BEHAVIOUR OF THERMAL DENSITY CURRENTS IN COLD RECEIVING WATER BODIES

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### A Thesis

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In cold climates, temperatures higher than the ambient have been observed near the bottom of water lakes in the vicinity of thermal discharges. Concern has <sup>4</sup> been expressed about the adverse effects of such abnormally warm water on the winter ecology of lake bottoms. It is expected that the existence of a density extremum in water at 4° C and the resulting nonlinear relation between density and temperature gives rise to densimetric flows which are markedly different from those in the linear range.

ABSTRACT

This thesis presents experimental and numerical investigations that give some insight into the phenomenon of the thermal bar and the manner in which it may influence nearshore transport processes in the vicinity of a thermal outfail in a cold climate. The investigations are restricted to an idealized model where the lock exchange mechanism is selected due to the fact that its behaviour is close to that expected in the prototype situation.

The experimental investigation provides dramatic proof that the existence of an extremum in the density-temperature relation has a profound influence on the behaviour of densimetric flows in general and lock exchange behaviour in particular. Three zones in the vicinity

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of a thermal bar are clearly demonstrated viz. (i) the thermal overflow region. (ii) the thermal bar, and (iii) the thermal underflow region. The experiments provide data on the horizontal scale at which sinking takes place.

A numerical model has been constructed to develop a means of modelling the behaviour \_of a thermal bar at the outfall of a steam electric generating station cooling water system. The numerical model employs a finite-difference scheme, where the resulting algebraic finite difference equations are solved using an alternating direction implicit method and a sparse-matrix package. The numerical model has been verified by comparing it to numerical solutions of four different cases of the idealized problem of steady laminar flow in .an enclosed rectangular cavity with differentially heated end walls. Moreover, additional acceleration techniques are introduced to improve the numerical solution procedure. The numerical model is employed to solve the actual problem of simulating lock exchange flows created between two water bodies having different temperatures around the temperature of maximum density (i.e. having temperatures above and below  $4^{\circ}$  C). The general behaviour which has been observed experimentally is also confirmed numerically. The sensitivity of the associated parameters is examined. The relative extension of the thermal bar is correlated with relevant system parameters. Difficulty was experienced in obtaining numerical results for the same (high) Rayleigh numbers as

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were used for the physical experiments. Despite this, an encouraging degree of consistency was observed between simulated and observed behaviour.

The important aspect of the study is to draw attention to the adverse effects of the sinking phenomenon (thermal bar), which may occur in the vicinity of man-made warm effluents as well as in natural bodies of water during the spring warming period. The study is significant. In terms of the horizontal scale at which sinking takes place, for the design of power station once-through-cooling water systems that must operate in cold climate winter conditions.

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### CHAPTER 1

SCOPE AND OBJECTIVES OF STUDY

### 1.1 <u>Introduction</u>

Despite increases in the overall efficiency of mechanical plants a large fraction of the heat input to steam electric generating stations is discarded. In stations with once-through-cooling water system this waste heat is discharged to the environment in the form of heated effluent with a temperature typically 10 C<sup>O</sup> above that of the receiving water. Under the temperate climate conditions which commonly obtain, this warm water forms a raft with a pronounced discontinuity in the vertical temperature distribution and a frontal system driven by the thermal density difference. When large stations are located adjacent to natural bodies of water these warm water discharges give rise to environmental concerns which are frequently more important than the technical problem of warm surface water being drawn into the cooling water intake with consequent reduction in the thermal and economic efficiency of the station.

#### 1.2 <u>Statement Of Problem</u>

In cold climates the behaviour of thermal density currents may be altered when the receiving water is close to the freezing point due to the fact that fresh water attains its maximum density at a temperature of  $4^{O}$  C. Temperatures higher than the ambient have been observed near the bottom of fresh water lakes in the vicinity of thermal discharges [Hoglund and Spigarelli- (1972), Pipes, Pritchard and Beer (1973)] and concern has been expressed about the adverse effects of such abnormally warm water on the winter ecology of lakes in cold climates. When the receiving water is warmer than 4° C the lighter fluid will spread out in the usual fashion shown in Figure 1.1(a), however when the ambient lake water is less than  $4^{\circ}$  C an entirely different behaviour is found [Metcalfe (1980)]. This may be seenfrom the well known non-monotonic nature of the density-temperature relation shown in Figure 1.1(b).

It may then be seen as demonstrated schematically in Figure 1.1(c) that thermal discharges warmer than  $8^{\circ}$  C will have positive buoyancy and spread as a surface layer but when an ambient temperature of close to  $0^{\circ}$  C is assumed then the plume will begin to sink when its temperature is brought to less than  $8^{\circ}$  C by cooling and mixing processes, since at this point its density is greater than the receiving fluid.

Thereafter there is the possibility that the warm water will spread over the bed as a density current subjecting life forms on the



lake bed to a transient and unseasonal increase in temperature. The sinking phenomenon, known as a thermal bar, may occur in the vicinity of man-made warm effluents and also in natural bodies of water during the spring warming period as reported by Rodgers (1968) and Spain, Wernert and Hubbard (1976). It seems likely that the formation of a thermal bar has a significant effect on exchange mechanisms in nearshore waters and is therefore of some importance in the problem of disposing of warm effluents in cold climates.

#### 1.3 <u>Objectives And Summary</u>

The purpose of this dissertation is to analyse the behaviour of the thermal density currents in water subject to a nonlinear relation between density and temperature which results from the existence of a density extremum at  $4^{\circ}$  C. This behaviour is markedly different from the well established behaviour in the linear range. The objectives of this study are :

- (.1) To prove that the existence of an extremum in the density-temperature relation has a profound influence on the behaviour of the thermal density currents.
- (2) To demonstrate the principal flow regime in an idealized case which is likely to occur in the vicinity of a thermal outfall.
- (3) To provide sufficient data on the horizontal extension at

which sinking takes place.

(4) To develop a mathematical model to simulate the mechanism and allow the sensitivity of the associated parameters to be examined.

The bibliography of the subject is presented in Chapter 2. The literature review of the environmental investigations indicates sufficient evidence of the existence of the sinking plumes. Moreover, it draws attention to such abnormally warm water which may cause disruption of temperature dependent life cycles of aquatic organisms within the area of elevated bottom temperature. To confirm Such behaviour, physical experiments are required to provide a valuable basis for continuing work on the analysis of such phenomena. To more closely approximate the prototype conditions at a thermal outfall a two-dimensional experiment was required which would allow inspection of the zones upstream and downstream of the thermai bar as well as the plunging region of the bar. Even a simple two-dimensional model of a thermal outfall presents a relatively complex phenomenon, combining both inertial and densimetric effects. It is worth mentioning that the phenomena of the sinking thermal plume are due to the buoyancy effect (i.e. existence of a density extremum), therefore, it was decided to restrict the experiments to an idealized model in which buoyancy is the only force governing the motion of the flow. Consequently, the literature review of the idealized models will be presented only for the buoyancy induced convective motion. The lock exchange flow

experiment was selected due to the fact that its behaviour is close to that expected in prototype situation.

The lock exchange flow experiments are illustrated in Chapter 3. The densimetric flows were created in a horizontal flume between two water bodies having the same surface elevation but which differ in temperature (i.e. density). The experimental temperatures adjusted to set up initial temperatures  $T_w$  and  $T_c$  for the warm and cold water respectively such that  $T_w > 8^{\circ}$  C and 4 >  $T_c > 0^{\circ}$  C. The initial flow pattern was similar to the classic lock exchange mechanism, the warmer, buoyant layer which was identified with a weak trace of fluorescene extending over the cold receiving water. Concurrently, a cold, dense wedge was propagated under the warm body of water. After the warm front had progressed for some distance, filaments of fluorescene dyed water could be seen extending downward from the interface through the colder water to the bed of the flume. As the volume of water entrained in this way increased, however, the warm front was arrested and a layer of dyed water was seen to propagate along the bed of the flume in the same direction as the original surface layer. It was found that the three zones of interest in the vicinity of a thermal bar were clearly demonstrated viz. (i) the thermal overflow region, (ii) the thermal bar, and (iii) the thermal underflow region. The experiments provide dramatic proof that the existence of an extremum in the density-temperature relation has a profound influence on the behaviour of densimetric flows. The ratio of the maximum extension of the warm front to the depth of the flow was

related to the density difference between the cold and warm water through a simple quadratic relation.

The thermodynamic simulation of the phenomena associated with the thermal bar is generally explained in Chapter 4. The governing differential equations of the fluid motion and heat transfer are described by a set of coupled, nonlinear partial differential equations. The system is transfered from the dimensional primitive form to the non-dimensional conservative form by using the vorticity-stream function approach and defining certain non-dimensional terms. Accordingly, the physical parameters involved in the problem collapse into two parameters, i.e. the Rayleigh number and the Prandtl "number, which are usually used to describe the flow patterns. It may be noted here that in certain related studies (e.g. cavity flows) the aspect ratio may be included as a third parameters. The governing differential equations are solved by using the finite difference method. The alternating direction implicit (ADI) method is selected to solve the parabolic forms and a sparse-matrix package is utilized to solve the elliptic form of the resulting partial differential equations. For simplicity, the time-dependent, nonlinear partial differential equations are usually solved by considering them to be linear in some small time interval, the magnitude of the time step being dictated by the requirement of numerical stability of the solution. To accelerate the solution procedure, ', large time steps are attempted, subject to numerical stability, along with the introduction inner corrector iterations within each time step to correct the

estimated values of the nonlinear terms.

In Chapter 5, the numerical model verifications and improvements are illustrated. The idealized problem of steady laminar flow in an enclosed rectangular cavity with differentially heated end walls was chosen for the purpose of numerical verification. The model is verified by comparing it to numerical (and thus indirectly experimental) solutions of four different cases of the idealized problem. The results of the four verification cases considered suggest that the numerical model is highly suitable for the solution of problems of this type. Moreover, significant numerical improvements to result following the introduction of additional are shown acceleration techniques and from modification of the finite difference approximations. The solution procedure is improved by using a variable step with step size control, averaging and extrapolation time techniques. It was found that if the advection term is approximated by the second upwind differencing method instead of by central-difference. approximation, high values of Rayleigh number can be simulated over a stable scheme.

Finally, the numerical analysis of the phenomena associated with the thermal bar's presented in Chapter 6. The numerical model was employed to solve the actual problem of simulating lock exchange flows created between two water bodies having different temperatures around the temperature of maximum density. Due to restrictions in terms of computer memory and computational costs, the numerical investigation was restricted to the simulation of similar cases for

which accurate and stable results could be achieved with reasonable computational cost. The effect of the nonlinear density-temperature relation on the behaviour of the thermal density currents was studied. Moreover, the general behaviour which had been observed experimentally was also confirmed numerically. The sensitivity of the associated parameters are examined by means of the numerical model. The relative extension of the thermal bar is correlated with relevant system parameters by an empirical relation and the numerical results are compared with the experimental results.

#### CHAPTER 2

### BIBLIOGRAPHY OF SUBJECT AREA

### 2.1 <u>Introduction</u>

The review of the available literature, related to-the phenomena of the sinking plumes presented in Chapter 1, can be covered by dividing the background area into the following main topics :

(1) Environmental investigations

(2) Idealized models for buoyancy induced convective motion

The first topic is classified into biological implications due to heated effluents, and actual field temperature measurements which indicate the existence of such phenomena. The second topic deals with previously published idealized models which are used to approximate the prototype conditions and provide some insight into these more difficult problems. Two idealized model types will be covered which are found to be related to the subject.

### 2.2 <u>Environmental Investigations</u>

#### 2.2.1 General

Large quantities of waste heat from steam electric generating stations are discharged in the form of circulating cooling water into natural water bodies such as lakes, rivers and oceans. This method of "once through" cooling is more economical than the "closed loop system" (i.e. cooling ponds, spray ponds, and cooling towers), and is employed provided the temperature standards set by various regulatory agencies can be achieved. Lately, some concerns have been expressed (based on field temperature measurements) about the adverse effects (biological implications) of warm water on the ecology of water bodies during winter when the ambient temperature of the lake water is near the freezing mark.

#### 2.2.2 Biological implications

It is commonly suggested that thermal discharges into natural water bodies cause disruption of temperature dependent life cycles of aquatic organisms, such as premature hatching of fish and insect eggs or reduction of viability of young.

The major fall spawning species in the Great Lakes that might be affected by a sinking plumes are the coregonids (whitefish, herring and chubs) and lake trout. The effect of controlled incubation temperature on the survival and development of whitefish (coregonus clupeaformis) and herring (coregonus artedii) have been studied by Price (1940) and Colby and Brooke (1970), respectively. It was found that a constant increase of  $4.5 \, \text{C}^{\circ}$  (from  $0.5^{\circ} \, \text{C}$  to  $5.0^{\circ} \, \text{C}$ ) resulted in the reduction of incubation time for whitefish eggs from 141 to 71 days and for herring eggs from 236 to 106 days. In general, increased incubation temperatures resulted in reduced incubation periods and incidence of abnormalities for both species.

The literature is very scant relative to the effects of thermal discharges on lake benthos and various studies have provided contradictory information. Coutant (1962) observed reductions in species diversity and biomass of benthos downstream from a thermal discharge in the Delawere River during summer months; the benthic population recovered to some extent during cooler seasons. Craven and Brown (1970) observed no detrimental effects of a heated effluent on lake benthos; biomass, numbers and species composition did not differ between heated and unheated areas.

Few studies have considered the effects of temperature elevations during winter and the possible influence on emergence of aquatic insects. Nebeker (1971) found a constant increase of 8 C<sup>o</sup> above ambient resulted in updating of stream insect emergence by 1-2 months.

A probable effect of the sinking thermal plumes is the stimulation of periphyton growth within the area of elevated bottom temperatures. Studies of periphyton production at Point Beach during the summer of 1971 by Spigarelli and Prepejchal (1972) revealed significantly higher biomass at near field plume stations than far

field. During the winter, stimulation of periphyton growth may be significant, relative to normal seasonal growth, considering the greater temperature increases at the bottom.

### 2.2.3 Field Temperature Measurements

The phenomenon of a sinking thermal plume was observed during March and April 1971 at the Point Beach Nuclear Power Plant, Wisconsin [Hoglund and Spigarelli (1972)]. Temperature recorders were placed on the bottom of Lake Michigan near the outfall of the plant. Analysis of the data revealed the warm water did interact with the bottom as long as the lake temperature was  $4^{\circ}$  C or less. When the ambient temperature exceeded  $4^{\circ}$  C, there was little indication of temperature perturbation on the bottom as a result of the thermal discharge.

The same behaviour was also observed during thermal plume surveys in Lake Ontario which had been carried out by Ontario Hydro, Metcalfe (1980), to investigate the behaviour of thermal plumes during cold weather in general and at the Pickering Generating Station in particular. The results of the thermal plume mapping from December 1979 to March 1980, indicated that the maximum extent of the plume on the bottom is larger than the maximum extent of the plume on the surface.

For more examples, sinking plumes were observed near the Oskarshamn Power Plant [Svensson (1980)] and the Wauhegan Generating Station [Pipes, Pritchard and Beer (1973)].

Unfortunately, these field measurements did not provide data

on the horizontal scale at which sinking takes place, but it gives sufficient evidence of the existence of the sinking plumes.

### 2.3 <u>Idealized Models For Buoyancy Induced Convective Motion</u>

#### 2.3.1 General

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When a heated fluid (such as warm water) is introduced into a cold environment (such as a natural water body), it will spread due to the initial momentum of the heated fluid (created by its discharge velocity), or due to the buoyancy forces (created by the temperature difference), or a combination of both. The important behaviour in this investigation is the spreading due to buoyancy forces. Consequently, the discussion for the previous idealized models will be presented only for the buoyancy induced convective motions.

The transport of heat or mass by buoyancy induced convective motions is a mechanism which finds relevance in many physical-systems; accordingly there have been numerous theoretical, experimental and numerical studies of various aspects of natural convective flows. Usually the direct modelling of these natural systems is very complex, however, the idealized cases of such convective motions do provide some insight into these more difficult problems. Large scale physical models of such phenomena were common in the 1950-1970 period and are described in more detail later in this section.

It is worth mentioning that three main types of basic models may be used to increase understanding of buoyancy induced convection

#### mechanisms :

(1) Rectangular heated cavity

(2) Lock exchange flow

(3) Buoyant plume

Only the first two are considered here.

### 2.3.2 Rectangular heated cavities

The fact that air is a good insulator has been appreciated and utilized in the construction of buildings for many years. For instance, in the construction of dwellings, it is common practice to build walls consisting of two thicknesses of brick separated by an unventilated air gap of a few inches. The same practice is used for double glazing insulation. Heating engineers have therefore been concerned with discovering how heat is transferred across the air gap, and with how the rate at which heat is transferred depends on the distance between the two vertical walls and on the temperature difference between them. Thus, considerable insight into the basic phenomena can be obtained from the idealized problem of laminar fluid flow in a two dimensional cavity with differentially heated end walls. Recently, the idea of this idealized model has been utilized for the motion set up in the epilimnion of lakes or reservoirs by non-uniform radiative heating.

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Generally, the convective motion of a fluid enclosed in a rectangular domain has been studied for several different boundary

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conditions, aspect ratios and temperature gradients according to the specified physical system (i.e. environmental application).

For а situation where а linear density-temperature relationship can be assumed, the steady-state problem for which the aspect ratio is greater than or equal to unity was solved analytically [Batchelor (1954); Gill (1966)], experimentally [Eckert and Carlson (1961); Elder (1965)] and numerically [Wilkes and Churchill (1966); de Vahl Davis (1968); Newell and Schmidt (1970); Quon (1972)]. Although occasionally included as part of a wider study [e.g. Cotton, Ayyaswamy and Clever (1974)], the small aspect ratio case was not treated in a detailed way until Cormack, Leal and Imberger (1974); Cormack, Leal and Seinfeld (1974); Imberger (1974) examined the problem from the analytical, numerical and experimental points of view respectively. The unsteady problem has been studied by Wilkes and Churchill (1966), who obtained some transient results and by Patterson and Imberger (1980), who examined the transient behaviour in detail by numerical studies and also presented a scaling analysis based on these numerical results. In most of these studies, the flow pattern phenomena are classified in terms of the well established non-dimensional parameters such as the Rayleigh number  $R_a$ , Prandtl number  $P_r$  and aspect ratio A; where

 $R_{a} = \frac{g(\Delta \rho)H^{3}}{\rho v \kappa}$   $P_{r} = \frac{v}{\kappa}$ 2.1
2.2

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2.3

for which

 $\Delta \rho$  = density difference, (=  $\rho_1 - \rho_2$ )  $M_z$  = depth of cavity

L = length of cavity

g = gravitational acceleration

v = kinematic viscosity

< = thermal conductivity

 $\rho$  = reference density [e.g.  $\rho_{ave} = (\rho_1 + \rho_2)/2$ ]

Using these parameters, Patterson and Imberger (1980) classified the flow pattern into a number of possible transient flow types depending on the value of  $R_a$  in relation to various combinations of  $P_r$  and A. These regimes were further combined to provide a broad classification of the flows into :

(1) conductive  $R_a < 1$ (2) transitional  $1 < R_a < P_r^2$ or (3) convective  $P_r^2 < R_a < P_r^4 A^{-4}$ and  $P_r^4 A^{-4} < R_a$ 

depending on the relative values of R<sub>a</sub>. More recently, Patterson (1984) and Ivey (1984) studied, analytically and experimentally respectively, the transient phase for

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which  $R_a > P_r^{+}A^{-+}$  where an oscillatory approach to final steady-state conditions exists for certain flow regimes.

The convective motion of an enclosed rectangular water cavity, in the region of maximum density, has been studied for several different geometries, boundary conditions and temperature gradients. The transient behaviour of water contained in a rigid rectangular container insulated and cooled from above to near freezing has been considered by Forbes and Cooper (1975). Vasseur and Robillard (1980) have studied the transient cooling of water, enclosed in a rectangular cavity with the wall temperature maintained at  $0^\circ\,$  C. Supercooling of water contained in a rectangular cavity subjected to convective boundary conditions has been investigated by Robillard and Vasseur (1981). Cooling of the cavity by a constant rate has been extended by Robiliard and Vasseur (1982). Generally, it was found numerically that the resulting flow motion is greatly influenced by the presence of a maximum density effect which slows down the initial circulation inside the cavity and subsequently reverses it. The resulting heat transfer is thus reduced in comparison to a standard situation without maximum density effect.

Moreover, Desai and Forbes (1971) and Watson (1972) have studied numerically the steady-state of the heat transfer and flow patterns in cold water in a rectangular enclosure with vertical boundaries maintained at different temperatures and with insulated horizontal boundaries. It was found that the fluid moves in two opposite vortices (cells) which have the same size when symmetrical
temperatures about 4° C are assumed. More recently, Hamblin and Ivey (1984) studied analytically, numerically and experimentally the same problem for cavities of small aspect ratios and symmetrical temperatures about the temperature of maximum density.

#### 2.3.3 Lock exchange flows

It has become a common practice to use estuaries and other bodies of water for the disposal of waste heat from thermal.generating . stations. Although such dumping may be safely carried out in many situations, it is important to take proper consideration of its impact on the biochemical processes that depend critically on the water temperature. This is particularly true when organic pollution is coupled with waste heat disposal. However, before outfall systems can be designed so that these limits are not exceeded, it is necessary to understand more fully the mechanisms by which these warm effluents are dispersed within the receiving body of water. The geometry of the estuary is complicated, and the flow is turbulent and generally coupled with the tidal cycle. Therefore, a complete dynamic model of an estuary, of course, would be very complex. Moreover, estuaries are usually shallow and have sufficiently strong vertical mixing to prevent the formation of density wedges, the result is a density distribution which is vertically uniform but varies in the horizontal direction. There was, however, concern lest local concentrations of heat should prove undesirable from the point of view of recirculation. injurious to the marine environment or should, in particular, affect

adversely the passage of migratory fish by the formation of a heat barrier. The lock exchange flow is usually selected as an analogous phenomenon, where knowledge exists regarding the varying pattern of diminution of front velocities of the density discontinuity over a wide range of scale and under controlled conditions.

The phenomenon of lock exchange flow is the classical case of unsteady non-uniform flow in the field of small density difference hydraulics. This phenomenon occurs when a lock gate or other such division separates bodies of still water with the same surface elevation but which differ slightly in density. While the opening of the gate may result in local disturbances, the predominent effect will be a continuing exchange pattern of flow which is caused by the density difference.

Experimental studies of thermal densimetric flow have been reported by a large number of investigators. Keulegan(1946, 1957) and Schijf and Schonfeld (1953) used salinity as the density difference agency, whereas Barr (1963, 1966, 1967) used both thermally induced and salinity density flows. More recently, Simpson (1982) has given a broad-ranging account of gravity currents, which includes a most comprehensive bibliography.

Barr (1963-A) discussed the significance of the densimetric Froude-Reynolds number  $F_{\Delta}R$  defined by

$$F_{\Delta}R = \sqrt{\frac{g(\Delta \rho)H^3}{\rho v^2}}$$

It has been customary to compare lock exchange flows on the basis of K.F\_AR where K is the initial velocity,  $u_0$ , coefficient :

$$K = \frac{u_0}{\sqrt{g(\Delta \rho)H/\rho}} = \frac{u_0}{u_{\Delta}}$$
 2.5

The results are usually shown on a diagram relating  $\clubsuit$   $F_{\Delta}R$  to L/H where L is the horizontal extent of the front measured from the barrier with  $u/u_0$  being a third parameter where u is the front velocity.

Barr (1963) and Barr and Hassan (1963) showed experimentally that the initial velocities of the fronts of the underflow and overflow appear uniform for a greater or lesser relative distance depending on the scale of the experiment, and are unaffected by the channel width to depth gatio (W/H) except for extreme cases where the channel width, W, is significantly less than H/2.

The results of such investigations have been applied to large scale physical models of lock exchange flow [Frazer, Barr and Smith (1967), (1968)] in order to estimate the appropriate degree of vertical scale exaggeration which is required to simulate surface spread correctly. New diagrams for lock exchange flow are utilized (plots of L/H against  $F_{\Delta}R$  for different values of the non-dimensional time  $t/t_{\Delta}$  where  $t_{\Lambda} = \sqrt{-H\rho/g(\Delta\rho)}$ ].

The various experimental investigations indicate that the expected relationship between the velocity of spread u of either the overflow front,  $u_1$ , or the underflow front,  $u_2$ , and the densimetric velocity  $u_A$  is :

$$\frac{u_i}{u_{\Delta}} = \frac{L_i/t}{\sqrt{g(\Delta \rho)H/\rho}} = \text{constant}, \quad i=1,2 \qquad 2.6$$

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This constant was found experimentally by Barr (1963), reported by Frazer, Barr and Smith (1967 and 1968) to be about 0.5 for underflow and 0.6 for overflow and by Yih (1980) to be 0.5 based on energy considerations. Equation 2.6 was found to be applicable in turbulent flows where the phenomenon is independent of viscosity [P. Ackers in discussion of Frazer, Barr and Smith (1968), reported by Elsayed (1978)]. The limit to the turbulent zone is represented approximately by :

$$F_{\Delta}R.\frac{H}{L_{i}} \equiv \frac{\sqrt{g(\Delta \rho)/\rho} H^{5/2}}{\nu L_{i}} \Rightarrow 150$$

2.4 <u>Discussion</u>

Review of the available literature, related to the problem presented in Chapter 1, has been covered in this chapter. The biological implications due to warm effluents indicate that thermal discharges into natural water bodies may cause disruption of temperature dependent life cycles of aquatic organisms. In addition, thermal plume surveys in cold climates give sufficient evidence of the exsistence of sinking plumes. Consequently, adverse effects on the winter ecology of a natural water body may be expected within the area of elevated bottom temperatures associated with the phenomenon of the sinking plumes. The idealized models for buoyancy induced convective motion provide insight into such "phenomena. Two types of idealized models have been presented, the rectangular heated cavities and the lock exchange flows. A fact of some significance is that the phenomenon of the thermal bar may be observed in both the rectangular cavity and lock exchange systems if asymmetrical or symmetrical temperatures around 4° C are assumed. However, the flow patterns in the two models are different. Lock exchange flow was selected to approximate the prototype due to the fact that its behaviour is close to that expected in a prototype situation. In the following chapter, the experimental investigation to examine such behaviour is restricted to a lock exchange flow in a two-dimensional, horizontal flume of rectangular cross-section.

