SIMULATION ANALYSIS OF THE IMPLICATIONS OF THE ALTERNATIVE LIFE-CYCLE MODELS

SIMULATION ANALYSIS OF THE IMPLICATIONS OF THE ALTERNATIVE LIFE-CYCLE MODELS

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ABSTRACT

This thesis deals with the important issue of the elasticity of saving with respect to the interest rate. Michael Boskin, and more recently, Lawrence Summers, have argued that saving is much more interest-elastic than economists have generally believed, and as a consequence, that the dynamic efficiency losses from capital income taxation are much higher than they were previously thought to be. In Summers' life-cycle simulation model an increase in the interest rate depresses the present value of future labour income and this leads to declines in the consumption of younger cohorts, more saving, and a higher capital stock. These results are established with a CES utility function. I argue that it is reasonable to introduce minimum consumption levels into the model and that when this is done, younger cohorts do not decrease their consumption by as much in response to an interest rate increase. I show that the interest elasticity of saving is significantly reduced and may even be negative.

In his analysis, Summers assumed an inelastic labour supply. I have relaxed this assumption by allowing both consumption and leisure to be decision variables of the individual. For a Cobb-Douglas utility function, I find the interest elasticity of saving to be 0.008, compared to 3.36 reported by Summers. Moreover, when a lower bound is imposed on consumption, the interest elasticity becomes negative.

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CHAPTER 1

INTRODUCTION

The interest elasticity of the aggregate saving rate is a key parameter for a number of important questions concerning the government's influence on capital formation. Implicitly, policy-makers rely heavily on the magnitude of this parameter to address questions regarding the desirability of capital income taxation and to assess the welfare implications of such a measure. For instance, if saving were very interest-elastic, shifting away from capital income taxation would lead to significant increase in capital formation.

Economists agree on the importance of the parameter and it has been given considerable attention in the literature. However, attempts at empirical estimation have produced conflicting results and made the interest elasticity of saving issue a controversial one. One controversy centers on whether the magnitude of the response of aggregate saving to changes in the interest rate is high or low. Depending on the answer to this question, policies are formulated which could have very important implications for capital formation and therefore for economic growth.

Prior to the publication of Boskin's (1978) controversial paper, most economists concluded that the interest elasticity of saving was likely to be small. Boskin estimated the parameter and reported a value of 0.4. His result was considered to be high, and since then the profession has moved away from a consensus view on this issue. Howrey and Hymans (1978) questioned Boskin's finding and using econometric estimation methods, demonstrated that his result is far from being robust.

In a challenge to the widely held view that the elasticity is low, Summers (1981) claimed to have formulated a realistic life-cycle model yielding a large and positive interest elasticity of saving. There are major differences between Summers' study and the previous studies. First, he used a multi-period life-cycle model to address the issue, in contrast to the static framework adopted by other authors. Second, he employed a simulation approach instead of econometric estimation to obtain the value of the elasticity parameter. His findings are much higher than those reported by Boskin.

Keeping the utility function employed by Summers, Evans (1983) showed that with negative time preferences, the interest elasticities are far lower. Furthermore, incorporating intergenerational transfers into Summers' model, he demonstrated that negative elasticities cannot be ruled out on a priori grounds. On this basis, Evans argued that Summers' findings were not robust.

The studies by Summers and Evans are rooted in the standard homothetic utility function, and in that regard they are identical. However, the 'true' utility function underlying the preferences of the individual is not known with any degree of certainty and therefore must be selected arbitrarily from the many possible functional forms of utility functions that exist in the literature. It is therefore reasonable to employ alternative functional forms of the utility function in considering the issue of the interest elasticity of saving. In the present study, I suggested a quasi-homothetic CES utility function which is more general than the standard homothetic CES function employed by the foregoing authors in addressing the same issue.

This thesis questions the claim put forward by Summers that "the theory when formulated realistically, implies interest elasticities well in excess of unity" (1981, p. 534). To examine this claim, first I replaced the standard homotheticity assumption adopted by Summers with a quasi-homotheticity assumption. My results clearly show that the claim does not stand up in the face of this change. Second, I relaxed the assumption of inelastic labour supply in allowing for consumption-leisure choice in a Stone-Geary utility function. In this setting, the interest elasticity is found to be much lower than the one reported by Summers. This demonstrates that the claim is not only sensitive to functional specification of the utility function but also to the assumption underlying labour supply.

The structure of the thesis is as follows. In chapter 2, I present reviews of some selected literature on the interest elasticity of saving. In chapter 3, a theoretical life-cycle model based on a quasi-homothetic utility function is developed. Simulation results based on the theoretical model are reported and discussed in chapter 4. The assumption of inelastic labour supply is then relaxed and the resulting alternative theoretical model and some simulation results based upon it are discussed in chapter 5. Finally, in chapter 6, a summary of the key findings of the thesis is provided and possible extensions of the present study are indicated.

CHAPTER 2

REVIEW OF SELECTED LITERATURE ON THE INTEREST ELASTICITY OF SAVING

The important role of aggregate saving in determing the long-run growth of the economy has long been recognized by economists. The effect of a change in the after-tax real interest rate on saving has attracted much attention primarily because the interest elasticity of saving has important policy implications. However, the empirical literature, reveals that the interest elasticity of saving is very controversial and far from being settled.

Section 2.1 of this chapter presents historical perspectives on the relationship between the interest rate and saving. Empirical evidence on the interest elasticity of the saving parameter is surveyed in Section 2.2. Possible sources of empirical inconsistencies are noted in Section 2.3. Selected econometric studies of saving, including those of Boskin (1978), Carlino (1983) and Zietz (1984), are discussed in Section 2.4.

2.1 PREVIOUS STUDIES OF SAVING BEHAVIOUR

There have been only a few empirical studies that have attempted to deal with the role of interest rate in aggregate consumption and saving behaviour. This was primarily due to the fact that the effect of the interest rate on aggregate consumption was considered to be insignificant, and as a result empirical work on saving focused on estimating the Keynesian-type of consumption function. In this context the inclusion of the interest rate as a variable in the aggregate consumption function appeared unnecessary or unrealistic. Keynes, in The General Theory of Employment, Interest and Money, noted

> ...the main conclusion suggested by experience... that the short-period influence of the rate of interest on individual spending out of a given income is secondary and relatively unimportant, except, perhaps, where unusually large changes are in question.
>
> (Keynes, 1963, p. 94)

Recently, however, empirical studies have led to the realization that the interest rate may indeed be a significant variable in determining aggregate consumption. Researchers have examined the effect of the interest rate on consumption employing one or more of the following interest rate definitions: a nominal interest rate, a nominal net-of-tax rate, a real rate and a real net-of-tax rate.

The models that are used to study the role of the interest rate in the determination of aggregate consumption or saving can be broadly divided into two categories. The first category specifies an aggregate consumption or saving function at one stage of the analysis and employs econometric methods to estimate it. The studies in this category include those of Hamburger (1967), Wright (1967, 1969), Houthakker-Taylor (1970), Taylor (1971), Heien (1972), Juster-Wachtel (1972b), Juster-Taylor (1975), Weber (1970, 1975), Blinder (1975), Springer (1975, 1977), Boskin (1978), Gylfason (1981), Carlino (1982) and Zietz (1984). The second category of models is based on multi-period life-cycle theory and uses a simulation approach to examine the issue. Some of the studies in this category are those of Summers (1981), Evans (1983) and Seidman (1983). The present study also falls into this second category.

2.2 EMPIRICAL EVIDENCE

A survey of the empirical evidence reveals inconsistent results with regard to the interest elasticity of saving. Houthakker and Taylor (1970), Weber (1970, 1975), and Springer (1975) reported negative interest elasticities of saving. Wright (1967, 1969), Taylor (1971), Heien (1972), Blinder (1975), Boskin (1978), and Gylfason (1981) found positive elasticities, but the magnitudes vary greatly from one study to another, ranging from as low as 0.03 to as high as 1.76[1]. Closer examination of these studies reveals differences in the dependent and independent variables of the estimating equations, in estimation methods, in data, in sample periods, and perhaps most important of all, in the definition of the interest rate variable. Economic theory indicates the real rate of interest to be the proper variable to be employed in examining the issue. However, in spite of this, all the studies mentioned in this section, except those by Blinder (1975), Boskin (1978), Howrey and Hymans (1978), Carlino (1982) and Zietz (1984) are based on a nominal interest rate variable.

Blinder (1975) obtained a very low savings elasticity of about 0.03, which is compatible with the widely held view that saving is relatively insensitive to interest rate changes. On the other hand, Boskin (1978) reported an interest elasticity of saving of 0.4, which is regarded to be high by many economists. Furthermore, Boskin concluded that his "results are striking: a variety of functional forms, estimation methods, and definitions of the real after-tax rate of return invariably lead to the conclusion of a substantial interest elasticity of saving" (1978, p. 53). His claim became a center of controversy and attracted the attention of many economists. Howrey and Hymans (1978) described Boskin's finding as "a novel and intriguing result that calls for replication and further scrutiny" (1978, p. 11). They performed sensitivity analysis and demonstrated that his results are not robust to minor changes in specification. Similarly, a study by Carlino (1982) found Boskin's claim hard to replicate.

A recent development in the empirical literature has been the emergence of multi-period life-cycle models of saving behaviour that depend on a simulation framework of analysis, rather than on econometric estimation. As mentioned earlier, the studies conducted by Summers (1981), Evans (1983) and Seidman (1983) are based on this approach.

Such studies have produced some inconsistent results. Summers reported interest elasticities of saving that are very high and claimed to have developed "a prima facie theoretical case for a high interest elasticity of saving" (1981, p. 537). On the other hand, Evans demonstrated that it is not impossible for the interest elasticity of saving to be negative.

The studies that employ a simulation approach to examine the relationship between the interest rate and saving will be discussed in chapter 4.

2.3 SOURCES OF INCONSISTENCIES

Empirical studies of the effects of the interest rate on consumption reveal inconsistent results with regard to the sign and magnitude of the interest elasticity of saving. Some of the possible reasons for this are discussed in the section below.

The empirical studies are characterized by inconsistencies in the definition of the interest rate variable, the functional form of the equation to be estimated, the nature of the data employed and the estimation method. These factors have been systematically documented in a tabular form by Gylfason (1981, Table 1, 234).

With respect to the interest rate variable, both nominal and real versions have been used. The study by Carlino (1982) attempts to resolve the controversy surrounding the interest elasticity of saving by focusing on the construction of the interest rate. The implication of alternative specifications of the consumption function is addressed by Zietz (1984). Although various studies have used either quarterly or annual data, to my knowledge no one has attempted to shed light on how the differences in the nature of the data could affect the interest-elasticity of saving.

2.4 ECONOMETRIC MODELS OF SAVING

There are a number of studies on savings that are based on econometric estimation methods. To shed some light on the general feature of these models, I will provide reviews of the studies conducted by Boskin (1978), Carlino (1982) and Zietz (1984).

Boskin

Boskin's main goal was to reconsider the widely held view that the interest elasticity of the saving rate is negligible, and in particular, to examine the so called 'Denison Law'. According to this 'law', the saving rate is essentially stable regardless of the changes in the tax system or other changes in the real after-tax rate of return on capital. Before Boskin tackled this issue, David and Scadding (1974) re-examined 'Denison's Law' and found evidence that supported the view that saving rates are constant.

To study the above mentioned issue, Boskin set forth an econometric model and specified his key equation as follows:

$$\ell n\ C_p = a_0 + a_1\ \ell n\ Y_p + a_2\ \ell n\ (Y_p)_{-1} + a_3 \ell n\ (W_p)_{-1}$$

$$+ a_4\ \ell n\ U^* + a_5\ (R - \Pi) + a_6 \Pi$$

where C_p = real per capita private consumption;

Y_p = real per capita private disposable income;

W_p = end-of-year real per capita wealth;

U^* = unemployment rate;

R = expected nominal after-tax return on capital;

Π = expected rate of inflation.

Boskin estimated different versions of the equation and found the effect of the interest rate on saving to be non-negligible, positive and robust. He stressed the robustness of the result by saying "...a variety of functional forms, estimation methods, and definitions of the real after-tax rate of return invariably lead to the conclusion of a substantial interest elasticity of saving" (1978, p. S3). His finding indicated a higher interest elasticity of saving than those reported by earlier studies[2]. Given the ad hoc specification of the model, Boskin's claim with regard to the insensitivity of his finding is questionable.

Howrey and Hymans (1978) performed sensitivity analysis on the equation estimated by Boskin, which was as follows (with estimated standard errors in brackets):

$$\begin{aligned}\ln C_p = & \underset{(1.34)}{-0.46} + \underset{(0.12)}{0.57} \ln Y_p + \underset{(0.08)}{0.18} \ln (Y_p)_{-1} \\ & + \underset{(0.07)}{0.26} \ln (W_p)_{-1} - \underset{(0.011)}{0.003} \ln U^* \\ & - \underset{(0.33)}{1.07} (R - \Pi) - \underset{(0.06)}{0.029} \Pi \\ & \qquad R^2 = 0.99\end{aligned}$$

Howrey and Hymans dropped the observation for 1934 from the $(R - \Pi)$ series and re-estimated Boskin's equation. As a result of this change, the coefficient of (-1.07) on the $(R - \Pi)$ variable became (-0.877) and the t-statistic changed from (-3.24) to (-1.62). They thus demonstrated the non-robustness of Boskin's findings. Furthermore, Howrey and Hymans replaced the $\ln U^*$ variable with U^*_{-2} (the unemployment rate with a two-year lag) and this time the t-statistic for the coefficient $(R - \Pi)$ was found to be (-0.90), which implies statistical insignificance, contrary to the result reached by Boskin. As a final experiment on the sensitivity of the Boskin result, Howrey and Hymans employed alternative interest rates other than the one adopted by Boskin and restricted the consumption regressions to postwar data (1947-69). This yielded a positive coefficient for the interest rate. Their finding led them to describe Boskin's finding as "....a novel and intriguing result that calls for replication and further scrutiny" (1978, p. 665).

Carlino

In 1982, Carlino attempted to address the causes that lead to divergent and often conflicting results that had emerged from the empirical studies of saving behaviour. He thought the inconsistency in the sign and statistical significance of the coefficient of the interest rate variable might have been related to the manner in which this variable is constructed. To explore this conjecture, he suggested an aggregate consumption function of the form:

$$C = b_0 + b_1Y^e + b_2(rY^e)$$

where

C = consumption;

Y^e = expected disposable income;

r = the interest rate.

He defined the interest rate variable in four different ways -- the nominal interest rate, the nominal net-of-tax rate, the real interest rate, and the real net-of-tax rate -- and estimated the equation given above for each definition. Carlino obtained a positive and statistically insignificant coefficient for the interest rate variable using the real net-of-tax rate. This result contradicts Boskin's finding of a negative and significant coefficient. Furthermore, when Carlino experimented with the nominal interest rate and the nominal net-of-tax rate, he found the estimated coefficient to be negative and significant in both cases. These findings are consistent with those of Heien (1972) and Wright (1967, 1969). However, economic theory indicates that a real rate of interest is the proper variable to be employed, and as a result, the findings of Heien and Wright should be viewed with caution.

Feldstein (1970) documented the fact that if the nominal before-tax interest rate is used, rather than the real after-tax rate, this may lead to downward bias in the interest elasticity estimates. Heien's and Wright's results may thus be regarded as subject to bias.

Carlino concluded that the sign and statistical significance of the interest rate coefficient are quite sensitive to how the interest rate variable is constructed.

Zietz

Zietz (1984) examined the interest elasticity of saving issue using alternative functional forms. He used three alternative specifications of the consumption function that have recently appeared in the literature and artificially nested them within a single model. The three functions he considered are those of Boskin (1978), Gylfason (1981) and Davidson, Hendry, Srba and Yeo (1978). The nested model adopted by Zietz is

$$\ell n\ C = b_0 + b_1\ \ell n\ Y_d + b_2\ \ell n\ (Y_d)_{-1} + b_3\ \ell n\ W^*$$

$$+ b_4 \ell n C_{-1} + b_5\ \ell n\ U^* + b_6 r + b_7 \Pi$$

where C = consumption;

Y_d = disposable income;

W^* = wealth at the beginning of the period;

U^* = unemployment rate;

r = real after-tax return; and

Π = expected rate of inflation.

Zietz characterized the three model specifications to be tested as follows:

B-Model: $b_4 = 0$

G-Model $b_1 + b_3 = 1, \quad b_2 + b_4 = 0$

$b_5 = 0$

DHSY-Model: $b_1 + b_2 + b_4 = 1, \quad b_3 = b_5 = 0$

where B stands for Boskin, G for Gylfason and DHSY for Davidson, Hendry, Srba and Yeo.

Using Bayesian sensitivity analysis he supported "the view that uncertainty with respect to the model specification adds to the empirical uncertainty that has become characteristic of the interest rate elasticity of savings" (1984, p. 324). Furthermore, he demonstrated that it is possible to obtain positive as well as negative parameter values, depending on the specification of the form of the consumption function.

FOOTNOTES

Chapter 2

1. For further detailed information with regard to the interest elasticity of saving, see Gylfason (1981).

2. Boskin reported an interest elasticity of saving of 0.4.

CHAPTER 3

THE BEHAVIOUR OF THE INDIVIDUAL CONSUMER: AN INTERTEMPORAL ALLOCATION PROBLEM

Summers (1981) and Evans (1983), in their examination of the interest elasticity of saving issue in the context of the life-cycle model, used a simulation approach. Both authors analyzed the intertemporal consumer behaviour by assuming that a representative individual maximizes a utility function subject to his (her) lifetime budget constraint, and both adopted the standard homothetic CES utility function (hereafter referred to as SHCES) for this purpose. In the literature, however, there are a number of functional forms which satisfy the key behavioural postulates and which are far more general than the SHCES[1]. Neither Summers nor Evans has addressed the issue of the interest elasticity of saving in relation to alternative functional forms of the utility function.

The purposes of this chapter are to develop a theoretical model based on a quasi-homothetic CES utility function (hereafter referred as QHCES) to examine the intertemporal allocation problem of the consumer, and to lay down a theoretical foundation for the simulation analysis to be discussed in the next chapter.

Section 3.1 attempts to familiarize the reader with the basic concept of the life-cycle hypothesis. Section 3.2 discusses the analytical approach to life-cycle models that is to be used subsequently. The initial specification of the utility function is dealt with in Section 3.3, while Section 3.4 develops a theoretical model based on an alternative specification.

3.1 THE LIFE-CYCLE HYPOTHESIS

The theory that individuals plan their economic lives on the basis of life-cycle considerations is frequently associated with Modigliani and Brumberg (1954). However, the issue of a multiperiod intertemporal allocation problem of consumption and its analysis in the context of utility maximization was introduced much earlier by Fisher (1907), Ramsey (1928), and

Hicks (1939). The life-cycle model of consumption and saving begins with the individual decision unit and, through appropriate aggregation, extends to either cross-section or time-series macro variables. The basic idea of the life-cycle hypothesis, as explained by Ando and Modigliani (1963), is as follows:

> The Modigliani and Brumberg model starts from the utility function of the individual consumer: his utility is assumed to be a function of his own aggregate consumption in current and future periods. The individual is assumed to maximize his utility subject to the resources available to him, his resources being the sum of the current and discounted future earnings over his lifetime and his current net worth. As a result of this maximization the current consumption of the individual can be expressed as a function of his resources and the rate of return on capital with parameters depending on age. The individual consumption functions thus obtained are then aggregated to arrive at the aggregate consumption function for the community.
>
> (Ando and Modigliani, 1963
> p. 56.)

Prior to Modigliani and Brumberg's study, estimation of consumption functions did not explicitly consider the important role played by expectations.

In some instances, attempts were made to capture expectations implicitly by introducing shift models or relating them to the past rather than the future, as pointed out by Somermeyer and Bannink (1973). However, Ando and Modigliani attempted to test the life-cycle hypothesis explicitly by estimating a consumption function using aggregate data. Since then, several studies have explored various dimensions of the life-cycle theory. For instance, decision variables such as leisure, fertility, and bequests have been incorporated into life-cycle models[2].

Although the life-cycle model of saving behaviour is based on solid theoretical ground, the empirical validity of it has been criticized as being dubious. Since the inception of the theory, numerous articles, either invoking or testing it, have been published. White (1978) argued that simple life-cycle models, such as the one developed by Ando and Modigliani, cannot account for realistic levels of aggregate saving, and therefore must be discarded:

> For a wide range of parametric values, the simulated values of aggregate savings fall significantly short of the observed levels. At best, the simulated values are about 60 per cent of the observed values.
>
> (White, 1978, p. 547.)

Similarly, Kotlikoff and Summers (1981) pointed out that life-cycle models account for a negligible fraction of aggregate capital formation in the U.S. If these findings are true, it implies that savings decisions that require policy response cannot be analyzed within the life-cycle framework.

Soderstorm (1982) challenged the claim put forward in White's paper and demonstrated that the life-cycle hypothesis can generate realistic levels of aggregate savings. Soderstorm incorporated uncertainty and unplanned bequests into the model developed by White and simulated reasonable values for aggregate savings.

The life-cycle theory has survived its criticism and it is widely used to analyze various issues such as the distribution of wealth, the incidence and optimality of taxation, and the effects of taxation, social security, and demographic changes.

3.2 ANALYTICAL APPROACH TO LIFE-CYCLE MODELS

Empirical studies based on life-cycle theory must take into account expectations about future incomes, future interest rates, and future conditions of the individual in the optimization procedure. It is therefore clear that the data required for estimation is difficult if not impossible, to obtain from actual observation. To solve the data requirement and related problems, researchers have adopted simulation techniques. An early attempt to use this approach in relation to life-cycle theory came from Tobin and Dolde (1971). In their pioneering paper, they examined various monetary influences (e.g., interest rate effects and the liquidity constraint) and a tax on consumption in which they undertook "semi-realistic" simulations rather than following the standard estimation procedures. Instead of postulating a macroeconomic consumption function, they based their model on the notion that individual households make lifetime consumption and saving plans conforming to life-cycle considerations. To arrive at total consumption, they assumed that each household optimizes a given multiperiod utility function subject to a lifetime budget constraint, and then aggregated the chosen consumption levels of the individuals. Tobin and Dolde argued that the methodology used in their study was a promising one and demonstrated that microeconomic simulation can provide insights into macroeconomic phenomena. Their work stimulated a number of important theoretical as well as policy oriented studies for which understanding of individual consumption-saving behaviour is crucial.

3.3 SPECIFICATION OF UTILITY FUNCTIONS

In the static consumer theory, the assumptions underlying the preferences of the individual are crucial, from both the theoretical and the empirical point of view. In particular, the important role played by the functional form of the utility function has been a subject of careful study in the static framework. At the intertemporal level, however, it appears that little or no attention has been given to this subject.

