OPTIMAL DEPLOYMENT OF CONTROLLER-
DETECTORS FOR THE HWR SYSTEM

by

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A major concern in the design of control systems for nuclear power reactors is where and how many controllers and detectors are to be deployed in the reactor to satisfy design criteria. In order to answer this concern, we have developed an analytical method in which emphasis is placed on the linear regulator theory and the least square estimation theory.

This work has four areas: realization of the measure of the optimality of the controller locations in terms of feedback gain in spatial control; calculation of the static set-points to compensate the excess reactivity of the reference state, evaluation of dynamic range of controllers for regulating neutron fluxes; and estimation of additional responses counteracting burn-up/fuelling induced random external disturbances.

The deviations of neutron, iodine and xenon distributions from the reference states were expanded with the referenced $\lambda$-modes. The order of amplitude vector space was reduced by assumption of the dominant mode concept. Performance indices were formed with reduced state vectors and separated control functions. Pontryagin's maximum principle was applied to deterministic components and the square-root filtering to stochastic components.
Problems were finally narrowed to solve a series of the algebraic matrix Lyapunov and Riccati equations whose solutions imply a linear transformation of adjoints to state vectors.

A computer code ODZCA was developed for designing CANDU zone control systems using the above theories. Analysis of the existing 600 MWe CANDU zone control compartments in terms of their locations and numbers led to the following conclusions. The effective region for both spatial and bulk control was very limited and, hence, the current vertical compartments occupied the region most effectively; also the range of spatial control assigned to individual compartments was coincident with the spatial control effective region found in the study.

Alternatively we propose a horizontal zone control system that has comparable performance with better predictability.
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I. BACKGROUND AND CONCEPTS

I.1 Introduction

A nuclear power reactor is directly controlled by adjustments of the neutron population varying over both time and space. The neutron population distribution is governed by the local/global multiplication factor changes, partly due to the build-up of fission products, such as $^{135}\text{Xe}$, partly because of refuelling and fuel burnup and partly as a result of temperature changes. All these changes have to be compensated by a reliable and accurate mechanism with certain predictable performance to maintain proper control of neutron distribution.

The most common and effective means for controlling criticality is the insertion or extraction of a specific neutron absorbing material from the reactor core. The types of control mechanism are usually divided into two categories; one is injection or extraction of liquid type absorber solution; the other method is insertion or withdrawal of solid type rods in specific locations.

The complexity of phenomena in nuclear reactions between neutrons and matter raises several difficulties in defining the function of controllers. The aspects of reactor physics considered in control design are briefly discussed as in the following:
(1) Delayed neutron production

Because the neutron life-time in conventional thermal reactors is in the $10^{-4}$ to $10^{-3}$ second range, no mechanical devices can sufficiently respond in this time domain. Thus the reactor control mechanism relies on the small fraction (for example, 0.0065 for $^{235}$U fission), of delayed neutrons born from certain fission products, commonly called 'precursors' (1)-(3). The production time constants of delayed neutrons are spread over 0.1 to 10 seconds.

(2) Xenon redistribution

$^{135}$Xe is a high neutron-absorbing fission product ($\sigma_a^{\text{xenon}} \approx 2.7 \times 10^6$ barns for 0.025 ev neutrons) decaying with a 9.2 hours half-life. Even when the reactor is controlled in the global sense, the localized variation of xenon may induce oscillations in the neutron population distribution. For $^{235}$U fuelled reactors, "oscillations are not possible for neutron fluxes below $3 \times 10^{11}$ neutron/cm$^2$-sec" (4)-(6).

Most of the current power reactors have neutron fluxes of the order $10^{13} - 10^{14}$ n/cm$^2$-sec well beyond the above threshold value. Therefore, the controllers in reactors operating at these high flux levels must be capable of eliminating the xenon-induced instability or oscillations (7)-(12).

(3) Burnup and refuelling

Fissile depletion and fertile conversion are the most slowly varying kinematics affecting the criticality factor. For reactors charged yearly with fresh fuel, the main concern of control is the long term criticality consideration. For CANDU reactors, the on-power fueling
concept requires that controllers respond to daily perturbed local and global flux levels.

(4) **Temperature Feedback**

Since power reactors are tied to electric generators through heat transport loops, changes in plant operation conditions cause disturbances in the physical characteristics of the reactor, e.g., shifting neutron energy spectrum, changing macroscopic reaction cross sections, etc. These feedback effects act over a time scale of the order of seconds and minutes.

(5) **Randomness of material motions**

Added to the statistical nature of neutron fluxes and their reactions with matter, void formation in the coolant, vibrations of structures, etc. cause the neutron population to fluctuate continuously. Control of this noise component should be eliminated from the objectives of control function.

Actually every transient occurring in nuclear reactors is a combination of the above physical phenomena and each component is not easily separable in a sufficiently accurate manner.

It appears that the most desirable method on which the reactor control system might rely would be to solve simultaneously all the dynamic equations describing the various phenomena and finding appropriate control actions to eliminate undesirable disturbances. But, even with modern computing systems, it is almost impossible to obtain.
of a complex nonlinear space-time dependent equation system in which a controller action is explicitly represented for the real-time applications. The common strategy used in power reactor control is, therefore, to minimize the neutron flux deviation from some predetermined reference shapes during any transients.

Because extreme difficulties arise in the design of control systems mainly due to the complexity and due to the infinite number of possible situations which can be anticipated during operation, the principle which is incorporated in conventional design method is to distribute the individual controllers with geometrical regularity and symmetry\(^{(5),(20)}\). After a large number of simulations for various significant situations which could occur during operation, designers usually choose a symmetric layout pattern which satisfies given control requirements and safety criteria.

Before designing the control system, the reactor designers usually set up quantitative criteria in terms of reactivity and power such as:

1. Limits on the total and individual reactivity worths and rates of controllers,
2. Limits on the global and the local power and their rates of change.

Obviously the reactivity is a measure of the potential for change in the neutron population and the associated power varies.
To control the global components of reactivity or power, it is preferrable that the controllers are distributed so as not to create flux distortions due to their presence. But to control the local components, the controllers should be placed where those local distortion can be effectively eliminated.

Modern nuclear power reactors have large-flat, high flux regions. In such circumstances, weak coupling of reactor sectors induces local criticality control problems. Therefore at least one controller must be assigned to control each sector of the reactor. But the deployment of as many controllers as necessary for local control is always accompanied by supporting structural materials, proportional to the number of controllers, which introduce parasitic neutron absorption into the core. From a neutron economy point of view, reduction in the amount of such material is desirable, which leads eventually to a reduction in the number of controllers.

From a safety aspect, a large number of controllers require more complicated electrical and mechanical circuitry, which inherently decreases the reliability of the system. But a system with a small number of controllers may not have sufficient redundancy built into it and, thus, even for a single controller failure, may possibly impact on reactor integrity.

These conflicting aspects acting in an inverse relationship to each
other, indicate that there should exist certain optimal conditions for deployment of the control system in the reactor as far as their numbers and locations are concerned.

I.2 History of Optimum Nuclear Reactor Control

Related to optimality in the deployment pattern of controllers in systems described by partial differential equations, Vidyasagar and Heggins\(^{(21)}\) have obtained conditions describing the existence of a control system. This system acts at a finite number of points for a distributed parameter system, which approximates a continuously distributed control function. Since then, numerous investigations have been made on the optimal discrete control of the distributed parameter system by the deterministic\(^{(22)}\)-(\(^{(25)}\)) or by the stochastic approach\(^{(26)}\)-(\(^{(28)}\)).

The optimal control design problem, however, has received little attention, at least in its practical utilization, in reactor system design despite its importance. The absence in application of optimal distribution parameter control theory in a full-sized industrial process has been criticized by W. H. Ray\(^{(29)}\). We are going to quote his reasons of the principal hindrances he identified:

(1) the lack of properly educated control engineers.

(2) a dearth of real-time experience,
(3) a communication gap between the potential user and the control theorist.

Even though a nuclear reactor is not such a simple dynamic system that the time characteristics of system variables are bounded in the same order of scale and that coefficient parameters can be expressed by analytical or algebraic functions, we have chosen to look at different aspects of the problem, because:

(1) most of basic design procedures for nuclear power reactors were established in the 60's when there was no concrete analytical method to attack the problem properly,

(2) large amount of design and operational experiences might be enough to confirm that the system designed with conventional methods would be optimum or suboptimum from the performance point of view.

Although the activity in practical application has not been sufficient, several excellent attempts have been made to demonstrate the feasibility of applying optimal control theory with applaudable results.

D. W. Wieberg (30) studied the slow transient problem, i.e., control of the xenon-induced spatial oscillation, with minimum number of controllers. The control of total power was separated from the spatial control and he assigned only a single controller to the control of the fundamental power shape. He derived the necessary
conditions for controllability with minimum number of controllers whose sufficiency was proved later by N. Suda\textsuperscript{(31)}. Wieberg's controllability theorems state that any finite number of modes of the reactor model can be returned to zero by the action of a control if:

(1) there are at least as many control rods as the maximum multiplicity of the spatial operator,

(2) all control rods are not on any possible mode of a combination of modes having the same eigenvalues.

However, such a minimum number of controllers may not be enough when we consider some practically important constraints, for example, control margin and redundancy.

W.M. Stacey\textsuperscript{(32)} derived an optimal continuous control function by solving the coupled non-linear neutron-iodine-xenon dynamic equations and their adjoint equations using nodal formulation and quasilinear, numerical algorithm. Although his proposed method makes the physical meaning of the problem clear, his control function is not regarded as a general control function in that it does not lead to a unique optimum number and locations of controllers for all possible transients. This is because the solution of the two-point-boundary-problem depends only on the given initial and final conditions, or on the transversality conditions for the unspecified terminal condition problem. A further difficulty arises because the resulting controllers must have a spatially continuous distribution, whereas in a real reactor they are discrete rods.

There are several good review papers covering new trends in
nuclear reactor control design using optimal control theory. J. Karppinen\(^{(33)}\) presented a broad review of spatial reactor control methods that have employed optimization techniques. He outlined the formulation of the spatial control algorithm, as an optimal control problem, and solution methods based on distributed parameter optimal control\(^{(34),(35)}\), variational calculus\(^{(36),(37)}\), dynamic programming\(^{(38),(39)}\), mathematical programming\(^{(40),(41)}\), the maximum principle\(^{(42)-(45)}\), linear-quadratic control theory\(^{(46)}\), and heuristic control techniques\(^{(47)}\).

S. Tzafestas\(^{(48)}\) surveyed the distributed parameter optimal control methods applied to nuclear reactor. After a brief introduction to the derivation of the nuclear reactor distributed parameter differential and integral models, he discussed the formulation of the linear-quadratic, the bilinear, the eigenvalue assignment and the stochastic control problems. He also mentioned the necessity of much further work for designing and implementing practical finite dimensional optimal controllers based on distributed parameter models of complete power reactor systems.

Apart from these pioneer-type or review-type papers, numerous research works appear in the literature, but the common conditions and strategies are specific problem-oriented ones:

(1) finding optimal manoeuvering strategies using existing control systems\(^{(49)-(51)}\),

(2) finding optimal control functions for given initial and/or final conditions\(^{(52)-(55)}\).
(3) dealing only with the spatial control problems or the reactivity control problems.

In Canada, because of the unique type of power reactors, research activities on optimal control problems have emphasized burnup control \(^{(56)}\), the xenon induced oscillation control and load-following strategies \(^{(57)},(58)}\) for the CANDU power reactors.

I.3 Fundamentals of CANDU Reactor Control

The CANDU-PBH reactor is a heavy water moderated, heavy water cooled, natural uranium fuelled reactor which utilizes the pressure tube concept \(^{(59)}\). The pressure tubes containing the fuel and the coolant run horizontally in the reactor core. Each pressure tube is isolated and insulated from the heavy water moderator by a concentric calandria tube and a gas annulus. Figure I-1 \(^{(59)}\) shows the standard CANDU reactor system.

The reactivity control mechanisms are vertically mounted from the reactor top and reside in the low pressure moderator. The layout of reactivity mechanisms is illustrated in Figure I-2 \(^{(59)}\). Natural uranium fuelled reactors have low excess reactivity and therefore parasitic neutron absorption should be as low as possible. This results in the necessity of on-power fuelling as a method of reactivity supply and

* CANada Deuterium Uranium Pressurized Heavy Water
FIGURE I-1 REACTOR ASSEMBLY

1. CALANDRIA
2. CALANDRIA - SIDE TUBESHEET
3. CALANDRIA TUBES
4. EMBEDMENT RING
5. FUELLING MACHINE - SIDE TUBESHEET
6. END SHIELD LATTICE TUBES
7. END SHIELD COOLING PIPES
8. INLET-OUTLET STRAINER
9. STEEL BALL SHIELDING
10. END FITTINGS
11. FEEDER PIPES
12. MODERATOR OUTLET
13. MODERATOR INLET
14. HORIZONTAL FLUX DETECTOR UNIT
15. ION CHAMBER
16. EARTHQUAKE RESTRAINT
17. CALANDRIA VAULT WALL
18. MODERATOR EXPANSION TO HEAD TANK
19. CURTAIN SHIELDING SLABS
20. PRESSURE RELIEF PIPES
21. RUPTURE DISC
22. REACTIVITY CONTROL UNIT NOZZLES
23. VIEWING PORT
24. SHUTOFF UNIT
25. ADJUSTER UNIT
26. CONTROL ABSORBER UNIT
27. ZONE CONTROL UNIT
28. VERTICAL FLUX DETECTOR UNIT
29. LIQUID INJECTION SHUTDOWN NOZZLE
30. BALL FILLING Pipe
consequently long-term control of reactivity and power shape. Therefore low reactivity worth of the control mechanisms is one of the requirements of the control system design.

The prompt neutron lifetime in a CANDU lattice is relatively long (\( \sim 0.9 \) m sec) and the fission delayed neutron fraction (\( \sim 0.005 \)) is augmented by the presence of photon neutrons that are produced by (n,\( \gamma \)) reactions with deuterium nuclei (60), (61). These two factors slow down a potential power excursion considerably and, thus, the control of global criticality factor is quite easy with a small amount and rate of controller's reactivity worth.

The size of the CANDU reactor core is comparatively large. In order to produce a large amount of power and reduce the reactor size, increasing the volume of the high power region and flattening the power shape in that region are required. These steps induce xenon instability or spatial power oscillation control problems (62) and require a xenon override capability of the control system (63), (64).

For the above reasons, the spatial control of the CANDU reactors has been repeatedly emphasized rather than bulk control. O. A. Trojan (65) mentioned the principles of CANDU spatial control and criteria in analytical methods associated with detector-controller response action.
Generally the CANDU control system is composed of three different kinds of mechanisms to regulate the reactor power distribution:

(1) **Zone Control Compartments**

The main method for controlling reactor power is by adjustment of the average $\text{H}_2\text{O}$ level in independently controllable compartments in regions of the reactor. Differential adjustment of levels in individual compartments is used for spatial control. Platinum in-core flux detectors provide the neutron flux feedback signals required by the digital control computers.

(2) **Mechanical Control Absorbers**

The reactivity range provided by the zone control system is usually adequate for most power maneuvers. However, certain situations, for example, a power setback, require additional negative reactivity that is provided by the mechanical control absorbers, normally residing out of core.

(3) **Adjuster Rod Banks**

Even with the on-power fueling to control the power shape, additional adjustments to achieve an adequately flattened power shape are required. Also to provide a sufficient amount of positive reactivity to override the xenon buildup during shutdown-restart periods, the adjuster rods normally fully inserted in the core are sequentially withdrawn in symmetrical bank modes.

The typical control modes for normal or certain upset operating conditions are shown in Figure I-3. Within ±3% power error,
FIGURE 1-3  REACTOR REGULATING SYSTEM CONTROL INTERFACE
the zone control system takes change of the global and the regional power control, if the average and the individual level of compartments are confined to the range of 20-70%. If the average level is below 20%, further control is complemented by the sequential withdrawal of the adjuster rod banks. If the average level is above 70%, the mechanical absorber banks are sequentially introduced into the reactor core until a sufficient negative reactivity is to be provided.

In order to control the flux and ultimately the power distribution in the reactor core, flux measurements are taken and an automatic on-line flux mapping system is used to provide an extensive flux map. The detection system is divided into two sub-systems according to their functions: a fast one of gamma-ray responding platinum detector assemblies for control and shutdown purposes, and a slow one of neutron responding vanadium detectors for mapping the neutron flux distribution and calibrating the platinum detectors.

The on-site control computer plays an integral role in assigning activation/deactivation of each control mechanism at the proper time. The functions and interactions of the reactor regulating system managed by the control computer are given in Figure I-4. This direct digital control system is also used for overall plant control, alarm annunciation and data display.
FIGURE 1-4 REACTOR REGULATING SYSTEM BLOCK DIAGRAM
I.4 Overview of Context

Theories and stages developed for the study on the optimal deployment of the CANDU control system are briefly discussed in this section. The main emphasis of the study is based on the linear quadratic regulator theory for the linear distributed parameter dynamic equations (28), (34), (67)-(71) separated into the bulk control and the spatial control objectives.

Assuming the separability of the system equations into the bulk and the spatial control components, the deviations of state function from a reference state are defined to satisfy approximated linear dynamic equations with the first order variation. The modal control theory (72)-(75) is available to expand the solution into an infinite series of the system eigenfunctions to be able to transform the distributed parameter state functions into the time invariant problems. We investigate the application of the λ-modes (76), (77) to reduce the order of the dynamic system whose eigenfunctions are difficult to generate. This linear time invariant approximation has advantages in its application to the CANDU controller deployment; that a unique asymptotic solution with small number of modes is guaranteed and, hence, approximately satisfies the property of finality (78), and that the optimality criteria can be derived by a regulator problem with Lyapunov's asymptotic stability (79), (80) rather than a servomechanism problem (81).
Optimality of controller locations is evaluated in terms of controller effectiveness, i.e., feedback gain due to placing a controller at a location, under the minimization conditions of a performance index constructed with iodine-xenon deviation and spatial control effort. In the time frame where the spatial control function shows the dynamic behaviour, neutron kinetics is assumed to be quasi-stationary and, thus, the reduced-order model includes the neutron balance equations implicitly. Selection of a low-order model using the dominant mode concept\(^{(82)-(84)}\) is also applied to the bulk control formulation because the iodine-xenon dynamics can be stationary in neutron kinetics problem.

As under certain observability and controllability conditions, there exists an optimum feedback control law to stabilize the dynamic system\(^{(85)-(88)}\), we introduce a boundary inside which a symmetrically placed pair of controllers can completely control the system.

The relationship between the dynamic variable representing iodine-xenon concentrations and their adjoint is expressed by a positive definite symmetric matrix which satisfies a certain matrix Riccati equation. The matrix Riccati equation is derived using the maximum principle applied on the Hamiltonian of the modified performance index and assuming a linear transformation between dynamic variables and their adjoints. Then the feedback gain of the optimum control law can be obtained as a function of the solution of the Riccati equation.

The numerical technique to solve the matrix Riccati equation is
based on the Kleinman's iterative scheme (89), which is equivalent to solving the matrix Lyapunov equation (90) at each iteration. The sufficient and necessary conditions for convergence are investigated according to Kleinman's theorems. Possible numerical truncation errors accumulated during numerous addition and multiplication operation in a digital machine is successfully reduced by the iterative correction with the residual Lyapunov equation. The convergence can be accelerated by introducing an over-relaxation parameter weighted on iterative solutions.

For every grid point in a controller domain, we can calculate the feedback gain, i.e., a measure of its control effectiveness, using the above strategy.

To determine the number of controllers, we apply similar procedures but with the time dependent neutron diffusion equations and with subsets of controller locations selected by the spatial control effectiveness and by the neutron importance distribution. Distinctive features of the dynamic equations formulated for computing bulk control are that they include inhomogeneous terms related to the reactivity source and that the feedback law can be modified by the output feedback concept (91), (92) that possibly extends the problem to the optimum detection system design.

As the first step, the inhomogeneous term describing the excess reactivity of the reference system is compensated by a pre-selected set.
of the static bulk controllers, using the calculus of variation to minimize the performance index subject to the steady state diffusion equation. The calculated depth of controllers will be used as the set-point of controllers to determine the dynamic range of them. The number of controllers in the set are roughly estimated from the maximum allowable controller material subtracted by the expected capability for the dynamic response. If we have reactors which are ideally fuelled, i.e., no flux variation due to the fuelling operation, the set of controllers determined in this way would be sufficient for our purpose.

However, in practice the on-power fuelling combined with the burnup process, has reciprocal effects where one inserts continuous disturbances both locally and globally and the other prevents the transients from going beyond acceptable limits. Thus, from the long-term view, these effects can be treated as a bounded random process whose mean is equivalent to the reference state. To utilize this idea in the formulation, we transform the time variable to the coordinate $t' = v_2 t$, where $v_2$ is the thermal neutron velocity. Then the two group time-dependent diffusion equations are reduced to the modal dynamic equations which dominate the thermal neutron behavior. The procedures are similar to those introduced to manipulate the spatial control problem.

The covariances of the disturbance vectors are obtainable from the burnup distribution of discharged fuel, if we assume the
proportionality between burnup and criticality factor. The formulation of the dynamic system equations takes advantages in the estimation of the stochastic system \(^\text{(93)-(95)}\) and in the exactness of the deterministic system. By introducing a matrix relationship between the state vectors and their adjoints, the control function and the governing equations are separable into two aspects. One deals with regulating the flux variation represented by the deterministic dynamic equation and the other deals with countering the random disturbances. Therefore the optimality of a controller set can be evaluated by the gain matrix obtained from the deterministic part plus by the covariance matrix derived from the stochastic part.

The dynamic range of the bulk controllers optimally selected by the above procedures is estimated using the maximum allowable local/global power error. And finally the range is modified by the additional expected response obtained from the variance matrix.

Table I-1 shows the time characteristics of system equations, which lead to concepts of dominant control objectives and corresponding optimization problems dealt in this thesis.
<table>
<thead>
<tr>
<th>Time Measure</th>
<th>System Equations</th>
<th>0</th>
<th>(&lt;\text{second})</th>
<th>Seconds</th>
<th>Hours</th>
<th>Days</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Criticality</td>
<td>Noise Analysis</td>
<td>Kinetics</td>
<td>Xenon Dynamics</td>
<td>Fuel Management</td>
<td></td>
</tr>
<tr>
<td>Neutron Diffusion</td>
<td>Static</td>
<td>Dynamic, Stochastic</td>
<td>Dynamic, Deterministic</td>
<td>Quasi-static</td>
<td>Quasi-static</td>
<td></td>
</tr>
<tr>
<td>Precursor</td>
<td>Static</td>
<td>Dynamic, Stochastic</td>
<td>Dynamic, Deterministic</td>
<td>Static</td>
<td>Static</td>
<td></td>
</tr>
<tr>
<td>Iodine/ Xenon</td>
<td>Static</td>
<td>Static</td>
<td>Static</td>
<td>Dynamic, Deterministic</td>
<td>Static</td>
<td></td>
</tr>
<tr>
<td>Burnup</td>
<td>Static</td>
<td>Static</td>
<td>Static</td>
<td>Static</td>
<td>Dynamic, but, if includes fuelling, then stochastic</td>
<td></td>
</tr>
<tr>
<td>Heat Transfer</td>
<td>Static</td>
<td>Static</td>
<td>Quasi-dynamic</td>
<td>Quasi-static</td>
<td>Static</td>
<td></td>
</tr>
<tr>
<td>Controllability</td>
<td>Controllable</td>
<td>Not Controllable</td>
<td>Controllable</td>
<td>Controllable</td>
<td>Controllable</td>
<td></td>
</tr>
</tbody>
</table>
II. FORMULATION OF LINEAR DYNAMIC SYSTEM

II.1 Reactor Dynamic Equations

II.1.1 Two-Group Diffusion Equations

For a description of neutron behaviour in a multiplying system, the two-group diffusion approximation (96), (97) to the Boltzmann transport equation has been proved to give sufficient accuracy for the neutronic design of the CANDU reactors.

The form of equations being illustrated here included the explicit expressions of the delayed neutron precursors and of the xenon poisoning to be able to describe the whole essence of the control objectives. The fuel burnup and refuelling obviously contribute to the burden of the control system, but the carefully controlled fuelling scheme can provide the minimum deviation from the ideal reference flux shape with small number of isolated ripples (13).

For a controlled system, the time dependent diffusion equations are written as:
\[
\frac{1}{v_1} \frac{\partial \phi_1(x,t)}{\partial t} = \nabla \cdot D_1(x,t) \nabla \phi_1(x,t) - \left( \Sigma_{a1}(x,t) + \Sigma_{Rem}(x,t) \right) \phi_1(x,t) \\
+ \sum_{i=1}^{I} \lambda_i C_i(x,t) + (1 - \beta) \nu \Sigma_{\ell2}(x,t) \phi_2(x,t) \\
\text{... (III.a)}
\]

\[
\frac{1}{v_2} \frac{\partial \phi_2(x,t)}{\partial t} = \nabla \cdot D_2(x,t) \nabla \phi_2(x,t) - \left( \Sigma_{a2}(x,t) + \sigma_x X(x,t) \right) \phi_2(x,t) \\
+ \Sigma_{Rem}(x,t) \phi_1(x,t) - \sum_{j=1}^{C} U(x,t) \phi_2(x,t) \delta(x-x_j) \\
\text{... (III.b)}
\]

where

\( \phi(x,t) \) : the neutron flux distribution function,
\( X(x,t) \) : the Xe\(^{135} \) nuclide concentration,
\( C_i(x,t) \) : the \( i \)-th group delayed neutron precursor distribution,
\( U(x,t) \) : the control function,
\( \nu \) : neutron velocity,
\( D(x,t) \) : diffusion coefficient,
\( \Sigma_a(x,t) \) : macroscopic absorption cross section,
\( \Sigma_{Rem}(x,t) \) : macroscopic removal cross section,
\( \nu \Sigma_{\ell2}(x,t) \) : macroscopic yield cross section,
\( \lambda_i \) : the \( i \)-th group precursor decay constant,
\( \beta \) : total fraction of delayed neutron, i.e., \( \beta = \sum_{i=1}^{I} \beta_i \).
\(\sigma_x\) : microscopic absorption cross section of Xe\(^{135}\),

\(x\) : space coordinates,

\(t\) : time coordinates,

\(i\) : number of delayed neutron groups,

\(I_c\) : number of controllers,

\(\delta(x-x_j)\) : Dirac delta function,

\[
\begin{cases}
1, & \text{if } x = x_j, \\
0, & \text{otherwise,}
\end{cases}
\]

and the subscripts 1 and 2 refer to the fast and the thermal neutron groups, respectively.

Assumptions involved in the equations are:

(1) Only thermal neutrons contribute to the fission process,

(2) Only fast neutrons are produced from fission and precursors,

(3) Only Xe\(^{135}\) is explicitly expressed as an absorptive isotope,

(4) Control function, equivalent to the effective cross section\(^{(61),(98)}\), contributes only to thermal neutrons at finite discrete points.

II.1.2. Precursor Kinetic Equations

Delayed neutrons are born from certain fission fragments which, after a \(\beta\)-decay, have excessive number of neutrons to be stable isotopes. With appropriate time delay, a neutron appears spontaneously. To express the rate of neutron emission from this source, for the CANDU reactors,
they are divided into 6 groups, according to the time scheme of their
decline processes, modified with photoneutron fractions (1)-(3). The kinetic equations for multi-group precursor time behaviour are;

\[ \frac{\partial C_i(x,t)}{\partial t} = \beta_i \nu \Sigma_{f2}(x,t) \phi_2(x,t) - \lambda_i C_i(x,t), \quad i = 1, 2, \ldots, 6. \]

\[ \ldots \ldots \ldots \] (II.2)

II.1.3 Xenon Poisoning

Among various fission fragments Xe$^{135}$ and Sm$^{149}$ have substantially high absorption cross sections and their concentrations in the reactor tend to change the flux distribution (5), (96). This poisoning effect appears with about 10 hours of time delay.

Since the absorption cross section of Sm$^{149}$ (=40,600 barns for 0.025 ev neutrons) is much less than that of Xe$^{135}$ and the half-life of Pm$^{149}$ ($\lambda=0.0128$ 1/hour), parent isotope of Sm$^{149}$, is much longer than Xe$^{135}$ and I$^{135}$ ($\lambda=0.0753$ 1/hour), the principal parent isotope of Xe$^{135}$, only the explicit dynamic equations for the Xe$^{135}$ chain will be treated in the study. For the Sm$^{149}$ chain, we assume that a constant value equivalent to the equilibrium concentration for the normal operation can be evaluated and included in the macroscopic absorption cross section of the thermal neutrons.
Considering the decay chain of $^{135}$Xe and $^{135}$I, after neglecting the intermediate stages, we approximate the coupled dynamic equations to be:

$$\frac{dI(x,t)}{dt} = \gamma_1 \phi_2(x,t) - \lambda_1 I(x,t)$$

and

$$\frac{dX(x,t)}{dt} = \gamma_2 \phi_2(x,t) + \lambda_1 I(x,t) - \sigma_X X(x,t) \phi_2(x,t) - \lambda_X X(x,t)$$

where

- $\gamma_1$, $\gamma_2$, $\lambda_1$, $\lambda_X$: the direct yield fraction of $^{135}$I and $^{135}$Xe from a fission event, respectively,
- $\lambda_I$, $\lambda_X$: the decay time constants of $^{135}$I and $^{135}$Xe, respectively.

II.1.4 Temperature Feedback

The phenomena associated with changes in operating conditions can be considered as following two categories. The first is the very localized perturbation, e.g., bubble formations in the coolant, vibrations of in-core materials, etc.; and the second is global or at least regional
changes, e.g., control bank manoeuvring power set back, etc.

The first category can be treated as random fluctuation in the neutron flux distribution, because the disturbance is hardly predictable. For the second category, a change in operating conditions is somehow intentional and directly affects the reactor power, and eventually the temperature of the system. The reactivity coefficient of power is the integrated parameter to evaluate this kind of perturbation. We define, for simplicity, the power coefficient, as a change in fission cross sections, as

$$\sum_{f2}(x,t) = \sum_{f2}^{\text{Ref}}(x) \left( 1 + \alpha_f(x) \frac{\phi_2(x,t) - \phi_2^{\text{Ref}}(x)}{\phi_2^{\text{Ref}}(x)} \right) \quad (II.4)$$

and

$$\alpha_f(x) = \left( \frac{\Delta \rho}{\Delta P} \right)_{P^{\text{Ref}}}$$

where $\alpha_f(x)$; power coefficient,

$(\Delta \rho/\Delta P)$; the ratio of changes in reactivity to power,

$P^{\text{Ref}}$; the reference power level,

II.1.5 Neutron Noise Field

Apart from the deterministic treatment of the reactor dynamics described up to here, the stochastic approach will be preferable in
some areas where the detector signal has an important role in determining control action\(^{(19),(97),(98)}\). As the reactor control automatically relies on the accumulated and evaluated reaction rates of the in-core neutron detectors during very small time intervals, the noise component is always included in the information.

One of the concerns in the design of a controller-detector pair system is how to separate this noise component from the control objectives. In other words, the optimal estimation of detector signals must be considered when we select the detector locations.

In incorporating the physical consideration of the noise components of the system dynamic equations, it is reasonable to assume that the noise is completely white, i.e., zero mean Gaussian\(^{(18)}\).

Fortunately except for the case of controlling the power excursion, the responding time and magnitude of controllers are not necessarily fast and large enough to cover every instantaneous noise-caused measurement fluctuation. Therefore in accounting for the uncertainties and difficulties associated with the use of a natural white noise \(\Gamma(x,t)\) it has become customary to introduce artificial Brownian motions which have statistical time characteristics very similar to those of white noise and are expanded with system eigenfunctions \(e_n(x)\)\(^{(99)}\):

\[
\Gamma(x,t) = \sum_{n=1}^{N} \beta_n(t) e_n(x) \quad \text{...(II.5)}
\]
where \( \beta_n(t) \) is the amplitude of noise component for \( e_n(t) \) having small time constant.

And the detector signal will be composed of the deterministic component and the stochastic component in the way of:

\[
\Phi(x, t) \delta(x - x_j) = (\phi_2(x, t) + \Gamma(x, t)) \delta(x - x_j), \quad j = 1, 2, \ldots, I_d
\]

where \( I_d \) is the number of in-core detectors.

The detectors are assumed sufficiently separated in the core and, thus, there are no cross-correlations between detector signals. The auto-correlation function of noise, \( \langle \Gamma(x, t) \Gamma(x_1, t + \tau) \rangle \) can be an arbitrary constant for each detector and has a form of \( \sum \delta(\tau) \), where \( \delta(\tau) \) is the Dirac delta function.

II.1.6 Boundary Conditions

State functions shown in Eqs. (II.1), (II.2), (II.3), (II.4), (II.5) and (II.6) have to satisfy the homogeneous boundary conditions

\[
\Phi(R, t) = \phi_2(R, t) = \Gamma(R, t) = x(R, t) = 0, \quad i = 1, 2, \ldots, 6
\]
and the control function must be defined inside the reactor, so

\[ U(R,t) = 0 \quad \text{...(II.8)} \]

where \( R \) is the extrapolated reactor boundary.

The locations of controllers must be unchanged during the reactor life-time and, therefore, the optimality is a time-invariant property which is independent from any kinds of perturbation locations. However, the initial conditions assigned to the state functions must not be beyond the allowable maximum deviation in safety aspects.

II.2 Linearized System

II.2.1 Definition of Perturbations

The reaction cross sections shown in the system equations (II.1), (II.2), (II.3) and (II.4) are slowly varying functions which change only significantly when the fuel burnup is important. Therefore, in the time domain of the control problems, it may be adequate to assume them as time invariant constants averaged over the control period \([t_0, t_f]\).

\[
\Sigma_{ij}(x) = \frac{\int_{t_0}^{t_f} \Sigma_{ij}(x,t) \phi_j(x,t) \, dt}{\int_{t_0}^{t_f} \phi_j(x,t) \, dt} \quad \text{...(II.9)}
\]
where the subscript \( j \) refers to the neutron energy group, and the subscript \( i \) to the reaction type, i.e., absorption, removal, fission and transport. \( t_0 \) and \( t_f \) are the instants in time when the control action is initiated and terminated, respectively.

In the early stage of the CANDU reactor design process, the reference state of the reactor was designated by defining an equilibrium burnup state. Because of on-power fuelling, the reactor burnup state is continually being changed by the time-dependent burnup process and by a stepwise refuelling process. If we average those variations over a fairly long time period, we can assume that the local burnup deviation is negligible. The average burnup state is defined as the time-averaged equilibrium burnup reference state \( \langle 100 \rangle \).

The system equations describing the reference state are:

\[
\nabla D_1^{\text{Ref}}(x) \nabla \phi_1^{\text{Ref}}(x) - \left( \Sigma_{a1}^{\text{Ref}}(x) + \Sigma_{\text{rem}}^{\text{Ref}}(x) \right) \phi_1^{\text{Ref}}(x) = 0,
\]

\[
\frac{1}{k_0} \left( (1 - \beta) \nu \Sigma_{\text{e2}}^{\text{Ref}}(x) \phi_2^{\text{Ref}}(x) + \sum_{i=1}^{6} \lambda_i c_i^{\text{Ref}}(x) \right) = 0,
\] \( \text{(II.10a)} \)

\[
\nabla D_2^{\text{Ref}}(x) \nabla \phi_2^{\text{Ref}}(x) - \left( \Sigma_{a2}^{\text{Ref}}(x) + \sigma_x x^{\text{Ref}}(x) \right) \phi_2^{\text{Ref}}(x) = 0,
\]

\[
\Sigma_{\text{rem}}^{\text{Ref}}(x) \phi_1^{\text{Ref}}(x) = 0,
\] \( \text{(II.10b)} \)
\[ \beta_2 \nu \sum_{i=2}^{\text{Ref}} (r) \phi_2^{\text{Ref}}(r) - \lambda_i^1 c_i^{\text{Ref}}(r) = 0, \quad i = 1, 2, \ldots, 6 \]  

... (II.10c)

\[ \gamma_i \sum_{i=2}^{\text{Ref}} (r) \phi_2^{\text{Ref}}(r) - \lambda_i^1 l_i^{\text{Ref}}(r) = 0, \]  

... (II.10d)

and

\[ \gamma_x \sum_{i=2}^{\text{Ref}} (r) \phi_2^{\text{Ref}}(r) + \lambda_i^1 l_i^{\text{Ref}}(r) - \sigma_x x^{\text{Ref}}(r) \phi_2^{\text{Ref}}(r) \]

\[ = -\lambda_x x^{\text{Ref}}(r) = 0. \]  

... (II.10e)

The effective multiplication factor \( k_0 \) of the design reference state is generally greater than 1.0 in order that an amount of excess reactivity remains for the control purpose.

If we look at differences between the general state functions in Eqs. (II.1), (II.2), (II.3), (II.4) and (II.5) and the reference state functions in Eqs. (II.10), the source of time variation is obviously a combination of differences in material properties, i.e., a perturbation, and controller action, i.e., the response. After a simplifying procedure that in which every material property change is smeared into a fractional change in the fission cross section, the critical system dynamic equations, whose dynamics are governed by arbitrary perturbations and corresponding control actions, can be written as follows.
\[
\frac{1}{v_1} \frac{\partial \phi_1(x,t)}{\partial t} = \nabla \cdot D_1(x) \nabla \phi_1(x,t) - \left[ \Sigma_{a_1}(x) + \Sigma_{rem}(x) \right] \phi_1(x,t) \\
+ \sum_{i=1}^{6} \lambda_1 c_{i}(x,t) + (1 - \beta) \nu \sum_{f_2}(x) \left[ 1 + \alpha_f(x) \right] \phi_2(x,t) \\
\left( \phi_2(x,t) - \phi_{2\text{Ref}}(x) \right) = \Delta(x, t_0) \phi_2(x,t) \\
\text{(II.1la)}
\]

\[
\frac{1}{v_2} \frac{\partial \phi_2(x,t)}{\partial t} = \nabla \cdot D_2(x) \nabla \phi_2(x,t) - \left[ \Sigma_{a_2}(x) + \sigma_x(x,t) \right] \phi_2(x,t) \\
+ \Sigma_{rem}(x) \phi_1(x,t) - \sum_{j=1}^{C} I_j(x,t) \phi_2(x,t) \delta(x - x_j) \\
\text{(II.1lb)}
\]

\[
\frac{\partial c_1(x,t)}{\partial t} = \beta_1 \nu \sum_{f_2}(x) \left[ 1 + \alpha_2(x) \right] \frac{\phi_2(x,t) - \phi_{2\text{Ref}}(x)}{\phi_{2\text{Ref}}(x)} \\
+ \Delta(x, t_0) \phi_2(x,t) - \lambda_1 c_1(x,t), \quad i = 1, 2, \ldots, 6 \\
\text{(II.1lc)}
\]

\[
\frac{\partial I(x,t)}{\partial t} = \gamma_x \sum_{f_2}(x) \left[ 1 + \alpha_f(x) \right] \frac{\phi_2(x,t) - \phi_{2\text{Ref}}(x)}{\phi_{2\text{Ref}}(x)} \\
+ \Delta(x, t_0) \phi_2(x,t) - \lambda_1 I(x,t) \\
\text{(II.1ld)}
\]
\[
\frac{\partial x(r,t)}{\partial t} = \gamma_x \sum_{\xi} \left[ 1 + \alpha_2(r) \frac{\phi_2(r,t) - \phi_2^{Ref}(r)}{\phi_2^{Ref}(r)} \right] + \Delta^{(r,t)} \phi_2(r,t) + \lambda_1 I_2(r,t) - \sigma_x x(r,t) \phi_2(r,t) - \lambda_x x(r,t)
\]

(II.11e)

where \( \Delta^{(r,t)} \) is a fixed perturbation inserted at \( t=t_0 \) and unchanged until \( t=t_f \).

Note that the superscript 'Ref' is dropped from material properties for convenience.

II.2.2 Separation of Control Function

As mentioned earlier, the control action is considered to be the combination of response to the precursor and the xenon buildup kinematics for a certain perturbation. For a fixed perturbation inserted at \( t=t_0 \), the former must be accomplished within seconds to compensate most of the reactivity change provided by the perturbation. On the other hand, the latter response is slowly activated to eliminate the belated regional subcriticality. Generally, the multiplication factor change during the slow transient is negligible compared to the fast transient's. Also, the fission reaction is important for the former and the absorption for the latter.
Considering the above facts, the dual control function may be separately defined:

\[
U(x, t) \delta(x - x_j) = \left[ U_B(x, t) + U_S(x, t) \right] \delta(x - x_j),
\]

\[j = 1, 2, \ldots, I_c \quad ..(II.12)\]

where

\[
U_B(x, t) = U_B(x, t) \left[ 1 - h(t - \tau) \right] \bigcap U_B(x, t) h(t - \tau)
\]

the bulk control function,

\[
\tau \quad \text{small time interval for the bulk control,}
\]

\[
h(t - \tau) \quad \text{the Heaviside step function,}
\]

\[
\begin{cases} 
  1, \text{ if } t \geq \tau, \\
  0, \text{ otherwise,}
\end{cases}
\]

\[
U_S(x, t) = U_S(x, t) h(t - \tau)
\]

the spatial control function, effective only for \( t \geq \tau \).

Although there is definitely some reactivity interface shared by both of the control functions, such a minor non-separable reactivity constraint could be easily relaxed by a controller maneuvering technique in practice.

Quantitatively, the bulk control function has the worth of the total reactivity difference between the reference state and the saturated perturbed state, and the spatial control function has the time variation of the regional reactivity equivalent to xenon concentration in the region compared to the saturated value.
Under the assumption of insignificance of the reactivity contribution of the xenon concentration difference between the reference state and the saturated states, the system equation for the bulk control can be approximated to:

\[ \frac{1}{v_1} \frac{\partial \phi_1(x,t)}{\partial t} = \nabla \cdot D_1(x) \nabla \phi_1(x,t) - \left[ \Sigma_{a1}(x) + \Sigma_{\text{Rem}}(x) \right] \phi_1(x,t) \]

\[ + \sum_{i=1}^{6} \lambda_i c_i(x,t) + (1 - \beta) \nu \sum_{i=2}^{n} \left[ 1 + \alpha_i(x) \right] \phi_1(x,t) \]

\[ \frac{\phi_2(x,t) - \phi_2^{\text{Ref}}(x)}{\phi_2^{\text{Ref}}(x)} + \Delta(x,\xi_0) \phi_2(x,t) \] ... (II.13a)

\[ \frac{1}{v_2} \frac{\partial \phi_2(x,t)}{\partial t} = \nabla \cdot D_2(x) \nabla \phi_2(x,t) - \left[ \Sigma_{a2}(x) + \sigma_x^{\text{Ref}}(x) \right] \phi_2(x,t) \]

\[ + \Sigma_{\text{Rem}}(x) \phi_1(x,t) - \left[ \sum_{j=1}^{n} u_B(x,t) \left\{ 1 - h(t - \tau) \right\} \right] \]

\[ + \left[ u_B(x, \tau) h(t - \tau) \right] \phi_2(x,t) \delta(x - \xi_j) \] ... (II.13b)

and

\[ \frac{\partial c_1(x,t)}{\partial t} = \beta_1 \nu \sum_{i=2}^{n} \left[ 1 + \alpha_i(x) \right] \frac{\phi_2(x,t) - \phi_2^{\text{Ref}}(x)}{\phi_2^{\text{Ref}}(x)} + \Delta(x,\xi_0) \phi_2(x,t) \]
\[ - \lambda_i C_i(r, t), \quad i = 1, 2, \ldots, 6 \quad \ldots \text{(II.13c)} \]

Emphasizing that our interest is the neutron population response to the control action, the fast mode of the system function, \( C_i(r, t) \) can be approximated to be stationary, i.e.,

\[
\frac{\partial C_i(r, t)}{\partial t} = 0 \quad \text{for } i = 1, 2, \ldots, 6.
\]

And eventually we have,

for \( 0 < t < 7 \),

\[
\frac{1}{\nu_1} \frac{\partial \phi_1(r, t)}{\partial t} = \nabla \cdot D_1(r) \nabla \phi_1(r, t) - \left[ \Sigma_{a1}(r) + \Sigma_{\text{Rem}}(r) \right] \phi_1(r, t)
\]

\[
+ \nu \Sigma_{\phi2}(r) \left[ 1 + \alpha_f(r) \frac{\phi_2(r, t) - \phi_2^{\text{Ref}}(r)}{\phi_2^{\text{Ref}}(r)} + \Delta(r, t_0) \right] \phi_2(r, t)
\]

\[
\ldots \text{(II.14a)}
\]

\[
\frac{1}{\nu_2} \frac{\partial \phi_2(r, t)}{\partial t} = \nabla \cdot D_2(r) \nabla \phi_2(r, t) - \left[ \Sigma_{a2}(r) + \sigma_x^{r\text{Ref}}(r) \right] \phi_2(r, t)
\]

\[
+ \Sigma_{\text{Rem}}(r) \phi_1(r, t) - \sum_{j=1}^{I_c} u_b(r, t) \phi_j(r, t) \delta(r - r_j)
\]

\[
\ldots \text{(II.14b)}
\]
and for $t \geq \tau$,

$$0 = \nabla \cdot D_{1}(x) \nabla \phi_1(x, \tau) - \left[ \sum_{a1}(x) + \sum_{\text{Rem}}(x) \right] \phi_1(x, \tau)$$

$$+ \nu \sum_{f2}(x) \left[ 1 + \alpha_{e}(x) \frac{\phi_2(x, \tau) - \phi_{2\text{Ref}}(x)}{\phi_{2\text{Ref}}(x)} \right] + \Delta(x, t_0) \phi_2(x, \tau)$$

... (II.15a)

$$0 = \nabla \cdot D_{2}(x) \nabla \phi_2(x, \tau) - \left[ \sum_{a2}(x) + \sigma_{x}^{\text{Ref}}(x) \right] \phi_2(x, \tau)$$

$$+ \sum_{\text{Rem}}(x) \phi_1(x, \tau) - \sum_{j=1}^{\mathbb{L} \mathcal{P}} U_{b}(x, \tau) \phi_2(x, \tau) \delta(x - x_{j})$$

... (II.15b)

In conclusion, we specify the bulk control problem as searching for the control function $U_{b}(x, t)$ to satisfy some stochastic properties of Eq. (II.14) for $0 < t < \tau$ and static equations (II.15) and condition of

$$\min \left\{ \phi(x, t) - \phi_{\text{Ref}}(x) \right\}$$

For the system where only spatial control is considered, and, thus, the neutron population being balanced in a global sense, the system equations should include the iodine-xenon dynamic chain;
\[
\frac{1}{v_1} \frac{\partial \phi_1(x, t)}{\partial t} = \nabla D_1(x) \nabla \phi_1(x, t) - \left[ \Sigma_{a1}(x) + \Sigma_{rem}(x) \right] \phi_1(x, t) \\
+ \frac{1}{\kappa(t)} \nu \Sigma_{f2}(x) \left[ 1 + \alpha_{f}(x) \right] \frac{\phi_2(x, t) - \phi_{2Ref}(x)}{\phi_{2Ref}(x)} \\
+ \Delta(x, t_0) \phi_2(x, t) 
\]

\[
\frac{1}{v_2} \frac{\partial \phi_2(x, t)}{\partial t} = \nabla D_2(x) \nabla \phi_2(x, t) - \left[ \Sigma_{a2}(x) + \sigma_{xRef}(x) \right] \phi_2(x, t) \\
- \Sigma_{rem}(x) \phi_1(x, t) - \sum_{j=1}^{Ic} U_s(x, t) \phi_2(x, t) \delta(x - x_j) 
\]

\[
\frac{\partial \Pi(x, t)}{\partial t} = \gamma_1 \Sigma_{f2}(x) \left[ 1 + \alpha_{f}(x) \right] \frac{\phi_2(x, t) - \phi_{2Ref}(x)}{\phi_{2Ref}(x)} \\
+ \Delta(x, t_0) \phi_2(x, t) - \lambda_1 \Pi(x, t) 
\]

and

\[
\frac{\partial x(x, t)}{\partial t} = \gamma_x \Sigma_{f2}(x) \left[ 1 + \alpha_{f}(x) \right] \frac{\phi_2(x, t) - \phi_{2Ref}(x)}{\phi_{2Ref}(x)} + \Delta(x, t_0) \phi_2(x, t) 
\]
\[ \lambda_1 I(x, t) + \sigma_x (x, t) \phi_2(x, t) - \lambda_x (x, t) \quad \ldots (II.16d) \]

Of course, in this case, the precursor kinetics are not necessary because the spatial transient is much slower than the precursor transient. The time dependent multiplication factor \( k(t) \) shown in Eq. (II.16a) represents the independence of the control function \( U_s(x, t) \) from the global reactivity contribution. Calculation of \( k(t) \) may impose another difficulty which may require computations of diffusion problems with various xenon concentrations. In order to overcome this, we can approximate \( k(t) \approx k(t_f) \) without losing any generality. If we have a bulk control system which is well-behaved for the large reactivity \( \rho(t_f) = 1 - 1/k(t_f) \), then, it is apparent that the controller will also be well-behaved for the small amount of reactivity \( \rho(t) = 1/k(t) - 1/k(t_f) \) through the trajectory of the entire transient.