## CHAPTER 3

## EXPERIMENTAL INVESTIGATION

#### 3.1 <u>Introduction</u>

This chapter describes a series of physical experiments which were carried out to investigate the phenomenon of the sinking plume which has been described in Chapter 1. The lock exchange flow experiment was selected as the basis of the study for a number of reasons. Firstly, the experiment is free from external inertial effects, motion being produced entirely by the density difference between two dissimilar but miscible bodies of water. A second reason is the large body of documented experimental work which has been carried out for lock exchange flows in which the density is a linear function of temperature. This has been discussed in Chapter 2.

In the tests described here, densimetric flows were produced between two bodies of water having asymmetrical temperatures around 4° C. In this range the relation between density and temperature is nonlinear with maximum density being attained at 4° C. To the author's

knowledge, this type of experiment had not previously been carried out at the time of writing.

The experiments were carried out to confirm the existence of the sinking plume phenomenon and to allow inspection of the zones upstream and downstream of the thermal bar as well as the plunging region of the bar. It was hoped also that the tests would provide some quantitative data relating to the maximum horizontal extension of the upper layer to the point at which sinking occurs.

The chapter contains a description of the experiments and both qualitative and quantitative results relating to the formation of the thermal bar. Some attempt is made at empirical correlation of the relative extension of the bar with relevant system parameters which provides a basis for the numerical simulation which is described in later chapters.

## 3.2 Lock exchange flow Experiment

## 3.2.1 General

The experiments were designed to be carried out at reasonable cost and within a limited time period for which the special facility was available and with the objective of providing qualitative and some quantitative evidence on which to base a numerical modelling strategy. The tests were made possible by obtaining the use of a flume in the Cold Room of the Hydraulics Laboratory in the Canada Centre for Inland Waters. This facility [Tsang (1977)] provides a temperature controlled

environment with a range from  $-30.0^{\circ}$  C to  $\pm 15.0^{\circ}$  C with accuracy of  $\pm 0.5^{\circ}$  C and uniformity of  $\pm 1.0^{\circ}$  C. The existing flume is horizontal, 10 m in length with a cross-section 0.6 m wide by 0.5 m deep. The base and rear wall of the flume are thermally insulated and the front is of transparent plastic with removable (hinged) insulating panels.

## 3.2.2 Theoretical considerations

The densimetric exchange flow experiments may be regarded as an idealized model of the motions set up in the cold receiving water bodies ( $T_a = 0^\circ$  C) by heated effluent ( $T_I > 8^\circ$  C). In such applications, inertia, buoyancy, and friction ought to be the principal forces governing the motion of the flow with Coriolis forces playing a secondary role. A non-dimensional number incorporating these forces is the Rayleigh namber,  $R_a$ . When  $R_a$  is defined in the usual way, it has an approximate range of 10<sup>4</sup> to 10<sup>6</sup>, when typical prototype values for turbulent diffusion; thermal diffusivity, friction, depth and density differences are assumed.

Some researchers have attempted to use synthetic liquid mixtures with volatile components (fluid consisting of alcohol with ethylene glycol in various proportions mixed with water) to simulate the nonliear density-temperature relationship [Turner (1966)]. In this case, with the availability of an experimental cold facility, it was possible (and indeed desirable) to use the same fluid (i.e. water) and temperatures in the laboratory as in the field setting. Thus a simulation of the field values of  $R_a$  would require water depths in the

laboratory ranging from 10 to 50 mm based upon molecular values of viscosity and conductivity. It was considered to be impractical to work with such a thin layer of fluid, or to control the temperature differences with the required accuracy to maintain sufficiently small density contrasts. The more reasonable working depths of 0.10 to 0.15 m and larger density differences result in corresponding Rayleigh numbers from  $10^7$  to  $10^8$ .

In the laboratory experiments the Rayleigh number is defined as

$$R_{a} = \frac{gH^{3}}{\omega_{o}\kappa_{o}} (\Delta \rho) / \rho_{ave} \qquad 3.1$$

where

Δp = density difference between cold receiving water and heated effluent

H = depth of water in the flume

P<sub>ave</sub> = average density

 $v_0$  = kinematic viscosity of water at 4<sup>0</sup> C

 $k_0$  = thermal conductivity of water at 4<sup>0</sup> C

## 3.2.3 Apparatus and procedure

To partially insulate the third dimension of the flume (i.e. the free surface) removable sheets of polystyrene were used. A removable plastic vertical barrier was fitted into grooves formed at the joints of the walls and in the base. The thin, flexible barrier was reinforced by providing a second, more robust barrier adjacent to it for the purpose of protecting the thin barrier from small differences in elevation across the barrier during the setup period when the two portions of the flume were being filled and mixed, and also to provide greater thermal insulation between the two dissimilar bodies of water. The stiff barrier was carefully removed some minutes prior to the start of the experiment to allow the small volume of water between the barriers to be made homogeneous.

The vertical barrier was located asymmetrically along the length of the channel, being at the quarter point, 2.5 m from one end. The shorter portion was used to contain the warm water. As a result, care had to be taken that the region of interest was not affected by the negative internal wave which reflected off the end wall of the flume. Care was also taken to seal the vertical barrier with grease to prevent minor leakage of the warm water which would have formed a thin, surface tension layer on top of the cold receiving water. Setting up of an experiment involved the initial cooling (but not freezing) of water in the whole length of the flume. Then the barriers were inserted and sealed and the shorter section was heated by mixing with hot water piped in from outside the room. Weirs were fitted at each end of the flume to provide a simple means of ensuring that the depth on both sides of the barrier was the same prior to the experiment. Figure 3.1(a) shows the general arrangement of the apparatus.

Water and air temperatures were monitored prior to the start of the test by means of calibrated thermisters connected to a data

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- Figure 3.1
- (a):
- (b):
- Sketch of apparatus. Arrangement of thermistors. Operation of rotary switch to read resistance by multimeter (i.e. probe #9). (c):

logger which forms part of the basic facility. To observe the vertical distribution of temperature during the tests it was necessary to devise a simple form of instrumentation. A number of thermisters were obtained from the Flow Systems Laboratory of Ontario Hydro. These thermisters were calibrated to an accuracy of 0.10 C by Ontario Hydro. The thermisters were arranged on a moveable carriage such that the measuring points were ranged over a vertical line, along the centre line of the flume and uniformly spaced over the depth of water used in the test. This arrangement is shown in Figure 3.1(b). The thermisters were connected through a rotary switch to a 3.5 digit multimeter to read the resistance in K-ohms as shown in Figure 3.1(c). During the tests the investigator manually operated the rotary switch, observing the reading on the multimeter and recording this on a pocket dictation machine together with remarks about the progress of the experiment. Although of limited capability, this intermediate level of technology proved adequate in that it was quickly and cheaply constructed and allowed tests to be monitored with a degree of confidence that may. have proved difficult to obtain with sophisticated equipment in such a hostile environment. With very little practice it was possible to record the array of ten temperatures in some 15 to 20 seconds. Movement of the surface front was sufficiently slow that a scanning time of this magnitude was acceptable.

To aid observation of the spreading front the warm, initially buoyant, layer was identified with a weak trace of fluorescene. In addition, vertical streaks of dye tracer with nearly neutral buoyancy

were injected at intervals. Due to the need to thermally insulate the front panel and the free surface, observation and photography of the lock exchange was made rather difficult and in most cases the photographic records were supported by audio recorded remarks during the course of the experiment.

## 3.3 <u>Experimental Results</u>

## 3.3.1 General

The range of the test parameters included two values of the depth H and a range of density differences corresponding to temperature increments between 11 and 18 degrees Celsius. The seven tests are summarized in Table 3.1. Generally, the densimetric Froude number given by ;

$$F_{\Delta} = \frac{u}{\sqrt{g(\Delta \rho)H/\rho}} ; \qquad 3.2$$

is normally a good descriptor of lock exchange behaviour, particularly the initial exchange velocity.

## 3.3.2 Observations

The tests were commenced when the two bodies of water were judged to be acceptably homogeneous and quiescent, following the removal of the stiff, thermally insulated secondary barrier. Then the primary barrier was removed by hand using a smooth, swift movement to

Table	3.	1
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Parameters used in experimental investigation

Test #	H, in meters	H, in T <sub>1</sub> , in meters deg. cel.		∆T, in deg. cel.	
1	0.10	18.20	0.00	18.20	
Ž	0.10	14.15	0.00	14.15	
3	0.10	12.15	0.30	11.85	
- 4	0.10	12.14	0.05	12.09	
5	0.15	10.80	0.25	10.55	
6	0.15	18.15	0.45	17.70	
7	0.15	12.45	0.70	11.75	

Note : 1 m ≈ 3.28 ft

 $^{\circ}C = 5(^{\circ}F - 32)/9$ 

minimize local disturbance. Almost immediately a classic lock exchange mechanism could be observed with the warmer, buoyant layer extending over the cold receiving water with an initial velocity close to that given by [Barr (1963)]

$$v_0 = C_v \sqrt{g(\Delta \rho)H/\rho}$$

where,  $C_v = 0.55$  for overflow.

Concurrently, a cold, dense wedge was propagated under the water [see Figure 3.2(a)]. As the buoyant raft warm body of progressed, the mixing process at the front discarded an interfacial layer of mixed water, with the frontal head being replenished by a surface flow of warm unadulterated water. After the warm front had progressed for some distance (L<sub>U,</sub> - measured from the barrier position), filaments of fluorescene dyed water could be seen extending downward from the interface through the colder water to the bed of the flume [Figure 3.2(b)]. This mixing process appeared to be initiated a short distance behind the head of the buoyant layer and at first appeared to cause only a small reduction in the rate of propagation of the warm surface layer. As the volume of water entrained in this way increased, however, the warm front was arrested and a layer of dyed water was seen to propagate along the bed of the flume in the same direction as the original surface layer [Figure 3.2(e)].

Two transitional stages between the onset of sinking and the fully developed sinking plume are shown in Figure 3.2(c) and 3.2(d).

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: Diagramatic illustration of stages in lock exchange experiments. Figure 3.2

During this transition it may be noted that the lower layer is reflected by the end-wall. After the reflection, the reverse travelling wave continues to supply warm water to the sinking zone.

As the warm but now diluted layer proceeded along the bed of the flume towards the end wall it had a temperature such that its density exceeded that of the layer  $T_2$ . Consequently, a second lock exchange mechanism was set up in that part of the flume containing the cold receiving water with the result that an overflowing layer of cold water,  $T_2$ , was forced over the plunging layer [Figure 3.2(f)].

Eventually, after a considerable time during which the various fronts were reflected from the end walls and had traversed the length of the flume, a three layer system was produced with distinct interfaces as shown in Figure 3.2(g).

During the progress of the experiment, the vertical temperature distribution was recorded along the centre line of the flume and a short distance behind the warm water overflow front. Sometimes the temperature measurements were repeated for the same location or recorded some distance behind the front. Some typical sets of observations for different relative extensions of the front are shown in Figure 3.3 while more temperature measurements are presented in Appendix II. With the benefit of hindsight, it would have been better to mount the array of ten angled thermister probes such that the supporting 7 mm diameter tubes did not intrude in the overflow front. Because of this there is some doubt as to the accuracy of the temperature measurements very close to the edge of the front, due to



Figure 3.3 : Typical temperaturo distributions for Test #5.

boundary layer effects around the probes.

## 3.3.3 Discussion

Table 3.2 summarizes the observed and calculated parameters for the seven tests. The temperature was based on observed thermister resistance using a calibration equation supplied by Ontario Hydro. From the computed temperature the density was estimated from [Cheng, Takeuchi and Gilpin (1978)] :

$$\rho = \rho_0 / (1 + b_1 T + b_2 T^2 + b_3 T^3 + b_4 T^3)$$

where

$$b_1 = -0.678964520*10^{-6}$$
 CP<sup>1</sup>

 $b_2 = +0.907294338*10^{-5} \circ C^{-2}$ 

 $b_3 = -0.964568125*10^{-7} \circ C^{-3}$ 

b<sub>4</sub> = +0.873702983\*10<sup>-9</sup> ° C<sup>-4</sup> ·

 $\rho_{\rm c} = 0.999839600 \, {\rm gm/cm}^3$ 

T = temperature in °C

The values of the density ratio  $\Delta \rho / \rho_{ave}$  and the Rayleigh number  $R_a$  were obtained by Eqs. 3.5 and 3.1 respectively.

$$\Delta \rho / \rho_{ave} = 2.(\rho_2 - \rho_1) / (\rho_1 + \rho_2) \qquad 3.5$$

In addition to the measured temperature profiles, Figure 3.3

3.4

Table 3.2

# Values calculated from experimental observations

Test #	ρ <sub>1</sub> , in gm/cm <sup>3</sup>	ρ <sub>2</sub> , in gm/cm <sup>3</sup>	(1) Δρ/ρ <sub>ave</sub>	(L <sub>u</sub> /H) <sub>max</sub>	. R <sub>a</sub> *10 <sup>-7</sup>
- 1	0.9985575	0.9998396	0.00128	42.5	6.33
2	0.9992225	0.9998396	0.00062	30.0	3.04
· 3	0.9994793	0.9998592	0.00038	23.5	• 1.87
4	0,9994805	0.9998430	0.00036	22.5	1.79
、5 、	0.9996243	0.9998560	0.00023	16.7	3.86
6	0.9985668	0.9998683	0.00130	43.3	21.69
7 -	0.9994440 *	0,9998827	0.00044	23.2	7.31

·· · · ·

Note : 1 Kg = 1000 gm = 2.208 Ib 1 m = 100 cm = 3.28 ft

(1) The accuracy of temperature measurements (i.e.  $0.1 \ C^{0}$ ) implies an error bar of 0.000005 (i.e. up to 5% error) in calculated density difference ratio  $\Delta p/p_{ave}$ .

shows the corresponding density distribution just behind the front and at various locations along the flume. Initially the interfacial layer, which contains water at maximum density, is quite thin but as the mixing process develops, the middle zone of maximum density water increases in thickness, reduces in forward velocity and is eventually "precipitated" through the cold water which is marginally less dense.

It was observed that the thickness of the middle zone was about one quarter of the total depth.

The extension  $(L_u)_{max}$  of the upper layer is the maximum distance attained by the buoyant overflow layer and is expressed as a relative distance using the depth H as a normalizing dimension.

It appears clear that the reversal of the lock exchange flow illustrated in (e) to (g) of Figure 3.3 is due entirely to the fact that the system is closed and does not correctly represent prototype conditions in a natural environment. Interest is therfore limited to the initial occurrence of the plunging bar and the arresting of the overflow wedge.

A number of arguments may be advanced as to why the warm surface wedge is arrested. The normal case of an arrested wedge occurs when the receiving water has a uniformly distributed velocity in opposition to and almost equal to the spread velocity; clearly this is not the case here. The existence of an adverse velocity gradient at the surface due either to wind shear or mass transport from gravity waves [Smith (1965)] can also prevent the replenishing flow of unadulterated water at the surface and cause arrest and progressive

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breakdown of the front. For normal lock exchange tests it has been shown [e.g. Keulegan (1957), Barr (1963 and 1967)] that the initial frontal velocity diminishes after some relative extension, particularly when the Rayleigh number is relatively small. In such cases, however, the diminution of velocity occurs gradually whereas in the tests described here, the warm wedge appears to be arrested more abruptly as a consequence of the enhanced sinking velocity due to the  $4^\circ$  C water produced by mixing/entrainment at the front.

The quasi-steady state [Figure 3.2(e)] which results following the formation of a thermal bar may be examined by means of simple dimensional analysis. Assuming a relationship involving the function :

$$f(L_{u}, \rho_{ave}, \Delta \rho, H) = 0 \qquad 3.6$$

and adopting the first two quantities of Eq. 3.6 as repeating variables it can be shown that the following rather obvious dimensionless groupings are obtained.

$$\pi(1) = L_U/H$$
;  $\pi(2) = \Delta \rho / \rho_{ave}$ 

Inclusion of g. v and k in Eq. 3.6 results in additional parameters one of which would be equivalent to the Rayleigh number of Eq. 3.1. Figure 3.4 examines the hypothesis that the maximum relative extension of the warm front  $(L_u/H)_{max}$  is a function of the density .

40

3.7



difference ratio  $(\Delta \rho / \rho_{ave})$  using a simple quadratic plot. The seven test results fall close to the straight line given by

$$(L_u/H)_{max} = 1200 \sqrt{(\Delta \rho)/\rho_{ave}}$$
 3.8

The range of Rayleigh numbers covered by the tests is necessarily somewhat limited since the density difference is small in the nonlinear range around 4° C. The scale of tests is also restricted by the physical length of the flume. Despite these limitations however the consistency of the results is encouraging. The large constant of proportionality in Eq. 3.8 is probably due to additional dependence on such parameters as the Prandtl number,  $P_r = v/\kappa$ , which is impossible' to vary in the laboratory experiments.

The evidence of Figure 3.4 is of some interest but remains far from conclusive concerning the prediction of the location of a thermal bar. It is clear that the occurrence of a thermal bar is caused by the formation of an interfacial layer with a density close to the maximum, of sufficient thickness and in the absence of a sustaining forward relative velocity. What is not yet clear is whether the formation of the layer with a temperature close to 4° C results from the mixing process at the front or from the cooling effect of the air under which the warm surface flow must pass in order to replenish the front and so sustain the frontal velocity.

The latter hypothesis is partly supported by the correlation between the maximum extension  $(L_u)_{max}$  and the initial temperature  $T_1$ 

for any depth H . If this was true, however, it would be expected that the same phenomenon would be significant when the ambient temperature is of the order of 15° C (say) and the density-temperature relation is nearly linear. In fact, Barr (1963) has reported negligible difference between lock exchange experiments in which the same density difference ratio has been produced respectively by salinity (conservative) and temperature (non-conservative) difference. On that basis it seems reasonable to assume that frontal mixing is the dominant process whereby the interfacial layer of maximum density is produced.

## 3.4 <u>Conclusion</u>

The experiments provide dramatic proof that the existence of an extremum in the density-temperature relation has a profound influence on the behaviour of densimetric flows in general and lock exchange behaviour in particular.

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The three zones of interest in the vicinity of a thermal bar were clearly demonstrated viz. (i) the thermal overflow region, (ii) the thermal bar, and (iii) the thermal underflow region. The results are qualitatively consistent with observations reported in the literature.

The quantitative results, although subject to some uncertainty, are sufficiently good to give a picture of a gradually thickening interfacial layer of maximum density which eventually

dominates the flow regime. The location of the thermal bar appears to be strongly linked to the initial density difference although the scope of the tests and limitations of equipment were such that other influences can not as yet be ruled out.

The experimental results [Marmoush, Smith and Hamblin (1984)] are significant for the design of power station cooling water systems which must operate in severe winter conditions. Moreover, it provides a valuable basis for continuing work on the numerical simulation of the phenomena associated with the thermal bar which will be presented in the following chapters.

#### CHAPTER 4

## THERMODYNAMIC SIMULATION

## 4.1 <u>Introduction</u>

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The governing set of differential equations of the fluid motion and heat transfer is the obvious starting point for °a comprehensive simulation. In general, a thermodynamic simulation involves the following steps:

- (i) derivation of the appropriate forms of the governingequations. (Sec. 4.2)
- (ii) formulation of the appropriate numerical schemes.(Sec. 4.3)
- (iii) definition of the initial and boundary conditions. (Sec. 4.4)
- (iv) determination of the solution procedure. (Sec. 4.5)
- (v) verifications and improvements of the numerical techniques.

Step (v) is described in Chapter 5 while the other steps, (i) to (iv), are reviewed in the following sections.

4.2

## Governing Differential Equations

The fluid motion and heat transfer are generally described by a set (system) of coupled (simultaneous) partial differential equations which are mathematical statements of the conservation of momentum, energy and mass. Moreover, the equation of state for the fluid of interest should be defined over the temperature range to be simulated. The system of equations, therefore, is comprised of :

(i) the Navier-Stokes equation (conservation of momentum);

(ii) the heat-transfer equation (conservation of energy);

(iii) the continuity equation (conservation of mass);

(iv) the equation of state for the fluid (density -temperature relationship).

4.2.1 Primitive equations

For a fluid element in a certain domain, the two-dimensional governing differential equations [Lamb (1945) and Schlichting (1968)] are:

				-					
9u		au		Эu		-1 ap			
	+	u —	+	v —	=	<u> </u>	÷	v ⊽ <sup>2</sup> u	Ai
Эt		Э×		ЗY		p_ 9x		-0	4.1

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \frac{-1}{\rho_0} \frac{\partial p}{\partial y} + v_0 \nabla^2 v - g \frac{\Delta p}{\rho_0}$$
4.2
$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \kappa_0 \nabla^2 T$$
4.3
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
4.4

where

 $\nabla^2$ 

D

Δρ

g

= two-dimensional Laplacian operator, a<sup>2</sup>/ax<sup>2</sup>+a<sup>2</sup>/ay<sup>2</sup>; = horizontal and vertical components of velocity field, respectively;

> = temperature;

= pressure;

= density difference;

= gravitational acceleration;

 $P_0, v_0, \kappa_0$  = fluid properties (density, kinematic viscosity and thermal conductivity, respectively) at some specified reference temperature,  $T_0$ ;

t = time;

\$.y = coordinates of elements in the horizontal and vertical directions, respectively.

The equations are written in an Eulerian frame of reference, i.e., a space-fixed reference within which the fluid flows [Bird, Steward and Lightfoot (1960), Mehaute (1976)]. In these equations, it is assumed

that the fluid is incompressible and also follows a Newtonian shear stress law whereby the viscous force is linearly related to the rate of strain. Moreover, the Boussinesq approximation is applied, [Gray and Giorgini (1976)], where :

- (i) the density is assumed constant, except when it directly causes buoyancy forces, i.e., gravitational forces;
- (ii) all other fluid properties are assumed constant, such as  $p_0$ ,  $v_0$ ,  $\kappa_0$ , and defined at a certain reference temperature, To;

(iii) the viscous dissipation is assumed negligible.

The equations are written in terms of the primitive variables, u, v, p, T,  $\Delta p$ , and are often referred to as the primitive form of the governing differential equations. Although it is possible to obtain numerical solutions for these equations, most successful numerical solutions have utilized the vorticity-stream function approach.

4.2.2 Appropriate equations

The equations of motion, Eqs. 4.1, 4.2 and 4.4, can be recast in another format, in which the pressure term is eliminated from the Navier-Stokes equations, 4.1 and 4.2, and therefore only one transport equation needs to be involved. The pressure; p, can be eliminated by first cross-differentiating Eq. 4.1 with respect to y and Eq. 4.2 with respect to x and then subtracting the two resultant equations. Moreover, by defining the x-y plane component of vorticity,  $\omega$ , as

$$m = \frac{9\lambda}{9\pi} - \frac{9\lambda}{9\lambda}$$

the following vorticity-transport equation is obtained

$$\frac{\partial \omega}{\partial t} + u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = v_0 \nabla^2 \omega + g \frac{\partial \Delta \rho}{\partial x \rho_0}$$
4.7

Defining the stream-function,  $\psi$ , by

$$\frac{\partial \psi}{\partial x} = -u$$
 and  $\frac{\partial \psi}{\partial x} = -u$  4.8

Eq. 4.6 may be rewritten as

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega$$

or

$$\nabla^2 \psi = -\omega \qquad 4.9(b)$$

It is noted that the continuity equation, 4.4, is identically satisfied by the introduction of the stream-function,  $\psi$ .

## 4.2.3 ¿ Conservative equations

The heat-transfer equation 4.3 and vorticity-transport equation 4.7 can also be recast in different but equivalent form to

4.6

4.9(a)

take advantage of the conservation property discussed by Roache (1982). Slightly modified versions of the continuity equation 4.4 are employed, with respect to the properties of temperature and vorticity, viz.

т( ~	9x 	÷	9À.) 9A.)	z	→ 7 30 30	+	т <mark>-</mark> ау	2	0	• •	•	4.10
ω (	9x 	+ 、	9À 9Å )	Ξ	υ6 	+	ω <del>-</del> 3ν	z	0	•		4.11

These are added to both sides of Eqs. 4.3 and 4.7, respectively, to yield the so-called "conservative form" of the heat-transfer and vorticity-transport equations, i.e.

$$\frac{\partial T}{\partial t} + \frac{\partial vT}{\partial x} + \frac{\partial vT}{\partial y} = \kappa_0 \nabla^2 T$$
4.12

It is noteworthy that the conservative form of the governing differential equations gives more accurate results than the non-conservative form [Torrance et al. (1972)].