There are several issues that one could address regarding the assumptions underlying preferences and the form of the utility function. Since the inception of utility theory in the intertemporal context, economists have been dealing with preferences that are additively separable over time. Studies that are based on this assumption are numerous and it is not my intention to give a full list of them. However, it worth mentioning a few: For instance, the studies by White (1978), Summers (1981), Aubry and Fleurent (1980), Auerbach and Kotlikoff (1983), Evans (1983), and Seidman (1983) are rooted in the assumption that individual preferences are additive and temporally separable. This assumption is very strong. The most obvious criticism of it arises from the separability rather than the additivity assumption. The assumption that preferences are separable in time implies that past and future consumption cannot influence current preferences, and as a result cannot appear as current state variables. It rules out the dependency of the marginal utility of consumption in the current period to that of any other period.

Recently, however, researchers seeking to explain macroeconomic phenomena have begun to develop life-cycle models in which preferences are assumed to be additive but not separable over time. Leading studies along this line of reasoning are those of Mankiw, Rotenberg and Summers (1982), Clark and Summers (1982), Barro and King (1982), Kydland and Prescott (1981), and Hotz, Kydland and Sedlacek (1982). The specification of preferences as additive but not separable over time has the desirable feature of allowing past consumption to influence current and future tastes. The marginal utility of consumption in the current period depends on the choice of consumption in other periods.

With regard to the functional form of the utility function, little or no attempt has been made to study its importance. The present study examines this issue in relation to the functional form of the utility function adopted in the Summers (1981) and Evans (1983) papers. Both of these authors formulated a multi-period life-cycle model in which a representative agent is assumed to maximize a SHCES utility function subject to his (her) lifetime budget constraint, and addressed the relationship between saving and the after-tax real interest rate. I have replaced the standard homotheticity assumption by the more general quasi-homotheticity assumption in an attempt to shed some light on the issue of functional form. A theoretical model in accordance with the quasi-homotheticity assumption is developed in Section 3.4.

3.4 AN OUTLINE OF THE PRESENT MODEL

In this section I outline a multiperiod life-cycle model that contains the essentials of Summers' model but extends it to allow for subsistence consumption levels. A utility function for an individual consumer that is particularly suited to the present purpose is obtained from a quasi-homothetic function of the form[3]:

$$(3.1)\qquad U_\tau = \begin{cases} \displaystyle\sum_{t=\tau}^{T} \frac{(C_t^*)^\delta}{\delta}(1+\rho)^{\tau-t} & (\text{if } \delta < 1 \text{ and } \neq 0) \\ \\ \displaystyle\sum_{t=\tau}^{T} \log C_t^*(1+\rho)^{\tau-t} & (\text{if } \delta = 0) \end{cases}$$

where

$$C_t^* = (C_t - \gamma_t) > 0,$$

$$\gamma_t \geq 0, \qquad \sigma = \frac{1}{1-\delta} \quad ;$$

and T = age to which the individual will live, which he (she) knows with certainty;

C_t = total consumption at age t;

γ_t = subsistence consumption at age t;

σ = intertemporal elasticity of substitution between discretionary consumption in any two periods;

ρ = rate of time preference;

τ = individual's present age.

U_τ = lifetime utility as perceived at age τ.

The utility function given by equation (3.1) is of the generalized CES form. This function has some desirable properties in that it does not restrict preferences to be homothetic from the origin and under proper specification of the subsistence consumption parameter it yields the SHCES function. In spite of this, the function appears to have attracted little attention from researchers dealing with the allocation of consumption and saving over time, as an alternative to the SHCES function. The utility function employed in Summers' paper is of the latter kind. The allowance for subsistence consumption in the life-cycle model used in the present study is intended to capture the notion that at some ages individuals may have consumption that cannot be redistributed or sacrificed over time for purposes of life-cycle optimization.

Following Summers, I assume each individual to be engaged in life-cycle consumption planning with no inheritance or bequest. Further, I assume that the individual's wage rate grows at an exogenous rate g per period until a fixed age of retirement. The individual maximizes lifetime utility in (3.1) subject to the following budget constraint:

$$(3.2) \qquad \sum_{t=\tau}^{T} C_t(1 + r)^{\tau-t} = a_\tau + \sum_{t=\tau}^{R} W_t(1 + r)^{\tau-t}$$

where r = after-tax rate of return;

W_t = wage rate net of tax;

a_τ = net worth of the individual at the beginning of period τ;

R = the number of years the individual spends in the work force.

The first-order conditions of the maximization problem yield the following:

$$(3.3) \qquad C^*_{t+1} = [(1 + r)/(1 + \rho)]^\sigma C^*_t$$

Equation (3.3) determines the shape of the discretionary consumption profile, but not its level. It is obvious that the partial elasticity of substitution between C^*_t and C^*_{t+1} becomes smaller, the stronger the desire of the individual to flatten his discretionary consumption profile. For a given value of σ, however, the rate of discretionary consumption increases with an increase in the effective interest rate and decreases with an increase in the rate of time preference.

To obtain an expression for discretionary consumption at $t = \tau$, equations (3.2) and (3.3) are solved simultaneously:

$$(3.4) \qquad C^*_\tau = \{ \sum_{t=\tau}^{T} [(1 + r)/(1 + \rho)]^{(t-\tau)\sigma} (1+r)^{\tau-t} \}^{-1} (Y_\tau - V_\tau)$$

where

$$Y_\tau = a_\tau + \sum_{t=\tau}^{R} W_t (1 + r)^{\tau-t}$$

and

$$V_\tau = \sum_{t=\tau}^{T} \gamma_t (1 + r)^{\tau-t}$$

Total consumption at $t = \tau$ is then given by $C_t = C^*_\tau + \gamma_\tau$. Using (3.4) and (3.3) it is possible to determine the individual's complete consumption age path.

The theoretical framework developed in this chapter will be used in chapter 4 to study the implications of alternative forms of utility function for the optimal paths of the decision variables of the individual as well as for the sign and magnitude of the interest elasticity of savings.

FOOTNOTES

Chapter 3

1. Quasi-homotheticity is more general than the ordinary homotheticity assumption. Although it has been widely used in consumer theory at the static level, only a few researchers have exploited it at the intertemporal level. Among them are Heien (1972), Betancourt (1973), Somermeyer and Bannink (1973), Ashenfelter and Ham (1979), and Biorn (1980).

2. For life-cycle models that include leisure, fertility, and bequests, see Ghez and Becker (1975), Denton and Spencer (1981), and Yaari (1964).

3. The utility function specified by equation (3.1) is based on Uzawa's (1960) first generalization of the CES production function which was then adopted in consumer theory and modified to incorporate subsistence Consumption. See Heien (1972).

CHAPTER 4

SOME SIMULATION RESULTS

This chapter attempts to shed some light on the implications of assuming a specific utility function to represent the behaviour of the individual and the associated optimal paths of the decision variables. Comparison of functional forms of utility functions is usually confined to static consumer theory. To my knowledge this issue has not been given explicit consideration in the intertemporal context.

Summers (1981) assumed that a representative individual maximizes a SHCES utility function subject to a lifetime budget constraint. In the present study the SHCES utility function is replaced by a QHCES function in order to examine the implications of alternative functional forms.

Using the equations and relationships discussed in chapter 3, I demonstrate that simulated optimal paths of consumption, net worth and saving ratios are sensitive to functional specification of the utility function at the micro level. My findings indicate also that the interest elasticity of saving is not robust to changes in the functional form of the utility function.

The lay-out of this chapter is as follows: In Section 4.1 the objectives of the micro simulation experiments are set forth. The specification of the parameter values of the utility function and the assumptions underlying the variables are discussed in Section 4.2. The micro simulation results relating to the individual's optimization problem are reported in Section 4.3. Aggregation of the micro relationship is dealt with in Section 4.4. The interest elasticities of saving that arise from SHCES and QHCES utility functions are compared in Section 4.5.

4.1 OBJECTIVES OF THE MICRO SIMULATION

Given the fact that a large number of utility functions have been suggested in the economics literature, the choice of a particular functional form is somewhat arbitrary. This raises some concern with regard to the sensitivity of empirical results to the functional specification of the utility function. In order to shed light on the importance of the role played by the form of the utility function, it is reasonable to to ask: What difference does it make if the preferences of a consumer are represented by a utility function of a particular form rather than by an alternative specification?

I will answer the above question by referring to the utility functions adopted in Summers' paper and in the present study. In chapter 3, I noted that Summers had based his analysis on the SHCES utility function. To examine the implications of functional forms, I proposed a QHCES utility function as an alternative specification. This functional form is more general than SHCES and under appropriate specification of its parameters it collapses to the SHCES. Because of this, it suffices to address the key question of functional form by focusing on the QHCES utility function.

One way of tackling the issue is to examine the optimal time paths of consumption, net worth and saving ratios that arise from optimization under the SHCES and QHCES specifications. Comparison of these paths provides an indication of the role played by functional form in determining the optimal time paths.

4.2 PARAMETERIZATION OF THE MODEL

The theoretical model discussed in the previous chapter can be used as the basis for a simulation analysis. A general approach to simulation involves the specification of a standard set of assumptions and parameter values. During the simulation process these assumptions and parameter values remain unaltered.

CONSTRUCTION OF THE LABOUR INCOME VARIABLE

To simulate the path of labour income, I assume that all individuals in the economy are identical except for age. Following Summers, it is assumed that adult individuals live for fifty years, that is, the years between ages twenty-one and seventy, inclusive. Furthermore, active participation in the labour force ends at age sixty, owing to mandatory retirement. All individuals who are in the work force at any given time are assumed to earn wage rates that are identical, regardless of their age. Wage rates for all ages are assumed to grow at 2 per cent per annum as a result of productivity growth. For convenience, the wage rate is normalized to unity.

PARAMETER VALUES OF THE UTILITY FUNCTION

The utility function given by equation (3.1) requires specification of its parameter values. This involves assignment of values to δ and γ. Following Summers, the parameter δ is set at alternative values in the range 0.5 to -2.0. As mentioned earlier, the subsistence level of consumption is assumed to be constant over the life of the individual. To generate values for the γ's, I follow the procedure described below. The procedure is unavoidably somewhat arbitrary but seems reasonable in the present experimental context.

I assume initially that the individual maximizes the SHCES utility function subject to the life time budget constraint (3.2). For this purpose population is assumed to grow at the rate of 1.5 per cent per annum ($n = 0.015$), the annual after-tax rate of return and the rate of time preference are set equal at 6 per cent, and the other parameters (namely δ and the rate of productivity growth) assume values already noted. This yields an optimal path of consumption $\overline{C}_t$ which is constant over the adult life span. Next, for convenience in setting values for the γ's, the individual's subsistence consumption is assumed to be a fraction of $\overline{C}_t$:

$$\gamma_t = \Psi \bar{C}_t \qquad \forall\ t$$

where $\Psi \geq 0$

In the subsequent simulations reported in this chapter, I experiment with Ψ values of 0.00, 0.33, 0.66 and 0.90.

The values of the γ's obtained in the above manner are then imposed as lower bounds on consumption in the subsequent experiments, and the QHCES utility function given by equation (3.1) is maximized subject to the budget constraint (3.2). Following Summers, the rate of time preference is set at 3 per cent per annum and the after-tax rate of return is in the range of 4 to 8 per cent per annum.

The simulated optimal paths of the individual decision variables that arise from the theoretical model and the foregoing assumptions are discussed below.

4.3 MICRO SIMULATION RESULTS

The implications of alternative functional forms of the utility function for the optimal paths of consumption, net worth, and saving ratios are examined using two sets of experiments.

Experiment I

The experiments that belong to this category are based on the following assumptions:

1. The parameter of the utility function δ is set at 0.5.

2. The value of Ψ is assumed to take values of 0.00, 0.33, 0.66 and 0.90.

3. The interest rate is set at 4 per cent per annum ($r = 0.04$) while the rate of time preference is fixed at 3 per cent ($\rho = 0.03$).

4. The wage rate is assumed to grow at the rate of 2 per cent per annum as a result of productivity growth ($g = 0.02$).

5. The individual is assumed to live for fifty years ($T = 50$) which includes forty years of active participation in the labour force ($R = 40$) and ten years of retirement.

Experiment II

The set of experiments that belongs to this category are based on the same assumptions as those of Experiment I, except that now the value of δ is set at 0.0 instead of 0.5.

RESULTS OF SIMULATION OF EXPERIMENT I

Experiment I is composed of four "sub-experiments," depending on the value of Ψ. The simulated optimal paths of the key variables are reported in Tables 4.1 to 4.4.

Labour Income

The individual is assumed to spend a fixed amount of time on work in each year. The individual's lifetime labour income is generated by multiplying the fixed work time by the wage rate available to the individual at each age. The resulting labour income increases monotonically with age, as shown in column 1 of Tables 4.1 to 4.4.

Consumption

The lifetime paths of optimal consumption simulated under the assumptions stated in Experiment I are reported in column 2 of Tables 4.1 to 4.4, and depicted in Figure 4.1. Although consumption increases with the age of the individual, closer inspection

TABLE 4.1

Simulated Individual Paths Using SHCES Utility Function
(Ψ = 0.0)

Economic Age	Labour Income	Consumption	Net Worth	Saving Ratio
1	1.000000	.877909	0.000000	.122091
2	1.020000	.895038	.122091	.126693
3	1.040400	.912502	.251937	.131345
4	1.061208	.930307	.389912	.136049
5	1.082432	.948458	.536410	.140802
6	1.104081	.966965	.691840	.145606
7	1.126162	.985832	.856630	.150458
8	1.148686	1.005067	1.031226	.155360
9	1.171659	1.024678	1.216094	.160309
10	1.195093	1.044671	1.411719	.165306
11	1.218994	1.065054	1.618610	.170350
12	1.243374	1.085835	1.837295	.175440
13	1.268242	1.107022	2.068325	.180576
14	1.293607	1.128622	2.312279	.185756
15	1.319479	1.150643	2.569755	.190981
16	1.345868	1.173094	2.841381	.196249
17	1.372786	1.195983	3.127810	.201559
18	1.400241	1.219319	3.429725	.206911
19	1.428246	1.243110	3.747837	.212304
20	1.456811	1.267365	4.082887	.217737
21	1.485947	1.292094	4.435648	.223209
22	1.515666	1.317305	4.806928	.228719
23	1.545980	1.343007	5.197567	.234266
24	1.576899	1.369212	5.608442	.239849
25	1.608437	1.395928	6.040467	.245467
26	1.640606	1.423165	6.494595	.251120
27	1.673418	1.450933	6.971820	.256805
28	1.706886	1.479243	7.473178	.262522
29	1.741024	1.508106	7.999749	.268270
30	1.775845	1.537532	8.552657	.274048
31	1.811362	1.567531	9.133076	.279854
32	1.847589	1.598117	9.742230	.285687
33	1.884541	1.629299	10.381391	.291546
34	1.922231	1.661089	11.051888	.297431
35	1.960676	1.693500	11.755106	.303339
36	1.999890	1.726543	12.492487	.309269
37	2.039887	1.760231	13.265532	.315221
38	2.080685	1.794576	14.075810	.321192
39	2.122299	1.829591	14.924952	.327182
40	2.164745	1.865290	15.814657	.333189
41	0.000000	1.901685	16.746698	-1.838895
42	0.000000	1.938790	15.514881	-2.124081
43	0.000000	1.976619	14.196687	-2.480776
44	0.000000	2.015186	12.787935	-2.939625
45	0.000000	2.054506	11.284266	-3.551706
46	0.000000	2.094593	9.681130	-4.408958
47	0.000000	2.135462	7.973782	-5.695262
48	0.000000	2.177129	6.157271	-7.839607
49	0.000000	2.219609	4.226433	-12.129326
50	0.000000	2.262917	2.175882	-25.000000
51			.000000	

NOTE: The computations assume T = 50, R = 40, g = 0.02,

n = 0.015, r = 0.04, ρ= 0.03, δ = 0.5, and Ψ = 0.0.

TABLE 4.2

Simulated Individual Paths Using QHCES Utility Function ($\psi = 0.33$)

Economic Age	Labour Income	Consumption	Net Worth	Saving Ratio
1	1.000000	1.001891	0.000000	-.001891
2	1.020000	1.013419	-.001891	.006378
3	1.040400	1.025172	.004615	.014811
4	1.061208	1.037155	.020028	.023403
5	1.082432	1.049371	.044882	.032149
6	1.104081	1.061826	.079739	.041042
7	1.126162	1.074523	.125184	.050076
8	1.148686	1.087469	.181830	.059250
9	1.171659	1.100667	.250320	.068552
10	1.195093	1.114122	.331326	.077977
11	1.218994	1.127840	.425549	.087520
12	1.243374	1.141826	.533725	.097173
13	1.268242	1.156085	.656622	.106930
14	1.293607	1.170622	.795044	.116784
15	1.319479	1.185442	.949831	.126728
16	1.345868	1.200552	1.121861	.136755
17	1.372786	1.215956	1.312052	.146858
18	1.400241	1.231661	1.521364	.157029
19	1.428246	1.247673	1.750799	.167262
20	1.456811	1.263997	2.001404	.177550
21	1.485947	1.280639	2.274275	.187885
22	1.515666	1.297606	2.570554	.198261
23	1.545980	1.314904	2.891436	.208669
24	1.576899	1.332540	3.238169	.219105
25	1.608437	1.350520	3.612055	.229560
26	1.640606	1.368850	4.014455	.240028
27	1.673418	1.387539	4.446789	.250502
28	1.706886	1.406592	4.910540	.260975
29	1.741024	1.426016	5.407256	.271442
30	1.775845	1.445820	5.938554	.281897
31	1.811362	1.466010	6.506121	.292332
32	1.847589	1.486594	7.111718	.302742
33	1.884541	1.507580	7.757181	.313122
34	1.922231	1.528975	8.444429	.323465
35	1.960676	1.550787	9.175463	.333767
36	1.999890	1.573026	9.952370	.344022
37	2.039887	1.595698	10.777328	.354225
38	2.080685	1.618812	11.652611	.364371
39	2.122299	1.642378	12.580588	.374457
40	2.164745	1.666403	13.563733	.384477
41	0.000000	1.690897	14.604624	-1.894455
42	0.000000	1.715869	13.497912	-2.178027
43	0.000000	1.741328	12.321960	-2.532977
44	0.000000	1.767284	11.073510	-2.989891
45	0.000000	1.793747	9.749166	-3.599743
46	0.000000	1.820725	8.345386	-4.454287
47	0.000000	1.848230	6.858477	-5.737030
48	0.000000	1.876272	5.284585	-7.876157
49	0.000000	1.904861	3.619696	-12.156222
50	0.000000	1.934008	1.859623	-25.000000
51			.000000	

NOTE: The computations assume $T = 50$, $R = 40$, $g = 0.02$, $n = 0.015$, $r = 0.04$, $\rho = 0.03$, $\delta = 0.5$, and $\psi = 0.33$.

TABLE 4.3

Simulated Individual Paths Using QHCES Utility Function

(Ψ = 0.66)

Economic Age	Labour Income	Consumption	Net Worth	Saving Ratio
1	1.000000	1.125872	0.000000	-.125872
2	1.020000	1.131799	-.125872	-.115112
3	1.040400	1.137842	-.242707	-.103960
4	1.061208	1.144003	-.349857	-.092425
5	1.082432	1.150283	-.446646	-.080518
6	1.104081	1.156687	-.532363	-.068250
7	1.126162	1.163215	-.606263	-.055633
8	1.148686	1.169870	-.667566	-.042681
9	1.171659	1.176656	-.715453	-.029408
10	1.195093	1.183574	-.749068	-.015830
11	1.218994	1.190627	-.767512	-.001903
12	1.243374	1.197817	-.769844	.012175
13	1.268242	1.205148	-.755081	.026507
14	1.293607	1.212622	-.722190	.041193
15	1.319479	1.220241	-.670092	.056034
16	1.345868	1.228009	-.597658	.071071
17	1.372786	1.235929	-.503706	.086262
18	1.400241	1.244004	-.386997	.101648
19	1.428246	1.252236	-.246240	.117147
20	1.456811	1.260628	-.080079	.132759
21	1.485947	1.269184	.112901	.148403
22	1.515666	1.277908	.334180	.164238
23	1.545980	1.286801	.585306	.180004
24	1.576899	1.295868	.867897	.195920
25	1.608437	1.305112	1.183644	.211706
26	1.640606	1.314536	1.534315	.227642
27	1.673418	1.324144	1.921757	.243471
28	1.706886	1.333940	2.347901	.259252
29	1.741024	1.343927	2.814764	.274970
30	1.775845	1.354108	3.324452	.290635
31	1.811362	1.364489	3.879166	.306143
32	1.847589	1.375071	4.481205	.321568
33	1.884541	1.385861	5.132971	.336864
34	1.922231	1.396861	5.836970	.352018
35	1.960676	1.408075	6.595819	.367018
36	1.999890	1.419508	7.412253	.381849
37	2.039887	1.431165	8.289124	.396503
38	2.080685	1.443048	9.229412	.410967
39	2.122299	1.455164	10.236225	.425233
40	2.164745	1.467516	11.312809	.439292
41	0.000000	1.480109	12.462550	-1.969114
42	0.000000	1.492948	11.480943	-2.250926
43	0.000000	1.506037	10.447233	-2.603914
44	0.000000	1.519382	9.359085	-3.058575
45	0.000000	1.532987	8.214067	-3.665736
46	0.000000	1.546857	7.009643	-4.516891
47	0.000000	1.560998	5.743171	-5.795020
48	0.000000	1.575415	4.411899	-7.927082
49	0.000000	1.590114	3.012960	-12.193951
50	0.000000	1.605099	1.543364	-25.000000
51			.000000	

NOTE: The computations assume T = 50, R = 40, g = 0.02, n = 0.015, r = 0.04, ρ = 0.03, δ = 0.5, and Ψ = 0.66.