Now we construct the separated control functions with their own governing equations. However, the set of controllers are restrained by reactivity constraints, which means that they are mutually dependent. We can eliminate these constraints either by relaxing restrictions on the state functions or by annexing conditions on the control functions.

1. For bulk control,

If we replace the strict condition

\[ \int_R dr \left\{ \sum_{j=1}^{l_c} \left( \phi_{2, \text{Ref}}^j (x) \right)^* U_B(x, t) \phi_2(x, t) \delta(x-x_j) \right\} = k \rho(t) \quad \ldots (II.17) \]
where

\[ \rho(t) = \int_R dx \left\{ (\phi_1^{\text{Ref}}(x))^* \left[ (\nabla \cdot D_1(x)) + \Sigma_{e_1}(x) - \Sigma_{\text{Rem}}(x) \right] \phi_1(x,t) \right. \]

\[ + \left. \phi_2^{\text{Ref}}(x)^* \Sigma_{\text{Rem}}(x) \phi_1(x,t) \right. \]

\[ + \left( \phi_1^{\text{Ref}}(x)^* \nu \Sigma_{e_2}(x) \right) \left\{ 1 + \alpha_x(x, \phi_2(x,t), \phi_2^{\text{Ref}}(x)) \frac{\phi_2(x,t) - \phi_2^{\text{Ref}}(x)}{\phi_2^{\text{Ref}}(x)} + \Delta(x,t_0) \right\} \phi_2(x,t) \]

\[ + \left( \phi_2^{\text{Ref}}(x)^* \left[ \nabla \delta(x) - \Sigma_{e_2}(x) - \alpha_x^{\text{Ref}}(x) \right] \phi_2(x) \right\} \kappa, \quad \ldots (\text{II-18}) \]

\[ \kappa \equiv \int_R dx \left\{ (\phi_1^{\text{Ref}}(x))^* \nu \Sigma_{e_2}(x) \left[ 1 + \alpha_x(x, \phi_2(x,t), \phi_2^{\text{Ref}}(x)) \frac{\phi_2(x,t) - \phi_2^{\text{Ref}}(x)}{\phi_2^{\text{Ref}}(x)} \right. \right. \]

\[ \left. + \left. \Delta(x,t_0) \phi_2(x,t) \right\} \right. \]

\[ \left( \phi_i^{\text{Ref}}(x) \right)^* ; \text{ the adjoint function of } \phi_i^{\text{Ref}}(x), \]

by the optimal estimation condition

\[ \min \left\{ \int_R dx \left[ \sum_{j=1}^{I_C} (\phi_1^{\text{Ref}}(x))^* u_j(x,t) \phi_2(x,t) \delta(x-x_j) \right] - \kappa \rho(t) \right\}^2 \]

\[ \ldots (\text{II-19}) \]

then, we can deal with the bulk control problem for the \( I_C \) mutually independent controllers.

(2) For spatial controllers:

\[ \int_R dx \left\{ \sum_{j=1}^{I_C} \left( \phi_2^{\text{Ref}}(x)^* u_j(x,t) \phi_2(x,t) \delta(x-x_j) \right) \right\} = 0 \quad \ldots (\text{II-20}) \]
the $I_c$-th controller can be expressed by other controllers,

$$U_s(I_c, t) = - \sum_{j=1}^{I_c-1} \frac{\omega(I_j, t)}{\omega(I_c, t)} U_s(I_j, t) \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \ quarters
functions in the equations are not significantly deviated from those of the reference state's. Thus we define the first order deviation functions:

\[ \phi_1(\xi,t) \approx \phi_{1,\text{Ref}}^\text{Ref}(\xi) + \eta_1(\xi,t) \]

\[ \phi_2(\xi,t) \approx \phi_{2,\text{Ref}}^\text{Ref}(\xi) + \eta_2(\xi,t) \]

\[ I(\xi,t) \equiv I_{\text{Ref}}^\text{Ref}(\xi) + \eta_3(\xi,t) \]

and

\[ \chi(\xi,t) \equiv \chi_{\text{Ref}}^\text{Ref}(\xi) + \eta_4(\xi,t) \]

Substituting Eq. (II.23) into Eqs. (II.14), (or Eqs. (II.15)) and (II.16), subtracting the reference state equations (II.10) and neglecting the higher order terms, we have:

for the bulk control,

\[ \frac{1}{v_1} \frac{\partial \eta_1(\xi,t)}{\partial t} = \nabla \cdot D_1(\xi) \nabla \eta_1(\xi,t) - [\Sigma_{a1}(\xi) + \Sigma_{\text{Rem}}(\xi)] \eta_1(\xi,t) \]

\[ + \nu \Sigma_{\xi2}(\xi) [1 + \alpha_\xi(\xi)] \eta_2(\xi,t) + \nu \Sigma_{\xi2}(\xi) [1 + \Delta(\xi,t_0) - \frac{1}{k_0}] \phi_{2,\text{Ref}}^\text{Ref}(\xi) \]

\[ \ldots \text{(II.14a')} \]

\[ \frac{1}{v_2} \frac{\partial \eta_2(\xi,t)}{\partial t} = \nabla \cdot D_2(\xi) \nabla \eta_2(\xi,t) - [\Sigma_{a2}(\xi) + \sigma \chi_{\text{Ref}}^\text{Ref}(\xi)] \eta_2(\xi,t) \]
\[ + \Sigma_{\text{Rem}}(\xi) \eta_1(\xi, t) - \sum_{j=1}^{I_2} U_b(\xi, t) \phi_{2, \text{Ref}}(\xi) \delta(\xi - \xi_j) \]

or

\[ 0 = \nabla \cdot D_1(\xi) \nabla \eta_1(\xi, \tau) - [ \Sigma_{a1}(\xi) + \Sigma_{\text{Rem}}(\xi) ] \eta_1(\xi, \tau) \]
\[ + \nu \Sigma_{\gamma}(\xi) [1 + \alpha_\gamma(\xi)] \eta_2(\xi, \tau) + \nu \Sigma_{\gamma}(\xi) [1 + \Delta(\xi, t_0) - \frac{1}{k_0}] \phi_{2, \text{Ref}}(\xi) \]

\[ (II.14b') \]

\[ 0 = \nabla \cdot D_2(\xi) \nabla \eta_2(\xi, \tau) - [ \Sigma_{a2}(\xi) + \sigma_{\chi, \text{Ref}}(\xi) ] \eta_2(\xi, \tau) \]
\[ + \Sigma_{\text{Rem}}(\xi) \eta_2^\prime(\xi, \tau) - \sum_{j=1}^{I_2} U_b(\xi, \tau) \phi_{2, \text{Ref}}(\xi) \delta(\xi - \xi_j) \]

\[ (II.15b') \]

and, for the spatial control,

\[ \frac{1}{v_1} \frac{\partial \eta_1(\xi, t)}{\partial t} = \nabla \cdot D_1(\xi) \nabla \eta_1(\xi, t) - [ \Sigma_{a1}(\xi) + \Sigma_{\text{Rem}}(\xi) ] \eta_1(\xi, t) \]
\[ + \frac{1}{k(t_f) \nu} \Sigma_{\gamma}(\xi) [1 + \alpha_\gamma(\xi)] \eta_2(\xi, t) \]

\[ (II.15a') \]
\[ + \nu \Sigma \xi_2(\xi) \left[ \frac{1}{k(t_f)} \left( 1 + \Delta(\xi, t_0) \right) - \frac{1}{k_0} \right] \phi_{2\text{Ref}}(\xi) \]

\[ \cdots (II.16a') \]

\[ \frac{1}{\nu_2} \frac{\partial \eta_2(\xi, t)}{\partial t} = \nabla \cdot D_2(\xi) \nabla \eta_2(\xi, t) - \left[ \Sigma_a(\xi) + \sigma_x \phi_{2\text{Ref}}(\xi) \right] \eta_2(\xi, t) \]

\[ - \sigma_2 \phi_{2\text{Ref}}(\xi) \eta_4(\xi, t) + \Sigma_{\text{Rem}}(\xi) \psi \eta_1(\xi, t) \]

\[ \frac{1}{\nu_2} \sum_{j=1}^{I_c-1} \left( \phi_{2\text{Ref}}(\xi_j) - \frac{\omega_{\text{Ref}}(\xi_j)}{\omega_{\text{Ref}}(\xi_c)} \phi_{2\text{Ref}}(\xi_c) \right) U_s(\xi_c, t) \delta(\xi - \xi_j) \]

\[ \cdots (II.16b') \]

\[ \frac{\partial \eta_3(\xi, t)}{\partial t} = \gamma_1 \Sigma \xi_2(\xi) \left( 1 + \alpha_{F}(\xi) \right) \eta_2(\xi, t) - \lambda_1 \eta_3(\xi, t) \]

\[ + \gamma_1 \Sigma \xi_2(\xi) \phi_{2\text{Ref}}(\xi) \Delta(\xi, t_0) \]

\[ \cdots (II.16c') \]

and \[ \frac{\partial \eta_4(\xi, t)}{\partial t} = \gamma_2 \Sigma \xi_2(\xi) \left( 1 + \alpha_{\partial}(\xi) \right) \eta_2(\xi, t) + \lambda_2 \eta_3(\xi, t) \]
II.2.4 Physical Significances of Linearized State Functions

Examining Eqs. (II.15') without the fixed perturbation term \( \Delta (\xi, t_0) \), the optimal controllers \( U_B(\xi, T) \) will be chosen such that the reactivity of the reference state is to be compensated by time-invariant controllers \( U_B(\xi, T) \) with minimization of static flux deviation \( \eta (\xi, T) \), i.e., controller induced perturbation. Therefore, the constant term \( \nu \sum f_2(\xi) \left( 1 - \frac{1}{k_0} \right) \phi_2(\xi) \) is the dominant source of reactivity. Thus one intent of the bulk controller design strategy is to search for the static controllers satisfying the static reactivity condition and the static flux constraint. This is sufficient to design the bulk controllers if the fuelling ideally controls the flux shape.

For non-ideal fuelling situations, let's transform the time variable in Eqs. (II.14') into \( t' = \nu_2 t \) and, then, the fixed perturbation
term $\Delta(\xi,t_0)$ can be considered as a time dependent impulsive function. Still we hope the perturbation can be expressed by a single term, i.e., fractional change in the fission cross section. If we take $\Delta(\xi,t)$ as a random parameter with the magnitude of fuelling effect, then, 'Eqs.(II.14)' represent the dynamic system whose optimal bulk controller can be designed to eliminate random fuelling perturbations. In this case, the time invariant reactivity term $\nu \Sigma_{f2}(1 - \frac{1}{k_0}) \phi_{\text{Ref}}(\xi)$ and correlated amount of $u_B(\xi,t) \phi_{\text{Ref}}(\xi)$ must be balanced as in the static problem Eqs.(II.15'). Therefore the bulk controller design problem is the optimum estimation of the linear stochastic system.

For the spatial control problem, the inhomogeneous terms in Eqs. (II.16') are neglected for the reasons of;

1. $\frac{1}{k(t_0)} \left( 1 + \Delta(\xi,t_0) \right) - \frac{1}{k_0} \approx 0$,

2. $\Delta(\xi,t_0)$ is very localized small fraction being neglected in the iodine-xenon production terms.

Now we have linear equations as the control objectives;

1. the bulk control for the non-ideal fuelling case,

$$\frac{1}{\mu} \frac{\partial \eta_1(\xi,t'/\nu_2)}{\partial t'} = \nabla \cdot D_1(\xi) \nabla \eta_1(\xi,t'/\nu_2) - \left[ \Sigma_{al}(\xi) + \Sigma_{\text{Rem}}(\xi) \right] \eta_1(\xi,t'/\nu_2)$$

$$\eta_1(\xi,t'/\nu_2) + \nu \Sigma_{f2}(\xi) \left[ 1 + \alpha_{f}(\xi) \right] \eta_2(\xi,t''/\nu_2)$$
\[
\frac{\partial \eta_2(\xi, t'/\nu_2)}{\partial t'} = \nabla \cdot D_2(\xi) \nabla \eta_2(\xi, t'/\nu_2) - \left[ \Sigma_{a_2}(\xi) + \sigma_{x}^{\text{Ref}}(\xi) \right] \eta_2(\xi, t'/\nu_2) \\
+ \Sigma_{\text{Rem}}(\xi) \eta_1(\xi, t'/\nu_2) - \sum_{j=1}^{L} \sum_{l=1}^{L} U_0(\xi, t'/\nu_2, I) \phi^\text{Ref}_2(\xi) \delta(\xi - \xi_j).
\]

where \( \mu = \nu_1/\nu_2 \) and \( t' = \nu_2 t \).

(2) the spatial control,

\[
\frac{1}{\nu_1} \frac{\partial \eta_1(\xi, t)}{\partial t} = \nabla \cdot D_1(\xi) \nabla \eta_1(\xi, t) - \left[ \Sigma_{a_1}(\xi) + \Sigma_{\text{Rem}}(\xi) \right] \eta_1(\xi, t) \\
+ \frac{1}{k(\eta_1)} \nu \Sigma_f(\xi) \left[ 1 + \alpha(\xi) \right] \eta_2(\xi, t).
\]

\[
\frac{1}{\nu_2} \frac{\partial \eta_2(\xi, t)}{\partial t} = \nabla \cdot D_2(\xi) \nabla \eta_2(\xi, t) - \left[ \Sigma_{a_2}(\xi) + \sigma_{x}^{\text{Ref}}(\xi) \right] \eta_2(\xi, t)
\]
\[ - \sigma_x \phi_2^{\text{Ref}}(\xi) \eta_4(\xi,t) + \Sigma_{\text{rem}}(\xi) \eta_1(\xi,t) \]

\[ \int_{\xi_{1c}}^{\xi_{c-1}} \left[ \phi_2^{\text{Ref}}(\xi) - \frac{\omega^{\text{Ref}}(\xi_{1c})}{\omega^{\text{Ref}}(\xi_{1c})} \phi_2^{\text{Ref}}(\xi_{1c}) \right] u_5(\xi,t) \delta(\xi - \xi_j) \]

\[ \text{(II.25b)} \]

\[ \frac{\partial \eta_3(\xi,t)}{\partial t} = \gamma_1 \Sigma_{\text{f2}}(\xi) \left[ 1 + \alpha_f(\xi) \right] \eta_2(\xi,t) - \lambda_1 \eta_3(\xi,t) \]

\[ \text{(II.25c)} \]

and

\[ \frac{\partial \eta_4(\xi,t)}{\partial t} = \gamma_4 \Sigma_{\text{f2}}(\xi) \left[ 1 + \alpha_f(\xi) \right] \eta_2(\xi,t) + \lambda_1 \eta_3(\xi,t) - \sigma_x \phi_2^{\text{Ref}}(\xi) \eta_2(\xi,t) - \sigma_x \phi_2^{\text{Ref}}(\xi) \eta_4(\xi,t) - \lambda_x \eta_4(\xi,t) \]

\[ \text{(II.25d)} \]

Boundary conditions equivalent to Eqs. (II.7) and (II.8) are:

\[ \eta_i(\xi,t) = 0, \ i=1, 2, 3 \text{ and } 4, \]
\[ U_s(R,t) = U_B(R,t) = 0, \]
\[ \Delta(R,t) = 0. \]

\((\text{II.26})\)
III. CONTROLLER LOCATION DETERMINATION BY PROPERTIES OF OPTIMALITY

IN SPATIAL CONTROL

III.1 Modal Control Theory for Reactor Dynamics

III.1.1 Properties of Space Modes

The linearized homogeneous time-invariant space-dynamic equations (II.25) have the simplified matrix form

$$\eta(x,t) = A\eta(x,t) + Bu(x,t)$$  \hspace{1cm} (III.1)

with a given set of boundary conditions. The solutions of equations $\eta(x,t)$ in the absence of controllers may be uniquely determined by the expansion with the system eigenfunctions (101), (102):

$$\eta(x,t) = \sum_{n=1}^{\infty} C_n e_n(x) \exp(\omega_n t)$$  \hspace{1cm} (III.2)

where the eigenvalues, $\omega_n$, and the eigenfunctions, $e_n(x)$, are obtained by

$$Ae_n(x) = \omega_ne_n(x), \quad n = 0, 1, 2, \ldots, \infty$$  \hspace{1cm} (III.3)
the \( C_n \) are given by the relationship

\[
C_n = \int_R \frac{dr}{r} \left\{ \mathbf{e}_n^*(r) \cdot \eta(r,0) \right\}, \quad n = 0, 1, 2, \ldots, \infty \quad (III.4)
\]

and \( \mathbf{e}_n^*(r) \), the eigenfunctions of the adjoint system satisfy the ortho-
normality conditions,

\[
\int_R \frac{dr}{r} \left\{ \mathbf{e}_n^*(r) \cdot \mathbf{w}(r) \cdot \mathbf{e}_n(r) \right\} = \delta_{mn} \quad (III.5)
\]

\( \mathbf{w}(r) \) is a weighting function, and
\( \delta_{mn} \) is Kronecker delta function.

It is assumed here that \( \mathbf{A} \) is such that \( \mathbf{w}_n \)'s are not degenerated.

The idea of the modal expansion technique is usually challenged
by several difficulties, when applied to the nuclear reactor system even
with the linearized form of equations. S. Kaplan\(^{(78)}\) examined several
expansion schemes which had been proposed for reactor space-time problems,
and attempted to devise expansion schemes which would have displayed the
finality in a given problem. He suggested the specific mode set which
is exactly the same as the eigenfunctions in Eq. (III.3). The so-called
"Kaplan modes" have the most proper characteristics to describe the xenon
spatial problems. But, generally, even if such modes would exist, it is
very difficult to generate the modes from system equations and to guarantee
their completeness.
Alternatively, if we have an orthonormal set of base functions \( \varphi_i(r) \), which can be easily generated and which are assured of convergence and completeness, we may expand \( e_n(r) \) in the form of (102), (103)

\[
e_n(r) = \sum_{i=0}^{L} a_i \varphi_i(r).
\]

... (III.6)

where the expansion coefficient \( a_i \) is given by

\[
a_i = \int_{R} \varphi_i^*(r) e_n(r) \, dr^{T}.
\]

... (III.7)

we, then, have

\[
A e_n(r) = A \left[ \sum_{i=0}^{L} a_i \varphi_i(r) \right].
\]

... (III.8)

The relationships between the properties of \( e_n(r) \) and \( \varphi_i(r) \) should be established by examining the submatrices of \( A \), terms of their physical significance. For example, the iodine components of eigenfunctions, \( e_n(r) \) are closely related only with their thermal flux components. And the xenon components are related only to the thermal flux and the iodine components. Thus, we can introduce the approximation that the iodine and xenon components can be expressed
only in terms of neutron flux expansion functions.

Those decoupled properties of the system matrix $A$ hint at the existence of a simple method which could possibly generate the base function, $\phi_i(r)$ by isolating the neutron diffusion equations from the system equations.

Also we assume that, if the open-loop solution of Eq. (III.1) has the form of Eq. (III.2), the eigenvalues and eigenfunctions of the closed-loop solution of Eq. (III.1) can be expressed by a linear combination of the open-loop system matrix $A^{(75)}$. The closed-loop system, then, should be stabilizable by controller feedback.

### III.1.2 \( \lambda \)-Modes as the Base Functions

The lack of existence and convergence of the Kaplan modes is possibly avoided when we use the $\lambda$-mode approximation\(^{(104)}\) which accounts explicitly for the spatial dependence of the neutron fluxes and uses conventional diffusion theory calculations.

We define the $\lambda$-modes as being the eigenfunctions which satisfy the reference state neutron balance equations;
\[- \nabla \cdot D_{1}(x) \nabla \varphi_{n,1}(x) + [\Sigma_{a1}(x) + \Sigma_{Rem}(x)] \varphi_{n,1}(x) = \frac{1}{k_{n}} \nu \Sigma_{f2}(x) \varphi_{n,2}(x) \quad \text{...(III.9a)}\]

\[- \nabla \cdot D_{2}(x) \nabla \varphi_{n,2}(x) + [\Sigma_{a2}(x) + \sigma_{x}^{\text{Ref}}(x)] \varphi_{n,2}(x) = \Sigma_{Rem}(x) \varphi_{n,1}(x) \quad \text{...(III.9b)}\]

or in matrix form

\[\mathcal{L} \varphi_{n} = \frac{1}{k_{n}} \mathcal{M} \varphi_{n}, \quad n = 0, 1, 2, \ldots \quad \text{(III.9c)}\]

where \(\mathcal{L}\) and \(\mathcal{M}\) are regarded as the destruction and production matrices, respectively, and \(1/k_{n} (\equiv \lambda_{n})\) is the \(n\)-th eigenvalue. Note that the fundamental mode, i.e., the mode corresponding to the largest eigenvalue, \(1/k_{0}\) is equivalent to the solution of Eqs. (II.10).

The adjoints of \(\lambda\)-eigenfunctions also satisfy the adjoint equations of Eqs. (III.9), which are:
\(- \nabla \cdot \mathbf{D}_1(\mathbf{r}) \nabla \varphi_{m,1}^*(\mathbf{r}) + \left[ \Sigma_{a1}(\mathbf{r}) + \Sigma_{\text{Rem}}(\mathbf{r}) \right] \varphi_{m,1}^*(\mathbf{r}) \)

\[ = \frac{1}{k_m} \Sigma_{\text{Rem}}(\mathbf{r}) \varphi_{m,2}^*(\mathbf{r}) \quad \text{(III.10a)} \]

\(- \nabla \cdot \mathbf{D}_2(\mathbf{r}) \nabla \varphi_{m,2}^*(\mathbf{r}) + \left[ \Sigma_{a2}(\mathbf{r}) + \sigma_x \chi^\text{Ref}(\mathbf{r}) \right] \varphi_{m,2}^*(\mathbf{r}) \)

\[ = \nu \Sigma_{f2}(\mathbf{r}) \varphi_{m,1}^*(\mathbf{r}), \quad m = 0, 1, 2, \ldots \quad \text{(III.10b)} \]

or in matrix form,

\[ \mathcal{L}^* \varphi_m^* = \frac{1}{k_m} M^* \varphi_m^*, \quad m = 0, 1, 2, \ldots \quad \text{(III.10c)} \]

Multiplying Eqs. (III.9a) and (III.9b) by \( \varphi_{m,1}^*(\mathbf{r}) \) and \( \varphi_{m,2}^*(\mathbf{r}) \), respectively, and similarly Eqs. (III.10a) and (III.10b) by \( \varphi_{n,1}^*(\mathbf{r}) \) and \( \varphi_{n,2}^*(\mathbf{r}) \), integrating over the reactor volume \( R \) and subtracting, we get the necessary relationship to satisfy the biorthogonality of modes:

\[ \frac{1}{k_n} \int_R \text{d} \mathbf{r} \left( \varphi_{m,1}^*(\mathbf{r}) \nu \Sigma_{f2}(\mathbf{r}) \varphi_{n,2}^*(\mathbf{r}) \right) \]
\[
\frac{1}{k_m} \int_R dx \left( \varphi_{m,2}(x) \sum_{rem}(x) \varphi_{n,1}(x) \right)
\]

... (III.11a)

and

\[
\int_R dx \left( \varphi_{m,1}(x) \nu \sum_{f_2}(x) \varphi_{n,2}(x) \right)
\]

\[
= \int_R dx \left( \varphi_{m,2}(x) \sum_{rem}(x) \varphi_{n,1}(x) \right)
\]

... (III.11b)

eventually

\[
\left( \frac{1}{k_n} - \frac{1}{k_m} \right) \int_R dx \left( \varphi_{m,2}(x) \sum_{rem}(x) \varphi_{n,1}(x) \right)
\]

\[
= \left( \frac{1}{k_n} - \frac{1}{k_m} \right) \int_R dx \left( \varphi_{m,1}(x) \nu \sum_{f_2}(x) \varphi_{n,2}(x) \right) = 0
\]

... (III.12)

Thus, if \( n \neq m \),

\[\vdots\]
\[ \int_{\mathbb{R}} d\tau \left| \varphi_{m,2}^\ast(\tau) \cdot \Sigma_{\text{Rem}}(\tau) \cdot \varphi_{n,1}(\tau) \right| \]

\[ = \int_{\mathbb{R}} d\tau \left| \varphi_{m,1}^\ast(\tau) \nu \Sigma_{f_2}(\tau) \varphi_{n,2}(\tau) \right| = 0 \quad \text{...}(\text{III.13a}) \]

and, if \( n = m \),

\[ \int_{\mathbb{R}} d\tau \left| \varphi_{n,2}^\ast(\tau) \cdot \Sigma_{\text{Rem}}(\tau) \cdot \varphi_{n,1}(\tau) \right| \]

\[ = \int_{\mathbb{R}} d\tau \left| \varphi_{n,1}^\ast(\tau) \nu \Sigma_{f_2}(\tau) \varphi_{n,2}(\tau) \right| = \frac{N_n^2}{N_n^2} \quad \text{...}(\text{III.13b}) \]

where \( \frac{N_n^2}{N_n^2} \) is a constant, normalization factor.

Now we have the normalizable biorthogonal \( \lambda \)-modes and their adjoints which will replace the \( \varphi_\lambda(\tau) \) and \( \varphi_\lambda^\ast(\tau) \)'s in Eqs. (III.9) and (III.10). These orthonormal modes are:

\[ \psi_{n,1}(\tau) = \frac{1}{N_n} \left[ \Sigma_{\text{Rem}}(\tau) \right]^{1/2} \phi_{n,1}(\tau) \]

\[ \psi_{m,2}(\tau) = \frac{1}{N_n} \left[ \nu \Sigma_{f_2}(\tau) \right]^{1/2} \phi_{n,2}(\tau) \quad \text{...}(\text{III.14}) \]

\[ \psi_{n,1}^\ast(\tau) = \frac{1}{N_n} \left[ \nu \Sigma_{f_2}(\tau) \right]^{1/2} \phi_{n,1}^\ast(\tau) \]
\[ \psi_{n,2}^* (\tau) = \frac{1}{n} \left[ \sum_{\text{Rem}} (\Phi) \right]^{1/2} \varphi_{n,2}^* (\tau) \]

Note that \( \nu \sum_{\ell=2} (\tau) \) and \( \epsilon \sum_{\text{Rem}} (\tau) \) are scalar functions and, thus, are interchangeable with \( \varphi_{n,1}^* (\tau) \) and \( \varphi_{n,2}^* (\tau) \). Therefore, the biorthonormality conditions are obtained as

\[ \int_{\Omega} d\tau \left| \psi_{m,2}^* (\tau) \psi_{n,1} (\tau) \right| = \int_{\Omega} d\tau \left| \psi_{m,1}^* (\tau) \psi_{n,2} (\tau) \right| = \delta_{mn}, \quad \text{...(III.15)} \]

It has been proved that the eigenfunction \( s_n (\tau) \) of the system equation (III.1) could be expanded in terms of \( \lambda \)-modes with reasonable accuracy for the CANDU reactor system (105), (106).

III.1.3 Modal Dynamic Equations

Rewriting Eq. (III.2) with \( \lambda \)-modes \( \varphi_i (\tau) \) of Eq. (III.14) we have

\[ \eta_1 (\tau, t) = \sum_{n=0}^{\infty} C_n \sum_{\ell=0}^{L} a_{1,1} \varphi_{1,1} (\tau) \exp (\omega_n t) \]
= \sum_{l=0}^{L} b_{1,1}(t) \varphi_{1,1}(x) = \sum_{n=0}^{L} b_{1,1}(t) \psi_{1,1}(x) / \left( \Sigma_{\text{Rem}}(x) \right)^{1/2} \ldots (III.16a)

where

b_{1,1}(t) = a_{1,1} \sum_{n=0}^{\infty} c_n \exp(\omega_n t),

and, similarly

\eta_2(x,t) = \sum_{l=0}^{L} b_{1,2}(t) \psi_{1,2}(x) / (\nu \Sigma_{f2}(x))^{1/2} \ldots (III.16b)

\eta_3(x,t) = \sum_{l=0}^{L} b_{1,3}(t) \psi_{1,2}(x) / (\nu \Sigma_{f2}(x))^{1/2} \ldots (III.16c)

\eta_4(x,t) = \sum_{l=0}^{L} b_{1,4}(t) \psi_{1,2}(x) / (\nu \Sigma_{f2}(x))^{1/2} \ldots (III.16d)

Substituting Eqs. (III.16) into the system equations (II.25), replacing the Laplacian terms by the λ-mode equations (III.9) and (III.10), dividing both sides of the thermal neutron diffusion equation by \left\{ \Sigma_{\text{Rem}}(x) \right\}^{1/2}, dividing the fast neutron diffusion equation by \left\{ \nu \Sigma_{f2}(x) \right\}^{1/2} and multiplying the iodine and xenon equations by \left\{ \nu \Sigma_{f2}(x) \right\}^{1/2}. We have:

\[ \frac{1}{\nu_1} \sum_{l=0}^{L} b_{1,1}(t) \Sigma_{\text{Rem}}(x) \nu \Sigma_{f2}(x) \left| \psi_{1,1}(x) \right|^{1/2} \psi_{1,1}(x) \]
\[
\frac{1}{V_2} \sum_{l=0}^{L} b_{1,2}(t) \left( \frac{1}{k(t_x)} \right) \left| 1 + \alpha_f(t) \right| \psi_{1,2}(t) \\
- \sum_{l=0}^{L} b_{1,1}(t) \left( \frac{1}{k_1} \right) \psi_{1,2}(t) 
\]  
\[\text{(III.17a)}\]

\[
\frac{1}{V_2} \sum_{l=0}^{L} b_{1,2}(t) \left( \left| \sum_{\text{Rem}(t)} \nu \sum_{\text{f}_2(t)} \right| \right) \frac{1}{2} \psi_{1,2}(t) 
\]

\[
= \sum_{l=0}^{L} p_{1,1}(t) \psi_{1,1}(t) - \sum_{l=0}^{L} b_{1,2}(t) \psi_{1,1}(t) \\
- \sum_{l=0}^{L} b_{1,4}(t) \sigma_x \phi^{\text{Ref}}(t) \nu \sum_{f_2(t)} \sum_{\text{Rem}(t)} \left| \frac{1}{2} \psi_{1,2}(t) \right| \\
- \sum_{l=0}^{L} \left( \phi^{\text{Ref}}(t) - \frac{\omega^{\text{Ref}(t)}}{\omega^{\text{Ref}(t_I)}} \phi^{\text{Ref}(t_{I,c})} \right) \left| \sum_{\text{Rem}(t)} \right| \frac{1}{2} \psi_{1,2}(t) \\
- U_s(t) \delta(t-t_3) 
\]  
\[\text{(III.17b)}\]

\[
= \sum_{l=0}^{L} b_{1,3}(t) \psi_{1,2}(t) = \sum_{l=0}^{L} b_{1,2}(t) \gamma_1 \sum_{f_2(t)} \left| 1 + \alpha_f(t) \right| \psi_{1,2}(t) \\n- \sum_{l=0}^{L} b_{1,3}(t) \lambda_1 \psi_{1,2}(t) 
\]  
\[\text{(III.17c)}\]
\[
\sum_{l=0}^{L} b_{1,4}(t) \psi_{1,2}(x) = \sum_{l=0}^{L} b_{1,2}(t) \gamma_{x} \Sigma_{f_{2}}(x) \{ 1 + \alpha_{x}(x) \} \psi_{1,2}(x) \\
+ \sum_{l=0}^{L} b_{1,3}(t) \lambda_{x} \psi_{1,2}(x) - \sum_{l=0}^{L} b_{1,2}(t) \sigma_{x}^{\text{Ref}}(x) \psi_{1,2}(x) \\
- \sum_{l=0}^{L} b_{1,4}(t) \sigma_{x} \phi_{2}^{\text{Ref}}(x) \psi_{1,2}(x) - \sum_{l=0}^{L} b_{1,4}(t) \lambda_{x} \psi_{1,2}(x) \\
\text{...(III.17d)}
\]

where

\[
b_{1,i}(t) = \frac{d}{dt}[b_{1,i}(t)].
\]

Multiplying \( \psi_{m,1}^{*}(x) \) on Eqs. (III.17a), (III.17c) and (III.17d) and \( \psi_{m,2}^{*}(x) \) on Eq. (III.17d), and integrating over the reactor volume, then by the biorthonormality condition Eq. (III.15), we have:

\[
\frac{1}{v_{1}} \sum_{l=0}^{L} b_{1,1}(t) \int_{R} dx \left\{ \psi_{m,1}^{*}(x) \left[ \Sigma_{\text{Rem}}(x) \nu \Sigma_{f_{2}}(x) \right]^{-\frac{1}{2}} \psi_{1,1}(x) \right\} \\
= \frac{1}{k(t_{f})} b_{m,2}(t) - \frac{1}{k_{m}} b_{m,1}(t)
\]
\[ + \sum_{l=0}^{L} b_{1,2}(t) \frac{1}{k(t)} \int_{\mathbb{R}} dx \left\{ \psi_{m,1}(x) \alpha_{f}(x) \psi_{1,2}(x) \right\} \]

... (III.17a')

\[ \frac{1}{\nu_{2}} \sum_{l=0}^{L} b_{1,2}(t) \int_{\mathbb{R}} dx \left\{ \psi_{m,2}(x) \left[ \Sigma_{\text{rem}}(x) \nu \Sigma_{f2}(x) \right]^{-\frac{1}{2}} \psi_{1,2}(x) \right\} \]

\[ = b_{m,1}(t) - b_{m,2}(t) - \sum_{l=0}^{L} b_{1,4}(t) \int_{\mathbb{R}} dx \left\{ \psi_{m,2}(x) \alpha_{x} \phi_{2,1}^{\text{ref}}(x) \right\} \]

\[ \left[ \Sigma_{\text{rem}}(x) \nu \Sigma_{f2}(x) \right]^{-\frac{1}{2}} \left\{ \sum_{j=1}^{L} \psi_{m,2}(x) \phi_{2,1}^{\text{ref}}(x) \right\} - \sum_{j=1}^{L} \psi_{m,2}(x) \phi_{2,1}^{\text{ref}}(x) \]

\[ \left[ \Sigma_{\text{rem}}(x) \nu \Sigma_{f2}(x) \right]^{-\frac{1}{2}} \left\{ \sum_{j=1}^{L} \psi_{m,2}(x) \phi_{2,1}^{\text{ref}}(x) \right\} - \sum_{j=1}^{L} \psi_{m,2}(x) \phi_{2,1}^{\text{ref}}(x) \]

\[ \frac{\omega \phi_{2,1}^{\text{ref}}(x)}{\phi_{2,1}^{\text{ref}}(x)} \left[ \Sigma_{\text{rem}}(x) \nu \Sigma_{f2}(x) \right]^{-\frac{1}{2}} \psi_{2,1}^{\text{ref}}(x) \left[ \Sigma_{\text{rem}}(x) \nu \Sigma_{f2}(x) \right]^{-\frac{1}{2}} \psi_{2,1}^{\text{ref}}(x) \]

... (III.17b')

\[ b_{m,3}(t) = -\lambda_{1} b_{m,3}(t) \]

\[ + \sum_{l=0}^{L} b_{1,2}(t) \int_{\mathbb{R}} dx \left\{ \psi_{m,1}(x) \gamma_{f} \Sigma_{f2}(x) \left[ 1 + \alpha_{f}(x) \right] \psi_{1,2}(x) \right\} \]

... (III.17c')

and

...
\[ b_{m,4}(t) = \lambda_1 b_{m,4}(t) - \lambda_x b_{m,4}(t) \]

\[ + \sum_{i=0}^{L} b_{1,2}(t) \int_R d\xi \left\{ \psi_{m,1}^*(\xi) \gamma_x \sum_{\xi} \frac{1}{v} \left[ 1 + \alpha(\xi) \right] \psi_{1,2}(\xi) \right\} \]

\[ - \sum_{i=0}^{L} b_{1,4}(t) \int_R d\xi \left\{ \psi_{m,1}^*(\xi) \sigma_x \text{Ref} \left( \xi \right) \psi_{1,2}(\xi) \right\} \]

\[ - \sum_{i=0}^{L} b_{1,4}(t) \int_R d\xi \left\{ \psi_{m,1}^*(\xi) \sigma_x \phi_2 \text{Ref} \left( \xi \right) \psi_{1,2}(\xi) \right\}. \]

...(III.17d')

For convenience, we express these equations in matrix form:

\[
\begin{bmatrix}
A_{11} & 0 & 0 & 0 \\
0 & A_{22} & 0 & 0 \\
0 & 0 & A_{33} & 0 \\
0 & 0 & 0 & A_{44}
\end{bmatrix}
\begin{bmatrix}
B_1 \\
B_2 \\
B_3 \\
B_4
\end{bmatrix}
= \begin{bmatrix}
A_{11} & A_{12} & 0 & 0 \\
21 & A_{22} & 0 & A_{24} \\
0 & A_{32} & A_{33} & 0 \\
0 & 0 & A_{32} & A_{44}
\end{bmatrix}
\begin{bmatrix}
B_1 \\
B_2 \\
B_3 \\
B_4
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
E_2 \\
0
\end{bmatrix}.
\]

...(III.18)

Where \( A_{ii} \) are \((L+1)\times(L+1)\) square matrices whose elements are

\[
\left[ A_{m\lambda}^{i} \right] = \frac{1}{v_i} \int_R d\xi \left\{ \psi_{m,1}^*(\xi) \left[ \Sigma_{\text{Re}m} \left( \xi \right) \nu \Sigma_{f2} \left( \xi \right) \right]^{-\frac{1}{2}} \psi_{1,1}(\xi) \right\}
\]
for \( i = 1 \) or 2,

\( \mathbf{I} \) is a \((L+1) \times (L+1)\) identity matrix,

\( \mathbf{B}_i \) is a \((L+1)\) dimensional column vector,

\[
[\mathbf{B}_i] = \begin{bmatrix} b_{0,i}(t), & b_{1,i}(t), & \ldots, & b_{L,i}(t) \end{bmatrix}^T,
\]

\( i = 1, 2, 3 \) and \( 4, \)

\( \mathbf{A}_{1i} \) are \((L+1) \times (L+1)\) square matrices whose elements are

\[
\begin{align*}
\left[\mathbf{A}_{11}\right] &= -\frac{1}{k_i} \delta_{1m}, \\
\left[\mathbf{A}_{12}\right] &= \frac{1}{k_i(t)} \int_{\mathcal{R}} \mathrm{d}x \left\{ \psi^*_m(x) \alpha_f(x) \psi_{1,2}^*(x) \right\} + \delta_{ml},
\end{align*}
\]

\( \mathbf{A}_{2i} \) are \((L+1) \times (L+1)\) square matrices whose elements are

\[
\begin{align*}
\left[\mathbf{A}_{21}\right] &= \delta_{ml}, \\
\left[\mathbf{A}_{22}\right] &= -\delta_{ml}, \\
\left[\mathbf{A}_{24}\right] &= -\int_{\mathcal{R}} \mathrm{d}x \left\{ \psi^*_m(x) \delta_x \phi^\text{Ref}_{2}(x) \left[ \Sigma_{\text{Rem}}(x) \psi_{1,2}^*(x) \right]^{\frac{1}{2}} \psi_{1,2}(x) \right\},
\end{align*}
\]

\( \mathbf{A}_{3i} \) are \((L+1) \times (L+1)\) square matrices whose elements are
\[
\begin{align*}
[A_{ml}^{32}] & = \int_\mathbb{R} d\xi \left\{ \psi_{m,1}^*(\xi) \gamma_1 \Sigma_{f2}(\xi) \left[ 1 + \alpha_{f}(\xi) \right] \psi_{1,2}(\xi) \right\}, \\
[A_{ml}^{33}] & = -\lambda_{I} \delta_{ml}, \\
\end{align*}
\]

\( A_{4i} \) are \((L+1) \times (L+1)\) square matrices whose elements are
\[
\begin{align*}
[A_{ml}^{41}] & = \int_\mathbb{R} d\xi \left\{ \psi_{m,1}^*(\xi) \left[ \gamma_3 \Sigma_{f2}(\xi) \left[ 1 + \alpha_f(\xi) \right] \right. \right. \\
& \quad \left. - \sigma_{x}^{\text{Ref}}(\xi) \right\} \psi_{1,2}(\xi), \\
[A_{ml}^{44}] & = -\lambda_{x} \delta_{ml} - \int_\mathbb{R} d\xi \left\{ \psi_{m,1}^*(\xi) \sigma_{x}^{\text{Ref}}(\xi) \right\} \psi_{1,2}(\xi),
\end{align*}
\]

\( E_2 \) is a \((L+1) \times (I_c-1)\) matrix whose elements are
\[
\begin{align*}
[E_{mj}] & = \psi_{m,2}^*(\xi_j) \left( \phi_{2}^{\text{Ref}}(\xi_j) \right. \\
& \quad \left. \left( \frac{\omega}{\omega_{\text{Ref}}(\xi_j)} \right) \phi_{2}^{\text{Ref}}(\xi_{I_c}) \right) \\
& \quad \times \left\{ \Sigma_{\text{Rem}}(\xi_j) \right\}^{1/2},
\end{align*}
\]

and
\( \Pi_s \) is a \((I_c-1)\) column vector
\[
\Pi_s = \left[u_{s}^1(t), u_{s}^2(t), \ldots, u_{s}^{I_c-1}(t)\right]^T.
\]

Here \( u_{s}^j(t) = u_{s}(\xi_j,t) \) and the superscript "T" refers to the transpose of
III.2 Optimality Criteria for Spatial Controllers

III.2.1 Reduced-Order Quadratic Linear Regulator

We now consider the dynamic system described by Eq. (III.18) in a simple form

$$\dot{B}(t) = AB(t) + E(t) \Pi_s(x, t)$$  \hspace{1cm} (III.19)

for \( \{ \pi \mid \pi_1, \pi_2, \ldots, \pi_{n-1} \} \), unknown number of locations of controllers with an arbitrary disturbance at time \( t = t_0 \) being any of \( B(t_0) \neq 0 \) but bounded \( B(t) \). The coefficient matrix \( E(t) \) is to be uniquely determined whenever the locations of controllers are known inside the reactor (cf. Eq. (III.18)).

The servomechanism problem\(^{(107)}\) can be applied to find an optimal control function \( \Pi_s(x, t) \) such that the state \( B(t) \) is returned to a desirable steady state, \( B^D \), not necessarily \( B^D = 0 \), as quickly as possible. However, if we choose the reference state as the most desirable state of the operating reactor, the deviation of the state functions may satisfy the natural terminal condition \( B(t_f) - B^D \approx 0 \).
Based on modern control theory (67)–(111), we should try to make the minimization of the transient error vector \( \varepsilon(t) = B(t) - B^D \) with minimum control effort. In the quadratic servomechanism problem, this is done by finding \( \Pi_\varepsilon(t) \) to minimize the performance index, or the cost functional,

\[
\mathcal{J}(\xi_1, \xi_2, \ldots, \xi_{Ic-1}) = \frac{1}{2} \left[ B(t_f) - B^D \right]^T Q_f [B(t_f) - B^D] \\
+ \frac{1}{2} \int_{t_0}^{t_f} \dot{\varepsilon}(t) \left[ B(t) - B^D \right]^T Q [B(t) - B^D] \dot{\varepsilon}(t) \, dt
\]

subject to the system equation (III.19) and initial conditions. Here \( Q, R \) and \( Q_f \) are constant positive diagonal matrices giving weighting to the cost of control and the loss of state vectors. Selection of such scaling matrices is quite empirical, based on experience rather than theoretical.

Our interest here lies in the fact that under certain observability and controllability conditions (85)–(88), the optimal controllers in terms of their locations minimize \( \mathcal{J}(\xi_1, \xi_2, \ldots, \xi_{Ic-1}) \) with a linear feedback law

\[
\Pi_\varepsilon(\xi, t) = -G(\xi) \varepsilon(t), \quad t_0 < t < t_f \quad \ldots (III.21)
\]
where $G(r)$ is the feedback gain and dependent on $A$ and $E(r)$.

Because the essence of spatial control is the quick suppression of the xenon-induced spatial power oscillation, it might be worthwhile to reduce the order of the state functions $B(t)$ in Eq. (III.20) to be able to deal only with iodine and xenon dynamics. This order-reduction technique is very effective for the dynamic system whose time constants are spread over a large time range [83], [84], [112]. For our system, the application of such a technique can be justified by the fact that:

1. The modal expression Eq. (III.18) doesn't satisfy the properties of finality with the finite number of modes, $(L+1)$. The reason is that, because of the time derivative term matrices, all the equations must be solved simultaneously and, thus, the solutions will be changed if the order of truncation modes, $(L+2)$ is changed.

2. Because the coefficient terms $\Lambda_{11}$ and $\Lambda_{22}$ of the matrix $\Lambda$ are negligible compared to the identity matrices in the iodine-xenon derivative terms, due to the multiplication of the inverse of the neutron velocities (e.g., $2.2 \times 10^5$ cm/sec for 0.025 ev neutrons).

3. According to the studies reported by several authors [33], [49], [55] the optimal xenon control is of the 'bang-bang' type. In such a control mode, however, there are no constraints on maximum power and the important flux limit criteria may be lost. But as far as the optimality in the controller locations is concerned, such manoeuvering-strategy-oriented difficulties won't be necessarily considered.
The advantages of using such a low-order model are not only in approaching the properties of finiteness, but reducing the numerical effort to solve the large matrix system.