## 4.2.4 Normalized equations

The governing differential equations can be normalized by defining the following non-dimensional terms

T.

$$X = \frac{x}{H} ; \qquad Y = \frac{y}{H} ; \qquad \tau = \frac{t\kappa_0}{H^2} ;$$
$$U = \frac{uH}{\kappa_0} ; \qquad V = \frac{vH}{\kappa_0} ; \qquad \Omega = \frac{\omega H^2}{\kappa_0} ; \qquad 4.14$$
$$\Psi = \frac{\Psi}{\kappa_0} ; \qquad \Theta = \frac{T-T_0}{\Delta T}$$

where the water depth, H. is the characteristic length. Due to the semi-infinite nature of the domain of interest it is not possible to normalize the equations in terms of any characteristic horizontal length. Substituting the terms of Eq. 4.14 into Eqs. 4.8, 4.9, 4.12 and 4.13, the normalized conservative governing differential equations become

$$\frac{\partial \Theta}{\partial \tau} + \frac{\partial U\Theta}{\partial X} + \frac{\partial V\Theta}{\partial Y} = \nabla^2 \Theta \qquad 4.15$$

$$\frac{\partial \Omega}{\partial \tau} + \frac{\partial U \Omega}{\partial \tau} + \frac{\partial V \Omega}{\partial Y} = P_r \nabla^2 \Omega + R_a P_r \frac{\partial F(\theta)}{\partial X}$$

$$\frac{\partial F(\theta)}{\partial x} = 4_a 16$$

$$U = -\frac{\partial \Psi}{\partial Y} \qquad V = -\frac{\partial \Psi}{\partial X} \qquad 4.18$$

where  $P_r$  is the Prandtl number,  $P_r = v_0/\kappa_0$ . The Rayleigh number,  $R_a$ and f(0) will be defined according to the specified equation of state. For example when the linear version of the equation of state 5.1 is used then  $R_a$  and f(0) are defined as  $g\alpha_0(\Delta T)H^3/v_0\kappa_0$  and 0,

respectively. On the other hand, they are defined as  $g\beta_0 (\Delta T)^2 H^3 / v_0 r_0$ and  $\theta^2$ , respectively when the nonlinear equation of state 5.9 is used.

#### 4.3 <u>Numerical Formulation</u>

In general, the governing differential equations are not amenable to analytical solution, and a solution has to be obtained numerically, for example, by using either a finite difference method or a finite element method. In the present study, the finite difference method is considered to be more appropriate for the following reasons :

- (i) its treatment is considerably simpler for the rectangular geometry involved;
  - (ii) it requires less computer storage;
- (iii) there is a vast literature on its application to other fluid mechanics problems.

As a result, a finite difference method was selected for the numerical simulation using a uniform size mesh,  $\Delta X$ ,  $\Delta Y$  over a rectangular computational domain as shown in Figure 4.1.

In a finite difference method, the derivative of any function, F, (such as  $\Omega$ ,  $\Theta$ ,  $\Psi$ , ..., ect) with respect to S (such as X, Y, and  $\tau$ ) are preplaced by approximations in terms of the differences of the values of the function at the neighbouring nodes. The general finite



1 -

difference expressions for first and second derivatives are obtained below for the uniform size mesh.

A function F may be expanded in a Taylor series expansion forward and backward from a point S to the neighbouring points S $\pm\Delta$ S to give

$$F_{s+\Delta s} = F_{s} + \frac{3F}{3S} \Big|_{s}\Delta S + \frac{1}{2} \frac{\partial^{2}F}{\partial S^{2}} \Big|_{s}\Delta S^{2} + \frac{1}{6} \frac{\partial^{3}F}{\partial S^{3}} \Big|_{s}\Delta S^{3} + O(\Delta S^{4})$$
 4.19  
$$F_{s-\Delta s} = F_{s} - \frac{\partial F}{\partial S} \Big|_{s}\Delta S + \frac{1}{2} \frac{\partial^{2}F}{\partial S^{2}} \Big|_{s}\Delta S^{2} - \frac{1}{6} \frac{\partial^{3}F}{\partial S^{3}} \Big|_{s}\Delta S^{3} + O(\Delta S^{4})$$
 4.20

The expression for the first derivative,  $\partial F/\partial S$ , is obtained for first-order accuracy using Eq. 4.19 as

$$\frac{\partial F}{\partial S} = \frac{\delta F}{\delta S} = \frac{F_{S+\Delta S} - F_{S}}{\Delta S} + O(\Delta S)$$
4.21

which is the known "forward-difference approximation". Subtracting Eq.4.19 and 4.20, a second-order accurate expression is obtained as

$$\frac{\partial F}{\partial S} = \frac{\delta F}{\delta S} = \frac{F_{S+\Delta S} - F_{S-\Delta S}}{2\Delta S} + O(\Delta S^2)$$
 4.22

which is the known "central-difference approximation". The expression for the second derivative,  $\partial^2 F/\partial S^2$  is obtained for second-order accuracy by adding Eq. 4.19 and 4.20, i.e.

$$\frac{\partial^2 F}{\partial S^2} \bigg|_{S} \simeq \frac{\delta^2 F}{\delta S^2} \bigg|_{S} = \frac{F_{S+\Delta S} - 2F_{S} + F_{S-\Delta S}}{\Delta S^2} + O(\Delta S^2)$$
 4.23
In the present problem; the time derivative,  $\partial/\partial \tau$ , is expressed by Eq. 4.21 and the spatial derivatives,  $\partial/\partial X$ ,  $\partial/\partial Y$ ,  $\partial^2/\partial X^2$ ,  $\partial^2/\partial Y^2$ , are expressed by Eqs. 4.22 or 4.23. Now, selecting an appropriate method for the numerical solution of each governing equation is a difficult task. There are a large number of available methods which have been applied to fluid dynamics and heat transfer problems. The selection of methods for each of the governing equations is discussed in the sections which follow.

## 4.3.1 Heat-transfer equation

For convenience the governing equation for heat-transfer is shown below.

$$\frac{\partial \Theta}{\partial \tau} + \frac{\partial U \Theta}{\partial X} + \frac{\partial V \Theta}{\partial Y} = \nabla^2 \Theta \qquad 4.15$$

The resulting difference equations produced by applying the heat-transfer equation 4.15 at each node (i,j), where i,j describes the location of the node, can be solved explicitly or implicitly for the unknown  $\theta_{i,j}^{n+1}$ , where n is time index. The solution process for an explicit scheme is a simple time-stepping procedure, where the algebraic difference equations are uncoupled. However, a Fourier stability analysis for this scheme shows that the time step is severely restricted by two limits, advection limit and diffusion limit [Roache (1982)]. For problems involving large integration times, these constraints on  $\Delta \tau$  may be unacceptable. The implicit scheme is

unconditionally stable, thus allowing a large time step. Yet, the solution process involves either a direct inversion of a large nonlinear sparse generally banded matrix or an iterative method of solution. Both approaches are very costly in terms of computer storage and execution time.

An alternative to the explicit and implicit methods is the alternating direction implicit. ADI, method. The ADI method [Peaceman and Rachford (1955), Douglas (1955)] is computationally stable and does not involve iteration or direct inversion of a large matrix. Instead, only the solution of tridiagonal matrices alternating along the rows or columns of the grid is required. The ADI method is also knows as the method of variable direction [Kuskova(1968)].

The parabolic, second order heat-transfer equation 4.15 is solved using the ADI method which advances the solution from time level  $f_1$  to time level n+1 in two steps. Using the interpretation indicated in Figure 4.2, the first step is the solution for the variation in the X-direction only, then followed by the second step where the solution is only for the Y-direction. Substituting for the unsteady, convection and diffusion terms by their finite difference approximations, 4.21-4.23, then the first step advances the solution in the X-direction to an intermediate level n+1/2. This result is given by

$$\frac{\Theta_{i,j}^{n+1/2} - \Theta_{i,j}^{n}}{\Delta \tau/2} + \frac{U_{i+1,j}\Theta_{i+1,j}^{n+1/2} - U_{i-1,j}\Theta_{i-1,j}^{n+1/2}}{2\Delta X}$$



# Figure 4.2

: Construction of ADI method in two dimensional rectangular domain.

$$\frac{\Theta_{i+1,j}^{n+1/2} - 2\Theta_{i,j}^{n+1/2} + \Theta_{i-1,j}^{n+1/2}}{\Delta X^{2}} + \frac{\Theta_{i,j+1}^{n} - 2\Theta_{i,j}^{n} + \Theta_{i,j-1}^{n}}{\Delta Y^{2}} + \frac{\Theta_{i,j+1}^{n} - 2\Theta_{i,j}^{n} + \Theta_{i,j-1}^{n}}{\Delta Y^{2}}$$

The second step advances the solution in the Y-direction to time level n+1 and is given by

$$\frac{\Theta_{i,j}^{n+1} - \Theta_{i,j}^{n+1/2}}{\Delta \tau/2} + \frac{U_{i+1,j}\Theta_{i+1,j}^{n+1/2} - U_{i-1,j}\Theta_{i-1,j}^{n+1/2}}{2\Delta X} + \frac{V_{i,j+1}\Theta_{i,j+1}^{n+1} - V_{i,j-1}\Theta_{i,j-1}^{n+1}}{2\Delta Y} \qquad 4.25$$

$$= \frac{\Theta_{i+1,j}^{n+1/2} - 2\Theta_{i,j}^{n+1/2} + \Theta_{i-1,j}^{n+1/2}}{\Delta X^{2}} + \frac{\Theta_{i,j+1}^{n+1} - 2\Theta_{i,j}^{n+1} + \Theta_{i,j-1}^{n+1}}{\Delta Y^{2}}$$

Rearranging the equations in a format appropriate for the use of the Gauss elimination technique [Richtmyer and Morton (1967); Peaceman and Rachford (1955); Von Rosenberg (1969); Ames (1969)] gives

$$a_{i}\theta_{i-1,j}^{n+1/2} + b_{i}\theta_{i,j}^{n+1/2} + c_{i}\theta_{i+1,j}^{n+1/2} = d_{i}$$
 4.26

for the first step advancement along the X-direction, and

$$a_{j}\theta_{i,j-1}^{n+1} + b_{j}\theta_{i,j}^{n+1} + c_{j}\theta_{i,j+1}^{n+1} = d_{j}$$
 4.27

for the second step advancement along the Y-direction. In these equations the coefficients are defined as follows.

$$a_{i} = -(R_{xx} + 0.5R_{x}U_{i-1,j})$$

$$4.28(a)$$

$$b_{i} = 2(1 + R_{xx})$$

$$4.28(b)$$

$$c_{i} = -(R_{xx} - 0.5R_{x}U_{i+1,j})$$

$$4.28(c)$$

$$d_{i} = (R_{yy} + 0.5R_{y}V_{i,j-1})\theta_{i,j-1}^{n} + 2(1 - R_{yy})\theta_{i,j}^{n}$$

$$+(R_{yy} - 0.5R_{y}V_{i,j+1})\theta_{i,j+1}^{n}$$

$$4.28(d)$$

$$a_{i} = -(R_{yy} + 0.5R_{y}V_{i,j-1})$$

$$4.28(d)$$

$$b_{j} = 2(1 + R_{yy}^{*})$$

$$4.29(b)$$

$$c_{j} = -(R_{yy} - 0.5R_{y}V_{i,j+1})$$
 4.29(c)

$$d_{j} = (R_{xx} + 0.5R_{x}U_{i-1,j})\theta_{i-1,j}^{n+1/2} + 2(1 - R_{xx})\theta_{i,j}^{n+1/2}$$

$$+(R_{xx} - 0.5R_{x}U_{i+1,j})\theta_{i+1,j}^{n+1/2}$$
 4.29(d)

and

$$R_{x} = \Delta \tau / \Delta X$$
,  $R_{xx} = \Delta \tau / \Delta X^{2}$   
 $R_{y} = \Delta \tau / \Delta Y$ ,  $R_{yy} = \Delta \tau / \Delta Y^{2}$ 
4.30

## 4.3.2 Vorticity-transport equation

For convenience the governing equation for vorticity -transport is shown below.

$$\frac{\partial\Omega}{\partial\tau} + \frac{\partial U\Omega}{\partial X} + \frac{\partial V\Omega}{\partial Y} = P_{r}\nabla^{2}\Omega + R_{a}P_{r}\frac{\partial f(\Theta)}{\partial X}$$
4.16

The parabolic, second order vorticity-transport equation 4.16 is also solved using the ADI method for the same reasons mentioned in Sec. 4.3.1. In the same manner as for the heat-transfer equation 4.15 the advancements along the X- and Y- direction are given by :

$$a_{i}\Omega_{i-1,j}^{n+1/2} + b_{i}\Omega_{i,j}^{n+1/2} + c_{i}\Omega_{i+1,j}^{n+1/2} = d_{i}$$
; 4.31

$$a_{j}\Omega_{i,j-1}^{n+1} + b_{j}\Omega_{i,j}^{n+1} + c_{j}\Omega_{i,j+1}^{n+1} = d_{j}$$
; 4.32

In these equations the coefficients are defined as follows.

$$a_i = -(P_r R_{xx} + 0.5 R_x U_{i-1,j})$$
 4.33(a)

$$b_i = 2(1 + P_r R_{xx})^{-1}$$
. 4.33(b)

$$c_i = -(P_r R_{xx} - 0.5 R_x U_{i+1,j})$$
 4.33(c)

$$d_{i} = (P_{r}R_{yy} + 0.5R_{y}V_{i,j-1})\Omega_{i,j-1}^{n} + 2(1 - P_{r}R_{yy})\Omega_{i,j}^{n}$$
$$+ (P_{r}R_{yy} - 0.5R_{y}V_{i,j+1})\Omega_{i,j+1}^{n}$$

$$+0.5R_{a}P_{r}R_{x}[f(\theta)_{i+1,j} - f(\theta)_{i-1,j}]$$
 4.33(d)

$$a_j = -(P_r R_{yy} + 0.5 R_y V_{i,j-1})$$
 4.34(a)

$$b_{j} = 2(1 + P_{r}R_{yy})$$
 4.34(b)

$$c_j = -(P_r R_{yy} - 0.5 R_y V_{i,j+1})$$
 4.34(c)

$$d_{j} = (P_{r}R_{xx} + 0.5R_{x}U_{j-1,j})\Omega_{j-1,j}^{n+1/2} + 2(1 - P_{r}R_{xx})\Omega_{j,j}^{n+1/2} + (P_{r}R_{xx} - 0.5R_{x}U_{j+1,j})\Omega_{j+1,j}^{n+1/2} + 0.5R_{a}P_{r}R_{x}[f(\theta)_{j+1,j} - f(\theta)_{j-1,j}] + 0.34(d)$$

It is worth mentioning that from a study of the literature it appears that the ADI method is currently the most popular approach to solve such problems over simple rectangular regions.

## 4.3.3 Stream-function equation

For convenience the governing equation for stream-function is shown below.

$$\nabla^2 \Psi = -\Omega \qquad 4.17$$

The stream-function equation, or Poisson equation, 4.17 represents a boundary-value or "Jury" problem. For the cases under consideration, it is solved for Dirichlet boundary conditions in which f

values of  $\Psi$  are known at all nodes on the boundary of the domain. The discretized form of Eq. 4.17 using second-order differences approximation 4.23 is the "5-point formula" [Thom and Apelt (1961)] :

$$\frac{\Psi_{i+1,j} - 2\Psi_{i,j} + \Psi_{i-1,j}}{\Delta X^2} + \frac{\Psi_{i,j+1} - 2\Psi_{i,j} + \Psi_{i,j-1}}{\Delta Y^2} = -\Omega_{i,j} \quad 4.35$$

The finite difference analog 4.35 of the stream-function equation 4.17 is solved over different rectangular domains using :

- (a) iterative method (successive over-relaxation, SOR, method)
- (b) direct methods
  - 1- Fourier-series method

2- sparse-matrix package

(a) There are many variations of iterative methods for the Poisson equation. The most popular method is the "successive over-relaxation", SOR, method [Frankel (1950); Young (1954)] where the solution is expressed as

 $\Psi_{i,j}^{k+1} = \Psi_{i,j}^{k} + \frac{\alpha}{2(1+\beta^{2})} \left[\Psi_{i+1,j}^{k} + \Psi_{i-1,j}^{k+1} + \beta^{2}(\Psi_{i,j+1}^{k} + \Psi_{i,j-1}^{k+1}) - 2(1+\beta^{2})\Psi_{i,j}^{k} + \Delta X^{2}\Omega_{i,j}\right]$   $-2(1+\beta^{2})\Psi_{i,j}^{k} + \Delta X^{2}\Omega_{i,j}]$  4.36

Δ

where  $\beta$  is the mesh aspect ratio,  $\beta = \Delta X/\Delta Y$ . This method makes use of a relaxation factor,  $\alpha$ , to speed the convergence and uses new values of the stream-function as they become available. Moreover, the optimum value,  $\alpha_0$ , of the over-relaxation parameter is used as

$$\alpha_0 = 2\xi(1 + \sqrt{1-\xi})/\xi]$$
; 4.37

which provides the maximum rate of convergence, where

$$\xi = \{ [\cos(\pi/(1-1)) + \beta^2 \cos(\pi/(J-1))] / [1 + \beta^2] \}^2$$
4.38

where I, J are the maximum number of nodes in the X- and Y- direction, respectively.

(b.1) For the direct method, Fourier-series methods are utilized which are based on the fact that an exact solution to the finite difference analog 4.35 can be expressed in terms of finite eigenfunction expansions [Dorr (1970)] as

$$\Psi_{i,j} = \sqrt{2/(N+1)} \sum_{p=1}^{N} H_{p,j} \sin(p\pi X_i/X)$$
 4.39

where

$$X_i = (i-1)\Delta X$$
  
N = I-2

The  $H_{p,j}$  are the solutions, for  $1 \leq p \leq N$ , of the tridiagonal difference equations

64

.41

4.43

$$(H_{p,j-1} - 2H_{p,j} + H_{p,j+1})/\Delta Y^{2} + \lambda_{p}H_{p,j} = -V_{p,j} \qquad 4.40$$

with,

$$H_{p,1} = H_{p,J} = 0$$

and

$$V_{p,j} = \sqrt{2/(N+1)} \sum_{q=1}^{N} \Omega_{q+1,j+1} \sin(q \pi p \Delta X/X)$$
 4

 $\lambda_{p} = [2/\Delta X^{2}][\cos(p\Pi \Delta X/X) - 1] \qquad 4.42$ 

(b.2) For the direct method, a "sparse-matrix package", courtesy of the National Water Research Institute, is utilized to solve the sparse linear equations produced from the discretized form, 4.35, of the stream-function equation 4.17, as a system of simultaneous linear algebraic equations.

$$[A][\Psi] = -[\Omega]$$

where A is an N by N nonsingular sparse matrix, and  $\Psi$ ,  $\Omega$  are vectors of length N, where N is equal to (1-2)(J-2), i.e. solving for all non-boundary nodes. The discussion on the initial design of this package is given by George and Liu (1979). One of the attractive features of this package is that, for a certain domain and different time steps, the sparse matrix A can be set-up, ordering and factorizing just once and thereafter using the factors repeatedly in

the calculations of  $\Psi$  for each different time step, i.e. different  $\Omega$ in Eq. 4.43. Moreover, the package allows the user to stop the calculation at some point, save the results on an external sequential file, and then restart the calculation at exactly that point some time later. This advantage is found to be very useful, in the present time integration problems.

A number of numerical experiments were carried out using rectangular domains of different sizes. The following observations were made :

- (i) The successive over-relaxation (SOR) method is reasonable for a small domain.
  - (ii) The Fourier-series method can be used for small to moderate size problems. The method required additional storage but yielded accurate results.
  - (iii) For large domains, when many equations must be solved which differ only in the right hand side, the sparse
     -matrix package is probably the best.

Consequently for the present case involving large integration times (i.e. many time steps), the exact solution of the stream-function equation 4.17 is obtained using the sparse-matrix package.

## 4.3.4 Velocity-field equation

For convenience the governing equation for velocity-field i

shown below.

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 $= -\frac{9\lambda}{24}$  ;

 $V = \frac{\Delta Y}{\Delta Y}$ 

The velocity-field equation 4.18 can be evaluated by the finite difference approximation of Eq. 4.22 for second-order accuracy as



4.4

## Initial And Boundary Conditions

In the solutions of any initial-value, boundary-value partial differential equations, it is necessary to specify the initial state of the flow domain and the conditions at its boundaries. It is worth mentioning that all common problems describing thermodynamic flow patterns involve solutions of the same partial differential equations in which the solutions are distinguished only by boundary and initial conditions, and by the thermal flow parameters  $R_a$  and  $P_r$ .

For the present problems, an irrotational fluid initially at rest (i.e.  $\Omega_{i,j} = 0$ ;  $\Psi_{i,j} = 0$ ;  $U_{i,j} = V_{i,j} = 0$ ) is assumed. The initial value of the non-dimensional temperature  $\theta_{i,j}$  will be specified according to each problem.

4.18

4.44

The boundary conditions are described as being of dominant importance in computational fluid thermodynamics [Cheng (1970)]. Host of the boundary conditions given are either Dirichlet conditions (specified function value) or the Neumann conditions (specified normal gradient). The various boundary conditions involved in the computational domains under investigations can be classified as follows :

(I) The boundary conditions at the free surface.

The free surface, say B1, is considered as a rigid slip wall where :

- i) a constant  $\Psi$  can be assigned, i.e.,  $\Psi |_{B1}^{*} = 0;$
- ii) the velocity component normal to the surface is zero, i.e.,  $V|_{B^1} = \vec{0}$ , where V represents the normal velocity component;
- iii) the velocity component along the surface, say U<sub>x</sub> is free to develop, i.e.  $\frac{\partial U}{\partial Y}|_{B1} = 0$ , when Y is the normal direction to the surface;
- iv) in the absence of applied external shear, vorticity must be zero at a free fluid surface, i.e.  $\Omega|_{B^1} = 0$ . This can be shown by combining the conditions  $V|_{B^1} = 0$  and  $\partial U/\partial Y|_{B^1} = 0$  with equations 4.17 and 4.18.

The solid surface, say  $B^2$ , is considered as rigid no-slip wall where the following conditions apply :

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 i) Since a solid impermeable wall is a streamline, any appropriate constant value of Y may be chosen, the conventional choice is

$$\Psi_{B2} = 0;$$

ii) No slip is permitted at the boundaries so that both velocity components are set to zero at all

iii) The second-order accurate boundary condition for

vorticity along a solid nd-slip wall is used, as described by Jensen (1959), Pearson (1965), and Briley (1970). This is given by

$$\Omega_{B2} = \frac{-7\Psi_{B2} + 8\Psi_{B2+\Delta n} - \Psi_{B2+2\Delta n}}{2\Delta n^{2}} + O(\Delta n^{2})$$

with a consistent expression for the parallel velocity component, say U, at and only at  $B^{2}+\Delta n$ 

$$- U|_{B^{2}+\Delta n} = \frac{-5\Psi|_{B^{2}}+4\Psi|_{B^{2}+\Delta n}+\Psi|_{B^{2}+2\Delta n}}{4\Delta n} + O(\Delta n^{2})$$

where  $\Delta n$  is the normal distance to the surface  $B^2$ .

The boundary condition for  $\theta$  is also specified according to the circumstances of each problem. When constant temperature is assigned over surface B<sup>3</sup>, then the specified given value  $\theta|_{B^3}$  is assumed. However, if an insulated surface, say B<sup>4</sup>, is assumed, then  $\partial\theta/\partial n|_{B^4} = 0$ . The adiabatic surface temperature  $\theta|_{B^4}$  is evaluated by subtracting the two expressions of the Taylor series expansion for the neighbouring temperatures at points B<sup>4</sup>+\Delta n and B<sup>4</sup>+2\Delta n, i.e.

 $4\left[\theta\right]_{B^{L}+\Delta n} = \theta\left[\frac{1}{B^{L}} + \frac{\partial \theta}{\partial n}\right]_{B^{L}}(\Delta n) + \frac{1}{2}\frac{\partial^{2}\theta}{\partial n^{2}}\Big|_{B^{L}}(\Delta n)^{2}\frac{1}{2}\frac{\partial^{2}\theta}{\partial n^{2}} + O(\Delta n^{3})\right] 4.45$   $\theta\left[\frac{1}{B^{L}+2\Delta n} = \theta\left[\frac{1}{B^{L}} + \frac{\partial \theta}{\partial n}\right]_{B^{L}}(2\Delta n) + \frac{1}{2}\frac{\partial^{2}\theta}{\partial n^{2}}\Big|_{B^{L}}(2\Delta n)^{2} + O(2\Delta n^{3}) 4.46$ 

Satisfying the adiabatic condition  $\partial \theta / \partial n|_{B^{+}} = 0$ , the following one sided difference expression is obtained

 $\Theta|_{B^{\mu}} = (4\Theta|_{B^{\mu}+\lambda_{\Pi}} - \Theta|_{B^{\mu}+2\Delta_{\Pi}})/3 + O(\Delta_{\Pi}^{3})$ 

#### 4.5 <u>Solution Procedure</u>

The heat-transfer equation 4.15 and the vorticity-transport equation 4.16 are parabolic, second order due to the viscous diffusion terms  $\nabla^2 \Theta$ ;  $P_{\rm T} \nabla^2 \Omega$ . In addition, the equations are nonlinear due to the quadratic convective inertia terms,  $\partial U \Theta / \partial X$ ;  $\partial V \Theta / \partial Y$ ;  $\partial U \Omega / \partial X$ ;  $\partial V \Omega / \partial Y$ , since U and V are functions of the vorticity  $\Omega$  via equation 4.17 and 4.18. Moreover, equation 4.16 and equation 4.15 are coupled by virtue of the buoyancy term,  $R_{\rm a} P_{\rm T} \partial F(\Theta) / \partial X$ . Both equations are time

dependent, thus posing a "marching" or initial-value problem, wherein the solution is stepped out from some initial condition.

