partial t}\right|_{G=G_{fix},N} = \frac{\omega_S \sin i \cos Q}{\cos\phi_{LatN}} \tag{A2.17}$$

Noting that:

$$\cos(\phi_{LongN} + \omega_E t - \phi_{oN}) = \frac{\cos Q}{[1-\sin^2 i \sin^2 Q]^{1/2}} \tag{A2.18}$$

By putting $\phi_{LongN} + \omega_E t - \phi_{oN} = \delta$, Eq. (A2.17) can be written as:

$$\left.\frac{\partial \phi_{LatN}}{\partial t}\right|_{G=G_{fix},N} = \omega_S \sin i \cos\delta \tag{A2.19}$$

Also,

$$\left.\frac{\partial \phi_{LongN}}{\partial t}\right|_{G=G_{fix},N},$$

can be obtained by differentiating Eq. (A2.5) as follows:

$$\cos(\phi_{LongN} + \omega_E t - \phi_{oN}) \left.\frac{\partial \phi_{LongN}}{\partial t}\right|_{G=G_{fix},N} = \left\{ \frac{\omega_S \cos Q}{[1-\sin^2 i \sin^2 Q]^{1/2}} \right.$$

$$\left. + \frac{(\frac{-1}{2}) \sin Q\,(-\sin^2 i)\,(2 \sin Q)\,\omega_S}{[1-\sin^2 i \sin^2 Q]} \right\} \cos i \tag{A2.20}$$

or

$$\left.\frac{\partial \phi_{LongN}}{\partial t}\right|_{G=G_{fix},N} = \omega_S \cos i \left(1 + \frac{\sin^2 i \sin^2 Q}{1-\sin^2 i \sin^2 Q} \right) \tag{A2.21}$$

The last Equation can be reduced to:

$$\left.\frac{\partial \phi_{LongN}}{\partial t}\right|_{G=G_{fix},N} = \omega_S \frac{\cos i}{\cos^2 \phi_{LatN}} \tag{A2.22}$$

Substituting Eq. (A2.19) and Eq. (A2.22) in Eq. (A2.15) this gives:

$$\left.\frac{\partial \cos \lambda}{\partial t}\right|_{G=G_{fix},N} = - \left[\cos \eta \sin \phi_{LatN} \sin i \cos \delta + \frac{\cos i}{\cos \phi_{LatN}} \sin \eta \right] \omega_S \tag{A2.23}$$

Substituting Eq. (A2.23) in Eq. (A2.14) we obtain the formula for the change in the space delay if the geostationary satellite is fixed

$$\left.\frac{\partial T}{\partial t}\right|_{G=G_{fix},N} = \frac{H_N H_G}{d.C} \omega_S \left[\sin i \cos \delta \cos \eta \sin \phi_{LatN} + \frac{\cos i \sin \eta}{\cos \phi_{LatN}} \right] \tag{A2.24}$$

APPENDIX 3

THE GRADIENT OF THE CHANGE IN THE SPACE DELAY

In this Appendix we calculate the gradient of the change in the space delay between the navigation satellite and the geostationary satellite assuming either G or N is fixed. The calculated gradient will be expressed in polar coordinates of the navigation satellite which has the directions of its altitude H_N, its subpoint latitude ϕ_{LatN} and longitude ϕ_{LongN}.

The gradient form for these particular coordinates will be

$$\nabla = \frac{\partial}{\partial H_N} a_{H_N} + \frac{1}{H_N} \frac{-\partial}{\partial \phi_{LatN}} a_{\phi_{LatN}} + \frac{1}{H_N \cos\phi_{LatN}} \frac{\partial}{\partial \phi_{LongN}} a_{\phi_{LongN}} \tag{A3.1}$$

where a_{HN}, $a\phi_{LatN}$, $a\phi_{LongN}$ are the unit vectors in the directions H_N, ϕ_{LatN} and ϕ_{LongN} respectively.

