FLEXIBILITY AND CRITICALNESS IN RESOURCE USE: THE CASE OF URBAN EXPANSION AND LAND NEEDS FOR AGRICULTURE

By

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ABSTRACT

This thesis reviews the debate over the conversion of agricultural land to urban uses, and assesses the usefulness of land and land-use classification techniques for establishing land-use priorities. It demonstrates that the establishment of land-use priorities cannot be based solely on an assessment of the supply of land of different degrees of suitability for agriculture, and outlines the type of information needed for land-use planning. It argues that in order to determine if and where agricultural land needs to be protected to ensure adequate food production capacity, policy-makers require information on the capacity of the land base to meet agricultural commodity requirements under a variety of future conditions. This includes data not only on the feasibility of meeting given demands, but also on how much production capacity exists and whether the fulfillment of given demands hinges upon particular land areas or types of land being available to agriculture.

Flexibility is interpreted as the amount of production capacity that remains after specified demands are met. This thesis assesses several measures of flexibility. Each measure is based upon the analysis of a linear set of inequalities which describe resource

availability and demand conditions. Two different approaches are examined. The first approach determines the extent to which production capacity exceeds or falls short of given demands. The second approach is based on the size of the feasible region defined by the linear set of inequalities.

Criticalness is interpreted as the extent to which the fulfillment of given demands is dependent upon different types of land being available to agriculture. This thesis has assessed several measures of land-use criticalness. Each of the criticalness measures is based on the analysis of a linear set of inequalities. Two types of measures are assessed. The first type rates the importance of assigning individual crops to each land type so as to meet given demands. This measure takes two forms: absolute minimum assignments and conditional minimum assignments. The second type rates the importance of each land type in meeting all the demands.

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CHAPTER 1

INTRODUCTION

1.1 Nature of the Problem

A major land-use issue in both developed and developing nations is whether the continued conversion of agricultural land to urban uses threatens the future production of adequate supplies of food and fibre (see, for example, Bentley, 1978, 1981; Centre for Agricultural Strategy, 1976; O.E.C.D Observer, 1976; Parker and Coyle, 1981; Vining *et al.*, 1977). Whereas it has been common practice to consider agricultural land as a reserve upon which urban uses could draw as needed, concern over the loss of agricultural land to urban expansion has arisen in the face of rising world demand for food, the importance of agricultural exports in maintaining a favourable balance of payments, and the prospects of increasing variation in climatic conditions (Bentley, 1978; Plaut, 1980). As well, some physical resources important to modern agriculture, particuarly fuels and fertilizers, may not be as plentiful in the future as in the past, which raises doubts about the capacity to maintain, let alone increase, current levels of productivity, and suggests the possible return to a more land extensive agriculture (Gibson, 1977; Jorling, 1978; Pimmentel and Pimmentel, 1980).

Despite the facts that land resources are finite, and that widespread concern exists over the spectre of increasing land-use conflicts, the effect of continued agricultural land conversion to urban uses upon the capacity to produce sufficient food and fibre remains a topic of heated debate. Some cite the now familiar litany of limited resources and rising population and maintain that if the conversion of agricultural land to urban uses continues unabated, the remaining agricultural land resources may not be sufficient to meet projected increases in the domestic demand for food and fibre let alone offset expected shortfalls of food in various regions of the world (Beaubien and Tabacnik, 1977; Brown, 1978; Collins, 1976; Diderickson and Sampson, 1976; Krueger, 1977). Others express faith in technology and the use of marginal agricultural lands, and argue that a future shortage of agricultural land is unlikely (Edwards, 1969; Hart, 1976; U.S.D.A., 1974; Urban Land Institute, 1982).

Pierce and Furuseth (1983) argue that one of the most critical long-term implications of increasing land needs is that the agricultural system in North America is losing its flexibility. Although they are optimistic about the adequacy of the land resource for the next thirty years, uncertainty about the capacity to meet future food needs, has led others to advocate farmland preservation policies as a means of retaining land-use flexibility (see, for example, Lapping, 1980; Miner and Chorich, 1979; Platt, 1977). Such policies are designed to keep as many agricultural land-use options open as possible as a hedge against future contingencies. It is argued that the magnitude of current conversions, together with concerns for such factors as energy availability and population growth, justifies a cautious approach to the use of agricultural land.

Flexibility is a characteristic that implies the ability to adapt to changed conditions. As more land is converted to urban uses, some degree of flexibility will be lost. However, although the area and quality of land available for agriculture largely determines its capacity to produce food, land cannot be pre-empted solely for this purpose. As noted by Blumenfeld (1978) and Clawson (1978), many farmland preservation advocates view the city as the enemy and ignore the fact that the real problem is increasing numbers of people and their activities. Many equally pressing and legitimate claims to the land resource exist, among them space for housing, industrial parks, transport networks and so forth. More people and more jobs will require more land, even if urban land is used more efficiently than in the past (Urban Land Institute, 1982). Efforts to preserve agricultural land inevitably impinge upon the availability of land for competing urban uses and therefore raise a serious issue over priorities for allocating land resources. The controversy over the use of agricultural land, both in Canada and elsewhere, suggests that there is a need to develop a better means of assessing the impact of agricultural land conversion on food production capacity, so that the need for public policy intervention can be established.

In order to determine if and where public intervention to protect agricultural land from urban encroachment is needed to ensure adequate food production capacity, policy-makers require information on the ability of the land base to meet future agricultural commodity requirements under a variety of future conditions. This not only includes an indication of whether sufficient land is available to meet given demands, but the extent to which available production capacity is used. Information is also needed on the extent to which the fulfillment of the demands hinges upon particular land areas or types of land being available to agriculture.

1.2 Purpose of Study

Many techniques have been developed to assist planners and policy-makers in the formulation of land-use policies. The purpose of this study is to identify and evaluate methods that can be used to address three issues: the feasiblity of meeting future food needs under specified conditions, the extent to which available production capacity is used in meeting the demands, and the extent to which the fulfillment of the demands is dependent upon particular land areas being available for production. The specific study objectives are:

1) to review the emergence of agricultural land-use conversion as a public policy issue;
2) to assess the role of land and land-use classification techniques in land-use planning;
3) to review the use of land-use programming models for analyzing the impact of urban expansion on food production capacity;
4) to outline the need for measures of land-use flexibility and criticalness; and
5) to assess alternative measures of land-use flexibility and criticalness.

1.3 Outline of Thesis

This thesis has eight chapters. Chapter 2 reviews the literature which analyses agricultural land conversion to urban uses particularly in Canada, and assesses the role of land and land-use classification techniques in land use planning. It also examines information needs for land-use planning and outlines the information requirements for measures of land-use flexibility and criticalness. Chapter 3 reviews the current state of conceptual and empirical work in agricultural land-use programming models, and asseses the usefulness of mathematical programming techniques for analyzing land-use flexibility and criticalness. Chapter 4 presents alternative measures of land-use flexibility. In Chapter 5 the analytical methods and properties of selected flexibility measures are examined for a hypothetical data set. Chapter 6 presents alternative measures of land-use criticalness. In Chapter 7 the analytical methods and the properties of selected criticalness measures are examined. Finally, in Chapter 8, the study's findings are summarized and conclusions are drawn.

CHAPTER 2

BACKGROUND

Many studies have investigated the process by which rural land is converted to urban and other nonagricultural uses and have attempted to assess the consequences of such conversion for the land-use system. This chapter reviews the debate over the conversion of agricultural land to urban uses and assesses the usefulness of land classification and land-use evaluation systems for establishing land-use priorities. This both establishes the context of the study and demonstrates the need for a more comprehensive and integrated approach to the analysis of agricultural land conversion than that facilitated by existing land classification or land-use evaluation systems alone.

2.1 Urban Centres and Agricultural Land

The growth of the world's population is characterized by two notable features. First, overall population growth has greatly accelerated, especially since about 1950, and it is anticipated that such growth will continue for a number of decades. Secondly, since the last century, the percentage of urban population in total population has progressed steadily in almost all nations (United Nations, 1980). Although the experts disagree as to what the exact future size and distribution of the world's population is likely to be, the general view is that the total population will increase and that the trend to higher levels of urbanization will continue. An indication of the magnitude of future population growth and urbanization which is expected, is given by the estimates developed by the Population Council of the United Nations (Figure 2.1). By 2025, the world's population is likely to almost double its 1980 level, and the urban population may increase to 62 percent of the total.

The combined trends of population growth and rising levels of urbanization are widely perceived as being accompanied by serious social, economic and environmental problems. Difficulties exist in matching the rapid growth of city populations with expansion of employment opportunities, housing and public services, while air, noise, and water pollution are now endemic to the urbanized areas of the world (Adams, 1976; Hanten *et al.*, 1980). The upsurge in population growth and urbanization also affects the rate of attrition of the world's nonrenewable resources, including the supply of agricultural land. There is growing uncertainty about whether sufficient agricultural land will be available to meet future food needs, and this uncertainty has focused attention on the competition for land between urban and agricultural uses.

Urbanization, in both developed and developing countries, tends to affect farmland of the best quality in terms of soil fertility, water supply and drainage (Brown, 1978; O.E.C.D., 1979). For many years, agricultural land has been considered a virtually inexhaustible reserve for further residential, commercial and industrial development (O.E.C.D., 1976). The main land-use problems consisted of providing for the orderly development of rural land or resolving conflicts between competing urban uses. Recently, however, several national and global factors have combined to prompt a reassessment of the manner in which land is allocated between urban and agricultural uses.

A major factor contributing to the concern over the conversion of agricultural land to urban uses is that between 1970 and 1980, the growth of world food production has slowed down in relation to population growth, while in many developing countries the rate of increase in food production has fallen behind that of population growth (Brown, 1981; FAO, 1983). Many countries in Central Africa and South Asia have experienced a growing divergence between domestic food consumption and production. These regions are characterized by high human fertility rates, low per capita incomes and low capital investment in agriculture. Third World nations have become increasingly dependent upon the industrialized nations as sources of food imports and food aid to maintain domestic consumption levels (Alexandratos *et al.*, 1983; Saouma, 1981). While there is some disagreement as to what constitutes minimum nutritional standards, an even larger gap may exist between current levels of food consumption and food needs in Third World Nations (North-South Institute, 1978).

The future of the world food situation is the subject of a growing volume of literature and debate (see, for example, Chou and Harmon, 1979; Hopkins *et al.*, 1982; Woods, 1981). These studies have attempted to assess the implications of such factors as lower fertility rates, increasing crop yields and rising per capita incomes on food production and consumption. Although many assumptions are built into such forecasts, the consensus is that the gap between food demand and supply in most developing countries will increase substantially in the next ten to fifteen years. The FAO projects that by the year 2000, sixty-five countries will be unable to feed their populations from their own land (Harrison, 1983). This, in turn, has focused attention on the means by which agricultural production capacity can be increased.

There are three main options for increasing the supply of food: new lands can be cultivated, agriculture on lands already under cultivation can be intensified by conventional means, and new sources and techniques of food production can be developed. Up until 1950, increases in food output largely came by expanding the area under cultivation. Since then, most have come from raising yields on existing cropland through the application of fertilizers and other nonland resources. By 1950, the bulk of the world's arable lands were already being cultivated, and it has been far less expensive and easier to intensify cultivation on existing cropland than to extend onto marginal lands (Brown, 1978; United Nations, 1980). Although some opportunity exists for expanding cropland area, the long-term impact of using potentially arable land is a matter of controversy.

Expanding the area under cultivation is limited by several factors. Unfavourable climatic conditions remove vast areas from consideration due to cold temperatures or inadequate moisture from rainfall in areas where irrigation is not practical. Mountainous, rocky and arid regions further reduce the cultivable area, while much of the remaining land is covered by soils which, while potentially arable, pose serious problems for agriculture. The greatest potential for expanding cropland area exists in

Africa and South America. Cultivating this land, however, will require diverting it from other uses and/or investing in other scarce resources to make it productive (Brown, 1978). Much of the potential cropland area is forested and has thin soils which deteriorate rapidly once deprived of their tree cover. Forests also represent a renewable source of building materials and energy (Nicholson and Nicholson, 1979). The other major component of potential cropland is grassland or savannah which is extensively used for pasture. These lands are highly susceptible to wind erosion once cultivated and could deteriorate very quickly (Pimmentel and Pimmentel, 1980). In such cases, present uses of land may be environmentally more sound than the agricultural alternative.

As cultivable land becomes scarcer in relation to population, pressure grows to increase its productivity. The developed nations have had great success in raising yields through the increased use of nonland inputs. Many believe that great opportunity exists for increasing food output through the wider use of more intensive production strategies - more fertilizer, better moisture control, denser strands, and crop varieties that provide higher yield potential (see, for example, Barker *et al*., 1979). There is evidence, however, that high technology agriculture is not the answer to the food production problems of developing nations.

The establishment of modern agricultural practices requires large inputs of energy, fertilizers and pesticides. Sharp increases in petroleum prices and an anticipated shortage of fossil fuels, however, raise doubts about whether the energy intensive methods of food production used in the more developed countries can be either sustained or transferred to other parts of the world (Pimmentel and Pimmentel, 1980; Wittwer, 1975). Chemical fertilizers do not enrich the soil in the long run, but are quickly leached out. Unless organic matter content is maintained so as to preserve moisture and nutrients, soils fertilized only with chemical fertilizers become increasingly powdery and subject to erosion. At the same time, restraints on the use of agricultural chemicals, enacted out of concern for environmental quality and the safety of human health, may further limit yield increasing practices (Timmons, 1979). Recent attempts to introduce new methods of crop cultivation and management have not enjoyed uninterrupted success. Failures have been traced to such factors as cultural predispositions to resist innovation as well as lack of infrastructural support (Awa, 1980). Furthermore, in the process of selectively breeding for certain plant qualities, other equally important qualities may be bred out of high-yielding varieties. The genetic homogeneity of most high-yielding crops makes large regions vulnerable to destruction by a single disease

(Enzer *et al.*, 1978).

The prospect for developing new sources and techniques of food production is uncertain. Although advances have been made in many areas including plant genetics, hydroponics and caprohage, no major breakthrough is likely to change the basic nature of the food and population problem (Hopkins *et al.*, 1982). In the developed nations, a large research effort focuses on new methods of keeping production costs in check and maintaining the variety, quality and convenience of food products that consumers have come to expect. In the underdeveloped world, the main issue is not developing new technology, which may be beyond its economic means and technical competence, but modifying existing technology appropriate for their circumstances (Woods, 1981).

Extensive food problems are not a new global feature. Throughout history, lack of food due to natural and man-made disasters has killed millions (Dando, 1980). In Northeast Africa, tribal warfare and superpower rivalry superimposed on a drought-stricken primitive agriculture, are now creating another tragedy. The world, however, is not close to universal famine. There is enough food now produced to feed the world's hungry (Alexandratos *et al*, 1983; Hopkins *et al.*, 1982). The world's serious nutritional problem presently arises from the uneven distribution of the food supply among countries, within countries and among families with different levels of income - not agricultural limits. Nevertheless, there is growing concern for the long-term availability of land and nonland resource inputs which delineate food production capacity. With the uncertainty surrounding the capacity to increase food production capacity by expanding cropland area, increasing crop yields or developing new food sources, land is aquiring greater significance as a production input.

The conversion of agricultural land to urban uses is now viewed by many as an emerging constraint on food production capacity (Bentley, 1978; Science Council of Canada, 1979; Briggs and Yurman, 1980). Although the acreages involved are small in relation to the total land area, collectively and cumulatively these land use shifts are reducing the world's supply of high quality farmlands. The preservation of agricultural land is now the focus of contemporary land policy in several nations including Britain (Best, 1976), Japan (Cathelinaud, 1980), Egypt (Parker and Coyle, 1981), The Netherlands (Miner and Chorich, 1979), and the North American breadbasket nations, the United States and Canada (Furuseth and Pierce, 1982).

In North America, concern over the impact upon food production capacity of agricultural land conversion to urban uses has been sufficiently strong for local, state and provincial governments to implement policies to protect agricultural land from urban encroachment (see, for reviews, Bryant and Russwurm, 1982; Lapping, 1980; Platt, 1977). Nevertheless, there is substantial disagreement among investigators as to the severity of the conversion problem. Many hold that the supply of agricultural land is extremely limited and that strict controls on urban development are needed (Bentley, 1981; Collins, 1976; Science Council of Canada, 1979). Others maintain that the loss of agricultural land to urban uses in North America will not detrimentally affect the capacity to meet future food demands, and that public policy intervention is not needed (Barron and Dickinson, 1976; Fischel, 1982; Hart, 1976). One factor contributing to this controversy, is the debate surrounding the role of the land market in deciding land use.

2.2 Market Failure

Land use decisions in North America are made within a modified free market system. Changes in land use are determined by factors of supply, demand and, to varying degrees, public policy. Many believe that prevailing market forces should be permitted to decide land use entirely, emphasizing the rights of the individual or corporate owner (see, for example, Gramm and Ekland, 1975; Pasour, 1972; Siegan, 1975). In a free market economy with private ownership of land and no restrictions on land use, market forces would allocate land to those uses yielding the highest economic returns. In such an environment, market forces would assure an optimum allocation of land resources with respect to the different needs of society and there would be no need for land-use planning (Walter-Jorgensen, 1978).

It could be argued that the differences in crop productivity between good farmland and mediocre farmland should be reflected in land prices, and that as the demand for food increases relative to the demand for housing, this discrepancy in prices will discourage the development of high quality agricultural land for nonagricultural uses. There are several reasons, however, why markets in land may not automatically achieve socially desirable outcomes. Private resource decision-making tends to focus on short term needs and benefits, which in the long run may not be in the overall public interest (Lee, 1981; Raup, 1976; Wood, 1976). Private land owners are concerned with particular parcels for particular uses, and have little reason to worry about the fixed nature of total supply. The failure of the market to allocate resources to their best social use is the result of external costs and benefits not being reflected in market prices.

At present, even the most favourable physical conditions for agriculture do not provide levels of return that enable farmers to pay for land what urban uses can offer (Healy and Short, 1979; Rodd, 1976). Market prices, however, do not reflect important externalities. For example, cultivating high quality agricultural land, may produce less water pollution or soil erosion than cultivating marginal land, but the social advantage associated with this is not reflected in relative land prices. As well, the market price of agricultural land does not take account of such long-term social considerations as possible food shortages twenty or thirty years hence.

The market assumes that resources are free to move between alternatives uses as needed. Yet, once land is committed to urban uses it is virtually impossible for it to be returned to agriculture because of high transformation costs (Harriss, 1980). By the time that economic demands for food have raised food production incentives, the suppply of high quality agricultural land may already be too small (Castle, 1982). Thus, the market presents a problem in that the relative ability of agriculture to compete for land may not be consistent with its importance in providing certain goods and/or amenities to society. In such cases, government intervention is needed to compensate for the effects of market failure (Brown, 1981).

Farmland preservation policies reflect the concern that the capacity to produce sufficient agricultural products is threatened. Although it is widely accepted that price distortions exist in the land market, the case for increased public intervention is not overwhelmingly strong. Despite the link between development patterns and farmland loss, it has not been adequately demonstrated that more land will be needed to produce food in the decades ahead than the market will make available (Frankena and Scheffman, 1980; Gardner, 1977). Arguments that it is better to err on the side of safety ignore the fact that this response could impose considerable and unnecessary costs on society, such as increasing the price of land for housing and other nonagricultural purposes (Siegan, 1976; Urban Development Institute, 1982).

Given both the critical importance of agriculture to the future well-being of society and this basic disagreement on the need for public policy intervention, it is essential that efforts be made to determine more precisely the extent to which a decrease in the current stock of agricultural land could impair the future production of adequate supplies of food. Before examining the types of information currently used in rural land-use planning, an overview of urbanization in Canada is presented along with a review of studies which have examined the conversion of agricultural land to urban uses. This discussion highlights the shortcomings of recent attempts to assess the significance of agricultural land losses to urbanization, and provides a foundation for the conceptual and empirical work presented later in the thesis.

2.3 The Canadian Context

For most of its history Canada has been a rural country. In 1901, thirty-four years after Confederation, 64 percent of the population was rural (Statistics Canada, 1981). Although the process of urbanization was somewhat slower to develop in Canada than in older industrial countries, since the end of the Second World War Canada's rate of urban growth has surpassed that of any other western industrial nation (Gertler and Crowley, 1977). In 1981, 76 percent of the Canadian population lived in an urban place (Statistics Canada, 1982). The many population forecasts which have been developed for Canada (see, for example, Barrett, 1982; Stone and Marceau, 1977; United Nations, 1982) all point to further increases in the total population and even higher levels of urban concentration.

Overall, Canada has a low population to land ratio — about 2.6 persons per square kilometre (Statistics Canada, 1981). Two factors, however, have evoked concern over the potential impact of continued urban growth upon agricultural production capacity. The first factor is that although endowed with a vast land area, the larger part of Canada is under arctic and subarctic type of climate and a great deal of the remainder is rocky or mountainous. This situation is documented by the Canada Land Inventory for agriculture. This classification system groups arable soils according to their potentials and limitations for sustained production of field crops. The inventory identifies seven agricultural classes, six of which have some measure of suitability for agriculture. Classes 1 to 3 are considered suitable for the sustained production of field crops, Class 4 land is physically marginal for sustained agriculture, while Classes 5 and 6 are only suitable for pasture (Hoffman, 1971).

A breakdown of the Canadian land resource by agricultural land capability class is presented in Table 2.1. Only 5 percent of Canada's total land area (922 million hectares), is free from severe physical limitations and capable of supporting crop production. A mere 0.5 percent, or about four million hectares, is rated as Capability Class 1. Although the Canada Land Inventory for agriculture does not provide expected yields for the different field crops Hoffman (1971), suggests that there is a 20 percent decrease in crop productivity between Class 1 and Class 2 land.

Some potential exists in Canada for producing crops on marginal agricultural lands. It is generally believed, however, that these lands will be difficult to develop. The climatic hazards encountered are much more severe than

TABLE 2.1

LAND CAPABILITY FOR AGRICULTURE IN CANADA

CLI Class	Hectares	Percent of Area of Canada
1	4,146,084	0.5
2	16,190,801	1.8
3	25,126,125	2.7
Subtotal	45,463,010	5.0
4	24,996,520	2.7
5	34,348,755	3.7
6	16,776,316	1.8
7	58,607,711	6.3
Organic	18,076,161	2.0
Other[1]	723,829,264	78.5
Total	922,097,737	100.0

[1] Includes unmapped areas; water areas; forest reserves; national parks; urban areas and provincial parks.

Source: McCuaig, J.D. and Manning, E.W. 1982. Agricultural Land-Use Change in Canada: Process and Consequences. Ottawa: Lands Directorate, Environment Canada.

than on most lands now being farmed, and they are remote from markets and sources of supplies; therefore production and marketing costs will be higher than on existing land (Beattie *et al.*, 1981; Geno and Geno, 1976). Furthermore, even with the application of the best farming technology available without regard to cost, such lands have production potentials of only one-quarter to one-eighth of the best land in Ontario (Bentley, 1980).