The stream-function equation 4.17, Poisson equation, is elliptic, i.e., it poses a "Jury" or boundary-value problem with the vorticity as the driving function.

The system (set) of the governing differential equations, Eqs. 4.15 to 4.18, comprises coupled (simultaneous), nonlinear, partial differential equations. For simplicity, the equations are usually solved by considering them to be linear in some small time interval, the magnitude of the time step being dictated by the requirement of numerical stability of the solution. In some cases, the nonlinear terms play an important role and this frequently requires the use of even smaller time intervals to ensure numerical stability and accuracy. In general, the solutions to such problems are computationally expensive in order to achieve a certain state (or steady-state) solution. To accelerate the solution procedure, larger time steps can be attempted, subject to numerical stability, along with the introduction of inner iterations within each time step to correct the estimated values of the nonlinear terms. This latter approach may reduce the overall computational time.

In order to solve these coupled nonlinear partial differential equations 4.15 to 4.18, the following iterative procedure is used.

. At the n<sup>th</sup> time step, the values of  $U_{i,j}$ ,  $V_{i,j}$ ,  $\Psi_{i,j}$ ,  $\theta_{i,j}$ , and  $\Omega_{i,j}$  are obtained for  $\tau_{n+1} = \tau_n + \Delta \tau_n$  by using an inner iteration

scheme. This inner iteration scheme for the time step n with increment  $\Delta \tau_n$  and inner iteration loop index k is as follows.

Procedure inner iteration :

<u>Step 1</u> :

Set :

<u>Step 2</u>: Obtain  $[\theta_{i,j}^{n+1}]_{(k)}$  by solving the heat-transfer equation 4.15 with  $\pi$  $[U_{i,j}^{n+1}]_{(k)}, [V_{i,j}^{n+1}]_{(k)}, \text{ and } \theta_{i,j}^{n}$ 

R + 1 -

 $[U_{i,i}^{n+1}]_{(0)} + U_{i,j}^{n}$ 

 $[v_{i,j}^{n+1}]_{(0)} + v_{i,j}^{n}$ 

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<u>Step 3</u> :

Obtain  $[\Omega_{i,j}^{n+1}]_{(k)}$  by solving the vorticity-transport equation 4.16 with  $[U_{i,j}^{n+1}]_{(k)}, [v_{i,j}^{n+1}]_{(k)}, [\Theta_{i,j}^{n+1}]_{(k)}, and \Omega_{i,j}^{n}$ .

<u>Step 4</u> :

Solve the stream-function equation 4.17 with  $[\Omega_{i,j}^{n+1}]_{(k)}$  to obtain  $[\Psi_{i,j}^{n+1}]_{(k)}$ 

<u>Step 5</u> :

Compute the velocity vectors  $[U_{i,j}^{n+1}]_{(k)}$ and  $[V_{i,j}^{n+1}]_{(k)}$  by using the velocity-field equation 4.18 Update the values of parameters on the boundaries, using the most recent values at the interior nodes adjacent to the boundaries according to Sec. 4.4

## <u>Step 7</u> :

Test if the following convergence criterion is satisfied,

$$\frac{\left[\Omega_{i,j}^{n+1}\right](k) - \left[\Omega_{i,j}^{n+1}\right](k-1)}{\left[\Omega_{i,j}^{n+1}\right](k)} \right|_{max} \leq \varepsilon.$$

If not, go to Step 9

#### Step.8 :

Set :  $U_{i,j}^{n+1} \leftarrow [U_{i,j}^{n+1}]_{(k)}$   $V_{i,j}^{n+1} \leftarrow [V_{i,j}^{n+1}]_{(k)}$   $\Psi_{i,j}^{n+1} \leftarrow [\Psi_{i,j}^{n+1}]_{(k)}$   $\Theta_{i,j}^{n+1} \leftarrow [\Theta_{i,j}^{n+1}]_{(k)}$ 

 $\Omega_{i,j}^{n+1} + [\Omega_{i,j}^{n+1}]_{(k)}$ 

## Exit from procedure inner iteration

 $\langle \rangle$ 

Set :

k + k+1

If k < k<sub>max</sub> (the maximum number of iterations permitted) then

Go back to Step 2.

Otherwise

Set CONV + FALSE\*

Exit <u>procedure</u> inner iteration.

End of procedure inner iteration.

## 4.6: <u>Summary</u>

In this chapter, the governing equations of the fluid motion and heat transfer are described for an incompressible fluid. The governing equations yield to a system of coupled, nonlinear partial differential equations which has been generalized to non-dimensional conservative form, where the solutions are distinguished by boundary and initial conditions, and by the thermal flow parameters Rayleigh number  $R_a$  and Prandtl number  $P_r$ .

The governing partial differential equations are solved by using the finite difference methods over a rectangular domain with

• The logical variable CONV is utilized by an external algorithm described in Sec. 5.3.2.

regular boundaries. The alternating direction implicit (ADI) method is selected to solve the parabolic equations (i.e. the heat-transfer and vorticity-transport equations) and a sparse-matrix package is utilized to solve the elliptic equation (i.e. stream-function equation) of the resulting partial differential equations.

The initial condition is specified and the boundary conditions are treated numerically in general circumstances where the selections are dependent on the specified problem boundary conditions.

Inner iterations procedure within each time step is utilized to correct the estimated values of the nonlinear terms. Consequently; larger time steps can be attempted subject to numerical stability.

The numerical applications of this mathematical model are presented in terms of verification procedure and simulation of the thermal bar phenomena, as described in the next two chapters.

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# CHAPTER 5

#### MODEL VERIFICATIONS AND IMPROVEMENTS

## 5.1 <u>Introduction</u>

The mathematical aspects of the proposed model have been presented in Chapter 4, including the numerical formulations as well as the solution procedure. The proposed (numerical model is programmed in Fortran 66 and the results reported henceforth were obtained from runs on a Cyber 170/730. Details of the computer programs are presented in Appendix I.

This chapter is concerned with the last stage of the simulation procedure which relates to the verification and improvement of the proposed model prior to its use for actual problem simulation.

The numerical model is verified by comparing it to numerical solutions of four different cases of an idealized problem of two-dimensional steady laminar flow in an enclosed rectangular cavity with, differentially heated end walls which has been discussed in

· Chapter 2.

Moreover, significant improvements in the computational efficiency of the numerical procedure have been achieved by introducing different acceleration techniques to the employed solution procedure and by modifying the finite difference approximation for the advection terms.

5.2 <u>Model Verifications</u>

5.2.1 General

The idealized problem of steady laminar flow in an enclosed rectangular cavity with differentially heated end walls has been chosen for the numerical verification of the model. The reason for this choice is due to the large body of documented experimental, numerical and analytical studies which have been carried out for heated end cavities. These have been discussed in Chapter 2. The blosed rectangular two-dimensional cavity is of length L and height H, with rigid non-slip boundaries on all sides, and contains a Newtonian fluid which is initially at rest and at temperature  $T_0$ . The upper and lower boundaries are insulated. At time t = 0, the left- and right-hand end walls are instantaneously heated and cooled respectively to temperatures  $T_0 + \Delta T$  and  $T_0 - \Delta T$ , and thereafter, maintained at these temperatures.

Two selected problems are solved by the proposed numerical model as a verification procedure. These two problems were treated by

Patterson and imberger (1980) and Hamblin and Ivey (1984) respectively, from the analytical, numerical and experimental points of view. The notation used in this chapter is consistent with that introduced in previous chapters and may differ from that used in the references cited.

## 5.2.2 Test problem # 1

This problem is analyzed by Patterson and Imberger (1980) to study the transient natural convection in a cavity of aspect ratio A =1 (A = height/length) with differentially heated end walls at temperatures greater than 8° C, as shown in Figure 5.1(a). For this problem, they use a linear version of the equation of state 4.5, i.e.

 $\rho = \rho_0 [1 - \alpha (T - T_0)]$ 

where

 $P_0$  = density of water at temperature  $T_0$ :

a = thermal expansion coefficient;

 $T_0 =$  an arbitrary reference temperature. Consequently, the non-dimensional governing differential equations 4.15 - 4.18 are modified to the form

> $\frac{\partial \Theta}{\partial \tau} + \frac{\partial U \Theta}{\partial X} + \frac{\partial V \Theta}{\partial Y} = P_{r} \nabla^{2} \Omega - P_{r} R_{a} \frac{\partial \Theta}{\partial X}$  5.2  $\frac{\partial \Omega}{\partial \tau} + \frac{\partial U \Omega}{\partial X} + \frac{\partial V \Omega}{\partial Y} = P_{r} \nabla^{2} \Omega - P_{r} R_{a} \frac{\partial \Theta}{\partial X}$  5.3

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5.1



(a)



Figure 5.1

Rectangular cavity notations, initial and boundary : conditions of test problem # 1 for: dimensional parameters.

- (a)
- (b) non-dimensional parameters.

$$\nabla^2 \Psi = -\Omega$$

where  $R_a$  is Rayleigh number, defined in this case (i.e. linear range) as  $g\alpha_0(\Delta T)H^3/\kappa_0\nu_0$ . Other terms are as defined in Sec. 4.2.4.

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The associated non-dimensional initial and boundary conditions, as shown in Figure 5.1(b), are

For 
$$\tau < 0$$
:  $U = V = \Psi = \Theta = \Omega = 0$ , everywhere 5.6

For  $\tau \ge 0$  :  $U = V = \Psi = 0$ , at X = 0, 1/A; Y = 0, 1  $\Omega = -\partial^2 \Psi / \partial X^2$ , at X = 0, 1/A  $\theta = +1$ , at X = 0  $\theta = -1$ , at X = 1/A  $\partial \theta / \partial Y = 0$ , at Y = 0, 1 $\Omega = -\partial^2 \Psi / \partial Y^2$ , at Y = 0, 1

For aspect ratio A = 1, Patterson and Imberger presented solutions of 7 numerically-simulated runs, classified in Table 5.1 as Run # 1 to Run # 7. In their solutions, the steady state is assumed to have been reached when the two values of the Nusselt number, N<sub>u</sub>, one at the heated end, N<sub>ue</sub>, and the other at the centre line, N<sub>uc</sub>, of the cavity become equal to within a prescribed tolerance. For this purpose, the Nusselt number, N<sub>u</sub>, is defined as

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5.5

Run #	1	2.	3	4	5	6	7
Ra	0.1	21.0	1.0*10	1_4*10	1.4*10 <sup>5</sup>	1.4*10	1.4*10 <sup>5</sup>
P <sub>r</sub>	7	7	7	7	7	2	2

Patterson and Imberger's, numerical runs

Table 5.1

Note : For all runs, A = 1

$$N_{U} = \frac{1}{2A} \int (U\Theta - \frac{1}{2A}) dY$$

In Eq. 5.8 the term U0 is a measure of the heat transfer by advection; on the other hand the gradient  $\partial \theta/\partial X$  is an indicator of the amount of heat flux by conduction. In a steady state situation, there can be no net addition or loss of heat in the system and thus the Nusselt number  $N_{\rm u}$  will remain constant with respect to time.

Two cases are solved by the numerical model of Chapter 4 which correspond to Run # 3 (i.e.  $R_a = 10^3$ ,  $P_r = 7$  and A = 1) and Run # 4 (i.e.  $R_a = 1.4 \cdot 10^4$ , Pr = 7 and A = 1). To reduce the computational cost while still simulating significantly high Rayleigh numbers. runs at the highest values of  $R_a$  and/or low  $P_r$  were not attempted. The results obtained for these two cases are compared with Patterson and Imberger's results by plotting contour lines for the same values of  $\Psi$  and  $\theta$  as shown in Figures 5.2 and 5.3 and by plotting the variation of  $\hat{N}_u$  with respect to time as shown in Figure 5.4. It is worth mentioning that Patterson and Imberger solved the non-dimensional governing differential equations in primitive form using finite difference methods. Due to the use of different normalizing parameters, the values obtained for  $\Psi$  ( $\Psi = \psi/\kappa_0$ ) differ by a factor of  $P_r$  ( $P_r = \psi_0/\kappa_0$ ) from their published values for  $\Psi$  ( $\Psi = \psi/\nu_0$ ).

5.2.3 Test problem # 2

This problem is analyzed by Hamblin and Ivey (1984), in order to study the convection near the temperature of maximum density due to

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5.8



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Figure 5.2

- Comparison between Patterson and Imberger's results and model results (shown as solid and dashed lines respectively) of Run #3 (see Table 5.1) for: •• ĒĒ
  - stream-function distribution,  $\Psi (\Delta \Psi = 0.70)$ . temperature distribution, 0 (  $\Delta 0 = 0.33$ ).





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Figure 5.3

(shown as solid and dashed lines respectively) of Run #4 (see Table 5.1) for: stream-function distribution,  $\psi (\Delta \psi = 1.40)$ . temperature distribution, 0 (  $\Delta 0 = 0.33$ ). Comparison between Patterson and Imberger's results and model results •• ΞĒ



Figure 5.4

Comparison between Patterson and Imberger's results and model results (shown, as solid and dashed lines respectively) for variation of N<sub>u</sub> with respect to time for:
 (a) Run #3 (see Table 5.1).
 (b) Run #4 (see Table 5.1).

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horizontal temperature differences in a cavity of aspect ratio, A  $\langle 1$ . The temperature range is specified to be between 0° and 8° C. It is convenient to employ the same nonlinear equation of state as used by Hamblin and Ivey(1984), i.e.

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5.9

$$= (\rho_0 [1 - \beta(T - T_0)^2)$$

where

 $P_0$  = density of water at 4<sup>0</sup> C (i.e. maximum density);

B = thermal expansion coefficient;

 $T_0$  = temperature of water at maximum density  $\rho_0$  (i.e. 4<sup>o</sup> C) Consequently, the non-dimensional governing differential equations 4.15 - 4.18 are modified to the form

$$\frac{\partial \Theta}{\partial \tau} + \frac{\partial U \Theta}{\partial X} + \frac{\partial V \Theta}{\partial Y} = \nabla^2 \Theta$$
 5.10

$$\frac{\partial \Omega}{\partial \tau} = \frac{\partial U \Omega}{\partial \tau} = \frac{\partial V \Omega}{\partial Y} = \frac{\partial \Theta^2}{\partial r} - \frac{\partial \Theta^2}{\partial r}$$
5.11

 $\nabla^2 \Psi \approx -\Omega$  5.12

$$U = -\frac{\partial \Psi}{\partial Y}, \qquad V = -\frac{\partial \Psi}{\partial X}$$
 5.13

where  $R_a$  is Rayleigh number, defined in this case (i.e. nonlinear range) as  $g\beta_0(\Delta T)^2 H^3/\kappa_0 v_0$ . It is worth mentioning that the set of lequations 5.2 - 5.5 solved by Patterson and Imberger differs from the

set of equations 5.10 - 5.13 only in the buoyancy term of the vorticity-transport equations 5.3 and 5.11 due to the difference, between the linear and nonlinear equations of state, 5.1; 5.9.

Due to the symmetrical behaviour, Hamblin and Ivey are able to solve for only half of the domain; as shown in Figure 5.5(a), where the associated non-dimensional initial and boundary conditions, are as shown in Figure 5.5(b), and are listed below.

For  $\tau < 0$  :  $U = V = \Psi = \Theta = \Omega = 0$ , everywhere

For  $\tau \ge 0$ :  $U = \Psi = 0$ , at X = 0, 1/A; Y = 0, 1  $\Omega \ge -\partial^2 \Psi/\partial X^2$ , at X = 1/A  $\theta = +1$ , at X = 1/A V = 0, -at X = 1/A V = 0, -at X = 1/A  $\partial V/\partial X = \theta = \Omega = 0$ , at X = 0  $V = \partial \theta/\partial Y = 0$ , at Y = 0, 1 $\Omega = -\partial^2 \Psi/\partial Y^2$ , at Y = 0, 1

Hamblin and Ivey present solutions for three cases, listed in Table 5.2 as Run # 1 to Run # 3. All cases are solved for  $P_r = 11.6$ , i.e. for water at 4° C. The steady state solution is assumed to have been reached when the values of Nusselt number,  $N_u$ , at the plane of symmetry (rigid slip wall) and at a warm (or cold) boundary (rigid no-slip wall) become equal. For this purpose the Nusselt number,  $N_u$ , is defined as :

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5.14



Figure 5.5

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(a)

Rectangular cavity notations, initial and boundary conditions of test problem #2 for : dimensional parameters.

(b) non-dimensional parameters.

Table 5.2

# Hamblin and Ivey's numerical runs

·····	•···		
Run #	1	2	• 3
Ra	2.00*10 <sup>3</sup>	1.24*10	1.44*10 <sup>5</sup>
A	0.200	0.118	0.118

# Note : For all runs, $P_r = 11.6$

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$$N_{U} = \frac{1}{A} \int \frac{1}{A} \frac{$$

where the heat transfer by advection term  $U\Theta$  is eliminated from the similar Eq. 5.8 due to the assumed rigid boundary (i.e. U = 0) at both vertical ends.

Two cases are solved by the numerical model of Chapter 4 which correspond to Run # 1 (i.e.  $R_a = 2.0*10^3$ ,  $P_r = 11.6$  and A = 0.2) and Run # 2 (i.e.  $R_a = 1.24*10^4$ ,  $P_r = 11.6$  and A = 0.118). Run # 3 was not simulated by the author because of the larger value of Rayleigh number which proved costly in terms of storage and computation time. The results obtained for the two runs are presented in Figures 5.6 and 5.7 by plotting contour lines for the same values of  $\Psi$  and  $\Theta$ . Detailed comparison between the results obtained using the author's model and those obtained by Hamblin and Ivey (private communication) showed a very satisfactory measure of agreement.

Hamblin and Ivey (1984) solved the appropriate, non-dimensional form of the governing differential equations using the finite element method, and, due to different normalizing methods, the values obtained for U and  $\Psi$  are larger by a factor of AR<sub>a</sub>. Figure 5.8 compares the values obtained using the present model for U at the middle of the cavity to those spublished by Hamblin and Ivey (1984) based on their normalizing parameters.

#### 5.2.4 Discussion

The analytical solution of the well known problems of steady

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5:16




: Computed solutions for test problem #2 (Run #2 in Table 5.2) for: (a) stream-function distribution,  $\Psi (\Delta \Psi = 0.4)$ . (b) temperature distribution,  $\theta (\Delta \theta = 0.1)$ .



Figure 5.8

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Comparison between Hamblin and Ivey's results and model results (shown as circles and lines respectively) for horizontal velocity, U, at middle of cavity. laminar flows in enclosed rectangular cavities with differentially , heated end walls [Patterson and Imberger (1980); Hamblin and Ivey (1984)] shows that heat conducted into the fluid from the wall, results in a vertical layer of heated fluid of non-dimensional thickness  $O(\delta_T)$ , where

 $\delta_T \cong R_a^{-1/4}$ 

The appropriate non-dimensional boundary layer length scale,  $\delta_{T}$ , due to conduction-convection balance is also identical with the same scale obtained by Braun. Ostrach and Heighway (1961) and Gill (1966) for the corresponding steady-state problems. For accurate spatial and temporal representation of the solution, a limit on the mesh size is enforced by the appropriate length scale,  $\delta_{T}$ , of such problems, in addition to any other stability criterion in use. For example, for convection dominated flows, it is essential that at least two mesh points are contained in the boundary layer at each vertical level. Hence, for a uniform grid of N<sub>X</sub> points in the X- direction, with spacing  $\Delta X$  which is half of  $\delta_{T}$ ,

 $N_X \ge 2R_a^{1/4}A^{-1}$ 

5.20

For the present verification runs it is sufficient to note that the grid spacing utilized has been determined initially according to the appropriate criterion 5.20 and then refined in order to achieve

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5.19

the numerical stability where necessary. It is worth mentioning that for a square domain, i.e.  $A^* \equiv 1$ , a uniform grid may be used [Patterson and Imberger (1980)] but for low aspect ratio domains, i.e.  $A \ll 1$ , a variable mesh is recommended in order to reduce the storage requirements [Hamblin and Ivey (1984)].

## 5.3 <u>Model Improvements</u>

#### 5.3.1 General

In Sec. 5.2, the proposed numerical model is verified, yielding numerical results which are consistent with the numerical solutions of two problem by other methods, thus allowing the numerical model to be perfected prior to actual simulated problem. The present section is concerned with numerical improvements which are dictated by higher values of Rayleigh number,  $R_a$ , over large domains. A << 1, where even modest improvements are of value in reducing the computational time. Such numerical improvements can be achieved either by introducing additional accelerating techniques throughout the solution procedure or by modifying the finite difference approximation utilized to relax the numerical stability constraints.

# 5.3.2 Alternative acceleration techniques

To enhance the overall computational time, use has been made of the following three computational techniques :

- (i) step-size control with variable time step;
- (ii) averaging; and
- (iii) extrapolation.

# Variable time step :

Since the behaviour of the solution changes with time, the use of different time steps,  $\Delta \tau_n$ , should be appropriate. The choice of smaller time intervals reduces the number of inner iterations per step but the number of time steps needed to reach the steady (or required) state solution increases proportionately. On the other hand, the choice of larger time intervals increases the number of inner iterations per step but decreases the number of time steps needed to reach the required state solution. Most of the mathematical models reported in the literature employ a constant time step over the time of integration.

For the proposed numerical model, step-size control with a variable time step is introduced where the value of the  $n^{th}$  time step,  $\Delta \tau_n$ , is calculated at the end of the previous time step n-1 as

$$\Delta \tau_{n} = [1 - (k-5)/k_{max}] \Delta \tau_{n-1}$$
 5.17

where k is the number of inner iterations required at the  $(n-1)^{th}$  time step. If k takes a value other than 5 the time step will be either increased (k < 5) or reduced (k > 5). The value of  $k_{max}$  is set at 10 iterations; if convergence is not achieved within  $k_{max}$  iterations

(i.e. CONV = FALSE), then

$$\Delta \tau_n = \Delta \tau_n/2$$

and the time step is repeated again. The logical variable CONV is defined by the algorithm described in Sec. 4.5.

#### Averaging :

While carrying out the numerical experiments it was observed that during the inner iteration, the errors in the successive approximations obtained for. U and V oscillated around the final solution values. Because of this, an averaging technique is introduced whereby the average of the values obtained by two successive iterations are used [Pearson (1965); Aziz and Hellums (1967)]. This is achieved by inserting the following step into the inner iteration procedure (Sec. 4.5)

<u>Step 5.1</u> :

Set : 
$$[U_{i,j}^{n+1}]_{(k)} + \frac{[U_{i,j}^{n+1}]_{(k)} + [U_{i,j}^{n+1}]_{(k-1)}}{2}$$
  
 $[V_{i,j}^{n+1}]_{(k)} + \frac{[V_{i,j}^{n+1}]_{(k)} + [V_{i,j}^{n+1}]_{(k-1)}}{2}$ 

#### Extrapolation :

At time step n (n  $\ge$  1), the solution values for the previous time step (n-1) are known. This information can be used for calculating the initial estimates  $[U_{i,j}^{n+1}]_{(0)}$  and  $[V_{i,j}^{n+1}]_{(0)}$  by using

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5.18

linear extrapolation [Briley (1970)]. This is achieved by replacing step 1 in the inner iteration procedure (Sec. 4.5) with

Step 1':

Set :  

$$k + 1$$
(
  
If n = 0, then set :  

$$[U_{i,j}^{n+1}]_{(0)} + U_{i,j}^{n}$$

$$[V_{i,j}^{n+1}]_{(0)} + v_{i,j}^{n}$$
Otherwise, set :  

$$[U_{i,j}^{n+1}]_{(0)} + U_{i,j}^{n} + \frac{\Delta \tau_{n}}{\Gamma} [U_{i,j}^{n} - U_{i,j}^{n-1}]$$

$$\Delta \tau_{n=1}$$

$$[v_{i,j}^{n+1}]_{(0)} + v_{i,j}^{n} + \frac{\Delta \tau_{n}}{\Delta \tau_{n-1}} [v_{i,j}^{n} - v_{i,j}^{n-1}]$$

To compare the different accelerating techniques without unreasonably increasing the computational cost, the Patterson and Imberger problem (Sec. 5.2.2) for  $R_a = 10^3$  was chosen as a test case. In order to compare the different computational methods, four runs were made as detailed in Table 5.3. The behaviour of each different run (technique) is demonstrated in Figure 5.9 and Table 5.4 which give an explicit comparison between the alternative acceleration techniques to ascertain the relative efficiency.