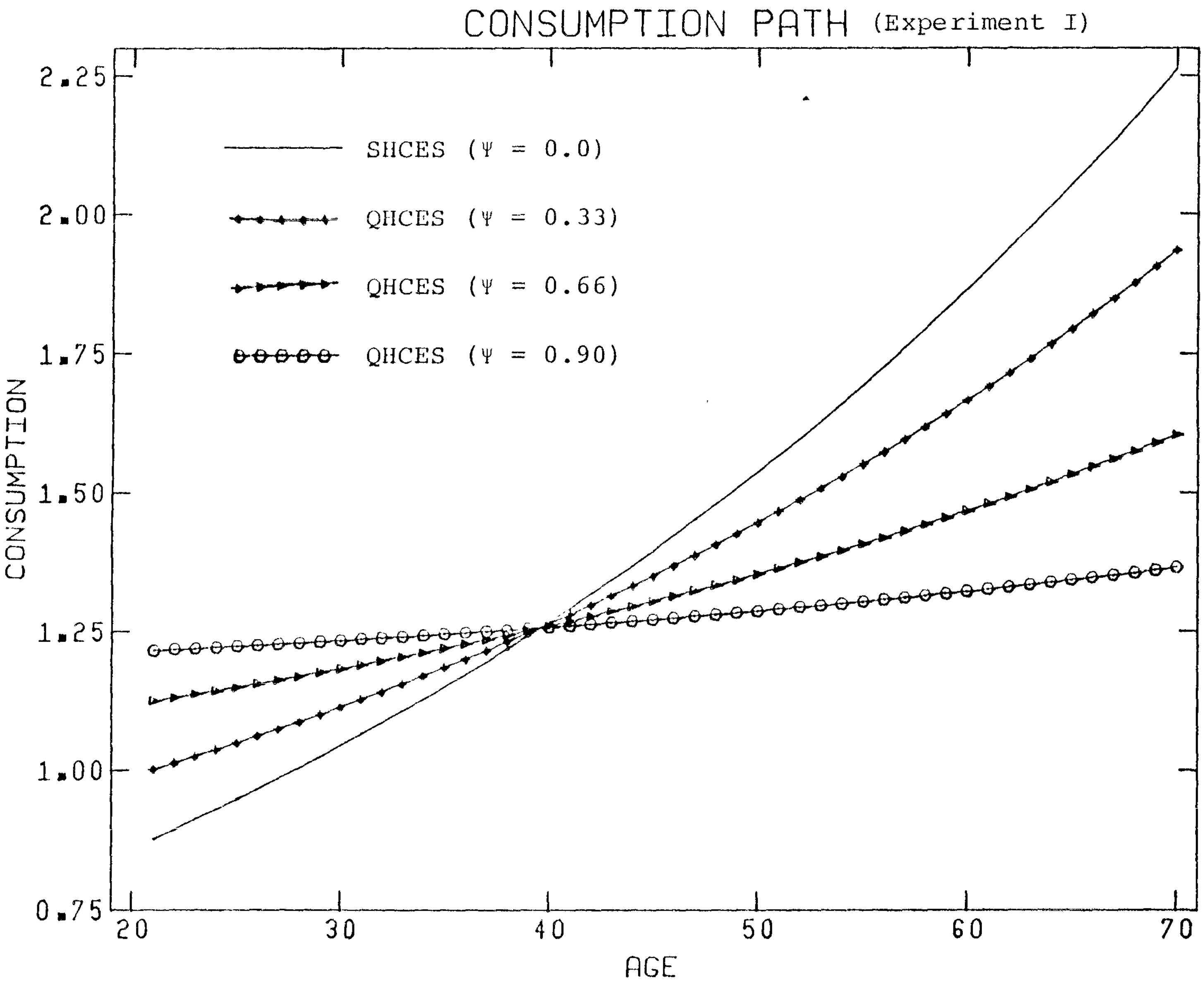
TABLE 4.4

Simulated Individual Paths Using QHCES Utility Function
(Ψ = 0.90)

Economic Age	Labour Income	Consumption	Net Worth	Saving Ratio
1	1.000000	1.216041	0.000000	-.216041
2	1.020000	1.217894	-.216041	-.204216
3	1.040400	1.219784	-.422577	-.191781
4	1.061208	1.221710	-.618864	-.178741
5	1.082432	1.223674	-.804121	-.165107
6	1.104081	1.225676	-.977527	-.150892
7	1.126162	1.227718	-1.138224	-.136109
8	1.148686	1.229799	-1.285308	-.120777
9	1.171659	1.231921	-1.417834	-.104915
10	1.195093	1.234084	-1.534808	-.088545
11	1.218994	1.236289	-1.635192	-.071692
12	1.243374	1.238538	-1.717895	-.054381
13	1.268242	1.240830	-1.781774	-.036642
14	1.293607	1.243167	-1.825633	-.018504
15	1.319479	1.245550	-1.848218	.000000
16	1.345868	1.247979	-1.848218	.018838
17	1.372786	1.250455	-1.824257	.037975
18	1.400241	1.252980	-1.774897	.057375
19	1.428246	1.255554	-1.698631	.077003
20	1.456811	1.258178	-1.593884	.096821
21	1.485947	1.260854	-1.459007	.116794
22	1.515666	1.263582	-1.292273	.136883
23	1.545980	1.266362	-1.091879	.157053
24	1.576899	1.269198	-.855937	.177268
25	1.608437	1.272088	-.582473	.197491
26	1.640606	1.275035	-.269423	.217688
27	1.673418	1.278040	.085371	.237825
28	1.706886	1.281103	.484164	.257871
29	1.741024	1.284225	.929315	.277793
30	1.775845	1.287409	1.423286	.297563
31	1.811362	1.290655	1.968653	.317153
32	1.847589	1.293964	2.568106	.336535
33	1.884541	1.297338	3.224455	.355686
34	1.922231	1.300777	3.940635	.374583
35	1.960676	1.304284	4.719715	.393205
36	1.999890	1.307859	5.564895	.411533
37	2.039887	1.311504	6.479521	.429550
38	2.080685	1.315220	7.467085	.447240
39	2.122299	1.319009	8.531234	.464590
40	2.164745	1.322871	9.675773	.481588
41	0.000000	1.326809	10.904678	-2.041834
42	0.000000	1.330823	10.014056	-2.322388
43	0.000000	1.334916	9.083795	-2.673895
44	0.000000	1.339089	8.112231	-3.126760
45	0.000000	1.343343	7.097631	-3.731661
46	0.000000	1.347681	6.038193	-4.579818
47	0.000000	1.352102	4.932040	-5.853668
48	0.000000	1.356611	3.777219	-7.978899
49	0.000000	1.361207	2.571697	-12.232573
50	0.000000	1.365892	1.313358	-25.000000
51			.000000	

NOTE: The computations assume T = 50, R = 40, g = 0.02, n = 0.015, r = 0.04, ρ = 0.03, δ = 0.5, and Ψ = 0.90.

FIGURE 4.1


CONSUMPTION PATH (Experiment I)
SHCES (Ψ = 0.0)
QHCES (Ψ = 0.33)
QHCES (Ψ = 0.66)
QHCES (Ψ = 0.90)
CONSUMPTION
0.75
1.00
1.25
1.50
1.75
2.00
2.25
AGE
20
30
40
50
60
70

of Figure 4.1 reveals noteworthy differences among the simulated paths. The slopes of the consumption paths become flatter the higher the level of subsistence consumption. Similarly, consumption at economic age 1 (i.e., calendar age 21) rises and consumption at economic age 50 (i.e., calendar age 70) as Ψ increases, as shown in the table below.

TABLE 4.5

SUMMARY RESULTS FOR CONSUMPTION LEVELS

Experiment	Value of Ψ	CONSUMPTION LEVELS Economic Age 1	CONSUMPTION LEVELS Economic Age 50
1	0.00	0.877909	2.262917
2	0.33	1.001891	1.934008
3	0.66	1.125872	1.605099
4	0.90	1.216041	1.365892

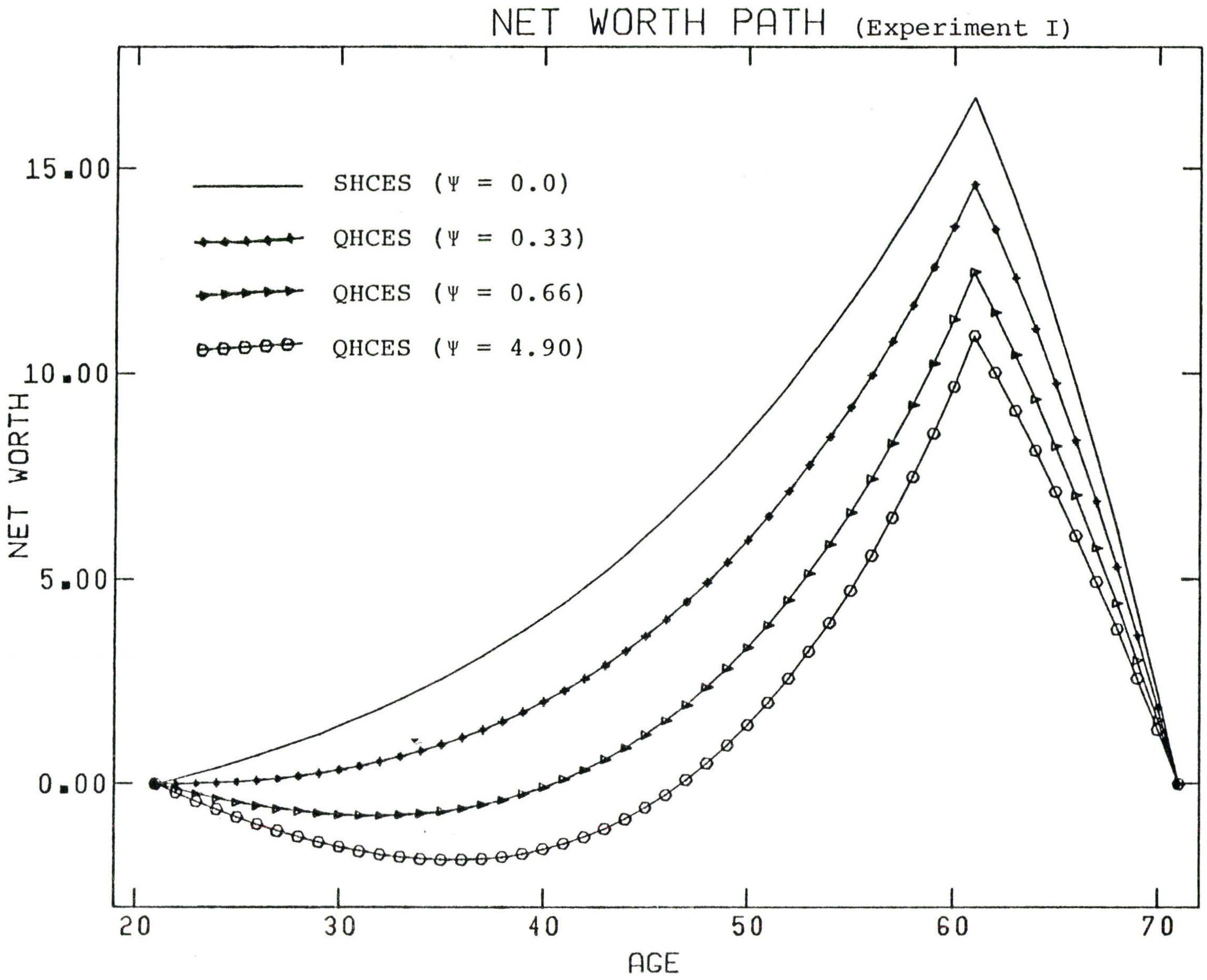
In Table 4.5, the consumption levels that result from a SHCES maximization problem are shown in row 1 while those based on a QHCES function are given in rows 2 to 4. It is clear that consumption at economic age 1 is relatively smaller in the SHCES cases than in the QHCES case. On the other hand, consumption at economic age 50 is relatively higher with SHCES than with QHCES.

The differences in the level as well as the shape of the consumption paths discussed above suggest that the choice of functional form of the utility function is of considerable importance.

Net Worth

The representative individual is assumed to begin his (her) economic life with zero net worth and to leave no bequest at the time of death. The net worth profiles that arise from the maximization of the SHCES and the QHCES functions are reported in Tables 4.1 to 4.4. These results are also shown in Figure 4.2. For the SHCES function, net worth is zero at economic age 1, then rises with age until retirement, and then declines to zero at the time of death. (See Table 4.1.) For the QHCES function, however, net worth is zero at economic age 1, then turns negative, and then positive as the individual approaches retirement age. It declines and becomes zero by the end of the life span. (See Tables 4.2 to 4.4.)

FIGURE 4.2


NET WORTH PATH (Experiment I)
SHCES (Ψ = 0.0)
QHCES (Ψ = 0.33)
QHCES (Ψ = 0.66)
QHCES (Ψ = 4.90)
15.00
10.00
5.00
0.00
NET WORTH
20
30
40
50
60
70
AGE

The simulated net worth profile that results from the SHCES function never becomes negative. This finding is consistent with the Modigliani-Brumberg model. On the other hand, QHCES functions yield net worth profiles that support the Tobin model[1].

Although the age at which maximum net worth occurs is the same regardless of the functional specification of the utility function, the age of the minimum net worth is not. It is clear from Figure 4.2 that the negative net worth associated with the QHCES function varies, depending on the assumed level of subsistence consumption. The smaller the value of Ψ, the shorter the period of negative net worth the individual experiences. As the value of Ψ becomes large, the period of negative net worth associated with the profile increases. This shows again the sensitivity of the profile to the functional specification of the utility function.

Saving Ratio

The saving ratio is defined as the ratio of saving to the total income of the individual (including interest income). The profile of the saving ratio that arises from the SHCES function is given in column 5 of Table 4.1. The profile exhibits a ratio that is positive and increases with the age of the individual until retirement, and thereafter becomes negative. With a QHCES function, the saving ratio becomes negative when the individual is young, then positive until retirement, and then negative again thereafter, as shown in column 5 of Tables 4.1 to 4.4. The period of negative saving that an individual experiences before retirement depends on the value of the parameter Ψ. The smaller is this value, the shorter the period of negative saving; conversely, the larger the value of Ψ, the longer the period of negative saving.

Comparison of the simulated saving ratios in column 5 of Tables 4.1 to 4.4 indicates that the level of the saving ratio, as well as the period of negative saving, is sensitive to the assumptions underlying the optimization problem.

RESULTS OF SIMULATION OF EXPERIMENT II

In Experiment II the value of δ is set to zero while maintaining assumptions 2 to 5 of Experiment I. The optimal paths of consumption, net worth and the saving ratio are simulated under these assumptions, and the results are reported in Tables 4.6 to 4.9. It is to be noted that when δ is zero, the SHCES and the QHCES collapse to the logarithmic and the Stone-Geary functions, respectively.

Consumption

Both the logarithmic and the Stone-Geary functions yield consumption profiles that increase with the age of the individual, as shown in column 3 of Tables 4.6 to 4.9. These results are depicted in Figure 4.3 and it is clear that the level as well as the slope of the profiles depends on the functional form of the utility function. The standard homothetic logarithmic function yields a steeper profile compared to the Stone-Geary function. This implies that the logarithmic function yields relatively low levels of consumption when the individual is young and relatively

TABLE 4.6

Simulated Individual Paths Using Logarithmic Utility Function ($\Psi = 0.0$)

Economic Age	Labour Income	Consumption	Net Worth	Saving Ratio
1	1.000000	1.059732	0.000000	-.059732
2	1.020000	1.070021	-.059732	-.051503
3	1.040400	1.080409	-.112142	-.042952
4	1.061208	1.090899	-.156637	-.034084
5	1.082432	1.101490	-.192593	-.024901
6	1.104081	1.112184	-.219355	-.015409
7	1.126162	1.122982	-.236232	-.005614
8	1.148686	1.133885	-.242501	.004479
9	1.171659	1.144893	-.237400	.014860
10	1.195093	1.156009	-.220130	.025524
11	1.218994	1.167232	-.189851	.036461
12	1.243374	1.178564	-.145683	.047661
13	1.268242	1.190007	-.086700	.059115
14	1.293607	1.201560	-.011933	.070812
15	1.319479	1.213226	.079636	.082741
16	1.345868	1.225005	.189074	.094890
17	1.372786	1.236898	.317501	.107246
18	1.400241	1.248907	.466088	.119797
19	1.428246	1.261032	.636066	.132530
20	1.456811	1.273275	.828723	.145430
21	1.485947	1.285637	1.045408	.158484
22	1.515666	1.298119	1.287535	.171678
23	1.545980	1.310722	1.556584	.184998
24	1.576899	1.323447	1.854105	.198427
25	1.608437	1.336296	2.181721	.211953
26	1.640606	1.349270	2.541130	.225559
27	1.673418	1.362370	2.934111	.239232
28	1.706886	1.375597	3.362524	.252956
29	1.741024	1.388952	3.828315	.266717
30	1.775845	1.402437	4.333519	.280501
31	1.811362	1.416053	4.880268	.294293
32	1.847589	1.429801	5.470787	.308078
33	1.884541	1.443683	6.107406	.321844
34	1.922231	1.457699	6.792560	.335577
35	1.960676	1.471851	7.528795	.349265
36	1.999890	1.486141	8.318772	.362893
37	2.039887	1.500570	9.165271	.376451
38	2.080685	1.515138	10.071199	.389926
39	2.122299	1.529849	11.039594	.403308
40	2.164745	1.544701	12.073628	.416585
41	0.000000	1.559697	13.176616	-1.959217
42	0.000000	1.574841	12.143982	-2.242020
43	0.000000	1.590131	11.054901	-2.595967
44	0.000000	1.605569	9.906966	-3.051617
45	0.000000	1.621157	8.697675	-3.659743
46	0.000000	1.636897	7.424425	-4.511803
47	0.000000	1.652789	6.084505	-5.790974
48	0.000000	1.668835	4.675097	-7.924067
49	0.000000	1.685038	3.193266	-12.192118
50	0.000000	1.701397	1.635959	-25.000000
51			.000000	

NOTE: The computations assume $T = 50$, $R = 40$, $g = 0.02$, $n = 0.015$, $r = 0.04$, $\rho = 0.03$, $\delta = 0.00$, and $\Psi = 0.00$.

TABLE 4.7

Simulated Individual Paths Using Stone-Geary Utility Function
(Ψ = 0.33)

Economic Age	Labour Income	Consumption	Net Worth	Saving Ratio
1	1.000000	1.124259	0.000000	-.124259
2	1.020000	1.131183	-.124259	-.114433
3	1.040400	1.138174	-.240412	-.104184
4	1.061208	1.145234	-.347803	-.093515
5	1.082432	1.152362	-.445741	-.082434
6	1.104081	1.159559	-.533500	-.070948
7	1.126162	1.166826	-.610318	-.059066
8	1.148686	1.174164	-.675394	-.046800
9	1.171659	1.181572	-.727888	-.034159
10	1.195093	1.189053	-.766917	-.021159
11	1.218994	1.196607	-.791554	-.007811
12	1.243374	1.204233	-.800828	.005868
13	1.268242	1.211934	-.793721	.019862
14	1.293607	1.219710	-.769162	.034154
15	1.319479	1.227561	-.726031	.048725
16	1.345868	1.235488	-.663154	.063558
17	1.372786	1.243492	-.579300	.078631
18	1.400241	1.251574	-.473179	.093925
19	1.428246	1.259734	-.343438	.109419
20	1.456811	1.267974	-.188664	.125091
21	1.485947	1.276294	-.007374	.140920
22	1.515666	1.284694	.201985	.156884
23	1.545980	1.293176	.441037	.172961
24	1.576899	1.301740	.711482	.189128
25	1.608437	1.310388	1.015100	.205304
26	1.640606	1.319119	1.353754	.221647
27	1.673418	1.327935	1.729391	.237955
28	1.706886	1.336837	2.144049	.254267
29	1.741024	1.345825	2.599860	.270563
30	1.775845	1.354901	3.099054	.286822
31	1.811362	1.364064	3.643960	.303025
32	1.847589	1.373317	4.237015	.319152
33	1.884541	1.382659	4.880768	.335187
34	1.922231	1.392092	5.577880	.351111
35	1.960676	1.401617	6.331134	.366908
36	1.999890	1.411234	7.143439	.382562
37	2.039887	1.420945	8.017832	.398058
38	2.080685	1.430749	8.957488	.413383
39	2.122299	1.440649	9.965723	.428524
40	2.164745	1.450645	11.046002	.443469
41	0.000000	1.460738	12.201941	-1.992840
42	0.000000	1.470930	11.229280	-2.274764
43	0.000000	1.481220	10.207522	-2.627765
44	0.000000	1.491610	9.134603	-3.082305
45	0.000000	1.502100	8.008378	-3.689153
46	0.000000	1.512693	6.826613	-4.539692
47	0.000000	1.523389	5.586984	-5.816686
48	0.000000	1.534188	4.287075	-7.946590
49	0.000000	1.545092	2.924370	-12.208763
50	0.000000	1.556102	1.496252	-25.000000
51			.000000	

NOTE: The computations assume T = 50, R = 40, g = 0.02, n = 0.015, r = 0.04, ρ = 0.03, δ = 0.00, and Ψ = 0.33

TABLE 4.8

Simulated Individual Paths Using Stone-Geary Utility Function
($\Psi = 0.66$)

Economic Age	Labour Income	Consumption	Net Worth	Saving Ratio
1	1.000000	1.188785	0.000000	-.188785
2	1.020000	1.192345	-.188785	-.177685
3	1.040400	1.195940	-.368681	-.166028
4	1.061208	1.199569	-.538968	-.153821
5	1.082432	1.203234	-.698888	-.141072
6	1.104081	1.206934	-.847645	-.127791
7	1.126162	1.210670	-.984404	-.113991
8	1.148686	1.214443	-1.108288	-.099686
9	1.171659	1.218252	-1.218376	-.084892
10	1.195093	1.222098	-1.313704	-.069628
11	1.218994	1.225981	-1.393257	-.053915
12	1.243374	1.229902	-1.455974	-.037774
13	1.268242	1.233861	-1.500741	-.021229
14	1.293607	1.237859	-1.526390	-.004307
15	1.319479	1.241895	-1.531698	.012967
16	1.345868	1.245971	-1.515383	.030564
17	1.372786	1.250086	-1.476101	.048654
18	1.400241	1.254241	-1.412445	.066607
19	1.428246	1.258437	-1.322943	.084992
20	1.456811	1.262673	-1.206051	.103577
21	1.485947	1.266950	-1.060155	.122332
22	1.515666	1.271269	-.883565	.141222
23	1.545980	1.275630	-.674510	.160217
24	1.576899	1.280033	-.431141	.179284
25	1.608437	1.284479	-.151520	.198391
26	1.640606	1.288968	.166377	.217508
27	1.673418	1.293501	.524070	.236605
28	1.706886	1.298077	.925574	.255651
29	1.741024	1.302698	1.371406	.274618
30	1.775845	1.307364	1.864588	.293480
31	1.811362	1.312076	2.407652	.312210
32	1.847589	1.316833	3.003244	.330782
33	1.884541	1.321636	3.654130	.349174
34	1.922231	1.326486	4.363200	.367364
35	1.960676	1.331382	5.133474	.385331
36	1.999890	1.336327	5.966106	.403050
37	2.039887	1.341319	6.870393	.420522
38	2.080685	1.346360	7.845777	.437713
39	2.122299	1.351450	8.891853	.454615
40	2.164745	1.356589	10.018376	.471214
41	0.000000	1.361778	11.227266	-2.032302
42	0.000000	1.367018	10.314578	-2.313315
43	0.000000	1.372308	9.360144	-2.665297
44	0.000000	1.377650	8.362241	-3.118663
45	0.000000	1.383044	7.319081	-3.724103
46	0.000000	1.388490	6.228800	-4.572862
47	0.000000	1.393969	5.089402	-5.847425
48	0.000000	1.399541	3.899052	-7.973596
49	0.000000	1.405147	2.655474	-12.228779
50	0.000000	1.410808	1.356546	-25.000000
51			.000000	

NOTE: The computations assume $T = 50$, $R = 40$, $g = 0.02$, $n = 0.015$, $r = 0.04$, $\rho = 0.03$, $\delta = 0.00$ and $\Psi = 0.66$.