Now, the gradient of the change in the space delay $\partial T/\partial t$ is given by:

$$\nabla \frac{\partial T}{\partial t} = \frac{\partial}{\partial t} \nabla T \tag{A3.2}$$

$$\nabla T = \frac{\partial T}{\partial H_N} a_{H_N} + \frac{1}{H_N}\left(\frac{-\partial T}{\partial \phi_{LatN}}\right) a_{\phi_{LatN}} + \frac{1}{H_N \cos\phi_{LatN}} \frac{\partial T}{\partial \phi_{LongN}} a_{\phi_{LongN}} \tag{A3.3}$$

Substituting for T from Eq. (A2.3) into Eq. (A3.3) and by considering only the radial component of the gradient which is given by:

$$\frac{\partial T}{\partial H_N} = \frac{\partial}{\partial H_N} (H_N^2 + H_G^2 - 2 H_N H_G \cos\lambda)^{1/2} \tag{A3.4}$$

The last equation after differentiation and substituting for range $d = (H_N^2 + H_G^2 - 2 H_N H_G \cos\lambda)^{1/2}$, will be

$$\frac{\partial T}{\partial H_N} = \frac{H_N - H_G \cos\lambda}{d \cdot C} \tag{A3.5}$$

Consider the component in the latitude direction and after substituting for $\cos\lambda$ such a component will be given by

$$-\frac{1}{H_N}\frac{\partial T}{\partial \phi_{LatN}} = -\frac{1}{H_N}\frac{\partial T}{\partial \phi_{LatN}}\frac{(H_N^2 + H_G^2 - 2\,H_N H_G \cos\phi_{LatN}\cos\eta)^{1/2}}{C} \qquad \text{(A3.6)}$$

or

$$-\frac{1}{H_N}\frac{\partial T}{\partial \phi_{LatN}} = -\frac{1}{H_N}\frac{(\frac{1}{2})(-\,2\,H_N H_G)(-\sin\phi_{LatN})\cos\eta}{C(H_N^2 + H_G^2 - 2\,H_N H_G \cos\phi_{LatN}\cos\eta)^{1/2}} \qquad \text{(A3.7)}$$

After simplification, the last equation can be written as:

$$-\frac{1}{H_N}\frac{\partial T}{\partial \phi_{LatN}} = -\frac{H_G}{d\cdot C}\sin\phi_{LatN}\cos\eta \qquad \text{(A3.8)}$$

Now the component in the longitude direction should be given by:

$$\frac{1}{H_N\cos\phi_{LatN}}\frac{\partial T}{\partial \phi_{LongN}} = \frac{1}{H_N\cos\phi_{LatN}}\frac{\partial T}{\partial \phi_{LongN}}$$

$$\left(\frac{H_N^2 + H_G^2 - 2H_N H_G \cos\phi_{LatN}\cos\eta}{C}\right) \qquad \text{(A3.9)}$$

Noting that $\eta = \phi_{LongN} - \phi_{LongG}$ the last equation after simplification is given by:

$$\frac{1}{H_N\cos\phi_{LatN}}\frac{\partial T}{\partial \phi_{LongN}} = \frac{H_G}{d\cdot C}\sin\eta \qquad \text{(A3.10)}$$

A3.1 The Gradient of the Change in the Space Delay if the Navigation Satellite is Fixed:

To find the gradient of the change in the space delay if the navigation satellite is fixed, we differentiate its components given by Eq. (A3.5), Eq. (A3.8) and Eq. (A3.10) with respect to time while holding Q constant.

Consider the radial component of the change in the space delay which is given by:

$$\frac{\partial}{\partial t}\frac{\partial T}{\partial H_N}\bigg|_{G,N=N_{fix}} = \frac{\partial}{\partial t}\frac{(H_N - H_G\cos\lambda)}{d\cdot C}\bigg|_{G,N=N_{fix}} \qquad \text{(A3.11)}$$

Noting that d and λ are the only dependent on time in Eq. (A3.11), this yields:

$$\frac{\partial}{\partial t}\frac{\partial T}{\partial H_N}\bigg|_{G,N=N_{fix}} = \frac{H_N - H_G\cos\lambda}{d^2}\left(-\frac{\partial d/C}{\partial t}\bigg|_{N=N_{fix}}\right)$$

$$+\left(\frac{-H_G}{d\cdot C}\right)\frac{\partial\cos\lambda}{\partial t}\bigg|_{G,N=N_{fix}} \tag{A3.12}$$