The non-dimensional time  $\tau$  is a measure of the degree of convergence and the total number of inner iterations  $\Sigma k$  is a good index of computation cost. The introduction of the different

# Table 5.3

Comparison of computational methods

Run #	Computational methods
1	<pre>1- variable step with step-size control; 2- inner iterations.</pre>
2	<ul> <li>1- variable step with step-size control;</li> <li>2- inner iterations;</li> <li>3- averaging.</li> </ul>
3	<ul> <li>1- variable step with step-size control;</li> <li>2- inner iterations;</li> <li>3- extrapolation.</li> </ul>
4	<ul> <li>1- variable step with step-size control;</li> <li>2- inner iterations;</li> <li>3- averaging;</li> <li>4- extrapolation.</li> </ul>



Table 5.4

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Compar	ison	between	alternative	acceleration	techniques
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z,

Run # 1		Run # 2		run # 3		run # 4	
τ	Σĸ	τ	Σĸ	τ	Σĸ	τ	Σk
0.0010	232	0.0010	326	0.0052	204	0.0095	165
0.0020	432	0.0020	526	0.0164	490	0.0421	467
0.0030	632	0.0033	717	0.0284	767	0.0768	729
0.0040	831	0.0052	909	0.0404	· 1084	0.1020	925
0.0052	1027	0.0079	1101	0.0528	1362		<b>_</b>
0.0084	1417	0.0183	1485	0.0778	1910		
0.0108	1612	0.0266	1681	0.0900	2185		
0.0138	1807	0.0367	1876	0.1022	2468		
0.0176	2002	0.0493	2072	· · · ·			•
0.0221	2200	0.0655	2265				
0.0271	2398	0.0886	2456				,
0.0327	2596	0.1020	2544				
0.0390	2793		<b>_</b>	l			
0.0545	3187						
0.0780	3593						
0.1019	4094					、	

acceleration techniques did not involve significant additional coding, nor was there any extra memory requirement. It may be reasonably assumed that the computational cost <u>per iteration</u> did not change significantly.

# 5.3.3 Alternative differencing methods

Stability of a finite difference scheme is concerned with the unbounded growth or controlled decay of any errors associated with the solution of the finite difference equations. In case of a computationally stable difference scheme, all disturbances remain bounded in the computation. Stability can be investigated analytically using various methods [Smith (1975)] such as the discrete perturbation stability analysis, Von Neumann stability analysis and Hirt's stability analysis. It is difficult to apply the methods of stability analysis to a set of coupled, nonlinear, partial differential equations, hence, the stability requirements for the linear model equation may give some insight to the stability requirements for the nonlinear model equations. Consequently, the two necessary conditions for stability of the advection-diffusion equation [Roache (1982)] are

- (i) Courant number,  $C \equiv u\Delta t/\Delta x \in (C)_{cr}$ ;
- (ii) Cell-Reynolds number,  $R_c \equiv u\Delta x/\gamma \in (R_c)_{cr}$ .

where  $\gamma$  is the diffusion coefficient which may be represented by the thermal conductivity  $\kappa_0$ , as in the heat-transfer equation 4.12 or by

the kinematic viscosity  $v_0$ , as in the vorticity-transport equation 4.13. The two numbers C and R<sub>c</sub> should be within certain critical limits, otherwise, unstable results may be obtained.

Obviously, maintaining a reasonable cell-Reynolds number,  $R_c$ , requires a small  $\Delta x$  which, in turn, restricts the time increments  $\Delta t$ to keep Courant number, C, within the critical limit  $[(C)_{cr} = 1]$ . To satisfy the first constraint,  $C \leq C_{cr}$ , the ADI method was employed to solve the time-dependent partial differential equations, as described before in Sec. 4.3. This method is unconditionally time stable. Consequently, the stability constraints involve only the cell-Reynolds number,  $R_c$ , where the critical limit  $[(R_c)_{cr}]$  is dependent on the differencing scheme used for the advection terms.

The accumulation of error due to the advection terms is independent of time derivatives and therefore cannot be removed by lowering the time step  $\Delta t$ . The problem can be overcome by employing some other finite difference method. Therefore, particular care has to be taken in choosing a differencing scheme, see Sec. 4.3. for the advection terms  $\partial AF/\partial S$  (where A symbolizes the advection. U or V), in both the heat-transfer equation 4.15 and the vorticity-transport equation 4.16. Adopting different difference methods, it was found that the second upwind differencing method (donor cell method) [Gentry, Martin and Daly (1966)] can be used with higher values of the critical cell-Reynolds number,  $R_{cr}$ , to achieve stable solutions. Other methods, including the central-difference method, used before in Sec 4.3, produce unstable results. For that reason, the second upwind

differencing method is adopted for the spatial derivative convection

terms as

а

$$AF/\partial S \cong \delta AF/\delta S = [A_aF_a - A_bF_b]/\Delta S + O(\Delta S^2)$$

where

А <sub>а</sub>	=	$[A_{s+\Delta s} + A_{s}]/2$					
А <sub>Ъ</sub>	=	$[A_{s} + A_{s-\Delta s}]$	/2				
Fa	- =	F <sub>s</sub>	for	A <sub>a</sub> ≩ O			
	=	F <sub>s+∆s</sub>	for	A <sub>a</sub> < 0			
Fь	=	F <sub>s-∆s</sub>	for	A <sub>b.</sub> ≩ 0			
	=	Fs	for	A <sub>b</sub> < 0			

This method is second-order accurate and possesses the transport property where the effect of a perturbation is advected only in the direction of the velocity.

For the actual problem of simulating densimetric lock exchange flow, the associated experimental values of Rayleigh number,  $R_a \sim 0(10^8)$  are required to be employed for numerical simulations. For such high values of  $R_a$ , the advection terms play an important role and this frequently requires the use of appropriate mesh size,  $\Delta X$  and  $\Delta Y$ to ensure the required accuracy and numerical stability. Due to the high order of the advection terms in the differential equations, the lowest-order terms in the Taylor series expansion (Eqs. 4.19 and 4.20) are not sufficient to accurately describe the advection terms if  $\Delta X$ and  $\Delta Y$  exceed certain values such that the higher-order terms and

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truncation errors, cause unstability in the solutions. To overcome such problem,  $\Delta X$  and  $\Delta Y$  should be small enough to achieve numerical stability and, consequently, the computer storage may restrict the required simulated values of  $R_a$  and A for a required domain.

#### 5.3.4 Discussion

For the iteration procedure, the step-size control with variable time step technique is used for additional flexibility where the appropriate time step is utilized according to the numerical stability requirements.

For the inner corrector iterations procedure, the averaging and extrapolation techniques—are employed. The averaging technique improves the solution by damping the oscillation of the nonlinear terms within the inner iterations. The extrapolation technique improves the solution by giving better estimated values for the nonlinear terms at the beginning of the inner iterations. It is interesting to note that although the extrapolation technique exhibits rapid convergence close to the initial conditions, this is not maintained as the solution approaches a steady state. The net benefit of the extrapolation technique compared to the averaging algorithm at the steady state is thus not very dramatic, and both methods involve approximately the same number of global iterations. Combining the techniques results in both rapid initial convergence and reasonably fast convergence close to the solution.

Moreover, using the second powind differencing method for the

spatial derivative convection terms instead of any other difference approximation allows the simulation of higher values of advection terms, over a stable scheme, when associated with high values of the Rayleigh numbers.

#### 5.4 <u>Conclusion</u>

The numerical model developed in Chapter 4 and improved by methods described in the present chapter, has been employed to solve the well known problem of steady laminar flow in an enclosed rectangular cavity with differentially heated end walls. The solution (i.e. steady state) was determined by iterating the set of coupled, nonlinear, partial differential equations which describe the thermodynamic behaviour of the present problem.

The numerical results obtained in this way have been compared to numerical solutions of four different cases of the idealized problem described in the literature. The excellent agreement obtained suggest that the numerical model has the ability to solve problems of this type.

Different accelerating techniques have been developed and employed to improve the efficiency of the solution procedure. It is observed that the combination of the three used techniques, i.e. step-size control, averaging and extrapolation, improves the outer and inner iterations over the iterating time steps. The actual magnitude of the improvement factor is a function of the time of integration.

Consequently, this technique is introduced to the numerical model and also recommended to be used when the solution of a system of coupled, nonlinear, partial differential equations is required.

The advection terms are reapproximated where the second upwind differencing method is used instead of the central difference approximation to simulate high values of Rayeigh number over a stable scheme.

The results of the verification studies and the testing of the numerical improvement schemes [Marmoush, Chakravarti and Smith (1985)] have confirmed the validity of the numerical model. The model can thus be applied with some confidence to the actual problem of simulating lock exchange flow which will be presented in the following chapter.

### CHAPTER 6

# NUMERICAL INVESTIGATION

#### 6.1 <u>Introduction</u>

This chapter describes a series of numerical investigations which are carried out to provide a more comprehensive study of the phenomenon of the sinking plume which had been confirmed experimentally as described in Chapter 3.

The mathematical model has been constructed to develop a means of modelling numerically the behaviour of a thermal bar at the outfall of a steam electric generating station cooling water system as explained in Chapter 4.

Moreover, the numerical model has been verified and additional improvements have been introduced prior to the actual numerical investigation as discussed in Chapter 5.

This chapter contains the numerical results which provide an explanation for the lock exchange flows created between two water bodies having asymmetrical different temperatures around the

temperature of maximum density. Some attempt is made at empirical correlation of the relative extension of the thermal bar with relevant system parameters. Comparison between the numerical and experimental results are discussed.

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# 6.2 <u>Numerical Considerations</u>

6.2.1 General

The main objectives of the numerical analysis for the lock exchange flows subject to the existence of a density extremum in water at  $4^{\circ}$  C are :

- 1- to confirm the general behaviour which had been observed experimentally.
- 2- to describe the existence of the three zones of interest in the vicinity of the thermal bar, namely, the thermal overflow region, the thermal bar and the thermal underflow region.
- 3- to allow the sensitivity of the associated parameters to be examined and to guide the formulation of scaling arguments.
- 4- to integrate the numerical and experimental results into a comprehensive framework.

# 6.2.2 Limitations

In the environmental applications, the Rayleigh number, defined by Eq. 3.1. has values ranging from  $10^4$  to  $10^6$ . Due to laboratory considerations discussed in Sec. 3.2.2 (specifically the use of molecular values of viscosity and conductivity), an experimental Rayleigh number with values ranging from  $10^7$  to  $10^8$  was used. For the computational simulation, two constraints should be considered which are expressed in terms of the numerical stability and accuracy (Sec. 5.3.3). Consequently, the mesh size should be fine enough to achieve the accurate and stable results.

With the available-computer facilities, the direct memory is not sufficiently large in order to provide the required storage to simulate the experimental values of Rayleigh numbers over such large domains. Additional virtual memory has been used to increase the storage size but the computational costs in terms of execution time restricted such trials.

It was found that the experimental flow patterns associated with large Rayleigh numbers and low aspect ratios (e.g.  $R_a = 10^7 - 10^8$ , A = 0.01 - 0.015) required very large array sizes to accomodate the physical domain and could not be simulated numerically. Therefore, it was decided to restrict the numerical investigation to the simulation of similar cases but with lower values of  $R_a$  (up to  $10^5$ ) to be within the environmental range, although still far from the experimental range.

#### 6.2.3 Discussion

The objectives of the numerical analysis can be achieved by simulating lower values of  $R_a$  under a certain condition. This condition is expressed in terms of the flow regime (i.e. flow type) associated with the specified value of  $R_a$ . The simulated cases using lower values of  $R_a$  may be reasonably assumed to represent the behaviour with higher values of  $R_a$  if both cases are in the same flow regime. Due to insufficient experience with the phenomenon of the sinking plume, the same classification established for the heated end cavities, Sec. 2.3.2, is assumed to be valid for such a phenomenon. Using these criteria it follows that in order to create flow patterns which are dominated by convection, it is necessary to ensure that the lowest value of  $R_a$  is greater than  $P_r^{2}$ .

It is worth mentioning that the numerical analysis will be for lower values of  $R_a$ , therefore, a direct comparison between the numerical and experimental results cannot be achieved. But, some measure of consistency can be demonstrated if correlation between the extension of the thermal bar with relevant system parameters is found to be valid in both experimental and numerical cases.

# 6.3 <u>Numerical Analysis</u>

#### 6.3.1 General

For buoyancy induced convective motions, the flow patterns are usually described and classified in terms of non-dimensional parameters. These parameters provide the criteria which are required for the application of the laws of similitude. For complete similarity these parameters should have the same values in both models and prototypes.

For example, the flow patterns produced in the heated end cavities are usually described by using the Rayleigh number,  $R_a = g(\Delta \rho)H^3/\rho\nu\kappa$ , or Grashof number,  $G_r = g(\Delta \rho)H^3/\rho\nu^2$ . The Rayleigh number provides a measure of the relative importance of the buoyancy, viscous and diffusive forces while the Grashof number relates buoyancy and viscous forces. The relation between the  $R_a$  and  $G_r$  is given by :

$$R_a = G_r P_r$$

where  $P_r$  is the Prandtl number,  $P_r = v/\kappa$ , which is the ratio between the momentum viscosity to the thermal conductivity.

On the other hand, the flow patterns produced in the lock exchange mechanisms are usually described by the densimetric Froude Reynolds number,  $F_{\Delta}R = \sqrt{g(\Delta\rho)H^3/\rho\nu^2}$ . This number can be considered as a combination between the Reynolds number,  $R = uH/\nu$ , [i.e. (inertial force)/(viscous force)] and the densimetric Froude number,  $F_{\Delta} = u/\sqrt{g(\Delta\rho)H/\rho}$ , [i.e. (inertial force)/ (gravitational force)]. However, the requirements of both densimetric Froude and Reynolds numbers criteria can be met together over a wide range of practical circumstances if the densimetric Froude Reynolds number is used.

When the motion is free from the external inertial effects

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6.1

(i.e. motion is produced entirely by the density differences) all these non-dimensional parameters can be related as

$$G_{r} = (F_{\Delta}R)^{2} = R_{a}/P_{r} \qquad 6.2$$

In the following investigation, the flow patterns will be presented with reference to the Rayleigh and Prandtl numbers.

Moreover, when the nonlinear density-temperature relation is assumed, three values of Rayleigh numbers are found to be involved. These three values are :

1	) th	e cold	Rayleigh	number.		(R_) -	_	ρ, -	ρ <sub>c</sub>	gH <sup>3</sup>
•				,		···a·c	-	٩	>	voro
2	) th	e warm	Rayleigh	number,	:	(R <sub>a</sub> )	=	ρ <sub>ι</sub> –	ρ <sub>₩</sub>	gH <sup>3</sup>
		-						٩	)	vo <sup>r</sup> o r
3	) th	e lock	exchange	Rayleigh	number,	(R <sub>a)1</sub>	=	ρ <sub>c</sub> -	р <sub>₩</sub>	gH <sup>-</sup>
								٩	)	voro

It should be noted that any one of the above definitions of  $R_a$  is dependent on the other two values. In the following investigation,  $(R_a)_c$  and  $(R_a)_l$  are considered the independent parameters while  $(R_a)_w$ is assumed to be the dependent one as

> $(R_{a})_{w} = (R_{a})_{c} + (R_{a})_{1}$ 6.3

It is worth mentioning that when symmetrical different temperatures

around 4° C are assumed,  $(R_a)_c = (R_a)_w$  and  $(R_a)_1 = 0$  and only one value of Rayleigh number need be involved to describe the behaviour. Moreover, when the linear density-temperature relation is used assuming  $T_o = T_c$ ,  $(R_a)_1 = (R_a)_w$  and  $(R_a)_c = 0$ , and again only one Rayleigh number can be used.

The numerical investigation is carried out for the lock exchange flows shown in Figure 6.1(a). The associated non-dimensional initial and boundary conditions, shown in Figure 6.1(b), are detailed in Eqs. 6.4 and 6.5.

For	τ < Ο΄:		$U = V = \Psi = \Omega =$	0	•	everywhere	
			Θ = Θ <sub>₩</sub>	· · ·	٠	at X <o< td=""><td></td></o<>	
		~	$\theta = (\theta_w + \theta_c)/2$	·.	•	at X=0	6.4
			θ ≈ θ <sub>C</sub>		•	at X>0 👻	

For  $\tau > 0$ :  $U = \frac{\partial V}{\partial X} = \Psi \approx \frac{\partial \Theta}{\partial X} = \Omega = 0$ , at  $X = -L_{W} + L_{C}$ 

 $\partial U/\partial Y = V = \Psi = \partial \Theta/\partial Y = \Omega \approx 0$ , at  $Y \approx 0, 1$  6.5

The lower boundary condition of  $\partial U/\partial Y = 0$  is judged to be acceptable since :

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 interest is concentrated in the penetration of the upper layer only;

(2) inclusion of a 'no-slip' boundary layer would greatly increase the computer storage requirements due to the



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# Figure 6.1:

Rectangular flume notations, initial and boundary conditions of lock exchange flow for: dimensional parameters. non-dimensional parameters.

(a) (b) increased node density close to the boundary.

Generally, for any numerical run, four dimensionless input parameters are required to be specified prior to the numerical calculations. These four parameters are  $\theta_w$ ,  $\theta_c$ ,  $P_r$  and  $R_a$  and can be calculated as shown in Table 6.1. The corresponding physical parameters  $[(R_a^{\bullet})_w, (R_a)_c, (R_a)_1]$ , the equation of state, and the buoyancy term which are defined according to the specified temperature-density range are also listed in Table 6.1.

6.3.2 Density-temperature relation

When the lower temperature is greater than or equal to  $4^{\circ}$  C it is common to assume the relationship between density and temperature to be linear. This may be appropriate for large temperature ranges, (e.g.  $4^{\circ}$  C to  $20^{\circ}$  C). The assumption may be less justifiable if the temperature range is small and just above  $4^{\circ}$  C (e.g.  $4^{\circ}$  C to  $10^{\circ}$  C).

Equations of state, 5.1 and 5.9, may be used for the linear and nonlinear cases respectively. The validity of the above assumption was tested by simulating two cases of the lock exchange mechanism as illustrated in Table 6.2, where the cold temperature is assumed to be  $4^{\circ}$  C.

For the first case (i.e. Run # 1), the linear equation of state is utilized and for the second case (i.e. Run # 2), the nonlinear version of equation of state is assumed. All other

• 			:
Temperature	range '	Linear	Nonlinear
- · ·	θ <sub>w</sub>	$0 < \frac{T_w - T_o}{T_w - T_c} \le 1$	$0 < \frac{T_w - 4}{T_w - T_c} \le 1$
Numerical model	θ <sub>c</sub>	$0 \leqslant \frac{T_c - T_o}{T_w - T_c} < 1$	$-1 < \frac{T_{c} - 4}{T_{w} - T_{c}} \le 0$
input parameters	P <sub>r</sub>	Pr	P <sub>r</sub>
	R <sub>a</sub>	$\frac{ga_{0}(\Delta T)H^{3}}{\rho_{0}v_{0}\kappa_{0}}$	gβ <sub>4</sub> (ΔT) <sup>2</sup> H <sup>3</sup>  ρ <sub>4</sub> ν <sub>4</sub> κ <sub>4</sub>
corresponding	(R <sub>a</sub> ) <sub>w</sub>	$R_a \cdot \Theta_w > 0$	$R_a \cdot \theta_w^2 > 0$
physical	(R <sub>a</sub> )c	r R <sub>a</sub> .θ <sub>c</sub> ≥ Ο	$-R_a \cdot \theta_c^2 \neq 0$
	. (R <sub>a</sub> )1	$R_a(\theta_w - \theta_c) > 0$	$R_a(\theta_w^2 - \theta_c^2) > 0$
Equation of s	state	$\frac{\Delta \rho}{\rho_4} = -\alpha(T-4)$	$\frac{\Delta \rho}{\rho_{4}} = -\beta(T-4)^{2}$
Buoyant term $R_a P_r \frac{\partial f(\theta)}{\partial X}$		R <sub>a</sub> P <sub>r</sub> 30 3X	$R_a P_r \frac{\partial \Theta^2}{\partial X}$

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Specified numerical and physical parameters

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Table 6.2

Effect of different versions of the equation of state

Run		Numerio	cal mode paramete	el ers	Corresponding physical parameters			Employed - gersion
#	, θ <sub>₩</sub>	, <sup>e</sup> c	Pr	Ra ÷	(R <sub>a</sub> ) <sub>w</sub>	(R <sub>a</sub> )c	(R <sub>a</sub> )1	of state 🛩
1 2	+1.0	0.0 0.0	11.6 11.6	10000. 10000.	10000. 10000.	0. 0.	10000.	Eq. 5.1 Eq. 5.9

7

parameters were assumed to be constant. Both cases were run for the same time,  $\tau = 0.16$ , and the results are shown in Figure 6.2 and 6.3. Both cases show the same classical lock exchange mechanism where the bouyant warm water extends as an upper layer while the heavier cold water penetrates in the opposite direction as a lower layer. The flow patterns and the temperature distributions are significantly different in the two cases. The result of Run # 1 (linear range) shows complete symmetry whereas those for Run # 2 (nonlinear range) show asymmetrical behaviour. The observed symmetrical behaviour of Run # 1 is due to the linear density-temperature relation where  $\partial p/\partial T$  is assumed constant which indicates that

# $(\partial \rho/\partial T)$ for warm water = $(\partial \rho/\partial T)$ for cold water 6.6

It follows that the effect of the heat loss on the upper layer has the same effect as the heat gain on the lower layer because no other external heat losses are assumed. Consequently, a completely symmetrical behaviour should be obtained.

On the other hand in the case of a nonlinear density -temperature relation, the extensions are expected to be asymmetrical as demonstrated by the observed behaviour of  $Run \int \# 2$ . From the relationship shown in Figure 1.1(b) it is clear that ( $\partial p/\partial T$ ) for warm water (e.g. > 8° C) is numerically greater than ( $\partial p/\partial T$ ) for cold water (i.e  $\equiv 4^{\circ}$  C). The effect of lock exchange is a dilution of the water at the warm and cold fronts. Consequently for a dilution which causes



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Figure 6.2

: Numerical results for Run #1, at  $\iota = 0.16$  for: (a) stream-function distribution,  $\psi (\Delta \psi = 2.0)$ , (b) temperature distribution,  $\theta (\Delta \theta = 0.1)$ .



: Numerical results for Run #2, at t = 0.16 for: (a) stream-function distribution,  $\psi (\Delta \psi = 2.0)$ . (b) temperature distribution,  $\theta (\Delta \theta = 0.1)$ .

Figure 6.3

an equal magnitude of temperature change of  $\pm \Delta T C^{\circ}$  at each front, the result will be a much greater reduction in the density difference at the warm front compared to the cold front and thus the driving force for the warm layer will be smaller than for the cold layer. This explains the much more pronounced diminution of velocity in the bouyant warm layer compared to that of the cold underflow.

By contrast, the linear case (Run # 1) exhibits no such asymmetry in either ( $\frac{\partial p}{\partial T}$ ) or in the diminution of front velocity.

Further supporting evidence can be obtained by comparing the numerical results obtained from Run # 1 (i.e. the linear case) with experimental results published by Barr (1967). The parameters of  $(R_a)_1 = 10000$  and  $\tau = 0.16$ , used in Run # 1, are equivalant to Barr's parameters of  $F_{\Delta}R = 30$  and  $t/\Delta t = 54$ . The numerically predicted relative extension of the warm layer of approximately 7.2 (Figure 6.2) is quite consistent with extrapolated lines of Barr's "congruency diagram".

# 6.3.3 General behaviour

The nonlinear density-temperature relation, Eq. 5.9, is adopted in the following numerical analysis. For asymmetrical temperature around 4° C, the general behaviour needs to be confirmed. The general behaviour will be discussed for a numerical run where the values of the lock exchange  $[(R_a)_1]$  and cold  $[(R_a)_c]$  Rayleigh numbers are assumed to be 15000 and 1000, respectively. These two chosen values of Rayleigh numbers correspond to warm  $(\theta_w)$  and cold  $(\theta_c)$ 

dimensionless temperatures of +0.8 and -0.2, respectively. The Prandtl number  $P_{\Gamma}$  is assumed to have a value of 11.6 which is the corresponding value for a reference temperature of 4° C. This run will be designated as Run # 8 in Sec. 6.4 and Table 6.4.

To reduce the computational cost in all simulated runs, a smaller domain, as shown in Figure 6.1, is initially used when the variations of the variables involved  $(\Omega, \Theta, \Psi, U, \Psi)$  occur only within a small distance from the barrier (i.e. X = 0). Thereafter, when the variation of any variable close to the vertical side boundaries reaches  $\pm 0.1\%$  from its maximum value within the domain at the same time step, the working domain is stretched by introducing additional nodes having the same specified initial conditions as the original domain and subject to the required boundaries technique shows a very significant saving in the computational cost in terms of storage and execution time. Generally, the numerical results are presented in this chapter over sufficient domains, as required to illustrate the obtained results, but in most cases, these domains are less in extent than the actual simulated domains.

The computational results obtained for this case (i.e. Run # 8) indicated that :

1) The initial behaviour is governed by the initial inertial force, and is typically like the classic lock exchange mechanism. The warm water extends as an upper layer, while the cold water extends in

the opposite direction as a lower layer, as shown in Figure 6.4. As the motion of both layers is governed by the inertial force, almost symmetrical behaviour for both layers around the barrier is to be expected.

2) As the relative extension distance between the upper and lower fronts increases, the heat transfer between the two layers causes a reduction in the buoyancy driving force. Due to the nonlinear density response to different temperatures in the warm and cold water layers, as discussed in Sec. 6.3.2, asymmetrical behaviour exists as shown in Figure 6.5.

3) Due to the continuous reduction of the horizontal inertia and mixing at the warm front, the diluted water near the front attains the temperature of maximum density ( $\theta = 0$ ,  $T = 4^{\circ}$  C). This water then sinks vertically from the upper layer and is entrained by the lower layer which results in an increase of the temperature of the lower layer. The location of this sinking phenomenon can be identified as the point at which the isotherm for  $\theta = 0$  ( $T = 4^{\circ}$  C) is vertical through the upper layer, as shown in Figure 6.6.

4) Due to the phenomenon described in (3) above, the sinking frontal extremity of the upper layer forms a closed convective cell as shown in Figure 6.7. Forward movement of the upper front ceases at this stage.