The approximation form of Eq. (III.18) is decoupled into:

\[
\begin{bmatrix}
A_{11} & A_{12} \\ \mu A_{21} & \mu A_{22}
\end{bmatrix}
\begin{bmatrix}
B_1 \\ B_2
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 \\ -1 & \mu A_{24}
\end{bmatrix}
\begin{bmatrix}
B_3 \\ B_4
\end{bmatrix}
+ \begin{bmatrix}
0 \\ \mu E_2
\end{bmatrix}
\Pi_s = 0
\]

... (III.22a)

and

\[
\begin{bmatrix}
\dot{B}_3 \\ \dot{B}_4
\end{bmatrix}
= \begin{bmatrix}
0 & -A_{32} \\ -1 & \mu A_{42}
\end{bmatrix}
\begin{bmatrix}
B_1 \\ B_2
\end{bmatrix}
+ \begin{bmatrix}
A_{33} & 0 \\ -1 & \mu A_{44}
\end{bmatrix}
\begin{bmatrix}
B_3 \\ B_4
\end{bmatrix}
\]

... (III.22b)

by taking \( A_{11} B_1 = A_{22} B_2 \approx 0 \) and defining \( \mu = \sqrt{V_1/V_2} \). Replacing \([B_1, B_2]^T\) in Eq. (III.22b) by the new expression of Eq. (III.22a),

\[
\begin{bmatrix}
B_1 \\ B_2
\end{bmatrix}
= \left( \begin{bmatrix}
A_{11} & A_{12} \\ \mu A_{21} & \mu A_{22}
\end{bmatrix}^{-1}
\right)
\left( \begin{bmatrix}
0 & 0 \\ -1 & \mu A_{24}
\end{bmatrix}
\begin{bmatrix}
B_3 \\ B_4
\end{bmatrix}
+ \begin{bmatrix}
0 \\ \mu E_2
\end{bmatrix}
\Pi_s \right)
\]

... (III.22a')

we have 2(L+1)-order dynamic equations reduced from 4(L+1)-order system:

\[
\begin{bmatrix}
\dot{B}_3 \\ \dot{B}_4
\end{bmatrix}
= \begin{bmatrix}
A_{33} & 0 \\ -1 & \mu A_{44}
\end{bmatrix}
\begin{bmatrix}
A_{32} & 0 \\ \mu A_{42} & \mu A_{21}
\end{bmatrix}^{-1}
\begin{bmatrix}
A_{11} & A_{12} \\ \mu A_{21} & \mu A_{22}
\end{bmatrix}
\begin{bmatrix}
0 & 0 \\ -1 & \mu A_{24}
\end{bmatrix}
\begin{bmatrix}
B_3 \\ B_4
\end{bmatrix}
+ \begin{bmatrix}
0 \\ \mu E_2
\end{bmatrix}
\Pi_s
\]

... (III.22b')
or simply

\[
\begin{bmatrix}
\dot{\bar{B}}_3 \\
\dot{\bar{B}}_4
\end{bmatrix} =
\begin{bmatrix}
\bar{A}_{33} & \bar{A}_{34} \\
\bar{A}_{43} & \bar{A}_{44}
\end{bmatrix}
\begin{bmatrix}
\bar{B}_3 \\
\bar{B}_4
\end{bmatrix} +
\begin{bmatrix}
\bar{E}_1 \\
\bar{E}_2
\end{bmatrix} \Pi_s
\]

or

\[
\bar{B}(t) = \bar{A} \bar{B}(t) + \bar{E} \Pi_s 
\]

...(III.23)

The inverse of the submatrix in Eqs. (III.22) obviously exists because the non-singularity of the submatrix is guaranteed by the diagonal dominance due to the orthonormal modes.

Further simplifications can be made on the performance index Eq. (III.20) with the points noted below.

1. The errors in iodine and xenon concentrations between the reference values and another desirable steady state at \( t = t_f' \) have no significance on the neutron flux shape as far as the two systems are essentially in the critical state. Therefore the desirable final state is equivalent to the reference state, i.e., \( B^D \approx B^{Ref} = 0 \).

2. Because we assumed the reactivity contribution by the spatial control to be negligible, the modal equation for the fundamental mode, \( l = 0 \), must be deleted \((113), (114)\) in order to conserve the non-singularity of the submatrix described above. Thus the order of \( B_3(t) \) and \( B_4(t) \) becomes \( L \) rather than \( (L+1) \).

Rewriting the performance index will be above modifications.
we have

\[ J(\xi_1, \xi_2, \ldots, \xi_{n-1}) = \]

\[ \frac{1}{2} \int_{t_0}^{t_f} dt \left[ H(t) \right]^T Q \left[ H(t) + \left[ \Pi_s(t) \right]^T R \left[ \Pi_s(t) \right] \right] \]

...(III.20')

subject to the system equation (III.23) with L-mode terms. And the problem becomes the quadratic linear regulator problem rather than the servomechanism problem.

III.2.2 Plausibility of the Initial-Condition-Free Formulations

A standard method of solving problems like Eq. (III.20') is to use Lagrange multipliers and the calculus of variations (37), (115), (119). Defining a vector Lagrange multiplier \( \Theta(\xi, t) \), where \( \{ \xi | \xi_1, \xi_2, \ldots, \xi_{n-1} \} \), adjoining the system equation (III.23) in order to form the scalar positive Lagrangian \( L \),

\[ L(\mathbf{B}, \Pi_s, \Theta) = J(\mathbf{B}, \Pi_s) + \Theta^T \mathbf{X} (\mathbf{B}, \Pi_s) \] ...(III.24)

where

\[ \mathbf{X} (\mathbf{B}, \Pi_s) = \mathbf{A} \mathbf{B}(t) + \mathbf{E} \Pi_s(t) - \mathbf{B}(t) \] ...(III.25a)
\[
\mathbf{f}(\mathbf{B}, \Pi_s) = \frac{1}{2} \left[ \mathbf{B}(t) \big] \mathbf{T} \mathbf{Q} \big[ \mathbf{B}(t) \big] + \left[ \Pi_s^{(E)} \bigg] \bigg) \right. \mathbf{T} \mathbf{R} \left[ \Pi_s(t) \bigg]\right.
\]

...(III.25b)

We now adjust \( \mathbf{B}(t) \) and \( \Pi_s(t) \) such that \( \mathbf{L} \) is a maximum or minimum.

This requires that

\[
\frac{\partial \mathbf{L}}{\partial \mathbf{B}} = \frac{\partial \mathbf{f}}{\partial \mathbf{B}} + \frac{\partial}{\partial \mathbf{B}} [\mathbf{X}^T \Theta] = \mathbf{0}
\]

...(III.26a)

\[
\frac{\partial \mathbf{L}}{\partial \Pi_s} = \frac{\partial \mathbf{f}}{\partial \Pi_s} + \frac{\partial}{\partial \Pi_s} [\mathbf{X}^T \Theta] = \mathbf{0}
\]

...(III.26b)

where,

\[
\left[ \frac{\partial \mathbf{L}}{\partial \Pi_s} \right]^T = \left[ \frac{\partial \mathbf{L}}{\partial \mathbf{u}_1}, \frac{\partial \mathbf{L}}{\partial \mathbf{u}_2}, \ldots, \frac{\partial \mathbf{L}}{\partial \mathbf{u}_{c-1}} \right]
\]

and

\[
\left[ \frac{\partial \mathbf{L}}{\partial \mathbf{B}} \right]^T = \left[ \frac{\partial \mathbf{L}}{\partial \mathbf{b}_{1,3}}, \frac{\partial \mathbf{L}}{\partial \mathbf{b}_{2,3}}, \ldots, \frac{\partial \mathbf{L}}{\partial \mathbf{b}_{1,4}}, \ldots, \frac{\partial \mathbf{L}}{\partial \mathbf{b}_{L,4}} \right]
\]

In order that \( \mathbf{J}(\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_{c-1}) \) be a minimum, not only Eqs. (III.26) must be satisfied but also the second variation of \( \mathbf{L} \) must be greater than zero.
The solution of Eqs. (III.26) with the minimality condition usually gives the optimum control law for a given initial and final condition sets of $B(t)$ and $\Theta(\xi, t)$, where \( \{ \xi | \xi_1, \xi_2, \ldots, \xi_{I-1} \} \), is the so-called 'two point boundary value problem'. But we are interested in where and how many discrete controllers are necessary to give the best performance of the reactor during the entire reactor lifetime, not in searching the optimum control function which is well-behaved against a specific initial perturbation. Therefore, the solution should be an initial-condition-free, i.e., time-independent one.

Let's follow the Pontryagin's maximum principle \((42)\) \((45)\). The Hamiltonian of the system equation (III.23) is defined as

\[
\mathcal{H}_n(B, \Pi_s, \Theta, t, \xi_1, \xi_2, \ldots, \xi_{I-1})
= f(B, \Pi_s, t, \xi_1, \xi_2, \ldots, \xi_{I-1}) + \Theta^T \tilde{A} B
+ \Theta^T \tilde{E} \Pi_s, \tag{III.27}
\]

The physical interpretation of the adjoint $\Theta$ can be analogued from Stacey \((32)\) as the incremental importance of a modal component of an iodine and xenon atom at time $t$ which are added to the importances of the reference state, varying with the controller locations.
Application of the maximum principle requires that, for optimal control, i.e., for minimizing $J(\tau_1, \tau_2, \ldots, \tau_{n-1})$:

$$\frac{\partial \mathcal{L}}{\partial \Pi_s} = 0 = \mathcal{R} \Pi_s(t) + \tilde{E}^T \Theta(x, t) \quad \ldots(III.28a)$$

$$\frac{\partial \mathcal{L}}{\partial B} = \dot{\Theta}(x, t) = \mathcal{Q} \mathcal{B} \mathcal{B}^T + \tilde{A}^T \Theta(x, t) \quad \ldots(III.28b)$$

and

$$\frac{\partial \mathcal{L}}{\partial \Theta} = \dot{B}(t) = \tilde{A} B(t) + \tilde{E}^T \Pi_s(t) \quad \ldots(III.28c)$$

and the given boundary conditions $B(t_f) = 0$ after a sufficient control period. The boundary conditions $\Theta(x, t_0)$ and $\Theta(x, t_f)$ must be investigated with the imposed conditions of whether $t_0$, $t_f$, $B(t_0)$ and $B(t_f)$ are specified or not, i.e., the natural boundary conditions or the transversality conditions.

From Eq. (III.28a), the optimum controllers with the optimal trajectory of a transient is satisfied by

$$\Pi_s(t) = -R^{-1} \tilde{E}^T \Theta(x, t) \quad \ldots(III.29)$$

and the closed-loop system equations in matrix form are
\[
\begin{bmatrix}
\dot{B}(t) \\
\dot{\Theta}(t)
\end{bmatrix} =
\begin{bmatrix}
\tilde{A} & -\tilde{E}_R^{-1} \tilde{E}^T \\
-\mathcal{Q} & -\tilde{A}^T
\end{bmatrix}
\begin{bmatrix}
B(t) \\
\Theta(t)
\end{bmatrix}
\]  
\[\cdots (III.30)\]

If we require that there exists a matrix \( P(\tau) \), where \( \{ \tau \mid \tau = \mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_{I-1}\} \), satisfies the relationship of

\[
\Theta(\tau, t) = P(\tau) B(t),
\]
\[\cdots (III.31)\]

the properties of the matrix \( P(\tau) \) can be determined by Eq. (III.30).

Taking the time-derivative of \( \Theta(\tau, t) \), we have

\[
\dot{\Theta}(\tau, t) = P(\tau) \dot{B}(t),
\]
\[\cdots (III.32)\]

and substituting Eqs. (III.31) and (III.32) into Eq. (III.30), we have

\[
B(t) = \tilde{A} - \tilde{E}_R^{-1} \tilde{E}^T P(\tau) B(t),
\]
\[\cdots (III.33a)\]

\[
P(\tau) B(t) = -\mathcal{Q} + \tilde{A}^T P(\tau) B(t),
\]
\[\cdots (III.33b)\]

and finally

\[
P(\pm) \tilde{A} + \tilde{A}^T P(\tau) - P(\tau) \tilde{E}_R^{-1} \tilde{E}^T P(\tau) + \mathcal{Q} = 0.
\]
\[\cdots (III.34)\]
Eq. (III.34) is the algebraic matrix Riccati equation and it was proven that, if matrices $\tilde{A}$, $\tilde{E}$, $R$ and $Q$ satisfy some properties to be discussed later, then, there exists a unique non-negative definite matrix solution $P(x)$ with $\{x \mid x_1, x_2, \ldots, x_{I_c-1}\}$ (120), (121).

By solving Eq. (III.34) with a given set of controllers located at $\{x_j\}$, $j = 1, 2, \ldots, (I_c-1)$, the optimum feedback gain, i.e., the effectiveness of controllers, can be derived as a function of the controller locations,

$$G(x) = \tilde{E}R^{-1}\tilde{E}^TP(x) \quad \ldots \text{(III.35)}$$

Note that matrix $\tilde{E}$ also depends on the controller locations (see Eq. (III.18) or (III.19)). And the system dynamics

$$\dot{B}(t) = [\tilde{A} - G(x)]B(t) \quad \ldots \text{(III.36)}$$

can be stabilizable with the optimum closed-loop feedback law, Eq. (III.21).

Generally, even though $P(x)$ is required to be non-negative definite, the solution of Eq. (III.34) may fail to be a unique solution if system matrices do not satisfy certain properties. Kalman derived some necessary and sufficient conditions providing for the existence of the unique solution (122). According to his theorem on the algebraic characterization of optimality,
"When the linear system equation (III.23) is completely controllable with the given controllers \( \Pi_s(t) \), then, there exists a positive-definite symmetric matrix, i.e., \( P(t) = P(t)^T \), which satisfies Eq. (III.34) and the optimal control gain Eq. (III.35) which stabilizes the system Eq. (III.36)."

In order to apply Kalman's theorem to determine the locations of the optimum discrete controllers, we have to investigate the controllability of the given control system using the controllability theorem (108), (109).

"The linear system equation (III.23) is completely controllable if and only if the rank of controller matrix \( \mathbf{C} \)

\[
\mathbf{C} = [\hat{E}(z), \hat{A}\hat{E}(z), \hat{A}^2\hat{E}(z), \ldots, \hat{A}^{2L-1}\hat{E}(z)]
\]  

...(III.37)

is equal to the dimension of the system matrix \( \hat{A} \), i.e., \( 2L \)."

It may be that we can find \( M \) for the partial controllability matrix

\[
\mathbf{C}_M = [\hat{E}(z), \hat{A}\hat{E}(z), \hat{A}^2\hat{E}(z), \ldots, \hat{A}^{M-1}\hat{E}(z)],
\]

where \( 1 \leq M \leq 2L \),  

...(III.37')

and that the smallest \( M \) be such that \( \mathbf{C}_M \) has full rank, i.e., \( 2L \), is called the controllability index of the given controlled system. The controllability index is a parameter to determine the minimum number of controllers.
III.3 Numerical Methods Involved in Spatial Controller Design

III.3.1 Generation of the \( \lambda \)-Modes

The potential use and efficiency of the higher harmonics of the steady state diffusion equation (III.9) is due to the fact that they can describe rather large perturbations using only a few terms. One successful method of calculation and application of these modes which has been reported is an iterative method based on a Fourier series expansion technique\(^{123}\).

The fundamental mode of the 2-group diffusion equation (III.9) is usually calculated in iterative ways\(^{103}\). An intermediate solution vector \( \phi^n_0 \) at the \( n \)-th iteration can be written:

\[
\phi^n_0 = M^{-1}L\phi^{n-1}_0 \tag{III.38}
\]

Expanding the eigenfunction \( \varphi_i \) of the operator \( M^{-1}L \) with the corresponding eigenvalue, \( \lambda_i \), and expansion coefficient, \( a_i \), we have

\[
\phi^n_0 = \sum_i a_i (\lambda_i)^n \varphi_i \tag{III.39}
\]

After a sufficient number of iterations, we have the approximate expression
\[ \phi_0^N = \lambda_0 \phi_0^{N-1} \equiv \lambda_0 \phi_0, \quad \ldots (\text{III.40}) \]

where, \( \lambda_0 \) is the largest eigenvalue of \( \mathcal{M}^{-1} \mathcal{L} \) and, \( \phi_0 \) is the corresponding eigenfunction, the so-called 'the fundamental mode'.

When we start with an unconverged flux vector at the iteration \( k < N \), the flux vector contains high harmonics. Thus, if we subtract the predetermined fundamental mode at every iteration where the coefficient can be determined by the biorthogonality conditions Eq. (III.13), the residual flux vector must converge to the first harmonic flux vector.

Defining the unconverged flux vector of the \( k \)-th iteration in the previous procedure, and then this vector will be the initial guess for another cycle of iteration,

\[ \phi_1^{k,0} = \phi_0^k - a_0^{k,0} \phi_0 = \sum_{i>0} a_i^{k,0} \phi_i, \quad \ldots (\text{III.41}) \]

where

\[ a_0^{k,0} = \frac{\int_R \mathrm{d}r \cdot \phi_0^* \mathcal{M} \phi_0^k}{\int_R \mathrm{d}r \cdot \phi_0^* \mathcal{M} \phi_0}, \quad \ldots (\text{III.42}) \]
Then for $l$-th iteration of the cycle $k$

$$\phi_1^l = M^{-1}L[\phi_1^{k,1-2} - a_0^{k,1-1} \varphi_0]$$

$$= (M^{-1}L)^2 [\phi_1^{k,1-2} - a_0^{k,1-2} \varphi_0] - a_0^{k,1-1} M^{-1}L \varphi_0$$

$$= (M^{-1}L)^l \phi_1^{k,0} - \sum_{m=0}^{l-1} a_0^{k,m} M^{-1}L \varphi_0$$

$$= (III.43)$$

By the relationship of Eq. (III.41), the first term on the right-hand side of Eq. (III.43) is converged to the eigenfunction $\phi_1$ whose eigenvalue $\lambda_1$ is the second largest. After a sufficient number of iterations, we have

$$\phi_1^L = \lambda_1 \varphi_1 - a_0 \lambda_0 \varphi_0$$

$$(III.44)$$

where

$$a_0 = \sum_{m=0}^{L-1} a_0^{k,m} (M^{-1}L) \varphi_0.$$

In general, up to the $i$-th mode, we have expressions
\[ a_i = \frac{\int_R dr \varphi_i^* \mathcal{M} \phi_i^{uc}}{\int_R dr \varphi_i^* \mathcal{M} \varphi_i} \]  \quad \text{(III.45)}

and

\[ \phi_{i+1}^{uc} = \phi_i^{uc} - \sum_{k=0}^{i} a_k \varphi_k \]  \quad \text{(III.46)}

where, \( \phi_i^{uc} \) is the intermediate unconverted flux obtained during the \( i \)-th mode iteration cycle and being used for the initial vector for the \( (i+1) \)-st mode calculation.

### III.3.2 Approximate Adjoint Modes

By similar procedures described in the previous section but with the adjoint operator of Eqs. (III.10), adjoint modes can be generated. Usually the rate of convergence for the higher modes is very slow and so the computing cost can be extremely large.

To avoid these difficulties, we propose an alternative method by utilizing the assumption that the shape of the adjoint modes is not too different from the \( \lambda \)-modes, i.e.,
\[ \varphi_{n,1}^{\star}(r) = \frac{1}{a_n^2} \varphi_{n,1}(r) \]  

\[ \varphi_{n,2}^{\star}(r) = \frac{1}{b_n^2} \varphi_{n,2}(r) \]

Then by the biorthogonality conditions Eqs. (III.13), \( a_n \) and \( b_n \) can be evaluated as follows:

\[ \int_{\mathbb{R}} dr \frac{1}{a_n^2} \varphi_{m,1}(r) \nu \sum \xi_2(r) \varphi_{n,2}^{\star}(r) \varphi_{n,1}^{\star}(r) \]

\[ = \int_{\mathbb{R}} dr \frac{1}{b_n^2} \varphi_{m,2}(r) \sum_{\text{Rem}} \varphi^{\star*}_{n,1}(r) \]

\[ = \delta_{mn} N_n \]

thus

\[ a_n^2 = \frac{1}{N_n^2} \int_{\mathbb{R}} dr \varphi_{m,1}(r) \nu \sum \xi_2(r) \varphi_{m,2}(r) \]

\[ b_n^2 = \frac{1}{N_n^2} \int_{\mathbb{R}} dr \varphi_{m,2}(r) \sum_{\text{Rem}} \varphi^{\star*}_{m,1}(r) \]
The normalization conditions Eq. (III.14) will be modified by $a_n$ and $b_n$ to satisfy the orthonormality condition Eq. (III.15):

$$\psi_{n,1}(x) = \frac{1}{a_n} \left[ \sum_{\text{Rem}} \right]^{\frac{1}{2}} \varphi_{n,1}(x)$$

$$\psi_{n,2}(x) = \frac{1}{b_n} \left[ \nu \sum_{f_2} \right]^{\frac{1}{2}} \varphi_{n,2}(x)$$

This approximation apparently causes some error in the system matrix construction, Eqs. (III.17) and (III.18). The degree of uncertainty would be small, if we remember that the thermal neutron group is dominant in the CANDU reactor and that the adjoint fluxes are comparable with the self-adjoint properties of the one-group representation.

III.3.3 Controller Location Selection Rules
The system matrix equation (III.18) can be constructed with modes generated by the method described in the previous sections. To illustrate the effectiveness of controllers in terms of their locations, we propose the following rules to minimize the computation effort to solve the matrix Riccati equation at every possible location;

1. **Reduction Rule**

The control of xenon has been achieved indirectly by controlling the neutron population and, the importance of a neutron or a xenon atom becomes small as the reactor boundary is approached. Therefore the actual boundary inside which the effectiveness of the controllers is to be computed would be reduced to within the reactor boundary.

2. **Dual Controller Rule**

The properties of eigenfunctions and associated eigenvalues of the reactor would be similar to those obtained from the 2-dimensional Helmholtz equations for the reactor geometry. The maximum multiplicity of Helmholtz eigenvalues for the rectangular geometry is no greater than 2, if we take the maximum number of modes less than 16\(^{(12)}\). Also all power reactors have some degree of asymmetry with respect to a certain orientation, and, hence, the degeneracy may be somehow relaxed. Therefore a pair of controllers will satisfy Wieberg's Theorem I.

\[\text{Wieberg's Theorem I}^{(30)}\]

The minimum number of controllers required for the complete controllability should be equal to the maximum multiplicity of eigenvalues of the system matrix.
(3) **Rejection Rule**

The controllability requirement in the modal control theory is the ability to change the modal amplitudes from arbitrary given values to the desirable values in all cases.

**Wieberg Theorem II**

All control rods are not on any possible nodes of a combination of modes having the same eigenvalues.

Wieberg theorem II is equivalent to the rank of the controllability matrix being equal to the order of the system matrix. Therefore a selected location, where the rank test has failed, must be rejected before computing the controller effectiveness.

(4) **Unfolding Rule**

The determined controller domain which would be smaller than the actual reactor boundary may have some symmetrical properties in some orientations. The control policy is to keep such symmetry during reactor operation. Also, for redundancy and reliability consideration, each pair of controllers will be unfolded with respect to the symmetry axis.

III.3.4: **Iterative Technique Solving the Matrix Riccati Equation.**

We reduce the domain where the solutions of Eq. (III.34) exist
with symmetrical pairs of controllers. Now we have to focus our attention to solve the matrix Riccati equations at every symmetric pair of points inside the domain. One of the existing methods which we will apply to our problem is the Kleinman's iterative scheme \(^{(69)}\).

**Kleinman's Theorem**

Let \(V_k, \ k = 0, 1, 2, \ldots\) be the unique positive definite solution of the linear algebraic equation

\[
A_k^T V_k + V_k A_k + H + Z_k^T R Z_k = 0
\]

where \(H\) is a positive definite symmetric matrix.

\[
Z_k = K^{-1} E^T V_{k-1}
\]

\[
A_k = A - E Z_k
\]

\(\{A, E\}\); a completely controllable linear system,

\[
\dot{X}(t) = AX(t) + EP(t)
\]

and where \(Z_0\) is chosen such that the matrix \(A_0 = A - E Z_0\) has eigenvalues with negative real parts. Then
\( (1) \quad K < V_{k-1} < V_k < \cdots < V_0. \)

\( (2) \quad \lim_{k \to \infty} V_k = K. \)

Writing the iterative form of Eq. (III.34) for the pair of points \( \{ \xi, \xi_1, \xi_{i-1} \} \),

\[
\begin{bmatrix}
\tilde{A} - \tilde{E}(\xi) R^{-1} \tilde{E}(\xi)^T P_{k-1}(\xi)
\end{bmatrix}^T P_k(\xi) + P_k(\xi) \begin{bmatrix}
\tilde{A} - \tilde{E}(\xi) R^{-1} \tilde{E}(\xi)^T P_{k-1}(\xi)
\end{bmatrix} + H_k = 0 \quad \text{...(III.50)}
\]

where \( H_k \) is a known matrix calculated at the previous iteration step,

\[
H_k = Q + P_{k-1}(\xi) \tilde{E}(\xi) R^{-1} \tilde{E}(\xi)^T P_{k-1}(\xi).
\]

Eq. (III.50) is a matrix Lyapunov equation which can be decomposed to \( 2L(2L+1)/2 \) linear equations and can be easily solved by ordinary Gauss elimination method. Note that the existence of a unique positive definite symmetric matrix solution \( P_k(\xi) \) of the Lyapunov matrix equation at every iteration step \( k \), if we have the initial guess \( P_0(\xi) \).
satisfying the Kleinman condition, is guaranteed by (125):

(1) \( H_k \) is a symmetric matrix,

\[
[H_k]^T = [Q + P_{k-1}\tilde{E}^{-1}\tilde{E}^TP_{k-1}]^T
= Q + P_{k-1}\tilde{E}^{-1}\tilde{E}^TP_{k-1} = H_k
\]

because \( P_{k-1} \) is a symmetric matrix, and matrices \( Q \) and \( R \) are taken to be diagonal matrices.

(2) \( \{ \tilde{A} - \tilde{E}^{-1}\tilde{E}^TP_{k-1} \} \) has eigenvalues with negative real parts because of the stabilizable feedback law (cf. Eq.III.36).

Since solving the \( 2L(2L+1)/2 \) linear equations expanded from the Lyapunov matrix equation, especially for large \( L \), requires numerous additions and multiplications in the computing step, we introduce an improved method suggested by R. H. Bartels and G. W. Stewart (126).

The algorithm is divided into several steps.

(1) By an orthogonal transformation, the matrix \( \tilde{A} \) in the Lyapunov equation \( P_k\tilde{A}_k + \tilde{A}_k^TP_k = H_k \) into an upper Schur form which is an upper triangular form whose diagonal parts are composed, at most, of \( 2\times2 \) blocks.

(2) The prime solution \( P^0_k \) is obtained by solving a set of linear equations that we partitioned to have the order at most 4.

(3) The first residual solution \( P^1_k \) is obtained by setting the residual matrix \( P_k^1\tilde{A}_k + \tilde{A}_k^TP_k^1 = H^1_k \), where \( H^1_k = H_k - P^0_k\tilde{A}_k - \tilde{A}_k^TP^0_k \).
(4) Repeating Step (3) until the residual matrix $H_k^n$ becomes numerically negligible,

(5) Then, the accurate solution would be

$$P_k = P_k^0 + P_k^1 + \ldots$$

Generally this iteration can be completed within 5 cycles.

In order to improve the convergence of the Kleinman iterative method, we suggest the following,

(1) The initial guess matrix $P_0$ is calculated by Kleinman's method\(^{(127)}\)

that shows an easy way to stabilize a linear dynamic system by a feedback law,

$$P_0^{-1} = \int_0^T \{ \exp(-\tilde{A} t) \tilde{E} R^{-1} \tilde{E}^T \exp(-\tilde{A}^T t) \} dt \quad \text{[III.51]}$$

where, $T$ is the arbitrary final time so chosen that $P_0$ becomes numerically nonsingular.

(2) For given $P_0^r$, the sequence $P_k$ is determined\(^{(128)}\) from,

$$P_k = P_{k-1} - \alpha F(P_{k-1}) \quad \text{[III.52]}$$

where

$$F(P_k) = P_k \tilde{A}_k + \tilde{A}_k^T P_k - P_k \tilde{E} R^{-1} \tilde{E}^T P_k + H \quad \text{[III.53]}$$
and the overrelaxation parameter $\alpha$ is chosen to avoid numerical instability.

III.4 Spatial Control Effectiveness Map

From Eqs. (III.29) and (III.31), the spatial control function, $\Pi_s(t)$, will be expressed by the state deviation vector $B(t)$ as

$$\Pi_s(t) = -R^{-1}\tilde{E}^T P(\tau) \cdot B(t)$$  ...(III.54)

To find the minimum control effort to eliminate arbitrary transient error $B(t)$, we take the norm of the control vector

$$\|\Pi_s(t)\| = \|R^{-1}\tilde{E}^T P(\tau) \cdot B(t)\|$$

$$<\|R^{-1}\tilde{E}^T P(\tau)\| \|B(t)\|$$

We have a requirement that

"if $\|R^{-1}\tilde{E}^T P(\tau)\|$ is minimum, then $\|\Pi_s(t)\|$ is minimum on the trajectory of $B(t)$."
defined earlier, the controller effectiveness inside the entire controller domain can be illustrated at least with the symmetrical pair of controller representation.

The algorithm we propose is as follows:

1. To construct a controller domain where a relatively high neutron importance would be expected,

2. To reduce the domain, by invoking symmetry, as much as possible in order to reduce the reactor volume requiring actual computation;

3. To construct a mesh, where the effectiveness will be illustrated,

4. To check the controllability of a controller pair composed of one controller inside the volume and the other at its symmetric location,

5. To solve the matrix Riccati equation describing the optimal condition for the pair controller system, \( P(r) \),

6. To calculate \( \| R^{-1} E^T P(r) \| \) for individual controllers of the pair,

7. To repeat procedures (4) to (6) until the computation has been completed over all mesh points.
IV. MULTIPLE CONTROLLER DEPLOYMENT USING OPTIMAL BULK CONTROL CONCEPT

IV.1 Long-Term Reactivity Compensation

If the reactor is ideally fuelled, we can assume that the neutron flux distribution may not be affected by fuelling-induced perturbations. The main function of the bulk controllers, in this case, is focused to the reactivity compensation under condition of minimizing controller induced flux distortion over control margin of the reactivity.

With Eqs. (II.15), without fuelling perturbation, i.e.,

\[ \Delta r(t_0) = 0 \]

the optimality condition for the controlled system is directly determined by minimization of the performance index,

\[ J = \frac{1}{2} \int_{R} dt \left\{ \eta^{T} Q \eta + u_{B}^{0T} R u_{B}^{0} \right\} \]

...(IV.1)

The \( \lambda \)-mode expression technique is applied to separate the space-dependency of the problem. By similar definitions and procedures discussed in the previous chapter but including the fundamental mode, we
have

\[
- \sum_{l=0}^{L} a_{l,1} \frac{1}{k_1} \left\{ \nu \Sigma_{f_2}(x) \right\} = \frac{1}{2} \psi_{1,2}(x) + \sum_{l=0}^{L} 1,2 \left\{ \nu \Sigma_{f_2}(x) \right\} \frac{1}{2} \left[ 1 + \alpha_f(x) \right] \psi_{1,2}(x) + \nu \Sigma_{f_2}(x) \left\{ 1 - \frac{1}{k_0} \phi_{2}(x) \right\} = 0, \quad \ldots \text{(IV.2a)}
\]

\[
- \sum_{l=0}^{L} a_{l,2} \left\{ \Sigma_{Rem}(x) \right\} \frac{1}{2} \psi_{1,1}(x) + \sum_{l=0}^{L} a_{l,1} \left\{ \Sigma_{Rem}(x) \right\} \frac{1}{2} \psi_{1,1}(x)
\]

\[
- \sum_{j=1}^{I} u_B^0(x) \phi_{2}(x) \delta(x-x_j) = 0 \quad \ldots \text{(IV.2b)}
\]

Multiplying \( \psi_{m,2}^*(x) \} \Sigma_{Rem}(x) \} \frac{1}{2} \) on Eq. (II.2a) and \( \psi_{m,1}^*(x) \}) \nu \Sigma_{f_2}(x) \} \frac{1}{2} \) on Eq. (IV.2b); and integrating over the reactor volume \( \mathbb{R} \), we have, by the biorthonormality conditions Eqs. (III.15);

\[
- \sum_{l=0}^{L} a_{l,1} \int_{\mathbb{R}} dx \psi_{m,2}^*(x) \left\{ \Sigma_{Rem}(x) \right\} \frac{1}{k_1} \left\{ \nu \Sigma_{f_2}(x) \right\} = \frac{1}{2} \psi_{1,2}(x)
\]

\[
- \sum_{l=0}^{L} a_{l,2} \int_{\mathbb{R}} dx \psi_{m,2}^*(x) \nu \Sigma_{f_2}(x) \frac{1}{2} \left[ 1 + \alpha_f(x) \right] \psi_{1,2}(x)
\]
\[ + \int_{\mathbb{R}} dr \, \tilde{\psi}^{*}_{m,2}(r) \left\{ \Sigma_{\text{Rem}}(r) \right\} \frac{1}{2} \left[ 1 - \frac{1}{\hbar c} \right] \nu \Sigma_{f_2}(r) \phi_{\text{Re}}^{ \text{Ref}}(r) = 0 \]

\[ \ldots (\text{IV.3a}) \]

\[ - \sum_{l=0}^{L} a_{l,2} \int_{\mathbb{R}} dr \, \tilde{\psi}^{*}_{m,1}(r) \nu \Sigma_{f_2}(r) \Sigma_{\text{Rem}}(r) \left\{ \frac{1}{2} \tilde{\psi}_{1,1}(r) \right\} \]

\[ + \sum_{l=0}^{L} a_{l,1} \int_{\mathbb{R}} dr \, \tilde{\psi}^{*}_{m,1}(r) \nu \Sigma_{f_2}(r) \Sigma_{\text{Rem}}(r) \left\{ \frac{1}{2} \tilde{\psi}_{1,1}(r) \right\} \]

\[ - \sum_{j=1}^{L} \psi^{*}_{m,1}(r_j) \nu \Sigma_{f_2}(r_j) \left\{ \frac{1}{2} \phi_{\text{Re}}^{ \text{Ref}}(r_j) \right\} = 0 \]

\[ \ldots (\text{IV.3b}) \]

in matrix form

\[
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
N_1 \\
N_2
\end{bmatrix}
+ \begin{bmatrix}
E_1 \\
0
\end{bmatrix}
+ \begin{bmatrix}
\cdots \\
-B_2
\end{bmatrix}
\Pi^0_B = 0
\]

\[ \ldots (\text{IV.4}) \]

In order to obtain the reduced order form, rewriting Eq. (IV.4) by

\[ A_{11} N_1 + A_{12} N_2 + E_1 = 0 \]

\[ \ldots (\text{IV.5a}) \]

\[ A_{21} N_1 + A_{22} N_2 + B_2 \Pi^0_B = 0 \]

\[ \ldots (\text{IV.5b}) \]
and, because $A_{11}$ is a non-singular diagonal matrix, from Eq. (IV.5a), we have

$$N_1 = -A_{11}^{-1} A_{12} N_2 - A_{11}^{-1} E_1. \quad \cdots (IV.6)$$

Substituting Eq. (IV.6) into Eq. (IV.5b), we obtain an $(L+1)$ dimensional matrix equation explicitly representing the thermal neutron balance:

$$[A_{22} - A_{21} A_{11}^{-1} A_{12}] N_2 - A_{21} A_{11}^{-1} E_1 + B_2 \Pi_B^0 = 0,$$

$$\cdots (IV.7a)$$

or

$$\tilde{A} N_2 + \tilde{E} + B_2 \Pi_B^0 = 0. \quad \cdots (IV.7b)$$

Note that submatrices and vectors shown in Eqs. (IV.4) ~ (IV.7) are:

$A_{11}$ is a $(L+1) \times (L+1)$ matrix whose elements are

$$[A_{11}] = \int \left\{ \psi_{m,2}(\xi) \left[ \frac{1}{\kappa_1} \Sigma_{\text{rem}}(\xi) \mu \Sigma_f^2(\xi) \right] \right\} \frac{1}{2} \psi_{1,2}(\xi)$$

$A_{12}$ is a $(L+1) \times (L+1)$ matrix whose elements are

$$[A_{12}] = \int \left\{ \psi_{m,2}(\xi) \left[ \Sigma_{\text{rem}}(\xi) \mu \Sigma_f^2(\xi) \right] \right\} \frac{3}{2} [1 + \alpha(\xi)]$$
$A_{21}$ is a $(L+1)\times(L+1)$ matrix whose elements are

$$
[A_{m1}^{21}] = - \int d\tau \left\{ \psi_{m,1}^*(\tau) \left\{ \Sigma_{Rem}(\tau) \frac{1}{2} \right\} \psi_{1,1}^*(\tau) \right\}
$$

$A_{22}$ is a $(L+1)\times(L+1)$ matrix whose elements are

$$
[A_{m1}^{22}] = - [A_{m1}^{21}]
$$

$E_1$ is a $(L+1)$ column vector whose elements are

$$
[B_m^1] = \left( 1 - \frac{1}{\kappa_0} \right) \int d\tau \left\{ \psi_{m,2}^*(\tau) \left\{ \Sigma_{Rem}(\tau) \frac{1}{2} \right\} \psi_{2,2}^*(\tau) \right\}
$$

$B_2$ is a $(L+1)\times_{\mathbb{C}}$ matrix whose elements are

$$
[B_{m3}^2] = \frac{1}{2} \psi_{m,1}^*(\tau_j) \left\{ \Sigma_{f_2}(\tau_j) \frac{1}{2} \right\} \phi_2^{Ref}(\tau_j)
$$

$\Pi_B$ is a column vector

$$
[\Pi_B^0]^T = [u_B^0(\tau_1), u_B^0(\tau_2), \ldots, u_B^0(\tau_c)]
$$

$N_1$ and $N_2$ are column vectors
we have
\[
\left[ N_i \right]^T = \left[ a_{0,i}, a_{1,i}, \ldots, a_{1,i} \right] \quad \text{for } i = 1 \text{ or } 2.
\]

After replacing the state vectors \( \eta(t) \) in the performance index by the modal expression and integrating over the reactor volume, \( \mathcal{R} \), the Hamiltonian of the steady state system becomes

\[
\mathcal{H} \{ N_2, \Pi_B^0, \xi_1, \xi_2, \ldots, \xi_{1c} \}
= \frac{1}{2} N_2^T \varnothing N_2 + \frac{1}{2} \Pi_B^0 \pi \Pi_B^0 \\
+ \Theta^T \left\{ \tilde{A} N_2 + \tilde{E} + B_2 \Pi_B^0 \right\}, \quad (IV.8)
\]

where \( \Theta^T \); the transpose of the adjoint vector.

By the Pontryagin's maximum principle, we have

\[
\frac{\partial \mathcal{H}}{\partial N_2} = 0 = \varnothing N_2 + \tilde{A}^T \Theta, \quad \quad (IV.9a)
\]

\[
\frac{\partial \mathcal{H}}{\partial \Theta} = 0 = \tilde{A} N_2 + \tilde{E} + B_2 \Pi_B^0, \quad \quad (IV.9b)
\]

and
\[
\frac{\partial \mathcal{L}}{\partial \Pi_B^0} = O = R \Pi_B^0 + B_2^T \Theta \ldots \text{(IV.9c)}
\]

Solving Eqs. (IV.9) to get the expression of \(\Pi_B^0\), we have finally
\[
\Pi_B^0 = -R B_2^T \tilde{\Phi}^{-1} \tilde{A} \Phi \tilde{A}^{-1} B_2 \tilde{Y} \quad \text{(IV.10)}
\]

For \(\mathcal{J}(N_2, \Pi_B^0, I_1, I_2, \ldots, I_{IC})\) to have a minimum, the second variation,
\[
\delta^2 \mathcal{J} = \frac{1}{2} \left[ \begin{array}{c} \delta N_2 \\ \delta \Pi_B^0 \end{array} \right]^T \left[ \begin{array}{cc} Q & 0 \\ 0 & R \end{array} \right] \left[ \begin{array}{c} \delta N_2 \\ \delta \Pi_B^0 \end{array} \right]
\]
\[
= \frac{1}{2} \delta N_2^T \Phi \delta N_2 + \frac{1}{2} \delta \Pi_B^0^T R \delta \Pi_B^0 \ldots \text{(IV.11)}
\]

must be non-negative, therefore, it is sufficient that \(Q\) and \(R\) be non-negative definite. But if we consider the variation of the system equation which is
\[ \tilde{A} \delta N_2 + B_2 \delta \Pi_B^0 = 0, \quad \text{(iv.12)} \]

then, replacing \( \delta \Pi_B^0 \) in Eq. (iv.11) by (iv.12), we have the necessary condition for \( \delta^2 J > 0 \), that 
\[ R + B_2^T \tilde{A}^T \tilde{Q} \tilde{A}^{-1} B_2 \]

is positive.

If we investigate the properties of matrices present in Eq. (iv.10), we find that the matrix \( \tilde{A} \) and the column vector \( \tilde{E} \) are known; \( Q \) and \( R \) are arbitrary diagonal scale matrices, thus, known; and \( B_2 \) can be determined whenever we know the locations of controllers.

Physically the controller vector \( \Pi_B^0 \) has a component of each controller \( u_B^0(x_j) \), where \( j = 1, 2, \ldots, I_c \), whose property is an amount of macroscopic absorption cross section for thermal neutrons. Therefore, if we have the maximum allowable control material for static bulk control purpose, \( \Sigma_{a2}^u \), the minimum number of controllers must satisfy

\[ u_B^0(x_j) \leq \Sigma_{a2}^u \quad \text{for } j = 1, 2, \ldots, I_c \]

and the minimum number of controllers, \( \text{Min.} | I_c | \) becomes

\[ \text{Min.} \{ I_c \} = \frac{R}{|| \Pi_B^0 ||} / \Sigma_{a2}^u \]

\[ \text{...(iv.13)} \]

where \( || \Pi_B^0 || \) refers to the norm of the static bulk controller vector.
\[
\| \Pi_B^0 \| = \left( \Pi_B^0 \Pi_B^0 \right)^{\frac{1}{2}} = \left[ \sum_{j=1}^{L} \left( u_B^0(x_j) / u_B^0(x_j) \right) \right]^{\frac{1}{2}}
\]

In this case, the flux mode deviation due to introducing the static bulk control function can be represented, from Eqs. (IV.9a) and (IV.10), by

\[
N_2 = -A^{-T} \left[ I - B_2 R^{-1} B_2^T \left( \tilde{A}_2^{-1} \tilde{A}^T + B_2 R^{-1} B_2^T \right)^{-1} E \right]
\]

IV.2.1 Stochastic Modal Control Model

The existence of optimum locations of the controllers in terms of minimum control effort for spatial control, has been demonstrated in the previous chapter. Also we investigated one of the requirements that the control system should meet, that is the reactivity compensation with minimum flux shape distortion. From the dynamics point of view, the controllers set to a static level should be able to respond against transient trajectories deviating from the desirable condition. In this chapter
we evaluate the dynamic range of control actions for manoeuvring reactor power and counteracting fuel burnup/refuelling induced bounded random disturbances. The maximum-minimum control range of an individual controller will give direct information to determine the optimum number of controllers in the system.

The time-dependent neutron diffusion problem with stochastic parameters, i.e., Eq. (II.24) can be written to a matrix form,

\[ V \dot{\eta}(x, t) = A \eta(x, t) + B u_B(x, t) + E + H \Delta(x, t), \]

... (IV.14)

where,

\[ V = \begin{bmatrix} \frac{1}{\mu} & 0 \\ 0 & 1 \end{bmatrix}, \]

\[ A = \begin{bmatrix} \nabla D_1(x) \nabla - \Sigma_{a1}(x) - \Sigma_{\text{rem}}(x) & \nu \Sigma_{\alpha_1}(x) \left[ 1 + \alpha_f(x) \right] \\ \Sigma_{\text{rem}}(x) & \nabla D_2(x) \nabla - \Sigma_{a2}(x) - \alpha_x x^{\text{Ref}}(x) \end{bmatrix}, \]

\[ B^T = \begin{bmatrix} \phi_{2\text{Ref}}^x(x) \delta(x-\Sigma_1) \\ \phi_{2\text{Ref}}^x(x) \delta(x-\Sigma_2) \\ \cdots \\ \phi_{2\text{Ref}}^x(x) \delta(x-\Sigma_c) \\ 0 \phi_{2\text{Ref}}^x(x) \delta(x-\Sigma_1) \end{bmatrix}, \]
If we assume that the detector signal is composed of the time-dependent thermal neutron flux deviation from the reference state and of a certain amount of noise, at the detector location \( k \), the output will be

\[
\tilde{\xi}(x, t) \delta(x - x_k) = \{\eta_2(x, t) \Gamma(x, t)\} \delta(x - x_k),
\]

\( k = 1, 2, \ldots, I_d \) \( \ldots(IV.15) \)

where \( I_d \) is the number of detectors, and, for convenience, \( I_d = I_c \), i.e., one-to-one correspondence between controllers and detectors.

We have assumed that the noise function can be expanded with the system eigenfunctions (cf. Eq. II.5) \( (99), (129) \). Hence, by a similar method introduced in the spatial controller design, the state functions in Eq. (IV.14) are expanded with the normalized \( \lambda \)-modes but including the fundamental mode, we have
\[ \eta_1(x,t) = \sum_{i=0}^{\infty} a_{1,1}(t) \psi_{1,1}(x) \left\{ \Sigma_{\text{rem}}(x) \right\}^{-\frac{1}{2}} \]

\[ \eta_2(x,t) = \sum_{i=0}^{\infty} a_{1,2}(t) \psi_{1,2}(x) \left\{ \nu \Sigma_{f_2}(x) \right\}^{-\frac{1}{2}} \]

\[ \Delta(x,t) = \sum_{i=0}^{\infty} n_{1,1}(t) \psi_{1,2}(x) \left\{ \nu \Sigma_{f_2}(x) \right\}^{-\frac{1}{2}} \]

\[ \Gamma(x,t) = \sum_{i=0}^{\infty} n_{1,2}(t) \psi_{1,2}(x) \left\{ \nu \Sigma_{f_2}(x) \right\}^{-\frac{1}{2}} \]

where i can be 1 or 2, and where \( n_{1,i} \) are amplitudes of modes for the state disturbance noise, and \( n_{1,2} \) are amplitudes of modes for the output uncertainty noise. Note that noise is important only for thermal neutrons.

