OPTIMAL DEPLOYMENT OF CONTROLLER-DETECTORS FOR THE HWR SYSTEM

by

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ABSTRACT

A major concern in the design of control systems for nuclear power reactors is where and how many controllers and detectors are to be deployed in the reactor to satisfy design criteria. In order to answer this concern, we have developed an analytical method in which emphasis is placed on the linear regulator theory and the least square estimation theory.

This work has four areas: realization of the measure of the optimality of the controller locations in terms of feedback gain in spatial control; calculation of the static set-points to compensate the excess reactivity of the reference state; evaluation of dynamic range of controllers for regulating neutron fluxes; and estimation of additional responses counteracting burn-up/fuelling induced random external disturbances.

The deviations of neutron, iodine and xenon distributions from the reference states were expanded with the referenced λ-modes. The order of amplitude vector space was reduced by assumption of the dominant mode concept. Performance indices were formed with reduced state vectors and separated control functions. Pontryagin's maximum principle was applied to deterministic components and the square-root filtering to stochastic components.

Problems were finally narrowed to solve a series of the algebraic matrix Lyapunov and Riccati equations whose solutions imply a linear transformation of adjoints to state vectors.

A computer code ODZCR was developed for designing CANDU zone control systems using the above theories. Analysis of the existing 600 MWe CANDU zone control compartments in terms of their locations and numbers led to the following conclusions. The effective region for both spatial and bulk control was very limited and, hence, the current vertical compartments occupied the region most effectively: also the range of spatial control assigned to individual compartments was coincident with the spatial control effective region found in the study.

Alternatively we propose a horizontal zone control system that has comparable performance with better predictability.

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I. BACKGROUND AND CONCEPTS

I.1 Introduction

A nuclear power reactor is directly controlled by adjustments of the neutron population varying over both time and space. The neutron population distribution is governed by the local/global multiplication factor changes, partly due to the build-up of fission products, such as Xe^{135}, partly because of refuelling and fuel burnup and partly as a result of temperature changes. All these changes have to be compensated by a reliable and accurate mechanism with certain predictable performance to maintain proper control of neutron distribution.

The most common and effective means for controlling criticality is the insertion or extraction of a specific neutron absorbing material from the reactor core. The types of control mechanism are usually divided into two categories; one is injection or extraction of liquid type absorber solution; the other method is insertion or withdrawal of solid type rods in specific locations.

The complexity of phenomena in nuclear reactions between neutrons and matter raises several difficulties in defining the function of controllers. The aspects of reactor physics considered in control design are briefly discussed as in the following:

(1) Delayed neutron production

Because the neutron life-time in conventional thermal reactors is in the 10^{-4} to 10^{-3} second range, no mechanical devices can sufficiently respond in this time domain. Thus the reactor control mechanism relies on the small fraction (for example, 0.0065 for U^{235} fission), of delayed neutrons born from certain fission products, commonly called 'precursors'[(1)-(3)]. The production time constants of delayed neutrons are spread over 0.1 to 10 seconds.

(2) Xenon redistribution

Xe^{135} is a high neutron-absorbing fission product ($\sigma_a^{xe} \approx 2.7 \times 10^6$ barns for 0.025 ev neutrons) decaying with a 9.2 hours half-life. Even when the reactor is controlled in the global sense, the localized variation of xenon may induce oscillations in the neutron population distribution. For U^{235} fuelled reactors, "oscillations are not possible for neutron fluxes below 3×10^{11} neutron/cm^2-sec"[(4)-(6)]. Most of the current power reactors have neutron fluxes of the order 10^{13} - 10^{14} n/cm^2·sec well beyond the above threshold value. Therefore, the controllers in reactors operating at these high flux levels must be capable of eliminating the xenon-induced instability or oscillations[(7)-(12)]

(3) Burnup and refuelling

Fissile depletion and fertile conversion are the most slowly varying kinematics affecting the criticality factor. For reactors charged yearly with fresh fuel, the main concern of control is the long term criticality consideration. For CANDU reactors, the on-power fuelling concept[13] requires that controllers respond to daily perturbed local and global flux levels.

(4) Temperature Feedback

Since power reactors are tied to electric generators through heat transport loops, changes in plant operation conditions cause disturbances in the physical characteristics of the reactor[14],[15], e.g., shifting neutron energy spectrum, changing macroscopic reaction cross sections, etc. These feedback effects act over a time scale of the order of seconds and minutes.

(5) Randomness of material motions

Added to the statistical nature of neutron fluxes and their reactions with matter, void formation in the coolant, vibrations of structures, etc.[16]-[18] cause the neutron population to fluctuate continuously. Control of this noise component should be eliminated from the objectives of control function[19].

Actually every transient occurring in nuclear reactors is a combination of the above physical phenomena and each component is not easily separable in a sufficiently accurate manner.

It appears that the most desirable method on which the reactor control system might rely would be to solve simultaneously all the dynamic equations describing the various phenomena and finding appropriate control actions to eliminate undesirable disturbances. But, even with modern computing systems, it is almost impossible to obtain the solution of a complex nonlinear space-time dependent equation system in which a controller action is explicitly represented for the real-time applications. The common strategy used in power reactor control is, therefore, to minimize the neutron flux deviation from some pre-determined reference shapes during any transients.

Because extreme difficulties arise in the design of control systems mainly due to the complexity and due to the infinite number of possible situations which can be anticipated during operation, the principle which is incorporated in conventional design method is to distribute the individual controllers with geometrical regularity and symmetry[(5),(20)]. After a large number of simulations for various significant situations which could occur during operation, designers usually choose a symmetric layout pattern which satisfies given control requirements and safety criteria.

Before designing the control system, the reactor designers usually set up quantitative criteria in terms of reactivity and power such as;

(1) Limits on the total and individual reactivity worths and rates of controllers,

(2) Limits on the global and the local power and their rates of change. Obviously the reactivity is a measure of the potential for change in the neutron population and the associated power varies.

To control the global components of reactivity or power, it is preferrable that the controllers are distributed so as not to create flux distortions due to their presence. But to control the local components, the controllers should be placed where those local distortion can be effectively eliminated.

Modern nuclear power reactors have large flat high flux regions. In such circumstances, weak coupling of reactor sectors induces local criticality control problems. Therefore at least one controller must be assigned to control each sector of the reactor. But the deployment of as many controllers as necessary for local control is always accompanied by supporting structural materials, proportional to the number of controllers, which introduce parasitic neutron absorption into the core. From a neutron economy point of view, reduction in the amount of such material is desirable, which leads eventually to a reduction in the number of controllers.

From a safety aspect, a large number of controllers require more complicated electrical and mechanical circuitry, which inherently decreases the reliability of the system. But a system with a small number of controllers may not have sufficient redundancy built into it and, thus, even for a single controller failure, may possibly impact on reactor integrity.

These conflicting aspects acting in an inverse relationship to each other, indicate that there should exist certain optimal conditons for deployment of the control system in the reactor as far as their numbers and locations are concerned.

I.2 History of Optimum Nuclear Reactor Control

Related to optimality in the deployment pattern of controllers in systems described by partial differential equations, Vidyasagar and Heggins[21] have obtained conditions describing the existence of a control system. This system acts at a finite number of points for a distributed parameter system, which approximates a continuously distributed control function. Since then, numerous investigations have been made on the optimal discrete control of the distributed parameter system by the deterministic[22]-[25] or by the stochastic approach[26]-[28].

The optimal control design problem, however, has received little attention, at least ~~in its~~ practical utilization, in reactor system design despite its importance. The absence in application of optimal distribution parameter control theory in a full-sized industrial process has been criticized by W. H. Ray[29]. We are going to quote his reasons of the principal hindrances he identified:

(1) the lack of properly educated control engineers.

(2) a dearth of real-time experience,

(3) a communication gap between the potential user and the control theorist.

Even though a nuclear reactor is not such a simple dynamic system that the time characteristics of system variables are bounded in the same order of scale and that coefficient parameters can be expressed by analytical or algebraic functions, we have chosen to look at different aspects of the problem, because;

(1) most of basic design procedures for nuclear power reactors were established in the 60's when there was no concrete analytical method to attack the problem properly,

(2) large amount of design and operational experiences might be enough to confirm that the system designed with conventional methods would be optimum or suboptimum from the performance point of view.

Although the activity in practical application has not been sufficient, several excellent attempts have been made to demonstrate the feasibility of applying optimal control theory with applaudable results.

D. W. Wieberg[30] studied the slow transient problem, i.e., control of the xenon-induced spatial oscillation, with minimum number of controllers. The control of total power was separated from the spatial control and he assigned only a single controller to the control of the fundamental power shape. He derived the necessary conditions for controllability with minimum number of controllers whose sufficiency was proved later by N. Suda[31]. Wieberg's controllability theorems state that any finite number of modes of the reactor model can be returned to zero by the action of a control if;

(1) there are at least as many control rods as the maximum multiplicity of the spatial operator,

(2) all control rods are not on any possible nodes of a combination of modes having the same eigenvalues.

However, such a minimum number of controllers may not be enough when we consider some practically important constraints, for example, control margin and redundancy.

W.M. Stacey[32] derived an optimal continuous control function by solving the coupled non-linear neutron-iodine-xenon dynamic equations and their adjoint equations using nodal formulation and quasilinear numerical algorithm. Although his proposed method makes the physical meaning of the problem clear, his control function is not regarded as a general control function in that it does not lead to a unique optimum number and locations of controllers for all possible transients. This is because the solution of the two-point-boundary-problem depends only on the given initial and final conditions, or on the transversality conditions for the unspecified terminal condition problem. A further difficulty arises because the resulting controllers must have a spatially continuous distribution, whereas in a real reactor they are discrete rods.

There are several good review papers covering new trends in

nuclear reactor control design using optimal control theory.
J. Karppinen[33] presented a broad review of spatial reactor control methods that have employed optimization techniques. He outlined the formulation of the spatial control algorithm, as an optimal control problem, and solution methods based on distributed parameter optimal control[34],[35], variational calculus[36],[37], dynamic programming [38],[39], mathematical programming[40],[41], the maximum principle[42]-[45], linear-quadratic control theory[46], and heuristic control techniques[47].
S. Tzafestas[48] surveyed the distributed parameter optimal control methods applied to nuclear reactor. After a brief introduction to the derivation of the nuclear reactor distributed parameter differential and integral models, he discussed the formulation of the linear-quadratic, the bilinear, the eigenvalue assignment and the stochastic control problems. He also mentioned the necessity of much further work for designing and implementing practical finite dimensional optimal controllers based on distributed parameter models of complete power reactor systems.

Apart from these pioneer-type or review-type papers, numerous research works appear in the literature, but the common conditions and strategies are specific problem-oriented ones;

(1) finding optimal manoeuvering strategies using existing control systems[49]-[51],

(2) finding optimal control functions for given initial and/or final conditions[52]-[55],

(3) dealing only with the spatial control problems or the reactivity control problems.

In Canada, because of the unique type of power reactors, research activities on optimal control problems have emphasized burnup control[56], the xenon induced oscillation control and load-following strategies[57],[58] for the CANDU power reactors.

I.3 Fundamentals of CANDU Reactor Control

The CANDU-PHW reactor* is a heavy water moderated, heavy water cooled, natural uranium fuelled reactor which utilizes the pressure tube concept[59]. The pressure tubes containing the fuel and the coolant run horizontally in the reactor core. Each pressure tube is isolated and insulated from the heavy water moderator by a concentric calandria tube and a gas annulus. Figure I-1[59] shows the standard CANDU reactor system.

The reactivity control mechanisms are vertically mounted from the reactor top and reside in the low pressure moderator. The layout of reactivity mechanisms is illustrated in Figure I-2[59]. Natural uranium fuelled reactors have low excess reactivity and therefore parasitic neutron absorption should be as low as possible. This results in the necessity of on-power fuelling as a method of reactivity supply and

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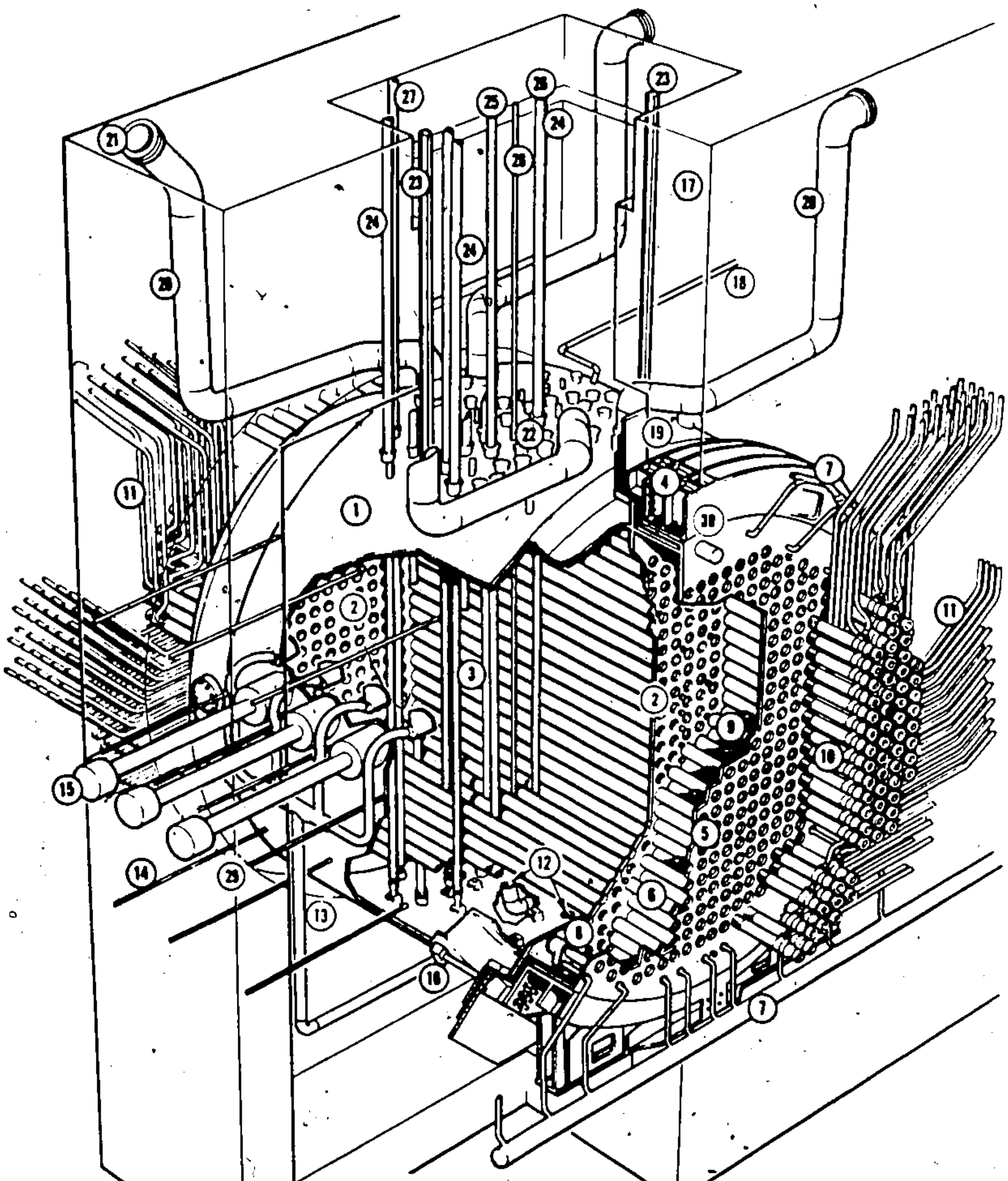
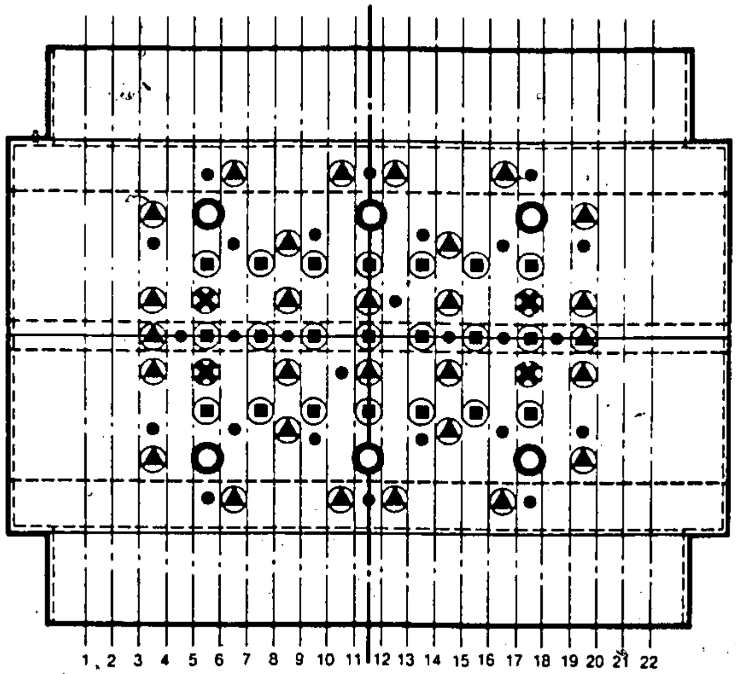
* CANada Deuterium Uranium Pressurized Heavy Water



1. CALANDRIA
2. CALANDRIA – SIDE TUBESHEET
3. CALANDRIA TUBES
4. EMBEDMENT RING
5. FUELLING MACHINE – SIDE TUBESHEET
6. END SHIELD LATTICE TUBES
7. END SHIELD COOLING PIPES
8. INLET-OUTLET STRAINER
9. STEEL BALL SHIELDING
10. END FITTINGS
11. FEEDER PIPES
12. MODERATOR OUTLET
13. MODERATOR INLET
14. HORIZONTAL FLUX DETECTOR UNIT
15. ION CHAMBER
16. EARTHQUAKE RESTRAINT
17. CALANDRIA VAULT WALL
18. MODERATOR EXPANSION TO HEAD TANK
19. CURTAIN SHIELDING SLABS
20. PRESSURE RELIEF PIPES
21. RUPTURE DISC
22. REACTIVITY CONTROL UNIT NOZZLES
23. VIEWING PORT
24. SHUTOFF UNIT
25. ADJUSTER UNIT
26. CONTROL ABSORBER UNIT
27. ZONE CONTROL UNIT
28. VERTICAL FLUX DETECTOR UNIT
29. LIQUID INJECTION SHUTDOWN NOZZLE
30. BALL FILLING PIPE

FIGURE I-1 REACTOR ASSEMBLY



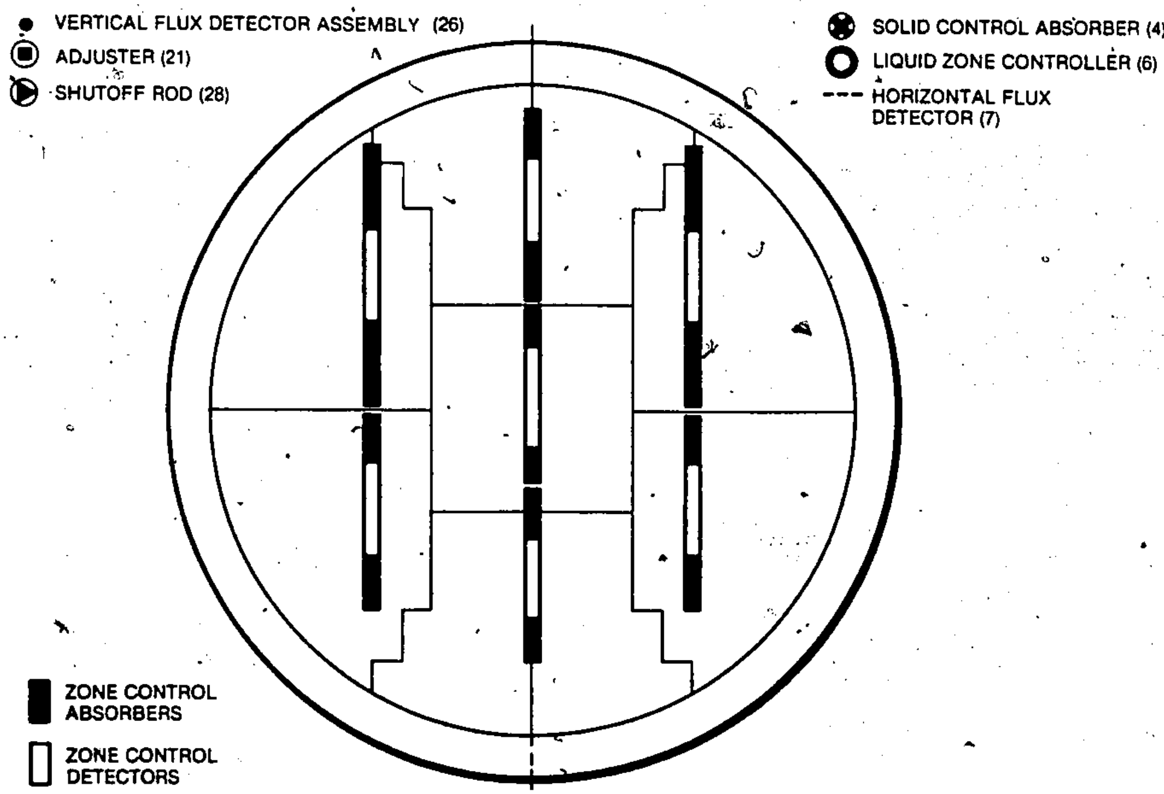


FIGURE I-2 REACTIVITY MECHANISM LAYOUT

consequently long-term control of reactivity and power shape. Therefore low reactivity worth of the control mechanisms is one of the requirements of the control system design.

The prompt neutron lifetime in a CANDU lattice is relatively long (~ 0.9 m sec) and the fission delayed neutron fraction (~ 0.005) is augmented by the presence of photoneutrons that are produced by (n,γ) reactions with deuterium nuclei [(60),(61)]. These two factors slow down a potential power excursion considerably and, thus, the control of global criticality factor is quite easy with a small amount and rate of controller's reactivity worth.

The size of the CANDU reactor core is comparatively large. In order to produce a large amount of power and reduce the reactor size, increasing the volume of the high power region and flattening the power shape in that region are required. These steps induce xenon instability or spatial power oscillation control problems[(62)] and require a xenon override capability of the control system[(63),(64)].

For the above reasons, the spatial control of the CANDU reactors has been repeatedly emphasized rather than bulk control. O. A. Trojan[(65)] mentioned the principles of CANDU spatial control and criteria in analytical methods associated with detector-controller response action.

Generally the CANDU control system is composed of three different kinds of mechanisms to regulate the reactor power distribution;

(1) Zone Control Compartments

The main method for controlling reactor power is by adjustment of the average H_2O level in independently controllable compartments in regions of the reactor. Differential adjustment of levels in individual compartments is used for spatial control. Platinum in-core flux detectors provide the neutron flux feedback signals required by the digital control computers.

(2) Mechanical Control Absorbers

The reactivity range provided by the zone control system is usually adequate for most power maneuvers. However certain situations, for example, a power setback, require additional negative reactivity that is provided by the mechanical control absorbers, normally residing out of core.

(3) Adjuster Rod Banks

Even with the on-power fuelling to control the power shape, additional adjustments to achieve an adequately flattened power shape are required. Also to provide a sufficient amount of positive reactivity to override the xenon buildup during shutdown-restart periods, the adjuster rods normally fully inserted in the core are sequentially withdrawn in symmetrical bank modes.

The typical control modes for normal or certain upset operating conditions are shown in Figure I-3. Within ±3% power error,

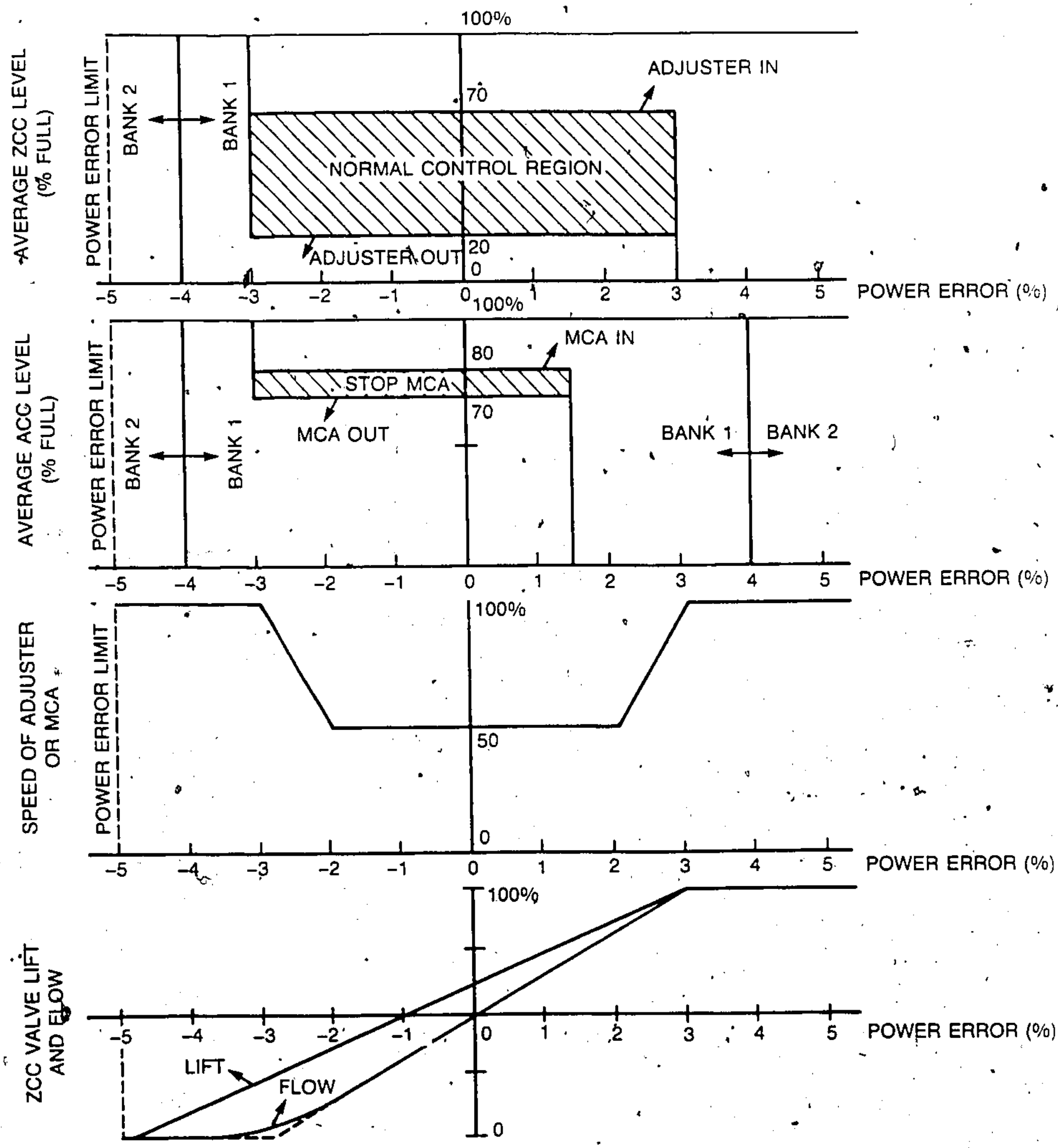


FIGURE I-3 REACTOR REGULATING SYSTEM CONTROL INTERFACE

the zone control system takes change of the global and the regional power control, if the average and the individual level of compartments are confined to the range of 20-70%. If the average level is below 20%, further control is complemented by the sequential withdrawal of the adjuster rod banks. If the average level is above 70%, the mechanical absorber banks are sequentially introduced into the reactor core until a sufficient negative reactivity is to be provided.

In order to control the flux and ultimately the power distribution in the reactor core, flux measurements are taken and an automatic on-line flux mapping system is used to provide an extensive flux map. The detection system is divided into two sub-systems according to their functions; a fast one of gamma-ray responding platinum detector assemblies for control and shutdown purposes, and a slow one of neutron responding vanadium detectors for mapping the neutron flux distribution and calibrating the platinum detectors.

The on-site control computer plays an integral role in assigning activation/deactivation of each control mechanism at the proper time. The functions and interactions of the reactor regulating system managed by the control computer are given in Figure I-4[59]. This direct digital control system is also used for overall plant control, alarm annunciation and data display[66].

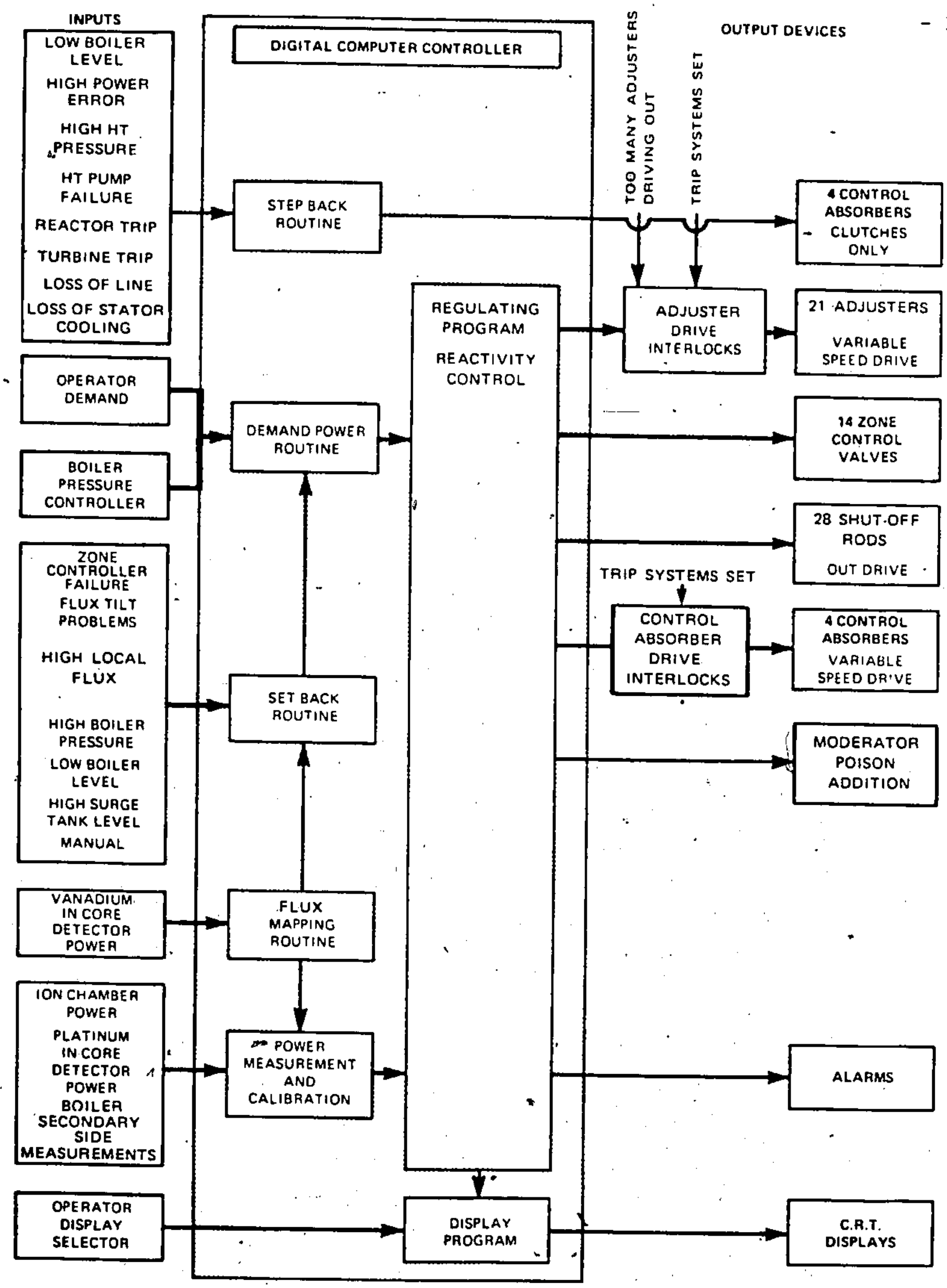


FIGURE I-4 REACTOR REGULATING SYSTEM BLOCK DIAGRAM

I.4 Overview of Context

Theories and stages developed for the study on the optimal deployment of the CANDU control system are briefly discussed in this section. The main emphasis of the study is based on the linear quadratic regulator theory for the linear distributed parameter dynamic equations [(28),(34),(67)-(71)] separated into the bulk control and the spatial control objectives.

Assuming the separability of the system equations into the bulk and the spatial control components, the deviations of state function from a reference state are defined to satisfy approximated linear dynamic equations with the first order variation. The modal control theory [(72)-(75)] is available to expand the solution into an infinite series of the system eigenfunctions to be able to transform the distributed parameter state functions into the time invariant problems. We investigate the application of the λ-modes [(76),(77)] to reduce the order of the dynamic system whose eigenfunctions are difficult to generate. This linear time invariant approximation has advantages in its application to the CANDU controller deployment; that a unique asymptotic solution with small number of modes is guaranteed and, hence, approximately satisfies the property of finality [(78)], and that the optimality criteria can be derived by a regulator problem with Lyapunov's asymptotic stability [(79),(80)] rather than a servomechanism problem [(81)].

Optimality of controller locations is evaluated in terms of controller effectiveness, i.e., feedback gain due to placing a controller at a location, under the minimization conditions of a performance index constructed with iodine-xenon deviation and spatial control effort. In the time frame where the spatial control function shows the dynamic behaviour, neutron kinetics is assumed to be quasi-stationary and, thus, the reduced-order model includes the neutron balance equations implicitly. Selection of a low-order model using the dominant mode concept[(82)-(84)] is also applied to the bulk control formulation because the iodine-xenon dynamics can be stationary in neutron kinetics problem.

As under certain observability and controllability conditions, there exists an optimum feedback control law to stabilize the dynamic system[(85)-(88)], we introduce a boundary inside which a symmetrically placed pair of controllers can completely control the system.

The relationship between the dynamic variable representing iodine-xenon concentrations and their adjoint is expressed by a positive definite symmetric matrix which satisfies a certain matrix Riccati equation. The matrix Riccati equation is derived using the maximum principle applied on the Hamiltonian of the modified performance index and assuming a linear transformation between dynamic variables and their adjoints. Then the feedback gain of the optimum control law can be obtained as a function of the solution of the Riccati equation.

The numerital technique to solve the matrix Riccati equation is based on the Kleinman's iterative scheme[89], which is equivalent to solving the matrix Lyapunov equation[90] at each iteration. The sufficient and necessary conditions for convergence are investigated according to Kleinman's theorems. Possible numerical truncation errors accumulated during numerous addition and multiplication operation in a digital machine is successfully reduced by the iterative correction with the residual Lyapunov equation. The convergence can be accelerated by introducing an over-relaxation parameter weighted on iterative solutions.

For every grid point in a controller domain, we can calculate the feedback gain, i.e., a measure of its control effectiveness, using the above strategy.

To determine the number of controllers, we apply similar procedures but with the time dependent neutron diffusion equations and with subsets of controller locations selected by the spatial control effectiveness and by the neutron importance distribution. Distinctive features of the dynamic equations formulated for computing bulk control are that they include inhomogeneous terms related to the reactivity source and that the feedback law can be modified by the output feedback concept[91],[92] that possibly extends the problem to the optimum detection system design.

As the first step, the inhomogeneous term describing the excess reactivity of the reference system is compensated by a pre-selected set of the static bulk controllers, using the calculus of variation to minimize the performance index subject to the steady state diffusion equation. The calculated depth of controllers will be used as the set-point of controllers to determine the dynamic range of them. The number of controllers in the set are roughly estimated from the maximum allowable controller material subtracted by the expected capability for the dynamic response. If we have reactors which are ideally fuelled, i.e., no flux variation due to the fuelling operation, the set of controllers determined in this way would be sufficient for our purpose.

However, in practice the on-power fuelling combined with the burnup process, has reciprocal effects where one inserts continuous disturbances both locally and globally and the other prevents the transients from going beyond acceptable limits. Thus, from the long-term view, these effects can be treated as a bounded random process whose mean is equivalent to the reference state. To utilize this idea in the formulation, we transform the time variable to the coordinate $t' = v_2 t$, where v_2 is the thermal neutron velocity. Then the two group time-dependent diffusion equations are reduced to the modal dynamic equations which dominate the thermal neutron behavior. The procedures are similar to those introduced to manipulate the spatial control problem.

The covariances of the disturbance vectors are obtainable from the burnup distribution of discharged fuel, if we assume the proportionality between burnup and criticality factor. The formulation of the dynamic system equations takes advantages in the estimation of the stochastic system[(93)-(95)] and in the exactness of the deterministic system. By introducing a matrix relationship between the state vectors and their adjoints, the control function and the governing equations are separable into two aspects. One deals with regulating the flux variation represented by the deterministic dynamic equation and the other deals with counteracting the random disturbances. Therefore the optimality of a controller set can be evaluated by the gain matrix obtained from the deterministic part plus by the covariance matrix derived from the stochastic part.

The dynamic range of the bulk controllers optimally selected by the above procedures is estimated using the maximum allowable local/global power error. And finally the range is modified by the additional expected response obtained from the variance matrix.

Table I-1 shows the time characteristics of system equations, which lead to concepts of dominant control objectives and corresponding optimization problems dealt in this thesis.

TABLE I-1 TIME CHARACTERISTICS OF SYSTEM EQUATIONS AND SPECIFICATIONS OF THE MODEL PROBLEMS

Time Measure / System Equations	0	$\ll$ Second	Seconds	Hours	Days
	Criticality	Noise Analysis	Kinetics	Xenon Dynamics	Fuel Management
Neutron Diffusion	Static	Dynamic, Stochastic	Dynamic, Deterministic	Quasi-static	Quasi-static
Precursor	Static	Dynamic, Stochastic	Dynamic, Deterministic	Static	Static
Iodine/ Xenon	Static	Static	Static	Dynamic, Deterministic	Static
Burnup	Static	Static	Static	Static	Dynamic, but, if includes fuelling, then stochastic
Heat Transfer	Static	Static	Quasi-dynamic	Quasi-static	Static
Controllability	Controllable	Not Controllable	Controllable	Controllable	Controllable
Model Problems	Determination of Static Set-points	Excluded	Determination of Dynamic Ranges	Calculation of Spatial Contr. Effectiveness	Determination of Extra Response for Random Pert.

II. FORMULATION OF LINEAR DYNAMIC SYSTEM

II.1 Reactor Dynamic Equations

II.1.1 Two-Group Diffusion Equations

For a description of neutron behaviour in a multiplying system, the two-group diffusion approximation[96],[97] to the Boltzmann transport equation has been proved to give sufficient accuracy for the neutronic design of the CANDU reactors.

The form of equations being illustrated here included the explicit expressions of the delayed neutron precursors and of the xenon poisoning to be able to describe the whole essence of the control objectives. The fuel burnup and refuelling obviously contribute to the burden of the control system, but the carefully controlled fuelling scheme can provide the minimum deviation from the ideal reference flux shape with small number of isolated ripples[13].

For a controlled system, the time dependent diffusion equations are written as;

$$\frac{1}{v_1}\frac{\partial\phi_1(\underline{r},t)}{\partial t} = \nabla\cdot D_1(\underline{r},t)\nabla\phi_1(\underline{r},t) - \left(\Sigma_{a1}(\underline{r},t) + \Sigma_{Rem}(\underline{r},t)\right)\phi_1(\underline{r},t)$$

$$+ \sum_{i=1}^{I} \lambda_i C_i(\underline{r},t) + (1-\beta)\nu\Sigma_{f2}(\underline{r},t)\ \ \phi_2(\underline{r},t) \qquad \text{...(II.1a)}$$

$$\frac{1}{v_2}\frac{\partial\phi_2(\underline{r},t)}{\partial t} = \nabla\cdot D_2(\underline{r},t)\nabla\phi_2(\underline{r},t) - \left(\Sigma_{a2}(\underline{r},t) + \sigma_x X(\underline{r},t)\right)\phi_2(\underline{r},t)$$

$$+\Sigma_{Rem}(\underline{r},t)\ \phi_1(\underline{r},t) - \sum_{j=1}^{I_c} U(\underline{r},t)\ \phi_2(\underline{r},t)\ \delta(\underline{r}-\underline{r}_j) \qquad \text{...(II.1b)}$$

where

$\phi(\underline{r},t)$	;	the neutron flux distribution function,
$X(\underline{r},t)$	;	the Xe^{135} nuclide concentration,
$C_i(\underline{r},t)$	;	the i-th group delayed neutron precursor distribution,
$U(\underline{r},t)$	;	the control function,
v	;	neutron velocity,
$D(\underline{r},t)$	;	diffusion coefficient,
$\Sigma_a(\underline{r},t)$	;	macroscopic absorption cross section,
$\Sigma_{Rem}(\underline{r},t)$	;	macroscopic removal cross section,
$\nu\Sigma_{f2}(\underline{r},t)$	;	macroscopic yield cross section,
λ_i	;	the i-th group precursor decay constant,
β	;	total fraction of delayed neutron, i.e., $(\beta = \sum_{i=1}^{I}\beta_i)$,

σ_x ; microscopic absorption cross section of Xe^{135},

$\underline{r}$; space coordinates,

t ; time coordinates,

I ; number of delayed neutron groups,

I_c ; number of controllers,

$\delta(\underline{r}-\underline{r}_j)$; Dirac delta function,

$$\begin{cases} = 1, & \text{if } \underline{r} = \underline{r}_j \\ = 0, & \text{otherwise,} \end{cases}$$

and the subscripts 1 and 2 refer to the fast and the thermal neutron groups, respectively.

Assumptions involved in the equations are;

(1) Only thermal neutrons contribute to the fission process,

(2) Only fast neutrons are produced from fission and precursors,

(3) Only Xe^{135} is explicitly expressed as an absorptive isotope,

(4) Control function, equivalent to the effective cross section[61],[98] contributes only to thermal neutrons at finite discrete points.

II.1.2. Precursor Kinetic Equations

Delayed neutrons are born from certain fission fragments which, after a β-decay, have excessive number of neutrons to be stable isotopes. With appropriate time delay, a neutron appears spontaneously. To express the rate of neutron emission from this source, for the CANDU reactors, they are divided into 6 groups, according to the time scheme of their decay processes, modified with photoneutron fractions[(1)-(3)].

The kinetic equations for multi-group precursor time behaviour are;

$$\frac{\partial C_i(\underline{r},t)}{\partial t} = \beta_i \nu \Sigma_{f2}(\underline{r},t)\, \phi_2(\underline{r},t) - \lambda_i C_i(\underline{r},t), \quad i = 1, 2, .., 6. \qquad \text{....... (II.2)}$$

II.1.3 Xenon Poisoning

Among various fission fragments Xe^{135} and Sm^{149} have substantially high absorption cross sections and their concentrations in the reactor tend to change the flux distribution[(5),(96)]. This poisoning effect appears with about 10 hours of time delay.

Since the absorption cross section of Sm^{149} (=40,800 barns for 0.025 ev neutrons) is much less than that of Xe^{135} and the half-life of Pm^{149} (λ=0.0128 1/hour), parent isotope of Sm^{149}, is much longer than Xe^{135} and I^{135} (λ=0.0753 1/hour), the principal parent isotope of Xe^{135}, only the explicit dynamic equations for the Xe^{135} chain will be treated in the study. For the Sm^{149} chain, we assume that a constant value equivalent to the equilibrium concentration for the normal operation can be evaluated and included in the macroscopic absorption cross section of the thermal neutrons.

Considering the decay chain of Xe^{135} and I^{135}, after neglecting the intermediate stages, we approximate the coupled dynamic equations to be;

$$\frac{\partial I(\underline{r},t)}{\partial t} = \gamma_I \Sigma_{f2}(\underline{r},t)\ \phi_2(\underline{r},t) - \lambda_I I(\underline{r},t) \qquad \ldots(\text{II.3a})$$

and

$$\frac{\partial X(\underline{r},t)}{\partial t} = \gamma_x \Sigma_{f2}(\underline{r},t)\ \phi_2(\underline{r},t) + \lambda_I I(\underline{r},t) - \sigma_x X(\underline{r},t)\,\phi_2(\underline{r},t) - \lambda_x X(\underline{r},t) \qquad \ldots(\text{II.3b})$$

where

γ_I, γ_x ; the direct yield fraction of I^{135} and Xe^{135} from a fission event, respectively,

λ_I, λ_x. ; the decay time constants of I^{135} and Xe^{135}, respectively.

II.1.4 Temperature Feedback

The phenomena associated with changes in operating conditions can be considered as following two categories. The first is the very localized perturbation, e.g., bubble formations in the coolant, vibrations of in-core materials, etc.; and the second is global or at least regional changes, e.g., control bank manoeuvering power set back, etc.

The first category can be treated as random fluctuation in the neutron flux distribution, because the disturbance is hardly predictable. For the second category, a change in operating conditions is somehow intentional and directly affects the reactor power, and eventually the temperature of the system. The reactivity coefficient of power is the integrated parameter to evaluate this kind of perturbation. We define, for simplicity, the power coefficient, as a change in fission cross sections, as

$$\tilde{\Sigma}_{f2}(\underline{r},t) = \Sigma_{f2}^{Ref}(\underline{r}) \left(1 + \alpha_f(\underline{r}) \frac{\phi_2(\underline{r},t) - \phi_2^{Ref}(\underline{r})}{\phi_2^{Ref}(\underline{r})} \right) \quad \text{(II.4)}$$

and

$$\alpha_f(\underline{r}) = \left(\frac{\Delta\rho}{\Delta P}\right) P^{Ref}$$

where $\alpha_f(\underline{r})$; power coefficient,

$(\Delta\rho/\Delta P)$; the ratio of changes in reactivity to power,

P^{Ref} ; the reference power level,

II.1.5 Neutron Noise Field

Apart from the deterministic treatment of the reactor dynamics described up to here, the stochastic approach will be preferrable in some areas where the detector signal has an important role in determining control action[19],[97],[98]. As the reactor control automatically relies on the accumulated and evaluated reaction rates of the in-core neutron detectors during very small time intervals, the noise component is always included in the information.

One of the concerns in the design of a controller-detector pair system is how to separate this noise component from the control objectives. In other words, the optimal estimation of detector signals must be considered when we select the detector locations.

In incorporating the physical consideration of the noise components of the system dynamic equations, it is reasonable to assume that the noise is completely white, i.e., zero mean Gaussian[18]. Fortunately except for the case of controlling the power excursion, the responding time and magnitude of controllers are not necessarily fast and large enough to cover every instantaneous noise-caused measurement fluctuation. Therefore in accounting for the uncertainties and difficulties associated with the use of a natural white noise $\Gamma(\underline{r},t)$ it has become customary to introduce artificial Brownian motions which have statistical time characteristics very similar to those of white noise and are expanded with system eigenfunctions $\underline{e}_n(\underline{r})$ [99];

$$\Gamma(\underline{r},t) = \sum_{n=1}^{N} \beta_n(t)\, \underline{e}_n(\underline{r}) \qquad \text{...(II.5)}$$

where $\beta_n(t)$; amplitude of noise component for $\underline{e}_n(\underline{r})$ having small time constant.

And the detector signal will be composed of the deterministic component and the stochastic component in the way of;

$$\Phi(\underline{r},t)\ \delta(\underline{r}-\underline{r}_j) = \left(\phi_2(\underline{r},t) + \Gamma(\underline{r},t)\right)\delta(\underline{r}-\underline{r}_j), \qquad j = 1, 2, \ldots, I_d \qquad \ldots(\text{II.6})$$

where I_d is number of in-core detectors.

The detectors are assumed sufficiently separated in the core and, thus, there are no cross-correlations between detector signals. The auto-correlation function of noise, $\langle \Gamma(\underline{r}_i,t)\ \Gamma(\underline{r}_i,t+\tau) \rangle$ can be an arbitrary constant for each detector and has a form of $\Xi_i\ \delta(\tau)$, where $\delta(\tau)$ is the Dirac delta function.

II.1.6 Boundary Conditions

State functions shown in Eqs. (II.1), (II.2), (II.3), (II.4), (II.5) and (II.6) have to satisfy the homogeneous boundary conditions

$$\begin{aligned}\phi_1(\underline{R},t) &= \phi_2(\underline{R},t) = I(\underline{R},t) = X(\underline{R},t) \\ &= \Gamma(\underline{R},t) = C_i(\underline{R},t) = 0, \quad i = 1, 2, \ldots, 6\end{aligned} \qquad (\text{II.7})$$

and the control function must be defined inside the reactor, so

$$U\ (\underline{R},t) = 0 \qquad \ldots(\text{II}.8)$$

where $\underline{R}$ is the extrapolated reactor boundary.

The locations of controllers must be unchanged during the reactor life-time and, therefore, the optimality is a time-invariant property which is independent from any kinds of perturbation locations. However, the initial conditions assigned to the state functions must not be beyond the allowable maximum deviation in safety aspects.

II.2 Linearized System

II.2.1 Definition of Perturbations

The reaction cross sections shown in the system equations (II.1), (II.2), (II.3) and (II.4) are slowly varying functions which change only significantly when the fuel burnup is important. Therefore, in the time domain of the control problems, it may be adequate to assume them as time invariant constants averaged over the control period $[t_0, t_f]$.

$$\Sigma_{ij}(\underline{r}) = \frac{\int_{t_0}^{t_f} \Sigma_{ij}(\underline{r},t)\ \phi_j(\underline{r},t)\ dt}{\int_{t_0}^{t_f} \phi_j(\underline{r},t)\ dt} \qquad \ldots(\text{II}.9)$$

where the subscript j refers to the neutron energy group, and the subscript i to the reaction type, i.e., absorption, removal, fission and transport. t_0 and t_f are the instants in time when the control action is initiated and terminated, respectively.

In the early stage of the CANDU reactor design process, the reference state of the reactor was designated by defining an equilibrium burnup state. Because of on-power fuelling, the reactor burnup state is continually being changed by the time-dependent burnup process and by a stepwise refuelling process. If we average those variations over a fairly long time period, we can assume that the local burnup deviation is negligible. The average burnup state is defined as <u>the time-averaged equilibrium burnup reference state</u>[100].

The system equations describing the reference state are;

$$\nabla\cdot D_1^{Ref}(\underline{r})\nabla\phi_1^{Ref}(\underline{r}) - \left(\Sigma_{a1}^{Ref}(\underline{r}) + \Sigma_{Rem}^{Ref}(\underline{r})\right)\phi_1^{Ref}(\underline{r})$$

$$\frac{1}{k_0}\left((1-\beta)\,\nu\Sigma_{f2}^{Ref}(\underline{r})\quad\phi_2^{Ref}(\underline{r}) + \sum_{i=1}^{6}\lambda_i\,C_i^{Ref}(\underline{r})\right) = 0, \qquad \text{...(II.10a)}$$

$$\nabla\cdot D_2^{Ref}(\underline{r})\nabla\phi_2^{Ref}(\underline{r}) - \left(\Sigma_{a2}^{Ref}(\underline{r}) + \sigma_x\,X^{Ref}(\underline{r})\right)\phi_2^{Ref}(\underline{r})$$

$$+\,\Sigma_{Rem}^{Ref}(\underline{r})\,\phi_1^{Ref}(\underline{r}) = 0, \qquad \text{...(II.10b)}$$

$$\beta_i \ \nu\Sigma_{f2}^{Ref}(\underline{r}) \ \ \phi_2^{Ref}(\underline{r}) \ - \ \lambda_i C_i^{Ref}(\underline{r}) = 0, \quad i = 1, 2, \ldots, 6 \qquad \ldots(II.10c)$$

$$\gamma_I \ \ \Sigma_{f2}^{Ref}(\underline{r}) \ \ \phi_2^{Ref}(\underline{r}) \ - \ \lambda_I \ I^{Ref}(\underline{r}) = 0\,, \qquad \ldots(II.10d)$$

and

$$\gamma_x \Sigma_{f2}^{Ref}(\underline{r}) \ \phi_2^{Ref}(\underline{r}) + \ \lambda_I I^{Ref}(\underline{r}) \ - \ \sigma_x X^{Ref}(\underline{r}) \ \phi_2^{Ref}(\underline{r})$$
$$- \lambda_x X^{Ref}(\underline{r}) = 0. \qquad \ldots(II.10e)$$

The effective multiplication factor k_0 of the design reference state is generally greater than 1.0 in order that an amount of excess reactivity remains for the control purpose.

If we look at differences between the general state functions in Eqs. (II.1), (II.2), (II.3), (II.4) and (II.5) and the reference state functions in Eqs. (II.10), the source of time variation is obviously a combination of differences in material properties, i.e., a perturbation, and controller action, i.e., the response. After a simplifying procedure that in which every material property change is smeared into a fractional change in the fission cross section, the critical system dynamic equations, whose dynamics are governed by arbitrary perturbations and corresponding control actions, can be written as follows.

$$\frac{1}{v_1}\frac{\partial\phi_1(\underline{r},t)}{\partial t} = \nabla\cdot D_1(\underline{r})\nabla\phi_1(\underline{r},t) - \left[\Sigma_{a1}(\underline{r}) + \Sigma_{Rem}(\underline{r})\right]\phi_1(\underline{r},t)$$

$$+ \sum_{i=1}^{6}\lambda_i C_i(\underline{r},t) + (1-\beta)\,\nu\Sigma_{f2}(\underline{r})\left[1 + \alpha_f(\underline{r})\right.$$

$$\left.\frac{\phi_2(\underline{r},t) - \phi_2^{Ref}(\underline{r})}{\phi_2^{Ref}(\underline{r})} + \Delta(\underline{r},t_0)\right]\phi_2(\underline{r},t) \qquad \text{...(II.11a)}$$

$$\frac{1}{v_2}\frac{\partial\phi_2(\underline{r},t)}{\partial t} = \nabla\cdot D_2(\underline{r})\nabla\phi_2(\underline{r},t) - \left[\Sigma_{a2}(\underline{r}) + \sigma_x X(\underline{r},t)\right]\phi_2(\underline{r},t)$$

$$+ \Sigma_{Rem}(\underline{r})\,\phi_1(\underline{r},t) - \sum_{j=1}^{I_c} U(\underline{r},t)\,\phi_2(\underline{r},t)\,\delta(\underline{r}-\underline{r}_j) \qquad \text{...(II.11b)}$$

$$\frac{\partial C_i(\underline{r},t)}{\partial t} = \beta_i\,\nu\Sigma_{f2}(\underline{r})\left[1 + \alpha_f(\underline{r})\,\frac{\phi_2(\underline{r},t) - \phi_2^{Ref}(\underline{r})}{\phi_2^{Ref}(\underline{r})}\right.$$

$$\left.+ \Delta(\underline{r},t_0)\right]\phi_2(\underline{r},t) - \lambda_i C_i(\underline{r},t), \quad i = 1, 2, \ldots, 6 \qquad \text{...(II.11c)}$$

$$\frac{\partial I(\underline{r},t)}{\partial t} = \gamma_I\,\Sigma_{f2}(\underline{r})\left[1 + \alpha_f(\underline{r})\,\frac{\phi_2(\underline{r},t) - \phi_2^{Ref}(\underline{r})}{\phi_2^{Ref}(\underline{r})}\right.$$

$$\left.+ \Delta(\underline{r},t_0)\right]\phi_2(\underline{r},t) - \lambda_I I(\underline{r},t) \qquad \text{...(II.11d)}$$

$$\frac{\partial X(\underline{r},t)}{\partial t} = \gamma_x \Sigma_{f2}(\underline{r}) \left[1 + \alpha_f(\underline{r}) \frac{\phi_2(\underline{r},t) - \phi_2^{Ref}(\underline{r})}{\phi_2^{Ref}(\underline{r})} \right.$$

$$\left. + \Delta(\underline{r},t_0) \right] \phi_2(\underline{r},t) + \lambda_I I(\underline{r},t) - \sigma_x X(\underline{r},t) \phi_2(\underline{r},t)$$

$$- \lambda_x X(\underline{r},t) \qquad \ldots \text{(II.11e)}$$

where $\Delta(\underline{r},t_0)$ is a fixed perturbation inserted at $t=t_0$ and unchanged until $t=t_f$.

Note that the superscript 'Ref' is dropped from material properties for convenience.

II.2.2 Separation of Control Function

As mentioned earlier, the control action is considered to be the combination of response to the precursor and the xenon buildup kinematics for a certain perturbation. For a fixed perturbation inserted at $t=t_0$, the former must be accomplished within seconds to compensate most of the reactivity change provided by the perturbation. On the other hand the latter response is slowly activated to eliminate the belated regional subcriticality. Generally the multiplication factor change during the slow transient is negligible compared to the fast transient's. Also, the fission reaction is important for the former and the absorption for the latter.

Considering the above facts, the dual control function may be separately defined;

$$U(\underline{r},t)\,\delta(\underline{r}-\underline{r}_j) = \left[\, U_B(\underline{r},t) + U_s(\underline{r},t) \right] \delta(\underline{r}-\underline{r}_j),$$

$$j = 1, 2, \ldots, I_C \qquad \ldots (II.12)$$

where $U_B(\underline{r},t) = U_B(\underline{r},t)\left[\, 1 - h(t-\tau) \right] + U_B(\underline{r},\tau)\, h(t-\tau)$

the bulk control function,

τ ; small time interval for the bulk control,

$h(t-\tau)$; the Heaviside step function,

$$\begin{cases} = 1, \text{ if } t \geq \tau , \\ = 0, \text{ otherwise}, \end{cases}$$

$U_s(\underline{r},t) = U_s(\underline{r},t)\; h(t-\tau)$

the spatial control function, effective only for $t \geq \tau$.

Although there is definitely some reactivity interface shared by both of the control functions, such a minor non-separable reactivity constraint could be easily relaxed by a controller manoeuvering technique in practice.

Quantitatively, the bulk control function has the worth of the total reactivity difference between the reference state and the saturated perturbed state, and the spatial control function has the time variation of the regional reactivity equivalent to xenon concentration in the region compared to the saturated value.

Under the assumption of insignificancy of the reactivity contribution of the xenon concentration difference between the reference state and the saturated states, the system equation for the bulk control can be approximated to;

$$\frac{1}{v_1}\frac{\partial \phi_1(\underline{r},t)}{\partial t} = \nabla\cdot D_1(\underline{r})\nabla\phi_1(\underline{r},t) - [\Sigma_{a1}(\underline{r}) + \Sigma_{Rem}(\underline{r})]\phi_1(\underline{r},t)$$

$$+ \sum_{i=1}^{6}\lambda_i C_i(\underline{r},t) + (1-\beta)\nu\Sigma_{f2}(\underline{r})\,[1 + \alpha_f(\underline{r})$$

$$\frac{\phi_2(\underline{r},t) - \phi_2^{Ref}(\underline{r})}{\phi_2^{Ref}(\underline{r})} + \Delta(\underline{r},t_0)]\,\phi_2(\underline{r},t) \qquad \ldots(II.13a)$$

$$\frac{1}{v_2}\frac{\partial \phi_2(\underline{r},t)}{\partial t} = \nabla\cdot D_2(\underline{r})\nabla\phi_2(\underline{r},t) - [\Sigma_{a2}(\underline{r}) + \sigma_x X^{Ref}(\underline{r})]\,\phi_2(\underline{r},t)$$

$$+ \Sigma_{Rem}(\underline{r})\,\phi_1(\underline{r},t) - [\sum_{j=1}^{I_c} U_B(\underline{r},t)\left\{1 - h(t-\tau)\right\}$$

$$+ U_B(\underline{r},\tau)\,h(t-\tau)]\,\phi_2(\underline{r},t)\,\delta(\underline{r}-\underline{r}_j) \qquad \ldots(II.13b)$$

and

$$\frac{\partial C_i(\underline{r},t)}{\partial t} = \beta_i\nu\Sigma_{f2}\,[1 + \alpha_f(\underline{r})\,\frac{\phi_2(\underline{r},t) - \phi_2^{Ref}(\underline{r})}{\phi_2^{Ref}(\underline{r})} + \Delta(\underline{r},t_0)]\,\phi_2(\underline{r},t)$$

$$- \lambda_i C_i(\underline{r},t), \quad i = 1, 2, \ldots, 6 \qquad \ldots(II.13c)$$

Emphasizing that our interest is the neutron population response to the control action, the fast mode of the system function, $C_i(\underline{r},t)$ can be approximated to be stationary, i.e.,

$$\frac{\partial C_i(\underline{r},t)}{\partial t} = 0 \quad \text{for } i = 1, 2, \ldots, 6.$$

And eventually we have;

for $0 \leq t < \tau$,

$$\frac{1}{v_1}\frac{\partial \phi_1(\underline{r},t)}{\partial t} = \nabla \cdot D_1(\underline{r}) \nabla \phi_1(\underline{r},t) - [\Sigma_{a1}(\underline{r}) + \Sigma_{Rem}(\underline{r})]\, \phi_1(\underline{r},t)$$

$$+ \nu\Sigma_{f2}(\underline{r}) \left[1 + \alpha_f(\underline{r}) \frac{\phi_2(\underline{r},t) - \phi_2^{Ref}(\underline{r})}{\phi_2^{Ref}(\underline{r})} + \Delta(\underline{r},t_0)\right] \phi_2(\underline{r},t)$$

$$\ldots(II.14a)$$

$$\frac{1}{v_2}\frac{\partial \phi_2(\underline{r},t)}{\partial t} = \nabla \cdot D_2(\underline{r}) \nabla\, \phi_2(\underline{r},t) - [\Sigma_{a1}(\underline{r}) + \sigma_x X^{Ref}(\underline{r})]\, \phi_2(\underline{r},t)$$

$$+ \Sigma_{Rem}(\underline{r})\, \phi_1(\underline{r},t) - \sum_{j=1}^{I_c} U_B(\underline{r},t)\, \phi_2(\underline{r},t)\, \delta(\underline{r}-\underline{r}_j)$$

$$\ldots(II.14b)$$

and for $t > \tau$,

$$0 = \nabla \cdot D_1(\underline{r}) \nabla \phi_1(\underline{r}, \tau) - [\Sigma_{a1}(\underline{r}) + \Sigma_{Rem}(\underline{r})] \phi_1(\underline{r}, \tau)$$

$$+ \nu \Sigma_{f2}(\underline{r}) \left[1 + \alpha_f(\underline{r}) \frac{\phi_2(\underline{r}, \tau) - \phi_2^{Ref}(\underline{r})}{\phi_2^{Ref}(\underline{r})} + \Delta(\underline{r}, t_0) \right] \phi_2(\underline{r}, \tau)$$

...(II.15a)

$$0 = \nabla \cdot D_2(\underline{r}) \nabla \phi_2(\underline{r}, \tau) - [\Sigma_{a2}(\underline{r}) + \sigma_x X^{Ref}(\underline{r})] \phi_2(\underline{r}, \tau)$$

$$+ \Sigma_{Rem}(\underline{r}) \phi_1(\underline{r}, \tau) - \sum_{j=1}^{I_C} U_B(\underline{r}, \tau) \phi_2(\underline{r}, \tau) \delta(\underline{r} - \underline{r}_j)$$

...(II.15b)

In conclusion, we specify the bulk control problem as searching for the control function $U_B(\underline{r}, t)$ to satisfy some stochastic properties of Eqs. (II.14) for $0 < t < \tau$ and static equations (II.15) and condition of Min. $\{ \underline{\phi}(\underline{r}, t) - \underline{\phi}^{Ref}(\underline{r}) \}$.

For the system where only spatial control is considered, and, thus, the neutron population being balanced in a global sense, the system equations should include the iodine-xenon dynamic chain;

$$\frac{1}{v_1}\frac{\partial\phi_1(\underline{r},t)}{\partial t} = \nabla\cdot D_1(\underline{r})\nabla\phi_1(\underline{r},t) - [\Sigma_{a1}(\underline{r}) + \Sigma_{Rem}(\underline{r})]\,\phi_1(\underline{r},t)$$

$$+ \frac{1}{k(t)}\,\nu\Sigma_{f2}(\underline{r})\left[1 + \alpha_f(\underline{r})\,\frac{\phi_2(\underline{r},t) - \phi_2^{Ref}(\underline{r})}{\phi_2^{Ref}(\underline{r})} + \right.$$

$$\left. + \Delta(\underline{r},t_0)\right]\phi_2(\underline{r},t) \qquad \text{...(II.16a)}$$

$$\frac{1}{v_2}\frac{\partial\phi_2(\underline{r},t)}{\partial t} = \nabla\, D_2(\underline{r})\nabla\phi_2(\underline{r},t) - [\Sigma_{a2}(\underline{r}) + \sigma_x X^{Ref}(\underline{r})]\,\phi_2(\underline{r},t)$$

$$- \Sigma_{Rem}(\underline{r})\,\phi_1(\underline{r},t) - \sum_{j=1}^{I_c} U_s(\underline{r},t)\,\phi_2(\underline{r},t)\,\delta(\underline{r}-\underline{r}_j)$$

$$\text{...(II.16b)}$$

$$\frac{\partial I(\underline{r},t)}{\partial t} = \gamma_I\Sigma_{f2}(\underline{r})\left[1 + \alpha_f(\underline{r})\,\frac{\phi_2(\underline{r},t) - \phi_2^{Ref}(\underline{r})}{\phi_2^{Ref}(\underline{r})}\right.$$

$$\left. + \Delta(\underline{r},t_0)\right]\phi_2(\underline{r},t) - \lambda_I I(\underline{r},t) \qquad \text{...(II.16c)}$$

and

$$\frac{\partial X(\underline{r},t)}{\partial t} = \gamma_x\Sigma_{f2}(\underline{r})\left[1 + \alpha_f(\underline{r})\,\frac{\phi_2(\underline{r},t) - \phi_2^{Ref}(\underline{r})}{\phi_2^{Ref}(\underline{r})} + \Delta(\underline{r},t_0)\right]\phi_2(\underline{r},t)$$

$$+ \lambda_I I(\underline{r},t) - \sigma_x X(\underline{r},t)\,\phi_2(\underline{r},t) - \lambda_x X(\underline{r},t) \qquad \ldots(\text{II.16d})$$

Of course, in this case, the precursor kinetics are not necessary because the spatial transient is much slower than the precursor transient. The time dependent multiplication factor k(t) shown in Eq.(II.16a) represents the independence of the control function $U_s(\underline{r},t)$ from the global reactivity contribution. Calculation of k(t) may impose another difficulty which may require computations of diffusion problems with various xenon concentrations. In order to overcome this, we can approximate $k(t) \approx k(t_f)$ without losing any generality. If we have a bulk control system which is well-behaved for the large reactivity $\rho(t_f) \doteq 1 - 1/k(t_f)$, then, it is apparent that the controller will also be well-behaved for the small amount of reactivity $\rho(t) = 1/k(t) - 1/k(t_f)$ through the trajectory of the entire transient.

Now we construct the separated control functions with their own governing equations. However, the set of controllers are restrained by reactivity constraints, which means that they are mutually dependent. We can eliminate these constraints either by relaxing restrictions on the state functions or by annexing conditions on the control functions.

(1) For bulk control;

If we replace the strict condition

$$\int_{\underline{R}} d\underline{r} \left\{ \sum_{j=1}^{I_c} \left(\phi_2^{\text{Ref}}(\underline{r})\right)^* U_B(\underline{r},t)\,\phi_2(\underline{r},t)\,\delta(\underline{r}-\underline{r}_j) \right\} = K\rho(t) \qquad \ldots(\text{II.17})$$

where

$$\rho(t) = \int_{\underline{R}} d\underline{r} \left\{ \left(\phi_1^{Ref}(\underline{r})\right)^* [\nabla \cdot D_1(\underline{r}) \nabla - \Sigma_{a1}(\underline{r}) - \Sigma_{Rem}(\underline{r})] \phi_1(\underline{r},t) \right.$$

$$+ \left(\phi_2^{Ref}(\underline{r})\right)^* \Sigma_{Rem}(\underline{r}) \phi_1(\underline{r},t)$$

$$+ \left(\phi_1^{Ref}(\underline{r})\right)^* \nu\Sigma_{f2}(\underline{r}) \left[1 + \alpha_f(\underline{r}) \frac{\phi_2(\underline{r},t) - \phi_2^{Ref}(\underline{r})}{\phi_2^{Ref}(\underline{r})} + \Delta(\underline{r},t_0)\right] \phi_2(\underline{r},t)$$

$$\left. + \left(\phi_2^{Ref}(\underline{r})\right)^* [\nabla \cdot D_2(\underline{r}) \nabla - \Sigma_{a2}(\underline{r}) - \sigma_x X^{Ref}(\underline{r})] \phi_2(\underline{r}) \right\} \Big/ \kappa , \quad \ldots \text{(II-18)}$$

$$\kappa \equiv \int_{\underline{R}} d\underline{r} \left\{ \left(\phi_1^{Ref}(\underline{r})\right)^* \nu\Sigma_{f2}(\underline{r}) \left[1 + \alpha_f(\underline{r}) \frac{\phi_2(\underline{r},t) - \phi_2^{Ref}(\underline{r})}{\phi_2^{Ref}(\underline{r})} + \Delta(\underline{r},t_0)\right] \phi_2(\underline{r},t) \right\}$$

$\left(\phi_i^{Ref}(r)\right)^*$; the adjoint function of $\phi_i^{Ref}(\underline{r})$,

by the optimal estimation condition

$$\text{Min} \left\{ \int_{\underline{R}} d\underline{r} \left[\sum_{j=1}^{I_c} \left(\phi_1^{Ref}(\underline{r})\right)^* U_B(\underline{r},t) \ \phi_2(\underline{r},t) \ \delta(\underline{r}-\underline{r}_j) \right] - \kappa\rho(t) \right\}^2 \quad \ldots \text{(II-19)}$$

then, we can deal with the bulk control problem for the I_c mutually independent controllers.

(2) For spatial controllers;

From

$$\int_{\underline{R}} d\underline{r} \left\{ \sum_{j=1}^{I_c} \left(\phi_2^{Ref}(\underline{r})\right)^* U_s(\underline{r},t) \ \phi_2(\underline{r},t) \ \delta(\underline{r}-\underline{r}_j) \right\} = 0 \quad \ldots \text{(II.20)}$$

the I_c-th controller can be expressed by other controllers,

$$U_s(\underline{r}_{I_c},t) = - \sum_{j=1}^{I_c-1} \frac{\omega(\underline{r}_j,t)}{\omega(\underline{r}_{I_c},t)} U_s(\underline{r}_j,t) \qquad \text{...(II-21)}$$

where $\omega(\underline{r}_j,t) = [\phi_2^{Ref}(\underline{r}_j)]^* \ \phi_2(\underline{r}_j,t)$

and we have (I_c-1) independent controllers in the form of

$$\sum_{j=1}^{I_c} U_s(\underline{r},t) \ \phi_2(\underline{r},t) \ \delta(\underline{r}-\underline{r}_j)$$

$$= \sum_{j=1}^{I_c-1} [\phi_2(\underline{r},t) - \frac{\omega(\underline{r},t)}{\omega(\underline{r}_{I_c},t)} \phi_2(\underline{r}_{I_c},t)] U_s(\underline{r},t) \ \delta(\underline{r}-\underline{r}_j)$$

...(II.22)

The reason why different criteria were used to relax the controller constraints will be explained later.

II.2.3 Linearization

Now we have derived two sets of coupled non-linear system equations (II.14), (or Eqs. (II.15)) and (II.16) with the (I_c-1) linearly independent controllers of Eqs.(II.20) and (II.22). As reactor designers and operators always want the neutron flux shape to be kept close to the pre-designed reference shape, we can assume the time-variations of state functions in the equations are not significantly deviated from those of the reference state's. Thus we define the first order deviation functions;

$$\left.\begin{aligned}\phi_1(\underline{r},t) &\cong \phi_1^{Ref}(\underline{r}) + \eta_1(\underline{r},t), \\ \phi_2(\underline{r},t) &\cong \phi_2^{Ref}(\underline{r}) + \eta_2(\underline{r},t), \\ I(\underline{r},t) &\cong I^{Ref}(\underline{r}) + \eta_3(\underline{r},t),\end{aligned}\right\} \qquad \ldots(II.23)$$

and $$X(\underline{r},t) \cong X^{Ref}(\underline{r}) + \eta_4(\underline{r},t).$$

Substituting Eq. (II.23) into Eqs. (II.14), (or Eqs. (II.15)) and (II.16), subtracting the reference state equations (II.10) and neglecting the higher order terms, we have;
for the bulk control,

$$\frac{1}{v_1}\frac{\partial \eta_1(\underline{r},t)}{\partial t} = \nabla\cdot D_1(\underline{r})\nabla\,\eta_1(\underline{r},t) - [\Sigma_{a1}(\underline{r}) + \Sigma_{Rem}(\underline{r})]\,\eta_1(\underline{r},t)$$

$$+ \nu\Sigma_{f2}(\underline{r})\,[1 + \alpha_f(\underline{r})]\,\eta_2(\underline{r},t) + \nu\Sigma_{f2}(\underline{r})\,[1 + \Delta(\underline{r},t_0) - \frac{1}{k_0}]\,\phi_2^{Ref}(\underline{r}) \qquad \ldots(II.14a')$$

$$\frac{1}{v_2}\frac{\partial \eta_2(\underline{r},t)}{\partial t} = \nabla\cdot D_2(\underline{r})\nabla\eta_2(\underline{r},t) - [\Sigma_{a2}(\underline{r}) + \sigma_x X^{Ref}(\underline{r})]\,\eta_2(\underline{r},t)$$

$$+ \quad \Sigma_{Rem}(\underline{r}) \quad \eta_1(\underline{r},t) \; - \; \sum_{j=1}^{I_c} U_B(\underline{r},t) \; \phi_2^{Ref}(\underline{r}) \; \delta(\underline{r}-\underline{r}_j)$$

...(II.14b')

, or

$$0 = \nabla \cdot D_1(\underline{r}) \; \nabla \; \eta_1(\underline{r},\tau) \; - \; [\; \Sigma_{a1}(\underline{r}) \; + \; \Sigma_{Rem}(\underline{r})] \; \eta_1(\underline{r},\tau)$$

$$+ \; \nu \, \Sigma_{f2}(\underline{r}) [1 + \; \alpha_f(\underline{r})] \quad \eta_2(\underline{r},\tau) \; + \; \nu \Sigma_{f2}(\underline{r}) \; [1 \; + \; \Delta(\underline{r},t_0) \; - \; \frac{1}{k_0}] \phi_2^{Ref}(\underline{r})$$

...(II.15a')

$$0 = \; \nabla \cdot D_2(\underline{r}) \; \nabla \; \eta_2(\underline{r},\tau) \; - \; [\; \Sigma_{a2}(\underline{r}) \; + \; \sigma_x X^{Ref}(\underline{r})] \; \eta_2(\underline{r},\tau)$$

$$+ \; \Sigma_{Rem}(\underline{r}) \quad \eta_1(\underline{r},\tau) \; - \; \sum_{j=1}^{I_c} U_B(\underline{r},\tau) \; \phi_2^{Ref}(\underline{r}) \; \delta(\underline{r}-\underline{r}_j)$$

...(II.15b')

and, for the spatial control,

$$\frac{1}{v_1} \frac{\partial \eta_1(\underline{r},t)}{\partial t} = \nabla \cdot D_1(\underline{r}) \, \nabla \; \eta_1(\underline{r},t) \; - \; [\; \Sigma_{a1}(\underline{r}) \; + \; \Sigma_{Rem}(\underline{r})] \; \eta_1(\underline{r},t)$$

$$+ \; \frac{1}{k(t_f)} \; \nu \, \Sigma_{f2}(\underline{r}) \; [1 \; + \; \alpha_f(\underline{r})] \quad \eta_2(\underline{r},t)$$

$$+ \ \nu\Sigma_{f2}(\underline{r})\left[\frac{1}{k(t_f)}\left(1 + \Delta(\underline{r},t_0)\right) - \frac{1}{k_0}\right]\phi_2^{Ref}(\underline{r})$$

...(II.16a')

$$\frac{1}{v_2}\ \frac{\partial\eta_2(\underline{r},t)}{\partial t} = \nabla\cdot D_2(\underline{r})\nabla\ \eta_2(\underline{r},t) - \left[\Sigma_{a2}(\underline{r}) + \sigma_x X^{Ref}(\underline{r})\right]\eta_2(\underline{r},t)$$

$$- \ \sigma_x\phi_2^{Ref}(\underline{r})\ \ \eta_4(\underline{r},t) + \Sigma_{Rem}(\underline{r})\ \eta_1(\underline{r},t)$$

$$- \sum_{j=1}^{I_c-1}\left(\phi_2^{Ref}(\underline{r}) - \frac{\omega^{Ref}(\underline{r}_j)}{\omega^{Ref}(\underline{r}_{I_c})}\ \phi_2^{Ref}(\underline{r}_{I_c})\right)U_s(\underline{r},t)\delta(\underline{r}-\underline{r}_j)$$

...(II.16b')

$$\frac{\partial\eta_3(\underline{r},t)}{\partial t} = \gamma_I\ \Sigma_{f2}(\underline{r})\left(1 + \alpha_f(\underline{r})\right)\ \eta_2(\underline{r},t) - \lambda_I\ \eta_3(\underline{r},t)$$

$$+ \ \gamma_I\ \Sigma_{f2}(\underline{r})\ \phi_2^{Ref}(\underline{r})\ \Delta(\underline{r},t_0)\ .$$ ...(II.16c')

and $$\frac{\partial\eta_4(\underline{r},t)}{\partial t} = \gamma_x\ \Sigma_{f2}(\underline{r})\left(1 + \alpha_f(\underline{r})\right)\ \eta_2(\underline{r},t) + \lambda_I\ \eta_3(\underline{r},t)$$

$$- \sigma_x X^{Ref}(\underline{r})\ \eta_2(\underline{r},t) - \sigma_x\ \phi_2^{Ref}(\underline{r})\ \eta_4(\underline{r},t)$$

$$- \lambda_x\ \eta_4(\underline{r},t) + \gamma_x\ \Sigma_{f2}(\underline{r})\ \phi_2^{Ref}(\underline{r})\ \Delta(\underline{r},t_0)$$

...(II.16d')

II.2.4 Physical Significances of Linearized State Functions

Examining Eqs. (II.15') without the fixed perturbation term $\Delta(\underline{r},t_0)$, the optimal controllers $U_B(\underline{r},\tau)$ will be chosen such that the reactivity of the reference state is to be compensated by time-invariant controllers $U_B(\underline{r},\tau)$ with minimization of static flux deviation $\eta(\underline{r},\tau)$, i.e., controller induced perturbation. Therefore, the constant term $\nu\Sigma_{f2}(\underline{r})\left(1 - \frac{1}{k_0}\right)\phi_2^{Ref}(\underline{r})$ is the dominant source of reactivity. Thus one intent of the bulk controller design strategy is to search for the static controllers satisfying the static reactivity condition and the static flux constraint. This is sufficient to design the bulk controllers if the fuelling ideally controls the flux shape.