5) The temperature gradient created between the cold water and the thermal bar drives a compensating convective cell in the cold region as shown in Figure 6.8.

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: Numerical results for Run #8, at t = 0.011 for: stream-function distribution,  $\psi (\Delta \Psi = 10)$ , temperature distribution,  $0 (\Delta 0 = 0.1)$ .

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Pigure 6.4

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Numerical results for Run #8, at  $\iota = 0.048$  for: stream-function distribution,  $\Psi (\Delta \Psi = 5)$ . temperature distribution,  $0 (\Delta 0 = 0.1)$ .

E)

. 125

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: Numerical results for Run #8, at t = 0.080 for (a) stream-function distribution,  $\Psi (\Delta \Psi = 4)$ . (b) temperature distribution,  $0 (\Delta 0 = 0.1)$ .

Figure 6.6


2

Numerical results for Run #8, at  $\iota = [0.134$  for: stream-function distribution,  $\psi (\Delta \psi = 3)$ , temperature distribution,  $\theta (\Delta \theta = 0.1)$ .

(P)



Figure 6.8 : Numerical results for Run #8, at  $\iota = 0.149$  for: (a) stream-function distribution,  $\Psi (\Delta \Psi = 3)$ . (b) temperature distribution,  $0 (\Delta 0 = 0.1)$ .

6) Thereafter, the simulated domain is divided into two convective cells of opposite rotational sense on either side of the location of the thermal bar. Each of these is governed by the gradient between the 'ambient' water temperature ( $\theta_w$  or  $\theta_c$ ) and the thermal bar temperature ( $\theta = 0$ ). Both of them transport ambient water to (through the upper layers) and from (through the lower layers) the thermal bar location.

7) The location of the thermal bar is dependent on the thermal balance between the heat-transfer by advection through the upper layers and the heat-transfer by conduction between the two convective cells at the thermal bar, as shown in Figure 6.9.

To reduce the computational costs, the numerical calculation is stopped when the thermal bar location shows an acceptably small variation with the elapsed time. It should be noted, however, that this point is not a final equilibrium state but it is considered to represent the point at which the horizontal extension is arrested by the vertical sinking. The maximum extension of the thermal bar is obtained by relating the upper penetration distance L/H for the isothermal line  $\theta = 0$  (which corresponds to the maximum density) to the elapsed time  $\tau$ , as plotted in Figure 6.10, by the following relation

$$/H = a(1 - e^{\sqrt{\tau}/b})^{c}$$

129

6.10





This correlation was found to be reasonable for all the numerical results obtained in this investigation, where the coefficients a, b and c are determined according to the best curve fitting of the numerical results for each run. For example, by fitting this equation to the obtained results for Run # 8, it is found that the best curve fitting occurs when a, b and c are equal to 6.16, 0.15 and 2.30 respectively where the relation yields to be

$$L/H = 6.16(1 - e^{\sqrt{\tau}/0.15})^{2.3}$$
 6.1

As  $\tau \neq -$ , (L/H) = (L/H)<sub>max</sub>  $\rightarrow$  6.16 which indicates that the maximum penetration distance for the thermal bar, measured from the barrier, is 6.2 times the working depth.

## 6.3.4 Discussion

The natural convection motions under investigation are produced due to differences in density (i.e. due to temperature gradient). Generally, the flow patterns are mainly dependent on the specified value of the Rayleigh number  $R_a$ , Prandtl number  $P_r$  and aspect ratio A.

When a linear density-temperature relation is assumed, only one specified value of Ra is sufficient to describe the flow pattern. Most of the numerical results published in the linear range are referred to one specified value of  $R_a$ . This value provides the criterion which is required for the application of the laws of

similitude. But, when the nonlinear density-temperature relation is used, two specified values of  $R_a$  must be employed to describe the relevant behaviour.

The aspect ratio A has a significant effect on the flow patterns particularly for the case of densimetric flows produced by the temperature gradients between the surrounding boundaries and enclosed fluid, as for the case of the heated end cavities. By contrast, the flow patterns produced in the lock exchange mechanism are created almost entirely by the density differences between the two water bodies. The assumption of zero heat flux at the boundaries is discussed further in Chapter 7.

The flow resistance in terms of Prandtl number  $P_r$  plays an important role particularly for flow patterns dominated by convection. The Prandtl number  $P_r$  is temperature dependent where it has a value of 11.6, 9.0 and 7.0 for a specified temperature of 4, 15 and 20° C, respectively. This variation with temperature is due mainly to the change in the kinematic viscosity.

Unlike the case of the conventional cavity convection problem, in which A,  $P_r$  and  $R_a$  specify the flow pattern, the lock exchange flow mechanism is a function of  $R_a$  and  $P_r$ . The aspect ratio A will be determined by these parameters as long as the numerical domains are essentially semi-infinite.

The general behaviour of the density currents where the maximum density exists is identical with the observed experimental behaviour for asymmetrical temperatures around  $4^{\circ}$  C. Moreover, the

numerical results show that the initial behaviour of the lock exchange flow is mainly governed by the inertial force where the lock exchange Rayleigh number  $(R_a)_1$  is sufficient to describe such behaviour. Thereafter, due to the effect of the maximum density, the behaviour of the two opposite convective cells is described by two specified values of  $R_a$ , namely  $(R_a)_1$  and  $(R_a)_c$ . Consequently, the maximum extension distance  $(L/H)_{max}$  will be a function of these two numbers.

#### 6.4 <u>Numerical Results</u>

#### 6.4.1 General

For a densimetric flow created between two water bodies having different temperatures around  $4^{\circ}$  C, the flow pattern produced can be described over three zones of interest. These three zones are demonstrated schematically in Figure 6.11. Figures 6.11(a) and 6.11(b) are expressed in terms of dimensional and non-dimensional parameters, respectively.

Over one of these zones, the densimetric flow is maintained by the density differences between the thermal bar (i.e.  $\theta \approx 0$ , or  $T = 4^{\circ}$  C) and the warm water body (i.e.  $\theta = \theta_{w}$ , or  $T = T_{w}$ ). Two warm layers having different temperatures, both more than  $4^{\circ}$  C ( $\theta > .0$ ) and less than  $T_{w}$  ( $\theta < \theta_{w}$ ) are established. The convective motion in this zone will be identified as a warm convective cell which transports the ambient warm water to the thermal bar, where the water approaches maximum density, sinks and by continuity returns as a reversed under





Figure 6.11

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Schematic diagram showing general behaviour of water around thermal bar for: dimensional parameters.

- (a) (b) non-dimensional parameters.

In the other zone, the densimetric flow is maintained by the density differences between the thermal bar (i.e  $\theta = 0$ , or  $T = 4^{\circ}$  C) and the cold water body (i.e.  $\theta = \theta_{C}$ , or  $T = T_{C}$ ). Two cold layers having different temperatures, both less than  $4^{\circ}$  C ( $\theta < 0$ ) and more than  $T_{C}$  ( $\theta > \theta_{C}$ ) are established. The convective motion in this zone will be identified as a cold convective cell which transports the ambient cold water to the thermal bar, where the water approaches maximum density, sinks and by continuity returns as a reversed under flow.

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It is worth mentioning that for both warm and cold convective cells, the water layers are vertically stable.

Between these two zones (i.e. the warm and cold convective cells), the thermal bar exists forming the third zone and showing a constant vertical temperature  $\theta = 0$  (i.e.  $T = 4^{\circ}$  C).

The maximum distance  $(L/H)_{max}$  between the location of the thermal bar and the original location of mixing (i.e. the barrier) is a function of the associated parameters  $(R_a)_1$ ,  $(R_a)_c$  and  $P_r$ . The maximum distance  $(L/H)_{max}$  is produced in two stages caused by a different balance of forces. In the first stage, the inertial force governing the extension of the flow is mainly dependent on the lock exchange Rayleigh number {i.e.  $L/H = f[(R_a)_1]$ }. Thereafter, any additional penetration distance (second stage) will be due to the difference in the density gradient around the thermal bar. It is difficult to separate the two different mechanisms because the density

flow.

gradient is usually set up by the inertial penetration. Moreover, the viscous effect (i.e. shear stress), expressed by  $P_r$ , may affect the maximum extension of the thermal bar. This viscous effect is expected to be more significant in the first stage than in the second stage.

Generally, the maximum extension of the thermal bar  $(L/H)_{max}$  can be assumed as

$$(L/H)_{max} = f[(R_a)_1, (R_a)_c, P_r]$$
 6.12

In the following sections, the sensitivity of each associated parameter will be examined. The numerical results of the sensitivity analysis will be used to correlate the relative extension of the thermal bar with these system parameters. Moreover, a comparison between the numerical and experimental results will be discussed.

#### 6.4.2 Sensitivity analysis

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All the tests in the present experimental investigation were carried out with cold water temperatures close to  $0^{\circ}$  C. Thus the effect of the variation of  $(R_a)_c$ , caused by different cold water temperatures, could not be examined based on the experimental data. By definition  $P_r$  is a function of temperature for any specific fluid and it is therefore subject to minor variations between warm and cold zones in the laboratory experiments. For calculation purposes, a constant value was assumed and defined at a reference temperature of  $4^{\circ}$  C. Consequently, the empirical relation, based on the experimental

results, showed the maximum extension of the thermal bar  $(L/H)_{max}$  to be a function of the initial density difference of the cold and warm water (Eq. 3.8). However, the large value of the constant of proportionality in this equation (1200) gives an indication that other system parameters should be included in this relation. The effect of these parameters is discussed in this section.

Generally, the sensitivity analysis was carried out to investigate the effect of the variation of each parameter on  $(L/H)_{max}$ . This was done by varying each parameter while keeping the other parameters constant. The same procedure as discussed in Sec. 6.3.3 was followed to determine  $(L/H)_{max}$  by using the time history of L/H to obtain the best curve fitting of Eq. 6.10. The coefficients of Eq. 6.10 obtained for all the numerical runs are listed in Table 6.3. The numerical results of the sensitivity analysis in terms of the stream-function and temperature distributions, as presented in this section, represent conditions at the end of the computational time.

The sensitivity of  $(L/H)_{max}$  to the variation of  $(R_a)_1$  was examined using 6 numerically simulated runs. listed in Table 6.4 as Run # 3 to Run # 8. The values of the input parameters were chosen to cover two ranges of  $(R_a)_1$ , with a constant value of  $(R_a)_c$  for each range. The numerical results are shown in Figures 6.12 to 6.17 for the last computational stage of these runs. The calculated values of  $(L/H)_{max}$  is also listed in Table 6.4. The relation between  $(L/H)_{max}$ and  $(R_a)_1$  is plotted in Figure 6.18 for the two values of  $(R_a)_c$ . It is noted that  $(L/H)_{max}$  is proportional to  $[(R_a)_1]^{+0.50}$  for both values

Table 6.3

RUN # а Ь С З. • 0.99 1.40. 0.97 4 2.09 0.70 1.35 5 3.51 0.37 1.75 6 1.79 0.70 1.10 7 3.29 0.25 1.74 . 8 6.16 0.15 2.30 -10 2.76 0.50 1.43 11 4.49 0.65 1.45 12 2.48 0.11 2.27 13 4.04 0.12 2.42 16 1.80 0.65 1.21 17 1.84 0.75 1.33 19 8.89 0.30 2.47

Calculated coefficients for numerical runs

Note :

RUN # 6 ≡ RUN # 9 ≅ RUN # 15 RUN # 8 ≡ RUN # 14 ≡ RUN # 18 139



Effect of variation of the lock exchange Rayleigh number  $(R_a)_1$  on the maximum relative extension  $(L/H)_{max}$ 

Run	1	Numerica	al mode arameter	l rs	Con	L (-) H		
#	θ <sub>₩</sub>	θc	۴r	Ra	(R <sub>a</sub> ) <sub>w</sub>	(R <sub>a</sub> ) <sub>c</sub>	(R <sub>a</sub> );	max
З	+0.6	-0.4	11.6	625.0	225.0	100.0	125.0	1.0
4	+0.7	-0.3	11.6	1111.1	544.4	100.0	444.4	2.1
5	+0.8	-0.2	11.6	2500.0	1600.0	100.0	1500.0	3.5
6	+0.6	-0.4	11.6	6250.0	2250.0	1000.0	1250.0	1.8
7	+0.7	-0.3	11.6	11111.1	5444.4	1000.0	4444.4	3.3
8	+0.8	-0.2	11.6	25000.0	16000.0	1000.0	15000.0	6.2



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r.







Figure 6.18 :

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Plot of maximum extension  $(L/H)_{max}$  against lock exchange Rayleigh number,  $(R_a)_1$ .

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of  $(R_a)_c$ . The constant of proportionality (i.e. the slope) decreases as  $(R_a)_c$  increases.

The same sensitivity procedure was repeated to study the effect of the variation of  $(R_a)_c$  for two different values of  $(R_a)_1$ , as listed in Table 6.5. The numerical results are shown in Figures 6.19 to 6.22 and Table 6.5. In these runs  $(L/H)_{max}$  was found to be proportional to  $[(R_a)_c]^{-9.36}$  as shown in Figure 6.23.

The effect of  $P_r$  on  $(L/H)_{max}$  was examined as listed in Table 6.6 for two values of  $(R_a)_1$ . The numerical results are shown in Figures 6.24 to 6.26 and Table 6.6. In this case, it was found, as shown in Figure 6.27, that for small values of  $(R_a)_1$ ,  $(L/H)_{max}$  is not sensitive to  $P_r$ , but for high values of  $(R_a)_1$ , it is proportional to  $P_r^{-0.15}$ .

### 6.4.3 Numerical correlations

The experimental parameters described in Chapter.3 are used to determine the numerical parameters listed in Table 6.7. The relation between the maximum extension of the upper layer  $(L/H)_{max}$  and the system parameters  $[(R_a)_1, (R_a)_c, P_r]$  was assumed to take the general form :

$$(L/H)_{max} = C [(R_a)_1]^a [(R_a)_c]^b [P_r]^c$$
 6.13

As a first trial to obtain the proper relation, the exponents of the system parameters (a, b, c) in Eq. 6.13 were assumed to have



Effect of variation of the cold Rayleigh

number  $(R_a)_c$  on the maximum relative extension  $(L/H)_{max}$ 

Run		Numerica input pa	al mode arameter	1 -\$	Corresponding physical parameters			د (-)
#	θ₩	θc	P <sub>r</sub>	Ra	(R <sub>a</sub> ) <sub>w</sub>	(R <sub>a</sub> ) <sub>c</sub>	(R <sub>a</sub> )]	max
9	+0.6	-0.4	11.6	6250.0	2250.0	.1000.0	1250.0	1.8
10	+0.7	<del>,</del> 0.3	11.6	3125.0	1531.3	281.3	1250.0	2.8
11	+0.8	-0.2	11.6	2083.3	1333.3	83.3	1250.0	4.5
12	+0.6	-0.4	11.6	75000.0	27000.0	12000.0	-15000.0	2.5
. 13	+0.7	-0.3	11.6	37500.0	18375.0	3375.0	15000.0	4.0
14	+0.8	-0.2	-11.6	25000.0	16000.0	1000.0	15000.0	6.2

Note :

Run # 9 ≡ Run # 6

Run # 14 ≡ Run # 8





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Figure 6.23

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Plot of maximum extension  $(L/H)_{max}$  against cold Rayleigh number,  $(R_a)_c$ .

Table	6.6
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# Effect of variation of the Prandtl number $P_r$ on the maximum relative extension (L/H)<sub>max</sub>

Run		Numeric	al mode aramete	l rs ,	Corresponding physical parameters			L (-)
#.	θ <sub>₩</sub>	θ <sub>C</sub>	۴ <sub>۲</sub>	Ra	(Ra)w	(R <sub>a</sub> ) <sub>c</sub>	(R <sub>a)1</sub>	
15	+0.6	-0.4	11.6	6250.0	2250.0	1000.0	1250.0	1.8
16	+0.6	-0.4	5.0	6250.0	2250.0	1000.0	1250.0	1.8
17	+0.6	-0.4	1.0	6250.0	2250.0	1000.0	1250.0	1.8
18	+0.8	-0.2	11.6	25000.0	16000.0	1000.0	15000.0	6.2
19	+0.8	-0.2	1.0	25000.0	16000.0	1000.0	15000.0	8.9

Note : \*

Run # 15 ≡ Run # 6

' Run # 18 ≡ Run # 8



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: Numerical results for Run #16, at  $\iota = 3.786$  for: stream-function distribution,  $\psi (\Delta \psi = 0.5)$ , temperature distribution,  $\theta (\Delta 0 = 0.1)$ .

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Figure 6.25



Numerical results for Run #19, at t = 0.236 for stream-function distribution,  $\Psi (\Delta \Psi = 5)$ , temperature distribution,  $\theta (\Delta 0 = 0, 1)$ .



Figure 6.27 :

Plot of maximum extension  $(L/H)_{max}$  against the Prandtl number,  $P_r$ .



# Experimental values of specified numerical

parameters .

Test	Nui	merical m put param	odel eters	Ca	L (-) H		
#	θ <sub>w.</sub>	, θ <sub>C</sub>	R <sub>a</sub> (*10 <sup>-7</sup> )	(R <sub>a</sub> ) <sub>W</sub> (*10 <sup>-7</sup> )	<sup>(R</sup> a <sup>)</sup> c. (*10 <sup>-7</sup> )	(R <sub>a</sub> ) <sub>1</sub> (*10 <sup>-7</sup> )	max
1	+0.780	-0.220	11.113	6.761	0.538	6.223	42.5
2	+0.717	-0.283	6.717	3.453	0.538	2.915	30.0
3	+0.688	-0.312	- 4.711	2.230	0.459	1.771	23.5
4	+0.673	0.327	4.904	2.221	0.524	1.697	22.5
5	+0.645	-0.355	12.603	5.243	1.588	3.655	16.7
6	+0.799	-0.201	35.473	22.646	1.433	21.213	43.3
7	+0.719	-0.281	15.633	8.082	1.234	6.847	23.2

Note :  $P_r = constant = 11.6$ 

the same corresponding values (+0.50, -0.36, -0.15) achieved in the sensitivity analysis of each individual parameter. Consequently, Eq.

$$(L/H)_{max} = C[(R_a)_1]^{+0.50}[(R_a)_c]^{-0.36}[P_r]^{-0.15}$$
 6.14

The scomputed results using Eq. 6.14 are compared with the numerical and experimental results in Figure 6.28 (a) and (b), respectively. The curve fitting of Eq. 6.14 shows that the constant of proportionality, C, has two different values (i.e. 0.85, 1.75) when the numerical and experimental parameters are used, respectively. The different values of C are to be expected since different values of the constants of proportionality were observed in the sensitivity analysis when  $(L/H)_{max}$  was correlated with a single parameter (i.e.  $(R_a)_1$ ,  $(R_a)_c$  or  $\mathsf{P}_{\mathsf{r}}$ ) over different ranges of the other two system parameters. It was also observed that Eq. 6.14 produces errors with a maximum value of 20% when the experimental parameters are used. The difference between the experimental results and the results predicted using Eq. 6.14 may be due in part to laboratory errors in measuring the maximum extension of the upper layer or in the temperature measurements. Based on the experimental measurement methods, the laboratory errors are not expected to be more than 10%. The second source of error may be due to the different flow mechanisms between the numerically simulated range  $(R_a up to 10^5)$  and the experimental range  $R_a$  up to 10<sup>8</sup>). Therefore, Eq. 6.14 may be valid within the numerical range of  $R_a$ , while another


relation (i.e. involving different exponents) should be used for the experimental range of  $R_a$ .

As an alternative approach, the constants in equation 6.13 (i.e. a, b, c and C) were assumed to be unknown. The optimum values of these constants were calculated by minimizing the following error function

$$Z_{\min} = \sum \{ (L/H)_{\max} - C[(R_a)_1]^a [(R_a)_c]^b [P_r]^c \}^2$$
 6.15

For the numerical system parameters, the optimum, relation was found to be

$$(L/H)_{max} = 0.522[(R_a)_1]^{+0.572}[(R_a)_c]^{-0.387}[P_r]^{-0.133}$$
 6.16

It is noted that the exponent values in Eq. 6.16 are almost the same as the corresponding values in Eq. 6.14.

A similar procedure was followed using the experimental values and assuming the Prandtl number is essentially constant. The Prandtl number was therefore eleminated from the optimization procedure. The experimental relation was found to be

$$(L/H)_{max} = 17.345 [(R_a)_1]^{+0.531} [(R_a)_c]^{-0.560}$$
 6.17

The constant of proportionality in Eq. 6.17 (i.e. 17.345) possibly includes the additional dependence on the Prandtl number,  $P_r$ , which is

was eliminated from the optimization procedure. It should be noted that both Eqs. 6.16 and 6.17 represent the best fit (optimal correlation) between  $(L/H)_{max}$  and the relevant system parameters based on the numerical and experimental results, respectively. It was found that most of the numerical and experimental results are within  $\pm 1.0$  s<sub>d</sub>. (where S<sub>d</sub> is the standard deviation of each set) from the lines represented by Eqs. 6.16 and 6.17, as shown in Figure 6.29 (a) and (b), respectively. Figure 6.30 shows the relation between the observed and predicted values of  $(L/H)_{max}$  when Eqs. 6.16 and 6.17 are used for the numerical and experimental parameters, respectively.

6.4.4 Scaling analysis

In this section an attempt is made to relate the estimated value of the maximum extension distance of the thermal bar to the relevant parameters by using a scaling analysis approach. A comprehensive analysis is difficult because of the several complex mechanisms governing the flow behaviour. Therefore, for illustration purposes, a simple case is considered in the following analysis.

Suppose the thickness of the thermal bar is denoted as 1 which is small compared to the vertical length scale H and the extension length  $L_u$  [Figure 6.11(a)]. Using the relation between the velocity components and the stream function, Eq. 4.8, it can be seen that the convection and diffusion terms in the heat-transfer equation, 4.12, are in the order of ( $\Psi\Delta T/1H$ ) and ( $\kappa\Delta T/1^2$ ), respectively. When the thermal bar, attains a fixed location, the balance between





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Figure 6.30

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convection and diffusion requires that

i.e.

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~ kH/1 \*

Similarly, in the vorticity-transport equation, 4.13, the convection, diffusion and buoyancy terms are in the order of  $(\Psi^2/1^3 H \sim \kappa^2 H/1^5)$ ,  $(\nu\Psi/1^4 \sim \nu\kappa H/1^5)$  and  $(g\Delta\rho/\rho I)$ , respectively, where  $\Delta\rho = \rho_4 - \rho_c$ . The balance between diffusion and buoyancy generates the motion at the thermal bar and gives

i.e.

νκΗ/1<sup>5</sup> ~ gΔρ/ρ]

1 ~ νκΗρ/gδρ

Using Eqs. 6.18 and 6.19, the vertical sinking velocity, v, at the thermal bar can be obtained as

 $v \sim \Psi/1 \sim \kappa H/1^2$  $v \sim \sqrt{[g(\Delta \rho/\rho)H^3/vk]} \kappa/H$ 

i.e.

$$v \sim [(R_a)_c]^{1/2} / H$$

Based on experimental observations Hamblin and Ivey (1984) estimated

6.19

6.20

6.18

the vertical sinking velocity, v, as : `

 $v = 0.3 [(R_a)c]^{1/2} k/H$ 

On the other hand, the horizontal velocity of the upper layer, u, can be estimated from the balance between the advection and buoyancy terms (where  $\Delta \rho = \rho_c - \rho_w$ ) in the horizontal equation of momentum, 4.1, which yields :

 $u^2 \sim g(\Delta \rho / \rho) H$ 

 $u \sim [(R_a)_1]^{1/2} \sqrt{\sigma_k}/H$ 

i.e.

Based on experimental observations, Barr (1963) estimated the horizontal velocity, u, as :

$$u = 0.6 \left[ \left( R_{a} \right)_{1} \right]^{1/2} \sqrt{v \kappa} / H$$
 6.23

The maximum extension distance (L<sub>u</sub>)<sub>max</sub> of the thermal bar is achieved when the continuity equation, 4.4, is satisfied

 $u/(L_u)_{max} \sim v/H^{-1}$ 

and combining with Eqs. 6.21 and 6.23 we obtain,

168

6.21

6.22

$$(L_{u}/H)_{max} \sim 2[P_{r}]^{1/2}[(R_{a})_{1}]^{1/2}/[(R_{a})_{c}]^{1/2}$$
 6.24

It should be noted that the above scaling analysis is carried out for a situation in which the inertial force dominates the extension of the -, upper layer. Consequently, Eq. 6.24 is reasonable for higher values of  $R_a$ . When the Prandtl number is assumed constant and defined at 4° C (i.e.  $P_r = 41.6$ ), Eq. 6.24 is modified to be

$$(L_u/H)_{max} \sim 6.812[(R_a)_1]^{+0.5}[(R_a)_c]^{-0.5}$$
 6.25

It is also noted that the exponent of the system parameters of Eq. 6.25 are close to the experimental relation, 6.17.

### 6.4.5 Discussion

The numerical simulation was carried out for a range of Rayleigh number between 625 and 75000. In this range, the relation between  $(L/H)_{max}$  and the system parameters can be expressed by Eq. 6.16 which provides good agreement with the numerical results.

The experimental tests were carried out for a range of Rayleigh number between  $4.711*10^7$  and  $35.473*10^7$ . In this range, Eq. 6.17 shows good agreement with the experimental results. Moreover, the scaling analysis confirmed that for high values of Rayleigh number, the exponents of  $(R_a)_1$  and  $(R_a)_c$  have the same values and should equal +0.5 and -0.5, respectively. Consequently, Eq. 6.17 can be used for high values of  $R_a$  based on the experimental results and the scaling

analysis.