The second factor contributing to the concern over agricultural land conversion in Canada, is that there is a close correspondence between the distribution of high quality agricultural land and urban centres. Nearly 57 percent of Canada's class 1 land for agriculture lies within 80 kilometres of its 23 census metropolitan areas (Neimanis, 1979). As Canadian urban centres expand, they will do so largely at the expense of the best soils.

Previous research efforts have examined the conversion of agricultural land to urban uses from several different perspectives. Gierman and Lenning (1980) distinguish the urban from the rural-oriented researchers. The urban-oriented researchers (see, for example, Bourne and Simmons, 1979; Spurr, 1976), are concerned with the growth of urban centres and the uses made of the land absorbed. These studies generally ignore the original use and capabilities of the land which is absorbed for urban purposes. The rural-oriented researchers (see, for example, Bryant *et al*, 1981; Krueger, 1978; Russwurm, 1977), are concerned with the identification and analysis of the impacts urban expansion has upon both the agricultural industry and food production capacity. The following section reviews the rural-oriented studies.

2.3.1 Agricultural Land Conversion

A large literature examines the extent of agricultural land conversion to urban uses and its impact on both the agricultural industry and the food production capacity of the Canadian land base. This section reviews these studies and the case which has been made for the protection of agricultural land from urban encroachment.

Nationally, total farmland area in Canada, which includes improved and unimproved land, experienced only modest fluctuations between 1921 and 1980 (Simpson-Lewis and Manning, 1981). The losses of farmland in eastern and central Canada have been balanced by gains in the west. However, when productivity potential is considered, Canadian productive capacity has decreased due to losses of high quality agricultural land (Manning and McCuaig, 1981). Not all of the farmland losses can be attributed to urban development. Much of the acreage lost in the eastern provinces is due to the abandonment of marginal agricultural lands. However, it is in the urban fringe zones of southern Ontario and Quebec where the most intensive agricultural production is concentrated, and where the loss of these lands from agriculture constitutes the greatest threat to regional and national productivity (McCuaig and Manning, 1982). Just over 50 percent of Canada's population lives in areas having the best 5 percent of the nation's farmland (Williams *et al.*, 1978).

Several studies have investigated the quantity and quality of land consumed by the growth of Canadian urban centres. Gierman (1977) provides an extensive account of the urbanization of rural land between 1966 and 1971, for all urban areas with populations greater than 25,000. Warren and Rump (1981) extended this analysis by undertaking a similar study for the years 1971-76. These studies indicate that there was a national decrease in the amount of rural land converted to urban use between 1971 and 1976 compared to the 1966-71 period, but that the ratio of population change to area of land consumed was lower suggesting a trend to lower average density of development. Overall, 62 percent of the land converted to urban uses was in agricultural land capability classes one to three (Warren and Rump, 1981).

The studies by Gierman (1977) and Warren and Rump (1981) were undertaken solely to determine the amount of rural land converted to to urban uses in Canada and to compare the amount and rates of conversion in cities across the country. Although it is acknowledged that the conversion of rural lands to urban uses "has implications" for food production capacity, the nature of these implications is not investigated. They do note, however, that much of the concern over the conversion of agricultural land to urban uses has focused upon Ontario. Southern Ontario contains over 50 percent of the nation's class one agricultural land, and Ontario had the highest provincial total of rural land converted to urban uses in both study periods (Warren and Rump, 1981).

As part of an effort to assess the impact of continued urban growth upon Ontario's agricultural production capacity, several attempts have made to estimate the magnitude of future rural to urban land conversions. Nowland (1975) adopts the land consumption estimates of Crerar (1962) and Hind-Smith (1962), and assumes that every 1,000 increase in urban population requires 80 hectares of land. Applying this to a provincial population of 11.4 million, he estimates that 518,969 hectares will be lost to urban uses in Ontario by the year 2001. He concludes that this represents a loss of 7.7 percent of the soils in agricultural land capability classes 1 to 3, and a 12 percent reduction in Ontario's food producing capacity. Tosine (1979) has revised Nowland's predictions using more recent and lower population projections. He estimates that at most 327,000 hectares of rural land will be converted to urban use by the year 2001, and that this will consume only 4.5 percent of all class 1 to 3 soils in Ontario. He concludes, however, that the loss of these soils to urban uses "may have repercussions" for Ontario's food producing capacity (Tosine, 1979:39).

Smit and Cocklin (1981a) have developed future urban land requirements for each of the urban centres in Ontario over 25,000 population in 1976. They estimate that as much as 130,253 hectares of rural land will be converted to urban uses by the year 2001. They later extended their analysis (Smit and Cocklin, 1981b) to examine the agricultural capability of the land they expect to be converted under alternative growth scenarios. The scenarios consider the effects of directing urban development onto and away from the better quality agricultural lands. They conclude that regardless of the assumptions made about the distribution of urban growth across agricultural land capability classes, the proportion of class one to three land required is relatively small. Only 3.9 percent of the province's class one to three land would be affected, and only 1.9 percent of the class one land in the province.

Forecasts of land requirements for urban purposes depend upon the assumptions made about future population growth and development densities. Given the uncertainty which surrounds any attempt to describe future conditions, it is unlikely that consensus on the likely average annual rate of transfer of land from agriculture to urban uses can be achieved. What is certain is that more agricultural land will be converted to urban uses in the future, and that much of this conversion will occur near Ontario's existing urban areas - areas which contain a high proportion of the province's good quality agricultural land. Of major concern, however, is not that there are widely varying estimates of the amount of rural land needed for urban purposes. Rather, it is the lack of a suitable framework for assessing the impact of the different estimates on the capacity to meet future food demands.

The studies by Nowland (1975) Tosine (1979) and Smit and Cocklin (1982a; 1982b) allude to the possible detrimental effects which continued agricultural losses to urban uses may have upon agricultural production capacity. Yet their analyses do not provide a basis for gauging the impact such losses. Although the proportion of high quality land likely to be needed for urban development may be relatively small, such losses could be important if the soil and climate requirements of some specialty crops and nonland resource factors are considered. Cocklin *et al.* (1983) have attempted to estimate the loss in production potential associated with expected agricultural land conversions for selected crops in Ontario. They express the productivity potential of each crop on lands expected to be converted to urban uses, as a proportion of the crop's total provinical productive potential. This measure, however, considers the potential productivity of each crop separately. It does not provide a summary measure of how much crop production capacity is lost.

Some researchers maintain that Canada's best agricultural land, in terms of soil and climate, must be designated for agricultural purposes only (see, for example, Beaubien and Tabacnik, 1977; Pearson, 1974; Science Council of Canada, 1976, 1979). There are, however, two quite separate objectives expressed by most farmland preservation advocates: the protection of the land resource itself and the maintenance of a viable farm economy. At the local level, a number of conflicts in the use and management of land have arisen as a result of the intrusion of urban activities into rural areas. The countryside has become a source of space for such varied land uses as outdoor recreation, nonfarm residential development and aggregate extraction. These uses, which rely upon urban proximity, have had a number of detrimental effects upon the local agricultural industry, including higher prices for agricultural land (Bryant and Russwurm, 1979), farm fragmentation (Rodd, 1976), and reduced levels of agricultural investment (Russwurm, 1977). As well, several public sector problems have arisen concerning the provision of social and physical services and fair taxation (Joseph and Smit, 1981). These factors alone have been considered sufficient grounds for the implementation of policies to protect agricultural land from urban encroachment (Pearson, 1974).

Restricting urban growth may eliminate some of the nuisance effects which urbanization has on the agricultural industry. Maintaining a viable farm economy, however, depends largely on profitability which is affected by a host of factors unrelated to land such as interest rates, foreign markets and management. Preserving the land resource will not necessarily ensure that landowners will decide to farm it, that economic factors such as the high cost of land and equipment will permit them to farm it, or that provincial or federal programmes will provide the right incentives to keep it in farming. Much of the support for farmland preservation derives from the fact that it has become the proxy for numerous social concerns such as protecting environmental quality, containing urban sprawl and maintaining scenic beauty. However, while these problems pose significant challenges for local planning authorities, it is one matter to preserve agricultural land as part of a programme to protect the local agricultural industry, and quite another to try and justify such measures by the need to ensure that the nation's future food requirements can be met. It is this latter question which is one of the most widely disputed land-use policy issues in Canada.

Concern about the rate of agricultural land conversion and the long-term implications of continued farmland losses on food production capacity, has spurred the development of policies to control farmland conversion. The provincial governments of Newfoundland, Quebec and British Columbia have enacted specific farmland preservation policies and in five of the other seven provinces, some consideration is given to farmland preservation (for a review of provincial policy initiatives, see Furuseth and Pierce, 1982). There is concern, however, that such policies ignore the existence of legitimate urban land needs. Pierce (1981:172) states:

> "The need for the preservation of agricultural land and the need for adequate space for urban expansion are two conflicting goals in need of reconciliation. To date the development of strategies for the containment of urban growth and the protection of rural-resource lands in Canada has not been supported by systematic national or provincial analyses of the land-conversion process."

Urban expansion in Canada continues on lands having high capability for agriculture. Despite the recent surge of interest in the nature and extent of agricultural land conversion, the data available serve only to highlight areas in which land-use conflicts may be pending. They do little to assist in assessing whether agricultural land preservation policies are needed to ensure that future food requirements can be met. Nevertheless, increasing pressure on farmland has brought about provincial guidelines for preserving agricultural land. These programmes use agricultural land capability systems to identify land areas which should be protected. The limitations of these and other land classification systems for developing agricultural land-use policies are reviewed in the following section.

2.4 Planning for Agricultural Land Use

Public concern over the use of land resources has been translated into a growing acceptance of government intervention in the land-use allocation process in the form of land-use planning. Land-use planning is essentially a process of establishing and operating public institutions that modify the market allocation of land (Supalla, 1976). An essential prerequisite for public policy intervention is the development of a land and land-use information support system, which will facilitate the assessment of land-use opportunities and needs. This section examines the information provided by land classification and land-use evaluation systems, and assesses the limitations of these techniques for evaluating the need for agricultural land preservation policies.

2.4.1 Soil and Land Classification

Classification refers to the process of giving names to a collection of objects which are thought to be similar to each other in some respect (Everitt, 1974). Placing objects into categories or classes performs an essential function in the process of making sense of the world around us. It is much easier to develop classification schemes for river systems, cities, regions of the world or land-use patterns, than to consider each component as a separate entity. The primary function of classification is to construct classes about which inductive generalizations can be made (Bailey *et al.*, 1978).

Almost all resources have been classified at one time or another, and most have been classified in several different ways. Thus, it must be stressed that classifications are developed with a particular purpose in mind. There is seldom the case in which a single or "absolute" classification can be developed for a set of objects, as the purposes of the classification may vary. The best classification is, quite simply, that which best serves the purposes for which it was designed (Sokal, 1974). Generally, the objective of scientific classification is to establish categories within which the degree of association among members is high and between which association is low.

Soil classification and survey involve the collection, organization and interpretation of the kinds and distribution of soils over the surface of the earth. The collection of this information is done in the soil survey, whereas its organization and interpretation are the work of soil classification. Classifications that describe soils and show their interrelationships are called taxonomic. Those that group soils according to their suitability for one or more uses are called interpretive (Steele, 1967).

The standard soil survey identifies the important characteristics of soil, classifies them into defined units and delimits their boundaries on maps. These maps are intended to suit the purposes of users with widely different problems and, therefore, contain considerable detail to show basic soil differences (Westerveld and van den Hurk 1973). It is useful to distinguish between the concepts of soil characteristics and soil qualities. Soil characteristics are those aspects that can be observed directly in the field or measured on samples in the laboratory; for example, texture, structure, stoniness. Soil qualities are those aspects which can be inferred from the observable characteristics; for example, water availability, suitability for root growth, workability (Way, 1973). In general, soil characteristics are mapped during the course of soil inventory.

Each soil has many characteristics which, in one way or another, affect its response to different types of manipulation and management. Since many uses, both agricultural and nonagricultural are made of the soil, each land use requires a separate interpretation of the basic information collected from the soil survey. The application of soil information to land-use planning is undertaken at two quite different scales. Detailed soil maps assist in the planning of specific locations for particular land uses. For instance, as an aid to urban development, soil maps indicate limitations of each soil class for housing, industrial sites, pipelines, and other common land uses (see, for example, Bartelli *et al.*, 1966; Hill, 1979; McCormack and Bartelli, 1977). Generalized soil maps are constructed from the detailed maps and are used to identify, very roughly, areas which are suitable for particular uses at the local or regional level (Bauer, 1973; Orbell, 1977).

Some argue that generalized soil maps provide a suitable basis for formulating land-use plans and initiating land-use policies. Indeed, Donahue *et al.* (1977:237) suggest that:

> "A soil survey is the <u>only</u> means of establishing a scientific basis for planning the most appropriate use of every acre or hectare of land."

This method of establishing land-use priorities is frequently applied to the resolution of conflict between urban and agricultural land uses. Essentially it is maintained that once high quality agricultural lands are identified, they should be exclusively reserved for agricultural use (see, for example, Jorling, 1978; Olson and Hardy, 1967; Orbell, 1977). It would appear, however, that the case for using soil surveys to make land-use decisions has generally been overstated. Obviously, the soil conditions in an area are an important element of the natural resource base and influence both urban and rural land uses. However, while it is acknowledged that an understanding of the physical resource base is an essential component of land-use planning, it is only a first step.

Resource inventories, such as the soil survey, do not indicate the need to establish particular land-use policies, nor do they provide criteria with which to decide which land use should be given priority in an area. Soil survey interpretation is designed to indicate alternatives or opportunities in the use of soils, not to provide recommendations for their use (Aandhal, 1958; Becker-Platen, 1979; Steele, 1967). Knowledge of soils is integral to improving the management and output from existing agricultural areas, as well as in developing new localities. This information, however, does not necessarily lead to sound land-use planning decisions. Soil maps are entirely physically based and do not consider the land demands of competing activities. This issue is addressed more fully in the following discussion of land capability indices.

2.4.2 Land Capability Indices

Land capability refers to the ability of land to accommodate a particular use or activity without permanent damage. Capability, therefore, refers to both the characteristics of the land and those of the activity or use in mind (Lang and Armour, 1980). Agricultural land capability maps showing the distribution of capability classes have been prepared in a number of countries, among them the United States (Klingebiel and Montgomery, 1973), Great Britain (Bibby and Mackney, 1969) and Canada (Hoffman, 1971). Although each capability system is geared specifically to the conditions in each country, many similarities exist between them. Generally, they assess agricultural land capability from known relationships between the growth and management of field crops and physical factors of soil, site and climate.

Some authors (see, for example, Davidson, 1980; Krueger, 1977; McCormack, 1971; Pearson, 1975), maintain that land capability for agriculture indices have an indispensable role to play in the making of land-use policy decisions. They point to the ability of such schemes to provide land-use planners and policy-makers with data for implementing such policies as directing urban development away from the better quality agricultural land. Indeed, some supporters of the Canadian agricultural land capability system would have legislation enacted that prevented any further transfer of Class 1 or 2 soils from agricultural to urban uses (see, for example, APPEAL, 1975).

The assumption underlying the call for the preservation of agricultural land, is that the best farmlands must be retained for agriculture in order to ensure that future food requirements can be met. In British Columbia, rapid population growth and urbanization, combined with concern for a limited farmland base, led to the establishment of the British Columbia Land Commission. The Commission has since designated Agricultural Land Reserves (A.L.R.'s) using agricultural land capability classes. The principal objective of the programme is to preserve land for farm use (Baxter, 1974). However, as noted by Rees (1977:34):

> "It might reasonably be argued that designation of A.L.R.'s almost solely on the basis of agricultural capability ratings is a misuse of these data, that other resource sectors and relevant socio-economic factors were not sufficiently taken into account in an integrated or comprehensive regional planning process as originally envisaged by the C.L.I.."

Although land capability classifications for agriculture, forestry and other land uses are useful for delimiting areas which are of high quality for their respectives uses, by themselves they are an inadequate data base for making many land-use allocation decisions. Land capability for agriculture systems were developed to group soils with similar management problems for interpretive purposes, not to define how much and which lands should be reserved for agriculture (Hilton, 1968; Singer, 1978). It does not follow that because an area is well suited to a particular activity it should necessarily be reserved for that use. Land which is of high quality for agriculture is frequently also of high quality for other uses, including urban development. Similarly, vast areas of land may be ideally suited for an activity for which there is little need or demand. As noted by Flaherty and Smit (1982), land capability maps highlight the conflicts between competing land uses which land-use planning is intended to solve, but they do not help to resolve this conflict by indicating the consequences of alternative courses of action.

2.4.3 Composite Land Capability

Single factor land capability systems assist in identifying land uses which might be incompatible with a land area, and those that are feasible alternatives. It has been demonstrated, however, that they do not provide land-use planners sufficient information with which to decide among alternative land uses. An approach which attempts to synthesize the information obtained from single purpose land capability classification systems is typified by the "design with nature" work of McHarg (1969). This method has been used in such diverse applications as selecting highway corridors (McHarg, 1968) and evaluating development options in coastal areas (Nehman *et al*., 1973).

The method consists of preparing a series of overlays for single factors such as topography, agricultural "value", recreational "value" and so forth. For each overlay, areas where the values are judged highest are shaded in dark tones and progressively lighter shading is used for lower values. The final map indicates areas least or most sensitive to the type or types of development or use under consideration. To make the task of overlaying several different factors easier, computer based mapping systems have been developed.

One of the more elaborate computer applications is the METLAND system (Fabos, 1979). This system evaluates the effects, benefits and costs of alternative land-use plans with respect to a set of landscape parameters such as agricultural productivity, visual amenity, flood hazard and so forth. It consists of an economic valuation procedure which assigns dollar values per acre to defined landscape parameter classes. The dollar value terms are deemed to provide a common denominator for the diverse set of landscape parameters under consideration (Fabos and Joyner, 1980). Alternative land-use plans are evaluated by producing a composite map of landscape value profiles for each plan.

All composite overlay techniques are dependent upon mapping systems, either hand drafted or computerized, and most place important variables into categories and overlay them. These methods have been criticized for several reasons, many of which are technical concerns regarding variable measurement or weighting, and the subjectivity inherent in the application of the different methods (see, for example, Gordon, 1978; Sneader, 1975). Quite apart from the technical problems associated with the use of composite overlay methods, however, are criticisms of a more conceptual nature concerning the types of problems for which these techniques are appropriate.

The major weakness of composite overlay techniques for assessing land-use priorities is that, like land-use capability systems, they are based entirely upon assessments of the suitability of land for selected uses. This information does not permit one to make predictive or normative statements about the allocation of land to different uses, because the demand for these uses is ignored (Hopkins, 1977). The importance of land for agriculture or housing, for example, would be assessed according to its inherent suitability for these uses, regardless of how much land is required to meet current or future demands for agricultural products or housing space. The limitations of composite techniques are succinctly stated by Gold (1974:286):

> "Just as there is something wrong with a market system that cannot provide adequate open space in congested cities, there is also something wrong with a planning tool that would plant corn on Wall Street simply because the land is agriculturally productive there."

Proponents of the composite overlay techniques argue that they provide a sound basis for making decisions about the allocation of land among different uses. Miller (1979), for example, argues that McHarg's method is suitable for identifying high quality agricultural lands which can then be set aside exclusively for farm use. This review has shown, however, that the establishment of land-use priorities cannot be based solely on an analysis of the supply of land of different degrees of suitability. The importance of allocating land to particular uses can only be determined by considering the demands for the different land uses which exist or which may develop in the future.

2.4.4 Plan Evaluation Techniques

The evaluation of land-use alternatives is a difficult undertaking, as multiple goals and criteria must usually be considered. Recently, attempts have been made to develop land-use planning techniques which incorporate such considerations in the generation and evaluation of alternate plans. Most of these multicriteria methods, as they are generally known, are based upon weighting systems which reflect the relative importance of decision criteria. Although several different methods now exist (see, for example, Massam and Wolfe, 1979; Nijkamp and Vos, 1976), only the basics of the approach are reviewed here.

One of the most widely known methods for evaluating alternative land-use plans, is the plan evaluation matrix which describes numerically how well specified objectives are met under each of the plans. The goals achievement matrix was originally developed by Hill (1968), and has since been revised and extended by a number of researchers (see, for example, Manning and Moncreif, 1979; Welch and Lewis, 1976). A common feature of these methods is that they indicate the extent to which a plan or set of alternative plans, will meet goals which have been agreed upon and set in advance. A matrix is defined in which a proposed plan is rated against a set of objectives or factors such as environmental or economic, to determine how each objective or factor is affected by the different plans. The matrix summarizes the information specified for a given problem, but does not make recommendations as to which of the plans considered is best. This task is left to the decision-makers.

Proponents of matrix plan evaluation methods argue that they offer an organizational framework which ensures that important aspects of a land-use planning problem are not omitted. Nevertheless, several criticisms have been levelled against these methods, concerning the subjectivity involved in calculating the measures of performance for different strategies or plans, and the trade-offs which must be made between comprehensiveness and matrix manageability (see, for example, Kettle and Whitbread, 1973; Poulton, 1982). The concern of this review, however, is not with the technical issues peculiar to each method, but with the more general problem of goal selection which is fundamental to all multicriteria methods.

The evaluation of alternative plans is undertaken on the basis of goals which have been set in advance. The question arises, however, as to whether or not the particular goal set selected is valid. For example, given the concern which exists over the conversion of agricultural land to urban uses, it may seem reasonable to include the goal of preserving good farmland. The matrix plan evaluation methods, however, are designed only to indicate how well the different plans conform to this objective. They do not, and were never intended to, address the important antecedent question of whether such a land-use goal is necessary or desirable.

Matrix plan evaluation methods are appropriate for problems in which planning goals have been previously formulated and the rationale for their selection justified. It is this latter problem of providing an adequate basis for the establishment of land-use goals, however, which underlies the debate over the possible conflict between urban expansion and land needs for agriculture. The following section outlines the type of information which is needed to assess the adequacy of the agricultural land base for meeting future land-use demands.

2.5 Information Needs for Land-Use Planning

Public concern over the use of land resources has increased in recent years. It is realized that land resources are relatively scarce and require improved management. Although planning is underpinned by the assumption that adequate information exists for public authorities to make decisions about the "best" use of land in the future (Wildavsky, 1974), land use and other resource policy-makers generally lack the the kinds of information they need to make sound decisions. They are typically confronted not only with a large number of resource use alternatives, but an inadequate understanding of their future implications (Gibson and Timmons, 1976; Sewell, 1973). There is a need, then, to develop techniques which can enhance the capacity of decision-makers to assess both the adequacy of the land resource for meeting future land-use demands and the alternative means by which these demands could be met. This section outlines the information and analysis that could provide a basis for addressing these issues.