For non-ideal fuelling situations, let's transform the time variable in Eqs. (II.14') into $t' = v_2 t$ and, then, the fixed perturbation term $\Delta(\underline{r},t_0)$ can be considered as a time dependent impulsive function. Still we hope the perturbation can be expressed by a single term, i.e., fractional change in the fission cross section. If we take $\Delta(\underline{r},t)$ as a random parameter with the magnitude of fuelling effect, then, Eqs.(II.14') represent the dynamic system whose optimal bulk controller can be designed to eliminate random fuelling perturbations. In this case, the time invariant reactivity term $\nu\Sigma_{f2}(1 - \frac{1}{k_0})\ \phi_2^{Ref}(\underline{r})$ and correlated amount of $U_B(\underline{r},t)\ \phi_2^{Ref}(\underline{r})$ must be balanced as in the static problem Eqs.(II.15'). Therefore the bulk controller design problem is the optimum estimation of the linear stochastic system.

For the spatial control problem, the inhomogeneous terms in Eqs. (II.16') are neglected for the reasons of;

(1) $\frac{1}{k(t_f)}\left(1 + \Delta(\underline{r},t_0)\right) - \frac{1}{k_0} \approx 0,$

(2) $\Delta(\underline{r},t_0)$ is very localized small fraction being neglected in the iodine-xenon production terms.

Now we have linear equations as the control objectives;

(1) the bulk control for the non-ideal fuelling case,

$$\frac{1}{\mu}\,\frac{\partial\eta_1(\underline{r},t'/v_2)}{\partial t'} = \nabla\cdot D_1(\underline{r})\,\nabla\,\eta_1(\underline{r},t'/v_2) - \left[\Sigma_{a1}(\underline{r}) + \Sigma_{Rem}(\underline{r})\right]\times$$

$$\eta_1(\underline{r},t'/v_2) + \nu\Sigma_{f2}(\underline{r})\left[1 + \alpha_f(\underline{r})\right]\eta_2(\underline{r},t'/v_2)$$

$$+ \ \nu \Sigma_{f2}(\underline{r}) \ \phi_2^{Ref}(\underline{r}) \ \Delta(\underline{r}, t'/v_2) \qquad \text{...(II.24a)}$$

$$\frac{\partial \eta_2(\underline{r}, t'/v_2)}{\partial t'} = \nabla \cdot D_2(\underline{r}) \nabla \ \eta_2(\underline{r}, t'/v_2) - \left[\Sigma_{a2}(\underline{r}) + \sigma_x X^{Ref}(\underline{r}) \right] \eta_2(\underline{r}, t'/v_2)$$

$$+ \ \Sigma_{Rem}(\underline{r}) \ \ \eta_1(\underline{r}, t'/v_2) - \sum_{j=1}^{I_c} U_B(\underline{r}, t'/v_2) \ \ \phi_2^{Ref}(\underline{r}) \ \delta(\underline{r}-\underline{r}_j)$$

$$\text{...(II.24b)}$$

where $\mu = v_1/v_2$ and $t' = v_2 t$.

(2) the spatial control,

$$\frac{1}{v_1} \frac{\partial \eta_1(\underline{r}, t)}{\partial t} = \nabla \cdot D_1(\underline{r}) \nabla \ \eta_1(\underline{r}, t) - \left[\Sigma_{a1}(\underline{r}) + \Sigma_{Rem}(\underline{r}) \right] \eta_1(\underline{r}, t)$$

$$+ \frac{1}{k(t_f)} \ \nu \Sigma_{f2}(\underline{r}) \left[1 + \alpha_f(\underline{r}) \right] \eta_2(\underline{r}, t) \quad \text{...(II.25a)}$$

$$\frac{1}{v_2} \frac{\partial \eta_2(\underline{r}, t)}{\partial t} = \nabla \cdot D_2(\underline{r}) \nabla \ \eta_2(\underline{r}, t) - \left[\Sigma_{a2}(\underline{r}) + \sigma_x X^{Ref}(\underline{r}) \right] \eta_2(\underline{r}, t)$$

$$- \sigma_x \, \phi_2^{Ref}(\underline{r}) \;\; \eta_4(\underline{r},t) \; + \; \Sigma_{Rem}(\underline{r}) \;\; \eta_1(\underline{r},t)$$

$$- \sum_{j=1}^{I_c - 1} \left[\phi_2^{Ref}(\underline{r}) - \frac{\omega^{Ref}(\underline{r}_j)}{\omega^{Ref}(\underline{r}_{I_c})} \; \phi_2^{Ref}(\underline{r}_{I_c}) \right] \dot{U}_s(\underline{r},t) \; \delta(\underline{r}-\underline{r}_j)$$

...(II.25b)

$$\frac{\partial \eta_3(\underline{r},t)}{\partial t} = \gamma_I \;\; \Sigma_{f2}(\underline{r}) \left[1 + \alpha_f(\underline{r}) \right] \; \eta_2(\underline{r},t) \; - \; \lambda_I \;\; \eta_3(\underline{r},t)$$

...(II.25c)

and $$\frac{\partial \eta_4(\underline{r},t)}{\partial t} = \gamma_x \;\; \Sigma_{f2}(\underline{r}) \left[1 + \alpha_f(\underline{r}) \right] \eta_2(\underline{r},t) \; + \; \lambda_I \;\; \eta_3(\underline{r},t)$$

$$- \; \sigma_x X^{Ref}(\underline{r}) \;\; \eta_2(\underline{r},t) \; - \; \sigma_x \, \phi_2^{Ref}(\underline{r}) \;\; \eta_4(\underline{r},t)$$

$$- \; \lambda_x \, \eta_4(\underline{r},t)$$ ...(II.25d)

Boundary conditions equivalent to Eqs. (II.7) and (II.8) are;

$\eta_i(\underline{R},t) = 0$, i=1, 2, 3 and 4,

$$U_s(\underline{R},t) = U_B(\underline{R},t) = 0, \qquad \ldots(\text{II}.26)$$

$$\Delta(\underline{R},t) = 0.$$

III. CONTROLLER LOCATION DETERMINATION BY PROPERTIES OF OPTIMALITY IN SPATIAL CONTROL

III.1 Modal Control Theory for Reactor Dynamics

III.1.1 Properties of Space Modes

The linearized homogeneous time-invariant space-dynamic equations (II.25) have the simplified matrix form

$$\dot{\eta}(\underline{r},t) = \mathbf{A}\,\eta(\underline{r},t) + \mathbf{B}\,u(\underline{r},t) \qquad \text{...(III.1)}$$

with a given set of boundary conditions. The solutions of equations $\eta(r,t)$ in the absence of controllers may be uniquely determined by the expansion with the system eigenfunctions[(101),(102)]:

$$\eta(\underline{r},t) = \sum_{n=1}^{\infty} c_n \underline{e}_n(\underline{r}) \exp(\omega_n t) \qquad \text{...(III.2)}$$

where the eigenvalues, ω_n, and the eigenfunctions, $e_n(r)$, are obtained by

$$\mathbf{A} e_n(\underline{r}) = \omega_n e_n(\underline{r}), \quad n = 0, 1, 2, \ldots, \infty \qquad \text{...(III.3)}$$

the C_n are given by the relationship

$$C_n = \int_{\underline{R}} d\underline{r} \left\{ \underline{e}_n^*(r) \; \eta(\underline{r},0) \right\} , \qquad n = 0, 1, 2, .., \infty \quad \text{(III.4)}$$

and $\underline{e}_n^*(\underline{r})$, the eigenfunctions of the adjoint system satisfy the ortho-normality conditions,

$$\int_{\underline{R}} d\underline{r} \left\{ \underline{e}_n^*(\underline{r}) \; w(\underline{r}) \; \underline{e}_n(r) \right\} = \delta_{mn} \qquad \text{...(III.5)}$$

$w(\underline{r})$ is a weighting function, and

δ_{mn} is Kronecker delta function.

It is assumed here that $\mathbf{A}$ is such that ω_n's are not degenerated

The idea of the modal expansion technique is usually challenged by several difficulties, when applied to the nuclear reactor system even with the linearized form of equations. S. Kaplan (78) examined several expansion schemes which had been proposed for reactor space-time problems, and attempted to devise expansion schemes which would have displayed the finality in a given problem. He suggested the specific mode set which is exactly the same as the eigenfunctions in Eq. (III.3). The so called 'Kaplan modes' have the most proper characteristics to describe the xenon spatial problems. But, generally, even if such modes would exist, it is very difficult to generate the modes from system equations and to guarantee their completeness.

Alternatively, if we have an orthonormal set of base functions $\varphi_i(\underline{r})$, which can be easily generated and which are assured of convergence and completeness, we may expand $\underline{e}_n(\underline{r})$ in the form of[102],[103]

$$\underline{e}_n(\underline{r}) = \sum_{i=0}^{L} a_i \ \varphi_i(\underline{r}). \qquad \text{...(III.6)}$$

where the expansion coefficient a_i is given by

$$a_i = \int_{\underline{R}} d\underline{r} \left\{ \varphi_i^*(\underline{r}) \right\}^T \underline{e}_n(\underline{r}) \qquad \text{...(III.7)}$$

we, then, have

$$\mathbf{A}\,\underline{e}_n(\underline{r}) = \mathbf{A}\left[\sum_{i=0}^{L} a_i \,\varphi_i(\underline{r})\right]$$

$$= \sum_{i=0}^{L} a_i \left[\mathbf{A}\,\varphi_i(\underline{r})\right] \qquad \text{...(III.8)}$$

The relationships between the properties of $\underline{e}_n(\underline{r})$ and $\varphi_i(\underline{r})$ should be established by examining the submatrices of $\mathbf{A}$, in terms of their physical significance. For example, the iodine components of eigenfunctions, $\underline{e}_n(\underline{r})$ are closely related only with their thermal flux components . And the xenon components are related only to the thermal flux and the iodine components. Thus, we can introduce the approximation that the iodi[illegible]xenon components can be expressed only in terms of neutron flux expansion functions.

Those decoupled properties of the system matrix $\mathbf{A}$ hint at the existence of a simple method which could possibly generate the base function, $\varphi_i(\underline{r})$ by isolating the neutron diffusion equations from the system equations.

Also we assume that, if the open-loop solution of Eq.(III.1) has the form of Eq. (III.2), the eigenvalues and eigenfunctions of the closed-loop solution of Eq. (III.1) can be expressed by a linear combination of the open-loop system matrix $\mathbf{A}$ [75]. The closed-loop system, then, should be stabilizable by controller feedback.

III.1.2 λ-Modes as the Base Functions

The lack of existence and convergence of the Kaplan modes is possibly avoided when we use the λ-mode approximation[104] which accounts explicitly for the spatial dependence of the neutron fluxes and uses conventional diffusion theory calculations.

We define the λ-modes as being the eigenfunctions which satisfy the reference state neutron balance equations;

$$-\nabla \cdot D_1(\underline{r})\,\nabla \varphi_{n,1}(\underline{r}) + [\Sigma_{a1}(\underline{r}) + \Sigma_{Rem}(\underline{r})]\,\varphi_{n,1}(\underline{r})$$

$$= \frac{1}{k_n}\,\nu\Sigma_{f2}(\underline{r})\quad \varphi_{n,2}(\underline{r}) \qquad \text{...(III.9a)}$$

$$-\nabla \cdot D_2(\underline{r})\,\nabla \varphi_{n,2}(\underline{r}) + [\Sigma_{a2}(\underline{r}) + \sigma_x X^{Ref}(\underline{r})]\,\varphi_{n,2}(\underline{r})$$

$$= \Sigma_{Rem}(\underline{r})\quad \varphi_{n,1}(\underline{r}) \qquad \text{...(III.9b)}$$

or in matrix form

$$\mathcal{L}\varphi_n = \frac{1}{k_n}\mathcal{M}\varphi_n\,, \qquad n = 0,\ 1,\ 2,.. \qquad \text{(III.9c)}$$

where $\mathcal{L}$ and $\mathcal{M}$ are regarded as the destruction and production matrices, respectively, and $1/k_n$ ($\equiv \lambda_n$) is the n-th eigenvalue. Note that the fundamental mode, i.e., the mode corresponding to the largest eigenvalue, $1/k_0$ is equivalent to the solution of Eqs. (II.10).

The adjoints of λ-eigenfunctions also satisfy the adjoint equations of Eqs. (III.9), which are;

$$- \nabla \cdot D_1(\underline{r}) \nabla \varphi^*_{m,1}(\underline{r}) + [\Sigma_{a1}(\underline{r}) + \Sigma_{Rem}(\underline{r})] \varphi^*_{m,1}(\underline{r})$$

$$= \frac{1}{k_m} \Sigma_{Rem}(\underline{r}) \varphi^*_{m,2}(\underline{r}) \qquad \text{...(III.10a)}$$

$$- \nabla \cdot D_2(\underline{r}) \nabla \varphi^*_{m,2}(\underline{r}) + [\Sigma_{a2}(\underline{r}) + \sigma_x X^{Ref}(\underline{r})] \varphi^*_{m,2}(\underline{r})$$

$$= \nu\Sigma_{f2}(\underline{r}) \varphi^*_{m,1}(\underline{r}), \quad m = 0, 1, 2, .. \qquad \text{(III.10b)}$$

or in matrix form,

$$\mathcal{L}^* \varphi^*_m = \frac{1}{k_m} \mathcal{M}^* \varphi^*_m , \quad m = 0, 1, 2, .. \qquad \text{(III.10c)}$$

Multiplying Eqs. (III.9a) and (III.9b) by $\varphi^*_{m,1}(\underline{r})$ and $\varphi^*_{m,2}(\underline{r})$, respectively, and similarly Eqs.(III.10a) and (III.10b) by $\varphi_{n,1}(\underline{r})$ and $\varphi_{n,2}(\underline{r})$, integrating over the reactor volume $\underline{R}$ and subtracting, we get the necessary relationship to satisfy the biorthogonality of modes;

$$\frac{1}{k_n} \int_{\underline{R}} d\underline{r} \; (\varphi^*_{m,1}(\underline{r}) \; \nu\Sigma_{f2}(\underline{r}) \; \varphi_{n,2}(\underline{r}))$$

$$= \frac{1}{k_m} \int_{\underline{R}} d\underline{r} \left(\varphi^*_{m,2}(r) \ \Sigma_{Rem}(\underline{r}) \ \varphi_{n,1}(\underline{r}) \right)$$

...(III.11a)

and

$$\int_{\underline{R}} d\underline{r} \left(\varphi^*_{m,1}(\underline{r}) \ \nu\Sigma_{f2}(\underline{r}) \ \varphi_{n,2}(\underline{r}) \right)$$

$$= \int_{\underline{R}} d\underline{r} \left(\varphi^*_{m,2}(\underline{r}) \ \Sigma_{Rem}(\underline{r}) \ \varphi_{n,1}(\underline{r}) \right)$$

...(III.11b)

eventually

$$\left(\frac{1}{k_n} - \frac{1}{k_m} \right) \int_{\underline{R}} d\underline{r} \left(\varphi^*_{m,2}(\underline{r}) \ \Sigma_{Rem}(\underline{r}) \ \varphi_{n,1}(\underline{r}) \right)$$

$$= \left(\frac{1}{k_n} - \frac{1}{k_m} \right) \int_{\underline{R}} d\underline{r} \left(\varphi^*_{m,1}(\underline{r}) \ \nu\Sigma_{f2}(\underline{r}) \ \varphi_{n,2}(\underline{r}) \right) = 0$$

...(III.12)

Thus, if $n \neq m$,

$$\int_{\underline{R}} d\underline{r} \left\{ \varphi^*_{m,2}(\underline{r}) \, \Sigma_{Rem}(\underline{r}) \; \varphi_{n,1}(\underline{r}) \right\}$$

$$= \int_{\underline{R}} d\underline{r} \left\{ \varphi^*_{m,1}(\underline{r}) \; \nu\Sigma_{f2}(\underline{r}) \; \varphi_{n,2}(\underline{r}) \right\} = 0 \quad \text{...(III.13a)}$$

and, if n = m,

$$\int_{\underline{R}} d\underline{r} \left\{ \varphi^*_{n,2}(\underline{r}) \; \Sigma_{Rem}(\underline{r}) \, \varphi_{n,1}(\underline{r}) \right\}$$

$$= \int_{\underline{R}} d\underline{r} \left\{ \varphi^*_{n,1}(\underline{r}) \; \nu\Sigma_{f2}(\underline{r}) \; \varphi_{n,2}(\underline{r}) \right\} = N_n^2 \quad \text{...(III.13b)}$$

where N_n^2 ; a constant, normalization factor.

Now we have the normalizable biorthogonal λ-modes and their adjoints which will replace the $\varphi_i(\underline{r})$ and $\varphi^*_j(\underline{r})$'s in Eqs. (III.9) and (III.10). These orthonormal modes are[105];

$$\left.\begin{aligned}
\psi_{n,1}(\underline{r}) &= \frac{1}{N_n} \left[\Sigma_{Rem}(\underline{r})\right]^{\frac{1}{2}} \; \varphi_{n,1}(\underline{r}) \\
\psi_{m,2}(\underline{r}) &= \frac{1}{N_n} \left[\nu\Sigma_{f2}(\underline{r})\right]^{\frac{1}{2}} \; \varphi_{n,2}(\underline{r}) \\
\psi^*_{n,1}(\underline{r}) &= \frac{1}{N_n} \left[\nu\Sigma_{f2}(\underline{r})\right]^{\frac{1}{2}} \; \varphi^*_{n,1}(\underline{r})
\end{aligned}\right\} \quad \text{...(III.14)}$$

and

$$\psi^*_{n,2}(\underline{r}) = \frac{1}{N_n}\left[\Sigma_{Rem}(\underline{r})\right]^{\frac{1}{2}} \varphi^*_{n,2}(r)$$

Note that $\nu\Sigma_{f2}(\underline{r})$ and $\Sigma_{Rem}(r)$ are scalar functions and, thus, are interchangable with $\varphi^*_{n,1}(\underline{r})$ and $\varphi^*_{n,2}(\underline{r})$. Therefore, the biorthonomality conditions are obtained as

$$\int_{\underline{R}} d\underline{r} \left\{ \psi^*_{m,2}(\underline{r}) \quad \psi_{n,1}(\underline{r}) \right\}$$

$$= \int_{\underline{R}} d\underline{r} \left\{ \psi^*_{m,1}(\underline{r}) \quad \psi_{n,2}(\underline{r}) \right\} = \delta_{mn}, \quad \ldots(\text{III.15})$$

It has been proved that the eigenfunction $\underline{e}_n(r)$ of the system equation (III.1) could be expanded in terms of λ-modes with reasonable accuracy for the CANDU reactor system[105],[106].

III.1.3 Modal Dynamic Equations

Rewriting Eq. (III.2) with the λ-modes, $\varphi_i(\underline{r})$, of Eq.(III.14) we have

$$\eta_1(\underline{r},t) = \sum_{n=0}^{\infty} c_n \sum_{l=0}^{L} a_{l,1} \quad \varphi_{l,1}(r) \exp(\omega_n t)$$

$$= \sum_{l=0}^{L} b_{l,1}(t)\ \varphi_{l,1}(\underline{r}) = N_n \sum_{l=0}^{L} b_{l,1}(t)\ \psi_{l,1}(\underline{r})/(\Sigma_{Rem}(\underline{r}))^{\frac{1}{2}} \qquad \text{...(III.16a)}$$

where

$$b_{l,1}(t) = a_{l,1} \sum_{n=0} C_n \exp(\omega_n t),$$

and, similarly

$$\eta_2(\underline{r},t) = N_n \sum_{l=0}^{L} b_{l,2}(t)\ \psi_{l,2}(\underline{r})/(\nu\Sigma_{f2}(\underline{r}))^{\frac{1}{2}} \qquad \text{...(III.16b)}$$

$$\eta_3(\underline{r},t) = N_n \sum_{l=0}^{L} b_{l,3}(t)\ \psi_{l,2}(\underline{r})/(\nu\Sigma_{f2}(\underline{r}))^{\frac{1}{2}} \qquad \text{...(III.16c)}$$

$$\eta_4(\underline{r},t) = N_n \sum_{l=0}^{L} b_{l,4}(t)\ \psi_{l,2}(\underline{r})/(\nu\Sigma_{f2}(\underline{r}))^{\frac{1}{2}} \qquad \text{...(III.16d)}$$

Substituting Eqs. (III.16) into the system equations (II.25), replacing the Laplacian terms by the λ-mode equations (III.9) and (III.10), dividing both sides of the thermal neutron diffusion equation by $\left\{\Sigma_{Rem}(\underline{r})\right\}^{1/2}$, dividing the fast neutron diffusion equation by $\left\{\nu\Sigma_{f2}(\underline{r})\right\}^{1/2}$ and multiplying the iodine and xenon equations by $\left\{\nu\Sigma_{f2}(r)\right\}^{1/2}$. We have;

$$\frac{1}{v_1} \sum_{l=0}^{L} \dot{b}_{l,1}(t) \left\{\Sigma_{Rem}(\underline{r})\,\nu\Sigma_{f2}(\underline{r})\right\}^{\frac{1}{2}} \psi_{l,1}(\underline{r})$$

$$= \sum_{l=0}^{L} b_{l,2}(t) \frac{1}{k(t_f)} \left| 1 + \alpha_f(\underline{r}) \right| \psi_{l,2}(\underline{r})$$

$$- \sum_{l=0}^{L} b_{l,1}(t) \frac{1}{k_1} \psi_{l,2}(\underline{r}) \qquad \ldots\text{(III.17a)}$$

$$\frac{1}{v_2} \sum_{l=0}^{L} \dot{b}_{l,2}(t) \left| \Sigma_{Rem}(\underline{r}) \; \nu\Sigma_{f2}(\underline{r}) \right|^{-\frac{1}{2}} \psi_{l,2}(\underline{r})$$

$$= \sum_{l=0}^{L} b_{l,1}(t) \psi_{l,1}(\underline{r}) - \sum_{l=0}^{L} b_{l,2}(t) \psi_{l,1}(\underline{r})$$

$$- \sum_{l=0}^{L} b_{l,4}(t) \sigma_x \phi_2^{Ref}(\underline{r}) \left| \nu\Sigma_{f2}(\underline{r}) \; \Sigma_{Rem}(\underline{r}) \right|^{-\frac{1}{2}} \psi_{l,2}(\underline{r})$$

$$- \sum_{j=1}^{I_c-1} \left\{ \phi_2^{Ref}(\underline{r}) - \frac{\omega^{Ref}(\underline{r}_j)}{\omega^{Ref}(\underline{r}_{I_c})} \phi_2^{Ref}(\underline{r}_{I_c}) \right\} \left| \Sigma_{Rem}(\underline{r}) \right|^{\frac{1}{2}} x$$

$$U_s(\underline{r},t) \; \delta(\underline{r}-\underline{r}_j) \qquad \ldots\text{(III.17b)}$$

$$\sum_{l=0}^{L} \dot{b}_{l,3}(t) \psi_{l,2}(\underline{r}) = \sum_{l=0}^{L} b_{l,2}(t) \gamma_I \Sigma_{f2}(\underline{r}) \left| 1 + \alpha_f(\underline{r}) \right| \psi_{l,2}(\underline{r})$$

$$- \sum_{l=0}^{L} b_{l,3}(t) \lambda_I \psi_{l,2}(\underline{r}) \qquad \ldots\text{(III.17c)}$$

and

$$\sum_{l=0}^{L} \dot{b}_{l,4}(t)\,\psi_{l,2}(\underline{r}) = \sum_{l=0}^{L} b_{l,2}(t)\,\gamma_x\,\Sigma_{f2}(\underline{r})\{1+\alpha_f(\underline{r})\}\,\psi_{l,2}(\underline{r})$$

$$+\sum_{l=0}^{L} b_{l,3}(t)\,\lambda_I\,\psi_{l,2}(\underline{r}) - \sum_{l=0}^{L} b_{l,2}(t)\,\sigma_x\,X^{Ref}(\underline{r})\,\psi_{l,2}(\underline{r})$$

$$-\sum_{l=0}^{L} b_{l,4}(t)\,\sigma_x\,\phi_2^{Ref}(\underline{r})\,\psi_{l,2}(\underline{r}) - \sum_{l=0}^{L} b_{l,4}(t)\,\lambda_x\,\psi_{l,2}(\underline{r})$$

...(III.17d)

where

$$\dot{b}_{l,i}(t) = \frac{d}{dt}\left[b_{l,i}(t)\right].$$

Multiplying $\psi^*_{m,1}(\underline{r})$ on Eqs. (III.17a), (III.17c) and (III.17d) and $\psi^*_{m,2}(\underline{r})$ on Eq. (III.17d), and integrating over the reactor volume, $\underline{R}$, then by the biorthonormality condition Eq. (III.15), we have;

$$\frac{1}{v_1}\sum_{l=0}^{L} \dot{b}_{l,1}(t)\int_{\underline{R}} d\underline{r}\left\{\psi^*_{m,1}(\underline{r})\left[\Sigma_{Rem}(\underline{r})\,\nu\Sigma_{f2}(\underline{r})\right]^{-\frac{1}{2}}\psi_{l,1}(\underline{r})\right\}$$

$$= \frac{1}{k(t)}\,b_{m,2}(t) - \frac{1}{k_m}\,b_{m,1}(t)$$

$$+ \sum_{l=0}^{L} b_{l,2}(t) \frac{1}{k(t_f)} \int_{R} d\underline{r} \left\{ \psi^{*}_{m,1}(\underline{r})\, \alpha_f(\underline{r})\, \psi_{l,2}(\underline{r}) \right\}$$

...(III.17a')

$$\frac{1}{v_2} \sum_{l=0}^{L} \dot{b}_{l,2}(t) \int_{R} d\underline{r} \left\{ \psi^{*}_{m,2}(\underline{r}) \left[\Sigma_{Rem}(\underline{r})\, \nu\Sigma_{f2}(\underline{r})\right]^{-\frac{1}{2}} \psi_{l,2}(\underline{r}) \right\}$$

$$= b_{m,1}(t) - b_{m,2}(t) - \sum_{l=0}^{L} b_{l,4}(t) \int_{R} d\underline{r} \left\{ \psi^{*}_{m,2}(\underline{r})\, \sigma_x\, \phi_2^{Ref}(\underline{r})\, x \right.$$

$$\left. \left[\Sigma_{Rem}(\underline{r})\, \nu\Sigma_{f2}(\underline{r})\right]^{-\frac{1}{2}} \right\} - \sum_{j=1}^{I_c-1} \psi^{*}_{m,2}(\underline{r}_j) \left\{ \phi_2^{Ref}(\underline{r}_j) - \right.$$

$$\left. \frac{\omega^{Ref}(\underline{r}_j)}{\omega^{Ref}(\underline{r}_{I_c})} \phi_2^{Ref}(\underline{r}_{I_c}) \right\} \left[\Sigma_{Rem}(\underline{r}_j)\right]^{-\frac{1}{2}} U_s^j(t), \quad ..(III.17b')$$

$$\dot{b}_{m,3}(t) = -\lambda_I b_{m,3}(t)$$

$$+ \sum_{l=0}^{L} b_{l,2}(t) \int_{R} d\underline{r} \left\{ \psi^{*}_{m,1}(\underline{r})\, \gamma_I\, \Sigma_{f2}(\underline{r}) \left[1 + \alpha_f(\underline{r})\right] \psi_{l,2}(\underline{r}) \right\},$$

...(III.17c')

and

$$b_{m,4}(t) = \lambda_I b_{m,3}(t) - \lambda_x b_{m,4}(t)$$

$$+ \sum_{l=0}^{L} b_{l,2}(t) \int_{\underline{R}} d\underline{r} \left\{ \psi^*_{m,1}(\underline{r}) \; \gamma_x \Sigma_{f2}(\underline{r}) \left[1 + \alpha_f(\underline{r}) \right] \psi_{l,2}(\underline{r}) \right\}$$

$$- \sum_{l=0}^{L} b_{l,2}(t) \int_{\underline{R}} d\underline{r} \left\{ \psi^*_{m,1}(\underline{r}) \; \sigma_x X^{Ref}(\underline{r}) \quad \psi_{l,2}(\underline{r}) \right\}$$

$$- \sum_{l=0}^{L} b_{l,4}(t) \int_{\underline{R}} d\underline{r} \left\{ \psi^*_{m,1}(\underline{r}) \; \sigma_x \phi_2^{Ref}(\underline{r}) \quad \psi_{l,2}(\underline{r}) \right\}.$$

...(III.17d')

For convenience, we express these equations in matrix form;

$$\left[\begin{array}{c|c|c|c} \Lambda_{11} & 0 & 0 & 0 \\ \hline 0 & \Lambda_{22} & 0 & 0 \\ \hline 0 & 0 & I & 0 \\ \hline 0 & 0 & 0 & I \end{array}\right] \left[\begin{array}{c} B_1 \\ \hline B_2 \\ \hline B_3 \\ \hline B_4 \end{array}\right] = \left[\begin{array}{c|c|c|c} A_{11} & A_{12} & 0 & 0 \\ \hline A_{21} & A_{22} & 0 & A_{24} \\ \hline 0 & A_{32} & A_{33} & 0 \\ \hline 0 & A_{42} & A_{33} & A_{44} \end{array}\right] \left[\begin{array}{c} B_1 \\ \hline B_2 \\ \hline B_3 \\ \hline B_4 \end{array}\right] + \left[\begin{array}{c} 0 \\ \hline E_2 \\ \hline 0 \\ \hline 0 \end{array}\right] \Pi_s$$

...(III.18)

where Λ_{ii} are (L+1)x(L+1) square matrices whose elements are

$$\left[\Lambda^{ii}_{ml}\right] = \frac{1}{v_i} \int_{\underline{R}} d\underline{r} \left\{ \psi^*_{m,i}(\underline{r}) \left[\Sigma_{Rem}(\underline{r}) \; \nu\Sigma_{f2}(\underline{r}) \right]^{-\frac{1}{2}} \; \psi_{l,i}(\underline{r}) \right\}$$

for i = 1 or 2,

I is a (L+1)x(L+1) identity matrix,

B_i is a (L+1) dimensional column vector,

$$\left[B_i\right] = \left[\, b_{0,i}(t),\ b_{1,i}(t), \ldots, b_{L,i}(t)\,\right]^T,$$

$$i = 1,\ 2,\ 3 \text{ and } 4,$$

A_{1i} are (L+1)x(L+1) square matrices whose elements are

$$\left[A^{11}_{ml}\right] = -\frac{1}{k_1}\,\delta_{lm},$$

$$\left[A^{12}_{ml}\right] = \frac{1}{k(t_f)} \int_{\underline{R}} d\underline{r} \left\{\psi^*_{m,1}(\underline{r})\ \alpha_f(\underline{r})\ \psi_{l,2}(\underline{r})\right\} + \delta_{ml},$$

A_{2i} are (L+1)x(L+1) square matrices whose elements are

$$\left[A^{21}_{ml}\right] = \delta_{ml},$$

$$\left[A^{22}_{ml}\right] = -\delta_{ml},$$

$$\left[A^{24}_{ml}\right] = -\int_{\underline{R}} d\underline{r} \left\{\psi^*_{m,2}(\underline{r})\ \sigma_x\,\phi_2^{Ref}(\underline{r})\left[\Sigma_{Rem}(r)\ \nu\Sigma_{f2}(\underline{r})\right]^{-\frac{1}{2}}\right.$$

$$\left. \times\ \psi_{l,2}(r)\right\},$$

A_{3i} are (L+1)x(L+1) square matrices whose elements are

$$\left[A_{ml}^{32}\right] = \int_{\underline{R}} d\underline{r} \left\{ \psi_{m,1}^{*}(\underline{r})\ \gamma_I\ \Sigma_{f2}(\underline{r})\ \left[1 + \alpha_f(\underline{r})\right] \psi_{1,2}(\underline{r}) \right\},$$

$$\left[A_{ml}^{33}\right] = -\lambda_I\, \delta_{ml},$$

A_{4i} are (L+1)x(L+1) square matrices whose elements are

$$\left[A_{ml}^{41}\right] = \int_{\underline{R}} d\underline{r} \left\{ \psi_{m,1}^{*}(\underline{r}) \Big(\gamma_x\ \Sigma_{f2}(\underline{r})\ \left[1 + \alpha_f(\underline{r})\right] - \sigma_x X^{Ref}(\underline{r}) \Big)\ \psi_{1,2}(\underline{r}) \right\}$$

$$\left[A_{ml}^{44}\right] = -\lambda_x \delta_{ml} - \int_{\underline{R}} d\underline{r} \left\{ \psi_{m,1}^{*}(\underline{r})\ \sigma_x\ \phi_2^{Ref}(\underline{r})\ \psi_{1,2}(\underline{r}) \right\},$$

E_2 is a (L+1)x(I_c-1) matrix whose elements are

$$\left[E_{mj}\right] = \psi_{m,2}^{*}(\underline{r}_j) \left(\phi_2^{Ref}(\underline{r}_j) - \frac{\omega^{Ref}(\underline{r}_j)}{\omega^{Ref}(\underline{r}_{I_c})}\, \phi_2^{Ref}(\underline{r}_{I_c}) \right) \times \left[\Sigma_{Rem}(\underline{r}_j)\right]^{\frac{1}{2}}$$

and

Π_s is a (I_c-1) column vector

$$\Pi_s = \left[U_s^1(t),\ U_s^2(t), \ldots,\ U_s^{I_c-1}(t) \right]^T.$$

Here $U_s^j(t) = U_s(\underline{r}_j, t)$ and the superscript "T" refers to the transpose of matrices.

III.2 Optimality Criteria for Spatial Controllers

III.2.1 Reduced-Order Quadratic Linear Regulator

We now consider the dynamic system described by Eq.(III.18) in a simple form

$$\Lambda \dot{\mathbf{B}}(t) = \mathbf{A}\mathbf{B}(t) + \mathbf{E}(\underline{r})\,\Pi_s(\underline{r},t) \qquad \ldots\text{(III.19)}$$

for $\{\underline{r} \mid \underline{r}_1, \underline{r}_2, \ldots, \underline{r}_{Ic-1}\}$, unknown number of locations of controllers with an arbitrary disturbance at time $t = t_0$ being any of $\mathbf{B}(t_0) \neq 0$ but bounded $\mathbf{B}(t)$. The coefficient matrix $\mathbf{E}(\underline{r})$ is to be uniquely determined whenever the locations of controllers are known inside the reactor (cf. Eq. III.18).

The servomechanism problem[107] can be applied to find an optimal control function $\Pi_s(\underline{r},t)$ such that the state $\mathbf{B}(t)$ is returned to a desirable steady state, $\mathbf{B}^D$, not necessarily $\mathbf{B}^D = 0$, as quickly as possible. However, if we choose the reference state as the most desirable state of the operating reactor, the deviation of the state functions may satisfy the natural terminal condition $\mathbf{B}(t_f) - \mathbf{B}^D \approx 0$.

Based on modern control theory[(67),(108)~(111)], we should try to make the minimization of the transient error vector $\epsilon(t) \equiv \mathbf{B}(t) - \mathbf{B}^D$ with minimum control effort. In the quadratic servomechanism problem, this is done by finding $\Pi_s(t)$ to minimize the performance index, or the cost functional,

$$\mathcal{J}(\underline{r}_1,\underline{r}_2,\ldots,\underline{r}_{I_c-1}) \equiv \frac{1}{2}\left[\mathbf{B}(t_f) - \mathbf{B}^D\right]^T Q_f\left[\mathbf{B}(t_f) - \mathbf{B}^D\right]$$

$$+ \frac{1}{2}\int_{t_0}^{t_f} dt \Big\{\left[\mathbf{B}(t) - \mathbf{B}^D\right]^T Q\left[\mathbf{B}(t) - \mathbf{B}^D\right]$$

$$+ \left[\Pi_s(\underline{r},t)\right]^T R\left[\Pi_s(\underline{r},t)\right]\Big\} \qquad \ldots(\text{III.20})$$

subject to the system equation (III.19) and initial conditions. Here Q, R and Q_f are constant positive diagonal matrices giving weighting to the cost of control and the loss of state vectors. Selection of such scaling matrices is quite empirical, based on experience rather than theoretical.

Our interest here lies in the fact that under certain observability and controllability conditions[(85)~(88)], the optimal controllers in terms of their locations minimize $\mathcal{J}(\underline{r}_1,\underline{r}_2,\ldots,\underline{r}_{I_c-1})$ with a linear feedback law

$$\Pi_s(\underline{r},t) = -G(\underline{r})\,\epsilon(t), \qquad t_0 < t < t_f \qquad \ldots(\text{III.21})$$

where $G(\underline{r})$ is the feedback gain and dependent on Λ and $E(\underline{r})$.

Because the essence of spatial control is the quick suppression of the xenon-induced spatial power oscillation, it might be worthwhile to reduce the order of the state functions $B(t)$ in Eq. (III.20) to be able to deal only with iodine and xenon dynamics. This order-reduction technique is very effective for the dynamic system whose time constants are spread over a large time range[(83),(84),(112)]. For our system, the application of such a technique can be justified by the fact that;

(1) The modal expression Eq. (III.18) doesn't satisfy the properties of finality with the finite number of modes, (L+1). The reason is that, because of the time derivative term matrices, all the equations must be solved simultaneously and, thus, the solutions will be changed if the order of truncation modes, (L+2) is changed.

(2) Because the coefficient terms Λ_{11} and Λ_{22} of the matrix Λ are negligible compared to the identity matrices in the iodine-xenon derivative terms, due to the multiplication of the inverse of the neutron velocities (e.g., 2.2×10^5 cm/sec for 0.025 ev neutrons).

(3) According to the studies reported by several authors[(33),(49),(55)] the optimal xenon control is of the 'bang-bang' type. In such a control mode, however, there are no constraints on maximum power and the important flux limit criteria may be lost. But as far as the optimality in the controller locations is concerned, such manoeuvering-strategy-oriented difficulties won't be necessarily considered.

The advantages of using such a low-order model are not only in approaching the properties of finality, but reducing the numerical effort to solve the large matrix system.

The approximation form of Eq. (III.18) is decoupled into;

$$\left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline \mu A_{21} & \mu A_{22} \end{array}\right] \left[\begin{array}{c} B_1 \\ \hline B_2 \end{array}\right] + \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & \mu A_{24} \end{array}\right] \left[\begin{array}{c} B_3 \\ \hline B_4 \end{array}\right] + \left[\begin{array}{c} 0 \\ \hline \mu E_2 \end{array}\right] \Pi_s = O \qquad \text{...(III.22a)}$$

and

$$\left[\begin{array}{c} \dot{B}_3 \\ \hline \dot{B}_4 \end{array}\right] = \left[\begin{array}{c|c} 0 & A_{32} \\ \hline 0 & A_{42} \end{array}\right] \left[\begin{array}{c} B_1 \\ \hline B_2 \end{array}\right] + \left[\begin{array}{c|c} A_{33} & 0 \\ \hline A_{33} & A_{44} \end{array}\right] \left[\begin{array}{c} B_3 \\ \hline B_4 \end{array}\right] \qquad \text{...(III.22b)}$$

by taking $\Lambda_{11} B_1 = \Lambda_{22} B_2 \approx 0$ and defining $\mu = V_1/V_2$. Replacing $[B_1, B_2]^T$ in Eq. (III.22b) by the new expression of Eq. (III.22a),

$$\left[\begin{array}{c} B_1 \\ \hline B_2 \end{array}\right] = -\left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline \mu A_{21} & \mu A_{22} \end{array}\right]^{-1} \left\{ \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & \mu A_{24} \end{array}\right] \left[\begin{array}{c} B_3 \\ \hline B_4 \end{array}\right] + \left[\begin{array}{c} 0 \\ \hline \mu E_2 \end{array}\right] \Pi_s \right\} \qquad \text{...(III.22a')}$$

we have 2(L+1)-order dynamic equations reduced from 4(L+1)-oredr system;

$$\left[\begin{array}{c} \dot{B}_3 \\ \hline \dot{B}_4 \end{array}\right] = \left\{ \left[\begin{array}{c|c} A_{33} & 0 \\ \hline A_{33} & A_{44} \end{array}\right] - \left[\begin{array}{c|c} 0 & A_{32} \\ \hline 0 & A_{42} \end{array}\right] \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline \mu A_{21} & \mu A_{22} \end{array}\right]^{-1} \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & \mu A_{24} \end{array}\right] \right\} \cdot$$

$$\left[\begin{array}{c} B_3 \\ \hline B_4 \end{array}\right] - \left[\begin{array}{c|c} 0 & A_{32} \\ \hline 0 & A_{42} \end{array}\right] \left[\begin{array}{c|c} A_{11} & {}_{12} \\ \hline \mu A_{21} & \mu A_{22} \end{array}\right]^{-1} \left[\begin{array}{c} 0 \\ \hline \mu E_2 \end{array}\right] \Pi_s \qquad \text{(III.22b')}$$

or simply

$$\begin{bmatrix} \dot{B}_3 \\ \dot{B}_4 \end{bmatrix} = \begin{bmatrix} \tilde{A}_{33} & \tilde{A}_{34} \\ \tilde{A}_{43} & \tilde{A}_{44} \end{bmatrix} \begin{bmatrix} B_3 \\ B_4 \end{bmatrix} + \begin{bmatrix} \tilde{E}_1 \\ \tilde{E}_2 \end{bmatrix} \Pi_s$$

or

$$\dot{\mathbf{B}}(t) = \tilde{A}\,\mathbf{B}(t) + \tilde{E}\,\Pi_s \qquad \ldots(\text{III.23})$$

The inverse of the submatrix in Eqs. (III.22) obviously exists because the non-singularity of the submatrix is guaranteed by the diagonal dominance due to the orthonormal modes.

Further simplifications can be made on the performance index Eq. (III.20) with the points noted below.

(1) The errors in iodine and xenon concentrations between the reference values and another desirable steady state at $t = t_f$, have no significance on the neutron flux shape as far as the two systems are essentially in the critical state. Therefore the desirable final state is equivalent to the reference state, i.e., $\mathbf{B}^D \cong \mathbf{B}^{Ref} = 0$.

(2) Because we assumed the reactivity contribution by the spatial control to be negligible, the modal equation for the fundamental mode, $l = 0$, must be deleted(113),(114) in order to conserve the non-singularity of the submatrix described above. Thus the order of $\mathbf{B}_3(t)$ and $\mathbf{B}_4(t)$ becomes L rather than (L+1).

Rewriting the performance index will be above modifications, we have

$$\mathcal{J}\,(\underline{r}_1,\underline{r}_2,\ldots,\underline{r}_{I_c-1}) =$$

$$\frac{1}{2}\int_{t_0}^{t_f} dt \left\{[\mathbf{B}(t)]^T\, Q\, [\mathbf{B}(t)] + [\Pi_s(t)]^T\, R[\Pi_s(t)]\right\} \qquad \ldots(\text{III.20}')$$

subject to the system equation (III.23) with L-mode terms. And the problem becomes the quadratic linear regulator problem rather than the servomechanism problem.

III.2.2 Plausability of the Initial-Condition-Free Formulations

A standard method of solving problems like Eq. (III.20') is to use Lagrange multipliers and the calculus of variations[37],[115] [119]. Defining a vector Lagrange multiplier $\Theta(\underline{r},t)$, where $\left\{\underline{r} \mid \underline{r}_1,\underline{r}_2,\ldots,r_{I_c-1}\right\}$, adjoining the system equation (III.23) in order to form the scalar positive Lagrangian $\mathcal{L}$,

$$\mathcal{L}(\mathbf{B},\Pi_s,\Theta) = f(\mathbf{B},\Pi_s) + \Theta^T\, \mathbf{X}(\mathbf{B},\Pi_s) \qquad \ldots(\text{III.24})$$

where

$$\mathbf{X}(\mathbf{B},\Pi_s) = \tilde{\mathbf{A}}\mathbf{B}(t) + \tilde{\mathbf{E}}\,\Pi_s(t) - \dot{\mathbf{B}}(t) \qquad \ldots(\text{III.25a})$$

$$f(B, \Pi_s) = \frac{1}{2}\left\{[B(t)]^T Q [B(t)] + [\Pi_s(t)]^T R [\Pi_s(t)]\right\} \quad \text{...(III.25b)}$$

We now adjust $B(t)$ and $\Pi_s(t)$ such that $\mathcal{L}$ is a maximum or minimum. This requires that

$$\frac{\partial \mathcal{L}}{\partial B} = \frac{\partial f}{\partial B} + \frac{\partial}{\partial B}[X^T\Theta] = 0 \quad \text{...(III.26a)}$$

$$\frac{\partial \mathcal{L}}{\partial \Pi_s} = \frac{\partial f}{\partial \Pi_s} + \frac{\partial}{\partial \Pi_s}[X^T\Theta] = 0 \quad \text{...(III.26b)}$$

where,

$$\left[\frac{\partial \mathcal{L}}{\partial \Pi_s}\right]^T = \left[\frac{\partial \mathcal{L}}{\partial u_s^1}, \frac{\partial \mathcal{L}}{\partial u_s^2}, \ldots, \frac{\partial \mathcal{L}}{\partial u_s^{I_c-1}}\right]$$

and

$$\left[\frac{\partial \mathcal{L}}{\partial B}\right]^T = \left[\frac{\partial \mathcal{L}}{\partial b_{1,3}}, \frac{\partial \mathcal{L}}{\partial b_{2,3}}, \ldots, \frac{\partial \mathcal{L}}{\partial b_{L,3}}, \frac{\partial \mathcal{L}}{\partial b_{1,4}}, \ldots, \frac{\partial \mathcal{L}}{\partial b_{L,4}}\right]$$

In order that $\mathcal{J}(\underline{r}_1, \underline{r}_2, \ldots, \underline{r}_{I_c-1})$ be a minimum, not only Eqs.(III.26) must be satisfied but also the second variation of $\mathcal{L}$ must be greater than zero.

The solution of Eqs. (III.26) with the minimality condition usually gives the optimum control law for a given initial and final condition sets of $\mathbf{B}(t)$ and $\Theta(\underline{r},t)$, where $\{\underline{r} \mid \underline{r}_1, \underline{r}_2, \ldots, \underline{r}_{I_c-1}\}$, is the so called 'two point boundary value problem'. But we are interested in where and how many discrete controllers are necessary to give the best performance of the reactor during the entire reactor life-time, not in searching the optimum control function which is well-behaved against a specific initial perturbation. Therefore the solution should be an initial-condition-free, i.e., time-independent ones.

Let's follow the Pontryagin's maximum principle (42) (45). The Hamiltonian of the system equation (III.23) is defined as

$$\mathcal{H}(\mathbf{B}, \Pi_s, \Theta, t, \underline{r}_1, \underline{r}_2, \ldots, \underline{r}_{I_c-1})$$

$$= f(\mathbf{B}, \Pi_s, t, \underline{r}_1, \underline{r}_2, \ldots, \underline{r}_{I_c-1}) + \Theta^T \tilde{A} \mathbf{B}$$

$$+ \Theta^T \tilde{E}\, \Pi_s, \qquad \ldots \text{(III.27)}$$

The physical interpretation of the adjoint Θ can be analogued from Stacey[32] as the incremental importance of a modal component of an iodine and xenon atom at time t which are added to the importances of the reference state, varying with the controller locations.

Application of the maximum principle requires that, for optimal control, i.e., for minimizing $\mathcal{J}(r_1, \underline{r}_2, \ldots, \underline{r}_{I_c - 1})$;

$$\frac{\partial \mathcal{H}}{\partial \Pi_s} = O = R\, \Pi_s(t) + \tilde{E}^T \Theta(\underline{r}, t) \qquad \text{...(III.28a)}$$

$$\frac{\partial \mathcal{H}}{\partial B} = - \dot{\Theta}(\underline{r}, t) = Q\, B(t) + \tilde{A}^T \Theta(\underline{r}, t) \qquad \text{...(III.28b)}$$

and

$$\frac{\partial \mathcal{H}}{\partial \Theta} = \dot{B}(t) = \tilde{A}\, B(t) + \tilde{E}^T \Pi_s(t) \qquad \text{...(III.28c)}$$

and the given boundary conditions $B(t_f) = 0$ after a sufficient control period. The boundary conditions $\Theta(\underline{r}, t_0)$ and $\Theta(\underline{r}, t_f)$ must be investigated with the imposed conditions of whether t_0, t_f, $B(t_0)$ and $B(t_f)$ are specified or not, i.e., the natural boundary conditions or the transversality conditions[81].

From Eq. (III.28a), the optimum controllers with the optimal trajectory of a transient is satisfied by

$$\Pi_s(t) = - R^{-1} \tilde{E}^T \Theta(\underline{r}, t) \qquad \text{...(III.29)}$$

and the closed-loop system equations in matrix form are

$$\begin{bmatrix}\dot{B}(t)\\ \dot{\Theta}(t)\end{bmatrix} = \begin{bmatrix}\widetilde{A} & -\widetilde{E}R^{-1}\widetilde{E}^T\\ -Q & -\widetilde{A}^T\end{bmatrix}\begin{bmatrix}B(t)\\ \Theta(\underline{r},t)\end{bmatrix} \qquad \text{...(III.30)}$$

If we require that there exists a matrix $P(\underline{r})$, where $\{\underline{r} \mid \underline{r}_1, \underline{r}_2, \ldots, \underline{r}_{I_c-1}\}$, satisfies the relationship of

$$\Theta(\underline{r},t) = P(\underline{r})\ B(t), \qquad \text{...(III.31)}$$

the properties of the matrix $P(r)$ can be determined by Eq. (III.30). Taking the time-derivative of $\Theta(\underline{r},t)$, we have

$$\dot{\Theta}(\underline{r},t) = P(\underline{r})\ \dot{B}(t), \qquad \text{...(III.32)}$$

and substituting Eqs. (III.31) and (III.32) into Eq. (III.30), we have

$$B(t) = \widetilde{A} - \widetilde{E}R^{-1}\widetilde{E}^T P(\underline{r}) B(t), \qquad \text{...(III.33a)}$$

$$P(\underline{r})\ B(t) = -Q + \widetilde{A}^T P(\underline{r})\ B(t), \qquad \text{...(III.33b)}$$

and finally,

$$P(\underline{r})\widetilde{A} + \widetilde{A}^T P(\underline{r}) - P(\underline{r})\widetilde{E}R^{-1}\widetilde{E}^T P(\underline{r}) + Q = 0. \qquad \text{...(III.34)}$$

Eq. (III.34) is the algebraic matrix Riccati equation and it was proven that, if matrices $\tilde{\mathbf{A}}$, $\tilde{\mathbf{E}}$, R and Q satisfy some properties to be discussed later, then, there exists a unique non-negative definite matrix solution $\mathbf{P}(\underline{r})$ with $\{ \underline{r} \mid \underline{r}_1, \underline{r}_2, \ldots, \underline{r}_{I_C-1} \}$ [120],[121].

By solving Eq. (III.34) with a given set of controllers located at $\{ \underline{r}_j \}$; $j = 1, 2, \ldots, (I_C-1)$, the optimum feedback gain, i.e., the effectiveness of controllers, can be derived as a function of the controller locations,

$$G(\underline{r}) = \tilde{\mathbf{E}} R^{-1} \tilde{\mathbf{E}}^T \mathbf{P}(\underline{r}) \qquad \ldots (III.35)$$

Note that matrix $\tilde{\mathbf{E}}$ also depends on the controller locations (see Eq. (III.18) or (III.19)). And the system dynamics

$$\dot{\mathbf{B}}(t) = [\tilde{\mathbf{A}} - G(\underline{r})]\, \mathbf{B}(t) \qquad \ldots (III.36)$$

can be stabilizable with the optimum closed-loop feedback law, Eq. (III.21).

Generally, even though $\mathbf{P}(\underline{r})$ is required to be non-negative definite, the solution of Eq. (III.34) may fail to be a unique solution if system matrices do not satisfy certain properties. Kalman derived some necessary and sufficient conditions providing for the existence of the unique solution[122]. According to his theorem on the algebraic characterization of optimality,

" When the linear system equation (III.23) is completely controllable with the given controllers $\Pi_s(t)$, then, there exists a positive-definite symmetric matrix, i.e., $\mathbf{P}(\underline{r}) = \mathbf{P}(\underline{r})^T$, which satisfies Eq. (III.34) and the optimal control gain Eq. (III.35) which stabilizes the system Eq. (III.36)."

In order to apply Kalman's theorem to determine the locations of the optimum discrete controllers, we have to investigate the controllability of the given control system using the controllability theorem (108),(109)

" The linear system equation (III.23) is completely controllable if and only if the rank of controller matrix $\mathcal{C}$

$$\mathcal{C} = [\tilde{\mathbf{E}}(\underline{r}),\ \tilde{\mathbf{A}}\tilde{\mathbf{E}}(\underline{r}),\ \hat{\mathbf{A}}^2\ \tilde{\mathbf{E}}(\underline{r}), \ldots,\ \tilde{\mathbf{A}}^{2L-1}\ \tilde{\mathbf{E}}(\underline{r})] \qquad \ldots(III.37)$$

is equal to the dimension of the system matrix $\mathbf{A}$, i.e., 2L."
It may be that we can find M for the partial controllability matrix

$$\mathcal{C}_M = [\tilde{\mathbf{E}}(\underline{r}),\ \tilde{\mathbf{A}}\tilde{\mathbf{E}}(\underline{r}),\ \tilde{\mathbf{A}}^2\tilde{\mathbf{E}}(\underline{r}), \ldots,\ \tilde{\mathbf{A}}^{M-1}\tilde{\mathbf{E}}(\underline{r})],$$

$$\text{where} \quad 1 < M < 2L, \qquad \ldots(III.37')$$

and that the smallest M be such, that $\mathcal{C}_M$ has full rank, i.e., 2L, is called the controllability index of the given controlled system. The controllability index is a parameter to determine the minimum number of controllers.

III.3 Numerical Methods Involved in Spatial Controller Design

III.3.1 Generation of the λ-Modes

The potential use and efficiency of the higher harmonics of the steady state diffusion equation (III.9) is due to the fact that they can describe rather large perturbations using only a few terms. One successful method of calculation and application of these modes which has been reported is an iterative method based on a Fourier series expansion technique[123].

The fundamental mode of the 2-group diffusion equation (III.9) is usually calculated in iterative ways[103]. An intermediate solution vector ϕ_0 at the n-th iteration can be written;

$$\phi_0^n = \mathcal{M}^{-1}\mathcal{L}\,\phi_0^{n-1} \qquad \text{...(III.38)}$$

Expanding the eigenfunction φ_i of the operator $\mathcal{M}^{-1}\mathcal{L}$ with the corresponding eigenvalue, λ_i, and expansion coefficient, a_i, we have

$$\phi_0^n = \sum_i a_i\,(\lambda_i)^n\,\varphi_i. \qquad \text{...(III.39)}$$

After a sufficient number of iterations, we have the approximate expression

$$\phi_0^N = \lambda_0 \ \phi_0^{N-1} \cong \lambda_0 \ \varphi_0, \qquad \text{...(III.40)}$$

where, λ_0 is the largest eigenvalue of $\mathcal{M}^{-1}\mathcal{L}$ and, φ_0 is the corresponding eigenfunction, the so called 'the fundamental mode'.

When we start with an unconverged flux vector at the iteration $k < N$, the flux vector contains high harmonics. Thus, if we subtract the predetermined fundamental mode at every iteration where the coefficient can be determined by the biorthogonality conditions Eq. (III.13), the residual flux vector must converge to the first harmonic flux vector.

Defining the unconverged flux vector of the k-th iteration in the previous procedure, and then this vector will be the initial guess for another cycle of iteration,

$$\phi_1^{k,0} = \phi_0^k - a_0^{k,0} \ \varphi_0 = \sum_{i>0} a_i^{k,0} \ \varphi_i, \qquad \text{...(III.41)}$$

where

$$a_0^{k,0} = \frac{\int_{\underline{R}} d\underline{r} \cdot \varphi_0^* \mathcal{M} \phi_0^k}{\int_{\underline{R}} d\underline{r} \ \ \varphi_0^* \mathcal{M} \ \varphi_0}. \qquad \text{...(III.42)}$$

Then for l-th iteration of the cycle k

$$\phi_1^l = \mathcal{M}^{-1}\mathcal{L}[\phi_1^{k,l-1} - a_0^{k,l-1}\ \varphi_0]$$

$$= (\mathcal{M}^{-1}\mathcal{L})^2\ [\phi_1^{k,l-2} - a_0^{k,l-2}\ \varphi_0] - a_0^{k,l-1}\mathcal{M}^{-1}\mathcal{L}\,\varphi_0$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$= (\mathcal{M}^{-1}\mathcal{L})^l\ \ \phi_1^{k,0} - \sum_{m=0}^{l-1} a_0^{k,m}\ \mathcal{M}^{-1}\mathcal{L}\ \ \varphi_0, \qquad \text{...(III.43)}$$

By the relationship of Eq. (III.41), the first term on the right-hand side of Eq. (III.43) is converged to the eigenfunction φ_1 whose eigenvalue λ_1 is the second largest. After a sufficient number of iterations, we have

$$\phi_1^L \cong \lambda_1\ \varphi_1 - a_0\,\lambda_0\ \varphi_0, \qquad \text{...(III.44)}$$

where

$$a_0 = \sum_{m=0}^{L-1} a_0^{k,m}\ (\mathcal{M}^{-1}\mathcal{L})\ \ \varphi_0.$$

In general, up to the i-th mode, we have expressions

$$a_i = \frac{\int_{\underline{R}} d\underline{r} \quad \varphi_i^* \mathcal{M} \phi_i^{uc}}{\int_{\underline{R}} d\underline{r} \quad \varphi_i^* \mathcal{M} \varphi_i} , \qquad \text{...(III.45)}$$

and

$$\phi_{i+1}^{uc} = \phi_i^{uc} - \sum_{n=0}^{i} a_i^k \varphi_i , \qquad \text{...(III.46)}$$

where, ϕ_i^{uc} is the intermediate unconverged flux obtained during the i-th mode iteration cycle and being used for the initial vector for the (i+1)-st mode calculation.

III.3.2 Approximate Adjoint Modes

By similar procedures described in the previous section but with the adjoint operator of Eqs. (III.10), adjoint modes can be generated. Usually the rate of convergence for the higher modes is very slow and so the computing cost can be extremely large.

To avoid these difficulties, we propose an alternative method by utilizing the assumption that the shape of the adjoint modes is not too different from the λ-modes, i.e.,

$$\left.\begin{aligned} \varphi^*_{n,1}(\underline{r}) &= \frac{1}{a_n^2}\ \varphi_{n,1}(\underline{r}) \\ \varphi^*_{n,2}(\underline{r}) &= \frac{1}{b_n^2}\ \varphi_{n,2}(\underline{r}) \end{aligned}\right\} \qquad \ldots\text{(III.47)}$$

Then by the biorthogonality conditions Eqs. (III.13), a_n and b_n can be evaluated as follows;

$$\int_{\underline{R}} d\underline{r}\ \frac{1}{a_m^2}\ \varphi_{m,1}(\underline{r})\ \nu\Sigma_{f2}(\underline{r})\ \varphi_{n,2}(\underline{r})$$

$$= \int_{\underline{R}} d\underline{r}\ \frac{1}{b_m^2}\ \varphi_{m,2}(\underline{r})\ \Sigma_{Rem}\ \varphi_{n,1}(\underline{r})$$

$$= N_n^2\ \delta_{mn}\ , \qquad \ldots\text{(III.48)}$$

thus

$$\left.\begin{aligned} a_m^2 &= \frac{1}{N_m^2}\int_{\underline{R}} d\underline{r}\ \ \varphi_{m,1}(\underline{r})\ \nu\Sigma_{f2}(\underline{r})\ \varphi_{m,2}(\underline{r}) \\ b_m^2 &= \frac{1}{N_m^2}\int_{\underline{R}} d\underline{r}\ \ \varphi_{m,2}(\underline{r})\ \Sigma_{Rem}(\underline{r})\ \ \varphi_{m,1}(\underline{r}) \end{aligned}\right\} \qquad \ldots\text{(III.49)}$$

The normalization conditions Eq. (III.14) will be modified by a_n and b_n to satisfy the orthonormality condition Eq. (III.15):

$$\psi_{n,1}(\underline{r}) = \frac{1}{a_n N_n}\left[\Sigma_{Rem}(\underline{r})\right]^{\frac{1}{2}} \varphi_{n,1}(\underline{r})$$

$$\left.\begin{aligned}
\psi_{n,2}(\underline{r}) &= \frac{1}{a_n N_n}\left[\nu\Sigma_{f2}(\underline{r})\right]^{\frac{1}{2}} \varphi_{n,2}(\underline{r}) \\
\psi^{*}_{n,1}(\underline{r}) &= \frac{1}{a_n N_n}\left[\nu\Sigma_{f2}(\underline{r})\right]^{\frac{1}{2}} \varphi_{n,1}(\underline{r})
\end{aligned}\right\} \quad \ldots\text{(III.14')}$$

$$\psi^{*}_{n,2}(\underline{r}) = \frac{a_n}{b_n^2 N_n}\left[\Sigma_{Rem}(\underline{r})\right]^{\frac{1}{2}} \varphi_{n,2}(\underline{r})$$

This approximation apparently causes some error in the system matrix construction, Eqs. (III.17) and (III.18). The degree of uncertainty would be small, if we remember that the thermal neutron group is dominant in the CANDU reactor and that the adjoint fluxes are comparable with the self-adjoint properties of the one-group representation.

III.3.3 Controller Location Selection Rules

The system matrix equation (III.18) can be constructed with modes generated by the method described in the previous sections. To illustrate the effectiveness of controllers in terms of their locations, we propose the following rules to minimize the computation effort to solve the matrix Riccati equation at every possible location;

(1) Reduction Rule

The control of xenon has been achieved indirectly by controlling the neutron population and, the importance of a neutron or a xenon atom becomes small as the reactor boundary is approached. Therefore the actual boundary inside which the effectiveness of the controllers is to be computed would be reduced to within the reactor boundary.

(2) Dual Controller Rule

The properties of eigenfunctions and associated eigenvalues of the reactor would be similar to those obtained from the 2-dimensional Helmholtz equations for the reactor geometry. The maximum multiplicity of Helmholtz eigenvalues for the rectangular geometry is no greater than 2, if we take the maximum number of modes less than 18[12]. Also all power reactors have some degree of asymmetry with respect to a certain orientation, and, hence, the degeneracy may be somehow relaxed. Therefore a pair of controllers will satisfy Wieberg's Theorem I.

> **Wieberg's Theorem I**[30]
>
> The minimum number of controllers required for the complete controllability should be equal to the maximum multiplicity of eigenvalues of the system matrix.

(3) Rejection Rule

The controllability requirement in the modal control theory is the ability to change the modal amplitudes from arbitrary given values to the desirable values in all cases.

> Wieberg Theorem II[30]
>
> All control rods are not on any possible nodes of a combination of modes having the same eigenvalues.

Wieberg theorem II is equivalent to the rank of the controllability matrix being equal to the order of the system matrix. Therefore a selected location, where the rank test has failed, must be rejected before computing the controller effectiveness.

(4) Unfolding Rule

The determined controller domain which would be smaller than the actual reactor boundary may have some symmetrical properties in some orientations. The control policy is to keep such symmetry during reactor operation. Also, for redundancy and reliability consideration, each pair of controllers will be unfolded with respect to the symmetry axis.

III.3.4 Iterative Technique Solving the Matrix Riccati Equation

We reduce the domain where the solutions of Eq. (III.34) exist with symmetrical pairs of controllers. Now we have to focus our attention to solve the matrix Riccati equations at every symmetric pair of points inside the domain. One of the existing methods which we will apply to our problem is the Kleinman's iterative scheme[89].

Kleinman's Theorem

Let V_k, $k = 0, 1, 2, \ldots$, be the unique positive definite solution of the linear algebraic equation

$$A_k^T V_k + V_k A_k + H + Z_k^T R Z_k = 0$$

where H is a positive definite symmetric matrix,

$$Z_k = R^{-1} E^T V_{k-1},$$

$$A_k = A - E Z_k,$$

$\{A, E\}$; a completely controllable linear system,

$$\dot{X}(t) = A X(t) + E \Pi(t),$$

and where Z_0 is chosen such that the matrix $A_0 = A - E Z_0$ has eigenvalues with negative real parts. Then

(1) $K < V_{k-1} < V_k < \cdots < V_0$,

(2) $\lim_{k \to \infty} V_k = K$.

Writing the iterative form of Eq. (III.34) for the pair of points $\{\underline{r} \mid \underline{r}_i, \underline{r}_{i-1}\}$,

$$\left[\tilde{A} - \tilde{E}(\underline{r}) R^{-1} \tilde{E}(\underline{r})^T P_{k-1}(\underline{r}) \right]^T P_k(\underline{r})$$

$$+ P_k(\underline{r}) \left[\tilde{A} - \tilde{E}(\underline{r}) R^{-1} \tilde{E}(\underline{r})^T P_{k-1}(\underline{r}) \right]$$

$$+ H_k = 0 \qquad \ldots(\text{III.50})$$

where H_k is a known matrix calculated at the previous iteration step,

$$H_k = Q + P_{k-1}(\underline{r}) \tilde{E}(\underline{r}) R^{-1} \tilde{E}(\underline{r})^T P_{k-1}(\underline{r}) .$$

Eq. (III.50) is a matrix Lyapunov equation which can be decomposed to 2L(2L+1)/2 linear equations and can be easily solved by ordinary Gauss elimination method. Note that the existence of a unique positive definite symmetric matrix solution $P_k(\underline{r})$ of the Lyapunov matrix equation at every iteration step k, if we have the initial guess $P_0(\underline{r})$

satisfying the Kleinman condition, is guaranteed by[125]:

(1) $\mathbf{H}_k$ is a symmetric matrix,

$$\left[\mathbf{H}_k\right]^T = \left[Q + \mathbf{P}_{k-1}\tilde{\mathbf{E}}R^{-1}\tilde{\mathbf{E}}^T\mathbf{P}_{k-1}\right]^T$$

$$= Q + \mathbf{P}_{k-1}\tilde{\mathbf{E}}R^{-1}\tilde{\mathbf{E}}^T\mathbf{P}_{k-1} = \mathbf{H}_k$$

because $\mathbf{P}_{k-1}$ is a symmetric matrix, and matrices Q and R are taken to be diagonal matrices.

(2) $\{\tilde{\mathbf{A}} - \tilde{\mathbf{E}}R^{-1}\tilde{\mathbf{E}}^T\mathbf{P}_{k-1}\}$ has eigenvalues with negative real parts because of the stabilizable feedback law (cf. Eq.III.36).

Since solving the 2L(2L+1)/2 linear equations expanded from the Lyapunov matrix equation, especially for large L, requires numerous additions and multiplications in the computing step, we introduce an improved method suggested by R. H. Bartels and G. W. Stewart[126]. The algorithm is divided into several steps.

(1) By an orthogonal transformation, the matrix $\tilde{\mathbf{A}}$ in the Lyapunov equation $\mathbf{P}_k\tilde{\mathbf{A}}_k + \tilde{\mathbf{A}}_k^T\mathbf{P}_k = \mathbf{H}_k$ into an upper Schur form which is an upper triangular form whose diagonal parts are composed, at most, of 2x2 blocks.

(2) The prime solution $\mathbf{P}_k^0$ is obtained by solving a set of linear equations that we partitioned to have the order at most 4.

(3) The first residual solution $\mathbf{P}_k^1$ is obtained by setting the residual matrix $\mathbf{P}_k^1\tilde{\mathbf{A}}_k + \tilde{\mathbf{A}}_k^T\mathbf{P}_k^1 = \mathbf{H}_k^1$, where $\mathbf{H}_k^1 = \mathbf{H}_k - \mathbf{P}_k^0\tilde{\mathbf{A}}_k - \tilde{\mathbf{A}}_k^T\mathbf{P}_k^0$.

(4) Repeating Step (3) until the residual matrix $\mathbf{H}_k^n$ becomes numerically negligible,

(5) Then, the accurate solution would be

$$\mathbf{P}_k = \mathbf{P}_k^0 + \mathbf{P}_k^1 + \ldots\ldots .$$

Generally this iteration can be completed within 5 cycles.

In order to improve the convergence of the Kleinman iterative method, we suggest the following,

(1) The initial guess matrix $\mathbf{P}_0$ is calculated by Kleinman's method[127] that shows an easy way to stabilize a linear dynamic system by a feedback law,

$$\mathbf{P}_0^{-1} \equiv \int_0^T \left\{ \exp(-\widetilde{\mathbf{A}}\,t)\, \widetilde{\mathbf{E}}\, R^{-1}\, \widetilde{\mathbf{E}}^T \exp(-\widetilde{\mathbf{A}}^T t) \right\} dt \quad \ldots (\text{III.51})$$

where, T is the arbitrary final time so chosen that $\mathbf{P}_0$ becomes numerically nonsingular.

(2) For given $\mathbf{P}_0$, the sequence $\mathbf{P}_k$ is determined[128] from,

$$\mathbf{P}_k = \mathbf{P}_{k-1} - \alpha\, F(\mathbf{P}_{k-1}) \quad \ldots (\text{III.52})$$

where

$$F(\mathbf{P}_k) = \mathbf{P}_k \widetilde{\mathbf{A}}_k + \widetilde{\mathbf{A}}_k^T \mathbf{P}_k - \mathbf{P}_k \widetilde{\mathbf{E}}\, R^{-1}\, \widetilde{\mathbf{E}}^T \mathbf{P}_k + \mathbf{H} \quad \ldots (\text{III.53})$$

and the overrelaxation parameter α is chosen to avoid numerical instability.

III.4 Spatial Control Effectiveness Map

From Eqs. (III.29) and (III.31), the spatial control function, $\Pi_s(t)$, will be expressed by the state deviation vector $B(t)$ as

$$\Pi_s(t) = - R^{-1} \tilde{E}^T P(\underline{r}) \cdot B(t) \qquad \text{...(III.54)}$$

To find the minimum control effort to eliminate arbitrary transient error $B(t)$, we take the norm of the control vector

$$\| \Pi_s(t) \| = \| R^{-1} \tilde{E}^T P(\underline{r}) B(t) \|$$

$$\leq \| R^{-1} \tilde{E}^T P(\underline{r}) \| \| B(t) \| .$$

We have a requirement that

" if $\| R^{-1} \tilde{E}^T P(\underline{r}) \|$ is minimum, then, $\| \Pi_s(t) \|$ is minimum on the trajectory of $B(t)$."

Also, if we follow the controller location selection rule defined earlier, the controller effectiveness inside the entire controller domain can be illustrated at least with the symmetrical pair of controller representation.

The algorithm we propose is as follows.

(1) To construct a controller domain where a relatively high neutron importance would be expected,

(2) To reduce the domain, by imvoking symmetry, as much as possible in order to reduce the reactor volume requiring actual computation,

(3) To construct a mesh, where the effectiveness will be illustrated,

(4) To check the controllability of a controller pair composed of one controller inside the volume and the other at its symmetric location,

(5) To solve the matrix Riccati equation describing the optimal condition for the pair controller system, $P(\underline{r})$,

(6) To calculate $\| R^{-1} \tilde{E}^{T} P(\underline{r}) \|$ for individual controllers of the pair,

(7) To repeat procedures (4) to (6) until the computation has been completed over all mesh points.

IV. MULTIPLE CONTROLLER DEPLOYMENT USING OPTIMAL BULK CONTROL CONCEPT

IV.1 Long-Term Reactivity Compensation

If the reactor is ideally fuelled, we can assume that the neutron flux distribution may not be affected by fuelling-induced-perturbations. The main function of the bulk controllers, in this case, is focused to the reactivity compensation under condition of minimizing controller induced flux distortion over control margin of the reactivity.