It is worth mentioning that the difference between equations 6.16 (numerical results) and 6.17 (experimental results) may be due to the large difference in the range of  $R_a$  used in each case which may result in two different flow regimes (i.e. inertial in the experimental range and viscous/diffusive in the numerical range).

6.5 <u>Conclusion</u>

A series of numerical investigations was carried out to provide a more comprehensive study of the phenomenon of the sinking plume. The numerical model was employed to simulate lock exchange flows created between two water bodies having different temperatures around the temperature of maximum density.

Due to restrictions in terms of available computer memory and computational costs, the experimental Rayleigh numbers of up to  $10^8$ could not be simulated numerically. Consequently, the numerical investigation was restricted to the simulation of similar cases with lower values of Rayleigh numbers up to  $10^5$  where accurate and stable results can be achieved with reasonable computational cost. The simulated cases using lower values of R<sub>a</sub> are assumed to also represent the general behaviour at higher values of R<sub>a</sub>.

The nonlinear density-temperature relation has a profound influence on the behaviour of the thermal density currents. For a dilution which causes a temperature change of  $\pm \Delta T^{O}$  C, the result will be a much greater reduction in the driving force of the warm water than in the cold water for the same amount of entrainment. Consequently, more pronounced diminution of velocity in the warm layer is expected compared to that of the cold layer.

When asymmetrical temperatures around 4° C are assumed, the warm water extends as an upper layer dominated by inertial force. As the buoyant layer attains the temperature of maximum density, it sinks vertically and forms a thermal bar. The flow patterns produced show three zones of interest viz. (i) the thermal overflow region (warm convective cell), (ii) the thermal bar, and (iii) the thermal underflow region (cold convective cell). The general behaviour is consistant with the experimental observations reported in Chapter 3.

The flow pattern around the thermal bar is related to three differently-defined values of Rayleigh number (where one of them is dependent on the other two values). It was found that the maximum extension of the upper layer (where the thermal bar attains a fixed location) can be related to the two independent values of Rayleigh number. It was also found that this relation does not depend on the range of Rayleigh numbers provided that the simulated cases are in the same flow regime.

On the other hand, it was found that the location of the thermal bar is also dependent on the Prandtl number. The weak sensitivity of the thermal bar location to the value of Prandtl number was found to increase with increasing Rayleigh numbers within the same flow regime.

The empirical relation, 6.16, obtained by using the numerical results over the simulated range of  $R_a$  up to  $10^5$  shows acceptable results. Due to the different mechanisms between the numerical and experimental range of Rayleigh number, another empirical relation, 6.17, was found to be applicable for the experimental tests. The latter relation was confirmed by the scaling analysis. It can be concluded that for the numerical range of  $R_a$  both advection and diffusion govern the extension of the upper layer. On the other hand, for the experimental range of  $R_a$  the extension of the upper layer is dominated by convection.

The numerical investigation [Marmoush, Hamblin and Smith (1985)] is sufficiently good to give a picture of the behaviour of densimetric flows in general and lock exchange flows in particular due to the nonlinear density-temperature relation. The empirical relation 6.16 can be used to predict the maximum penetration distance for the warm effluent in a typical field application since the values of the governing parameters would be in the same range as the values used in the numerical solution.

# CHAPTER 7

## SUMMARY AND CONCLUSIONS

This chapter includes a brief summary of this investigation with reference to the general findings. It also includes a discussion as to the extent to which the objectives of the thesis have been achieved and the scope for future research.

The behaviour of densimetric or density currents produced due to heated effluent at the outfall of steam electric generating station cooling water systems where the linear relation between density and temperature can be assumed have received considerable attention by many researchers. This heated effluent forms a raft with a pronounced discontinuity in the vertical temperature distribution and a frontal system driven by the thermal density difference. Usually the direct modelling of these natural system is very complex, however, the idealized models are used to provide some insight into these more difficult problems.

In cold climates, the existence of a density extremum in water at  $4^{\circ}$  C and the resulting nonlinear relation between density and

temperature gives rise to densimetric or density currents which are markedly different from those in the linear range. Temperatures higher than the ambient have been observed near the bottom of lakes in the vicinity of thermal discharges and concern has been expressed about the adverse effects of such abnormally warm water on the winter ecology of lake bottoms. It seems likely that the observed warm water at the bottom is due to the sinking phenomenon which is known as a thermal bar. However, the phenomenon of the thermal bar and the manner in which it may influence nearshore transport processes in the vicinity of a thermal outfall in a cold climate merits further study. The present-investigation addresses this problem from the experimental and numerical points of view.

The first step of the present study comprises a literature review of :

- (i) The environmental investigations in terms of biological implications and actual field temperature measurements near thermal outfalls in winter conditions.
- (ii) The idealized models which are usually used to approximate the prototype conditions.

The idealized model of lock exchange flow was selected as the basis of the study due to the fact that its behaviour is close to that expected in the prototype situation.

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The lock exchange flow system which was set up experimentally

and simulated numerically comprises a horizontal flume containing two water bodies having asymmetrical different temperatures around the temperature of maximum density and initially separated by a removable vertical barrier. Three physical parameters are found to be involved, i.e. the lock exchange Rayleigh number, the cold Rayleigh number and the Prandtl number. Both experimental and numerical results show that the initial flow patterns are similar to the classical lock exchange mechanism, the warm, buoyant layer extending over the cold receiving water. Concurrently, a cold, dense wedge is propagated under the warm body of water. Moreover, the initial symmetrical behaviour is mainly governed by the inertial force where the lock exchange Rayleigh number is sufficient to describe such behaviour. After the warm front has propagated for some distance, the buoyant layer attains the temperature of maximum density and then sinks vertically forming the thermal bar. Both experimental and numerical investigations provide the existence of an extremum in the dramatic proof that density-temperature relation has a profound influence on the behaviour of densimetric flows in general and lock exchange behaviour in particular. It is found that three zones of interest in the vicinity of a thermal bar are clearly demonstrated viz. (i) the thermal overflow region (warm convective cell), (ii) the thermal bar and (iii) the thermal underflow region (cold convective cell).

The experimental investigation was carried out for high values of Rayleigh numbers (up to  $10^8$ ) dictated by the need to use reasonable working parameters. Moreover, all of the experiments

employed a cold water temperature close to  $0^{\circ}$  C and it was difficult to control the Prandtl number throughout the experimental investigation. Consequently, the experimental relation for the maximum extension distance of the thermal bar was obtained as a function of the initial density difference between the cold and warm water.

On the other hand, the numerical investigation was carried out for smaller values of Rayleigh numbers (up to 10<sup>5</sup>) due to practical limitations on the available computational resources. To ensure similar behaviour, the numerical simulations were computed for the same flow regime for which the experiments had been carried out, i.e. convection dominated the flow patterns. The sensitivity of the maximum extension distance of the thermal bar to the range of the relevant parameters was examined. For the numerical results, sensitivity of the thermal bar location to the Prandtl number was found to increase with increasing Rayleigh number within the same flow regime. The sensitivity of the other parameters was not significantly different over the range of Rayleigh numbers explored.

Based on the experimental and numerical results, two empirical relations were found to be applicable for predicting the maximum penetration distance of the warm effluent according to the range of the Rayleigh number. The empirical relation based on the numerical results can be employed in a typical field application since the values of the governing parameters would be in the same range as the values used in the numerical solution. The empirical relation based on the experimental results can be employed for large values of the Rayleigh number. Both empirical relations show good correlation within the specified range of Rayleigh numbers.

In general, the investigations are sufficiently good to give a picture of the behaviour of the thermal density currents in general and lock exchange flows in particular due to the nonlinear density-temperature relation.

This study represents a preliminary stage of a more comprehensive study to develop a means of modeling the behaviour of a thermal density currents at the outfall of a steam electric generating station. Further investigations are needed which can be classified as follows.

I- Local extension of the investigation presented here.

This can be carried out either experimentally or numerically for the idealized model of the lock exchange mechanism. Smaller values of the Rayleigh numbers require to be employed in laboratory tests for two reasons. Firstly, the experiments must be within the typical field application ( $R_a = 10^4$  to  $10^6$ ). Secondly, the numerical simulation of these smaller values of Rayleigh numbers can be carried out with reasonable computational cost. Moreover, variable temperatures for the cold water allow the effect of cold Rayleigh number to be examined. To achieve laboratory tests with such requirements, sophisticated equipment may be needed. It is worth mentioning that recently available micro-computers can provide large storage where the numerical solution for high values of Rayleigh

number can be obtained with computational cost playing a secondary role. The sensitivity analysis shows that it is usefull to define the Prandtl number as a function of temperature instead of using a constant value which can incur an error of up to 7%. Moreover, the large amount of numerical results obtained in this investigation will be usefull to determine the limits of the different mechanisms responsible for achieving a fixed location of the thermal bar and also provide the basic data for the scaling analysis of such behaviour.

2- Global extension of the investigation presented here.

The numerical model was employed for the idealized case of the thermal density currents where simplifying assumptions were made. Examples of these are the absence of the effect of external forces such as the heat exchange between the water and air, bottom friction and wind stresses. This indicates the need for a more sophisticated turbulence model to better represent the spatial distribution of turbulent diffusion instead of the assumption of laminar molecular diffusion. The numerical model can be modified to simulate the actual prototype situation where the thermal currents are dependent on the discharge charactrestics and the hydrologic and meteorologic conditions prevailing in the receiving water bodies.

APPENDIX I

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LISTING OF COMPUTER PROGRAMME

PROGRAM THRDMC (OUTPUT, TAPE6=OUTPUT, TAPE10, TAPE20, TAPE30) PURPOSE : Solving system (set) of coupled (simultaneous) : partial differential equations. SPECIFIED PARAMETERS : PR,RA,DX,DY,NX,NY,TOL INPUTS : TAPE10, TAPE20 OUTPUT : TAPE10 DIFINITION : PR = the Prandtl number RA = the Rayleigh number DX. = horizontal mesh increament DY = vertical mesh increament NX = number of nodes in X-direction NY = number of nodes in Y-direction TOL = allowable tollerance TAPE10 = sparse matrix package file containing the restart system TAPE20 = data file containing the initial or previous values of the variables;U(I,J)\* V(I,J), PSI(I,J), OMEGA(I,J), THETA(I,J) OTHER DECKS REQUIRED : ENERGY, VORTCTY, STREAM, MOTION COMMON /VARIAB/ THETA(21,11),U(21,11),PSI(21,11), 1 . . OMEGA(21,11),V(21,11) COMMON /INVERT/ A(21),B(21),TRANS(21),BETA(21), 1 C(21),D(21),GAMMA(21) COMMON /PARAMT/ RA,NX,DX,RX,NX1,NX2,RXX,DX2,DX4,DX52, 1 PR.NY,DY,RY,NY1,NY2,RYY,DY2,DY4,DYS2,DTAUS COMMON /WORKAR/ WORK1(21,11), WORK2(21,11) COMMON /SPKUSR/ 'MSGLVL, IERR, MAXS, NEQNS REAL \$(2242) Specified parameters PR = 11.6 RA = 1000.0TOL = 1.0E - 4DX = 0.10

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= 0.10 DY NX = 21 NY = 11 = 2.0\*DX DX2 DY2 = 2.0\*DY DX4  $= 4.0^{+}DX$ DY4 = 4.0\*DY DXS = DX\*\*2 DYS = DY\*\*2 DXS2 = DX\*DX2 DYS2  $= DY^*DY2$ . NX1 = NX - INY1 = NY-1NX2 = NX-2NY2 = NY-2 С C C Start sparse package package Ĉ ...... ..... С CALL SPRSPK MAXS = 2242 . MSGLVL = 0с' C C \* Restart system \* Ċ С CALL RESTRT (10,S) С С С Print statistics \* \*\*\*\*\*\*\*\*\*\*\*\* С С CALL PSTATS С С С \* Read the values of all variables \* С \* С REWIND 20 READ(20,\*) ICOUNT, ITERAT, TAUS, DTAUS, DTAUO DO 1 I=1,NX DO 1 J=1,NY READ(20,\*) U(I,J),V(I,J),PSI(I,J),OMEGA(I,J),THETA(I,J) CONTINUE 1 REWIND 30 DO 2 I=1,NX DO 2 J=1,NY

C	2	READ(20,*) UO,VO WRITE(30,*) UO,VO CONTINUE
		* Outer iterations * -
	3	DO 15 L=1.10 WRITE(6.330) ICOUNT.ITERAT.TAUS WRITE(6.340) DTAUS RX = DTAUS/DX RY = DTAUS/DY RXX = RX/DX RYY = RY/DY DTAUR = DTAUS/DTAUO REWIND 30 DO 4 I=1.NX DO 4 J=1.NY READ(30.*) UD.VO
	•	* Linear extrapolation technique *
	4	U(I,J) = UO+DTAUR*(U(I,J)-UO) V(I,J) = VO+DTAUR*(V(I,J)-VO) CONTINUE REWIND 30 DO 5 I=I,NX
	5	DO 5 J=1,NY WRITE(30,*) OMEGA(1,J),U(1,J),V(1,J) CONTINUE CALL ENERGY CALL VORTCTY CALL STREAM (S) CALL MOTION
CCCCC		* Inner iteration *
		DO 9 K=1,3 ERRMAX = -1.0E10 VALMAXI = -1.0E10 VALMAX2 = -1.0E10 VALMAX3 = -1.0E10 ITERAT = ITERAT+1 REWIND 30

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DO 6 I=1,NXDO 6 J=1.NY WORK2(I,J) = OMEGA(I,J)READ(30,\*) OMEGA(I,J).,UP,VP С 0000 Averaging technique \* \*\*\*\*\*\*\*\*\*\*\* U(I,J) = (U(I,J)+UP)/2.0V(I,J) = (V(I,J)+VP)/2.06 CONTINUE REWIND 30 DO 7 I=1,NX DO 7 J=I,NY ... WRITE(30,\*) OMEGA(I,J),U(I,J),V(I,J) 7 CONTINUE CALL VORTCTY CALL STREAM (S) CALL MOTION DO 8 I=I,NX DO 8 J=1,NY VALABSI = ABS(WORK2(I,J)) VALABS2 = ABS(U(I,J))VALABS3 = ABS(V(I,J)ERRABS = ABS(OMEGA(I,J)-WORK2(I,J)) VALMAX1 = AMAX1(VALMAX1,VALABS1) VALMAX2 = AMAX1(VALMAX2,VALABS2) VALMAX3 = AMAX1(VALMAX3,VALABS3) ERRMAX = AMAX1(ERRMAX,ERRABS) 8 CONTINUE С C C Percentage of error \* С -----C POE = ERRMAX/VALMAX1 WRITE(6,350) K,POE WRITE(6,360) VALMAX2, VALMAX3 IF (POE.LE.TOL) GO TO 12 9 CONTINUE С C C C Repeat the time step \* С DTAUS = 0.50\*DTAUS REWIND 20 READ(20,\*) IA,IA,AA,AA,AA

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I=1,NX DO 10 DO 10 "J=1,NY READ(20,\*) U(I,J),V(I,J),PSI(I,J),OMEGA(I,J),THETA(I,J) 10 CONTINUE REWIND 30 DO 11 \_ I=1,NX DO 11 J=1,NY READ(20,\*) UO,VQ WRITE(30,\*) UO,VO 11 CONTINUE GO TO 3 12 TAUS = TAUS +DTAUS ICOUNT = ICOUNT + 1DTAUO' = DTAUS,С С С \* Variable time step with step size control techni .C .......... С DTAUS = (1.0+(5-K)/10.0)\*DTAUS REWIND 20 REWIND 30 READ(20,\*) IA, IA, AA, AA, AA DO 13 . I=1,NX DO 13 J=1,NY READ(20,\*) UO, VO, PSIO, OMEGAO, THETAO WRITE(30,\*) UO,VO 13 CONTINUE С С С • Store the solution at the end of time step С С REWIND 20 WRITE(20,\*) ICOUNT, ITERAT, TAUS, DTAUS, DTAUO DO 14 I=J,NX DO 14 J=1,NY WRITE(20,\*) U(I,J),V(I,J),PSI(I,J),OMEGA(I,J),THETA(I,J) 14 CONTINUE С Ç С Solve for another time step • ¢ С 15 CONTINUE REWIND 30 DO 16 I=1,NX DO 16 J=1,NY READ(30,\*) UO,VO

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WRITE(20,\*): U0,VO 16 CONTINUE С С С С Print the required informations С WRITE(6,330) ICOUNT, ITERAT, TAUS WRITE(6,200) DO 17 I=1,NX WRITE(6,250) I, (THETA(I,J), J=1,NY) 17 WRITE(6,210) DO 18 I=1,NX 18 WRITE(6,250) 1,(OMEGA(I,J),J=1,NY) WRITE(6,220) DO 19 I=1,NX 19 WRITE(6,250) I, (" PSI(I,J), J=1, NY) WRITE(6,230) DO 20 I=1,NX 20 WRITE(6,250) I.( U(I,J), J=1,NY)WRITE(6,240) DO 21 I=1,NX WRITE(6,250).1.( 21 V(I,J), J=1,NYFORMAT(1H1,//,40X, 'THE TEMPERATURE DISTRIBUTION, THETA', ... 200 - 77) 210 FORMAT(1H1,//,40X, 'THE VORTICITY DISTRIBUTION, OMEGA', 11) FORMAT(1H1,//,40X, 'THE STREAM FUNCTION DISTRIBUTION, PSI', 220 11) FORMAT(1H1,//,40X, 'THE HORIZONTAL VELOCITY DISTRIBUTION, U',//) -230 FORMAT(1H1.//.40X, 'THE VERTICAL VELOCITY DISTRIBUTION, V', //) 240 250 FORMAT(1X,13,1X,11E12.4) 330 FORMAT(41X, 15, 9X, 15, 12X, E10.4) FORMAT(87X,E10.4) 340 .350 FORMAT(110X,12,9X,E10.4) 360 FORMAT(4X,E10\_4,5X,E10\_4) STOP

END

	: Solution of the heat-transfer equation
METHOD	: Alternating direction inplicit, ADI, method.
GENERAL	<ul> <li>The function derivatives are replaced by the finite-difference approximations where :</li> <li>(A) The unsteady term is expressed by the forward-difference approximation.</li> <li>(B) The advection terms are expressed by the second-upwind approximation.</li> <li>(C) The diffusion terms are expresses by the central-difference approximation.</li> </ul>
PROCEDURE	<ul> <li>(1) Advance the solution in the X-direction from the begining to intermediate value of the time step. Satisfy the boundary conditions on the lift and right sides of the working domain.</li> <li>(2) Advance the solution in the Y-direction from the intermediate to the end value of the time step. Satisfy the boundary conditions on the lower and upper sides the working domain.</li> </ul>
INPUTS .	: RX,RY,RXX,RYY,NXI,NYI,U(I,J),V(I,J),THATA(I,
OUTPUTS	: THETA(I,J)
	RX = DTAU/DX ; RY = DTAU/DY
DIFINITION .	<pre>RXX = DTAU/(DX*DX); RYY = DTAU/(DY*C NX1 = NX-1 ; NY1 = NY-1 U(I,J) = horizontal component of velocity V(I,J) = vertical component of velocity THETA(I,J) = temperature where :</pre>
DIFINITION	<pre>RXX = DTAU/(DX*DX); RYY = DTAU/(DY*C NX1 = NX-1 ; NY1 = NY-1 U(I,J) = horizontal component of velocity V(I,J) = vertical component of velocity THETA(I,J) = temperature where : NX = number of nodes in X-directic NY = number of nodes in Y-directic DX = horizontal mesh increament DY = vertical mesh increament DTAU = time increament I,J = location of node over domain</pre>

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C(1) = -0.0IF(J.NE.2) GO TO 4 3 С С С Satisfy the boundary condition on the lower side C \*\*\*\*\*\*\*\*\*\*\*\*\*\* G D(I) = +(RYY+RY\*FD\*VD)\*(4.0\*THETA(1,J)-THETA(1,J+1))/3.01 +(2.0-2.0\*RYY-RY\*FU\*VU+RY\*(1.0-FD)\*VD)\*THETA(I.J) 1 +(RYY-RY\*(1.0-FU)\*VU)\*THETA(1,J+1) GO TO 6 4 IF(J.EQ.NY1) GO TO 5 D(I) = +(RYY+RY\*FD\*VD)\*THETA'(I,J-1)+(2.0-2.0\*RYY-RY\*FU\*VU+RY\*(1.0-FD)\*VD)\*THETA(I.J) 1 +(RYY-RY\*(1.0-FU)\*VU)\*THETA(1,J+1) 1 GO TO 6 С С C \* Satisfy the boundary condition on the upper side č \*\*\*\*\*\*\* \*\*\*\*\*\*\*\*\* С 5. D(I) = +(RYY+RY\*FD\*VD)\*THETA(I,J-I)+(2.0-2.0\*RYY-RY\*FU\*VU+RY\*(1.0-FD)\*VD)\*THETA(I,J) 1 +(RYY-RY\*(1.0-FU)\*VU)\*(4.0\*THETA(1,J)\_THETA(1,J-1))/3.0 1 6 CONTINUE С С С Solve the tridiagonal matrix along row С С CALL TRIDAG (2,NXI) С С С \* Store the intermediate solution in the working array WORKI \* С С DO 7 I=2,NX1 WORK1(I,J) = TRANS(I)CONTINUE С C С \* Solve for another row \* С С 8 CONTINUE C С С \* Advance the solution in the Y-direction С

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С DO 16 I=2,NX1 DO 14 . J=2,NY1 UL = (U(I,J)+U(I-1,J))/2.0UR = (U(I,J)+U(I+I,J))/2.0 $\dot{V}D = (V(I,J)+V(I,J-I))/2.0$ VU = (V(I,J)+V(I,J+1))/2.0FL = UPWIND(UL)FR = UPWIND(UR)FD = UPWIND(VD)FU = UPWIND(VU)IF(J.NE.2) GO TO 9 С C C Satisfy the boundary condition on the lower side \* Ĉ С A(J) = -0.0B(J) = +(2.0+2.0\*RYY+RY\*FU\*VU-RY\*(1.0-FD)\*VD) --(RYY+RY\*FD\*VD)\*(+4.0/3.0) 1  $C(J) = -(RYY-RY^*(1_0-FU)^*VU)$ 1 -(RYY+RY\*FD\*VD)\*(-1.0/3.0) GO TO 11 9 IF(J.EQ.NY1) GO TO 10 A(J) = -(RYY+RY\*FD\*VD)B(J) = +(2.0+2.0\*RYY+RY\*FU\*VU-RY\*(1.0-FD)\*VD)  $C(J) = -(RYY - RY^*(I \cdot O - FU)^*VU)$ GO TO 11 С С С Satisfy the boundary condition on the upper side • С \*\*\*\*\*\*\*\*\*\*\*\*\* С  $10 \quad A(J) = -(RYY+RY*FD*VD)$ 1 -(RYY-RY\*(1.0-FU)\*VU)\*(-1.0/3.0) B(J) = +(2.0+2.0\*RYY+RY\*FU\*VU-RY\*(1.0-FD)\*VD)-(RYY-RY\*(1.0-FU)\*VU)\*(+4.0/3.0) 1 C(J) = -0.011 IF(I.NE.2) GO TO 12 С С С Satisfy the boundary condition on the lift side \* С D(J) = +(RXX+RX\*FL\*UL)\*(4.0\*WORK1(I,J)-WORK1(I+1,J))/3.0 +(2.0-2.0\*RXX-RX\*FR\*UR+RX\*(1.0-FL)\*UL)\*WORKI(I,J) 1 1 +(RXX-RX\*(1.0-FR)\*UR)\*WORKI(I+1.J) GO TO 14 12 IF(I.EQ.NX1) GO TO 13

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D(J) = +(RXX+RX*FL*UL)*WORK1(I-1,J)
     1
             +(2.0-2.0*RXX-RX*FR*UR+RX*(1.0-FL)*UL)*WORKI(1.J)
     1
             +(RXX-RX*(1.0-FR)*UR)*WORK1(I+1,J)
      GO TO 14
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С
      * Satisfy the boundary condition on the right side *
С
          ........
С
  13 D(J) = +(RXX+RX*FL*UL)*WORK1(I-I,J)
             +(2.0-2.0*RXX-RX*FR*UR+RX*(1.0-FL)*UL)*WORK1(I,J)
     1
     l
             +(RXX-RX*(1.0-FR)*UR)*(4.0*WORK1(I,J)-WORK1(I-1,J))/3.0
     CONTINUE
  14
C
С
¢
        Solve the tridiagonal matrix along column *
С
С
      CALL TRIDAG (2,NYI)
С
С
С
      * Store the final solution in the array THETA'*
С
        С
      DO 15 J=2,NY1
      THETA(I,J) = TRANS(J)
  15 - CONTINUE
С
С
¢
       Solve for another column
С
С
     CONTINUE
  16
С
С
С
      • Update the values of THETA on the lift and right sides
С
С
      DO 17 · I=1,NX
      THETA(1,1) = (4.0*THETA(1,2)-THETA(1,3))/3.0
      THETA(I,NY) = (4.0*THETA(I,NY1)-THETA(I,NY2)/3.0
  17
      CONTINUE
С
С
С
        Update the values of THETA on the lower and upper sides '
С
С
      DO 18
               J=1,NY
      THETA(1,J) = (4.0^{THETA}(2,J) - THETA(3,J))/3.0
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## SUBROUTINE VORTCTY