The controversy over the impact of agricultural land conversion on food production capacity largely stems from two sources. A major factor is that assessments of the gravity of the problem vary greatly owing to differences in assumptions about future levels of population growth, food needs (both domestic and foreign) and resource availability. Of greater concern to this thesis, however, is the lack of a suitable framework for evaluating the impacts of the different estimates. Many of the techniques used by land-use planners and policy-makers are simply inadequate for assessing the long-term implications of agricultural land losses on the ability to meet future food needs. There is a need, then, not just to identify and quantify the major variables which influence land use, but to develop a means to integrate land and nonland resource factors so that the long-term consequences of agricultural land conversion to urban uses can be identified.

With the increased awareness of the need for a systematic consideration of the factors which affect land use, attention has focused on the development of mathematical models to explain and predict land-use activity. Broadly, such models consist of one or more statements, expressed in mathematical terms, which describe the interrelationships among variables. The assessment of future food production capacity, subject to constraints on such factors as crop productivity, energy use and land availability, is consistent with mathematical programming techniques. These methods have been widely applied in the analysis of resource allocation problems, as they provide a framework for identifying the conditions under which different levels of demand can be met and those under which a resource scarcity problem arises. Many agricultural land-use programming models have been developed to assess food production potential and the implied resource base necessary to reach this potential. These models are reviewed in Chapter 3.

Information about the adequacy of a given land base for meeting future food needs is an essential prerequisite for formulating land-use policy. In assessing the need for particular land-use policies and/or identifying the areas in which they should be enacted, however, additional information may be needed. Two related concerns identified earlier in this thesis are the amount of production capacity used in meeting given demands, and the extent to which the fulfillment of expected food needs is dependent upon particular land areas being available to agriculture. These issues are discussed in the following sections under the headings of land-use flexibility and criticalness respectively.

2.5.1 Land-Use Flexibility

The concept of flexibility has appeared in microeconomic theory in relation to choice of plant and investment portfolio (see, for example, Goldman, 1974, 1978; Kreps, 1979). In this context, flexibility refers to the tendency of policy-makers to seek reversable or adaptable decisions (Marschak and Nelson, 1962). It is argued that in an uncertain world, the extent to which alternative plans or policies foreclose future options is an important consideration in selecting a preferred course of action from a set of alternatives.

A similar notion of flexibility has appeared in the land-use planning literature. Friend and Jessop (1971) note that land-use planning is essentially a process of generating and evaluating alternatives under conditions of uncertainty. The development of alternative strategies is based upon future estimates of such factors as population growth, resource availability, technology and political priorities. Consequently, there is concern that decisions taken on the basis of such forecasts be sufficiently flexible to take account of future divergences between observed and forecast conditions (Lundqvist, 1982).

Concern over the conversion of agricultural land to urban uses has arisen due to uncertainty surrounding the capacity to meet future food needs. This concern is further compounded by the fact that returning urban land to agricultural uses is difficult due to the high costs of reclamation. It is the twin presence of uncertainty and irreversibility that makes flexibility an attractive feature of a planned course of action. In its final report to the United States Department of Agriculture, for example, the Committee on Prime Lands (1975:6) states:

> "Public interest will be served by maintaining a maximum flexibility of options with respect to future land use needs in a changing and uncertain world."

The uncertainty about the effect of urban expansion on the ability to meet future food needs has contributed to calls for the protection of the most productive agricultural lands as a means of retaining land-use flexibility. This begs the question, however, of the degree of flexibility society is currently afforded in its use of land. Proponents of programmes to preserve farmland in order to maintain land-use flexibility, have little knowledge of how much flexibility presently exists, let alone the extent to which changes in food needs and/or resource availability will constrain future land-use options. It is this information, however, which could prove useful in evaluating the need to choose a particular course of action such as preserving agricultural land.

As society's food needs grow and as more agricultural land is converted to urban uses, some degree of land-use flexibility is likely to be lost. A means is required, then, to gauge the extent to which available production capacity is used, and to indicate the effect of such changes as reductions in the amount of land available to agriculture due to urban encroachment, or increases in food demands, upon the range of land-use options. Several methods have been developed to assess the amount of flexibility associated with particular decisions (see, for example, Marschak and Nelson, 1962; Merkhofer, 1977; Rosenhead, 1980). These techniques are designed, however, to address the question: which of the alternative plans or policies under consideration is most adaptive to changes in future conditions? As such, the interpretation of flexibility on which these methods are based is not congruent with that adopted in this thesis.

The amount of flexibility afforded society in its use of agricultural land is affected by a wide variety of factors. Among these are the availability of land and nonland resources, the level of population growth and technological advances in agriculture. Many efforts have been made to incorporate these and other considerations into agricultural land-use programming models. The suitability of mathematical programming techniques for assessing the amount of flexibility available in the use of land under given conditions, is examined in Chapter 3. A related concern of this thesis is to assess the importance of different types of land in meeting specified crop demands. This notion of land-use criticalness is examined in the following section.

2.5.2 Land-Use Criticalness

The potential for growing crops is not uniformly distributed over the land base. Some areas are more limited than others in both the range of crops which can be grown and the anticipated yields for those which can be produced. As indicated in section 2.4.2, interpretive soil classification systems have been developed to identify homogeneous parcels of land and to rate these parcels according to their suitability for different uses, especially agriculture. Although many argue that high quality agricultural land is a strategic resource and that it is crucial that as large an acreage as possible should be reserved for agriculture, the identification of critical agricultural areas cannot be based solely upon an assessment of the supply of land of different degrees of suitability for agriculture.

Classification maps are convenient for summarizing the potential agricultural use of specific areas. Alone, however, they contribute little towards the fundamental issue of whether or not agriculture should have priority over other land uses. The extent to which particular types of land are of critical importance for agriculture must be defined in terms of two sets of factors. One set is associated with the increasing demand for food. The other set consists of factors that limit agriculture's capacity for responding to increased food needs. A measure of land-use criticalness is needed which would indicate whether there are land areas of critical importance for agriculture in the sense that the fulfillment of agricultural commodity demands hinges upon particular land areas or types of land being available to agriculture.

2.6 Summary

For the forseeable future, there is no end in sight to increasing population levels and to escalating needs for agricultural production. Historically, increases in agricultural production have come from increasing the lands under cultivation and from raising yields per hectare. With these options now viewed as increasingly limited and expensive, concern has arisen over the adequacy of the current stock of agricultural land for meeting future food needs.

The preservation of agricultural land is now the focus of direct and indirect government policies in several nations. Despite the fact that land resources are finite, however, the need for public intervention continues to be a source of controversy. Several attempts have been made to determine the nature and extent of rural to urban land conversion and to assess the consequences of such conversion for food production. To date, however, no effective method has been developed for evaluating the significance of agricultural land losses so that the need for public policy intervention can be established.

In order to determine if and where agricultural land needs to be protected to ensure adequate food production capacity, policy-makers require information on the capacity of the land base to meet agricultural commodity requirements under a variety of future conditions. This includes data not only on the ability of land to meet the demands (feasibility), but also on how much production capacity is used in meeting these demands (flexibility). Information is also needed on the extent to which the fulfillment of given demands hinges upon particular land areas or types of land being available to agriculture (criticalness). Chapter 3 examines the suitability of mathematical programming models for addressing these issues.

CHAPTER 3

LAND-USE PROGRAMMING MODELS

This chapter reviews the use of mathematical programming models in research on urban and regional planning and summarizes the current state of conceptual and empirical work in agricultural land-use programming. It also examines the suitability of programming techniques for evaluating the capacity of land and related resources to meet future food needs, and for assessing agricultural land-use flexibility and criticalness. For a review of mathematical programming theory, the reader should consult a standard text such as Hillier and Lieberman (1980) or Wu and Coppins (1981).

3.1 Land-Use Planning Models

A problem common to all land-use planning is the difficulty of gaining an adequate understanding of the many activities, locations and interactions which characterize the land-use system (Turner, 1975). The growing acceptance that the intuitive approach to the resolution of complex problems may not always be sufficient in itself has brought about the application of more systematic methods (Chadwick, 1971).

A distinction is usually made between positive and normative models. Positive analysis concerns what is, what was, and what will be the consequences of any change in circumstances. Many mathematical models have been developed to explain or predict land-use activity. Many models, such as the Lowry model which determines population and non-basic employment from exogenously determined basic employment, or the gravity model which is used in land-use and travel demand studies to describe patterns of travel or activity location among zones, are of a non-optimizing type. They generate land-use configurations for alternative sets of assumptions about future conditions, but do not evaluate the desirability of each pattern.

It is often the case that planning agencies are interested not only in forecasts of future land development, but in identifying a land-use alternative which best conforms to a desired state represented by development goals (Openshaw, 1978). Normative models are concerned with what "ought to be", consistent with the criterion of optimality specified in the objective function and the set of constraints incorporated. This modelling approach essentially relinquishes the study of existing spatial patterns, although discrepancies between actual and optimal patterns can be compared as a measure of spatial inefficiency (Beaumont, 1979).

Both single and multiobjective programming methods have been utilized in land-use planning. Of the two programming frameworks, single-objective models are by far the most developed and widely used. The following section presents the general form of single-objective optimization models.

3.1.1 Single Objective Models

Single objective programming techniques allocate limited resources among competing activities, so as to achieve a stated objective or goal. They identify the "best" solution available in terms of the objective and information specified to define a system (Williams, 1978). The basic structure of a single objective optimization model is either to maximize or minimize an objective function while satisfying a set of constraining conditions. The objective function is a mathematical statement of the overall goal of the analysis, stated in terms of the decision variables which are the unknowns in the problem. The set of constraints represent conditions which must be satisfied in determining levels for the decision variables. The mathematical statement of the general optimization problem is the following. Find the values of the decision variables (x), that optimize the function:

$$\text{Min. or Max. } f(x_1, x_2, \ldots, x_n) \quad (3.1),$$

subject to:

$$g_1(x_1, x_2, \ldots, x_n) \leq b_1 \quad (3.2),$$
$$g_2(x_1, x_2, \ldots, x_n) \leq b_2$$
$$\vdots \qquad \vdots$$
$$g_m(x_1, x_2, \ldots, x_n) \leq b_m$$
$$x \geq 0 \quad (3.3).$$

There are many methods for solving this problem. Each solution method embodies a set of assumptions about the nature of the objective function to be optimized (linear versus nonlinear) and its constraints (linear versus nonlinear, continuous versus integer). Since linear programming methods are the most widely used, they provide the basis for the following discussion.

It may or may not be possible to satisfy all the constraints simultaneously. If the solution set is empty, there is no feasible solution and a resource scarcity problem exists. In many problems, however, an infinite number of feasible solutions exists. All combinations of the decision variables which satisfy the constraints are contained in the feasible region. These combinations are candidates for the optimal solution, which is the feasible solution having the largest or smallest value of the objective function, depending on whether the problem is one of maximization or minimization (Wu and Coppins, 1981).

Assuming feasibility, the optimal value of the objective function is unique. However, the set of decision variables which yield the optimal value of the objective function is not necessarily unique. Two conditions must be met in order for alternative optimal solutions to exist. First, the objective function must be parallel to a constraint which forms an edge or boundary of the feasible solution space. Second, the constraint must form a boundary in the direction of optimal movement for the objective function (Paris, 1981).

Programming models are usually formulated to assist [illegible]ing a course of action. The coefficients [illegible] although treated as constants, are often based [illegible]ctions of future conditions; this introduces uncertainty. Once an optimal solution is found, sensitivity or post-optimality analysis is usually undertaken to examine the manner in which the values of the decision variables change as the parameters of the model change. At its simplest, it consists of calculating the rate of change of the objective function with respect to a given parameter. Parameters which cannot be altered much without changing the optimal solution can be estimated more closely, and a solution selected which changes minimally over the likely values of these parameters (Hillier and

Lieberman, 1981).

Optimal solutions of linear programming problems provide calculations of dual activity values for both constraints and solution variables. Dual values pertaining to constraints indicate the amount by which the objective function could be further increased or decreased if a given constraint was relaxed by one unit. Only binding constraints report non-zero dual values since non-binding constraints have no effect on the value of the objective function. Dual values pertaining to decision variables indicate the effect of changing the level of any variable on the objective function. The optimal solution of a linear programming problem reports a non-zero dual value for any variable that is not allocated in the optimal solution. Duals for solution variables are zero, as these variables already yield optimal values of the objective function.

3.1.2 Programming Methods and Land-Use Planning

One of the traditional concerns of land-use planning is that of searching for an activity or land-use configuration over space that meets certain prescribed goals, and at the same time is feasible in terms of the constraints imposed by local topography, economic conditions, and any number of planning constraints which might be identified (Coelho and Williams, 1978). This section examines the rationale for the use of programming methods in land-use planning, and some of the difficulties associated with their use and interpretation.

Depending upon the assumptions made, programming models can be used in different ways. In some studies, the primary concern may be to isolate the effects of individual parameters in order to gain a better understanding of the interactions among plan objectives and constraints, rather than to identify a final solution to a problem. In other analyses, programming models are used to predict the characteristics of a system under different public policies, levels of population, demand and other parameters (see, for example, Herbert and Stevens 1960; Walsh *et al.*, 1981). Whether a programming model should be considered and used for normative or positive analysis should be judged not solely from its structure but rather from the manner in which the results are interpreted.

As a set of procedures that identify an optimum configuration of land uses, it may be tempting to view a particular plan as being the most desirable one for implementation. This straightforward interpretation of "the optimal solution" is common in business and operations research applications. These studies, however, are typically concerned with problems such as the efficient scheduling of industrial machine time in which the constraints are relatively obvious and the objective function is not subject to dispute. This is not the case in the kinds of problems with which land-use planners and many other public sector decision-makers are usually concerned. Most public sector planning situations differ from the more traditional types of programming problems in two important respects: first, they deal with extremely complex phenomena, and, second, a single overriding objective is always difficult, if not impossible, to specify (Liebman, 1976).

The land-use system consists of a large number of interrelated variables and many competing interests, which lend to it a great deal of complexity. In any modelling exercise, there are inevitably important trade-offs to be made between the realism of the representation and the tractability of the resulting model (Williams, 1979). The empirical programming problem is also complicated by the existence of quasi or slightly sub-optimal solutions which may only slightly change the value of the objective function. This aspect of spatial allocation models is important, as several only slightly sub-optimal solutions might exist, yet the results could have very different spatial implications for the system under study (Beaumont, 1979).

The basic requirement for the successful application of programming methods, is a well defined objective. The overall objective of planning is clearly related to improving social and economic welfare. However, the specification of a meaningful public welfare function is a question which no social scientist would claim to have solved (Ben-Shahar *et al.*, 1969). There are many different definitions and measures of public welfare which could be employed, depending upon the manner in which a problem is perceived by the analysts or decision-makers. Such definitions depend on the type and scale of the problem, and the political, economic and social framework within which planning operates (Roseth, 1970). As noted by Parry-Lewis (1969:11):

> "The objective function is objective only in the sense that it expresses an objective - not in the sense that it has been objectively chosen."

A common concern underlying the search for an objective function, is to select a criterion with which to identify the best course of action from all possible alternatives. Selecting an appropriate objective function is a major and often controversial task (see, for example, Openshaw and Whitehead, 1978, 1980; Willis and Thompson, 1981). The absence of a single widely accepted measure of public welfare implies that the same problem can be modelled in many different ways; ways that will doubtlessly yield very different solutions. Together with the information loss associated with identifying and reducing the more crucial aspects of a problem to a limited number of equations, the optimal solution is unlikely to be "the" best solution for a planning problem, as it is optimal only for the problem as it is presented in the model, and not as it exists in reality (Roy, 1976; Hopkins, 1977). This has led to the development of methods capable of considering multiple objectives, or which identify a range of feasible planning alternatives rather than a single "optimal" solution.

Programming techniques do not represent a panacea for the analysis of land use issues. Nevertheless, their ability to integrate a broad range of planning considerations makes them a tool that can be used to assess whether urban development will compromise the capacity to meet future food needs. However, although the search for the best solution out of all possible alternatives is suitable for many land use and public policy issues, it may not be the most appropriate approach for assessing how much flexibility is available in the use of land or in determining whether some land areas are critically important for agriculture. These issues are addressed in the following reivew of agricultural land-use programming models.

3.2 Agricultural Land-Use Programming Models

Agricultural land-use programming models have been developed in several countries to provide information on changes in agricultural land use in response to changes in technology, economic conditions and agricultural policies, in order that the regional and national consequences of the adjustments can be determined (see, for example, Buckwell and Thompson, 1978; Condos and Cappi, 1978; Duloy and Norton, 1973; Framingham *et al.*, 1978; Monypenny and Walker, 1976). The widespread application of mathematical programming techniques to the analysis of agricultural production patterns has produced a vast literature. Although the specification of individual models varies, so as to account for different agricultural systems and research emphases, a number of common charcteristics in model formulation are evident.

To date, agricultural land-use programming studies have emphasized the identification of economically efficient agricultural production patterns for farm commodities, subject to constraints on export and domestic food demand, land and nonland resource availability. Perhaps the best known land-use programming studies in agriculture are the large interregional programming models developed in Iowa at the Center for Agricultural and Rural Development (CARD). The works of Heady and Egbert (1962, 1964), were the forerunners of a family of models concerned with the national allocation of agricultural resources in the United States. The original models have been expanded to include a wider range of agricultural commodities, soil loss restricions, water use and other factors related to agricultural production (see, for example, Boggess and Heady, 1981; Heady and Nicol, 1976; Heady and Srivastava, 1975).

The main objective of the CARD models is determine whether the United States can meet future domestic food needs and export requirements, under various settings of resource availability. The models indicate the combination of production activities which minimize the total national cost of producing and transporting a predetermined quantity of agricultural commodities. Costs include labour, machinery, pesticides, fertilizers, water, energy and land rent. This cost minimization procedure is subject to a set of constraints corresponding to land, water and energy supplies by regions. Two sets of regions are used in the analyses - producing areas and market regions. Producing areas represent internally homogeneous conditions with respect to production possibilities. Factors considered to determine these production possibilities are soil type, climate, historic yields and production costs (Chowdhury and Heady, 1979). Market regions are contiguous producing areas aggregated into major marketing areas of the United States.

The CARD models simulate production equilibrium in that the supply price of each crop commodity must cover the cost of producing that commodity in each rural area. Market equilibrium is simulated in that the quantity of each commodity supplied must equal the demand for that commodity in each consuming area. The basic structure of the models can be summarized as:

$$\text{Min. } f(c)=\sum_i\sum_j c_{ij}X_{ij} + \sum_m\sum_f\sum_r T_{mfr}Z_{mfr} \quad (3.4),$$

where:

c_{ij} is the cost per acre of producing the j-th crop in the i-th producing area;

X_{ij} is the number of acres of the j-th crop in production in the i-th producing area;

T_{mfr} is the cost of transporting one ton of the r-th commodity to (from) the m-th demand region from (to) the f-th demand region ($m \neq f$);

Z_{mfr} is the tons of the r-th commodity transported from (to) the m-th demand region to (from) the f-th demand region.

Production of the commodities is restricted by the total cropland available in each producing area:

$$\sum_j X_{ij} \leq L_i \quad (3.5),$$

where:

L_i is the total acreage of land available for crop production in the i-th region;

and is subject to regional requirement constraints of the form:

$$X_{ir}Y_{ir} \pm Z_{mr} \geq D_{mr} \quad (3.6),$$

where:

D_{mr} is the tons of the r-th commodity demand in the m-th consuming region.

Y_{ir} is the yield in tons of the r-th commodity in the i-th producing area.

Finally, there are the usual nonnegativity assumptions of:

$$X_{ij} \geq 0; \; Z_{mr} \geq 0 \quad (3.7).$$

For computational reasons, most CARD models have used linear programming methods. However, alternative model formulations based upon quadratic programming have also been developed (see, for example, Meister *et al.*, 1978).

The CARD models are used for normative planning and impact analysis rather than positive prediction. The spatial configuration of crops identified is the most cost-efficient agricultural production pattern that meets the nation's food and fibre requirements at prescribed levels of environmental quality, consumer demand, export levels and other policy or market and technology parameters (see, for example, Dvoskin and Heady, 1976; Heady and

Short, 1981; Vocke and Heady, 1978). By varying the assumptions, production capacity under conditions of limited land, water and other factors can be assessed.

The effect of agricultural land conversion on food production capacity has been examined within the CARD programming framework. Spaulding and Heady (1977) have estimated the acreage of agricultural land needed for urban and other nonagricultural purposes in the continental United States to the year 2000. This study suggests that the largest acreage likely to be converted from agriculture to other uses by the turn of the century would represent a 2.2 percent reduction in the nation's existing agricultural land base. It concludes that overall the nation's productive capacity in agriculture would not be impaired significantly by the loss of these lands. Nevertheless, it notes that the impacts at regional and local levels may well be significant. In particular, the withdrawl of agricultural land for nonagricultural purposes would make land relatively more scarce and commodity prices would rise to reflect the increased scarcity of land. As well, a shift in the location of crop and livestock production to other areas would be required, which would subject arable lands in nonurban regions to more intensive use through increased dependence on such land substitutes as irrigation water and chemicals.

The effect of converting agricultural land to urban and other nonagricultural uses upon agricultural production capacity is influenced by factors other than the physical removal of land from production. For example, if water or chemical shortages developed, or if higher environmental quality controls were imposed, the effect of conversion would be severe if nonurban areas could not make up the loss in production due to these other resource limitations. These types of issues could be readily addressed within the CARD programming framework. In addition, however, Spaulding and Heady (1977:92) note that:

> "... there are many areas of critical concern to ecologists, environmentalists, and others that, if destroyed, could not be replaced at any cost."

The term "areas of critical concern" can be applied to a wide variety of physical, economic and social conditions. The common characteristic, however, is that the significance of particular land areas extends beyond the boundaries of the communities in which they happen to be located. This is not to suggest that critical areas, however defined, should be preserved once they are identified. However, it seems important that land use policy-makers have information to indicate which land-use decisions warrant special consideration as they affect regional or national interests.

Spaulding and Heady acknowledge that some land areas may be of critical concern from an agricultural perspective because of unique site characteristics. They note that the production of specialty crops, such as citrus fruits, cannot be readily relocated because of unique climatic requirements, and that the loss of some agricultural areas to other uses could have a detrimental effect on production capacity. Their study, however, does not provide information about which croplands are irreplacable. Nor does it indicate the extent to which the loss of particular land areas would reduce production capacity. Without such evidence, it is difficult to argue that agricultural land needs to be preserved.

Heady and Spaulding (1979) have examined the consequences of alternative land-use programmes in the North Central Region of the United States upon the interregional distribution of food production and upon the supply and export capacity of American agriculture. This region contains over 64 percent of the nation's prime cropland. They considered land-use scenarios that included an alternative in which agricultural land capability classes one and two had to remain in agricultural use, as well as a base solution in which no agricultural lands were protected from nonagricultural pressures.