Substituting Eq. (IV.16) into Eq. (II.24), and replacing the Laplacian terms by the \( \lambda \)-mode equations (III.9), we have, for the stochastic neutron diffusion equation,

\[ \frac{1}{\mu} \sum_{i=0}^{\infty} a_{1,1}(t) \psi_{1,1}(x) \left\{ \Sigma_{\text{rem}}(x) \right\}^{-\frac{1}{2}} \]

\[ = - \sum_{i=0}^{\infty} a_{1,1}(t) \frac{1}{k_1} \left\{ \nu \Sigma_{f_2}(x) \right\}^{\frac{1}{2}} \psi_{1,2}(x) \]
\begin{align*}
+ \sum_{l=0}^{L} a_{1,2}(t) \left\{ \Sigma_{l_{2}}(\varepsilon) \right\} \frac{1}{2} \left( 1 + \alpha_{l}(\varepsilon) \right) \psi_{1,2}(\varepsilon) + \left( 1 - \frac{1}{k_0} \right) \nu \Sigma_{l_{2}}(\varepsilon) \phi_{2}^{Ref}(\varepsilon) \\
+ \sum_{l=0}^{L} n_{1,1}(t) \left\{ \nu \Sigma_{l_{2}}(\varepsilon) \right\} \phi_{2}^{Ref}(\varepsilon) \psi_{1,2}(\varepsilon), \quad \ldots&(\text{IV.17a}) \\
= - \sum_{l=0}^{L} a_{1,2}(t) \left\{ \Sigma_{\text{Rem}}(\varepsilon) \right\} \frac{1}{2} \psi_{1,1}(\varepsilon) \\
+ \sum_{l=0}^{L} a_{1,1}(t) \left\{ \Sigma_{\text{Rem}}(\varepsilon) \right\} \frac{1}{2} \psi_{1,1}(\varepsilon) - \sum_{j=1}^{I_C} u_{j}(\varepsilon, t) \phi_{2}^{Ref}(\varepsilon) \delta(\varepsilon - \varepsilon_{j}), \quad \ldots&(\text{IV.17b})
\end{align*}

and, for the detector signal equation (IV.15);

\begin{align*}
\xi(\varepsilon, t) \delta(\varepsilon - \varepsilon_{k}) = \sum_{l=0}^{L} \left( a_{1,2}(t) + n_{1,2}(t) \right) \psi_{1,2}(\varepsilon) \left\{ \nu \Sigma_{l_{2}}(\varepsilon) \right\} \frac{1}{2} \delta(\varepsilon - \varepsilon_{k}), \quad k = 1, 2, \ldots, I_C \quad \ldots&(\text{IV.17c})
\end{align*}

Multiplying \( \left\{ \Sigma_{\text{Rem}}(\varepsilon) \right\} \frac{1}{2} \psi_{m,2}^{*}(\varepsilon) \) by Eq. (IV.17a) and
\[ \left\{ \Sigma_{\ell 2} (x) \right\} \frac{1}{2} \psi_{m, 1}^* (x) \text{ by Eq. (IV.17b), and integrating over the reactor volume, } R, \text{ then, by the orthogonality condition Eq. (III.15), we have the modal dynamic equations,} \]

\[
\dot{a}_{m, 1} (t) = - \sum_{l=0}^{L} a_{l, 1} (t) \frac{1}{k_1} \int_{R} d\tau \psi_{m, 2}^* (\tau) w(\tau) \psi_{1, 2} (\tau)
+ \sum_{l=0}^{L} a_{l, 2} (t) \int_{R} d\tau \psi_{m, 2}^* (\tau) w(\tau) \left[ 1 + \alpha_2 (\tau) \right] \psi_{1, 2} (\tau)
+ \sum_{l=0}^{L} n_{l, 1} (t) \int_{R} d\tau \psi_{m, 2}^* (\tau) w(\tau) \phi^{\text{Ref}}_2 (\tau) \psi_{1, 2} (\tau)
+ \int_{R} d\tau \left( 1 - \frac{1}{k_0} \right) \psi_{m, 2}^* (\tau) w(\tau) \nu_{\Sigma_{\ell 2}} (\tau) \left\{ \frac{1}{2} \phi^{\text{Ref}}_2 (\tau) \right\}
\]

\( \text{(IV.18a)} \)

\[
\dot{a}_{m, 2} (t) = - \sum_{l=0}^{L} a_{l, 2} (t) \int_{R} d\tau \psi_{m, 1}^* (\tau) w(\tau) \psi_{1, 1} (\tau)
+ \sum_{l=0}^{L} a_{l, 1} (t) \int_{R} d\tau \psi_{m, 1}^* (\tau) w(\tau) \psi_{1, 1} (\tau)
- \sum_{j=1}^{J} \nu_{\Sigma_{\ell 2}} (\tau_j) \left\{ \frac{1}{2} \phi^{\text{Ref}}_2 (\tau_j) u_3 (t) \right\}, \quad \text{(IV.18b)}
\]
and for the detector signal

\[ \xi_k(t) = \sum_{i=0}^{\infty} \left\{ a_{1,2}^{(i)}(t) + n_{1,2}^{(i)}(t) \right\} \psi_{1,2}^{(i)}(x, \xi_k) \mu \Sigma_{\xi_k}(x, \xi_k) \frac{1}{2}, \]

... (IV.19)

where,

\[ w(x) = \left\{ \Sigma_{\text{rem}}(x) \mu \Sigma_{\xi_2}(x) \right\} \frac{1}{2}. \]

In matrix form, Eqs. (IV.18) and (IV.19) become

\[
\begin{bmatrix}
\begin{array}{c}
A_{11} & 0 \\
0 & 1
\end{array}
\end{bmatrix}
\begin{bmatrix}
X_1^{(t)} \\
X_2^{(t)}
\end{bmatrix}
= \begin{bmatrix}
A_1 & A_2 \\
A_2 & A_3
\end{bmatrix}
\begin{bmatrix}
X_1(t) \\
X_2(t)
\end{bmatrix}
+ \begin{bmatrix}
E_1 \\
0
\end{bmatrix}
\]

\[ \begin{bmatrix}
0 \\
B_2
\end{bmatrix}
+ \begin{bmatrix}
H_1 \\
0
\end{bmatrix}
\Gamma_1(t) \]

... (IV.20a)

and

\[ Y_2(t) = M_2 X_2(t) + M_2 \Gamma_2(t), \]

... (IV.20b)

where \( A_{11} \) is a \((L+1) \times (L+1)\) diagonal matrix whose elements are
$$
abla_{111} = \frac{1}{\mu} = \frac{v_2}{v_1}
$$

$H$ is a $(L+1) \times (L+1)$ square matrix whose elements are

$$
[H_{ml}^{1}] = \int d\xi \left\{ \psi_{n,2}^{*}(\xi) \left\{ \Sigma_{\text{rem}}(\xi) \psi_{l,2}(\xi) \right\} \right\} \frac{1}{2} \phi_{2}^{\text{Ref}}(\xi) \psi_{1,2}(\xi)
$$

$M^2$ is a $I_C \times (L+1)$ matrix whose elements are

$$
[M_{km}^{2}] = \psi_{m,2}(\xi_{k}) \psi_{n,2}(\xi_{k}),
$$

and $\Gamma_1$ and $\Gamma_2$ are $(L+1)$ column vectors

$$
\Gamma_1^{T} = \left[ n_{0,1}(t), n_{1,1}(t), \ldots, n_{L,1}(t) \right]
$$

for $i = 1$ or 2.

Because $\mu$ is the ratio of the fast to thermal neutron velocity ($v_1/v_2 \approx 20$), by the dominant mode concept described in Chapter III, we can formulate the reduced-order modal approximation with the quasi-static treatment of the fast neutron diffusion equation, i.e., $\nabla_{111} \approx 0$;

$$
O = A_{11}X_1(t) + A_{12}X_2(t) + H_1 \Gamma_1(t) + E_1 \quad \text{...(IV.21a)}
$$

$$
\dot{X}_2(t) = A_{21}X_1(t) + A_{22}X_2(t) + B_2 \Pi_B(t) \quad \text{...(IV.21b)}
$$
Replacing $X_1(t)$ in Eq. (IV.21b) by the expression derived from Eq. (IV.21b), we have the system dynamic equation dominant for thermal neutron behaviours,

$$
\dot{X}_2(t) = \begin{bmatrix} A_{22} - A_{21} A_{11}^{-1} A_{12} \end{bmatrix} X_2(t) + B_2 \Pi_B(t)
- A_{21} A_{11}^{-1} H_1 \Gamma_1(t) - A_{21} A_{11}^{-1} E_1,
$$

or, in simple matrix form,

$$
\dot{X}_2(t) = \tilde{A} X_2(t) + B_2 \Pi_B(t) + \tilde{H} \Gamma_1(t) + \tilde{E}, \quad \ldots (IV.22)
$$

and the output

$$
Y_2(t) = M_2 X_2(t) + M_2 \Gamma_2(t). \quad \ldots (IV.19b)
$$

Thus the problem is specified to find the optimal control function, $\Pi_B(t)$, for the time-invariant linear inhomogeneous stochastic system.

Comparing Eq. (IV.22) with (IV.7a), we recognize that the inhomogeneous term $A_{21} A_{11}^{-1} E_1$ is the static reactivity source term that can be eliminated by the static bulk control function $\Pi_B^0(t)$. Thus, defining new state and control vectors

$$
\tilde{X}(t) = X_2(t) - N_2
$$
\[ \mu(t) = \Pi_B(t) - \Pi_B^0, \quad \ldots \text{(IV.23)} \]

and subtracting Eq. (IV.7a) from (IV.22), we have a new purely stochastic system

\[ \dot{\xi}(t) = \tilde{A} \xi(t) + B_1 \mu(t) + \tilde{H} \Gamma_1(t), \quad \ldots \text{(IV.24a)} \]

and a new output description

\[ \xi(t) = M_2 \xi(t) + M_2 \Gamma_2(t). \quad \ldots \text{(IV.24b)} \]

Eqs. (IV.24) mean that, for the random fuelling perturbations, the control function \( \mu(t) \) is responding to the thermal flux variation \( \dot{\xi}(t) \) of the basis of measurement statistics \( \xi(t) \). Process \( \Gamma_1(t) \)'s are assumed to be stationary uncorrelated zero-mean white Gaussian such that their covariances are:

\[
\begin{align*}
\text{cov} \left[ \Gamma_1(t), \Gamma_1(\tau) \right] &= \varrho_2(t) \delta(t-\tau), \\
\text{cov} \left[ \Gamma_2(t), \Gamma_2(\tau) \right] &= \varpi_2(t) \delta(t-\tau), \\
\text{and} \quad \text{cov} \left[ \Gamma_1(t), \Gamma_2(t) \right] &= 0.
\end{align*}
\]

\[ \ldots \text{(IV.25)} \]

Determination of \( \varrho_2(t) \) and \( \varpi_2(t) \) depend on the physical assessments of the noise fields. Therefore we specify \( \Gamma_1(t) \) as the random perturbation.
in the fission cross section and \( \Gamma_2(t) \) as the uncertainty in the detector readings. We note that there definitely exists noise in the neutron field and in the detector field, but the neutron noise in high power reactors is negligible compared to other sources, e.g., mechanical vibrations \(^{18}\).

IV.2.2 External Disturbances in Linear Regulator

If we carefully investigate the physical meaning of Eqs. (IV.24), the optimally selected controllers, \( \mu(t) \), must completely counteract the effect of the disturbances, \( \Gamma_1(t) \), in spite of the uncertainty in measurements, \( M_2 \Gamma_2(t) \). Suppose it turns out that we cannot achieve an exact cancellation of the disturbance, then, we might be able to attempt to design controllers which minimize the effects.

Because the neutron noise in the system was neglected and the measurement uncertainties were only considered for the efficiencies of the detectors, we should be able to construct the performance index functional in terms of the state vectors rather than the output deviations,

\[
J = E \left\{ \int_{t_0}^{t_f} \text{dt} \left[ \xi(t)^T Q_1 \xi(t) + \mu(t)^T R_1 \mu(t) \right] \right\} \quad \text{... (IV.26)},
\]

where \( E \) refers to the expectation value of \( \{ \cdot \} \), and \( R_1 \) and \( Q_1 \) are
supposed to be positive definite diagonal matrices.

By the useful matrix formula:

\[
\mathcal{E} \{ X^T \mathcal{A} X \} = \mathcal{E} \{ (X - \overline{X})^T \mathcal{A} (X - \overline{X}) \} + \overline{X}^T \mathcal{A} \overline{X}
\]

\[
= \mathcal{E} \{ \text{tr} \ (X - \overline{X}) \mathcal{A} (X - \overline{X})^T \} + \overline{X}^T \mathcal{A} \overline{X}
\]

\[
= \mathcal{E} \{ \text{tr} \ \mathcal{A} [ (X - \overline{X}) (X - \overline{X})^T ] \} + \overline{X}^T \mathcal{A} \overline{X}
\]

\[
= \text{tr} \ \mathcal{A} \mathcal{E} \{ (X - \overline{X}) (X - \overline{X})^T \} + \overline{X}^T \mathcal{A} \overline{X}
\]

if \( \overline{X} = 0 \), then

\[
\mathcal{E} \{ X^T \mathcal{A} X \} = \text{tr} \ \mathcal{A} \mathcal{E} \{ XX^T \} = \text{tr} \ \mathcal{A} \text{ cov} \{ X \},
\]

\[\text{... (IV.27)}\]

we have

\[
\mathcal{J} = \int_{t_0}^{t_f} dt \ \text{tr} \ \left\{ \phi_1 \mathcal{E} \left[ \xi(t) \xi(t)^T \right] \right\}
\]

\[
+ \phi_1 \mathcal{E} \left[ \mu(t) \mu(t)^T \right], \quad \text{... (IV.28)}
\]
\[ \begin{align*}
  &= -Q_1 \xi(t) - \tilde{A}^T \{ P \xi(t) + \Omega(t) \}.
\end{align*} \]

Therefore, we have,

\[
\begin{align*}
  \left[ P \tilde{A} + \tilde{A}^T P - PB_2 R_1^{-1} B_2^T P + Q_1 \right] \cdot \xi(t) \\
  + \dot{\Omega}(t) + \left[ \tilde{A}^T - PB_2 R_1^{-1} B_2^T \right] \Omega(t) + PH \Gamma_1(t) &= 0.
\end{align*}
\]

\[\text{(IV.36)}\]

In order to satisfy the condition Eq. (IV.36) for arbitrary deviation function \( \xi(t) \) and \( \Gamma_1(t) \), we may be able to separate it into; for the regulation of the system,

\[
P \tilde{A} + \tilde{A}^T P - PB_2 R_1^{-1} B_2^T P + Q_1 = 0, \quad \text{(IV.37a)}
\]

and for the counteraction of the noise,

\[
\Omega(t) = - \left[ \tilde{A}^T - PB_2 R_1^{-1} B_2^T \right] \Omega(t) + PH \Gamma_1(t).
\]

\[\text{(IV.37b)}\]
\[
\frac{\partial \mathcal{L}}{\partial \dot{\xi}} = - \dot{\theta}(t) = \Omega_1 \xi(t) + \bar{A}^T \theta(t), \quad \text{(IV.33)}
\]

and, in addition to these, we have the system equation (IV.24a).

In order to determine a closed-loop state feedback control law, we can assume

\[
\theta(t) = P \dot{\xi}(t) + \Omega(t). \quad \text{(IV.34)}
\]

If we substitute Eq. (IV.34) into Eqs. (IV.32), (IV.33) and (IV.24a), and determine the requirements for the solution. The procedures are:

\[
\mu(t) = - R^{-1} \bar{B}_2^T \left\{ P \dot{\xi}(t) + \Omega(t) \right\}, \quad \text{(IV.35)}
\]

\[
\dot{\theta}(t) = P \dot{\xi}(t) + \dot{\Omega}(t)
\]

\[
= P \bar{A} \xi(t) - B_2 R^{-1} B_2^T \left( P \dot{\xi}(t) + \Omega(t) \right)
\]

\[
+ \bar{H} \Gamma_1(t) + \dot{\Omega}(t)
\]
\[- \dot{q}_1 \xi(t) - \tilde{A}^T \{ P \dot{x}(t) + \Omega(t) \} \]

Therefore, we have

\[
\left[ P \tilde{A} + \tilde{A}^T P - P B_2 R_1^{-1} B_2^T P + Q_1 \right] \xi(t) \\
+ \dot{\Omega}(t) + \left[ \tilde{A}^T - P B_2 R_1^{-1} B_2^T \right] \Omega(t) + P \tilde{H} \Gamma_1(t) = 0 .
\]

\[\text{... (IV.36)}\]

In order to satisfy the condition Eq. (IV.36) for arbitrary deviation function \( \xi(t) \) and \( \Gamma_1(t) \), we may be able to separate it into; for the regulation of the system,

\[
P \tilde{A} + \tilde{A}^T P - P B_2 R_1^{-1} B_2^T P + Q_1 = 0 . \quad \text{... (IV.37a)}
\]

and for the counteraction of the noise,

\[
\Omega(t) = - \left[ \tilde{A}^T - P B_2 R_1^{-1} B_2^T \right] \Omega(t) + P \tilde{H} \Gamma_1(t).
\]

\[\text{... (IV.37b)\]}\]
The control function obtained in Eq. (IV.35) is separated into the regulation of the system,

\[ \mu_R(t) = - R_1^{-1} B_2^T P \xi(t) \]  \hspace{1cm} \text{(IV.38a)}

and the counteraction of the random noise \( \Gamma_1(t) \),

\[ \mu_C(t) = - R_1^{-1} B_2^T \Omega(t) \]  \hspace{1cm} \text{(IV.38b)}

We shall note here that, if the optimal design of the detector locations is preferred, the cost functional Eqs. (IV.26) and (IV.28), must be constructed using the expression of

\[ \mu(t) = - P \xi(t) = - P M_2 \xi(t) \]

and the optimum condition would be investigated by modifying Levine's method.(139)

IV.2.3 Dynamic Range of Bulk Controllers for Flux Regulation

For a pre-selected set of controllers, Eq. (IV.10) gives their static set-points, and from Eq. (IV.38a) their dynamic ranges.
for regulating the flux variations can be evaluated, if one takes the norm of the control function below,

\[ \| \mu_{R(t)} \| = \| R_1^{-1} B_2^T P \xi(t) \| \]

\[ < \| R_1^{-1} B_2^T P \| \| \xi(t) \|. \quad \text{(IV.39)} \]

The definition of norm follows the conventional one that is:

for a column vector,

\[ \| X \| = \sqrt{X^T X} = \left( x_1^2 + x_2^2 + \ldots + x_n^2 \right)^{\frac{1}{2}}, \]

and for a matrix,

\[ \| A \| = \left( \text{tr} A^T A \right)^{\frac{1}{2}}. \]

Even though one determines a set of controllers satisfying Eq. (IV.10) and whose gain \( R_1^{-1} B_2^T P \) was chosen to be minimum, the maximum response range is still unknown until a limit in \( \xi(t) \) is defined. Fortunately, nuclear power reactors have been designed with limits on power error because of safety concerns.

Estimation of \( \| \xi(t) \| \) has to follow the definition of the power error in a global sense,
\[ \epsilon_g(t) = \frac{\int_{R} dt \sum_{f_2(x)} \phi_2(x,t) - \int_{R} dt \sum_{f_2(x)} \phi_2^{Ref}(x)}{\int_{R} dt \sum_{f_2(x)} \phi_2^{Ref}(x)} \quad \cdots (IV.40) \]

and of the power tilt in a regional sense\(^{(140)}\),

\[ \epsilon_x(t) = \frac{\left| \int_{R_1} dt \sum_{f_2(x)} \phi_2(x,t) - \int_{R-R_1} dt \sum_{f_2(x)} \phi_2^{Ref}(x) \right|}{\int_{R} dt P_{f_2(x)} \phi_2^{Ref}(x)} \quad \cdots (IV.41) \]

If we assume that the reactor power is proportional to the thermal neutron flux and if we take \( R_1 = R - R_1 = \frac{1}{2} \times R \), the total power error would be

\[ \epsilon_{t}(t) = \epsilon_g(t) + \epsilon_x(t) \]

\[ R \left( a_{0,2}(t) + \sum_{_{_{f=1}}}^{L} a_{1,2}(t) \right) / a_0^{Ref} \quad \cdots (IV.42) \]

where \( a_0^{Ref} \) is the fundamental mode amplitude of the reference state, and

\[ g_1 = \left( \int_{R_1} dt \psi_{1,2}(x) - \int_{R-R_1} dt \psi_{1,2}(x) \right) / \int_{R} dt \psi_{0,2}(x) \]
thus, \( 0 \leq q_i \leq 1 \) for \( 0 \leq i \leq L \).

Taking \( A_0^{Ref} = 1 \), i.e., normalized to the total reactor power, and remembering that \( a_{1,2}(t) \) are elements of the state vector \( \xi(t) \), then, by the maximum power error criterion \( \epsilon_{max} \),

\[
\| g \cdot \xi(t) \| \leq \epsilon_{max}
\]

or

\[
\text{Max} \| \xi(t) \| \leq \epsilon_{max} \quad \text{for} \quad \text{Max} \| g \| \leq \epsilon_{max} \quad \text{...}(IV.43)
\]

where \( g^T = [g_0, g_1, \ldots, g_L] \), and \( g_0 \int_1 = 1 \) for the fundamental mode.

Substituting Eq. (IV.43) into Eq. (IV.39), we have the dynamic limit of the bulk controllers, excluding the response to external random disturbances due to fueling and burnup:

\[
\| \mu_R(t) \| \leq \| R_1^{-1} B_2^T P \| \epsilon_{max} \quad \text{...}(IV.44)
\]

IV.2.4 Most Probable Control Actions Limited on Random Local Reactivity perturbations
The control function derived to Eq. (IV.36b) is expected to counteract the random external disturbance \( \Gamma_1(t) \) which is considered to be a random reactivity source. To estimate the maximum behaviour of this control system, we have to solve the dynamic equation (IV.37b) with arbitrary initial condition of \( \Gamma_1(t_0) \) and with the fixed natural terminal condition of \( \Omega(t_L) = 0 \). Furthermore, the disturbance vector \( \Gamma_1(t) \) is a random input continuously disturbing the process unpredictably.

To overcome these difficulties, we assume that the dynamic range of controllers for counteracting these disturbances would be estimated adequately by the second moments of the random state vector \( \Gamma_1(t) \) and the associated adjoint vector \( \Omega(t) \).

Because we assume that \( \Gamma_1(t) \) is a zero-mean white Gaussian, its adjoint \( \Omega(t) \) is apparently a zero-mean white Gaussian (81). Thus, the first moment should be

\[
\mathbb{E}[\Omega(t)] = \bar{\Omega}(t) = 0, \quad \cdots (IV.45)
\]

and the covariance matrix \( K \) is

\[
K(t_1, t_2) \equiv \text{cov} \{ \Omega(t_1), \Omega(t_2) \} = \mathbb{E}\left[\left[(\Omega(t_1) - \mathbb{E}[\Omega(t_1)])(\Omega(t_2) - \mathbb{E}[\Omega(t_2)])^T\right]\right].
\]
\[ \mathcal{E}\{\Omega(t_1) \Omega(t_2)^T\} = \mathcal{E}\{\Omega(t_1)\} \mathcal{E}\{\Omega(t_2)^T\} \quad \ldots \text{(IV.46)} \]

where \( \mathcal{E} \) is the expectation value of \( \{\cdot\} \).

If we define the transition matrix \( \Psi(t,t') \) satisfying the differential equation

\[ \frac{\partial \Psi(t,t_b)}{\partial t} = \left[ \hat{A}^T - \mathbf{P} \mathbf{B}_2 \mathbf{R}_1^{-1} \mathbf{B}_2^T \right] \Psi(t,t_0), \quad \ldots \text{(IV.47)} \]

with the initial condition

\[ \Psi(t_0,t_0) = I; \quad \text{identity matrix}, \quad \ldots \text{(IV.48)} \]

then, the solution of Eq. (IV.37b) will be in the form of

\[ \Omega(t) = \Psi(t,t_0) \Omega(t_0) \]

\[ + \int_{t_0}^{t} dt' \Psi(t,t') \mathbf{P} \mathbf{H} \mathbf{T}_1(t'). \quad \ldots \text{(IV.49)} \]

Substituting Eq. (IV.49) into Eq. (IV.46), we have
\[ K(t_1, t_2) = \mathcal{E} \left\{ \left[ \Psi(t_1, t_0) \Omega(t_0) + \int_{t_0}^{t_1} \Psi(t_1, t') \tilde{P} \tilde{H} \Gamma_1(t') \, dt' \right]^T \left[ \Psi(t_2, t_0) \Omega(t_0) + \int_{t_0}^{t_2} \Psi(t_2, t') \tilde{P} \tilde{H} \Gamma_1(t') \, dt' \right] \right\} \]

\[
= \mathcal{E} \left\{ \Psi(t_1, t_0) \Omega(t_0) \Omega^T(t_0) \tilde{V}(t_2, t_0) \right\} \\
+ \mathcal{E} \left\{ \Psi(t_1, t_0) \Omega(t_0) \int_{t_0}^{t_2} \Gamma_1(t')^T \tilde{P} \tilde{H}^T \tilde{V}(t_2, t') \, dt' \right\} \\
+ \mathcal{E} \left\{ \int_{t_0}^{t_1} \Psi(t_1, t') \tilde{P} \tilde{H} \Gamma_1(t') \, dt' \Omega^T(t_0) \tilde{V}(t_2, t_0) \right\} \\
+ \mathcal{E} \left\{ \int_{t_0}^{t_1} \int_{t_0}^{t_2} \Psi(t_1, t') \tilde{P} \tilde{H} \Gamma_1(t') \Gamma_1(t_2)^T \tilde{H}^T \tilde{V}(t_2, t_2) \, dt' \, dt_2 \right\} \\
= \Psi(t_1, t_0) \mathcal{E} \left\{ \Omega(t_0) \Omega^T(t_0) \right\} \Psi^T(t_2, t_0) \\
+ \mathcal{E} \left\{ \int_{t_0}^{t_2} \Omega(t_0) \Gamma_1(t')^T \tilde{H}^T \tilde{P} \Psi^T(t_2, t') \, dt' \right\}
\[ + \int_{t_0}^{t_1} \Psi(t_1', t') P \tilde{H} \varepsilon \left\{ \Gamma_1(t') \Omega(t_0')^T \right\} dt' \Psi^T(t_2, t_0) \]

\[ + \int_{t_0}^{t_1} \int_{t_0}^{t_2} \Psi(t_1, t_1') P \tilde{H} \varepsilon \left\{ \Gamma_1(t_1') \Gamma_1(t_2')^T \right\} \tilde{H}^T P \Psi^T(t_2, t_1') dt' dt_1' \]

... (IV.50)

Obviously there is no correlation between \( \Gamma_1(t) \) and \( \Omega(t_0) \) for \( t \geq t_0 \), i.e.,

\[ \varepsilon \left\{ \Omega(t_0), \Gamma_1(t')^T \right\} = 0. \]...

Recalling Eq. (IV.25), we have

\[ K(t_1, t_2) = \Psi(t_1, t_0) K(t_0, t_0) \Psi^T(t_2, t_0) \]

\[ + \int_{t_0}^{t_1} \int_{t_0}^{t_2} \Psi(t_1, t_1') P \tilde{H} \Omega(t_1') \delta(t_1' - t_2') \tilde{H}^T P \Psi^T(t_2, t_1') dt_1' dt_2' \]

\[ = \Psi(t_1, t_0) K(t_0, t_0) \Psi^T(t_2, t_0) \]
\[
+ \int_{t_0}^{t} \min \left\{ t_1, t_2 \right\} \psi(t_1, t') \, \mathbf{P} \mathbf{H} \, \mathbf{Q}_2(t') \, \mathbf{H}^T \mathbf{P} \psi(t_2, t') \, dt'.
\]

\ldots(IV.52)

To avoid difficulties met in solving the integral equation (IV.52), we try to transform it to a differential form:

\[
\frac{d K(t)}{dt} = \frac{\partial \psi(t, t_0)}{\partial t} \, K(t_0) \, \psi^T(t, t_0) + \psi(t, t_0) \, K(t_0) \, \frac{\partial \psi^T(t, t_0)}{\partial t}
\]

\[+ \psi(t, t) \, \mathbf{P} \mathbf{H} \, \mathbf{Q}_2(t) \, \mathbf{H}^T \mathbf{P} \psi^T(t, t)\]

\[+ \int_{t_0}^{t} \frac{\partial \psi(t, t_1)}{\partial t} \, \mathbf{P} \mathbf{H} \, \mathbf{Q}_2(t') \, \mathbf{H}^T \mathbf{P} \psi^T(t, t') \, dt'\]

\[+ \int_{t_0}^{t} \psi(t, t') \, \mathbf{P} \mathbf{H} \, \mathbf{Q}_2(t') \, \mathbf{H}^T \mathbf{P} \frac{\partial \psi^T(t, t')}{\partial t} \, dt'\]

\ldots(IV.53)

Using Eq. (IV.47) and its transpose equation.
\[
\frac{\partial \Psi^T(t, t_0)}{\partial t} = -\Psi^T(t, t_0) \left[ A^T - B_2 \bar{r}_1^{-1} B_2^T \right]^T
\]

... (IV.54)

Eq. (IV.53) can be written to

\[
\frac{d}{dt} K(t) = -\left[ A^T - P B_2 \bar{r}_1^{-1} B_2^T \right] \Psi(t, t_0) K(t_0) \Psi^T(t, t_0)
\]

\[
- \Psi(t, t_0) K(t_0) \Psi^T(t, t_0) \left[ A^T - P B_2 \bar{r}_1^{-1} B_2^T \right]^T
\]

\[
P \bar{H} \Omega_2(t) \bar{H}^T P
\]

\[
- \left[ A^T - P B_2 \bar{r}_1^{-1} B_2^T \right] \int_{t_0}^{t} \Psi(t, t_0) P \bar{H} \Omega_2(t') \bar{H}^T P \Psi^T(t, t') dt'
\]

\[
- \int_{t_0}^{t} \Psi(t, t') P \bar{H} \Omega_2(t') \bar{H}^T P \Psi^T(t, t') dt' \left[ A^T - P B_2 \bar{r}_1^{-1} B_2^T \right]^T
\]

... (IV.55)

The integration parts in the last two terms can be replaced by Eq. (IV.52). And we finally have a simple reduced form,
\[
\frac{dK(t)}{dt} = -\left[\begin{array}{c}
\hat{A}^T \quad PB_2 R_1^{-1} B_2^T
\end{array}\right]K(t)
- K(t)\left[\begin{array}{c}
\hat{A}^T \quad PB_2 R_1^{-1} B_2^T
\end{array}\right]^T - P\hat{H}_2(t)\hat{H}^TP,
\]

which is a differential Lyapunov equation.

In addition to the time-invariance of the system equation (IV.37b), if we consider the noise statistics as being stationary, then, we will have significant practical advantages in applying the steady-state approximation in Eq. (IV.56) (46), (141). This is realizable without losing any generality, because \(Q(t)\) can be obtained from the burnup distribution of discharged fuel bundles, which is essentially independent of the reactor condition at any particular moments.

Thus, we invoke the algebraic matrix Lyapunov equation, to determine the most probable dynamic range of the counteracting controllers, \(\mu_c(t)\), by replacing \(Q(t)\) by a constant diagonal matrix \(Q\) and by taking \(\dot{K}(t) = 0\);

\[
\left[\begin{array}{c}
\hat{A}^T \quad PB_2 R_1^{-1} B_2^T
\end{array}\right]K + K\left[\begin{array}{c}
\hat{A}^T \quad PB_2 R_1^{-1} B_2^T
\end{array}\right]^T + P\hat{H}_2 Q \hat{H}^TP = 0.
\]
If we assume that there exists a positive-definite symmetric matrix solution $K$ of Eq. (IV.57) with the requirements mentioned in Section III.3.4, then, we can investigate the relationship between $K$ and the most probable dynamic range of $\mu_c(t)$. Apparently from the steady-state operation of the reactor, the mean of counteraction for the zero-mean Gaussian-type noise disturbances would be approximately zero, i.e.,

$$\mu_c(t) \approx 0.$$ 

and, thus, the covariance of $\mu_c(t)$ will be, from Eq. (IV.38b),

$$E[\mu_c(t) \mu_c(t)^T]$$

$$= R_1^{-1} B_2^T E[\Omega(t) \Omega(t)^T] B_2 R_1^{-T}$$

$$= R_1^{-1} B_2^T K B_2 R_1^{-T}. \quad \cdots (IV.58)$$

The standard deviation of individual controller action will be simply obtained from the diagonal elements of matrix in Eq. (IV.58),

$$\sigma = \left[ \text{diag.} \{ R_1^{-1} B_2^T K B_2 R_1^{-T} \} \right]^{1/2}$$

where $\sigma$ is vector expression of the standard deviation, i.e., the most probable range of individual controller.
IV.3 Strategies for Determination of Optimum Controller Set

The procedures involved in determining the number of controllers consist of three steps. From the results of Chapter III, we have developed a method to generate a map of controller effectiveness. One can select a number of high control gain nodes whose expected total reactivity worth would be roughly equal to the excess reactivity of the reference model. The number of controllers at this stage is approximately estimated by the amount of absorbing material allowed for the static set-point of each controller. During this initialization procedure, one must check the cost loss that is a measure of flux deviation due to the static controller deployment. It seems that there would be number of freedoms to accomplish the procedure, but the probable locations for controllers are restricted by avoiding fuel sites and other instrumentation nodes. Even a relatively inexperienced designer can easily find a set of locations after a few trials.

The second step is the computation of the dynamic range of the pre-selected controller set. If the boundary of a controller dynamic range is beyond the maximum allowable absorbing material limit, then, we have to return to the first step with an additional number of controllers. At this stage, the dynamic controllers respond only to the neutron flux regulation, and not to external random reactivity supply. Also, we can compute the optimality of the controller set by solving the matrix Riccati...
equation (IV.38a), and then by comparing with solutions for other possible controller sets.

The final step is mainly to improve the reliability and adequacy of the designed control system. Additional reactivity sources with local perturbation are always possible in operating reactors. Our control system must accommodate those external disturbances at any instance. The probable dynamic range of controllers is evaluated for the system with a probable disturbance range. If there is a controller whose action boundary is larger than the limit criteria, we must return to the first step and re-select different controller locations.
V. OPTIMALITY OF THE 600 MWe CANDU ZONE CONTROLLERS

V.1 Two-Dimensional Reactor Model

The thrust of this chapter is to apply the theory we have developed up to here and assure ourselves of the optimal performance of our control system. The reactor model we deal with is the two-dimensional model of the standard 600 MWe CANDU-PHW reactor illustrated in Figure V-1.

The material properties\(^{(15)}\) are evaluated by flux-weighted, averaging over the reactor length and tabulated in Table V-1. The total number of different materials equals 12 for the reflected core and three types of the adjuster rods.

The \(\lambda\)-modes for the same model were generated with seven harmonics plus the fundamental mode. Because of excessive computing time to generate high harmonics, we determined to use an approximation of integrating the existing 3-dimensional numerical mode functions\(^{(123)}\) over the reactor length instead of directly generating 2-dimensional modes. The properties of the first eight modes are given in Table V-2. The degeneracy of the complemented azimuthal modes is eliminated after
<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$\Sigma a_1$ $(x10^{-4})$</th>
<th>$\Sigma a_2$ $(x10^{-3})$</th>
<th>$\Sigma Rem$ $(x10^{-3})$</th>
<th>$\nu \Sigma f_2$ $(x10^{-3})$</th>
<th>$H^*$</th>
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<td>2</td>
<td>Reflector</td>
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<td>1.0x10^{-7}</td>
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<td>10.1241</td>
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<td>3</td>
<td>Outer Core</td>
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<td>0.9413</td>
<td>7.6387</td>
<td>3.9795</td>
<td>7.39171</td>
<td>4.6116</td>
<td>0.2089695</td>
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<tr>
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<td>Inner Core</td>
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<td>0.9412</td>
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<td>3.9976</td>
<td>7.39203</td>
<td>4.5960</td>
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<td>0.9413</td>
<td>7.6371</td>
<td>3.9886</td>
<td>7.39187</td>
<td>4.6038</td>
<td>0.2081178</td>
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<td>7</td>
<td>Adjuster I + Outer Core</td>
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<td>0.9413</td>
<td>7.6387</td>
<td>3.9837</td>
<td>7.39171</td>
<td>4.6196</td>
<td>0.2089876</td>
</tr>
<tr>
<td>8</td>
<td>Adjuster I + Inner Core</td>
<td>1.2739</td>
<td>0.9412</td>
<td>7.6355</td>
<td>4.0018</td>
<td>7.39203</td>
<td>4.5964</td>
<td>0.2072843</td>
</tr>
<tr>
<td>9</td>
<td>Adjuster III + Outer Core</td>
<td>1.2739</td>
<td>0.9413</td>
<td>7.6387</td>
<td>3.9978</td>
<td>7.39272</td>
<td>4.6129</td>
<td>0.2090351</td>
</tr>
<tr>
<td>10</td>
<td>Adjuster III + $\frac{1}{4}$ Inner Core + $\frac{1}{2}$ Outer Core</td>
<td>1.2739</td>
<td>0.9413</td>
<td>7.6379</td>
<td>4.0023</td>
<td>7.39179</td>
<td>4.6090</td>
<td>0.2086093</td>
</tr>
<tr>
<td>11</td>
<td>Adjuster III + $\frac{1}{4}$ Inner Core + $\frac{1}{2}$ Outer Core</td>
<td>1.2739</td>
<td>0.9413</td>
<td>7.6371</td>
<td>4.0069</td>
<td>7.39187</td>
<td>4.6051</td>
<td>0.2081834</td>
</tr>
<tr>
<td>12</td>
<td>Adjuster III + Inner Core</td>
<td>1.2739</td>
<td>0.9412</td>
<td>7.6355</td>
<td>4.0159</td>
<td>7.39203</td>
<td>4.5973</td>
<td>0.2073318</td>
</tr>
<tr>
<td>13</td>
<td>Adjuster IV + Outer Core</td>
<td>1.2739</td>
<td>0.9413</td>
<td>7.6387</td>
<td>3.9864</td>
<td>7.39171</td>
<td>4.6122</td>
<td>0.2090014</td>
</tr>
<tr>
<td>14</td>
<td>Adjuster IV + $\frac{1}{2}$ Inner Core + $\frac{1}{2}$ Outer Core</td>
<td>1.2739</td>
<td>0.9413</td>
<td>7.6371</td>
<td>3.9955</td>
<td>7.39187</td>
<td>4.6044</td>
<td>0.2081497</td>
</tr>
</tbody>
</table>

* Heat output factor per bundle, $Kw/(10^{11} n/cm^2 \cdot sec)$

**TABLE V-1 NUCLEAR PROPERTIES OF REACTOR MATERIALS**
<table>
<thead>
<tr>
<th>No</th>
<th>Name</th>
<th>Eigenvalue $\left( \frac{1}{\phi_m} \right)$</th>
<th>Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fundamental</td>
<td>1.002330</td>
<td><img src="image" alt="Circle" /></td>
</tr>
<tr>
<td>2</td>
<td>First Azimuthal</td>
<td>0.985047</td>
<td><img src="image" alt="Circle" /></td>
</tr>
<tr>
<td>3</td>
<td>First Azimuthal</td>
<td>0.984677</td>
<td><img src="image" alt="Circle" /></td>
</tr>
<tr>
<td>4</td>
<td>Second Azimuthal</td>
<td>0.958128</td>
<td><img src="image" alt="Circle" /></td>
</tr>
<tr>
<td>5</td>
<td>Second Azimuthal</td>
<td>0.955536</td>
<td><img src="image" alt="Circle" /></td>
</tr>
<tr>
<td>6</td>
<td>First Radial</td>
<td>0.925435</td>
<td><img src="image" alt="Circle" /></td>
</tr>
<tr>
<td>7</td>
<td>Third Azimuthal</td>
<td>0.922456</td>
<td><img src="image" alt="Circle" /></td>
</tr>
<tr>
<td>8</td>
<td>Third Azimuthal</td>
<td>0.920478</td>
<td><img src="image" alt="Circle" /></td>
</tr>
</tbody>
</table>

Table V-2 Properties of the 2-Dimensional $\lambda$-Modes
the third digit of eigenvalues, mostly because the geometry of the reactor model was not completely symmetric and because of the vertical arrangement of the incore control mechanism.

Distributions of generated mode functions for the fast and the thermal neutrons are normalized to the fundamental mode flux at the center of core and are plotted in Figs. V-2 to -5, respectively. The figures show better symmetricity with respect to the Y-axis than with the X-axis. This supports the results that the degeneracy of eigenvalues were destroyed.

Also, with the controversial errors involved in the finite difference formulation of the X-Y geometry in order to generate harmonics having $R-\theta$ properties, the axial integration may cause undesirable distortions in mode shapes especially for the radial modes. The effects will be discussed later in the section analyzing various perturbation simulations.

Input parameters used in calculations are tabulated in Table V-3. The only material property considered for controllers was the effective thermal neutron absorption cross section of the existing CANDU zone controllers. The magnitude is equivalent to the amount of homogenized neutron absorbing material giving the proper current-to-flux ratio at the core-controller interface and the same reaction rate inside controllers. The MULTICELL code (143) was available to provide the
FIGURE V.4: THERMAL FLUX MODE DISTRIBUTIONS (VIEW 2)
FIGURE V-5  THERMAL FLUX MODE DISTRIBUTIONS (VIEW 1)
<table>
<thead>
<tr>
<th>Descriptions (unit)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neutron Yield, ( \nu )</td>
<td>2.6062</td>
</tr>
<tr>
<td>Microscopic Xenon Absorption Cross-section, ( \sigma_a^{xe} ) (cm(^2))</td>
<td>1.261 \times 10^{-18}</td>
</tr>
<tr>
<td>Direct Yield Fraction of Iodine, ( \gamma_I )</td>
<td>6.44 \times 10^{-2}</td>
</tr>
<tr>
<td>Direct Yield Fraction of Xenon, ( \gamma_x )</td>
<td>2.30 \times 10^{-3}</td>
</tr>
<tr>
<td>Iodine Decay Constant, ( \lambda_I ) (sec(^{-1}))</td>
<td>2.94 \times 10^{-5}</td>
</tr>
<tr>
<td>Xenon Decay Constant, ( \lambda_x ) (sec(^{-1}))</td>
<td>2.10 \times 10^{-5}</td>
</tr>
<tr>
<td>Neutron Velocity (cm/sec) ; Thermal</td>
<td>2.676 \times 10^{5}</td>
</tr>
<tr>
<td>; Fast</td>
<td>5.335 \times 10^{6}</td>
</tr>
<tr>
<td>Macroscopic Absorption Cross-section of Controller Material (cm(^{-1}))</td>
<td>1.23 \times 10^{-4}</td>
</tr>
<tr>
<td>Unit Controller Area (cm(^2))</td>
<td>8.1653 \times 10^{2}</td>
</tr>
</tbody>
</table>

**TABLE V-3 INPUT PARAMETERS USED IN CALCULATIONS**
effective cross section. Finally smearing of material properties over the reactor length was necessary to obtain 2-dimensional controller properties.

V.2 Stability and Spatial Control: Effectiveness

One of the important characteristics of the reactor for the design of the control system is whether all the reactor modes are stable or not. If some modes are unstable, the priority in design considerations should be given to relocating all the unstable eigenvalues to the stable region. However, we note that this is clearly a weaker requirement than complete modal controllability.

To examine the stability of the model reactor, the code, ODZCR was used to compute eigenvalues of the iodine/xenon dynamic system implicitly including neutron flux modes at various power levels. Iodine/xenon levels at different power states were obtained by steady state equilibrium condition of the neutron flux level associated with that power level. The feedback effect due to the power level changes was not considered in those uncontrolled reactor system equations in the study.

As the steady state distributions could be calculated, the condition of spatial stability implied that all of the inverse time characteristics of eigenvalues, i.e.,
\[ \omega = a + ib \text{ (hr}^{-1}) \]

of the iodine/xenon mode, should have a negative real part \( a \leq 0 \) for stability. Otherwise, the system would either increase exponentially with an e-folding time of \( 1/a \), or oscillate divergently with a period \( 2\pi/b \) and an e-folding time characteristic.

The calculated eigenvalues are listed in Table V-4. They show that the system is unstable with respect to the first two couples of eigenvalues for reactor power above 70%. In examining the system equation, we find that these eigenvalues are associated with the first two azimuthal modes, i.e. the top-to-bottom and the side-to-side \( \lambda \)-modes, respectively. Thus, the e-folding time and the period of the top-to-bottom mode for uncontrolled transient are approximately 8.9 and 19.9 hours.

Figure V-6 gives a more accurate threshold power level, above which the oscillatory diverging power would occur. At 50% power level, a power tilt oriented from top-to-bottom will cause to the reactor to be unstable. A 3-dimensional diffusion calculation for the reactor has shown the threshold power to be about 40 to 60%\(^{(140)}\).

To calculate the spatial control effectiveness of the uncontrolled system, the program ODZCR requires several preliminary simulations; for determining the minimum number of controllers required to stabilize and control the entire system; for choosing appropriate
<table>
<thead>
<tr>
<th>Power</th>
<th>100</th>
<th>80</th>
<th>70</th>
<th>40</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>1*</td>
<td>3.1267</td>
<td>8.8235</td>
<td>1.6955</td>
<td>8.3804</td>
<td>0.9864</td>
</tr>
<tr>
<td>2</td>
<td>3.1267</td>
<td>-8.8235</td>
<td>1.6955</td>
<td>-8.3804</td>
<td>0.9864</td>
</tr>
<tr>
<td>3</td>
<td>2.7428</td>
<td>8.9338</td>
<td>1.3980</td>
<td>8.4181</td>
<td>0.7321</td>
</tr>
<tr>
<td>4</td>
<td>2.7428</td>
<td>-8.9338</td>
<td>1.3980</td>
<td>-8.4181</td>
<td>0.7321</td>
</tr>
<tr>
<td>5</td>
<td>-15.777</td>
<td>0.0</td>
<td>-12.744</td>
<td>0.0</td>
<td>-11.195</td>
</tr>
<tr>
<td>6</td>
<td>-16.167</td>
<td>0.0</td>
<td>-13.068</td>
<td>0.0</td>
<td>-11.488</td>
</tr>
<tr>
<td>7</td>
<td>-16.691</td>
<td>0.0</td>
<td>-13.471</td>
<td>0.0</td>
<td>-11.830</td>
</tr>
<tr>
<td>12</td>
<td>-4.5466</td>
<td>0.0</td>
<td>-4.6214</td>
<td>0.0</td>
<td>-4.6853</td>
</tr>
<tr>
<td>13</td>
<td>-4.6527</td>
<td>0.0</td>
<td>-4.7850</td>
<td>0.0</td>
<td>-4.8528</td>
</tr>
<tr>
<td>14</td>
<td>-4.7059</td>
<td>0.0</td>
<td>-4.7314</td>
<td>0.0</td>
<td>-4.7999</td>
</tr>
</tbody>
</table>

Notes: * In the order of magnitudes of absolute eigenvalues
** Multiplied by $10^{-5}$
FIGURE V-6  THRESHOLD POWER AGAINST UNSTABLE MODES
Q and R matrices in Eq. (III.20) in order that the Riccati equation behaves numerically well; and for determining \( \alpha \) in Eq. (III.52) which promotes the speed of convergence.

Those studies concluded that a pair of controllers symmetrically located in the reactor satisfied the controllability requirements for the unstable system, and that numerical stability was guaranteed with

\[
\begin{align*}
R &= \text{the unit matrix}, \\
Q &= \text{a diagonal matrix whose elements are } 0.002, \text{ and} \\
\alpha &= 0.001.
\end{align*}
\]

Thus, the results of the calculation imply an equal importance of every mode amplitude loss and of control penalty paid by individual controllers.

An interesting point indicated in the simulations was that, even though eigenvalues of complemented azimuthal \( \lambda \)-modes could be distinguished after the third digit in their numerical values, the analytical properties of degeneracy were hardly relaxed, which leads to the result that a single controller won't satisfy the Wieberg's Theorem I for the minimum controller criterion.

The controller domain was arbitrarily chosen with preference given to high neutron flux regions and peaks of mode shapes.
The only possible sites for locating the controllers were on grid points which match the lattice spacing of 28.575 cm, because fuel channels occupy the in-between space.

The computations were scanned over each nodal point inside the controller domain with a pair of controllers symmetrically deployed with respect to the Y-axis or to the center of the reactor. Results of both cases are illustrated in Figs. V-7 and -8, respectively. At nodes where the Riccati equations were not fully converged to the given convergence criterion $10^{-2}$ after 25 iterations, the effectiveness distributions were deleted in the figures.