With Eqs. (II.15), without fuelling perturbation, i.e., $\Delta(\underline{r}, t_0) = 0$, the optimality condition for the controlled system is directly determined by minimization of the performance index,

$$\mathcal{J}(\eta, U_B, \underline{r}_1, \underline{r}_2, \ldots, \underline{r}_{I_C})$$

$$\equiv \frac{1}{2} \int_{\underline{R}} d\underline{r} \left\{ \eta^T Q \eta + U_B^{0^T} R \, U_B^0 \right\} \qquad \ldots(\text{IV.1})$$

The λ-mode expression technique is applied to separate the space-dependency of the problem. By similar definitions and procedures discussed in the previous chapter but including the fundamental mode, we have

$$- \sum_{l=0}^{L} a_{l,1} \frac{1}{k_1} \left\{ \nu\Sigma_{f2}(\underline{r}) \right\}^{\frac{1}{2}} \psi_{1,2}(\underline{r}) + \sum_{l=0}^{L} {}_{1,2} \left\{ \nu\Sigma_{f2}(\underline{r}) \right\}^{\frac{1}{2}} \left[1+\alpha_f(\underline{r}) \right]$$

$$\psi_{1,2}(\underline{r}) + \nu\Sigma_{f2}(\underline{r}) \left\{ 1 - \frac{1}{k_0} \right\} \phi_2^{Ref}(\underline{r}) = 0 , \qquad \text{...(IV.2a)}$$

$$- \sum_{l=0}^{L} a_{1,2} \left\{ \Sigma_{Rem}(\underline{r}) \right\}^{\frac{1}{2}} \psi_{1,1}(\underline{r}) + \sum_{l=0}^{L} a_{1,1} \left\{ \Sigma_{Rem}(\underline{r}) \right\}^{\frac{1}{2}} \psi_{1,1}(\underline{r})$$

$$- \sum_{j=1}^{I_C} U_B^0(\underline{r}) \; \phi_2^{Ref}(\underline{r}) \; \delta(\underline{r}-\underline{r}_j) = 0 \qquad \text{...(IV.2b)}$$

Multiplying $\psi^*_{m,2}(\underline{r}) \left\{ \Sigma_{Rem}(\underline{r}) \right\}^{\frac{1}{2}}$ on Eq. (II.2a) and $\left\{ \psi^*_{m,1}(\underline{r}) \, \nu\Sigma_{f2}(\underline{r}) \right\}^{\frac{1}{2}}$ on Eq. (IV.2b); and integrating over the reactor volume $\underline{R}$, we have, by the biorthonormality conditions Eqs. (III.15);

$$- \sum_{l=0}^{L} a_{1,1} \int_{\underline{R}} d\underline{r} \; \psi^*_{m,2}(\underline{r}) \; \frac{1}{k_1} \left\{ \Sigma_{Rem}(\underline{r}) \; \nu\Sigma_{f2}(\underline{r}) \right\}^{\frac{1}{2}} \psi_{1,2}(\underline{r})$$

$$- \sum_{l=0}^{L} a_{1,2} \int_{\underline{R}} d\underline{r} \; \psi^*_{m,2}(\underline{r}) \left\{ \nu\Sigma_{f2}(\underline{r}) \right\}^{\frac{1}{2}} \left[1 + \alpha_f(\underline{r}) \right] \psi_{1,2}(\underline{r})$$

$$+ \int_{\underline{R}} d\underline{r}\ \psi^*_{m,2}(\underline{r}) \left\{\Sigma_{Rem}(\underline{r})\right\}^{\frac{1}{2}} \left[1 - \frac{1}{k_0}\right] \nu\Sigma_{f2}(\underline{r})\ \phi_2^{Ref}(\underline{r}) = 0$$

...(IV.3a)

$$- \sum_{l=0}^{L} a_{l,2} \int_{\underline{R}} d\underline{r}\ \psi^*_{m,1}(\underline{r}) \left\{\nu\Sigma_{f2}(\underline{r})\ \Sigma_{Rem}(\underline{r})\right\}^{\frac{1}{2}} \psi_{l,1}(\underline{r})$$

$$+ \sum_{l=0}^{L} a_{l,1} \int_{\underline{R}} d\underline{r}\ \psi^*_{m,1}(\underline{r}) \left\{\nu\Sigma_{f2}(\underline{r})\ \Sigma_{Rem}(\underline{r})\right\}^{\frac{1}{2}} \psi_{l,1}(\underline{r})$$

$$- \sum_{j=1}^{I_C} \psi^*_{m,1}(\underline{r}_j) \left\{\nu\Sigma_{f2}(\underline{r}_j)\right\}^{\frac{1}{2}} \phi_2^{Ref}(\underline{r}_j) = 0, \qquad \text{...(IV.3b)}$$

in matrix form

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} + \begin{bmatrix} E_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \Pi_B^0 = O$$

...(IV.4)

In order to obtain the reduced order form, rewriting Eq. (IV.4) by

$$A_{11}\ N_1 + A_{12}\ N_2 + E_1 = O, \qquad \text{...(IV.5a)}$$

$$A_{21}\ N_1 + A_{22}\ N_2 + B_2\Pi_B^0 = O, \qquad \text{...(IV.5b)}$$

and, because A_{11} is a non-singular diagonal matrix, from Eq. (IV.5a), we have

$$N_1 = - A_{11}^{-1} A_{12} N_2 - A_{11}^{-1} E_1. \qquad \text{...(IV.6)}$$

Substituting Eq. (IV.6) into Eq. (IV.5b), we obtain an (L+1) dimensional matrix equation explicitly representing the thermal neutron balance;

$$[A_{22} - A_{21} A_{11}^{-1} A_{12}] N_2 - A_{21} A_{11}^{-1} E_1 + B_2 \Pi_B^0 = O, \qquad \text{...(IV.7a)}$$

or

$$\tilde{A} N_2 + \tilde{E} + B_2 \Pi_B^0 = O. \qquad \text{...(IV.7b)}$$

Note that submatrices and vectors shown in Eqs. (IV.4) $\sim$ (IV.7) are;

A_{11} is a (L+1)x(L+1) matrix whose elements are

$$\left[A_{ml}^{11}\right] = - \int_{\underline{R}} d\underline{r} \left\{ \psi_{m,2}^{*}(\underline{r}) \frac{1}{k_1} \left\{ \Sigma_{Rem}(\underline{r}) \nu\Sigma_{f2}(\underline{r}) \right\}^{\frac{1}{2}} \psi_{1,2}(\underline{r})\right.$$

A_{12} is a (L+1)x(L+1) matrix whose elements are

$$\left[A_{ml}^{12}\right] = \int_{\underline{R}} d\underline{r} \left\{ \psi_{m,2}^{*}(\underline{r}) \left\{ \Sigma_{Rem}(\underline{r}) \nu\Sigma_{f2}(\underline{r}) \right\}^{\frac{1}{2}} [1 + \alpha_f(\underline{r})]\right.$$

$$\times \; \psi_{1,2}(\underline{r}) \Big\} ,$$

$\mathbf{A}_{21}$ is a (L+1)x(L+1) matrix whose elements are

$$\left[A^{21}_{ml}\right] = - \int_{\underline{R}} d\underline{r} \left\{ \psi^{*}_{m,1}(\underline{r}) \left\{ \Sigma_{Rem}(\underline{r}) \; \nu\Sigma_{f2}(\underline{r}) \right\}^{\frac{1}{2}} \psi_{1,1}(\underline{r}) \right\} ,$$

$\mathbf{A}_{22}$ is a (L+1)x(L+1) matrix whose elements are

$$\left[A^{22}_{ml}\right] = - \left[A^{21}_{ml}\right] ,$$

$\mathbf{E}_1$ is a (L+1) column vector whose elements are

$$\left[E^{1}_{m}\right] = \left(1 - \frac{1}{k_0}\right) \int_{\underline{R}} d\underline{r} \left\{ \psi^{*}_{m,2}(\underline{r}) \left\{ \Sigma_{Rem}(\underline{r}) \right\}^{\frac{1}{2}} \nu\Sigma_{f2}(\underline{r}) \; \phi^{Ref}_{2}(\underline{r}) \right\}$$

$\mathbf{B}_2$ is a (L+1)xI_c matrix whose elements are

$$\left[B^{2}_{mj}\right] = - \psi^{*}_{m,1}(\underline{r}_j) \left\{ \nu\Sigma_{f2}(\underline{r}_j) \right\}^{\frac{1}{2}} \phi^{Ref}_{2}(\underline{r}_j) ,$$

Π_B is a column vector

$$\left[\Pi^{0}_{B}\right]^{T} = \left[U^{0}_{B}(\underline{r}_1), U^{0}_{B}(\underline{r}_2), \ldots, U^{0}_{B}(\underline{r}_{I_c}) \right]$$

$\mathbf{N}_1$ and $\mathbf{N}_2$ are column vectors we have

$$[N_i]^T = [a_{0,i}, a_{1,i}, \ldots, a_{1,i}] \quad \text{for } i = 1 \text{ or } 2.$$

After replacing the state vectors $\eta(t)$ in the performance index by the modal expression and integrating over the reactor volumn, $\underline{R}$, the Hamiltonian of the steady state system becomes

$$\mathcal{H}(N_2, \Pi_B^0, \underline{r}_1, \underline{r}_2, \ldots, \underline{r}_{I_C})$$

$$= \frac{1}{2} N_2^T Q N_2 + \frac{1}{2} \Pi_B^{0\,T} R\, \Pi_B^0$$

$$+ \Theta^T \left\{ \tilde{A} N_2 + \tilde{E} + B_2 \Pi_B^0 \right\}, \quad \ldots \text{(IV.8)}$$

where Θ^T ; the transpose of the adjoint vector.

By the Pontryagin's maximum principle, we have

$$\frac{\partial \mathcal{H}}{\partial N_2} = O = Q N_2 + \tilde{A}^T \Theta, \quad \ldots \text{(IV.9a)}$$

$$\frac{\partial \mathcal{H}}{\partial \Theta} = O = \tilde{A} N_2 + \tilde{E} + B_2 \Pi_B^0, \quad \ldots \text{(IV.9b)}$$

and

$$\frac{\partial \mathcal{H}}{\partial \Pi_B^0} = O = R\,\Pi_B^0 + B_2^T\,\Theta \quad \ldots(IV.9c)$$

Solving Eqs. (IV.9) to get the expression of Π_B^0, we have finally

$$\Pi_B^0 = -\left\{ R + B_2^T\,\tilde{A}^T\,Q\,\tilde{A}^{-1}\,B_2 \right\}^{-1} B_2^T\,\tilde{A}^{-T}\,Q\,\tilde{A}^{-1}\tilde{E}$$

$$= -R^{-1}B_2^T\left\{ \tilde{A}\,Q^{-1}\,\tilde{A}^T + B_2\,R^{-1}B_2^T \right\}^{-1}\tilde{E}\,,$$

$$\ldots(IV.10)$$

For $\mathcal{J}(N_2, \Pi_B^0, \underline{r}_1, \underline{r}_2, \ldots, \underline{r}_{I_C})$ to have a minimum, the second variation,

$$\delta^2\mathcal{J} = \frac{1}{2}\left[\delta N_2^T\,\delta\Pi_B^0\right]\begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix}\begin{bmatrix} \delta N_2 \\ \delta\Pi_B^0 \end{bmatrix},$$

$$= \frac{1}{2}\,\delta N_2^T\,Q\,\delta N_2 + \frac{1}{2}\,\delta\Pi_B^{0\,T}\,R\,\delta\Pi_B^0$$

$$\ldots(IV.11)$$

must be non-negative, therefore, it is sufficient that Q and R be non-negative definite. But if we consider the variation of the system equation which is

$$\tilde{A}\,\delta N_2 + B_2\,\delta\Pi_B^0 = 0, \qquad \ldots\text{(iv.12)}$$

then, replacing $\delta\Pi_B^0$ in Eq. (IV.11) by (IV.12), we have the necessary condition for $\delta^2\mathcal{J} > 0$, that $R + B_2^T\,\tilde{A}^T\,Q\,\tilde{A}^{-1}\,B_2$ is positive.

If we investigate the properties of matrices present in Eq. (IV.10), we find that the matrix $\tilde{A}$ and the column vector $\tilde{E}$ are known; Q and R are arbitrary diagonal scale matrices, thus, known; and B_2 can be determined whenever we know the locations of controllers.

Physically the controller vector Π_B^0 has a component of each controller $U_B^0(\underline{r}_j)$, where j = 1, 2,..., I_c, whose property is an amount of macroscopic absorption cross section for thermal neutrons. Therefore, if we have the maximum allowable control material for static bulk control purpose, Σ_{a2}^u, the minimum number of controllers must satisfy

$$U_B^0(\underline{r}_j) \leq \Sigma_{a2}^u \qquad \text{for } j = 1, 2, \ldots, I_c$$

and the minimum number of controllers, Min. $\{I_c\}$ becomes

$$\text{Min.}\left\{I_c\right\} = \|\Pi_B^0\| / \Sigma_{a2}^u, \qquad \ldots\text{(IV.13)}$$

where $\|\Pi_B^0\|$ refers to the norm of the static bulk controller vector,

$$\| \Pi_B^0 \| = \left(\Pi_B^{0\,T} \Pi_B^0 \right)^{\frac{1}{2}}$$

$$= \left[\sum_{j=1}^{I_C} \left(U_B^0(\underline{r}_j) \; U_B^0(\underline{r}_j) \right) \right]^{\frac{1}{2}}$$

In this case, the flux mode deviation due to introducing the static bulk control function can be represented, from Eqs. (IV.9a) and (IV.10), by

$$N_2 = - \tilde{A}^{-1} \left[I - B_2 R^{-1} B_2^T \left(\tilde{A} Q^{-1} \tilde{A}^T + B_2 R^{-1} B_2^T \right)^{-1} \right] \tilde{E}$$

IV.2 Bulk Control with Burnup/Fuelling Induced Disturbances

IV.2.1 Stochastic Modal Control Model

The existence of optimum locations of the controllers in terms of minimum control effort for spatial control, has been demonstrated in the previous chapter. Also we investigated one of the requirements that the control system should meet, that is the reactivity compensation with minimum flux shape distortion. From the dynamics point of view, the controllers set to a static level should be able to respond against transient trajectories deviating from the desirable condition. In this chapter we evaluate the dynamic range of control actions for manoeuvering reactor power and counteracting fuel burnup/refuelling induced bounded random disturbances. The maximum-minimum control range of an individual controller will give direct information to determine the optimum number of controllers in the system.

The time-dependent neutron diffusion problem with stochastic parameters, i.e., Eq. (II.24) can be written to a matrix form,

$$V\dot{\eta}(\underline{r},t) = A\eta(\underline{r},t) + B\,u_B(\underline{r},t) + E + H\,\Delta(\underline{r},t), \qquad \text{...(IV.14)}$$

where ,

$$V \equiv \begin{bmatrix} \frac{1}{\mu} & 0 \\ 0 & 1 \end{bmatrix},$$

$$A \equiv \begin{bmatrix} \nabla D_1(\underline{r})\nabla - \Sigma_{a1}(\underline{r}) - \Sigma_{Rem}(\underline{r}) & \nu\Sigma_{f2}(\underline{r})\{1+\alpha_f(\underline{r})\} \\ \Sigma_{Rem}(\underline{r}) & \nabla D_2(\underline{r})\nabla - \Sigma_{a2}(\underline{r}) - \sigma_x X^{Ref}(\underline{r}) \end{bmatrix},$$

$$B^T \equiv \begin{bmatrix} 0 & \phi_2^{Ref}(\underline{r})\ \delta(\underline{r}-\underline{r}_1) \\ 0 & \phi_2^{Ref}(\underline{r})\ \delta(\underline{r}-\underline{r}_2) \\ \cdots & \cdots\cdots\cdots\cdots \\ 0 & \phi_2^{Ref}(\underline{r})\ \delta(\underline{r}-\underline{r}_{Ic}) \end{bmatrix},$$

$$\mathbf{E} \equiv \begin{bmatrix} (1-\frac{1}{k_0})\,\nu\Sigma_{f2}(\underline{r})\ \phi_2^{Ref}(\underline{r}) \\ \\ 0 \end{bmatrix}$$

and

$$\mathbf{H} \equiv \begin{bmatrix} \nu\Sigma_{f2}(\underline{r})\ \phi_2^{Ref}(\underline{r}) \\ \\ 0 \end{bmatrix}$$

If we assume that the detector signal is composed of the time-dependent thermal neutron flux deviation from the reference state and of a certain amount of noise, at the detector location k, the output will be

$$\xi(\underline{r},t)\ \delta(\underline{r}-\underline{r}_k) = \left\{\eta_2(\underline{r},t) + \Gamma(\underline{r},t)\right\} \delta(\underline{r}-\underline{r}_k),$$

$$k = 1, 2, \ldots, I_d \qquad \ldots (IV.15)$$

where I_d is the number of detectors, and, for convenience, $I_d = I_c$, i.e., one-to-one correspondence between controllers and detectors.

We have assumed that the noise function can be expanded with the system eigenfunctions (cf. Eq. II.5) (99),(129). Hence, by a similar method introduced in the spatial controller design, the state functions in Eq. (IV.14) are expanded with the normalized λ-modes but including the fundamental mode, we have

$$\eta_1(\underline{r},t) = \sum_{l=0}^{L} a_{l,1}(t)\, \psi_{l,1}(\underline{r}) \left\{\Sigma_{Rem}(\underline{r})\right\}^{-\frac{1}{2}}$$

$$\left.\begin{aligned} \eta_2(\underline{r},t) &= \sum_{l=0}^{L} a_{l,2}(t)\, \psi_{l,2}(\underline{r}) \left\{\nu\Sigma_{f2}(\underline{r})\right\}^{-\frac{1}{2}} \\ \Delta(\underline{r},t) &= \sum_{l=0}^{L} n_{l,1}(t)\, \psi_{l,2}(\underline{r}) \left\{\nu\Sigma_{f2}(\underline{r})\right\}^{-\frac{1}{2}} \end{aligned}\right\} \quad \ldots (IV.16)$$

$$\Gamma(\underline{r},t) = \sum_{l=0}^{L} n_{l,2}(t)\, \psi_{l,2}(\underline{r}) \left\{\nu\Sigma_{f2}(\underline{r})\right\}^{-\frac{1}{2}}$$

where i can be 1 or 2, and where $n_{l,1}$ are amplitudes of modes for the state disturbance noise, and $n_{l,2}$ are amplitudes of modes for the output uncertainty noise. Note that noise is important only for thermal neutrons.

Substituting Eq. (IV.16) into Eq. (II.24), and replacing the Laplacian terms by the λ-mode equations (III.9), we have, for the stochastic neutron diffusion equation,

$$\frac{1}{\mu} \sum_{l=0}^{L} \dot{a}_{l,1}(t)\, \psi_{l,1}(\underline{r}) \left\{\Sigma_{Rem}(\underline{r})\right\}^{-\frac{1}{2}}$$

$$= - \sum_{l=0}^{L} a_{l,1}(t) \frac{1}{k_l} \left\{\nu\Sigma_{f2}(\underline{r})\right\}^{\frac{1}{2}} \psi_{l,2}(\underline{r})$$

$$+ \sum_{l=0}^{L} a_{l,2}(t) \left\{ \Sigma_{f2}(\underline{r}) \right\}^{\frac{1}{2}} \Big(1 + \alpha_f(\underline{r})\Big) \psi_{l,2}(\underline{r}) + (1 - \frac{1}{k_0}) \nu\Sigma_{f2}(\underline{r})\, \phi_2^{Ref}(\underline{r})$$

$$+ \sum_{l=0}^{L} n_{l,1}(t) \left\{ \nu\Sigma_{f2}(\underline{r}) \right\}^{\frac{1}{2}} \phi_2^{Ref}(\underline{r})\, \psi_{l,2}(\underline{r}), \quad \ldots \text{(IV.17a)}$$

$$\sum_{l=0}^{L} \dot{a}_{l,2}(t)\, \psi_{l,2}(\underline{r}) \left\{ \nu\Sigma_{f2}(\underline{r}) \right\}^{-\frac{1}{2}}$$

$$= - \sum_{l=0}^{L} a_{l,2}(t) \left\{ \Sigma_{Rem}(\underline{r}) \right\}^{\frac{1}{2}} \psi_{l,1}(\underline{r})$$

$$+ \sum_{l=0}^{L} a_{l,1}(t) \left\{ \Sigma_{Rem}(\underline{r}) \right\}^{\frac{1}{2}} \psi_{l,1}(\underline{r}) - \sum_{j=1}^{I_c} U_B(\underline{r},t)\, \phi_2^{Ref}(\underline{r})\, \delta(\underline{r}-\underline{r}_j),$$

$$\ldots \text{(IV.17b)}$$

and, for the detector signal equation (IV.15),

$$\xi(\underline{r},t)\, \delta(\underline{r}-\underline{r}_k) = \sum_{l=0}^{L} \Big(a_{l,2}(t) + n_{l,2}(t) \Big) \psi_{l,2}(\underline{r}) \left\{ \nu\Sigma_{f2}(\underline{r}) \right\}^{-\frac{1}{2}} \delta(\underline{r}-\underline{r}_k),$$

$$k = 1, 2, \ldots, I_c \quad \ldots \text{(IV.17c)}$$

Multiplying $\left\{ \Sigma_{Rem}(\underline{r}) \right\}^{\frac{1}{2}} \psi^*_{m,2}(\underline{r})$ by Eq. (IV.17a) and

$\left\{\Sigma_{f2}(\underline{r})\right\}^{\frac{1}{2}}\psi^*_{m,1}(\underline{r})$ by Eq. (IV.17b), and integrating over the reactor volume, $\underline{R}$, then, by the orthogonality condition Eq.(III.15), we have the modal dynamic equations,

$$\frac{1}{\mu}\dot{a}_{m,1}(t) = -\sum_{l=0}^{L} a_{l,1}(t)\,\frac{1}{k_l}\int_{\underline{R}} d\underline{r}\ \psi^*_{m,2}(\underline{r})W(\underline{r})\ \psi_{l,2}(\underline{r})$$

$$+\sum_{l=0}^{L} a_{l,2}(t)\int_{\underline{R}} d\underline{r}\ \psi^*_{m,2}(\underline{r})W(\underline{r})\left\{1+\overset{\circ}{\alpha}_f(\underline{r})\right\}\ \psi_{l,2}(\underline{r})$$

$$+\sum_{l=0}^{L} n_{l,1}(t)\int_{\underline{R}} d\underline{r}\ \psi^*_{m,2}(\underline{r})W(\underline{r})\,\phi_2^{Ref}(\underline{r})\ \psi_{l,2}(\underline{r})$$

$$+\int_{\underline{R}} d\underline{r}\left(1-\frac{1}{k_0}\right)\psi^*_{m,2}(\underline{r})W(\underline{r})\left\{\nu\Sigma_{f2}(\underline{r})\right\}^{\frac{1}{2}}\phi_2^{Ref}(\underline{r}),$$

...(IV.18a)

$$\dot{a}_{m,2}(t) = -\sum_{l=0}^{L} a_{l,2}(t)\int_{\underline{R}} d\underline{r}\ \psi^*_{m,1}(\underline{r})W(\underline{r})\ \psi_{l,1}(\underline{r})$$

$$+\sum_{l=0}^{L} a_{l,1}(t)\int_{\underline{R}} d\underline{r}\ \psi^*_{m,1}(\underline{r})W(\underline{r})\ \psi_{l,1}(\underline{r})$$

$$-\sum_{j=1}^{I_C}\ \psi^*_{m,j}(\underline{r}_j)\left\{\nu\Sigma_{f2}(\underline{r}_j)\right\}^{\frac{1}{2}}\phi_2^{Ref}(\underline{r}_j)\,U_B^j(t),\quad \text{(IV.18b)}$$

and for the detector signal

$$\xi_k(t) = \sum_{l=0}^{L} \left\{ a_{l,2}(t) + n_{l,2}(t) \right\} \psi_{l,2}(\underline{r}_k) \left\{ \nu\Sigma_{f2}(\underline{r}_k) \right\}^{\frac{1}{2}} , \qquad \text{...(IV.19)}$$

where,

$$W(\underline{r}) = \left\{ \Sigma_{Rem}(\underline{r})\, \nu\Sigma_{f2}(\underline{r}) \right\}^{\frac{1}{2}} .$$

In matrix form, Eqs. (IV.18) and (IV.19) become

$$\left[\begin{array}{c|c} \Lambda_{11} & 0 \\ \hline 0 & I \end{array}\right] \left[\begin{array}{c} \dot{X}_1(t) \\ \hline \dot{X}_2(t) \end{array}\right] = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] \left[\begin{array}{c} X_1(t) \\ \hline X_2(t) \end{array}\right] + \left[\begin{array}{c} E_1 \\ \hline 0 \end{array}\right]$$

$$+ \left[\begin{array}{c} 0 \\ \hline B_2 \end{array}\right] \Pi_B(t) + \left[\begin{array}{c} H_1 \\ \hline 0 \end{array}\right] \Gamma_1(t) , \qquad \text{...(IV.20a)}$$

and

$$Y_2(t) = M_2 X_2(t) + M_2 \Gamma_2(t) , \qquad \text{...(IV.20b)}$$

where Λ_{11} is a (L+1)x(L+1) diagonal matrix whose elements are

$$\left[\Lambda_{11}^{11}\right] = \frac{1}{\mu} = \frac{v_2}{v_1},$$

H_1 is a $(L+1)\times(L+1)$ square matrix whose elements are

$$\left[H_{ml}^{1}\right] = \int_{\underline{R}} d\underline{r} \left\{ \psi_{m,2}^{*}(\underline{r}) \left\{ \Sigma_{Rem}(\underline{r})\, \nu\Sigma_{f2}(\underline{r}) \right\}^{\frac{1}{2}} \phi_2^{Ref}(\underline{r})\; \psi_{1,2}(\underline{r}) \right\}$$

M_2 is a $I_c\; x(L+1)$ matrix whose elements are

$$\left[M_{km}^{2}\right] = \nu\Sigma_{f2}(\underline{r}_k)\;\; \psi_{m,2}(\underline{r}_k),$$

and Γ_1 and Γ_2 are $(L+1)$ column vectors

$$\Gamma_i^{T} = \left[n_{0,i}(t),\; n_{1,i}(t), \ldots,\; n_{L,i}(t) \right],$$

for $i = 1$ or 2.

Because μ is the ratio of the fast-to-thermal neutron velocity ($v_1/v_2 \approx 20$), by the dominant mode concept described in Chapter III, we can formulate the reduced-order modal approximation with the quasi-static treatment of the fast neutron diffusion equation, i.e., $\Lambda_{11} \approx 0$;

$$O = A_{11} X_1(t) + A_{12} X_2(t) + H_1 \Gamma_1(t) + E_1 \quad \ldots\text{(IV.21a)}$$

$$\dot{X}_2(t) = A_{21} X_1(t) + A_{22} X_2(t) + B_2 \Pi_B(t). \quad \ldots\text{(IV.21b)}$$

Replacing $X_1(t)$ in Eq. (IV.21b) by the expression derived from Eq. (IV.21b), we have the system dynamic equation dominant for thermal neutron behaviours,

$$\dot{X}_2(t) = [A_{22} - A_{21}\, A_{11}^{-1}\, A_{12}]\, X_2(t) + B_2 \Pi_B(t)$$

$$- A_{21}\, A_{11}^{-1} H_1\, \Gamma_1(t) - A_{21}\, A_{11}^{-1} E_1 ,$$

or, in simple matrix form,

$$\dot{X}_2(t) = \tilde{A} X_2(t) + B_2 \Pi_B(t) + \tilde{H}\,\Gamma_1(t) + \tilde{E} \quad , \ldots \text{(IV.22)}$$

and the output

$$Y_2(t) = M_2\, X_2(t) + M_2\, \Gamma_2(t) . \qquad \ldots \text{(IV.19b)}$$

Thus the problem is specified to find the optimal control function, $\Pi_B(t)$, for the time-invariant linear inhomogeneous stochastic system.

Comparing Eq. (IV.22) with (IV.7a), we recognize that the inhomogeneous term $A_{21}\, A_{11}^{-1}\, E_1$ is the static reactivity source term that can be eliminated by the static bulk control function $\Pi_B^0(\underline{r})$. Thus, defining new state and control vectors

$$\zeta(t) = X_2(t) - N_2 ,$$

$$\mu(t) = \Pi_B(t) - \Pi_B^0 , \qquad \text{...(IV.23)}$$

and subtracting Eq. (IV.7a) from (IV.22), we have a new purely stochastic system

$$\dot{\zeta}(t) = \widetilde{A}\,\zeta(t) + B_2\,\mu(t) + \widetilde{H}\,\Gamma_1(t) , \qquad \text{...(IV.24a)}$$

and a new output description

$$\xi(t) = M_2\,\zeta(t) + M_2\,\Gamma_2(t) . \qquad \text{...(IV.24b)}$$

Eqs. (IV.24) mean that, for the random fuelling perturbations, the control function $\mu(t)$ is responding to the thermal flux variation $\zeta(t)$ on the basis of measurement statistics $\xi(t)$ [130] [135]. Process $\Gamma_i(t)$'s are assumed to be stationary uncorrelated zero-mean white Gaussian such that their covariances are;

$$\left.\begin{aligned} &\mathrm{cov}\left[\Gamma_1(t),\ \Gamma_1(\tau)\right] = Q_2(t)\,\delta(t-\tau), \\ &\mathrm{cov}\left[\Gamma_2(t),\ \Gamma_2(\tau)\right] = R_2(t)\,\delta(t-\tau), \\ \text{and}\quad &\mathrm{cov}\left[\Gamma_1(t),\ \Gamma_2(t)\right] = 0. \end{aligned}\right\} \qquad \text{...(IV.25)}$$

Determination of $Q_2(t)$ and $R_2(t)$ depend on the physical assessments of the noise fields. Therefore we specify $\Gamma_1(t)$ as the random perturbation in the fission cross section and $\Gamma_2(t)$ as the uncertainty in the detector readings. We note that there definitely exists noise in the neutron field and in the detector field, but the neutron noise in high power reactors is negligible compared to other sources, e.g., mechanical vibrations[18].

IV.2.2 External Disturbances in Linear Regulator

If we carefully investigate the physical meaning of Eqs. (IV.24), the optimally selected controllers, $\mu(t)$, must completely counteract the effect of the disturbances, $\Gamma_1(t)$, in spite of the uncertainty in measurements, $M_2\ \Gamma_2(t)$. Suppose it turns out that we cannot achieve an exact cancellation of the disturbance, then, we might be able to attempt to design controllers which minimize the effects.

Because the neutron noise in the system was neglected and the measurement uncertainties were only considered for the efficiencies of the detectors, we should be able to construct the performance index functional in terms of the state vectors rather than the output deviations,

$$\mathcal{J} = \mathcal{E}\left\{\int_{t_0}^{t_f} dt \left[\zeta(t)^T Q_1\ \zeta(t) + \mu(t)^T R_1\ \mu(t)\right]\right\}, \quad \ldots (IV.26)$$

where $\mathcal{E}$ referes to the expectation value of $\{\ .\ \}$, and R_1 and Q_1 are supposed to be positive definite diagonal matrices.

By the useful matrix formula;

$$\mathcal{E}\left\{X^T \mathcal{S} X\right\} = \mathcal{E}\left\{(X-\overline{X})^T \mathcal{S} (X-\overline{X})\right\} + \overline{X}^T \mathcal{S} \overline{X}$$

$$= \mathcal{E}\left\{\mathrm{tr}\ (X-\overline{X}) \mathcal{S} (X-\overline{X})^T\right\} + \overline{X}^T \mathcal{S} \overline{X}$$

$$= \mathcal{E}\left\{\mathrm{tr}\ \mathcal{S}(X-\overline{X})(X-\overline{X})^T\right\} + \overline{X}^T \mathcal{S} \overline{X}$$

$$= \mathrm{tr}\left\{\mathcal{S}\, \mathcal{E}\left[(X-\overline{X})(X-\overline{X})^T\right]\right\} + \overline{X}^T \mathcal{S} \overline{X}$$

if $\overline{X} = 0$, then

$$\mathcal{E}\left\{X^T \mathcal{S} X\right\} = \mathrm{tr}\left\{\mathcal{S}\, \mathcal{E}\left[X X^T\right]\right\} = \mathrm{tr}\left\{\mathcal{S}\ \mathrm{cov}\left[X\right]\right\}, \qquad \text{...(IV.27)}$$

we have

$$\mathcal{J} = \int_{t_0}^{t_f} dt\ \ \mathrm{tr}\left\{ Q_1 \mathcal{E}\left[\zeta(t)\ \zeta(t)^T\right] + R_1 \mathcal{E}\left[\mu(t)\ \mu(t)^T\right]\right\}; \qquad \text{...(IV.28)}$$

$$= - Q_1 \zeta(t) - \widetilde{A}^T \{ P\zeta(t) + \Omega(t) \} .$$

Therefore, we have

$$\left[P\widetilde{A} + \widetilde{A}^T P - PB_2 R_1^{-1} B_2^T P + Q_1 \right] \zeta(t)$$

$$+ \dot{\Omega}(t) + \left[\widetilde{A}^T - PB_2 R_1^{-1} B_2^T \right] \Omega(t) + P\widetilde{H} \Gamma_1(t) = 0 .$$

...(IV.36)

In order to satisfy the condition Eq. (IV.36) for arbitrary deviation function $\zeta(t)$ and $\Gamma_1(t)$, we may be able to separate it into; for the regulation of the system,

$$P\widetilde{A} + \widetilde{A}^T P - PB_2 R_1^{-1} B_2^T P + Q_1 = 0, \quad \text{...(IV.37a)}$$

and for the counteraction of the noise,

$$\dot{\Omega}(t) = - \left[\widetilde{A}^T - PB_2 R_1^{-1} B_2^T \right] \Omega(t) + P\widetilde{H} \Gamma_1(t) .$$

...(IV.37b)

$$\frac{\partial \mathcal{H}}{\partial \zeta} = - \dot{\theta}(t) = Q_1 \zeta(t) + \tilde{A}^T \theta(t), \qquad \ldots(IV.33)$$

and, in addition to these, we have the system equation (IV.24a).

In order to determine a closed-loop state feedback control law, we can assume

$$\theta(t) = P \zeta(t) + \Omega(t). \qquad \ldots(IV.34)$$

If we substitute Eq. (IV.34) into Eqs. (IV.32), (IV.33) and (IV.24a), and determine the requirements for the solution. The procedures are;

$$\mu(t) = - R_1^{-1} B_2^T \{ P\zeta(t) + \Omega(t) \}, \qquad \ldots(IV.35)$$

$$\dot{\theta}(t) = P \dot{\zeta}(t) + \dot{\Omega}(t)$$

$$= P\{\tilde{A}\zeta(t) - B_2 R_1^{-1} B_2^T \left(P\zeta(t) + \Omega(t)\right)$$

$$+ \tilde{H} \Gamma_1(t) \} + \dot{\Omega}(t)$$

$$= - Q_1 \zeta(t) - \tilde{A}^T \{ P\zeta(t) + \Omega(t) \} .$$

Therefore, we have

$$\left[P\tilde{A} + \tilde{A}^T P - PB_2 R_1^{-1} B_2^T P + Q_1 \right] \zeta(t)$$

$$+ \dot{\Omega}(t) + \left[\tilde{A}^T - PB_2 R_1^{-1} B_2^T \right] \Omega(t) + P\tilde{H} \Gamma_1(t) = 0 .$$

...(IV.36)

In order to satisfy the condition Eq. (IV:36) for arbitrary deviation function $\zeta(t)$ and $\Gamma_1(t)$, we may be able to separate it into; for the regulation of the system,

$$P\tilde{A} + \tilde{A}^T P - PB_2 R_1^{-1} B_2^T P + Q_1 = 0, \quad \text{...(IV.37a)}$$

and for the counteraction of the noise,

$$\Omega(t) = - \left[\tilde{A}^T - PB_2 R_1^{-1} B_2^T \right] \Omega(t) + P\tilde{H} \Gamma_1(t) .$$

...(IV.37b)

The control function obtained in Eq. (IV.35) is separated[136] [138] into the regulation of the system,

$$\mu_R(t) = -R_1^{-1} B_2^T P \; \zeta(t) \qquad \ldots(IV.38a)$$

and the counteraction of the random noise $\Gamma_1(t)$,

$$\mu_C(t) = -R_1^{-1} B_2^T \; \Omega(t). \qquad \ldots(IV.38b)$$

We shall note here that, if the optimal design of the detector locations is preferred, the cost functional Eqs. (IV.26) and (IV.28) must be constructed using the expression of

$$\mu(t) = -\; P\xi(t) = -PM_2\,\zeta(t)$$

and the optimum condition would be investigated by modifying Levine's method[139].

IV.2.3 Dynamic Range of Bulk Controllers for Flux Regulation

For a pre-selected set of controllers, Eq.(IV.10) gives their static set-points, and from Eq. (IV.38a) their dynamic ranges for regulating the flux variations can be evaluated, if one takes the norm of the control function below,

$$\|\mu_R(t)\| = \| R_1^{-1} B_2^T P \ \zeta(t)\|$$

$$< \| R_1^{-1} B_2^T P \| \| \zeta(t)\|. \qquad \ldots(IV.39)$$

The definition of norm follows the conventional one that is; for a column vector,

$$\|X\| \equiv \sqrt{X^T X} = (x_1^2 + x_2^2 + \ldots + x_N^2)^{\frac{1}{2}},$$

and for a matrix,

$$\| \mathscr{A} \| \equiv (tr\ \mathscr{A}^T \mathscr{A})^{\frac{1}{2}}.$$

Even though one determines a set of controllers satisfying Eq. (IV.10) and whose gain $R_1^{-1} B_2^T P$ was chosen to be minimum, the maximum response range is still unknown until a limit in $\zeta(t)$ is defined. Fortunately nuclear power reactors have been designed with limits on power error because of safety concerns.

Estimation of $\|\zeta(t)\|$ has to follow the definition of the power error in a global sense,

$$\epsilon_g(t) = \frac{\int_{\underline{R}} d\underline{r}\ \Sigma_{f2}(\underline{r})\ \phi_2(\underline{r},t) - \int_{\underline{R}} d\underline{r}\ \Sigma_{f2}(\underline{r})\ \phi_2^{Ref}(\underline{r})}{\int_{\underline{R}} d\underline{r}\ \Sigma_{f2}(\underline{r})\ \phi_2^{Ref}(\underline{r})} \qquad \text{...(IV.40)}$$

and of the power tilt in a regional sense[140],

$$\epsilon_r(t) = \frac{\left| \int_{\underline{R}_1} d\underline{r}\ \Sigma_{f2}(\underline{r})\ \phi_2(\underline{r},t) - \int_{\underline{R}-\underline{R}_1} d\underline{r}\ \Sigma_{f2}(\underline{r})\ \phi_2^{Ref}(\underline{r}) \right|}{\int_{\underline{R}} d\underline{r}\ P_{f2}(\underline{r})\ \phi_2^{Ref}(\underline{r})} \qquad \text{...(IV.41)}$$

If we assume that the reactor power is proportional to the thermal neutron flux and if we take $\underline{R}_1 = \underline{R} - \underline{R}_1 = \frac{1}{2}\underline{R}$, the total power error would be

$$\epsilon_T(t) = \epsilon_g(t) + \epsilon_r(t)$$

$$\cong \left(a_{0,2}(t) + \left| \sum_{l=1}^{L} g_l\ a_{1,2}(t) \right| \right) \Big/ A_0^{Ref}, \qquad \text{...(IV.42)}$$

where A_0^{Ref} is the fundamental mode amplitude of the reference state, and

$$g_l = \left(\int_{\underline{R}_1} d\underline{r}\ \psi_{1,2}(\underline{r}) - \int_{\underline{R}-\underline{R}_1} d\underline{r}\ \psi_{1,2}(\underline{r}) \right) \Big/ \int_{\underline{R}} d\underline{r}\ \psi_{0,2}(\underline{r}),$$

thus, $0 \leq g_l \leq 1$ for $0 \leq l \leq L$.

Taking $A_0^{Ref} = 1$, i.e., normalized to the total reactor power, and remembering that $a_{1,2}(t)$ are elements of the state vector $\zeta(t)$, then, by the maximum power error criterion ϵ_{max},

$$\| \mathbf{g} \cdot \zeta(t) \| < \epsilon_{max}$$

or

$$\mathrm{Max} \; \| \zeta(t) \| < \epsilon_{max} / \mathrm{Min} \| \mathbf{g} \| < \epsilon_{max} \, , \qquad \ldots \text{(IV.43)}$$

where $\mathbf{g}^T = [g_0, g_1, \ldots, g_L]$, and $g_0 = 1$ for the fundamental mode.

Substituting Eq. (IV.43) into Eq. (IV.39), we have the dynamic limit of the bulk controllers, excluding the response to external random disturbances due to fuelling and burnup;

$$\| \mu_R(t) \| \leq \| R_1^{-1} \mathbf{B}_2^T \mathbf{P} \| \epsilon_{max} \qquad \ldots \text{(IV.44)}$$

IV.2.4 Most Probable Control Actions Limited on Random Local Reactivity perturbations

The control function derived to Eq. (IV.38b) is expected to counteract the random external disturbance $\Gamma_1(t)$ which is considered to be a random reactivity source. To estimate the maximum behaviour of this control system, we have to solve the dynamic equation (IV.37b) with arbitrary initial condition of $\Gamma_1(t_0)$ and with the fixed natural terminal condition of $\Omega(t_f) = 0$. Furthermore, the disturbance vector $\Gamma_1(t)$ is a random input continuously disturbing the process unpredictably.

To overcome these difficulties, we assume that the dynamic range of controllers for counteracting these disturbances would be estimated adequately by the second moments of the random state vector $\Gamma_1(t)$ and the associated adjoint vector $\Omega(t)$.

Because we assume that $\Gamma_1(t)$ is a zero-mean white Gaussian, its adjoint $\Omega(t)$ is apparently a zero-mean white Gaussian[81]. Thus, the first moment should be

$$\mathcal{E}\{\Omega(t)\} = \bar{\Omega}(t) = 0, \qquad \text{...(IV.45)}$$

and the covariance matrix $\mathbf{K}$ is

$$\mathbf{K}(t_1,t_2) \equiv \mathrm{cov}\ \{\ \Omega(t_1),\ \Omega(t_2)\}$$

$$= \mathcal{E}\left\{\left[\Omega(t_1) - \mathcal{E}\{\Omega(t_1)\}\right]\left[\Omega(t_2) - \mathcal{E}\{\Omega(t_2)\}\right]^T\right\}$$

$$= \mathcal{E}\left\{ \Omega(t_1)\ \Omega(t_2)^T \right\} , \qquad \text{...(IV.46)}$$

where $\mathcal{E}$ is the expectation value of $\{\cdot\}$.

If we define the transition matrix $\Psi(t,t')$ satisfying the differential equation

$$\frac{\partial \Psi(t,t_0)}{\partial t} = -\left[\tilde{A}^T - PB_2\, R_1^{-1} B_2^{\ T} \right] \Psi(t,t_0) \ , \qquad \text{...(IV.47)}$$

with the initial condition

$$\Psi(t_0,t_0) = I \ ; \quad \text{identity matrix}, \qquad \text{...(IV.48)}$$

then, the solution of Eq. (IV.37b) will be in the form of

$$\Omega(t) = \Psi(t,t_0)\ \Omega(t_0)$$

$$+ \int_{t_0}^{t} dt'\ \Psi(t,t')\, P\tilde{H}\ \Gamma_1(t') . \qquad \text{...(IV.49)}$$

Substituting Eq. (IV.49) into Eq. (IV.46), we have

$$K(t_1,t_2) = \mathcal{E}\left\{\left[\Psi(t_1,t_0)\,\Omega(t_0) + \int_{t_0}^{t_1} \Psi(t_1,t')\; P\tilde{H}\Gamma_1(t')\,dt'\right]\right.$$

$$\left.\left[\Psi(t_2,t_0)\,\Omega(t_0) + \int_{t_0}^{t_2} \Psi(t_2,t')\; P\tilde{H}\,\Gamma_1(t')\,dt''\right]^T\right\}$$

$$= \mathcal{E}\left\{\Psi(t_1,t_0)\,\Omega(t_0)\,\Omega^T(t_0)\;\Psi^T(t_2,t_0)\right\}$$

$$+ \mathcal{E}\left\{\Psi(t_1,t_0)\,\Omega(t_0)\int_{t_0}^{t_2} \Gamma_1(t')^T\; P\tilde{H}^T\Psi^T(t_2,t')\,dt'\right\}$$

$$+ \mathcal{E}\left\{\int_{t_0}^{t_1} \Psi(t_1,t')\; P\tilde{H}\,\Gamma_1(t')\,dt'\,\Omega^T(t_0)\,\Psi^T(t_2,t_0)\right\}.$$

$$+ \mathcal{E}\left\{\int_{t_0}^{t_1}\int_{t_0}^{t_2} \Psi(t_1,t_1')\; P\tilde{H}\,\Gamma_1(t_1')\;\Gamma_1(t_2')^T\;\tilde{H}^T P\Psi^T(t_2,t_2')\;dt_1'dt_2'\right\}$$

$$= \Psi(t_1,t_0)\;\mathcal{E}\left\{\Omega(t_0)\,\Omega(t_0)^T\right\}\;\Psi^T(t_2,t_0)$$

$$+ \Psi(t_1,t_0)\int_{t_0}^{t_2} \mathcal{E}\left\{\Omega(t_0)\;\Gamma_1(t')^T\right\}\tilde{H}^T\,P\,\Psi^T(t_2,t')\,dt'$$

$$+ \int_{t_0}^{t_1} \Psi(t_1,t')\mathbf{P}\widetilde{\mathbf{H}}\ \mathcal{E}\left\{\Gamma_1(t')\,\Omega(t_0)^T\right\} dt'\,\Psi^T(t_2,t_0)$$

$$+ \int_{t_0}^{t_1}\int_{t_0}^{t_2} \Psi(t_1,t')\mathbf{P}\widetilde{\mathbf{H}}\ \mathcal{E}\left\{\Gamma_1(t_1')\,\Gamma_1(t_2')^T\right\} \widetilde{\mathbf{H}}^T\mathbf{P}\Psi^T(t_2,t_2')\ dt_1'dt_2'$$

...(IV.50)

Obviously there is no correlation between $\Gamma_1(t)$ and $\Omega(t_0)$ for $t \geq t_0$, i.e.,

$$\mathcal{E}\left\{\Omega(t_0)\ \Gamma_1(t')^T\right\} = 0. \qquad \text{...(IV.51)}$$

Recalling Eq. (IV.25), we have

$$\mathbf{K}(t_1,t_2) = \Psi(t_1,t_0)\,\mathbf{K}(t_0,t_0)\,\Psi^T(t_2,t_0)$$

$$+ \int_{t_0}^{t_1}\int_{t_0}^{t_2} \Psi(t_1,t_1')\ \mathbf{P}\widetilde{\mathbf{H}}\ Q_2(t_1')\ \delta(t_1'-t_2')\ \widetilde{\mathbf{H}}^T\mathbf{P}\Psi^T(t_2,t_2')\ dt_1'dt_2'$$

$$= \Psi(t_1,t_0)\,\mathbf{K}(t_0,t_0)\,\Psi^T(t_2,t_0)$$

$$+ \int_{t_0}^{\mathrm{Min}\{t_1,t_2\}} \Psi(t_1,t')\, P\hat{H}\, Q_2(t')\, \tilde{H}^T P \Psi^T(t_2,t')\, dt' .$$

...(IV.5 2)

To avoid difficulties met in solving the integral equation (IV.52), we try to transform it to a differential form;

$$\frac{d\,K(t)}{d\,t} = \frac{\partial \Psi(t,t_0)}{\partial t} K(t_0)\, \Psi^T(t,t_0) + \Psi(t,t_0)\, K(t_0) \frac{\partial \Psi^T(t,t_0)}{\partial t}$$

$$+ \Psi(t,t)\, P\tilde{H}\, Q_2(t)\, \tilde{H}^T P \Psi^T(t,t)$$

$$+ \int_{t_0}^{t} \frac{\partial \Psi(t,t')}{\partial t} P\tilde{H}\, Q_2(t')\, \tilde{H}^T P \Psi^T(t,t')\, dt'$$

$$+ \int_{t_0}^{t} \Psi(t,t')\, P\tilde{H}\, Q_2(t')\, \tilde{H}^T P \frac{\partial \Psi^T(t,t')}{\partial t} dt' .$$

...(IV.53)

Using Eq. (IV.47) and its transpose equation

$$\frac{\partial \Psi^T(t,t_0)}{\partial t} = - \Psi^T(t,t_0) \left[\tilde{A}^T - B_2 R_1^{-1} B_2^T \right]^T, \qquad \ldots \text{(IV.54)}$$

Eq. (IV.53) can be written to

$$\frac{d\, K(t)}{d\ t} = -\left[\tilde{A}^T - P B_2 R_1^{-1} B_2^T\right] \Psi(t,t_0)\, K(t_0)\, \Psi^T(t,t_0)$$

$$- \Psi(t,t_0)\, K(t_0)\, \Psi^T(t,t_0) \left[\tilde{A}^T - P B_2 R_1^{-1} B_2^T\right]^T$$

$$- P \tilde{H} Q_2(t)\, \tilde{H}^T P$$

$$- \left[\tilde{A}^T - P B_2 R_1^{-1} B_2^T\right] \int_{t_0}^{t} \Psi(t,t_0)\, P \tilde{H}\, Q_2(t')\, \tilde{H}^T P \Psi^T(t,t')\, dt'$$

$$- \int_{t_0}^{t} \Psi(t,t')\, P \tilde{H}\, Q_2(t')\, \tilde{H}^T P \Psi^T(t,t')\, dt' \left[\tilde{A}^T - P B_2 R_1^{-1} B_2^T\right]^T \qquad \ldots \text{(IV.55)}$$

The integration parts in the last two terms can be replaced by Eq. (IV.52) and we finally have a simple reduced form,

$$\frac{d\,K(t)}{d\,t} = -\left[\tilde{A}^T - PB_2\,R_1^{-1}B_2^T\right]K(t)$$

$$- K(t)\left[\tilde{A}^T - PB_2\,R_1^{-1}B_2^T\right]^T - P\tilde{H}\,Q_2(t)\,\tilde{H}^T P\,,$$

....(IV.56)

which is a differential Lyapunov equation.

In addition to the time-invariance of the system equation (IV.37b), if we consider the noise statistics as being stationary, then, we will have significant practical advantages in applying the steady-state approximation in Eq. (IV.56)[(46),(141)]. This is realizable without losing any generality, because $Q_2(t)$ can be obtained from the burnup distribution of discharged fuel bundles, which is essentially independent of the reactor condition at any particular moments.

Thus, we invoke the algebraic matrix Lyapunov equation, to determine the most probable dynamic range of the counteracting controllers, $\mu_c(t)$, by replacing $Q_2(t)$ by a constant diagonal matrix Q and by taking $\dot{K}(t) = 0$;

$$-\left[\tilde{A} - PB_2\,R_1^{-1}B_2^T\right]^T K + K\left[\tilde{A} - B_2\,R_1^{-1}B_2^T P\right]$$

$$+ P\tilde{H}\,Q\,\tilde{H}^T P = 0\,.$$

...(IV.57)

If we assume that there exists a positive-definite symmetric matrix solution $\mathbf{K}$ of Eq. (IV.57) with the requirements mentioned in Section III.3.4, then, we can investigate the relationship between $\mathbf{K}$ and the most probable dynamic range of $\mu_c(t)$. Apparently from the steady-state operation of the reactor, the mean of counteraction for the zero-mean Gaussian-type noise disturbances would be approximately zero, i.e., $\overline{\mu}_c(t) \approx 0.$ and, thus, the covariance of $\mu_c(t)$ will be, from Eq. (IV.38b),

$$\mathcal{E}\{\mu_c(t)\ \mu_c(t)^T\}$$

$$= R_1^{-1}\mathbf{B}_2^T\ \mathcal{E}\left\{\Omega(t)\ \Omega(t)^T\right\}\mathbf{B}_2\ R_1^{-T}$$

$$= R_1^{-1}\mathbf{B}_2^T\mathbf{K}\,\mathbf{B}_2\ R_1^{-T}. \qquad \text{...(IV.58)}$$

The standard deviation of individual controller action will be simply obtained from the diagonal elements of matrix in Eq. (IV.58),

$$\sigma = \left[\text{diag.}\left\{R_1^{-1}\mathbf{B}_2^T\mathbf{K}\mathbf{B}_2\ R_1^{-T}\right\}^{\frac{1}{2}}\right]^T$$

where σ is vector expression of the standard deviation, i.e., the most probable range of individual controller.

IV.3 Strategies for Determination of Optimum Controller Set

The procedures involved in determining the number of controllers consist of three steps. From the results of Chapter III, we have developed a method to generate a map of controller effectiveness. One can select a number of high control gain nodes whose expected total reactivity worth would be roughly equal to the excess reactivity of the reference model. The number of controllers at this stage is approximately estimated by the amount of absorbing material allowed for the static set-point of each controller. During this initialization procedure, one must check the cost loss that is a measure of flux deviation due to the static controller deployment. It seems that there would be number of freedoms to accomplish the procedure, but the probable locations for controllers are restricted by avoiding fuel sites and other instrumentation nodes. Even a relatively inexperienced designer can easily find a set of locations after a few trials.

The second step is the computation of the dynamic range of the pre-selected controller set. If the boundary of a controller dynamic range is beyond the maximum allowable absorbing material limit, then, we have to return to the first step with an additional number of controllers. At this stage, the dynamic controllers respond only to the neutron flux regulation, and not to external random reactivity supply. Also we can compute the optimality of the controller set by solving the matrix Riccati equation (IV.38a), and then by comparing with solutions for other possible controller sets.

The final step is mainly to improve the reliability and adequacy of the designed control system. Additional reactivity sources with local perturbation are always possible in operating reactors. Our control system must accommodate those external disturbances at any instance. The probable dynamic range of controllers is evaluated for the system with a probable disturbance range. If there is a controller whose action boundary is larger than the limit criteria, we must return to the first step and re-select different controller locations.

V. OPTIMALITY OF THE 600 MWE CANDU ZONE CONTROLLERS

V.1 Two-Dimensional Reactor Model

The thrust of this chapter is to apply the theory we have developed up to here and assure ourselves of the optimal performance of our control system. The reactor model we deal with is the two-dimensional model of the standard 600 MWe CANDU-PHW reactor illustrated in Figure V-1.

The material properties[15] are evaluated by flux-weighted, averaging over the reactor length and tabulated in Table V-1. The total number of different materials equals 12 for the reflected core and three types of the adjuster rods.

The λ-modes for the same model were generated with seven harmonics plus the fundamental mode. Because of excessive computing time to generate high harmonics, we determined to use an approximation of integrating the existing 3-dimensional numerical mode functions[123] over the reactor length instead of directly generating 2-dimensional modes. The properties of the first eight modes are given in Table V-2. The degeneracy of the complemented azimuthal modes is eliminated after

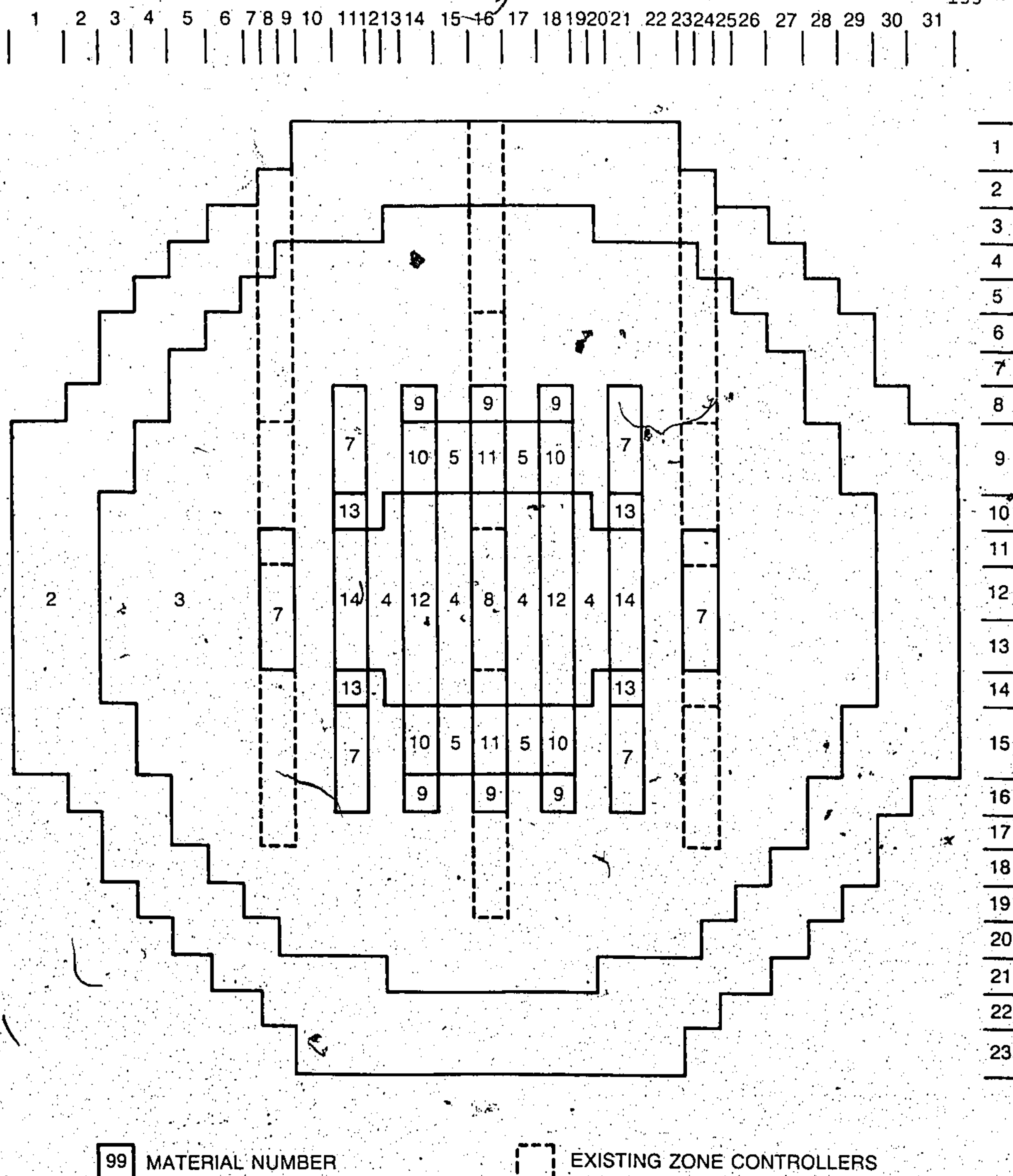


FIGURE V-1 2-DIMENSIONAL REACTOR MODEL

No.	Description	D_1	D_2	Σ_{a1} $(x10^{-4})$	Σ_{a2} $(x10^{-3})$	Σ_{Rem} $(x10^{-3})$	$\nu\Sigma_{f2}$ $(x10^{-3})$	H*
2	Reflector	1.3270	0.8787	$1.0x10^{-7}$	0.083415	10.1241	0.0	0.0
3	Outer Core	1.2739	0.9413	7.6387	3.9795	7.39171	4.6116	0.2089695
4	Inner Core	1.2739	0.9412	7.6355	3.9976	7.39203	4.5960	0.2072662
5	$\frac{1}{2}$ Inner + $\frac{1}{2}$ Outer Core	1.2739	0.9413	7.6371	3.9886	7.39187	4.6038	0.2081178
7	Adjuster I + Outer Core	1.2739	0.9413	7.6387	3.9837	7.39171	4.6196	0.2089876
8	Adjuster I + Inner Core	1.2739	0.9412	7.6355	4.0018	7.39203	4.5964	0.2072843
9	Adjuster III + Outer Core	1.2739	0.9413	7.6387	3.9978	7.39272	4.6129	0.2090351
10	Adjuster III + $\frac{1}{4}$ Inner Core + $\frac{3}{4}$ Outer Core	1.2739	0.9413	7.6379	4.0023	7.39179	4.6090	0.2086093
11	Adjuster III + $\frac{1}{2}$ Inner Core + $\frac{1}{2}$ Outer Core	1.2739	0.9413	7.6371	4.0069	7.39187	4.6051	0.2081834
12	Adjuster III + Inner Core	1.2739	0.9412	7.6355	4.0159	7.39203	4.5973	0.2073318
13	Adjuster IV + Outer Core	1.2739	0.9413	7.6387	3.9864	7.39171	4.6122	0.2090014
14	Adjuster IV + $\frac{1}{2}$ Inner Core + $\frac{1}{2}$ Outer Core	1.2739	0.9413	7.6371	3.9955	7.39187	4.6044	0.2081497

* Heat output factor per bundle, $Kw/(10^{11}\overline{n/cm^2}.sec)$

TABLE V-1 NUCLEAR PROPERTIES OF REACTOR MATERIALS

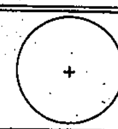
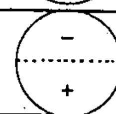
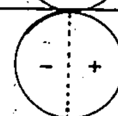
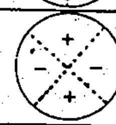
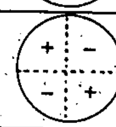
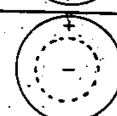
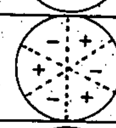
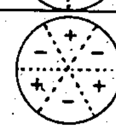
No	Name	Eigenvalue $\left(\frac{1}{\lambda_m}\right)$	Shape
1	Fundamental	1.002330	
2	First Azimuthal	0.985047	
3	First Azimuthal	0.984677	
4	Second Azimuthal	0.958128	
5	Second Azimuthal	0.955536	
6	First Radial	0.925435	
7	Third Azimuthal	0.922456	
8	Third Azimuthal	0.920478	

Table V-2 Properties of the 2-Dimensional λ- Modes the third digit of eigenvalues, mostly because the geometry of the reactor model was not completely symmetric and because of the vertical arrangement of the incore control mechanism.

Distributions of generated mode functions for the fast and the thermal neutrons are normalized to the fundamental mode flux at the center of core and are plotted in Figs. V-2 to -5, respectively. The figures show better symmetricity with respect to the Y-axis than with the X-axis. This supports the results that the degeneracy of eigenvalues were destroyed.

Also, with the controversial errors involved in the finite difference formulation of the X-Y geometry in order to generate harmonics having R-θ properties, the axial integration may cause undesirable distortions in mode shapes especially for the radial modes. The effects will be discussed later in the section analyzing various perturbation simulations.

Input parameters used in calculations are tabulated in Table V-3. The only material property considered for controllers was the effective thermal neutron absorption cross section of the existing CANDU zone controllers. The magnitude is equivalent to the amount of homogenized neutron absorbing material giving the proper current-to-flux ratio at the core-controller interface and the same reaction rate inside controllers. The MULTICELL code[143] was available to provide the

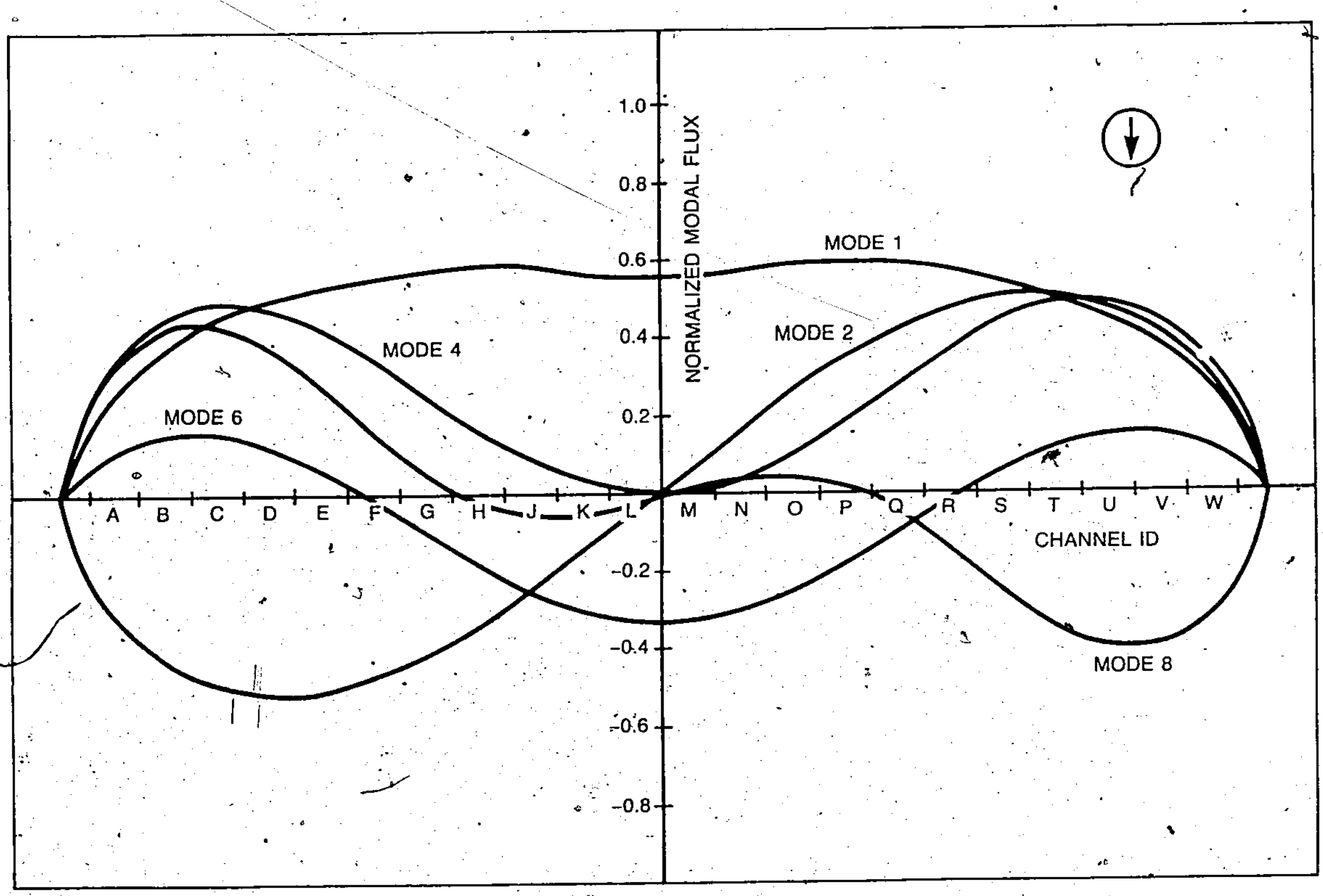


FIGURE V-2 FAST FLUX MODE DISTRIBUTIONS (VIEW 1)

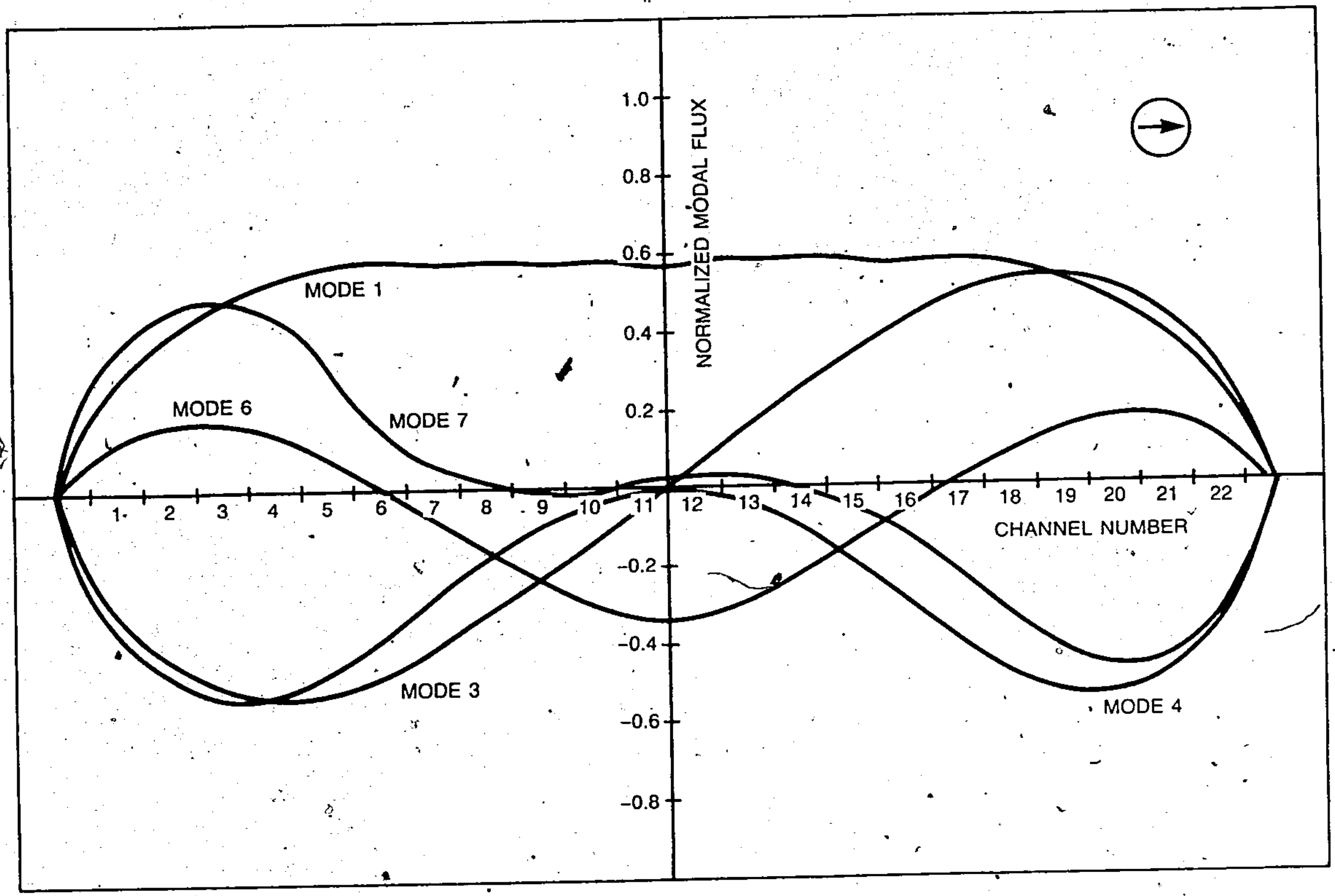


FIGURE V-3 FAST FLUX MODE DISTRIBUTIONS (VIEW 2)

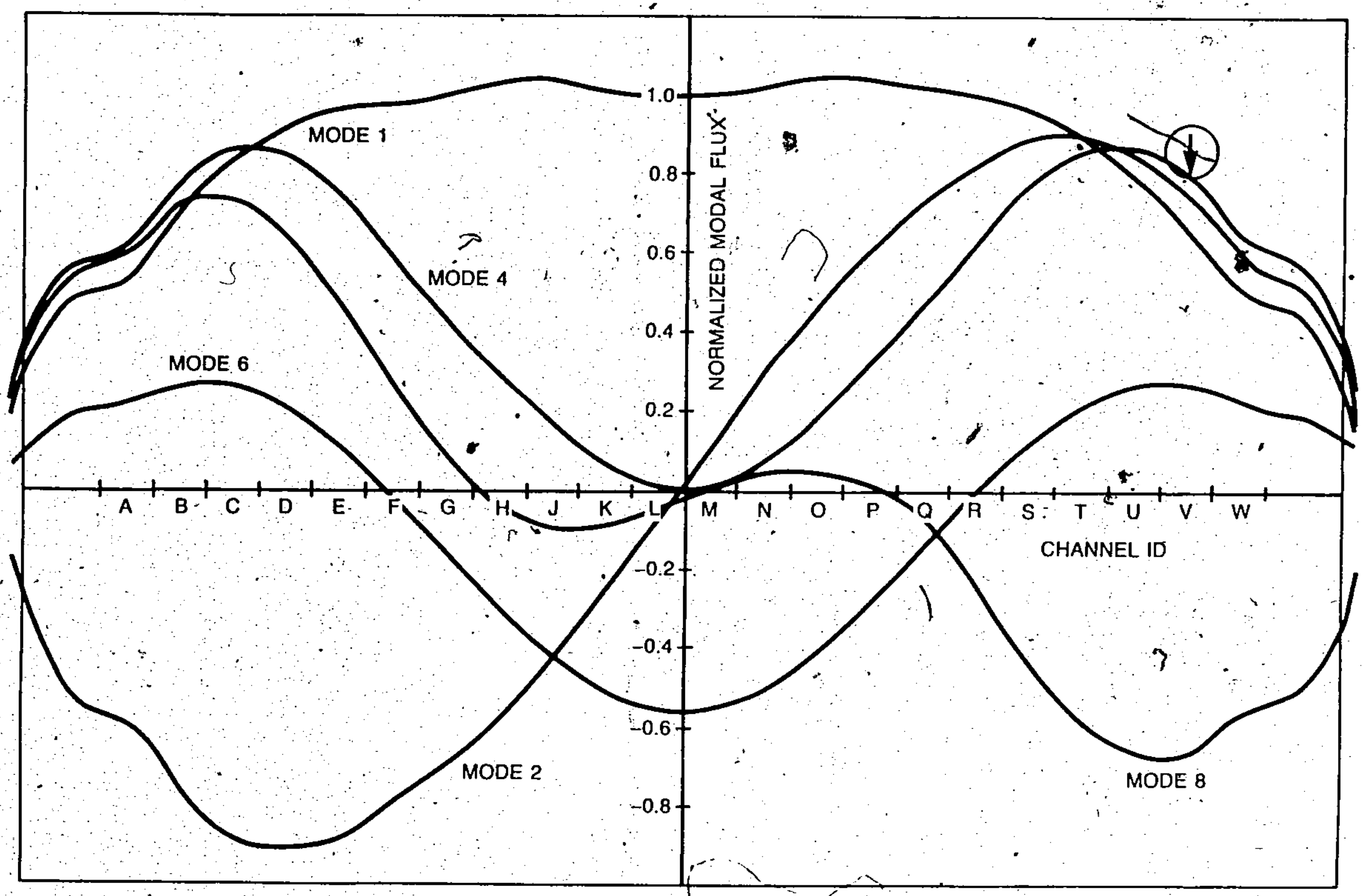


FIGURE V-4 THERMAL FLUX MODE DISTRIBUTIONS (VIEW 2)

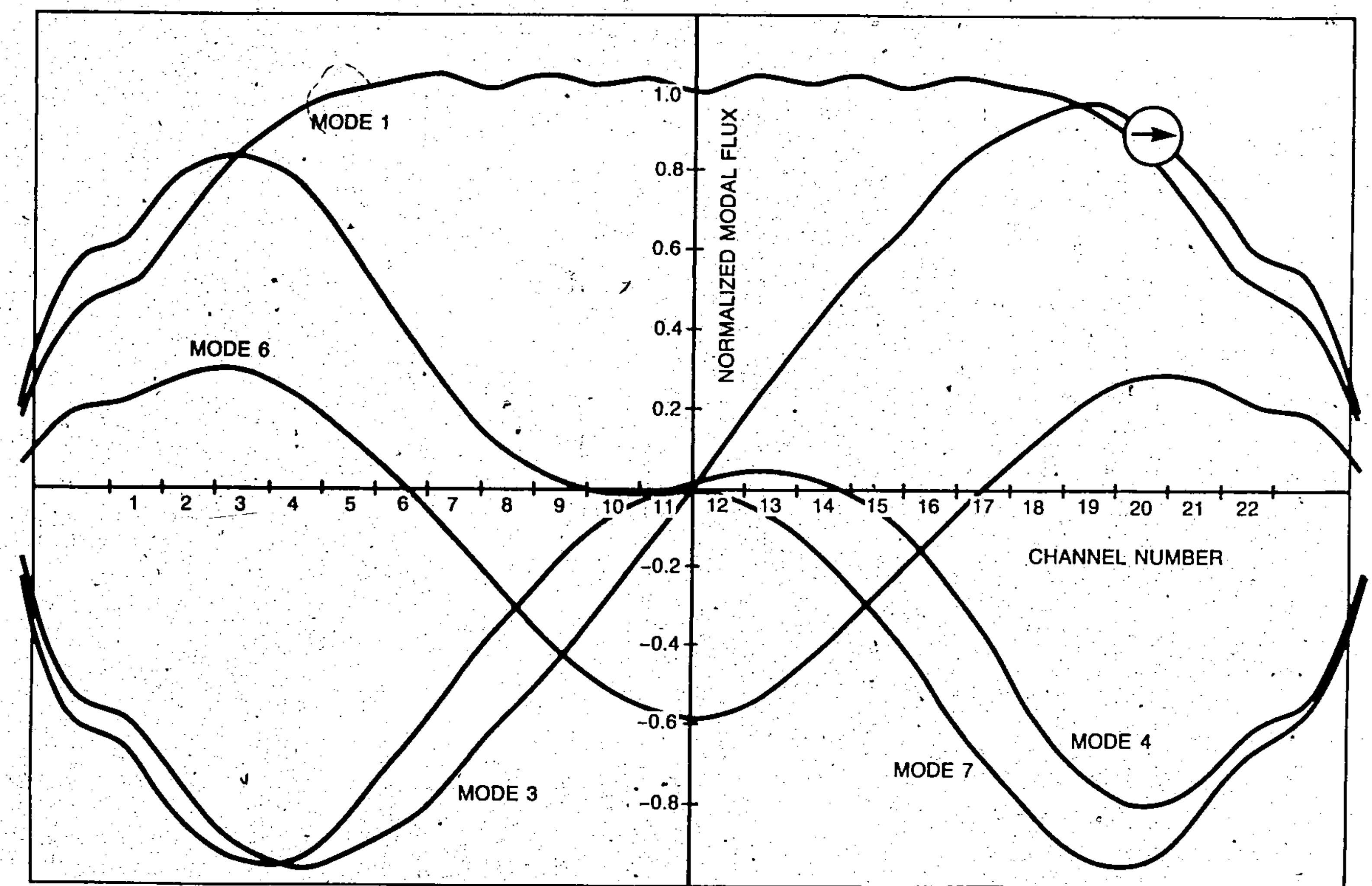


FIGURE V-5 THERMAL FLUX MODE DISTRIBUTIONS (VIEW 1)

Descriptions (unit)	Value
Neutron Yield, ν	2.6062
Microscopic Xenon Absorption Cross-section, σ_a^{xe} (cm^2)	1.261×10^{-18}
Direct Yield Fraction of Iodine, γ_I	6.44×10^{-2}
Direct Yield Fraction of Xenon, γ_x	2.30×10^{-3}
Iodine Decay Constant, λ_I (sec^{-1})	2.94×10^{-5}
Xenon Decay Constant, λ_x (sec^{-1})	2.10×10^{-5}
Neutron Velocity (cm/sec) ; Thermal	2.676×10^{5}
: Fast	5.335×10^{6}
Macroscopic Absorption Cross-section of Controller Material (cm^{-1})	1.23×10^{-4}
Unit Controller Area (cm^2)	8.1653×10^{2}

TABLE V-3 INPUT PARAMETERS USED IN CALCULATIONS effective cross section. Finally smearing of material properties over the reactor length was necessary to obtain 2-dimensional controller properties.