From Eq. (A2.3) we can say

$$\frac{\partial T}{\partial t}\bigg|_{G,N=N_{fix}} = \frac{-H_N H_G}{d\cdot C}\;\frac{\partial\cos\lambda}{\partial t}\bigg|_{G,N=N_{fix}} \tag{A3.13}$$

Noting that

$$\frac{\partial d/C}{\partial t}\bigg|_{G,N=N_{fix}} = \frac{\partial T}{\partial t}\bigg|_{G,N=N_{fix}}$$

in Eq. (A3.12) and by substituting Eq. (A3.13) in Eq. (A3.12) and after simplification we get:

$$\frac{\partial}{\partial t}\frac{\partial T}{\partial H_N}\bigg|_{G,N=N_{fix}} = \left(\frac{H_G\cos\lambda - H_N}{d^2} + \frac{1}{H_N}\right)\frac{\partial T}{\partial t}\bigg|_{G,N=N_{fix}} \tag{A3.14}$$

Consider the component in a_ϕ direction of the change int he space delay, which is given by:

$$\frac{\partial}{\partial t}\left(\frac{-1}{H_N}\;\frac{\partial T}{\partial\phi_{LatN}}\right)\bigg|_{G,N=N_{fix}} = \frac{\partial}{\partial t}\left(\frac{-H_G}{d\cdot C}\sin\phi_{LatN}\cos\eta\right)\bigg|_{G,N=N_{fix}} \tag{A3.15}$$

We notice here that ϕ_{LatN} is const. [Q = const.], therefore d and η are the only time dependent in Eq. (A3.15) thus we have:

$$\frac{\partial}{\partial t}\left(\frac{-1}{H_N}\;\frac{\partial T}{\partial\phi_{LatN}}\right)\bigg|_{G,N=N_{fix}} = -H_G\sin\phi_{LatN}\left(\frac{\cos\eta}{d^2}\cdot\frac{-\partial d/C}{\partial t}\bigg|_{G,N=N_{fix}}\right.$$

$$\left.+\frac{-\sin\eta}{d\cdot C}\;\frac{d\eta}{dt}\bigg|_{G,N=N_{fix}}\right) \tag{A3.16}$$

Knowing that $\eta = \phi_{LongN} - \phi_{LongG}$, therefore from Eq. (A2.11) we have:

$$\frac{d\eta}{dt}\bigg|_{G,N=N_{fix}} = \frac{\partial\phi_{LongN}}{\partial t}\bigg|_{G,N=N_{fix}} = -\omega_E \tag{A3.17}$$

Substituting Eq. (A3.17) in Eq. (A3.16) and after simplification, this yields:

$$\frac{\partial}{\partial t}\left(\frac{-1}{H_N}\frac{\partial T}{\partial \phi_{LatN}}\right)\bigg|_{G,N=N_{fix}} = \frac{H_G \sin\phi_{LatN}}{d}\left|\frac{\cos\eta \left.\frac{\partial T}{\partial t}\right|_{G,N=N_{fix}}}{d} - \frac{\omega_E \sin\eta}{C}\right| \tag{A3.18}$$

The component of the gradient in the longitude direction can be obtained by differentiating Eq. (A3.10)

$$\frac{\partial}{\partial t}\frac{\cdot 1}{H_N \cos\phi_{LatN}}\frac{\partial T}{\partial \phi_{LongN}}\bigg|_{G,N=N_{fix}} = \frac{\partial}{\partial t}\left(\frac{H_G}{d\cdot C}\sin\eta\right)\bigg|_{G,N=N_{fix}} \tag{A3.19}$$

η and d are the only time dependent variables, therefore, the last equation becomes:

$$\frac{\partial}{\partial t}\frac{1}{H_N \cos\phi_{LatN}}\frac{\partial T}{\partial \phi_{LongN}}\bigg|_{G,N=N_{fix}} = \frac{-H_G}{d^2}\sin\eta\,\frac{\partial d/C}{\partial t}\bigg|_{G,N=N_{fix}}$$

$$+\frac{H_G}{d\cdot C}\cos\eta\,(-\omega_E) \tag{A3.20}$$

or

$$\frac{\partial}{\partial t}\frac{1}{H_N \cos\phi_{LatN}}\frac{\partial T}{\partial \phi_{LongN}}\bigg|_{G,N=N_{fix}} = \frac{-H_G}{d}\left|\frac{\sin\eta}{d}\frac{\partial T}{\partial t}\bigg|_{G,N=N_{fix}} + \frac{\omega_E \cos\eta}{C}\right| \tag{A3.21}$$

The gradient can be calculated by the vector addition of its components given byEq. (A3.14), Eq. (A3.18) and Eq. (A3.21).