This study also concludes that the supply of agricultural land in the United States is so large relative to domestic food demands that the United States is afforded considerable flexibility in its selection of land-use alternatives. That is, many different land-use options could be pursued without detrimentally affecting its ability to meet future food and fibre requirements. Many crops can be shifted from one area to another and, within an area, crops can be substituted for each other. They also note that because of the productivity differences between prime cropland and cropland in other capability classes, production capacity will be greater under the prime lands alternative than under any other alternative. How much larger production capacity would be, however, is not investigated. Their results simply indicate that some more flexibility exists in the prime lands alternative. Furthermore, as in their previous study, the authors do not investigate the extent to which the attainment of food supply objectives is dependent upon particular land areas being available for agriculture. Excess production capacity is interpreted as sufficent evidence that farmland protection policies are not required. Yet this ignores the fact that some crops are produced on lands where special conditions are required for production and that these crop land areas may comprise a significant part of the total production of some commodities. Under such conditions, it can be argued that these areas are critically important for agriculture.

The studies by Heady and Spaulding identify and integrate what are perceived to be the more important components of the agricultural land-use system. They demonstrate the suitability of a mathematical programming framework for investigating the capacity of land and related resources to accommodate anticipated land-use demands. However, although they raise the issues of how much flexibility exists in agricultural land use and whether there are land areas of critical importance for agriculture, they do not measure the range of land-use options available under different sets of conditions nor identify critical agricultural areas.

The assessment of whether continued urban development will compromise the ability to meet future food needs, does not require that a "best" or recommended use be identified for land areas. Rather, it requires information on the capacity of land and related resources to meet future urban and agricultural land-use needs, the land-use options available under different sets of conditions and the extent to which the fulfillment of agricultural commodity demands hinges upon particular land areas being available to agriculture. Traditional single-objective programming models in agriculture work quite well in assessing the ability to meet future food needs under various settings of resource availability. They have yet, however, to be used to examine the issues of land-use flexibility and criticalness as defined in this thesis. These issues are further examined in the following review of multiobjective programming methods.

3.3 Multiobjective Programming Models

In many resource planning problems it is often difficult, if not impossible, to specify a single overriding objective, as several conflicting and noncommensurable objectives can be identified. In developing a regional land-use plan, for example, policy-makers may wish to maximize economic efficiency which is measured in monetary units and at the same time minimize environmental pollution which is measured in terms of pollutant concentration. Since the imposition of a single-objective modelling framework on such problems is viewed as overly restrictive and unrealistic, interest has developed in applying multiobjective programming methods (see, for example, Barber, 1976; Das and Haimes, 1979; Loucks, 1977).

Multiobjective programming represents a generalization of single-objective approaches to resource allocation problems. It is suitable for investigating the relationships among a set of desired goals and the resources available to achieve them. The general multiobjective programming problem with n decision variables, m constraints and k objectives is the following:

$$\text{Max. } \underline{Z}(\underline{x}) = \text{Max. } [Z_1(\underline{x}), \; Z_2(\underline{x}), \; \ldots, \; Z_k(\underline{x})] \qquad (3.8)$$

$$g_i(\underline{x}) \leq 0 \quad (i = 1, 2, \ldots, m) \qquad (3.9),$$

$$x_j \geq 0 \quad (j = 1, 2, \ldots, n) \qquad (3.10).$$

Strictly speaking, a vector cannot be optimized. A solution which maximizes one objective will not, in general, maximize any of the other objectives (Cohon and Marks, 1975). With the inclusion of a vector of objectives the notion of an optimal solution is dropped and the concept of a set of nondominated solutions (Goicoechea *et al.*, 1982) or noninferior solutions (Cohon and Marks, 1975) is adopted. Cohon (1978:70) states:

> "A feasible solution to a multiobjective programming problem is noninferior if there exists no other feasible solution that will yield an improvement in one objective without causing a degradation in at least one other objective."

A solution belonging to the noninferior set which is preferred by decision-makers is the best compromise solution.

Two classes of techniques, generating methods and preference oriented methods, have been used in the analysis of multiobjective problems. Generating techniques use the vector of objective functions to identify the subset of nondominated solutions in the feasible region. These methods deal strictly with the constraints of the problem and do not consider the preferences of a decision-maker (Ignizio, 1982). The weighting and constraint methods are the most common of the generating techniques. The weighting method identifies the noninferior set by assigning a relative weight to each objective to convert the objective vector to a scalar which is the weighted sum of the separate objective functions (Cohon and Marks, 1975). These relative weights can be varied to generate a wide range of plans which reflect different priorities. The constraint method also determines the noninferior set by solving a scalar version of the original problem, but allows the analyst to specify bounds on the objectives in a sequential manner (Goicoechea *et al.*, 1982). Once the set of nondominated solutions is identified the decision-maker can select a preferred solution by examining the trade-offs between the levels of the different objectives.

If there are several objectives the nondominated set identified by generating techniques may be quite large. Since this may represent an unwieldy number of solutions for decision-makers to compare, the second class of techniques incorporate the preferences of decision-makers within the formal solution process and identify a single preferred solution. Cohon and Marks (1975) classify these on the basis of whether the decision-maker's preferences are specified *a priori*, as with goal programmming, or are progressively articulated by interactive methods.

Goal programming requires that the decision-maker set goals or targets for each objective. The preferred solution is then defined as the one that minimizes the deviations from established goals. The aim is to satisfy each goal while not constraining the solution to the achievement of any one goal (Lee, 1972). In addition to setting goal levels, however, goal weights must be specifed. Goals are achieved according to priority, with the higher priority goals being given precedence in the allocation of resources over those of lower priority. Weights may also be needed to equalize priority for goals because the unit of measure, as well as the relative values of each goal, might implicitly weight the goal set.

The prior selection of relative weightings for objectives introduces an element of subjectivity into multiobjective programming, which raises the question of how these weighting factors should be defined. Nijkamp and Rietveld (1976) argue that an appropriate set of goal levels and weights can be approximated subjectively by professionals or by the political process. Yet, this requires knowledge about decision-maker's preferences – preferences that may be extremely difficult to establish. It also assumes that the preferences of decision-makers or the public are a suitable basis for policy making. It ignores the problem that both parties may lack adequate information with which to to set goal priorities, let alone identify an appropriate set of goals.

A major problem in applying multiobjective programming methods is that of specifying the set of objectives. Haimes *et al.* (1979), for example, have developed a multiobjective programming model for the integrated planning of water and land resources in a river basin. One of the five objectives adopted is to protect agricultural land and thereby ensure the ability to increase agricultural production. The goal of preserving agricultural land, however, is introduced in the absence of evidence to support its legitimacy. As noted in Chapter 2, the need for farmland preservation policies is the subject of heated debate. Although multiobjective programming techniques allow decision-makers to consider a variety of different land-use goals, they do little to aid the assessment of whether a particular goal is necessary or desirable. It is the antecedent problem of providing an adequate basis for establishing particular land-use goals which underlies the debate over the possible conflict between urban expansion and land needs for agriculture.

An attempt has been made to develop a measure of flexibility within a multiobjective programming framework. Morse and Lieb (1980) introduce flexibility as an evaluative criterion with which to prune the nondominated set of solutions in a multiobjective programming model to a preferred solution. They argue that in the absence of any other criteria for preferring one solution over another, that the solution affording the most flexibility, in the sense of being most adaptable to future changes, should be selected. Their approach to the analysis of flexibility is consistent with that of the measures noted in section 2.5.1 (that is, which of the alternatives under consideration is most adaptive to changes in the conditions expressed in the model). This thesis, however, is concerned with measures of the range of land-use options that exist under different conditions, rather than measures that identify the most flexible option.

The problem of selecting land-use goals *a priori* limits the use of multiobjective programming formulations to assess land-use flexibility and criticalness. As well, the emphasis in most applications has been to identify a preferred solution. There has, however, been a growing interest in developing techniques to generate alternative solutions for policy-makers to consider, rather than identifying the solution which is, in some sense, optimal. These methods are reviewed in the following section.

3.3 Modelling to Generate Alternatives

The constraints in a programming model may identify an infinite number of feasible solutions. The standard approach in land-use research is to specify an objective function to select one optimal or best solution from the feasible set. This solution, however, is optimal only in a very restricted sense. Several solutions may exist that are nearly as good as the mathematical optimum, yet specify very different decisions. Indeed, some of these solutions may be better than the mathematical optimum, if unmodelled factors are taken into account (Chang *et al.* 1982). This section reviews programming techniques which identify solutions other than the mathematical optimum, and examines their suitability for the analysis of land-use flexibility and criticalness.

The concern that the best solution to a mathematical programming problem may fall within the feasible space defined by a single or multiobjective model, has prompted the development of techniques that select solutions from the feasible region which are not the mathematical optimum. This approach represents a major philosophical shift in the use of programming methods to the formulation of a set of planning alternatives rather than the identification of a single preferred solution. Many argue that the proper role of the analyst is not to present decision-makers with "the" answer to a problem, but to provide insight and understanding which supplements that of the decision-makers (Liebman, 1976).

One way of providing greater insight is to present decision-makers with a set of alternatives that are feasible under the given conditions. They can then devise their own methods of evaluating the trade-offs amongst the different options in selecting a preferred alternative from this set. Within a mathematical programming context, this approach involves selecting a set of solutions from the feasible region for consideration.

The optimal solution in a linear programming problem is not always unique. If the objective function is parallel to one of the binding constraints, an entire set of optimal solutions exists (Williams, 1978). Such a condition frequently occurs in large linear models. In some applications, alternate optima are interpreted as an indication of flexibility in the specification of the decision variables (Wu and Coppins, 1981). That is, the decision-maker is free to incorporate subjective or secondary considerations in choosing one optimal model solution over another. This interpretation of flexibility, however, is not congruent with that adopted in this thesis which is concerned with measuring the full range of feasible options defined by a set of constraints, rather than identifying strict alternate optima with respect to modelled objectives.

One general purpose method of generating a set of feasible alternatives is to select solutions which are near the optimum in a mathematical sense. Church and Huber (1979), for example, use a measure of difference based on one initial model solution to obtain near optima. This approach is appealing if the objective of the analysis is to identify solutions which are only slightly sub-optimal for comparison. A major limitation of this approach for assessing flexibility as defined in this study, is that it restricts its search for alternative solutions to a relatively small part of the feasible region. It does not ennumerate all the feasible alternatives nor does it provide a measure of the range of options that are feasible.

An approach that indicates the degree to which the decision variables can vary, is to generate model solutions at random (see, for example, Southeastern Wisconsin Regional Planning Commission, 1973). This method selects combinations of the decision variables at random and determines whether or not they satisfy the conditions specified in the constraint set (Brooks, 1958). A number of very different feasible solutions can be identified in this manner as the search for different solutions is not confined to one part of the feasible space. In practice, however, it is difficult to generate a set of values of decision variables at random such that the solution is feasible if there are many mathematical constraints (Brill, 1979). More importantly for the purposes of this study, it does not provide a means of measuring the range of options which are open.

One criterion proposed for selecting a set of alternatives for consideration by decision-makers, is that they should be as different from each other as possible, subject to some loose constraint on quality with respect to modelled objectives. Several methods of modelling to generate alternatives (MGA) have been proposed. Brill *et al*. (1981) have developed the Hop, Skip, and Jump method. It identifies solutions that are good with respect to the objectives included in the model, and yet are significantly different from one another with respect to the decisions specified. Solutions that are "maximally" different from each other are obtained by optimizing a function that is a surrogate for difference. Since there is no perfect measure of difference among planning alternatives, a number of different measures can be used to suit the problem of interest.

Chang *et al*. (1982) have developed an efficient random generation method for identifying a set of alternatives. The method uses the original constraints of the model to ensure that the solution obtained is feasible. Targets are set for the objectives included in the model to further reduce the space to include only solutions that are good with respect to modelled objectives. The objective function is formed by selecting at random a specified number of decision-variables. The solutions obtained, however, are not random in the strictest sense that all possible solutions are equally likely to be selected. Only extreme points of the feasible space, as further constrained by the targets, are considered. It is expected, however, that the alternatives generated will be different from each other with respect to the values of the decision variables, if such differences are possible, and that these alternatives may also be different with respect to objectives not included in the model. Although this represents an innovative approach to the identification of feasible land-use alternatives, it does not adhere to the definition of land-use flexibility adopted in this thesis.

The assumption underlying the development of MGA methods is that, in general, better planning decisions can be made if widely different, but nonetheless feasible alternatives are available for inspection by decision-makers. The solutions derived can be further evaluated using criteria which may not have been expressed in the original model (see, for example, Hill and Werczberger, 1978). These methods have developed out of the awareness that the optimal solutions identified by single or multiobjective models are optimal only in a restricted sense. The MGA techniques are well suited to problems in which decision-makers must select a specific plan or course of action for implementation such as locating public facilities. By generating a number of feasible alternatives, these methods come close to the concern of this thesis, which is to determine the range of options open. As well, it might be possible to develop a measure of overall flexibility based on the difference between maximally different solutions. The MGA techniques, however, were not explicitly developed for this purpose, and no such measure is available.

3.4 Summary

Considerable interest exists in the application of mathematical programming techniques to problems of agricultural land use and food supply. The existing models are capable of analyzing a wide range of issues given concern for such factors as land quality, environmental impact and nonland resource use in the agricultural sector. These models have emphasized the identification of economically efficient agricultural production patterns, under different scenarios of resource availability. Although the issues of how much flexibility exists in agricultural land use and whether or not there are land areas critically important for meeting future food needs are raised, conventional uses of programming methods have not directly investigated the issues of land-use flexibility and criticalness as defined in thesis.

Mathematical programming techniques are well suited to assessing the capacity of land and related resources to meet future food needs. Although they have not been used to address the issues of land-use flexibility and criticalness as defined in this study, they nonetheless provide a framework within which such measures could be developed. Alternative measures of land-use flexibility and criticalness based upon mathematical programming considerations are presented in Chapters 4 and 6 respectively.

CHAPTER 4

ALTERNATIVE MEASURES OF LAND-USE FLEXIBILITY

The concept of flexibility adopted in this study refers to the extent to which available production capacity is used and the range of land-use options open under different sets of conditions. This chapter reviews alternative methods of measuring the amount of flexibility available in the use of agricultural land.

4.1 Land-Use Flexibility

The amount of flexibility afforded society in its use of agricultural land is affected by a wide variety of factors. Among these are the availability of land and nonland resources such as fuels and fertilizers, the level of population growth and technological advances in agriculture. Many efforts have been made to incorporate these and other considerations into agricultural land-use programming models. While these analyses indicate the feasibility of meeting specified food production levels, they have focused on identifying land-use configurations that are in some sense optimal. The issue of how much flexibility is available under specified conditions has not been addressed.

Expressing the conditions that affect the ability to meet future food needs in the form of constraints is a useful starting point for the analysis of land-use flexibility. For a given constraint set, it is possible to determine whether the demands placed upon available resources can be met. By varying the levels of resources and demands, the impact of expected changes in conditions on the capacity to meet demands can be assessed. What is required, however, is a means of measuring the amount of flexibility defined by a given constraint set. This measure should gauge how much production capacity is used in meeting given demands, and indicate the effect of such changes as increases in crop demands or reductions in the amount of land available to agriculture due to urban encroachment, upon the amount of flexibility available.

To set the notation for the ensuing review of alternative measures of land-use flexibility, consider the following rudimentary set of constraints:

$$\sum_{i} a_{ij} \leq A_j, \quad 1 \leq j \leq m \qquad (4.1),$$

$$\sum_{j} a_{ij} y_{ij} \geq Q_i, \quad 1 \leq i \leq n \qquad (4.2),$$

$$a_{ij},\ y_{ij} \geq 0 \qquad (4.3),$$

where:

A_j = the amount of land type j available,

Q_i = the amount of crop i required,

y_{ij} = the productivity of crop i on land type j,

a_{ij} = the amount of land of type j allocated to crop i.

Any number of constraints could be specified in a similar manner to reflect more accurately a given land-use system. This simple constraint set, however, is sufficient to illustrate how alternative assumptions about future conversions of agricultural land to urban uses or changes in food production requirements could be incorporated into the analysis. It is also adequate for assessing the role which the information provided by each of several possible flexibility measures could play in land-use planning.

4.2 Minimum Area

An obvious means of assessing the adequacy of a given land base for meeting future food needs under different conditions, is to determine the minimum amount of land needed to meet specified crop targets. This section considers the use of this information as a measure of land-use flexibility.

A "minimum area" objective function indicates whether or not different levels of crop demands can be satisfied and the minimum acreage of land required to do so. This objective function takes the form:

$$\text{Min. } Z = \sum_{i}\sum_{j} a_{ij} \qquad (4.4),$$

subject to constraints 4.1 to 4.3.

Given the minimum land area required to satisfy the constraints, a measure of land-use flexibility can be expressed as:

$$F(1) = 1 - [(\text{Min. } Z^{*})/A] \qquad (4.5),$$

where:

$A = \sum_{j} A_j$ (the total land area available).

As the value of F(1) approaches unity the amount of flexibility available increases. This simply indicates that a relatively small amount of the available land will satisfy the constraints. As F(1) approaches zero, less and less flexibility is available as larger amounts of land are required. A value of zero indicates that all of the available land must be used. If alternate optima do not exist, the acreage of each land type allocated to each crop must take the value prescribed by the corresponding solution variable in the model. If specified demands exceed production capacity, the system is infeasible and the flexibility measure is not defined.

An important issue in the analysis of land-use flexibility, relates to the sensitivity of the measure to changes in conditions such as a decrease in land area, or an increase in crop demand. The dual activity values in this model indicate the effect of relaxing the constraints by one unit on the minimum area allocated. Although the rate of change in F(1) with respect to a unit change in the area of a land type or the amount of a particular crop demanded can be calculated, a measure of land-use flexibility based upon the minimum land area needed to satisfy crop demands is of limited value. By attempting to satisfy crop demands with as little land as possible, this model allocates higher quality agricultural lands to crops at the expense of lower quality lands. Thus, it may suggest that a great deal of flexibility exists, but this could be entirely dependent upon the best lands being available to agriculture. For example, a value of F(1) equal to 0.5 would be misleading if the remaining agricultural land is unsuitable for some crops or is capable of only very limited production - say 10 percent of the total production attainable on the other 50 percent of the land base. In this situation production capacity for some crops could be very close to being exhausted which means that little flexibility exists. A measure of flexibility is needed which would provide a better indication of the extent to which available production capacity is utilized in meeting given demands.

4.3 Maximum Production

A somewhat different approach to measuring land-use flexibility is to determine the maximum amount of food that can be produced under different conditions, rather than identifying the minimum amount of land needed to satisfy expected food needs. This approach has been adopted by the Food and Agriculture Organization of the United Nations for a global assessment of countries with the greatest long-term food production potential (see Naiken, 1977). One of the objectives of their study is to estimate the human carrying capacity of the world. Carrying capacity is defined as the maximum number of persons which available land can support. A linear programming model is employed to determine the maximum amount of food that can be produced in calorie equivalents. This is then converted into human equivalents using average calorie requirements per person, for comparison with current or projected population totals. This model takes the form:

$$\text{Max. } C = \sum_i \sum_j a_{ij} y_{ij} c_i \quad (4.6),$$

subject to constraints 4.1 and 4.3, where:

c_i = calorie conversion factor for crop i.

A measure of flexibility based on carrying capacity can be expressed as:

$$F(2) = \text{Max. } C^{*}/CPOP \quad (4.7),$$

where:

CPOP = the calorie requirement of the population.

Human carrying capacity has some appeal as a measure of how much flexibility society is afforded in its use of agricultural land. It can be argued that the greater the extent to which available calories exceed population needs, the more flexibility there is in meeting these requirements. The methodology adopted by Naiken (1977), however, has two noteworthy shortcomings. Firstly, it assesses the food production potential of countries on the basis of crops yielding the largest amount of food in terms of calories. No account is taken of the fact that maximum food potential may not correspond with local dietary or agricultural patterns in a given nation. Crop production targets, such as those expressed in constraint 4.2, are omitted in Naiken's model. Thus, it could base its assessment of Canada's food production potential on the basis of corn production rather than wheat. This approach also ignores the fact that people value food not only for its calorie content but also for its taste and other nutrients. Calories per capita per day is actually a measure of food energy. It is, however, a poor measure of dietary quality. People living on a diet consisting largely of starchy foods, such as yams or cassava, are likely to be malnourished due to protein deficiency despite an adequate caloric intake (Chou and Harmon, 1979). A measure of flexibility based on calorie production, then, is likely to overstate human carrying capacity under given conditions.

4.4 Maximum Proportion of Targets

The amount of flexibility available is a function of the productive capacity of the land base and the level of land-use demands or targets which must be met. Another way of measuring flexibility, then, is to determine the extent to which production capacity falls short of, or exceeds specified production targets. This section describes a method of calculating this quantity, and its interpretation as a measure of land-use flexibility.

Given a set of conditions expressed in the form of inequalities 4.1 to 4.3, Smit *et al.* (1984) suggest that the question of what proportion of these targets is attainable can be addressed by specifying inequalitiy 4.2 somewhat differently as:

$$\sum_{j} a_{ij} y_{ij} - PQ_i \geq 0 \quad (1 \leq i \leq n) \quad (4.8),$$

where:

P = the proportion of each and all production targets that must be met.

Since P is a solution variable, the following nonnegativity constraint is implicit:

$$P \geq 0 \quad (4.9).$$

Given this revised set of constraints, the difference between the production capacity of available resources and targeted production levels is obtained by finding the maximum value of P. A measure of flexibility can be expressed directly as:

$$F(3) = \text{Max. } P \qquad (4.10).$$

If the maximum value of F(3) is less than one, production capacity is not sufficient to meet the specified targets. One advantage of this approach over conventional models, then, is that if all demands cannot be satisfied, this model provides a measure of the shortfall in production capacity. A value of F(3) equal to one indicates that all the targets are attainable, but only just. A value of F(3) greater than one indicates the degree to which production capacity exceeds the targets.

The rate of change in F(3) with respect to unit changes in land area or crop demands is given directly by the dual activity values in the model. They measure the effect of unit changes in land area or crop demands upon the maximum value of P. Thus:

$$\partial F(3) / \partial A_j = \partial P / \partial A_j \qquad (4.11),$$

and:

$$\partial F(3) / \partial Q_i = \partial P / \partial Q_i \qquad (4.12).$$

They allow the factors that limit the maximum size of P to be identified and ranked.

The F(3) measure of flexibility provides a better indication of how much production capacity is used under given conditions than the measures based on the minimum area needed to meet given crop demands, or the maximum amount of food in calorie equivalents that can be produced by a given resource base. It is, however, a conservative measure if the maximum value of P in the model is not unique. Under these conditions all the land available is not allocated, and it is possible to produce more than the indicated proportion for one or more of the other crops. The properties of this measure are examined further in Chapter 5.