V.2 Stability and Spatial Control Effectiveness

One of the important characteristics of the reactor for the design of the control system is whether all the reactor modes are stable or not. If some modes are unstable, the priority in design considerations should be given to relocating all the unstable eigenvalues to the stable region. However, we note that this is clearly a weaker requirement than complete modal controllability.

To examine the stability of the model reactor, the code, ODZCR was used to compute eigenvalues of the iodine/xenon dynamic system implicitly including neutron flux modes at various power levels. Iodine/xenon levels at different power states were obtained by steady state equilibrium condition of the neutron flux level associated with that power level. The feedback effect due to the power level changes was not considered in those uncontrolled reactor system equations in the study.

As the steady state distributions could be calculated, the condition of spatial stability implied that all of the inverse time characteristics of eigenvalues, i.e.,

$$\omega = a + ib(hr^{-1})$$

of the iodine/xenon mode, should have a negative real part ($a \leq 0$) for stability. Otherwise, the system would either increase exponentially with an e-folding time of $1/a$, or oscillate divergently with a period $2\pi/b$ and an e-folding time characteristic.

The calculated eigenvalues are listed in Table V-4. They show that the system is unstable with respect to the first two couples of eigenvalues for reactor power above 70%. In examining the system equation, we find that these eigenvalues are associated with the first two azimuthal modes, i.e. the top-to-bottom and the side-to-side λ-modes, respectively. Thus, the e-folding time and the period of the top-to-bottom mode for uncontrolled transient are approximately 8.9 and 19.9 hours.

Figure V-6 gives a more accurate threshold power level, above which the oscillatory diverging power would occur. At 50% power level, a power tilt oriented from top-to-bottom will cause to the reactor to be unstable. A 3-dimensional diffusion calculation for the reactor has shown the threshold power to be about 40 to 60%[140].

To calculate the spatial control effectiveness of the uncontrolled system, the program ODZCR requires several preliminary simulations; for determining the minimum number of controllers required to stabilize and control the entire system; for choosing appropriate

TABLE V-4 EIGENVALUES OF IODINE/XENON SYSTEM

% Power	100		80		70		40		20	
	Real**	Imagin.	Real	Imagin.	Real	Imagin.	Real	Imagin.	Real	Imagin.
1*	3.1267	8.8235	1.6955	8.3804	0.9864	8.0518	-0.4077	7.1341	-1.0830	6.5108
2	3.1267	-8.8235	1.6955	-8.3804	0.9864	-8.0518	-0.4077	-7.1341	-1.0830	-6.5108
3	2.7428	8.9338	1.3980	8.4181	0.7321	8.0615	-0.5760	7.1036	-1.2087	6.4684
4	2.7428	-8.9338	1.3980	-84181	0.7321	-8.0615	-0.5760	-7.1036	-1.2087	-6.4684
5	-15.777	0.0	-12,744	0.0	-11.195	0.0	-7.8702	0.0	-4.8505	3.6267
6	-16.167	0.0	-13.068	0.0	-11.488	0.0	-8.1238	0.0	-4.8505	-3.6267
7	-16.691	0.0	-13.471	0.0	-11.830	0.0	-8.3345	0.0	-4.9908	3.4209
8	-7.5742	4.5313	-6.6776	4.4314	-6.2266	4.3132	-5.3150	3.9196	-4.9908	-3.4209
9	-7.5742	-4.5313	-6.6776	-4.4314	-6.2266	-4.3132	-5.3150	-3.9196	-5.7653	8.3280
10	-7.9652	3.8105	-6.9844	3.9319	-6.4915	3.9021	-5.4965	3.6537	-5.7653	-8.3280
11	-7.9652	-3.8105	-6.9844	-3.9319	-6.4915	-3.9021	-5.4965	-3.6537	-5.8129	6.7019
12	-4.5466	0.0	-4.6214	0.0	-4.6853	0.0	-5.0051	0.0	-5.8129	-6.7019
13	-4.6527	0.0	-4.7850	0.0	-48528	0.0	-5.1970	0.0	-5.9549	7.5051
14	-4.7059	0.0	-4.7314	0.0	-4.799	0.0	-5.1502	0.0	-5.9549	-7.4051

Notes ; * In the order of magnitudes of absolute eigenvalues

** Multiplied by 10^{-5}

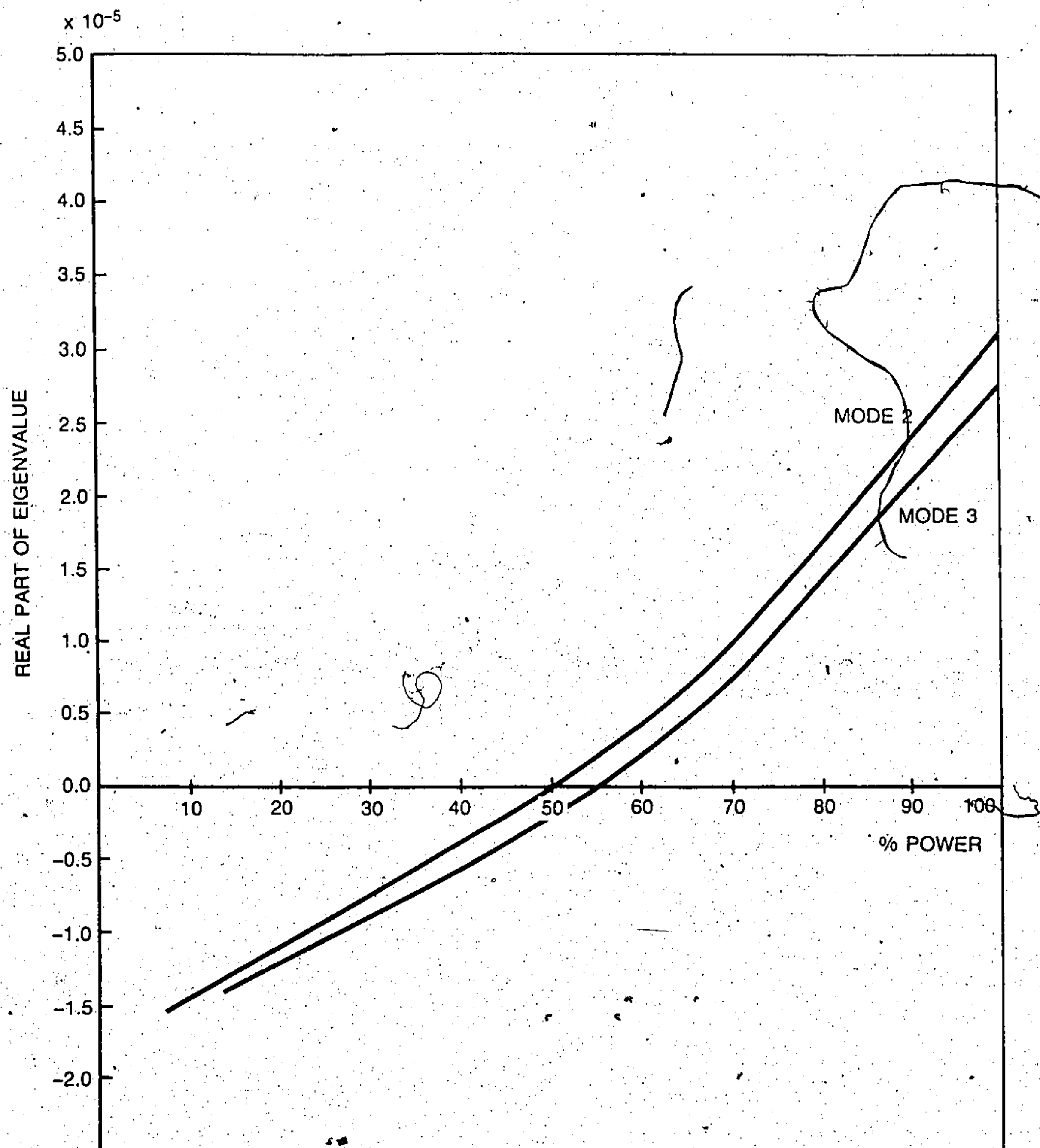


FIGURE V-6 THRESHOLD POWER AGAINST UNSTABLE MODES

Q and R matrices in Eq. (III.20) in order that the Riccati equation behaves numerically well; and for determining α in Eq. (III.52) which promotes the speed of convergence.

Those studies concluded that a pair of controllers symmetrically located in the reactor satisfied the controllability requirements for the unstable system, and that numerical stability was guaranteed with

R as the unit matrix ,

Q a diagonal matrix whose elements are 0.002, and

α = 0.001.

Thus, the results of the calculation imply an equal importance of every mode amplitude loss and of control penalty paid by individual controllers.

An interesting point indicated in the simulations was that, even though eigenvalues of complemented azimuthal λ -modes could be distinguished after the third digit in their numerical values, the analytical properties of degeneracy were hardly relaxed, which leads to the result that a single controller won't satisfy the Wieberg's Theorem I for the minimum controller criterion.

The controller domain was arbitrarily chosen with preference given to high neutron flux regions and peaks of mode shapes.

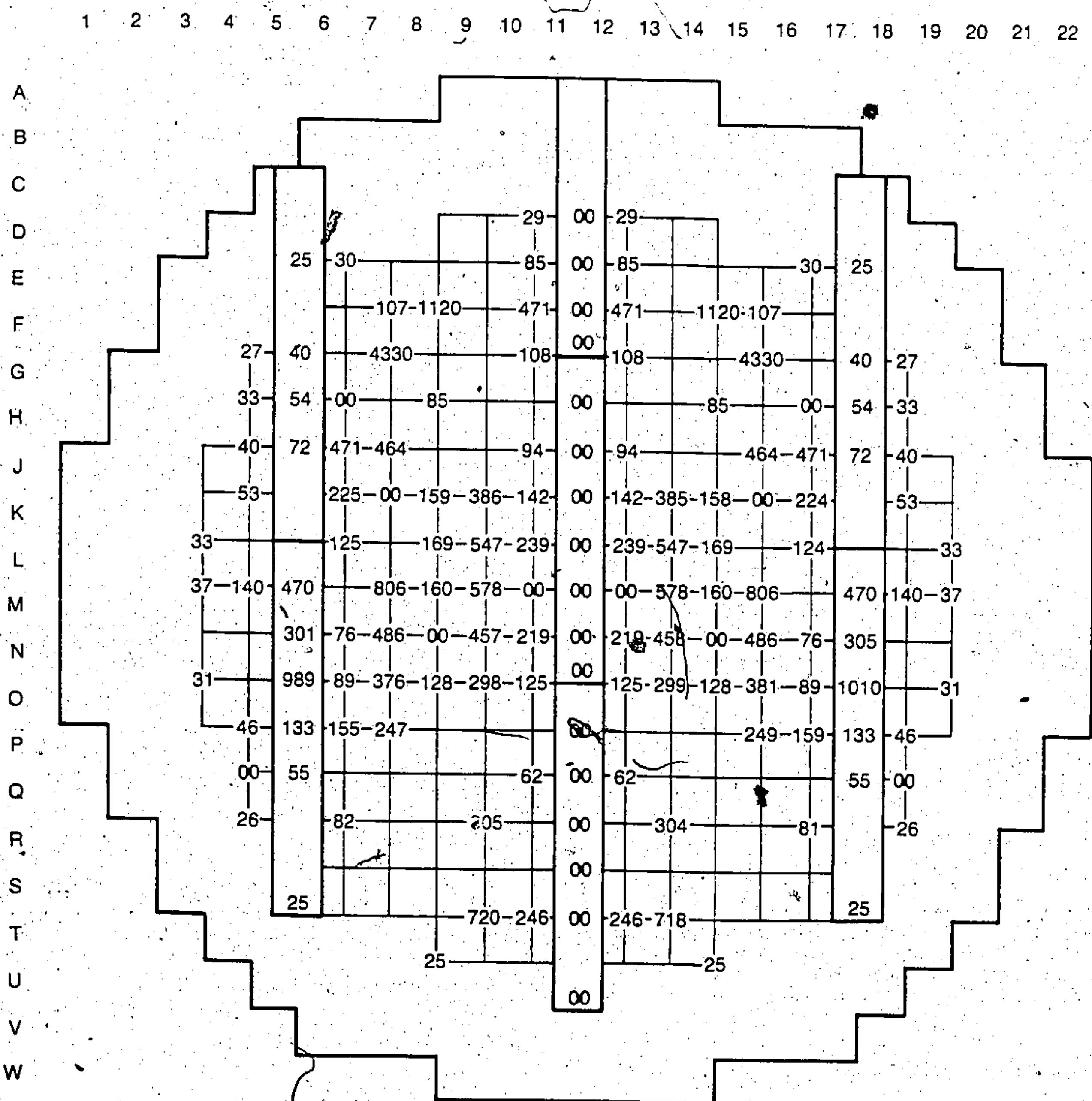
The only possible sites for locating the controllers were on grid points which match the lattice spacing of 28.575 cm, because fuel channels occupy the in-between space.

The computations were scanned over each nodal point inside the controller domain with a pair of controllers symmetrically deployed with respect to the Y-axis or to the center of the reactor. Results of both cases are illustrated in Figs. V-7 and -8, respectively. At nodes where the Riccati equations were not fully converged to the given convergence criterion 10^{-2} after 25 iterations, the effectiveness distributions were deleted in the figures.

Reminding that the spatial control effectiveness was quantified by

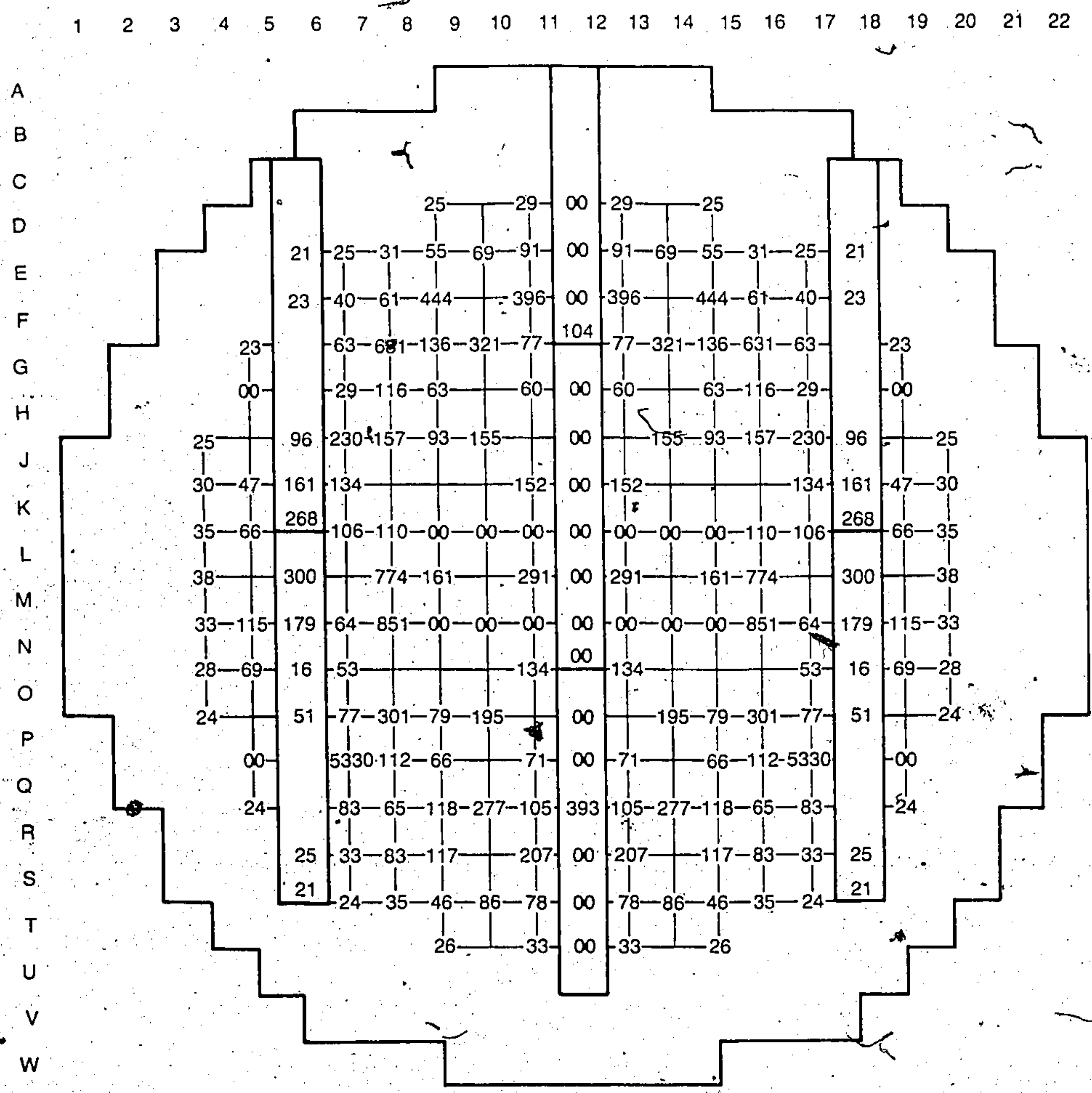
$$
\begin{aligned}
\|\Pi_s(t)\| &= \| -R^{-1}\mathbf{E}^T \ \Theta(\underline{r},t) \| \\
&= \| -R^{-1}\mathbf{E}^T \mathbf{P}(\underline{r}_j) \cdot \mathbf{B}(t) \| \\
&\leq \| -R^{-1}\mathbf{E}^T \mathbf{P}(\underline{r}_j) \| \ \|\mathbf{B}(t)\|
\end{aligned}
$$

in Eqs. (II.29) and (III.31), the minimum of $\| -R^{-1}\mathbf{E}^T \ \mathbf{P}(\underline{r}_j) \|$ should be equivalent to a minimum of $\|\Pi_s(t)\|$ for any kind of perturbation transient $\mathbf{B}(t)$. Therefore, the smaller values of $\| -R^{-1}\mathbf{E}^T \mathbf{P}(\underline{r}_j) \|$ shown in Figs. V-7 and -8 represent the more effective locations for the spatial control.



NOTE: BLANK-NOT COMPLETELY
CONVERGED AFTER 25 ITERATIONS

FIGURE V-7. SPATIAL CONTROL EFFECTIVENESS MAP
(Y — SYMMETRIC DEPLOYMENT)



NOTE: BLANK-NOT COMPLETELY
CONVERGED AFTER 25 ITERATIONS

FIGURE V-8 SPATIAL CONTROL EFFECTIVENESS MAP
(℄ — SYMMETRIC DEPLOYMENT)

In comparing Figures V-7 and V-8, we find that the regions of low value of the inverse of control effectiveness are coincident. Further, if controllers are deployed at the boundary of the given controller domain, the best spatial control is expected with an order of magnitude greater than for those at the center of the core where only a few sites have a reasonably good effectiveness, and that the nodes along the Y-axis are the worst sites for spatial control. These facts mean that the controller domain boundary was fortunately chosen so as to be adjacent to the areas where the peaks of most high harmonics, as shown in Figs. V-4 and -5, are located. Also, because of the nearly perfect Y-symmetry of the system and harmonics, the controllers located along the Y-axis cannot control modes 3, 5, and 7 of Table V-2, whose magnitudes at the controller sites are zero. Also the rank of the controllability matrix of Eq. (III.37) is less than the rank of the system matrix.

It can be stated that in spite of the fact that the most effective region for bulk reactivity control is the central region of the reactor, where we expect a high neutron importance, the spatial controllers should be located where the peaks of the harmonics occur. But, it should be noted that, given the existing shape and magnitude of the neutron flux distribution, one cannot assign separately spatial control and bulk control functions to different controllers. Designers must be careful to deploy controllers whose dynamic range responds to both effects. The range must be limited by hardware considerations.

V.3 Evaluation of the CANDU 600 MWE Zone Control System

During normal operation, the CANDU zone control system has the prime responsibility for both of bulk and spatial control. This section investigates the optimality of the CANDU 600 MWe zone control system in terms of dynamic response ranges with respect to variation of locations in each vertical compartment. Because the code ODZCR treats controllers as points in the reactor, the analyses include uncertainty due to neglecting neutron flux variations along the controller length which should be considered when handling volumetric controllers.

A reference location of the control system was arbitrarily selected which would have similar average responses as the existing vertical controllers, as estimated from design data. Analysis followed procedures described in Chapter IV with various control systems whose individual controller had the same locations corresponding to the reference location, except for a single controller which was vertically shifted between the top and the bottom boundaries of the compartment.

Figure V-9 illustrates the locations of controllers selected for the study. The grid points marked by a circle are the reference locations designating the corresponding compartments. The numbers shown inside the squares indicate the particular case of study. Therefore, the case I-2 means that the controller I and its symmetrical counter part II were shifted from their reference locations to the marked locations.

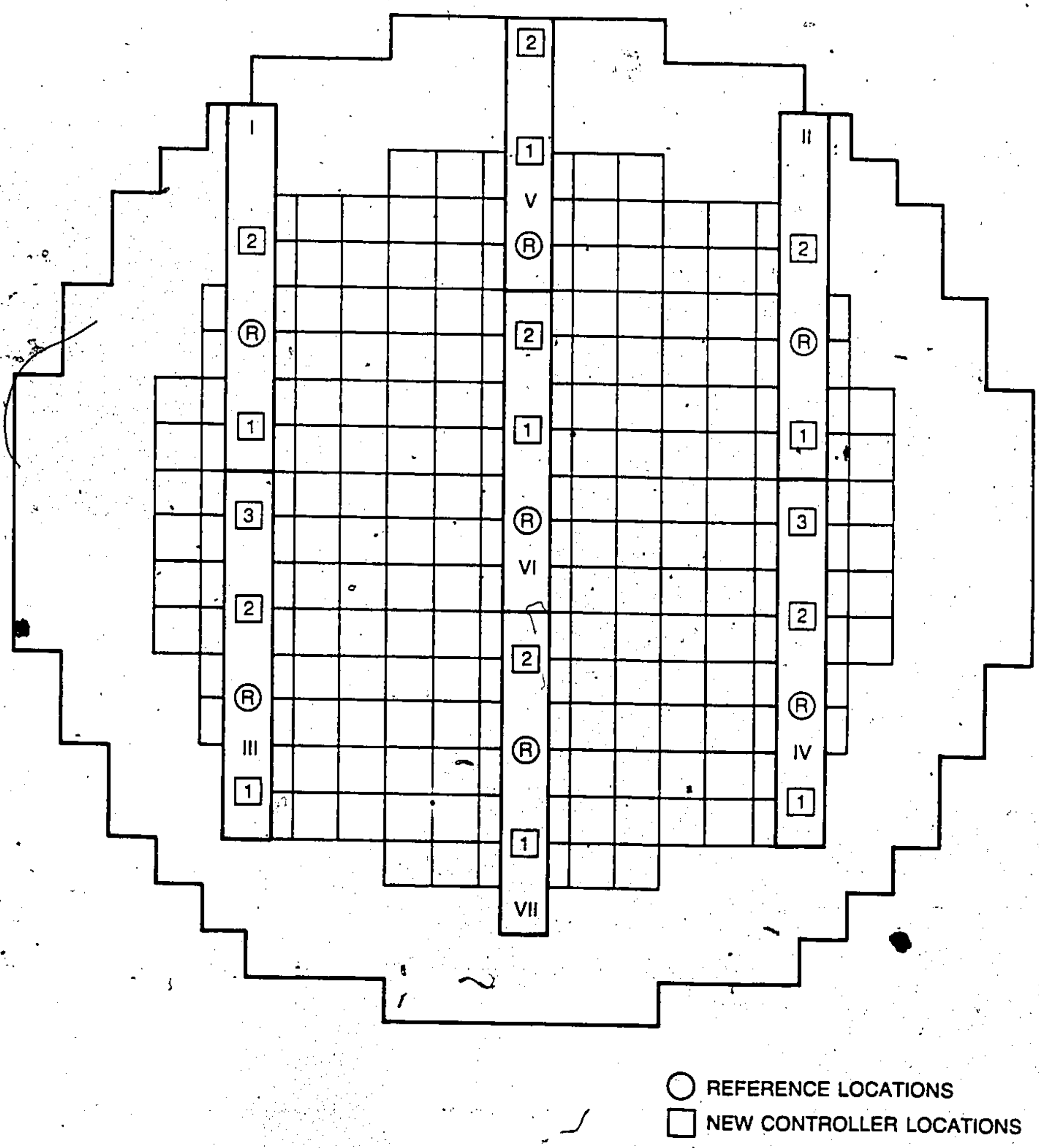


FIGURE V-9 LOCATIONS ASSIGNED FOR CONTROLLERS IN THE VERTICAL REFERENCE ZONE CONTROL SYSTEM

Thus, the results of the vertical shifting exercise would be useful to measure the optimality of deployment policy in terms of the spatial control effectiveness and of the degree of reactivity compensation with minimum neutron flux distortions.

In order to evaluate additional controller actions to compensate for local burnup/fuelling effects, the coveriance matrix Q_2 in Eq. (IV.25) was, again, chosen to be a constant diagonal matrix. Figures V-10 shows that the burnup distribution of discharged fuel bundles would fit fairly well a Gaussian distribution. Also, the multiplication factor variation over burnup intervals equivalent to a standard deviation of the above Gaussian distribution is approximately linear. From this linearity, the coveriance of the random vector $\Gamma_1(t) = \Delta\nu\Sigma_{f2}(t) / \nu\Sigma_{f2}^{Ref}$ can be replaced by the variance on the k_∞ coordinate* instead of the burnup or the time coordinate, such that diagonal elements of the covariance matrix Q_2 are equal to the square of the standard deviation of bundle distribution on the k_∞ coordinate, estimated to be 1.06×10^{-4}.

Table V-5 gives the depth of the control systems to compensate for the excess reactivity originally introduced in the reference state of the reactor. After comparing the thermal neutron flux distribution of the reference state, Figure V-11, the magnitudes of control costs required for the static set-point determination were found to depend not only on the fundamental mode shape but were also affected by high harmonics shapes. Figures V-12, -13 and -14 show the significances of those effects

* k_∞ is the infinite multiplication factor.

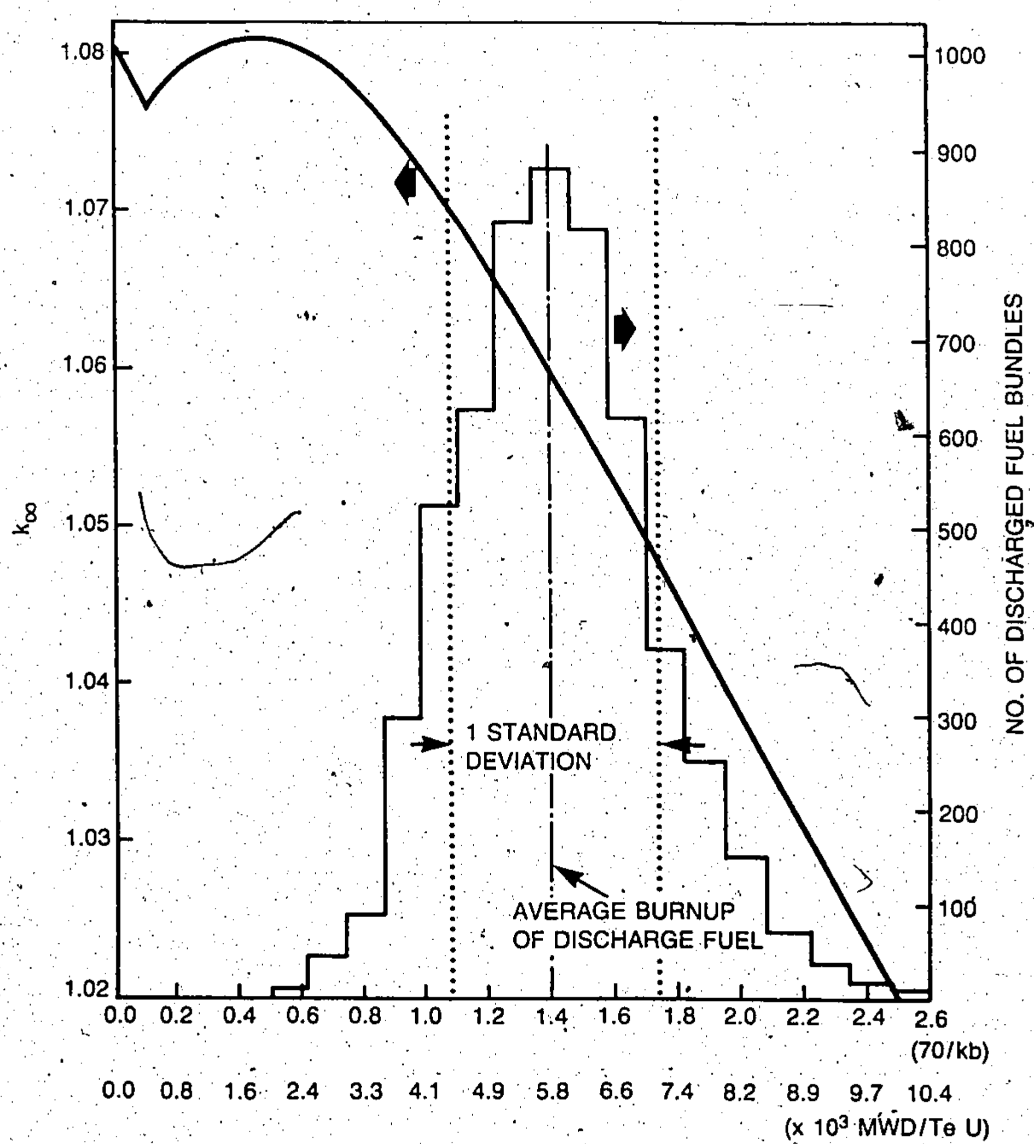


FIGURE V-10 REACTIVITY PARAMETER AND DISCHARGED FUEL DISTRIBUTION VS BURNUP

TABLE V-5 STATIC CONTROL SET-POINTS VARIED BY DIFFERENT VERTICAL CONTROLLER LOCATIONS**

Cases	Amp. Loss $(x10^{-2})$	Control Cost $(x10^{-2})$	R.M.S. Error(%) in Shape	Control. Volume*	Levels (%)						
					I	II	III	IV	V	VI	VII
Refer.	1.7226	0.16516	0.00072	24.0288	49.2	49.1	45.7	45.6	57.7	22.6	54.6
I-1	0.00325	0.16805	0.00004	22.9444	52.4	52.4	27.6	27.5	82.6	1.1	73.3
I-2	49.238	0.21286	0.00255	26.7462	71.3	71.6	50.2	50.5	13.1	49.0	43.6
III-1	3.3649	0.23947	0.00067	28.5171	51.3	51.4	80.7	80.8	46.8	56.2	8.9
III-2	0.16541	0.16328	0.00031	22.3394	36.0	36.0	41.2	41.2	73.4	3.2	76.5
III-3	0.18882	0.22352	0.00020	21.4995	9.5	9.5	55.8	55.9	93.5	-13.9+	91.5
V-1	2.8566	0.17812	0.00097	24.5675	52.8	52.7	42.8	42.6	50.5	35.0	54.6
V-2	3.3269	0.20754	0.00103	27.2927	54.5	54.4	41.3	41.2	89.8	39.1	55.0
VI-1	1.0950	0.16182	0.00058	23.8934	47.6	47.5	47.1	47.1	50.8	25.3	56.2
VI-2	0.42879	0.16191	0.00036	23.6632	45.1	45.0	49.0	49.0	24.6	46.2	55.8
VII-1	0.13911	0.17210	0.00021	24.6295	45.6	45.6	50.1	50.0	57.2	42.0	42.2
VII-2	6.0699	0.27338	0.00138	23.4388	54.7	54.6	39.6	39.4	62.5	-40.4+	106.5

Notes; * Total volume occupied by controllers; 52.0 lattice sites.

\+ (-) sign in controller levels means that the controller is not required at the location.

** To compensate for an initial reactivity, 2.325mk.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
A									501	519	523	523	518	501								
B						435	550	612	669	702	707	707	702	168	611	550	435					
C					489	595	713	772	832	860	860	860	860	831	771	712	594	489				
D				518	632	748	857	908	955	970	952	952	969	954	907	856	747	631	517			
E			502	640	762	869	960	998	1025	1029	1007	1006	1028	1024	996	958	867	761	639	501		
F			614	753	862	950	1013	1034	1025	1023	1006	1006	1022	1024	1033	1011	948	860	752	613		
G		551	702	839	926	995	1031	1043	1035	1036	1035	1035	1036	1034	1042	1029	994	925	838	700	550	
H		626	779	901	951	1003	1039	1048	1041	1044	1044	1044	1043	1040	1047	1037	1001	949	899	777	624	
J	508	680	837	946	974	1000	1030	1038	1040	1043	1041	1041	1042	1039	1037	1029	998	972	944	836	678	507
K	545	725	880	980	990	1005	1032	1037	1037	1036	1030	1030	1035	1036	1036	1031	1003	988	978	878	724	544
L	569	751	905	1006	1028	1037	1042	1041	1035	1025	1003	1002	1025	1034	1040	1041	1035	1026	1004	904	749	568
M	571	754	911	1016	1043	1051	1051	1047	1037	1023	998	997	1022	1036	1045	1050	1050	1041	1014	909	753	570
N	549	734	895	1008	1044	1057	1057	1052	1042	1030	1007	1007	1030	1041	1051	1056	1056	1042	1006	893	732	548
O	513	690	855	977	1035	1059	1060	1057	1049	1045	1039	1039	1044	1048	1056	1058	1058	1033	976	853	688	513
P		633	790	920	992	1044	1059	1064	1052	1054	1054	1054	1053	1051	1063	1058	1042	990	919	789	632	
Q		553	703	837	908	979	1033	1050	1047	1053	1055	1055	1053	1046	1049	1032	978	907	836	702	552	
R			608	737	823	915	1004	1035	1036	1045	1046	1046	1045	1036	1035	1003	914	822	736	607		
S			493	623	726	836	951	998	1036	1047	1028	1028	1046	1035	998	950	835	725	622	493		
T				510	627	745	858	913	965	982	964	964	981	964	913	857	745	626	510			
U					504	611	725	784	843	969	858	858	869	843	783	724	610	504				
V						472	571	629	684	718	727	726	718	683	629	570	472					
W									520	542	555	555	542	520								

FIGURE V-11 THERMAL NEUTRON FLUX DISTRIBUTION (NORMALIZED BY FLUX AT CENTER)

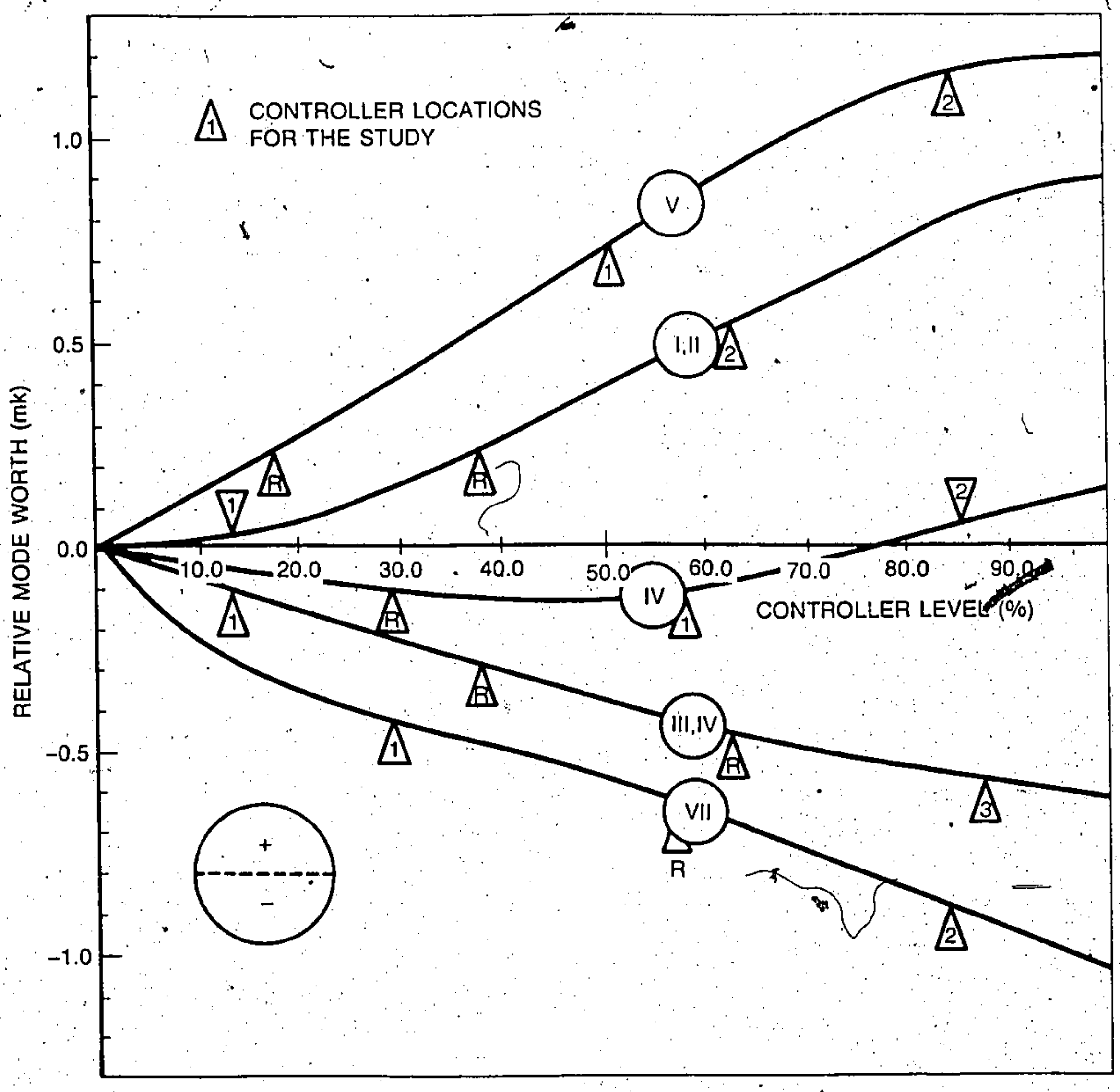


FIGURE V-12 RELATIVE CONTROLLER MODE WORTH AT DIFFERENT LEVELS (1-ST AZIMUTHAL MODE)

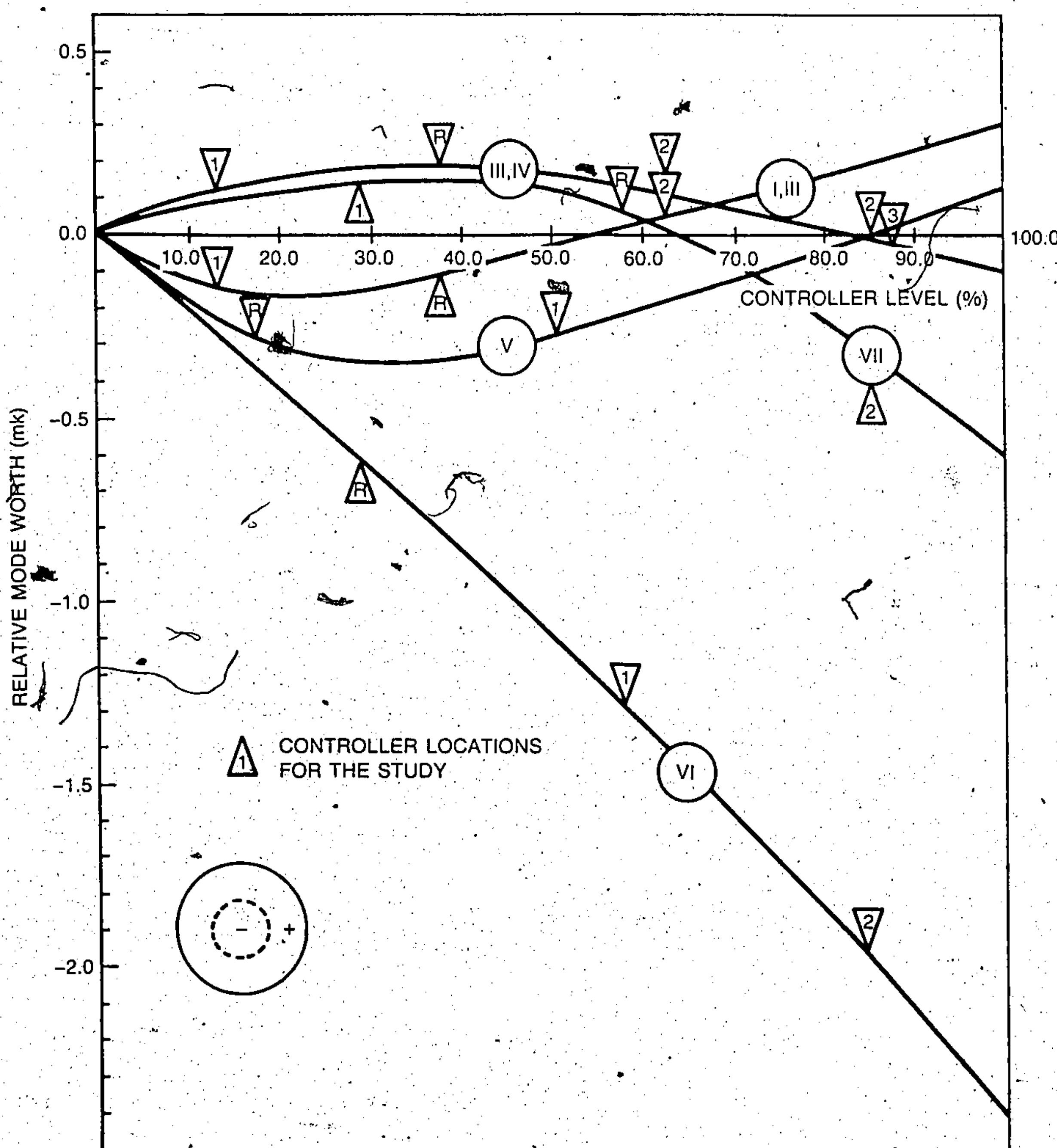


FIGURE V-13 RELATIVE CONTROLLER MODE WORTH AT DIFFERENT LEVEL (1-ST RADIAL MODE

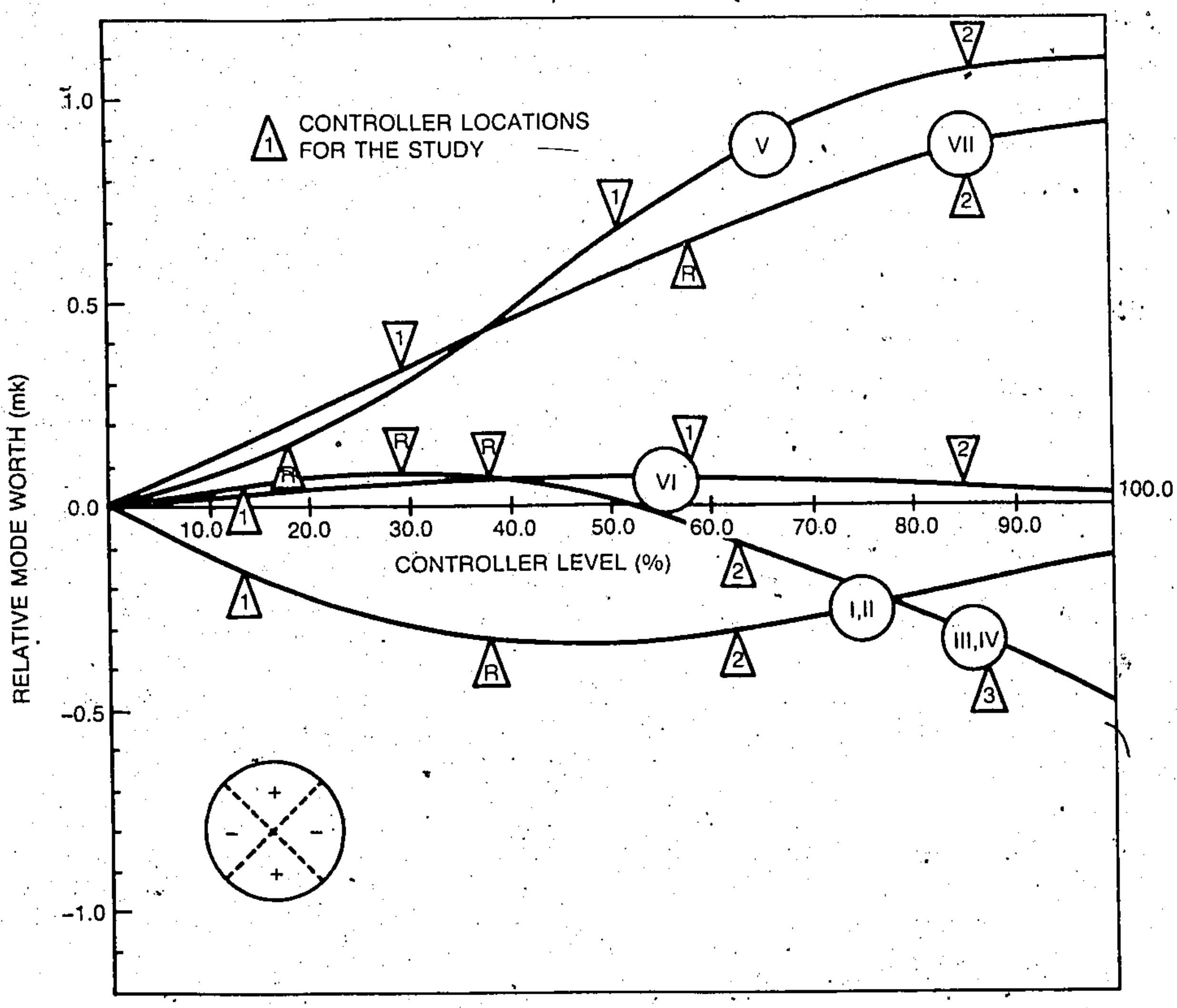


FIGURE V-14 RELATIVE CONTROLLER MODE WORTH AT DIFFERENT LEVELS (2-ND AZIMUTHAL MODE)

in terms of controller location. For example, if controller VI moves in a vertical direction, because of a nearly flat thermal flux distribution inside the compartment, the principal contribution to the resultant reactivity change is made by the first radial mode worth.

Judging by the smaller control penalty and minimum loss of mode amplitudes, Cases I-1, III-2 and VII-1 are not only acceptable but also "optimum in reactivity control".

Even though a control system is optimum in the sense of the reactivity control, it may not be acceptable if its dynamic behaviour corresponding to state transients would be poor. Table V-6 gives the maximum variations of controller levels corresponding to an allowable power range of ±4%, that is the power error to be controlled by the zone controller system (±3% (cf. Figure I-3) plus an additional ±1% error for redundancy). Among the control systems selected by the bulk control criterion, Case VII-1 is the most desirable one because of the smallest dynamic range. Note that an important contribution to determining the dynamic range is the flux shape deviations and, therefore, the spatial control effectiveness maps should explain the part of the results. For example, Case I-2, has a smaller dynamic range (14.3), than Case I-1, (23.3) and the controller I in the former system has a smaller inverse effectiveness (23.0), than the latter's (161.0) in Figure V-8.

Another condition by which to judge the optimality of the system

	Average Level Change (%)	Individual Controller Level Change (%)						
		I	II	III	IV	V	VI	VII
Reference	25.4	27.0	26.7	25.9	26.1	25.4	25.3	20.7
I-1	23.3	24.5	24.5	25.6	25.9	26.8	12.4	22.6
I-2	14.3	12.6	12.6	14.8	14.8	20.5	8.7	17.1
III-1	18.7	18.3	18.9	13.1	14.1	21.5	26.7	20.5
III-2	22.8	26.0	25.8	25.5	24.5	26.6	10.6	19.3
III-3	18.4	18.0	18.1	19.1	19.9	25.2	11.7	17.6
V-1	25.6	28.8	28.4	25.2	24.9	31.4	22.7	18.2
V-2	26.4	31.5	31.2	27.4	27.4	22.3	23.6	18.8
VI-1	25.8	27.0	26.7	26.7	27.0	24.9	25.8	21.5
VI-2	24.9	28.8	28.6	27.5	27.6	27.4	15.6	17.5
VII-1	13.0	10.2	10.1	10.3	10.4	16.8	23.4	11.6
VII-2	23.6	24.3	24.1	23.8	24.0	23.1	23.0	22.6

TABLE V-6 DYNAMIC RANGES OF CONTROLLERS FOR 4% POWER CHANGE is the most probable dynamic range corresponding to randomly distributed perturbations, which are mainly caused by refuelling. Table V-7 compares the effects for various cases. Case VII-1 has still a relatively small range of change $\pm 0.096\%$. Generally the magnitudes of the controller responses for these types of perturbations are $<$ 1% of those expected in controlling global perturbations, as expected from design experience[142].

To verify the optimal design rules they were applied to the existing CANDU zone control system, by simulating various transients. The initial reactor conditions assumed were such that the reactor had reached an equilibrium state with a 5% mode perturbation. Then the control system was activated to return the flux distribution to the desirable reference state instantaneously. Every transient was examined with an hour interval. Table V-8 to -14 show the transient behaviour of modes for the thermal neutron and the xenon concentrations. The neutron flux modes are well-controlled in most of the single mode excitation transients with a residual error of 0.02%. The residual errors shown in mode 3, 6, 7 excitation cases reached a maximum 0.5%. Pressumably controllers located at nodes where the mode fluxes are nearly zero would have less control effectiveness and introduce more controller-induced perturbations. From the spatial control effectiveness map for the Y-symmetric deployment (Fig. V-8) and the characteristics of the spatial modes (Table V-2), the reference controllers I, II, III and IV are expected to have a poor effectiveness on modes 6 and 7, and controllers V and VII on modes 3, 5 and 7.

	Average Change (%)	Individual Controller Level Changes (%)						
		I	II	III	IV	V	VI	VII
Reference	0.106	0.102	0.105	0.105	0.102	0.138	0.071	0.122
I-1	0.143	0.120	0.124	0.093	0.092	0.173	0.245	0.179
I-2	0.108	0.096	0.095	0.112	0.112	0.150	0.066	0.130
III-1	0.102	0.110	0.109	0.081	0.079	0.146	0.074	0.128
III-2	0.136	0.131	0.132	0.148	0.149	0.173	0.076	0.148
III-3	0.128	0.079	0.080	0.120	0.117	0.143	0.212	0.161
V-1	0.093	0.072	0.081	0.099	0.100	0.134	0.065	0.106
V-2	0.090	0.074	0.085	0.104	0.104	0.081	0.068	0.109
VI-1	0.109	0.104	0.106	0.106	0.104	0.139	0.085	0.124
VI-2	0.121	0.112	0.117	0.115	0.110	0.154	0.116	0.129
VII-1	0.096	0.094	0.095	0.094	0.093	0.127	0.065	0.112
VII-2	0.095	0.096	0.098	0.096	0.093	0.126	0.065	0.093

TABLE V-7 DYNAMIC RANGES RESPONDING TO RANDOM BURNUP DEVIATIONS

TABLE V- 8 CHANGES IN STATE FUNCTIONS AND CORRESPONDING CONTROLLER RESPONSES

(Ref. Case; Mode 2 Excited; 100 % Initial Power)

		Transient Time (hrs.)												
		<0.0	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0
Controller Level (%)	I	0.0	2.35	2.31	2.35	2.41	2.49	2.56	2.61	2.69	2.75	2.81	2.86	2.90
	II	0.0	1.76	1.83	1.80	1.72	1.61	1.50	1.42	1.29	1.20	1.11	1.03	0.95
	III	0.0	-1.79	-1.83	-1.82	-1.77	-1.72	-1.66	-1.61	-1.54	-1.49	-1.44	-1.39	-1.35
	IV	0.0	-2.40	2.33	-2.39	-2.50	-2.63	-2.77	-2.86	-3.01	-3.12	-3.22	-3.32	-3.40
	V	0.0	4.21	4.23	4.26	4.28	4.29	4.30	4.30	4.30	4.30	4.29	4.20	4.28
	VI	0.0	-0.22	-0.31	-0.24	-0.08	0.10	0.28	0.41	0.61	0.77	0.91	1.03	1.15
	VII	0.0	-3.47	-3.37	-3.49	-3.70	-3.93	-4.17	-4.33	-4.59	-4.78	-4.95	-5.11	-5.25
Thermal Neutron Mode	1	-0.005	-0.003	0.0	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
	2	5.000	-0.001	0.0	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.0
	3	0.0	-0.018	0.009	0.009	0.014	0.014	0.014	0.013	0.012	0.012	0.011	0.010	0.009
	4	0.0	-0.006	0.0	0.003	0.005	0.005	0.005	0.005	0.004	0.004	0.004	0.003	0.003
	5	0.0	-0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	6	0.0	-0.014	0.0	0.007	0.010	0.011	0.011	0.010	0.010	0.009	0.008	0.008	0.007
	7	0.0	-0.005	0.0	0.003	0.004	0.004	0.004	0.004	0.004	0.003	0.003	0.003	0.003
	8	0.0	-0.003	0.0	0.002	0.002	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002
Xenon Mode	1	0.004	0.004	0.002	0.0	-0.002	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003	-0.002	-0.002
	2	-1.966	-1.962	0.049	1.304	1.506	1.485	1.386	1.267	1.148	1.036	0.933	0.841	0.757
	3	0.0	0.0	0.001	-0.001	-0.003	-0.004	-0.004	-0.004	-0.003	-0.003	-0.002	-0.001	-0.001
	4	-0.007	-0.007	-0.004	0.0	0.002	0.004	0.004	0.005	0.004	0.004	0.004	0.004	0.004
	5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	6	-0.008	-0.008	-0.010	-0.046	0.001	0.004	0.006	0.007	0.008	0.008	0.008	0.008	0.008
	7	0.016	-0.016	-0.027	-0.011	0.004	0.014	0.019	0.021	0.022	0.022	0.022	0.021	0.020
	8	0.043	0.043	0.085	0.034	-0.014	-0.043	-0.059	-0.066	-0.068	-0.068	-0.067	-0.065	-0.062

TABLE V- 9 CHANGES IN STATE FUNCTIONS AND CORRESPONDING CONTROLLER RESPONSES

(Ref. Case; Mode 3 Excited; 100 % Initial Power)

		Transient Time (hrs.)												
		<0.0	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0
Controller Level (%)	I	0.0	2.63	2.30	2.76	3.48	4.27	5.04	5.76	6.41	7.01	7.55	8.04	8.49
	II	0.0	-2.31	-1.52	-2.60	-4.34	-6.22	-8.05	-9.76	-11.33	-12.76	-14.06	-15.23	-16.03
	III	0.0	2.34	1.87	2.52	3.57	4.70	5.80	6.83	7.78	8.64	9.42	10.13	10.77
	IV	0.0	-2.63	-1.97	-2.88	-4.32	-5.88	-7.40	-8.82	-10.13	-11.32	-12.40	-13.37	-14.26
	V	0.0	-0.33	0.17	-0.51	-1.60	-2.79	-3.95	-5.03	-6.02	-6.93	-7.75	-8.49	-9.17
	VI	0.0	-0.03	-0.80	0.25	1.93	3.76	5.55	7.22	8.75	10.14	11.41	12.56	13.59
	VII	0.0	0.18	0.73	-0.02	-1.21	-2.52	-3.78	-4.96	-6.04	-7.03	-7.93	-8.74	-9.48
Thermal Neutron Mode	1	-0.003	-0.039	0.007	0.024	0.029	0.030	0.028	0.026	0.024	0.022	0.020	0.018	0.016
	2	0.0	-0.003	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001
	3	5.000	-0.257	0.047	0.160	0.195	0.197	0.187	0.173	0.158	0.143	0.130	0.117	0.106
	4	0.0	-0.062	0.011	0.038	0.046	0.047	0.045	0.041	0.038	0.034	0.031	0.028	0.025
	5	0.0	-0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	6	0.0	-0.172	0.031	0.107	0.130	0.131	0.125	0.115	0.105	0.096	0.087	0.078	0.071
	7	0.0	0.073	0.013	0.045	0.055	0.056	0.053	0.049	0.045	0.040	0.037	0.033	0.030
	8	0.0	0.045	0.008	0.028	0.034	0.035	0.033	0.030	0.028	0.025	0.023	0.021	0.019
Xenon Mode	1	0.001	0.001	0.002	-0.008	-0.013	-0.012	-0.010	-0.007	-0.004	-0.002	0.0	0.002	0.003
	2	0.0	0.0	0.0	0.0	0.0	-0.001	-0.001	0.0	0.0	0.0	0.0	0.0	0.0
	3	-1.970	-1.966	0.501	1.284	1.462	1.431	1.334	1.222	1.111	1.008	0.913	0.828	0.750
	4	0.0	0.0	0.003	-0.006	-0.012	-0.015	-0.014	-0.012	-0.010	-0.008	-0.006	-0.004	-0.002
	5	0.003	0.003	0.002	0.0	-0.001	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002	-0.002
	6	0.0	0.0	0.009	-0.018	-0.035	-0.040	-0.038	-0.032	-0.026	-0.019	-0.013	-0.008	-0.004
	7	0.018	0.018	0.039	0.006	-0.020	-0.035	-0.040	-0.041	-0.039	-0.036	-0.033	-0.030	-0.027
	8	0.005	0.005	0.008	-0.003	-0.011	-0.015	-0.015	-0.014	-0.012	-0.011	-0.009	-0.007	-0.006

TABLE V-10 CHANGES IN STATE FUNCTIONS AND CORRESPONDING CONTROLLER RESPONSES

(Ref. Case; Mode 4 Excited; 100 % Initial Power)

		Transient Time (hrs.)												
		<0.0	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0
Controller Level (%)	I	0.0	-0.36	-0.38	-0.35	-0.29	-0.22	-0.16	-0.11	-0.06	-0.01	0.03	0.07	0.11
	II	0.0	0.36	0.37	0.36	0.34	0.31	0.28	0.26	0.23	0.21	0.19	0.17	0.16
	III	0.0	0.92	0.87	0.94	1.06	1.19	1.31	1.43	1.53	1.63	1.71	1.79	1.86
	IV	0.0	0.09	0.09	0.09	0.08	0.06	0.05	0.04	0.03	0.01	0.0	-0.01	-0.02
	V	0.0	-0.78	-0.66	-0.85	-1.14	-1.41	-1.75	-2.03	-2.28	-2.51	-2.72	-2.91	-3.08
	VI	0.0	-0.43	-0.55	-0.38	-0.13	0.14	0.41	0.65	0.88	1.08	1.26	1.43	1.58
	VII	0.0	-0.62	-0.49	-0.69	-1.00	-1.34	-1.66	-1.96	-2.24	-2.49	-2.71	-2.91	-3.10
Thermal Neutron Mode	1	-0.013	-0.004	0.001	0.003	0.003	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.001
	2	0.0	-0.001	0.0	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.0	0.0
	3	0.0	-0.012	0.002	0.007	0.009	0.010	0.008	0.008	0.007	0.006	0.006	0.005	0.005
	4	5.000	-0.015	0.003	0.010	0.011	0.011	0.011	0.010	0.009	0.008	0.007	0.007	0.006
	5	0.0	-0.003	0.001	0.002	0.003	0.003	0.002	0.002	0.002	0.002	0.002	0.001	0.001
	6	0.0	-0.016	0.003	0.010	0.012	0.012	0.012	0.011	0.010	0.009	0.008	0.007	0.006
	7	0.0	-0.003	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001
	8	0.0	-0.002	0.0	0.001	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001
Xenon Mode	1	0.007	0.007	0.001	-0.003	-0.005	-0.006	-0.006	-0.005	-0.005	-0.004	-0.004	-0.003	-0.003
	2	-0.006	-0.006	-0.005	0.0	0.003	0.005	0.006	0.006	0.005	0.005	0.005	0.004	0.004
	3	0.0	0.0	0.001	-0.001	-0.002	-0.003	-0.003	-0.002	-0.002	-0.002	-0.001	-0.001	0.0
	4	-2.056	-2.052	0.445	1.320	1.557	1.550	1.454	1.333	1.209	1.092	0.984	0.886	0.798
	5	0.0	0.0	0.0	-0.001	-0.001	-0.001	-0.001	-0.001	0.0	0.0	0.0	0.0	0.0
	6	-0.006	-0.006	-0.007	-0.004	-0.001	0.001	0.003	0.004	0.004	0.005	0.005	0.005	0.005
	7	0.001	0.001	0.003	0.001	-0.001	-0.002	-0.003	-0.003	-0.003	-0.003	-0.002	-0.002	-0.002
	[illegible]	-0.003	-0.003	-0.004	-0.002	0.001	0.002	0.003	0.003	0.003	0.003	0.003	0.003	0.003

TABLE V- 11. CHANGES IN STATE FUNCTIONS AND CORRESPONDING CONTROLLER RESPONSES

(Ref. Case; Mode 5 Excited; 100 % Initial Power)

		Transient Time (hrs.)												
		<0.0	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0
Controller Level (%)	I	0.0	1.51	1.53	1.56	1.59	1.60	1.61	1.61	1.61	1.61	1.61	1.60	1.60
	II	0.0	-1.22	-1.23	-1.36	-1.49	-1.61	-1.70	-1.77	-1.83	-1.88	-1.93	-1.96	-1.99
	III	0.0	-1.02	-1.05	-0.92	-0.76	-0.60	-0.46	-0.33	-0.22	-0.12	-0.04	0.04	0.10
	IV	0.0	1.40	1.43	1.26	1.06	0.87	0.70	0.55	0.42	0.30	0.20	0.12	0.04
	V	0.0	-0.40	-0.38	-0.50	-0.64	-0.77	-0.88	-0.98	-1.07	-1.14	-1.20	-1.26	-1.31
	VI	0.0	0.03	0.02	0.19	0.39	0.58	0.74	0.88	1.00	1.11	1.20	1.28	1.35
	VII	0.0	-0.30	-0.29	-0.42	-0.57	-0.71	-0.84	-0.94	-1.04	-1.12	-1.19	-1.25	-1.31
Thermal Neutron Mode	1	-0.007	-0.004	0.002	0.003	0.003	0.003	0.002	0.002	0.002	0.002	0.001	0.001	0.001
	2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	3	0.0	-0.023	0.013	0.020	0.020	0.017	0.015	0.013	0.011	0.009	0.008	0.007	0.006
	4	0.0	-0.007	0.003	0.006	0.006	0.005	0.005	0.004	0.003	0.003	0.003	0.002	0.002
	5	5.000	0.001	0.0	0.0	0.0	-0.001	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	6	0.0	-0.016	0.009	0.014	0.014	0.012	0.010	0.009	0.008	0.007	0.006	0.005	0.004
	7	0.0	-0.007	0.004	0.006	0.006	0.005	0.004	0.004	0.003	0.003	0.002	0.002	0.002
	8	0.0	-0.004	0.002	0.004	0.003	0.003	0.003	0.002	0.002	0.002	0.001	0.001	0.001
Xenon Mode	1	-0.002	-0.002	0.0	0.0	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	3	0.003	0.003	0.002	-0.004	-0.007	-0.007	-0.006	-0.005	-0.004	-0.003	-0.002	-0.002	-0.001
	4	0.0	0.0	0.0	-0.001	-0.002	-0.002	-0.001	-0.001	-0.001	0.0	0.0	0.0	0.0
	5	-2.049	-2.045	0.448	1.321	1.557	1.549	1.452	1.330	1.206	1.089	0.981	0.883	0.795
	6	0.0	0.0	-0.001	-0.003	-0.004	-0.004	-0.003	-0.002	-0.001	-0.001	0.0	0.0	0.001
	7	0.0	-0.012	-0.014	-0.004	0.004	0.009	0.011	0.012	0.013	0.013	0.012	0.012	0.011
	8	0.0	-0.005	-0.006	-0.002	0.001	0.003	0.005	0.005	0.005	0.005	0.005	0.005	0.005

TABLE V-12 CHANGES IN STATE FUNCTIONS AND CORRESPONDING CONTROLLER RESPONSES

(Ref. Case; Mode 6 Excited; 100 % Initial Power)

		Transient Time (hrs.)												
		<0.0	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0
Controller Level (%)	I	0.0	-9.50	-9.51	-9.51	-9.51	-9.51	-9.50	-9.50	-9.49	-9.48	-9.48	-9.47	-9.46
	II	0.0	-9.29	-9.26	-9.[illegible]	-9.39	-9.48	-9.56	-9.64	-9.71	-9.78	-9.84	-9.89	-9.94
	III	0.0	-7.88	-7.90	-7.89	-7.86	-7.82	-7.79	-7.75	-7.72	-7.68	-7.65	-7.63	-7.60
	IV	0.0	-7.64	-7.61	-7.66	-7.74	-7.81	-7.89	-7.96	-8.02	-8.08	-8.13	-8.18	-8.22
	V	0.0	-13.18	-13.13	-13.21	-13.33	-13.46	-13.58	-13.68	-13.80	-13.89	-13.97	-14.05	-14.12
	VI	0.0	-18.56	-18.68	-18.52	-18.25	-17.97	-17.68	-17.42	-17.17	-16.94	-16.74	-16.55	-16.63
	VII	0.0	-14.57	-14.51	-14.60	-14.73	-14,88	-15.02	-15.14	-15.26	-15.37	-15.47	-15.55	-15.63
Thermal Neutron Mode	1	-2.008	-0.003	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001
	2	0.0	-0.001	0.0	0.0	0.001	0.001	0.001	0.0	0.0	0.0	0.0	0.0	0.0
	3	0.0	-0.010	0.002	0.006	0.008	0.008	0.008	0.007	0.007	0.006	0.006	0.005	0.005
	4	0.0	-0.008	0.001	0.005	0.006	0.006	0.005	0.005	0.005	0.004	0.004	0.004	0.003
	5	0.0	-0.001	0.0	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	6	5.000	-0.014	0.003	0.008	0.010	0.010	0.010	0.009	0.008	0.008	0.007	0.006	0.006
	7	0.0	-0.003	0.0	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001
	8	0.0	-0.002	0.0	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
Xenon Mode	1	0.566	0.565	0.025	-0.283	-0.431	-0.491	-0.506	-0.499	-0.483	-0.462	-0.439	-0.416	-0.393
	2	-0.007	-0.007	-0.011	-0.004	0.002	0.006	0.008	0.009	0.009	0.009	0.009	0.009	0.008
	3	0.0	0.0	0.001	-0.001	-0.002	-0.003	-0.003	-0.002	-0.002	-0.002	-0.001	-0.001	-0.001
	4	-0.005	-0.005	-0.006	-0.003	0.001	0.003	0.004	0.005	0.005	0.005	0.005	0.005	0.005
	5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	6	-1.888	-1.884	0.376	1.185	1.422	1.435	1.364	1.265	1.162	1.062	0.970	0.885	0.807
	7	-0.004	-0.004	0.0	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.0	0.0
	8	0.007	0.007	0.004	0.0	-0.003	-0.005	-0.006	-0.006	-0.005	-0.005	-0.005	-0.004	-0.004

TABLE V-13 CHANGES IN STATE FUNCTIONS AND CORRESPONDING CONTROLLER RESPONSES

(Ref. Case; Mode 7 Excited; 100 % Initial Power)

		Transient Time (hrs.)												
		<0.0	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0
Controller Level (%)	I	0.0	-20.14	-19.54	-20.51	-22.62	-24.90	-27.11	-29.16	-31.04	-32.73	-34.26	-35.64	-36.88
	II	0.0	-8.68	-10.18	-7.75	-2.49	3.20	8.71	13.82	18.50	22.73	26.54	29.98	33.07
	III	0.0	8.56	9.43	8.01	4.94	1.61	-1.61	-4.60	-7.33	-9.80	-12.03	-14.04	-15.84
	IV	0.0	20.77	19.54	21.55	25.92	30.64	35.22	39.47	43.35	46.86	50.03	52.88	55.45
	V	0.0	12.10	11.11	12.72	16.19	19.94	23.57	26.94	30.03	32.82	35.34	37.61	39.66
	VI	0.0	-0.71	0.80	-1.67	-7.01	-12.78	-15.40	-23.57	-28.31	-32.60	-36.48	-39.97	-43.11
	VII	0.0	-5.52	-6.59	-4.83	-1.05	3.03	6.99	10.66	14.02	17.06	19.80	22.27	24.49
Thermal Neutron Mode	1	-0.005	0.123	-0.023	-0.077	-0.093	-0.093	-0.088	-0.080	-0.073	-0.066	-0.060	-0.005	-0.005
	2	0.0	0.010	-0.002	-0.006	-0.007	-0.007	-0.007	-0.006	-0.006	-0.005	-0.005	-0.004	-0.004
	3	0.0	0.815	-0.152	-0.509	-0.614	-0.617	-0.582	-0.534	-0.485	-0.438	-0.395	-0.355	-0.319
	4	0.0	0.196	-0.037	-0.122	-0.147	-0.148	-0.139	-0.128	-0.116	-0.105	-0.094	-0.085	-0.077
	5	0.0	0.002	0.0	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
	6	0.0	0.547	-0.102	-0.342	-0.411	-0.413	-0.390	-0.358	-0.325	-0.294	-0.265	-0.238	-0.214
	7	5.000	0.231	-0.043	-0.145	-0.174	-0.175	-0.165	-0.156	-0.138	-0.124	-0.112	-0.101	-0.091
	8	0.0	0.144	-0.027	-0.090	-0.109	-0.109	-0.103	-0.095	-0.086	-0.078	-0.070	-0.063	-0.057
Xenon Mode	1	0.002	0.002	-0.008	0.023	0.036	0.035	0.027	0.018	0.009	0.002	-0.003	-0.007	-0.010
	2	-0.017	-0.017	-0.028	-0.009	0.008	0.018	0.022	0.024	0.024	0.024	0.022	0.021	0.020
	3	0.020	0.020	-0.003	0.095	0.154	0.166	0.150	0.122	0.091	0.061	0.034	0.012	-0.007
	4	0.002	0.001	-0.007	0.020	0.039	0.044	0.042	0.036	0.028	0.021	0.014	0.008	0.003
	5	-0.013	-0.013	-0.014	-0.003	0.006	0.011	0.013	0.014	0.014	0.013	0.012	0.012	0.011
	6	-0.003	-0.003	-0.026	0.058	0.111	0.127	0.119	0.101	0.080	0.059	0.040	0.024	0.010
	7	-2.120	-2.116	0.397	1.351	1.640	1.653	1.558	1.428	1.291	1.161	1.041	0.931	0.834
	8	0.003	0.003	0.009	0.025	0.032	0.031	0.025	0.018	0.012	0.006	0.0	-0.004	-0.007

TABLE V-14 CHANGES IN STATE FUNCTIONS AND CORRESPONDING CONTROLLER RESPONSES

(Ref. Case; Mode 8 Excited; 100 % Initial Power)

		Transient Time (hrs.)												
		<0.0	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0
Controller Level (%)	I	0.0	22.95	23.35	22.77	21.88	20.92	19.99	19.13	18.35	17.64	17.01	16.44	15.93
	II	0.0	29.07	27.98	29.51	31.90	34.46	36.93	39.21	41.29	43.17	44.87	46.39	47.75
	III	0.0	-28.11	-27.49	-28.40	-29.81	-31.33	-32.80	-34.16	-35.39	-36.51	-37.51	-38.41	-39.22
	IV	0.0	-21.08	-21.91	-20.69	-18.79	-16.74	-14.77	-12.95	-11.28	-9.78	-8.43	-7.22	-6.13
	V	0.0	-18.86	-19.52	-18.54	-17.03	-15.42	-13.86	-12.43	-11.12	-9.93	-8.87	-7.91	-7.05
	VI	0.0	-0.62	0.44	-1.11	-3.51	-6.09	-8.58	-10.89	-12.99	-14.88	-16.59	-18.13	-19.51
	VII	0.0	7.99	7.22	8.34	10.09	11.97	13.79	15.47	17.01	18.40	19.65	20.77	21.78
Thermal Neutron Mode	1	-0.001	0.055	-0.011	-0.035	-0.041	-0.041	-0.039	-0.036	-0.032	-0.029	-0.026	-0.023	-0.021
	2	0.0	0.005	-0.001	-0.003	-0.004	-0.004	-0.003	-0.003	-0.003	-0.002	-0.002	-0.002	-0.002
	3	0.0	0.361	-0.072	-0.228	-0.273	-0.272	-0.256	-0.234	-0.212	-0.191	-0.172	-0.154	-0.138
	4	0.0	0.088	-0.018	-0.056	-0.066	-0.066	-0.062	-0.057	-0.052	-0.047	-0.042	-0.038	-0.034
	5	0.0	0.001	0.0	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
	6	0.0	0.243	-0.048	-0.154	-0.183	-0.183	-0.172	-0.158	-0.143	-0.129	-0.116	-0.104	-0.093
	7	0.0	0.102	-0.020	-0.065	-0.077	-0.077	-0.073	-0.066	-0.060	-0.054	-0.049	-0.044	-0.039
	8	5.000	0.064	-0.013	-0.040	-0.048	-0.048	-0.045	-0.041	-0.038	-0.034	-0.030	-0.027	-0.025
Xenon Mode	1	0.001	0.001	-0.003	0.010	0.016	0.015	0.012	0.008	0.004	0.001	-0.001	-0.003	-0.005
	2	0.048	0.048	0.085	0.033	-0.015	-0.044	-0.059	-0.065	-0.068	-0.067	-0.065	-0.062	-0.060
	3	0.005	0.005	-0.010	0.038	0.069	0.077	0.072	0.060	0.046	0.033	0.021	0.011	0.003
	4	-0.003	-0.003	-0.009	0.008	0.019	0.023	0.023	0.021	0.017	0.014	0.010	0.008	0.005
	5	-0.005	-0.005	-0.006	-0.001	0.003	0.005	0.006	0.006	0.006	0.006	0.006	0.005	0.005
	6	0.008	0.007	-0.006	0.026	0.046	0.051	0.046	0.038	0.029	0.020	0.012	0.005	0.0
	7	0.003	0.003	0.011	0.021	0.024	0.021	0.016	0.010	0.005	0.0	-0.004	-0.007	-0.009
	8	-2.125	-2.121	0.400	1.330	1.605	1.615	1.524	1.400	1.271	1.147	1.033	0.929	0.835

The global reactivity contribution of the control action on the flux tilt were negligible except for mode 6, i.e. the first radial mode excitation case which contributed approximately -0.57mk. This was caused by the axially integrated 2-dimensional mode approximation mentioned in Section V.1, which had eventually affected the principal biorthogonality between the fundamental and the first radial mode. The differences in the first radial mode shape at various cross-sections are plotted in Figure V-15. Thus, the actual control action must be equivalent to the action shown in Table V-12 minus an action taken to correct for the artificial fundamental mode error of -2.0%.

It should be noted, that generally λ-mode approximations of the state functions seldom satisfy the property of finality, but, according to the tables, the coupling between thermal flux modes are almost completely negligible and between xenon modes are up to 2% of the initial deviations.

Similar calculations have been done for different control systems previously introduced. The prompt control action to eliminate the excitations on modes 2, 5 and 7 are given in Table V-15. As mentioned earlier, controllers I, II, III and IV are located near nodes where mode 7 has zero value and, thus, in order to suppress the excitation of mode 7, very large efforts were required. From among the other cases, Case VII-1 shows a slightly superior performance.