Reminding that the spatial control effectiveness was quantified by

$$
\| \Pi_s(t) \| = \| -R^{-1}E^T \Theta(\xi,t) \|
$$

$$
\Theta = \| -R^{-1}E^TP(\xi_j) \cdot B(t) \|
$$

$$
\leq \| -R^{-1}E^TP(\xi_j) \| \| B(t) \|
$$

in Eqs. (II.29) and (III.31), the minimum of $\| -R^{-1}E^TP(\xi_j) \|$ should be equivalent to a minimum of $\| \Pi_s(t) \|$ for any kind of perturbation transient $B(t)$. Therefore, the smaller values of $\| -R^{-1}E^TP(\xi_j) \|$ shown in Figs. V-7 and -8 represent the more-effective locations for the spatial control.
NOTE: BLANK—NOT COMPLETELY CONVERGED AFTER 25 ITERATIONS

FIGURE V-7. SPATIAL CONTROL EFFECTIVENESS MAP
(Y — SYMMETRIC DEPLOYMENT)
FIGURE V-8 SPATIAL CONTROL EFFECTIVENESS MAP
(© — SYMMETRIC DEPLOYMENT)

NOTE: BLANK — NOT COMPLETELY
CONVERGED AFTER 25 ITERATIONS
In comparing Figures V-7 and V-8, we find that the regions of low value of the inverse of control effectiveness are coincident. Further, if controllers are deployed at the boundary of the given controller domain, the best spatial control is expected with an order of magnitude greater than for those at the center of the core where only a few sites have a reasonably good effectiveness, and that the nodes along the Y-axis are the worst sites for spatial control. These facts mean that the controller domain boundary was fortunately chosen so as to be adjacent to the areas where the peaks of most high harmonics, as shown in Figs. V-4 and -5, are located. Also, because of the nearly perfect Y-symmetry of the system and harmonics, the controllers located along the Y-axis cannot control modes 3, 5, and 7 of Table V-2, whose magnitudes at the controller sites are zero. Also the rank of the controllability matrix of Eq. (III.37) is less than the rank of the system matrix.

It can be stated that in spite of the fact that the most effective region for bulk reactivity control is the central region of the reactor, where we expect a high neutron importance, the spatial controllers should be located where the peaks of the harmonics occur. But, it should be noted that, given the existing shape and magnitude of the neutron flux distribution, one cannot assign separately spatial control and bulk control functions to different controllers. Designers must be careful to deploy controllers whose dynamic range responds to both effects. The range must be limited by hardware considerations.
V.3 Evaluation of the CANDU 600 MWe Zone Control System

During normal operation, the CANDU zone control system has the prime responsibility for both of bulk and spatial control. This section investigates the optimality of the CANDU 600 MWe zone control system in terms of dynamic response ranges with respect to variation of locations in each vertical compartment. Because the code ODZCR treats controllers as points in the reactor, the analyses include uncertainty due to neglecting neutron flux variations along the controller length which should be considered when handling volumetric controllers.

A reference location of the control system was arbitrarily selected which would have similar average responses as the existing vertical controllers, as estimated from design data. Analysis followed procedures described in Chapter IV with various control systems whose individual controller had the same locations corresponding to the reference location, except for a single controller which was vertically shifted between the top and the bottom boundaries of the compartment.

Figure V-9 illustrates the locations of controllers selected for the study. The grid points marked by a circle are the reference locations designating the corresponding compartments. The numbers shown inside the squares indicate the particular case of study. Therefore, the case I-2 means that the controller I and its symmetrical counter part II were shifted from their reference locations to the marked locations.
FIGURE V-9  LOCATIONS ASSIGNED FOR CONTROLLERS IN THE VERTICAL REFERENCE ZONE CONTROL SYSTEM
Thus, the results of the vertical shifting exercise would be useful to measure the optimality of deployment policy in terms of the spatial control effectiveness and of the degree of reactivity compensation with minimum neutron flux distortions.

In order to evaluate additional controller actions to compensate for local burnup/fuelling effects, the covariance matrix $Q_2$ in Eq. (IV.25) was, again, chosen to be a constant diagonal matrix. Figures V-10 shows that the burnup distribution of discharged fuel bundles would fit fairly well a Gaussian distribution. Also, the multiplication factor variation over burnup intervals equivalent to a standard deviation of the above Gaussian distribution is approximately linear. From this linearity, the covariance of the random vector $\Gamma_1(t) = \frac{\Delta\psi}{\sum_{f2}} / \psi^{\text{Ref}}$ can be replaced by the variance on the $k_\infty$ coordinate instead of the burnup or the time coordinate, such that diagonal elements of the covariance matrix $Q_2$ are equal to the square of the standard deviation of bundle distribution on the $k_\infty$ coordinate, estimated to be $1.06 \times 10^{-4}$.

Table V-5 gives the depth of the control systems to compensate for the excess reactivity originally introduced in the reference state of the reactor. After comparing the thermal neutron flux distribution of the reference state, Figure V-11, the magnitudes of control costs required for the static set-point determinations were found to depend not only on the fundamental mode shape but were also affected by high harmonics shapes. Figures V-12, -13 and -14 show the significances of those effects.

* $k_\infty$ is the infinite multiplication factor.
FIGURE V-10 REACTIVITY PARAMETER AND DISCHARGED FUEL DISTRIBUTION VS BURNUP
### TABLE V-5  STATIC CONTROL SET-POINTS VARIED BY DIFFERENT VERTICAL CONTROLLER LOCATIONS

<table>
<thead>
<tr>
<th>Caseg</th>
<th>Amp. Loss (x10^-2)</th>
<th>Control Cost (x10^-2)</th>
<th>R.M.S. Error (%) in Shape</th>
<th>Control Volume</th>
<th>Levels (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td><strong>Ref</strong></td>
<td>1.7226</td>
<td>0.16516</td>
<td>0.00072</td>
<td>24.0288</td>
<td>49.2</td>
</tr>
<tr>
<td>I-1</td>
<td>0.00325</td>
<td>0.16805</td>
<td>0.00004</td>
<td>22.9444</td>
<td>52.4</td>
</tr>
<tr>
<td>I-2</td>
<td>49.238</td>
<td>0.21286</td>
<td>0.00255</td>
<td>26.7462</td>
<td>71.3</td>
</tr>
<tr>
<td>III-1</td>
<td>3.3649</td>
<td>0.23947</td>
<td>0.00067</td>
<td>28.5171</td>
<td>51.3</td>
</tr>
<tr>
<td>III-2</td>
<td>0.16541</td>
<td>0.16328</td>
<td>0.0031</td>
<td>22.3394</td>
<td>36.0</td>
</tr>
<tr>
<td>III-3</td>
<td>0.18882</td>
<td>0.22352</td>
<td>0.0020</td>
<td>21.4995</td>
<td>9.5</td>
</tr>
<tr>
<td>V-1</td>
<td>2.8566</td>
<td>0.17812</td>
<td>0.00097</td>
<td>24.5675</td>
<td>52.8</td>
</tr>
<tr>
<td>V-2</td>
<td>3.3269</td>
<td>0.20754</td>
<td>0.00103</td>
<td>27.2927</td>
<td>54.5</td>
</tr>
<tr>
<td>VI-1</td>
<td>1.0950</td>
<td>0.16182</td>
<td>0.00558</td>
<td>23.8934</td>
<td>47.6</td>
</tr>
<tr>
<td>VI-2</td>
<td>0.42879</td>
<td>0.16191</td>
<td>0.00036</td>
<td>23.6632</td>
<td>45.1</td>
</tr>
<tr>
<td>VII-1</td>
<td>0.13911</td>
<td>0.17210</td>
<td>0.00021</td>
<td>24.6295</td>
<td>45.6</td>
</tr>
<tr>
<td>VII-2</td>
<td>6.0699</td>
<td>0.27338</td>
<td>0.00138</td>
<td>23.4388</td>
<td>54.7</td>
</tr>
</tbody>
</table>

Notes:  
* Total volume occupied by controllers; 52.0 lattice sites.  
+ (-) sign in controller levels means that the controller is not required at the location.  
** To compensate for an initial reactivity, 2.325mk.
FIGURE V-11 THERMAL NEUTRON FLUX DISTRIBUTION (NORMALIZED BY FLUX AT CENTER)
FIGURE V-12  RELATIVE CONTROLLER MODE WORTH AT DIFFERENT LEVELS (1-ST AZIMUTHAL MODE)
FIGURE V-13  RELATIVE CONTROLLER MODE WORTH AT DIFFERENT LEVEL
(1-ST RADIAL MODE
FIGURE V-14  RELATIVE CONTROLLER MODE WORTH AT DIFFERENT LEVELS (2-ND AZIMUTHAL MODE)
In terms of controller location. For example, if controller VI moves in a vertical direction, because of a nearly flat thermal flux distribution inside the compartment, the principal contribution to the resultant reactivity change is made by the first radial mode worth.

Judging by the smaller control penalty and minimum loss of mode amplitudes, Cases I-1, III-2 and VII-1 are not only acceptable but also "optimum in reactivity control".

Even though a control system is optimum in the sense of the reactivity control, it may not be acceptable if its dynamic behaviour corresponding to state transients would be poor. Table V-6 gives the maximum variations of controller levels corresponding to an allowable power range of ±4%, that is the power error to be controlled by the zone controller system (±3% (cf. Figure I-3) plus an additional ±1% error for redundancy). Among the control systems selected by the bulk control criterion, Case VII-1 is the most desirable one because of the smallest dynamic range. Note that an important contribution to determining the dynamic range is the flux shape deviations and, therefore, the spatial control effectiveness maps should explain the part of the results. For example, Case I-2, has a smaller dynamic range (14.3), than Case I-1, (23.3) and the controller I in the former system has a smaller inverse effectiveness (23.0), than the latter's (161.0) in Figure V-8.

Another condition by which to judge the optimality of the system
<table>
<thead>
<tr>
<th>Average Level Change</th>
<th>Individual Controller Level Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>Reference</td>
<td>25.4</td>
</tr>
<tr>
<td>I-1</td>
<td>23.3</td>
</tr>
<tr>
<td>I-2</td>
<td>14.3</td>
</tr>
<tr>
<td>III-1</td>
<td>18.7</td>
</tr>
<tr>
<td>III-2</td>
<td>22.8</td>
</tr>
<tr>
<td>III-3</td>
<td>18.4</td>
</tr>
<tr>
<td>V-1</td>
<td>25.6</td>
</tr>
<tr>
<td>V-2</td>
<td>26.4</td>
</tr>
<tr>
<td>VI-1</td>
<td>25.8</td>
</tr>
<tr>
<td>VI-2</td>
<td>24.9</td>
</tr>
<tr>
<td>VII-1</td>
<td>13.0</td>
</tr>
<tr>
<td>VII-2</td>
<td>23.6</td>
</tr>
</tbody>
</table>

**Table V-6 Dynamic Ranges of Controllers for 4% Power Change**
is the most probable dynamic range corresponding to randomly distributed perturbations, which are mainly caused by refuelling. Table V-7 compares the effects for various cases. Case VII-l has still a relatively small range of change $\pm 0.096\%$. Generally the magnitudes of the controller responses for these types of perturbations are $< 1\%$ of those expected in controlling global perturbations, as expected from design experience (142).

To verify the optimal design rules they were applied to the existing CANDU zone control system, by simulating various transients. The initial reactor conditions assumed were such that the reactor had reached an equilibrium state with a 5\% mode perturbation. Then the control system was activated to return the flux distribution to the desirable reference state instantaneously. Every transient was examined with an hour interval. Table V-8 to -14 show the transient behaviour of modes for the thermal neutron and the xenon concentrations. The neutron flux modes are well-controlled in most of the single mode excitation transients with a residual error of 0.02\%. The residual errors shown in mode 3, 6, 7 excitation cases reached a maximum 0.5\%. Presumably controllers located at nodes where the mode fluxes are nearly zero would have less control effectiveness and introduce more controller-induced perturbations. From the spatial control effectiveness map for the Y-symmetric deployment (Fig. V-8) and the characteristics of the spatial modes (Table V-2), the reference controllers I, II, III and IV are expected to have a poor effectiveness on modes 6 and 7, and controllers V and VII on modes 3, 5 and 7.
<table>
<thead>
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TABLE V-8 CHANGES IN STATE FUNCTIONS AND CORRESPONDING CONTROLLER RESPONSES

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**Table V-9 Changes in State Functions and Corresponding Controller Responses**  
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**TABLE V-11. Changes in State Functions and Corresponding Controller Responses**

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### TABLE V-13  CHANGES IN STATE FUNCTIONS AND CORRESPONDING CONTROLLER RESPONSES

(Ref. Case; Mode '7' Excited; 100% Initial Power)

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<th>Controller-Level (%)</th>
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</table>

TABLE V-14 CHANGES IN STATE FUNCTIONS AND CORRESPONDING CONTROLLER RESPONSES
(Ref. Case: Mode 8 Excited; 100 % Initial Power)
The global reactivity contribution of the control action on the flux tilt were negligible except for mode 6, i.e. the first radial mode excitation case which contributed approximately \(-0.57\text{mk}\). This was caused by the axially integrated 2-dimensional mode approximation mentioned in Section V.1, which had eventually affected the principal biorthogonality between the fundamental and the first radial mode. The differences in the first radial mode shape at various cross-sections are plotted in Figure V-15. Thus, the actual control action must be equivalent to the action shown in Table V-12 minus an action taken to correct for the artificial fundamental mode error of \(-2.0\%\).

It should be noted, that generally \(\lambda\)-mode approximations of the state functions seldom satisfy the property of finality, but, according to the tables, the coupling between thermal flux modes are almost completely negligible and between xenon modes are up to \(2\%\) of the initial deviations.

Similar calculations have been done for different control systems previously introduced. The prompt control action to eliminate the excitations on modes 2, 5 and 7 are given in Table V-15. As mentioned earlier, controllers I, II, III and IV are located near nodes where mode 7 has zero value and, thus, in order to suppress the excitation of mode 7, very large efforts were required. From among the other cases, Case VII-1 shows a slightly superior performance.
FIGURE V-15 1-ST RADIAL MODE SHAPES AT DIFFERENT Z-SECTIONS
<table>
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<th>Excited Mode</th>
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<th>Controller Levels (%)</th>
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<td>-2.06</td>
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<tr>
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**TABLE V-15** CONTROLLER LEVELS FOR INITIAL PROMPT ACTION
In the simulation routine, SIMUL, of ODZCR a neutron balance is achieved by appropriate control action. However, the xenon concentration information for the above neutron balance is obtained from a previous time step. Therefore, the calculated control system response may in fact be slightly greater or smaller than required depending upon the xenon transient behaviour.

Table V-16 lists the maximum rates of controller responses which occurred during the first 11 hour transient period. Although the rates are relatively small, \( \leq 0.2\% \text{hour} \) in most of the cases, except for the reference case, control system VII-l shows minimum actions in controlling the same kind of perturbation.

Another interesting result is shown in Table V-17, that the optimally chosen control system controls the reactor with minimum action and that the residual error in states is kept to a minimum. Since the residual error is caused by a controller-induced perturbation, less control action obviously introduces less perturbation.

V.4 Remarks

Since the CANDU zone control system consists of vertically arranged compartments having an extensive length, their control action varies spatially with the absorbed level in the compartment. Our control system on the other hand is a point system, and it is impossible,
<table>
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<th>Rate of Controller Level (%/hr.)</th>
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TABLE V-16 MAXIMUM RATES OF CONTROLLER LEVEL CHANGES DURING TRANSIENT
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<td>0.001</td>
<td>0.0</td>
<td>0.001</td>
<td>0.0</td>
<td>0.001</td>
<td>0.0</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>Reference</td>
<td>-0.004</td>
<td>0.0</td>
<td>-0.023</td>
<td>-0.007</td>
<td>0.001</td>
<td>-0.016</td>
<td>-0.007</td>
<td>-0.004</td>
<td></td>
</tr>
<tr>
<td>I-1</td>
<td>0.0</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.008</td>
<td></td>
</tr>
<tr>
<td>I-2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>III-1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.001</td>
<td>0.001</td>
<td>0.003</td>
<td>0.0</td>
<td>0.004</td>
<td>-0.001</td>
<td></td>
</tr>
<tr>
<td>III-2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.005</td>
<td>0.007</td>
<td></td>
</tr>
<tr>
<td>VII-1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>

TABLE V-17 MAXIMUM UNCONTROLLED ERROR IN THERMAL FLUX MODES DURING TRANSIENT
to duplicate with it the real system. But the general conclusion would be as follows.

(1) The controllers should always be located in the high thermal flux regions of the core. They should also be located as close as possible to the peaks of the important harmonics. Even though the first radial mode does not seem to be at first glance important, nevertheless, by placing a controller in the center of the reactor core, we can also control many local perturbations.

(2) Cross-coupling among individual controllers may be strong enough, that a change of one controller location can cause a complete change in the effectiveness of the entire system. When we examine Table V-15, -16 and -17, we can see this cross-coupling effect change not only the magnitude but also the direction of control action.

The main emphasis of the above is that the most effective region for both bulk and spatial control is very limited for the 600 MWe CANDU reactors as shown in Figure V-16. The area, between the concentric circles, which was determined from the spatial control effectiveness maps, Figures V-7 and -8, and from the thermal flux mode shapes, Figures V-4 and -5, is the most effective region for spatial control. The area inside the dashed line, where the thermal neutron flux is at least 99% of the value at the reactor center, is the most effective region for the bulk control. Hence, the overlap of these areas must be the locations for deploying individual controllers. The following conclusions can be drawn from the figure.
(1) The existing zone control system of the 600 MWe CANDU is deployed where the vertically mounted controllers can achieve the best performance.

(2) The dashed areas in Fig. V-16, which identify the 20% to 70% controller levels, which correspond to the normal operating range of Fig. I-3 fall within our spatial control effective region.

(3) The central controller falls outside the spatial control effective region.

(4) The reason why Case VII-1 shows a better performance than others is because controller VII was shifted to the most effective region from its reference point.
VI. ALTERNATIVES IN THE ZONE CONTROL SYSTEM DESIGN USING OPTIMALITY CRITERIA

We examined optimality of the existing 600 MWe CANDU zone control system whose vertical orientation resides mostly in the most effective region for the bulk and spatial control. No better place can be found where the larger fraction of controllers is placed in the intersection area in Figure V-16 for the system.

However, if we change the orientation of the controllers parallel to the X-axis from its original geometry, a larger intersection mainly in the top and bottom sections of the reactor may be assigned for the control system. Advantages in the new system may be not only a matter of performance but also a simplification in hardware design by removing the driving mechanism from the crowded control mechanism deck on top of the reactor. This alternative may possibly eliminate uncertainty factors in prediction of controller response to perturbations, whose differential behaviour was shown to be strongly dependent on the individual controller level during transients.

Figure VI-1 shows the horizontally oriented zone control system, where only its orientation is changed from the original one. The method of assigning the case numbers is similar to the previous vertical model's.
Table VI-1 shows the calculated optimality parameters and the controller static set-point. Magnitudes of parameters are similar to the vertical control system's as expected. As we move to the center line of the reactor core, controllers I, II, III and IV tend to require less control efforts, but are accompanied by a heavier amplitude loss.

The dynamic ranges for regulation and control are shown in Tables VI-2 and VI-3. Their ranges did not change very much even though the controller locations were changed. The reason is that the controllers occupy large fractions of the control effective region, and that the magnitudes of the dynamic ranges for most cases are comparable with those of the best case in the vertical control system.

We considered another alternative by reducing the number of controllers in the system. Generally we found that, without paying heavy penalties in the amplitude loss, i.e., in the flux shape distortion, it was quite difficult to find controllers whose performances were comparable to the seven controller system. Some of the cases which showed reasonably good performances are listed in Table VI-4. The 6 controller system has two controllers along the center line of the reactor rather than 3 controllers of the 7 controller system. Thus, there is no controller in the reactor center. Another system studied had 5 controllers with only one controller along the center line at the reactor center.
<table>
<thead>
<tr>
<th>Cases</th>
<th>Amp. Loss (x10^-2)</th>
<th>Control Cost (x10^-2)</th>
<th>R.M.S. (%) Error in Flux Shape</th>
<th>Controller Volume</th>
<th>Levels (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>I</td>
</tr>
<tr>
<td>Reference</td>
<td>3.6011</td>
<td>0.18979</td>
<td>0.00073</td>
<td>24.1274</td>
<td>48.1</td>
</tr>
<tr>
<td>I-1</td>
<td>4.9669</td>
<td>0.16112</td>
<td>0.00086</td>
<td>22.2953</td>
<td>39.4</td>
</tr>
<tr>
<td>I-2</td>
<td>0.8894</td>
<td>0.30012</td>
<td>0.00036</td>
<td>31.6534</td>
<td>101.2</td>
</tr>
<tr>
<td>III-1</td>
<td>1.3558</td>
<td>1.0680</td>
<td>0.00045</td>
<td>46.1117</td>
<td>57.9</td>
</tr>
<tr>
<td>III-2</td>
<td>3.6238</td>
<td>0.27097</td>
<td>0.00074</td>
<td>28.3818</td>
<td>55.1</td>
</tr>
<tr>
<td>III-3</td>
<td>0.02881</td>
<td>0.15872</td>
<td>0.00007</td>
<td>22.7648</td>
<td>32.5</td>
</tr>
<tr>
<td>V-1</td>
<td>11.701</td>
<td>0.18489</td>
<td>0.00132</td>
<td>23.9566</td>
<td>43.6</td>
</tr>
<tr>
<td>V-2</td>
<td>6.0024</td>
<td>0.20973</td>
<td>0.00095</td>
<td>24.7243</td>
<td>50.5</td>
</tr>
<tr>
<td>VI-1</td>
<td>0.0590</td>
<td>0.19277</td>
<td>0.00099</td>
<td>23.9475</td>
<td>51.6</td>
</tr>
<tr>
<td>VI-2</td>
<td>9.0462</td>
<td>0.19171</td>
<td>0.00116</td>
<td>23.8205</td>
<td>44.0</td>
</tr>
<tr>
<td>VII-1</td>
<td>0.9352</td>
<td>0.17295</td>
<td>0.00037</td>
<td>23.7647</td>
<td>50.8</td>
</tr>
<tr>
<td>VII-2</td>
<td>0.0168</td>
<td>0.24336</td>
<td>0.00005</td>
<td>23.6404</td>
<td>54.7</td>
</tr>
</tbody>
</table>

Notes:  
* Total volume occupied by controllers  
+ Controller size is too small  
++ Controller is not necessary

TABLE VI-1 STATIC SET-POINTS OF HORIZONTAL CONTROL SYSTEM
<table>
<thead>
<tr>
<th>Reference</th>
<th>Average Level Change (%)</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-2</td>
<td>13.68</td>
<td>8.18</td>
<td>8.33</td>
<td>16.17</td>
<td>15.56</td>
<td>22.16</td>
<td>8.60</td>
<td>18.92</td>
</tr>
<tr>
<td>III-1</td>
<td>13.20</td>
<td>15.18</td>
<td>14.62</td>
<td>6.96</td>
<td>5.94</td>
<td>23.56</td>
<td>9.26</td>
<td>29.47</td>
</tr>
<tr>
<td>III-3</td>
<td>14.42</td>
<td>13.04</td>
<td>12.54</td>
<td>15.22</td>
<td>15.06</td>
<td>20.48</td>
<td>7.99</td>
<td>17.73</td>
</tr>
<tr>
<td>V-1</td>
<td>13.30</td>
<td>12.64</td>
<td>12.15</td>
<td>14.51</td>
<td>13.84</td>
<td>15.38</td>
<td>7.74</td>
<td>17.17</td>
</tr>
<tr>
<td>V-2</td>
<td>12.86</td>
<td>11.74</td>
<td>11.44</td>
<td>13.75</td>
<td>13.23</td>
<td>17.25</td>
<td>7.31</td>
<td>16.08</td>
</tr>
</tbody>
</table>

**Table VI-2 Dynamic Ranges of Horizontal Control System for 4% Power Change**
<table>
<thead>
<tr>
<th>Reference</th>
<th>Average Level Change (%)</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-1</td>
<td>0.097</td>
<td>0.094</td>
<td>0.101</td>
<td>0.107</td>
<td>0.104</td>
<td>0.147</td>
<td>0.157</td>
<td>0.128</td>
</tr>
<tr>
<td>I-2</td>
<td>0.104</td>
<td>0.098</td>
<td>0.097</td>
<td>0.095</td>
<td>0.092</td>
<td>0.0132</td>
<td>0.059</td>
<td>0.0113</td>
</tr>
<tr>
<td>III-1</td>
<td>0.089</td>
<td>0.069</td>
<td>0.072</td>
<td>0.023</td>
<td>0.023</td>
<td>0.156</td>
<td>0.208</td>
<td>0.109</td>
</tr>
<tr>
<td>III-2</td>
<td>0.097</td>
<td>0.095</td>
<td>0.093</td>
<td>0.082</td>
<td>0.073</td>
<td>0.151</td>
<td>0.067</td>
<td>0.130</td>
</tr>
<tr>
<td>III-3</td>
<td>0.099</td>
<td>0.059</td>
<td>0.031</td>
<td>0.095</td>
<td>0.094</td>
<td>0.235</td>
<td>0.179</td>
<td>0.124</td>
</tr>
<tr>
<td>V-1</td>
<td>0.101</td>
<td>0.096</td>
<td>0.093</td>
<td>0.110</td>
<td>0.106</td>
<td>0.116</td>
<td>0.058</td>
<td>0.131</td>
</tr>
<tr>
<td>V-2</td>
<td>0.098</td>
<td>0.091</td>
<td>0.088</td>
<td>0.104</td>
<td>0.100</td>
<td>0.130</td>
<td>0.055</td>
<td>0.123</td>
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<tr>
<td>VI-1</td>
<td>0.106</td>
<td>0.095</td>
<td>0.092</td>
<td>0.109</td>
<td>0.105</td>
<td>0.149</td>
<td>0.073</td>
<td>0.129</td>
</tr>
<tr>
<td>VI-2</td>
<td>0.090</td>
<td>0.057</td>
<td>0.059</td>
<td>0.075</td>
<td>0.072</td>
<td>0.128</td>
<td>0.166</td>
<td>0.089</td>
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<tr>
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<td>0.108</td>
<td>0.099</td>
<td>0.097</td>
<td>0.114</td>
<td>0.110</td>
<td>0.156</td>
<td>0.060</td>
<td>0.126</td>
</tr>
<tr>
<td>VII-2</td>
<td>0.102</td>
<td>0.058</td>
<td>0.061</td>
<td>0.077</td>
<td>0.074</td>
<td>0.131</td>
<td>0.174</td>
<td>0.162</td>
</tr>
</tbody>
</table>

**TABLE VI-3: Dynamic Ranges Responding to Random Burnup Deviations for Horizontal Controllers**
<table>
<thead>
<tr>
<th>Cases</th>
<th>Penalty ($10^{-2}$)</th>
<th>R.M.S. Error in Thermal Flux (%)</th>
<th>Avg. Static Set-Point (%)</th>
<th>Avg. Dynamic Range for Regulation (%)</th>
<th>Avg. Range for Counteraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-1</td>
<td>635.38</td>
<td>0.17014</td>
<td>0.00932</td>
<td>49.15</td>
<td>15.04</td>
</tr>
<tr>
<td>6-2</td>
<td>26799.0</td>
<td>0.16163</td>
<td>0.06070</td>
<td>46.78</td>
<td>13.50</td>
</tr>
<tr>
<td>5-1</td>
<td>113510.0</td>
<td>0.19828</td>
<td>0.12556</td>
<td>57.20</td>
<td>14.03</td>
</tr>
</tbody>
</table>

Note: * The first number in the case identification indicates the number of controllers in the system.

+ Volume of each controller is 8 lattice sites.

**TABLE VI-4 OPTIMALITY COMPARISON FOR REDUCED CONTROL SYSTEM**
VII. CONCLUSION AND DISCUSSION

An analytical method to determine the optimum design of the CANDU control system layout was developed using modal control theory extended to the linear regulator and the least-square-root estimator. Since the dynamic model describing phenomena occurring in nuclear power reactors are not as simple as ordinary dynamic problems, we decomposed and reduced the system to be able to handle the dominant phenomena only. This reduced-order model has proven to be in agreement with available reactor stability design data for the CANDU 600 MWe reactors.

Separation of the spatial control from the bulk reactivity control was useful to determine the effectiveness of distributed discrete controllers and reached the conclusion that the most effective region for the spatial control was coincident with peaks of the spatial modes.

The number of controllers might be determined by the estimated dynamic range of individual controllers with constraints on maximum power error to be controlled and on the most probable response to random material property disturbances.

After evaluating the proposed method with the 2-dimensional CANDU 600 MWe reactor model, we concluded that the following.
(1) The existing system is expected to have the best performance as far as its vertical orientation is concerned.

(2) The range designated for spatial control was selected to agree with the most effective region for the purpose.

(3) The vertical control system could be replaced by a horizontal one with the advantages of better predictable performance and the hardware design considerations.

We have derived the optimum conditions for deploying control systems and have applied them to CANDU reactors. As many aspects of control requirements as possible were included, but we still need some further modifications for practical and more extensive applications. Such modifications should probably include the following.

(1) The procedure should be to deal with 3-dimensional models. The incore lay-out of the reactor control system has a symmetrical pattern in the Z-coordinate, and, thus, we can extend the location selection rules to the problem. In the modal approach, increasing the number of dimensions of the problem means adding a number of modes pertaining to the added direction. This causes an increase in the order of the system matrix, which eventually leads to a rapid increase in computing time. For example, to solve the matrix Riccati equation, the CPU time roughly proportional to the cube of the order of the system matrix. Possible ways for reducing the computing time could be to use a coarse-mesh model for the controller domain, or to change the algorithm which would be more efficient to compute the Riccati equation.
(2) Introduce direct feedback from the detector readings. During the theoretical formulation, we assumed that transients were detected by exact deviations of the individual mode amplitudes. In principle, the reactor control system should be able to control disturbances and follow transients completely, regardless of what kind of information is obtained. In practice, the control system relies on the estimated detector output, and the uncertainty in control actions induces forward disturbances in the reactor. To reduce the degree of control-action-induced perturbation, the design based on output feedback is preferred. But the problem to determine the optimum locations of detectors, which is expected not only to be more complex in formulating, but to introduce severe penalties in computing costs, because we may have to solve coupled Riccati equation for every possible combination of controller-detector pairs.

(3) Implementing volumetric controllers could be incorporated. One of the assumptions introduced in the study are point controllers rather than realistic volumetric ones. Thus the neutron fluxes were kept constant inside the controllers. Definitely this is not correct for controllers whose dimensions are large. But when a controller model with volumetric controllers is developed, the algorithm must include a criticality search by varying the controller levels.

However, the study shows the possibility of the application of the optimum control theory to layout the reactor control system. In addition, the effectiveness of control can be quantitatively evaluated,
which is very useful for reactor designers, to decide which system will have a better performance.
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(62) The same as (9).


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APPENDIX

ODZCR—OPTIMAL DEPLOYMENT OF THE CANDU ZONE CONTROL SYSTEM

A. GENERAL DESCRIPTIONS

The program ODZCR (Optimal Deployment of the Zone Controllers) is modeled for the study on the CANDU zone control system deployment. It is capable to study the stability of uncontrolled reactors; the spatial control effectiveness distribution calculation with a symmetrically placed pair of controllers; the dynamic range determination of individual controller at the pre-selected locations; and the simulation of transients initiated by certain mode amplitude excitations.

Limits made on program utilization are as follows, even though these limitations can be easily extended.

1. The number of grid points in the X-coordinate must be less than 31 and in the Y-coordinate 23.
2. Number of modes should be not greater than 10.
3. Number of different material properties is limited to 25.

With the above limitation, the maximum CPU size required is approximately 220000B.

Computation time is strongly dependent on the typed problem,
because most of the time is spent in computing the Lyapunov matrix
and the Riccati equations. Approximately a single iteration to solve
an 8 by 8 dimensioned Lyapunov equation requires 0.05 second CRU time
on the CDC-6600. This time use is approximately proportional to \( N^3 \),
where \( N \) is the dimension of the matrix.

B. STRUCTURE OF PROGRAM ODZCR

ODZCR is composed of 16 subroutines and 7 external functions.
A flow diagram of the main routine is shown in Figure B-1. Details of
each subroutine are described as follows.

B.1 Subroutine HEAHD

The routine prints head on each page of output, and contains
title, page number, date and computing time consumed up to present.

Called by:

Main, SMTRX, COTMX, SPCON, BKCON, SIMULL

Calling:

SECOND, DATE

B.2 Subroutine NEFST

NEFST produces neutron, iodine and xenon distributions of the
reference state and normalizes mode functions.

Called by:

Main

B.3 Subroutine XMAP (AA)
The subroutine illustrates the distribution of state functions of the given reactor configuration and also indicates their maximum value.

Arguments:

AA 2-dimensional real number array stating a state function distribution

Called by:

Main, BKCON

B.4 Subroutine SMTRX (NPROB)

The subroutine generates complete system matrixes, not in reduced form, for each problem type and, if NPROB = 1, the coefficient matrix for random material perturbations.

Arguments:

NPROB Index for the problem type,

\[
\begin{align*}
&= 1 \quad \text{Dynamic range determination,} \\
&= 2 \quad \text{Spatial control effectiveness calculation,} \\
&= 3 \quad \text{Transient simulation.}
\end{align*}
\]

Called by:

Main, BKCON, SIMUL

B.5 Subroutine GINV (A, U, AFLAG, ATEMP, MR, NR, NC, ERROR)

The routine gives the generalized inverse for any MR by NR matrix A, including the special case when MR = NR and rank (A) = NR. The algorithm yields the ordinary least squares transformation \((A^T A)^{-1} A^T\) and the inverse of A can be denoted by \(A^{-1}\).
\[ A^{-1} = (A^T A)^{-1} A^T. \]

**Arguments:**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MR</td>
<td>The first dimension of array A,</td>
</tr>
<tr>
<td>NR</td>
<td>The number of rows in A,</td>
</tr>
<tr>
<td>NC</td>
<td>The number of columns in A,</td>
</tr>
<tr>
<td>U</td>
<td>Bookkeeping array,</td>
</tr>
<tr>
<td>AFLAG, ATEMP</td>
<td>Temporary working storage,</td>
</tr>
<tr>
<td>ERROR</td>
<td>Maximum error in elements of ((AA^{-1} - I)),</td>
</tr>
<tr>
<td></td>
<td>where (I) is an identity.</td>
</tr>
</tbody>
</table>

**Called by:**

- Main, CNTRLB, SFCON, BKCON, SIMUL
- Calling:
  - DOT, DOT4

**Function CNTRLB (A, B, NN, N, N1, N2)**

This subroutine checks the controllability of the system with a given pair of controllers. If the formed controllability matrix has rank \(N\), equivalent to the rank \((A)\), CNTRLB is set to 1.0. Otherwise, \(\text{CNTRLB} = 0.0\).

**Arguments:**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>NN</td>
<td>The first dimension of (A,)</td>
</tr>
<tr>
<td>N</td>
<td>Order of (A,)</td>
</tr>
<tr>
<td>N1</td>
<td>Number of controllers. Set to 2,</td>
</tr>
<tr>
<td>N2</td>
<td>Column dimension of controllability matrix,</td>
</tr>
<tr>
<td>A</td>
<td>System matrix,</td>
</tr>
<tr>
<td>B</td>
<td>Controller coefficient matrix produced with a pair of controllers.</td>
</tr>
</tbody>
</table>

**Called by:**

- SFCON
B.7 Subroutine EIGVAL (NM, N, A, B, CA, ALFR, ALFI, BETA, MATZ, Z)

The routine computes eigenvalues of a generalized real matrix

\[(Ax = \omega x)\] already transformed into the Schur form. The reduced form \(A\), hence, is upper triangular except for possible 2x2 diagonal blocks

corresponding to pairs of complex eigenvalues.

**Arguments:**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>NM</td>
<td>The first dimension of A,</td>
</tr>
<tr>
<td>N</td>
<td>Order of A,</td>
</tr>
<tr>
<td>A</td>
<td>(A_{ii}), upper triangular form of A. see p. 211,</td>
</tr>
<tr>
<td>B</td>
<td>Working storage array, but (B(N, 1)) contains information mentioned on subroutine SCHUR,</td>
</tr>
<tr>
<td>CA</td>
<td>Original matrix A,</td>
</tr>
<tr>
<td>ALFR</td>
<td>Vector array for real parts of eigenvalues,</td>
</tr>
<tr>
<td>ALFI</td>
<td>Vector array for imaginary parts of eigenvalues, and if an eigenvalue is purely real, corresponding element of ALFI set to zero,</td>
</tr>
<tr>
<td>BETA</td>
<td>Normalization factors for eigenvalues,</td>
</tr>
<tr>
<td>MATZ</td>
<td>Logical input set TRUE if transformation matrix will be saved,</td>
</tr>
<tr>
<td>Z</td>
<td>Transformation matrix.</td>
</tr>
</tbody>
</table>

Called by:

MAIN

Calling:

SCHUR

B.8 Subroutine EIGVEC (NM, N, A, B, ALFR, ALFI, BETA, Z)

If subroutines SCHUR and EIGVAL have been used to reduce the matrix of the system and to accumulate the transformation matrix,
the eigenvectors of a real general matrix system can also be computed
by EIGVEC. The eigenvectors of the system are determined by a back
substitution process, then transformed to the eigenvectors of the
original system using the information in Z and finally normalized to
the length of the vector.

Arguments:
Same as described in EIGVAL

Called by:
Main

Calling:
DOT, DOT3

B.9 Subroutine CONVX (MR, NMAX, NS, A, WR, WI)
The routine is used for computing the transition matrix of
a linear dynamic system in the form of

\[ \Psi(t, t_0) = F_1 e^{\omega_1(t-t_0)} + \ldots + F_N e^{\omega_N(t-t_0)} \]

Each matrix \( F_1, F_2, \ldots, F_N \) has dimension of the system matrix \( A \). Once
called, \( F_i \) is generated for the specified \( i \).

Arguments:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MR</td>
<td>The first dimension of ( A ), Order of ( A )</td>
</tr>
<tr>
<td>NMAX</td>
<td>Eigenvalue number ( i ) for which ( F_i ) is computed</td>
</tr>
<tr>
<td>NS</td>
<td></td>
</tr>
</tbody>
</table>
A System matrix,
WR Real parts of eigenvalues,
WI Imaginary parts of eigenvalues.

Called by:
Main

Calling:
HEAD

B.10 Subroutine SPCON

This routine is called only when NPROB = 2. The routine generates the spatial control effectiveness distribution inside the defined controller domain. A pair of controllers that were accepted by the controllability criterion, are symmetrically deployed with respect to the Y-axis or to the center of the reactor. The as built matrix Riccati equations are solved at every pair of grid points.

Algorithm inside the routine has the sequence of:

(1) read-in of controller material, domain and grid structure data,
(2) for the Y-symmetry option, constructing meshes for the first half core and, for the center symmetry option, meshes for the first quarter of the core,
(3) interpolating of mode fluxes and controller coefficient matrix elements at the mesh point,
(4) checking the controllability of the pair controllers,
(5) computing the matrix Riccati equation, if the pair passed the controllability test,
(6) producing the gain matrix for the pair,

(7) unfolding and mapping the norm of the gain matrix of individual controllers in the entire controller domain.

Called by:

Main

Calling:

AXPX, CNTRLB, DOT2, DOT3, DOT4, GINV, HEAD, SECOND

and XINT

B.11 Subroutine AXPX (A, U, C, B, N, NA, NC, NU, NB, EPS)

This routine solves the Lyapunov matrix equation,

\[ A^T X + XA = C \]

where A and C are real matrices of dimensions N x N and C is symmetric. A is already transformed into an upper Schur form by an orthogonal transformation U. In principle, the algorithm takes advantages of solving symmetry matrix system, using a recursive procedure that at each step solves at most 4 x 4 linear equation system obtained from partitioning A into 2 x 2 submatrices. An iterative procedure to reduce round-off errors is also incorporated.

Arguments:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Order of A</td>
</tr>
<tr>
<td>NA</td>
<td>The first dimension of array A,</td>
</tr>
<tr>
<td>NC</td>
<td>&quot;</td>
</tr>
<tr>
<td>NU</td>
<td>&quot;</td>
</tr>
<tr>
<td>NB</td>
<td>&quot;</td>
</tr>
<tr>
<td>A</td>
<td>Matrix A in an upper Schur form,</td>
</tr>
<tr>
<td></td>
<td>dimension of A must be at least N+1 by N+1,</td>
</tr>
<tr>
<td>U</td>
<td>Transformation matrix,</td>
</tr>
</tbody>
</table>
C Coefficient matrix. On return, C contains the solution X,  
  Working storage array,  
B EPS Index. If, EPS < 0, subroutine  
  SCHUR won't be called. If EPS#1.0  
  but > 0, iteration for reducing round-off  
  errors will be extended to 10. Otherwise,  
  iteration is limited to 2.

Called by:

  SPCON, BKCON

Calling:

  DOT2, DOT3, DOT4, SCHUR, SYMSLV

B.12 Subroutine SYMSLV (A, C, N, NA, NC)

Subroutine SYMSLV called by AXFXA produces recursively  
partitioned equations at most 4x4. Starting of partitioning is at the  
upper left-handed corner of the transposed triangular matrix A^T.

Arguments:  

  Defined as in AXFXA  

Called by:

  AXFXA

Calling:

  DOT2, DOT3, SYSSLV

B.13 Subroutine SYSSLV

The routine solves ordinary linear equation system whose  
unknown vector has a dimension of at most 4.

Called by:

  SYMSLV
Subroutine SCHUR (NM, NB, N, A, B, MATX, Z, IERR)

Subroutine SCHUR reduces the generalized real matrix to an equivalent upper Schur form, i.e., a upper block triangular matrix whose block diagonal submatrices have dimension of at most 2 by 2. The similarity transformation matrix $Z$, accumulated transformation, and the transformed matrix $\tilde{A}$ satisfy

$$\tilde{A} = Z^T A Z.$$

**Arguments:**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>NM</td>
<td>The first dimension of $A$, $\text{B}$</td>
</tr>
<tr>
<td>NB</td>
<td>Order of $A$, $\text{B}$</td>
</tr>
<tr>
<td>N</td>
<td>Real generalized matrix. On return, array $A$ contains $\tilde{A}$.</td>
</tr>
<tr>
<td>A</td>
<td>Working storage. Initially an identity matrix. On return, $B(N, 1)$ is used for the tolerance to determine negligible elements of $\tilde{A}$.</td>
</tr>
<tr>
<td>B</td>
<td>Logical input set TRUE if the transformation matrix is to be saved,</td>
</tr>
<tr>
<td>MATX</td>
<td>Transformation matrix,</td>
</tr>
<tr>
<td>Z</td>
<td>Index used for error return. Set to the row number for which the transformation is failed.</td>
</tr>
<tr>
<td>IERR</td>
<td></td>
</tr>
</tbody>
</table>

**Called by:**

EIGVAL, AXPHA

**Calling:**

DOT, DOT2

Function XINT (X, Y, XI, YJ, FXY)

XINT is written to interpolate 2-dimensional data. Algorithm follows a linear interpolation to get value, $F$ inside a square whose corner values are known as $F_{i-1,j-1}$, $F_{i,j-1}$, $F_{i-1,j}$, and $F_{i,j}$.
\[
P = \frac{(X - X_{i-1}) (Y - Y_{j-1}) (P_{i,j} - P_{i-1,j} - P_{i,j-1} + P_{i-1,j-1})}{(X_i - X_{i-1}) (Y_j - Y_{j-1})}
+ \frac{(X - X_{i-1}) (P_{i,j-1} - P_{i-1,j-1})}{(X_i - X_{i-1})}
+ \frac{(Y - Y_{j-1}) (P_{i-1,j} - P_{i-1,j-1})}{(Y_j - Y_{j-1})}
+ P_{i-1,j-1}
\]

**Arguments:**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X)</td>
<td>X-coordinate of the point where (P) is interpolated,</td>
</tr>
<tr>
<td>(Y)</td>
<td>Y-coordinate of the point where (P) is interpolated,</td>
</tr>
<tr>
<td>(X_{i-1})</td>
<td>Array contains (X_{i-1}) and (X_i),</td>
</tr>
<tr>
<td>(Y_{j-1})</td>
<td>Array contains (Y_{j-1}) and (Y_j),</td>
</tr>
<tr>
<td>(P_{i,j})</td>
<td>2-dimensional array contains (P_{i-1,j-1}), (P_{i-1,j}), (P_{i,j-1}) and (P_{i,j})</td>
</tr>
</tbody>
</table>

**Called by:**

SFCON and BMTRX

**B. 16 Subroutine BMTRX (LSUM)**

The subroutine constructs the controller coefficient matrix, \(B_2\) in Eq. (IV.7b). Material properties and mode fluxes at controller locations are evaluated by using the interpolation technique.
Arguments:

On return, ISUM contains the total number of controllers included in the control system.

Called by:

BKCON

Calling:

XINT

B.17 Subroutine BKCON (NPJOB)

BKCON calculates the dynamic ranges of the control system corresponding to reactivity, power error and random perturbations. The algorithm follows the theory described in Chapter IV. Solution of the matrix Riccati equation is obtained by iteration, based on Kleinman's scheme

\[
P_k \left( A - BR^{-1}B^T F_{k-1} \right) = \left( A - BR^{-1}B^T F_{k-1} \right)^T P_k + P_k BR^{-1}B^T F_{k-1}
\]

\[+ Q = 0\]

modified by using a finite-difference approximation to the time-dependent Riccati equation,

\[F_{k-1} = P_{k-1} - \alpha F(P_{k-1})\]

where

\[F(P_{k-1}) = P_{k-1}A + A^T P_{k-1} - P_{k-1}BR^{-1}B^T P_{k-1} + Q\]

Arguments:

NPJOB is the problem type set to 1 in the main program in order to call BKCON.
Called by:
   Main

Calling:
    AXFXA, BMTRX, DOT, DOT2, DOT3, DOT4, GINV, HEAD,
    SECOND, SMTRX, XMAP

B.18 Subroutine SIMUL (NPROB)

The routine simulates the transient initiated by exciting mode
amplitudes and corresponding control actions. Information related to
control actions should be prepared before running the problem type and
provided by TAPE2. TAPE2 can be generated by running the program with
NPROB=1.

Called by:
   Main

Calling:
    DOT2, DOT4, GINV, HEAD, SMTRX

B.19 Matrix Operation Routines

<table>
<thead>
<tr>
<th>Functions</th>
<th>Arguments</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DOT</td>
<td>MR, N1, N2, A, JC, KC</td>
<td>A'A'</td>
</tr>
<tr>
<td>DOT1</td>
<td>MR, NR, N1, N2, A, JC, KC</td>
<td>AA'T</td>
</tr>
<tr>
<td>DOT2</td>
<td>MR, NR, N1, N2, A, B, JC, KC</td>
<td>A'B</td>
</tr>
<tr>
<td>DOT3</td>
<td>MR, NR, N1, N2, A, B, JC, KC</td>
<td>AB</td>
</tr>
<tr>
<td>DOT4</td>
<td>MR, NR, N1, N2, A, B, JC, KC</td>
<td>AB'T</td>
</tr>
</tbody>
</table>
FIGURE B-1  PROGRAM ODZOR MAIN FLOW DIAGRAM
C. INPUT DATA DESCRIPTION

Most of the input data are primarily prepared by means of punched cards or keyboard. Numerical values of \( \lambda \)-modes are accepted through a magnetic tape or disk file. Also data transfer from the dynamic range determination option to the simulation option may be achieved by any data storage facilities of the computer.

C-I INPUT CARDS

Total 16 different sets of data cards would be necessary for running the entire program. The first 7 sets of cards are independent of problem types. The next 3 sets are only useful for the spatial control effectiveness computation. The next 4 sets are required for the dynamic range-determination option. The last 2 sets should be provided for the simulation of transients initially excited by modal amplitude errors. Table C-1 shows a sample of input data for NPRO=2.

C-I.1 Title Card (18A4)

The information contained in this card is printed at the top of each page of output. If the user does not desire a title to be printed, a blank card must be inserted.

C-I.2 Problem Control Card (615)

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>NMAT</td>
<td>Number of material property types,</td>
</tr>
<tr>
<td>6-10</td>
<td>NRX</td>
<td>Number of grid points in the X-coordinates,</td>
</tr>
<tr>
<td>11-15</td>
<td>NRY</td>
<td>Number of grid points in the Y-coordinates,</td>
</tr>
<tr>
<td>16-20</td>
<td>XMOD</td>
<td>Number of ( \lambda )-modes including the fundamental mode,</td>
</tr>
</tbody>
</table>
21-25 NSTABL  Control index for the stability calculation, 
    } = 1 Yes, 
    } = 1 No, 
26-30 NPROB  Control index for the problem type, 
    } = 1 Dynamic range determination, 
    } = 2 Spatial control effectiveness calculation, 
    } = 3 Simulation.