4.5 Volume of the Feasible Region

Given inequalities 4.1 to 4.3, a land supply problem arises if the amount of land available is not sufficient to meet all the crop demands specified. If only one feasible solution exists there is no flexibility in allocating land among the specified uses. Alternatively, an infinite number of feasible solutions may exist which indicates that all or some of the decision variables can vary to some degree. This in turn suggests that there is some degree of flexibility available in allocating land to the various crops in meeting the demands.

The range over which all or some of the decision variables can vary, is determined by the size of the feasible region defined by the set of linear inequalities. An indication of how much flexibility is available, then, is given by the size of the feasible region. One measure of the feasible region's size is its volume. The volume of the feasible region defined by inequalities 4.1 to 4.3, is difficult to interpret as a measure of land-use flexibility, however, without either some benchmark against which to compare the number obtained or some scale on which to place it. In conventional programming applications, the feasible region is defined in a solution space bounded only by the nonnegativity constraints. In order to establish a benchmark against which to compare the volume of the feasible region under different conditions, it is necessary to define an upper limit for the solution space within which the feasible region is defined.

One way of establishing a base is to determine the conditions under which the volume of the feasible region is as large as possible, and use this as a benchmark against which to compare the volume of feasible regions defined by different levels of crop demands and/or land availability. For the set of constraints defined by inequalities 4.1 to 4.3, the volume of the feasible region is at a maximum when the crop demands are omitted and only the land availability and nonnegativity constraints are considered. The effect of introducing different levels of crop demands, using the original land area constraints or under different scenarios of land availability, can be judged relative to this base.

Denote the solution space defined by the nonnegativity and land availability constraints as Q and the feasible region defined by the complete set of constraints as R. One measure of flexibility, then, is the ratio of volume of the feasible region (R) to the base solution space (Q). This can be expressed as:

$$F(4) = Vol(R)/Vol(Q) \quad (4.13).$$

The value of F(4) ranges between zero and one. A value close to zero indicates that region R is relatively small, and that there is little flexibility available. This suggests that a large part of the available production capacity must be used to meet the demands. Alternatively, a value close to one indicates that region R is relatively large and that the introduction of crop demands has had little effect on the amount of flexibility available. This can be taken as an indication that there is little cause for concern over the manner in which land is used.

The ratio of region R's volume to that of region Q's, is conceptually appealing as a measure of flexibility. The constraints that define region Q, also delimit maximum production capacity. The volume of region R relative to this base indicates how much production capacity remains after meeting given demands. The following subsections examine two methods of measuring the volume of region R relative to that of region Q.

4.5.1 Monte Carlo Method

The feasible region defined by inqualities 4.1 to 4.3 is a convex polytope in mn-dimensional space. Chapman *et al.* (1984) note that although efficient methods are available for calculating the volume of convex polytopes for small mn (see, for example, Cohen and Hickey 1979), they are not computationally feasible for large problems such as those typically encountered in the analysis of a land-use system. They suggest, however, that the volume of the feasible region relative to a base solution space can be estimated using a Monte-Carlo method.

Each feasible solution is defined by the co-ordinates of a point in mn-dimensional space. The co-ordinates of the feasible points give the allocations of crops to land types that satisfy the constraints. One measure of volume, then, is the total number of feasible points that exists under a given set of conditions. The Monte-Carlo method estimates the volume of the feasible region (R) relative to the largest feasible region possible (Q), by generating random points within region Q. The proportion of points that satisfy all of the constraints is then calculated. This is done as follows.

Given that there are n crops, for each land type j such that $1 \leq j \leq m$, let:

$$a_{1j},\ a_{2j},\ \ldots,\ a_{nj} \qquad (4.14),$$

be random numbers between zero and one. Multiply each by A_j, and sort into ascending order to obtain:

$$b_{1j} \leq b_{2j} \leq \ldots \leq b_{nj} \qquad (4.15).$$

Set:

$$c_{1j} = b_{rj} \qquad (4.16),$$

$$c_{2j} = b_{2j} - b_{1j},$$

and in general:

$$c_{ij} = b_{ij} - b_{i-1,j} \quad (1 \leq i \leq n) \qquad (4.17).$$

Repeat this for each land type j, and consider the point:

$$(c_{11}, c_{21}, \ldots, c_{n1}, c_{12}, c_{22}, \ldots, c_{n2}, \ldots, c_{1m}, c_{2m}, \ldots, c_{nm}) \qquad (4.18).$$

Because of the constraint that:

$$\sum_{i} b_{ij} \leq A_j \quad (1 \leq j \leq m) \qquad (4.19),$$

this point will lie inside the region defined by inequalities 4.1 and 4.3 (region Q). Generating a large number of points within region Q and determining how many also satisfy inequality 4.2, provides a measure of the volume of region R relative to that of Q.

Chapman *et al.* (1984:60) state that the Monte Carlo method can be used to compare the amount of flexibility available as crop targets or other conditions vary. In actual fact, however, the Monte Carlo method of measuring the volume of region R relative to that of region Q, has a serious shortcoming for the analysis of land-use flexibility. It is necessary to assume that the area of each type of land is fixed, and that only the yields or the level of demand for the different crops vary from one scenario to the next. This assumption must be made as the Monte Carlo method cannot measure the volumes of regions R and Q independently. Region Q is defined by the nonnegativity and land availability constraints. If the amount of land decreases, a new base region Q is established and the volume of region R is estimated relative to this base. Since the Monte Carlo method does not establish a constant base against which decreases in land area can be compared, it is of limited value for the analysis of land-use flexibility.

4.5.2 Lasserre's Algorithm

Lasserre (1983) gives a formula for the volume of a convex polyhedron in dimensions of R^n and an algorithm for evaluating the formula. This section presents the formula.

Each inequality in 4.1 to 4.3 defines a face of the feasible region R in the following way. If the ith inequality is replaced by an equality and all others are left unaltered, the feasible region for this new system is a face of R (the ith face). This being a body in (mn−1) dimensional space, it will have an (mn−1) dimensional volume. A zero volume for the i-th face means that the i-th constraint is redundant and can be omitted from inequalities 4.1 to 4.3, and the feasible region R remains unchanged.

Lasserre's formula for the volume of the region defined by inequalities 4.1 to 4.3 is:

$$1/mn \left[\sum_j (A_j/n^{1/2} \, Vol(S_j)) + \sum_i (Q_i/(\sum p_{ij}^2)^{1/2} \, Vol(T_i) \right] \quad (4.20),$$

where $Vol(S_k)$ is the volume of feasible region for the system:

$$\sum_i a_{ik} = A_k \quad (4.21),$$

$$\sum_i a_{ij} \leq A_j \qquad (1 \leq j \leq m,\ j=k) \qquad (4.22),$$

$$\sum_j a_{ij}y_{ij} \geq Q_i \qquad (1 \leq i \leq n) \qquad (4.23),$$

$$a_{ij} \geq 0 \qquad (4.24),$$

and $Vol(T_k)$ is the volume for the system:

$$\sum_i a_{ij} \leq A_j \qquad (1 \leq j \leq m) \qquad (4.25),$$

$$\sum_j a_{kj}y_{kj} = Q_k \qquad (4.26),$$

$$\sum_j a_{ij}y_{ij} \geq Q_i \qquad (1 \leq i \leq n,\ i \neq k) \qquad (4.27),$$

$$a_{ij} \geq 0 \qquad (4.28).$$

If equation 4.21 is used to eliminate one of the decision variables in the inequalities 4.22 to 4.24, a system of (mn+mn-1) inequalities in (mn-1) variables results. $Vol(S_k)$ may be obtained from the volume of the feasible region of this new system, and this new system has one less inequality and one less variable than inequalities 4.1 to 4.3. Similar remarks apply to $Vol(T_k)$. Formula 4.20 establishes an iteration, the volume of R being expressed as a combination of volumes of regions defined by fewer variables and constraints. This eventually reduces to a system in a single unknown which can be found.

Lasserre's algorithm for calculating the volume of region R relative to region Q, has two advantages over the Monte Carlo method proposed by Chapman *et al.* (1984). Firstly, the volumes of regions R and Q can be calculated independently of each other so that scenarios that specify a decrease in land area can be compared to the base volume.

Secondly, Lasserre's algorithm provides a more accurate measure of the volume of a convex polytope. Unfortunately, Lasserre's algorithm has proved difficult and expensive to implement.

4.6 Radius of an Inscibed Sphere

As an alternative means of measuring the size of the feasible region defined by a linear set of inequalities, Chapman *et al.* (1984), propose the radius of the largest sphere that can be inscribed inside the feasible space. This section describes this method and its interpretation as a measure of land-use flexibility.

The feasible region defined by inequalities 4.1 to 4.3, has a boundary consisting of a set of faces which are determined by the linear constraints. If e is a point in region R, the distance from e to each face can be measured. Assuming there are s faces, denote these distances as:

$$d_1(e),\ d_2(e),\ \ldots,\ d_s(e) \qquad (4.29).$$

As e moves within region R, these distances will vary. A point is in the middle of R if it is no nearer to a given face than any other. The distance to the nearest face is defined as:

$$\min_{e}.\ [d_1(e),\ d_2(e),\ \ldots,\ d_s(e)] \qquad (4.30).$$

The epicentre is the point in R at which this minimum is maximized. This can be expressed as:

$$\max.\ \min.\ [d_1(e),\ d_2(e),\ \ldots,\ d_n(e)] \quad (4.31).$$

The location of the epicentre and the radius of the largest sphere that can be inscribed within the feasible region defined by inequalities 4.1 to 4.3, can be calculated via linear programming. The constraints expressed in inequalities 4.1 and 4.2, however, are measured in different units. Thus, the distance between a given point and a face is not directly comparable to the distance to other faces, as it is affected by the different measurement units. This problem is overcome by normalizing the inequalities to obtain:

$$\sum_i a_{ij}/X \leq A_j/X \qquad (1 \leq j \leq m) \qquad (4.32),$$

$$\sum_j a_{ij}y_{ij}/Y_i \geq Q_i/Y_i \qquad (1 \leq i \leq n) \qquad (4.33),$$

$$a_{ij} \geq 0. \qquad (1 \leq i \leq n,\ 1 \leq j \leq m) \qquad (4.34),$$

where:

$$X = (n)^{1/2} \qquad (4.35),$$

$$Y_i = \left(\sum_j y_{ij}^2\right)^{1/2} \qquad (4.36).$$

The co-ordinates of the epicentre and the radius of the largest inscribed sphere are found by introducing a variable (Z) to represent the distance from a feasible point to a face and writing inequalities:

$$Z + \sum_i a_{ij}/X \leq A_j/X \qquad (1 \leq j \leq m) \qquad (4.37),$$

$$Z - \sum_j a_{ij}y_{ij}/Y_i \leq -Q_i/Y_i \quad (1 \leq i \leq n) \qquad (4.38),$$

$$Z - a_{ij} \leq 0 \qquad (1 \leq i \leq n,\ 1 \leq j \leq m) \quad (4.39),$$

$$Z,\ a_{ij} \geq 0 \qquad (4.40).$$

The maximum value of Z is the radius of the largest sphere that can be inscribed within the feasible region, and the a_{ij} values at the point of optimization give the co-ordinates of the epicentre. As with the measure of flexibility based upon the ratio of the volume of the feasible region to a base solution space, a benchmark for the radius can be established by considering the nonnegativity and land availability constraints alone. Denoting the maximum value of Z for the complete set of constraints as r_R and the maximum value of Z when only the nonnegativity and land availability constraints are considered as r_Q, this flexibility measure can be expressed as:

$$F(5) = r_R/r_Q \qquad (4.41).$$

The value of F(5) ranges between zero and one. A value close to zero indicates that the radius of the sphere inscribed within region R is small in relation to that inscribed within region Q. This suggests that a large part of the available production capacity must be used to meet the demands. Similarly, a value of approaching one indicates that a great deal of flexibility exists.

Chapman *et al.* (1984:60) state that the ratio value of zero is obtained when the constraints are incompatible. In actual fact, however, the value of F(5) can be zero under three quite different sets of circumstances. The first, and most obvious case, is if the constraints cannot be satisfied. The second case is if the feasible region consists of a single point. The third case is if the feasible region consists of a plane. The radius represents the maximum minimum distance from a feasible point to a face of the feasible region. In the latter two cases, this distance is zero. Although the value of the objective function in both cases is zero, the co-ordinates of the epicentre are given as usual. Whether case two or case three arises in any given scenario can be determined by examining the dual activity values in the model.

The dual activity values in this model indicate the effect that a unit change in either the production targets for different crops or the amount of the different land types available, has upon the size of the radius. Since the base radius is constant, the rate of change in F(5) with respect to unit changes in land area or the amount of each crop demanded can be expressed as:

$$\partial F(5) / \partial A_j = 1/r_0 \ . \ \partial r_R / \partial A_j \qquad (4.42),$$

and:

$$\partial F(5) / \partial Q_i = 1/r_0 \ . \ \partial r_R / \partial Q_i \qquad (4.43).$$

The radius of the largest sphere that can be inscribed within region R relative to that inscribed within region Q, is conceptually appealing as a measure of flexibility. The constraints that define region Q also delimit maximum production capacity. The radius of the largest sphere that can be inscribed within region R relative to this base, indicates how much production capacity remains after the demands have been met. An important feature of this measure, is that it is readily calculated using standard linear programming packages. One characteristic of this measure, however, is that it is sensitive to the shape of the feasible region. This property is examined in Chapter 5.

4.7 Summary

Expressing the conditions that delineate crop production capacity in the form of constraints is a useful starting point for the analysis of land-use flexibility. For a given constraint set it is possible to determine whether sufficient resources are available to meet the demads. By varying the levels of resources, the impact of expected changes in conditions on the capacity to meet demands can be assessed. The set of constraints also contain information about the amount of flexibility that is available. This chapter has examined alternative measures land-use flexibility based on the analysis of a linear set of inequalities. Of these the F(3) measure of flexibility, which indicates the amount by which production capacity exceeds or falls short of specified demands, and the F(5) measure of flexibility, which is based on the ratio of the radius of the largest inscribed sphere that can be inscribed inside region R relative to that inscribed in region Q, closely conform to the definition of flexibility adopted in this thesis. Chapter 5 examines the analytical methods and the properties of these two measures.

CHAPTER 5

AN ASSESSMENT OF SELECTED LAND-USE FLEXIBILITY MEASURES

Alternative measures of land-use flexibility were presented in Chapter 4. Each measure is based upon the analysis of a linear set of inequalities. In this chapter the analytical methods and the properties of these measures are examined for a hypothetical data set.

5.1 Land-Use Scenarios

A scenario contains a structured set of assumptions about the environment. Hence, a land-use scenario contains a set of assumptions about key variables thought to influence some aspect of land-use supply or demand. Scenarios indicate to policy-makers and analysts, likely or possible developments so that the "what if" implications of proposed or expected changes, either endogenous or exogenous, can be determined (Mitchell *et al.*, 1979). Scenario analysis is well suited to the examination of urban expansion and land needs for agriculture, as assessments of the gravity of the problem vary greatly in their assumptions about future food needs and food production capacity.

Any number of constraints could be specified to represent a particular land use system. As noted in section 3.2, agricultural land-use programming models specify a wide range of factors that affect agricultural production in the form of constraints. The overriding consideration in specifying the number and form of constraints is the nature of the research questions for which the set is developed. This thesis uses a small hypothetical problem to compare the properties of the selected measures of land-use flexibility and criticalness. This assessment, however, will indicate the limitations of these measures for the analysis of more realistic systems. The basic requirement for the purposes of this thesis is that each constraint be linear.

The set of constraints employed is represented by inequalities 4.1 to 4.3. The analysis considers demand for four different crops and three types of land of varying productivity. The numerical description of the problem is presented in Table 5.1. This table specifies the yields of each crop on the different land types, two levels of demand for the different crops, and the amount of each type of land available. The analysis illustrates how the measures of flexibility could be used to address the question of agricultural land conversion to urban uses by examining the

TABLE 5.1

Land Use Scenario Data

Crop Yields (Metric Tonnes per Hectare)

	Land 1	Land 2	Land 3
Crop 1	12.1	7.0	0.0
Crop 2	9.8	8.1	4.9
Crop 3	8.0	5.8	3.6
Crop 4	2.7	2.5	1.0

Crop Demands (Metric Tonnes)

	Crop 1	Crop 2	Crop 3	Crop 4
Low	2000	5000	7500	9000
High	7000	10000	12000	17000

Land Availability (Hectares)

	Land 1	Land 2	Land 3
Base	2000	6000	12000

impact of decreases in the area of the three land types upon the amount of flexibility available.

5.2 Maximum Proportion of Targets

The first measure of flexibility examined is based on the maximum proportion of crop targets attainable. Figure 5.1 presents the values for the F(3) measure of flexibility as the area of land type 1 is reduced and the area of land types 2 and 3 are held at their base levels. The highest possible value for F(3) under low crop demand conditions is 2.6. This indicates that it is possible to produce 2.6 times as much of each crop than the quantity actually demanded. As the area of land type 1 is gradually reduced to zero, the value of F(3) falls to 2.1. This indicates that the loss of all land type 1 to other uses would not have a large impact on crop production capacity.

With a shift from low to high crop demands, the value of F(3) falls from 2.6 to 1.35. As the area of land type 1 is reduced to zero, F(3) drops to 1.10. This indicates that it would be possible to meet the high demands even if all land type 3 was devoted to other uses. Nevertheless, the combined effect of higher crop demands and reduced land area is to substantially reduce the amount of flexibility available, as the demand for each crop can be exceeded by only 10 percent.

Figure 5.2 presents the F(3) measure of flexibility as the area of land type 2 is reduced. In the low crop demand scenarios, the value of F(3) declines from 2.6 to 1.4. Although land type 1 is the best agricultural land, as it supports higher crop yields for all crops, the loss of all land type 2 has a larger impact on flexibility than the loss of all land type 1. This is attributable to the larger acreage that is lost. Under high crop demand conditions, reducing the area of land type 2 by half to 3000 hectares, lowers the value of F(3) from 1.35 to 1.04. If less than 2600 hectares of land type 2 are available, the demand for all crops cannot be met. F(3) has a value of 0.7 once all land type 2 is removed, which indicates that only 70 percent of the demand for each crop can be met under these conditions.

Figure 5.3 presents the F(3) measure of flexibility as the area of land type 3 is reduced. Under low crop demand conditions, the value of F(3) falls from 2.6 to 1.65. This indicates that despite the relatively large acreage involved, the loss of all land type 3 does not remove the ability to meet the demands. In fact, the remaining land resources are capable of exceeding each of the demands by 65 percent. The loss of all land type 3 has a smaller effect on the amount of flexibility available than the loss of all land type 2, even though the area of land type 3 is twice that of land type 2. This difference is attributable to the higher crop yields that exist for all crops on land type 2.

Under high demand conditions, the value of F(3) falls from 1.35 in the base land area scenario to 1.01 when only 4000 hectares of land type 3 are available. This indicates that all the demands can be met, but only just. Further reductions in the area of land type 3 remove the capacity to meet all the high level crop demands. Once all land type 3 is removed, F(3) has a value of 0.8. This indicates that the production of each crop is restricted to 80 percent of the amount actually demanded.

An important feature of F(3) as a measure of flexibility, is that if sufficient resources are not available to meet given demands, it indicates the amount by which production capacity falls short of the demands. Standard linear programming models simply indicate that a feasible solution cannot be found. As noted in section 4.4, however, this measure of flexibility is a conservative one if the maximum value of F(3) is not unique. Under these conditions the value of F(3) can be exceeded for some crops, as some of the available land is not allocated in the solution. Whether more production capacity exists under given conditions than the value of F(3) suggests, is indicated by the dual activity values (partial derivatives) in the model.

Tables A1 to A6 in Appendix A present the partial derivatives for the scenarios presented in Figures 5.1 to

5.3. In each scenario, all the constraints have nonzero partial derivatives. This indicates that the maximum value of F(3) in each scenario is unique, and that all the available land is allocated in each solution. Although the maximum value of F(3) is unique in the reported scenarios, it is important to demonstrate that this measure of flexibility can be adjusted to estimate the extra capacity available if some of the partial derivatives are zero.

The maximum value of F(3) in the model is determined by the binding constraints. In the reported scenarios, all the constraints are binding. Consider the case, however, if crop 1 could only be grown on land type 1, and only 1 hectare of land type 1 was available. Under these conditions it would not be possible to meet the demand for crop 1, and the maximum value of F(3) would be less than 1. The F(3) value, however, would not apply to all crops as most of land types 2 and 3 would not be allocated in the solution. The F(3) measure of flexibility can be extended, however, to estimate the extra production capacity available once the intitial value of F(3) is calculated. The partial derivatives identify the land type(s) that are at their upper limit and the binding crop(s). For the binding crop(s) adjust the right hand side(s) as follows:

$$Q_i^* = (Q_i P) \qquad (5.1),$$

and set the demand constraint for this crop as an equality. Now calculate the maximum value of F(3) for this new set of constraints, and continue iteratively as the other land type reach their upper limit, to determine the maximum proportion of other crop targets attainable.

5.3 Radius of an Inscribed Sphere

The second measure of flexibility examined is F(5) which is based on the radius of the largest sphere that can be inscribed inside the feasible region defined by inequalities 4.1, 4.2 and 4.3 (region R), relative to the feasible region defined by inequalities 4.1 and 4.3 alone (region Q). Inequality 4.2 is the constraint in which the demand for each crop is specified.

Figure 5.4 presents the F(5) measure of flexibility as the area of land type 1 is reduced. The highest value for F(5) under low crop demand and base land area conditions is 1.0. This means that the introduction of low crop demands has not reduced the size of the radius of the sphere inscribed in region R from that which could be inscribed inside region Q. As the area of land type 1 is gradually reduced to zero, however, the value of F(5) falls sharply to 0.0. This suggests that there is no flexibility available under these conditions, which contrasts with the value of F(3) for this scenario (Figure 5.1). The F(3) measure indicates that 2.1 times the quantity of each crop demanded could be produced under the same conditions.

With a shift from low to high crop demands the value of F(5) drops from 1.0 to .83 in the base land area scenario. Thus, the introduction of high crop demands causes an immediate reduction in the radius of the sphere inscribed inside region R relative to that inscribed in region Q. As the area of land type 1 is reduced, F(5) falls sharply. Below 1000 hectares of land type 1, F(5) has the same values as in the low demand scenarios. Once all land type 1 removed, the value of F(5) is zero. As above, however, this measure of how much production capacity remains is very different from that of F(3) for the same scenario. The F(3) measure suggests that 10 percent more than the quantity of each crop demanded could be produced under the same conditions. Although the two measures of flexibility are not directly comparable, this discrepancy merits further discussion.