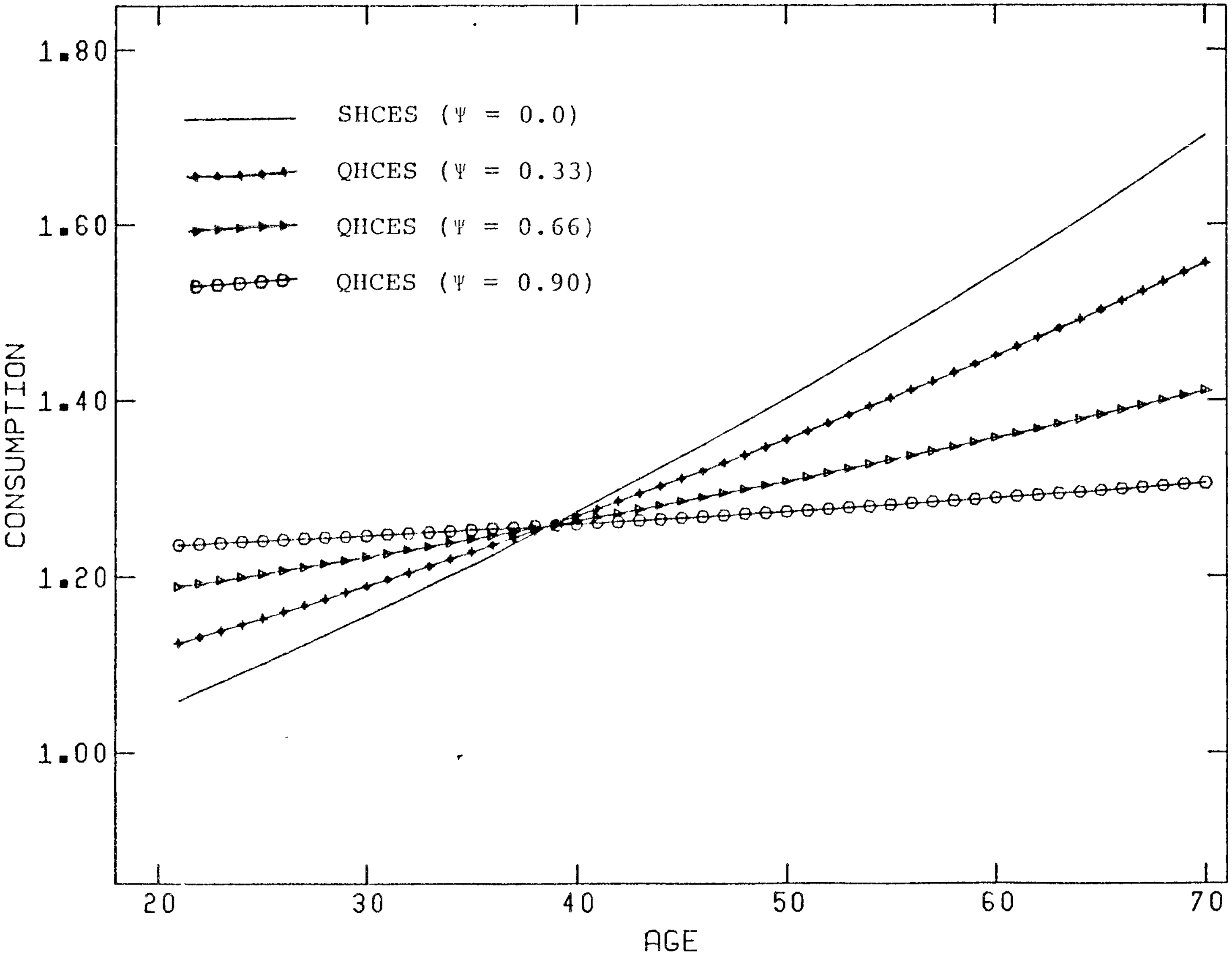
TABLE 4.9

Simulated Individual Paths Using Stone-Geary Utility Function
($\Psi = 0.90$)

Economic Age	Labour Income	Consumption	Net Worth	Saving Ratio
1	1.000000	1.235713	0.000000	-.235713
2	1.020000	1.236827	-.235713	-.223888
3	1.040400	1.237951	-.461969	-.211395
4	1.061208	1.239085	-.677998	-.198240
5	1.082432	1.240231	-.882995	-.184430
6	1.104081	1.241388	-1.076114	-.169977
7	1.126162	1.242557	-1.256407	-.154896
8	1.148686	1.243736	-1.423120	-.139202
9	1.171659	1.244927	-1.575095	-.122916
10	1.195093	1.246130	-1.711367	-.106061
11	1.218994	1.247344	-1.830859	-.088661
12	1.243374	1.248571	-1.932444	-.070745
13	1.268242	1.249809	-2.014937	-.052342
14	1.293607	1.251059	-2.077102	-.033480
15	1.319479	1.252321	-2.117638	-.014211
16	1.345868	1.253595	-2.135185	.005447
17	1.372786	1.254882	-2.128319	.025450
18	1.400241	1.256181	-2.095548	.045759
19	1.428246	1.257493	-2.035310	.066334
20	1.456811	1.258818	-1.945969	.087133
21	1.485947	1.260155	-1.825815	.108117
22	1.515666	1.261506	-1.673055	.129242
23	1.545980	1.262869	-1.485817	.150468
24	1.576899	1.264246	-1.262139	.171754
25	1.608437	1.265636	-.999972	.193060
26	1.640606	1.267040	-.697170	.214345
27	1.673418	1.268457	-.351491	.235574
28	1.706886	1.269889	.039410	.256707
29	1.741024	1.271333	.477985	.277710
30	1.775845	1.272793	.966795	.298550
31	1.811362	1.274266	1.508519	.319194
32	1.847589	1.275753	2.105955	.339613
33	1.884541	1.277255	2.762029	.359779
34	1.922231	1.278772	3.479796	.379666
35	1.960676	1.280303	4.262448	.399250
36	1.999890	1.281849	5.113319	.418510
37	2.039887	1.283410	6.035892	.437427
38	2.080685	1.284986	7.033805	.455984
39	2.122299	1.286578	8.110856	.474105
40	2.164745	1.288185	9.271011	.491958
41	0.000000	1.289807	10.518412	-2.065595
42	0.000000	1.291446	9.049341	-2.345943
43	0.000000	1.293100	8.743866	-2.697163
44	0.000000	1.294770	7.800523	-3.149627
45	0.000000	1.296457	6.817774	-3.753960
46	0.000000	1.298160	5.794027	-4.601285
47	0.000000	1.299879	4.727629	-5.873845
48	0.000000	1.301616	3.616854	-7.996876
49	0.000000	1.303369	2.459913	-12.246084
50	0.000000	1.305139	1.254941	-25.000000
51			.000000	

NOTE: The computations assume $T = 50$, $R = 40$, $g = 0.02$, $n = 0.015$, $r = 0.04$, $\rho = 0.03$, $\delta = 0.00$, and $\Psi = 0.90$.

FIGURE 4.3



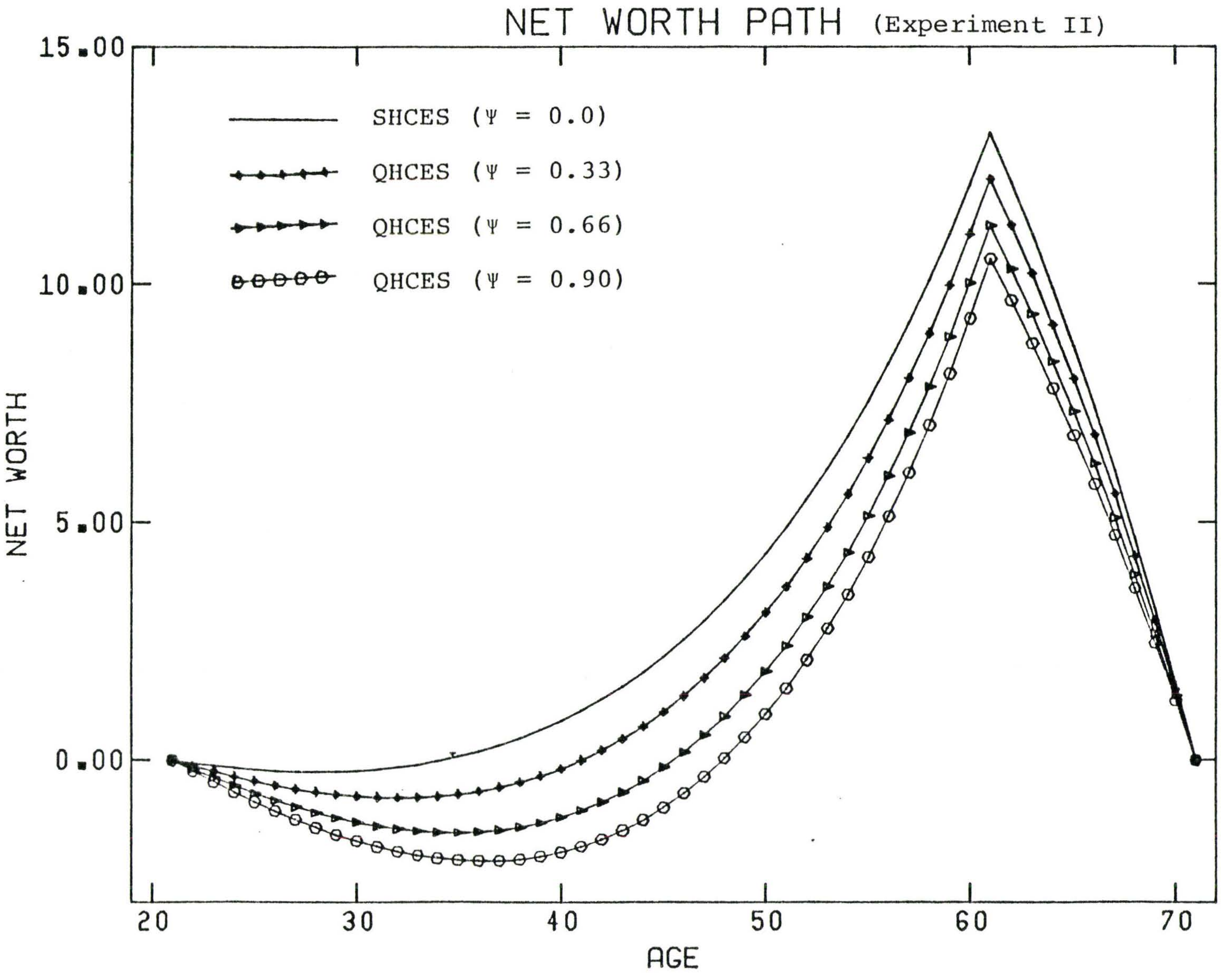
CONSUMPTION PATH (Experiment II)

high levels when old, compared to the Stone-Geary function. Furthermore, the age-consumption profile is sensitive to the value of Ψ. Figure 4.3 demonstrates that the slope of the profile becomes flatter as the value of Ψ becomes larger. This result is consistent with the findings of Experiment I.

Net Worth

Setting the parameter δ to zero yields a net worth profile that is consistent with the predictions of the Tobin Model, regardless of whether the utility function is specified as a logarithmic or as a Stone-Geary function. However, the implications of assuming alternative functional forms are reflected in the profile levels as well as the periods of negative net worth. From Figure 4.4 it is clear that the period of negative net worth associated with the logarithmic function is shorter than that associated with the Stone-Geary function. Moreover, as the value of Ψ increases from 0.0 to 0.9, the period of negative net worth persists for a longer period. The age of the minimum net worth as well as its value varies, depending on the form of function employed.

FIGURE 4.4


NET WORTH PATH (Experiment II)
SHCES (Ψ = 0.0)
QHCES (Ψ = 0.33)
QHCES (Ψ = 0.66)
QHCES (Ψ = 0.90)
15.00
10.00
5.00
0.00
NET WORTH
20
30
40
50
60
70
AGE

Saving Ratios

Both the logarithmic and the Stone-Geary utility functions yield individual saving ratios that are negative before retirement, as shown in column 5 of Tables 4.6 to 4.9. The period of negative saving is shorter in the case of the logarithmic function than in the case of the Stone-Geary function. The level as well as the slope of the profile is also sensitive to the functional form of the utility function.

4.4 AGGREGATION OF THE MICRO-RELATIONSHIPS

Once consumption, net worth and income of all individuals of all ages are known, the corresponding aggregate values for any period t are calculated by aggregating each variable over all persons alive in that period. Assuming population to grow at a constant rate n per annum, aggregation proceeds as follows:

$$C_t^{**} = \sum_{\tau=1}^{T} (1 + n)^{T-\tau} C_\tau \tag{4.1}$$

Aggregate consumption C_t^{**} is calculated in equation (4.1) by multiplying the consumption of an individual of age τ by the relative number of persons of each age and aggregating them over all cohorts.

The aggregate values of net worth A_t^{**} and total income Y_t^{**} are computed in a similar way using equations (4.2) and (4.3):

$$(4.2) \qquad A_t^{**} = \sum_{\tau=1}^{T} (1 + n)^{T-\tau} a_\tau$$

and

$$(4.3) \qquad Y_t^{**} = \sum_{\tau=1}^{R} (1 + n)^{T-\tau} w_\tau + \sum_{\tau=1}^{T} (1 + n)^{T-\tau} r\ a_\tau$$

Aggregate saving is calculated by subtracting total consumption from total income as shown in equation (4.4):

$$(4.4) \qquad S_t^{**} = Y_t^{**} - C_t^{**}$$

4.5 THE INTEREST ELASTICITY OF SAVING

In a challenge to the widely held view that the interest elasticity of saving is low, Summers (1981) claimed to have formulated a realistic life-cycle model yielding a large and positive interest elasticity of saving. In fact, he made the strong claim that "the theory when formulated realistically implies interest elasticities well in excess of unity" (1981, p. 534). This claim has been questioned by Evans (1983), who showed that with negative time preferences the interest elasticities are lower.

In this section, first I shall examine the sensitivity of Summers' results to alternative functional forms of the utility function. Second, I shall point out some errors in Evans' paper and provide correct results for the saving elasticities, saving rates, and wealth-income ratios.

Summers' Results

As mentioned earlier, the theoretical life-cycle model based on a QHCES function is far more general than Summers' model, which is based on the SHCES function. One of the advantages of a QHCES function is that it yields the SHCES function as a special case when subsistence consumption is specified to be zero. The interest elasticity of saving and the saving rates obtained using these functions can be compared. In order for Summers' claim to be credible, his results must be robust to plausible changes in functional form. To put it differently, the incorporation of subsistence consumption into Summers' model should not lead to results that are significantly different from those which he reported.

The interest elasticities and saving rates corresponding to those of Summers can be read from the first row of each block in Table 4.10 (the rows with $\Psi = 0$) for various values of r. Within each block, it is clear that the elasticity declines as the subsistence level increases. The results in Table 4.10 indicate that Summers' conclusion is by no means insensitive to changes in functional specification of the utility function and that negative interest elasticities of saving are not implausible.

The incorporation of subsistence consumption into Summers' model implies that the individual is constrained in his ability to trade off current and future consumption in response to interest rate changes. The intertemporal substitution effect is limited to discretionary consumption in the case of the QHCES utility function, and thus may be weaker than the corresponding effect with the SCHES function. The income effect, on the other hand, depends on whether the individual dissaves or not when young, and the size of his net worth which, in turn, depends on the level of subsistence consumption. With significant dissaving in youth, the income effect of an interest rate change, which works to lower consumption,may not be obvious in the

TABLE 4.10

INTEREST ELASTICITIES OF SAVING AND SAVING RATES

	Values of		Value of r					
			0.04		0.06		0.08	
Block	δ	ψ	Saving Elasticity	Saving Rate	Saving Elasticity	Saving Rate	Saving Elasticity	Saving Rate
#1	0.5	0.00	3.97	0.11	2.50	0.24	2.60	0.29
		0.33	3.86	0.08	2.48	0.19	2.57	0.25
		0.66	3.55	0.04	2.42	0.11	2.50	0.17
		0.90	2.05	0.02	2.01	0.03	1.91	0.05
#2	0.0	0.00	3.55	0.06	2.14	0.13	2.06	0.18
		0.33	3.33	0.03	2.11	0.09	2.05	0.14
		0.66	2.67	0.02	2.01	0.05	2.00	0.07
		0.90	-0.66	0.01	-0.53	0.01	1.19	0.01
#3	-0.5	0.00	3.25	0.04	1.94	0.09	1.76	0.13
		0.33	2.90	0.03	1.90	0.06	1.75	0.09
		0.66	1.86	0.01	1.72	0.03	1.73	0.04
		0.90	-3.65	0.004	*	*	*	*
#4	-1.0	0.00	2.96	0.03	1.82	0.06	1.60	0.09
		0.33	2.49	0.02	1.78	0.04	1.60	0.06
		0.66	1.06	0.01	1.45	0.02	1.58	0.02
		0.90	-7.05	0.002	*	*	*	*
#5	-2.0	0.00	2.43	0.02	1.65	0.04	1.44	0.06
		0.33	1.70	0.01	1.51	0.02	1.44	0.03
		0.66	-0.60	0.01	0.47	0.01	1.43	0.01
		0.90	-15.67	0.001	*	*	*	*

NOTES: The computations assume $T = 50$, $R = 40$, $g = 0.02$, $n = 0.015$, and $\rho = 0.03$. The saving rate is calculated relative to total income.

What Summers calls γ in his paper is identical to δ in the present study.

Individual subsistence consumption is a function of Ψ, the latter being defined as in Section 4.2.

Asterisks indicate negative saving rates.

TABLE 4.11: INTEREST ELASTICITIES OF SAVING, SAVING RATES, AND WEALTH-INCOME RATIOS (Corresponding to Table 1 of Evans)

	(1): ρ = -0.03			(2): ρ = 0.00			(3) ρ = +0.03		
Value of μ	Saving Elasticity	Saving Rate	Wealth/ Income	Saving Elasticity	Saving Rate	Wealth/ Income	Saving Elasticity	Saving Rate	Wealth/ Income
1.0	0.71	0.31	9.17	0.99	0.21	6.03	3.55	(0.06)	(1.64)
	(0.70)	(0.31)	(9.17)	(1.11)*	(0.34)*	(9.83)*	(3.55)	(0.06)	(1.64)
2.0	0.49	0.19	5.46	0.74	0.11	3.26	2.83	0.03	0.82
	(0.48)	(0.19)	(5.46)	(0.74)	(0.11)	(3.25)	(3.24)*	(0.04)*	(1.09)*
3.0	0.35	0.13	3.81	0.56	0.08	2.21	2.35	0.02	0.54
	(0.41)*	(0.15)*	(4.49)*	(0.56)	(0.08)	(2.21)	(2.97)*	(0.03)*	(0.82)*
4.0	0.23	0.10	2.91	0.41	0.06	1.67	1.86	0.01	0.40
	(0.34)*	(0.13)*	(3.81)*	(0.40)	(0.06)	(1.67)	(1.90)*	(0.01)	(0.40)

NOTES: The computations assume T = 50, R = 40, g = 0.02, n = 0.015, and r = 0.04. The saving rates and the wealth-income ratios are calculated relative to total income. Figures in parentheses are those reported by Evans (1983, Table 1, p.401). Asterisks indicate errors in his reported results.

TABLE 4.12: INTEREST ELASTICITIES OF SAVING, SAVING RATES, AND WEALTH-INCOME RATIOS (Corresponding to Table 2 of Evans)

	(1): ρ = -0.03			(2): ρ = 0.00			(3): ρ = +0.03		
Value of μ	Saving Elasticity	Saving Rate	Wealth/ Income	Saving Elasticity	Saving Rate	Wealth/ Income	Saving Elasticity	Saving Rate	Wealth/ Income
1.0	0.73	0.21	10.47	0.91	(0.15)	7.40	1.87	0.06	3.16
	(0.72)	(0.21)	(10.47)	(0.90)	(0.15)	(7.40)	(1.91)*	(0.06)	(3.16)
2.0	0.47	0.13	6.84	0.57	0.09	4.71	1.04	0.05	2.37
	(0.46)	(0.13)	(6.84)	(0.56)	(0.09)	(4.71)	(1.04)	(0.05)	(2.37)
3.0	0.30	0.10	5.24	0.37	0.07	3.70	0.62	0.04	2.10
	(0.30)	(0.10)	(5.24)	(0.35)*	(0.07)	(3.70)	(0.61)	(0.04)	(2.10)
4.0	0.19	0.09	4.38	0.23	0.06	3.18	0.37	0.04	1.97
	(0.19)	(0.09)	(4.38)	(0.23)	(0.06)	(3.18)	(0.36)	(0.04)	(1.97)
5.0	0.11	0.08	3.84	0.13	0.06	2.86	0.20	0.04	1.89
	(0.11)	(0.08)	(3.84)	(0.13)	(0.06)	(2.86)	(0.20)	(0.04)	(1.89)

NOTE: The computations assume $T = 50$, $R = 40$, $g = 0.01$, $n = 0.01$, and $r = 0.04$. The saving rates and the wealth-income ratios are calculated relative to total income. Figures in parentheses are those reported by Evans (1983, Table 2, p. 401). Asterisks indicate errors in his reported results.

presence of subsistence consumption. Thus, the net effect of a change in interest rate on consumption is much less certain in the QHCES than in the SHCES case.

Evans' Results

Parameter values are assigned to the theoretical model presented in chapter 3, based on the same assumptions as those made by Evans[2].

In an attempt to duplicate the results reported by Evans in his Tables 1 and 2 (p. 401), I repeated his experiments and both my results and his are reported in Tables 4.11 and 4.12 of the present study. Except for minor errors, presumably the result of rounding, the results in Table 2 of Evans are consistent with mine. However, Table 1 of Evans contains a number of substantial errors, as indicated in my Table 4.11 by asterisks[3]. The magnitude of the errors ranges from 0.02 to 0.62 in the interest elasticity of saving, 0.01 to 0.13 in the saving rate, and 0.27 to 3.80 in the wealth/income ratio.

My corrections demonstrate that the interest elasticity of saving is likely to be less than what Evans suggested.

4.5 ANALYSIS OF TRANSITION PATHS

In studying the interest elasticity of saving, Summers confined his analysis to steady-state situations. The transitional adjustment of the aggregate saving ratio resulting from a shock that ultimately leads to a new steady state can provide valuable information. Observation of an aggregate variable during transition can provide insight into the behaviour of the variable that may not be obvious from an examination of the steady state alone. Evans incorporated this type of analysis in his 1983 paper. My objective is to examine the effect of an interest rate shock on the aggregate saving ratio and to examine the transition path that occurs following the shock under alternative assumptions about the functional form of the utility function. By comparing the resulting adjustment paths, it is possible to shed further light on the important role of functional form.

The economy is initially in a steady state. This stiuation is then disturbed by introducing an increase of 2 percentage points in the interest rate from 4 to 6 percent, and maintaining the rate at the new level. The initial steady states, as well as the transition paths that lead to the new steady states, are reported in Tables 4.13 and 4.14.

Table 4.13 shows how the aggregate saving ratio adjusts from its initial steady state value following an interest rate shock when the utility function embodying the preferences of the individual is logarithmic, as shown in column 2, and when it is Stone-Geary, as shown in columns 3 to 5. Regardless of the functional specification of the utility function, the adjustment process from the initial steady state is gradual. It is complete only when all the cohorts that were alive at the time of the shock have died. (This finding has been noted also by Evans, 1983.) This, 50 years are required before a new steady state is obtained.