Note: If NPROB = 2, NSTABL should be 1.

C-1.3 Inverse of \( \lambda \)-mode Eigenvalues (8F10.0)

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>XK(1)</td>
<td>( 1/\lambda_0 ), inverse of the fundamental mode eigenvalue,</td>
</tr>
<tr>
<td>11-20</td>
<td>XK(2)</td>
<td>( 1/\lambda_1 ), inverse of the second mode eigenvalue,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>71-80</td>
<td>XK(8)</td>
<td>( 1/\lambda_7 ), inverse of the eighth mode eigenvalue.</td>
</tr>
</tbody>
</table>

Note: The sequence of data should be consistent with the sequence of modes stored on TAPE 1. If \( NMOD > 8 \), another card will be necessary for those modes.

C-1.4 Geometry Distribution Cards (F10.5, 3412)

The data consists of the following:

1. A set of cards specifying the mesh spacing in the \( X \)-coordinates,
2. One blank card,
3. A set of cards specifying the mesh spacing in the \( Y \)-coordinates,
4. One blank card.

Each mesh spacing card has the following information,
C-I.5 Dynamic Parameter Cards (6E12.6)

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-12</td>
<td>XNU</td>
<td>( \nu ) neutron yield rate per fission,</td>
</tr>
<tr>
<td>13-24</td>
<td>SIGXE</td>
<td>( \sigma_N ) microscopic xenon thermal</td>
</tr>
<tr>
<td>25-36</td>
<td>ZLAMI</td>
<td>absorption cross section (cm(^2)),</td>
</tr>
<tr>
<td>37-48</td>
<td>ZLAMX</td>
<td>( \lambda_I ) iodine and xenon decay</td>
</tr>
<tr>
<td>49-60</td>
<td>GAM1</td>
<td>( \lambda_X ) constants (sec(^{-1})),</td>
</tr>
<tr>
<td>61-72</td>
<td>GAMX</td>
<td>( \gamma_I ) iodine and xenon yield</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \gamma_X ) fractions per fission,</td>
</tr>
</tbody>
</table>

Card 2:

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-12</td>
<td>( V_1 )</td>
<td>fast and thermal neutron</td>
</tr>
<tr>
<td>13-24</td>
<td>( V_2 )</td>
<td>velocities (cm/sec),</td>
</tr>
<tr>
<td>25-36</td>
<td>PWR</td>
<td>Total reactor power (MW-th).</td>
</tr>
</tbody>
</table>

C-I.6 Material Locations Cards (515)

These cards give the initial and final coordinate numbers in the X- or Y-directions for each material number. The overlay method is used, that is if the same volume is specified by two or more cards, the material number assigned by the latter is used. The final coordinate numbers must exceed the initial coordinate numbers.

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>J1</td>
<td>The initial X-coordinate number,</td>
</tr>
<tr>
<td>6-10</td>
<td>J2</td>
<td>The final X-coordinate number,</td>
</tr>
<tr>
<td>11-15</td>
<td>J3</td>
<td>The initial Y-coordinate number,</td>
</tr>
<tr>
<td>16-20</td>
<td>J4</td>
<td>The final Y-coordinate number,</td>
</tr>
<tr>
<td>21-25</td>
<td>J5</td>
<td>The material number, which occupies the volume bounded by the above</td>
</tr>
</tbody>
</table>
Note: The last of these cards must be followed by a blank card.

C-I.7 Material Property Cards (6E12.6)

The data consist of NMAT sets of pair cards.

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-12</td>
<td>DLP1</td>
<td>D1, fast and thermal neutron diffusion coefficients (cm),</td>
</tr>
<tr>
<td>13-24</td>
<td>DLP2</td>
<td>D2, fast and thermal neutron diffusion coefficients (cm),</td>
</tr>
<tr>
<td>25-36</td>
<td>SIGA1</td>
<td>( \Sigma_{a1} ), fast and thermal neutron macroscopic absorption</td>
</tr>
<tr>
<td>37-48</td>
<td>SIGA2</td>
<td>( \Sigma_{a2} ), cross-sections (cm(^{-1}))</td>
</tr>
<tr>
<td>49-60</td>
<td>SGR</td>
<td>( \Sigma_{Rem} ), macroscopic removal cross-section for fast neutron (cm(^{-1}))</td>
</tr>
<tr>
<td>61-72</td>
<td>SIGF</td>
<td>( \Sigma_{eff} ), macroscopic yield cross-section for thermal neutron (cm(^{-1}))</td>
</tr>
</tbody>
</table>

Card 2:

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-12</td>
<td>ALFA</td>
<td>( a_f ), power feedback coefficient,</td>
</tr>
<tr>
<td>13-24</td>
<td>HIFAC</td>
<td>N, flux-to-power conversion ratio (cf. POWDERPUFS-V).</td>
</tr>
</tbody>
</table>

C-I.8 Geometry Data Card for SPCON: (315)

This card is necessary only if NPROG = 2.

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>NSX</td>
<td>X-coordinate of the reactor center,</td>
</tr>
<tr>
<td>6-10</td>
<td>NSY</td>
<td>Y-coordinate of the reactor center,</td>
</tr>
<tr>
<td>11-15</td>
<td>NZC</td>
<td>Control index for deploying policy of controller pairs,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>{ = 1; center symmetry,</td>
</tr>
<tr>
<td></td>
<td></td>
<td># 1; Y-symmetry.</td>
</tr>
</tbody>
</table>

C-I.9 Geometry Card 1 for BKCON: (I5, 2F10.5)
This card is only needed when NPROB $\geq$ 2.

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>NODY</td>
<td>Number of nodes in Y-coordinates inside the controller domain, $Y$ in the controller domain; $Y_c$ in the controller domain.</td>
</tr>
<tr>
<td>6-10</td>
<td>DCX</td>
<td>$X$ in the controller domain.</td>
</tr>
<tr>
<td>11-15</td>
<td>DCY</td>
<td>$Y_c$ in the controller domain.</td>
</tr>
</tbody>
</table>

C-I.10  Geometry Card 2 for BKCON (16T5)

This card is only required when NPROB = 2.

The number of data must be equal to NQDY + 1.

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>NODX(1)</td>
<td>Number of nodes in X-coordinate at the first Y-node,</td>
</tr>
<tr>
<td>6-10</td>
<td>NODX(2)</td>
<td>Number of nodes in X-coordinates at the second Y-node,</td>
</tr>
</tbody>
</table>

C-I.11  Controller Specification Card (6E12,6)

The card is only required if NFROB = 1.

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-12</td>
<td>USIG2</td>
<td>$\Sigma_a^U$, macroscopic thermal absorption cross section of controller material (cm$^{-1}$),</td>
</tr>
<tr>
<td>13-24</td>
<td>VUNT</td>
<td>Unit volume occupied by controllers (cf. MULTICELL),</td>
</tr>
<tr>
<td>25-36</td>
<td>UFRC</td>
<td>Fraction of controller volume required for keeping reference reactor critical, ($\geq 0.5$),</td>
</tr>
<tr>
<td>37-48</td>
<td>TILT</td>
<td>Maximum fractional power error expected to be able to be controlled by controllers,</td>
</tr>
<tr>
<td>49-60</td>
<td>SIGMA</td>
<td>Standard deviation of discharged bundle distribution on k coordinates,</td>
</tr>
</tbody>
</table>

C-I.12  State Vector Scale Factor; QFACO (E12.6)
This data is only necessary if \( \text{NPORB} = 1 \). The recommended value of \( \text{QFACO} \) is \( 2.0 \times 10^{-2} \), (cf. Eq. IV.1).

C-I.13 Controller Deployment Data Cards

The card set is required only if \( \text{NPORB} = 1 \).

Card Format (15); The data \( \text{LCON} \) is the number of controllers whose geometrical properties will be provided in following Card 2.

Card 2 Format (4(2F9, 4, I2))

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-9</td>
<td>( \text{XL}(1) )</td>
<td>X-coordinates of the controller number = 1,</td>
</tr>
<tr>
<td>10-18</td>
<td>( \text{YL}(1) )</td>
<td>Y-coordinates of the controller number = 1,</td>
</tr>
<tr>
<td>19-20</td>
<td>( \text{NSIGN}(1) )</td>
<td>Symmetricity index</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( = 2 ), if another controller will be located by Y-symmetric point,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( = 1 ), otherwise.</td>
</tr>
<tr>
<td>21-29</td>
<td>( \text{XL}(2) )</td>
<td>As same as before, but for the controller number</td>
</tr>
<tr>
<td>30-36</td>
<td>( \text{YL}(2) )</td>
<td></td>
</tr>
<tr>
<td>39-40</td>
<td>( \text{NSIGN}(2) )</td>
<td>( = \text{NSIGN}(1) + 1 )</td>
</tr>
</tbody>
</table>

Note: Thus, the actual total number of controllers should be equal to

\[
\text{MCON} = \sum_{i=1}^{\text{LCON}} \text{NSIGN}(i)
\]

C-I.14 Controller Size Data Card (8F10.5).

These data are necessary only if \( \text{NPORB} = 1 \). Number of data should be equal to \( \text{MCON} \).

<table>
<thead>
<tr>
<th>Column</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
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<tr>
<td>1-10</td>
<td>( \text{UWOL}(1) )</td>
<td>Multiple of ( \text{VWNT} ) for the controller number = 1.</td>
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</tbody>
</table>
11-20.  UVO(2)  Multiple of UNT for the controller number = 2.

C-I.15  Control Data for SIMUL (8E10.4)

This card should be appeared only if NPROB = 3.

<table>
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<th>Description</th>
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<tr>
<td>1-10</td>
<td>TMAX</td>
<td>Maximum time for transient calculation (hrs.)</td>
</tr>
<tr>
<td>11-20</td>
<td>PLVL</td>
<td>Initial power level (%)</td>
</tr>
<tr>
<td>21-30</td>
<td>PRAT</td>
<td>Maximum allowable power rate (%/sec)</td>
</tr>
<tr>
<td>31-40</td>
<td>DTL</td>
<td>Δt, time increment for transient calculation (hrs)</td>
</tr>
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</table>

C-I.16  Initial Mode Amplitude Excitation Data (8E10.4)

The card is required only if NPROB = 3.

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<tr>
<td>1-10</td>
<td>AMP(2,2)</td>
<td>Fractional increment of the thermal flux mode 2</td>
</tr>
<tr>
<td>11-20</td>
<td>AMP(2,3)</td>
<td>Fractional increment of the thermal flux mode 3</td>
</tr>
</tbody>
</table>

Note: The amplitude increment of the fundamental mode should be excluded in the data. The initial power level was assumed to contribute directly to the fundamental mode state. Hence, total number of data should be equal to NMOD-1.

C-II.  INPUT TAPES

Tape 1 is commonly called in all types of calculations. The tape stores the numerical data of each λ-mode sequentially in a single record. The structure of information is as follows;
NDUM ; integer number indicating the mode number,
(A(I,J), I=1, NRX), J=1, NRY) ; fast mode flux.
(B(I,J), I=1, NRX), J=1, NRY) ; thermal mode flux.

Tape 2 should be generated during the dynamic range
determination calculation and should be attached for the simulation
of transient. The structure of information stored in TAPE 2 is free
format and its contents are;

MCON ; Total number of controllers,
XL, YL, NSIGN ; cf. Card C-I.13,
BMTX ; Controller matrix, B_2 in Eq. (IV.20a),
UB, UNIT, UVOL ; Static set-points, \sum_{a_2} \times VUNT, volume
of controllers,
GMTX ; Gain matrix, i.e., \mathbf{R}^{-1} \mathbf{B}_2^T \mathbf{P} in Eq. (IV.39).
| Table C-1: A Sample Input Data Set |

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| 0.0   | 0.2090351 | 7.6426E-04 | 4.0000E-03 | 7.39243E-03 | 4.6363E-03 | 0.0

TABLE C-1 Cont'd
D. SAMPLE OUTPUT

Following pages list the output obtained in a spatial control effectiveness calculation. The first 4 pages describe the reactor model and the reference state's distributions and, hence, should be shown on all three options of NPROB. The next 10 pages contain the stability information and, thus appear only when NSTABL = 1, except for the system matrix. The next page gives the iteration information during computing the pointwise Riccati equations. The last page gives the geometry of controller domain and the calculated effectiveness distribution.
### Material Map

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*Note: The image contains additional text and diagrams, but they are not clearly visible or legible in the provided image.*
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<th>TACSP</th>
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FLUX SCALE FACTOR = 0.444920E+10
K-EFF FOR REFERENCE STATE = 0.000000E+00
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<th>System Matrix (2x2, 3x3)</th>
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<tr>
<td>1.121 - 0.0760E-03, 1.121E-03, 2.124E-03, 2.124E-03, 1.121E-03, 1.121E-03</td>
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**Matrix Details:**
- The table above represents a system matrix with entries for various systems, denoted by different labels (1.121, 1.212, etc.).
- Each entry is a complex number, indicating real and imaginary parts.

**Notes:**
- The matrix entries are likely related to system effectiveness computations in a CANDU 6C or similar reactor configuration.
### Controller Effectiveness Computation for Cancu F.C. (Y-Symmetric)

#### Under-Relief System Matrix \( A(14,14) \)

| 0.94936E-04 | 6.86694E-08 | 3.23776E-03 | 1.31239E-03 | 2.91398E-07 | 4.63135E-06 | 1.83685E-05 | 3.45734E+02 | 3.45734E+02 |
| 3.46434E-06 | 1.84936E+00 | 3.18567E+00 | 5.18456E+00 | 3.52591E+00 | 1.25239E+00 | 1.58106E+00 | 1.58106E+00 | 1.58106E+00 |
| 5.97634E+00 | 2.14571E+00 | 1.35341E+00 | 1.42255E+00 | 1.99456E+00 | 1.34130E+00 | 1.26125E+00 | 1.26125E+00 | 1.26125E+00 |

#### Controller Effectiveness Computation for Cancu F.C. (Y-Symmetric)

**Stability Information**

**Eigenvalues in the Form of Real \( \lambda_{real} \) + Imag \( \lambda_{imag} \)**

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## Transition Matrices for the Mode \( \pi = 1 \)

**In the Form of AFR**

**Matrix AFR (14 x 14)**

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| 3553718 |
### Transition Matrices for the MCE = 6

**In the Form of AFR-EXP(-1.57765E-003)**

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### LLLA Effectiveness Computation FCG CANDU 1.2 (E-Symmetric)

**Page 29**

| Transition Matrices for the MCE = 6
<table>
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<td>Matrix AFR (16,16)</td>
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### Section Matrices for the Mode = 1
The form of \( \lambda_{126711-25-1} \) for \( (AFR, COS(\theta), 0.62355T = 0.01) \), \( \pm 1 \):

<table>
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<th>Matrix A</th>
<th>(14 \times 14)</th>
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<tr>
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<td>0.24840 ( \pm 0 )</td>
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<tr>
<td>0.24840</td>
<td>0.24840 ( \pm 0 )</td>
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**Matrix A**

\[ \begin{bmatrix} \lambda_{126711-25-1} \end{bmatrix} \]
E. PROGRAM SOURCE LIST

/JOBT
/NONEQ
/OPT,B1200-0P277/CYBER,T77,1006. S.K. OH (EXT 462)
/SENDTO,TP.
/FTN.
/EXIT(S)
/EOH

**** BLOCK FOR DIFFUSION APPROXIMATION
**** QUADRATIC PERFORMANCE INDICES FOR SPACE-TIME DISTRIBUTED PARAMETER
**** SYSTEM

COMMON /STF1/ XF(31,23), XT(31,23), XI(31,23), XE(31,23)
COMMON /XCN/ DIF1(25), DIF2(25), SIGA1(25), SIGA2(25),
1 SIGR(25), SIGF(25), ALFA(25), HFAC(25),
2 XK(10), V1, V2, XNU,
3 GAM, GAMX, SIGXE, ZLAM,
4 ZLAMX, XKS
COMMON /GEM/ MAT(31,23), DX(31), DY(33), X(31), Y(23)
COMMON /HESS/ TITLE(30), NPAGE, NMOD, MM2, MM4,
1 NRX, NRY, NTIT, PWR, BLANK,
2 CX, CY, NSX, NSY, INSTB, BFAC, SFAC,
3 OFAC, AMP1
COMMON /SYSN/ SMX11(20,20), SMX12(20,20), SMX21(21,21),
1 SMX22(20,20), SMX(20,20)
COMMON /SYM/ FF(31,23,8), PS(31,23,8), "FNORM(B), TNORM(B)
DIMENSION IR(2), JJJ(31), WORK(20), WR(20), WI(20),
1 TRMX(20,20), TNRX(20,20), CMNX(20,20)

READ (5,1000) TITLE
1000 FORMAT(19A4)
NPAGE=0
CALL HEA
READ (5,1001) NMAT, NRX, NRY, NMOD, NSTAB, NPROB
1001 FORMAT(16IS)
WRITE (6,1050) NMAT, NRX, NRY, NMOD, NSTAB, NPROB
1050 FORMAT(*5X,'NO OF MATERIAL PROPERTIES = ',IS,'/5X,'NO OF MESH PO-
1 NTS IN X-AXIS = ',IS,'/5X,'NO OF MESH POINTS IN Y-AXIS = ',IS,'/5X,
2X,'NO OF MODES = ',IS,'/5X,'STABILITY CHECK (YES = 1) = ',IS,'/5X,
3 X,'PROBLEM TYPE (=1; BULK CONTROLLER, =2; SPATIAL CONTROLLER, =3;
4 SIMULATION) = ',IS,'/5X,
M4=4*NMOD-1)
M2=2*NMOD
M22=2*(NMOD-1)
DO 10 I=1,M2

C
WR(I) = 0.0
WI(I) = 0.0
WORK(I) = 0.0
TNRX(I) = 0.0

DO 10 J=1,N2
TRMX(I,J) = 0.0
TNRX(I,J) = 0.0
10 CONTINUE
READ (5,1002) (XK(I),I=1,NMOD)

1002 FORMAT (8F10.0)
IH(1) = IHX
IB(2) = IHY
KXS = XK(1)
NRX1 = NFX - 1
NRY1 = NRY - 1
III = 0
20 CONTINUE
III = III + 1
IF (III-2) 21,27,28
21 WRITE (6,1003) II(III)

1003 FORMAT (/H*6X* MESH SPACING (*A1,*DIRECTION*)*,/)
22 READ (5,1004) XYZ, N1,N2, NJJ,J(JJJ(J),J=1,NJJ)
IF (XYZ.EQ.0.0) GO TO 20
WRITE (6,1005) XYZ, (JJJ(J),J=1,NJJ)

1004 FORMAT (F10.5,3412)
1005 FORMAT (1X,F10.5,30(1X,I2))
IF (NJJ.NE.0) GO TO 23
N1 = 1
N2 = NJJ
23 DO 26 I=N1,N2
I,K = I
IF (NJJ.NE.0) IJK = JJJ(I)
IF (III-1) 24,24,25
24 DX(IJK) = XYZ
GO TO 26
25 DY(IJK) = XYZ
26 CONTINUE
GO TO 22
27 WRITE (6,1006) DX
GO TO 21
28 WRITE (6,1006) DY
X(1) = 0.5*DX(1)
Y(1) = 0.5*DY(1)
DO 35 I=2,NRX
35 X(I) = X(I-1) + 0.5*(DX(I-1) + DX(I))
DO 36 J=2,NRY
36 Y(J) = Y(J-1) + 0.5*(DY(J-1) + DY(J))
CX = X(NRX) + X(1))/2.0
CY = Y(NRY) + Y(1))/2.0
CX = X(1) + X(NRX))/2.0
CY = Y(1) + Y(NRY))/2.0
C READ IN IODINE/XENON PARAMETERS
READ (5,1007) XNU,SIGXE,ZLAM1,ZLAMX,GAMI,GAMX
READ (5,1007) V1,V2,PWR
1006 FORMAT(1X,'(1X,12F10.5))
1007 FORMAT(6E12.6)
  30 READ(5,1001) J1,J2,J3,J4,J5
   DO 31 I=J1,J2
   DO 31 J=J3,J4
31   MAT(I,J)=J5
   IF (J1.NE.0) GO TO 30
   CALL HEAD
   WRITE (6,1009)
1009 FORMAT(1X,'**** MATERIAL MAP ****',/)
   DO 32 J=1,NRY
   WRITE (6,1008) (MAT(I,J),I=1,NRX)
1008 FORMAT(9X,35I3)
32   CONTINUE
C  READ-IN MATERIAL PROPERTIES
   DO 33 I=1,NMAT
   READ (5,1007) DIF1(I),DIF2(I),SIGA1(I),SIGA2(I),SIGR(I),SIGF(I)
   READ (5,1007) ALFA(I),HFAC(I)
33   CONTINUE
   CALL HEAD
   WRITE (6,1010)
1010 FORMAT(1X,'REACTOR STRUCTURES')
   WRITE (6,1011) PWR
1011 FORMAT(10X,'REACTION POWER = '1.12.6,' (MW-TH)')
   WRITE (6,1012)
1012 FORMAT(1X,'NEUTRON KINETICS DATA',/,'13X,'V1
   WRITE (6,1014) V1, V2
   WRITE (6,1013)
1013 FORMAT('IODINE/XENON PARAMETERS',/,'11X,'XNU
   WRITE (6,1014) XNU, SIGE
   WRITE (6,1015)
1014 FORMAT(1X,’MATERIAL PROPERTIES’S/’ ID,’ DIF1 — DIF2
   WRITE (6,1016) ID, DIF1, DIF2
1015 FORMAT(1X,’1SIGA1 ’ SIGA2 ’ SIGR ’ SIGF ’ ALFA ’ HFAC ’
   DO 37 I=1,NMAT
   SIGF(I)=SIGF(I)/XNU
   HFAC(I)=HFAC(I)*1.16
   IF (SIGF(I).LE.0.0) GO TO 37
   SIGA2(I)=SIGA2(I)-1.20E-04
37   CONTINUE
   DO 34 I=1,NMAT
   WRITE (6,1016) I,DIF1(I),DIF2(I),SIGA1(I),SIGA2(I),SIGR(I),SIGF(I)
1016 FORMAT(13,1X,’ALFA(I),HFAC(I) ’)
34   CONTINUE
C  CALCULATE FLUX,IODINE,XENON DISTRIBUTION AT REFERENCE STATE
   CALL REFS
   CALL HEAD
   WRITE (6,1026)
1026 FORMAT(10X,’FAST FLUX DISTRIBUTION’)
   CALL XMAP(1)
   CALL HEAD
WRITE (6,1017)
1017 FORMAT(10X,'THERMAL FLUX DISTRIBUTION')
   CALL XMAP(XT)
   CALL HEAD
   WRITE (6,1018)
1018 FORMAT(10X,'IODINE DISTRIBUTION')
   CALL XMAP(XI)
   CALL HEAD
   WRITE (6,1019)
1019 FORMAT(10X,'XENON DISTRIBUTION')
   CALL XMAP(XE)
   CALL HEAD
   WRITE (6,1020)
1020 FORMAT(10X,'MODE INFORMATION'/)
   WRITE (6,1021)
1021 FORMAT(10X,'MOD NO . K-EFF . FNORM . TNORM'/)
   DO 40 I=1,NMOD
      WRITE (6,1022) I,XK(I),FNORM(I),TNORM(I)
40 CONTINUE
   WRITE (6,1023) SFAC,XKS
1023 FORMAT(/,10X,'FLUX SCALE FACTOR = ',E12.6,/
   1 10X,'K-EFF FOR REFERENCE STATE = ',E12.6)
   IF (NSTABL.NE.1 .AND. NPROB.NE.2) GO TO 70
C COMPUTE SYSTEM MATRIX EIGEN-STATES
   CALL SMTRX(2)
C INVERSE OF SUBMATRIX (THE FIRST QUARTER OF SYSTEM MATRIX)
   CALL GINV(SMX11,TRMX,WR,WI,20,MM2,MM2,ERROR)
C TRANSFORMATION OF FLUX VECTORS INTO IODINE/XENON VECTORS
   DO 50 L=1,MM2
      DO 60 N=1,MM2
         SMXX(L,N)=DOT4(21,20,1,MM2,SMX21,SMX11,L,N)
      60 CONTINUE
   50 CONTINUE
   DO 60 L=1,MM2
      DO 60 N=1,MM2
         SMXX(L,N)=SMXX(L,N)-DOT3(20,20,1,MM2,SMXN,SMX12,L,N)
      60 CONTINUE
C CALL HEAD
   WRITE (6,1024) MM2,MM2
1024 FORMAT(/,30X,'ORDER-REDUCED SYSTEM MATRIX ('',12''','','12'')'/)
   DO 61 L=1,MM2
      WRITE (6,1045) (SMX22(L,N),N=1,MM2)
61 CONTINUE
C XENON STABILITY CHECK
   CALL HEAD
   WRITE (6,1040)
1040 FORMAT(/,30X,'STABILITY INFORMATION'/)
   CALL EIGVAL(20,MM2,CSMX,TRMX,SMX22,WR, WI, TNRX, TRUE, TRMX)
   CALL EIGVEC(20,MM2,CSMX,TRMX,WR,WI,TNRY,TRMX)
   DO 65 I=1,MM2
      IF (WI(I).EQ.0.0) GO TO 62
         WORK(I)=1.7453293E-03/ABS(WI(I))
         IF (WR(I).LT.ALPHA) GO TO 65
62 CONTINUE
   CONTINUE
WRITE (6,1041)
WRITE (6,1044) (WR(I),WI(I),WORK(I),I=1,MM2)
1041 FORMAT(3X,'EIGENVALUES IN THE FORM OF REAL(LAMDA)+IMAG(LAMDA)',
1/13X,'REAL PART IMAG PART PERIOD(HR)'),/
1044 FORMAT(10X,3E12.5)
CALL HEAD
WRITE (6,1042)
1042 FORMAT(//,30X,'EIGENVECTORS(NORMALIZED BY THE NORM OF SET)'),/
130X,'FOR COMPLEX VECTORS - AT (I)-TH LINE = REAL; AT (I+1)-TH',
2/30X,'LINE = IMAG','/',
DO 66 J=1,MM2
WRITE (6,1045) (TRMX(I,J),I=1,MM2)
1045 FORMAT(10E12.6)
66 CONTINUE
J=0
DO 67 I=1,MM2
IF (WI(I).EQ.0.0) GO TO 67
IF (I.EQ.J) GO TO 67
J=I+1
IF (J.GT/MM2) GO TO 67
TEMP=WI(I)
WI(I)=WI(J)
WI(J)=TEMP
67 CONTINUE
DO 68 I=1,MM2
IN=MM2-I+1
IF (IN.EQ.1) GO TO 69
IN1=IN/2
IN2=(IN-1)/2
IF (WI(IN).NE.0.0.AND.IN1.NE.IN2) GO TO 68
69 CONTINUE
CALL DOTMX(20,MM2,IN,SMX22,WR,WH)
68 CONTINUE
70 CONTINUE
IF (NPROB.EQ.0) GO TO 90
IF (NPROB-2) 75,80,85
75 CALL BKCON(NPROB)
GO TO 90
80 CALL SPCON
GO TO 90
85 CALL SIMUL(NPROB),
90 CONTINUE
STOP
END
SUBROUTINE HEAD
COMMON /MISC/ T(18), NP
IF (NP) 1,1,2
1 CALL SECOND(A1)
CALL DATE(NOW)
2 CALL SECOND(A2)
A2=A2-A1
NP=NP+1
WRITE (6,1000) T,NP,NOW,A2
SUBROUTINE REFST

C**** CALCULATION OF SYSTEM FUNCTIONS AT REFERENCE STATE
C**** USING THE FUNDAMENTAL MODES OF NEUTRON FLUXES AS THE
C**** DISTRIBUTIONS AT THE STATE.

COMMON /STFN/ XF(31,23), XT(31,23), XI(31,23), XE(31,23)
COMMON /GEOM/ MAT(31,23), DX(31), DY(23), X(31), Y(23)
COMMON /XSCN/ DIF1(25), DIF2(25), SIGA1(25), SIGA2(25),
  SIGR(25), SIGF(25), ALFA(25), HFAC(25),
  V1, V2, XNU,
COMMON /MISC/ TITLE(19), NPAGE, NMOD, M2, M4,
  NRX, NRY, NTTR, NMRX, NMY, NMRY, NMRZ,
2   DUM(6), SFAC, DUM1, AMPO
COMMON /XYMOD/ FF(31,23,8), PS(31,23,8), FNORM(8), TNORM(8)
DIMENSION A(31,23), B(31,23)

DO 10 MOD=1,NMOD
  READ (1,1000) NDUM
  READ (1,1000) ((A(I,J),I=1,31),J=1,23)
  READ (1,1000) ((B(I,J),I=1,31),J=1,23)
1000 FORMAT(8E10.4)
1001 FORMAT(I5)
  SUM1=SUM2=0.0
  IF (MOD,NE,1) GO TO 13
  NSUM=TSUM=0.0
13 CONTINUE
  FNORM(MOD)=0.0
  TNORM(MOD)=0.0
C NORMALIZATION FACTOR FOR MODES
  DO 11 I=1,NRX
    DO 11 J=1,NRY
      IF (MOD,NE,1) GO TO 21
      XF(I,J)=A(I,J)
      XT(I,J)=B(I,J)
11 CONTINUE
  FF(I,J,MOD)=0.0
  PS(I,J,MOD)=0.0
  DU=DX(I)*DY(J)
  IDX=MAT(I,J)
  SUM2=SUM2+B(I,J)*SIGR(IDX)*A(I,J)*DV
  SUM1=SUM1+A(I,J)*XNU*SIGF(IDX)*B(I,J)*DV
  IF (MOD,NE,1) GO TO 11
  SUM=SUM+D(I,J)*HFAC(J)*DV
  TSUM=TSUM+D(I,J)*DV
11 CONTINUE
  SUM1=SQRT(SUM1)
  DO 12 I=1,NRX
    DO 12 J=1,NRY
      }
IDX=MAT(I,J)
PF(I,J,MOD)=A(I,J)*SQRT(SIGR(IDX))/SUM1
FS(I,J,MOD)=B(I,J)*SQRT(XNU*SIGF(IDX))/SUM1

12 CONTINUE
F NORM(MOD)=1.0/SUM1
TNORM(MOD)=SUM1/SUM2.

10 CONTINUE
C FLUX SCALE UP TO FULL POWER LEVEL
SFAC=6.804422E+15*PWR/SUM
RATI=GM1/3AM1
RAT2=GM1*GMX
SUM=SUM0=SUMF=0.0
DO 20 I=1,NRX
DO 10 J=1,NRY
          DX(I,J)=DX(I,J)*IY(J)
          XX=SIGF(MAT(I,J))
          XF(I,J)=XF(I,J)*SFAC
          XT(I,J)=XT(I,J)*SFAC
          SUM=SUM+XT(I,J)*DIV
          Y1=XX*XT(I,J)
          XI(I,J)=RATI*YY
          XE(I,J)=XE(I,J)/(SIGX*XT(I,J)+ZLAMX)
          SUMP=SUMP+PS(I,J,1)*PS(I,J,1)*DIV
          SUMD=SUMD+PS(I,J,1)*PS(I,J,1)*PS(I,J,1)*DIV
      20 CONTINUE
AMPO=SUM/TSUM
RETURN
END

SUBROUTINE XMAP(AA)
C*** SHOW STATE FUNCTION MAP.
C
COMMON /GEOM/ MAT(31,23), IX(31), IY(23), X(31), Y(23)
COMMON /MISC/ TITLE(18), NPAGE, NMOD, M2, M4,
              MRR, NRY, NRT, FWR, BLANK
DIMENSION AA(31,23), IA(31,23)
AAAMAX=0.0
DO 10 I=1,NRX
DO 10 J=1,NRY
          IF (AA(I,J).GE.AAAMAX) AAAMAX=AA(I,J)
      10 CONTINUE
AMAX=AAAMAX/10000.0
BEX=ALOG10(AMAX)+0.9999
INDEX=IFIX(BEX)
MULT=10**INDEX
DO 20 I=1,NRX
DO 20 J=1,NRY
    IA(I,J)=AA(I,J)/MULT
      20 CONTINUE
WRITE(6,1000) INDEX
1000 FORMAT(10X,'MULTIPLIED BY E',I5,'
WRITE(6,1001) (Y(J),J=1,NRY)
1001 FORMAT(5X,'Y',20F5.1)
1004 FORMAT(5X, 'X = ') DO 50 II=1,NRX
I=NRX-II+1 WRITE (6,1002) X(I),(IA(I,J),J=1,NRX)
1002 FORMAT(5X,FS,1,23IS)
50 CONTINUE WRITE (6,1003) AAMAX
1003 FORMAT(10X,'MAXIMUM VALUE = ',E12.6) RETURN

SUBROUTINE SMTRX(NPROB)

C**** FORMULATION OF SYSTEM MATRIX
C**** DIMENSION OF MATRIX IS (4*MOD)*(4*MOD)

COMMON /STFN/ XF(31,23), XT(31,23), XI(31,23), XE(31,23)
COMMON /GRED/ MAT(31,23), DX(31), DY(23), X(31), Y(23)
COMMON /XSCN/ DFI1(25), DFI2(25), SIGA1(25), SIGA2(25),
1 SIGR(25), SIGF(25), ALFA(25), HFA(25),
2 X(10), V1, v2, XNU,
3 GAM, GAMX, SIGX, SIGE, ZLAM,
4 ZLAMX, XKS
COMMON /HSCH/ TITLE(13), NFAGE, NMOD, M2, M4,
1 NRX, NRY, NIT, FWK, BLANK
COMMON /SYSM/ SMX1(20,20), SMX2(20,20), SMX3(21,21),
1 SMX22(20,20), SMX20(20,20)
COMMON /XYMO/ FF(31,23,8), FS(31,23,8), FNORM(8), TNORM(8)

IF (NPROB.EQ.1) MMX=MMOD
IF (NPROB.EQ.2) MMX=2*MMOD-1
IF (NPROB.EQ.3) MMX=2*MMOD

DO 10 L=1,MMX
DO 10 N=1,MMX
SMX1(L,N)=0.0
SMX12(L,N)=0.0
SMX13(L,N)=0.0
SMX15(L,N)=0.0
5AX(L,N)=0.0
10 CONTINUE

V0=V2/V1
NFRST=1
IF (NPROB.EQ.2) NFRST=NPROB
N=SMN-NFRST+1
DO 20 M=NFRST,MMOD
L1=M-NFRST+1
L2=PLAST4L1
TLN=TNORM(M)/FNORM(M)
DO 23 N=NFRST,MMOD,
1 N=SMN-NFRST+1
N2=SMN1
20=SMN1
DO 24 I=1,NRX
DO 24 J=1,NRY
IDX=MAT(I,J)
IF (SIGF(IDX).EQ.0.0.OR.SIGR(IDX).EQ.0.0) GO TO 24
DV=DX(I)*DY(J)
ALN=1.0/SORT(SIGR(IDX))
BLN=1.0+ALFA(IDX)
CLN=SIGXE*XT(I,J)
DLN=BLN*SIGF(IDX)
ENL=SIGXE*XE(I,J)
SMX11(L1,N1)=SMX11(L1,N1)-AB1*PF(I,J,M)*PS(I,J,N)*DV/XK(N)
SMX11(L1,N2)=SMX11(L1,N2)+AB1*PF(I,J,M)*BLN*PS(I,J,N)*DV/XKS
SMX11(L2,N1)=SMX11(L2,N1)+V*AR2*PS(I,J,M)*TLN*FF(I,J,N)*DV
SMX11(L2,N2)=SMX11(L2,N2)-SMX11(L2,N1)
SMX12(L2,N2)=SMX12(L2,N2)-AB2*PS(I,J,M)*TLN*ALN*CLN*PS(I,J,N)*

1

V*DV
SMX21(L1,N2)=SMX21(L1,N2)-AB1*PF(I,J,M)*GAMI*DLN*PS(I,J,N)*DV
SMX22(L1,N1)=SMX22(L1,N1)-AB1*PF(I,J,M)*ZLAM*PS(I,J,N)*DV
SMX21(L2,N2)=SMX21(L2,N2)+AB1*PF(I,J,M)*(GAMX*DLN-ENL)*PS(I,J,N)*

1

DV
SMX22(L2,N1)=SMX22(L2,N1)
SMX22(L2,N2)=SMX22(L2,N2)-AB1*PF(I,J,M)*(CLN+ZLAM)*PS(I,J,N)*DV
GO TO 24

26 CONTINUE
ALN=SOR(T(XNU*SIGF(IDX))*SIGR(IDX))
BLN=ALN*(1.0+ALFA(IDX))
SMX11(L1,N1)=SMX11(L1,N1)-TLN*AB2*PS(I,J,M)*ALN*PS(I,J,N)*DV

1

XK(N)
SMX12(L1,N1)=SMX12(L1,N1)+TLN*AB2*PS(I,J,M)*BLN*PS(I,J,N)*DV
SMX21(L1,N1)=SMX21(L1,N1)-AB1*PF(I,J,M)*ALN*PF(I,J,N)*DV
SMX22(L1,N1)=SMX22(L1,N1)
SMX21(L1,N1)=SMX21(L1,N1)+TLN*AB2*PS(I,J,M)*XT(I,J)

1

PS(I,J,N)*ALN*DV
GO TO 24

27 CONTINUE
ALN=SOR(T(XNU*SIGF(IDX))
BLN=SOR(T(SIGR(IDX))
CLN=(1.0+ALFA(IDX))
DLN=SIGXE*XT(I,J)
ENL=SIGXE*XE(I,J)
FN=SIGF(IDX)
SMX11(L1,N1)=SMX11(L1,N1)-TLN*AB2*PS(I,J,M)*ALN*BLN

1

PS(I,J,N)*DV/XK(N)
SMX11(L1,N2)=SMX11(L1,N2)+TLN*AB2*PS(I,J,M)*ALN*BLN*CLN*

1

PS(I,J,N)*DV
SMX12(L2,N2)=SMX12(L2,N2)-TLN*AB2*PS(I,J,M)*BLN*DV

1

PS(I,J,N)*DV
SMX11(L2,N1)=SMX11(L2,N1)-AB1*PF(I,J,M)*ALN*BLN*PF(I,J,N)*DV
SMX11(L2,N2)=SMX11(L2,N2)-SMX21(L2,N1)
SMX21(L1,N2)=SMX21(L1,N2)+AB1*PF(I,J,M)*GAMI*CLN*FLN*

1

PS(I,J,N)*DV
SMX21(L2,N2)=SMX21(L2,N2)-AB1*PF(I,J,M)*(GAMX*CLN*FLN-ENL)*

1

PS(I,J,N)*DV
SMX22(L1,N1)=SMX22(L1,N1)-AB1*PF(I,J,M)*ZLAM*PS(I,J,N)*DV
20 U(I,1)=U(I,1)*FAC
   AFLAG(1)=1.0
   N=27
   TOL=(10.0*0.5**N)**2
   DO 100 J=2,NC
   DOTT=DOT(MR,1,MR,A,J,J)
   JM1=J-1
   DO 50 L=1,2
   DO 30 K=1,JM1
30 ATEMP(K)=DOT(MR,1,NR,A,J,K)
   DO 45 K=1,JM1
   DO 35 I=1,NR
35 A(I,J)=A(I,J)-ATEMP(K)*A(I,K)*AFLAG(K)
   DO 40 I=1,NC
40 U(I,J)=U(I,J)-ATEMP(K)*U(I,K)
   CONTINUE
50 CONTINUE
   DOTN=DOT(MR,1,NR,A,J,J)
   IF ((DOTN/DOTT)-TOL) 55,55,70
55 DO 60 I=1,JM1
60 ATEMP(I)=DOT(NC,1,I,U,I,J)
   DO 65 I=1,NR
   A(I,J)=0.0
   DO 65 K=1,JM1
65 A(I,J)=A(I,J)-ATEMP(K)*AFLAG(K)
   AFLAG(J)=0.0
   FAC=DOT(NC,1,NC,U,J,J)
   FAC=1.0/SORT(FAC)
   GO TO 75
70 AFLAG(J)=1.0
   FAC=1.0/SORT(DOTN)
75 DO 80 I=1,NR
80 A(I,J)=A(I,J)*FAC
   DO 85 I=1,NC
85 U(I,J)=U(I,J)*FAC
100 CONTINUE
   DO 130 J=1,NC
   DO 130 I=1,NR
   A(I,J)=DOT4(MR,NC,J,NC,A,U,I,J)
130 CONTINUE
   DO 94 I=1,NC
   DO 94 J=1,NC
   U(I,J)=DOT4(MR,20,1,NC,A,B,I,J)
94 CONTINUE
   ERROR=0.0
   DO 95 I=1,NC
   DO 95 J=1,NC
   IF (I-J) 97,96,97
96 ERROR=ABS(U(I,J)-1.0)
   GO TO 98
97 ERROR=ABS(U(I,J))
98 ERROR=A(MAX1(ERROR,ERR)
95 CONTINUE
RETURN
END
SUBROUTINE GINV(A,U,AFLAG,ATEMP,MR,NR,NC,ERROR)

C**** CALCULATES THE GENERALIZED INVERSE OF MATRIX A AND STORE THE
C**** TRANSPPOSE OF U IN A

C**** REF - P381-P387, VOL.9, NO.5, MAY(1966) OF COMMUNICATIONS OF ACM

DIMENSION A(MR,NC), U(NC,NC), AFLAG(NC), ATEMP(NC)
DIMENSION B(20,20)

1 DO 1 I=1,MR
2 DO 1 J=1,NC
3 B(I,J)=A(I,J)
1 CONTINUE

5 U(I,J)=0.0
10 U(I,I)=1.0

FAC=DOT(MR,1,NK,A,1,1)
FAC=1.0/SORT(FAC)
15 A(I,1)=A(I,1)*FAC
DO 20 I=1,NC
FUNCTION CNTRLB(A,B,NN,N,N1,N2)
C**** CHECK THE CONTROLLABILITY OF SYSTEM WITH A GIVEN CONTROLLER
C**** SYSTEM IS CONTROLLABLE - CNTRLB=1.0
C
DIMENSION A(NN,N), W(14,28), WORK(14,28), U(28,28), B(NN,N1),
1 AF(28), AT(28), AB(14,14)
C
DO 5 I=1,N
DO 5 J=1,N
AB(I,J)=A(I,J)
5 CONTINUE
DO 10 I=1,N
DO 10 K=1,N
WORK(I,K)=B(I,K)
10 AA(I,K)=WORK(I,K)
DO 11 J=2,N
DO 11 L=1,N
J1=2*(J-2)+L
J2=2*(J-1)+L
DO 12 K=1,N
AA(K,J2)=DOT3(14,14,1,N,AB,AA,K,J1)
12 WORK(K,J2)=AA(K,J2)
11 CONTINUE
11 CONTINUE
CALL GINV(AA,U,AF,AT,14,N,N2,ERROR)
CNTRLB=0.0
IF (ERROR.LT.1.0E-5) CNTRLB=1.0
RETURN
END

SUBROUTINE EIGUAL(NM,N,A,B,CA,ALFR,ALFI,BETA,MATZ,Z)
C**** EIGEN VALUE AND EIGEN VECTOR COMPUTATION FOR GENERAL MATRIX
C**** REF - MATRIX EIGENSYSTEM Routines, EISPACK, B.S.GARROW ET AL.
C
DIMENSION A(NM,N), B(NM,N), Z(NM,N), ALFR(N), ALFI(N),
1 BETA(N), CA(NM,N)
INTEGER EN
LOGICAL MATZ
C
DO 5 I=1,NM
DO 5 J=1,NM
A(I,J)=CA(I,J)
5 CONTINUE
DO 6 I=1,N
DO 6 J=1,N
B(I,J)=0.0
6 CONTINUE
B(I,I)=1.0
6 CONTINUE
CALL SCHUR(NM,NM,N,A,B,MATZ,Z,IERR)
IF (IERR.EQ.0) GO TO 10
WRITE (6,1000)
1000 FORMAT(//,10X,**ERROR STOP - FAIL TO GET THE SCHUR FORM**)
STOP

10 EPSB=B(N,1)

C COMPUTE EIGENVALUES OF QUASI-TRIANGULAR MATRIX

ISW=1
DO 90 NN=1,N
EN=N+1-NN
NA=EN-1
IF (ISW.EQ.2) GO TO 87
IF (EN.EQ.1) GO TO 62
IF (A(EN+NA),NE,0.0) GO TO 63
62 ALFR(EN)=A(EN,EN)
IF (B(EN,EN),LT,0.0) ALFR(EN)=-ALFR(EN)
BETA(EN)=ABS(B(EN,EN))
ALFI(EN)=0.0
GO TO 90
63 IF (ABS(B(NA,NA)),LE,EPSB) GO TO 72
IF (ABS(B(EN,EN)),GT,EPSB) GO TO 64
A1=A(EN,EN)
A2=A(EN+NA)
BN=0.0
GO TO 67
64 AN=ABS(A(NA,NA))+ABS(A(NA,EN))+ABS(A(EN,NA))+ABS(A(EN,EN))
BN=ABS(B(NA,NA))+ABS(B(NA,EN))+ABS(B(EN,EN))
A11=A(NA,NA)/AN
A12=A(NA,EN)/AN
A21=A(EN,NA)/AN
A22=A(EN,EN)/AN
B11=B(NA,NA)/BN
B12=B(NA,EN)/BN
B22=B(EN,EN)/BN
E=A11/B11
F1=A22/B22
S=A21/(B11*B22)
T=(A22-E*B22)/B22
IF (ABS(E),LE,ABS(EI)) GO TO 65
E=EI
T=(A11-E*B11)/B11
65 C=0.5*(T-S*B12)
D=C+C+S*(A12-E*B12)
IF (D,LT,0.0) GO TO 76
E=E+(C+SIGN(SQRT(D),C))
A11=A11-E*B11
A12=A12-E*B12
A22=A22-E*B22
IF (ABS(A11)+ABS(A12),LT,ABS(A21)+ABS(A22)) GO TO 66
A1=A12
A2=A11
GO TO 67
66 A1=A22
A2=A21
67 S=ABS(A1)+ABS(A2)
U1=A1/S
U2=A2/S
R=SIGN(SQRT(U1*U1+U2*U2)-U1)
V1 = -(U1 + R) / R
V2 = -U2 / R
U2 = V2 / V1
DO 68 I = 1, EN
   T = A(I, EN) + U2 * A(I, NA)
   A(I, EN) = A(I, EN) + T * V1
   A(I, NA) = A(I, NA) + T * V2
   T = B(I, EN) + U2 * B(I, NA)
   B(I, EN) = B(I, EN) + T * V1
   B(I, NA) = B(I, NA) + T * V2
68 CONTINUE
IF (.NOT. MATZ) GO TO 71
DO 69 I = 1, N
   T = Z(I, EN) + U2 * Z(I, NA)
   Z(I, EN) = Z(I, EN) + T * V1
   Z(I, NA) = Z(I, NA) + T * V2
69 CONTINUE
71 IF (BN .EQ. 0.0) GO TO 75
    IF (AN .LT. ABS(E) * BN) GO TO 72
   A1 = B(NA, NA)
   A2 = B(EN, NA)
   GO TO 73
72 A1 = A(NA, NA)
   A2 = A(EN, NA)
73 S = ABS(A1) + ABS(A2)
    IF (S .EQ. 0.0) GO TO 75
    U1 = A1 / S
   U2 = A2 / S
   R = SIGN(SORT(U1 * U1 + U2 * U2), U1)
V1 = -(U1 + R) / R
V2 = -U2 / R
U2 = V2 / V1
DO 74 J = NA, N
   T = A(NA, J) + U2 * A(EN, J)
   A(NA, J) = A(NA, J) + T * V1
   A(EN, J) = A(EN, J) + T * V2
   T = B(NA, J) + U2 * B(EN, J)
   B(NA, J) = B(NA, J) + T * V1
   B(EN, J) = B(EN, J) + T * V2
74 CONTINUE
75 A(EN, NA) = 0.0
   B(EN, NA) = 0.0
   ALFR(NA) = A(NA, NA)
   ALFR(EN) = A(EN, EN)
   IF (B(NA, NA) .LT. 0.0) ALFR(NA) = -ALFR(NA)
   IF (B(EN, EN) .LT. 0.0) ALFR(EN) = -ALFR(EN)
   BETA(NA) = ABS(B(NA, NA))
   BETA(EN) = ABS(B(EN, EN))
   ALFI(NA) = 0.0
   ALFI(EN) = 0.0
GO TO 87
76 E = E + C
   E1 = SORT(-D)
   A11R = A11 - E * B11
A11I = EI*B11
A12I = A12-EI*B12
A12I = EI*B12
A22I = A22-EI*B22
A22I = EI*B22
IF (ABS(A11R)+ABS(A11I)+ABS(A12R)+ABS(A12I)+ABS(A21)+ABS(A22)+ABS(A22I)) \n1 \nABS(A22I) \nGO TO 77
A1 = A12R
A11 = A12I
A2 = -A11R
A21 = -A11I
GO TO 78
77 A1 = A22R
A11 = A22I
A2 = -A21
A21 = 0.0
78 CZ = SQRT(A1*A1+A11*A11)
IF (CZ.EQ.0.0) GO TO 79
SZR = (A1*A2+A11*A21)/CZ
SZI = (A1*A21-A11*A2)/CZ
R = SQRT((CZ*CZ+SZR*SZR+SZI*SZI)
CZ = CZ/R
SZR = SZR/R
SZI = SZI/R
GO TO 80
79 SZR = 1.0
SZI = 0.0
80 IF (AN.LT.(ABS(E)+EI)*RN) GO TO 81
A1 = CZ*B11+SZR*B12
A11 = SZI*B12
A2 = SZR*B22
A21 = SZI*B22
GO TO 82
81 A1 = CZ*A11+SZR*A12
A11 = SZI*A12
A2 = CZ*A21+SZR*A22
A21 = SZI*A22
82 CO = SQRT(A1*A1+A11*A11)
IF (CO.EQ.0.0) GO TO 83
SOR = (A1*A2+A11*A21)/CO
SQR = (A1*A21-A11*A2)/CO
R = SQRT((CO*CO+SOR*SOR+SQR*SQR)
CO = CO/R
SOR = SQR/R
SQR = SQI/R
GO TO 84
83 SQR = 1.0
SQR = 0.0
84 SSR = SQR*SZR+SQI*SZI
SQR = SQR*SZ1-SQR*SZI
I = 1
TR = CO*CZ*A11+CO*SZR*A12+SQR*CZ*A21+SSR*A22
TI = CO*SZI*A12-SQR*CZ*A21+SQR*A22
DR = CO*CZ*B11+CO*SZR*B12+SSR*B22
DI=C0*SZI*B12+SSI*B22
GO TO 86