The gradient and its components represented by the above equations is plotted in Fig. (A3.1) for the case of a 12 hour navigation satellite with inclination angle 30°,

$$\phi_{oN} = 0° \text{ and } \phi_{LongG} = 45°.$$

The same case is plotted using Eq. (5.62) and Eq. (5.65) and Eq. (5.68) in Fig. (A3.2) which shows that the two sets of equations are representing the same gradient.

A3.II <u>The Gradient of the Change in the Space Delay if the Geostationary Satellite is Fixed:</u>

Here, we follow the same steps as in Section (A3.I), except we consider that the geostationary satellite is fixed, ie, $\omega_E t = \text{const}$.

To find the components of the gradient under the previous condition we differentiate Eq. (A3.5), Eq. (A3.8), and Eq. (A3.10) with respect to time, this yields:

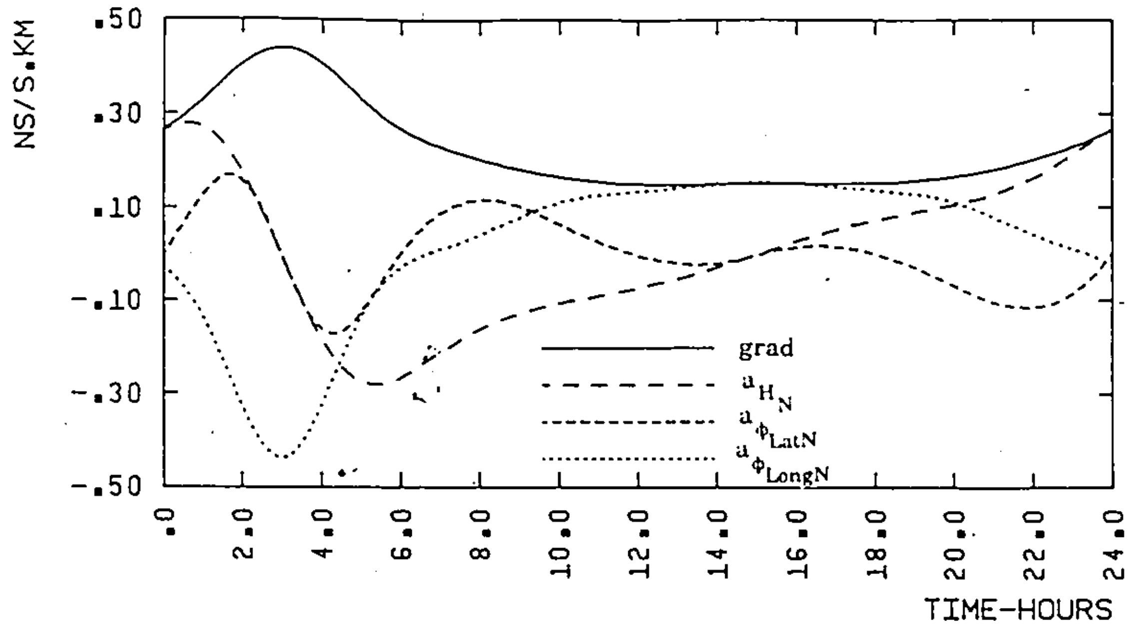

Fig. A3.1 The gradient of the change in the space delay and its components represented by Eq. (A3.14), Eq. (A3.18) and Eq. (A3.21).