In the case of the logarithmic utility function, the initial steady state aggregate saving ratio is about 5.6 percent. Following the shock, this value increases to 21.3 percent, as shown by row 2 in column 2 of Table 4.13. During the transition period, the saving ratio gradually declines until it achieves a new steady state level of 13.4 percent.

In the case of a Stone-Geary utility function, the initial steady state aggregate saving ratio depends on the value of the parameter Ψ, as shown by row 1 and columns 3 to 5. When Ψ is specified to be 0.33, the effect of a 2 percentage point increase in the interest

TABLE 4.13

ADJUSTMENT PATH FROM ONE STEADY STATE TO ANOTHER FOLLOWING A CHANGE IN THE INTEREST RATE (from 4 to 6 percent per year)

(δ = 0.0)

	SAVING RATES			
	Logarithmic Utility Function	STONE-GEARY UTILITY FUNCTION		
Period		Ψ=0.33	Ψ=0.66	Ψ=0.90
0	.056091	.038328	.019785	.005779
1	.212649	.140862	.064398	.005581
2	.210528	.139670	.063700	.004913
3	.208426	.138485	.063027	.004298
4	.206348	.137313	.062383	.003742
5	.204297	.136158	.061776	.003251
6	.202273	.135027	.061210	.002831
7	.200297	.133926	.060692	.002487
8	.198358	.132862	.060229	.002228
9	.196466	.131840	.059827	.002059
10	.194627	.130867	.059494	.001989
11	.192848	.129952	.059236	.002024
12	.191105	.129068	.059029	.002138
13	.189381	.128201	.058855	.002311
14	.187665	.127337	.058698	.002527
15	.185951	.126468	.058550	.002775
16	.184230	.125588	.058400	.003045
17	.182500	.124690	.058244	.003328
18	.180756	.123772	.058077	.003619
19	.178996	.122832	.057894	.003912
20	.177226	.121868	.057692	.004201
21	.175438	.120880	.057469	.004484
22	.173636	.119867	.057224	.004757
23	.171822	.118830	.056955	.005017
24	.169995	.117770	.056662	.005262
25	.168160	.116689	.056345	.005488
26	.166318	.115587	.056003	.005696
27	.164473	.114467	.055637	.005883
28	.162627	.113332	.055248	.006048
29	.160784	.112184	.054836	.006191
30	.158949	.111027	.054404	.006311
31	.157126	.109864	.053953	.006407
32	.155320	.108698	.053485	.006480
33	.153536	.107534	.053001	.006530
34	.151779	.106376	.052505	.006557
35	.150055	.105229	.052000	.006562
36	.148372	.104098	.051488	.006546
37	.146734	.102988	.050972	.006511
38	.145151	.101906	.050458	.006457
39	.143629	.100857	.049948	.006386
40	.142177	.099849	.049447	.006304
41	.140803	.098887	.048960	.006209
42	.139516	.097981	.048493	.006105
43	.138327	.097136	.048049	.005995
44	.137244	.096363	.047636	.005883
45	.136280	.095670	.047260	.005773
46	.135444	.095066	.046927	.005668
47	.134750	.094561	.046644	.005574
48	.134209	.094165	.046420	.005494
49	.133835	.093690	.046261	.005436
50	.133640	.093746	.046178	.005403

NOTE: The computations are based on the assumption that $T = 50$, $R = 40$, $g = 0.02$, $n = 0.015$, $\delta = 0.00$, and $\rho = 0.03$.

rate is to raise the steady state saving ratio from 3.8 percent to 14.1 percent. Similarly, when Ψ is set to 0.66, the steady state saving ratio increases from 2.0 to 6.4 percent. However, when Ψ is assigned a value of 0.90, the steady state saving ratio (0.58 percent, initially) shows a slight decline instead of an increase.

The adjustment process following the shock that is associated with the Stone-Geary utility function depends on the value of Ψ. When this parameter is specified to take values of 0.33 and 0.66, the resulting transition paths exhibit gradual declines in the saving ratio. The new steady state ratio is higher than the initial one. When Ψ is specified to be 0.90, the saving rate declines and remains below its initial steady state value for ten periods. Thereafter, the saving rate gradually increases, attains its maximum value, and then starts to decline.

Comparison of column 2 with columns 3 to 5 reveals three important facts. First, the response of the aggregate saving ratio to the shock varies, depending on whether the utility function is logarithmic or Stone-Geary. In general, it is true that the responsiveness is greater in the logarithmic than in the Stone-Geary case. If Ψ is large, it is possible for the saving ratio in period 1 to be smaller than the initial steady state value in the case of the Stone-Geary function, as demonstrated in column 5 of Table 4.13. Second, the adjustment path that follows the shock does not have to be characterized by a continuous decline in the saving ratio. Third, it is possible for the aggregate saving ratio to be lower in the new steady state than in the initial one, depending on the utility function employed.

To gain further insight into the implications of alternative functional forms for saving, I repeated the experiment using the SHCES and the QHCES functions. The initial state as well as the transitional saving ratios are reported in Table 4.14. The results confirm again that responsiveness to the shock depends on the form of the utility function.

The above analysis demonstrates that an identical shock to the interest rate may generate different transitional as well as ultimate saving ratios when the utility functions differ. This additional information about the implications of functional form is missing in studies that confine their analysis to steady state situations.

TABLE 4.14

ADJUSTMENT PATH FROM ONE STEADY STATE TO ANOTHER FOLLOWING A CHANGE IN THE INTEREST RATE (from 4 to 6 percent per year)

(δ = 1.00)

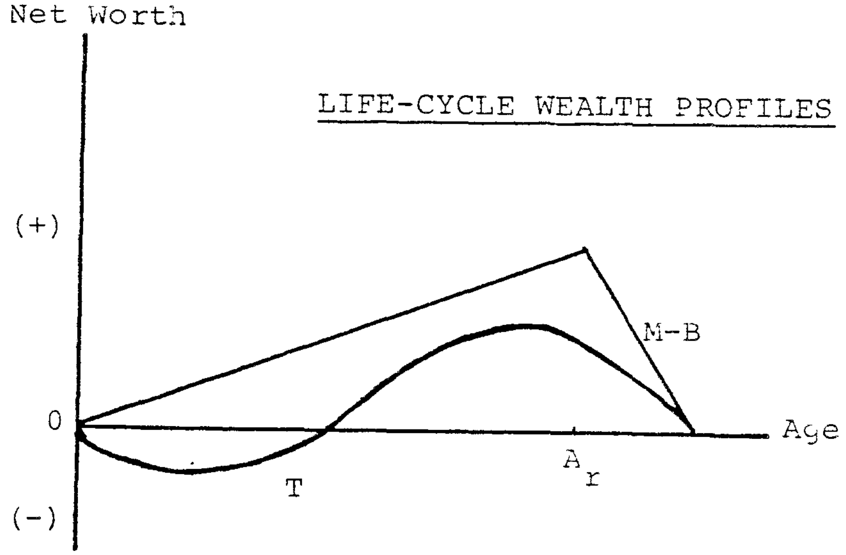
	SAVING RATES			
		QHCES		
Period	SHCES	Ψ= 0.33	Ψ=0.66	Ψ=0.90
0	.139466	.276583	.140778	.012673
1	.408539	.286433	.148419	.036191
2	.405631	.285626	.148449	.035676
3	.402647	.284698	.148403	.035167
4	.399595	.283659	.148291	.034667
5	.396482	.282520	.148123	.034187
6	.393316	.281290	.147907	.033737
7	.390104	.279982	.147654	.033329
8	.386855	.278606	.147376	.032974
9	.383578	.277173	.147084	.032685
10	.380281	.275696	.146791	.032478
11	.376975	.274187	.146511	.032366
12	.373639	.272627	.146222	.032325
13	.370262	.271003	.145908	.032338
14	.366834	.269307	.145556	.032391
15	.363349	.267531	.145159	.032473
16	.359803	.265672	.144709	.032575
17	.356193	.263727	.144203	.032690
18	.352520	.261697	.143635	.032812
19	.348783	.259579	.143004	.032935
20	.344983	.257376	.142308	.033054
21	.341122	.255088	.141546	.033166
22	.337203	.252719	.140717	.033267
23	.333229	.250271	.139821	.033353
24	.329204	.247747	.138859	.033421
25	.325132	.245152	.137832	.033470
26	.321020	.242490	.136742	.033495
27	.316872	.239766	.135591	.033496
28	.312697	.236987	.134382	.033473
29	.308502	.234159	.133117	.033415
30	.304295	.231289	.131801	.033333
31	.300086	.228386	.130437	.033220
32	.295885	.225458	.129031	.033075
33	.291704	.222515	.127588	.032902
34	.287555	.219568	.126114	.032696
35	.283454	.216628	.124617	.032461
36	.279414	.213709	.123105	.032197
37	.275453	.210824	.121585	.031907
38	.271589	.207990	.120068	.031592
39	.267841	.205222	.118564	.031255
40	.264233	.202539	.117086	.030901
41	.260787	.199961	.115645	.030534
42	.257530	.197509	.114258	.030159
43	.254489	.195207	.112939	.029783
44	.251694	.193080	.111705	.029414
45	.249179	.191156	.110576	.029060
46	.246979	.189464	.109572	.028730
47	.245131	.188036	.108716	.028438
48	.243677	.186907	.108031	.028195
49	.242660	.186114	.107546	.028015
50	.242126	.185696	.107287	.027917

NOTE: The computations are based on the assumption that $T = 50$, $R = 40$, $g = 0.02$, $n = 0.015$, $\delta = 0.100$, and $\rho = 0.03$.

FOOTNOTES

Chapter 4

1. In the literature, there are two well known life-cycle net worth profiles, usually referred to as the Modigliani-Brumberg Model and the Tobin Model. According to the former, the net worth profile starts at zero, rises with age until retirement, and then declines to zero; according to the Tobin model, net worth starts from zero, is negative at young ages, becomes positive as the individual approaches retirement, and declines after retirement. Wolff (1979) illustrated these theories graphically as follows:



where

M-B = Modigliani-Brumberg Model

T = Tobin Model

A_r = Retirement Age.

2. These assumptions are given in the notes to Tables 4.11 and 4.12 of the present study.

3. In addition to the substantial errors there are also some minor differences between Evans' figures and mine, which are probably the result of rounding. The latter are not indicated by asterisks.

CHAPTER 5

CONSUMPTION-LEISURE CHOICES AND THE INTEREST ELASTICITY OF SAVING

The theoretical life-cycle model developed in chapter 3 assumes that an individual maximizes his consumption of market goods subject to a lifetime budget constraint. Moreover, it assumes that the individual labour supply is fixed and that therefore the individual's income stream is exogenously determined.

An alternative way of modelling the life-cycle behaviour of the individual is to view earnings as being an outcome of the labour supply decision. To capture this notion I incorporate consumption and leisure into the utility function. This allows the individual to choose his hours of work depending on the exogenous wage rates he faces over the life-cycle.

In examining the interest elasticity of saving, Summers as well as Evans focused on a life-cycle model in which labour supply is assumed to be fixed. This chapter attempts to address further Summers' claim of high interest elasticity of saving by relaxing the assumption of exogenous labour income. My findings clearly show that Summers' claim is fragile under the assumption of endogenous labour income.

This chapter is organized as follows. In Section 5.1, the existence of steady states is discussed in relation to functional forms of the utility function. The reasons for confining my analysis to the Stone-Geary utility is also briefly mentioned. A theoretical life-cycle model based on the Stone-Geary utility function is developed in Section 5.2. In Section 5.3 the parameters of the model are specified. The micro-simulation results of the experiments are discussed in Section 5.4. Aggregation of the micro relationship is considered in Section 5.5. The interest elasticity of saving, which is the key issue of the chapter, is analyzed in Section 5.6.

5.1 UTILITY FUNCTIONS AND STEADY STATES

In an article that appeared in the Journal of International Economics (1983), Auerbach, Kotlikoff and Skinner pointed out conditions under which productivity growth resulting from technical progress is compatible with a steady state when labour supply is endogenous. Although they did not doubt the existence of a steady state when productivity growth is analyzed in the context of fixed labour supply, as in the work of Auerbach and Kotlikoff (1981), they made it clear that this conclusion does not hold when the assumption of inelastic labour supply is relaxed unless the utility function is Cobb-Douglas. The requirement is stated in their article as follows:

> It is impossible, in general, to retain this element once labour supply is endogenous; the steady rise of wage rates over time is not compatible with a steady state unless $\rho = 1$, i.e., unless the utility function of contemporoneous consumption and leisure is Cobb-Douglas.
>
> (Auerbach, Kotlikoff and Skinner, 1983, p. 88.)

Their finding shows that a CES utility function is not compatible with the notion of a steady state under the conditions mentioned above. The authors did not address the issue in reference to the Stone-Geary utility function. If the subsistence consumption of successive generations for any age grows at the rate of technical progress, it is possible to show that the Stone-Geary utility function is consistent with a steady state.

As mentioned above, it is not possible to employ a CES utility function if leisure is a choice variable, and because of this, I confined my analysis to the Stone-Geary function.

5.2 THE BASIC MODEL

In this section a life-cycle model which takes into account the consumption-leisure choice of the individual is considered. The basic model assumes that the individual chooses consumption and leisure so as to maximize a Stone-Geary utility function of the form:

$$(5.1) \qquad U_\tau = \sum_{t=\tau}^{T} (\alpha_t \log C_t^* + \alpha_t^\ell \log L_t^*)(1 + \rho)^{\tau-t}$$

where

$$C_t^* = (C_t - \gamma_t) > 0,$$

$$L_t^* = (L_t - \gamma_t^\ell) > 0$$

and T = age to which the individual will live, which he (she) knows with certainty;

C_t = total consumption at age t;

L_t = total leisure at age t;

γ_t = subsistence consumption at age t;

γ_t^ℓ = subsistence leisure at age t;

α_t = intensity parameter for consumption at age t;

α_t^ℓ = intensity parameter for leisure at age t;

ρ = rate of time preference;

τ = individual's present age;

U_τ = lifetime utility as perceived at age τ.

The Stone-Geary utility function given by equation (5.1) does not restrict preferences to be homothetic from the origin. Furthermore, by specifying the subsistence consumption and leisure parameters to zero, the function reduces to the standard logarithmic function. It is also possible to set subsistence leisure to zero while at the same time maintaining positive subsistence consumption.

The individual is assumed to maximize (5.1) subject to a budget constraint of the form:

$$\sum_{t=\tau}^{T} C_t(1 + r)^{\tau-t} + \sum_{t=}^{R} W_t L_t(1 + r)^{\tau-t} \tag{5.2}$$

$$= a_\tau + \sum_{t=\tau}^{R} W_t H(1 + r)^{\tau-t}$$

where H = total annual time endowment available to the individual for allocation (assumed independent of age);

a_τ = net worth of the individual at the beginning of period τ;

R = the number of years the individual spends in the work force;

r_t = after-tax rate of return;

W_t = wage rate net of tax.

$$(5.3) \qquad C_\tau^* = \alpha_\tau \left[\frac{S_\tau}{D_{1\tau} + D_{2\tau}}\right]$$

and

$$(5.4) \qquad L_\tau^* = \frac{\alpha_\tau^\ell}{W_\tau} \left[\frac{S_\tau}{D_{1\tau} + D_{2\tau}}\right]$$

where $S_\tau = P_\tau - E_\tau$

$$P_\tau = a_\tau + \sum_{t=\tau}^{R} W_t H (1 + r)^{\tau-t}$$

$$E_\tau = \sum_{t=\tau}^{T} \gamma_t (1 + r)^{\tau-t} + \sum_{t=\tau}^{R} \gamma_t^\ell W_t (1 + r)^{\tau-t}$$

$$D_{1\tau} = \sum_{t=\tau}^{T} \alpha_t (1 + \rho)^{\tau-t}$$

$$D_{2\tau} = \sum_{t=\tau}^{R} \alpha_t^\ell (1 + \rho)^{\tau-t}$$

The symbol S_τ stands for supernumerary expenditure, P for potential income and E for expenditure on subsistence consumption and leisure.

Equations (5.3) and (5.4) represent the demand for consumption goods and leisure, respectively, over and above subsistence requirements.

Rewriting equation (5.4) in the following way

$$(5.5) \qquad L_\tau = \gamma_\tau^\ell + \frac{\alpha_\tau^\ell}{W_\tau}\left[\frac{S_\tau}{D_{1\tau} + D_{2\tau}}\right]$$

and subtracting the right hand side of equation (5.5) from the total time endowment H yields a labour supply equation.

$$(5.6) \qquad L_\tau^S = (H - \gamma_\tau^\ell) - \frac{\alpha_\tau^\ell}{W_\tau}\left[\frac{S_\tau}{D_{1\tau} + D_{2\tau}}\right]$$

Multiplying (L_τ^S) by the wage rate W_τ yields the labour income of an individual of age τ.

5.3 PARAMETERIZATION OF THE MODEL

The major components of the model that require careful attention in specifying the parameter values are the wage profile, the utility function, the discount factors and the rate of population growth. In all the experiments that follow, population is assumed to grow at the rate of 1.5 per cent per annum.

Although the individual age-wage profile is assumed to be exogenous, it is derived on the basis of some empirical work. The parameters of the utility function are also specified on the basis of earlier empirical findings. I will discuss these issues in detail.

Specification of the Wage Function

Studies of the wage function that exist in the literature include Hurd (1977), Holm (1970), Blinder (1973), Heckman (1974), Blomquist (1978), Welch (1979), and Nagatani and Wales (1981). These studies share a common view with regard to the cross-sectional age-wage profile. For instance, Hurd's and Holm's findings suggest that wage rates for white males reach a maximum in the age interval 45-54. Heckman studies male individuals with 10-12 years of schooling and found the wage profile to reach a maximum at ages 48-50. These studies seem to suggest the existence of a peak in the cross-sectional wage profile. Taking this finding into consideration, it is possible to construct the lifetime wage function for the individual. Blinder suggested a methodology that leads to the derivation of this function.

> ...if the wage rate is considered to be a function of both age A (due to seniority, experience, and the like), and calendar time t (due to technological improvements not embodied in the worker), $W = W(A,t)$, the desired growth rate is
>
> $$m = \frac{1}{W} \left.\frac{\partial W}{\partial A}\right|_{t=\text{const.}} + \frac{1}{W} \frac{\partial W}{\partial t} \left.\frac{\partial W}{\partial t}\right|_{A = \text{const.}}$$
>
> The first term on the right-hand side is the effect of age on wages, calendar time held constant. Empirically, this would be the age trend observed in cross-sectional studies at a point in time; call it m_1.
>
> The second term is the general growth of wage levels over time, age held constant; call it m_2. Then $m = m_1 + m_2$.
>
> (Blinder, 1974, p. 99.)

A hypothetical wage-profile which characterizes the wage rate as a function of both age and calendar time is shown in Figure 5.1. (Adapted from Blomquist, (1978).)

HYPOTHETICAL CROSS-SECTION WAGE PROFILES IN TWO CONSECUTIVE YEARS

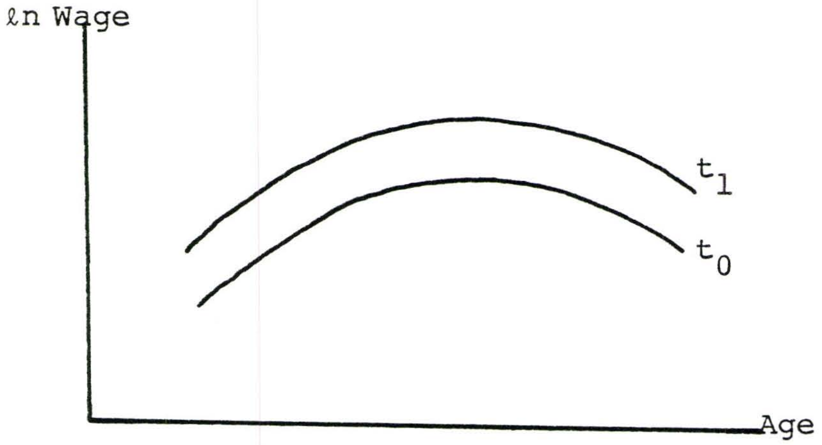


FIGURE 5.1

Keeping all factors other than age constant the wage rate of a cross-section in the economy in any calendar year t_0 is depicted by the curve t_0. Moving along this curve from left to right, i.e., from young to old cohorts, the wage rate rises at first, then attains its maximum and starts to decline. The change in the slope of the curve is assumed to be the result of age-related characteristics such as seniority, experience, etc.

As calendar time progresses so does the age of the individual. However, keeping the age factor constant, the wage rate function of a cross-section shifts up as indicated in Figure 5.1. At any age, the vertical gap between the curves is assumed to be the result of productivity associated with technological improvement rather than productivity embodied in the worker. The implication is that an individual who started to work in calendar year t_1 at age 'j', i.e., one year later than the individual who started in year t_0 when he was age 'j', enjoys a higher wage rate by an amount equal to 'g'. The wage path of the representative individual is obtained by joining the relevant points on successive cross sectional profiles.

Given the cross sectional wage function, it is possible to derive the individual age-wage profile using the procedure described above. For instance, Auerbach, Kotlikoff and Skinner (1983) adopted a cross-section regression of weekly labour earnings estimated by Welch (1979) to determine the individual wage function. They replaced the experience and experience-squared variables by age and age-squared of the individual, respectively, in Welch's equation:

$$e_t = 4.47 + 0.33t - 0.00067t^2$$

where e_t is earnings of high school graduates, t and t^2 are the individual's age and age-squared variables. Using this equation, the authors derived an individual wage function that peaks at the economic age of 30. This implies a rate of productivity growth associated with technological improvement of 0.0014835 per annum. In the simulation experiments, I employed this value for the parameter 'g'[1].

PARAMETERS OF THE UTILITY FUNCTION

The utility function given below requires specification of its parameters α, α^{ℓ}, γ, ρ and γ^{ℓ}. The leisure intensity parameter α^{ℓ} depends on the labour endowment available to the individual per year and the typical person is assumed to spend about 2,000 hours in work. Blinder (1974) assumed the total endowment of time potentially available for work to be 5864 hours per year while Auerbach, Kotlikoff and Skinner assumed it to be 5000 hours. The value of the parameter α^{ℓ} is computed accordingly, yielding a value of 0.66 in the case of Blinder and 0.6 in the case of Auerbach, Kotlikoff and Skinner. Assigning a value of 0.6 to α^{ℓ} leaves a value of 0.4 for α.

In the case of the Cobb-Douglas utility function, the subsistence consumption and leisure parameters, γ and γ^{ℓ}, are set to zero in equation (5.1) In the case of a Stone-Geary utility function, however, the parameter γ is computed in the same fashion as mentioned in chapter 4[2]. However, γ^{ℓ} is calculated as a fraction of some arbitrary value of leisure: $\gamma^{\ell} = \Omega\bar{L}$, where $\Omega \geq 0$ and $\bar{L}$ is total leisure[3].