85 I=2
TR=SSR*A11-SQR*CZ*A12-C0*SZR*A21+C0*CZ*A22
TI=SSR*A11-SQ1*CZ*A12+C0*SZI*A21
DR=SSR*B11-SQR*CZ*B12+C0*CZ*B22
DI=SSR*B11-SQ1*CZ*B12

86 T=TI*DR-TR*DI
J=NA
IF (T.LT.0.0) J=EN
R=SQR(T*DR+DI*DI)
BETA(J)=BN*R
ALFR(J)=AN**(TR*DR+TI*DI)/R
ALFI(J)=AN**T/R
IF (I.EQ.1) GO TO 85

87 ISW=3-ISW
90 CONTINUE
RETURN
END

SUBROUTINE EIGVEC(NM,N,A,B,ALFR,ALFI,BETA,Z)
C**** COMPUTES EIGEN-VECTORS
C
DIMENSION A(NM,N), B(NM,N), ALFR(N), ALFI(N), BETA(N), Z(NM,N)
INTEGER EN,ENM2
C
EPSB=B(N,1)
ISW=1
DO 10 NN=1,N
EN=NN+1-NN
NA=EN-1
IF (ISW.EQ.2) GO TO 11
IF (ALFI(EQ).NE.0.0) GO TO 30
M=EN
B(EN+EN)=1.0
IF (NA.EQ.0) GO TO 10
ALFM=ALFR(M)
BETM=BETA(M)
DO 20 II=1,NA
I=EN-II
W=BETM*A(I,I)-ALFM*B(I,I)
R=0.0
DO 21 J=EN-1
R=R+(BETM**A(I,J)-ALFM*B(I,J))**2(J,EN)
IF (I.EQ.1.OR.ISW.EQ.2) GO TO 22
IF (BETM**A(I+1,EN).EQ.0.0) GO TO 22
ZZ=W
S=R
GO TO 25

21 J=J+1
20 II=II+1
22 I=I+1
IF (ISW.EQ.2) GO TO 23
T=W
IF (W.EQ.0.0) T=EPSB
B(I,EN)=:R/T
GO TO 20
23 X=BETM*A(I,I+1)-ALFM*B(I,I+1)
Y=BETM*A(I+1,I)
Q=W*ZZ-X*Y
T=(X*S-ZZ*R)/Q
B(I,EN)=T
IF (ABS(X).LE.ABS(ZZ)) GO TO 24
B(I+1,EN)=(-R-W*T)/X
GO TO 25
24 B(I+1,EN)=(-S-Y*T)/ZZ
25 ISW=3-ISW
20 CONTINUE
GO TO 10
30 M=NA
ALMR=ALFR(N)
ALMI=ALFI(M)
BETM=BETA(M)
Y=BETM*A(EN,NA)
R(NA,NA)=ALMI*B(EN,EN)/Y
B(NA,EN)=(ALMR*B(EN,EN)-BETM*A(EN,EN))/Y
R(EN,NA)=0.0
B(EN,EN)=1.0
ENM2=NA-1
IF (ENM2.EQ.0) GO TO 11
DO 40 II=1,ENM2
I=NA-II
W=BETM*A(I,I)-ALMR*B(I,I)
W1=-ALMI*B(I,I)
RA=0.0
SA=0.0
DO 41 J=H,EN
X=BETM*A(I,J)-ALMR*B(I,J)
X1=-ALMI*B(I,J)
RA=RA+X*B(J,NA)-X1*B(J,EN)
SA=SA+X*B(J,EN)+X1*B(J,NA)
41 CONTINUE
IF (I.EQ.1.OR.ISW.EQ.2) GO TO 42
IF (BETM*A(I,I-1).EQ.0.0) GO TO 42
ZZ=W
Z1=W1
R=RA
S=SA
ISW=2
GO TO 40
42 M=I
IF (ISW.EQ.2) GO TO 47
TR=-RA
TI=-SA
44 DR=W
DI=W1
45 IF (ABS(DI).GT.ABS(DR)) GO TO 46
RR=DI/DR
D=DR+DI*RR
T1=(TR+TI*RR)/D  
T2=(TI-TR*RR)/D  
IF (ISW-1) 50, 50, 48
46 RR=DR/DI  
D=DR*RR+DI  
T1=(TR*RR+TI)/D  
T2=(TI*RR-TR)/D  
IF (ISW-1) 50, 50, 48 -  
47 X=BEM*A(I,I+1)-ALMR*B(I,I+1)  
X1=-ALMI*B(I,I+1)  
Y=BEM*A(I+1,I)  
TR=Y*RA-W*RR+WI*SA  
TI=W*RA-W*RR+WI*SA  
DR=W*ZZ-ZI*X1-XY  
DI=W*ZZ-ZI*X1*Y  
IF (DR.EQ.0.0.AND.D1.EQ.0.0) DR=EPSB  
GO TO 45  
48 B(I+1,NA)=T1  
B(I+1,EN)=T2  
ISW=1  
IF (ABS(Y).GT.ABS(W)+ABS(W1)) GO TO 49  
TR=-RA-X*SB(I+1,EN)+X1*SB(I+1,NA)  
TI=-SA-X*SB(I+1,EN)-X1*SB(I+1,NA)  
GO TO 44  
49 T1=(-RA-ZZ*SB(I+1,NA)+Z1*SB(I+1,EN))/Y  
T2=(-RA-ZZ*SB(I+1,EN)-Z1*SB(I+1,NA))/Y  
50 B(I,NA)=T1  
B(I,EN)=T2  
40 CONTINUE  
11 ISW=3-ISW  
10 CONTINUE  
DO 60 JJ=1,N  
J=N+1-JJ  
60-61 I=1,N  
Z(I,J)=D03(NM,NM,1,J,Z,B,I,J)  
61 CONTINUE  
60 CONTINUE  
C NORMALIZATION TO NORM OF EIGEN-VECTOR  
DO 70 J=1,N  
IF (ISW.EQ.2) GO TO 73  
IF (ALFI(J,NE.O.O)) GO TO 76  
D=D0T(NM,1,N,Z,J,J)  
D=SORT(D)  
DO 72 I=1,N  
72 Z(I,J)=Z(I,J)/D  
GO TO 70  
73 D=D0T(NM,1,N,Z,J-1,J-1)+D0T(NM,1,N,Z,J,J)  
D=SORT(D)  
DO 75 I=1,N  
Z(I,J-1)=Z(I,J-1)/D  
Z(I,J)=Z(I,J)/D  
75 CONTINUE  
76 ISW=3-ISW  
70 CONTINUE  
RETURN  
END
SUBROUTINE COTMX(MR,NMAX,NS,A,WR,CI)
C**** CALCULATES THE COEFFICIENT MATRIX OF TRANSITION MATRIX
DIMENSION A(MR,NMAX), WR(MR), CI(MR)
COMPLEX CA(14,14), C1A(14,14), C2A(14,14), C1B(14,14)
COMMON AFR(20,20), AF1(20,20)
C
CALL HEAD
WRITE (6,1000) 'NS
1000 FORMAT(//,10X,'TRANSITION MATRICES FOR THE MODE =',
112,/) DO 10 L=1,NMAX
20 CW(L)=CMPLX(WR(L),WI(L)) DO 10 M=1,NMAX
30 CA(L,M)=CMPLX(A(L,M),0.0) 10 CONTINUE DO 11 M=1,NMAX
40 DENOM(M)=CW(NS)-CW(M) 11 CONTINUE L=NS DO 30 N=1,NMAX
50 IF (N-L) 31,30,32
31 IF (N-1) 35,35,36
32 IF (L-1) 33,33,34
33 IF (N-L-1) 35,35,36
34 IF (N-L-1) 35,40,40 DO 36 LL=1,NMAX
60 DO 36 MM=1,NMAX
70 CA1(LL,MM)=CA(LL,MM) 36 CONTINUE DO 37 LL=1,NMAX
80 CA1(LL,LL)=CA(LL,LL)-CW(N) DO 37 MM=1,NMAX
90 CA1(LL,MM)=CA1(LL,MM)/DENOM(N) 37 CONTINUE GO TO 30
100 CONTINUE DO 41 LL=1,NMAX
110 DO 41 MM=1,NMAX
120 CA2(LL,MM)=CA(LL,MM) 41 CONTINUE DO 42 LL=1,NMAX
130 CA2(LL,LL)=CA(LL,LL)-CW(N) DO 42 MM=1,NMAX
140 CA2(LL,MM)=CA2(LL,MM)/DENOM(N) 42 CONTINUE DO 43 LL=1,NMAX
150 DO 43 MM=1,NMAX
160 TCA(LL,MM)=(0.0,0.0) DO 43 NN=1,NMAX
170 TCA(LL,MM)=TCA(LL,MM)+CA1(LL,NN)*CA2(NN,MM) 43 CONTINUE DO 44 LL=1,NMAX

DO 44 MM=1,NMAX
CA1(LL,MM)=JCA1(LL,MM)
44 CONTINUE
STOP CONTINUE
IF (AIMAG(CW(L))) 50,60,50
50 CONTINUE
WRITE (6,1001) WR(L),W(L),WI(L)
1001 FORMAT(1X,"IN THE FORM OF EXP(\"*/1PE12.5\"*T)*AFR*COS(\"*/1PE12.5\"*T)\" + - I*AFI*SIGN(\"*/1PE12.5\"*T))")
DO 51 M=1,NMAX
DO 51 N=1,NMAX
AFR(M,N)=REAL(CA1(M,N))**2.0
AFI(M,N)=AIMAG(CA1(M,N))**2.0
51 CONTINUE
WRITE (6,1002) NMAX,NMAX
1002 FORMAT(1X,"MATRIX AFR (\"*/I2\"*,\"*/I2\"*))")
DO 52 M=1,NMAX
WRITE (6,1003) (AFR(M,N),N=1,NMAX)
52 CONTINUE
CALL HEAD
1003 FORMAT(5X,10E12.5)
WRITE (6,1004) NMAX,NMAX
1004 FORMAT(1X,"MATRIX AFI (\"*/I2\"*,\"*/I2\"*)")
DO 53 M=1,NMAX
WRITE (6,1003) (AFI(M,N),N=1,NMAX)
53 CONTINUE
RETURN
60 CONTINUE
WRITE(6,1005) WR(L)
1005 FORMAT(1X,"IN THE FORM OF AFR*EXP("*/1PE12.5\"*T")")
WRITE (6,1006) NMAX,NMAX
1006 FORMAT(1X,"MATRIX AFR (\"*/I2\"*,\"*/I2\"*,\"*/I2\"*)")
DO 61 M=1,NMAX
DO 61 N=1,NMAX
AFR(M,N)=REAL(CA1(M,N))
AFI(M,N)=0.0
61 CONTINUE
DO 62 M=1,NMAX
WRITE (6,1003) (AFR(M,N),N=1,NMAX)
62 CONTINUE
RETURN
END

SUBROUTINE SPCON
C*** DESIGN THE SPATIAL CONTROLLERS
C***
COMMON /STKN/ XF(31,23), XT(31,23), XD(31,23),XE(31,23)
COMMON /XSCN/ DIF1(25), DIF2(25), SIGA1(25),SIGA2(25)
1 SGR(25), SIGF(25), ALFA(25), HFAC(25)
2 XK(10), V1, V2, XNU
3 GAM, GAMX, SIGXE, ZLAM1
4 ZLAMX, XKS
COMMON /GEOG/ MAT(31,23), DX(31), DY(23), X(31), Y(23)
COMMON /MISC/ TITLE(18), NPAGE, NMOD, MM2, MM4,
**COMPUTE THE EFFECTIVENESS OF SPACIAL CONTROL**

```c
C
EF=1.0E-10  
KMAX=2  
VU=V2/V1  
NRX1=NRX-1  
NYX1=NYY-1  
READ (5,1010) NSX,NSY,NZC  
1010 FORMAT(16IS5)  
CALL HEADL  
WRITE (6,1100)  
1100 FORMAT(1//,3X,'SPATIAL CONTROL INFORMATION'//)  
WRITE (6,1101) NSX,NSY,NZC  
1101 FORMAT(10X,'SYMMETRY AXIS FOR X = ',IS5,'/10X,'SYMMETRY AXIS FOR Y = ',IS5,'/10X,'SYMMETRICITY OF CONTROLLERS (=1; Y-SYMMETRIC, =2; C E NTRAL SYMMETRIC) = ',IS5,'/10X,'KMAX=1  
WRITE (6,1005)  
1005 FORMAT('10X,'WARNING - SINGLE CONTROLLER SYSTEM MAY NOT BE COMPLETELY CONTROLLABLE'//)  
5 CONTINUE  
C 
CONSTRUCT CONTROLLER DOMAIN  
READ (5,1000) NODY,DCX,DCY  
1000 FORMAT(IS5,2F10.5)  
WRITE (6,1001) NODY,DCX,DCY  
1001 FORMAT(IS5,'NO OF CONTROLLER NODES IN Y-DIRECTION = ',IS5,'/10X,'NO DE SIZE IN X-DIRECTION = ',IS5,'/10X,'NODE SIZE IN Y-DIRECTION = '  
2*F10.5,'/10X,'  
WRITE (6,1004)  
1004 FORMAT(30X,'CONTROLLER LOCATIONS WHERE THE EFFECTIVENESS ARE COMMIPUTED'//)  
NODY1=NODY+1  
READ (5,1010) (NODY(NY),NY=1,NODY1)  
WRITE (6,1002)  
1002 FORMAT(17X,'X-DIR= 1  2  3  4  5  6  7  8  9  10',//,'Y-DIR = 2',/  
2*NY,  
NODY2=2*NODY+1  
DO 10 NY=1,NODY2  
YNOD(NY)=CY+(NY-NODY1)*DCY  
10 CONTINUE
```
IF (NY.LE.NODY1) NNX=NXDX(NY)
IF (NY.GT.NODY1) NNX=NXDX(NODY2-ny+1)
DO 11 NX=1,NNX
XNOD(nx,ny)=CX+(nx-nnx)*DCX
11 CONTINUE
WRITE (6,1003) XNOD(NY),XNOD(nx,ny),NX=1,NNX
1003 FORMAT(10X,11(F9.4,1X))
10 CONTINUE
CALL HEAD
NNX=2*NODX(NODY1)-1
DO 12 NX=1,NNX
12 CONTINUE
DO 13 LF=1,MMH
13 CONTINUE
QSC=0.0
DO 14 L=1,MMH
14 CONTINUE
QSC=SQR(T(QSC))
QSC=QSC/MMH
DO 15 L=1,MMH
15 CONTINUE
SMX22(L,M)=SMX22(L,M)/QSC
14 CONTINUE
MMH=MMH-1
NSX1=NSX+1
NSY1=NSY+1
C COMPUTES THERMAL FLUXES AT THE CENTRE OF CORE
XI(1)=X(NSX)
XI(2)=X(NSX1)
YJ(1)=Y(NSY)
YJ(2)=Y(NSY1)
P(1,1)=XT(NSX,NSY)
P(1,2)=XT(NSX,NSY1)
P(2,1)=XT(NSX1,NSY)
P(2,2)=XT(NSX1,NSY1)
XTOC=INT(CX*CY,XI,YJ,P)
IDX1=MAT(NSX,NSY)
IDX2=MAT(NSX,NSY1)
IDX3=MAT(NSX1,NSY)
IDX4=MAT(NSX1,NSY1)
DV1=(CX-X(NSX))*(QY-Y(NSY))
DV2=(CX-X(NSX))*(CY(NSY1)-CY)
DV3=(X(NSX1)-CX)*(Y(NSY1)-CY)
DV4=(X(NSX1)-CX)*Y(NSY1)-CY)
SIGK=(SIGR(IDX1)*XF(NSX,NSY)*DV1+%SIGR(IDX2)*XF(NSX1,NSY)*DV2+
1 SIGR(IDX3)*XF(NSX1,NSY)*DV3+%SIGR(IDX4)*XF(NSX1,NSY1)*DV4)/
2 (XF(NSX,NSY)*DV1+XF(NSX1,NSY)*DV2+XF(NSX1,NSY1)*DV3+
3 XF(NSX1,NSY1)*DV4)
SIGFD=(SIGF(IDX1)*P(1,1)+DV1+SIGF(IDX2)*P(1,2)+DV2+
1 SIGF(IDX3)*P(2,1)+DV3+SIGF(IDX4)*P(2,2)+DV4)/
1 (P(1,1)+P(1,2)+P(2,1)+P(2,2)+DV1+DV2+DV3+DV4)
WFACT=SQR(T(SIGK/SIGFD/XNU)
WRITE (6,1013) QSC
1013 FORMAT(/,10X,"SCALE FACTOR FOR SYSTEM MATRIX = ",E12.5,/) 
C COMPUTES ELEMENTS OF CONTROLLER MATRIX AT THE MESH 
WRITE (6,1008) 
1008 FORMAT(/,30X,"CONVERGENCE INFORMATION",/10X, 
1'X-NODE Y-NODE ITER.NO. CONVERGEN CYAP.UVDD.ERR IT.TIME 
1(SEC) INV.ERR R-FACTOR Q-FACTOR",/) 
NYMAX=NODY2 
IF (NXC.EQ.2) NYMAX=NODY1. 
DO 20 NY=1,NYMAX 
NY1=NY 
IF (NXC.EQ.2) NY1=2*NODY1-NY. 
IF (NY.LE.NODY1) NNX=NODX(NY) 
IF (NY.GT.NODY1) NNX=NODX(NODY2-NY+1) 
DO 21 J=1,NRY1 
IF (YNOD(NY),LT,Y(J),OR,YNOD(NY),GE,Y(J+1)) GO TO 21 
JJ=J 
J(J)=Y(JJ) 
YJ(J)=Y(JJ+1) 
YJO=YNOD(NY) 
21 CONTINUE 
DO 20 NX=1,NNX 
NX1=2*NNX-NX 
IF (YNOD(NY),EQ,CY,AND,XNOD(NX,NY),EQ,CX) GO TO 42 
DO 22 I=1,NRX1 
IF (XNOD(NX+NY),LT,X(I),OR,XNOD(NX,NY),GE,X(I+1)) GO TO 22 
II=I 
XI(I)=X(I) 
XI(2)=X(I)+1 
XIO=XNOD(NX,NY) 
22 CONTINUE 
CALL SECOND(SEC1) 
DO 23 KK=1,KMAX 
IF (KK.EQ.1) GO TO 230 
II=2*NSX-II-1 
XI(1)=X(I) 
XI(2)=X(I+1) 
XIO=2.0*CX-XIO 
IF (NXC.NE.2) GO TO 230 
JJ=2*NSY-JJ-1 
YJ(J)=Y(JJ) 
YJ(J)=Y(JJ+1) 
YJO=2.0*CY-YJO 
230 CONTINUE 
P(1,1)=XT(IJ,JJ) 
P(1,2)=XT(IJ,JJ+1) 
P(2,1)=XT(IJ+1,JJ) 
P(2,2)=XT(IJ+1,JJ+1) 
XT0=XINT(XIO,YJO,XI,YJ,P) 
IDX1=HAT(IJ,JJ) 
IDX2=HAT(IJ,JJ+1) 
IDX3=HAT(IJ+1,JJ) 
IDX4=HAT(IJ+1,JJ+1) 
DV1=XT0*(YJ-YJ) 
DV2=XT0*(YJ-YJ)
\[ DV3 = (X(2) - X(0)) \times (Y(J) - Y(1)) \]
\[ DV4 = (X(2) - X(0)) \times (Y(J) - Y(0)) \]
\[ SIGRD = (SIGRI(IX1) \times X(I) + J) + DV1 + SIGRI(IX2) \times X(I) + J + J1) + DV2 \]
\[ SIGFD = (SIGFI(IK1) \times X(I) + J) + DV1 + SIGFD(IK2) \times X(I) + J + J1) + DV2 \]

\[ X(I) + J + DV1 + X(I) + J + DV2 + X(I) + J + DV3 \]
\[ X(I) + J + DV1 + X(I) + J + DV2 + X(I) + J + DV3 \]

DO 24 M = 2, NMOD
M1 = M - 1
P(1, 1) = PS(I, J, M)
P(1, 2) = PS(I, J, M + 1)
P(2, 1) = PS(I, J, M)
P(2, 2) = PS(I, J, M + 1)

PCN(M1, KK) = INT(X(I0, Y(J0, X, Y, P)) / SQRT(XNU * SIGFD))

24 CONTINUE

WFAC = SQRT(SIGRD / SIGFD / XNU)

DO 25 M = 1, MM1
M1 = M + M1
BMX(M, KK) = 0.0
AMOD = PCN(M, KK) * TNORM(M + 1) * WFAC / FNORM(M + 1)
TEMP = WFAC * XT0 * XT0 / (WFAC * XT0 * XT0)
BMX(M, KK) = AMOD + VV * (XT0 - TEMP) / (AMOD * SQRT(SIGRD))

25 CONTINUE

C CHECK CONTROLLABILITY FOR A GIVEN CONTROLLER

DO 26 L = 1, MM2
RMTX(L, KK) = 0.0
DO 26 M = 1, MM2
RMTX(L, KK) = RMTX(L, KK) - SMX(L, M) * BMX(M, KK)

26 CONTINUE

C CALCULATES RMTX = TRANS(RMTX) / RFAC

DO 27 L = 1, MM2
DO 27 M = 1, MM2
GCTX(L, M) = DOT4(20, 20, 1, KMAX, RMTX, RMTX, L, M)
GCTX(M, L) = GCTX(L, M)

27 CONTINUE

DO 270 L = 1, MM2
DO 270 M = 1, MM2
SMX(L, M) = 0.0

270 CONTINUE

C DO 271 L = 1, MM1
L1 = MM1 + L
DO 271 L = 1, MM1
M1 = MM1 + L
SMX(L, M) = DOT4(10, 10, 10, KMAX, PCN, PCN, L, M)
SMX(L, M) = SMX(L, M)

271 CONTINUE
NORMALIZATION OF MATRICES.
RNORM=QNORM=0.0
DO 272 L=1,MM2
RNORM=RNORM+DOT4(20,20,1,MM2,GMTX,GMTX,L,L)
QNORM=QNORM+DOT4(20,20,1,MM2,SMX11,SMX11,L,L)
272 CONTINUE
QFAC=SQRT(QNORM)/MM2
RFAC=SQRT(RNORM)/MM2
DO 273 L=1,MM2
DO 273 M=1,MM2
GMTX(L,M)=GMTX(L,M)/RFAC
SMX11(L,M)=SMX11(L,M)*QFAC
273 CONTINUE

INITIAL SOLUTION OF THE RICCATI EQUATION
IL=1
THET=1.0E-2
XTIME=5.0E-1
DO 28 L=1,MM2
DO 28 M=1,MM2
GMTF(L,M)=GMTX(L,M)*XTIME
DMTX(L,M)=GMTF(L,M)*XTIME
28 CONTINUE
50 CONTINUE
ERR=0.0
XIL=IL+1.0
DO 51 L=1,MM2
DO 51 M=L,MM2
GMTY(L,M)=-(DOT3(20,20,1,MM2,SMX22,DMTX,L,M)+
1.0DOT4(20,20,1,MM2,DMTX,SMX22,L,M))/XIL
51 CONTINUE
DO 52 L=1,MM2
DO 52 M=1,MM2
GMTF(L,M)=GMTF(L,M)+GMTY(L,M)
DMTX(L,M)=GMTY(L,M)*XTIME
XERR=ABS(GMTF(L,M)/GMTF(L,M))
IF (XERR.GE.ERR) ERR=XERR
52 CONTINUE
IF (ERR.LE.EP) GO TO 53
IL=IL+1
IF (IL.LE.30) GO TO 53
GO TO 50
53 CONTINUE
CALL GINV(GMTF,DMTX,BMTX,BMTXY,20,MM2,MM2,ERROR)
IT=0
DO 54 L=1,MM2
DO 54 M=1,MM2
AFR(L,M)=GMTF(L,M)
54 CONTINUE

ITERATION FOR THE RICCATI EQUATION
100 CONTINUE
EPS=1.0
DO 30 L=1,MM2
DO 30 M=1,MM2
$\text{SMX12}(L,M) = \text{SMX22}(L,M) \cdot \text{DOT3}(20,20,1,M2,\text{GMTX},\text{GMTP},L,M)$

$\text{GMTY}(L,M) = - \text{SMX11}(L,M)$

DO 31 LL = 1, M2
CC = 0.0
DO 37 MM = 1, M2
CC = CC + GMTX(LL, MM) * GMTP(MM, M)
37 CONTINUE

$\text{GMTY}(L,M) = \text{GMTY}(L,M) - \text{GMTP}(LL,L) \cdot CC$

31 CONTINUE

30 CONTINUE

CALL AXPA(SMX12, GMTP, GMTY, DMTX, MM2, 20, 20, 20, 20 + EPS)
CON = 0.0
DO 32 L = 1, MM2
DO 32 M = 1, MM2
$\text{GMTY}(L,M) = \text{GMTY}(L,M) \cdot RSC$
IF (AFR(L,M), EQ, 0.0) GO TO 32
ENR = GMTY(L,M) / AFR(L,M)
CTR = ABS(1.0 - ERR)
CON = MAX(CON, CTR)
32 CONTINUE

IF (CON.LE.1.0E-2) GO TO 40
DO 34 L = 1, MM2
DO 34 M = 1, MM2
AFR(L,M) = GMTY(L,M)
DMTX(LL,M) = 0.0
DO 35 LL = 1, MM2
DMTX(LL,M) = DMTX(LL,M) - GMTY(LL,L) \cdot DOT3(20,20,1,M2,\text{GMTX},\text{GMTY},LL,M)
35 CONTINUE

$\text{DMTX}(L,M) = \text{DMTX}(L,M) + \text{DOT3}(20,20,1,M2,\text{GMTY},\text{SMX22},L,M) +$

$1 \cdot \text{SMX11}(L,M) + \text{DOT2}(20,20,1,M2,\text{SMX22},\text{GMTY},L,M)$

34 CONTINUE

IT = IT + 1
DO 36 L = 1, MM2
DO 36 M = 1, MM2
GMTP(LL,M) = GMTY(LL,M) - THET * DMTX(LL,M)
36 CONTINUE

IF (IT.LE.50) GO TO 100
40 CONTINUE

CALL SECON(SEC2)
SEC = SEC2 - SEC1
WRITE (6, 1009) NX, NY, IT, CON, EPS, SEC, ERROR, RFAC, QFAC
1009 FORMAT(11X, I5, 5X, I5, 5X, I5, 5X, 6E12.5)
IF (CON.GT.1.0E-2) GO TO 44
XI(NX, NY) = 0.0
XI(NX1, NY1) = 0.0
DO 41 K = 1, KMAX
XI(K) = 0.0
DO 43 L = 1, MM2
TEMP = DOT2(20, 20, 1, MM2, RMTX, GMTP, K, L) / RFAC
XI(K) = XI(K) + TEMP * IEMP
43 CONTINUE
XI(K) = SORT(XI(K))
41 CONTINUE
XI(NX, NY) = XI(1)
XD(NX1,NY1)=XI(KMAX)
IF (NX.EQ.NX1.AND.NY.EQ.NY1) XD(NX1,NY1)=XI(1)
IF (NZC.NE.2) GO TO 20
XD(NX1,NY)=XI(1)
XD(NX,NY1)=XI(KMAX)
IF (NY.EQ.NY1) XD(NX,NY1)=XI(1)
GO TO 20
42 XD(NX,NY)=-1.0E+10
XD(NX1,NY1)=1.0E+10
GO TO 20
44 XD(NX,NY)=-1.0
XD(NX1,NY1)=-1.0
20 CONTINUE
C PRINT THE EFFECTIVENESS MAP
CALL HEAD
WRITE (6,1014)
1014 FORMAT(//'CONTROLLER EFFECTIVENESS MAP'//''))
NX=NODX(NODY1)
DO 70 NX=1,NNX
RMTXX(NX)=NOD(NNX,NODY1)+DCXX(NX-1)
70 CONTINUE
WRITE (6,1015) (XNOD(NX,NODY1),NX=1,NNX),(RMTXX(NX),NX=2,NNX)
1015 FORMAT(10X,12F9.2)
NX=NODX(NODY1)
NX2=2*NXX-1
DO 71 NY=1,NODY2
IF (NY.LT.NODY1) NNX=NODX(NY)
IF (NY.GT.NODY1) NNX=NODX(NODY2-NY+1)
NDX=NX-NNX
IF (NDX.LE.0.0) GO TO 74
NNX=2*NNX-1
DO 72 NX=1,NNX
ND=NXX+NDX-NX+1
ND1=NNX-NX+1
XD(ND,NY)=XD(ND1,NY)
72 CONTINUE
DO 73 NX=1,NDX
XD(NX,NY)=1.0E+10
73 CONTINUE
74 CONTINUE
WRITE (6,1016) YNOD(NY),(XD(NX,NY),NX=1,NX2)
1016 FORMAT(F9.2,1X,12E9.3,/) 
71 CONTINUE
RETURN
END

SUBROUTINE AXPX(A,U,C,B,N,NA,NC,NU,NB,EPS)
C*** SOLVES THE REAL MATRIX EQUATION TRANS(A)*X+B*EPS=C.
C*** REF - SOLUTION OF THE MATRIX EQUATION AX+XB=C, BARTELS AND STEWART
C*** VOL. 15, NO. 9, COMMUN. OF ACM (1972)
C
DIMENSION A(NA+1),U(NU+1),C(NC+1),B(NB+1)
DIMENSION D(14+14)
N1=N+1
NM1=N-1
IC=1
DO 5 I=1,N
DO 6 J=1,N
R(I,J)=0.0
D(I,J)=0.0
6 CONTINUE
R(I,I)=1.0
CONTINUE
5 CONTINUE
IF (EPS.LT.0.0) GO TO 17
CALL SCHUR(NA,NB,N,A,B,TRUE,U,IERR)
IF (IERR.EQ.0) GO TO 15
WRITE (6,1000)
1000 FORMAT(10X,'***ERROR STOP. FAILED TO GET THE SCHUR FORM*)
STOP
15 CONTINUE
ICMAX=2
IF (EPS.NE.1.0) ICMAX=10
17 CONTINUE
C(I,I)=C(I,I)/2.0
DO 20 I=1,N
20 CONTINUE
DO 30 J=1,N
DO 30 J=1,N
A(N1,J)=DOT(B(NC,NU,I,N,C,U,I,J)
30 CONTINUE
DO 35 J=1,N
35 C(I,J)=A(N1,J)
40 CONTINUE
DO 50 J=1,N
50 CONTINUE
A(I,N1)=DOT2(B(NU,NC,1,N,U,C,I,J)
55 CONTINUE
DO 55 I=1,N
55 C(I,J)=A(I,N1)
60 CONTINUE
DO 70 I=1,N
DO 70 J=1,N
C(I,J)=C(I,J)+C(J,I)
CC(I,J)=C(I,J)
70 CONTINUE
C SOLVE THE TRANSFORMED EQUATIONS
200 CONTINUE
DO 75 I=1,N
DO 75 J=1,N
B(I,J)=C(I,J)
CALL SYMSLV(A,0,N,NA,NC)
CMAX=0.0
DO 76 I=1,N
76 DO 76 J=1,N
B(I,J)=B(I,J)+C(I,J)
TEMP=B(I,J)-DOT2(NA,NC,1,N,RA,C,I,J)-DOT3(NC,NA,1,N,RA,C,A,I,J)
IF (B(I,J),EQ,0.0) GO TO 71
ERR=ABS(TEMP/B(I,J))
71 B(I,J)=TEMP
CMAX=AMAX1(CMAX,ERR)
76 CONTINUE
IF (CMAX.LE.1.0E-5) GO TO 300
IC=IC+1
DO 77 I=1,N
DO 77 J=1,N
C(I,J)=B(I,J)
77 CONTINUE
IF (IC.LE.ICMAX) GO TO 200
300 CONTINUE
EPS=CMAX
DO 78 I=1,N
DO 78 J=1,N
C(I,J)=D(I,J)
78 CONTINUE
DO 80 I=1,N
C(I,I)=C(I,I)/2.0
80 CONTINUE
DO 100 I=1,N
DO 90 J=1,N
A(N1,J)=DOT4(NC,NU,I,N,C,I,J)
90 CONTINUE
DO 95 J=1,N
95 C(I,J)=A(N1,J)
100 CONTINUE
DO 110 J=1,N
DO 110 I=1,N
A(I,N1)=DOT3(NU,NC,1,N,UC,I,J)
110 CONTINUE
DO 115 I=1,N
115 C(I,J)=A(I,N1)
120 CONTINUE
DO 130 I=1,N
DO 130 J=1,N
C(I,J)=C(I,J)+C(J,I)
130 CONTINUE
RETURN
END

SUBROUTINE SYMSLV(A,NC,NA,NC)
C**** SOLVES THE TRANSFORMED MATRIX EQUATION
C
DIMENSION A(NA,i), C(NC,1)
INTEGER I, K, L, M, N, NLC, NLS, NLD
COMMON /SLYBLK/ T(S,5), P(5), NSYS
C
L=1
10 DL=1
   IF (L.EQ.N) GO TO 20
   IF (A(L+1,L),NE.0,0) DL=2
20 LL=L+DL-1
   K=L
30 KM1=K-1
   DK=1
   IF (K.EQ.N) GO TO 35
   IF (A(K+1,K),NE.0,0) DK=2
35 KK=K+DK-1
   DO 40 J=1,LL
   C(I,J)=C(I,J)-DOT2(NA,NC,L,KM1,A,C,I,J)
40 CONTINUE
45 IF (DL,EQ.2) GO TO 60
   IF (DK,EQ.2) GO TO 50
   T(1,1)=A(K,K)+A(L,L)
   T(I,1)=A(K,K)
   T(2,1)=A(K,K)
   T(2,2)=A(K,K)+A(L,L)
   C(K,L)=C(K,L)
   P(2)=C(K,K)
   NSYS=2
   CALL SYSSLV
   C(K,L)=P(1)
   C(K,L)=P(2)
50 GO TO 90
55 C(K,L)=C(K,L)/T(1,1)
GO TO 10
60 IF (DK,EQ.2) GO TO 70
   T(1,1)=A(K,K)+A(L,L)
   T(1,2)=A(L,L)
   T(2,1)=A(L,L)
   T(2,2)=A(K,K)+A(L,L)
   P(1)=C(K,L)
   P(2)=C(K,L)
   NSYS=2
   CALL SYSSLV
   C(K,L)=P(1)
   C(K,L)=P(2)
65 GO TO 90
70 IF (K,NE,L) GO TO 80
   T(1,1)=A(L,L)
   T(1,2)=A(L,L)
75 GO TO 110
80 IF (K.EQ.N) GO TO 10
   T(I,1)=A(L,L)
   T(I,2)=A(L,L)
85 CONTINUE
90 WRITE (6,1000)
1000 FORMAT(10X,**ERROR STOP - FAILED TO SOLVE LYAPUNOV EQUATION**) STOP
GO TO 90
105 C(K,L)=C(K,L)/T(1,1)
GO TO 110
110 STOP
L(1, 3)=0.0
L(1, 4)=A(L, LL)
L(2, 1)=A(KK, K)
L(2, 2)=A(KK, K)+A(L, L)
L(2, 3)=0.0
L(3, 1)=A(LL, LL)
L(3, 2)=0.0
L(3, 3)=A(K, K)+A(LL, LL)
L(3, 4)=T(1, 2)
T(4, 1)=0.0
T(4, 2)=T(3, 1)
T(4, 3)=T(2, 1)
T(4, 4)=A(KK, KK)+A(LL, LL)
P(1)=C(K, L)
P(2)=C(KK, L)
P(3)=C(K, LL)
P(4)=C(KK, LL)
NSYS=4
CALL SYSSLV
C(K, L)=P(1)
C(KK, L)=P(2)
C(K, LL)=P(3)
C(KK, LL)=P(4)
90 K=K+DK
IF (K .LE. N) GO TO 30
LDL=L+DL
IF (LDL .GT. N) RETURN
DO 120 J=LDL, N
DO 100 I=L, LL
C(I,J)=C(J,I)
100 CONTINUE
DO 110 I=J, N
SUBROUTINE SYSLV
C**** SOLVES THE LINEAR SYSTEM AX=B OF ORDER N LESS THAN 5
C
COMMON /SLUBLK/ A(5,5), B(5), N

1 NM1=N-1
N=N+1
DO 80 K=1,N
KM1=K-1
IF.(K.EQ.1) GO TO 20
DO 10 I=K,N
DO 11 J=1,KM1
A(I,K)=A(I,K)-A(I,J)*A(J,K)
11 CONTINUE
10 CONTINUE
20 IF (K.EQ.N) GO TO 100
KP1=K+1
XMAX=ABS(A(K,K))
INTR=K
DO 30 I=KP1,N
AA=ABS(A(I,K))
IF.(AA.LE.XMAX) GO TO 30
XMAX=AA
INTR=I
30 CONTINUE
IF.(XMAX.NE.0.0) GO TO 35
WRITE(*,1000)
1000 FORMAT(10X, 'ERROR STOP - FAILED TO SOLVE LINEAR EQ. OBTAINED FROM S HUR FORM/')
STOP
35 A(N1,K)=INTR
IF.(INTR.EQ.K) GO TO 50
DO 40 J=1,N
TEMP=A(K,J)
A(K,J)=A(INTR,J)
A(INTR,J)=TEMP
40 CONTINUE
50 DO 75 J=KP1,N
IF.(K.EQ.1) GO TO 70
DO 71 JJ=1,KM1
A(K,J)=A(K,J)-A(K, JJ)*A(JJ,J)
71 CONTINUE
70 A(K,J)=A(K,J)/A(K,K)
75 CONTINUE
80 CONTINUE
100 DO 110 J=1,NM1
   INTR=A(N1,J)
   IF (INTR.EQ.J) GO TO 110
   TEMP=B(J)
   B(J)=B(INTR)
   B(INTR)=TEMP
110 CONTINUE
   B(I)=B(I)/A(I,1)
   DO 120 I=2,N
      IM1=I-1
      DO 125 J=1,IM1
         B(I)=B(I)-A(I,J)*B(J)
      125 CONTINUE
      B(I)=B(I)/A(I,1)
   120 CONTINUE
   DO 130 I=1,NM1
      I=NM1-II+1
      I1=I+1
      DO 135 J=1,I
         B(I)=B(I)-A(I,J)*B(J)
      135 CONTINUE
   CONTINUE
   RETURN
END

FUNCTION DOT(MR,N1,N2,A,JC,KC)
C**** COMPUTES THE INNER PRODUCT OF COLUMNS JC AND KC OF MATRIX A
C
   DIMENSION A(MR,1)
   DOT=0.0
   DO 5 I=N1,N2
      5 DOT=DOT+A(I,JC)*A(I,KC)
   RETURN
END

FUNCTION DOT1(MR,NR,N1,N2,A,JC,KC)
C**** COMPUTES THE SQUARE OF MATRIX A FOR COLUMN JC AND ROW KC
C
   DIMENSION A(MR,NR)
   DOT1=0.0
   DO 5 I=N1,N2
      5 DOT1=DOT1+A(JC,I)*A(I,KC)
   RETURN
END

FUNCTION DOT2(MR,NR,N1,N2,A,B,JC,KC)
C**** COMPUTES THE INNER PRODUCT OF MATRICES A AND B FOR COLUMN JC AND
C**** KC
   DIMENSION A(MR,1), B(NR,1)
   DOT2=0.0
   DO 5 I=N1,N2
      5 DOT2=DOT2+A(I,JC)*B(I,KC)
   RETURN
END
FUNCTION DOT3(MR,NR,N1,N2,A,B,JC,KC)
C**** COMPUTES THE PRODUCT OF MATRICES AND B FOR COLUMN JC AND ROW KC
C
DIMENSION A(MR,N2), B(NR,1)
DOT3=0.0
DO 5 I=N1,N2
  5 DOT3=DOT3+A(JC,I)*B(I,KC)
RETURN
END

FUNCTION DOT4(MR,NR,N1,N2,A,B,JC,KC)
C**** PRODUCT OF MATRICES AND A FOR COLUMN JC AND B FOR COLUMN KC
DIMENSION A(MR,N2), B(NR,N2)
DOT4=0.0
DO 5 I=N1,N2
  5 DOT4=DOT4+A(JC,I)*B(KC,I)
RETURN
END

SUBROUTINE SCHUR(NM,NB,N,A,B,MATZ,Z,IERR)
C**** MATRIX TRANSFORMATION - A-REAL SCHUR FORM) AND B(UPPER TRIANGULAR
C**** FORM) BY ORTHOGONAL TRANSFORMATION
DIMENSION A(NM,N), B(NB,N), Z(NM,N)
INTEGER EN, ENM2, ENORM
LOGICAL MATZ
C
TRANSFORMATION OF MATRIX INTO UPPER HESSENBORG FORM
IF (.NOT.MATZ) GO TO 3
DO 2 I=1,N
  2 J=1,N
  Z(I,J)=0.0
1 CONTINUE
  Z(I,I)=1.0
2 CONTINUE
3 IF (N.LE.1) GO TO 27
   NM1=N-1
   DO 10 L=1,NM1
     L1=L+1
     S=0.0
     DO 4 I=L1,N
4 CONTINUE
     S=S+ABS(B(I,L))
     IF (S.EQ.0.0) GO TO 10
     S=S+ABS(B(L,L))
     DO 5 I=L,N
5 CONTINUE
     B(I,L)=B(I,L)/S
     R=DOT(NM1,L,N,B,I,L)
     R=SIGN(SQRT(R),B(L,L))
     B(L,L)=B(L,L)+R
     RHO=R*B(L,L)
     DO 8 J=L1,N
8 CONTINUE
     T=DOT(NM1,L,N,B,L,J)
     T=T/RHO
     DO 7 I=L,N
7 CONTINUE

R(I,J) = B(I,J) + T*B(I,L)
CONTINUE
CONTINUE
DO 12 J=1,N
T = DOT2(NM,NM,L,N,B,A,L,J)
T = -T/RHO
DO 11 I=L,N
A(I,J) = A(I,J) + T*B(I,L)
CONTINUE
11 CONTINUE
B(L,L) = -S*R
DO 13 I=L1,N,
B(I,L) = 0.0
CONTINUE
13 CONTINUE
IF (N.EQ.2) GO TO 27
NM2 = N-2
DO 20 K=1,NM2
NK1 = NK1-K
DO 21 LB=1,NK1
L = N-LB
L1 = L1+1
S = ABS(A(L1,K)) + ABS(A(L1,K))
IF (S.EQ.0.0) GO TO 21
U1 = A(L1,K)/S
U2 = A(L1,K)/S
R = SIGN(SQRT(U1*U1+U2*U2),U1)
V1 = -(U1*R)/R
V2 = -U2/R
U2 = U2/V1
DO 22 J=K,N
T = A(L,J) + U2*A(L1,J)
A(L,J) = A(L,J) + T*V1
A(L1,J) = A(L1,J) + T*V2
CONTINUE
22 CONTINUE
A(L1,K) = 0.0
DO 23 J=1,N
T = B(L,J) + U2*B(L1,J)
B(L,J) = B(L,J) + T*V1
B(L1,J) = B(L1,J) + T*V2
23 CONTINUE
S = ABS(B(L1,L1)) + ABS(B(L1,L1))
IF (S.EQ.0.0) GO TO 21
U1 = B(L1,L1)/S
U2 = B(L1,L1)/S
R = SIGN(SQRT(U1*U1+U2*U2),U1)
V1 = -(U1*R)/R
V2 = -U2/R
U2 = U2/V1
DO 24 I=1,L1
T = B(I,L1) + U2*B(I,L1)
B(I,L1) = B(I,L1) + T*V1
B(I,L1) = B(I*L1) + T*V2
CONTINUE
K(L, L) = 0.0
DO 25 I = 1, N
    T = A(I, L1) + U2 * A(I, L)
    A(I, L1) = A(I, L1) + T * V1
    A(I, L) = A(I, L) + T * V2
25 CONTINUE
IF (.NOT. MATZ) GO TO 21
DO 26 I = 1, N
    T = Z(I, L1) + U2 * Z(I, L)
    Z(I, L1) = Z(I, L1) + T * V1
    Z(I, L) = Z(I, L) + T * V2
26 CONTINUE
21 CONTINUE
20 CONTINUE