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• • PURPOSE	: Solution of the vortigity transport equation
* * METHOD	: Alternating direction implicit, ADI, method.
GENERAL	<ul> <li>The function derivatives are replaced by the finite-difference approximations where :</li> <li>(A) The unsteady term is expresses by the forward-difference approximation.</li> <li>(B) The advection terms are expressed by the second-upwind approximation.</li> <li>(C) The diffusion terms are expressed by the central-difference approximation.</li> <li>(D) The buoyant term is expressed by the central-difference approximation.</li> </ul>
PROCEDU	<ul> <li>RE : (1) Advance the solution in the X-direction from the begining to intermediate value of the time step. Satisfy the boundary conditions on the lift and right sides of the working domain.</li> <li>(2) Advance the solution in the Y-direction from the intermediate to the end value of the time step. Satisfy the boundary conditions on the lower and upper sides of the working domain.</li> </ul>
INPUTS	<pre>PR,RA,RX,RY,RXX,RYY,NX1,NY1,U(I,J),V(I,J), THETA(I,J),OMEGA(I,J)</pre>
• OUTPUTS	: OMEGA(I,J)
• DIFINIT	<pre>ION : PR = THE PRANDTL NUMBER RA =-THE RAYLEIGH NUMBER RX = DTAU/DX ; RY = DTAU/DY RXX = DTAU/(DX*DX) ; RYY = DTAU/(DY*DY) NXI = NX-1 ; NYI = NY-1 U(I,J) = horizontal component of velocity V(I,J) = vertical component of velocity THETA(I,J) = temperature OMEGA(I,J) = vorticity* where :</pre>
•	NX = number of nodes in X-direction * NY = number of nodes in Y-direction * DX = horizontal mesh increament * DY = vertical mesh increament *

DTAU = time increament I,J = location of node over domain OTHER DECKS REQUIRED : TRIDAG, UPWIND COMMON /VARIABY THETA(21,11),U(21,11),PSI(21,11), 1 OMEGA(21,11),V(21,11) COMMON /INVERT/ A(21),B(21),TRANS(21),BETA(21), 1 C(21),D(21),GAMMA(21) COMMON /PARAMT/ RA,NX,DX,RX,NX1,NX2,RXX,DX2,DX4,DXS2, 1 PR,NY,DY,RY,NY1,NY2,RYY,DY2,DY4,DYS2,DTAUS COMMON /WORKAR/ WORK1(21,11),WORK2(21,11) Advance the solution in the X-direction DO 5 J=2,NY1 DO 3 I=2,NX1 UL = (U(I,J)+U(I-1,J))/2.0UR = (U(I,J)+U(I+1,J))/2.0VD = (V(I,J)+V(I,J-1))/2.0VU = (V(I,J)+V(I,J+1))/2.0FL = UPWIND(UL) FR = UPWIND(UR) FD = UPWIND(VD)FU = UPWIND(VU)IF(I.NE.2) GO TO I \* Satisfy the boundary condition on the lift side A(I) = -0.0B(I) = +(2.0+2.0\*PR\*RXX+RX\*FR\*UR-RX\*(1.0-FL)\*UL)  $C(I) = -(PR*RXX-RX*(I_0-FR)*UR)$ D(I) = +(PR\*RYY+RY\*FD\*VD)\*OMEGA(I,J-1) +(2.0-2.0\*PR\*RYY-RY\*FU\*VU+RY\*(1.0-FD)\*VD)\*OMEGA(I,J) +(PR\*RYY-RY\*(1.0-FU)\*VU)\*OMEGA(1.J+1) -0.5\*PR\*RA\*RX\*(THETA(I+1,J)\*\*2.0-THETA(I-1,J)\*\*2.0) +(PR\*RXX+RX\*FL\*UL)\*OMEGA(I-1,J) GO TO 3 IF(I.EQ.NX1) GO TO 2  $A(I) = -(PR^*RXX+RX^*FL^*UL)$ B(1) = +(2.0+2.0\*PR\*RXX+RX\*FR\*UR-RX\*(1.0-FL)\*UL) C(I) = -(PR\*RXX-RX\*(I.0-FR)\*UR)

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	•	D(I) = +(PR*RYY+RY*FD*VD)*OMEGA(I,J-1) 1 +(2.0-2.0*PR*RYY-RY*FU*VU+RY*(1.0-FD)*VD)*OMEGA(I,J) 1 +(PR*RYY-RY*(1.0-FU)*VU)*OMEGA(I,J+I) 1 -0.5*PR*RA*RX*(THETA(I+1,J)**2.0-THETA(I-1,J)**2.0) GO TO 3	- 
C C	•	******	
	•	* Satisfy the boundary condition on the right side *	
	2 3	A(I) = -(PR*RXX+RX*FL*UL) B(I) = +(2.0+2.0*PR*RXX+RX*FR*UR-RX*(1.0-FL)*UL) C(I) = -0.0 D(I) = +(PR*RYY+RY*FD*VD)*OMEGA(I,J-1) 1 +(2.0-2.0*PR*RYY-RY*FU*VU+RY*(1.0-FD)*VD)*OMEGA(I,J) 1 +(PR*RYY-RY*(1.0-FU)*VU)*OMEGA(I,J+1) 1 -0.5*PR*RA*RX*(THETA(I+1,J)**2.0-THETA(I-1,J)**2.0) 1 +(PR*RXX-RX*(1.0-FR)*UR)*OMEGA(I+1,J) CONTINUE	· )
		*********	
		* Solve the tridiagonal matrix along row *	
		CALL TRIDAG (2,NX1)	
Ċ		***************************************	•
000	-	* Store the intermediate solution in the working array WORK!	*
	4.	DO 4 1=2,NX1 WORK1(1,J) = TRANS(1) CONTINUE	1
č		***************************************	
		* Solve for another row *	
	5	CONTINUE	
		***************************************	
		* Advance the solution in the Y-direction *	
Ç		DO 10 1=2,NX1	
		DO 8 $J=2,NY1$	
	•	UR = (U(I,J)+U(I-1,J))/2. UR = (U(I,J)+U(I+1,J))/2. VD = (V(I,J)+V(I,J-1))/2.	-

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VU = (V(I,J)+V(I,J+1))/2.FL = UPWIND(UL)FR = UPWIND(UR) FD = UPWIND(VD)FU = UPWIND(VU)IF(J.NE.2) GO TO 6 \* Satisfy the boundary condition on the lower side \* A(J) = -0.0B(J) = +(2.0+2.0\*PR\*RYY+RY\*FU\*VU-RY\*(1.0-FD)\*VD) C(J) = -(PR\*RYY-RY\*(1.0-FU)\*VU) $D(J) = \pm (PR*RXX+RX*FL*UL)*WORK1(I-I,J)$ ~ +(2.0-2.0\*PR\*RXX-RX\*FR\*UR+RX\*(1.0-FL)\*UL)\*WORK1(I,J) 1 +(PR\*RXX-RX\*(1.0-FR)\*UR)\*WORK1(I+1,J) 1 -0.5\*PR\*RA\*RX\*(THETA(I+1,J)\*\*2.0-THETA(I-1,J)\*\*2.0) 1-+(PR\*RYY+RY\*FD\*VD)\*OMEGA(I.J-1) GO TO 8 6 IF(J\_EQ\_NY1) GO TO 7 A(J) = -(PR\*RYY+RY\*FD\*VD)B(J) = +(2.0+2.0\*PR\*RYY+RY\*FU\*VU-RY\*(1.0-FD)\*VD)C(J) = -(PR\*RYY-RY\*(1.0-FU)\*VU)D(J) = +(PR\*RXX+RX\*FL\*UL)\*WORK1(I-1,J)+(2.0-2.0\*PR\*RXX-RX\*FR\*UR+RX\*(1.0-FL)\*UL)\*WORK1(1,J) +(PR\*RXX-RX\*(1.0-FR)\*UR)\*WORK1(I+1,J) ţ. -0.5\*PR\*RA\*RX\*(THETA(I+1,J)\*\*2.0-THETA(I-1,J)\*\*2.0) . 1 GO TO 8 \* Satisfy the boundary condition on the upper side \* \*\*\*\*\*\*\*\*\*\* A(J) = -(PR\*RYY+RY\*FD\*VD)7 B(J) = +(2.0+2.0\*PR\*RYY+RY\*FU\*VU-RY\*(1.0-FD)\*VD) C(J) = -0.0 $D(J) = +(PR^*RXX+RX^*FL^*UL)^*WORK1(I-I,J)$ +(2.0-2.0\*PR\*RXX-RX\*FR\*UR+RX\*(1.0-FL)\*UL)\*WORKI(1,J) +(PR\*RXX-RX\*(1.0-FR)\*UR)\*WORK1(I+1,J) -0.5\*PR\*RA\*RX\*(THETA(I+1,J)\*\*2.0 -THETA(I-1,J)\*\*2.0) +(PR\*RYY-RY\*(1\_0-FU)\*VU)\*OMEGA(1,J+1) 8 CONTINUE \* Solve the tridiagonal matrix along column \* CALL TRIDAG (2,NYI)

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Store the final solution in the array OMEGA \* 7 DO 9 J=2,NY1 OMEGA(I,J) = TRANS(J)9

CONTINUE

\* Solve for another column \*

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CONTINUE 10 RETURN END

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SUBROUTINE STREAM (S)

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: Solution of the stream-function equation.
   PURPOSE
   METHOD

    Sparse matrix package.

               : The sparse matrix package is initialized in
   GENERAL
                 main program. In this subroutine, the R.H.S.
                 of the linear system will be specified and the
                 corresponding solution is obtained.
   INPUTS ·
               : NX1,NY1,S(MAXS),OMEGA(I,J)
   OUTPUT
               : PSI(I,J)
   DIFINITION
               : NX1
                            = NX-1; NY1 = NY-1
                 S(MAXS)
                            = array initialized in the main
                              program
                 PSI(I,J)
                            = stream-function
                 OMEGA(I,J) = vorticity
                   where :
                          NX
                               = number of nodes in X-direction
                          NY
                               = number of nodes in Y-direction
                          I,J = location of node over domain
                          MAXS = size of storage required by
                                 the sparse matrix package
  OTHER DECKS
    REQUIRED : SPARSE MATRIX PACKAGE
COMMON /VARIAB/ THETA(21,11),U(21,11),PSI(21,11),
1
                  OMEGA(21,11),V(21,11)
COMMON /PARAMT/
                 RA,NX,DX,RX,NX1,NX2,RXX,DX2,DX4,DX52,
1
                  PR,NY,DY,RY,NY1,NY2,RYY,DY2,DY4,DYS2,DTAUS
COMMON /SPKUSR/
                 MSGLVL, IERR, MAXS, NEONS
REAL
                  S(2242)
       K = 1
DO 2
       I = 2,NX1
DO 1
       J = -2, NY1
         = OMEGA(I,J)
VALUE
  Input to R.H.S. of the linear system *
CALL INBI (K,VALUE,S)
```

		K = K+1
	1	CONTINUE
_	2	CONTINUE -
C C		*****
č		• Celus deture *
		- Solve driver -
C		**************
С		
		CALL SOLVE3 (S)
		K = 1
		DO 4 $I = 2, NX1$
		DO 3 J = 2.NY1
		PSI(I,J) = S(K)
		K = K+1
	З.	CONTINUE
	4	CONTINUE
		RETURN

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END
SUBROUTINE MOTION С С С С PURPOSE : Solution of the velocity-field equation. С С : Central-difference method. METHOD С t INPUTS : DX2,DY2,NX1,NY1,PSI(I,J) ~ C С OUTPUT : U(I,J),V(I,J) C C \* = 2.0\*DX DIFINITION : DX2 DY2 = 2.0\*DYC NX1 = NX - INY1 = NY-1; С = horizontal component of velocity U(I,J)/ C V(I,J)= vertical component of velocity С PSI(I,J) = stream-function С where : C C = number of nodes in X-direction NX -NY = number of nodes in Y-direction С DX = horizontal mesh increament С DY = vertical mesh increament ¢ I,J = location of node over domain C C C COMMON /VARIAB/ THETA(21,11),U(21,11),PSI(21,11), 1 OMEGA(21,11),V(21,11) COMMON /PARAMT/ RA,NX,DX,RX,NX1,NX2,RXX,DX2,DX4,DXS2, 1 PR,NY,DY,RY,NY1,NY2,RYY,DY2,DY4,DYS2,DTAUS С С С Calculation of the interior velocity field " С С DO 2 I=2,NX1 DO 1 J=2,NYI U(I,J) = -(PSI(I,J+I)-PSI(I,J-I))/DY2V(1,J) = +(PSI(1+1,J)-PSI(1-1,J))/DX2CONTINUE 1 2 CONTINUE С C C Calculation of velocities over the horizontal boundaries С С DO 3 I=2,NX1 · U(I,1) = -2.0\*PSI(I,2)/DY2 U(I,NY) = +2.0\*PSI(I,NYI)/DY2

# 3 CONTINUE \* Calculation of velocities over the vertical boundaries DO 4 J=2,NY1 V(1,J) = +2.0\*PSI(2,J)/DX2 V(NX,J) = -2.0\*PSI(NX1,J)/DX2

CONTINUE RETURN END

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· SUBROUTINE TRIDAG (IF, IL)
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	************	*******						
• PURPOSE	: Solution of the	tridiagonal matrix. *						
	🗧 Guassian elimina	uassian elimination method. *						
INPUTS	: IF,IL,A(I),B(I)	.C(I).D(I)						
	: TRANS(I)	•						
DIFINITION	: IF IL A(I),B(I),C(I) D(I) TRANS(I) BETA(I),GAMMA(I) where :	<pre>= number of first equation = number of last equation = array containing tridiagonal matrix coefficients = array containing the given R.H.S. of linear equations = array containing solution of linear equations = working arrays I = the number of equations *</pre>						
COMMON /INVER I BETA(IF) = B GAMMA(IF) = D IFPI = IF+1 DO 1 I=IFP BETA(I) = B( GAMMA(I) = (D CONTINUE TRANS(IL) = G LAST = IL-IF DO 2 K=1,LA I = IL-K TRANS(I) = GA CONTINUE RETURN END	<pre>RT/ A(21),B(21),TR/ C(21),D(21),GAN H(IF) D(IF)/BETA(IF) P1,IL I)-A(I)*C(I-1)/BETA D(I)-A(I)*GAMMA(I-1) GAMMA(IL) ST MMA(I)-C(I)*TRANS()</pre>	ANS(21).BETA(21), MMA(21) A(I-1) ))/BETA(I) I+1)/BETA(I)						

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	• PURPOSE	: Test for	the signs of t	the velocities	for the
	•	use of t	he second-upwir	nd method.	•
	**********	********	***********	***********	*********
	IF(X.GE.0.0) UPWIND = 0.0	GO TO 1	<b>?</b>		
	GO TO 2		>		
1	UPWIND = 1.0			•	
2	CONTINUE RETURN END				

## APPENDIX II

## EXPERIMENTAL TEMPERATURE MEASUREMENTS

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Table II!

Temperature measurements for experiment # 1

L/H h/H	5.0	35.0	37.5	40.0	41.5	37.5	35.0	65.0
0.05	10 <b>.</b> 2	12.2	10.1	8.8	3.5	7.4	9.7	0.1
0.15	15.0	3.7	3.8	3.1	1.4	2.2	8.0	- 0.0
0.25	12.9	2.8	2.9	. 2.5	.1.4	2.6	6.3	0.2
0. <del>3</del> 5	5.4	1.6	1.8	1.6	0.8	2.2	4.7	. 0.3
0.45	1.7	1.1	0.9	0.7	0.4	2.2	3.8	0.6
0.55	0.6	0.9	0.7	0.4	0.4	2.1	3.5	1.1
0.65	0.6	0.9	0.6	0.3	0.5	2.2	3.4	1.6
0.75	0.8	0.7	0.5.	0:3	0.8	2.5	3.3	2.0
0.85	1.0	0.7	0.6	0.3	1.0	2.6	3.3	2.2
0.95	1.5	0.9	0.4	0.3	1.4	2.8	3.3	2.3

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Note : h = depth measured from surface

L = distance measured from barrier

H = total depth of water

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Each column describes a temperature profile measured in degrees celsius at increasing values of elapsed time. Decreasing values of L/H imply measurement some distance behind the front.

				11	<u> </u>			
L/H h/H	15.0	25.0 •	37.5	40.0	25.0	25.5	26.0	28:5
0.05	10.7	8.8	0.3	0.3	6.9	8.4	8.0	0.4
0.15	6.1	5.3	0.3	0.3	4.8	5.4	2.8	0.5
0.25	4.2	3.4	0.3	0.3	4.4	3.8	3.0	· 0.9
0.35	2.4	2.5	0.3	0.3	4.1	3.0	2.7	1.2
0.45	2.0	2.1	. 0.4	0.4	3.3	3.0	2.8	2.0
0.55	1.5	1.8	0.8	0.6	3.2	3.1	3.0	2.6
0.65	0.9	. 1.7	1.3	0.9	2.8	3.1	3.1	2.7
0.75	0.8	1.5	1.8	.1.5	3.0	3.0	3.1	3.0
0.85	1.3	1.3	1.9	1.7	3.3	3.1	3.2	3.1
0.95	1.7	1.3	2.0	1.8	3.3	3.3	3.3	-3.1
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Temperature measurements for experiment # 2

Table II.2

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Note : h = depth.measured from surface

L = distance measured from barrier

H = total depth of water

Each column describes a temperature profile measured in degrees celsius at increasing values of elapsed time. Decreasing values of L/H imply measurement some distance behind the front.

Temperature measurements for experiment # 3

			_						
	L/H h/H	17.0	20.0	22.5	25.0	30.0	22.5	20.0	15.0
	0.05	11.0	10.1	4.8	0.9	0.2	0.3	4.7	8.1
•	0.15	9.0	6.8	2.0	0.7	0.2	0.3	1.5	8.0
	0.25	6.9	4.7	1.8	0.9	0.3	0.4	1.0	7.4
	0.35	3.3	2.8	1-1	0.8	0.3	0.7	1.2	6.2
	0.45	2.4	. 1.8	0.9	0.7	0.4	1.3	1.8	5.1
	0.55	2.1	1.6	0.8	0,9	0.7	1.8	2.1	3,9
	0.65	1.9	1.5	0.9	1.2	1.1	2.1	2.3	3.8
	0.75	1.8	1.6	1.4	1.8	1.6	2.6	2.6	3.8
	0.85	1.8	1.8	1.6	2.0.	1.8	2.7	2.7	3.7
	0.95 .	118	1.8	1.6	1.9	1.8	2.8	2.8	3.8
				_					

Note : h = depth measured from surface

L = distance measured from barrier

H = total depth of water

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Each column describes a temperature profile measured in degrees celsius at increasing values of elapsed time. Decreasing values of L/H imply measurement some distance behind the front.

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Temperature measurements for experiment # 4

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. L/H . h/H	10.0	17.5	22.5	25.0	22.5	20.0	15.0	25.0
0.05	10.2	10.8	<b>-</b> 5.1	4.0	7.4	7.6	7.9	11
0.15	5.7	9.4	2.6	2.7	2.9	3.6	7.8	0.8
0.25	2.4	5.3	2.6	2.0	3.0	2.8.	_ 5.8	0.9
0.35	1.9	3.1	1.9	1.6	2.4	2.3	4.5	0.8
0.45	1.3	. 2.6	1.4	1.1	1.8	2.4	4.0	1.5
0.55	0.9	2.0	1.2	1.1	, 1.8	2.7	3.5	2.2
0.65	0.9	1.9	1.0	0.9	1.8	2.8	3.4	2.7
0.75	0.9	1.7	0.9	0.9	2:3	2.7	3.3	3.0
0.85	1.3	1.7	0.9	0.9	2.5	2.9	3.3	3.2
0.95	1.9	2.0	0.9	1.0	2.7	3.1	3.3	3.1

Note : h = depth measured from surface

L = distance measured from barrier

H = total depth of water

Each column describes a temperature profile measured in degrees celsius at increasing values of elapsed time. Decreasing values of L/H imply measurement some distante behind the front.

Temperature measurements for experiment # 5

			_	•	• ••			
L/H h/H	8.3	8.3	10.0	16.7	23.3	26.7	10.7	3.3
0.05	9.4	9.6	9.8	6.8	0.5	0.3	8.4	7.4
0.15	. 8.4	0.017	6.5	1.7	0.3	0.2	7.0	6.3
0.25	4.1	5.8	4.1	0.6	0.3	0.3	2.5	2.5
0.35	2.5	3.2	2.6	0.3	0.2	0.2	2.1	2.4
0.45	111.	1.7	2.5	0.3	0.3	0.2	2.4	2.8
0.55	0.3	1.5	.1.2	.0.3	0.3	0.2	2.8	2.9
0.65	0_1	0.8	1.1	0.5	0.9	0.7	3.1	3.3
0.75	0.1	0.4	1.0	0.9	1.2	1.4	3.2	3.4
0.85	0.5	0.7	0.9	0.9	1.7	1.8	3.2	3.5
0.95	0.9	1.0	0.9	1.6	1.9	1.9	3.2	3.5
<u>I</u> <u> </u>								

Note : h = depth measured from surface

L = distance measured from barrier

H = total depth of water

Each column describes a temperature profile measured in degrees celsius at increasing values of elapsed time. Decreasing values of L/H imply measurement some distance behind the front.

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Temperature measurements for experiment # 6

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L/H h/H	16.7	16.7	23.3	30.0	36.7	43.3	43.3	30.0
0.05	15.1	16.8	17.0	15.9	16.0	2.8	0.4	9.2
0,15	9.4	17.1	17.0	16.0	11.6	1.0	0.6	5.1
0.25	3.4	8.2	8.1	9.4	4.8	0.9	0.9	4.1
0.35	1.8	2.8	3.5	4.9	. 2.8	1.0	1.2	3.6
0.45	0.8	1.7	2.2	3.0	1.8	1.2	1.6	3.2
0.55	0.3	1.6	1.8	2.1	1.5	1.4	1.8	3.0
0.65	0.2	1.8	1.6	1.8	1.4	1.7	2.0	3.1
0.75	0.2	1.5	1.6	1.7	1.5	1.7	2.1	C 3.1
0.85	0.3	1.7	1.8	1.7	1.6	1.8	2.2	3.1
0.95	0.3	2.0	1.7	1.8	1.8	1.8	2.3	3.1

Note : h = depth measured from surface

L = distance measured from barrier

H = total depth of water

Each column describes a temperature profile measured in degrees celsius at increasing values of elapsed time. Decreasing values of L/H imply measurement some distance behind the front.

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L/H 10.0 15.0 20.0 23.3 23.3 23.3 10.0 10.0 h/H 0.05 10.0 10.7 11.4 4.1 1.5 0.3 9.6 9.6 0.15 10.5 · 4.2 9.4 1.8 0.9 0.3 8.0 8.8 0.25 5.6 3.1 3.5 1.1 0.9 0.4 5.7 5.4 0.95 3.2 2.0 1.6 0.9 0.7 0.4 5.1 4.3 0.45 2.2 1.6 0.9 1.1 0.8 0.7 4.4 4.1 0.55 1.8 1.2 1.0 1.3 0.9 1.6 4.1 4.0 · 0.65 1.8 1.2 1.1 1.8 1.6 2.1 4.0 3.9 0.75 1.5 1.3 1.9 1.3 1.9 2.5 3.8 3.8 0.851 0.5 1.7 1.5 2.1 2.0 2.8 3.5 3.6 0.95 0.4 1.7 1.6 2.1 2.2. 2.9 3.5 3.5

Temperature measurements for experiment # 7

Note : h = depth measured from surface ,

L = distance measured from barrier

H = total depth of water

Each column describes a temperature profile measured in degrees celsius at increasing values of elapsed time. Decreasing values of L/H imply measurement some distance behind the front.

## APPENDIX III

## NOMENCLATURE

#### Dimensional Notations

9	=	gravitational acceleration
h	=	depth measured from surface
t	=	time
u	=	horizontal velocity
v	=	vertical velocity
×	=	horizontal coordinate
У	=	vertical coordinate
, P	=	, pressure
T	=	temperature .
L	÷	total length
н	z	total depth
W	=	total width
ß	=	thermal expansion coefficient
ĸ	=	thermal conductivity
ν	=0	kinematic viscosity
ρ	=	density
ψ	.=	stream function
ω	=	vorticity
Δ	=	increment/difference
$\nabla^2$	-	Lanlacian operator

## Dimensionless Notations

A	=	aspect ratio
F	=	function
к	=	initial velocity coefficient
U	=	horizontal velocity
٧	2	vertical velocity
x	=	horizontal coordinate
Y	=-	vertical coordinate
θ	=	temperature
τ	=	time
Ψ	2	stream function
Ω	=	vorticity
c,	=	overflow coefficient
F <sub>A</sub> .	=`	densimetric Froude number
FΔR	=	densimetric Froude-Reynolds number
Nu	=	Nusselt number
٩	=	Prandtl number
R <sub>a</sub>	=	Rayleigh number
(R <sub>a</sub> ) <sub>c</sub>	=	cold Rayleigh number
(R <sub>a</sub> ) <sub>w</sub>	=	warm Rayleigh number
(R <sub>a</sub> ) <sub>1</sub>	=	lock exchange Rayleigh number

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### Subscripts

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0	reference/initial
i	horizontal location
j	vertical location
W	warm water/upper layer
с	cold water/lower layer
а	receiving water bodies
I	heated effluent
0	defined at 0° C
4	defined at 4 <sup>0</sup> C
8	defined at 8° C
۵	densimetric
ave	average
max	maximum

### Superscripts

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n	time index
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k	iteration index

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