As noted in section 4.6, the value of F(5) can be zero under three quite different conditions. The most obvious case is if the set of constraints cannot be satisfied, and no feasible solution exists. The other cases are if the feasible region consists of a single point or a plane. In the latter cases, a zero value for F(5) does not mean that the demands cannot be met, but that the maximum minimum distance from a point to each face of the feasible region is zero. The dual activity values (partial derivatives) in the model indicate which case exists in a given scenario.

Tables A7 and A8 in Appendix A present the partial derivatives for scenarios that decrease the area of land type 1 under low and high crop demand conditions respectively. In both tables, a nonzero partial derivative is reported only for land type 1. This indicates that the area of land type 1 is binding on the maximum size of the radius, and that the inscribed sphere could be placed in different positions within region R. Because the maximum length of the radius is determined by land type 1, the radius does not provide an accurate measure of the size of region R. This, in turn, means that the F(5) measure of flexibility does not accurately indicate how much production capacity remains after meeting the specified demands.

Figure 5.5 presents the F(5) measure of flexibility as the area of land type 2 is reduced. This figure illustrates a marked difference in the behaviour of F(5) between the low and high crop demand scenarios. As the area of land type 2 is reduced from 6000 to 2000 hectares under low crop demand conditions, the value of F(5) remains unchanged at 1.0. Further reductions in the area of land type 2, however, cause F(5) to fall sharply to 0.0. A strict interpretation of this measure is that introducing the low crop demands and removing 4000 hectares of land type 2 has no effect on the amount of flexibility available, while the loss of a further 2000 hectares of

2000 hectares of land type 2 available, however, the epicentre is unique as "the marble" touches each face of the feasible region. Once the area of land type 2 is reduced to 1000 hectares, only one of the seven partial derivatives is nonzero. The binding constraint, however, is land type 2, which means that the smallest dimension of "the matchbox" is now determined by land type 2. Thus, as the area of land type 2 is reduced further, the radius of the inscribed sphere also decreases.

In the high crop demand scenarios, F(5) falls sharply from 0.8 to 0.1 when only 3000 hectares of land type 2 are available. Table A10 reports the partial derivatives for the scenarios that reduce the area of land type 2 under high demand conditions. In each scenario, all the constraints have nonzero partial derivatives, which indicates that the inscribed sphere touches each face of region R.

Figure 5.6 presents the F(5) measure of flexibility as the area of land type 3 is reduced. The results are similar to those presented in Figure 5.5. In the low demand scenarios, the value of F(5) is unchanged at 1.0 as the area of land type 3 is reduced from 12000 to 4000 hectares. Further reductions in the area of land type 3, however, cause the value of F(5) to fall sharply to 0.0 when all of land type 3 is removed. As above, a strict interpretation of this measure is that the loss of 8000 hectares of land type 3 has no effect on the amount of flexibility available, while the loss of a further 4000 hectares removes all flexibility. This finding is at odds with the F(3) measure of flexibility, which suggests that the demand for each crop could be exceeded by 65 percent under the same conditions.

Table A10 in Appendix A presents the partial derivatives for the scenarios that reduce the area of land type 3 under low demand conditions. In the first three scenarios, the only constraint with a nonzero partial derivative is land type 1, which indicates that the inscribed sphere can be placed in different positions within region R. As above, the loss of land type 3 has no effect on the radius until each face of region R comes into contact with the "marble". Once only 2000 hectares of land type 3 are available, all the constraints have nonzero partial derivatives which indicates that the inscribed sphere can be placed in only one position in region R. Below this, the only constraint with a nonzero partial derivative in land type 3, which indicates that land type 3 now determines the maximum size of the radius. Reducing the area of land type 3 has the effect of squeezing the inscribed sphere, which is reflected in the decreasing value of the radius. In the high demand scenarios, F(5) falls sharply from 0.8 to 0.02 as the area of land type 3 is reduced to zero. Table A12 reports the partial derivatives for the corresponding scenarios. In each scenario, all the constraints have nonzero partial derivatives, which indicates that the inscribed sphere touches each face of region R.

The F(3) and F(5) measures of flexibility are not directly comparable. It is useful, however, to contrast the information provided by the two measures. The F(3) measure of flexibility provides a more useful measure of flexibility than F(5) for two reasons. The first is a practical consideration. A measure of flexibility that indicates the extent to which production capacity exceeds or falls short of specified demands, is relatively easy for land-use planners and policy-makers to grasp. The F(5) measure of flexibility, however, does not yield an absolute value that has a simple, applied interpretation. The ratio of the radius of the largest sphere that can be inscribed within region R relative to that inscribed within a base region Q, is a difficult concept for those not well acquainted with mathematical programming to comprehend.

The second reason for prefering F(3), is that this measure provides an accurate indication of how much flexibility is defined by a given set of inequalities. In the low demand scenarios presented in Figures 5.4 to 5.6, the F(5) measure of flexibility is zero whenever all the land type of interest is removed. Strictly speaking, this means that there is no flexibility in the use of land. The

F(3) measure of flexibility (Figures 5.1 to 5.3) indicates, however, that although the amount of flexibility declines as the area of each land type falls to zero, the loss of these lands still leaves some production capacity in the system, hence some flexibility. The F(5) measure is incapable of indicating this, as it is sensitive to the shape of the feasible region. Its greatest value may lie in identifying the most limiting factor and measuring the effect of changes in parameters.

5.4 Summary

The information required of a measure of flexibility, as defined in this thesis, is that it should indicate how much production capacity is used in meeting given demands, and indicate the effect of such changes in conditions as increases in crop demands or reductions in the amount of land available to agriculture upon the amount of flexibility available. This chapter has examined two measures of flexibility using a hypothetical data set. Although both measurs are conceptually appealing, the analysis indicates that the F(5) measure of flexibility is much more sensitive to the shape of the feasible region than Chapman *et al.* (1984) suggest. In many of the scenarios examined, it does not provide an accurate measure of the size of the feasible region, nor does it indicate the effect of changes in conditions. The F(3) measure of flexibility, however, performs well in all the scenarios examined, and can be extended to take account of the special case if the maximum value of P is not unique.

CHAPTER 6

ALTERNATIVE MEASURES OF LAND-USE CRITICALNESS

The concept of land-use criticalness adopted in this thesis refers to the extent to which particular areas or types of land are critically important, in the sense that the fulfillment of agricultural commodity requirements hinges upon these lands being available to agriculture. This chapter presents alternative measures of land-use criticalness.

6.1 Land-Use Criticalness

The potential for growing crops is not uniformly distributed over the land base. Some areas are more limited than others in both the range of crops that can be grown and their anticipated yields. Many argue that high quality land is a strategic resource and that it is crucial that as large an acreage as possible should be reserved for agriculture. This thesis has shown, however, that the identification of critical agricultural areas cannot be based solely upon an assessment of the supply of land of different degrees of suitability for agriculture.

The extent to which particular land types are critically important for agriculture must be defined in terms of two sets of factors. One set consists of such factors as population growth and consumer tastes which affect the demand for food. The other set consists of factors that limit agriculture's capacity for responding to increased food demands, such as weather conditions, the availability of energy, fertilizers and pesticides, and expected crop yields. A useful starting point for the assessment of land-use criticalness is to express relevant factors in the form of constraints. The constraints indicate whether sufficient resources are available to meet food needs under specified conditions. Assuming that the constraints can be satisfied, the research problem becomes one of measuring how important particular areas or types of land are in satisfying different levels of crop demands.

Depending upon the treatment of land uses, the relative importance of different types of land can be specified in different ways. This thesis examines two alternatives. The first option is to rate the necessity of assigning individual crops to each land type so as to meet given crop demands. That is, how important is it that crop i be allocated to land type j? The second option is to specify a more general measure of criticalness, by rating the importance of each land type in meeting a set of crop demands. That is, how important is land type j in meeting the demand for all crops? For ease of presentation this review of land-use criticalness measures adopts the notation and constraints represented by inequalities 4.1 to 4.3.

6.2 Absolute Minimum Allocation

An area of land can be considered to be of critical importance for a given crop if the attainment of specified crop production targets requires that a large proportion of it be allocated to that crop. One measure of the critical importance of a land type (j) for a given crop (i), then, is the smallest value of a_{ij} found in the entire feasible region R. This value is the area of land type j that must be allocated to crop i in order to satisfy the constraints. Expressing the absolute minimum value of each decision variable as a proportion of the total area of the respective land type available, this measure of land-use criticalness measure can be specified as:

$$C_{ij}(1) = (\text{min. } a_{ij})/A_j \quad (1 \leq i \leq n,\ 1 \leq j \leq m) \quad (6.1).$$

This ratio provides a measure of requisiteness of particular crop assignments. A zero value indicates that it is not necessary to assign any of a given land type to a particular crop in order to satisfy the constraints. A nonzero value indicates the proportion of a particular land type that must be assigned to a given crop in order to satisfy the constraints.

The absolute minimum value of a decision variable is easy to calculate using linear programming, and is readily interpreted. One problem with this measure of land-use criticalness, however, is that the smallest value of each decision variable is identified without reference to the other decision variables in the model. A particular minimum allocation may satisfy the constraints only if some other decision variables are set at an extreme value. These extreme values represent allocations that may be close to being infeasible. Thus, the absolute minimum value of individual decision variables may understate the importance of some crop assignments in meeting the demands.

6.3 Conditional Minimum Allocation

An alternative means of measuring how critical different types of land are for specific crops, is to identify minimum assignments for individual decision variables that do not rely upon extreme values of other decision variables. One way of doing this is to determine the minimum value of a decision variable conditional upon all the remaining decision variables being held at some central or average value. The average values of the decision variables, then, are given by the co-ordinates of a central point in the feasible region. A central point T in region R can be defined as:

$$T = (t_1,\ t_2,\ \ldots,\ t_{mn} \in R) \qquad (6.2).$$

The general form of a conditional minimum allocation measure of criticalness can be expressed as:

$$C_{ij} = \min[a_{ij};\ (t_1,\ t_2,\ \ldots,\ a_{ij}\ \ldots,\ t_{mn} \in R)]/A_j \quad (6.3),$$

where:

C_{ij} = the proportion of land type j that must be allocated to crop i in order to satisfy the constraints, conditional on other decision variables being set at central values given by the co-ordinates of point T.

This measure is equivalent to setting all the decision variables, except the one of interest, to the value of the designated interior point (T). The smallest feasible value that the selected decision variable can assume while the others are held at their central values is then found. Geometrically, this corresponds to drawing a staight line through point T, parallel to the a_{ij} axis, and finding the lowest value for the point of intersection with the boundary of region R. This can be found using a simple iterative process, often called the bisection method (see, for example, Burden *et al.*, 1978). Given a point A outside region R and a point B inside region R, the method undertakes to decrease the interval between these points by finding the midpoint M=(A+B)/2 of the line segment A,B. If M is inside region R, set B=M, otherwise set A=M. Apply the same process to the new line segment and repeat to the required accuracy. The research problem, then, is to identify an interior point in region R that is, in some sense, central.

6.3.1 Centroid

One interior point that lies at the centre of region R, is the centre of gravity or centroid. Region R is a homogeneous body in mn-dimensional space. The centroid is the point at which this mass could be replaced by a point mass. It may also be thought of as the "average" point in region R, in that the co-ordinates of the centroid are the expected values of the decision variables when points are sampled randomly from region R.

The co-ordinates of the centroid can be found using the Monte Carlo method of estimating the volume of region R relative to that of region Q (section 4.4.1). This method generates points uniformly within region Q, which is defined by inequalities 4.1 and 4.3 only, and identifies points that also satisfy inequality 4.2. The three inequalities together define region R. The average of the co-ordinates falling in region R tends towards the co-ordinates of the centroid.

6.3.2 Epicentre

Another choice of a central point in region R is the centre of the sphere of largest radius that can be enclosed in the feasible region. This point was defined in section 4.6 as the epicentre. It represents the "safest" point in region R in the sense that any other point is nearer to some face and hence closer to being infeasible. The co-ordinates of the epicentre are given by the values of the solution variables in the linear programming formulation that calculates the radius of the largest inscribed sphere (section 4.6).

One problem not addressed by Chapman *et al.* (1984) is that the epicentre may not be unique. Alternate optima do not pose a problem for measuring the radius of the largest inscribed sphere. Regardless of the epicentre's location, the maximum value of the radius is always the same. However, alternate optima may be important in the analysis of land-use criticalness.

The conditional minimum measure of land-use criticalness requires that the decision variables be set at the co-ordinates of an interior point which is, in some sense, central. If the epicentre is unique, this requirement is satisfied. If epicentre is not unique, however, the inscribed sphere can be placed in different locations within region R. Depending upon the extreme values that the co-ordinates of the epicentre could take, the values of the conditional minimum assignments could be misleading.

Conceptually, region R can be portrayed as a matchbox of smallest dimension D and the inscribed sphere as a marble of diameter D placed in the matchbox. If the marble does not touch each face of the matchbox, it can adopt any one of an infinite number of positions within the matchbox. Since the conditional minimum values based on the epicentre may vary according to its location, it would be useful to determine the point at which the marble is in the centre of the matchbox. Giscondi *et al.* (1984) overcome the problem of finding a central location for the epicentre, by finding all the extreme positions in which the marble can be placed, and taking the average of the co-ordinates of the epicentres. In the matchbox example, the epicentre has four extremal positions corresponding to placing the marble in the four corners. The average of the co-ordinates of the four extreme points is both a unique interior point and the geometric centre of the matchbox.

The average epicentre can be found through an extension of the simplex method of solving linear programmes. A fundamental property upon which the simplex method is based is that the optimal solution is a "corner-point feasible solution" (Hillier and Lieberman, 1980). Like the simplex method, Giscondi *et al.*'s (1984) algorithm evaluates the objective function at a vertex or extreme point of region R. All edges of region R emanating from this vertex are then examined and the rate of change in the objective function in the direction of each edge is calculated. For a maximization problem, the algorithm selects the adjacent vertex having the highest value of the objective function. This process is continued until a vertex is found at which the rate of change of the objective function along any edge emanating from that vertex is less than or equal to zero. This is where linear programming usually stops, as an optimal solution for the problem has been found.

If all of the constraints have nonzero partial derivatives at the optimal point the solution is unique. If one or more of the partial derivatives are zero, however, it is possible to move to another vertex without changing the value of the objective function. This means that the set of decision variables yielding the optimal value of the objective function is not unique. The simplex method simply indicates that alternate optima exist. Giscondi *et al.*'s (1984) algorithm, however, prepares a list of all the vertices adjacent to the optimal point at which the objective function is unchanged. The algorithm then selects an adjacent vertex having an equal value of the objective function and seeks all the vertices adjacent to this point that have an equal value of the objective function. Some of these vertices will already be on the list. Those which are not are added to the bottom of the list. The algorithm then selects the next vertex on the list, and repeats its search for adjacent vertices having the same value of the objective function. Once the complete list has been examined, and no new vertices found, all extremal points at which the objective function is optimized have been identified. The values of the extreme point co-ordinates are then averaged to find the average epicentre.

6.4 Minimum Proportions

A very different approach to assessing the importance of particular crop assignments in meeting crop demands has been adopted by the Land Evaluation Group at the University of Guelph. Smit *et al.* (1981) use an objective function to select from the feasible region the solution that assigns as little of each crop as possible to the different land types. If the minimum possible allocation for a particular crop represents a large proportion of a given land type, then that land type is considered important for that crop.

The task of identifying crop assignments which are minimally necessary in order to satisfy demands can be addressed in several different ways. Smit *et al.* (1981) specify four criteria for selecting an objective function. These are:

1) the sum of the decision variables tends to zero as the constraints are relaxed,
2) individual decision variables tend to zero

as the constraints are relaxed,

3) individual decision variables tend to be equal as differences in constraint characteristics are minimized,

4) decreases in large decision variables that cause smaller decision variables to increase occur before or in lieu of decreases in small decision variables that cause large decision variables to increase.

Smit *et al.* (1981) examine four objective functions: minimum area, minimum proportions of land allocated, minimum sum of the allocated areas squared and the minimum sum of the squared proportions of land allocated. One of the objective functions that satisfied all of the criteria listed above is the sum of the squared proportions of land allocated. This objective function can be specified as:

$$\text{Min. } Z = \sum_{j} \sum_{i} p_{ij}^{2} \qquad (6.4),$$

where:

$$p_{ij} = a_{ij}/A_{j} \qquad (6.5).$$

The solution variables in this model indicate the proportion of the different land types that must be allocated to each crop. Bond *et al.* (1981) interpret these allocations directly as a measure of how critical the particular assignments are in meeting different levels of crop demands. This interpretation of land-use criticalness, however, is somewhat different than that adopted in this thesis. This objective function (equation 5.4), assigns as little as possible of each crop to each land type while simultaneously minimizing the differences among the assignments. The larger the proportion allocated, the more important that asssignment is in meeting the demands. This function, however, attempts to make the assignments as equal as possible and not as small as possible. It may, then, overstate the importance of some crop assignments.

6.5 Criticalness of Land Types

The measures of land-use criticalness examined in the preceding sections, indicate the importance of particular crop assignments in meeting given demands. For land-use planning, however, it may also be useful to consider the importance of the different land types in meeting the demand for the entire set of crops. Such an aggregate measure of criticalness provides a rating of land types that incorporates both soil qualities and the level of demands.

One measure of how important different land types are in meeting the demands is given by the partial derivatives for the land constraits calculated for the F(3) and F(5) measures of flexibility (sections 4.4 and 4.6). The partial derivatives, for F(3) indicate the effect of a unit change in area upon the maximum value of F(3). The partial derivatives for F(5) are calculated from the dual activty values in the model, which give the effect of a unit change in area on the size of the radius. These measures can be expressed as:

$$\partial F(3) / \partial A_j = \partial P / \partial A_j \qquad (6.6),$$

and:

$$\partial F(5) / \partial A_j = 1/r_a \, . \, \partial r_a / \partial A_j \qquad (6.7).$$

By examining the partial derivatives, the land types can be ranked in order of importance.

A second measure of the criticalness of a land type in meeting the set of demands is the minimum amount of each land type needed to meet the demands. The absolute minimum values of the decision variables (section 6.2) are additive over the crops for each land type. Thus, a measure of the importance of a land type in meeting the demands can be defined as:

$$C_j(1) = \sum_i C_{ij}(1) \qquad (6.9),$$

where:

C_j = the proportion of land type j that must be available in order to meet the demands.

6.5 Summary

A useful starting point for the assessment of land-use criticalness is to express relevant factors in the form of constraints. For a given consraint set, it is possible to determine whether the demands placed upon available resources can be met. Assuming that the constraints can be satisfied, the research problem becomes one of measuring how important particular areas or types of land are in satisfying different levels of crop demands. This chapter has examined two types of land-use criticalness measures. The first type rates the necessity of assigning individual crops to each land type so as to meet given demands. This measure takes two forms: absolute minimum assignments and conditional minimum assignments. The second option is to specify a more general measure of criticalness, by rating the importance of each land type in meeting all the demands. Chapter 7 examines the properties of these measures in greater detail.

CHAPTER 7

AN ASSESSMENT OF SELECTED LAND-USE CRITICALNESS MEASURES

Alternative measures of land-use criticalness were presented in Chapter 6. Each measure is based upon the analysis of a linear set of inequalities. In this chapter the analytical methods and the properties of these measures are examined for a hypothetical data set.

7.1 Absolute Minimum Allocations

The first measure of land-use criticalness examined is based on the smallest value of each decision variable. This measure, denoted as $C_{ij}(1)$ is the minimum proportion of land type j which must be allocated to each crop i in order to sastisfy the constraints.

Table 7.1 presents the values of $C_{ij}(1)$ as the area of land type 1 is reduced. Under low crop demand and base land area conditions, all the criticalness measures have zero values. This indicates that no one land type is essential for any given crop in meeting the demands. The values of the criticalness measures do not

TABLE 7.1

CRITICALNESS MEASURES BASED ON ABSOLUTE MINIMUM VALUES OF

DECISION VARIABLES: DECREASING AREA OF LAND TYPE 1, LOW AND HIGH CROP DEMANDS

	Area of Land Type 1									
	2000		1500		1000		500		0	
	Low	High	Low	High	Low	High	Low	High	Low	High
$C_{11}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-
$C_{21}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-
$C_{31}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-
$C_{41}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-
$C_{12}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.02	0.05	0.12
$C_{22}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$C_{32}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$C_{42}(1)$	0.0	0.0	0.0	0.06	0.0	0.15	0.0	0.24	0.0	0.42
$C_{13}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$C_{23}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$C_{33}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$C_{43}(1)$	0.0	0.0	0.0	0.0	0.0	0.07	0.0	0.19	0.0	0.37

Key

Criticalness Measure: $C_{ij}(1)$ = the proportion of land type j that must be allocated to crop i in order to satisfy the constraints.

change until all land type 1 is removed. In this scenario, $C_{12}(1)$ has a value of 0.05. This indicates that 5 percent of land type 2 must be allocated to crop 1 in order to satisfy the constraints. It should be noted that land type 2 is the only land type remaining for which crop 1 has nonzero yields.

With a shift from low to high crop demands, all the $C_{ij}(1)$ values remain zero. As the area of land type 2 is reduced, however, three of the criticalness measures take nonzero values. Once all land type 1 is removed, the demands cannot be met unless 12 percent of land type 2 is devoted to crop 1, a further 42 percent to crop 4, and 37 percent of land type 3 to crop 4. These results indicate that although the demands can be met, the loss of land type 1 places restrictions on the use of the remaining land.

Table 7.2 presents the values of $C_{ij}(1)$ as the area of land type 2 is reduced. Under low crop demand conditions, all the criticalness measures have zero values until the area of land type 2 is reduced to 1000 hectares. In this scenario the demands cannot be met unless 13 percent of land type 3 is devoted to crop 4. Once all land type 2 removed, it is necessary to allocate 34 percent of land type 3 to crop 4, and 8 percent of land type 1 to crop 1 in order to meet the demands. The latter assignment is

TABLE 7.2

CRITICALNESS MEASURES BASED ON ABSOLUTE MINIMUM VALUES OF

DECISION VARIABLES: DECREASING AREA OF LAND TYPE 2, LOW AND HIGH CROP DEMANDS

Area of Land Type 2

	5000		4000		3000		2000		1000		0	
	Low	High	Low	High	Low	High	Low	High[1]	Low	High[1]	Low	High[1]
$C_{11}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-	0.08	-
$C_{21}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-	0.0	-
$C_{31}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-	0.0	-
$C_{41}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-	0.0	-
$C_{12}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-	-	-
$C_{22}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-	-	-
$C_{32}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-	-	-
$C_{42}(1)$	0.0	0.0	0.0	0.02	0.0	0.62	0.0	-	0.0	-	-	-
$C_{13}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-	0.0	-
$C_{23}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-	0.0	-
$C_{33}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-	0.0	-
$C_{43}(1)$	0.0	0.05	0.0	0.26	0.0	0.47	0.0	-	0.13	-	0.34	-

Key

Criticalness Measure: $C_{ij}(1)$ = the proportion of land type j that must be allocated to crop i in order to satisfy the constraints.