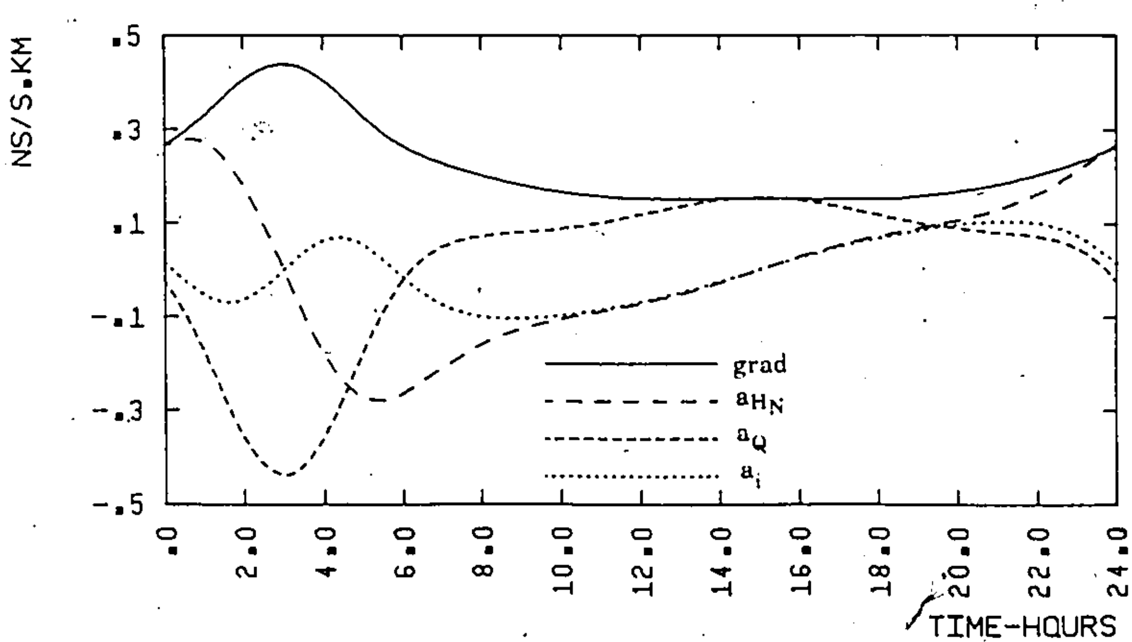

Fig. A3.2 The gradient of the change in the space delay and its components represented by Eq. (A3.62), Eq. (A3.65) and Eq. (A3.68).

$$\frac{\partial}{\partial t}\frac{\partial \Gamma}{\partial H_N}\bigg|_{G=G_{fix},N} = \frac{\partial}{\partial t}\frac{H_N - H_G\cos\lambda}{d\cdot C}\bigg|_{G=G_{fix},N} \quad (A3.22)$$

Noting that both of d and λ are time dependent this yields:

$$\frac{\partial}{\partial t}\frac{\partial \Gamma}{\partial H_N}\bigg|_{G=G_{fix},N} = \frac{H_N - H_G\cos\lambda}{d^2}\left(-\frac{\partial d/C}{\partial t}\right)\bigg|_{G=G_{fix},N}$$

$$+\left(\frac{-H_G}{d\cdot C}\right)\frac{\partial\cos\lambda}{\partial t}\bigg|_{G=G_{fix},N} \quad (A3.23)$$

Substituting for

$$\frac{\partial\cos\lambda}{\partial t}\bigg|_{G=G_{fix},N}$$

from Eq. (A2.14) and after simplification we have:

$$\frac{\partial}{\partial t}\frac{\partial \Gamma}{\partial H_N}\bigg|_{G=G_{fix},N} = \frac{H_G\cos\lambda - H_N}{d^2}\frac{\partial\Gamma}{\partial t}\bigg|_{G=G_{fix},N} + \frac{1}{H_N}\frac{\partial\Gamma}{\partial t}\bigg|_{G=G_{fix},N} \quad (A3.24)$$

or

$$\frac{\partial}{\partial t}\frac{\partial \Gamma}{H_N}\bigg|_{G=G_{fix},N} = \left(\frac{H_G\cos\lambda - H_N}{d^2} + \frac{1}{H_N}\right)\frac{\partial\Gamma}{\partial t}\bigg|_{G=G_{fix},N} \quad (A3.25)$$

The last equation gives the gradient radial component of the change in space delay if G is fixed.