5.4 SIMULATION EXPERIMENTS

The interest elasticity of saving has been examined by Summers under the assumption that individuals maximize consumption given their labour income. In chapter 4 of this thesis, I demonstrated that Summers' results are sensitive to the form of the utility function when labour income is considered to be exogenous. In this section the implications of relaxing the assumption of fixed labour supply for the life-cycle patterns of the individual decision variables are examined at the micro level. This helps to understand and explain the differences in the interest elasticity of saving that arise at the aggregate level.

In the micro simulation experiments, the optimal lifetime paths of labour supply, consumption, labour income, net worth and the saving rate that result from the maximization of Cobb-Douglas and Stone-Geary utility functions are considered. In the case of the Cobb-Douglas function, three experiments are performed, depending on whether the values assigned to the rate of time preference and the interest rate are close, substantially different, or identical to each other.

Simulation Experiments Using A Cobb-Douglas Utility Function

Case 1: ρ close to r

In simulating the optimal paths of the decision variables of the individual, the rate of time preference and the interest rate are assumed to take values that do not differ markedly. Based on this, two experiments are performed.

Experiment 1

In this experiment, the rate of time preference is set at 0.03 and and the interest rate at 0.035 per annum. The other parameter values assumed in the experiment are shown in the note to Table 5.1

In constructing the age-wage profile, the wage rate is normalized to unity for an individual of economic age 1. The wage rate increases with age and attains its maximum value at economic age 30. Thereafter, it declines until the age of retirement. The wage rate is assumed to be zero during the retirement period.

TABLE 5.1

SIMULATED PATHS OF THE KEY MICRO VARIABLES: EXPERIMENT 1

Age	Wage Rate	Earnings	Labour Supply	Consumption	Net Worth	Saving Rate
1	1.000000	.425370	.425370	.383087	0.000000	.099402
2	1.008377	.430957	.427377	.384947	.042283	.109820
3	1.016478	.436255	.429183	.386815	.089773	.119668
4	1.024302	.441262	.430793	.388693	.142355	.128969
5	1.031847	.445977	.432213	.390580	.199907	.137743
6	1.039113	.450399	.433446	.392476	.262301	.146011
7	1.046097	.454526	.434497	.394381	.329404	.153789
8	1.052800	.458357	.435369	.396296	.401078	.161092
9	1.059219	.461690	.436066	.398219	.477177	.167934
10	1.065353	.465124	.436592	.400152	.557549	.174328
11	1.071200	.468058	.436947	.402095	.642035	.180284
12	1.076761	.470690	.437136	.404047	.730469	.185811
13	1.082032	.473020	.437159	.406008	.822679	.190918
14	1.087013	.475045	.437018	.407979	.918484	.195611
15	1.091703	.476764	.436715	.409960	1.017696	.199896
16	1.096100	.478175	.436252	.411950	1.120120	.203777
17	1.100203	.479279	.435628	.413949	1.225550	.207256
18	1.104010	.480072	.434844	.415959	1.333773	.210336
19	1.107521	.480554	.433900	.417978	1.444568	.213016
20	1.110733	.480723	.432798	.420007	1.557704	.215295
21	1.113646	.480577	.431535	.422046	1.672939	.217172
22	1.116258	.480116	.430112	.424095	1.790023	.218643
23	1.118568	.479338	.428528	.426153	1.908695	.219703
24	1.120575	.478242	.426782	.428222	2.028684	.220345
25	1.122276	.476825	.424873	.430301	2.149708	.220561
26	1.123672	.475087	.422799	.432390	2.271472	.220341
27	1.124759	.473026	.420558	.434489	2.393670	.219674
28	1.125538	.470641	.418148	.436598	2.515986	.218547
29	1.126006	.467930	.415566	.438717	2.638088	.216944
30	1.126162	.464892	.412811	.440847	2.759634	.214847
31	1.126005	.461525	.409878	.442987	2.880265	.212235
32	1.125534	.457828	.406765	.445137	2.999612	.209086
33	1.124747	.453799	.403468	.447298	3.117289	.205374
34	1.123642	.449437	.399983	.449470	3.232895	.201069
35	1.122218	.444741	.396305	.451652	3.346014	.196137
36	1.120475	.439709	.392431	.453844	3.456214	.190542
37	1.118410	.434339	.388354	.456047	3.563046	.184240
38	1.116021	.428630	.384070	.458261	3.666044	.177183
39	1.113309	.422580	.379572	.460486	3.764725	.169317
40	1.110270	.416189	.374854	.462721	3.858585	.160581
41	0.000000	0.000000	0.000000	.464967	3.947104	-2.365702
42	0.000000	0.000000	0.000000	.467224	3.620285	-2.687352
43	0.000000	0.000000	0.000000	.469492	3.279771	-3.089941
44	0.000000	0.000000	0.000000	.471771	2.925070	-3.608157
45	0.000000	0.000000	0.000000	.474062	2.555676	-4.299817
46	0.000000	0.000000	0.000000	.476363	2.171063	-5.268987
47	0.000000	0.000000	0.000000	.478675	1.770688	-6.723800
48	0.000000	0.000000	0.000000	.480999	1.353986	-9.149901
49	0.000000	0.000000	0.000000	.483334	.920377	-14.004222
50	0.000000	0.000000	0.000000	.485680	.469256	-28.571429
51					.000000	

NOTE: The computations assume $\alpha = 0.4$; $\alpha^{\ell} = 0.6$; $\rho = 0.03$; $r = 0.035$; $g = 0.0014835$; $T = 50$; $R = 40$; and $n = 0.015$.

Given the wage profile, the resulting optimal path of labour supply is shown in column 4 of Table 5.1. In this case the profile takes the shape of an inverted 'U'. The peak work effort occurs at an economic age of 13 and has a value of 0.437159. These results are not significantly different from those reported by Auerbach, Kotlikoff and Skinner (1983). After age 13, the labour supply declines until age 40. Thereafter, it is assumed to be zero.

The earning profile, which is the product of the wage rate and the hours of work supplied, is given in column 3 of Table 5.1. The profile we observe is similar to that of the labour supply. The peak of the earning profile occurs at economic age 20, rather than at age 13. Because of mandatory retirement at age 40, the earning path is assumed to be zero during the retirement period. One important result to note is that the age of peak effort comes before the age of peak earnings which, in turn, comes before the age of peak wage rate. These findings are consistent with the theoretical results of Heckman:

> If wage rates rise to a peak and decline after a certain age... it is possible to observe peaks for hours worked and the consumption of goods only at the boundaries

> of the life cycle. If these peaks occur at an interior age, and $r > \rho$, the peak age for hours worked comes before the age of peak wage rates and the age of peak earnings occurs between these peak ages.
>
> (Heckman, 1974, p. 193.)

The consumption profile monotonically increases with the age of the individual. The points of minimum and maximum consumption occur at age 1 and age 50, respectively, as shown in column 5 of Table 5.1. This finding is also consistent with that of Heckman mentioned above.

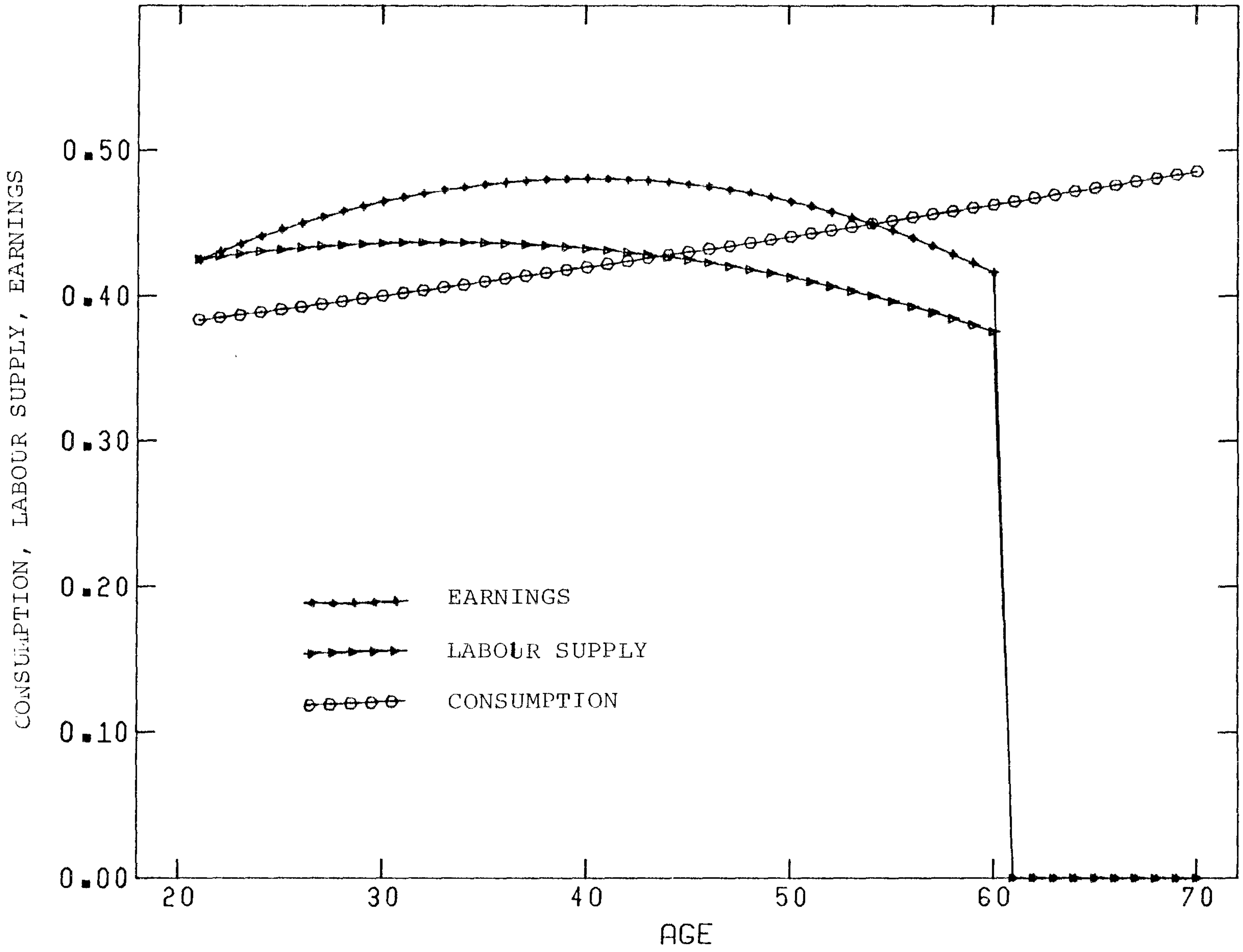
The net worth profile follows the path predicted by the Modigliani and Brumberg model. As shown in column 6 of Table 5.1, the net worth profile never becomes negative.

The path of the saving rate reaches its maximum value at age 25 and gradually declines until age 40. This is followed by a sharp decline during the retirement period, as reported in column 7 of Table 5.1.

The consumption, labour supply, and earnings profiles that result from Experiment 1 are presented graphically in Figure 5.2. The differences in ages at which the variables attain their minimum and maximum levels are apparent from this figure.

FIGURE 5.2

CONSUMPTION, LABOUR SUPPLY, AND EARNINGS PROFILES: EXPERIMENT 1



Experiment 2

In this experiment the rate of time preference and the interest rate are assigned values of 0.035 and 0.03 per annum respectively. The assumptions with regard to the remaining parameters are identical to those of Experiment 1. The results of Experiment 2 are reported in Table 5.2.

Comparison of Tables 5.1 and 5.2 reveals great differences in the optimal paths of the endogenous variables. In Experiment 2, the labour supply increases steadily with age and the point of peak work effort occurs at economic age 40, rather than in the interior age of the working span, as in Experiment 1. Earnings increase with age and the profile does not reveal a unique peak, but rather it remains almost flat once it attains its maximum. As shown in column 3 of Table 5.2, earnings are almost constant beginning about age 38-40. In contrast, in Experiment 1, there is a unique age at which earnings attain a maximum.

Since $\rho > r$, the consumption profile monotonically decreases with age and the peak age of consumption thus occurs at age 1. On the other hand, the age of minimum consumption occurs at age 50, which is the end

TABLE 5.2

SIMULATED PATHS OF THE KEY MICRO VARIABLES: EXPERIMENT 2

Age	Wage Rate	Earnings	Labour Supply	Consumption	Net Worth	Saving Rate
1	1.000000	.328218	.328218	.447855	0.000000	-.364506
2	1.008377	.339840	.337017	.445691	-.119637	-.325473
3	1.016478	.351170	.345478	.443538	-.229178	-.288239
4	1.024302	.362208	.353615	.441396	-.328318	-.252688
5	1.031847	.372952	.361441	.439263	-.417355	-.218715
6	1.039113	.383401	.368970	.437141	-.496186	-.186222
7	1.046097	.393553	.376211	.435029	-.564912	-.155122
8	1.052800	.403408	.383177	.432928	-.623232	-.125332
9	1.059219	.412964	.389876	.430836	-.671449	-.096776
10	1.065353	.422220	.396326	.428755	-.709465	-.069385
11	1.071200	.431175	.402516	.426684	-.737283	-.043093
12	1.076761	.439827	.408472	.424622	-.754911	-.017841
13	1.082032	.448175	.414198	.422571	-.762354	.006427
14	1.087013	.456219	.419699	.420530	-.759520	.029763
15	1.091703	.463956	.424983	.418498	-.746720	.052215
16	1.096100	.471385	.430057	.416476	-.723564	.073829
17	1.100203	.478506	.434925	.414464	-.690465	.094645
18	1.104010	.485317	.439595	.412462	-.647137	.114703
19	1.107521	.491816	.444070	.410470	-.593597	.134040
20	1.110733	.498003	.448355	.408487	-.530161	.152690
21	1.113646	.503876	.452456	.406513	-.456549	.170685
22	1.116258	.509434	.456377	.404550	-.372383	.188055
23	1.118568	.514676	.460120	.402595	-.279185	.204829
24	1.120575	.519599	.463690	.400650	-.175480	.221033
25	1.122276	.524204	.467090	.398715	-.061795	.236691
26	1.123672	.528489	.470323	.396789	.061841	.251828
27	1.124759	.532451	.473392	.394872	.195396	.266465
28	1.125538	.536091	.476298	.392964	.338338	.280623
29	1.126006	.539407	.479045	.391066	.492130	.294323
30	1.126162	.542397	.481633	.389177	.655235	.307582
31	1.126005	.545061	.484066	.387296	.828113	.320418
32	1.125534	.547396	.486343	.385425	1.010721	.332848
33	1.124747	.549401	.488467	.383564	1.203012	.344886
34	1.123642	.551076	.490437	.381711	1.404341	.356549
35	1.122218	.552419	.492256	.379867	1.616454	.367850
36	1.120475	.553428	.493922	.378031	1.837500	.378802
37	1.118410	.554102	.495437	.376205	2.069021	.389418
38	1.116021	.554440	.496800	.374388	2.307959	.399710
39	1.113309	.554440	.498011	.372579	2.557249	.409689
40	1.110270	.554101	.499069	.370779	2.815828	.419366
41	0.000000	0.000000	0.000000	.368988	3.083625	-2.988683
42	0.000000	0.000000	0.000000	.367206	2.807145	-3.360367
43	0.000000	0.000000	0.000000	.365432	2.524154	-3.825795
44	0.000000	0.000000	0.000000	.363666	2.234447	-4.425147
45	0.000000	0.000000	0.000000	.361909	1.937315	-5.225387
46	0.000000	0.000000	0.000000	.360161	1.634040	-6.347047
47	0.000000	0.000000	0.000000	.358421	1.322900	-8.031197
48	0.000000	0.000000	0.000000	.356690	1.004166	-10.840329
49	0.000000	0.000000	0.000000	.354966	.677601	-16.461916
50	0.000000	0.000000	0.000000	.353252	.342363	-33.333333
51					.000000	

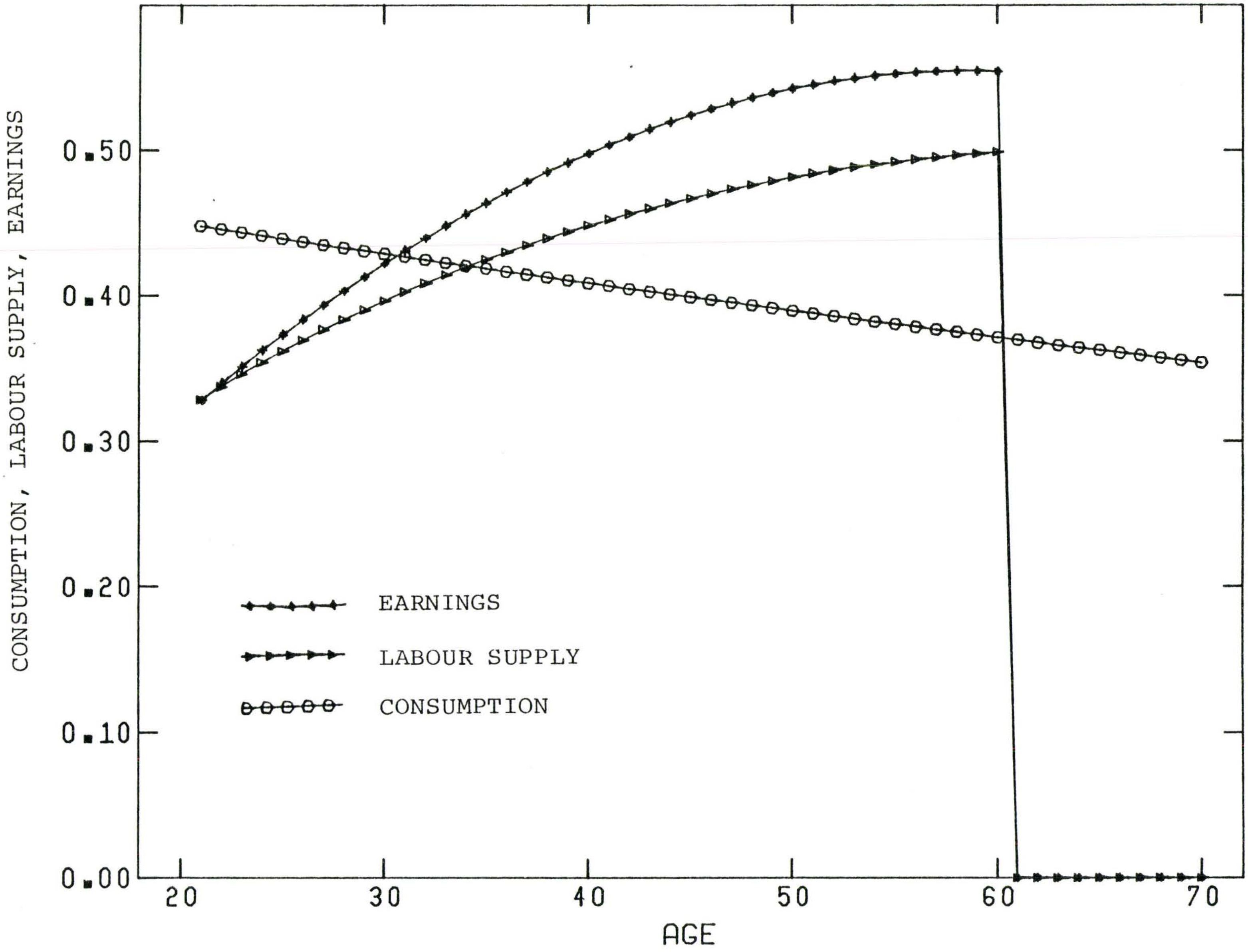
NOTE: The computations assume $\alpha = 0.4$; $\alpha^{\ell} = 0.6$; $\rho = 0.035$; $r = 0.03$; $g = 0.0014835$; $T = 50$; $R = 40$; and $n = 0.015$.

point of the life-cycle, as shown in column 5 of Table 5.2. Comparison of the net worth profiles reveals major differences between Experiment 1 and Experiment 2. With ρ being greater than r, the resulting net worth profile begins with zero at age 1, then becomes negative until age 25, then positive and increasing until age 40, then positive but declining, and finally zero at age 50, as shown in column 6 of Table 5.2. This result is consistent with the predictions of the Tobin model while the result of Experiment 1 follows the predictions of the Modigliani-Brumberg model. From column 7 of Table 5.2, it is clear that the saving rate becomes negative up to age 12, then positive and increasing up to age 40. The retirement period, however, is marked by a sharp decline in the saving rate. In Experiment 1, the saving rate peaks at age 25 (compared to age 40 in Experiment 2) and never becomes negative from age 1 to age 40.

The optimal paths that arise from Experiment 2 are presented graphically in Figure 5.3.

FIGURE 5.3

CONSUMPTION, LABOUR SUPPLY, AND EARNINGS PROFILES: EXPERIMENT 2



Case 2: $\rho = r$

In Case 2, the time preference and interest rates are restricted to having the same values. Other than this, the assumed parameter values are identical to those of Case 1.

Experiment 3

In this experiment both the rate of time preference and the interest rate are set at 0.03 per cent per annum. The results of the experiment are reported in Table 5.3 and the profiles of the relevant decision variables are depicted in Figure 5.4.

One important result to note is that the labour supply and the earnings profiles attain their maximum points at the age for which the wage rate is a maximum, which is age 30. (If the time preference and interest rates were not equal, we would expect the peak age of labour supply to be different from the peak age of earnings.)