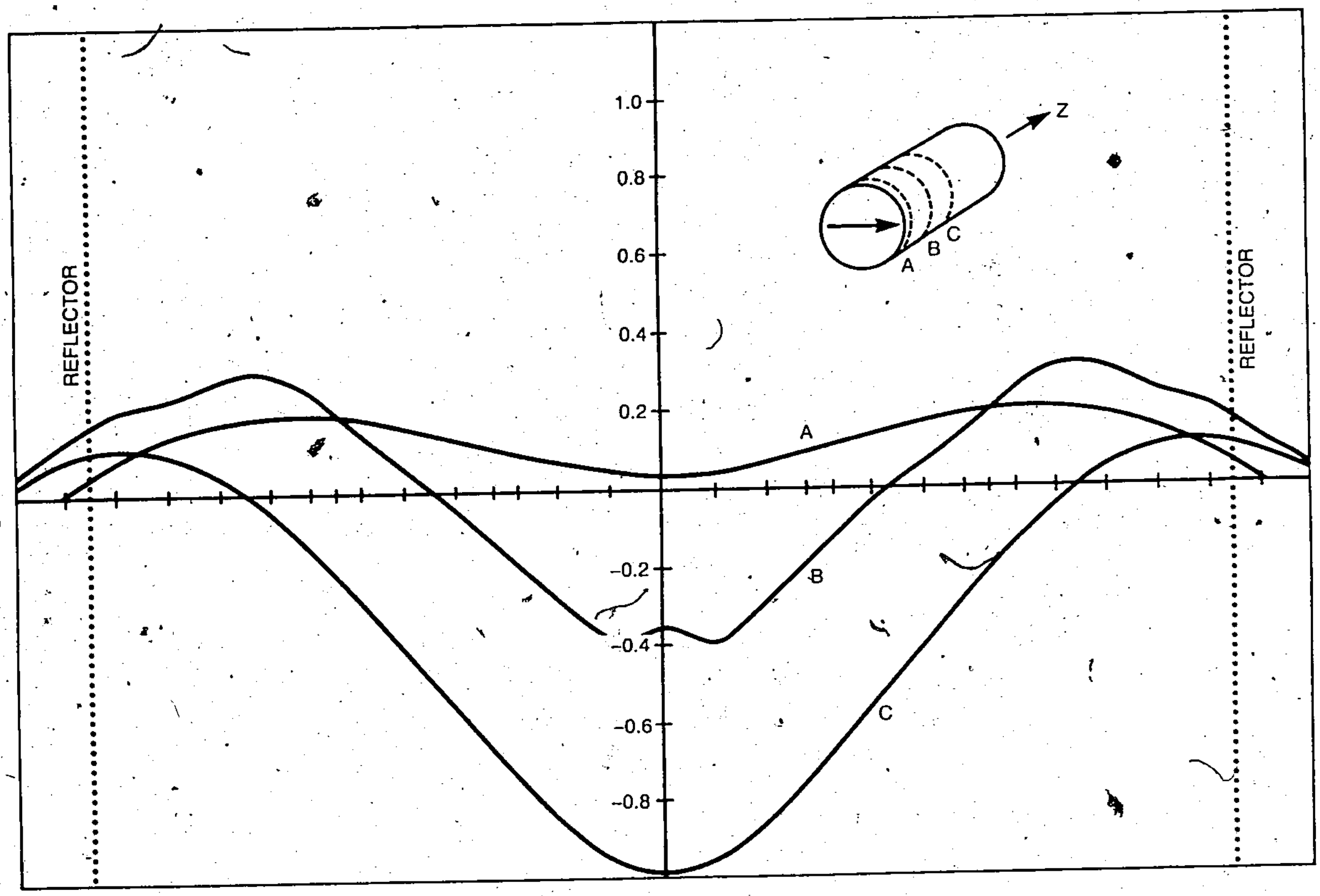


FIGURE V-15 1-ST RADIAL MODE SHAPES AT DIFFERENT Z-SECTIONS

Excited Mode	Control system	Controller Levels (%)							
		Average	I	II	III	IV	V	VI	VII
2	Reference	-0.02	2.35	1.76	-1.79	-2.40	4.21	-0.22	-3.47
	I-1	-0.08	1.95	1.76	-1.91	-2.21	5.31	-0.13	-4.58
	I-2	0.31	-1.13	-0.81	1.85	2.11	-4.97	0.15	4.30
	III-1	-0.09	-2.06	-1.74	1.20	2.03	-5.23	0.05	4.41
	III-2	-0.14	-1.76	-1.56	0.93	1.32	-4.68	0.13	4.01
	VII-1	-0.13	2.80	1.07	-1.24	-2.94	3.93	-0.03	-3.93
5	Reference	0.02	1.51	-1.22	-1.05	[illegible].43	-0.38	0.02	-0.30
	I-1	-0.02	0.97	-1.00	-1.26	1.24	0.00	-0.05	-0.02
	I-2	-0.03	-0.69	0.67	1.16	-1.20	-0.05	-0.05	-0.05
	III-1	0.02	-1.19	1.27	1.05	-1.03	0.0	0.02	0.02
	III-2	-0.03	-1.13	1.08	0.72	-0.73	-0.04	-0.05	-0.05
	VII-1	0.01	1.31	-1.11	-1.05	1.33	-0.26	0.05	-0.29
7	Reference	0.64	-20.14	-8.68	8.56	20.77	12.10	-0.71	-5.52
	I-1	-0.17	-0.02	0.60	-0.89	-0.50	-0.33	-0.03	-0.06
	I-2	-0.03	0.32	0.06	-0.48	-0.03	-0.028	0.03	0.16
	III-1	-0.34	-0.95	-0.98	0.41	-0.04	-0.08	-0.26	-0.44
	III-2	0.01	0.48	0.59	-0.18	-0.64	-0.46	0.01	0.20
	VII-1	-0.05	-0.10	-0.85	0.79	0.04	0.07	-0.01	-0.31

TABLE V-15 CONTROLLER LEVELS FOR INITIAL PROMPT ACTION

In the simulation routine, SIMUL, of ODZCR a neutron balance is achieved by appropriate control action. However, the xenon concentration information for the above neutron balance is obtained from a previous time step. Therefore, the calculated control system response may in fact be slightly greater or smaller than required depending upon the xenon transient behaviour.

Table V-16 lists the maximum rates of controller responses which occurred during the first 11 hour transient period. Although the rates are relatively small, $\lesssim 0.2\%$/hour in most of the cases, except for the reference case, control system VII-1 shows minimum actions in controlling the same kind of perturbation.

Another interesting result is shown in Table V-17, that the optimally chosen control system controls the reactor with minimum action and that the residual error in states is kept to a minimum. Since the residual error is caused by a controller-induced perturbation, less control action obviously introduces less perturbation.

V.4 Remarks

Since the CANDU zone control system consists of vertically arranged compartments having an extensive length, their control action varies spatially with the absorber level in the compartment. Our control system on the other hand is a point system, and it is impossible,

Excited Mode	Control System	Maximum Rate of Controller Level (%/hr.)						
		I	II	III	IV	V	VI	VII
2	Reference	0.07	-0.09	0.06	-0.14	0.03	0.18	-0.23
	I-1	0.05	0.04	-0.05	-0.03	0.09	0.04	-0.11
	I-2	0.03	0.02	-0.03	-0.03	0.13	0.02	-0.11
	III-1	0.05	0.02	0.61	-0.05	0.12	0.01	-0.09
	III-2	0.03	0.03	-0.03	-0.03	0.10	0.05	-0.14
	VII-1	0.04	0.03	-0.03	-0.04	0.13	-0.03	-0.08
5	Reference	0.03	-0.13	0.16	0.20	-0.14	0.20	-0.15
	I-1	-0.07	0.12	0.03	-0.05	-0.03	-0.01	0.01
	I-2	-0.07	0.06	0.09	-0.07	0.01	0.01	-0.03
	III-1	-0.05	0.04	0.07	-0.04	-0.01	0.01	-0.03
	III-2	-0.07	0.09	0.05	-0.07	-0.02	0.01	-0.01
	VII-1	-0.06	0.06	0.06	-0.04	-0.01	0.01	-0.01
7	Reference	-2.28	5.69	-3.34	4.72	3.75	-5.77	4.08
	I-1	-0.03	0.01	-0.05	0.04	-0.01	0.01	0.01
	I-2	0.02	-0.02	-0.02	0.02	0.01	-0.01	0.01
	III-1	-0.01	0.01	0.03	-0.04	0.01	0.01	-0.01
	III-2	-0.02	0.05	0.01	-0.03	-0.01	0.01	-0.01
	VII-1	-0.01	0.01	-0.01	0.01	0.01	-0.01	-0.01

TABLE V-16 MAXIMUM RATES OF CONTROLLER LEVEL CHANGES DURING TRANSIENT

Excited Mode	Control System	Maximum Thermal Flux Mode Amplitude Error (%)							
		1	2	3	4	5	6	7	8
2	Reference	-0.003	-0.001	-0.018	-0.006	-0.001	-0.014	-0.005	-0.003
	I-1	0.0	-0.001	0.0	0.001	0.0	0.0	0.001	0.001
	I-2	0.0	0.001	0.0	0.0	0.0	0.0	0.0	-0.001
	III-2	0.0	0.001	0.0	0.0	0.0	0.0	0.0	0.001
	III-2	0.0	0.001	0.0	0.001	0.0	0.001	0.0	0.001
	VII-1	0.0	0.001	0.0	0.001	0.0	0.001	0.0	0.001
5	Reference	-0.004	0.0	-0.023	-0.007	0.001	-0.016	-0.007	-0.004
	I-1	0.0	0.001	0.001	0.001	0.001	0.001	-0.007	0.008
	I-2	0.0	0.0	0.0	0.0	0.001	0.0	0.006	0.002
	III-1	0.0	0.0	0.001	0.001	-0.003	0.0	0.004	-0.007
	III-2	0.0	0.0	0.0	0.0	0.0	0.0	-0.005	0.007
	VII-1	0.0	0.0	0.0	0.0	0.001	0.0	0.0	0.0
7	Reference	0.12[illegible]	0.010	0.815	0.196	0.002	0.547	0.231	0.144
	I-1	0.0	0.001	-0.004	0.0	-0.007	0.0	-0.014	-0.020
	I-2	0.0	0.0	0.0	0.0	-0.003	0.0	-0.014	-0.006
	III-1	0.0	0.001	0.002	0.0	0.003	0.001	-0.010	0.017
	III-2	0.0	0.002	-0.001	0.0	0.005	0.0	-0.015	0.022
	VII-1	0.0	-0.001	0.002	0.0	0.0	0.0	0.013	0.006

TABLE V-17 MAXIMUM UNCONTROLLED ERROR IN THERMAL FLUX MODES DURING TRANSIENT to duplicate with it the real system. But the general conclusion would be as follows.

(1) The controllers should always be located in the high thermal flux regions of the core. They should also be located as close as possible to the peaks of the important harmonics. Even though the first radial mode does not seem to be at first glance important, nevertheless, by placing a controller in the center of the reactor core, we can also control many local perturbations.

(2) Cross-coupling among individual controllers may be strong enough, that a change of one controller location can cause a complete change in the effectiveness of the entire system. When we examine Table V-15, -16 and -17, we can see this cross-coupling effect change not only the magnitude but also the direction of control action.

The main emphasis of the above is that the most effective region for both bulk and spatial control is very limited for the 600 MWe CANDU reactors as shown in Figure V-16. The area, between the concentric circles, which was determined from the spatial control effectiveness maps, Figures V-7 and -8, and from the thermal flux mode shapes, Figures V-4 and -5, is the most effective region for spatial control. The area inside the dashed line, where the thermal neutron flux is at least 99% of the value at the reactor center, is the most effective region for the bulk control. Hence, the overlap of these areas must be the locations for deploying individual controllers. The following conclusions can be drawn from the figure.

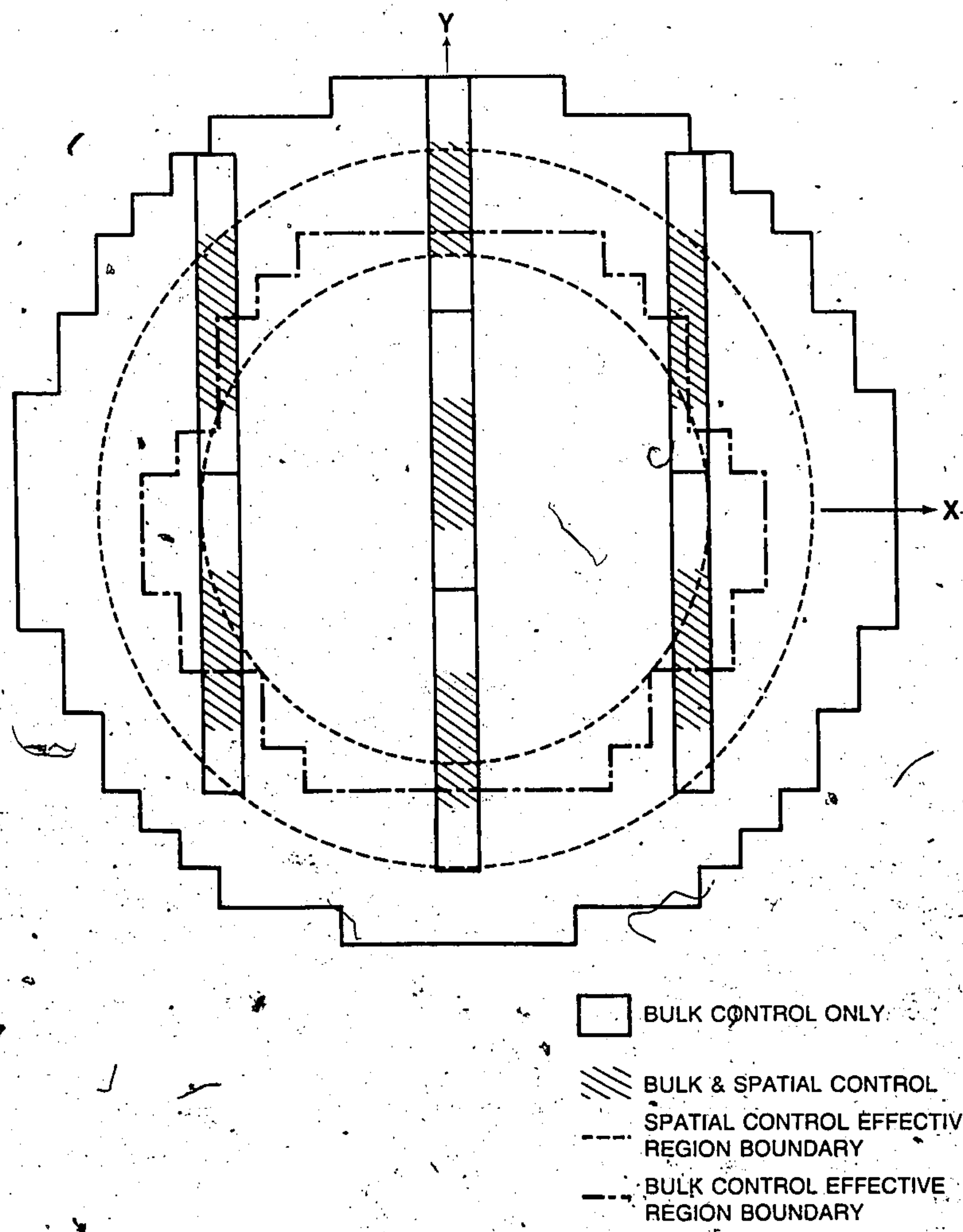


FIGURE V-16 DOMAINS FOR BULK AND SPATIAL CONTROLLER DEPLOYMENT

(1) The existing zone control system of the 600 MWe CANDU is deployed where the vertically mounted countrollers can achieve the best performance.

(2) The dashed areas in Fig. V-16, which identify the 20% to 70% controller levels, which correspond to the normal operating range of Fig. I-3 fall within our spatial control effective region.

(3) The central controller falls outside the spatial control effective region.

(4) The reason why Case VII-1 shows a better performance than others is, because controller VII was shifted to the most effective region from its reference point.

VI. ALTERNATIVES IN THE ZONE CONTROL SYSTEM DESIGN USING OPTIMALITY CRITERIA

We examined optimality of the existing 600 MWe CANDU zone control system whose vertical orientation resides mostly in the most effective region for the bulk and spatial control. No better place can be found where the larger fraction of controllers is placed in the intersection area in Figure V-16 for the system.

However, if we change the orientation of the controllers parallel to the X-axis from its original geometry, a larger intersection mainly in the top and bottom sections of the reactor may be assigned for the control system. Advantages in the new system may be not only a matter of performance but also a simplification in hardware design by removing the driving mechanism from the crowded control mechanism deck on top of the reactor. This alternative may possibly eliminate uncertainty factors in prediction of controller response to perturbations, whose differential behaviour was shown to be strongly dependent on the individual controller level during transients.

Figure VI-1 shows the horizontally oriented zone control system, where only its orientation is changed from the original one. The method of assigning the case numbers is similar to the previous vertical model's.

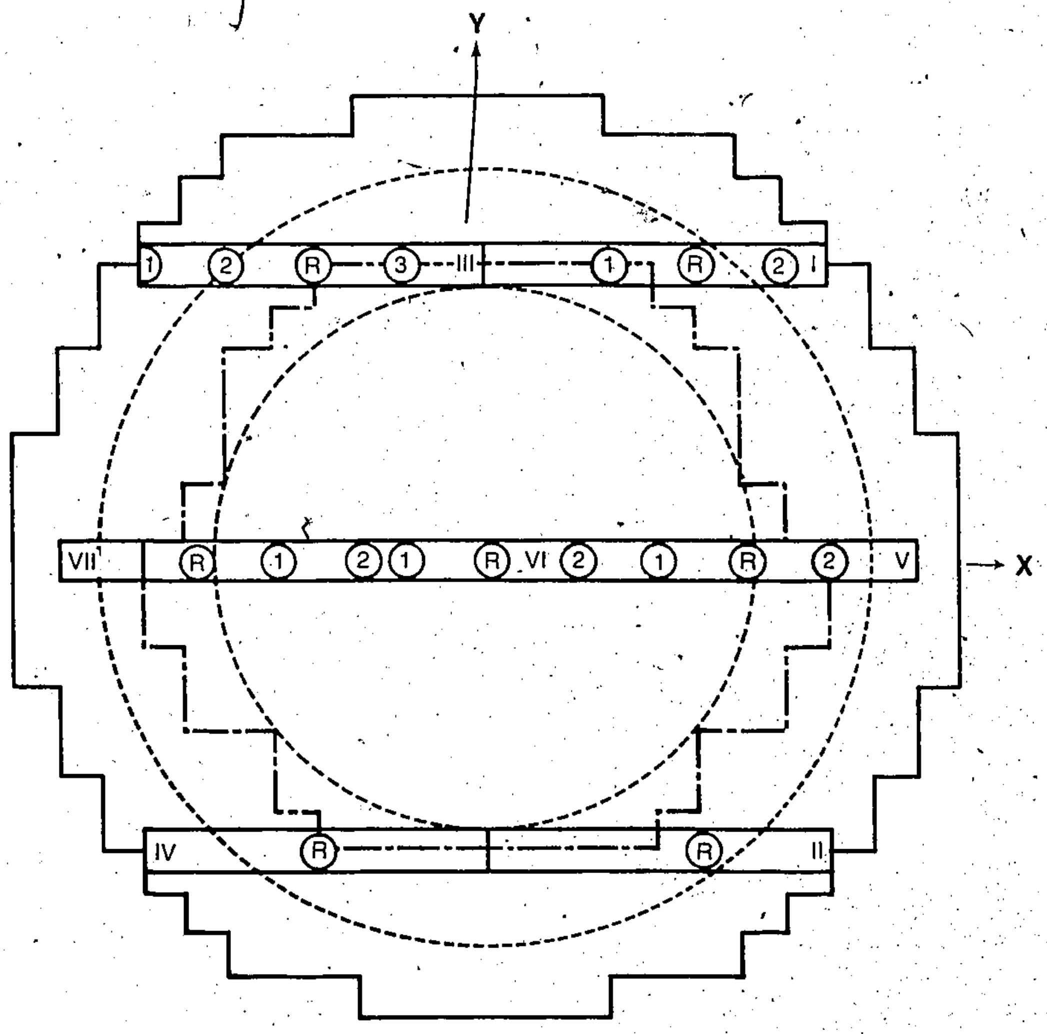


FIGURE VI-1 DEPLOYMENT OF HORIZONTAL ZONE CONTROL SYSTEM

Table VI-1 shows the calculated optimality parameters and the controller static set-point. Magnitudes of parameters are similar to the vertical control system's as expected. As we move to the center line of the reactor core, controllers I, II, III and IV tend to require less control efforts, but are accompanied by a heavier amplitude-loss.

The dynamic ranges for regulation and control are shown in Tables VI-2 and VI-3. Their ranges did not change very much even though the controller locations were changed. The reason is that the controllers occupy large fractions of the control effective region, and that the magnitudes of the dynamic ranges for most cases are comparable with those of the best case in the vertical control system.

We considered another alternative by reducing the number of controllers in the system. Generally we found that, without paying heavy penalties in the amplitude loss, i.e., in the flux shape distortion, it was quite difficult to find controllers whose performances were comparable to the seven controller system. Some of the cases which showed reasonably good performances are listed in Table VI-4. The 6 controller system has two controllers along the center line of the reactor rather than 3 controllers of the 7 controller system. Thus, there is no controller in the reactor center. Another system studied had 5 controllers with only one controller along the center line at the reactor center.

Cases	Amp. Loss ($\times 10^{-2}$)	Control Cost ($\times 10^{-2}$)	R.M.S. (%) Error in Flux Shape	Controller Volume*	Levels (%) I	II	III	IV	V	VI	VII
Reference	3.6011	0.18979	0.00073	24.1274	48.1	49.6	46.6	48.3	39.3	56.1	34.7
I-1	4.9669	0.16112	0.00086	22.2953	39.4	39.9	38.9	40.4	60.7	42.3	42.9
I-2	0.8894	0.38012	0.00036	31.6534	101.2+	102.4+	49.2	51.2	1.5	74.8	28.6
III-1	1.3558	1.0680	0.00045	46.1117	57.9	59.9	192.1+	203.5+	20.5	91.5	-37.2++
III-2	3.6238	0.27907	0.00074	28.3818	55.1	56.9	71.0	79.9	28.3	72.7	8.0
III-3	0.02881	0.15872	0.00007	22.7648	32.5	33.3	48.3	48.8	56.4	38.3	52.3
V-1	11.701	0.18489	0.00132	23.9566	43.6	45.3	49.0	51.0	70.4	29.3	36.6
V-2	6.0024	0.20973	0.00095	24.7243	50.5	52.1	45.3	46.8	36.5	65.4	34.1
VI-1	0.0590	0.19277	0.00009	23.9475	51.6	53.6	43.0	44.2	42.8	63.4	22.2
VI-2	9.0462	0.19171	0.00116	23.8205	44.0	44.8	49.3	51.3	24.0	65.1	38.3
VII-1	0.9352	0.17295	0.00037	23.7647	50.8	52.6	43.6	45.4	39.9	39.6	45.8
VII-2	0.0168	0.24336	0.00005	23.6404	54.7	57.2	38.9	41.4	43.7	-15.2++	95.8

Notes; * Total volume occupied by controllers, 52 lattice area

\+ Controller size is too small

++ Controller is not necessary

TABLE VI-1 STATIC SET-POINTS OF HORIZONTAL CONTROL SYSTEM

	Average Level Change (%)	Individual Controller Level Change (%)						
		I	II	III	IV	V	VI	VII
Reference	13.50	12.05	11.93	14.14	13.81	19.51	7.57	16.63
I-1	12.73	10.70	11.10	9.98	9.67	16.62	20.95	11.97
I-2	13.68	8.18	8.33	16.17	15.56	22.16	8.60	18.92
III-1	13.20	15.18	14.62	6.36	5.94	23.56	9.26	29.47
III-2	12.22	8.80	9.21	7.25	6.66	19.18	24.12	13.71
III-3	14.42	13.04	12.54	15.22	15.06	20.48	7.99	17.73
V-1	13.30	12.64	12.15	14.51	13.84	15.38	7.74	17.17
V-2	12.86	11.74	11.44	13.75	13.23	17.25	7.31	16.08
VI-1	13.90	12.25	11.95	14.38	13.84	19.69	9.67	16.79
VI-2	13.17	12.20	11.90	14.32	13.79	13.78	9.60	16.72
VII-1	14.16	12.80	12.67	14.99	14.63	20.69	8.02	16.51
VII-2	13.18	12.51	12.19	14.66	14.10	20.06	7.78	11.81

TABLE VI-2 DYNAMIC RANGES OF HORIZONTAL CONTROL SYSTEM FOR 4% POWER CHANGE

	Average Level Change (%)	Individual Controller Level Change (%)						
		I	II	III	IV	V	VI	VII
Reference	0.118	0.094	0.101	0.107	0.104	0.147	0.157	0.128
I-1	0.097	0.098	0.097	0.095	0.092	0.0132	0.059	0.0113
I-2	0.104	0.064	0.063	0.122	0.118	0.168	0.064	0.145
III-1	0.089	0.069	0.072	0.023	0.023	0.156	0.208	0.109
III-2	0.097	0.095	0.093	0.082	0.073	0.151	0.067	0.130
III-3	0.099	0.059	0.031	0.095	0.094	0.135	0.179	0.124
V-1	0.101	0.096	0.093	0.110	0.106	0.116	0.058	0.131
V-2	0.098	0.091	0.088	0.104	0.100	0.130	0.055	0.123
VI-1	0.106	0.095	0.092	0.109	0.105	0.149	0.073	0.129
VI-2	0.090	0.057	0.059	0.075	0.072	0.128	0.166	0.089
VII-1	0.108	0.099	0.097	0.114	0.110	0.156	0.060	0.126
VII-2	0.102	0.058	0.061	0.077	0.074	0.131	0.174	0.162

TABLE VI-3 DYNAMIC RANGES RESPONDING TO RANDOM BURNUP DEVIATIONS FOR HORIZONTAL CONTROLLERS

Cases*	Penality ($x10^{-2}$)		R.M.S. Error in Thermal Flux (%)	Avg. Static Set-Point (%)+	Avg. Dynamic Range for Regulation (%)	Avg. Range for (%) Counteraction
	Paid in Amplitude	Paid in Control				
6-1	635.38	0.17014	0.00932	49.15	15.04	0.114
6-2	26799.0	0.16163	0.06070	46.78	13.50	0.106
5-1	113510.0	0.19828	0.12556	57.20	14.03	0.107

Note ; * The first number in the case identification indicates the number of controllers in the system.

+ Volume of each controller is 8 lattice sites.

TABLE VI-4 OPTIMALITY COMPARISON FOR REDUCED CONTROL SYSTEM

VII. CONCLUSION AND DISCUSSION

An analytical method to determine the optimum design of the CANDU control system layout was developed using modal control theory extended to the linear regulator and the least-square-root estimator. Since the dynamic model describing phenomena occurring in nuclear power reactors are not as simple as ordinary dynamic problems, we decomposed and reduced the system to be able to handle the dominant phenomena only. This reduced-order model has proven to be in agreement with available reactor stability design data for the CANDU 600 MWe reactors.

Separation of the spatial control from the bulk reactivity control was useful to determine the effectiveness of distributed discrete controllers and reached the conclusion that the most effective region for the spatial control was coincident with peaks of the spatial modes.

The number of controllers might be determined by the estimated dynamic range of individual controllers with constraints on maximum power error to be controlled and on the most probable response to random material property disturbances.

After evaluating the proposed method with the 2-dimensional CANDU 600 MWe reactor model, we concluded that the following.

(1) The existing system is expected to have the best performance as far as its vertical orientation is concerned.

(2) The range designated for spatial control was selected to agree with the most effective region for the purpose.

(3) The vertical control system could be replaced by a horizontal one with the advantages of better predictable performance and the hardware design considerations.

We have derived the optimum conditions for deploying control systems and have applied them to CANDU reactors. As many aspects of control requirements as possible were included, but we still need some further modifications for practical and more extensive applications. Such modifications should probably include the following.

(1) The procedure should be to deal with 3-dimensional models. The incore lay-out of the reactor control system has a symmetrical pattern in the Z-coordinate, and, thus, we can extend the location selection rules to the problem. In the modal approach, increasing the number of dimensions of the problem means adding a number of modes pertaining to the added direction. This causes an increase in the order of the system matrix, which eventually leads to a rapid increase in computing time. For example, to solve the matrix Riccati equation, the CPU time roughly proportional to the cube of the order of the system matrix. Possible ways for reducing the computing time could be to use a coarse-mesh model for the controller domain, or to change the algorithm which would be more efficient to compute the Riccati equation.

(2) Introduce direct feedback from the detector readings. During the theoretical formulation, we assumed that transients were detected by exact deviations of the individual mode amplitudes. In principle, the reactor control system should be able to control disturbances and follow transients completely, regardless of what kind of information is obtained. In practice, the control system relies on the estimated detector output, and the uncertainty in control actions induces forward disturbances in the reactor. To reduce the degree of control-action-induced perturbation, the design based on output feedback is preferred. But the problem to determine the optimum locations of detectors, which is expected not only to be more complex in formulating, but to introduce severe penalties in computing costs, because we may have to solve coupled Riccati equation for every possible combination of controller-detector pairs.

(3) Implementing volumetric controllers could be incorporated. One of the assumptions introduced in the study are point controllers rather than realistic volumetric ones. Thus the neutron fluxes were kept constant inside the controllers. Definitely this is not correct for controllers whose dimensions are large. But when a controller model with volumetric controllers is developed, the algorithm must include a criticality search by varying the controller levels.

However, the study shows the possibility of the application of the optimum control theory to layout the reactor control system. In addition, the effectiveness of control can be quantitatively evaluated, which is very useful for reactor designers, to decide which system will have a better performance.

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APPENDIX

ODZCR-OPTIMAL DEPLOYMENT OF THE CANDU ZONE CONTROL SYSTEM

A. GENERAL DESCRIPTIONS

The program ODZCR (Optimal Deployment of the Zone ControlleRs) is modelled for the study on the CANDU zone control system deployment. It is capable to study the stability of uncontrolled reactors; the spatial control effectiveness distribution calculation with a symmetrically placed pair of controllers; the dynamic range determination of individual controller at the pre-selected locations; and the simulation of transients initiated by certain mode amplitude excitations.

Limits made on program utilization are as follows, even though these limitations can be easily extended.

(1) The number of grid points in the X-coordinate must be less than 31 and in the Y-coordinate 23.

(2) Number of modes should be not greater than 10.

(3) Number of different material properties is limited to 25.

With the above limitation, the maximum CPU size required is approximately 220000B.

Computation time is strongly depended on the typed problem, because most of the time is spent in computing the Lyapunov matrix and the Riccati equations. Approximately a single iteration to solve an 8 by 8 dimensioned Lyapunov equation requires 0.05 second CPU time on the CDC-6600. This time use is approximately proportional to N^3, where N is the dimension of the matrix.

B. STRUCTURE OF PROGRAM ODZCR

ODZCR is composed of 16 subroutines and 7 external functions. A flow diagram of the main routine is shown in Figure B-1. Details of each subroutine are described as follows.

B.1 Subroutine HEAD

The routine prints head on each page of output, and contains title, page number, date and computing time consumed up to present.

Called by;

Main, SMTRX, COTMX, SPCON, BKCON, SIMULL

Calling;

SECOND, DATE

B.2 Subroutine REFST

REFST produces neutron, iodine and xenon distributions of the reference state and normalizes mode functions.

Called by;

Main

B.3 Subroutine XMAP (AA)

The subroutine illustrates the distribution of state functions of the given reactor configuration and also indicates their maximum value.

Arguments;

AA : 2-dimensional real number array stating a state function distribution

Called by;

Main, BKCON

B.4 Subroutine SMTRX (NPROB)

The subroutine generates complete system matrixes, not in reduced form, for each problem type and, if NPROB = 1, the coefficient matrix for random material perturbations.

Arguments;

NPROB Index for the problem type,

= 1 Dynamic range determination,

= 2 Spatial control effectiveness calculation,

= 3 Transient simulation.

Called by;

Main, BKCON, SIMUL

B.5 Subroutine GINV (A, U, AFLAG, ATEMP, MR, NR, NC, ERROR)

The routine gives the generalized inverse for any MR by NR matrix A, including the special case when MR = NR and rank (A) = NR. The algorithm yields the ordinary least squares transformation $(A^TA)^{-1}A^T$ and the inverse of A can be denoted by A^{-1},

$$A^{-1} = (A^T A)^{-1} A^T.$$

Arguments;

Variable	Description
MR	The first dimension of array A,
NR	The number of rows in A,
NC	The number of columns in A,
U	Bookkeeping array,
AFLAG, ATEMP	Temporary working storage,
ERROR	Maximum error in elements of $(AA^{-1} - I)$, where I is an identity.

Called by;

Main, CNTRLB, SPCON, BKCON, SIMUL

Calling;

DOT, DOT4

B.6 Function CNTRLB (A, B, NN, N, N1, N2)

This subroutine checks the controllability of the system with a given pair of controllers. If the formed controllability matrix has rank N, equivalent to the rank (A), CNTRLB is set to 1.0. Otherwise, CNTRLB = 0.0.

Arguments;

Variable	Description
NN	The first dimension of A,
N	Order of A,
N1	Number of controllers. Set ~~to~~ 2,
N2	Column dimension of controllability matrix,
A	System matrix,
B	Controller coefficient matrix produced with a pair of controllers.

Called by;

SPCON

Calling;

GINV, DOT3

B.7 Subroutine EIGVAL (NM, N, A, B, CA, ALFR, ALFI, BETA, MATZ, Z)

The routine computes eigenvalues of a generalized real matrix ($Ax = \omega x$) already transformed into the Schur form. The reduced form A, hence, is upper triangular except for possible 2x2 diagonal blocks corresponding to pairs of complex eigenvalues.

Arguments;

Variable	Description
NM	The first dimension of A,
N	Order of A,
A	$\tilde{A}$, upper triangular form of A, see p. 211,
B	Working storage array, but B(N, 1) contains information mentioned on subroutine SCHUR,
CA	Original matrix A,
ALFR	Vector array for real parts of eigenvalues,
ALFI	Vector array for imaginary parts of eigenvalues, and if an eigenvalue is purely real, corresponding element of ALFI set to zero,
BETA	Normalization factors for eigenvalues,
MATZ	Logical input set TRUE if transformation matrix will be saved,
Z	Transformation matrix.

Called by;

MAIN

Calling;

SCHUR

B.8 Subroutine EIGVEC (NM, N, A, B, ALFR, ALFI, BETA, Z)

If subroutines SCHUR and EIGVAL have been used to reduce the matrix of the system and to accumulate the transformation matrix, the eigenvectors of a real general matrix system can also be computed by EIGVEC. The eigenvectors of the system are determined by a back substitution process, then transformed to the eigenvectors of the original system using the information in Z and finally normalized to the length of the vector.

Arguments;

Same as described in EIGVAL

Called by;

Main

Calling;

DOT, DOT3

B.9 Subroutine COMTX (MR, NMAX, NS, A, WR, WI)

The routine is used for computing the transition matrix of a linear dynamic system in the form of

$$\Psi(t, t_0) = F_1 e^{\omega_1(t-t_0)} + F_2 e^{\omega_2(t-t_0)} + \dots\dots\dots + F_N e^{\omega_N(t-t_0)}$$

Each matrix F_1, F_2, ... F_N has dimension of the system matrix A. Once called, F_i is generated for the specified i.

Arguments;

Variable	Description
MR	The first dimension of A,
NMAS	Order of A,
NS	Eigenvalue number i for which F_i is computed,

A	System matrix,
WR	Real parts of eigenvalues,
WI	Imaginary parts of eigenvalues.

Called by;

Main

Calling;

HEAD

B.10 Subroutine SPCON

This routine is called only when NPROB = 2. The routine generates the spatial control effectiveness distribution inside the defined controller domain. A pair of controllers that were accepted by the controllability criterion, are symmetrically deployed with respect to the Y-axis or to the center of the reactor. The as built matrix Riccati equations are solved at every pair of grid points.

Algorithm inside the routine has the sequence of;

(1) read-in of controller material, domain and grid structure data,

(2) for the Y-symmetry option, constructing meshes for the first half core and, for the center symmetry option, meshes for the first quarter of the core,

(3) interpolating of mode fluxes and controller coefficient matrix elements at the mesh point,

(4) checking the controllability of the pair controllers,

(5) computing the matrix Riccati equation, if the pair passed the controllability test,

(6) producing the gain matrix for the pair,

(7) unfolding and mapping the norm of the gain matrix of individual controllers in the entire controller domain.

Called by;

Main

Calling;

AXPXA, CNTRLB, DOT2, DOT3, DOT4, GINV, HEAD, SECOND and XINT

B.11 Subrountine AXPXA (A, U, C, B, N, NA, NC, NU, NB, EPS)

This routine solves the Lyapunov matrix equation,

$$A^T X + XA = C$$

where A and C are real matrices of dimensions N x N and C is symmetric. A is already transformed into a upper Schur form by on orthogonal transformation U. In principle, the algorithm takes advantages of solving symmetry matrix system, using a recursive procedure that at each step solves at most 4 x 4 linear equation system obtained from partitioning A into 2 x 2 submatrices. An iterative procedure to reduce round-off errors is also incorporated.

Arguments;

Variable	Description
N	Order of A
NA	The first dimension of array A,
NC	" C,
NU	" U,
NB	" B,
A	Matrix A in an upper Schur form, dimenison of A must be at least N+1 by N+1,
U	Transformation matrix,

C	Coefficient matrix. On return, C contains the solution X,
B	Working storage array,
EPS	Index. If, EPS < 0, subroutine SCHUR won't be called. If EPS$\neq$1.0 but > 0, iteration for reducing round-off errors will be extended to 10. Otherwise, iteration is limited to 2.

Called by;

SPCON, BKCON

Calling;

DOT2, DOT3, DOT4, SCHUR, SYMSLV

B.12 Subroutine SYMSLV (A, C, N, NA, NC)

Subroutine SYMSLV called by AXPXA produces recursively partitioned equations at most 4x4. Starting of partitioning is at the upper left-handed corner of the transposed triangular matrix A^T.

Arguments;

Defined as in AXPXA

Called by;

AXPXA

Calling;

DOT2, DOT3, SYSSLV

B.13 Subroutine SYSSLV

The routine solves ordinary linear equation system whose unknown vector has a dimension of at most 4.

Called by;

SYMSLV

B.14 Subroutine SCHUR (NM, NB, N, A, B, MATX, Z, IERR)

Subroutine SCHUR reduces the generalized real matrix to an equivalent upper Schur form, i.e., a upper block triangular matrix whose block diagonal submatrices have dimension of at most 2 by 2. The similarity transformation matrix Z, accumulated transformation, and the transformed matrix A satisfy

$$\tilde{A} = Z^{T}AZ.$$

Arguments;

Variable	Description
NM	The ~~first~~ dimension of A,
NB	" B,
N	Order of A,
A	Real generalized matrix. On return, array A contains $\tilde{A}$.
B	Working storage. Initially an identity matrix. On return, B(N, 1) is used for the tolerance to determine negligible elements of $\tilde{A}$
MATZ	Logical input set TRUE if the transformation matrix is to be saved,
Z	Transformation matrix,
IERR	Index used for error return. Set to the row number for which the transformation is failed.

Called by;

EIGVAL, AXPXA

Calling;

DOT, DOT2

B.15 Function XINT (X, Y, XI, YJ, PXY)

XINT is written to interpolate 2-dimensional data. Algorithm follows a linear interpolation to get value, ~~P~~ inside a square whose corner values are known as $P_{i-1,j-1}$, $P_{i,j-1}$, $P_{i-1,j}$ and $P_{i,j}$;

$$P = \frac{(X - X_{i-1})(Y - Y_{j-1})(P_{i,j} - P_{i-1,j} - P_{i,j-1} + P_{i-1,j-1})}{(X_i - X_{i-1})(Y_j - Y_{j-1})}$$

$$+ \frac{(X - X_{i-1})(P_{i,j-1} - P_{i-1,j-1})}{(X_i - X_{i-1})}$$

$$+ \frac{(Y - Y_{j-1})(P_{i-1,j} - P_{i-1,j-1})}{(Y_j - Y_{j-1})}$$

$$+ P_{i-1,j-1}.$$

Arguments;

Variable	Description
X	X-coordinate of the point where P is interpolated,
Y	Y-coordinate of the point where P is interpolated,
XI	Array contains X_{i-1} and X_i,
YJ	Array contains Y_{j-1} and Y_j,
PXY	2-dimensional array contains $P_{i-1,j-1}$, $P_{i-1,j}$, $P_{i,j-1}$ and $P_{i,j}$.

Called by;

SPCON and BMTRX

B. 16 Subroutine BMTRX (LSUM)

The subroutine constructs the controller coefficient matrix, B_2 in Eq. (IV.7b). Material properties and mode fluxes at controller locations are evaluated by using the interpolation technique.

Arguments;

On return, LSUM contains the total number of controllers included in the control system.

Called by;

BKCON

Calling;

XINT

B.17 Subroutine BKCON (NPROB)

BKCON calculates the dynamic ranges of the control system corresponding to reactivity, power error and random perturbations. The algorithm follows the theory described in Chapter IV. Solution of the matrix Riccati equation is obtained by iteration, based on Kleinman's scheme

$$P_k (A - BR^{-1}B^T\bar{P}_{k-1}) + (A - BR^{-1}B^T\bar{P}_{k-1})^T P_k + \bar{P}_{k-1}BR^{-1}B^T\bar{P}_{k-1} + Q = 0$$

modified by using a finite-difference approximation to the time-dependent Riccati equation,

$$\bar{P}_{k-1} = P_{k-1} - \alpha F(P_{k-1})$$

where

$$F(P_{k-1}) = P_{k-1}A + A^T P_{k-1} - P_{k-1}BR^{-1}B^T P_{k-1} + Q$$

Arguments;

NPROB is the problem type set to 1 in the main program in order to call BKCON.

Called by;

Main

Calling;

AXPXA. BMTRX, DOT, DOT2, DOT3, DOT4, GINV, HEAD, SECOND, SMTRX, XMAP

B.18 Subroutine SIMUL (NPROB)

The routine simulates the transient initiated by exciting mode amplitudes and corresponding control actions. Information related to control actions should be prepared before running the problem type and provided by TAPE2. TAPE2 can be generated by running the program with NPROB=1.

Called by;

Main

Calling;

DOT2, DOT4, GINV, HEAD, SMTRX

B.19 Matrix Operation Routines

Functions	Arguments	Description
DOT	MR, N1, N2, A, JC, KC	A^TA
DOT1	MR, NR, N1, N2, A, JC, KC	AA^T
DOT2	MR, NR, N1, N2, A, B, JC, KC	A^TB
DOT3	MR, NR, N1, N2, A, B, JC, KC	AB
DOT4	MR, NR, N1, N2, A, B, JC, KC	AB^T

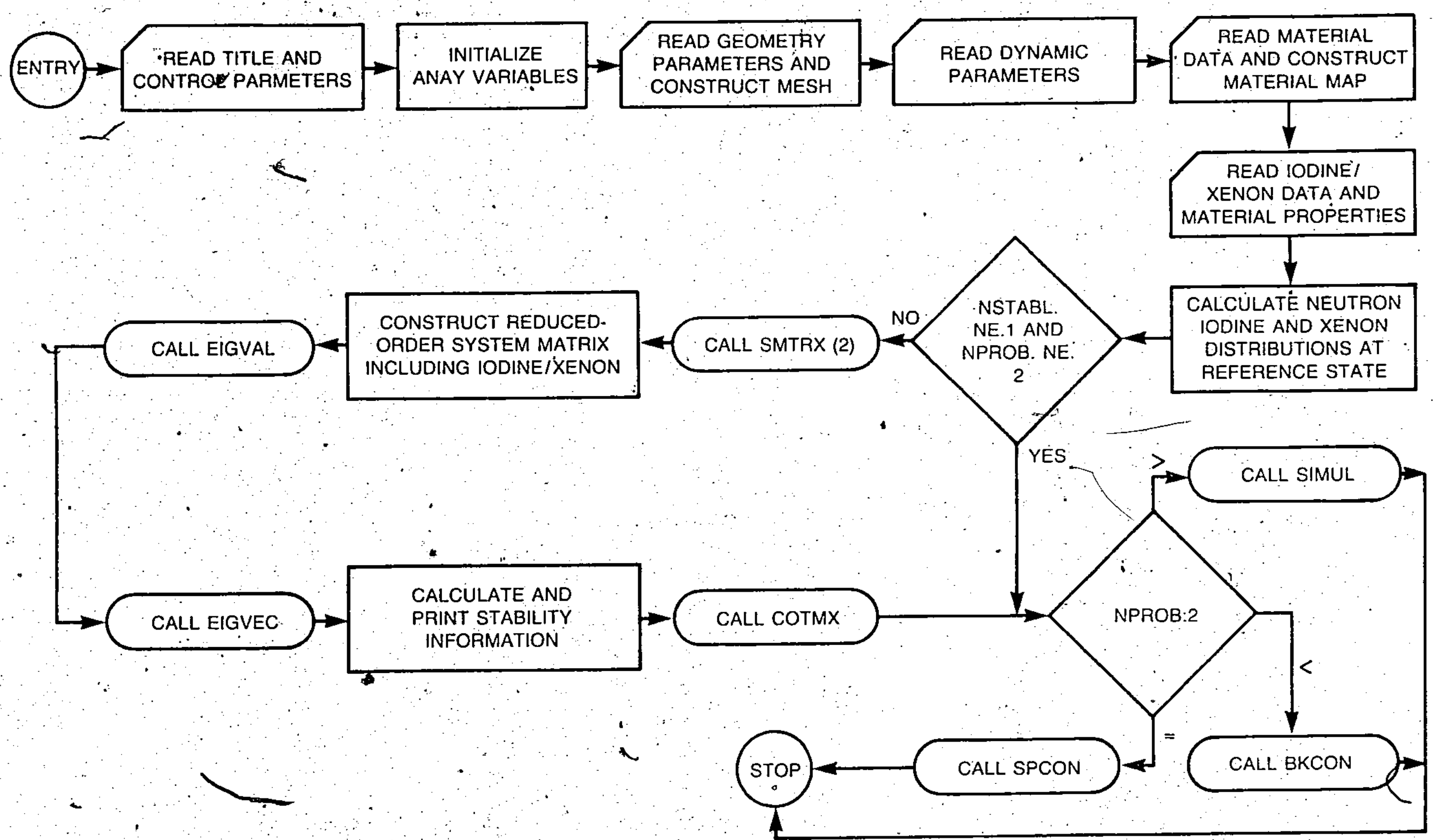

FIGURE B-1 PROGRAM ODZOR MAIN FLOW DIAGRAM

C. INPUT DATA DESCRIPTION

Most of the input data are primarily perpared by means of punched cards or keyboard. Numberical values of λ-modes are accepted through a magnetic tape or disk file. Also data transfer from the dynamic range determination option to the simulation option may be achieved by any data storage facilities of the computer.

C-I INPUT CARDS

Total 16 different sets of data cards would be necessary for running the entire program. The first 7 sets of cards are independent of problem types. The next 3 sets are only useful for the spatial control effectiveness computation. The next 4 sets are required for the dynamic range determination option. The last 2 sets should be provided for the simulation of transients initially excited by modal amplitude errors. Table C-1 shows a sample of input data for NPRC=2.

C-I.1 Title Card (18A4)

The information contained in this card is printed at the top of each page of output. If the user does not desire a title to be printed, a blank card must be inserted.

C-I.2 Problem Control Card (6I5)

Column	Variable	Description
1-5	NMAT	Number of material property types,
6-10	NRX	Number of grid points in the X-coordinates,
11-15	NRY	Number of grid points in the Y-coordinates,
16-20	NMOD	Number of λ-modes including the fundamental mode,

21-25	NSTABL	Control index for the stability calculation, { = 1 Yes, = 1 No,
26-30	NPROB	Control index for the problem type, = 1 Dynamic range determination, = 2 Spatial control effectiveness calculation, = 3 Simulation.

Note If NPROB=2, NSTABL should be 1.

C-I.3 Inverse of λ-mode Eigenvalues (8F10.0)

Column	Variable	Description
1-10	XK(1)	$1/\lambda_0$, inverse of the fundamental mode eigenvalue,
11-20	XK(2)	$1/\lambda_1$, inverse of the second mode eigenvalue,
.	.	.
.	.	.
.	.	.
71-80	XK(8)	$1/\lambda_7$, inverse of the eighth mode eigenvalue.

Note The sequence of data should be consistent with the sequence of modes stored on TAPE 1. If NMOD > 8, another card will be necessary for those modes.

C-I.4 Geometry Distribution Cards (F10.5, 34I2)

The data consists of the following;

(1) A set of cards specifying the mesh spacing in the X-coordinates,

(2) One blank card,

(3) A set of cards specifying the mesh spacing in the Y-coordinates,

(4) One blank card.

Each mesh spacing card has the following information;

Column	Variable	Description
1-10	XYZ	Mesh spacing in cm,
11-12	JJJ(1)	The first coordinate to which XYZ applies,
13-14	JJJ(2)	The second coordinate to which XYZ applies,
.	.	.
.	.	.
.	.	.

C-I.5 Dynamic Parameter Cards (6E12.6)

	Column	Variable	Description
Card 1;	1-12	XNU	ν, neutron yield rate per fission,
	13-24	SIGXE	σ_x, microscopic xenon thermal absorption cross section (cm^2),
	25-36	ZLAMI	λ_I } iodine and xenon decay constants (sec^{-1}),
	37-48	ZLAMX	λ_x
	49-60	GAMI	γ_I } iodine and xenon yield fractions per fission,
	61-72	GAMX	γ_x
Card 2;	1-12	V_1	v_1 } fast and thermal neutron velocities (cm/sec),
	13-24	V_2	v_2
	25-36	PWR	Total reactor power (MW-th).

C-I.6 Material Locations Cards (5I5)

These cards give the initial and final coordinate numbers in the X- or Y-directions for each material number. The overlay method is used, that is if the same volume is specified by two or more cards, the material number assigned by the latter is used. The final coordinate numbers must exceed the initial coordinate numbers.

Column	Variable	Description
1-5	J1	The initial X-coordinate number,
6-10	J2	The final X-coordinate number,
11-15	J3	The initial Y-coordinate number,
16-20	J4	The final Y-coordinate number,
21-25	J5	The material number, which occupies the volume bounded by the above

		rectangle J5 should not be greater than NMAT.

Note The last of these cards must be followed by a blank card.

C-I.7 Material Property Cards (6E12.6)

The data consist of NMAT sets of pair cards.

	Column	Variable	Description
Card 1;	1-12	D1F1	D_1, fast and thermal neutron diffusion coefficients (cm),
	13-24	D1F2	D_2
	25-36	SIGA1	Σ_{a1}, fast and thermal neutron macroscopic absorption cross sections (cm^{-1}),
	37-48	SIGA2	Σ_{a2}
	49-60	SIGR	Σ_{Rem}, macroscopic removal cross-section for fast neutron (cm^{-1}),
	61-72	SIGF	$\nu\Sigma_{f2}$, macroscopic yield cross-section for thermal neutron (cm^{-1}),
Card 2;	1-12	ALFA	α_f, power feedback coefficient,
	13-24	HFAC	H, flux-to-power conversion ratio (cf. POWDERPUFS-V).

C-I.8 Geometry Data Card for SPCON (3I5)

This card is necessary only if NPROB = 2.

Column	Variable	Description
1-5	NSX	X-coordinate of the reactor center,
6-10	NSY	Y-coordinate of the reactor center,
11-15	NZC	Control index for deploying policy of controller pairs, = 1; center symmetry, ≠ 1; Y-symmetry.

C-I.9 Geometry Card 1 for BKCON (I5, 2F10.5)

This card is only needed when NPROB ≧ 2.

Column	Variable	Description
1-5	NODY	Number of nodes in Y-coordinates inside the controller domain,
6-10	DCX	X_c in the controller domain,
11-15	DCY	Y_c in the controller domain.

C-I.10 Geometry Card 2 for BKCON (16I5)

This card is only required when NPROB = 2.

The number of data must be equal to NODY + 1.

Column	Variable	Description
1-5	NODX(1)	Number of nodes in X-coordinate at the first Y-node,
6-10	NODX(2)	Number of nodes in X-coordinates at the second Y-node,

.
.
.

C-I.11 Controller Specification Card (6E12.6)

The card is only required if NPROB = 1.

Column	Variable	Description
1-12	USIG2	Σ^U_{a2}, macroscopic thermal absorption cross section of controller material (cm^{-1}),
13-24	VUNT	Unit volume occupied by controllers (cf, MULTICELL),
25-36	UFRC	Fraction of controller volume required for keeping reference reactor critical, ($\simeq$0.5).
37-48	TILT	Maximum fractional power error expected to be able to be controlled by controllers,
49-60	SIGMA	Standard deviation of discharged bundle distribution on k_∞ coordinates,

C-I.12 State Vector Scale Factor, QFACO (E12.6)

This data is only necessary if NPROB = 1. The recommended value of QFACO is 2.0×10^{-2}, (cf. Eq. IV.1).

C-I.13 Controller Deployment Data Cards

The card set is required only if NPROB = 1.

Card Format (I5); The data LCON is the number of controllers whose geometrical properties will be provided in following Card 2.

Card 2 Format (4(2F9.4, I2))

Column	Variable	Description
1-9	XL(1)	X-coordinates of the controller number = 1,
10-18	YL(1)	Y-coordinates of the controller number = 1,
19-20	NSIGN(1)	Symmetricity index = 2, if another controller will be located by Y-symmetric point, = 1, otherwise.
21-29	XL(2)	As same as before, but for the
30-38	YL(2)	controller number
39-40	NSIGN(2)	= NSIGN(1) + 1
.	.	.
.	.	.
.	.	.

Note; Thus, the actual total number of controllers should be equal to

$$\text{MCON} = \sum_{i=1}^{\text{LCON}} \text{NSIGN}(i)$$

C-I.14 Controller Size Data Card (8F10.5)

These data are necessary only if NPROB = 1. Number of data should be equal to MCON.

Column	Variable	Description
1-10	UVOL(1)	Multiple of VUNT for the controller number = 1.

11-20	UVOL(2)	Multiple of VUNT for the controller number = 2,
.	.	.

C-I.15 Control Data for SIMUL (8E10.4)

This card should be appeared only if NPROB = 3.

Column	Variable	Description
1-10	TMAX	Maximum time for transient calculation (hrs.),
11-20	PLVL	Initial power level (%),
21-30	PRAT	Maximum allowable power rate (%/sec),
31-40	DT1	ΔT, time increment for transient calculation (hrs).

C-I.16 Initial Mode Amplitude Excitation Data (8E10.4)

The card is required only if NPROB = 3.

Column	Variable	Description
1-10	AMP(2,2)	Fractional increment of the thermal flux mode 2,
11-20	AMP(2,3)	Fractional increment of the thermal flux mode 3 .
.	.	

Note The amplitude increment of the fundamental mode should be excluded in the data. The initial power level was assumed to contribute directly to the fundamental mode state. Hence, total number of data should be equal to NMOD-1.

C-II. INPUT TAPES

Tape 1 is commonly called in all types of calculations. The tape stores the numerical data of each λ-mode sequentially in a single record. The structure of information is as follows;

NDUM ; integer number indicating the mode number,

((A(I,J), I=1, NRX), J=1, NRY) ; fast mode flux.

((B(I,J), I=1, NRX), J=1, NRY) ; thermal mode flux.

Tape 2 should be generated during the dynamic range determination calculation and should be attached for the simulation of transient. The structure of information stored in TAPE 2 is free format and its contents are;

MCON	; Total number of controllers,
XL, YL, NSIGN	; cf. Card C-I.13,
BMTX	; Controller matrix, B_2 in Eq. (IV.20a),
UB, UUNIT, UVOL	; Static set-points, Σ^U_{a2} x VUNT, volume of controllers,
GMTX	; Gain matrix, i.e., $R^{-1}B^T_2P$ in Eq. (IV.39).

```
/EOR
 SPATIAL CONTROL EFFECTIVENESS DISTRIBUTION CALCULATION
   23   31   23    8    1    2
 1.002330  0.985047  0.984677  0.958128  0.955536  0.925435  0.922456  0.920478
  39.95        2 131
  28.575.     19 2 3 4 5 61011141516171821222627282930
  14.2875     10 7 8 912131920232425
  0.0
  39.95        2 123
  28.575      17 2 3 4 5 6 7 81011141617181920212?
  57.15        2 915
  42.8625      21213
  0.0
     2.6062    1.261E-18    2.94E-05    2.10E-05    6.44E-02    2.3E-03
  5.335E+06    2.676E+05   0.94301E-3    2055.0
   1   31    1   23    1
  10   22    1   23    2
   8   24    2   22    2
   6   26    3   21    2
   5   27    4   20    2
   4   28    5   19    2
   3   29    6   18    2
   2   30    8   16    2
   1   31    9   15    2
  13   19    3   21    3
   9   23    4   20    3
   7   25    5   19    3
   6   26    6   18    3
   5   27    7   17    3
   4   28    9   15    3
   3   29   10   14    3
  14   18    9   15    4
  13   19   10   14    4
  11   21   11   13    4
  11   11   11   13    5
  21   21   11   13    5
  15   17    9    9    5
  15   17   15   15    5
   8    9   11   13    6
  23   24   11   13    6
  11   11    8    9    6
  11   11   15   16    6
  16   16   11   13    7
  14   14    8    8    8
  18   18    8    8    8
  14   14   16   16    8
  18   18   16   16    8
  16   16   16   16    8
  16   16    8    8    8
  14   14    9    9    9
  14   14   15   15    9
  18   18    9    9    9
  18   18   15   15    9
  16   16    9    9   10
  16   16   15   15   10
```

TABLE C-1 A SAMPLE INPUT DATA SET

14	14	10	14	11
18	18	10	14	11
11	11	10	10	12
11	11	14	14	12
21	21	10	10	12
21	21	14	14	12
11	11	11	13	13
21	21	11	13	13
8	9	15	17	14
8	9	9	10	14
16	16	6	7	14
16	16	17	19	14
23	24	9	10	14
23	24	15	17	14
16	16	8	8	15
16	16	11	13	16
8	9	2	3	17
8	8	4	4	17
16	16	1	2	17
23	24	2	3	17
24	24	4	4	17
8	9	5	8	18
8	9	14	14	18
9	9	4	4	18
16	16	3	5	8
23	24	5	8	18
23	24	14	14	18
24	24	4	4	18
8	9	12	13	19
23	24	12	13	19
16	16	9	9	20
16	16	15	15	20
16	16	10	10	21
16	16	14	14	21
16	16	16	16	22
8	9	11	11	23
23	24	11	11	23

1.3170	0.8787	1.0E+10	1.0E+10		
1.3170	0.8787	1.0E-11	8.3415E-05	1.01241E-02	0.0
0.0	0.0				
1.2739	0.9413	7.6387E-04	3.9795E-03	7.39171E-03	4.6116E-03
0.0	0.2089695				
1.2739	0.9412	7.6355E-04	3.9976E-03	7.39203E-03	4.5960E-03
0.0	0.2072662				

TABLE C-1 Cont'd

```
 1.2739      0.9413  7.6371E-04  3.9886E-03 7.39187E-03  4.6038E-03
 0.0      0.2081178
 1.2739      0.9413  7.6387E-04  3.9837E-03 7.39171E-03  4.6196E-03
 0.0      0.2089876
 1.2739      0.9412  7.6355E-04  4.0018E-03 7.39203E-03  4.5964E-03
 0.0      0.2072843
 1.2739      0.9413  7.6387E-04  3.9978E-03 7.39171E-03  4.6129E-03
 0.0      0.2090351
 1.2739      0.9413  7.6379E-04  4.0023E-03 7.39179E-03  4.6090E-03
 0.0      0.2086093
 1.2739      0.9413  7.6371E-04  4.0069E-03 7.39187E-03  4.6051E-03
 0.0      0.2081834
 1.2739      0.9412  7.6355E-04  4.0159E-03 7.39203E-03  4.5973E-03
 0.0      0.2073318
 1.2739      0.9413  7.6387E-04  3.9864E-03 7.39171E-03  4.6122E-03
 0.0      0.2090014
 1.2739      0.9413  7.6371E-04  3.9955E-03 7.39187E-03  4.6044E-03
 0.0      0.2081497
 1.2739      0.9410  7.6426E-04  3.9966E-03 7.39243E-03  4.6283E-03
 0.0      0.2089695
 1.2739      0.9409  7.6426E-04  4.0149E-03 7.39243E-03  4.6296E-03
 0.0      0.2090351
 1.2739      0.9409  7.6394E-04  4.0189E-03 7.39275E-03  4.6131E-03
 0.0      0.2072843
 1.3150      0.8804     1.0E-11  8.5190E-05 1.01255E-02  0.0
 0.0      0.0
 1.2759      0.9430  7.6324E-04  3.9813E-03 7.39308E-03  4.5991E-03
 0.0      0.2089695
 1.2759      0.9430  7.6324E-04  3.9855E-03 7.39308E-03  4.6071E-03
 0.0      0.2089876
 1.2759      0.9430  7.6308E-04  4.0087E-03 7.39324E-03  4.5926E-03
 0.0      0.2081834
 1.2759      0.9429  7.6292E-04  4.0036E-03 7.39340E-03  4.5839E-03
 0.0      0.2072843
 1.2759      0.9430  7.6324E-04  3.9996E-03 7.39308E-03  4.6004E-03
 0.0      0.2090351
 1.2739      0.9410  7.6426E-04  4.0008E-03 7.39243E-03  4.6363E-03
 0.0      0.2089876
16    12     1
 9       57.15     28.575
 2     2     2     2     2     2     2     2     2
```

TABLE C-1 Cont'd

D. SAMPLE OUTPUT

Following pages list the output obtained in a spatial control effectiveness calculation. The first 4 pages describe the reactor model and the reference state's distributions and, hence, should be shown on all three options of NPROB. The next 10 pages contain the stability information and, thus appear only when NSTABL = 1, except for the system matrix. The next page gives the iteration information during computing the pointwise Riccati equations. The last page gives the geometry of controller domain and the calculated effectiveness distribution.

NO OF MATERIAL PROPERTIES = 23
NO OF MESH POINTS IN X-AXIS = 31
NO OF MESH POINTS IN Y-AXIS = 23
NO OF NODES = 8
STABILITY CHECK (YES = 1) = 1
PROBLEM TYPE (=1; BULK CONTROLLER, =2; SPATIAL CONTROLLER, =3; SIMULATION) = 2

MESH SPACING (X-DIRECTION)

39.95000 1 31
28.57500 2 3 4 5 6 10 11 14 15 16 17 18 21 22 26 27 28 29 30
14.28750 7 8 9 12 13 19 20 23 24 25

39.95000 28.57500 28.57500 28.57500 28.57500 28.57500 14.28750 14.28750 14.28750 28.57500 28.57500 14.28750
14.28750 28.57500 28.57500 28.57500 28.57500 28.57500 14.28750 14.28750 28.57500 28.57500 14.28750 14.28750
14.28750 28.57500 28.57500 28.57500 28.57500 28.57500 39.95000

MESH SPACING (Y-DIRECTION)

39.95000 1 23
28.57500 2 3 4 5 6 7 8 10 11 14 16 17 18 19 20 21 22
57.15000 9 15
42.86250 12 13

39.95000 28.57500 28.57500 28.57500 28.57500 28.57500 28.57500 28.57500 57.15000 28.57500 28.57500 42.86250
42.86250 28.57500 57.15000 28.57500 28.57500 28.57500 28.57500 28.57500 28.57500 28.57500 28.57500 39.95000

3553702 7/5

**** MATERIAL MAP ****

1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	17	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	17	17	2	2	2	2	2	2	17	2	2	2	2	2	2	17	17	1	1	1	1	1	1	1
1	1	1	1	1	2	2	17	17	2	2	2	3	3	3	8	3	3	3	2	2	2	17	17	2	2	1	1	1	1	1
1	1	1	1	2	2	2	17	18	3	3	3	3	3	3	8	3	3	3	3	3	3	3	18	2	2	2	1	1	1	1
1	1	1	2	2	2	3	18	18	3	3	3	3	3	3	8	3	3	3	3	3	3	18	18	3	2	2	2	1	1	1
1	1	2	2	2	3	3	18	18	3	3	3	3	3	3	14	3	3	3	3	3	3	18	18	3	3	2	2	2	1	1
1	1	2	2	3	3	3	18	18	3	3	3	3	3	3	14	3	3	3	3	3	3	18	18	3	3	3	2	2	1	1
1	2	2	2	3	3	3	18	18	3	6	3	3	8	3	15	3	8	3	3	3	3	18	18	3	3	3	2	2	2	1
2	2	2	3	3	3	3	14	14	3	6	3	3	9	5	20	5	9	3	3	3	3	14	14	3	3	3	3	2	2	2
2	2	3	3	3	3	3	14	14	3	12	3	4	11	4	21	4	11	4	3	12	3	14	14	3	3	3	3	3	2	2
2	2	3	3	3	3	3	23	23	3	13	4	4	11	4	16	4	11	4	4	13	3	23	23	3	3	3	3	3	2	2
2	2	3	3	3	3	3	19	19	3	13	4	4	11	4	16	4	11	4	4	13	3	15	19	3	3	3	3	3	2	2
2	2	3	3	3	3	3	19	19	3	13	4	4	11	4	16	4	11	4	4	13	3	19	19	3	3	3	3	3	2	2
2	2	3	3	3	3	3	18	18	3	12	3	4	11	4	21	4	11	4	3	12	3	18	18	3	3	3	3	3	2	2
2	2	2	3	3	3	3	14	14	3	6	3	3	9	5	20	5	9	3	3	3	3	14	14	3	3	3	3	2	2	2
1	2	2	2	3	3	3	14	14	3	6	3	3	8	3	22	3	8	3	3	3	3	14	14	3	3	3	2	2	2	1
1	1	2	2	3	3	3	14	14	3	3	3	3	3	3	14	3	3	3	3	3	3	14	14	3	3	3	2	2	1	1
1	1	2	2	2	3	3	3	3	3	3	3	3	3	3	14	3	3	3	3	3	3	3	3	3	3	2	2	2	1	1
1	1	1	2	2	2	3	3	3	3	3	3	3	3	3	14	3	3	3	3	3	3	3	3	3	2	2	2	1	1	1
1	1	1	1	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	2	2	2	2	1	1	1	1
1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	3	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1
1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1

3553703 /5

ACTOR STRUCTURES
REACTOR POWER = .205500E+04 (MW-TH)

UTRON KINETICS DATA

V1	V2	LIFETIME
5.335000E+06	2.676000E+05	9.430100E-04

DINE/XENON PARAMETERS

XNU	SIGXE	ZLAMI	ZLAMX	GAMI	GAMX
2.606200E+00	1.261000E-18	2.540000E-05	2.100000E-05	6.440000E-02	2.300000E-03

TERIAL PROPERTIES

DIF1	DIF2	SIGA1	SIGA2	SIGF1	SIGF2	ALFA	HFAC1
1.31700E+00	8.78700E-01	1.00000E+10	1.00000E+10	0.	0.	0.	0.
1.31700E+00	8.78700E-01	1.00000E-11	8.34150E-05	1.01241E-02	0.	0.	0.
1.27390E+00	9.41300E-01	7.63670E-04	3.85950E-03	7.39171E-03	1.76947E-03	0.	2.42405E-01
1.27390E+00	5.41200E-01	7.63550E-04	3.87760E-03	7.39203E-03	1.76349E-03	0.	2.40429E-01
1.27390E+00	9.41300E-01	7.63710E-04	3.86840E-03	7.39187E-03	1.76648E-03	0.	2.41417E-01
1.27390E+00	9.41300E-01	7.63870E-04	3.86370E-03	7.39171E-03	1.77254E-03	0.	2.42426E-01
1.27390E+00	9.41200E-01	7.63550E-04	3.88180E-03	7.39203E-03	1.76364E-03	0.	2.40450E-01
1.27390E+00	9.41300E-01	7.63870E-04	3.87780E-03	7.39171E-03	1.76597E-03	0.	2.42481E-01
1.27390E+00	9.41300E-01	7.63790E-04	3.88230E-03	7.39179E-03	1.76846E-03	0.	2.41957E-01
1.27390E+00	9.41300E-01	7.63710E-04	3.88690E-03	7.391[illegible]E-03	1.76696E-03	0.	2.41493E-01
1.27390E+00	9.41200E-01	7.63550E-04	3.89590E-03	7.39203E-03	1.76399E-03	0.	2.40505E-01
1.27390E+00	9.41300E-01	7.63870E-04	3.86640E-03	7.39171E-03	1.76570E-03	0.	2.42442E-01
1.27390E+00	5.41300E-01	7.63710E-04	3.87550E-03	7.39187E-03	1.76671E-03	0.	2.41454E-01
1.27390E+00	9.41000E-01	7.64260E-04	3.87660E-03	7.39243E-03	1.77588E-03	0.	2.42455E-01
1.27390E+00	9.40900E-01	7.64260E-04	3.89490E-03	7.39243E-03	1.77638E-03	0.	2.42481E-01
1.27390E+00	9.40900E-01	7.63940E-04	3.89890E-03	7.39275E-03	1.77035E-03	0.	2.40450E-01
1.31500E+00	8.68400E-01	1.00000E-11	6.51900E-05	1.01255E-02	0.	0.	0.
1.27590E+00	9.43000E-01	7.63240E-04	3.86130E-03	7.39308E-03	1.76468E-03	0.	2.42405E-01
1.27590E+00	9.43000E-01	7.63240E-04	3.86550E-03	7.39308E-03	1.76775E-03	0.	2.42426E-01
1.27590E+00	9.43000E-01	7.63080E-04	3.69870E-03	7.39324E-03	1.76218E-03	0.	2.41495E-01
1.27590E+00	9.42900E-01	7.62920E-04	3.88360E-03	7.39340E-03	1.75864E-03	0.	2.42457E-01
1.27590E+00	9.43000E-01	7.63240E-04	3.87960E-03	7.39308E-03	1.76518E-03	0.	2.42451E-01
1.27390E+00	9.41000E-01	7.64260E-04	3.88080E-03	7.39243E-03	1.77895E-03	0.	2.42426E-01

3553704 /5

FAST FLUX DISTRIBUTION

(MULTIPLIED BY E 11)

Y = 20.0 54.2 82.8111.4140.0168.5197.1225.7268.6311.4343.0375.7418.6454.3497.2540.0576.6597.2625.7654.3682.9711.57

X =	20.0	54.2	82.8	111.4	140.0	168.5	197.1	225.7	268.6	311.4	343.0	375.7	418.6	454.3	497.2	540.0	576.6	597.2	625.7	654.3	682.9	711.57
745.7	0	0	0	0	0	0	0	0	0	3	4	4	4	3	0	0	0	0	0	0	0	0
711.5	0	0	0	0	0	0	0	1	13	56	67	71	69	57	10	1	0	0	0	0	0	0
682.9	0	0	0	0	0	1	6	15	84	497	564	595	576	458	64	15	6	1	0	0	0	0
654.3	0	0	0	0	2	13	57	107	594	752	810	847	[illegible]	757	553	105	55	13	[illegible]	0	0	0
625.7	0	0	0	2	16	96	475	624	829	945	958	1036	[illegible]	956	928	611	459	92	17	2	0	0
597.2	0	0	2	17	104	510	704	837	991	1084	1129	1171	1165	1108	951	811	672	457	[illegible]	16	2	0
575.7	0	0	9	70	439	672	634	552	1066	1140	1176	1229	1231	1167	1069	907	783	678	42	67	9	0
561.4	0	1	20	147	558	763	515	1024	1103	1158	1187	1261	1269	1233	1104	956	843	720	[illegible]	139	19	1
547.2	0	4	42	366	651	636	983	1064	1145	1189	1211	1281	1292	1266	1151	1014	911	795	[illegible]	372	40	3
525.7	0	7	62	527	757	934	1068	1156	1237	1256	1268	1297	1306	1252	1238	1124	1127	[illegible]	734	512	60	6
497.2	0	11	87	635	867	1035	1155	1224	1286	1267	1262	1270	1278	1264	1288	1206	1129	[illegible]	[illegible]	619	65	11
475.7	2	24	180	734	928	1088	1197	1257	1308	1294	1285	1286	1254	1306	1312	1245	1176	1066	[illegible]	687	176	24
461.4	3	47	444	755	961	1114	1216	1266	1311	1297	1269	1268	1266	1307	1316	1262	1201	1095	[illegible]	737	433	46
440.0	4	64	553	807	1000	1143	1232	1262	1296	1288	1281	1277	1255	1257	1305	1262	1223	1125	[illegible]	787	578	[illegible]
411.4	4	70	590	839	1022	1150	1231	1268	1313	1312	1257	1288	1258	1320	[illegible]	1283	1225	1133	[illegible]	614	573	[illegible]
382.9	5	73	603	852	1028	1117	1184	1211	1298	1306	1254	1239	[illegible]	1317	1314	1265	1192	1097	[illegible]	816	582	69
354.3	4	70	590	839	1022	1150	1231	1268	1313	1311	1257	1288	1258	1320	[illegible]	1283	1229	1133	[illegible]	614	573	[illegible]
325.7	4	64	553	807	1000	1143	1231	1262	1299	1288	1261	1277	1255	1257	1315	1262	1223	1125	[illegible]	787	538	[illegible]
304.3	3	47	444	755	961	1114	1216	1266	1311	1297	1269	1289	1266	1307	1316	1262	1201	1095	[illegible]	737	433	46
290.0	2	24	180	724	928	1068	1197	1257	1305	1294	1285	1286	1254	1305	1312	1246	1178	1066	[illegible]	687	176	24
268.6	[illegible]	11	87	634	867	1035	1155	1224	1286	1267	1262	1273	1278	1264	1268	1206	1129	[illegible]	[illegible]	615	65	11
240.0	0	7	62	527	757	934	1068	1158	1237	1256	1268	1297	1306	1252	1238	1124	1027	[illegible]	[illegible]	512	60	6
218.5	0	4	42	386	650	638	983	1084	1149	1189	1211	1291	[illegible]	1265	1151	1018	911	795	[illegible]	372	40	3
204.3	0	1	20	147	556	763	515	1024	1103	1158	1187	1261	1269	1233	1104	956	843	720	[illegible]	135	19	1
190.0	0	0	9	70	439	672	634	952	1066	1140	1176	1229	1231	1187	1065	907	763	636	42	67	9	0
168.5	0	0	2	17	104	509	704	837	991	1084	1129	1171	1165	1108	951	815	672	457	[illegible]	16	2	0
140.0	0	0	0	2	16	96	475	624	829	945	958	1036	1122	956	828	611	459	92	17	2	0	0
111.4	0	0	0	0	2	13	57	107	593	752	810	847	830	757	553	105	55	13	2	0	0	0
82.8	0	0	0	0	0	1	6	15	84	497	564	595	578	458	64	15	6	1	0	0	0	0
54.2	0	0	0	0	0	0	0	1	10	56	67	71	69	57	10	1	0	0	0	0	0	0
20.0	0	0	0	0	0	0	0	0	0	3	4	4	4	3	0	0	0	0	0	0	0	0

MAXIMUM VALUE = .132531E+15

3553705 /5

NTROLLER EFFECTIVENESS COMPUTATION FOR CANDU 6?? (Y-SYMMETRIC)   PAGE 5 , 84-06-26   TIME 1.358 SEC

THERMAL FLUX DISTRIBUTION

(MULTIPLIED BY E 1

Y = 20.0 54.2 82.8111.4140.0168.5197.1225.7266.6311.434..0375.7[illegible].6454.3[illegible]6557.2625.7654.3682.9711.57