Now, consider the component of the change in the space delay in the latitude direction which can be obtained by differentiating Eq. (A3.8), this gives:

$$\frac{\partial}{\partial t}\frac{-1}{H_N}\frac{\partial \Gamma}{\partial \phi_{LatN}}\bigg|_{G=G_{fix},N} = \frac{\partial}{\partial t}\bigg|_{G=G_{fix},N}\left(\frac{-H_G}{d\cdot C}\sin\phi_{LatN}\cos\eta\right) \quad (A3.26)$$

We notice here that d, ϕ_{LatN} and $\eta = \phi_{LongN} - \phi_{LongG}$ are time dependent, therefore Eq. (A3.26) will be:

$$\frac{\partial}{\partial t}\frac{-1}{H_N}\frac{\partial \Gamma}{\partial \phi_{LatN}}\bigg|_{G=G_{fix},N} = \frac{H_G\sin\phi_{LatN}}{d^2}\cos\eta\,\frac{\partial d/C}{\partial t}\bigg|_{G=G_{fix},N}$$

$$-\frac{H_G}{d\cdot C}\left(\cos\phi_{LatN}\cos\eta\,\frac{\partial\phi_{Lat_N}}{\partial t}\cdot\right|_{G=G_{fix},N}$$

$$-\sin\phi_{LatN}\sin\eta\,\frac{\partial\phi_{LongN}}{\partial t}\Bigg|_{G=G_{fix},N}\Bigg) \qquad (A3.27)$$

Substituting in Eq. (A3.27) for

$$\frac{\partial\phi_{LatN}}{\partial t}\Bigg|_{G=G_{fix},N}$$

and

$$\frac{\partial\phi_{LongN}}{\partial t}\Bigg|_{G=G_{fix},N}$$

from Eq. (A2.20) and (A2.22) respectively and after simplification we get:

$$\frac{\partial}{\partial t}\,\frac{-1}{H_N}\,\frac{\partial\Gamma}{\partial\phi_{LatN}}\Bigg|_{G=G_{fix},N} = \frac{H_G\sin\phi_{LatN}}{d^2}\cos\eta\,\frac{\partial T}{\partial t}\Bigg|_{G=G_{fix},N}$$

$$+\frac{H_G\omega_S}{d\cdot C}\sin i\left[\frac{\sin\delta\sin\eta}{\cos\phi_{LatN}}-\cos\phi_{LatN}\cos\delta\cos\eta\right] \qquad (A3.28)$$

The gradient component in the longitude direction can be obtained by differentiating Eq. (A3.10) as follows:

$$\frac{\partial}{\partial t}\,\frac{1}{H_N\cos\phi_{LatN}}\,\frac{\partial\Gamma}{\partial\phi_{LongN}}\Bigg|_{G=G_{fix},N} = \frac{\partial}{\partial t}\,\frac{H_G}{d\cdot C}\sin\eta\Bigg|_{G=G_{fix},N} \qquad (A3.29)$$

or

$$\frac{\partial}{\partial t}\,\frac{1}{H_N\cos\phi_{LatN}}\,\frac{\partial\Gamma}{\partial\phi_{LongN}}\Bigg|_{G=G_{fix},N} = \frac{H_G}{d^2}\sin\eta\left(\frac{-\partial d/C}{\partial t}\Bigg|_{G=G_{fix},N}\right)$$

$$+\frac{H_G}{d^2}\cos\eta\,\frac{\partial\phi_{LongN}}{\partial t}\Bigg|_{G=G_{fix},N} \qquad (A3.30)$$

Substituting from Eq. (A3.22) in Eq. (A3.30) and after simplification we have:

$$\frac{\partial}{\partial t}\,\frac{1}{H_N\cos\phi_{LatN}}\,\frac{\partial T}{\partial\phi_{LongN}}\Bigg|_{G=G_{fix},N} =$$

$$\frac{H_G}{d}\left[\frac{\omega_S \cos\eta \cos i}{C \cdot \cos^2 \phi_{LatN}} - \frac{\sin\eta}{d}\;\frac{\partial\Gamma}{\partial t}\;\Big|_{G=G_{fix},N}\right] \tag{A3.31}$$