Since ρ is equal to r, optimal consumption remains constant at a value of 0.413419 over the

TABLE 5.3

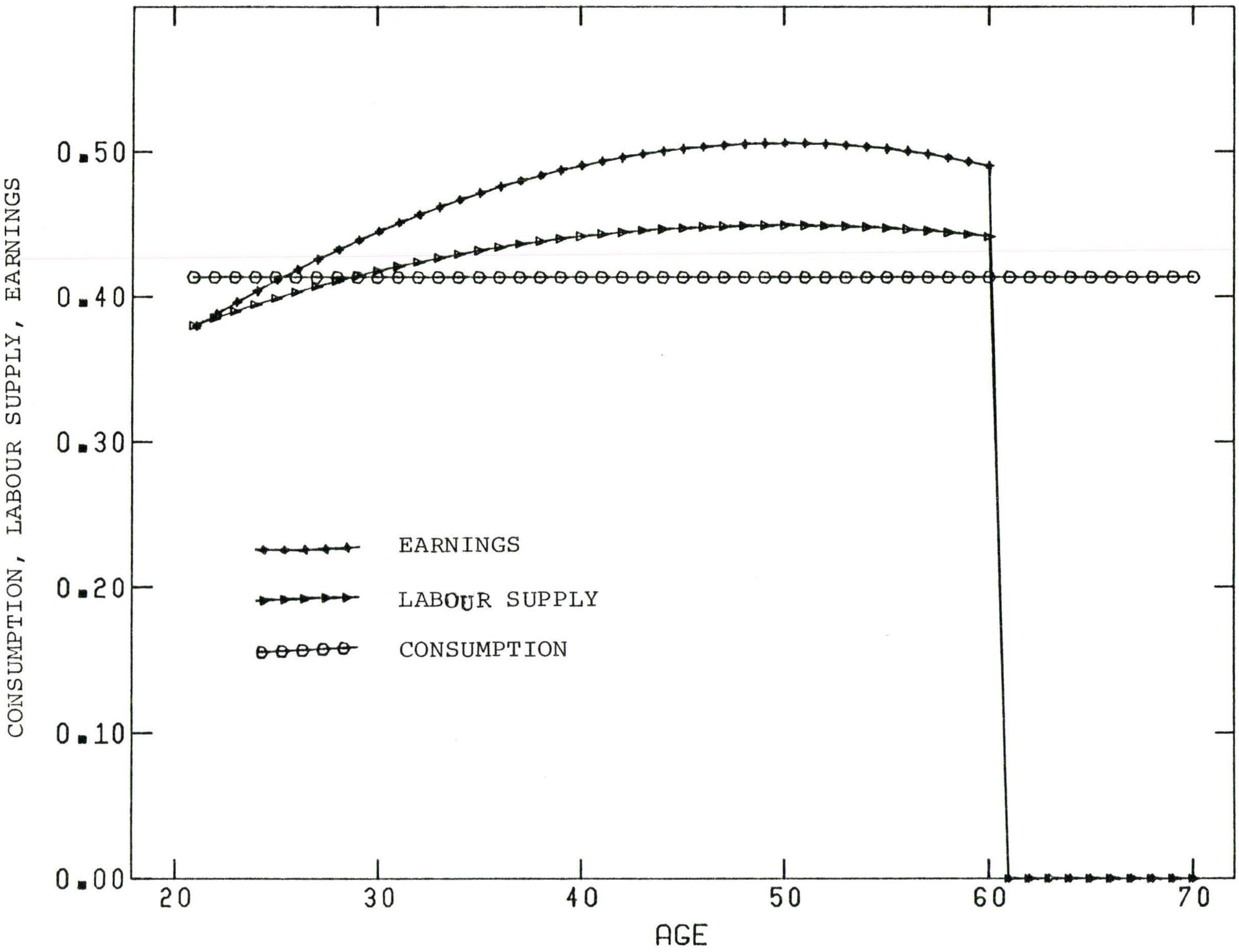
SIMULATED PATHS OF THE KEY MICRO VARIABLES: EXPERIMENT 3

Age	Wage Rates	Earnings	Labour Supply	Consumption	Net Worth	Saving Rate
1	1.000000	.379871	.379871	.413419	0.000000	-.088313
2	1.008377	.388246	.385023	.413419	-.033548	-.067599
3	1.016478	.396349	.389924	.413419	-.059725	-.047805
4	1.024302	.404173	.394584	.413419	-.078587	-.028878
5	1.031847	.411718	.399011	.413419	-.090190	-.010773
6	1.039113	.418984	.403213	.413419	-.094597	.006553
7	1.046097	.425969	.407198	.413419	-.091870	.023141
8	1.052800	.432671	.410972	.413419	-.082076	.039027
9	1.059219	.439090	.414541	.413419	-.065286	.054245
10	1.065353	.445224	.417912	.413419	-.041574	.068827
11	1.071200	.451072	.421090	.413419	-.011016	.082802
12	1.076761	.456632	.424079	.413419	.026306	.096196
13	1.082032	.461903	.426885	.413419	.070308	.109035
14	1.087013	.466885	.429511	.413419	.120901	.121341
15	1.091703	.471574	.431962	.413419	.177994	.133137
16	1.096100	.475971	.434241	.413419	.241489	.144442
17	1.100203	.480074	.436351	.413419	.311286	.155275
18	1.104010	.483882	.438294	.413419	.387279	.165653
19	1.107521	.487392	.440075	.413419	.469360	.175591
20	1.110733	.490605	.441694	.413419	.557414	.185104
21	1.113646	.493518	.443155	.413419	.651622	.194205
22	1.116258	.496130	.444458	.413419	.750961	.202907
23	1.118568	.498440	.445605	.413419	.856200	.211222
24	1.120575	.500446	.446598	.413419	.966907	.219159
25	1.122276	.502148	.447437	.413419	1.082941	.226728
26	1.123672	.503543	.448123	.413419	1.204158	.233938
27	1.124759	.504630	.448656	.413419	1.330407	.240796
28	1.125538	.505409	.449038	.413419	1.461530	.247309
29	1.126006	.505877	.449267	.413419	1.597366	.253484
30	1.126162	.506034	.449343	.413419	1.737745	.259326
31	1.126005	.505877	.449267	.413419	1.882492	.264839
32	1.125534	.505405	.449036	.413419	2.031424	.270027
33	1.124747	.504618	.448650	.413419	2.184353	.274892
34	1.123642	.503513	.448108	.413419	2.341083	.279439
35	1.122218	.502090	.447408	.413419	2.501409	.283667
36	1.120475	.500346	.446548	.413419	2.665123	.287577
37	1.118410	.498281	.445526	.413419	2.832003	.291170
38	1.116021	.495893	.444340	.413419	3.001825	.294444
39	1.113309	.493180	.442986	.413419	3.174354	.297397
40	1.110270	.490142	.441462	.413419	3.349346	.300028
41	0.000000	0.000000	0.000000	.413419	3.526549	-2.907684
42	0.000000	0.000000	0.000000	.413419	3.218926	-3.281129
43	0.000000	0.000000	0.000000	.413419	2.902075	-3.748546
44	0.000000	0.000000	0.000000	.413419	2.575718	-4.350212
45	0.000000	0.000000	0.000000	.413419	2.239570	-5.153250
46	0.000000	0.000000	0.000000	.413419	1.893338	-6.278486
47	0.000000	0.000000	0.000000	.413419	1.536719	-7.967568
48	0.000000	0.000000	0.000000	.413419	1.169402	-10.784345
49	0.000000	0.000000	0.000000	.413419	.791065	-16.420361
50	0.000000	0.000000	0.000000	.413419	.401378	-33.333333
51					.000000	

NOTE: The computations assume $\alpha = 0.4$; $\alpha^{\ell} = 0.6$; $\rho = 0.03$; $r = 0.03$; $g = 0.0014835$; $T = 50$; $R = 40$; and $n = 0.015$.

FIGURE 5.4

CONSUMPTION, LABOUR SUPPLY, AND EARNINGS PROFILES: EXPERIMENT 3



the entire life-span of the individual. The net worth profile in this case follows thepredictions of the Tobin model.

Case 3: r significantly higher than ρ

In this case, the rate of time preference and the interest rate are assumed to take values that differ greatly. It is important to understand how this assumption affects the optimal paths of the endogenous variables.

Experiment 4

In this particular experiment, the interest rate is set at 4 percent per annum and the rate of time preference at 3 percent. The remaining parameter values are shown in the note to Table 5.4.

Unlike the previous experiments, the age of maximum work effort occurs when the individual is young. Thereafter labour supply declines as the individual ages. Although this may not seem realistic from a theoretical point of view, it is plausible. Weiss (1972) pointed out

TABLE 5.4

SIMULATED PATHS OF THE KEY MICRO VARIABLES: EXPERIMENT 4

Age	Wage Rates	Earnings	Labour Supply	Consumption	Net Worth	Saving Rate
1	1.000000	.465825	.465825	.356117	0.000000	.235514
2	1.008377	.469015	.465119	.359574	.109708	.240449
3	1.016478	.471880	.464230	.363065	.223538	.244906
4	1.024302	.474416	.463161	.366590	.341294	.248896
5	1.031847	.476623	.461913	.370149	.462772	.252426
6	1.039113	.478498	.460487	.373743	.587757	.255505
7	1.046097	.480040	.458887	.377372	.716022	.258137
8	1.052800	.481247	.457111	.381035	.847332	.260327
9	1.059219	.482117	.455163	.384735	.981437	.262076
10	1.065353	.482648	.453040	.388470	1.118076	.263383
11	1.071200	.482838	.450745	.392242	1.256977	.264249
12	1.076761	.482686	.448276	.396050	1.397852	.264668
13	1.082032	.482190	.445633	.399895	1.540403	.264637
14	1.087013	.481347	.442816	.403777	1.684313	.264147
15	1.091703	.480157	.439824	.407697	1.829256	.263189
16	1.096100	.478616	.436654	.411656	1.974385	.261752
17	1.100203	.476724	.433306	.415652	2.120341	.259823
18	1.104010	.474478	.429777	.419688	2.266747	.257385
19	1.107521	.471877	.426066	.423762	2.412208	.254419
20	1.110733	.468918	.422170	.427877	2.556911	.250904
21	1.113646	.465600	.418086	.432031	2.700125	.246815
22	1.116258	.461921	.413511	.436225	2.841699	.242123
23	1.118568	.457878	.409343	.440460	2.981062	.236796
24	1.120575	.453470	.404676	.444737	3.117722	.230797
25	1.122276	.448694	.399807	.449055	3.251163	.224083
26	1.123672	.443550	.394733	.453414	3.380850	.216609
27	1.124759	.438034	.389447	.457816	3.506219	.208318
28	1.125538	.432146	.383946	.462261	3.626686	.199150
29	1.126006	.425882	.378224	.466749	3.741638	.189034
30	1.126162	.419241	.372274	.471281	3.850436	.177891
31	1.126005	.412221	.366091	.475856	3.952414	.165629
32	1.125534	.404620	.359669	.480476	4.046375	.152142
33	1.124747	.397035	.352999	.485141	4.133093	.137310
34	1.123642	.388865	.346076	.489851	4.210310	.120992
35	1.122218	.380308	.338889	.494607	4.277737	.103026
36	1.120475	.371361	.331432	.499409	4.334547	.083221
37	1.118410	.362023	.323695	.504258	4.379881	.061354
38	1.116021	.352291	.315667	.509153	4.412342	.037162
39	1.113309	.342164	.307340	.514097	4.432493	.010332
40	1.110270	.331639	.298701	.519088	4.437360	-.019513
41	0.000000	0.000000	0.000000	.524128	4.427925	-1.959217
42	0.000000	0.000000	0.000000	.529216	4.080314	-2.242020
43	0.000000	0.000000	0.000000	.534354	3.714935	-2.595987
44	0.000000	0.000000	0.000000	.539542	3.329178	-3.051617
45	0.000000	0.000000	0.000000	.544780	2.922303	-3.659743
46	0.000000	0.000000	0.000000	.550070	2.494935	-4.511863
47	0.000000	0.000000	0.000000	.555410	2.044663	-5.790974
48	0.000000	0.000000	0.000000	.560802	1.571039	-7.924067
49	0.000000	0.000000	0.000000	.566247	1.073078	-12.192118
50	0.000000	0.000000	0.000000	.571745	.549754	-25.000000
51					.000000	

NOTE: The computations assume $\alpha = 0.4$; $\alpha^{\ell} = 0.6$; $\rho = 0.03$; $r = 0.04$; $g = 0.0014\overset{6}{8}35$; $T = 50$; $R = 40$; and $n = 0.015$.

that "when ρ is significantly below (above) r then the optimal amount of labour supply tends to decrease (increase) with age".

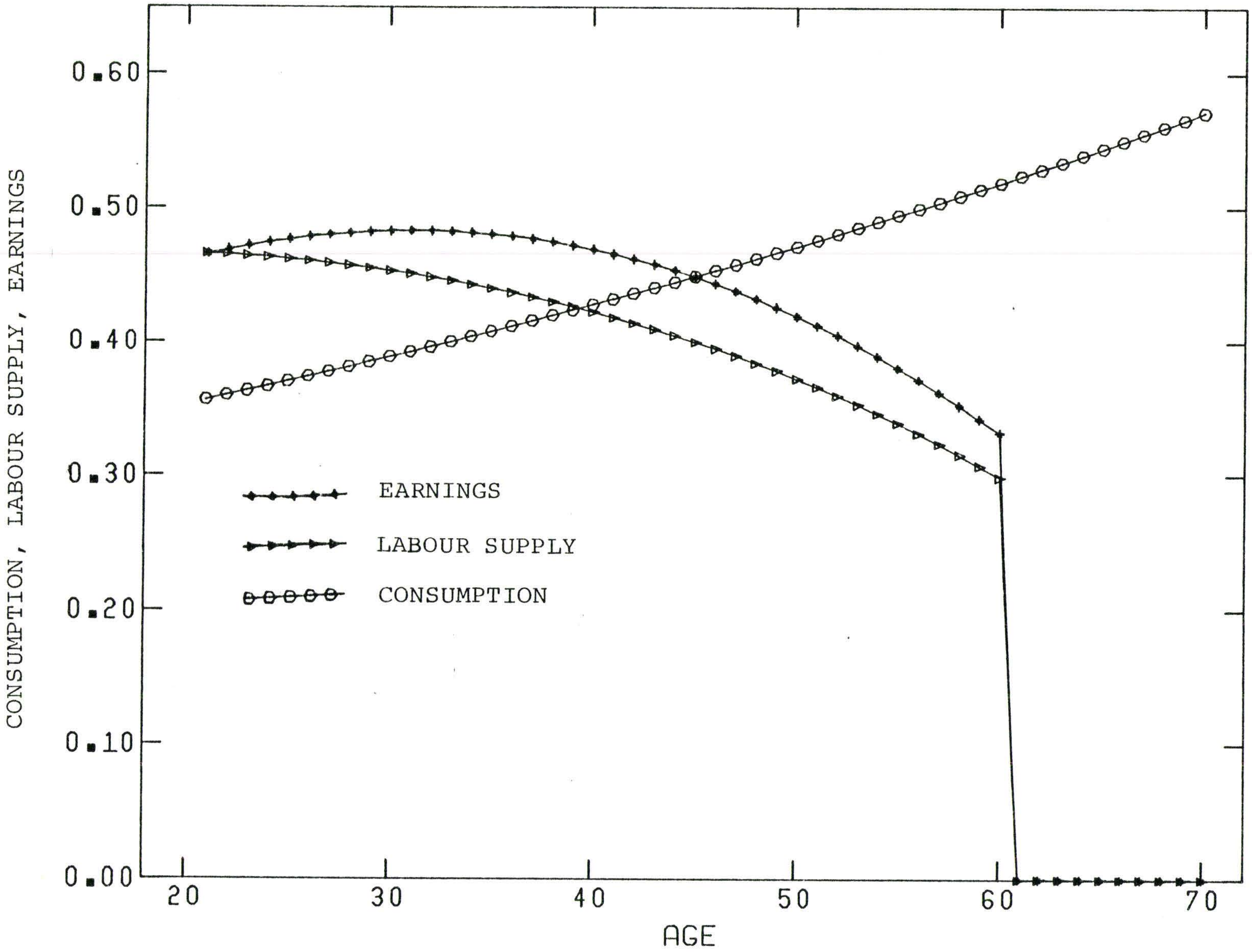
The optimal earnings profile peaks at age 11 and begins to decrease thereafter, as shown in column 3 of Table 5.4. On the other hand, the consumption profile increases with the age of the individual while the net worth profile shows no negative value during the entire life span. The path of the saving rate peaks at age 12 and then declines steadily as the age of the individual increase. These results are represented graphically in Figure 5.5.

SIMULATION EXPERIMENTS USING THE STONE-GEARY UTILITY FUNCTION

In this section I will examine the optimal plans that arise from maximizing a Stone-Geary utility function or some variant of it when ρ and r do not significantly differ from one another. Various experiments are conducted depending on the assumptions underlying the subsistence levels of consumption and leisure. For completeness, I also experiment with the

FIGURE 5.5

CONSUMPTION, LABOUR SUPPLY, AND EARNINGS PROFILES: EXPERIMENT 4



Stone-Geary function when r is significantly higher than ρ.

Experiment 5

In this experiment the rate of time preference and the interest rate are set at 3.0 and 3.5 per cent, respectively. I will consider a utility function in which subsistence consumption is non-zero but with no non-zero lower bound for leisure. This function is a special case of the Stone-Geary utility function given by equation (5.1). The parameter Ψ is assigned a value of 0.33 and assumptions made regarding the other parameters of the model are shown as a note to Table 5.5.

The results of the experiment can be compared with those based on the Cobb-Douglas function (Experiment 1). In both of the experiments, maximum work effort occurs at age 13. The earning profile exhibits an inverted 'U' shape in both experiments. The age of peak earnings in Experiment 5 occurs at age 21, compared to age 20 in Experiment 1. The consumption profile monotonically increases with age in both experiments. The net worth profiles resulting from the two experiments follow the

TABLE 5.5

SIMULATED PATHS OF THE KEY MICRO VARIABLES: EXPERIMENT 5

Age	Wage Rates	Earnings	Labour Supply	Consumption	Net Worth	Saving Rate
1	1.000000	.503725	.503725	.464718	0.000000	.077436
2	1.008377	.509692	.505458	.466324	.039007	.087531
3	1.016478	.515373	.507018	.467938	.083740	.097174
4	1.024302	.520764	.508409	.469560	.134106	.106379
5	1.031347	.525865	.509635	.471189	.190003	.115162
6	1.039113	.530674	.510699	.472827	.251329	.123536
7	1.046097	.535191	.511607	.474472	.317973	.131512
8	1.052600	.539413	.512361	.476126	.389821	.139102
9	1.059219	.543340	.512963	.477787	.466752	.146315
10	1.065353	.546969	.513416	.479457	.548641	.153161
11	1.071200	.550301	.513724	.481134	.635357	.159647
12	1.076761	.553332	.513886	.482820	.726761	.165781
13	1.082032	.556063	.513906	.484514	.822710	.171570
14	1.087013	.558491	.513785	.486216	.923054	.177018
15	1.091703	.560615	.513523	.487926	1.027636	.182130
16	1.096100	.562434	.513123	.489645	1.136292	.186912
17	1.100203	.563946	.512584	.491372	1.248850	.191364
18	1.104010	.565150	.511907	.493108	1.365134	.195491
19	1.107521	.566045	.511092	.494852	1.484956	.199293
20	1.110733	.566629	.510140	.496604	1.608124	.202772
21	1.113646	.566901	.509049	.498365	1.734433	.205927
22	1.116258	.566859	.507820	.500134	1.863674	.208758
23	1.118568	.566502	.506453	.501912	1.995627	.211262
24	1.120575	.565828	.504945	.503699	2.130064	.213438
25	1.122276	.564837	.503296	.505494	2.266746	.215282
26	1.123672	.563526	.501504	.507298	2.405425	.216789
27	1.124759	.561894	.499569	.509111	2.545842	.217954
28	1.125538	.559941	.497487	.510932	2.687731	.218771
29	1.126006	.557663	.495258	.512763	2.830809	.219232
30	1.126162	.555061	.492878	.514602	2.974788	.219328
31	1.126005	.552131	.490345	.516450	3.119364	.219049
32	1.125534	.548874	.487657	.518308	3.264223	.218383
33	1.124747	.545288	.484809	.520174	3.409037	.217318
34	1.123642	.541370	.481799	.522049	3.553468	.215838
35	1.122218	.537120	.478623	.523933	3.697160	.213928
36	1.120475	.532536	.475277	.525827	3.839747	.211568
37	1.118410	.527617	.471756	.527730	3.980847	.208738
38	1.116021	.522361	.468056	.529642	4.120064	.205414
39	1.113309	.516766	.464172	.531563	4.256985	.201571
40	1.110270	.510832	.460097	.533493	4.391183	.197179
41	0.000000	0.000000	0.000000	.535433	4.522213	-2.382878
42	0.000000	0.000000	0.000000	.537383	4.145057	-2.704120
43	0.000000	0.000000	0.000000	.539342	3.752752	-3.106256
44	0.000000	0.000000	0.000000	.541310	3.344757	-3.623952
45	0.000000	0.000000	0.000000	.543288	2.920513	-4.314993
46	0.000000	0.000000	0.000000	.545275	2.479443	-5.283382
47	0.000000	0.000000	0.000000	.547272	2.020949	-6.737134
48	0.000000	0.000000	0.000000	.549279	1.544410	-9.161610
49	0.000000	0.000000	0.000000	.551296	1.049185	-14.012898
50	0.000000	0.000000	0.000000	.553322	.534611	-28.571429
51					.000000	

NOTE: The computations assume $\alpha = 0.4$; $\alpha^{\ell} = 0.6$; $\Psi = 0.33$; $\rho = 0.03$; $r = 0.035$; $g = 0.0014835$; $T = 50$; $R = 40$; and $n = 0.015$.

Modigliani-Brumberg prediction. The rates of saving also have similar age patterns.

Although the age patterns of the choice variables are similar in the two experiments, this does not imply that the generated values are identical. One would expect the differences to be reflected at the aggregate level.

Experiment 6

All the assumptions of Experiment 5 are retained here, except for the parameter Ψ, which is now specified to be 0.66 instead of 0.33. Because of the latter change, the resulting consumption profile exhibits a higher level at every age of the individual, compared to the results obtained in Experiment 5. This is evident from a comparison of columns 5 of Tables 5.6 and 5.5. The labour supply and the earnings profiles have also increased at every age, compared to those of Experiment 5. The intuitive explanation for this is that the imposition of a lower bound on consumption leads to a higher level of consumption at every age, compared to the case of no lower bound. To finance the

TABLE 5.6

SIMULATED PATHS OF THE KEY MICRO VARIABLES: EXPERIMENT 6

Age	Wage Rate	Earnings	Labour Supply	Consumption	Net Worth	Saving Rate
1	1.000000	.582080	.582080	.546349	0.000000	.061384
2	1.008377	.588428	.583540	.547702	.035730	.071186
3	1.016478	.594490	.584853	.549061	.077707	.080623
4	1.024302	.600265	.586024	.550426	.125356	.089708
5	1.031847	.605752	.587056	.551799	.180100	.098451
6	1.039113	.610950	.587953	.553178	.240357	.106859
7	1.046097	.615856	.588718	.554563	.306542	.114943
8	1.052800	.620470	.589352	.555956	.378564	.122710
9	1.059219	.624790	.589859	.557355	.456327	.130168
10	1.065353	.628815	.590241	.558761	.539734	.137323
11	1.071200	.632544	.590500	.560173	.628679	.144182
12	1.076761	.635974	.590637	.561593	.723053	.150750
13	1.082032	.639106	.590654	.563020	.822741	.157033
14	1.087013	.641937	.590551	.564453	.927524	.163034
15	1.091703	.644466	.590331	.565893	1.037575	.168759
16	1.096100	.646692	.589994	.567341	1.152463	.174211
17	1.100203	.648614	.589540	.568795	1.272151	.179392
18	1.104010	.650229	.588970	.570257	1.396495	.184306
19	1.107521	.651537	.588284	.571725	1.525345	.188955
20	1.110733	.652536	.587482	.573201	1.658543	.193339
21	1.113646	.653224	.586564	.574684	1.795327	.197461
22	1.116258	.653601	.585529	.576174	1.937325	.201321
23	1.118568	.653665	.584377	.577671	2.082560	.204918
24	1.120575	.653415	.583107	.579175	2.231444	.208253
25	1.122276	.652849	.581718	.580687	2.383784	.211324
26	1.123672	.651965	.580210	.582206	2.539378	.214130
27	1.124759	.650763	.578580	.583733	2.698015	.216669
28	1.125538	.649241	.576827	.585267	2.859475	.218938
29	1.126006	.647397	.574950	.586808	3.023531	.220934
30	1.126162	.645230	.572946	.588357	3.189942	.222652
31	1.126005	.642738	.570813	.589914	3.353463	.224088
32	1.125534	.639921	.568549	.591478	3.528333	.225237
33	1.124747	.636776	.566151	.593049	3.700786	.226091
34	1.123642	.633303	.563616	.594628	3.874340	.226645
35	1.122218	.629499	.560942	.596215	4.048306	.226889
36	1.120475	.625363	.558124	.597810	4.223280	.226815
37	1.118410	.620895	.555159	.599412	4.398649	.226413
38	1.116021	.616091	.552043	.601022	4.574084	.225671
39	1.113309	.610952	.548771	.602640	4.749246	.224577
40	1.110270	.605475	.545340	.604266	4.923782	.223116
41	0.000000	0.000000	0.000000	.605900	5.097323	-2.396178
42	0.000000	0.000000	0.000000	.607541	4.669330	-2.717120
43	0.000000	0.000000	0.000000	.609191	4.225733	-3.118918
44	0.000000	0.000000	0.000000	.610848	3.764443	-3.636226
45	0.000000	0.000000	0.000000	.612514	3.285550	-4.326798
46	0.000000	0.000000	0.000000	.614187	2.787823	-5.294593
47	0.000000	0.000000	0.000000	.615869	2.271210	-6.747530
48	0.000000	0.000000	0.000000	.617559	1.734333	-9.170749
49	0.000000	0.000000	0.000000	.619257	1.177393	-14.019676
50	0.000000	0.000000	0.000000	.620964	.599365	-28.571429
51					.000000	

NOTE: The computations assume $\alpha = 0.4$; $\alpha^{\ell} = 0.6$; $\Psi = 0.66$; $\rho = 0.03$; $r = 0.035$; $g = 0.0014835$; $T = 50$; $R = 40$; and $n = 0.15$.

higher consumption levels, the individual provides more hours of work, and hence earns more labour income. In the case of exogenous labour income, the incorporation of subsistence consumption restricts the degree of life-cycle redistribution that is possible. However, with endogenous labour income, the individual can increase his work effort to finance his expenditures and overcome the rigidity introduced by the subsistence consumption.