X =																						
745.7	0	0	0	0	0	0	0	0	191	295	335	359	345	295	151		0	0	0	0	0	0
711.5	0	0	0	0	0	0	0	201	621	670	966	1050	1014	872	621	201	0	0	0	0	0	0
682.9	0	0	0	0	0	156	341	655	1125	1021	1104	1164	1132	1025	1124	641	334	152	0	0	0	0
654.3	0	0	0	0	200	534	663	11[illegible]	1169	1375	1473	1538	1507	1364	1166	1171	841	517	193	0	0	0
625.7	0	0	0	207	565	975	991	1221	1513	1709	1803	1872	1847	1729	1555	1194	958	938	562	155	0	0
597.2	0	0	178	569	996	1050	1292	1525	1791	1959	2140	2114	2103	2001	1791	1473	1236	1004	955	547	172	0
575.7	0	0	363	834	979	1243	1505	1715	1931	2060	2122	2207	2211	2135	1933	1641	1422	1186	541	806	352	0
561.4	0	118	527	977	1060	1371	1631	1826	1996	2095	2132	2230	2242	2197	1958	1732	1528	1314	1146	951	515	115
547.2	0	256	699	909	1193	1496	1751	1933	2081	2152	2175	2264	2241	2253	2084	1842	1651	1442	1155	866	664	2[illegible]1
525.7	115	480	954	1047	1378	1685	1926	2088	2233	2265	2285	2332	2348	2324	2236	2030	1856	1629	1337	1015	931	479
497.2	223	682	1214	1244	1575	1870	2064	2194	2304	2236	2227	2241	2255	2267	2307	2161	2038	1824	1535	1213	1163	615
475.7	274	768	1294	1353	1663	1965	2161	2265	2359	2325	2319	2312	2326	2347	2364	2245	2128	1925	1644	1319	1261	779
461.4	301	868	1089	1400	1740	2014	2195	2279	2360	2333	2319	2317	2331	2351	2367	2267	2169	1978	1702	1366	1061	647
440.0	333	972	1102	1470	1807	2064	2221	2238	2301	2285	2272	2265	2279	2299	2312	2238	2205	2033	1767	1433	1073	547
411.4	356	1040	1153	1522	1645	2076	2223	2284	2361	2361	2329	2324	2341	2376	2362	2308	2220	2047	1795	1479	1123	1014
382.9	362	1054	1162	1524	1632	2021	2141	2149	2265	2307	2251	2226	2245	2325	2293	2218	2152	1987	175[illegible]	147[illegible]	1137	1[illegible]3
354.3	356	1040	1153	1522	1645	2076	2223	2264	2361	2361	2339	2324	2341	2376	2362	2308	2220	2047	1795	1475	1123	1014
325.7	333	972	1102	1470	1807	2064	2221	2238	2301	2285	2272	2265	2279	2299	2312	2238	2205	2033	1767	1433	1073	547
304.3	301	868	1089	1400	1740	2013	2195	2279	2360	2333	2319	2317	2331	2351	2367	2267	2169	1978	1702	1366	1061	647
290.0	274	786	1294	1353	16[illegible]	1965	2161	2265	2359	2325	2319	2312	2326	2347	2364	2245	2126	1925	1644	1319	1261	749
268.6	223	662	1214	1244	1575	1870	2084	2194	2304	2236	2227	2241	2255	2266	2307	2161	2038	1624	1535	1213	1163	6[illegible]
240.0	115	480	954	1047	1378	1685	1926	2088	2233	2265	2285	2332	2348	2324	2236	2031	1856	1629	1337	1019	931	469
218.5	0	256	699	909	1193	1498	1751	1933	2081	2152	2175	2264	2241	2253	2083	1842	1651	1442	1155	866	674	[illegible]
204.3	0	118	527	977	1060	1371	1631	1826	1996	2095	2131	2230	2242	2197	1958	1732	1528	1314	1146	951	515	115
190.0	0	0	363	834	979	1242	1505	1715	1931	2060	2122	2207	2211	2135	1932	1641	1421	1186	541	806	352	0
168.5	0	0	178	568	956	1050	1292	1525	1791	1959	2140	2114	2103	2001	1791	1473	1235	1004	955	547	172	0
140.0	0	0	0	207	565	974	991	1221	1510	1709	1803	1872	1847	1726	1505	1194	954	978	562	199	0	0
111.4	0	0	0	0	200	534	663	11[illegible]	1169	1375	1473	1539	1507	1384	1166	1171	841	517	197	0	0	0
82.8	0	0	0	0	0	156	341	655	1125	1021	1104	1164	1132	1025	1124	641	334	152	0	0	0	0
54.2	0	0	0	0	0	0	0	201	621	870	986	1050	1014	872	621	201	0	0	0	0	0	0
20.0	0	0	0	0	0	0	0	0	191	295	335	359	345	255	151		0	0	0	0	0	0

MAXIMUM VALUE = .238240E+15

3553706 /5

NTROLLER EFFECTIVENESS COMPUTATION FOR CANDU 6?? (Y-SYMMETRIC)   PAGE 6   84-06-26   TIME [illegible].419 SEC

IODINE DISTRIBUTION

(MULTIPLIED BY E 11

Y = 20.0 54.2 82.8111.4140.0168.5197.1225.7266.6311.434[illegible]0375.7418.6454.3497.2540.0[illegible].6557.2625.7654.3682.9711.57

X =																						
745.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0		0	0	0
711.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0		0	0	0
682.9	0	0	0	0	0	0	0	0	0	3960	4282	4512	4390	3974	0		0	0		0	0	0
654.3	0	0	0	0	0	0	0	0	4611	5333	5710	5963	5843	5366	4607		0	0		0	0	0
625.7	0	0	0	0	0	0	3842	4732	5653	6625	6990	7256	7161	6703	5849	463[illegible]	3715	0		0	0	0
597.2	0	0	0	0	0	4071	5008	5893	6944	7593	7909	8194	8152	7758	6944	5712	4791	3894		0	0	0
575.7	0	0	0	0	3795	4816	5236	6655	7466	7985	8226	8557	8572	8278	7452	6364	5512	4597	[illegible]	0	0	0
561.4	0	0	0	3779	4176	5301	6304	7059	7767	8152	8308	8635	8683	8496	7775	6734	5547	5094	[illegible]	0	0	0
547.2	0	0	0	3523	4614	5791	6771	7472	8097	8374	8478	8773	8835	8713	8107	7166	6425	5592	[illegible]	3435	0	0
525.7	0	0	0	4061	5341	6532	7467	8095	8656	8780	8758	9040	9133	9010	8669	7871	7195	6315	[illegible]	3951	0	0
497.2	0	0	0	4624	6106	7249	8180	8516	8932	8670	8619	8674	8729	8786	8941	8375	7901	7072	[illegible]	4702	0	0
475.7	0	0	0	5244	6524	7616	8378	8782	9145	9014	8921	8931	8986	9057	9166	8724	8249	7464	[illegible]	5115	0	0
461.4	0	0	4221	5430	6747	7606	8510	8835	9147	9015	8561	8952	9105	9083	9177	8787	8409	7667	[illegible]	5297	4116	0
440.0	0	0	4272	5699	7007	8002	8612	8675	8912	8831	8760	8755	8806	8886	8959	8675	8542	7880	[illegible]	5556	4162	0
411.4	0	0	4470	5900	7154	8055	8618	8854	9138	9120	9036	8990	9043	9179	9216	8947	8607	7937	[illegible]	5735	4354	0
382.9	0	0	4507	5912	7106	7862	8332	8362	8746	8689	8728	8631	8716	8956	8851	8576	8372	7731	[illegible]	5748	4409	0
354.3	0	0	4470	5900	7154	8055	8619	8854	9138	9120	9036	8990	9043	9179	9216	8947	8607	7937	[illegible]	5735	4354	0
325.7	0	0	4272	5697	7007	8002	8609	8675	8912	8831	8780	8755	8806	8886	8959	8675	8548	7880	[illegible]	5556	4162	0
304.3	0	0	4221	5430	6747	7604	8510	8835	9147	9015	8561	8952	9105	9083	9177	8787	8409	7667	[illegible]	5297	4116	0
290.0	0	0	0	5244	6524	7618	8378	8782	9145	9014	8921	8931	8986	9057	9164	8704	8249	7464	[illegible]	5115	0	0
268.6	0	0	0	4824	6106	7245	8180	8521	8946	8668	8619	8674	8729	8786	8958	8392	7901	7072	[illegible]	4702	0	0
240.0	0	0	0	4061	5341	6532	7467	8095	8656	8780	8758	9040	9133	9010	8669	7865	7195	6315	[illegible]	3951	0	0
218.5	0	0	0	3514	4614	5791	6771	7472	8097	8374	8478	8770	8833	8710	8105	7166	6425	5592	[illegible]	343[illegible]	0	0
204.3	0	0	0	0	4176	5295	6304	7059	7764	8150	8306	8635	8683	8496	7773	6735	5944	5094	[illegible]	0	0	0
190.0	0	0	0	0	3795	4816	5836	6650	7466	7985	8226	8557	8572	8278	7456	6364	5510	4597	[illegible]	0	0	0
168.5	0	0	0	0	0	4071	5006	5893	6944	7593	7909	8194	8152	7758	6941	5712	4791	3894		0	0	0
140.0	0	0	0	0	0	0	3842	4732	5653	6625	6950	7256	7161	6701	5849	463[illegible]	3715	0		0	0	0
111.4	0	0	0	0	0	0	0	0	4611	5333	5710	5963	5843	5366	4605		0	0		0	0	0
82.8	0	0	0	0	0	0	0	0	0	3960	4282	4512	4390	3974	0		0	0		0	0	0
54.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0		0	0	0
20.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0		0	0	0

MAXIMUM VALUE = .921855E+15

3553707 /5

XENON DISTRIBUTION

(MULTIPLIED BY E 1

Y = 20.0 54.2 82.8111.4140.0168.5197.1225.7 [illegible] 65.?311.434 [illegible] 375.7419.?454.3 [illegible] 6557.2677.7654.3682.9711.5

X =																						
745.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
711.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
682.9	0	0	0	0	0	0	0	0	0	8047	8133	8198	8159	8051	0	0	0	0	0	0	0	0
654.3	0	0	0	0	0	0	0	0	8210	8349	8409	8445	8426	8354	8205	0	0	0	0	0	0	0
625.7	0	0	0	0	0	0	8013	8230	8430	8526	8566	8594	8585	8537	8429	8214	7974	0	0	0	0	0
597.2	0	0	0	0	0	8076	[illegible]	8435	8563	8626	8653	8676	8672	8640	8563	8409	8248	8028	0	0	0	0
575.7	0	0	0	0	7959	8253	8427	8531	8616	8659	8678	8703	8704	8682	8617	8497	8378	8207	7953	0	0	0
561.4	0	0	0	7975	8087	8323	8469	8554	8670	8701	8727	8700	8704	8676	8671	8565	8470	8307	8074	0	0	0
547.2	0	0	0	7910	8151	8400	8523	8553	8697	8718	8743	8709	8714	8691	8696	8614	8533	8391	8184	7875	0	0
525.7	0	0	0	8076	8350	8517	8614	8661	8710	8718	8723	8735	8729	8733	8710	8650	8588	8491	8323	8045	0	0
497.2	0	0	0	8255	8464	8594	8667	8695	8728	8712	8694	8698	8712	8720	8729	8690	8652	8576	8443	8225	0	0
475.7	0	0	0	8333	8516	8628	8690	8718	8742	8734	8700	8701	8704	8739	8743	8713	8682	8614	8495	8310	0	0
461.4	0	0	8118	8365	8542	8644	8699	8722	8742	8706	8713	8712	8705	8710	8744	8719	8692	8632	8525	8342	8090	0
440.0	0	0	8131	8407	8570	8660	8706	8713	8722	8696	8693	8651	8655	8700	8725	8715	8702	8650	8553	8365	8113	0
411.4	0	0	8178	8436	8585	8665	8707	8723	8728	8713	8707	8704	8706	8717	8733	8729	8706	8655	8565	8412	8151	0
382.9	0	0	8189	8440	8582	8678	8715	8721	8682	8677	8717	8710	8716	8681	8689	8694	8718	8667	8576	8414	8164	0
354.3	0	0	8178	8436	8585	8665	8707	8723	8728	8713	8707	8704	8708	8717	8733	8729	8706	8655	8565	8412	8151	0
325.7	0	0	8131	8407	8570	8660	8706	8713	8722	8696	8693	8691	8695	8700	8725	8713	8702	8650	8553	8365	8103	0
304.3	0	0	8118	8365	8542	8644	8699	8722	8742	8706	8703	8712	8705	8710	8744	8719	8692	8632	8525	8342	8090	0
290.0	0	0	0	8333	8516	8628	8690	8716	8742	8734	8700	8701	8704	8739	8743	8713	8660	8614	8495	8310	0	0
268.6	0	0	0	8255	8464	8594	8667	8714	8743	8711	8694	8698	8702	8720	8744	8705	8652	8576	8443	8229	0	0
240.0	0	0	0	8076	8350	8517	8614	8668	8710	8718	8723	8735	8729	8733	8710	8650	8586	8491	8323	8045	0	0
218.5	0	0	0	7869	8151	8400	8523	8593	8697	8718	8740	8709	8714	8691	8658	8614	8533	8391	8184	7875	0	0
204.3	0	0	0	0	8087	8323	8469	8554	8670	8701	8727	8700	8704	8676	8673	8565	8470	8307	8074	0	0	0
190.0	0	0	0	0	7959	8253	8427	8531	8616	8659	8678	8703	8704	8682	8616	8497	8378	8207	7953	0	0	0
168.5	0	0	0	0	0	8078	8290	8435	8563	8626	8653	8676	8672	8640	8563	8409	8248	8028	0	0	0	0
140.0	0	0	0	0	0	0	8013	8236	8429	8528	8568	8594	8585	8537	8429	8214	7974	0	0	0	0	0
111.4	0	0	0	0	0	0	0	0	8210	8349	8409	8445	8426	8354	8219	0	0	0	0	0	0	0
82.8	0	0	0	0	0	0	0	0	0	8047	8133	8188	8159	8051	0	0	0	0	0	0	0	0
54.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

MAXIMUM VALUE = .874462E+14

MODAL INFORMATION

MOD NO	K-EFF	FNORM	TNORM
1	.10023E+01	.10106E-03	.62396E-04
2	.98505E+00	.16021E-03	.96324E-04
3	.98468E+00	.16196E-03	.99442E-04
4	.95813E+00	.16563E-03	.10346E-03
5	.95554E+00	.16664E-03	.10299E-03
6	.92544E+00	.36234E-03	.22124E-03
7	.92246E+00	.16360E-03	.10973E-03
8	.92048E+00	.20611E-03	.12511E-03

FLUX SCALE FACTOR = .544052E+12
K-EFF FOR REFERENCE STATE = .100233E+01

SYSTEM MATRIX (2*14,2*14)

```
.1015E+01 .2308E-03 .1144E-02-.2396E-03 .4936E-02 .1356E-01-.4475E-01 .9977E+[illegible] [illegible]E-03-.1[illegible]93E-02 .2266E-03-.4[illegible]57E-
-.2486E-01 .4109E-010.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.
.2293E-03-.1016E+01-.1604E-03-.2088E-03-.2470E-02-.1845E-01-.2347E-02-.2254E-03 .9977E+00 .1534E-02 .1551E-03 .2[illegible]E-
.1696E-01 .2156E-020.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.
.1133E-02-.1457E-03-.1044E+01 .2124E-03 .2474E-02-.1504E-02 .2242E-02-.1114E-02 .1431E-03 .9977E+00-.2024E-03-.2744E-
-.1354E-02-.2059E-020.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.
.2064E-03-.2042E-03 .1582E-03-.1047E+01 .4855E-03 .5245E-02 .2483E-02 .2328E-03 .20[illegible]E-03-.1895E-03 .9977E+00-.3747E-
.4027E-02-.2280E-020.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.
.5021E-02-.2699E-03 .2505E-02 .4166E-03-.1161E+01-.1572E-02-.3257E-04-.4934E-02 .26[illegible]E-03-.2294E-02-.3514E-03 .9977E+
.1446E-02 .2808E-040.   0.   0.   0.   0.   0.   0.   0.   0.   0.
 0.   0.   0.
.1318E-01-.1791E-01-.1469E-02 .5075E-02-.161[illegible]E-02-.1084E+01-.9724E-02-.1295E-01 .1759E-01 .1405E-02-.4838E-02 .1495E-
.9977E+00 .6930E-020.   0.   0.   0.   0.   0.   0.   0.   0.   0.
[illegible]
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ORDER-REDUCED SYSTEM MATRIX (14,14)

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.294000E-04 .668094E-08 .322170E-07-.673673E-08 .134299E-06 .367696E-06-.121093E-05-.391146E-03 .103713E-06 .162428E

.231436E-06 .204797E-05 .449843E-05-.125764E-04
.664156E-06-.294000E-04-.451906E-08-.586662E-08-.672020E-08-.500476E-06-.635259E-07 .174361E-06-.381457E-03-.101964E

.460424E-06-.120636E-06-.534639E-05-.124176E-05
.328237E-07-.421628E-08-.294000E-04 .596572E-08 .673056E-07-.407786E-07 .606782E-07 .989744E-06-.820966E-07-.142910E
.588623E-07 .581595E-06-.234051E-06 .351508E-06

.597630E-08-.591050E-08 .558318E-08-.294000E-04 .110430E-07 .142255E-06 .671996E-07-.914600E-07-.246732E-06 .552955E

.135341E-03 .705869E-07 .126277E-05 .551796E-06
.145407E-06-.781308E-08 .705583E-07 .115352E-07-.294000E-04-.426260E-07-.827377E-08 .226100E-05-.121974E-06 .570428E
.767920E-07-.824671E-04-.575464E-08-.286304E-06
.361712E-06-.516449E-06-.413890E-07 .142562E-06-.440556E-07-.294000E-04-.263146E-06 .575456E-05-.751657E-05 .278530E
.113651E-05-.259722E-07-.748153E-04-.652191E-06

.125610E-05-.676214E-07 .595302E-07 .681797E-07 .206531E-08-.264127E-06-.294000E-04-.184741E-04-.110765E-05 .427767E
.516390E-06-.231464E-06-.640551E-06-.731448E-04
.294000E-04-.668094E-08-.322170E-07 .673673E-08-.134299E-06-.367696E-06 .121053E-05 .922979E-04-.102760E-06-.209066E
.920483E-07 .369654E-06 .121680E-05-.494062E-05

.664156E-06 .294000E-04 .451906E-08 .586662E-08 .672020E-08 .500476E-06 .635259E-07-.770432E-07 .643129E-04 .314406E
.361674E-07 .242505E-07-.188725E-05-.816668E-07

.328237E-07 .421628E-08 .294000E-04-.596572E-08-.673056E-07 .407786E-07-.606782E-07 .155079E-06 .184136E-07-.122955E

.139564E-08 .668713E-06-.311957E-06 .578691E-06
.597630E-08 .591050E-08-.558318E-08 .294000E-04-.110430E-07-.142255E-06-.671996E-07 .550354E-07-.173020E-06-.132465E

.129935E-03 .240579E-07 .160393E-05 .743308E-06

.145407E-06 .781308E-08-.705563E-07-.115352E-07 .294000E-04 .426260E-07 .827377E-08 .163232E-06 .152276E-07 .514160E
.191096E-07-.184463E-03 .959571E-07-.682558E-06

.361712E-06 .516449E-06 .413890E-07-.142562E-06 .440556E-07 .294000E-04 .263146E-06 .435158E-07 .592136E-07-.280305E
.171170E-05 .995202E-07-.175907E-03-.179260E-05
.125610E-05 .676214E-07-.595302E-07-.681757E-07-.206531E-08 .264127E-06 .294000E-04 .429140E-06-.237191E-06 .510345E
.777754E-06-.704838E-06-.161162E-05-.177201E-03
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STABILITY INFORMATION

EIGENVALUES IN THE FORM OF REAL(LAMDA)+IMAG(LAMDA)

REAL PART	IMAG PART	PERIOD(HR)
.31267E-04	.88235E-04	.19780E+02
.31267E-04	-.88235E-04	.19780E+02
.27428E-04	.89338E-04	.19536E+02
.27428E-04	-.89338E-04	.19536E+02
-.15777E-03	0.	0.
-.16167E-03	0.	0.
-.16691E-03	0.	0.
-.75742E-04	.45313E-04	.38517E+02
-.75742E-04	-.45313E-04	.38517E+02
-.79652E-04	.38105E-04	.45804E+02
-.79652E-04	-.38105E-04	.45804E+02
-.45966E-04	0.	0.
-.46527E-04	0.	0.
-.47059E-04	0.	0.

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EIGENVECTORS(NORMALIZED BY THE NORM OF SET)
FOR COMPLEX VECTORS - AT (I)-TH LINE = REAL; AT (I+1)-TH
LINE = IMAG
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.010E+00-.624066E-02-.183561E-02 .238036E-03-.431856E-02-.112512E-01 .357704E-01-.262379E+00 .131977E-02-.531066E-03

.4915E-04-.784838E-03-.160991E-02 .456264E-02

.7276E+00 .161236E-02 .541462E-03-.152652E-03 .159645E-02 .548838E-02-.174443E-01-.293822E-01 .111313E-02 .502283E-03

2701E-04 .109347E-02 .285773E-02-.885516E-02

.7137E-02-.960262E+00-.186095E-03-.331820E-03-.240978E-03-.158362E-01-.238029E-02-.849727E-04 .161416E+00-.221711E-04

.9172E-03-.373847E-04-.411160E-02-.872782E-03

.6828E-02 .781370E-01-.600598E-04-.252551E-03-.488332E-04-.548336E-02-.114689E-02 .251764E-02 .213214E+00 .306767E-04

.9641E-04 .352693E-04 .128319E-02-.280551E-04
.6272E-01-.303818E-01-.816436E-02 .166150E-01-.318879E-01-.421790E+00 .259838E+00 .145829E-02-.101467E-02-.496097E-02
.5069E-01-.506076E-01-.727573E+00 .462428E+00
.0012E-01 .267404E-01-.325207E-02-.243112E-01 .548306E-01 .268436E+00 .405667E+00 .282181E-02-.605176E-05-.225933E-02

.141E-01-.904637E-01 .466961E+00 .726768E+00

.2274E-03 .141629E-02-.129619E-01-.202563E-02 .510737E+00 .402335E-02 .580591E-01-.843458E-03-.204653E-03-.848562E-02

.5750E-02 .651008E+00 .576046E-02 .106137E+00
.5741E-02-.549624E-04-.303123E+00-.850287E-03-.317737E-02 .101333E-02-.155756E-02-.998712E-03 .633164E-04-.370609E+00
.2654E-03-.334079E-02 .846558E-03-.160729E-02
.3913E-02 .853582E-06-.856742E+00 .866677E-02 .233618E-02-.213519E-02 .256641E-02-.102578E-02 .785277E-04-.182349E+00
.2696E-02 .103607E-02-.851795E-03 .407697E-02

.0258E-03-.592496E-03 .294142E-02 .843661E+00-.743092E-03-.812990E-02-.377626E-02 .287159E-04-.191611E-04 .405674E-03

.6499E+00-.157989E-03-.623821E-02-.270449E-02
.1654E-03-.174429E-02-.225688E-02-.836565E+00 .715557E-03 .342718E-03 .474535E-03 .204201E-03 .287400E-03-.177596E-02

.8260E+00 .268265E-03-.342026E-02-.141336E-02
.7203E-01-.424425E-02 .598110E-03 .241540E-04 .328521E-01-.409544E+00 .684130E+00-.896824E-02 .668155E-03 .116466E-02

.9375E-04 .574857E-02-.933620E-01 .198078E+00

.7699E-02-.135820E-01 .615157E-03-.221921E-02 .135492E+00-.871572E+00-.416202E+00 .111584E-02 .366141E-02 .425534E-03

.6650E-02-.289391E-01-.197625E+00-.929085E-01

.6729E-02 .216546E-02-.944288E-03 .146565E-02 .567044E+00 .142496E+00 .231500E-01 .150658E-02-.649151E-03 .111625E-02
.4132E-03 .206579E+00 .330291E-01 .368917E-02
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TRANSITION MATRICES FOR THE MODE = 14

IN THE FORM OF AFR*EXP(-4.70591E-05)T

MATRIX AFR (14,14)

-.15345E-04	.84486E-05	-.17816E-04	-.16961E-04	-.60252E-02	-.86182E-03	-.16279E-03	.64346E-04	-.32274E-04	.32521E-0
.11596E-04	.36175E-02	.50327E-03	.82293E-04						
.64016E-05	-.34684E-05	.74375E-05	.73805E-05	.25153E-02	.35977E-03	.67958E-04	-.26800E-04	.13422E-04	-.13577E-0
-.85331E-05	-.15102E-02	-.21010E-03	-.34355E-04						
-.27912E-05	.15360E-05	-.32434E-05	-.33875E-05	-.10966E-02	-.15688E-03	-.29634E-04	.11687E-04	-.56745E-05	.59215E-0
.36517E-05	.65853E-03	.91615E-04	.14981E-04						
.43334E-05	-.23875E-05	.50345E-05	.47927E-05	.17026E-02	.24354E-03	.46002E-04	-.18140E-04	.91188E-05	-.91902E-0
-.55793E-05	-.10223E-02	-.14222E-03	-.23255E-04						
.26588E-02	-.15751E-02	.32214E-02	.31619E-02	.11233E+01	.16066E+00	.30348E-01	-.11967E+01	.60166E-02	-.60629E-0
-.39445E-02	-.67440E+00	-.93822E-01	-.15342E-01						
.42125E-03	-.23209E-03	.46941E-03	.46592E-03	.16551E+00	.23674E-01	.44719E-02	-.17634E-02	.88647E-03	-.89338E-0
-.55123E-03	-.99374E-01	-.13825E-01	-.22606E-02						
.66436E-04	-.37706E-04	.75510E-04	.75654E-04	.26890E+01	.39461E-02	.72651E-03	-.28646E-03	.14402E-03	-.14514E-0
-.94428E-04	-.16144E-01	-.22460E-02	-.36726E-03						
.44307E-05	-.24471E-05	.51607E-05	.49130E-05	.17453E-02	.24964E-03	.47154E-04	-.18652E-04	.93484E-05	-.94204E-0
-.61250E-05	-.10479E-02	-.14576E-03	-.23836E-04						
-.19155E-05	.10608E-05	-.22302E-05	-.21232E-05	-.75425E-03	-.10788E-03	-.20378E-04	.80765E-05	-.39881E-05	.40711E-0
.26467E-05	.45285E-03	.62995E-04	.10302E-04						
.32596E-05	-.18181E-05	.38336E-05	.36498E-05	.12966E-02	.18545E-03	.35031E-04	-.13814E-04	.69442E-05	-.69976E-0
-.45531E-05	-.77846E-03	-.10830E-03	-.17709E-04						
.25546E-05	-.14074E-05	.29675E-05	.28255E-05	.10037E-02	.14357E-03	.27119E-04	-.10694E-04	.53758E-05	-.54177E-0
-.35251E-05	-.60263E-03	-.83834E-04	-.13709E-04						
.61186E-03	-.33712E-03	.71088E-03	.67676E-03	.24041E+00	.34387E-01	.64955E-02	-.25614E-02	.12876E-02	-.12977E-0
-.84426E-03	-.14434E+00	-.20081E-01	-.32836E-02						
.57645E-04	-.53603E-04	.11344E-03	.10800E-03	.38365E-01	.54875E-02	.10365E-02	-.40873E-03	.20547E-03	-.20708E-0
-.13472E-03	-.23034E-01	-.32045E-02	-.52359E-03						
.15066E-04	-.63347E-05	.13356E-04	.12716E-04	.45174E-02	.64615E-03	.12205E-03	-.48116E-04	.24195E-04	-.24383E-0
-.15864E-04	-.27122E-02	-.37733E-03	-.61701E-04						

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TRANSITION MATRICES FOR THE MODE = 13

IN THE FORM OF AFR*EXP(-4.65274E-05)T

MATRIX AFR (14,14)

.22260E-04	-.59103E-04	-.10809E-04	.71961E-04	-.10321E-02	.65340E-02	.31416E-02	.89879E-04	.22665E-03	.15170E-0
-.14228E-03	.61682E-03	-.37707E-02	-.16855E-02						
-.51206E-04	-.12597E-03	-.23751E-04	.15814E-03	-.22687E-02	.14362E-01	.66962E-02	.19791E-03	.49624E-03	.33345E-0
-.22463E-03	.13558E-02	-.82884E-02	-.37349E-02						
.23169E-05	.58941E-05	.10761E-05	-.71642E-05	.10275E-03	-.65050E-03	-.30283E-03	-.89646E-05	-.22563E-04	-.15121E-0
.16163E-04	-.61408E-04	.37540E-03	.16762E-03						
.83666E-05	-.21227E-04	-.38807E-05	.25845E-04	-.37068E-03	.23467E-02	.10925E-02	.32337E-04	.81395E-04	.54482E-0
-.36735E-04	.22153E-03	-.13543E-02	-.60535E-03						
.51062E-03	.12960E-02	.23693E-03	-.15779E-02	.22632E-01	-.14328E+00	-.66700E-01	-.19743E-02	-.49697E-02	-.33264E-0
.22426E-02	-.13525E-01	.82664E-01	.36959E-01						
-.32859E-02	-.83368E-02	-.15241E-02	.10150E-01	-.14558E+00	.92165E+00	.42906E+00	.12700E-01	.31965E-01	.21397E-0
.14427E-01	.87005E-01	-.53188E+00	-.23775E+00						
.15691E-02	-.39811E-02	-.72761E-03	.48471E-02	-.69519E-01	.44012E+00	.20489E+00	.60647E-02	.15266E-01	.10218E-0
.68895E-02	.41547E-01	-.25399E+00	-.11353E+00						
.42225E-05	.10673E-04	.19512E-05	-.12995E-04	.18638E-03	-.11800E-02	-.54931E-03	-.16175E-04	-.40930E-04	-.27395E-0
.18471E-04	-.11139E-03	.68094E-03	.30438E-03						
.13629E-04	.35081E-04	.64144E-05	-.42720E-04	.61270E-03	-.39789E-02	-.18058E-02	-.53451E-04	-.13462E-03	-.90055E-0
.60720E-04	-.36617E-03	.22385E-02	.10006E-02						
.16045E-05	.40703E-05	.74451E-06	-.49556E-05	.71078E-04	-.44996E-03	-.20948E-03	-.61958E-05	-.15608E-04	-.10456E-0
.70440E-05	-.42479E-04	.25968E-03	.11608E-03						
-.13597E-04	-.34497E-04	-.63067E-05	.42002E-04	-.60241E-03	.38137E-02	.17754E-02	.52552E-04	.13228E-03	.88541E-0
.59699E-04	.36002E-03	-.22009E-02	-.98378E-03						
.10910E-03	.27681E-03	.50600E-04	-.33703E-03	.48338E-02	-.30602E-01	-.14246E-01	-.42169E-03	-.10615E-02	-.71047E-0
.47904E-03	-.28688E-02	.17660E-01	.78942E-02						
.74506E-03	-.18903E-02	-.34558E-03	.23016E-02	-.33018E-01	.20898E+00	.97288E-01	.28797E-02	.72487E-02	.48518E-0
.32714E-02	.19728E-01	-.12061E+00	-.53908E-01						
.25626E-03	-.88869E-03	-.16247E-03	.10826E-02	-.15519E-01	.96247E-01	.45737E-01	.13535E-02	.34076E-02	.22809E-0
.15380E-02	.92746E-02	-.56657E-01	-.25343E-01						

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TRANSITION MATRICES FOR THE MODE = 12

IN THE FORM OF AFR*EXP(-4.54655E-05)T

MATRIX AFR (14,14)

-.52942E-03	.11627E-03	.16326E-03	.13270E-04
-.65232E-05	-.98890E-03	.11012E-01	-.22343E-01
.10455E-03	-.13287E-04	-.20618E-04	-.14925E-05
.55676E-06	.11125E-03	-.12386E-02	.25135E-02
-.14734E-04	.18749E-05	.29054E-05	.21038E-06
-.13512E-06	-.15677E-04	.17457E-03	-.35421E-03
-.55503E-06	.75731E-07	.11734E-06	.84235E-08
-.55275E-06	-.63310E-06	.70498E-05	-.14304E-04
-.60929E-03	.10298E-03	.15959E-03	.11555E-04
-.74217E-05	-.86109E-03	.95886E-02	-.19455E-01
.10065E-01	-.12838E-02	-.15895E-02	-.14405E-03
.92521E-04	.10735E-01	.11953E+00	.24254E+00
-.21766E-01	.27715E-02	.42556E-02	.31098E-03
-.19974E-03	-.23174E-01	.25806E+00	-.52359E+00
.22952E-03	-.26113E-04	-.43567E-04	-.31545E-05
.20260E-05	.23507E-03	-.26176E-02	.53110E-02
-.21404E-04	.27253E-05	.42205E-05	.33564E-06
-.19629E-06	-.22774E-04	.25360E-03	-.51456E-03
-.29164E-04	.37137E-05	.57545E-05	.41670E-06
-.26763E-06	-.31052E-04	.34577E-03	-.70156E-03
.10578E-05	-.13464E-06	-.20856E-06	-.15082E-07
.95632E-06	.11254E-05	-.12532E-04	.25428E-04
-.14161E-03	.18021E-04	.27926E-04	.20220E-05
-.12987E-05	-.15068E-03	.16776E-02	-.34044E-02
.22999E-02	-.29267E-03	-.45354E-03	-.32839E-04
.21092E-04	.24471E-02	-.27250E-01	.55290E-01
-.40756E-02	.62093E-03	.96224E-03	.69672E-04
-.44748E-04	-.51916E-02	.57813E-01	-.11730E+00

.17551E-02	-.18997E-01	.49066E-01	.35455E-02	-.44847E-03	-.27120E-
-.19744E-03	.21371E-02	-.45065E-02	-.39929E-03	.50432E-04	.30506E-
.27824E-04	-.30117E-03	.63567E-03	.56264E-04	-.71097E-05	-.42987E-
.11236E-05	-.12162E-04	.25646E-04	.22724E-05	-.28701E-06	-.17362E-
.15283E-02	-.16542E-01	.34882E-01	.30907E-02	-.39051E-03	-.23615E-
-.19052E-01	.20622E+00	-.43485E+00	-.38529E-01	.48682E-02	.29439E-
.41130E-01	-.44519E+00	.93876E+00	.83176E-01	-.10510E-01	-.63553E-
-.41721E-03	.45158E-02	-.95224E-02	-.84375E-03	.10661E-03	.64465E-
.40421E-04	-.43751E-03	.92257E-03	.81744E-04	-.10305E-04	-.62457E-
.55112E-04	-.59652E-03	.12579E-02	.11145E-03	-.14082E-04	-.85154E-
-.19975E-05	.21620E-04	-.45591E-04	-.40394E-05	.51039E-06	.30864E-
.26743E-03	-.28946E-02	.61038E-02	.54081E-03	-.68333E-04	-.41322E-
-.43432E-02	.47011E-01	-.99131E-01	-.87833E-02	.11098E-02	.67110E-
.92147E-02	-.99736E-01	.21032E+00	.18635E-01	-.23545E-02	-.14238E-

TRANSITION MATRICES FOR THE MODE = 11

IN THE FORM OF EXP(-7.96517E-05*T)*(AFR*COS(3.81047E-05)T +/- I*AFI*SIN(3.81047E-05)T)

MATRIX AFR (14,14)

.25736E-14	.43307E-14	.14006E-11	.98552E-07
.17952E-06	.10882E-12	.95916E-12	.46430E-13
.12109E-13	.50493E-12	.80556E-11	.12871E-07
.56120E-05	.19270E-12	.31635E-06	.62757E-09
.26735E-12	.23221E-12	.15807E-09	.97621E-05
.13353E-04	.11669E-10	.57033E-09	.70571E-10
.14316E-07	.10621E-07	.62584E-05	.25531E+00
.16172E-06	.46154E-06	.35156E-03	.63618E-04
.22451E-13	.65684E-14	.12471E-10	.67606E-06
.66455E-06	.87694E-12	.10141E-09	.15186E-10
.62486E-12	.88608E-11	.91406E-09	.52601E-04
.20961E-03	.41217E-10	.95416E-08	.23451E-08
.20067E-12	.16762E-11	.10449E-05	.11853E-04
.42128E-04	.98828E-11	.13407E-08	.34811E-09
.29590E-15	.46004E-14	.16482E-12	.52973E-09
.41463E-07	.68245E-14	.36586E-10	.71276E-11
.16365E-14	.10175E-13	.98517E-12	.11625E-07
.64022E-07	.45510E-13	.11035E-09	.21065E-10
.46026E-13	.11711E-12	.28217E-10	.59996E-06
.34069E-06	.14930E-11	.15326E-08	.36620E-09
.62635E-09	.23993E-07	.54284E-06	.32117E-08
.24975E+00	.15650E-07	.16255E-03	.32121E-04
.16610E-14	.22936E-15	.10702E-11	.40933E-07
.75832E-08	.67003E-13	.28750E-10	.50236E-11
.10444E-12	.10968E-10	.43786E-10	.22098E-04
.19166E-03	.92314E-11	.31406E-07	.67179E-08
.21607E-13	.20061E-11	.92307E-11	.42201E-05
.25407E-04	.16301E-11	.56106E-06	.12033E-08

.81525E-13	.28680E-12	.84123E-13	.67239E-13	.57520E-13	.66209E-
.62950E-12	.42753E-09	.83214E-10	.11531E-11	.67966E-14	.63159E-
.93646E-11	.38690E-13	.39171E-12	.54115E-11	.51844E-11	.62843E-
.52741E-06	.28846E-04	.51984E-05	.12131E-07	.15333E-06	.12227E-
.75050E-12	.17291E-11	.19372E-12	.29695E-12	.36631E-12	.41942E-
.21888E-10	.31216E-08	.66905E-09	.65443E-10	.27660E-10	.40295E-
.56419E-11	.52864E-09	.11482E-09	.13460E-10	.62800E-11	.66786E-
.13265E-13	.45165E-11	.66752E-12	.74685E-14	.27972E-15	.47949E-
.67055E-13	.12232E-10	.23134E-11	.80650E-14	.62466E-14	.26142E-
.18527E-11	.19822E-09	.34677E-10	.83916E-14	.32061E-12	.24011E-
.40893E-07	.21342E-04	.41356E-05	.50258E-07	.39865E-12	.19415E-
.66897E-13	.18755E-11	.32255E-12	.69809E-14	.21791E-13	.22240E-
.15087E-11	.55969E-08	.12160E-08	.52481E-10	.11668E-10	.21324E-
.33297E-12	.10838E-08	.22042E-09	.97415E-11	.22287E-11	.38466E-

MATRIX AFI (14,14)

.12720E-15	.23918E-13	.10052E-12	.77319E-08
.21146E-06	.34142E-15	.12000E-09	.24255E-10
.59502E-13	.24392E-12	.31738E-10	.28151E-05
.74031E-05	.27009E-11	.42873E-10	.16623E-10
.27073E-14	.28009E-11	.31266E-11	.16923E-05
.39363E-04	.50963E-13	.12417E-07	.25383E-06
.30178E-06	.17044E-06	.13745E-05	.43942E+00
.31504E+01	.21687E-06	.42456E-03	.92311E-04
.52533E-17	.22824E-12	.95838E-14	.22678E-06
.34165E-05	.28466E-13	.90405E-09	.18683E-05
.59241E-12	.57497E-11	.36242E-09	.25764E-05
.44332E-04	.15240E-10	.53019E-07	.13224E-07
.16499E-12	.15202E-11	.65381E-10	.21643E-06
.13426E-04	.24683E-11	.12346E-07	.24016E-08
.60113E-15	.47826E-14	.31386E-12	.34926E-07
.12160E-06	.29433E-13	.35916E-11	.94429E-12
.15397E-14	.23036E-13	.77999E-12	.11384E-06
.51323E-06	.82830E-13	.31824E-10	.75071E-11
.42001E-13	.61516E-12	.10702E-10	.21655E-05
.12367E-04	.13367E-11	.12025E-08	.20975E-09
.29263E-06	.15374E-07	.15454E-05	.14667E+00
.43446E+00	.13627E-06	.56155E-05	.17451E-05
.16426E-15	.21158E-13	.66513E-13	.39539E-07
.36415E-06	.15643E-13	.62790E-10	.13376E-10
.56359E-12	.20070E-12	.54979E-09	.20079E-04
.24507E-05	.33925E-10	.16414E-07	.28898E-08
.17466E-12	.48607E-13	.99973E-10	.35397E-05
.30563E-06	.61098E-11	.31875E-08	.56430E-09

.96072E-14	.17752E-10	.34966E-11	.72679E-13	.40379E-14	.12261E-11
.17862E-11	.50647E-10	.11713E-10	.25266E-11	.14927E-11	.19969E-09
.42011E-12	.19368E-08	.38406E-09	.96141E-11	.88815E-12	.21408E-09
.57325E-07	.86898E-04	.17786E-04	.68583E-06	.22957E-06	.38122E-04
.70566E-14	.14923E-09	.29575E-10	.86441E-12	.10984E-12	.23287E-10
.25246E-10	.62082E-08	.11835E-08	.69364E-11	.13919E-11	.28579E-10
.46695E-11	.15124E-08	.29033E-09	.23748E-11	.11941E-12	.11494E-14
.17005E-13	.14753E-11	.32142E-12	.39032E-13	.16506E-13	.26559E-11
.40050E-13	.89766E-11	.18965E-11	.15647E-12	.60235E-13	.89443E-11
.50964E-12	.27939E-09	.57943E-10	.36194E-11	.11455E-11	.17543E-09
.86106E-07	.37968E-05	.65435E-06	.14477E-06	.78817E-07	.10741E-04
.21004E-14	.11782E-10	.23858E-11	.98976E-13	.21030E-13	.37170E-11
.34521E-10	.11349E-08	.19759E-09	.28479E-11	.10692E-10	.10699E-08
.62957E-11	.22832E-09	.40026E-10	.43764E-12	.16852E-11	.18608E-09

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TRANSITION MATRICES FOR THE MODE = 5

IN THE FORM OF EXP(-7.57421E-05*T)*(AFR*COS(4.53133E-05)T +/- I*AFI*SIN(4.53133E-05)T)

MATRIX AFR (14,14)

.72125E-11	.34415E-13	.88265E-08	.41624E-10
.13417E-10	.19603E-08	.30618E-09	.94140E-09
.55730E-14	.73621E-17	.11236E-08	.10195E-14
.16066E-12	.15691E-11	.19676E-12	.69328E-12
.15797E-06	.43475E-08	.25002E+00	.97643E-05
.13453E-04	.60019E-04	.13697E-04	.33136E-04
.22518E-13	.46021E-13	.82860E-05	.15765E-09
.63245E-09	.22157E-10	.32682E-10	.32557E-10
.42225E-11	.39587E-14	.75454E-05	.45403E-10
.75625E-09	.10157E-08	.76054E-10	.38258E-09
.11251E-12	.12662E-13	.34442E-05	.50696E-10
.20605E-09	.16932E-10	.62120E-12	.15322E-11
.46273E-12	.15718E-13	.55146E-05	.71110E-10
.47763E-05	.89085E-10	.25510E-12	.20037E-10
.11042E-11	.94580E-14	.14155E-06	.15721E-10
.24404E-11	.31239E-09	.54886E-10	.15567E-09
.74044E-14	.56483E-16	.65936E-09	.93740E-13
.79966E-14	.20850E-11	.35932E-12	.10315E-11
.11056E-06	.52497E+09	.12205E-09	.60243E-06
.33161E-06	.32452E-04	.49447E-05	.15346E-04
.51928E-12	.13781E-13	.54414E-06	.27599E-10
.24061E-10	.27000E-09	.54947E-10	.14237E-09
.61871E-11	.80095E-15	.35654E-05	.57932E-11
.43354E-09	.15753E-08	.16735E-09	.65914E-09
.24473E-12	.10254E-14	.79868E-06	.71695E-11
.75519E-10	.56149E-10	.29415E-11	.19239E-10
.11312E-11	.61803E-15	.17321E-05	.19163E-10
.17934E-09	.27482E-09	.21990E-10	.10550E-09

.19763E-09	.49231E-10	.11112E-09	.80196E-11	.19934E-13	.14272E-0
.23770E-12	.34796E-13	.10510E-12	.56285E-14	.10370E-15	.24188E-0
.14869E-05	.17653E-05	.21127E-05	.65658E-05	.62903E-07	.57008E-0
.33596E-10	.20950E-11	.13151E-11	.15800E-09	.76191E-11	.12261E-0
.29636E-09	.18299E-10	.95467E-10	.10031E-09	.29030E-11	.42689E-0
.36408E-10	.32756E-13	.59826E-11	.59529E-10	.11549E-11	.83231E-0
.60904E-10	.78736E-12	.16947E-10	.90861E-10	.16999E-11	.16177E-0
.22307E-10	.80776E-11	.15361E-10	.66296E-11	.18739E-13	.10606E-0
.15741E-12	.53422E-13	.10471E-12	.36862E-13	.76485E-16	.82300E-0
.33493E-05	.78509E-06	.18466E-05	.96790E-07	.62076E-09	.24595E+0
.11601E-10	.74975E-11	.11140E-10	.16425E-10	.12118E-12	.19266E-0
.30783E-09	.32563E-10	.12007E-09	.34946E-10	.15593E-11	.36617E-0
.21869E-10	.89207E-12	.62134E-11	.11659E-10	.79335E-12	.35162E-0
.75124E-10	.50762E-11	.24931E-10	.22332E-10	.67545E-12	.10528E-0

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MATRIX AFI (14,14)

.35769E-12	.21101E-13	.64874E-05	.89502E-10
.55106E-09	.62658E-10	.12486E-12	.10676E-10
.50850E-15	.40358E-16	.44381E-08	.11125E-12
.30262E-12	.16023E-12	.65485E-13	.12185E-12
.39014E-06	.23300E-10	.26142E+00	.55925E-06
.30989E-04	.10064E-03	.10416E-04	.41622E-04
.10160E-10	.26650E-13	.45162E-06	.18768E-10
.12621E-05	.27260E-08	.37911E-09	.12480E-08
.65551E-11	.78011E-13	.20422E-05	.14421E-05
.67217E-10	.20010E-08	.37620E-09	.10232E-08
.35019E-11	.16778E-13	.68514E-09	.18845E-10
.12495E-10	.10666E-08	.16161E-09	.53334E-09
.60143E-11	.31781E-13	.43185E-07	.41075E-10
.51445E-11	.16590E-08	.26181E-09	.79455E-09
.36809E-12	.13389E-14	.11218E-05	.97806E-11
.16734E-09	.84979E-10	.46845E-11	.29452E-10
.20154E-14	.10586E-16	.72839E-08	.65421E-13
.69440E-12	.45733E-12	.21759E-13	.15315E-12
.40316E-08	.37416E-09	.10521E+00	.15213E-05
.88108E-05	.63957E-06	.16115E-07	.73835E-07
.97317E-12	.97335E-16	.10795E-05	.47801E-11
.11705E-09	.24077E-05	.21623E-10	.95627E-10
.26055E-11	.63678E-13	.39796E-05	.14650E-09
.22209E-09	.79776E-09	.18840E-09	.44665E-09
.79177E-12	.65756E-14	.90736E-07	.10784E-10
.13681E-11	.22360E-09	.36996E-10	.11111E-09
.15590E-11	.19110E-13	.57640E-06	.36434E-10
.21256E-10	.45124E-05	.68857E-10	.23283E-09

.81250E-10	.36355E-12	[illegible]E-10	.10953E-09	.22049E-11	.17203E-
.10728E-14	.68071E-14	.23720E-14	.94992E-13	.13044E-14	.28451E-
.20470E-04	.20606E-05	.78340E-05	.26871E-05	.11226E-06	.24859E+
.34151E-09	.62277E-10	.16861E-09	.58902E-15	.37337E-12	.30204E-
.11541E-09	.53452E-10	.91073E-10	.73220E-10	.39186E-12	.39303E-
.11148E-09	.25737E-10	.61013E-10	.27932E-11	.24894E-13	.84204E-
.15770E-09	.40750E-10	.91247E-10	.96925E-11	.35837E-14	.10839E-
.31931E-10	.13733E-11	.92098E-11	.16227E-10	.41395E-12	.50001E-
.18960E-12	.70453E-14	.52458E-13	.10938E-12	.26943E-14	.31150E-
.11674E-05	.19518E-08	.19956E-06	.18066E-05	.35412E-07	.26143E+
.58115E-10	.46504E-11	.20443E-10	.12950E-10	.43497E-12	.77004E-
.16074E-10	.23878E-10	.26663E-10	.10155E-09	.10247E-11	.54774E-
.16315E-10	.57613E-11	.11084E-10	.44323E-11	.11256E-13	.81485E-
.23980E-10	.12192E-10	.19965E-10	.19432E-10	.11515E-12	.69197E-

TRANSITION MATRICES FOR THE MODE = 7

IN THE FORM OF AFR*EXP(-1.66911E-04)T

MATRIX AFR (14,14)

-.94897E-07	.79169E-07	-.24176E-05	-.34560E-06
.67711E-06	-.86623E-03	-.62433E-05	-.10279E-03
.20600E-06	-.17190E-06	.52497E-05	.75041E-06
-.19045E-05	.18808E-02	.13556E-04	.22319E-03
-.10658E-05	.15732E-05	-.48046E-04	-.69674E-05
.74300E-04	-.17214E-01	-.12407E-03	-.20427E-02
-.29473E-06	.24568E-06	-.75055E-05	-.10734E-05
.27241E-05	-.26903E-02	-.19391E-04	-.31925E-03
.74305E-04	-.61990E-04	.18931E-02	.27061E-03
-.60678E-03	.67626E+00	.46886E-02	.80487E-01
.50534E-06	-.46833E-06	.14913E-04	.21317E-05
-.54102E-05	.53430E-02	.38511E-04	.63404E-03
.04467E-05	-.70469E-05	.21521E-03	.30762E-04
-.76072E-04	.77103E-01	.55572E-03	.91495E-02
.12271E-06	-.10237E-06	.31264E-05	.44690E-06
-.11342E-05	.11201E-02	.80733E-05	.13292E-03
-.29774E-07	.24839E-07	-.75856E-06	-.10843E-06
.27519E-06	-.27178E-03	-.19565E-05	-.32251E-04
-.12345E-05	.10299E-05	-.31452E-04	-.44960E-05
.11411E-04	-.11269E-01	-.81221E-04	-.13372E-02
-.15365E-06	.12835E-06	-.39158E-05	-.56030E-06
.14220E-05	-.14044E-02	-.10122E-04	-.16665E-03
.12361E-03	-.10329E-03	.31544E-02	.45090E-03
-.11443E-02	.11301E+01	.81458E-02	.13411E+00
.63066E-06	-.69917E-06	.21352E-04	.30521E-05
-.77461E-05	.76500E-02	.55137E-04	.90779E-03
.54416E-04	-.12882E-04	.39342E-03	.50236E-04
-.14272E-03	.14095E+00	.10155E-02	.16726E-01

.16521E-03	.18991E-05	.21959E-04	.48162E-06	.33805E-06	.85430E-(
-.40214E-03	-.41235E-05	-.47767E-04	-.10455E-05	-.73400E-06	-.18551E-
.36804E-02	.37738E-04	.43717E-03	.95711E-05	.67177E-05	.16977E-
.57522E-03	.58981E-05	.68325E-04	.14959E-05	.10499E-05	.26532E-
-.14502E+00	-.14870E-02	-.17226E-01	-.37712E-03	-.26470E-03	-.66893E-
-.11424E-02	-.11714E-04	-.13565E-03	-.29718E-05	-.20851E-05	-.52695E-
-.16485E-01	-.16904E-03	-.19582E-02	-.42870E-04	-.30090E-04	-.76042E-
-.23945E-03	-.24557E-05	-.28447E-04	-.62281E-06	-.43713E-06	-.11047E-
.58109E-04	.59584E-06	.69023E-05	.15111E-06	.10606E-06	.26804E-
.24094E-02	.24706E-04	.28619E-03	.62657E-05	.43978E-05	.11114E-
.30027E-03	.30789E-05	.35666E-04	.78085E-06	.54806E-06	.13850E-
-.24164E+00	-.24777E-02	-.28702E-01	-.62836E-03	-.44104E-03	-.11146E-
-.16356E-02	-.16771E-04	-.19426E-03	-.42535E-05	-.29654E-05	-.75447E-
-.30137E-01	-.30901E-03	-.35797E-02	-.78371E-04	-.55007E-04	-.13901E-

TRANSITION MATRICES FOR THE MODE = 6

IN THE FORM OF AFR*EXP(-1.61667E-04)T

MATRIX AFR (14,14)

.3509E-03	-.14441E-03	.4731E-04	.47353E-03	.20845E-02	-.10630E-01	-.16240E-01	-.15960E-02	-.56812E-03	-.14737E-03
.6495E-02	-.94650E-02	.47025E-01	.72555E-01						
.3576E-03	-.56507E-04	.15166E-04	.19185E-03	.84454E-03	-.43069E-02	-.65795E-02	-.64670E-03	-.23017E-03	-.57685E-04
.6005E-03	-.36347E-02	.15055E-01	.29411E-01						
.6511E-04	.71154E-05	-.23311E-05	-.22332E-04	-.10271E-03	.52379E-03	.80017E-03	.78649E-04	.27993E-04	.70148E-05
.2260E-04	.46637E-03	-.23172E-02	-.35766E-02						
.2343E-03	.53192E-04	-.17426E-04	-.17442E-03	-.76782E-03	.39156E-02	.59816E-02	.58795E-03	.20926E-03	.52440E-04
.7406E-03	.34664E-02	-.17321E-01	-.26735E-01						
.7036E-03	.11997E-03	-.39303E-04	-.39335E-03	-.17317E-02	.88312E-02	.13491E-01	.13260E-02	.47196E-03	.11827E-03
.3699E-02	.78630E-02	-.35061E-01	-.60306E-01						
.3629E-02	-.56732E-03	.19241E-03	.19259E-02	.84780E-02	-.43235E-01	-.66049E-01	-.64919E-02	-.23106E-02	-.57902E-03
.7066E-02	-.38495E-01	.15126E+00	.29524E+00						
.5596E-02	-.88758E-03	.25076E-03	.25105E-02	.12812E-01	-.65338E-01	-.99815E-01	-.98108E-02	-.34919E-02	-.87503E-03
.1355E-01	-.58175E-01	.26903E+00	.44616E+00						
.4327E-04	-.61740E-05	.20227E-05	.20245E-04	.89121E-04	-.45449E-03	-.69431E-03	-.68244E-04	-.24289E-04	-.60867E-05
.6501E-04	-.40467E-03	.20105E-02	.31036E-02						
.7466E-07	.13252E-07	-.43401E-08	-.43440E-07	-.19126E-06	.97536E-06	.14900E-05	.14645E-06	.52137E-07	.13062E-07
.5130E-06	.86843E-06	-.43146E-05	-.66605E-05						
.1471E-04	.49433E-05	-.16155E-05	-.16210E-04	-.71356E-04	.36389E-03	.55591E-03	.54640E-04	.19448E-04	.48734E-05
.6447E-04	.32400E-03	-.16097E-02	-.24849E-02						
.1305E-04	.35036E-04	-.11475E-04	-.11485E-03	-.50577E-03	.25793E-02	.39403E-02	.38729E-03	.13784E-03	.34543E-04
.0101E-03	.22965E-02	-.11415E-01	-.17613E-01						
.5929E-03	.15793E-03	-.64844E-04	-.64904E-03	-.28571E-02	.14570E-01	.22259E-01	.21878E-02	.77868E-03	.19513E-03
.2602E-02	.12973E-01	-.64454E-01	-.99497E-01						
.3709E-02	-.10217E-02	.33472E-03	.33502E-02	.14748E-01	-.75210E-01	-.11490E+00	-.11293E-01	-.40194E-02	-.10072E-02
.1667E-01	-.66965E-01	.32271E+00	.51355E+00						
.6696E-02	-.15901E-02	.52095E-03	.52143E-02	.22953E-01	-.11706E+00	-.17882E+00	-.17576E-01	-.62558E-02	-.15677E-02
.0156E-01	-.10422E+00	.51781E+00	.79934E+00						

TRANSITION MATRICES FOR THE MODE = 5

IN THE FORM OF AFR*EXP(-1.57769E-04)T

MATRIX AFR (14,14)

-.9469E-03	.19104E-03	.1589[illegible]E-03	-.41377E-03	.15509E-02	.20417E-01	-.12721E-01	-.22730E-02	.55116E-03	-.41867E-03
.3860E-02	-.66105E+02	-.89763E-01	.56604E-01						
.6835E-03	-.72711E-04	-.66477E-04	.16365E-03	-.55025E-03	-.77706E-02	.48415E-02	.86510E-03	-.36201E-03	.15934E-03
.2750E-03	.25159E-02	.34171E-01	-.21543E-01						
.6155E-04	-.15539E-04	-.16252E-04	.41266E-04	-.15862E-03	-.20862E-02	.13010E-02	.23247E-03	-.57262E-04	.42819E-04
.4175E-03	.67609E-03	.91826E-02	-.57852E-02						
.0300E-03	.39764E-04	.33073E-04	-.84025E-04	.32275E-03	.42495E-02	-.26477E-02	-.47310E-03	.19798E-03	-.67140E-04
.0648E-03	-.13759E-02	-.18687E-01	.11781E-01						
.9769E-03	-.76315E-04	-.63475E-04	.16126E-03	-.61951E-03	-.81558E-02	.50815E-02	.90796E-03	-.37996E-03	.16724E-03
.5365E-03	.26406E-02	.35866E+01	-.22611E-01						
.6149E-02	-.10094E-02	-.83967E-03	.21331E-02	-.81945E-02	-.10789E+00	.67214E-01	.12010E-01	-.50256E-02	.22121E-02
.73233E-02	.34928E-01	.47439E+00	-.29908E+00						
.6109E-02	.62185E-03	.51722E-03	-.13140E-02	.50481E-02	.66457E-01	-.41406E-01	-.73987E-02	.30961E-02	-.13626E-02
.6114E-02	-.21517E-01	-.29224E+00	.18425E+00						
.4066E-05	.34900E-05	.29028E-05	-.73748E-05	.28332E-04	.37299E-03	-.23239E-03	-.41524E-04	.17376E-04	-.76482E-05
.5319E-04	-.12076E-03	-.16402E-02	.10340E-02						
.2504E-05	-.24283E-05	-.20198E-05	.51313E-05	-.19713E-04	-.25952E-03	.16169E-03	.28892E-04	-.12090E-04	.53216E-05
.7617E-04	.84025E-04	.11412E-02	-.71946E-03						
.4755E-04	-.11673E-04	-.98751E-05	.25088E-04	-.96381E-04	-.12688E-02	.79055E-03	.14126E-03	-.59112E-04	.26019E-04
.6134E-04	.41082E-03	.55797E-02	-.35177E-02						
.2149E-04	.25150E-04	.20916E-04	-.53145E-04	.20416E-03	.26876E-02	-.16746E-02	-.29923E-03	.12522E-03	-.55115E-04
.0246E-03	-.87024E-03	-.11819E-01	.74516E-02						
.1495E-03	-.12159E-03	-.10114E-03	.25694E-03	-.98708E-03	-.12995E-01	.80964E-02	.14467E-02	-.60540E-03	.26647E-03
.6214E-03	.42074E-02	.57144E-01	-.36027E-01						
.5106E-02	-.17412E-02	-.14483E-02	.36794E-02	-.14135E-01	-.18609E+00	.11594E+00	.20717E-01	-.86694E-02	.38159E-02
.2632E-01	.60250E-01	.81631E+00	-.51591E+00						
.6666E-02	.11067E-02	.92049E-03	-.23386E-02	.89840E-02	.11827E+00	-.73690E-01	-.13167E-01	.55100E-02	-.24253E-02
.2066E-02	-.38294E-01	-.52010E+00	.32790E+00						

TRANSITION MATRICES FOR THE MODE = 3

IN THE FORM OF EXP(2.7427PE-05*T)*(AFR*COS(-8.93376E-05)T +/- I*AFI*SIN(-8.933 E-05)T)

MATRIX AFR (14,14)

.92852E-10	.30023E-05	.78354E-13	.13166E-10	.17759E-12	.42554E-06	.12057E-09	.15610E-07	.15095E-03	.61175E-11
.13066E-09	.68788E-11	.96010E-05	.17174E-09						
.23155E-05	.25015E+00	.52971E-08	.19924E-07	.41103E-08	.54465E-05	.11054E-06	.65661E-04	.20856E-07	.37992E-07
27461E-05	.58484E-07	.47373E-03	.12556E-04						
.70324E-16	.47621E-08	.81060E-17	.38349E-14	.20789E-19	.11985E-11	.31961E-13	.58944E-11	.27596E-07	.31683E-14
.93411E-13	.33065E-14	.57716E-11	.25014E-12						
.47085E-12	.47654E-08	.27884E-15	.31430E-13	.55213E-15	.10206E-10	.29152E-12	.35849E-10	.38139E-06	.18542E-12
.26491E-12	.15153E-13	.59506E-12	.28166E-12						
.44672E-13	.10021E-07	.64313E-16	.44739E-14	.24135E-16	.13698E-11	.35157E-13	.79162E-11	.22854E-07	.43250E-14
.46144E-12	.45519E-14	.14514E-10	.51763E-12						
.45777E-12	.32680E-04	.34803E-13	.29604E-10	.23573E-14	.92817E-08	.24900E-09	.44457E-07	.22378E-03	.23630E-10
.67242E-09	.24463E-10	.38159E-07	.17335E-08						
.17840E-11	.55063E-06	.41081E-15	.93347E-12	.35996E-14	.29713E-09	.81935E-11	.12511E-08	.87700E-05	.66125E-12
.14672E-10	.62259E-12	.46592E-09	.25781E-10						
.92594E-10	.90354E-06	.14326E-12	.13203E-11	.17004E-12	.45642E-09	.14481E-10	.80265E-09	.30771E-04	.37325E-12
.33576E-12	.11985E-12	.37722E-06	.40463E-10						
.76003E-06	.76250E-05	.46872E-09	.16327E-07	.68574E-09	.54101E-05	.16003E-06	.15546E-04	.25005E+00	.78293E-08
.53625E-07	.52791E-06	.32636E-05	.11492E-08						
.21070E-13	.49594E-05	.36034E-16	.10026E-15	.38383E-16	.37157E-13	.13120E-14	.20184E-13	.40694E-08	.67936E-17
.19613E-14	.22550E-17	.14393E-11	.23559E-13						
.37043E-13	.20460E-07	.14764E-15	.83706E-14	.61723E-16	.25537E-11	.65054E-13	.15169E-10	.40014E-07	.83070E-14
.32117E-12	.96488E-14	.31265E-10	.10546E-11						
.33170E-13	.98893E-05	.58653E-16	.87008E-16	.60253E-16	.34082E-13	.13021E-14	.27203E-14	.50765E-08	.12171E-18
.50771E-14	.23583E-16	.26440E-11	.47529E-13						
.10264E-09	.62195E-05	.20561E-12	.28346E-13	.18442E-12	.35707E-11	.34620E-15	.57039E-09	.43948E-05	.36135E-12
.63230E-10	.81255E-12	.13480E-07	.30726E-09						
.10115E-11	.17458E-06	.26340E-14	.30223E-13	.17720E-14	.88073E-11	.20507E-12	.72165E-10	.47857E-07	.40626E-13
22068E-11	.54674E-13	.30246E-09	.88515E-11						

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MATRIX AFI (14,14)

.1640E-08	.26474E-04	.19813E-11	.59318E-11	.21212E-11	.21864E-08	.76578E-10	.13390E-08	.23154E-03	.47163E-12
10022E-09	.77027E-13	.77792E-07	.12553E-08						
.2668E-04	.10126E+00	.19256E-07	.20775E-06	.23295E-07	.71351E-04	.22355E-05	.13640E-03	.45600E+01	.64406E-07
.3697E-06	.24443E-07	.46519E-03	.44551E-05						
.1727E-12	.97164E-08	.10036E-14	.55457E-14	.11291E-14	.19657E-11	.64879E-13	.24308E-11	.16026E-06	.10506E-14
.5994E-13	.11013E-15	.32911E-10	.45062E-12						
.5164E-11	.61074E-07	.43216E-14	.11227E-13	.45826E-14	.41630E-11	.14985E-12	.20092E-11	.47297E-06	.64281E-15
.5060E-12	.36403E-15	.17531E-09	.29056E-11						
.6134E-12	.12869E-07	.15352E-14	.10357E-13	.17610E-14	.36327E-11	.11795E-12	.52445E-11	.77419E-06	.23451E-14
.7657E-13	.44991E-15	.46644E-10	.54057E-12						
45337E-06	.73461E-04	.72955E-11	.39294E-10	.82909E-11	.13958E-07	.46227E-09	.16689E-07	.11561E-02	.71568E-11
.6652E-09	.63196E-12	.24592E-06	.34148E-08						
.1092E-09	.20927E-05	.16425E-12	.77913E-12	.20255E-12	.28054E-09	.94864E-11	.26879E-09	.25531E-04	.10899E-12
.3319E-11	.11903E-14	.65962E-06	.88191E-10						
94322E-11	.14893E-05	.23923E-13	.23003E-12	.16568E-13	.66514E-10	.15241E-11	.57328E-09	.57962E-06	.32400E-12
.6403E-10	.44462E-12	.26206E-06	.75382E-10						
75654E-06	.27078E-01	.14234E-06	.12167E-08	.14260E-08	.50765E-06	.21087E-07	.24596E-07	.10133E+00	.61133E-10
.5647E-06	.11109E-08	.65083E-04	.13096E-05						
.568E-14	.46500E-10	.16702E-17	.25119E-15	.39208E-17	.81385E-13	.23162E-14	.29172E-12	.29695E-08	.15127E-15
.2675E-14	.12575E-15	.95463E-14	.27026E-14						
.6642E-11	.24598E-07	.30005E-14	.20623E-13	.24523E-14	.72935E-11	.23628E-12	.10739E-10	.54468E-06	.48228E-14
.1764E-13	.96427E-15	.90120E-10	.11268E-11						
4991E-14	.11786E-10	.11233E-16	.56135E-15	.17791E-16	.18462E-12	.53913E-14	.57270E-12	.79092E-08	.29148E-15
.7612E-14	.21257E-15	.29624E-13	.91917E-15						
.9687E-09	.80488E-06	.28502E-12	.45227E-11	.36340E-12	.15316E-08	.46983E-10	.34697E-08	.86756E-04	.16655E-11
.7501E-11	.84506E-12	.52872E-08	.32819E-10						
.2995E-10	.11415E-06	.17541E-13	.15684E-12	.20745E-13	.54251E-10	.17261E-11	.93852E-10	.36987E-05	.43506E-13
2715E-13	.13459E-13	.46894E-09	.51266E-11						

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SITION MATRICES FOR THE MODE = 1

THE FORM OF EXP(3.12671E-05*T)*(AFR*COS(-8.82353E-05)T +/- I*AFI*SIN(-8.82353E-05)T)

MIX AFR (14,14)

.E+00	.32041E-05	.70245E-08	.59983E-07
E-07	.47549E-04	.30711E-03	.27553E-02
E-05	.91912E-10	.31384E-10	.13659E-12
E-10	.37323E-05	.16095E-06	.72609E-08
E-06	.77132E-11	.26822E-11	.12517E-13
E-11	.33163E-10	.14424E-09	.66440E-09
E-07	.34138E-14	.12918E-13	.20842E-14
E-13	.22367E-11	.12947E-10	.99925E-10
E-05	.82093E-11	.89265E-11	.28751E-12
E-11	.40447E-09	.21291E-08	.14152E-07
E-04	.36090E-1[illegible]	[illegible]E-10	.22662E-11
1E-10	.30254E-08	.16211E-07	.11095E-06
E-03	.36568E-05	.55982E-09	.22885E-10
E-09	.30565E-07	.16376E-06	.11207E-05
E-07	.86206E-06	.13103E-06	.59306E-09
E-07	.21554E-07	.85911E-06	.24350E-04
E-06	.92656E-10	.88614E-11	.22735E-12
E-11	.90388E-10	.81572E-09	.10555E-07
E-06	.90532E-12	.35695E-14	.26554E-13
7E-13	.23094E-10	.14475E-09	.12500E-08
E-09	.15771E-12	.14567E-13	.43213E-15
E-14	.17252E-12	.15213E-11	.15267E-10
E-06	.10172E-10	.11023E-12	.13950E-12
E-13	.10592E-05	.70088E-09	.64954E-08
E-05	.93383E-10	.20085E-11	.99682E-12
E-13	.71264E-09	.48336E-06	.46198E-07
E-04	.97509E-09	.24226E-10	.97048E-11
4E-12	.66104E-06	.46551E-07	.44915E-06

.52836E-06	.33894E-05	.48950E-04	.41966E-07	.14937E-03	.17356E-04
.18230E-09	.11965E-08	.12316E-07	.65765E-04	.16615E-07	.91131E-09
.15693E-10	.10300E-09	.10612E-08	.56254E-05	.14365E-06	.79146E-10
.17356E-12	.11323E-11	.12826E-10	.33536E-07	.22932E-10	.17471E-11
.71722E-10	.46951E-09	.50421E-08	.20164E-04	.77552E-08	.50194E-09
.47202E-09	.30886E-08	.33401E-07	.12651E-03	.52438E-07	.34757E-08
.47718E-08	.31223E-07	.33765E-06	.12832E-02	.53003E-06	.35127E-07
.47890E-06	.31580E-05	.30179E-04	.25047E+00	.21149E-04	.11107E-05
.21326E-10	.14136E-09	.12346E-08	.15807E-04	.87326E-09	.11105E-10
.54610E-12	.35332E-11	.45293E-10	.26981E-07	.10892E-09	.10750E-10
.33625E-13	.22302E-12	.19282E-11	.25823E-07	.12975E-11	.13433E-13
.46031E-12	.29050E-11	.51952E-10	.62834E-07	.22156E-09	.20932E-10
.44572E-12	.26150E-11	.59943E-10	.28077E-05	.87577E-09	.16533E-09
.10029E-11	.51608E-11	.51280E-09	.29677E-04	.68414E-08	.14008E-08

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ATRIX AFI (14,14)

07E+00	.29148E-04	.27645E-05	.75911E-07
24E-06	.25247E-04	.26242E-03	.33776E-02
40E-04	.13798E-08	.94691E-10	.55996E-11
37E-10	.31202E-06	.24297E-07	.27166E-06
93E-05	.11992E-09	.82523E-11	.51967E-12
56E-11	.26982E-05	.21027E-08	.23547E-07
92E-07	.27832E-11	.22561E-12	.95346E-14
70E-13	.43780E-11	.36105E-10	.42867E-09
47E-05	.76061E-09	.58655E-10	.30015E-11
72E-11	.14680E-06	.11757E-07	.13548E-06
92E-04	.54251E-08	.41245E-05	.26504E-10
74E-10	.99381E-08	.79928E-07	.92499E-06
72E-03	.54828E-07	.41665E-08	.25724E-05
21E-09	.10045E-06	.80787E-06	.93491E-05
76E-01	.15471E-05	.77075E-07	.95354E-08
33E-06	.57447E-05	.41904E-04	.43644E-03
19E-06	.11222E-10	.37650E-13	.19574E-12
715E-12	.15619E-09	.10130E-08	.91496E-08
594E-07	.17715E-10	.15916E-11	.51064E-13
712E-12	.21106E-10	.18304E-09	.22821E-08
311E-08	.12332E-13	.12127E-10	.27686E-15
01E-15	.23111E-12	.14721E-11	.12969E-10
227E-06	.52962E-10	.52983E-11	.12373E-12
208E-11	.43940E-10	.41052E-09	.54804E-08
277E-06	.28153E-09	.30110E-10	.56640E-12
94E-11	.17732E-05	.17664E-08	.24920E-07
012E-05	.24830E-06	.27202E-09	.47137E-11
043E-10	.13982E-08	.14325E-07	.20682E-06

.65880E-05	.43676E-04	.24055E-0[illegible]	.45241E+01	.26611E-03	.32343E-05
.14142E-09	.94442E-09	.72226E-06	.15623E-03	.22844E-08	.25736E-10
.12383E-10	.82667E-10	.63343E-09	.13757E-04	.20265E-09	.20877E-11
.43939E-12	.29217E-11	.24169E-10	.39259E-06	.12655E-10	.26428E-13
.10175E-09	.67806E-09	.54316E-08	.99898E-04	.23752E-08	.11[illegible]35E-12
.72925E-09	.48563E-08	.39106E-07	.70430E-03	.17697E-07	.12450E-12
.73690E-08	.49072E-07	.39515E-06	.71174E-02	.17879E-06	.12147E-11
.55432E-07	.37559E-06	.21222E-05	.11926E+00	.47632E-07	.43536E-06
.15239E-11	.97580E-11	.14459E-09	.34876E-08	.46079E-05	.55009E-10
.35497E-11	.23555E-10	.20195E-09	.26071E-05	.13021E-09	.11107E-11
.37785E-14	.24355E-13	.32946E-12	.58074E-10	.48581E-12	.94790E-1[illegible]
.13427E-10	.88541E-10	.78626E-09	.95262E-05	.58920E-09	.92492E-11
.82023E-10	.54281E-09	.49779E-08	.54756E-04	.39390E-06	.76286E-10
.75965E-09	.50256E-08	.45410E-07	.49665E-03	.37568E-07	.85[illegible]07E-09

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SCALE FACTOR FOR SYSTEM MATRIX = .51091E-04

CONVERGENCE INFORMATIONS

X-NODE	Y-NODE	ITER.NO.	CONVERGEN	LYAPUNOV ERR	IT.TIME(SEC)	INV.ERROR	R-FACTOR	G-FACTOR
1	1	26	.49723E+00	.35919E-09	.92537E+01	.79521E+00	.11637E-01	.18412E-02
2	1	26	.12546E+00	.17323E-10	.92471E+01	[illegible]	.89955E-02	.11169E-02
3	1	22	.73636E-02	.10559E-09	.79671E+01	.54047E+00	.83556E-02	[illegible]
1	2	26	.65679E-01	.37176E-11	[illegible]	.76520E+00	.15231E-01	.15064E-02
2	2	5	.65116E-02	.60177E-11	.23570E+01	.88640E+00	.15731E-01	.18416E-02
3	2	26	.72295E+00	.80296E-11	.93161E+01	.90414E+00	.54976E-02	.16655E-02
4	2	26	.95531E-01	.11153E-10	.82401E+01	.84154E+00	[illegible]	.17537E-02
5	2	26	.76273E+00	.10887E-09	.92460E+01	.80924E+00	.10354E-02	.17923E-02
6	2	12	.25367E-02	.17841E-09	.46741E+01	.87045E+00	.99974E-03	.19394E-02
1	3	26	.67575E+01	.12291E-10	.93261E+01	.95836E+00	.10590E-01	.16785E-02
2	3	26	.88124E-01	.50793E-11	.93410E+01	[illegible]	.40222E-02	.17648E-02
3	3	23	.29525E-04	.62602E-11	.85073E+01	.19279E+00	[illegible]	.17477E-02
4	3	15	.94945E-02	.82501E-09	[illegible]	[illegible]	.70768E-05	.14594E-02
5	3	26	[illegible]	.59993E-10	.92141E+01	.82133E+00	[illegible]	[illegible]
6	3	6	.11154E-03	.10363E-10	[illegible]	[illegible]	[illegible]	[illegible]
1	4	16	.15287E-02	.55811E-10	.61250E+01	.72715E+00	[illegible]	[illegible]
2	4	26	[illegible]	.11171E-10	.92510E+01	.94554E+00	[illegible]	.16369E-02
3	4	26	.13005E+01	.16141E-10	.92990E+01	.99634E+00	.43652E-03	[illegible]
4	4	11	.65241E-04	.17104E-10	.43920E+01	.73849E+00	[illegible]	.14429E-02
5	4	26	.19563E+00	.32925E-10	.93440E+01	.54718E+00	[illegible]	.11241E-02
6	4	26	.16525E+00	.19583E-10	.92640E+01	.66310E+00	.65930E-04	.83673E-03
7	4	18	.22998E-06	.12933E-09	.67460E+01	.95310E+00	.62915E-03	[illegible]
1	5	10	.15842E-02	.37976E-10	.42670E+01	.93254E+00	[illegible]	[illegible]
2	5	24	.87337E-02	.36885E-11	[illegible]	[illegible]	[illegible]	.16067E-02
3	5	6	.45184E-02	.25720E-10	[illegible]	[illegible]	.21122E-10	.14816E-02
4	5	26	.75852E+00	.14811E-10	[illegible]	[illegible]	[illegible]	.21972E-02
5	5	7	[illegible]	.47895E-11	[illegible]	.70472E+00	[illegible]	[illegible]
6	5	28	.12401E+00	.85934E-11	[illegible]	.89052E+00	[illegible]	.66672E-03
7	5	26	.98136E-01	.90739E-11	.93490E+01	.91753E+00	.14411E-02	.72769E-03
1	6	26	[illegible]	.10892E-09	.93410E+01	.76475E+00	[illegible]	.16939E-02
2	6	12	[illegible]	.62796E-10	[illegible]	.81986E+00	[illegible]	[illegible]
3	6	18	.94551E-02	.47709E-10	[illegible]	.93015E+00	.17121E-02	.13045E-02
4	6	10	.31950E-02	.20735E-10	.40640E+01	.58355E+00	[illegible]	[illegible]
5	6	6	.54041E-02	.16120E-10	[illegible]	.91157E+00	.43181E-04	[illegible]
6	6	26	.13158E+00	.36254E-10	.93770E+01	.99917E+00	[illegible]	.66059E-03
7	6	26	[illegible]	.13182E-10	.92330E+01	[illegible]	.15352E-03	.55623E-03
8	6	4	.99025E-02	.13005E-10	.20290E+01	.93205E+00	.87451E-03	.79300E-03
1	7	26	.22304E+00	.54771E-11	.93450E+01	.76604E+00	.10035E-01	[illegible]
2	7	5	.55447E-02	.30444E-10	[illegible]	[illegible]	.29415E-02	[illegible]
3	7	26	.33609E-01	.76645E-09	[illegible]	[illegible]	.90726E-03	[illegible]
4	7	17	.54322E-02	.73173E-10	[illegible]	[illegible]	[illegible]	[illegible]
5	7	5	[illegible]	.26001E-10	.23410E+01	.49541E+00	.11721E-08	[illegible]
6	7	6	[illegible]	.85674E-12	[illegible]	.66072E+00	[illegible]	.57094E-03
7	7	11	.71926E-02	.26565E-11	[illegible]	[illegible]	[illegible]	.67553E-03
8	7	6	.75699E-02	.30992E-11	[illegible]	[illegible]	[illegible]	[illegible]
1	8	[illegible]	[illegible]	.25556E-10	[illegible]	.91585E+00	[illegible]	.16279E-02
2	8	26	[illegible]	.59852E-11	.92350E+01	[illegible]	[illegible]	[illegible]
3	8	26	.52641E+00	.73359E-11	[illegible]	[illegible]	[illegible]	.16709E-02
4	8	23	.27333E-02	.84937E-11	[illegible]	[illegible]	[illegible]	.75873E-03
5	8	26	.73605E+00	.12433E-10	[illegible]	[illegible]	.56245E-06	[illegible]
6	8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
7	8	6	[illegible]	.15896E-10	[illegible]	.94446E+00	[illegible]	[illegible]
8	8	4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	9	6	[illegible]	[illegible]	[illegible]	.49747E+00	[illegible]	[illegible]
2	9	5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3	9	26	[illegible]	.24307E-09	.92360E+01	[illegible]	.35077E-04	.11265E-02
4	9	26	.26252E+00	.72659E-11	[illegible]	[illegible]	[illegible]	.77132E-03
5	9	6	.95674E-02	.33878E-10	[illegible]	[illegible]	[illegible]	[illegible]
6	9	[illegible]	.71809E-02	.47713E-10	[illegible]	[illegible]	[illegible]	[illegible]
7	9	12	.93703E-02	.52922E-09	[illegible]	[illegible]	[illegible]	[illegible]
1	10	26	.72537E+00	[illegible]	[illegible]	[illegible]	.64934E-02	.15066E-02
2	10	26	[illegible]	.28917E-10	[illegible]	[illegible]	[illegible]	.16511E-02
3	10	23	.17586E-06	.14656E-10	[illegible]	[illegible]	[illegible]	[illegible]
4	10	6	[illegible]	.16223E-10	[illegible]	.97741E+00	[illegible]	[illegible]
5	10	16	.64192E-02	[illegible]	[illegible]	.97053E+00	[illegible]	[illegible]
7	10	4	.86605E-02	[illegible]	[illegible]	.94236E+00	[illegible]	[illegible]
8	10	6	.91074E-02	[illegible]	[illegible]	[illegible]	[illegible]	.11961E-02
1	11	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.16144E-02
2	11	26	.18454E+00	.10977E-10	[illegible]	[illegible]	.13219E-02	.16513E-02
3	11	7	[illegible]	.14772E-10	[illegible]	[illegible]	[illegible]	.12636E-02
4	11	16	.74234E-02	[illegible]	[illegible]	[illegible]	.15571E-02	.93847E-03
5	11	6	.56827E-02	.21426E-10	[illegible]	[illegible]	[illegible]	.61647E-03
6	11	6	.12389E-02	.15944E-10	[illegible]	.87555E+00	.57575E-03	[illegible]
7	11	7	[illegible]	.24816E-10	[illegible]	[illegible]	[illegible]	[illegible]
8	11	6	.95811E-02	.73423E-11	[illegible]	[illegible]	.51837E-03	.56423E-03
1	12	26	[illegible]	.19745E-10	[illegible]	[illegible]	.12657E-01	.17074E-02
2	12	8	[illegible]	.47012E-10	[illegible]	.82126E+00	[illegible]	[illegible]
3	12	26	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.11739E-02
4	12	12	[illegible]	.15576E-08	[illegible]	.97661E+00	[illegible]	.11291E-02
5	12	16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6	12	26	[illegible]	[illegible]	.92711E+01	[illegible]	[illegible]	[illegible]
7	12	26	[illegible]	.16275E-10	[illegible]	[illegible]	.24446E-03	.59253E-03
8	12	26	[illegible]	.22649E-11	[illegible]	[illegible]	[illegible]	.76645E-03
2	13	8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3	13	26	[illegible]	.55062E-10	[illegible]	[illegible]	[illegible]	.14562E-02
4	13	26	[illegible]	.13327E-10	[illegible]	[illegible]	[illegible]	.11534E-02
5	13	26	[illegible]	.13542E-11	[illegible]	[illegible]	[illegible]	.91162E-03
6	13	26	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.67460E-03
7	13	5	[illegible]	[illegible]	[illegible]	[illegible]	.19244E-02	.74166E-03
1	14	16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.14781E-02
2	14	26	[illegible]	.54663E-11	[illegible]	.72459E+00	.75411E-02	.15041E-02
3	14	7	[illegible]	[illegible]	[illegible]	.97626E+00	[illegible]	.15896E-02
4	14	26	[illegible]	[illegible]	[illegible]	[illegible]	.49325E-04	.14287E-02
5	14	26	[illegible]	.26663E-10	[illegible]	[illegible]	[illegible]	.11391E-02
6	14	22	[illegible]	.13092E-10	[illegible]	.82775E+00	[illegible]	[illegible]
7	14	26	[illegible]	.56033E-11	[illegible]	[illegible]	[illegible]	[illegible]
1	15	26	.47656E-01	[illegible]	[illegible]	[illegible]	.12663E-01	.15024E-02
2	15	26	[illegible]	.26971E-10	[illegible]	[illegible]	[illegible]	.17741E-02
3	15	26	.49325E+00	.56207E-10	[illegible]	[illegible]	.15867E-02	.17356E-02
4	15	26	.65311E+00	.44614E-10	[illegible]	[illegible]	[illegible]	.14859E-02
5	15	20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6	15	4	[illegible]	.20137E-10	[illegible]	.93453E+00	.12037E-03	.13642E-02
1	16	26	.32179E+00	.82395E-10	[illegible]	.78959E+00	.15557E-01	.14087E-02
2	16	26	[illegible]	[illegible]	[illegible]	[illegible]	.12319E-01	[illegible]
3	16	26	[illegible]	.90946E-11	[illegible]	[illegible]	[illegible]	.16968E-02
4	16	26	.19654E+00	.10739E-09	[illegible]	[illegible]	.33922E-02	.17872E-02
5	16	26	.65675E+00	.37365E-10	[illegible]	.79347E+00	[illegible]	.16152E-02
6	16	26	.53185E+00	.84256E-10	[illegible]	[illegible]	[illegible]	.19279E-02
1	17	24	.74148E-04	.33241E-10	.67920E+01	.80323E+00	[illegible]	.16636E-02
2	17	26	[illegible]	.44167E-10	[illegible]	[illegible]	.10611E-01	[illegible]
3	17	26	.63774E-01	.24945E-10	[illegible]	.82074E+00	.10316E-01	.23003E-02

SPATIAL CONTROL INFORMATION