TRANSFORMATION OF UPPER HESSENBERG MATRIX INTO QUASI-UPPER
TRIANGULAR FORM
IERR = 0
ANORM = 0.0
BNORM = 0.0
DO 30 I = 1, N
    ANI = 0.0
    IF (I .NE. 1) ANI = ABS(A(I, I-1))
    BNI = 0.0
    DO 31 J = 1, N
        ANI = ANI + ABS(A(I, J))
        BNI = BNI + ABS(B(I, J))
    31 CONTINUE
    IF (ANI .GT. ANORM) ANORM = ANI
    IF (BNI .GT. BNORM) BNORM = BNI
30 CONTINUE
IF (ANORM .EQ. 0.0) ANORM = 1.0
IF (BNORM .EQ. 0.0) BNORM = 1.0
IF (E + EPS .LE. 1.0) GO TO 32
EPSA = EPS * ANORM
EPSB = EPS * BNORM
LOR1 = 1
ENORM = N
EN = N
33 IF (EN .LE. 2) GO TO 70
    IF (.NOT. MATZ) ENORM = EN
    ITS = 0
    NA = EN - 1
    ENM2 = NA - 1
34 ISH = 2
    DO 35 LL = 1, EN
        LM1 = EN - LL
        L = LM1 + 1
        IF (L .EQ. 1) GO TO 37
        IF (ABS(A(L, LM1)) .LE. EPSA) GO TO 36
    35 CONTINUE
36 A(L, LM1) = 0.0
IF (L.LT.NA) GO TO 37
EN=LM
GO TO 33
37 LD=L
40 LI=LI+1
B1=B(L,L)
IF (ABS(B11).GT.EPSB) GO TO 42
B(L,L)=0.0
S=ABS(A(L,L))+ABS(A(L1,L))
U1=AX(L,L)/S
U2=AX(L1,L)/S
R=SIGN(SQR(A(U1*U1+U2*U2)),U1)
V1=(-U1+R)/R
V2=-U2/R
U2=V2/V1
DO 41 J=L1.ENORM
T=A(L,J)+U2*A(L1,J)
A(L,J)+A(L1,J)+V1
A(L1,J)+A(L1,J)+V1
T=B(L,J)+U2*B(L1,J)
B(L,J)+B(L1,J)+V1
B(L1,J)+B(L1,J)+V1
41 CONTINUE
IF (L.NE.1).A(L,LM1)=A(L,LM1)
LM1=L
L=L1
GO TO 36
42 A11=A(L,L)/B11
A21=A(L1,L)/B11
IF (ISN.EQ.1) GO TO 44
IF (ITS.EQ.50) GO TO 61
IF (ITS.EQ.10) GO TO 50
B22=B(L1,L1)
IF (ABS(B22).LT.EPSB) B22=EPSB
B33=B(NA,NA)
IF (ABS(B33).LT.EPSB) B33=EPSB
B44=B(EN,EN)
IF (ABS(B44).LT.EPSB) B44=EPSB
A33=A(NA,NA)/B33
A34=A(NA,EN)/B44
A43=A(EN,NA)/B33
A44=A(EN,EN)/B44
B34=B(NA,EN)/B44
T=0.5*(A43*B34-A33*A44)
R=T+T+A34*A43-A33*A44
IF (R.LT.0.0) GO TO 45
ISH=1
R=SQR(R)
SH=-R
S=-T-R
IF (ABS(S-A44).LT.ABS(SH-A44)) SH=S
DO 43 LL=LD,ENM2
L=ENM2+LL-LL
IF (L.EQ.LD) GO TO 44
LM1=L-1
L1=L+1
T=A(L,L)
IF (ABS(B(L,L)) .GT. EPSB) T=T-SH*B(L,L)
IF (ABS(A(L,L1)) .LE. ABS(T/A(L,L)) * EPSA) GO TO 40
43 CONTINUE
44 A1=A11-SH
     A2=A21
     IF (L.NE.LD) A(L,LM1)=-A(L,LM1)
     GO TO 51
45 A12=A(L,L1)/B22
     A22=A(L1+1)/B22
     B12=B(L,L1)/B22
     A3=A(L1+1,L1)/B22
     GO TO 51
50 A1=0.0
     A2=1.0
     A3=1.1605
51 ITS=ITS+1
     IF (.NOT.MATZ) LOR1=LD
     DO 60 K=L,NA
     OTLÅS=K.AE.NA.AND.ISH.EQ.2
     K1=K+1
     K2=K+1
     KM1=MAX0(K-1,L)
     LL=MIN0(EN,K1+ISH)
     IF (NOTLÅS) GO TO 54
     IF (K.EQ.L) GO TO 52
     A1=A(K,KM1)
     A2=A(K1,KM1)
52 S=ABS(A1)+ABS(A2)
     IF (S.EQ.0.0) GO TO 34
     U1=A1/S
     U2=A2/S
     R=SIGN(SORT(U1*U1+U2*U2),U1)
     V1=-(U1+R)/R
     V2=-U2/R
     U2=V2/V1
     DO 53 J=KM1,ENORM
     T=A(K,J)+U2*A(K1,J)
     A(K,J)=A(K,J)+T*V1
     A(K1,J)=A(K1,J)+T*V2
     T=B(K,J)+U2*B(K1,J)
     B(K,J)=B(K,J)+T*V1
     B(K1,J)=B(K1,J)+T*V2
53 CONTINUE
     IF (K.NE.L) A(K1,KM1)=0.0
     GO TO 59
54 IF (K.EQ.L) GO TO 55
     A1=A(K,KM1)
     A2=A(K1,KM1)
     A3=A(K2,KM1)
55 S=ABS(A1)+ABS(A2)+ABS(A3)
   IF (S.EQ.0.0) GO TO 60
   U1=A1/S
   U2=A2/S
   U3=A3/S
   R=SIGN(SQRT(U1*U1+U2*U2+U3*U3),U1)
   V1=-(U1+R)/R
   V2=-U2/R
   V3=-U3/R
   U2=V2/V1
   U3=V3/V1
   DO 56 J=NM1,ENORM
     T=A(K,J)+U2*A(K1,J)+U3*A(K2,J)
     A(K,J)=A(K,J)+T*V1
     A(K1,J)=A(K1,J)+T*V2
     A(K2,J)=A(K2,J)+T*V3
     T=B(K,J)+U2*B(K1,J)+U3*B(K2,J)
     B(K,J)=B(K,J)+T*V1
     B(K1,J)=B(K1,J)+T*V2
     B(K2,J)=B(K2,J)+T*V3
   56 CONTINUE
   IF (K.EQ.L) GO TO 57
   A(K1,KM1)=0.0
   A(K2,KM1)=0.0
  57 S=ABS(B(K2,K2))+ABS(B(K2*K1))+ABS(B(K2*K))
   IF (S.EQ.0.0) GO TO 59
   U1=B(K2,K2)/S
   U2=B(K2,K1)/S
   U3=B(K2,K)/S
   R=SIGN(SQRT(U1*U1+U2*U2+U3*U3),U1)
   V1=-(U1+R)/R
   V2=-U2/R
   V3=-U3/R
   U2=V2/V1
   U3=V3/V1
   DO 58 I=LR1,LL
     T=A(I,K2)+U2*A(I,K1)+U3*A(I,K)
     A(I,K2)=A(I,K2)+T*V1
     A(I,K1)=A(I,K1)+T*V2
     A(I,K)=A(I,K)+T*V3
     T=B(I,K2)+U2*B(I,K1)+U3*B(I,K)
     B(I,K2)=B(I,K2)+T*V1
     B(I,K1)=B(I,K1)+T*V2
     B(I,K)=B(I,K)+T*V3
  58 CONTINUE
   B(K2,K)=0.0
   B(K2,K1)=0.0
   IF (.NOT.MATZ) GO TO 59
   DO 46 I=1,N
     T=Z(I,K2)+U2*Z(I,K1)+U3*Z(I,K)
     Z(I,K2)=Z(I,K2)+T*V1
     Z(I,K1)=Z(I,K1)+T*V2
     Z(I,K)=Z(I,K)+T*V3
  46 CONTINUE
S = ABS(B(K1,K1)) + ABS(B(K1,K))
IF (S.EQ.0.0) GO TO 60
U1 = B(K1,K1)/S
U2 = B(K1,K)/S
R = SIGN(SQRT(U1*U1+U2*U2), U1)
V1 = -(U1+R)/R
V2 = -U2/R
DO 47 I = LDR1, LL
T = A(I,K1) + U2*A(I,K)
A(I,K1) = A(I,K1) + T*V1
A(I,K) = A(I,K) + T*V2
70 CONTINUE
B(K1,K) = 0.0
IF (N.DT.MATZ) GO TO 60
DO 48 I = 1, N
T = Z(I,K1) + U2*Z(I,K)
Z(I,K1) = Z(I,K1) + T*V1
Z(I,K) = Z(I,K) + T*V2
47 CONTINUE
60 CONTINUE
GO TO 34
48 CONTINUE
61 IERR = EN
70 IF (N.GT.1) B(N,1) = EPSB
RETURN
END

FUNCTION XINT(X,Y,XI,YJ,FXY)
C*** 2-DIMENSIONAL LINEAR INTERPOLATION ROUTINE
DIMENSION XI(2), YJ(2), FXY(2,2)
C
XX = X - XI(1)
XX1 = XI(2) - XI(1)
YY = Y - YJ(1)
YY1 = YJ(2) - YJ(1)
FXX = XX*FFXYY = 0.0
IF (XX1.NE.0.0) AND (YY1.NE.0.0) FXY = XX1*YY1*FXY(2,2) - FXY(1,2) -
1 FXY(2,1)*FXY(1,1))/XX1*YY1)
IF (XX1.NE.0.0) FXX = XX1*FXY(2,1) - FXY(1,1))/XX1
IF (YY1.NE.0.0) FYY = YY1*FXY(1,2) - FXY(1,1))/YY1
XINT = FXX + FXX + FXY(1,1)
END

SUBROUTINE BMTRX(LSUM)
COMMON /STFN/ XF(31,23), XT(31,23), XB(31,23), XE(31,23)
COMMON /XSCN/ DIF1(25), DIF2(25), SIGA1(25), SIGA2(25),
1 SIGR(25), SIGF(25), ALFA(25), HFA(25),
2 XK(10), V1, V2, XNU,
3 GAM1, GAMX, SIGXE, ZLAMX,
4 ZLAMX, XKS
COMMON /MISC/ TITLE(18), NPAG7, NMOD, M2, M4,
1 NRX, NRY, NTT, PWR, BLANK,
2 CX, CY, NSX, NSY, IUNSTB, RFAC, SFAC,
3 QFAC, AMP0
COMMON /NCON/ GGMX(10), NSIGN(10), YL(10), XL(10)
COMMON /ECON/ BMTX(10,20), CMTX(10,20), UB(20)
COMMON /DEOM/ MAT(31,23), DX(31), DY(23), X(31), Y(23)
COMMON /XYMOD/ FF(31,23,8), FS(31,23,8), FNORM(8), TNORM(8)
DIMENSION XI(2), YJ(2), F(2,2)

READ (5,1000) LCON
READ (5,1001) (XL(K),YL(K),NSIGN(K),K=1,LCON)
1000 FORMAT(16I5)
1001 FORMAT(4(F9.4,F9.4,F9.2))
WRITE (6,2000)
2000 FORMAT(//,30X,'CONTROLLER DATA FOR THE FIRST HALF OF CORE',//)
WRITE (6,2001) LCON
2001 FORMAT(10X,'NUMBER OF CONTROLLER = ',I5,10X,'X-CORD, Y-CORD, SYM,IDX',//)
WRITE (6,2002) (XL(K),YL(K),NSIGN(K),K=1,LCON)
2002 FORMAT(9X,F9.4,1X,F9.4,5X,I2)
LSUM=0.0
DO 10 K=1,LCON
K1=LSUM+1
LSUM=LSUM+NSIGN(K)
K2=LSUM
DO 11 KK=K1,K2
DO 14 L=1,NMOD
14 BMTX(L,KK)=0.0
XN=XL(K)
YN=YL(K)
IF (NSIGN(K),EQ.2,AND.KK,EQ.K2) XN=2*CX-XN
DO 12 I=2,NRX
DO 12 J=2,NRY
IF (XN,L.T,X(I-1).OR.XN.GE.X(I).OR.YN,L.T,Y(J-1).OR.YN.GE.Y(J))
1   GO TO 12
XI(I)=X(I-1)
XJ(I)=X(I)
YJ(I)=Y(I-1)
YJ(I)=Y(J)
DU1=(XN-X(I-1))*(YN-Y(J-1))
DU2=(XN-X(I-1))*(Y(J)-YN)
DU3=(X(I)-XN)*(YN-Y(J-1))
DU4=(X(I)-XN)*(Y(J)-YN)
P(1,1)=XT(I-1,J-1)
P(1,2)=XT(I-1,J)
P(2,1)=XT(I,J-1)
P(2,2)=XT(I,J)
XT=XINT(XN,YN,XI,YJ,P)
IDX1=MAT(I-1,J-1)
IDX2=MAT(I-1,J)
IDX3=MAT(I,J-1)
IDX4=MAT(I,J)
SIGRD=(SIGR(IDX1)*XF(I-1,J-1)*DV1+SIGR(IDX2)*XF(I-1,J)*DV2+  
1 SIGR(IDX3)*XF(I,J-1)*DV3+SIGR(IDX4)*XF(I,J)*DV4)/  
2 (XF(I-1,J-1)*DV1+XF(I-1,J)*DV2+XF(I,J-1)*DV3+XF(I,J)*DV4).  
SIGFD=(SIGF(IDX1)*XT(I-1,J-1)*DV1+SIGF(IDX2)*XT(I-1,J)*DV2+  
1 SIGF(IDX3)*XT(I,J-1)*DV3+SIGF(IDX4)*XT(I,J)*DV4)/  
2 (XT(I-1,J-1)*DV1+XT(I-1,J)*DV2+XT(I,J-1)*DV3+XT(I,J)*DV4)  
BLE=NXU*SIGFD  
CLN=SQRT(SIGRD)  
DO 13 L=1,NMOD  
P(1,1)=PF(I-1,J-1,L)  
P(1,2)=PF(I-1,J,L)  
P(2,1)=PF(I,J-1,L)  
P(2,2)=PF(I,J,L)  
PFCL=INT(XN,YN,XI,YJ,P)  
BMTX(L,KK)=PFCL*BLN*XT/(AMPO*CLN)  
P(1,1)=PS(I-1,J-1,L)  
P(1,2)=PS(I-1,J,L)  
P(2,1)=PS(I,J-1,L)  
P(2,2)=PS(I,J,L)  
CMTX(L,KK)=INT(XN,YN,XI,YJ,P)/SQRT(BLN)  
13 CONTINUE  
12 CONTINUE  
11 CONTINUE  
10 CONTINUE  
RETURN  
END  

SUBROUTINE BKCON(NPROB)  
C**** CALCULATES THE STATIC BULK CONTROLLERS WITH GROUPS OF LOWER GAIN  
C**** UNDER THE CRITERIA OF MAXIMUM CONTROL MATERIAL FOR A CONTROLLER  
COMMON /STFN/ XF(31,23), XT(31,23), XI(31,23), XE(31,23)  
COMMON /MISC/ TITLE(10), NPAGE, NMOD, M2, M4,  
1 NNX, NRY, NTT, PWX, BLANK,  
2 CX, CY, NSX, NSY, IUNSTB, RFAC, SFAC,  
3 QFAC, AMPO  
COMMON /GEOM/ MAT(31,23), DX(31), DY(23), X(31), Y(23)  
COMMON /XSCN/ DIF1(25), DIF2(25), SIGA1(25), SIGA2(25),  
1 SIGR(25), SIGF(25), ALFA(25), HFAC(25),  
2 XK(10), V1, V2, XNU,  
3 GAMI, GAMX, SIGXE, ZLAM1,  
4 ZLAMX, XKS  
COMMON /SYSM/ SMX11(20,20), SMX12(20,20), SMX21(21,21),  
1 SMX22(20,20), SMX(20,20)  
COMMON /XYMO/ PF(31,23,8), CS(31,23,8), FNORM(8), TNORM(8)  
COMMON /BCON/ BMTX(10,20), CMTX(10,20), UB(20)  
COMMON /NCOM/ GGMX(10), NSIGN(10), YL(10), XL(10)  
DIMENSION TRMX(10,10), W1(10), W2(10), TRMY(10,10),  
1 DMATX(10,10), OMATX(10,10), PMATX(10,10), TRM1(10,10),  
2 PMTN(10), UVOL(20), UC(20), SMXX(10,10)  
C  
EP=1.0  
5 EP=EP/2.0  
IF (1.0+EP.GT.1.0) GO TO 5  
REWIND 2
READ (5,1000) USIG2,VUNT,UFRC,TILT,SIGMA
1000 FORMAT(6E12.6)
WRITE (6,2000)
2000 FORMAT(/,3X,'BULK CONTROLLER DESIGN',/)
WRITE (6,2001) USIG2,VUNT,UFRC,TILT,SIGMA
2001 FORMAT(10X,'MACROSCOPIC ABSORPTION X-SECTION OF CONTROLLER MATERI
AL = ',E12.6,' (1/CM)',/)
10X,'UNIT VOLUME FOR CONTROLLER MODEL = ',E12.6,' (CM**2)',/)
10X,'EXPECTED FRACTION OF CONTROLLER FOR STATIC SET-POINT = ',E12.6,
10X,'FRACTIONAL MAXIMUM POWER TILT = ',E12.6,/)
READ (5,1000) QFACO
WRITE (6,2002) QFACO
2002 FORMAT(10X,'INITIAL SCALE FACTOR FOR AMP. LOSS = ',E12.6,/)
UUNIT=USIG2*VUNT
CALL BMTX(MCON)
WRITE(2) MCON
WRITE(2) XL,YL,NSIGN
WRITE(2) BMTX
READ (5,1001) (UVOL(K),K=1,MCON)
1001 FORMAT(BF10.5)
WRITE(6,2002) (UVOL(K),K=1,MCON)
2002 FORMAT(/,10X,'MAXIMUM MULTIPLES OF LATTICE VOLUME FOR CONTROLLER',
10X,'*CONTROL 1 2 3 4 5 6 7 8 9 10*',/)
310F10.5,/
CALL SMTX(NPROB)
CALL GINV(SMX11,DMTX,W1,W2,20,NMOD,NMOD,ERROR)
DO 10 L=1,NMOD
DO 10 M=1,NMOD
TMRX(L,M)=DOT4(W1,M,1,NMOD,SMX21,SMX11,L,M)
10 CONTINUE
DO 12 L=1,NMOD
DO 12 M=1,NMOD
SMX22(L,M)=SMX22(L,M)+DOT3(W1,M,1,NMOD,TMRX,SMX12,L,M)
12 CONTINUE
CALL HEAD
WRITE (6,2023) NMOD,NMOD
2023 FORMAT(/,10X,'REDUCED-SYSTEM MATRIX ("I2","I2")',/)
DO 17 L=1,NMOD
WRITE (6,2024) (SMX22(L,M),M=1,NMOD)
17 CONTINUE
WRITE (6,2034)
2034 FORMAT(/,10X,'CONTROLLER MATRIX',/)
DO 170 K=1,MCON
WRITE (6,2024) (BMTX(K,L),L=1,NMOD)
170 CONTINUE
WRITE (6,2033)
2033 FORMAT(/,10X,'OBSERVER MATRIX',/)
DO 171 K=1,MCON
WRITE (6,2024) (CMTX(K,L),L=1,NMOD)
171 CONTINUE
2024 FORMAT(5X,8E12.5)
C TMRX - MATRIX (A21)*INV(A11)
C SMX22 - MATRIX ((A22)-(TMRX)*(A12))
DK0=1.0, 0.0/XK(1)
DO 13 L=1,NMOD
W1(L)=0.0
TLN=TNORM(L)/F Norman(L)
DO 13 I=1,NRX
DO 13 J=1,NRY
IDX=MAT(I,J)
ALN=XNU*SIGF(IDX)
BLN=SORT(ALN)
CLN=SIGR(IDX)
DV=DX(I)*DY(J)
W1(L)=W1(L)+DK0*TLN*PS(I,J,L)*BLN*CLN*XT(I,J)*DV/AMPO
13 CONTINUE
C F-MATRIX CONSTRUCTION
DO 15 L=1,NMOD
DO 15 M=1,NMOD
DMTX(L,M)=DOT3(10,20,1,NMOD,TMRX,SMX22,L,M)
15 CONTINUE
DO 16 L=1,NMOD
DO 16 M=1,NMOD
SMX22(L,M)=DMTX(L,M)/AMPO
16 CONTINUE
DO 14 L=1,NMOD
FMTN(L)=0.0
DO 14 M=1,NMOD
FMTN(L)=FMTN(L)-TMRX(L,M)*W1(M)
14 CONTINUE
C COMPUTES THE STATIC SET-POINTS OF CONTROLLERS
CALL HEAD
DO 18 L=1,NMOD
DO 118 M=L,NMOD
DMTX(L,L)=DOT4(10,10,1,NMOD,DMTX,DMTX,L,L)
SMX11(L,L)=0.0
DMTX(M,L)=DMTX(L,M)
SMX11(M,L)=SMX11(L,M)
118 CONTINUE
SMX11(L,L)=1.0
18 CONTINUE
QSC=QNorm=RNorm=0.0
DO 19 L=1,NMOD
QSC=QSC+DOT4(20,20,1,NMOD,SMX22,SMX22,L,L)
QNorm=QNorm+DOT4(20,20,1,NMOD,SMX11,SMX11,L,L)
RNorm=RNorm+DOT4(10,10,1,NMOD,DMTX,DMTX,L,L)
19 CONTINUE
QSC=SORT(QSC)/NMOD
QFAC=QFAC/ NMOD/SORT(QNorm)
RFAC=SORT(RNorm)/NMOD
DO 180 L=1,NMOD
DO 180 M=1,NMOD
SMX11(L,M)=SMX11(L,M)*QFAC
DMTX(M,M)=DMTX(L,M)/RFAC
FMTN(M,M)=SMX11(L,M)
180 CONTINUE
CALL GINV(PMTX,TRMX,W1,W2,10,NMOD,NMOD,ERROR)
DO 20 L=1,NMOD
DO 20 M=1,NMOD
TRMX(L,M)=MTMX(L,M)
DO 200 LL=1,NMOD
CC=0.0
DO 201 MM=1,NMOD
CC=CC+MTMX(M,LL)*SMX22(L,LL)MM
201 CONTINUE
TRMX(L,M)=TRMX(L,M)+SMX22(L,LL)CC
200 CONTINUE
20 CONTINUE:
CALL GINV(TRMX,TRMY,W1,W2,10,NMOD,NMOD,ERROR)
DO 22 K=1,MCON
UB(K)=0.0
DO 22 L=1,NMOD
SUM=0.0
DO 23 M=1,NMOD
SUM=SUM+MTMX(M,K)TRMX(L,M)
23 CONTINUE
UB(K)=UB(K)+PMTN(L)*SUM/RFAC
22 CONTINUE
DO 21 K=1,MCON
UB(K)=UB(K)*100.0/(UUNIT*UVOL(K))
21 CONTINUE
WRITE (6,205)
2005 FORMAT(//,10X,'STATIC CONTROL SET-POINTS (X FILLED)*',//)
WRITE (6,2015)
WRITE (6,2006) (UB(K),K=1,MCON)
2006 FORMAT(6X,'LEVEL (X)*',10F10.5,/)!
DO 29 K=1,MCON
UB(K)=UB(K)*UUNIT*UVOL(K)/100.0
29 CONTINUE
WRITE (2) UB,UUNIT,UVOL
C.
THERMAL FLUX MODE DEVIATION DUE TO DEPLOY THE CONTROLLERS
DO 28 L=1,NMOD
DO 28 M=1,NMOD
TRMX(L,M)=SMX22(L,M)
28 CONTINUE
CALL GINV(TRMX,TRMY,W1,W2,10,NMOD,NMOD,ERROR)
DO 30 L=1,NMOD
W1(L)=0.0
DO 31 M=1,NMOD
SUM=0.0
DO 32 K=1,MCON
SUM=SUM+MTMX(M,K)*UB(K)
32 CONTINUE
W1(L)=W1(L)-TRMX(M,L)*(PMTN(M)-SUM)
31 CONTINUE
30 CONTINUE
WRITE (6,2007)
2007 FORMAT(//,10X,'THERMAL FLUX MODE ERROR */',10X,'MODE ERROR*//')
WRITE (6,2008) (L,W1(L),L=1,NMOD)
2008 FORMAT(7X,15,10X,E12.6)
RVOL=XTSO=0.0
DO 34 I1=NRX
DO 34 J1=NTY
DV=DX(I)*DY(J)
IF (XT(I,J),NE,0.0) RVOL=RVOL+DV
XI(I,J)=XT(I,J)/Amp0
DO 33 L1=NMOD
XI(I,J)=XI(I,J)+WI(L)*PS(I,J,L)
33 CONTINUE
XI(I,J)=XI(I,J)*AMP0
IF (.XT(I,J),EQ,0.0) GO TO 34
XTSO=XTSO+((XI(I,J)-XT(I,J))/XT(I,J))**2.0*DV
34 CONTINUE
XTSO=SQRT(XTSo/RVOL)*100.0
SUMF=SUMF=SUMU=0.0
DO 35 L1=NMOD
CC=0.0
DO 36 M1=NMOD
CC=CC+SMX1(L,M)*WI(M)
36 CONTINUE
SUMF=SUMF+WI(L)*CC
35 CONTINUE
TUVOl=0.0
DO 37 K1=MCON
SUMU=SUMU+UB(K)*UQ(K)*RFAC
TUVOl=TUVOl+UB(K)
37 CONTINUE
SUMF=SUMF/2.0
SUMU=SUMU/2.0
TUVOl=TUVOl/UUNIT
SUMF=SUMF+SUMU
WRITE (6,2031) SUMF, SUMU, SUMF, XTSO, TUVOl
2031 FORMAT(/,10X,"PENALTY IN FLUX SHAPE = ",E12.5/,10X,
1*PENALTY IN CONTROL EFFORT = ",E12.5/,10X,
2*TOTAL PENALTY PAID BY THE COST FUNCTIONAL = ",E12.5/,10X,
3*STANDARD DEVIATION OF THERMAL Flux DISTRIBUTION = ",F12.5",/(X)
4/10X,"TOTAL NUMBER OF LATTICE OCCUPIED BY CONTROLLERS = ",
5F12.5/,/
CALL HEAD
WRITE (6,2009)
2009 FORMAT(10X,"THERMAL FLUX DISTRIBUTION WITH THE STATIC CONTROLLER I")
CALL XMAP(XI)
CALL HEAD
WRITE (6,2030)
2030 FORMAT(/,30X,"FLUX REGULATION INFORMATION",//)
WRITE (6,2031) GSC, RFAC, RFAC
2032 FORMAT(/,10X,"SCALE FACTOR FOR SYSTEM MATRIX = ",E12.5/,10X,
1*SCALE FACTOR FOR THE MODE AMPLITUDE LOSS = ",E12.5/,10X,
2*SCALE FACTOR FOR CONTROL COST = ",E12.5/,/
C COMPUTES THE DYNAMIC RANGE OF CONTROLLERS FOR FLUX REGULATION
C INITIAL SOLUTION OF THE RICCATI EQUATION
IL=1
THET=1.0E-2
XTIME=1.0E-2
DO 38 L=1,NMOD
DO 38 M=1,NMOD
PMTX(L,M)=DMTX(L,M)**TIME
TRMX(L,M)=PMTX(L,M)**TIME
38 CONTINUE
DO 39 L=1,NMOD
DO 39 M=1,NMOD
SMX22(L,M)=SMX22(L,M)/GSC.
39 CONTINUE
80 CONTINUE
ERR=0.0
XIL=IL+1.0
DO 81 L=1,NMOD
DO 81 M=1,NMOD
TRMY(L,M)=(DOT3(20,10,1,NMOD,SMX22,TRMX,L,M)+
1
DOT4(10,20,1,NMOD,TRMX,SMX22,L,M))/XIL.
TRMY(L,M)=TRMY(L,M)
81 CONTINUE
DO 82 L=1,NMOD
DO 82 M=1,NMOD
PMTX(L,M)=PMTX(L,M)+TRMY(L,M)
TRMX(L,M)=TRMY(L,M)**TIME
XERR=ABS(TRMY(L,M)/PMTX(L,M))
IF (XERR,GE,ERR) ERR=XERR
82 CONTINUE
IF (ERR.LE.EP) GO TO 83
IL=IL+1
IF (IL.GE.50) GO TO 83
GO TO 80
83 CONTINUE
CALL GINV(PMTX,TRMY,W1,W2,10,NMOD,NMOD,ERROR)
IF (ERROR,GE,1.0E-5) WRITE (6,9999) ERROR
9999 FORMAT(10X,*INVERSE ERROR = *E12.6)
IT=1
DO 85 L=1,NMOD
DO 85 M=1,NMOD
TRMX(L,M)=PMTX(L,M)
85 CONTINUE
WRITE (6,2025)
2025 FORMAT(//,8X,*IT CONVERGENCE CRITERION ERR IT TIME*)
100 CONTINUE
EPS=1.0
CALL SECOND(SEC1)
DO 42 L=1,NMOD
DO 42 M=1,NMOD
SMXX(L,M)=SMX22(L,M)-DOT3(10,10,1,NMOD,DMTX,PMTX,L,M)
QMTX(L,M)=-SMX11(L,M)
DO 43 LL=1,NMOD
CC=0.0
40 CONTINUE
QMTX(L,M)=QMTX(LL,M)*PMTX(MM,L)*CC
43 CONTINUE
42 CONTINUE
CALL AXPXA(SMXX, TRMX, QMTX, TRMY, NMOD, 10, 10, 10, 10, EPS)
CON=0.0
DO 44 L=1, NMOD
DO 44 M=1, NMOD
QMTX(L, M)=QMTX(L, M)*OSC
IF (TRMX(L, M), EQ, 0.0) GO TO 44
ERR=QMTX(L, M)/TRMX(L, M)
ERROR=ABS(1.0-ERR)
CON=AMAX1(CON, ERROR)
44 CONTINUE
CALL SECOND(SEC2)
SEC=SEC2-SEC1
WRITE (6,2026) IT, CON, EPS, SEC
2026 FORMAT(8X,12,3E12.5)
IF (CON.LE.1.0E-3) GO TO 50
DO 440 L=1, NMOD
DO 440 M=1, NMOD
TRMY(L, M)=0.0
TRMX(L, M)=QMTX(L, M)
DO 441 LL=1, NMOD
TRMX(L, M)=TRMX(L, M)-QMTX(LL, L)*DOT3(10, 10, 1, NMOD, DMTX, QMTX, LL, M)
441 CONTINUE
TRMX(L, M)=TRMX(L, M)+DOT3(10, 20, 1, NMOD, QMTX, SMX22, L, M)+
1 SMX11(L, M)*DOT2(20, 10, 1, NMOD, SMX32, QMTX, L, M)
440 CONTINUE
IT=IT+1
DO 46 L=1, NMOD
DO 46 M=1, NMOD
PMTX(L, M)=QMTX(L, M)-THET*TRMY(L, M)
46 CONTINUE
IF (IT.LE.21) GO TO 100
WRITE (6,2010)
2010 FORMAT(*10X, 'WARNING - RICCATI EQUATION ILL-CONDITIONED', '')
C COMPUTES NORM OF CONTROLLER RANGE FOR FLUX REGULATION
50 CONTINUE
DO 45 L=1, NMOD
DO 45 M=1, NMOD
PMTX(L, M)=QMTX(L, M)/OSC
45 CONTINUE
CALL HEAD
WRITE(6,2027)
2027 FORMAT(*10X, 'SOLUTION MATRIX OF THE RICCATI EQUATION (P)', '')
DO 47 L=1, NMOD
WRITE (6,2024) (PMTX(L, M), M=1, NMOD)
47 CONTINUE
DO 51 L=1, NMOD
DO 51 K=1, MCON
TRMX(L, K)=DOT2(10, 10, 1, NMOD, BMTX, PMTX, K, L)/RFAC
51 CONTINUE
WRITE(6,2028)
2028 FORMAT(*10X, 'MODAL FEEDBACK GAIN MATRIX (-INV(R)*TRAN(B)*(P)', '')
DO 48 K=1, MCON
WRITE (6,2024) (TRMX(L, K), L=1, NMOD)
48 CONTINUE
WRITE(2) TRMX
UNORM=0.0
DO 52 K=1,MCON
    UNORM=UNORM+DOT(10,1,NMOD,TRMX,K,K)
    CONTINUE
    UNORM=SORRT(UNORM)
    UDEL=UNORM/NMOD
DO 53 K=1,MCON
    CC=0.0
    W1(K)=0.0
DO 531 L=1,NMOD
    CC=TRMX(L,K)*TRMX(L,K)
    W1(K)=W1(K)+CC
    CONTINUE
    W1(K)=SORT(W1(K))*TILT
CONTINUE
WRITE (6,2011) UNORM
2011 FORMAT(/,10X,'NORM OF (-INV(K)*TRAN(B)*(F))-MATRIX = ','E12.6)
WRITE (6,2012)
2012 FORMAT(/,10X,'DYNAMIC RANGE OF CONTROLLERS FOR REGULATION (FILL
        1ED)*/)
WRITE (6,2015)
2015 FORMAT(/,5X,'CONTROLLER ',15X,'1 ',6X,'2 ',9X,'3 ',10X,'4')
    DO 54 K=1,MCON
        UTEMP=100.0/UNIT/UVOL(K)
        UC(K)=(UB(K)+W1(K))*UTEMP
    CONTINUE
54 CONTINUE
WRITE (6,2013) (UC(K),K=1,MCON)
2013 FORMAT(4X,'UPPER LIMIT',10F10.5)
    DO 55 K=1,MCON
        UTEMP=100.0/UNIT/UVOL(K)
        UC(K)=(UB(K)-W1(K))*UTEMP
    CONTINUE
55 CONTINUE
WRITE (6,2014) (UC(K),K=1,MCON)
2014 FORMAT(4X,'LOWER LIMIT',10F10.5)
C COUNTERACTING REACTIVITY DISTURBANCES
    DO 56 L=1,NMOD
        DO 56 M=1,NMOD
            TRMX(L,M)=DOT3(10,20,1,NMOD,QMTX,SMMX,L,M)
        CONTINUE
56 CONTINUE
    DO 57 L=1,NMOD
        DO 57 M=1,NMOD
            SMMX(L,M)=TRMX(L,M)*SIGMA
        CONTINUE
57 CONTINUE
EPS=2.0
    DO 58 L=1,NMOD
        DO 58 M=1,NMOD
            SMXX(L,M)=SMXX(L,M)-DOT3(10,10,1,NMOD,DMTX,QMTX,L,M)
        CONTINUE
58 CONTINUE
    DO 59 L=1,NMOD
        DO 59 M=1,NMOD
            SMXX(L,M)=SMXX(L,M)*QSC
            QMTX(L,M)=-DOT4(20,20,1,NMOD,SMNX,SMMX,L,M)
        CONTINUE
59 CONTINUE
CALL AXPX(3,TRMX,QMTX,TRMY,NMOD,10,10,10,EPS).
CALL HEAD
WRITE (6,2029)
2029 FORMAT(/,10X,*SOLUTION MATRIX OF LYAPUNOV EQUATION (K)*,/)
DO 60 L=1,NMOD
WRITE (6,2024) (GMTX(L,M),M=1,NMOD)
60 CONTINUE
DO 61 K=1,MCON
SUMWW=0.0
DO 62 L=1,NMOD
SUMWW=SUMWW+BMTX(L,K)*DOT3(10,10,1,NMOD,GMTX,BMTX,L,K)/(RFAC*1)
END CONTINUE
W2(K)=ABS(SUMWW)
W2(K)=SORT(W2(K))
61 CONTINUE
WRITE (6,2017)
2017 FORMAT(/,10X,*DYNAMIC RANGE OF CONTROLLERS FOR COUNTERACTION (X F 11LLED)*,/)
WRITE (6,2015)
DO 63 K=1,MCON
UTEMP=100.0/UNIT(UVOL(K))
UC(K)=(UB(K)+W1(K)+W2(K))*UTEMP
63 CONTINUE
WRITE (6,2013) (UC(K),K=1,MCON)
DO 64 K=1,MCON
UTEMP=100.0/UNIT(UVOL(K))
UC(K)=(UB(K)-W1(K)-W2(K))*UTEMP
64 CONTINUE
WRITE (6,2014) (UC(K),K=1,MCON)
RETURN
END

SUBROUTINE SIMUL(NPROB)
C**** SIMULATION STUDY FOR A TRANSIENT WITH A GIVEN CONTROL SYSTEM
COMMON /STFN/ XF(31,23), XT(31,23), XI(31,23), XE(31,23)
COMMON /XSCN/ DIF1(25), DIF2(25), SIGA1(25), SIGA2(25),
  1 SIGR(25), SIGF(25), ALFA(25), HFA(25),
  2 XK(10), U1, V2, XNU,
  3 GAM1, GAMX, SIGE, ZLAM1,
  4 ZLAMX, XKS,
COMMON /GEOM/ MAT(31,23), DX(31), DY(23), X(31), Y(23)
COMMON /MISC/ JITLE(18), NPAGE, NMOD, M2, M4,
  1 NHK, NRY, NTT, PWR, BLANK,
  2 CX, CY, NSX, NSY, IUNSTB,
  3 RFAC, SFAC, QFAC, AMP0
COMMON /SYSH/ SMX1(20,20), SMX12(20,20), SMX21(21,21),
  1 SMX22(20,20), SMX2(20,20)
COMMON /XYMOD/ PF(31,23,8), PS(31,23,8), FNORM(8), TNORM(8)
COMMON /MCON/ GMX(10), NSIGN(10), YL(10), XL(10)
COMMON /KCON/ BMTX(10,20), CMTX(10,20), UB(20)
DIMENSION AMP(16,2), AMP1(16), AMX(10,10), BMX(10,10),
  1 TRMX(10,10), DMTX(10,10), WI(10), W2(10),
  2 US(10), GMXX(10,10), UVOL(10), CMX(10,10),
  3 AMPF(16)
C
TIME=0.0
LPAGE=1
REWRITE 2
READ (2) MCON
READ (2) XL,YL,NSIGN
READ (2) BMTX
READ (2) UB,UNIT,UVOL
READ (2) GMTX
READ (5,1001) TMAX,PLVL,PRAT,DT1
1001 FORMAT(8E10.4)
2000 FORMAT(30X,'TRANSIENT SIMULATION WITH A GIVEN CONTROL SYSTEM',/)
    CALL SMTXRX(MERDB)
    CALL HEAD
    WRITE (6,2000)
    WRITE (6,2002) TMAX,PLVL,PRAT,DT1
2002 FORMAT(10X,'TRANSIENT PERIOD = ',1PE10.4,' (HRS)',/)
    10X
    INITIAL POWER LEVEL = ',1PE10.4,' (%)',/)
    10X
    20X ALLOWABLE POWER CHANGE RATE = ',1PE10.4,' (%)SEC',/)
    10X
    TIME INCREMENT = ',1PE10.4,' (HRS)',/)
    NMOD1=NMOD+1
    NMOD11=NMOD1+1
    NMOD2=2*NMOD
C
READ INITIAL PERTURBED MODE AMPLITUDE INCREMENTS
DO 10 L=1,NMOD2
   DO 10 I=1,2
      AMP(L,I)=0.0
10 CONTINUE
2003 FORMAT(//,10X,'MODAL AMPLITUDE INCREMENT',/)
    3X,'IDENT MODE=1',/)
    1   2   3   4   5   6   7   8   9   10
2004 FORMAT(1X,'FAST FLX ','B(1PE10.3)
2005 FORMAT(1X,'THML FLX ','B(1PE10.3)
C
INITIAL AMPLITUDES FOR IODINE AND XENON
SUM=SUM=0.0
DO 5 I=1,NRX
   DO 5 J=1,NRY
      DX(I,J)*DY(J)
      SUM=SUM+PS(I,J,1)*DV
5 CONTINUE
L=1,NMOD
IF (L.EQ.1) GO TO 5
AMPL=AMP(L,1)
SUM=SUM+AMPL*PS(I,J,L)*DV
5 CONTINUE
PTMP=SUM/SUM
RTEMP=PLVL/100.0
AMPL(NMOD1,1)=RTEMP*(1.0-PTMP)-1.0
DO 6 L=2,NMOD
   L=NMOD+L
   AMP(L,1)=RTEMP*AMP(L,1)
6 CONTINUE
DO 7 L=1,NMOD
DO 7 M=1,NMOD
AMX(L,M)=SMX11(L,M)
7 CONTINUE
CALL GINV(AMX,TRMX,W1,W2,10,NMOD,NMOD,ERROR)
DO 11 L=1,NMOD
DO 11 M=1,NMOD
M1=NMOD+M
TRMX(L,M)=SMX11(L,M1)
11 CONTINUE
DO 12 L=1,NMOD
DO 12 M=1,NMOD
M1=NMOD+M
AMP(L,1)=AMP(L,1)-DOT2(10,10,1,NMOD,AMX,TRMX,L,M)*AMP(M1,1)
12 CONTINUE
DO 13 L=1,NMOD
DO 13 M=1,NMOD
CMTX(L,M)=DOT4(10,10,1,MCON,BMTX,GHTX,L,M)
13 CONTINUE
DO 24 L=1,NMOD
L1=NMOD+L
DO 24 M=1,NMOD
M1=NMOD+M
DO 25 N=1,NMOD
N1=NMOD+N
DO 25 NN=1,NMOD
CMX(L,M)=SMX11(L1,M1)-SMX11(L1,N)*AMX(NN,N)*SMX11(NN,M1)
25 CONTINUE
BMTX(L,M)=CMX(L,M)+CMTX(L,M)
24 CONTINUE
CALL GINV(BMX,TRMX,W1,W2,10,NMOD,NMOD,ERROR)
CALL GINV(CMX,TRMX,W1,W2,10,NMOD,NMOD,ERROR)
DO 14 L=1,NMOD
DO 14 M=1,NMOD
DHTX(L,M)=SMX22(L,M)
14 CONTINUE
CALL GINV(DMTX,TRMX,W1,W2,10,NMOD,NMOD,ERROR)
DO 15 L=1,NMOD
DO 15 M=1,NMOD
M1=NMOD+M
TRMX(L,M)=SMX21(L,M1)
15 CONTINUE
DO 16 L=1,NMOD
DO 16 M=1,NMOD
M1=NMOD+M
AMP(L,2)=AMP(L,2)-DOT2(10,10,1,NMOD,DHTX,TRMX,L,M)*AMP(M1,1)
16 CONTINUE
DO 20 L=1,NMOD
L1=NMOD+L
DO 20 M=1,NMOD
M1=NMOD+M
DHTX(L,M)=SMX22(L1,M1)
20 CONTINUE
CALL GINV(DMTX,TRMX,W1,W2,10,NMOD,NMOD,ERROR)
DO 21 L=1,NMOD
   L=L+MOD
   DO 21 M=1,NMOD
      M=M+MOD
      DO 22 N=1,NMOD
         N=N+MOD
         AMP(L1,2)=AMP(L1,2)-DMTX(M,L)*(SMX21(M1,N1)*AMP(N1,1)+SMX22(M,N1)*AMP(N,2))
      22 CONTINUE
   21 CONTINUE
C INITIAL CONTROLLER LEVELS ASSUMED TO BE FROZEN
DO 26 K=1,HCON
   US(K)=0.0
26 CONTINUE
WRITE (6,2008) TIME
WRITE (6,2009) (US(K),K=1,HCON)
WRITE (6,2003)
WRITE (6,2004) (AMP(L,1),L=1,NMOD)
WRITE (6,2005) (AMP(L,1),L=NMOD1,NMOD2)
WRITE (6,2006) (AMP(L,2),L=1,NMOD)
WRITE (6,2007) (AMP(L,2),L=NMOD1,NMOD2)
2004 FORMAT(1X,'IDNE CON *8(1PE10.3))
2007 FORMAT(1X,'XENN CON *8(1PE10.3))
2008 FORMAT(/,10X,'CONTROLLERS LEVEL AT TIME (X FILLED) = ',F10.5, '(H
1RS)*/,2X, 'IDENT = 1 ',2, 3, 4, 5, 6, 7, 8, 9, 10*/)
2009 FORMAT(2X,'LEVEL*',3X,10F10.5)
C TRANSIENT CALCULATION
DO 27 K=1,HCON
   US(K)=US(K)*UNIT*UVOL(K)/100.0
27 CONTINUE
BT=ABS(100.0-PLVL)/FRAT
100 CONTINUE
BT=BT*3600.0
IF (TIME.NE.0.0) GO TO 30
BT=DT0
IF (BT.EQ.0.0) BT=5.0
30 CONTINUE
TIME=TIME+DT
XTIME=TIME/3600.
IF (XTIME.GT.TIME_MAX) GO TO 999
NSTEP=60
DSTEP=DT/NSTEP
IF (TIME.GT.5.0) GO TO 37
NSTEP=1
DSTEP=DT
WRITE (6,2001)
2001 FORMAT(/,'10X,'***** CONTROLLERS ARE ACTIVATED*')
37 CONTINUE
DO 50 IT=1,NSTEP
DO 35 K=1,MCON
DO 36 M=1,NMOD
M1=NMOD+M
US(K)=US(K)+GTX(M+K)*AMP(M1,1)
36 CONTINUE
US(K)=US(K)*100.0/(UNIT*UVOL(K))
35 CONTINUE
DO 31 L=1,NMOD
L1=NMOD+L
AMP0(L)=AMP(L,1)
AMP0(L)=AMP(L1,1)
AMP(L)=AMP(L2,1)
AMP(L)=AMP(L1,2)
DO 31 M=1,NMOD
M1=NMOD+M
TRMX(L,M)=SMX12(L1,M1)
31 CONTINUE
DO 32 L=1,NMOD
L1=NMOD+L
AMP(L)=0.0
AMP(L)=0.0
DO 32 M=1,NMOD
M1=NMOD+M
AMP(L1,1)=AMP(L1,1)-DOT2(10,10,1,NMOD,SMX,TRMX,L,M)*AMP1(M1)
32 CONTINUE
DO 33 L=1,NMOD
DO 33 M=1,NMOD
M1=NMOD+M
TRMX(L,M)=SMX11(L,M1)
33 CONTINUE
DO 34 L=1,NMOD
DO 34 M=1,NMOD
M1=NMOD+M
AMP(L,1)=AMP(L,1)-DOT2(10,10,1,NMOD,AMX,TRMX,L,M)*AMP(M1,1)
34 CONTINUE
DO 40 L=1,NMOD
L1=NMOD+L
DO 40 M=1,NMOD
M1=NMOD+M
ATEMP=(AMP0(M1)+AMP(M1,1))/2.0
AMP(L,2)=AMP(L,2)+(SMX21(L,M1)*ATEMP+SMX22(L,M1)*AMP1(M))
1 *DTSTEP
AMP(L1,2)=AMP(L1,2)+(SMX21(L1,M1)*ATEMP+SMX22(L1,M1)*AMP1(M1))
1 +SMX22(L1,M1)*AMP1(M1)*DTSTEP
41 CONTINUE
40 CONTINUE
IF (IT.NE. NSTEP), GO TO 42
WRITE (6,2008) XTIME
WRITE (6,2009) (US(K), K=1, MCON)
WRITE (6,2010)
WRITE (6,2004) (AMF(L,1), L=1, NMOD)
WRITE (6,2005) (AMF(L,1), L=NM0D1, NMOD2)
WRITE (6,2006) (AMF(L,2), L=1, NMOD)
WRITE (6,2007) (AMF(L,2), L=NM0D1, NMOD2)
42 CONTINUE
   DO 43 K=1, MCON.
   US(K)=US(K)*UUNIT*UVOL(K)/100.0
43 CONTINUE
50 CONTINUE
   LPAGE=LPAGE+1
   XPAGE=LPAGE/3.0
   MPAGE=XPAGE
   YPAGE=MPAGE
   DPAGE=XPAGE-YPAGE
   IF (DPAGE.EQ.0.0) CALL HEAD
   GO TO 100
999 RETURN
END