[1] No Feasible Solution.

explained by the fact that land type 1 is the only land type remaining for which crop 1 has nonzero yields. Although the loss of all land type 2 does not remove the ability to meet low level demands, it does place more restrictions on the use of the remaining lands than the loss of all land type 1.

Under high crop demand conditions, the demand for crop 4 is an important factor in placing restrictions on the use of the available land. Although only 5 percent of land type 3 must be allocated to crop 4 when 5000 hectares of land type 2 are available, once the area of land type 2 is reduced to 3000 hectares, it is necessary to allocate 62 percent of land type 2 to crop 4 and a further 47 percent of land type 3 to crop 4.

Table 7.3 presents the values of $C_{ij}(1)$ as the area of land type 3 is reduced. In the low demand scenarios, all the criticalness measures are zero until only 2000 hectares of land type 3 are available. In this scenario, 11 percent of land type 2 must be devoted to crop 4. This increases to 24 percent once all land type 3 is removed. In the high demand scenarios, $C_{42}(1)$ is the only criticalness measure that is nonzero until the area of land type 3 is reduced to 4000 hectares. In this scenario, six of the $C_{ij}(1)$ measures are nonzero. Fully 94 percent of land type 2 must be allocated to crop 4

TABLE 7.3

CRITICALNESS MEASURES BASED ON ABSOLUTE MINIMUM VALUES OF

DECISION VARIABLES: DECREASING AREA OF LAND TYPE 3, LOW AND HIGH CROP DEMANDS

	Area of Land Type 3											
	10000		8000		6000		4000		2000		0	
	Low	High	Low	High	Low	High	Low	High	Low	High[1]	Low	High[1]
$C_{11}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.24	0.0	-	0.0	-
$C_{21}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-
$C_{31}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.03	0.0	-	0.0	-
$C_{41}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.22	0.0	-	0.0	-
$C_{12}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-
$C_{22}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-
$C_{32}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-
$C_{42}(1)$	0.0	0.06	0.0	0.24	0.0	0.53	0.0	0.94	0.11	-	0.24	-
$C_{13}(1)$	0.0	0,0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-	-
$C_{23}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.20	0.0	-	-	-
$C_{33}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.29	0.0	-	-	-
$C_{43}(1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-	-

<u>Key</u>

Criticalness Measure: $C_{ij}(1)$ = the proportion of land type j that must be allocated to crop i in order to satisfy the constraints.

[1] No Feasible Solution.

in order to meet the demands. These results indicate the the land base is under great pressure, and that high crop demands together with the loss of 8000 hectares of land type 3, place severe restrictions on the use of the remaining land.

As noted in section 5.2, this measure of criticalness may understate the importance of some crop allocations. A particular minimum allocation may satisfy the constraints only if some of the other decision variables are set at extreme assignments. The following section examines land-use criticalness measures based on conditional minimum assignments.

7.2 Conditional Minimum Allocations

The conditional minimum measure of land-use criticalness requires that all the decision variables, except the one of interest, be set at the co-ordinates of an interior point that is central in region R. Depending upon the selection of the central point, this measure can be calculated in different ways. This section presents the conditional minimum measures of criticalness calculated using the co-ordinates of the centroid and the average epicentre.

7.2.1 Centroid

The first conditional minimum measure of criticalness examined is based upon the centroid. This measure, denoted as $C_{ij}(2)$, indicates the proportion of land type j that must be allocated to crop i, conditional upon the other decision variables being set at the central values given by the co-ordinates of the centroid. This measure of criticalness, then, has a somewhat different interpretation than that based on the absolute minimum value of a decision variable. It indicates important crop assignments. That is, it indicates assignments that may restrict the use of land if small changes in conditions occur. The values of this measure, however, cannot be less than those based on the absolute minimum allocations presented in the preceding section.

Table 7.4 presents the values of $C_{ij}(2)$ for the low demand scenarios as the area of land type 1 is reduced. Differences between the absolute and conditional minimum measures of criticalness are immediately apparent. In the first scenario, three crop assignments are identified as important while the corresponding values of $C_{ij}(1)$ in Table 7.1 are all zero. Thus, although it is not necessary to allocate particular types of land to different crops, these crop assignments are important in meeting the demands. As the area of land type 1 is

TABLE 7.4

CONDITIONAL MEASURES OF CRITICALNESS BASED ON CENTROID:

DECREASING AREA OF LAND TYPE 1, LOW CROP DEMANDS

	Area of Land Type 1				
	2000	1500	1000	500	0
$C_{11}(2)$	0.0	0.0	0.0	0.0	-
$C_{21}(2)$	0.0	0.0	0.0	0.0	-
$C_{31}(2)$	0.0	0.0	0.0	0.0	-
$C_{41}(2)$	0.0	0.0	0.0	0.0	-
$C_{12}(2)$	0.0	0.0	0.0	0.02	0.05
$C_{22}(2)$	0.0	0.0	0.0	0.0	0.0
$C_{32}(2)$	0.0	0.0	0.0	0.0	0.0
$C_{42}(2)$	0.25	0.28	0.30	0.32	0.33
$C_{13}(2)$	0.0	0.0	0.0	0.0	0.0
$C_{23}(2)$	0.0	0.0	0.0	0.0	0.0
$C_{33}(2)$	0.03	0.05	0.06	0.07	0.07
$C_{43}(2)$	0.13	0.13	0.15	0.14	0.16

Key

Criticalness Measure: $C_{ij}(2)$ = proportion of land type j that must be allocated to crop i in order to satisfy the constraints, conditional on other decision variables being set at central values given by the co-ordinates of the centroid.

reduced, the importance of these assignments increases consistently. Once all land type 1 is removed, the assignment of crop 1 to land type 2 also becomes important, as this is the only land remaining on which crop 1 has nonzero yields.

Table 7.5 presents the values of $C_{ij}(2)$ for the low crop demand scenarios as the area of land type 2 is reduced. Initially, the same crop assignments as in Table 7.4 are identified as important. As the area of land type 2 decreases, however, the values of the three criticalness measures do not increase consistently. The value of $C_{42}(2)$ declines. In so far as it might be expected that the importance of a given land type should increase as its area declines, this finding is counter-intuitive. However, these measures indicate important assignments - not the ones that are absolutely necessary. The reduction in the area of land type 2 affects the size and shape of the feasible region. This also means that the co-ordinates of the centroid change. The values of the criticalness measure are subsequently affected by the change in co-ordinates. In Table 7.4, a similar situation exists. The crop assignments on land types 2 and 3 increased in importance as the area of land type 1 was reduced.

Table 7.6 presents the values of $C_{ij}(2)$ for

TABLE 7.5

CONDITIONAL MEASURES OF CRITICALNESS BASED ON CENTROID:

DECREASING AREA OF LAND TYPE 2, LOW CROP DEMANDS

	Area of Land Type 2					
	5000	4000	3000	2000	1000	0
$C_{11}(2)$	0.0	0.0	0.0	0.0	0.01	0.08
$C_{21}(2)$	0.0	0.0	0.0	0.0	0.04	0.0
$C_{31}(2)$	0.0	0.0	0.02	0.0	0.06	0.22
$C_{41}(2)$	0.0	0.0	0.0	0.17	0.14	0.08
$C_{12}(2)$	0.0	0.0	0.0	0.0	0.0	-
$C_{22}(2)$	0.0	0.0	0.0	0.01	0.0	-
$C_{32}(2)$	0.0	0.0	0.0	0.0	0.0	-
$C_{42}(2)$	0.28	0.24	0.19	0.05	0.0	-
$C_{13}(2)$	0.0	0.0	0.0	0.0	0.0	0.0
$C_{23}(2)$	0.0	0.0	0.0	0.01	0.03	0.05
$C_{33}(2)$	0.06	0.05	0.06	0.06	0.03	0.05
$C_{43}(2)$	0.19	0.30	0.37	0.44	0.51	0.65

Key

Criticalness Measure: $C_{ij}(2)$ = proportion of land type j that must be allocated to crop i in order to satisfy the constraints, conditional on other decision variables being held at central values given by the co-ordinates of the centroid.

TABLE 7.6

CONDITIONAL MEASURES OF CRITICALNESS BASED ON CENTROID:

DECREASING AREA OF LAND TYPE 3, LOW CROP DEMANDS

	Area of Land Type 3					
	10000	8000	6000	4000	2000	0
$C_{11}(2)$	0.0	0.0	0.0	0.0	0.0	0.0
$C_{21}(2)$	0.0	0.0	0.0	0.0	0.0	0.0
$C_{31}(2)$	0.0	0.0	0.0	0.11	0.18	0.18
$C_{41}(2)$	0.0	0.0	0.0	0.0	0.0	0.02
$C_{12}(2)$	0.0	0.0	0.0	0.0	0.0	0.0
$C_{22}(2)$	0.0	0.0	0.0	0.01	0.04	0.04
$C_{32}(2)$	0.0	0.0	0.0	0.03	0.08	0.07
$C_{42}(2)$	0.32	0.37	0.43	0.45	0.50	0.52
$C_{13}(2)$	0.0	0.0	0.0	0.0	0.0	-
$C_{23}(2)$	0.0	0.0	0.0	0.0	0.0	-
$C_{33}(2)$	0.06	0.07	0.11	0.04	0.0	-
$C_{43}(2)$	0.05	0.0	0.0	0.0	0.0	-

Key

Criticalness Measure: $C_{ij}(2)$ = proportion of land type j that must be allocated to crop i in order to satisfy the constraints, conditional on other decision variables being set at central values given by the co-ordinates of the centroid.

the low demand scenarios as the area of land type 3 is reduced. Again, the same $C_{ij}(2)$ values as in Tables 7.4 and 7.6 are identified as important. The behaviour of the criticalness measures in these scenarios is also similar to those in Table 7.5. The assignment of crop 4 to land type 2 increases in importance as the area of land type 3 is reduced, while the criticalness of crop 3 on land type 3 decreases.

The conditional minimum measures of criticalness based upon the centroid are not reported for the high crop demand scenarios. Although the centroid is a unique and central point in region R, it is difficult to calculate using the Monte Carlo method if the land base is under great pressure. As indicated in section 4.3.1, the Monte Carlo method calculates the centroid by generating random points in region Q, determining how many points fall in region R, and then averaging the point co-ordinates falling in region R to obtain the centroid. If region R is small relative to region Q, however, a large number of points within region Q must be generated in order to find points in region R. This quickly becomes a prohibitively expensive exercise. In this analysis, the centroid could not be calculated for the high crop demand and base land area scenario by generating 100,000 random points. The expense incurred in finding the centroid under conditions in which region R is small compared to region Q, poses a serious limitation for the practical application of this procedure.

7.2.2 Epicentre

The second conditional minimum measure of criticalness examined is based on the epicentre. As noted in section 6.3.2, however, one problem not addressed by Chapman *et al.* (1984), is that the epicentre may not be unique. Depending upon the extreme values that the co-ordinates of the epicentre can take, the values of the conditional minimum assignments could be misleading. Tables A7 to A12 in Appendix A, present partial derivatives for the F(5) measure of flexibility, which is based on the radius of the largest inscribed sphere. In the majority of low demand scenarios, many of the partial deriviatives are zero. This indicates that the inscribed sphere could be placed in different locations in region R. Thus, the conditional minimum criticalness measures based on the epicentre in these scenarios could be misleading. The average epicentre proposed by Giscondi *et al.* (1984), however, overcomes this problem. The difference between conditional minimum measures based on the co-ordinates of the epicentre and those based on the co-ordinates of the average epicentre is illustrated in Appendix B.

Since the co-ordinates of the epicentre are often not unique, this section reports the conditional criticalness measures based on the average epicentre. This measure, denoted as $C_{ij}(3)$, indicates the proportion of land type j that must be allocated to crop i, conditional upon the other decision variables being set at the central values given by the co-ordinates of the average epicentre.

Table 7.7 presents the values of $C_{ij}(3)$ as the area of land type 1 is reduced. Under low crop demand conditions, the same crop assignments as in Table 7.4 are identified as important. This is as expected, as the values of the measures are similar as the centroid and average epicentre are both central points in region R, albeit not identical. As the area of land type 1 decreases, these measures increase consistently. Table 7.7 also reports the values of $C_{ij}(3)$ for the high crop demand scenarios. Unlike the the case for the centroid, the co-ordinates of the average epicentre are relatively inexpensive to calculate using Giscondi *et al.*'s (1984) algorithm. In the base land area scenario, five crop assigments are identifed as important. Once the area of land type 1 is reduced to zero, the measures for six crop assignments have nonzero values.

Table 7.8 presents the values of $C_{ij}(3)$ as the area of land type 2 is reduced. Under low demand conditions two crop assignments are important. This increases to seven once all land type 2 is removed. Under high demand conditions, six crop assignments are important

TABLE 7.7

CONDITIONAL MEASURES OF CRITICALNESS BASED ON AVERAGE EPICENTRE:

DECREASING AREA OF LAND TYPE 1, LOW AND HIGH CROP DEMANDS

	Area of Land Type 1									
	2000		1500		1000		500		0	
	Low	High[1]	Low	High[1]	Low	High[1]	Low	High	Low	High
$C_{11}(3)$	0.0	0.15	0.0	0.09	0.0	0.0	0.0	0.0	-	-
$C_{21}(3)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-
$C_{31}(3)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-
$C_{41}(3)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-
$C_{12}(3)$	0.0	0.0	0.0	0.05	0.0	0.12	0.02	0.14	0.05	0.17
$C_{22}(3)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.08	0.0	0.06
$C_{32}(3)$	0.0	0.0	0.02	0.0	0.04	0.0	0.04	0.0	0.06	0.11
$C_{42}(3)$	0.34	0.66	0.35	0.67	0.37	0.67	0.39	0.63	0.41	0.60
$C_{13}(3)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$C_{23}(3)$	0.0	0.09	0.0	0.10	0.0	0.12	0.0	0.07	0.0	0.11
$C_{33}(3)$	0.02	0.19	0.0	0.21	0.0	0.22	0.02	0.24	0.04	0.17
$C_{43}(3)$	0.10	0.44	0.13	0.46	0.15	0.48	0.17	0.58	0.19	0.66

Key

Criticalness Measure: $C_{ij}(3)$ = the proportion of land type j that must be allocated to crop i in order to satisfy the constraints, conditional on other decision variables being set at central values given by the co-ordinates of the epicentre.

[1] Epicentre is Unique.

TABLE 7.8

CONDITIONAL MEASURES OF CRITICALNESS BASED ON AVERAGE EPICENTRE:

DECREASING AREA OF LAND TYPE 2, LOW AND HIGH CROP DEMANDS

	Area of Land Type 2											
	5000		4000		3000		2000		1000		0	
	Low	High[1]	Low	High[1]	Low	High[1]	Low[1]	High[2]	Low	High[2]	Low	High[2]
$C_{11}(3)$	0.0	0.23	0.0	0.26	0.0	0.28	0.0	-	0.03	-	0.08	-
$C_{21}(3)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-	0.02	-
$C_{31}(3)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.10	-	0.13	-
$C_{41}(3)$	0.0	0.11	0.0	0.36	0.0	0.61	0.0	-	0.0	-	0.09	-
$C_{12}(3)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-	-	-
$C_{22}(3)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-	-	-
$C_{32}(3)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-	-	-
$C_{42}(3)$	0.07	0.03	0.03	0.81	0.0	0.92	0.0	-	0.0	-	-	-
$C_{13}(3)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-	0.0	-
$C_{23}(3)$	0.0	0.11	0.0	0.13	0.0	0.16	0.0	-	0.0	-	0.02	-
$C_{33}(3)$	0.0	0.21	0.0	0.24	0.01	0.27	0.07	-	0.06	-	0.08	-
$C_{43}(3)$	0.44	0.47	0.51	0.50	0.57	0.54	0.60	-	0.64	-	0.66	-

Key

Criticalness Measure: $C_{ij}(3)$ = the proportion of land type j that must be allocated to crop i in order to satisfy the constraints, conditional on other decision variables being set at central values given by the co-ordinates of the epicentre.

[1] Epicentre is Unique.
[2] No Feasible Solution.

conditions two crop assignments are important. This increases to seven once all land type 2 is removed. Under high demand conditions, six crop assignments are important in the base land area scenario. As the area of land type 2 decreases, the criticalness measures all increase consistently.

Table 7.9 presents the values of $C_{ij}(3)$ as the area of land type 3 is reduced. Under low demand conditions, only two crop assignments are important. As the area of land type 3 decreases, the criticalness measures increase consistently. Under high demand conditions, six crop assignments are identified as important in the base land area scenario. All but two increase consistently as the area of land type 3 is reduced.

The measures of land-use criticalness that are conditional upon all the decision variables, except the one of interest, being held at their central or average value, are more difficult to interpret than the absolute minimum values of the decision variables. Combined with the information provided by the absolute minimum measures, however, they provide a better indication of which crop assignments are important in meeting given demands. The preceding analysis indicates, however, that the centroid and epicentre as originally proposed by Chapman *et al.* (1984), are not appropriate for many problems. The

TABLE 7.9

CONDITIONAL MEASURES OF CRITICALNESS BASED ON AVERAGE EPICENTRE

DECREASING AREA OF LAND TYPE 3, LOW AND HIGH CROP DEMANDS

	Area of Land Type 3											
	10000		8000		6000		4000		2000		0	
	Low	High[1]	Low	High[1]	Low	High[1]	Low	High[1]	Low[1]	High[2]	Low	High[2]
$C_{11}(3)$	0.0	0.23	0.0	0.25	0.0	0.26	0.0	0.29	0.0	-	0.0	-
$C_{21}(3)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-
$C_{31}(3)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.31	0.03	-	0.05	-
$C_{41}(3)$	0.0	0.06	0.0	0.26	0.0	0.46	0.0	0.38	0.0	-	0.18	-
$C_{12}(3)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-
$C_{22}(3)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.01	-	0.06	-
$C_{32}(3)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.06	-	0.14	-
$C_{42}(3)$	0.39	0.76	0.43	0.84	0.46	0.91	0.49	0.99	0.53	-	0.49	-
$C_{13}(3)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-	-
$C_{23}(3)$	0.0	0.12	0.0	0.19	0.0	0.29	0.0	0.50	0.0	-	-	-
$C_{33}(3)$	0.02	0.25	0.03	0.34	0.05	0.50	0.08	0.48	0.0	-	-	-
$C_{43}(3)$	0.0	0.35	0.0	0.23	0.0	0.03	0.0	0.0	0.0	-	-	-

Key

Criticalness Measure: $C_{ij}(3)$ = the proportion of land type j that must be allocated to crop i in order to satisfy the constraints, conditional on other decision variables being set at central values given by the co-ordinates of the epicentre.

[1] Epicentre is Unique.
[2] No Feasible Solution.

centroid is expensive to calculate if the land-use system is under heavy pressure, while the co-ordinates of the epicentre are often not unique. Given these considerations, the conditional minimum measure of criticalness based on the average epicentre appears to be the most appropriate measure.

7.3 Criticalness of a Land Type

The preceding measures of land-use criticalness indicate the relative importance of particular crop assignments in meeting given demands. As noted in section 6.5, however, it may also be useful to consider the importance of the different land types in meeting the demand for the entire set of crops. One measure of how important different land types are in meeting given crop demands is given by the partial derivatives on the land availability constraints for the F(3) and F(5) measures of flexibility. The partial derivatives for the two flexibility measures are presented in Appendix A.

The second measure of criticalness of a land type presented in section 5.5, is the minimum amount of the land type needed to satisfy the demands. This measure, denoted as C_j is the proportion of a given land type that must be available in order to satisfy the constraints. Table 7.10 presents the values of C_j as the area of land type 1 is reduced. Under low demand conditions,

conditions, all the measures are zero until all land type 1 is removed. In this scenario, 5 percent of land type 2 must be assigned to crop 1 in order to satisfy the constraints. This is explained by the fact that land type 2 is the only land type remaining on which crop 1 has nonzero yields. With all of land type 1 removed under high demand conditions, 54 percent of land type 2 and 37 percent of land type 3 must be allocated.

Table 7.11 presents the values of C_j as the area of land type 2 is reduced. Under low demand conditions, all the measures are zero until all land type 2 is removed. In this scenario 8 percent of land type 1 must be allocated to crop 1, as this is the only land type remaining on which the yield for crop 1 is nonzero. Under high demand conditions, increasing amounts of land types 2 and 3 must be available in order to satisfy the demands as the area of land type 2 decreases. In the last scenario examined, 62 percent of land type 2 must be allocated along with 47 percent of land type 3.

Table 7.12 presents the values of C_j as the area of land type 3 is reduced. As above, in the low demand scenarios all the measures are zero until all land type 3 is removed. In this scenario, however, it is land type 2 which must be reserved and not land type 1 which is has the higher yields. It must be stressed, however, that these measures are based on the minimum assignments of a

TABLE 7.10

CRITICALNESS MEASURES FOR LAND TYPES:

DECREASING AREA OF LAND TYPE 1, LOW AND HIGH CROP DEMANDS

	Area of Land Type 1									
	2000		1500		1000		500		0	
	Low	High	Low	High	Low	High	Low	High	Low	High
C_1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	-
C_2	0.0	0.0	0.0	0.06	0.0	0.15	0.0	0.26	0.05	0.54
C_3	0.0	0.0	0.0	0.0	0.0	0.07	0.0	0.19	0.0	0.37

Key

Criticalness Measure: C_j = the proportion of land type j that must be allocated in order to satisfy the constraints.

TABLE 7.11

CRITICALNESS MEASURES FOR LAND TYPES:

DECREASING AREA OF LAND TYPE 2, LOW AND HIGH CROP DEMANDS

	Area of Land Type 2											
	5000		4000		3000		2000		1000		0	
	Low	High	Low	High	Low	High	Low	High[1]	Low	High[1]	Low	High[1]
C_1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	-	0.08	-
C_2	0.0	0.0	0.0	0.02	0.0	0.62	0.0	-	0.0	-	-	-
C_3	0.0	0.05	0.0	0.26	0.0	0.47	0.0	-	0.13	-	0.34	-

Key

Criticalness Measure: C_j = the proportion of land type j that must be allocated in order to satisfy the constraints.

[1] No Feasible Solution.

TABLE 7.12

CRITICALNESS MEASURES FOR LAND TYPES:

DECREASING AREA OF LAND TYPE 3, LOW AND HIGH CROP DEMANDS

	Area of Land Type 3											
	10000		8000		6000		4000		2000		0	
	Low	High	Low	High	Low	High	Low	High	Low	High[1]	Low	High[1]
C_1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.49	0.0	-	0.0	-
C_2	0.0	0.06	0.0	0.24	0.0	0.53	0.0	0.94	0.11	[illegible]	0.24	-
C_3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.49	0.0	-	-	-

Key

Criticalness Measure: C_j = the proportion of land type j that must be allocated in order to satisfy the constraints.