```
SYMMETRY AXIS FOR X =      1
SYMMETRY AXIS FOR Y =     12
SYMMETRICITY OF CONTROLLERS (=1: Y-SYMMETRIC, =2: CENTRE SYMMETRIC) =     1

NO OF CONTROLLER NODES IN Y-DIRECTION =     8
NODE SIZE IN X-DIRECTION =    28.57500
NODE SIZE IN Y-DIRECTION =    28.57500
```

CONTROLLER LOCATIONS WHERE THE EFFECTIVENESSES ARE COMPUTED

Y-DIR = \ X-DIR=	1	2	3	4	5	6	7	8	9
154.2500	297.1250	325.7000	354.2750	382.8500					
162.6250	211.4000	239.9750	268.5500	297.1250	325.7000	354.2750	382.8500		
211.4000	211.4000	239.9750	268.5500	297.1250	325.7000	354.2750	382.8500		
239.9750	182.8250	211.4000	239.9750	268.5500	297.1250	325.7000	354.2750	382.8500	
268.5500	162.8250	211.4000	239.9750	268.5500	297.1250	325.7000	354.2750	382.8500	
297.1250	154.2500	182.8250	211.4000	239.9750	268.5500	297.1250	325.7000	354.2750	382.8500
325.7000	154.2500	182.8250	211.4000	239.9750	268.5500	297.1250	325.7000	354.2750	382.8500
354.2750	154.2500	162.8250	211.4000	239.9750	268.5500	297.1250	325.7000	354.2750	382.8500
382.8500	154.2500	182.8250	211.4000	239.9750	268.5500	297.1250	325.7000	354.2750	382.8500
411.4250	154.2500	162.8250	211.4000	239.9750	268.5500	297.1250	325.7000	354.2750	382.8500
440.0000	154.2500	162.8250	211.4000	239.9750	268.5500	297.1250	325.7000	354.2750	382.8500
468.5750	154.2500	182.8250	211.4000	239.9750	268.5500	297.1250	325.7000	354.2750	382.8500
497.1500	182.8250	211.4000	239.9750	268.5500	297.1250	325.7000	354.2750	382.8500	
525.7250	162.8250	211.4000	239.9750	268.5500	297.1250	325.7000	354.2750	382.8500	
554.3000	211.4000	239.9750	268.5500	297.1250	325.7000	354.2750	382.8500		
582.8750	211.4000	239.9750	268.5500	297.1250	325.7000	354.2750	382.8500		
611.4500	297.1250	325.7000	354.2750	382.8500					

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CONTROLLER EFFECTIVENESS MAP

```
          154.25    182.83    211.43    239.98    268.55    297.13    325.7.    354.28    382.85    411.43    440.00    468
          497.15    525.73    554.30    582.86    611.45
 154.25   .100E+11  .100E+11  .100E+11  .100E+11  .100E+11-.100E+01-.100E+01  .252E+02  .100E+11  .252E+02-.100E+01-.100E
********  .100E+11  .100E+11  .100E+11  .100E+11
 182.83   .100E+11  .100E+11-.100E+01  .302E+02-.100E+01-.100E+01-.100E+01  .850E+02  .100E+11  .851E+02-.100E+01-.100E

  -1.00   .302E+02-.100E+01  .100E+11  .100E+11
 211.40   .100E+11  .100E+11-.100E+01-.100E+01  .167E+03  .112E+04-.100E+01  .471E+03  .100E+11  .472E+03-.100E+01  .112E

 166.97  -.100E+01-.100E+01  .100E+11  .100E+11
 239.98   .100E+11  .273E+02-.100E+01-.100E+01  .432E+04-.100E+01-.100E+01  .109E+03  .100E+11  .109E+03-.100E+01-.100E

4018.65  -.100E+01-.100E+01  .272E+02  .100E+11
 268.55   .100E+11  .327E+02  .543E+02  .674E+06-.100E+01  .851E+02-.100E+01-.100E+01  .100E+11-.100E+01-.100E+01  .851E

  -1.00   .672E+06  .540E+02  .325E+02  .100E+11
 297.13  -.100E+01  .401E+02  .715E+02  .471E+03  .464E+03-.100E+01-.100E+01  .540E+02  .100E+11  .541E+02-.100E+01-.100E

 472.41   .501E+03  .711E+02  .399E+02-.100E+01
 325.70  -.100E+01  .530E+02-.100E+01  .225E+03  .661E+05  .159E+03  .386E+03  .142E+03  .100E+11  .142E+03  .386E+03  .159E

96920.73  .224E+03-.100E+01  .528E+02-.100E+01
 354.28   .330E+02-.100E+01-.100E+01  .125E+03-.100E+01  .169E+03  .547E+03  .239E+03  .100E+11  .239E+03  .547E+03  .169E

  -1.00   .124E+03-.100E+01-.100E+01  .330E+02
 382.85   .366E+02  .140E+03-.100E+01-.100E+01  .606E+03  .160E+03  .578E+03  .100E+11  .100E+11  .100E+11  .576E+03  .160E

 806.25  -.100E+01-.100E+01  .140E+03  .366E+02
 411.43  -.100E+01-.100E+01  .301E+03  .755E+02  .456E+03  .100E+11  .457E+03  .219E+03  .100E+11  .219E+03  .456E+03  .100E

 486.40   .761E+02  .305E+03-.100E+01-.100E+01
 440.00   .305E+02-.100E+01  .989E+03  .887E+02  .776E+03  .128E+03  .298E+03  .125E+03  .100E+11  .125E+03  .299E+03  .128E

 360.65   .885E+02  .101E+04-.100E+01  .306E+02
 468.58  -.100E+01  .460E+02-.100E+01  .155E+03  .247E+03-.100E+01-.100E+01-.100E+01  .100E+11-.100E+01-.100E+01-.100E

 248.96   .159E+03-.100E+01  .462E+02-.100E+01
 497.15   .100E+11  .100E+11  .546E+02-.100E+01-.100E+01-.100E+01-.100E+01  .623E+02  .100E+11  .623E+02-.100E+01-.100E

  -1.00  -.100E+01  .546E+02  .100E+11  .100E+11
 525.73   .100E+11  .263E+02-.100E+01  .619E+02-.100E+01-.100E+01  .305E+03-.100E+01  .100E+11-.100E+01  .304E+03-.100E

  -1.00   .807E+02-.100E+01  .263E+02  .100E+11
 554.30   .100E+11  .100E+11-.100E+01-.100E+01-.100E+01-.100E+01  .720E+03  .246E+03  .100E+11  .246E+03  .716E+03-.100E

  -1.00  -.100E+01-.100E+01  .100E+11  .100E+11
 582.88   .100E+11  .100E+11-.100E+01-.100E+01-.100E+01-.100E+01-.100E+01-.100E+01  .100E+11-.100E+01-.100E+01-.100E

  -1.00  -.100E+01-.100E+01  .100E+11  .100E+11
 611.45   .100E+11  .100E+11  .100E+11  .100E+11  .100E+11  .252E+02-.100E+01-.100E+01  .100E+11-.100E+01-.100E+01  .251E
********  .100E+11  .100E+11  .100E+11  .100E+11
```

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E. PROGRAM SOURCE LIST

```
/JOB
/NOSEQ
OHOPT,B1200-OP277/CYBER,T77,IO100.    S.K. OH (EXT 462)
SENDTO,TP.
FTN.
EXIT(S)
/EOR

      PROGRAM  ODZCR(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE1,TAPE2)
C**** OPTIMAL DEPLOYMENT OF CANDU ZONE CONTROLLERS
C**** USING LAMBDA MODES OF REFERENCE STATE OF 2-D REACTOR
C**** 2-GROUP MODEL FOR DIFFUSION APPROXIMATION
C**** QUADRATIC PERFORMANCE INDICES FOR SPACE-TIME DISTRIBUTED PARAMETER
C**** SYSTEM
      COMMON /STFN/  XF(31,23),   XT(31,23),   XI(31,23),   XE(31,23)
      COMMON /XSCN/  DIF1(25),    DIF2(25),    SIGA1(25),    SIGA2(25),
     1               SIGR(25),    SIGF(25),    ALFA(25),    HFAC(25),
     2               XK(10),      V1,          V2,          XNU,
     3               GAMI,        GAMX,        SIGXE,       ZLAMI,
     4               ZLAMX,       XKS
      COMMON /GEOM/  MAT(31,23),   DX(31),   DY(23),   X(31),   Y(23)
      COMMON /MISC/  TITLE(18),   NPAGE,       NMOD,        MM2,      MM4,
     1               NRX,         NRY,         NTT,         PWR,      BLANK,
     2              CX,     CY,     NSX,     NSY,   IUNSTB,   RFAC,   SFAC,
     3              QFAC,      AMPO
      COMMON /SYSM/  SMX11(20,20),   SMX12(20,20),   SMX21(21,21),
     1               SMX22(20,20),   SMMX(20,20)
      COMMON /XYMOD/ PF(31,23,8),   PS(31,23,8),   FNORM(8),   TNORM(8)
      DIMENSION  IB(2),    JJJ(31),    WORK(20),    WR(20),    WI(20),
     1            TRMX(20,20),   TMRX(20,20),   TNRX(20),    CSMX(20,20)
      COMMON     AFR(20,20),   AFI(20,20)
      DATA BLANK/4H    /
      DATA DX,DY/54*0.0/
C
      READ (5,1000) TITLE
 1000 FORMAT(18A4)
      NPAGE=0
      CALL HEAD
      READ (5,1001) NMAT,NRX,NRY,NMOD,NSTABL,NPROB
 1001 FORMAT(16I5)
      WRITE (6,1050) NMAT,NRX,NRY,NMOD,NSTABL,NPROB
 1050 FORMAT(//,5X,'NO OF MATERIAL PROPERTIES = ',I5,/,5X,'NO OF MESH PO
     1INTS IN X-AXIS = ',I5,/,5X,'NO OF MESH POINTS IN Y-AXIS = ',I5,/,5
     2X,'NO OF MODES = ',I5,/,5X,'STABILITY CHECK (YES = 1) = ',I5,/,5X,
     3'PROBLEM TYPE (=1; BULK CONTROLLER, =2; SPATIAL CONTROLLER, =3;
     4SIMULATION) = ',I5,/)
      MM4=4*(NMOD-1)
      M4=4*NMOD
      M2=2*NMOD
      MM2=2*(NMOD-1)
      DO 10 I=1,M2
```

```
      WR(I)=0.0
      WI(I)=0.0
      WORK(I)=0.0
      TNRX(I)=0.0
      DO 10 J=1,M2
      TRMX(I,J)=0.0
      TMRX(I,J)=0.0
   10 CONTINUE
      READ (5,1002) (XK(I),I=1,NMOD)
 1002 FORMAT(8F10.0)
      IB(1)=1HX
      IB(2)=1HY
      XKS=XK(1)
      NRX1=NRX-1
      NRY1=NRY-1
      III=0
   20 CONTINUE
      III=III+1
      IF (III-2) 21,27,28
   21 WRITE (6,1003) IB(III)
 1003 FORMAT(//,6X,'MESH SPACING ( ',A1,'-DIRECTION )',/)
   22 READ (5,1004) XYZ,N1,N2,NJJ,(JJJ(J),J=1,NJJ)
      IF (XYZ.EQ.0.0) GO TO 20
      WRITE (6,1005) XYZ,(JJJ(J),J=1,NJJ)
 1004 FORMAT(F10.5,34I2)
 1005 FORMAT(1X,F10.5,30(1X,I2))
      IF (NJJ.EQ.0) GO TO 23
      N1=1
      N2=NJJ
   23 DO 26 I=N1,N2
      IJK=I
      IF (NJJ.NE.0) IJK=JJJ(I)
      IF (III-1) 24,24,25
   24 DX(IJK)=XYZ
      GO TO 26
   25 DY(IJK)=XYZ
   26 CONTINUE
      GO TO 22
   27 WRITE (6,1006) DX
      GO TO 21
   28 WRITE (6,1006) DY
      X(1)=0.5*DX(1)
      Y(1)=0.5*DY(1)
      DO 35 I=2,NRX
   35 X(I)=X(I-1)+0.5*(DX(I-1)+DX(I))
      DO 36 J=2,NRY
   36 Y(J)=Y(J-1)+0.5*(DY(J-1)+DY(J))
      CX=(X(NRX)+X(1))/2.0
      CY=(Y(NRY)+Y(1))/2.0
      CX=(X(1)+X(NRX))/2.0
      CY=(Y(1)+Y(NRY))/2.0
C     READ-IN IODINE/XENON PARAMETERS
      READ (5,1007) XNU,SIGXE,ZLAMI,ZLAMX,GAMI,GAMX
      READ (5,1007) V1,V2,PWR
```

```
 1006 FORMAT(/,(1X,12F10.5))
 1007 FORMAT(6E12.6)
   30 READ(5,1001) J1,J2,J3,J4,J5
      DO 31 I=J1,J2
      DO 31 J=J3,J4
   31 MAT(I,J)=J5
      IF (J1.NE.0) GO TO 30
      CALL HEAD
      WRITE (6,1009)
 1009 FORMAT(///,30X,'**** MATERIAL MAP ****',//)
      DO 32 J=1,NRY
      WRITE (6,1008) (MAT(I,J),I=1,NRX)
 1008 FORMAT(8X,35I3)
   32 CONTINUE
C     READ-IN MATERIAL PROPERTIES
      DO 33 I=1,NMAT
      READ (5,1007) DIF1(I),DIF2(I),SIGA1(I),SIGA2(I),SIGR(I),SIGF(I)
      READ (5,1007) ALFA(I),HFAC(I)
   33 CONTINUE
      CALL HEAD
      WRITE (6,1010)
 1010 FORMAT(1X,'REACTOR STRUCTURES')
      WRITE (6,1011) PWR
 1011 FORMAT(10X,' REACTOR POWER = ',E12.6,' (MW-TH)'/)
      WRITE (6,1012)
 1012 FORMAT(/,' NEUTRON KINETICS DATA ',/,13X,'V1             V2',/)
      WRITE (6,1014) V1,V2
      WRITE (6,1013)
 1013 FORMAT(' IODINE/XENON PARAMETERS'/,11X,'XNU             SIGXE
     1 ZLAMI          ZLAMX         GAMI          GAMX',/)
      WRITE (6,1014) XNU,SIGXE,ZLAMI,ZLAMX,GAMI,GAMX
      WRITE (6,1015)
 1014 FORMAT(7X,1P8E13.6)
 1015 FORMAT(//,' MATERIAL PROPERTIES'/' ID    DIF1          DIF2
     1SIGA1        SIGA2         SIGR1         SIGF2         ALFA          HFAC1'
     2,/)
      DO 37 I=1,NMAT
      SIGF(I)=SIGF(I)/XNU
      HFAC(I)=HFAC(I)*1.16
      IF (SIGF(I).LE.0.0) GO TO 37
      SIGA2(I)=SIGA2(I)-1.20E-04
   37 CONTINUE
      DO 34 I=1,NMAT
      WRITE (6,1016) I,DIF1(I),DIF2(I),SIGA1(I),SIGA2(I),SIGR(I),SIGF(I)
     1              ,ALFA(I),HFAC(I)
 1016 FORMAT(I3,1X,1P8E12.5)
   34 CONTINUE
C     CALCULATE FLUX,IODINE,XENON DISTRIBUTION AT REFERENCE STATE
      CALL REFST
      CALL HEAD
      WRITE (6,1026)
 1026 FORMAT(10X,'FAST FLUX DISTRIBUTION')
      CALL XMAP(XF)
      CALL HEAD
```

```
      WRITE (6,1017)
 1017 FORMAT(10X,'THERMAL FLUX DISTRIBUTION')
      CALL XMAP(XT)
      CALL HEAD
      WRITE (6,1018)
 1018 FORMAT(10X,'IODINE DISTRIBUTION')
      CALL XMAP(XI)
      CALL HEAD
      WRITE (6,1019)
 1019 FORMAT(10X,'XENON DISTRIBUTION')
      CALL XMAP(XE)
      CALL HEAD
      WRITE (6,1020)
 1020 FORMAT(10X,'MODAL INFORMATION'//)
      WRITE (6,1021)
 1021 FORMAT(10X,'MOD NO        K-EFF         FNORM         TNORM',/)
      DO 40 I=1,NMOD
      WRITE (6,1022) I,XK(I),FNORM(I),TNORM(I)
 1022 FORMAT(10X,I4,5X,3E12.5)
   40 CONTINUE
      WRITE (6,1023) SFAC,XKS
 1023 FORMAT(//,10X,'FLUX SCALE FACTOR = ',E12.6,/,
     110X,'K-EFF FOR REFERENCE STATE = ',E12.6,/)
      IF (NSTABL.NE.1.AND.NPROB.NE.2) GO TO 70
C     COMPUTE SYSTEM MATRIX EIGEN-STATES
      CALL SMTRX(2)
C     INVERSE OF SUBMATRIX(THE FIRST QUARTER OF SYSTEM MATRIX)
      CALL GINV(SMX11,TRMX,WR,WI,20,MM2,MM2,ERROR)
C     TRANSFORMATION OF FLUX VECTORS INTO IODINE/XENON VECTORS
      DO 50 L=1,MM2
      DO 50 N=1,MM2
      SMMX(L,N)=DOT4(21,20,1,MM2,SMX21,SMX11,L,N)
   50 CONTINUE
      DO 60 L=1,MM2
      DO 60 N=1,MM2
      SMX22(L,N)=SMX22(L,N)-DOT3(20,20,1,MM2,SMMX,SMX12,L,N)
   60 CONTINUE
      CALL HEAD
      WRITE (6,1024) MM2,MM2
 1024 FORMAT(//,30X,'ORDER-REDUCED SYSTEM MATRIX (',I2,',',I2,')',/)
      DO 61 L=1,MM2
      WRITE (6,1045) (SMX22(L,M),M=1,MM2)
   61 CONTINUE
C     XENON STABILITY CHECK
      CALL HEAD
      WRITE (6,1040)
 1040 FORMAT(//,30X,'STABILITY INFORMATION',//)
      CALL EIGVAL(20,MM2,CSMX,TMRX,SMX22,WR,WI,TNRX,.TRUE.,TRMX)
      CALL EIGVEC(20,MM2,CSMX,TMRX,WR,WI,TNRX,TRMX)
      DO 65 I=1,MM2
      IF (WI(I).EQ.0.0) GO TO 62
      WORK(I)=1.7453293E-03/ABS(WI(I))
   62 IF (WR(I).LT.ALPA) GO TO 65
   65 CONTINUE
```

```
      WRITE (6,1041)
      WRITE (6,1044) (WR(I),WI(I),WORK(I),I=1,MM2)
 1041 FORMAT(30X,'EIGENVALUES IN THE FORM OF REAL(LAMDA)+IMAG(LAMDA)',
     1//,13X,' REAL PART    IMAG PART    PERIOD(HR)',/)
 1044 FORMAT(10X,3E12.5)
      CALL HEAD
      WRITE (6,1042)
 1042 FORMAT(//,30X,'EIGENVECTORS(NORMALIZED BY THE NORM OF SET)'/,
     130X,'FOR COMPLEX VECTORS - AT (I)-TH LINE = REAL; AT (I+1)-TH'
     2/,30X,'LINE = IMAG',//)
      DO 66 J=1,MM2
      WRITE (6,1045) (TRMX(I,J),I=1,MM2)
 1045 FORMAT(10E12.6)
   66 CONTINUE
      J=0
      DO 67 I=1,MM2
      IF (WI(I).EQ.0.0) GO TO 67
      IF (I.EQ.J) GO TO 67
      J=I+1
      IF (J.GT.MM2) GO TO 67
      TEMP=WI(I)
      WI(I)=WI(J)
      WI(J)=TEMP
   67 CONTINUE
      DO 68 I=1,MM2
      IN=MM2-I+1
      IF (IN.EQ.1) GO TO 69
      IN1=IN/2
      IN2=(IN-1)/2
      IF (WI(IN).NE.0.0.AND.IN1.NE.IN2) GO TO 68
   69 CONTINUE
      CALL COTMX(20,MM2,IN,SMX22,WR,WI)
   68 CONTINUE
   70 CONTINUE
      IF (NPROB.EQ.0) GO TO 90
      IF (NPROB-2) 75,80,85
   75 CALL BKCON(NPROB)
      GO TO 90
   80 CALL SPCON
      GO TO 90
   85 CALL SIMUL(NPROB)
   90 CONTINUE
      STOP
      END

      SUBROUTINE HEAD
      COMMON /MISC/  T(18),  NP
      IF (NP) 1,1,2
    1 CALL SECOND(A1)
      CALL DATE(NOW)
    2 CALL SECOND(A2)
      A2=A2-A1
      NP=NP+1
      WRITE (6,1000) T,NP,NOW,A2
```

```
 1000 FORMAT(1H1,18A4,11H     PAGE  ,I2,3X,A10,6H  TIME,F10.3,4H SEC//)
      RETURN
      END

      SUBROUTINE REFST
C**** CALCULATION OF SYSTEM FUNCTIONS AT REFERENCE STATE
C**** USING THE FUNDAMENTAL MODES OF NEUTRON FLUXES AS THE
C**** DISTRIBUTIONS AT THE STATE
C
      COMMON /STFN/  XF(31,23),  XT(31,23),  XI(31,23),  XE(31,23)
      COMMON /GEOM/  MAT(31,23),  DX(31),  DY(23),  X(31),  Y(23)
      COMMON /XSCN/  DIF1(25),   DIF2(25),   SIGA1(25),   SIGA2(25),
     1               SIGR(25),   SIGF(25),   ALFA(25),    HFAC(25),
     2               XK(10),     V1,         V2,          XNU,
     3               GAMI,       GAMX,       SIGXE,       ZLAMI,
     4               ZLAMX,      XKS
      COMMON /MISC/  TITLE(18),  NPAGE,      NMOD,        M2,      M4,
     1               NRX,        NRY,        NTT,         PWR,     BLANK
     2              ,DUM(6),  SFAC,   DUM1,   AMPO
      COMMON /XYMOD/ PF(31,23,8),  PS(31,23,8),  FNORM(8),  TNORM(8)
      DIMENSION  A(31,23),  B(31,23)
C
      DO 10 MOD=1,NMOD
      READ (1,1001) NDUM
      READ (1,1000) ((A(I,J),I=1,31),J=1,23)
      READ (1,1000) ((B(I,J),I=1,31),J=1,23)
 1000 FORMAT(8E10.4)
 1001 FORMAT(I5)
      SUM1=SUM2=0.0
      IF (MOD.NE.1) GO TO 13
      SUM=TSUM=0.0
   13 CONTINUE
      FNORM(MOD)=0.0
      TNORM(MOD)=0.0
C     NORMALIZATION FACTOR FOR MODES
      DO 11 I=1,NRX
      DO 11 J=1,NRY
      IF (MOD.NE.1) GO TO 21
      XF(I,J)=A(I,J)
      XT(I,J)=B(I,J)
   21 CONTINUE
      PF(I,J,MOD)=0.0
      PS(I,J,MOD)=0.0
      DV=DX(I)*DY(J)
      IDX=MAT(I,J)
      SUM2=SUM2+B(I,J)*SIGR(IDX)*A(I,J)*DV
      SUM1=SUM1+A(I,J)*XNU*SIGF(IDX)*B(I,J)*DV
      IF (MOD.NE.1) GO TO [illegible]
      SUM=SUM+B(I,J)*HFA[illegible]DV
      TSUM=TSUM+B(I,J)*DV
   11 CONTINUE
      SUM1=SQRT(SUM1)
      DO 12 I=1,NRX
      DO 12 J=1,NRY
```

```
      IDX=MAT(I,J)
      PF(I,J,MOD)=A(I,J)*SQRT(SIGR(IDX))/SUM1
      PS(I,J,MOD)=B(I,J)*SQRT(XNU*SIGF(IDX))/SUM1
   12 CONTINUE
      FNORM(MOD)=1.0/SUM1
      TNORM(MOD)=SUM1/SUM2
   10 CONTINUE
C     FLUX SCALE UP TO FULL POWER LEVEL
      SFAC=6.804422E+15*PWR/SUM
      RAT1=GAMI/ZLAMI
      RAT2=GAMI+GAMX
      SUM=SUMQ=SUMP=0.0
      DO 20 I=1,NRX
      DO 20 J=1,NRY
      DV=DX(I)*DY(J)
      XX=SIGF(MAT(I,J))
      XF(I,J)=XF(I,J)*SFAC
      XT(I,J)=XT(I,J)*SFAC
      SUM=SUM+XT(I,J)*DV
      YY=XX*XT(I,J)
      XI(I,J)=RAT1*YY
      XE(I,J)=RAT2*YY/(SIGXE*XT(I,J)+ZLAMX)
      SUMP=SUMP+PS(I,J,1)*PS(I,J,1)*DV
      SUMQ=SUMQ+PS(I,J,1)*PS(I,J,1)*PS(I,J,1)*DV
   20 CONTINUE
      AMPO=SUM/TSUM
      RETURN
      END

      SUBROUTINE XMAP(AA)
C**** SHOW STATE FUNCTION MAP
C
      COMMON /GEOM/  MAT(31,23),  DX(31),  DY(23),  X(31),  Y(23)
      COMMON /MISC/  TITLE(18),   NPAGE,      NMOD,        M2,      M4,
     1               NRX,         NRY,        NTT,         PWR,     BLANK
      DIMENSION  AA(31,23),  IA(31,23)
      AAMAX=0.0
      DO 10 I=1,NRX
      DO 10 J=1,NRY
      IF (AA(I,J).GE.AAMAX) AAMAX=AA(I,J)
   10 CONTINUE
      AMAX=AAMAX/10000.0
      DEX=ALOG10(AMAX)+0.9999
      INDEX=IFIX(DEX)
      MULT=10**INDEX
      DO 20 I=1,NRX
      DO 20 J=1,NRY
      IA(I,J)=AA(I,J)/MULT
   20 CONTINUE
      WRITE (6,1000) INDEX
 1000 FORMAT(100X,'(MULTIPLIED BY E',I5,' )'//)
      WRITE (6,1001) (Y(J),J=1,NRY)
 1001 FORMAT(5X,' Y = ',23F5.1,//)
      WRITE (6,1004)
```

```
 1004 FORMAT(5X,' X = ')
      DO 50 II=1,NRX
      I=NRX-II+1
      WRITE (6,1002) X(I),(IA(I,J),J=1,NRY)
 1002 FORMAT(5X,F5.1,23I5)
   50 CONTINUE
      WRITE (6,1003) AAMAX
 1003 FORMAT(10X,'MAXIMUM VALUE = ',E12.6)
      RETURN
      END

      SUBROUTINE SMTRX(NPROB)
C**** FORMULATION OF SYSTEM MATRIX
C**** DIMENSION OF MATRIX IS (4*MOD)*(4*MOD)
C
      COMMON /STFN/   XF(31,23),   XT(31,23),   XI(31,23),   XE(31,23)
      COMMON /GEOM/   MAT(31,23),   DX(31),   DY(23),   X(31),   Y(23)
      COMMON /XSCN/   DIF1(25),    DIF2(25),    SIGA1(25),    SIGA2(25),
     1                SIGR(25),    SIGF(25),    ALFA(25),     HFAC(25),
     2                XK(10),      V1,          V2,           XNU,
     3                GAMI,        GAMX,        SIGXE,        ZLAMI,
     4                ZLAMX,       XKS
      COMMON /MISC/   TITLE(18),   NPAGE,       NMOD,         M2,       M4,
     1                NRX,         NRY,         NIT,          PWR,      BLANK
      COMMON /SYSM/   SMX11(20,20),  SMX12(20,20),  SMX21(21,21),
     1                SMX22(20,20),  SMMX(20,20)
      COMMON /XYMOD/ PF(31,23,8),  PS(31,23,8),  FNORM(8),  TNORM(8)
      IF (NPROB.EQ.1) MMX=NMOD
      IF (NPROB.EQ.2) MMX=2*(NMOD-1)
      IF (NPROB.EQ.3) MMX=2*NMOD
      DO 10 L=1,MMX
      DO 10 N=1,MMX
      SMX11(L,N)=0.0
      SMX12(L,N)=0.0
      SMX21(L,N)=0.0
      SMMX(L,N)=0.0
      SMX22(L,N)=0.0
   10 CONTINUE
      VV=V2/V1
      NFRST=1
      IF (NPROB.EQ.2) NFRST=NPROB
      NLAST=NMOD-NFRST+1
      DO 20 M=NFRST,NMOD
      L1=M-NFRST+1
      L2=NLAST+L1
      TLN=TNORM(M)/FNORM(M)
      DO 23 N=NFRST,NMOD
      N1=N-NFRST+1
      N2=NLAST+N1
      DO 24 I=1,NRX
      DO 24 J=1,NRY
      IDX=MAT(I,J)
      IF (SIGF(IDX).EQ.0.0.OR.SIGR(IDX).EQ.0.0) GO TO 24
      DV=DX(I)*DY(J)
```

```
      AB=XNU*SIGF(IDX)/SIGR(IDX)
      AB1=SQRT(AB)
      AB2=1.0/AB1
      IF (NPROB-2) 26,25,27
   25 CONTINUE
      ALN=1.0/SQRT(SIGR(IDX)*XNU*SIGF(IDX))
      BLN=1.0+ALFA(IDX)
      CLN=SIGXE*XT(I,J)
      DLN=BLN*SIGF(IDX)
      ELN=SIGXE*XE(I,J)
      SMX11(L1,N1)=SMX11(L1,N1)-AB1*PF(I,J,M)*PS(I,J,N)*DV/XK(N)
      SMX11(L1,N2)=SMX11(L1,N2)+AB1*PF(I,J,M)*BLN*PS(I,J,N)*DV/XKS
      SMX11(L2,N1)=SMX11(L2,N1)+VV*AB2*PS(I,J,M)*TLN*PF(I,J,N)*DV
      SMX11(L2,N2)=-SMX11(L2,N1)
      SMX12(L2,N2)=SMX12(L2,N2)-AB2*PS(I,J,M)*TLN*ALN*CLN*PS(I,J,N)*
     1            VV*DV
      SMX21(L1,N2)=SMX21(L1,N2)+AB1*PF(I,J,M)*GAMI*DLN*PS(I,J,N)*DV
      SMX22(L1,N1)=SMX22(L1,N1)-AB1*PF(I,J,M)*ZLAMI*PS(I,J,N)*DV
      SMX21(L2,N2)=SMX21(L2,N2)+AB1*PF(I,J,M)*(GAMX*DLN-ELN)*PS(I,J,N)*
     1            DV
      SMX22(L2,N1)=-SMX22(L1,N1)
      SMX22(L2,N2)=SMX22(L2,N2)-AB1*PF(I,J,M)*(CLN+ZLAMX)*PS(I,J,N)*DV
      GO TO 24
   26 CONTINUE
      ALN=SQRT(XNU*SIGF(IDX)*SIGR(IDX))
      BLN=ALN*(1.0+ALFA(IDX))
      SMX11(L1,N1)=SMX11(L1,N1)-TLN*AB2*PS(I,J,M)*ALN*PS(I,J,N)*DV/
     1            XK(N)
      SMX12(L1,N1)=SMX12(L1,N1)+TLN*AB2*PS(I,J,M)*BLN*PS(I,J,N)*DV
      SMX21(L1,N1)=SMX21(L1,N1)-AB1*PF(I,J,M)*ALN*PF(I,J,N)*DV
      SMX22(L1,N1)=-SMX21(L1,N1)
      SMMX(L1,N1)=SMMX(L1,N1)+TLN*AB2*PS(I,J,M)*XT(I,J)
     1           *PS(I,J,N)*ALN*DV
      GO TO 24
   27 CONTINUE
      ALN=SQRT(XNU*SIGF(IDX))
      BLN=SQRT(SIGR(IDX))
      CLN=(1.0+ALFA(IDX))
      DLN=SIGXE*XT(I,J)
      ELN=SIGXE*XE(I,J)
      FLN=SIGF(IDX)
      SMX11(L1,N1)=SMX11(L1,N1)-TLN*AB2*PS(I,J,M)*ALN*BLN*
     1            PS(I,J,N)*DV/XK(N)
      SMX11(L1,N2)=SMX11(L1,N2)+TLN*AB2*PS(I,J,M)*ALN*BLN*CLN*
     1            PS(I,J,N)*DV
      SMX12(L2,N2)=SMX12(L2,N2)-TLN*AB2*PS(I,J,M)*BLN*DLN*
     1            PS(I,J,N)*DV
      SMX11(L2,N1)=SMX11(L2,N1)-AB1*PF(I,J,M)*ALN*BLN*PF(I,J,N)*DV
      SMX11(L2,N2)=-SMX11(L2,N1)
      SMX21(L1,N2)=SMX21(L1,N2)+AB1*PF(I,J,M)*GAMI*CLN*FLN*
     1            PS(I,J,N)*DV
      SMX21(L2,N2)=SMX21(L2,N2)+AB1*PF(I,J,M)*(GAMX*CLN*FLN-ELN)*
     1            PS(I,J,N)*DV
      SMX22(L1,N1)=SMX22(L1,N1)-AB1*PF(I,J,M)*ZLAMI*PS(I,J,N)*DV
```

```
 20 U(I,1)=U(I,1)*FAC
    AFLAG(1)=1.0
    N=27
    TOL=(10.0*0.5**N)**2
    DO 100 J=2,NC
    DOTT=DOT(MR,1,NR,A,J,J)
    JM1=J-1
    DO 50 L=1,2
    DO 30 K=1,JM1
 30 ATEMP(K)=DOT(MR,1,NR,A,J,K)
    DO 45 K=1,JM1
    DO 35 I=1,NR
 35 A(I,J)=A(I,J)-ATEMP(K)*A(I,K)*AFLAG(K)
    DO 40 I=1,NC
 40 U(I,J)=U(I,J)-ATEMP(K)*U(I,K)
 45 CONTINUE
 50 CONTINUE
    DOTN=DOT(MR,1,NR,A,J,J)
    IF ((DOTN/DOTT)-TOL) 55,55,70
 55 DO 60 I=1,JM1
 60 ATEMP(I)=DOT(NC,1,I,U,I,J)
    DO 65 I=1,NR
    A(I,J)=0.0
    DO 65 K=1,JM1
 65 A(I,J)=A(I,J)-A(I,K)*ATEMP(K)*AFLAG(K)
    AFLAG(J)=0.0
    FAC=DOT(NC,1,NC,U,J,J)
    FAC=1.0/SQRT(FAC)
    GO TO 75
 70 AFLAG(J)=1.0
    FAC=1.0/SQRT(DOTN)
 75 DO 80 I=1,NR
 80 A(I,J)=A(I,J)*FAC
    DO 85 I=1,NC
 85 U(I,J)=U(I,J)*FAC
100 CONTINUE
    DO 130 J=1,NC
    DO 130 I=1,NR
    A(I,J)=DOT4(MR,NC,J,NC,A,U,I,J)
130 CONTINUE
    DO 94 I=1,NR
    DO 94 J=1,NR
    U(I,J)=DOT4(MR,20,1,NC,A,B,I,J)
 94 CONTINUE
    ERROR=0.0
    DO 95 I=1,NR
    DO 95 J=1,NR
    IF (I-J) 97,96,97
 96 ERR=ABS(U(I,J)-1.0)
    GO TO 98
 97 ERR=ABS(U(I,J))
 98 ERROR=AMAX1(ERROR,ERR)
 95 CONTINUE
    RETURN
    END
```

```
      SMX22(L2,N1)=-SMX22(L1,N1)
      SMX22(L2,N2)=SMX22(L2,N2)-AB1*PF(I,J,M)*(DLN+ZLAMX)*
     1             PS(I,J,N)*DV
   24 CONTINUE
   23 CONTINUE
   20 CONTINUE
      CALL HEAD
      WRITE (6,2000) MMX,MMX
 2000 FORMAT(///,30X,"SYSTEM MATRIX (2*",I2,",2*",I2,")",/)
      DO 30 L=1,MMX
      WRITE (6,2001) (SMX11(L,M),M=1,MMX),(SMX12(L,M),M=1,MMX)
   30 CONTINUE
      LINE=2*MMX/12
      LINE=LINE*MMX
      IF (LINE.LE.45) GO TO 34
      CALL HEAD
      WRITE (6,2003)
 2003 FORMAT(//,30X,"(MATRIX CONTINUED)",/)
   34 CONTINUE
 2001 FORMAT(5X,12E10.4)
      DO 31 L=1,MMX
      WRITE (6,2001) (SMX21(L,M),M=1,MMX),(SMX22(L,M),M=1,MMX)
   31 CONTINUE
      IF (NPROB.NE.1) GO TO 35
      WRITE (6,2002) NMOD,NMOD
 2002 FORMAT(/,10X,"NOISE MATRIX(",I2,",",I2,")",/)
      DO 33 L=1,NMOD
      WRITE (6,2001) (SMMX(L,M),M=1,NMOD)
   33 CONTINUE
   35 CONTINUE
      RETURN
      END

      SUBROUTINE GINV(A,U,AFLAG,ATEMP,MR,NR,NC,ERROR)
C**** CALCULATES THE GENERALIZED INVERSE OF MATRIX A AND STORE THE
C**** TRANSPOSE OF IT IN A
C**** REF - P381-P387, VOL.9, NO.5, MAY(1966) OF COMMUNICATIONS OF ACM
C
      DIMENSION A(MR,NC), U(NC,NC), AFLAG(NC), ATEMP(NC)
      DIMENSION   B(20,28)
C
      DO 1 I=1,MR
      DO 1 J=1,NC
      B(I,J)=A(I,J)
    1 CONTINUE
      DO 10 I=1,NC
      DO 5 J=1,NC
    5 U(I,J)=0.0
   10 U(I,I)=1.0
      FAC=DOT(MR,1,NR,A,1,1)
      FAC=1.0/SQRT(FAC)
      DO 15 I=1,NR
   15 A(I,1)=A(I,1)*FAC
      DO 20 I=1,NC
```

```
      FUNCTION CNTRLB(A,B,NN,N,N1,N2)
C**** CHECK THE CONTROLLABILITY OF SYSTEM WITH A GIVEN CONTROLLER
C**** SYSTEM IS CONTROLLABLE - CNTRLB=1.0
C
      DIMENSION  A(NN,N), AA(14,28), WORK(14,28), U(28,28), B(NN,N1),
     1           AF(28),  AT(28),  AB(14,14)
C
      DO 5 I=1,N
      DO 5 J=1,N
      AB(I,J)=A(I,J)
    5 CONTINUE
      DO 10 I=1,N
      DO 10 K=1,N1
      WORK(I,K)=B(I,K)
   10 AA(I,K)=WORK(I,K)
      DO 11 J=2,N
      DO 11 L=1,N1
      J1=2*(J-2)+L
      J2=2*(J-1)+L
      DO 12 K=1,N
      AA(K,J2)=DOT3(14,14,1,N,AB,AA,K,J1).
      WORK(K,J2)=AA(K,J2)
   12 CONTINUE
   11 CONTINUE
      CALL GINV(AA,U,AF,AT,14,N,N2,ERROR)
      CNTRLB=0.0
      IF (ERROR.LT.1.0E-5) CNTRLB=1.0
      RETURN
      END

      SUBROUTINE EIGVAL(NM,N,A,B,CA,ALFR,ALFI,BETA,MATZ,Z)
C**** EIGEN VALUE  AND EIGEN VECTOR COMPUTATION FOR GENERAL MATRIX
C**** REF - MATRIX EIGENSYSTEM ROUTINES, EISPACK, B.S.GARBOW,ET.AL.
C
      DIMENSION  A(NM,N),  B(NM,N),  Z(NM,N),  ALFR(N),  ALFI(N),
     1           BETA(N),  CA(NM,N)
      INTEGER  EN
      LOGICAL  MATZ
C
      DO 5 I=1,NM
      DO 5 J=1,N
      A(I,J)=CA(I,J)
    5 CONTINUE
      DO 6 I=1,N
      DO 7 J=1,N
      B(I,J)=0.0
    7 CONTINUE
      B(I,I)=1.0
    6 CONTINUE
      CALL SCHUR(NM,NM,N,A,B,MATZ,Z,IERR)
      IF (IERR.EQ.0) GO TO 10
      WRITE (6,1000)
 1000 FORMAT(/,10X,'****ERROR STOP - FAIL TO GET THE SCHUR FORM'//)
```

```
      STOP
   10 EPSB=B(N,1)
C     COMPUTE EIGENVALUES OF QUASI-TRIANGULAR MATRIX
      ISW=1
      DO 90 NN=1,N
      EN=N+1-NN
      NA=EN-1
      IF (ISW.EQ.2) GO TO 87
      IF (EN.EQ.1) GO TO 62
      IF (A(EN,NA).NE.0.0) GO TO 63
   62 ALFR(EN)=A(EN,EN)
      IF (B(EN,EN).LT.0.0) ALFR(EN)=-ALFR(EN)
      BETA(EN)=ABS(B(EN,EN))
      ALFI(EN)=0.0
      GO TO 90
   63 IF (ABS(B(NA,NA)).LE.EPSB) GO TO 72
      IF (ABS(B(EN,EN)).GT.EPSB) GO TO 64
      A1=A(EN,EN)
      A2=A(EN,NA)
      BN=0.0
      GO TO 67
   64 AN=ABS(A(NA,NA))+ABS(A(NA,EN))+ABS(A(EN,NA))+ABS(A(EN,EN))
      BN=ABS(B(NA,NA))+ABS(B(NA,EN))+ABS(B(EN,EN))
      A11=A(NA,NA)/AN
      A12=A(NA,EN)/AN
      A21=A(EN,NA)/AN
      A22=A(EN,EN)/AN
      B11=B(NA,NA)/BN
      B12=B(NA,EN)/BN
      B22=B(EN,EN)/BN
      E=A11/B11
      EI=A22/B22
      S=A21/(B11*B22)
      T=(A22-E*B22)/B22
      IF (ABS(E).LE.ABS(EI)) GO TO 65
      E=EI
      T=(A11-E*B11)/B11
   65 C=0.5*(T-S*B12)
      D=C*C+S*(A12-E*B12)
      IF (D.LT.0.0) GO TO 76
      E=E+(C+SIGN(SQRT(D),C))
      A11=A11-E*B11
      A12=A12-E*B12
      A22=A22-E*B22
      IF (ABS(A11)+ABS(A12).LT.ABS(A21)+ABS(A22)) GO TO 66
      A1=A12
      A2=A11
      GO TO 67
   66 A1=A22
      A2=A21
   67 S=ABS(A1)+ABS(A2)
      U1=A1/S
      U2=A2/S
      R=SIGN(SQRT(U1*U1+U2*U2),U1)
```

```
      V1=-(U1+R)/R
      V2=-U2/R
      U2=V2/V1
      DO 68 I=1,EN
      T=A(I,EN)+U2*A(I,NA)
      A(I,EN)=A(I,EN)+T*V1
      A(I,NA)=A(I,NA)+T*V2
      T=B(I,EN)+U2*B(I,NA)
      B(I,EN)=B(I,EN)+T*V1
      B(I,NA)=B(I,NA)+T*V2
   68 CONTINUE
      IF (.NOT.MATZ) GO TO 71
      DO 69 I=1,N
      T=Z(I,EN)+U2*Z(I,NA)
      Z(I,EN)=Z(I,EN)+T*V1
      Z(I,NA)=Z(I,NA)+T*V2
   69 CONTINUE
   71 IF (BN.EQ.0.0) GO TO 75
      IF (AN.LT.ABS(E)*BN) GO TO 72
      A1=B(NA,NA)
      A2=B(EN,NA)
      GO TO 73
   72 A1=A(NA,NA)
      A2=A(EN,NA)
   73 S=ABS(A1)+ABS(A2)
      IF (S.EQ.0.0) GO TO 75
      U1=A1/S
      U2=A2/S
      R=SIGN(SQRT(U1*U1+U2*U2),U1)
      V1=-(U1+R)/R
      V2=-U2/R
      U2=V2/V1
      DO 74 J=NA,N
      T=A(NA,J)+U2*A(EN,J)
      A(NA,J)=A(NA,J)+T*V1
      A(EN,J)=A(EN,J)+T*V2
      T=B(NA,J)+U2*B(EN,J)
      B(NA,J)=B(NA,J)+T*V1
      B(EN,J)=B(EN,J)+T*V2
   74 CONTINUE
   75 A(EN,NA)=0.0
      B(EN,NA)=0.0
      ALFR(NA)=A(NA,NA)
      ALFR(EN)=A(EN,EN)
      IF (B(NA,NA).LT.0.0) ALFR(NA)=-ALFR(NA)
      IF (B(EN,EN).LT.0.0) ALFR(EN)=-ALFR(EN)
      BETA(NA)=ABS(B(NA,NA))
      BETA(EN)=ABS(B(EN,EN))
      ALFI(NA)=0.0
      ALFI(EN)=0.0
      GO TO 87
   76 E=E+C
      EI=SQRT(-D)
      A11R=A11-E*B11
```

```
      A11I=EI*B11
      A12R=A12-E*B12
      A12I=EI*B12
      A22R=A22-E*B22
      A22I=EI*B22
      IF (ABS(A11R)+ABS(A11I)+ABS(A12R)+ABS(A12I).LT.ABS(A21)+ABS(A22R)+
     1    ABS(A22I)) GO TO 77
      A1=A12R
      A1I=A12I
      A2=-A11R
      A2I=-A11I
      GO TO 78
   77 A1=A22R
      A1I=A22I
      A2=-A21
      A2I=0.0
   78 CZ=SQRT(A1*A1+A1I*A1I)
      IF (CZ.EQ.0.0) GO TO 79
      SZR=(A1*A2+A1I*A2I)/CZ
      SZI=(A1*A2I-A1I*A2)/CZ
      R=SQRT(CZ*CZ+SZR*SZR+SZI*SZI)
      CZ=CZ/R
      SZR=SZR/R
      SZI=SZI/R
      GO TO 80
   79 SZR=1.0
      SZI=0.0
   80 IF (AN.LT.(ABS(E)+EI)*BN) GO TO 81
      A1=CZ*B11+SZR*B12
      A1I=SZI*B12
      A2=SZR*B22
      A2I=SZI*B22
      GO TO 82
   81 A1=CZ*A11+SZR*A12
      A1I=SZI*A12
      A2=CZ*A21+SZR*A22
      A2I=SZI*A22
   82 CQ=SQRT(A1*A1+A1I*A1I)
      IF (CQ.EQ.0.0) GO TO 83
      SQR=(A1*A2+A1I*A2I)/CQ
      SQI=(A1*A2I-A1I*A2)/CQ
      R=SQRT(CQ*CQ+SQR*SQR+SQI*SQI)
      CQ=CQ/R
      SQR=SQR/R
      SQI=SQI/R
      GO TO 84
   83 SQR=1.0
      SQI=0.0
   84 SSR=SQR*SZR+SQI*SZI
      SSI=SQR*SZI-SQI*SZR
      I=1
      TR=CQ*CZ*A11+CQ*SZR*A12+SQR*CZ*A21+SSR*A22
      TI=CQ*SZI*A12-SQI*CZ*A21+SSI*A22
      DR=CQ*CZ*B11+CQ*SZR*B12+SSR*B22
```

```
      DI=CQ*SZI*B12+SSI*B22
      GO TO 86
   85 I=2
      TR=SSR*A11-SQR*CZ*A12-CQ*SZR*A21+CQ*CZ*A22
      TI=-SSI*A11-SQI*CZ*A12+CQ*SZI*A21
      DR=SSR*B11-SQR*CZ*B12+CQ*CZ*B22
      DI=-SSI*B11-SQI*CZ*B12
   86 T=TI*DR-TR*DI
      J=NA
      IF (T.LT.0.0) J=EN
      R=SQRT(DR*DR+DI*DI)
      BETA(J)=BN*R
      ALFR(J)=AN*(TR*DR+TI*DI)/R
      ALFI(J)=AN*T/R
      IF (I.EQ.1) GO TO 85
   87 ISW=3-ISW
   90 CONTINUE
      RETURN
      END

      SUBROUTINE EIGVEC(NM,N,A,B,ALFR,ALFI,BETA,Z)
C**** COMPUTES EIGEN-VECTORS
C
      DIMENSION  A(NM,N),  B(NM,N),  ALFR(N),  ALFI(N),  BETA(N),
     1           Z(NM,N)
      INTEGER  EN,ENM2
C
      EPSB=B(N,1)
      ISW=1
      DO 10 NN=1,N
      EN=N+1-NN
      NA=EN-1
      IF (ISW.EQ.2) GO TO 11
      IF (ALFI(EN).NE.0.0) GO TO 30
      M=EN
      B(EN,EN)=1.0
      IF (NA.EQ.0) GO TO 10
      ALFM=ALFR(M)
      BETM=BETA(M)
      DO 20 II=1,NA
      I=EN-II
      W=BETM*A(I,I)-ALFM*B(I,I)
      R=0.0
      DO 21 J=M,EN
   21 R=R+(BETM*A(I,J)-ALFM*B(I,J))*B(J,EN)
      IF (I.EQ.1.OR.ISW.EQ.2) GO TO 22
      IF (BETM*A(I,I-1).EQ.0.0) GO TO 22
      ZZ=W
      S=R
      GO TO 25
   22 M=I
      IF (ISW.EQ.2) GO TO 23
      T=W
      IF (W.EQ.0.0) T=EPSB
```

```
      B(I,EN)=-R/T
      GO TO 20
   23 X=BETM*A(I,I+1)-ALFM*B(I,I+1)
      Y=BETM*A(I+1,I)
      Q=W*ZZ-X*Y
      T=(X*S-ZZ*R)/Q
      B(I,EN)=T
      IF (ABS(X).LE.ABS(ZZ)) GO TO 24
      B(I+1,EN)=(-R-W*T)/X
      GO TO 25
   24 B(I+1,EN)=(-S-Y*T)/ZZ
   25 ISW=3-ISW
   20 CONTINUE
      GO TO 10
   30 M=NA
      ALMR=ALFR(M)
      ALMI=ALFI(M)
      BETM=BETA(M)
      Y=BETM*A(EN,NA)
      B(NA,NA)=-ALMI*B(EN,EN)/Y
      B(NA,EN)=(ALMR*B(EN,EN)-BETM*A(EN,EN))/Y
      B(EN,NA)=0.0
      B(EN,EN)=1.0
      ENM2=NA-1
      IF (ENM2.EQ.0) GO TO 11
      DO 40 II=1,ENM2
      I=NA-II
      W=BETM*A(I,I)-ALMR*B(I,I)
      W1=-ALMI*B(I,I)
      RA=0.0
      SA=0.0
      DO 41 J=M,EN
      X=BETM*A(I,J)-ALMR*B(I,J)
      X1=-ALMI*B(I,J)
      RA=RA+X*B(J,NA)-X1*B(J,EN)
      SA=SA+X*B(J,EN)+X1*B(J,NA)
   41 CONTINUE
      IF (I.EQ.1.OR.ISW.EQ.2) GO TO 42
      IF (BETM*A(I,I-1).EQ.0.0) GO TO 42
      ZZ=W
      Z1=W1
      R=RA
      S=SA
      ISW=2
      GO TO 40
   42 M=I
      IF (ISW.EQ.2) GO TO 47
      TR=-RA
      TI=-SA
   44 DR=W
      DI=W1
   45 IF (ABS(DI).GT.ABS(DR)) GO TO 46
      RR=DI/DR
      D=DR+DI*RR
```

```
      T1=(TR+TI*RR)/D
      T2=(TI-TR*RR)/D
      IF (ISW-1) 50,50,48
   46 RR=DR/DI
      D=DR*RR+DI
      T1=(TR*RR+TI)/D
      T2=(TI*RR-TR)/D
      IF (ISW-1) 50,50,48
   47 X=BETM*A(I,I+1)-ALMR*B(I,I+1)
      X1=-ALMI*B(I,I+1)
      Y=BETM*A(I+1,I)
      TR=Y*RA-W*R+W1*S
      TI=Y*SA-W*S-W1*R
      DR=W*ZZ-W1*Z1-X*Y
      DI=W*Z1+W1*ZZ-X1*Y
      IF (DR.EQ.0.0.AND.DI.EQ.0.0) DR=EPSB
      GO TO 45
   48 B(I+1,NA)=T1
      B(I+1,EN)=T2
      ISW=1
      IF (ABS(Y).GT.ABS(W)+ABS(W1)) GO TO 49
      TR=-RA-X*B(I+1,NA)+X1*B(I+1,EN)
      TI=-SA-X*B(I+1,EN)-X1*B(I+1,NA)
      GO TO 44
   49 T1=(-R-ZZ*B(I+1,NA)+Z1*B(I+1,EN))/Y
      T2=(-S-ZZ*B(I+1,EN)-Z1*B(I+1,NA))/Y
   50 B(I,NA)=T1
      B(I,EN)=T2
   40 CONTINUE
   11 ISW=3-ISW
   10 CONTINUE
      DO 60 JJ=1,N
      J=N+1-JJ
      DO 61 I=1,N
      Z(I,J)=DOT3(NM,NM,1,J,Z,B,I,J)
   61 CONTINUE
   60 CONTINUE
C     NORMALIZATION TO NORM OF EIGEN-VECTOR
      DO 70 J=1,N
      IF (ISW.EQ.2) GO TO 73
      IF (ALFI(J).NE.0.0) GO TO 76
      D=DOT(NM,1,N,Z,J,J)
      D=SQRT(D)
      DO 72 I=1,N
   72 Z(I,J)=Z(I,J)/D
      GO TO 70
   73 D=DOT(NM,1,N,Z,J-1,J-1)+DOT(NM,1,N,Z,J,J)
      D=SQRT(D)
      DO 75 I=1,N
      Z(I,J-1)=Z(I,J-1)/D
      Z(I,J)=Z(I,J)/D
   75 CONTINUE
   76 ISW=3-ISW
   70 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE COTMX(MR,NMAX,NS,A,WR,WI)
C**** CALCULATES THE COEFFICIENT MATRIX OF TRANSITION MATRIX
      DIMENSION    A(MR,NMAX),  WR(MR),   WI(MR)
      COMPLEX      CA(14,14),   CA1(14,14),    CA2(14,14),
     1             TCA(14,14),  DENOM(14),     CW(14)
      COMMON       AFR(20,20),  AFI(20,20)
C
      CALL HEAD
      WRITE (6,1000) NS
 1000 FORMAT(//,10X,'TRANSITION MATRICES FOR THE MODE = ',
     1I2,/)
      DO 10 L=1,NMAX
      CW(L)=CMPLX(WR(L),WI(L))
      DO 10 M=1,NMAX
      CA(L,M)=CMPLX(A(L,M),0.0)
   10 CONTINUE
      DO 11 M=1,NMAX
      DENOM(M)=CW(NS)-CW(M)
   11 CONTINUE
      L=NS
      DO 30 N=1,NMAX
      IF (N-L) 31,30,32
   31 IF (N-1) 35,35,40
   32 IF (L-1) 33,33,34
   33 IF (N-L-1) 35,35,40
   34 IF (N-L-1) 35,40,40
   35 DO 36 LL=1,NMAX
      DO 36 MM=1,NMAX
      CA1(LL,MM)=CA(LL,MM)
   36 CONTINUE
      DO 37 LL=1,NMAX
      CA1(LL,LL)=CA(LL,LL)-CW(N)
      DO 37 MM=1,NMAX
      CA1(LL,MM)=CA1(LL,MM)/DENOM(N)
   37 CONTINUE
      GO TO 30
   40 CONTINUE
      DO 41 LL=1,NMAX
      DO 41 MM=1,NMAX
      CA2(LL,MM)=CA(LL,MM)
   41 CONTINUE
      DO 42 LL=1,NMAX
      CA2(LL,LL)=CA(LL,LL)-CW(N)
      DO 42 MM=1,NMAX
      CA2(LL,MM)=CA2(LL,MM)/DENOM(N)
   42 CONTINUE
      DO 43 LL=1,NMAX
      DO 43 MM=1,NMAX
      TCA(LL,MM)=(0.0,0.0)
      DO 43 NN=1,NMAX
      TCA(LL,MM)=TCA(LL,MM)+CA1(LL,NN)*CA2(NN,MM)
   43 CONTINUE
      DO 44 LL=1,NMAX
```

```
      DO 44 MM=1,NMAX
      CA1(LL,MM)=TCA(LL,MM)
   44 CONTINUE
   30 CONTINUE
      IF (AIMAG(CW(L))) 50,60,50
   50 CONTINUE
      WRITE (6,1001) WR(L),WI(L),WI(L)
 1001 FORMAT(10X,'IN THE FORM OF EXP(',1PE12.5,'*T)*(AFR*COS(',
     11PE12.5,')T +/- I*AFI*SIN(',1PE12.5,')T)',/)
      DO 51 M=1,NMAX
      DO 51 N=1,NMAX
      AFR(M,N)=REAL(CA1(M,N))**2.0
      AFI(M,N)=AIMAG(CA1(M,N))**2.0
   51 CONTINUE
      WRITE (6,1002) NMAX,NMAX
 1002 FORMAT(10X,'MATRIX AFR (',I2,',',I2,')',//)
      DO 52 M=1,NMAX
      WRITE (6,1003) (AFR(M,N),N=1,NMAX)
   52 CONTINUE
      CALL HEAD
 1003 FORMAT(5X,10E12.5)
      WRITE (6,1004) NMAX,NMAX
 1004 FORMAT(/,10X,'MATRIX AFI (',I2,',',I2,')',//)
      DO 53 M=1,NMAX
      WRITE (6,1003) (AFI(M,N),N=1,NMAX)
   53 CONTINUE
      RETURN
   60 CONTINUE
      WRITE (6,1005) WR(L)
 1005 FORMAT(10X,'IN THE FORM OF AFR*EXP(',1PE12.5,')T',/)
      WRITE (6,1006) NMAX,NMAX
 1006 FORMAT(10X,'MATRIX AFR (',I2,',',I2,')',//)
      DO 61 M=1,NMAX
      DO 61 N=1,NMAX
      AFR(M,N)=REAL(CA1(M,N))
      AFI(M,N)=0.0
   61 CONTINUE
      DO 62 M=1,NMAX
      WRITE (6,1003) (AFR(M,N),N=1,NMAX)
   62 CONTINUE
      RETURN
      END

      SUBROUTINE SPCON
C**** DESIGN THE SPATIAL CONTROLLERS
C****
      COMMON /STFN/  XF(31,23),  XT(31,23),  XD(31,23),  XE(31,23)
      COMMON /XSCN/  DIF1(25),   DIF2(25),   SIGA1(25),   SIGA2(25),
     1               SIGR(25),   SIGF(25),   ALFA(25),   HFAC(25),
     2               XK(10),     V1,         V2,         XNU,
     3               GAMI,       GAMX,       SIGXE,      ZLAMI,
     4               ZLAMX,      XKS
      COMMON /GEOM/  MAT(31,23),  DX(31),  DY(23),  X(31),  Y(23)
      COMMON /MISC/  TITLE(18),  NPAGE,      NMOD,       MM2,        MM4,
```

```
     1                NRX,          NRY,          NTT,          PWR,     BLANK,
     2                CX,     CY,    NSX,    NSY,   IUNSTB,   RFAC,  SFAC,
     3                QFAC,    AMPO
      COMMON /SYSM/  SMX11(20,20),  SMX12(20,20),  SMX21(21,21),
     1               SMX22(20,20),  SMMX(20,20)
      COMMON /XYMOD/ PF(31,23,8),  PS(31,23,8),  FNORM(8),  TNORM(8)
      COMMON /DCON/  YNOD(20),   XNOD(15,20),   NODY,   NODX(20),
     1               NODY1,   NODY2
      COMMON     AFR(20,20),    AFI(20,20)
      DIMENSION    P(2,2),   PCN(10,2),   DMTX(20,20),   RMTX(20,2),
     1            GMTP(20,20),   GMTX(20,20),   GMTY(20,20),   XI(2),
     2            YJ(2),   BMTX(20,2),   BMTXX(20),    BMTXY(20),
     3            GMTN(20)
C**** COMPUTE THE EFFECTIVENESS OF SPATIAL CONTROL
C
      EP=1.0E-10
      KMAX=2
      VV=V2/V1
      NRX1=NRX-1
      NRY1=NRY-1
      READ (5,1010) NSX,NSY,NZC
 1010 FORMAT(16I5)
      CALL HEAD
      WRITE (6,1100)
 1100 FORMAT(///,30X,'SPATIAL CONTROL INFORMATION'//)
      WRITE (6,1101) NSX,NSY,NZC
 1101 FORMAT(10X,'SYMMETRY AXIS FOR X = ',I5,/,10X,'SYMMETRY AXIS FOR Y
     1= ',I5,/,10X,'SYMMETRICITY OF CONTROLLERS (=1; Y-SYMMETRIC, =2; CE
     2NTRE SYMMETRIC) = ',I5,/)
      IF (NZC.NE.0) GO TO 5
      KMAX=1
      WRITE (6,1005)
 1005 FORMAT(/,10X,'WARNING - SINGLE CONTROLLER SYSTEM MAY NOT BE COMPLE
     1TELY CONTROLLABLE',/)
    5 CONTINUE
C     CONSTRUCT CONTROLLER DOMAIN
      READ (5,1000) NODY,DCX,DCY
 1000 FORMAT(I5,2F10.5)
      WRITE (6,1001) NODY, DCX,DCY
 1001 FORMAT(10X,'NO OF CONTROLLER NODES IN Y-DIRECTION = ',I5,/,10X,'NO
     1DE SIZE IN X-DIRECTION = ',F10.5,/,10X,'NODE SIZE IN Y-DIRECTION =
     2 ',F10.5,//)
      WRITE (6,1004)
 1004 FORMAT(30X,'CONTROLLER LOCATIONS WHERE THE EFFECTIVENESSES ARE COM
     1PUTED',/)
      NODY1=NODY+1
      READ (5,1010) (NODX(NY),NY=1,NODY1)
      WRITE (6,1002)
 1002 FORMAT(17X,'X-DIR=   1                2                3                4                5
     1          6           7            8             9             10',//,10X,'Y-DIR = '
     2,/)
      NODY2=2*NODY+1
      DO 10 NY=1,NODY2
      YNOD(NY)=CY+(NY-NODY1)*DCY
```

```
      IF (NY.LE.NODY1) NNX=NODX(NY)
      IF (NY.GT.NODY1) NNX=NODX(NODY2-NY+1)
      DO 11 NX=1,NNX
      XNOD(NX,NY)=CX+(NX-NNX)*DCX
   11 CONTINUE
      WRITE (6,1003) YNOD(NY),(XNOD(NX,NY),NX=1,NNX)
 1003 FORMAT(10X,11(F9.4,1X))
   10 CONTINUE
      CALL HEAD
      NNX=2*NODX(NODY1)-1
      DO 12 NX=1,NNX
      DO 12 NY=1,NODY2
      XD(NX,NY)=1.0E+10
   12 CONTINUE
      QSC=0.0
      DO 13 L=1,MM2
      QSC=QSC+DOT4(20,20,1,MM2,SMX22,SMX22,L,L)
   13 CONTINUE
      QSC=SQRT(QSC)
      QSC=QSC/MM2
      DO 14 L=1,MM2
      DO 14 M=1,MM2
      SMX22(L,M)=SMX22(L,M)/QSC
   14 CONTINUE
      MM1=NMOD-1
      NSX1=NSX+1
      NSY1=NSY+1
C     COMPUTES THERMAL FLUXES AT THE CENTRE OF CORE
      XI(1)=X(NSX)
      XI(2)=X(NSX1)
      YJ(1)=Y(NSY)
      YJ(2)=Y(NSY1)
      F(1,1)=XT(NSX,NSY)
      F(1,2)=XT(NSX,NSY1)
      F(2,1)=XT(NSX1,NSY)
      F(2,2)=XT(NSX1,NSY1)
      XTOC=XINT(CX,CY,XI,YJ,F)
      IDX1=MAT(NSX,NSY)
      IDX2=MAT(NSX,NSY1)
      IDX3=MAT(NSX1,NSY)
      IDX4=MAT(NSX1,NSY1)
      DV1=(CX-X(NSX))*(CY-Y(NSY))
      DV2=(CX-X(NSX))*(Y(NSY1)-CY)
      DV3=(X(NSX1)-CX)*(CY-Y(NSY))
      DV4=(X(NSX1)-CX)*(Y(NSY1)-CY)
      SIGRD=(SIGR(IDX1)*XF(NSX,NSY)*DV1+SIGR(IDX2)*XF(NSX,NSY1)*DV2+
     1      SIGR(IDX3)*XF(NSX1,NSY)*DV3+SIGR(IDX4)*XF(NSX1,NSY1)*DV4)/
     2       (XF(NSX,NSY)*DV1+XF(NSX,NSY1)*DV2+XF(NSX1,NSY)*DV3+
     3      XF(NSX1,NSY1)*DV4)
      SIGFD=(SIGF(IDX1)*F(1,1)*DV1+SIGF(IDX2)*F(1,2)*DV2+
     1      SIGF(IDX3)*F(2,1)*DV3+SIGF(IDX4)*F(2,2)*DV4)/
     1      (F(1,1)*DV1+F(1,2)*DV2+F(2,1)*DV3+F(2,2)*DV4)
      WFACO=SQRT(SIGRD/SIGFD/XNU)
      WRITE (6,1013) QSC
```

```
 1013 FORMAT(/,10X,'SCALE FACTOR FOR SYSTEM MATRIX = ',E12.5,/)
C     COMPUTES  ELEMENTS OF CONTROLLER MATRIX AT THE MESH
      WRITE (6,1008)
 1008 FORMAT(/,30X,'CONVERGENCE INFORMATIONS',//,10X,
     1'  X-NODE     Y-NODE   ITER.NO.     CONVERGEN  LYAPUVOV ERR   IT.TIME
     1(SEC) INV.ERROR  R-FACTOR     Q-FACTOR',/)
      NYMAX=NODY2
      IF (NZC.EQ.2) NYMAX=NODY1
      DO 20 NY=1,NYMAX
      NY1=NY
      IF (NZC.EQ.2) NY1=2*NODY1-NY
      IF (NY.LE.NODY1) NNX=NODX(NY)
      IF (NY.GT.NODY1) NNX=NODX(NODY2-NY+1)
      DO 21 J=1,NRY1
      IF (YNOD(NY).LT.Y(J).OR.YNOD(NY).GE.Y(J+1)) GO TO 21
      JJ=J
       J(1)=Y(JJ)
      YJ(2)=Y(JJ+1)
      YJ0=YNOD(NY)
   21 CONTINUE
      DO 20 NX=1,NNX
      NX1=2*NNX-NX
      IF (YNOD(NY).EQ.CY.AND.XNOD(NX,NY).EQ.CX) GO TO 42
      DO 22 I=1,NRX1
      IF (XNOD(NX,NY).LT.X(I).OR.XNOD(NX,NY).GE.X(I+1)) GO TO 22
      II=I
      XI(1)=X(II)
      XI(2)=X(II+1)
      XI0=XNOD(NX,NY)
   22 CONTINUE
      CALL SECOND(SEC1)
      DO 23 KK=1,KMAX
      IF (KK.EQ.1) GO TO 230
      II=2*NSX-II-1
      XI(1)=X(II)
      XI(2)=X(II+1)
      XI0=2.0*CX-XI0
      IF (NZC.NE.2) GO TO 230
      JJ=2*NSY-JJ-1
      YJ(1)=Y(JJ)
      YJ(2)=Y(JJ+1)
      YJ0=2.0*CY-YJ0
  230 CONTINUE
      P(1,1)=XT(II,JJ)
      P(1,2)=XT(II,JJ+1)
      P(2,1)=XT(II+1,JJ)
      P(2,2)=XT(II+1,JJ+1)
      XT0=XINT(XI0,YJ0,XI,YJ,P)
      IDX1=MAT(II,JJ)
      IDX2=MAT(II,JJ+1)
      IDX3=MAT(II+1,JJ)
      IDX4=MAT(II+1,JJ+1)
      DV1=(XI0-XI(1))*(YJ0-YJ(1))
      DV2=(XI0-XI(1))*(YJ(2)-YJ0)
```

```
      DV3=(XI(2)-XI0)*(YJ0-YJ(1))
      DV4=(XI(2)-XI0)*(YJ(2)-YJ0)
      SIGRD=(SIGR(IDX1)*XF(II,JJ)*DV1+SIGR(IDX2)*XF(II,JJ+1)*DV2+
     1      SIGR(IDX3)*XF(II+1,JJ)*DV3+SIGR(IDX4)*XF(II+1,JJ+1)*DV4)/
     2      (XF(II,JJ)*DV1+XF(II,JJ+1)*DV2+XF(II+1,JJ)*DV3+
     3      XF(II+1,JJ+1)*DV4)
      SIGFD=(SIGF(IDX1)*XT(II,JJ)*DV1+SIGF(IDX2)*XT(II,JJ+1)*DV2+
     1      SIGF(IDX3)*XT(II+1,JJ)*DV3+SIGF(IDX4)*XT(II+1,JJ+1)*DV4)/
     2      (XT(II,JJ)*DV1+XT(II,JJ+1)*DV2+XT(II+1,JJ)*DV3+
     3      XT(II+1,JJ+1)*DV4)
      DO 24 M=2,NMOD
      M1=M-1
      P(1,1)=PS(II,JJ,M)
      P(1,2)=PS(II,JJ+1,M)
      P(2,1)=PS(II+1,JJ,M)
      P(2,2)=PS(II+1,JJ+1,M)
      PCN(M1,KK)=XINT(XI0,YJ0,XI,YJ,P)/SQRT(XNU*SIGFD)
   24 CONTINUE
      WFAC=SQRT(SIGRD/SIGFD/XNU)
      DO 25 M=1,MM1
      M1=MM1+M
      BMTX(M,KK)=0.0
      AMOD=-PCN(M,KK)*TNORM(M+1)*WFAC/FNORM(M+1)
      TEMP=WFAC*XTO**2/(WFACO*XTOC)
      BMTX(M1,KK)=AMOD*VV*(XTO-TEMP)/(AMPO*SQRT(SIGRD))
   25 CONTINUE
C     CHECK CONTROLLABILITY FOR A GIVEN CONTROLLER
      DO 26 L=1,MM2
      RMTX(L,KK)=0.0
      DO 26 M=1,MM2
      RMTX(L,KK)=RMTX(L,KK)-SMMX(L,M)*BMTX(M,KK)
   26 CONTINUE
   23 CONTINUE
      MXX=KMAX*MM2
      CHECK=CNTRLB(SMX22,RMTX,20,MM2,KMAX,MXX)
      IF (CHECK.EQ.0.0) GO TO 42
C     CALCULATES RMTX*TRANS(RMTX)/RFAC
      DO 27 L=1,MM2
      DO 27 M=L,MM2
      GMTX(L,M)=DOT4(20,20,1,KMAX,RMTX,RMTX,L,M)
      GMTX(M,L)=GMTX(L,M)
   27 CONTINUE
      DO 270 L=1,MM2
      DO 270 M=1,MM2
      SMX11(L,M)=0.0
  270 CONTINUE
      DO 271 L=1,MM1
      L1=MM1+L
      DO 271 M=L,MM1
      M1=MM1+M
      SMX11(L,M)=DOT4(10,10,1,KMAX,PCN,PCN,L,M)
      SMX11(L1,M1)=SMX11(L,M)
      SMX11(M,L)=SMX11(L,M)
      SMX11(M1,L1)=SMX11(L,M)
  271 CONTINUE
```