[1] No Feasible Solution.

crop to a land type. From Table 7.4, it is apparent that it is the demand for crop 4 that imposes this restriction. In the high demand scenario with only 4000 hectares of land type 3 available, 49 percent of land type 1, 94 percent of land type 2 and 49 percent of land type 3 must be available in order to meet the demands.

7.4 Summary

This chapter has examined alternative measures of land-use criticalness using a hypothetical data set. Each of the crticalness measures is based on the analysis of a linear set of inequalities. Two types of measures are assessed. The first type rates the necessity of assigning individual crops to each land type so as to meet given demands. This measure takes two forms: absolute minimum assignments and conditional minimum assignments. The second type rates the importance of each land type in meeting all the demands.

The absolute minimum value of a decision variable is a useful measure of criticalness as it is has a staightforward interpretation, and provides the base information for the second type of criticalness measure. It is also readily calculated using linear programming. Nevertheless, one of the limitations of this measure is that it is calculated without reference to the other decision variables and may hold only if they are set at extreme values. The calculation of a measure of criticalness that is conditional upon other decision variables being set at their average values represented by a central point in region R provides a means of overcoming this limitation. Of the central interior points examined, the average epicentre is the only point that is suitable for all cases.

CHAPTER 8

SUMMARY AND CONCLUSIONS

8.1 Implications

People are uncertain whether the loss of agricultural land to urban uses will impair the ability to meet future food demands. Much of the controversy over the need to protect agricultural land from urban encroachment derives from two sources. Firstly, there are significant differences in the assumptions that have been developed pertaining to future food needs and food production capacity. Secondly, there has been a lack of suitable procedures for assessing the impact of agricultural land conversion on food production capacity.

This thesis has addressed the second issue. It has assessed the usefulness of existing land and land-use classification techniques for establishing land-use priorities, and demonstrated that the establishment of land-use priorities cannot be based solely on an assessment of the supply of land of different degrees of suitability for agriculture. It has argued that in order to determine if and where agricultural land needs to be protected from urban encroachment so as to ensure adequate food production capacity, policy-makers require information on the ability of the land base to meet expected food needs under a variety of future conditions. There is a need, however, not just to identify and quantify the major variables that delineate food production capacity, but to develop a means of integrating land and nonland factors so that the long-term consequences of agricultural land conversion to urban uses can be identified.

Information about the adequacy of a given land base for meeting future food needs is an essential prerequisite for formulating land-use policy. In assessing both the need for agricultural land preservation policies and in identifying the areas in which such policies should be enacted, however, additional information is needed. This thesis has argued that policy-makers require data on the amount of flexibility that is available, which is defined as how much production capacity exists under given conditions, and whether there are land areas that are critically important for agriculture in the sense that the fulfillment of expected food needs is dependent upon particular land areas being available to agriculture.

The assessment of production capacity subject to constraints on crop productivity and land availability, is consistent with mathematical programming techniques. Considerable interest exists in the application of mathematical programming techniques to problems of agricultural land use and food supply. These methods have been widely applied in the analysis of resource allocation problems, as they provide a framework for identifying the conditions under which different levels of demand can be met and those under which a resource scarcity problem arises. Existing agricultural land-use programming models are capable of analyzing a wide range of issues given concern for such factors as land quality, environmental impact and nonland resource use in the agricultural sector. These models, however, have emphasized the identification of economically efficient agricultural production patterns. Although the issues of how much flexibility exists in agricultural land use, and whether there are land areas critically important for meeting future food needs are raised, conventional uses of programming methods have not directly investigated the issues of land-use flexibility and criticalness as defined in thesis.

Expressing the conditions that affect the ability to meet future food needs in the form of constraints is a useful starting point for the analysis of land-use flexibility and criticalness. A means is required, however, for assessing how much flexibility is defined by a given constraint set, and determining the degree to which the satisfaction of given demands is dependent upon particular types of land. This thesis has assessed alternative measures of land-use flexibility based upon the analysis of a linear set of inequalities that describe resource availability and demand conditions. The analytical methods and properties of two measures of flexibility have been further examined using a hypothetical data set.

The amount of flexibility available is a function of the productive capacity of the land, and the level of land-use demands that must be met. The F(3) measure of flexibility, which indicates the extent to which production capacity exceeds or falls short of specified demands provides a measure of flexibility that is readiliy calculated using linear programming, and one which is readily interpreted. Although a measure of flexibility based on the size of the feasible region relative to a base region is conceptually appealing, the implementation of suitable measure has proved difficult. In particular, the F(5) measure, which is based on the radius of the largest sphere that can be inscribed inside region R relative to that inscribed within a base region Q, is sensitive to the shape of the feasible region. As such, there are many conditions under which it does not provide an accurate measure of the size of the feasible region. This in turn means that it does not provide a true indication of how much flexibility is available.

This thesis has also assessed alternative measures of land-use criticalness. Two types of measures are assessed. The first type rates the importance of assigning individual crops to each land type so as to meet given demands. This measure takes two forms: absolute minimum assignments, and conditional minimum assignments. The minimum proportion of a particular land type that must be allocated to each crop in order to meet given demands, is conceptually appealing as a measure of land-use criticalness. It not only has a straightforward interpretation, but is easily calculated using linear programming. Information about the need to assign particular crops to different types of land in order to meet given demands has important implications for land-use planning. This type of information provides a basis for enacting legislation to preserve land for agricultural use, as it indicates both the area and type of land that must be available in order to meet given demands. This measure of criticalness, however, may understate the importance of some crop allocations as it could depend upon extreme assignments of the other decision variables. One way of overcoming this problem is to calculate a criticalness measure that is not dependent upon extreme assignments.

The measures of land-use criticalness that are conditional upon all the decision variables, except the one of interest, being held at their central or average value, require that a central point in the feasible region be identified. The analysis indicates that the centroid and epicentre as originally proposed by Chapman *et al.* (1984), are not appropriate in many situations. The algorithm developed by Giscondi *et al* (1984), however, provides a relatively inexpensive means of finding a unique central point in the feasible region.

Two issues arise in the practical application of these methods. First, as in any model based approach to problem solving, there are inevitably many trade-offs that must be made between the realism of the representation and the tractability of the resulting model. The amount of flexibility that exists, and the extent to which land areas are critically important for agriculture, are affected by a wide variety of factors. In applying the measures of land-use flexibility and criticalness examined in this thesis, a major task is to identify and quantify the major variables that delineate food production capacity.

One factor which has restricted the wider application of programming methods is that the solution of large linear programming models is expensive. Karmakar, however, has devised an algorithm that solves linear programming problems substantially faster than the currently used simplex method (IMSL Directions, 1985). The simplex algorithm is an exponential time algorithm, meaning that the time the algorithm generally takes to solve a problem increases as an exponential function of the size of the problem, rendering it extremely time consuming for problems. Karmakar's algorithm, however, is a polynomial time algorithm, which can find a solution in the time that is only a polynomial function of the number of variables.

The second issue that arises is the nature of the constraints. The measures of land-use flexibility and criticalness examined in this thesis are based on the analysis of a linear set of inequalities. One limitation of these measures, then, is that they are not designed to consider nonlinear relationships. Nevertheless, the same general principles of flexibility and criticalness apply to the nonlinear case.

8.2 Conclusions

How society makes use of its land resources is a matter of growing public concern and hence of interest to all levels of government. Increasingly, public policy-makers are being called upon to resolve conflicts in the use of land and to establish land-use priorities. The demands and needs of our presently increasing population require that social decisions on land use be based on the best and most reliable information that decision-makers can obtain. Establishing priorities, however, has proved to be a difficult, complex, and often contentious task. Decisions resulting in irrevocable commitment of land use continue to be based more on political expediency or intuitive judgement than informed knowledge of the long-term consequences of alternative courses of action. Although many methods have been developed to assist land-use policy-makers and planners in the formulaltion of land-use policies, policy-makers generally lack the type of information needed to make sound decisions. The dilemma facing policy-makers is aptly summarized by Singer *et al* (1979:225) who note:

> "There are three answers to the question about preserving agricultural land; yes, no, and maybe."

The problem of evaluating and planning for alternative uses of land is a complex one because of the large number of variables involved and numerous interrelationships among them. Land-use policy decisions that foreclose future options must be considered carefully, however, as society's goals and constraints such as population, technology, energy availability and climate may change over time. In order to plan effectively for the use of land in the future, there is a need to develop the ability to assess land resources with respect to alternative sets of conditions and goals. Resolving the various conflicts in land use that exist or which may develop in the future, requires knowledge of the needs for agricultural products and other social needs or goals from the use of land, as well as information on the amount and nature of the land available to satisfy these needs and the alternative means by which they could be met.

This thesis has demonstrated the need for a more comprehensive and integrated approach to the analysis of urban expansion and land needs for agriculture. The measures of land-use flexibility and criticalness examined represent an attempt to bridge the gap between inventories of land resources on the one hand, and information on expected food needs and land-use goals on the other.

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APPENDIX A

PARTIAL DERIVATIVES FOR THE F(3) AND F(5) MEASURES OF FLEXIBILITY

An important issue associated with the measurement of land-use flexibility is the relative importance of the factors that limit the amount of flexibility available. Although the F(3) and F(5) measures may indicate a great deal of flexibility exists, they could be sensitive to small changes in some of the constraints. One means of identifying the factors that have the largest impact on the amount of flexibility available, is simply to vary individual parameters as was done in Chapter 5. The F(3) and F(5) measures of flexibility, however, also provide information about their sensitivity to changing conditions. Both measures are calaculated via linear programming, which reports dual activity values (partial derivatives) for the constraints. They indicate the effect that relaxing the constraints by one unit would have on the objective function in the respective model.

The partial derivatives for the F(3) measure of flexibility are presented in Tables A1 to A6. As noted in section 4.4, these are equal to the dual activivity values in the model. The F(5) measure of flexibility, however, is defined as the ratio of the radius of the largest sphere inscribed in region R to that inscribed in region Q. The dual activity values provided by the linear programming formulation pertain to the radius of the sphere inscribed in region R. Since the radius of the sphere inscribed in region Q is a constant, the rate of change in F(5) with respect to unit changes in land area can be exressed as:

$$\partial F(5) / \partial A_j = 1/r_0 \cdot \partial r_R / \partial A_j.$$

The partial derivatives for F(5) are presented in Tables A7 to A12.

TABLE A1

PARTIAL DERIVATIVES FOR F(3) MEASURE OF FLEXIBILITY:

DECREASING AREA OF LAND TYPE 1, LOW DEMANDS

Area of Land Type 1 (Hectares)

	2000	1500	1000	500	0
L1	-0.00022	-0.00022	-0.00022	-0.00022	-0.00034
L2	-0.00020	-0.00020	-0.00020	-0.00020	-0.00020
L3	-0.00008	-0.00008	-0.00008	-0.00008	-0.00008
Q1	0.00002	0.00002	0.00002	0.00002	0.00003
Q2	0.00002	0.00002	0.00002	0.00002	0.00002
Q3	0.00002	0.00002	0.00002	0.00002	0.00002
Q4	0.00008	0.00008	0.00008	0.00008	0.00008

Key

L = Land Type.
Q = Crop.

TABLE A2

PARTIAL DERIVATIVES FOR F(3) MEASURE OF FLEXIBILITY:

DECREASING AREA OF LAND TYPE 1, HIGH DEMANDS

Area of Land Type 1 (Hectares)

	2000	1500	1000	500	0
L1	-0.00011	-0.00011	-0.00011	-0.00017	-0.00017
L2	-0.00010	-0.00010	-0.00010	-0.00010	-0.00010
L3	-0.00004	-0.00004	-0.00004	-0.00004	-0.00004
Q1	0.00001	0.00001	0.00001	0.00001	0.00001
Q2	0.00001	0.00001	0.00001	0.00001	0.00001
Q3	0.00001	0.00001	0.00001	0.00001	0.00001
Q4	0.00004	0.00004	0.00004	0.00004	0.00004

Key

L = Land Type.
Q = Crop.

TABLE A3

PARTIAL DERIVATIVES FOR F(3) MEASURE OF FLEXIBILITY:

DECREASING AREA OF LAND TYPE 2, LOW DEMANDS

	Area of Land Type 2 (Hectares)					
	5000	4000	3000	2000	1000	0
L1	-0.00022	-0.00022	-0.00022	-0.00022	-0.00022	-0.00022
L2	-0.00020	-0.00020	-0.00020	-0.00020	-0.00020	-0.00020
L3	-0.00008	-0.00008	-0.00008	-0.00008	-0.00008	-0.00008
Q1	0.00002	0.00002	0.00002	0.00002	0.00002	0.00002
Q2	0.00002	0.00002	0.00002	0.00002	0.00002	0.00002
Q3	0.00002	0.00002	0.00002	0.00002	0.00002	0.00002
Q4	0.00008	0.00008	0.00008	0.00008	0.00008	0.00008

<u>Key</u>

L = Land Type.
Q = Crop.

TABLE A4

PARTIAL DERIVATIVES FOR F(3) MEASURE OF FLEXIBILITY:

DECREASING AREA OF LAND TYPE 2, HIGH DEMANDS

	Area of Land Type 2 (Hectares)					
	5000	4000	3000	2000	1000	0
L1	-0.00011	-0.00011	-0.00011	-0.00011	-0.00011	-0.00011
L2	-0.00010	-0.00010	-0.00010	-0.00010	-0.00010	-0.00010
L3	-0.00004	-0.00004	-0.00004	-0.00004	-0.00004	-0.00004
Q1	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001
Q2	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001
Q3	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001
Q4	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004

<u>Key</u>

L = Land Type.
Q = Crop.

TABLE A5

PARTIAL DERIVATIVES FOR F(3) MEASURE OF FLEXIBILITY:

DECREASING AREA OF LAND TYPE 3, LOW DEMANDS

	Area of Land Type 3 (Hectares)					
	10000	8000	6000	4000	2000	0
L1	-0.00022	-0.00022	-0.00020	-0.00020	-0.00020	-0.00022
L2	-0.00020	-0.00020	-0.00019	-0.00019	-0.00019	-0.00018
L3	-0.00008	-0.00008	-0.00009	-0.00009	-0.00009	-0.00011
Q1	0.00002	0.00002	0.00002	0.00002	0.00002	0.00002
Q2	0.00002	0.00002	0.00002	0.00002	0.00002	0.00002
Q3	0.00002	0.00002	0.00003	0.00003	0.00003	0.00003
Q4	0.00008	0.00008	0.00008	0.00008	0.00008	0.00007

<u>Key</u>

L = Land Type.
Q = Crop.

TABLE A6

PARTIAL DERIVATIVES FOR F(3) MEASURE OF FLEXIBILITY:

DECREASING AREA OF LAND TYPE 3, HIGH DEMANDS

	Area of Land Type 3 (Hectares)					
	10000	8000	6000	4000	2000	0
L1	-0.00011	-0.00011	-0.00011	-0.00011	-0.00011	-0.00011
L2	-0.00010	-0.00010	-0.00010	-0.00010	-0.00010	-0.00009
L3	-0.00004	-0.00004	-0.00004	-0.00005	-0.00005	-0.00006
Q1	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001
Q2	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001
Q3	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001
Q4	0.00004	0.000[illegible]	0.00004	0.00004	0.00004	0.00004

<u>Key</u>

L = Land Type.
Q = Crop.

TABLE A7

PARTIAL DERIVATIVES FOR F(5) MEASURE OF FLEXIBILITY:

DECREASING AREA OF LAND TYPE 1, LOW DEMANDS

	Area of Land Type 1 (Hectares)				
	2000	1500	1000	500	0
L1	-0.001	-0.001	-0.001	-0.001	-0.003
L2	0.0	0.0	0.0	0.0	0.0
L3	0.0	0.0	0.0	0.0	0.0
Q1	0.0	0.0	0.0	0.0	0.0
Q2	0.0	0.0	0.0	0.0	0.0
Q3	0.0	0.0	0.0	0.0	0.0
Q4	0.0	0.0	0.0	0.0	0.0

Key

L = Land Type.
Q = Crop.

TABLE A8

PARTIAL DERIVATIVES FOR F(5) MEASURE OF FLEXIBILITY:

DECREASING AREA OF LAND TYPE 1, HIGH DEMANDS

	Area of Land Type 1 (Hectares)				
	2000	1500	1000	500	0
L1	-0.00067	-0.00067	-0.00067	-0.001	-0.003
L2	-0.00038	-0.00038	-0.00038	0.0	0.0
L3	-0.00015	-0.00015	-0.00015	0.0	0.0
Q1	-0.00038	-0.00038	-0.00038	0.0	0.0
Q2	-0.00021	-0.00021	-0.00021	0.0	0.0
Q3	-0.00022	-0.00022	-0.00022	0.0	0.0
Q4	-0.00029	-0.00029	-0.00029	0.0	0.0

Key

L = Land Type.
Q = Crop.

TABLE A9

PARTIAL DERIVATIVES FOR F(5) MEASURE OF FLEXIBILITY:

DECREASING AREA OF LAND TYPE 2, LOW DEMANDS

Area of Land Type 2 (Hectares)

	5000	4000	3000	2000	1000	0
L1	-0.001	-0.001	-0.001	-0.00067	0.0	0.0
L2	0.0	0.0	0.0	-0.00038	-0.001	-0.003
L3	0.0	0.0	0.0	-0.00015	0.0	0.0
Q1	0.0	0.0	0.0	-0.00038	0.0	0.0
Q2	0.0	0.0	0.0	-0.00021	0.0	0.0
Q3	0.0	0.0	0.0	-0.00022	0.0	0.0
Q4	0.0	0.0	0.0	-0.00029	0.0	0.0

Key

L = Land Type.
Q = Crop.

TABLE A10

PARTIAL DERIVATIVES FOR F(5) MEASURE OF FLEXIBILITY:

DECREASING AREA OF LAND TYPE 2, HIGH DEMANDS

Area of Land Type 2 (Hectares)

	5000	4000	3000
L1	-0.00054	-0.00054	-0.00054
L2	-0.00050	-0.00050	-0.00050
L3	-0.00020	-0.00020	-0.00020
Q1	-0.00031	-0.00031	-0.00031
Q2	-0.00028	-0.00028	-0.00028
Q3	-0.00029	-0.00029	-0.00029
Q4	-0.00038	-0.00038	-0.00038

Key

L = Land Type.
Q = Crop.

7

TABLE A11

PARTIAL DERIVATIVES FOR F(5) MEASURE OF FLEXIBILITY:

DECREASING AREA OF LAND TYPE 3, LOW DEMANDS

Area of Land Type 3 (Hectares)

	10000	8000	6000	4000	2000	0
L1	-0.001	-0.001	-0.001	0.001	-0.00058	0.0
L2	0.0	0.0	0.0	0.0	-0.00042	0.0
L3	0.0	0.0	0.0	0.0	-0.00026	0.003
Q1	0.0	0.0	0.0	0.0	-0.00034	0.0
Q2	0.0	0.0	0.0	0.0	-0.00036	0.0
Q3	0.0	0.0	0.0	0.0	-0.00038	0.0
Q4	0.0	0.0	0.0	0.0	-0.00032	0.0

Key

L = Land Type.
Q = Crop.

TABLE A12

PARTIAL DERIVATIVES FOR F(5) MEASURE OF FLEXIBILITY:

DECREASING AREA OF LAND TYPE 3, HIGH DEMANDS

Area of Land Type 3 (Hectares)

	10000	8000	6000	4000
L1	-0.00054	-0.00054	-0.00054	-0.00053
L2	-0.00050	-0.00050	-0.00050	-0.00049
L3	-0.00020	-0.00020	-0.00020	-0.00024
Q1	-0.00031	-0.00031	-0.00031	-0.00031
Q2	-0.00028	-0.00028	-0.00028	-0.00033
Q3	-0.00029	-0.00029	-0.00029	-0.00035
Q4	-0.00038	-0.00038	-0.00038	-0.00037

Key

L = Land Type.
Q = Crop.

APPENDIX B

CONDITIONAL CRITICALNESS MEASURES BASED ON THE EPICENTRE: THE EFFECT OF ALTERNATE OPTIMA

As noted in section 5.3.2, the co-ordinates of the epicentre may not be unique. Alternate optima do not pose a problem for measuring the radius of the largest inscribed sphere, as regardless of the epicentre's location, the maximum value of the radius will always be the same. However, alternate optima may be important in the analysis of land-use criticalness. The conditional minimum measure of land-use criticalness requires that the decision variables be set at the co-ordinates of an interior point which is, in some sense, central. If the epicentre is unique, this requirement is satisfied. If the co-ordinates of the epicentre are not unique, however, its location can vary. Depending upon the extreme values that the epicentre's co-ordinates can take, the values of the conditional minimum assignments could be misleading.

Standard linear programming packages, such as I.B.M.'s MPSX, identify only one extreme location for the epicentre if it is not unique. The algorithm developed by Giscondi *et al.* (1984), however, identifies all the extreme positions that the epicentre could adopt, and then averages them to obtain a unique, central interior point. An illustration of the difference between conditional minimum measures calculated when the co-ordinates of the epicentre are not unique, and those based on the average epicentre is presented in Table B1. This table presents the measures of criticalness based on the epicentre and average epicentre, for scenarios which decrease the area of land type 2 while land types 1 and 3 are held at their base levels. The criticalness measures based on the average epicentre are taken from Table 7.8. The two sets of measures are quite different from each other. The greatest discrepancy occurs in the initial scenario. As the area of land type 2 is reduced, the two measures converge until the epicentre is unique.

TABLE B1

CONDITIONAL MINIMUM MEASURES OF CRITICALNESS BASED ON EPICENTRE AND AVERAGE EPICENTRE: DECREASING AREA OF LAND TYPE 2, LOW DEMANDS

	Area of Land Type 2											
	5000		4000		3000		2000		1000		0	
	A	B	A	B	A	B	A[1]	B	A	B	A	B
c_{11}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.03	0.03	0.0	0.08
c_{21}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	0.0	0.0	0.02
c_{31}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.37	0.10	0.47	0.13
c_{41}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.02	0.0	0.45	0.09
c_{12}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	0.0	-	-
c_{22}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	0.0	-	-
c_{32}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	0.0	-	-
c_{42}	0.42	0.07	0.45	0.03	0.26	0.0	0.0	-	0.0	0.0	-	-
c_{13}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.0	0.0	0.0	0.0
c_{23}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-	0.03	0.0	0.08	0.02
c_{33}	0.07	0.0	0.07	0.0	0.07	0.01	0.07	-	0.0	0.06	0.0	0.08
c_{43}	0.04	0.44	0.20	0.51	0.41	0.57	0.60	-	0.65	0.64	0.55	0.66

<u>Key</u>

A : Epicentre.
B : Average Epicentre.
Criticalness Measure: c_{ij} = proportion of land type j that must be allocated to crop i in order to satisfy the constraints, conditional on other decision variables being set at the central values given by either the co-ordinates of the epicentre or the average epicentre.

[1] Epicentre is Unique.