```
C     NORMALIZATION OF MATRICES
      RNORM=QNORM=0.0
      DO 272 L=1,MM2
      RNORM=RNORM+DOT4(20,20,1,MM2,GMTX,GMTX,L,L)
      QNORM=QNORM+DOT4(20,20,1,MM2,SMX11,SMX11,L,L)
  272 CONTINUE
      QFAC=SQRT(QNORM)/MM2
      RFAC=SQRT(RNORM)/MM2
      DO 273 L=1,MM2
      DO 273 M=1,MM2
      GMTX(L,M)=GMTX(L,M)/RFAC
      SMX11(L,M)=SMX11(L,M)*QFAC
  273 CONTINUE
C     INITIAL SOLUTION OF THE RICCATI EQUATION
      IL=1
      THET=1.0E-2
      XTIME=5.0E-1
      DO 28 L=1,MM2
      DO 28 M=1,MM2
      GMTP(L,M)=GMTX(L,M)*XTIME
      DMTX(L,M)=GMTP(L,M)*XTIME
   28 CONTINUE
   50 CONTINUE
      ERR=0.0
      XIL=IL+1.0
      DO 51 L=1,MM2
      DO 51 M=L,MM2
      GMTY(L,M)=-(DOT3(20,20,1,MM2,SMX22,DMTX,L,M)+
     1            DOT4(20,20,1,MM2,DMTX,SMX22,L,M))/XIL
      GMTY(M,L)=GMTY(L,M)
   51 CONTINUE
      DO 52 L=1,MM2
      DO 52 M=1,MM2
      GMTP(L,M)=GMTP(L,M)+GMTY(L,M)
      DMTX(L,M)=GMTY(L,M)*XTIME
      XERR=ABS(GMTY(L,M)/GMTP(L,M))
      IF (XERR.GE.ERR) ERR=XERR
   52 CONTINUE
      IF (ERR.LE.EP) GO TO 53
      IL=IL+1
      IF (IL.GE.30) GO TO 53
      GO TO 50
   53 CONTINUE
      CALL GINV(GMTP,DMTX,BMTXX,BMTXY,20,MM2,MM2,ERROR)
      IT=0
      DO 54 L=1,MM2
      DO 54 M=1,MM2
      AFR(L,M)=GMTP(L,M)
   54 CONTINUE
C     ITERATION FOR THE RICCATI EQUATION
  100 CONTINUE
      EPS=1.0
      DO 30 L=1,MM2
      DO 30 M=1,MM2
```

```
      SMX12(L,M)=SMX22(L,M)-DOT3(20,20,1,MM2,GMTX,GMTP,L,M)
      GMTY(L,M)=-SMX11(L,M)
      DO 31 LL=1,MM2
      CC=0.0
      DO 37 MM=1,MM2
      CC=CC+GMTX(LL,MM)*GMTP(MM,M)
   37 CONTINUE
      GMTY(L,M)=GMTY(L,M)-GMTP(LL,L)*CC
   31 CONTINUE
   30 CONTINUE
      CALL AXPXA(SMX12,GMTP,GMTY,DMTX,MM2,20,20,20,20,EPS)
      CON=0.0
      DO 32 L=1,MM2
      DO 32 M=1,MM2
      GMTY(L,M)=GMTY(L,M)*QSC
      IF (AFR(L,M).EQ.0.0) GO TO 32
      ERR=GMTY(L,M)/AFR(L,M)
      CTRC1=ABS(1.0-ERR)
      CON=AMAX1(CON,CTRC1)
   32 CONTINUE
      IF (CON.LE.1.0E-2) GO TO 40
      DO 34 L=1,MM2
      DO 34 M=1,MM2
      AFR(L,M)=GMTY(L,M)
      DMTX(L,M)=0.0
      DO 35 LL=1,MM2
      DMTX(L,M)=DMTX(L,M)-GMTY(LL,L)*DOT3(20,20,1,MM2,GMTX,GMTY,LL,M)
   35 CONTINUE
      DMTX(L,M)=DMTX(L,M)+DOT3(20,20,1,MM2,GMTY,SMX22,L,M)+
     1        SMX11(L,M)+DOT2(20,20,1,MM2,SMX22,GMTY,L,M)
   34 CONTINUE
      IT=IT+1
      DO 36 L=1,MM2
      DO 36 M=1,MM2
      GMTP(L,M)=GMTY(L,M)-THET*DMTX(L,M)
   36 CONTINUE
      IF (IT.LE.50) GO TO 100
   40 CONTINUE
      CALL SECOND(SEC2)
      SEC=SEC2-SEC1
      WRITE (6,1009) NX,NY,IT,CON,EPS,SEC,ERROR,RFAC,QFAC
 1009 FORMAT(11X,I5,5X,I5,5X,I5,5X,6E12.5)
      IF (CON.GT.1.0E-2) GO TO 44
      XD(NX,NY)=0.0
      XD(NX1,NY1)=0.0
      DO 41 K=1,KMAX
      XI(K)=0.0
      DO 43 L=1,MM2
      TEMP=-DOT2(20,20,1,MM2,RMTX,GMTP,K,L)/RFAC
      XI(K)=XI(K)+TEMP*TEMP
   43 CONTINUE
      XI(K)=SQRT(XI(K))
   41 CONTINUE
      XD(NX,NY)=XI(1)
```

```
      XD(NX1,NY1)=XI(KMAX)
      IF (NX.EQ.NX1.AND.NY.EQ.NY1) XD(NX1,NY1)=XI(1)
      IF (NZC.NE.2) GO TO 20
      XD(NX1,NY)=XI(1)
      XD(NX,NY1)=XI(KMAX)
      IF (NY.EQ.NY1) XD(NX,NY1)=XI(1)
      GO TO 20
   42 XD(NX,NY)=1.0E+10
      XD(NX1,NY1)=1.0E+10
      GO TO 20
   44 XD(NX,NY)=-1.0
      XD(NX1,NY1)=-1.0
   20 CONTINUE
C     PRINT THE EFFECTIVENESS MAP
      CALL HEAD
      WRITE (6,1014)
 1014 FORMAT(///,30X,'CONTROLLER EFFECTIVENESS MAP',//)
      NNX=NODX(NODY1)
      DO 70 NX=1,NNX
      BMTXX(NX)=XNOD(NNX,NODY1)+DCX*(NX-1)
   70 CONTINUE
      WRITE (6,1015) (XNOD(NX,NODY1),NX=1,NNX),(BMTXX(NX),NX=2,NNX)
 1015 FORMAT(10X,12F9.2)
      NXX=NODX(NODY1)
      NX2=2*NXX-1
      DO 71 NY=1,NODY2
      IF (NY.LE.NODY1) NNX=NODX(NY)
      IF (NY.GT.NODY1) NNX=NODX(NODY2-NY+1)
      NDX=NXX-NNX
      IF (NDX.LE.0.0) GO TO 74
      NNX=2*NNX-1
      DO 72 NX=1,NNX
      ND=NNX+NDX-NX+1
      ND1=NNX-NX+1
      XD(ND,NY)=XD(ND1,NY)
   72 CONTINUE
      DO 73 NX=1,NDX
      XD(NX,NY)=1.0E+10
   73 CONTINUE
   74 CONTINUE
      WRITE (6,1016) YNOD(NY),(XD(NX,NY),NX=1,NX2)
 1016 FORMAT(F9.2,1X,12E9.3,/)
   71 CONTINUE
      RETURN
      END

      SUBROUTINE AXPXA(A,U,C,B,N,NA,NC,NU,NB,EPS)
C**** SOLVES THE REAL MATRIX EQUATION TRANS(A)*X+X*A=C.
C**** REF - SOLUTION OF THE MATRIX EQUATION AX+XB=C, BARTELS AND STEWART
C****       VOL. 15, NO. 9, COMMUN. OF ACM (1972)
C
      DIMENSION  A(NA,1),  U(NU,1),  C(NC,1),  B(NB,1)
      DIMENSION  D(14,14)
```

```
C
      N1=N+1
      NM1=N-1
      IC=1
      DO 5 I=1,N
      DO 6 J=1,N
      B(I,J)=0.0
      D(I,J)=0.0
    6 CONTINUE
      B(I,I)=1.0
    5 CONTINUE
      IF (EPS.LT.0.0) GO TO 17
      CALL SCHUR(NA,NB,N,A,B,.TRUE.,U,IERR)
      IF (IERR.EQ.0) GO TO 15
      WRITE (6,1000)
 1000 FORMAT(10X,"***ERROR STOP - FAILED TO GET THE SCHUR FORM"/)
      STOP
   15 CONTINUE
      ICMAX=2
      IF (EPS.NE.1.0) ICMAX=10
   17 CONTINUE
      DO 20 I=1,N
      C(I,I)=C(I,I)/2.0
   20 CONTINUE
      DO 40 I=1,N
      DO 30 J=1,N
      A(N1,J)=DOT2(NC,NU,I,N,C,U,I,J)
   30 CONTINUE
      DO 35 J=1,N
   35 C(I,J)=A(N1,J)
   40 CONTINUE
      DO 60 J=1,N
      DO 50 I=1,N
      A(I,N1)=DOT2(NU,NC,1,N,U,C,I,J)
   50 CONTINUE
      DO 55 I=1,N
   55 C(I,J)=A(I,N1)
   60 CONTINUE
      DO 70 I=1,N
      DO 70 J=I,N
      C(I,J)=C(I,J)+C(J,I)
      C(J,I)=C(I,J)
   70 CONTINUE
C     SOLVE THE TRANSFORMED EQUATIONS
  200 CONTINUE
      DO 75 I=1,N
      DO 75 J=1,N
   75 B(I,J)=C(I,J)
      CALL SYMSLV(A,B,N,NA,NC)
      CMAX=0.0
      DO 76 I=1,N
      DO 76 J=1,N
      D(I,J)=D(I,J)+C(I,J)
```

```
      TEMP=B(I,J)-DOT2(NA,NC,1,N,A,C,I,J)-DOT3(NC,NA,1,N,C,A,I,J)
      IF (B(I,J).EQ.0.0) GO TO 71
      ERR=ABS(TEMP/B(I,J))
   71 B(I,J)=TEMP
      CMAX=AMAX1(CMAX,ERR)
   76 CONTINUE
      IF (CMAX.LE.1.0E-5) GO TO 300
      IC=IC+1
      DO 77 I=1,N
      DO 77 J=I,N
      C(I,J)=B(I,J)
      C(J,I)=C(I,J)
   77 CONTINUE
      IF (IC.LE.ICMAX) GO TO 200
  300 CONTINUE
      EPS=CMAX
      DO 78 I=1,N
      DO 78 J=I,N
      C(I,J)=D(I,J)
      C(J,I)=C(I,J)
   78 CONTINUE
      DO 80 I=1,N
      C(I,I)=C(I,I)/2.0
   80 CONTINUE
      DO 100 I=1,N
      DO 90 J=1,N
      A(N1,J)=DOT4(NC,NU,I,N,C,U,I,J)
   90 CONTINUE
      DO 95 J=1,N
   95 C(I,J)=A(N1,J)
  100 CONTINUE
      DO 120 J=1,N
      DO 110 I=1,N
      A(I,N1)=DOT3(NU,NC,1,N,U,C,I,J)
  110 CONTINUE
      DO 115 I=1,N
  115 C(I,J)=A(I,N1)
  120 CONTINUE
      DO 130 I=1,N
      DO 130 J=I,N
      C(I,J)=C(I,J)+C(J,I)
      C(J,I)=C(I,J)
  130 CONTINUE
      RETURN
      END

      SUBROUTINE SYMSLV(A,C,N,NA,NC)
C**** SOLVES THE TRANSFORMED MATRIX EQUATION
C
      DIMENSION  A(NA,1), C(NC,1)
      INTEGER  DK, DL
      COMMON /SLVBLK/  T(5,5),  P(5),  NSYS
```

```
C
      L=1
   10 DL=1
      IF (L.EQ.N) GO TO 20
      IF (A(L+1,L).NE.0.0) DL=2
   20 LL=L+DL-1
      K=L
   30 KM1=K-1
      DK=1
      IF (K.EQ.N) GO TO 35
      IF (A(K+1,K).NE.0.0) DK=2
   35 KK=K+DK-1
      IF (K.EQ.L) GO TO 45
      DO 40 I=K,KK
      DO 40 J=L,LL
      C(I,J)=C(I,J)-DOT2(NA,NC,L,KM1,A,C,I,J)
   40 CONTINUE
   45 IF (DL.EQ.2) GO TO 60
      IF (DK.EQ.2) GO TO 50
      T(1,1)=A(K,K)+A(L,L)
      IF (T(1,1).NE.0.0) GO TO 55
      WRITE (6,1000)
 1000 FORMAT(10X,"*ERROR STOP - FAILED TO SOLVE LYAPUNOV EQ."/)
      STOP
   55 C(K,L)=C(K,L)/T(1,1)
      GO TO 90
   50 T(1,1)=A(K,K)+A(L,L)
      T(1,2)=A(KK,K)
      T(2,1)=A(K,KK)
      T(2,2)=A(KK,KK)+A(L,L)
      P(1)=C(K,L)
      P(2)=C(KK,L)
      NSYS=2
      CALL SYSSLV
      C(K,L)=P(1)
      C(KK,L)=P(2)
      GO TO 90
   60 IF (DK.EQ.2) GO TO 70
      T(1,1)=A(K,K)+A(L,L)
      T(1,2)=A(LL,L)
      T(2,1)=A(L,LL)
      T(2,2)=A(K,K)+A(LL,LL)
      P(1)=C(K,L)
      P(2)=C(K,LL)
      NSYS=2
      CALL SYSSLV
      C(K,L)=P(1)
      C(K,LL)=P(2)
      GO TO 90
   70 IF (K.NE.L) GO TO 80
      T(1,1)=A(L,L)
      T(1,2)=A(LL,L)
```

```
      T(1,3)=0.0
      T(2,1)=A(L,LL)
      T(2,2)=A(L,L)+A(LL,LL)
      T(2,3)=T(1,2)
      T(3,1)=0.0
      T(3,2)=T(2,1)
      T(3,3)=A(LL,LL)
      P(1)=C(L,L)/2.0
      P(2)=C(LL,L)
      P(3)=C(LL,LL)/2.0
      NSYS=3
      CALL SYSSLV
      C(L,L)=P(1)
      C(LL,L)=P(2)
      C(L,LL)=P(2)
      C(LL,LL)=P(3)
      GO TO 90
   80 T(1,1)=A(K,K)+A(L,L)
      T(1,2)=A(KK,K)
      T(1,3)=A(LL,L)
      T(1,4)=0.0
      T(2,1)=A(K,KK)
      T(2,2)=A(KK,KK)+A(L,L)
      T(2,3)=0.0
      T(2,4)=T(1,3)
      T(3,1)=A(L,LL)
      T(3,2)=0.0
      T(3,3)=A(K,K)+A(LL,LL)
      T(3,4)=T(1,2)
      T(4,1)=0.0
      T(4,2)=T(3,1)
      T(4,3)=T(2,1)
      T(4,4)=A(KK,KK)+A(LL,LL)
      P(1)=C(K,L)
      P(2)=C(KK,L)
      P(3)=C(K,LL)
      P(4)=C(KK,LL)
      NSYS=4
      CALL SYSSLV
      C(K,L)=P(1)
      C(KK,L)=P(2)
      C(K,LL)=P(3)
      C(KK,LL)=P(4)
   90 K=K+DK
      IF (K.LE.N) GO TO 30
      LDL=L+DL
      IF (LDL.GT.N) RETURN
      DO 120 J=LDL,N
      DO 100 I=L,LL
      C(I,J)=C(J,I)
  100 CONTINUE
      DO 110 I=J,N
```

```
      C(I,J)=C(I,J)-DOT3(NC,NA,L,LL,C,A,I,J)-DOT2(NA,NC,L,LL,A,C,I,J)
      C(J,I)=C(I,J)
  110 CONTINUE
  120 CONTINUE
      L=LDL
      GO TO 10
      END

      SUBROUTINE SYSSLV
C**** SOLVES THE LINEAR SYSTEM AX=B OF ORDER N LESS THAN 5
C
      COMMON /SLVBLK/  A(5,5),  B(5),  N
C
    1 NM1=N-1
      N1=N+1
      DO 80 K=1,N
      KM1=K-1
      IF (K.EQ.1) GO TO 20
      DO 10 I=K,N
      DO 11 J=1,KM1
      A(I,K)=A(I,K)-A(I,J)*A(J,K)
   11 CONTINUE
   10 CONTINUE
   20 IF (K.EQ.N) GO TO 100
      KP1=K+1
      XMAX=ABS(A(K,K))
      INTR=K
      DO 30 I=KP1,N
      AA=ABS(A(I,K))
      IF (AA.LE.XMAX) GO TO 30
      XMAX=AA
      INTR=I
   30 CONTINUE
      IF (XMAX.NE.0.0) GO TO 35
      WRITE (6,1000)
 1000 FORMAT(10X,'*ERROR STOP - FAILED TO SOLVE LINEAR EQ. OBTAINED FROM
     1 SHUR FORM'/)
      STOP
   35 A(N1,K)=INTR
      IF (INTR.EQ.K) GO TO 50
      DO 40 J=1,N
      TEMP=A(K,J)
      A(K,J)=A(INTR,J)
      A(INTR,J)=TEMP
   40 CONTINUE
   50 DO 75 J=KP1,N
      IF (K.EQ.1) GO TO 70
      DO 71 JJ=1,KM1
      A(K,J)=A(K,J)-A(K,JJ)*A(JJ,J)
   71 CONTINUE
   70 A(K,J)=A(K,J)/A(K,K)
   75 CONTINUE
   80 CONTINUE
```

```
  100 DO 110 J=1,NM1
      INTR=A(N1,J)
      IF (INTR.EQ.J) GO TO 110
      TEMP=B(J)
      B(J)=B(INTR)
      B(INTR)=TEMP
  110 CONTINUE
      B(1)=B(1)/A(1,1)
      DO 120 I=2,N
      IM1=I-1
      DO 125 J=1,IM1
      B(I)=B(I)-A(I,J)*B(J)
  125 CONTINUE
      B(I)=B(I)/A(I,I)
  120 CONTINUE
      DO 130 II=1,NM1
      I=NM1-II+1
      I1=I+1
      DO 135 J=I1,N
      B(I)=B(I)-A(I,J)*B(J)
  135 CONTINUE
  130 CONTINUE
      RETURN
      END

      FUNCTION DOT(MR,N1,N2,A,JC,KC)
C**** COMPUTES THE INNER PRODUCT OF COLUMNS JC AND KC OF MATRIX A
C
      DIMENSION  A(MR,1)
      DOT=0.0
      DO 5 I=N1,N2
    5 DOT=DOT+A(I,JC)*A(I,KC)
      RETURN
      END

      FUNCTION DOT1(MR,NR,N1,N2,A,JC,KC)
C**** COMPUTES THE SQUARE OF MATRIX A FOR COLUMN JC AND ROW KC
C
      DIMENSION  A(MR,NR)
      DOT1=0.0
      DO 5 I=N1,N2
    5 DOT1=DOT1+A(JC,I)*A(I,KC)
      RETURN
      END

      FUNCTION DOT2(MR,NR,N1,N2,A,B,JC,KC)
C**** COMPUTES THE INNER PRODUCT OF MATRICES AND B FOR COLUMN JC AND
C**** KC
      DIMENSION  A(MR,1),  B(NR,1)
      DOT2=0.0
      DO 5 I=N1,N2
    5 DOT2=DOT2+A(I,JC)*B(I,KC)
      RETURN
      END
```

```
      FUNCTION DOT3(MR,NR,N1,N2,A,B,JC,KC)
C**** COMPUTES THE PRODUCT OF MATRICES AND B FOR COLUMN JC AND ROW KC
C
      DIMENSION  A(MR,N2),  B(NR,1)
      DOT3=0.0
      DO 5 I=N1,N2
    5 DOT3=DOT3+A(JC,I)*B(I,KC)
      RETURN
      END

      FUNCTION DOT4(MR,NR,N1,N2,A,B,JC,KC)
C**** PRODUCT OF MATRICES AND A FOR COLUMN JC AND B FOR COLUMN KC
      DIMENSION  A(MR,N2),  B(NR,N2)
      DOT4=0.0
      DO 5 I=N1,N2
    5 DOT4=DOT4+A(JC,I)*B(KC,I)
      RETURN
      END

      SUBROUTINE SCHUR(NM,NB,N,A,B,MATZ,Z,IERR)
C**** MATRIX TRANSFORMATION - A(REAL SCHUR FORM) AND B(UPPER TRIANGULAR
C**** FORM) BY ORTHOGONAL TRANSFORMATION
      DIMENSION  A(NM,N),  B(NB,N),  Z(NM,N)
      INTEGER EN, ENM2,  ENORM
      LOGICAL  MATZ,NOTLAS
C     TRANSFORMATION OF MATRIX INTO UPPER HESSENBERG FORM
      IF (.NOT.MATZ) GO TO 3
      DO 2 I=1,N
      DO 1 J=1,N
      Z(I,J)=0.0
    1 CONTINUE
      Z(I,I)=1.0
    2 CONTINUE
    3 IF (N.LE.1) GO TO 27
      NM1=N-1
      DO 10 L=1,NM1
      L1=L+1
      S=0.0
      DO 4 I=L1,N
      S=S+ABS(B(I,L))
    4 CONTINUE
      IF (S.EQ.0.0) GO TO 10
      S=S+ABS(B(L,L))
      DO 5 I=L,N
    5 B(I,L)=B(I,L)/S
      R=DOT(NM,L,N,B,L,L)
      R=SIGN(SQRT(R),B(L,L))
      B(L,L)=B(L,L)+R
      RHO=R*B(L,L)
      DO 8 J=L1,N
      T=DOT(NM,L,N,B,L,J)
      T=-T/RHO
      DO 7 I=L,N
```

```
      B(I,J)=B(I,J)+T*B(I,L)
    7 CONTINUE
    8 CONTINUE
      DO 12 J=1,N
      T=DOT2(NM,NM,L,N,B,A,L,J)
      T=-T/RHO
      DO 11 I=L,N
      A(I,J)=A(I,J)+T*B(I,L)
   11 CONTINUE
   12 CONTINUE
      B(L,L)=-S*R
      DO 13 I=L1,N
      B(I,L)=0.0
   13 CONTINUE
   10 CONTINUE
      IF (N.EQ.2) GO TO 27
      NM2=N-2
      DO 20 K=1,NM2
      NK1=NM1-K
      DO 21 LB=1,NK1
      L=N-LB
      L1=L+1
      S=ABS(A(L,K))+ABS(A(L1,K))
      IF (S.EQ.0.0) GO TO 21
      U1=A(L,K)/S
      U2=A(L1,K)/S
      R=SIGN(SQRT(U1*U1+U2*U2),U1)
      V1=-(U1+R)/R
      V2=-U2/R
      U2=V2/V1
      DO 22 J=K,N
      T=A(L,J)+U2*A(L1,J)
      A(L,J)=A(L,J)+T*V1
      A(L1,J)=A(L1,J)+T*V2
   22 CONTINUE
      A(L1,K)=0.0
      DO 23 J=L,N
      T=B(L,J)+U2*B(L1,J)
      B(L,J)=B(L,J)+T*V1
      B(L1,J)=B(L1,J)+T*V2
   23 CONTINUE
      S=ABS(B(L1,L1))+ABS(B(L1,L))
      IF (S.EQ.0.0) GO TO 21
      U1=B(L1,L1)/S
      U2=B(L1,L)/S
      R=SIGN(SQRT(U1*U1+U2*U2),U1)
      V1=-(U1+R)/R
      V2=-U2/R
      U2=V2/V1
      DO 24 I=1,L1
      T=B(I,L1)+U2*B(I,L)
      B(I,L1)=B(I,L1)+T*V1
      B(I,L)=B(I,L)+T*V2
   24 CONTINUE
```

```
      B(L1,L)=0.0
      DO 25 I=1,N
      T=A(I,L1)+U2*A(I,L)
      A(I,L1)=A(I,L1)+T*V1
      A(I,L)=A(I,L)+T*V2
   25 CONTINUE
      IF (.NOT.MATZ) GO TO 21
      DO 26 I=1,N
      T=Z(I,L1)+U2*Z(I,L)
      Z(I,L1)=Z(I,L1)+T*V1
      Z(I,L)=Z(I,L)+T*V2
   26 CONTINUE
   21 CONTINUE
   20 CONTINUE
C     TRANSFORMATION OF UPPER HESSENBERG MATRIX INTO QUASI-UPPER
   27 CONTINUE
C     TRIANGULAR FORM
      IERR=0
      ANORM=0.0
      BNORM=0.0
      DO 30 I=1,N
      ANI=0.0
      IF (I.NE.1) ANI=ABS(A(I,I-1))
      BNI=0.0
      DO 31 J=I,N
      ANI=ANI+ABS(A(I,J))
      BNI=BNI+ABS(B(I,J))
   31 CONTINUE
      IF(ANI.GT.ANORM) ANORM=ANI
      IF (BNI.GT.BNORM) BNORM=BNI
   30 CONTINUE
      IF (ANORM.EQ.0.0) ANORM=1.0
      IF (BNORM.EQ.0.0) BNORM=1.0
      EP=1.0
   32 EP=EP/2.0
      IF (1.0+EP.GT.1.0) GO TO 32
      EPSA=EP*ANORM
      EPSB=EP*BNORM
      LOR1=1
      ENORM=N
      EN=N
   33 IF (EN.LE.2) GO TO 70
      IF (.NOT.MATZ) ENORM=EN
      ITS=0
      NA=EN-1
      ENM2=NA-1
   34 ISH=2
      DO 35 LL=1,EN
      LM1=EN-LL
      L=LM1+1
      IF (L.EQ.1) GO TO 37
      IF (ABS(A(L,LM1)).LE.EPSA) GO TO 36
   35 CONTINUE
   36 A(L,LM1)=0.0
```

```
      IF (L.LT.NA) GO TO 37
      EN=LM1
      GO TO 33
   37 LD=L
   40 L1=L+1
      B11=B(L,L)
      IF (ABS(B11).GT.EPSB) GO TO 42
      B(L,L)=0.0
      S=ABS(A(L,L))+ABS(A(L1,L))
      U1=A(L,L)/S
      U2=A(L1,L)/S
      R=SIGN(SQRT(U1*U1+U2*U2),U1)
      V1=-(U1+R)/R
      V2=-U2/R
      U2=V2/V1
      DO 41 J=L,ENORM
      T=A(L,J)+U2*A(L1,J)
      A(L,J)=A(L,J)+T*V1
      A(L1,J)=A(L1,J)+T*V2
      T=B(L,J)+U2*B(L1,J)
      B(L,J)=B(L,J)+T*V1
      B(L1,J)=B(L1,J)+T*V2
   41 CONTINUE
      IF (L.NE.1) A(L,LM1)=-A(L,LM1)
      LM1=L
      L=L1
      GO TO 36
   42 A11=A(L,L)/B11
      A21=A(L1,L)/B11
      IF (ISH.EQ.1) GO TO 44
      IF (ITS.EQ.50) GO TO 61
      IF (ITS.EQ.10) GO TO 50
      B22=B(L1,L1)
      IF (ABS(B22).LT.EPSB)  B22=EPSB
      B33=B(NA,NA)
      IF (ABS(B33).LT.EPSB) B33=EPSB
      B44=B(EN,EN)
      IF (ABS(B44).LT.EPSB) B44=EPSB
      A33=A(NA,NA)/B33
      A34=A(NA,EN)/B44
      A43=A(EN,NA)/B33
      A44=A(EN,EN)/B44
      B34=B(NA,EN)/B44
      T=0.5*(A43*B34-A33-A44)
      R=T*T+A34*A43-A33*A44
      IF (R.LT.0.0) GO TO 45
      ISH=1
      R=SQRT(R)
      SH=-T+R
      S=-T-R
      IF (ABS(S-A44).LT.ABS(SH-A44)) SH=S
      DO 43 LL=LD,ENM2
      L=ENM2+LD-LL
      IF (L.EQ.LD) GO TO 44
```

```
      LM1=L-1
      L1=L+1
      T=A(L,L)
      IF (ABS(B(L,L)).GT.EPSB) T=T-SH*B(L,L)
      IF (ABS(A(L,LM1)).LE.ABS(T/A(L1,L))*EPSA) GO TO 40
   43 CONTINUE
   44 A1=A11-SH
      A2=A21
      IF (L.NE.LD) A(L,LM1)=-A(L,LM1)
      GO TO 51
   45 A12=A(L,L1)/B22
      A22=A(L1,L1)/B22
      B12=B(L,L1)/B22
      A1=((A33-A11)*(A44-A11)-A34*A43+A43*B34*A11)/A21+A12-A11*B12
      A2=(A22-A11)-A21*B12-(A33-A11)-(A44-A11)+A43*B34
      A3=A(L1+1,L1)/B22
      GO TO 51
   50 A1=0.0
      A2=1.0
      A3=1.1605
   51 ITS=ITS+1
      IF (.NOT.MATZ) LOR1=LD
      DO 60 K=L,NA
       OTLAS=K.NE.NA.AND.ISH.EQ.2
      K1=K+1
      K2=K1+1
      KM1=MAX0(K-1,L)
      LL=MIN0(EN,K1+ISH)
      IF (NOTLAS) GO TO 54
      IF (K.EQ.L) GO TO 52
      A1=A(K,KM1)
      A2=A(K1,KM1)
   52 S=ABS(A1)+ABS(A2)
      IF (S.EQ.0.0) GO TO 34
      U1=A1/S
      U2=A2/S
      R=SIGN(SQRT(U1*U1+U2*U2),U1)
      V1=-(U1+R)/R
      V2=-U2/R
      U2=V2/V1
      DO 53 J=KM1,ENORM
      T=A(K,J)+U2*A(K1,J)
      A(K,J)=A(K,J)+T*V1
      A(K1,J)=A(K1,J)+T*V2
      T=B(K,J)+U2*B(K1,J)
      B(K,J)=B(K,J)+T*V1
      B(K1,J)=B(K1,J)+T*V2
   53 CONTINUE
      IF (K.NE.L) A(K1,KM1)=0.0
      GO TO 59
   54 IF (K.EQ.L) GO TO 55
      A1=A(K,KM1)
      A2=A(K1,KM1)
      A3=A(K2,KM1)
```

```
 55 S=ABS(A1)+ABS(A2)+ABS(A3)
    IF (S.EQ.0.0) GO TO 60
    U1=A1/S
    U2=A2/S
    U3=A3/S
    R=SIGN(SQRT(U1*U1+U2*U2+U3*U3),U1)
    V1=-(U1+R)/R
    V2=-U2/R
    V3=-U3/R
    U2=V2/V1
    U3=V3/V1
    DO 56 J=KM1,ENORM
    T=A(K,J)+U2*A(K1,J)+U3*A(K2,J)
    A(K,J)=A(K,J)+T*V1
    A(K1,J)=A(K1,J)+T*V2
    A(K2,J)=A(K2,J)+T*V3
    T=B(K,J)+U2*B(K1,J)+U3*B(K2,J)
    B(K,J)=B(K,J)+T*V1
    B(K1,J)=B(K1,J)+T*V2
    B(K2,J)=B(K2,J)+T*V3
 56 CONTINUE
    IF (K.EQ.L) GO TO 57
    A(K1,KM1)=0.0
    A(K2,KM1)=0.0
 57 S=ABS(B(K2,K2))+ABS(B(K2,K1))+ABS(B(K2,K))
    IF (S.EQ.0.0) GO TO 59
    U1=B(K2,K2)/S
    U2=B(K2,K1)/S
    U3=B(K2,K)/S
    R=SIGN(SQRT(U1*U1+U2*U2+U3*U3),U1)
    V1=-(U1+R)/R
    V2=-U2/R
    V3=-U3/R
    U2=V2/V1
    U3=V3/V1
    DO 58 I=LOR1,LL
    T=A(I,K2)+U2*A(I,K1)+U3*A(I,K)
    A(I,K2)=A(I,K2)+T*V1
    A(I,K1)=A(I,K1)+T*V2
    A(I,K)=A(I,K)+T*V3
    T=B(I,K2)+U2*B(I,K1)+U3*B(I,K)
    B(I,K2)=B(I,K2)+T*V1
    B(I,K1)=B(I,K1)+T*V2
    B(I,K)=B(I,K)+T*V3
 58 CONTINUE
    B(K2,K)=0.0
    B(K2,K1)=0.0
    IF (.NOT.MATZ) GO TO 59
    DO 46 I=1,N
    T=Z(I,K2)+U2*Z(I,K1)+U3*Z(I,K)
    Z(I,K2)=Z(I,K2)+T*V1
    Z(I,K1)=Z(I,K1)+T*V2
    Z(I,K)=Z(I,K)+T*V3
 46 CONTINUE
```

```
   59 S=ABS(B(K1,K1))+ABS(B(K1,K))
      IF (S.EQ.0.0) GO TO 60
      U1=B(K1,K1)/S
      U2=B(K1,K)/S
      R=SIGN(SQRT(U1*U1+U2*U2),U1)
      V1=-(U1+R)/R
      V2=-U2/R
      U2=V2/V1
      DO 47 I=LOR1,LL
      T=A(I,K1)+U2*A(I,K)
      A(I,K1)=A(I,K1)+T*V1
      A(I,K)=A(I,K)+T*V2
      T=B(I,K1)+U2*B(I,K)
      B(I,K1)=B(I,K1)+T*V1
      B(I,K)=B(I,K)+T*V2
   47 CONTINUE
      B(K1,K)=0.0
      IF (.NOT.MATZ) GO TO 60
      DO 48 I=1,N
      T=Z(I,K1)+U2*Z(I,K)
      Z(I,K1)=Z(I,K1)+T*V1
      Z(I,K)=Z(I,K)+T*V2
   48 CONTINUE
   60 CONTINUE
      GO TO 34
   61 IERR=EN
   70 IF (N.GT.1) B(N,1)=EPSB
      RETURN
      END

      FUNCTION XINT(X,Y,XI,YJ,PXY)
C**** 2-DIMENSIONAL LINEAR INTERPOLATION ROUTINE
      DIMENSION  XI(2),  YJ(2),  PXY(2,2)
C
      XX=X-XI(1)
      XX1=XI(2)-XI(1)
      YY=Y-YJ(1)
      YY1=YJ(2)-YJ(1)
      PPX=PPY=PPXY=0.0
      IF (XX1.NE.0.0.AND.YY1.NE.0.0) PPXY=XX*YY*(PXY(2,2)-PXY(1,2)-
     1     PXY(2,1)+PXY(1,1))/(XX1*YY1)
      IF (XX1.NE.0.0) PPX=XX*(PXY(2,1)-PXY(1,1))/XX1
      IF (YY1.NE.0.0) PPY=YY*(PXY(1,2)-PXY(1,1))/YY1
      XINT=PPXY+PPX+PPY+PXY(1,1)
      END

      SUBROUTINE BMTRX(LSUM)
      COMMON /STFN/  XF(31,23),   XT(31,23),   XD(31,23),   XE(31,23)
      COMMON /XSCN/  DIF1(25),    DIF2(25),    SIGA1(25),    SIGA2(25),
     1               SIGR(25),    SIGF(25),    ALFA(25),    HFAC(25),
     2               XK(10),      V1,          V2,          XNU,
     3               GAMI,        GAMX,        SIGXE,       ZLAMI,
     4               ZLAMX,       XKS
```

```
      COMMON /MISC/  TITLE(18),  NPAGE,      NMOD,      M2,    M4,
     1               NRX,        NRY,        NTT,       PWR,   BLANK,
     2               CX,    CY,  NSX,   NSY, IUNSTB,  RFAC,  SFAC,
     3              QFAC,    AMPO
      COMMON /NCON/  GGMX(10),  NSIGN(10),  YL(10),  XL(10)
      COMMON /BCON/ BMTX(10,20),  CMTX(10,20),  UB(20)
      COMMON /GEOM/  MAT(31,23),  DX(31),  DY(23),  X(31),  Y(23)
      COMMON /XYMOD/ PF(31,23,8),  PS(31,23,8),  FNORM(8),  TNORM(8)
      DIMENSION   XI(2),YJ(2),P(2,2)
C
      READ (5,1000) LCON
      READ (5,1001) (XL(K),YL(K),NSIGN(K),K=1,LCON)
 1000 FORMAT(16I5)
 1001 FORMAT(4(F9.4,F9.4,I2))
      WRITE (6,2000)
 2000 FORMAT(//,30X,'CONTROLLER DATA FOR THE FIRST HALF OF CORE ',/)
      WRITE (6,2001) LCON
 2001 FORMAT(10X,'NUMBER OF CONTROLLER = ',I5,///,10X,
     1 'X-COORD    Y-COORD    SYM-IDX',/)
      WRITE (6,2002) (XL(K),YL(K),NSIGN(K),K=1,LCON)
 2002 FORMAT(9X,F9.4,1X,F9.4,5X,I2)
      LSUM=0.0
      DO 10 K=1,LCON
      K1=LSUM+1
      LSUM=LSUM+NSIGN(K)
      K2=LSUM
      DO 11 KK=K1,K2
      DO 14 L=1,NMOD
   14 BMTX(L,KK)=0.0
      XN=XL(K)
      YN=YL(K)
      IF (NSIGN(K).EQ.2.AND.KK.EQ.K2) XN=2*CX-XN
      DO 12 I=2,NRX
      DO 12 J=2,NRY
      IF (XN.LT.X(I-1).OR.XN.GE.X(I).OR.YN.LT.Y(J-1).OR.YN.GE.Y(J))
     1        GO TO 12
      XI(1)=X(I-1)
      XI(2)=X(I)
      YJ(1)=Y(J-1)
      YJ(2)=Y(J)
      DV1=(XN-X(I-1))*(YN-Y(J-1))
      DV2=(XN-X(I-1))*(Y(J)-YN)
      DV3=(X(I)-XN)*(YN-Y(J-1))
      DV4=(X(I)-XN)*(Y(J)-YN)
      P(1,1)=XT(I-1,J-1)
      P(1,2)=XT(I-1,J)
      P(2,1)=XT(I,J-1)
      P(2,2)=XT(I,J)
      XTC=XINT(XN,YN,XI,YJ,P)
      IDX1=MAT(I-1,J-1)
      IDX2=MAT(I-1,J)
      IDX3=MAT(I,J-1)
      IDX4=MAT(I,J)
```

```
      SIGRD=(SIGR(IDX1)*XF(I-1,J-1)*DV1+SIGR(IDX2)*XF(I-1,J)*DV2+
     1       SIGR(IDX3)*XF(I,J-1)*DV3+SIGR(IDX4)*XF(I,J)*DV4)/
     2      (XF(I-1,J-1)*DV1+XF(I-1,J)*DV2+XF(I,J-1)*DV3+XF(I,J)*DV4)
      SIGFD=(SIGF(IDX1)*XT(I-1,J-1)*DV1+SIGF(IDX2)*XT(I-1,J)*DV2+
     1       SIGF(IDX3)*XT(I,J-1)*DV3+SIGF(IDX4)*XT(I,J)*DV4)/
     2      (XT(I-1,J-1)*DV1+XT(I-1,J)*DV2+XT(I,J-1)*DV3+XT(I,J)*DV4)
      BLN=XNU*SIGFD
      CLN=SQRT(SIGRD)
      DO 13 L=1,NMOD
      P(1,1)=PF(I-1,J-1,L)
      P(1,2)=PF(I-1,J,L)
      P(2,1)=PF(I,J-1,L)
      P(2,2)=PF(I,J,L)
      PFCL=XINT(XN,YN,XI,YJ,P)
      BMTX(L,KK)=-PFCL*BLN*XTC/(AMPO*CLN)
      P(1,1)=PS(I-1,J-1,L)
      P(1,2)=PS(I-1,J,L)
      P(2,1)=PS(I,J-1,L)
      P(2,2)=PS(I,J,L)
      CMTX(L,KK)=XINT(XN,YN,XI,YJ,P)/SQRT(BLN)
   13 CONTINUE
   12 CONTINUE
   11 CONTINUE
   10 CONTINUE
      RETURN
      END
      SUBROUTINE BKCON(NPROB)
C**** CALCULATES THE STATIC BULK CONTROLLERS WITH GROUPS OF LOWER GAIN
C**** UNDER THE CRITERIA OF MAXIMUM CONTROL MATERIAL FOR A CONTROLLER
      COMMON /STFN/  XF(31,23),  XT(31,23),  XI(31,23),  XE(31,23)
      COMMON /MISC/  TITLE(18),  NPAGE,      NMOD,       M2,     M4,
     1               NRX,        NRY,        NTT,        PWR,    BLANK,
     2              CX,  CY,  NSX,  NSY,  IUNSTB,  RFAC,  SFAC,
     3              QFAC,   AMPO
      COMMON /GEOM/  MAT(31,23),  DX(31),  DY(23),  X(31),  Y(23)
      COMMON /XSCN/  DIF1(25),    DIF2(25),    SIGA1(25),   SIGA2(25),
     1               SIGR(25),    SIGF(25),    ALFA(25),    HFAC(25),
     2               XK(10),      V1,          V2,          XNU,
     3               GAMI,        GAMX,        SIGXE,       ZLAMI,
     4               ZLAMX,       XKS
      COMMON /SYSM/  SMX11(20,20),  SMX12(20,20),  SMX21(21,21),
     1               SMX22(20,20),  SMMX(20,20)
      COMMON /XYMOD/ PF(31,23,8),  PS(31,23,8),  FNORM(8),  TNORM(8)
      COMMON /BCON/ BMTX(10,20),  CMTX(10,20),  UB(20)
      COMMON /NCON/  GGMX(10),  NSIGN(10),  YL(10),  XL(10)
      DIMENSION  TRMX(10,10),  W1(10),  W2(10),  TMRX(10,10),
     1          DMTX(10,10),  QMTX(10,10),  PMTX(10,10), TRMY(10,10),
     2          PMTN(10),  UVOL(20),  UC(20),  SMXX(10,10)
C
      EP=1.0
    5 EP=EP/2.0
      IF (1.0+EP.GT.1.0) GO TO 5
       REWIND 2
```

```
      READ (5,1000) USIG2,VUNT,UFRC,TILT,SIGMA
 1000 FORMAT(6E12.6)
      WRITE (6,2000)
 2000 FORMAT(//,30X,'BULK CONTROLLER DESIGN',/)
      WRITE (6,2001) USIG2,VUNT,UFRC,TILT,SIGMA
 2001 FORMAT(10X,'MACROSCOPIC ABSORPTION X-SECTION OF CONTROLLER MATERIA
     1L = ',E12.6,' (1/CM)',/,10X,'UNIT VOLUME FOR CONTROLER MODEL = ',
     2E12.6,' (CM**2)',/,10X,'EXPECTED FRACTION OF CONTROLLER FOR STATIC
     3 SET-POINT = ',E12.6,          /,10X,'FRACTIONAL MAXIMUM POWER TILT
     4 = ',F10.5,/,10X,'STANDARD DEVIATION FOR MATERIAL PERTURB = ',
     5E12.6,/)
      READ (5,1000) QFACO
      WRITE (6,2002) QFACO
 2002 FORMAT(10X,'INITIAL SCALE FACTOR FOR AMP. LOSS = ',E12.6,/)
      UUNIT=USIG2*VUNT
C
      CALL BMTRX(MCON)
      WRITE(2) MCON
      WRITE(2) XL,YL,NSIGN
      WRITE(2) BMTX
      READ (5,1001) (UVOL(K),K=1,MCON)
 1001 FORMAT(8F10.5)
      WRITE (6,2022) (UVOL(K),K=1,MCON)
 2022 FORMAT(//,10X,'MAXIMUM MULTIPLES OF LATTICE VOLUME FOR CONTROLLER'
     1,/,5X,'CONTROLLER      1            2            3            4            5
     2      6            7            8            9           10',/,6X,'MULTIPLES',
     310F10.5,//)
      CALL SMTRX(NPROB)
      CALL GINV(SMX11,DMTX,W1,W2,20,NMOD,NMOD,ERROR)
      DO 10 L=1,NMOD
      DO 10 M=1,NMOD
      TMRX(L,M)=DOT4(21,20,1,NMOD,SMX21,SMX11,L,M)
   10 CONTINUE
      DO 12 L=1,NMOD
      DO 12 M=1,NMOD
      SMX22(L,M)=SMX22(L,M)-DOT3(10,20,1,NMOD,TMRX,SMX12,L,M)
   12 CONTINUE
      CALL HEAD
      WRITE (6,2023) NMOD,NMOD
 2023 FORMAT(/,10X,'REDUCED-SYSTEM MATRIX (',I2,',',I2,')',//)
      DO 17 L=1,NMOD
      WRITE (6,2024) (SMX22(L,M),M=1,NMOD)
   17 CONTINUE
      WRITE (6,2034)
 2034 FORMAT(//,10X,'CONTROLLER MATRIX',//)
      DO 170 K=1,MCON
      WRITE (6,2024) (BMTX(L,K),L=1,NMOD)
  170 CONTINUE
      WRITE (6,2033)
 2033 FORMAT(//,10X,'OBSERVER MATRIX',//)
      DO 171 K=1,MCON
      WRITE (6,2024) (CMTX(L,K),L=1,NMOD)
  171 CONTINUE
 2024 FORMAT(5X,8E12.5)
```

```
C     TMRX - MATRIX (A21)*INV(A11)
C     SMX22 - MATRIX ((A22)-(TMRX)*(A12))
      DKO=1.0-1.0/XK(1)
      DO 13 L=1,NMOD
      W1(L)=0.0
      TLN=TNORM(L)/FNORM(L)
      DO 13 I=1,NRX
      DO 13 J=1,NRY
      IDX=MAT(I,J)
      ALN=XNU*SIGF(IDX)
      BLN=SQRT(ALN)
      CLN=SIGR(IDX)
      DV=DX(I)*DY(J)
      W1(L)=W1(L)+DKO*TLN*FS(I,J,L)*BLN*CLN*XT(I,J)*DV/AMPO
   13 CONTINUE
C     F-MATRIX CONSTRUCTION
      DO 15 L=1,NMOD
      DO 15 M=1,NMOD
      DMTX(L,M)=-DOT3(10,20,1,NMOD,TMRX,SMMX,L,M)
   15 CONTINUE
      DO 16 L=1,NMOD
      DO 16 M=1,NMOD
      SMMX(L,M)=DMTX(L,M)/AMPO
   16 CONTINUE
      DO 14 L=1,NMOD
      PMTN(L)=0.0
      DO 14 M=1,NMOD
      PMTN(L)=PMTN(L)-TMRX(L,M)*W1(M)
   14 CONTINUE
C     COMPUTES THE STATIC SET-POINTS OF CONTROLLERS
      CALL HEAD
      DO 18 L=1,NMOD
      DO 118 M=L,NMOD
      DMTX(L,M)=DOT4(10,10,1,MCON,BMTX,BMTX,L,M)
      SMX11(L,M)=0.0
      DMTX(M,L)=DMTX(L,M)
      SMX11(M,L)=SMX11(L,M)
  118 CONTINUE
      SMX11(L,L)=1.0
   18 CONTINUE
      QSC=QNORM=RNORM=0.0
      DO 19 L=1,NMOD
      QSC=QSC+DOT4(20,20,1,NMOD,SMX22,SMX22,L,L)
      QNORM=QNORM+DOT4(20,20,1,NMOD,SMX11,SMX11,L,L)
      RNORM=RNORM+DOT4(10,10,1,NMOD,DMTX,DMTX,L,L)
   19 CONTINUE
      QSC=SQRT(QSC)/NMOD
      QFAC=QFACO*NMOD/SQRT(QNORM)
      RFAC=SQRT(RNORM)/NMOD
      DO 180 L=1,NMOD
      DO 180 M=1,NMOD
      SMX11(L,M)=SMX11(L,M)*QFAC
      DMTX(L,M)=DMTX(L,M)/RFAC
      PMTX(L,M)=SMX11(L,M)
  180 CONTINUE
```

```
      CALL GINV(PMTX,TRMX,W1,W2,10,NMOD,NMOD,ERROR)
      DO 20 L=1,NMOD
      DO 20 M=1,NMOD
      TRMX(L,M)=DMTX(L,M)
      DO 200 LL=1,NMOD
      CC=0.0
      DO 201 MM=1,NMOD
      CC=CC+PMTX(MM,LL)*SMX22(M,MM)
  201 CONTINUE
      TRMX(L,M)=TRMX(L,M)+SMX22(L,LL)*CC
  200 CONTINUE
   20 CONTINUE
      CALL GINV(TRMX,TRMY,W1,W2,10,NMOD,NMOD,ERROR)
      DO 22 K=1,MCON
      UB(K)=0.0
      DO 22 L=1,NMOD
      SUM=0.0
      DO 23 M=1,NMOD
      SUM=SUM+BMTX(M,K)*TRMX(L,M)
   23 CONTINUE
      UB(K)=UB(K)+PMTN(L)*SUM/RFAC
   22 CONTINUE
      DO 21 K=1,MCON
      UB(K)=UB(K)*100.0/(UUNIT*UVOL(K))
   21 CONTINUE
      WRITE (6,2005)
 2005 FORMAT(//,10X,'STATIC CONTROL SET-POINTS (% FILLED)',/)
      WRITE (6,2015)
      WRITE (6,2006) (UB(K),K=1,MCON)
 2006 FORMAT(6X,'LEVEL (%)',10F10.5,/)
      DO 29 K=1,MCON
      UB(K)=UB(K)*UUNIT*UVOL(K)/100.0
   29 CONTINUE
      WRITE (2) UB,UUNIT,UVOL
C     THERMAL FLUX MODE DEVIATION DUE TO DEPLOY THE CONTROLLERS
      DO 28 L=1,NMOD
      DO 28 M=1,NMOD
      TRMX(L,M)=SMX22(L,M)
   28 CONTINUE
      CALL GINV(TRMX,TRMY,W1,W2,10,NMOD,NMOD,ERROR)
      DO 30 L=1,NMOD
      W1(L)=0.0
      DO 31 M=1,NMOD
      SUM=0.0
      DO 32 K=1,MCON
      SUM=SUM+BMTX(M,K)*UB(K)
   32 CONTINUE
      W1(L)=W1(L)-TRMX(M,L)*(PMTN(M)-SUM)
   31 CONTINUE
   30 CONTINUE
      WRITE (6,2007)
 2007 FORMAT(/,10X,'THERMAL FLUX MODE ERROR ',/,10X,'MODE              AMPLTD
     1.ERROR',/)
      WRITE (6,2008) (L,W1(L),L=1,NMOD)
 2008 FORMAT(7X,I5,10X,E12.6)
```

```
      RVOL=XTSQ=0.0
      DO 34 I=1,NRX
      DO 34 J=1,NRY
      DV=DX(I)*DY(J)
      IF (XT(I,J).NE.0.0) RVOL=RVOL+DV
      XI(I,J)=XT(I,J)/AMPO
      DO 33 L=1,NMOD
      XI(I,J)=XI(I,J)+W1(L)*PS(I,J,L)
   33 CONTINUE
      XI(I,J)=XI(I,J)*AMPO
      IF (XT(I,J).EQ.0.0) GO TO 34
      XTSQ=XTSQ+((XI(I,J)-XT(I,J))/XT(I,J))**2.0*DV
   34 CONTINUE
      XTSQ=SQRT(XTSQ/RVOL)*100.0
      SUMT=SUMF=SUMU=0.0
      DO 35 L=1,NMOD
      CC=0.0
      DO 36 M=1,NMOD
      CC=CC+SMX11(L,M)*W1(M)
   36 CONTINUE
      SUMF=SUMF+W1(L)*CC
   35 CONTINUE
      TUVOL=0.0
      DO 37 K=1,MCON
      SUMU=SUMU+UB(K)*UB(K)*RFAC
      TUVOL=TUVOL+UB(K)
   37 CONTINUE
      SUMF=SUMF/2.0
      SUMU=SUMU/2.0
      TUVOL=TUVOL/UUNIT
      SUMT=SUMF+SUMU
      WRITE (6,2031) SUMF,SUMU,SUMT,XTSQ,TUVOL
 2031 FORMAT(//,10X,'PENALTY IN FLUX SHAPE = ',E12.5,/,10X,
     1'PENALTY IN CONTROL EFFORT = ',E12.5,/,10X,
     2'TOTAL PENALTY PAID BY THE COST FUNCTIONAL = ',E12.5,/,10X,
     3'STANDARD DEVIATION OF THERMAL FLUX DISTRIBUTION = ',F12.5,' (%)',
     4/,10X,'TOTAL NUMBER OF LATTICE OCCUPIED BY CONTROLLERS = ',
     5F12.5,/)
      CALL HEAD
      WRITE (6,2009)
 2009 FORMAT(10X,'THERMAL FLUX DISTRIBUTION WITH THE STATIC CONTROLLER
     1',/)
      CALL XMAP(XI)
      CALL HEAD
      WRITE (6,2030)
 2030 FORMAT(/,30X,'FLUX REGULATION INFORMATION',//)
      WRITE (6,2032) QSC,QFAC,RFAC
 2032 FORMAT(/,10X,'SCALE FACTOR FOR SYSTEM MATRIX = ',E12.5,/,10X,
     1'SCALE FACTOR FOR THE MODE AMPLITUDE LOSS = ',E12.5,/,10X,
     2'SCALE FACTOR FOR CONTROL COST = ',E12.5,/)
C     COMPUTES THE DYNAMIC RANGE OF CONTROLLERS FOR FLUX REGULATION
C     INITIAL SOLUTION OF THE RICCATI EQUATION
      IL=1
      THET=1.0E-2
      XTIME=1.0E-2
```

```
      DO 38 L=1,NMOD
      DO 38 M=1,NMOD
      PMTX(L,M)=DMTX(L,M)*XTIME
      TRMX(L,M)=PMTX(L,M)*XTIME
   38 CONTINUE
      DO 39 L=1,NMOD
      DO 39 M=1,NMOD
      SMX22(L,M)=SMX22(L,M)/QSC
   39 CONTINUE
   80 CONTINUE
      ERR=0.0
      XIL=IL+1.0
      DO 81 L=1,NMOD
      DO 81 M=L,NMOD
      TRMY(L,M)=-(DOT3(20,10,1,NMOD,SMX22,TRMX,L,M)+
     1           DOT4(10,20,1,NMOD,TRMX,SMX22,L,M))/XIL
      TRMY(M,L)=TRMY(L,M)
   81 CONTINUE
      DO 82 L=1,NMOD
      DO 82 M=1,NMOD
      PMTX(L,M)=PMTX(L,M)+TRMY(L,M)
      TRMX(L,M)=TRMY(L,M)*XTIME
      XERR=ABS(TRMY(L,M)/PMTX(L,M))
      IF (XERR.GE.ERR) ERR=XERR
   82 CONTINUE
      IF (ERR.LE.EP) GO TO 83
      IL=IL+1
      IF (IL.GE.50) GO TO 83
      GO TO 80
   83 CONTINUE
      CALL GINV(PMTX,TRMY,W1,W2,10,NMOD,NMOD,ERROR)
      IF (ERROR.GE.1.0E-5) WRITE (6,9999) ERROR
 9999 FORMAT(10X,'INVERSE ERROR = ',E12.6)
      IT=1
      DO 85 L=1,NMOD
      DO 85 M=1,NMOD
      TMRX(L,M)=PMTX(L,M)
   85 CONTINUE
      WRITE (6,2025)
 2025 FORMAT(//,8X,'IT  CONVERGEN  LYAPUNOV ERR   IT.TIME',/)
  100 CONTINUE
      EPS=1.0
      CALL SECOND(SEC1)
      DO 42 L=1,NMOD
      DO 42 M=1,NMOD
      SMXX(L,M)=SMX22(L,M)-DOT3(10,10,1,NMOD,DMTX,PMTX,L,M)
      QMTX(L,M)=-SMX11(L,M)
      DO 43 LL=1,NMOD
      CC=0.0
      DO 41 MM=1,NMOD
      CC=CC+DMTX(LL,MM)*PMTX(MM,M)
   41 CONTINUE
      QMTX(L,M)=QMTX(L,M)-PMTX(LL,L)*CC
   43 CONTINUE
   42 CONTINUE
```

```
      CALL AXPXA(SMXX,TRMX,QMTX,TRMY,NMOD,10,10,10,10,EPS)
      CON=0.0
      DO 44 L=1,NMOD
      DO 44 M=1,NMOD
      QMTX(L,M)=QMTX(L,M)*QSC
      IF (TMRX(L,M).EQ.0.0) GO TO 44
      ERR=QMTX(L,M)/TMRX(L,M)
      ERROR=ABS(1.0-ERR)
      CON=AMAX1(CON,ERROR)
   44 CONTINUE
      CALL SECOND(SEC2)
      SEC=SEC2-SEC1
      WRITE (6,2026) IT,CON,EPS,SEC
 2026 FORMAT(8X,I2,3E12.5)
      IF (CON.LE.1.0E-3) GO TO 50
      DO 440 L=1,NMOD
      DO 440 M=1,NMOD
      TRMY(L,M)=0.0
      TMRX(L,M)=QMTX(L,M)
      DO 441 LL=1,NMOD
      TRMY(L,M)=TRMY(L,M)-QMTX(LL,L)*DOT3(10,10,1,NMOD,DMTX,QMTX,LL,M)
  441 CONTINUE
      TRMY(L,M)=TRMY(L,M)+DOT3(10,20,1,NMOD,QMTX,SMX22,L,M)+
     1            SMX11(L,M)+DOT2(20,10,1,NMOD,SMX22,QMTX,L,M)
  440 CONTINUE
      IT=IT+1
      DO 46 L=1,NMOD
      DO 46 M=1,NMOD
      PMTX(L,M)=QMTX(L,M)-THET*TRMY(L,M)
   46 CONTINUE
      IF (IT.LE.21) GO TO 100
      WRITE (6,2010)
 2010 FORMAT(/,10X,'WARNING - RICCATI EQUATION ILL-CONDITIONED',/)
C     COMPUTES NORM OF CONTROLLER RANGE FOR FLUX REGULATION
   50 CONTINUE
      DO 45 L=1,NMOD
      DO 45 M=1,NMOD
      PMTX(L,M)=QMTX(L,M)/QSC
   45 CONTINUE
      CALL HEAD
      WRITE(6,2027)
 2027 FORMAT(//,10X,'SOLUTION MATRIX OF THE RICCATI EQUATION (P)',/)
      DO 47 L=1,NMOD
      WRITE (6,2024) (PMTX(L,M),M=1,NMOD)
   47 CONTINUE
      DO 51 L=1,NMOD
      DO 51 K=1,MCON
      TRMX(L,K)=-DOT2(10,10,1,NMOD,BMTX,PMTX,K,L)/RFAC
   51 CONTINUE
      WRITE(6,2028)
 2028 FORMAT(//,10X,'MODAL FEEDBACK GAIN MATRIX (-INV(R)*TRAN(B)*(P)',/)
      DO 48 K=1,MCON
      WRITE (6,2024) (TRMX(L,K),L=1,NMOD)
   48 CONTINUE
      WRITE(2) TRMX
```

```
      UNORM=0.0
      DO 52 K=1,MCON
      UNORM=UNORM+DOT(10,1,NMOD,TRMX,K,K)
   52 CONTINUE
      UNORM=SQRT(UNORM)
      UDEL=UNORM/NMOD
      DO 53 K=1,MCON
      CC=0.0
      W1(K)=0.0
      DO 531 L=1,NMOD
      CC=TRMX(L,K)*TRMX(L,K)
      W1(K)=W1(K)+CC
  531 CONTINUE
      W1(K)=SQRT(W1(K))*TILT
   53 CONTINUE
      WRITE (6,2011) UNORM
 2011 FORMAT(//,10X,'NORM OF (-INV(R)*TRAN(B)*(P))-MATRIX = ',E12.6)
      WRITE (6,2012)
 2012 FORMAT(//,10X,'DYNAMIC RANGE OF CONTROLLERS FOR REGULATION (% FILL
     1ED)',/)
      WRITE (6,2015)
 2015 FORMAT(/,5X,'CONTROLLER     1         2         3         4
     1  5         6         7         8         9        10',/)
      DO 54 K=1,MCON
      UTEMP=100.0/UUNIT/UVOL(K)
      UC(K)=(UB(K)+W1(K))*UTEMP
   54 CONTINUE
      WRITE (6,2013) (UC(K),K=1,MCON)
 2013 FORMAT(4X,'UPPER LIMIT',10F10.5)
      DO 55 K=1,MCON
      UTEMP=100.0/UUNIT/UVOL(K)
      UC(K)=(UB(K)-W1(K))*UTEMP
   55 CONTINUE
      WRITE (6,2014) (UC(K),K=1,MCON)
 2014 FORMAT(4X,'LOWER LIMIT',10F10.5)
C     COUNTERACTING REACTIVITY DISTURBANCES
      DO 56 L=1,NMOD
      DO 56 M=1,NMOD
      TRMX(L,M)=DOT3(10,20,1,NMOD,QMTX,SMMX,L,M)
   56 CONTINUE
      DO 57 L=1,NMOD
      DO 57 M=1,NMOD
      SMMX(L,M)=TRMX(L,M)*SIGMA
   57 CONTINUE
      EPS=2.0
      DO 58 L=1,NMOD
      DO 58 M=1,NMOD
      SMXX(L,M)=SMX22(L,M)-DOT3(10,10,1,NMOD,DMTX,QMTX,L,M)
   58 CONTINUE
      DO 59 L=1,NMOD
      DO 59 M=1,NMOD
      SMXX(L,M)=SMXX(L,M)*QSC
      QMTX(L,M)=-DOT4(20,20,1,NMOD,SMMX,SMMX,L,M)
   59 CONTINUE
      CALL AXPXA(SMXX,TRMX,QMTX,TRMY,NMOD,10,10,10,10,EPS)
```

```
      CALL HEAD
      WRITE (6,2029)
 2029 FORMAT(//,10X,'SOLUTION MATRIX OF LYAPUNOV EQUATION (K)',/)
      DO 60 L=1,NMOD
      WRITE (6,2024) (QMTX(L,M),M=1,NMOD)
   60 CONTINUE
      DO 61 K=1,MCON
      SUMWW=0.0
      DO 62 L=1,NMOD
      SUMWW=SUMWW+BMTX(L,K)*DOT3(10,10,1,NMOD,QMTX,BMTX,L,K)/(RFAC*
     1      RFAC)
   62 CONTINUE
      W2(K)=ABS(SUMWW)
      W2(K)=SQRT(W2(K))
   61 CONTINUE
      WRITE (6,2017)
 2017 FORMAT(//,10X,'DYNAMIC RANGE OF CONTROLLERS FOR COUNTERACTION (% F
     1ILLED)',/)
      WRITE (6,2015)
      DO 63 K=1,MCON
      UTEMP=100.0/UUNIT/UVOL(K)
      UC(K)=(UB(K)+W1(K)+W2(K))*UTEMP
   63 CONTINUE
      WRITE (6,2013) (UC(K),K=1,MCON)
      DO 64 K=1,MCON
      UTEMP=100.0/UUNIT/UVOL(K)
      UC(K)=(UB(K)-W1(K)-W2(K))*UTEMP
   64 CONTINUE
      WRITE (6,2014) (UC(K),K=1,MCON)
      RETURN
      END

      SUBROUTINE SIMUL(NPROB)
C**** SIMULATION STUDY FOR A TRANSIENT WITH A GIVEN CONTROL SYSTEM
      COMMON /STFN/  XF(31,23), XT(31,23), XI(31,23), XE(31,23)
      COMMON /XSCN/ DIF1(25),   DIF2(25),  SIGA1(25), SIGA2(25),
     1              SIGR(25),   SIGF(25),  ALFA(25),  HFAC(25),
     2              XK(10),     V1,        V2,        XNU,
     3              GAMI,       GAMX,      SIGXE,     ZLAMI,
     4              ZLAMX,      XKS
      COMMON /GEOM/ MAT(31,23), DX(31),    DY(23),    X(31),     Y(23)
      COMMON /MISC/ TITLE(18),  NPAGE,     NMOD,      M2,        M4,
     1              NRX,        NRY,       NTT,       PWR,       BLANK,
     2              CX,         CY,        NSX,       NSY,       IUNSTB,
     3              RFAC,    SFAC,   QFAC,    AMPO
      COMMON /SYSM/ SMX11(20,20), SMX12(20,20), SMX21(21,21),
     1              SMX22(20,20), SMMX(20,20)
      COMMON /XYMOD/ PF(31,23,8), PS(31,23,8),  FNORM(8), TNORM(8)
      COMMON /NCON/ GGMX(10),  NSIGN(10),  YL(10),   XL(10)
      COMMON /BCON/ BMTX(10,20),  CMTX(10,20),  UB(20)
      DIMENSION     AMP(16,2),  AMP1(16),  AMX(10,10),  BMX(10,10),
     1              TRMX(10,10),  DMTX(10,10),  W1(10),   W2(10),
     2              US(10),    GMTX(10,10),   UVOL(10),  CMX(10,10),
     3              AMPO(16)
```

```
C
      TIME=0.0
      LPAGE=1
      REWIND 2
      READ (2) MCON
      READ (2) XL,YL,NSIGN
      READ (2) BMTX
      READ (2) UB,UUNIT,UVOL
      READ (2) GMTX
      READ (5,1001) TMAX,PLVL,PRAT,DT1
 1001 FORMAT(8E10.4)
 2000 FORMAT(30X,'TRANSIENT SIMULATION WITH A GIVEN CONTROL SYSTEM',//)
      CALL SMTRX(NPROB)
      CALL HEAD
      WRITE (6,2000)
      WRITE (6,2002) TMAX,PLVL,PRAT,DT1
 2002 FORMAT(10X,'TRANSIENY PERIOD = ',1PE10.4,' (HRS)',/,10X,
     1'INITIAL POWER LEVEL = ',1PE10.4,' (%)',/,10X,
     2'ALLOWABLE POWER CHANGE RATE = ',1PE10.4,' (%/SEC)',/,10X,
     3'TIME INCREMENT = ',1PE10.4,' (HRS)',/)
      NMOD1=NMOD+1
      NMOD11=NMOD1+1
      NMOD2=2*NMOD
C     READ INITIAL PERTURBED MODE AMPLITUDE INCREMENTS
      DO 10 L=1,NMOD2
      DO 10 I=1,2
      AMP(L,I)=0.0
   10 CONTINUE
      READ (5,1001) (AMP(L,1),L=NMOD11,NMOD2)
 2003 FORMAT(/,10X,'MODAL AMPLITUDE INCREMENT',//,3X,'IDENT   MODE=1
     1     2          3          4          5          6          7          8
     2',/)
 2004 FORMAT(1X,'FAST FLX ',8(1PE10.3))
 2005 FORMAT(1X,'THML FLX ',8(1PE10.3))
C     INITIAL AMPLITUDES FOR IODINE AND XENON
      SUMO=SUM=0.0
      DO 5 I=1,NRX
      DO 5 J=1,NRY
      DV=DX(I)*DY(J)
      SUMO=SUMO+PS(I,J,1)*DV
      DO 5 L=1,NMOD
      L1=NMOD+L
      IF (L.EQ.1) GO TO 5
      ATEMP=AMP(L1,1)
      SUM=SUM+ATEMP*PS(I,J,L)*DV
    5 CONTINUE
      PTEMP=SUM/SUMO
      RTEMP=PLVL/100.0
      AMP(NMOD1,1)=RTEMP*(1.0-PTEMP)-1.0
      DO 6 L=2,NMOD
      L1=NMOD+L
      AMP(L1,1)=RTEMP*AMP(L1,1)
    6 CONTINUE
```

```
      DO 7 L=1,NMOD
      DO 7 M=1,NMOD
      AMX(L,M)=SMX11(L,M)
    7 CONTINUE
      CALL GINV(AMX,TRMX,W1,W2,10,NMOD,NMOD,ERROR)
      DO 11 L=1,NMOD
      DO 11 M=1,NMOD
      M1=NMOD+M
      TRMX(L,M)=SMX11(L,M1)
   11 CONTINUE
      DO 12 L=1,NMOD
      DO 12 M=1,NMOD
      M1=NMOD+M
      AMP(L,1)=AMP(L,1)-DOT2(10,10,1,NMOD,AMX,TRMX,L,M)*AMP(M1,1)
   12 CONTINUE
      DO 13 L=1,NMOD
      DO 13 M=1,NMOD
      CMTX(L,M)=DOT4(10,10,1,MCON,BMTX,GMTX,L,M)
   13 CONTINUE
      DO 24 L=1,NMOD
      L1=NMOD+L
      DO 24 M=1,NMOD
      M1=NMOD+M
      DO 25 N=1,NMOD
      N1=NMOD+N
      DO 25 NN=1,NMOD
      CMX(L,M)=SMX11(L1,M1)-SMX11(L1,N)*AMX(NN,N)*SMX11(NN,M1)
   25 CONTINUE
      BMX(L,M)=CMX(L,M)+CMTX(L,M)
   24 CONTINUE
      CALL GINV(BMX,TRMX,W1,W2,10,NMOD,NMOD,ERROR)
      CALL GINV(CMX,TRMX,W1,W2,10,NMOD,NMOD,ERROR)
      DO 14 L=1,NMOD
      DO 14 M=1,NMOD
      DMTX(L,M)=SMX22(L,M)
   14 CONTINUE
      CALL GINV(DMTX,TRMX,W1,W2,10,NMOD,NMOD,ERROR)
      DO 15 L=1,NMOD
      DO 15 M=1,NMOD
      M1=NMOD+M
      TRMX(L,M)=SMX21(L,M1)
   15 CONTINUE
      DO 16 L=1,NMOD
      DO 16 M=1,NMOD
      M1=NMOD+M
      AMP(L,2)=AMP(L,2)-DOT2(10,10,1,NMOD,DMTX,TRMX,L,M)*AMP(M1,1)
   16 CONTINUE
      DO 20 L=1,NMOD
      L1=NMOD+L
      DO 20 M=1,NMOD
      M1=NMOD+M
      DMTX(L,M)=SMX22(L1,M1)
   20 CONTINUE
```

```
      CALL GINV(DMTX,TRMX,W1,W2,10,NMOD,NMOD,ERROR)
      DO 21 L=1,NMOD
      L1=NMOD+L
      DO 21 M=1,NMOD
      M1=NMOD+M
      DO 22 N=1,NMOD
      N1=NMOD+N
      AMP(L1,2)=AMP(L1,2)-DMTX(M,L)*(SMX21(M1,N1)*AMP(N1,1)+
     1                             SMX22(M,N1)*AMP(N,2))
   22 CONTINUE
   21 CONTINUE
C     INITIAL CONTROLLER LEVELS ASSUMED TO BE FROZEN
      DO 26 K=1,MCON
      US(K)=0.0
   26 CONTINUE
      WRITE (6,2008) TIME
      WRITE (6,2009) (US(K),K=1,MCON)
      WRITE (6,2003)
      WRITE (6,2004) (AMP(L,1),L=1,NMOD)
      WRITE (6,2005) (AMP(L,1),L=NMOD1,NMOD2)
      WRITE (6,2006) (AMP(L,2),L=1,NMOD)
      WRITE (6,2007) (AMP(L,2),L=NMOD1,NMOD2)
 2006 FORMAT(1X,'IDNE CON ',8(1PE10.3))
 2007 FORMAT(1X,'XENN CON ',8(1PE10.3))
 2008 FORMAT(//,10X,'CONTROLLERS LEVEL AT TIME (% FILLED) = ',F10.5,' (H
     1RS)',/,2X, 'IDENT =       1         2         3         4         5
     2         6         7         8         9        10',/)
 2009 FORMAT(2X,'LEVEL',3X,10F10.5)
C     TRANSIENT CALCULATION
      DO 27 K=1,MCON
      US(K)=US(K)*UUNIT*UVOL(K)/100.0
   27 CONTINUE
      DTO=ABS(100.0-PLVL)/PRAT
  100 CONTINUE
      DT=DT1*3600.0
      IF (TIME.NE.0.0) GO TO 30
      DT=DTO
      IF (DTO.EQ.0.0) DT=5.0
   30 CONTINUE
      TIME=TIME+DT
      XTIME=TIME/3600.
      IF (XTIME.GT.TMAX) GO TO 999
      NSTEP=60
      DTSTEP=DT/NSTEP
      IF (TIME.GT.5.0) GO TO 37
      NSTEP=1
      DTSTEP=DT
      WRITE (6,2001)
 2001 FORMAT(//,10X,'***** CONTROLLERS ARE ACTIVATED')
   37 CONTINUE
```

```
      DO 50 IT=1,NSTEP
      DO 35 K=1,MCON
      DO 36 M=1,NMOD
      M1=NMOD+M
      US(K)=US(K)+GMTX(M,K)*AMP(M1,1)
   36 CONTINUE
      US(K)=US(K)*100.0/(UUNIT*UVOL(K))
   35 CONTINUE
      DO 31 L=1,NMOD
      L1=NMOD+L
      AMPO(L)=AMP(L,1)
      AMPO(L1)=AMP(L1,1)
      AMP1(L)=AMP(L,2)
      AMP1(L1)=AMP(L1,2)
      DO 31 M=1,NMOD
      M1=NMOD+M
      TRMX(L,M)=SMX12(L1,M1)
   31 CONTINUE
      DO 32 L=1,NMOD
      L1=NMOD+L
      AMP(L,1)=0.0
      AMP(L1,1)=0.0
      DO 32 M=1,NMOD
      M1=NMOD+M
      AMP(L1,1)=AMP(L1,1)-DOT2(10,10,1,NMOD,BMX,TRMX,L,M)*AMP1(M1)
   32 CONTINUE
      DO 33 L=1,NMOD
      DO 33 M=1,NMOD
      M1=NMOD+M
      TRMX(L,M)=SMX11(L,M1)
   33 CONTINUE
      DO 34 L=1,NMOD
      DO 34 M=1,NMOD
      M1=NMOD+M
      AMP(L,1)=AMP(L,1)-DOT2(10,10,1,NMOD,AMX,TRMX,L,M)*AMP(M1,1)
   34 CONTINUE
      DO 40 L=1,NMOD
      L1=NMOD+L
      DO 41 M=1,NMOD
      M1=NMOD+M
      ATEMP=(AMPO(M1)+AMP(M1,1))/2.0
      AMP(L,2)=AMP(L,2)+(SMX21(L,M1)*ATEMP+SMX22(L,M)*AMP1(M))
     1                    *DTSTEP
      AMP(L1,2)=AMP(L1,2)+(SMX21(L1,M1)*ATEMP+SMX22(L1,M)*AMP1(M)
     1           +SMX22(L1,M1)*AMP1(M1))*DTSTEP
   41 CONTINUE
   40 CONTINUE
```

```
      IF (IT.NE.NSTEP) GO TO 42
      WRITE (6,2008) XTIME
      WRITE (6,2009) (US(K),K=1,MCON)
      WRITE (6,2003)
      WRITE (6,2004) (AMP(L,1),L=1,NMOD)
      WRITE (6,2005) (AMP(L,1),L=NMOD1,NMOD2)
      WRITE (6,2006) (AMP(L,2),L=1,NMOD)
      WRITE (6,2007) (AMP(L,2),L=NMOD1,NMOD2)
   42 CONTINUE
      DO 43 K=1,MCON
      US(K)=US(K)*UUNIT*UVOL(K)/100.0
   43 CONTINUE
   50 CONTINUE
      LPAGE=LPAGE+1
      XPAGE=LPAGE/3.0
      MPAGE=XPAGE
      YPAGE=MPAGE
      DPAGE=XPAGE-YPAGE
      IF (DPAGE.EQ.0.0) CALL HEAD
      GO TO 100
  999 RETURN
      END
```