Experiment 7

It is natural to extend the idea of a lower bound on consumption discussed in connection with Experiments 5 and 6 to include also a lower bound on leisure. To do this, I adopt a Stone-Geary utility function. The parameters Ψ and Ω are assigned a value of 0.33 each. Assumptions with regard to the remaining parameters of the model are shown as a note to Table 5.7.

To understand the role of the form of the utility function in influencing the optimal paths of the decision variables, I compare the results that arise from a Cobb-Douglas function with those based on a

TABLE 5.7

SIMULATED PATHS OF THE KEY MICRO VARIABLES: EXPERIMENT 7

Age	Wage Rate	Earnings	Labour Supply	Consumption	Net Worth	Saving Rate
1	1.000000	.447576	.447576	.414151	0.000000	.074681
2	1.008377	.452806	.449044	.415511	.033425	.084729
3	1.016478	.457787	.450366	.416878	.071390	.094339
4	1.024302	.462517	.451544	.418252	.115315	.103527
5	1.031847	.466996	.452582	.419633	.163516	.112306
6	1.039113	.471222	.453485	.421020	.216705	.120689
7	1.046097	.475193	.454254	.422414	.274492	.128686
8	1.052800	.478910	.454892	.423815	.336879	.136308
9	1.059219	.482370	.455402	.425222	.403765	.143564
10	1.065353	.485573	.455786	.426636	.475045	.150465
11	1.071200	.488517	.456046	.428058	.550508	.157016
12	1.076761	.491201	.456184	.429486	.630339	.163225
13	1.082032	.493624	.456201	.430921	.714116	.169098
14	1.087013	.495785	.456098	.432363	.801914	.174642
15	1.091703	.497682	.455877	.433812	.893300	.179859
16	1.096100	.499315	.455538	.435268	.988436	.184754
17	1.100203	.500681	.455081	.436731	1.087078	.189331
18	1.104010	.501781	.454507	.438201	1.189076	.193592
19	1.107521	.502612	.453817	.439678	1.294273	.197538
20	1.110733	.503174	.453010	.441163	1.402507	.201170
21	1.113646	.503465	.452087	.442655	1.513505	.204489
22	1.116258	.503484	.451046	.444154	1.627391	.207494
23	1.118568	.503229	.449887	.445660	1.743680	.210184
24	1.120575	.502700	.448609	.447173	1.862278	.212557
25	1.122276	.501896	.447212	.448694	1.982084	.214609
26	1.123672	.500815	.445695	.450223	2.105590	.216337
27	1.124759	.499455	.444055	.451758	2.229378	.217735
28	1.125538	.497816	.442292	.453302	2.355620	.218800
29	1.126006	.495896	.440403	.454852	2.482582	.219522
30	1.126162	.493695	.438387	.456410	2.610516	.219895
31	1.126005	.491210	.436241	.457976	2.739169	.219910
32	1.125534	.488441	.433964	.459549	2.868274	.219556
33	1.124747	.485386	.431552	.461130	2.997555	.218821
34	1.123642	.482044	.429002	.462719	3.126725	.217692
35	1.122218	.478414	.426311	.464315	3.255486	.216155
36	1.120475	.474495	.423476	.465920	3.383527	.214192
37	1.118410	.470284	.420494	.467531	3.510525	.211785
38	1.116021	.465782	.417359	.469151	3.636146	.208914
39	1.113309	.460986	.414068	.470779	3.760042	.205553
40	1.110270	.455895	.410616	.472414	3.881350	.201679
41	0.000000	0.000000	0.000000	.474058	4.001196	-2.385115
42	0.000000	0.000000	0.000000	.475709	3.667180	-2.706305
43	0.000000	0.000000	0.000000	.477369	3.319922	-3.108383
44	0.000000	0.000000	0.000000	.479036	2.958647	-3.626014
45	0.000000	0.000000	0.000000	.480712	2.583164	-4.316974
46	0.000000	0.000000	0.000000	.482395	2.192363	-5.285264
47	0.000000	0.000000	0.000000	.484087	1.787218	-6.738878
48	0.000000	0.000000	0.000000	.485787	1.365683	-9.163143
49	0.000000	0.000000	0.000000	.487496	.927695	-14.014034
50	0.000000	0.000000	0.000000	.489212	.472369	-28.571429
51					.000000	

NOTE: The computations assume $\alpha = 0.4$; $\alpha^{\ell} = 0.6$; $\Psi = 0.33$; $\Omega = 0.33$; $\rho = 0.03$; $r = 0.035$; $g = 0.0014835$; $T = 50$; $R = 40$; and $n = 0.015$.

Stone-Geary function, as shown in Tables 5.1 and 5.7. Although the optimal paths of the endogenous variables have similar patterns in both cases, it is clear that the individual is willing to consume relatively more and to supply relatively more hours of labour at every age if his preferences are represented by a Stone-Geary function. Other differences are reflected in the ages at which work effort, earnings and saving rates peak.

Experiment 8

This experiment is based on the assumption that the interest rate is significantly higher than the rate of time preference. The values of these parameters are set at 0.04 and 0.03, respectively. Other than this, the assumptions are identical as those of Experiment 7. The results are reported in Table 5.8.

In this case, the optimal path of labour supply steadily decreases with age, while the earnings profile peak at age 14. The consumption profile increases with age and the net worth profile never becomes negative. Comparison of Tables 5.4 and 5.8 reveals that the Stone-Geary utility function yields relatively higher levels

TABLE 5.8

SIMULATED PATHS OF THE KEY MICRO VARIABLES: EXPERIMENT 8

Age	Wage Rate	Earnings	Labour Supply	Consumption	Net Worth	Saving Rate
1	1.000000	.476445	.476445	.394904	0.000000	.171144
2	1.008377	.479915	.475928	.397439	.081541	.177446
3	1.016478	.483108	.475277	.399998	.167279	.183344
4	1.024302	.486023	.474493	.402581	.257080	.188846
5	1.031847	.488660	.473578	.405190	.350306	.193959
6	1.039113	.491015	.472533	.407824	.448307	.198690
7	1.046097	.493088	.471359	.410484	.549430	.203044
8	1.052800	.494877	.470058	.413170	.654011	.207025
9	1.059219	.496381	.468630	.415881	.761479	.210636
10	1.065353	.497599	.467074	.418619	.872354	.213879
11	1.071200	.498528	.465391	.421384	.986747	.216754
12	1.076761	.499167	.463582	.424175	1.103360	.219263
13	1.082032	.499514	.461645	.426994	1.222486	.221402
14	1.087013	.499569	.459580	.429840	1.343006	.223170
15	1.091703	.499330	.457386	.432713	1.467392	.224563
16	1.096100	.498794	.455063	.435615	1.592704	.225577
17	1.100203	.497961	.452609	.438544	1.719591	.226205
18	1.104010	.496829	.450322	.441502	1.847792	.226440
19	1.107521	.495396	.447302	.444489	1.977030	.226272
20	1.110733	.493661	.444446	.447505	2.107018	.225692
21	1.113646	.491622	.441452	.450550	2.237455	.224687
22	1.116258	.489277	.438319	.453624	2.368025	.223243
23	1.118568	.486626	.435043	.456729	2.498399	.221345
24	1.120575	.483665	.431623	.459864	2.628231	.218975
25	1.122276	.480395	.428054	.463029	2.757162	.216111
26	1.123672	.476812	.424334	.466224	2.884815	.212731
27	1.124759	.472916	.420460	.469451	3.010796	.208810
28	1.125538	.468705	.416427	.472709	3.134693	.204317
29	1.126006	.464177	.412233	.475999	3.256076	.199221
30	1.126162	.459330	.407872	.479320	3.374497	.193484
31	1.126005	.454163	.403340	.482674	3.489486	.187064
32	1.125534	.448674	.398632	.486061	3.600554	.179915
33	1.124747	.442861	.393743	.489480	3.707189	.171985
34	1.123642	.436724	.388666	.492933	3.808858	.163213
35	1.122218	.430259	.383400	.496419	3.905003	.153532
36	1.120475	.423466	.377934	.499939	3.995043	.142865
37	1.118410	.416342	.372263	.503493	4.078372	.131125
38	1.116021	.408886	.366378	.507081	4.154356	.118212
39	1.113309	.401096	.360274	.510765	4.222335	.104011
40	1.110270	.392971	.353942	.514363	4.281520	.088390
41	0.000000	0.000000	0.000000	.518058	4.331493	-1.990064
42	0.000000	0.000000	0.000000	.521788	3.986595	-2.272056
43	0.000000	0.000000	0.000000	.525554	3.624375	-2.625134
44	0.000000	0.000000	0.000000	.529357	3.243797	-3.079761
45	0.000000	0.000000	0.000000	.533196	2.844192	-3.686712
46	0.000000	0.000000	0.000000	.537073	2.424763	-4.537378
47	0.000000	0.000000	0.000000	.540983	1.984680	-5.814546
48	0.000000	0.000000	0.000000	.544940	1.523080	-7.944713
49	0.000000	0.000000	0.000000	.548931	1.039062	-12.207374
50	0.000000	0.000000	0.000000	.552961	.531593	-25.000000
51					.000000	

NOTE: The computations assume $\alpha = 0.4$; $\alpha^{\ell} = 0.6$; $\Psi = 0.33$; $\Omega = 0.33$; $\rho = 0.03$; $r = 0.04$; $g = 0.0014835$; $T = 50$; $R = 40$; and $n = 0.015$.

of consumption, labour supply and earnings at every age, compared to the Cobb-Douglas function. Differences with respect to the age of peak earnings, net worth and saving rates are also observed.

5.5 AGGREGATION OF THE MICRO-RELATIONSHIP WHEN LABOUR SUPPLY IS ENDOGENOUS

When the assumption of inelastic labour supply is relaxed, the aggregation procedure of the micro variables differs slightly from the case of the exogenous labour supply discussed in Section 4.4 of chapter 4.

Once consumption, net worth and income of all individuals of all ages are known and population is assumed to grow at a constant rate n, aggregate values of consumption and net worth are computed according to equations (4.1) and (4.2). Aggregate total income, however, is calculated as follows:

$$(5.7) \qquad Y_t^{**} = \sum_{\tau=1}^{R} (1 + n)^{T-\tau} W_\tau L_\tau^S + \sum_{\tau=1}^{T} (1 + n)^{T-\tau} r \, a_\tau$$

Equation (5.7) states that aggregate total income is the sum of labour income plus total interest income. Note that the first component on the right hand side of equation (5.7) is different from that in equation (4.3).

Aggregate saving is calculated by subtracting total consumption (4.1) from total income (5.7).

5.6 THE INTEREST ELASTICITY OF SAVING WHEN LABOUR SUPPLY IS ENDOGENOUS

In earlier chapters, I discussed the interest elasticity of saving issue in the context of fixed labour income. Using this framework, I demonstrated that Summers' claim of high interest elasticity of saving is sensitive to alternative functional forms.

In this section I will attempt to address the interest elasticity of saving issue at the macro level by restricting the form of the utility function to be a Cobb-Douglas but allowing the labour supply to be endogenous in the optimization problem. The simulated elasticities and aggregate saving rates corresponding to Experiments 1 to 4 were reported in Tables 5.1 to 5.4, and are now brought together for comparison in Table 5.9.

TABLE 5.9

INTEREST ELASTICITIES OF AGGREGATE SAVING AND SAVING RATES

Experiment	r	ρ	Interest Elasticity of Saving	Saving Rate
1	0.035	0.030	0.27	0.06
2	0.030	0.035	-0.21	0.01
3	0.030	0.030	0.17	0.04
4	0.040	0.030	0.36	0.08

NOTE: The computations assume $\alpha = 0.4$; $\alpha^{\ell} = 0.6$; $T = 50$; $R = 40$; $g = 0.0014835$; and $n = 0.015$. The saving rates are calculated relative to total income.

The interest elasticities of saving are found to be positive and small for Experiments 1, 3 and 4. When the rate of time preference is assumed to be greater than the interest rate, as in Experiment 2, the elasticity becomes negative.

When a Stone-Geary utility function is specified, as in Experiments 7 and 8, the resulting interest elasticities are 0.27 and 0.34, respectively. These findings are not significantly different from those obtained using the Cobb-Douglas specification of Experiments 1 and 4. In both cases, the interest elasticity of saving is low.

Summers reported an interest elasticity of 3.36 for the Cobb-Douglas utility function with exogenous labour supply. To examine the sensitivity of his result to the assumption underlying the labour supply and the functional form of the utility function, various experiments were conducted.

First, following Summers, I assumed $g = 0.02$, $r = 0.04$, $\rho = 0.03$, and $n = 0.015$ for all subsequent experiments[4]. Based on these assumptions, the values of α and α^{ℓ} that yield reasonable values for the labour supply are 0.6 and 0.4, respectively. With subsistence consumption and leisure set at zero, the

Cobb-Douglas utility function yields an interest elasticity of saving of 0.008. This is far lower than the interest elasticity of 3.36 reported by Summers for the Cobb-Douglas utility function with exogenous labour supply. Thus Summers' result is shown to be sensitive to the assumption underlying the labour supply.

The effect of an increase in the interest rate is to reduce current consumption in favour of the future, regardless of whether labour supply is exogenous or endogenous. However, from equation (5.6), an increase in the interest rate leads to an increase in the hours of work and as a result labour income and consumption of the individual increase. Because of these two competing effects the net effect on consumption in this case is not as large as in the case of exogenous labour supply.

Second, a lower bound was imposed on leisure while setting subsistence consumption to zero. Three experiments were conducted with values of $\Omega = 0.33$, $\Omega = 0.66$ and $\Omega = 0.90$. The resulting elasticities remained unchanged at 0.008. One important thing to note is that, as the lower bound on leisure increases, the individual responds by lowering his consumption at every age in such a way as to leave his saving ratio unaffected. Because of the substitutability of consumption and leisure, the interest elasticity of aggregate saving remains at .008, regardless of the value of Ω.

Third, a lower bound was imposed on consumption while setting subsistence leisure to zero. I experimented with values of 0.33, 0.66 and 0.90 for the paramter Ψ and the resulting elasticities are found to be -0.28, -1.17 and -6.77, respectively. These results indicate that the interest elasticity of saving is sensitive to the form of the utility function.

In light of the above findings, Summers' claim of a high interest elasticity of saving appears fragile. This result is consistent with the findings of chapter 4, where I suggested that it is not implausible for the interest elasticity of saving to be negative.

FOOTNOTES

Chapter 5

1. Blinder (1973) specified a cross-section wage function in which the dependent variable is the logarithm of wages and the explanatory variables are age and age squared. Attempts to derive the age-wage profile of the individual from Blinder's equation yield a wage path that peaks at an age later than what is generally considered to be the average, i.e., later than age 50.

2. To obtain a value for subsistence consumption γ, first I calculated total consumption by assuming $\gamma = \rho = 0.035$ and $g = 0.00$. This yields consumption levels that are constant over the entire life span of the individual. Next, I computed subsistence consumption by multiplying this value by a fraction Ψ

3. The value of leisure $\bar{L}$ is set arbitrarily at 0.4. Subsistence leisure, γ^{ℓ}, is assumed to be a fraction Ω of 0.4, and computed accordingly.

4. A productivity growth of 0.02 per annum yields an individual wage function that monotonically increases with age.

APPENDIX

to Chapter 5

This appendix provides the derivation of the demand equation (5.3).

The individual consumer maximizes (5.1) subject to (5.2). Construction of a Lagrangian from (5.1) and (5.2) and differentiation with respect to C_t and L_t yields the following first-order conditions:

$$(\alpha_t/C_t^*)(1 + \rho)^{\tau-t} = \lambda(1 + r)^{\tau-t} \tag{A5.1}$$

$$(\alpha_t^\ell/L_t^*)(1 + \rho)^{\tau-t} = \lambda W_t(1 + r)^{\tau-t} \tag{A5.2}$$

$$P_\tau - \sum_{t=\tau}^{T} C_t(1 + r)^{\tau-t} + \sum_{t=\tau}^{R} W_t L_t(1 + r)^{\tau-t} = 0 \tag{A5.3}$$

where

$$P_\tau = a_\tau + \sum_{t=\tau}^{R} W_t H(1 + r)^{\tau-t}$$

and λ is the Lagrange multiplier.

From (A5.1) the relationship between consumption in period t and τ is given by

$$C_t^* = [(1 + \ell)/(1 + r)]^{\tau-t}\, C_\tau^* \tag{A5.4}$$

and the expression that relates leisure in different periods is given by

$$L_t^* = (\alpha_t^{\ell}/\alpha_\tau^{\ell})(W_\tau/W_t)[(1 + \rho)/(1 + r)]^{\tau-t} L_\tau^* \tag{A5.5}$$

Combination of consitions (A5.1) and (A5.2) yields:

$$L_t^* = (\alpha_t^{\ell}/\alpha_t)(C_t^*/W_t) \tag{A5.6}$$

Since $C_t = C_t^* + \gamma_t$ and $L_t = L_t^* + \gamma_t^{\ell}$, substitute these expressions in (A5.3) and eliminate L_t^* using (A5.6). Rearranging the resulting expression yields:

$$S_\tau = P_\tau - E_\tau \tag{A5.7}$$

where

$$S_\tau = \sum_{t=\tau}^{T} C_t^*(1 + r)^{\tau-t} + \sum_{t=\tau}^{R} (\alpha_t^{\ell}/\alpha_t) C_t^*(1 + r)^{\tau-t} \tag{A5.8}$$

and

$$E_\tau = \sum_{t=\tau}^{T} \gamma_t(1 + r)^{\tau-t} + \sum_{t=\tau}^{R} \gamma^{\ell} W_t(1 + r)^{\tau-t}$$

Substituting (A5.4) in (A5.8) and solving for C_τ^*, we obtain

$$(A5.9) \qquad C_\tau^* = \alpha_\tau \left[\frac{S_\tau}{D_{1\tau} + D_{2\tau}} \right]$$

where

$$D_{1\tau} = \sum_{t=\tau}^{T} \alpha_t (1 + \ell)^{\tau - t}$$

and

$$D_{2\tau} = \sum_{t=\tau}^{T} \alpha_t^\ell (1 + \rho)^{\tau - t}$$

Note that expression (A5.9) is identical to expression (A5.3). This completes the derivation of the demand equation for C_τ^*.

A similar approach yields expressions for the demand for leisure and the labour supply as given by equations (A5.4) and (A5.6).

CHAPTER 6

SUMMARY AND OUTLOOK FOR FURTHER RESEARCH

This concluding chapter provides a brief summary of the findings of the study and indicates possible directions for further research. A major purpose of the thesis has been to assess the claim put forward by Summers that he had developed "a prima facie theoretical case for a high interest elasticity of savings " (1981a, p. 537). That claim had already been challenged by Evans: using the CES utility function employed by Summers, Evans demonstrated that the claim of high interest elasticity does not stand up to changes in the rate of time preference as well as to other changes in parameter values.

To examine the sensitivity of Summers' findings I assumed alternative functional forms of the utility function while keeping other parameter assumptions identical to his. In particular, I assumed that individuals base their utility maximization behaviour on a QHCES function. This function is more general than the SHCES function employed by both Summers and Evans. In chapter 3, this utility function was specified in equation (3.1). The obvious advantage of the function is that it takes into account the effect of subsistence consumption on consumer behaviour. The imposition of a lower bound on consumption introduces some degree of distributional rigidity into the life-cycle model when the labour supply of the individual is assumed to be perfectly inelastic.

Using the above approach, I demonstrated in chapter 4 that Summers' claim is far from robust. In fact, it is not implausible that the interest elasticity of saving would be negative. In the same chapter, I pointed out a number of errors in the saving elasticities and the wealth income ratios in the work of Evans, which can have substantial policy implications. My corrections show that the interest elasticity of saving is likely to be less than what Evans or Summers suggested.

It is important to point out that Summers' analysis of the issue is confined to comparing steady states. He entirely ignored the analysis of the transition path that leads from an initial steady state to a new one. Evans, however, pointed out that "for policy purposes, the transition may be of as much importance as the steady state analysis" (1983, p. 402). Comparison of the saving rates outside the steady state shows clearly the important role played by alternative functional forms in determining the behaviour of saving during the adjustment process.

The above results are obtained on the assumption that individuals maximize utility of consumption of market goods given their incomes. It is natural to extend the model to allow for consumption-leisure choice in the life-cycle context. This approach relaxes the assumption of exogenous labour income adopted by both Summers and Evans. On this basis, I addressed the interest elasticity of saving issue in chapter 5. The results indicate that the relaxation of the assumption leads to interest elasticities of saving that are substantially lower than those reported by Summers. In fact, my findings suggest that elasticities with negative values should not be ruled out.

The interest elasticity of saving is found to be sensitive to the functional form of the utility function when labour income is considered to be exogenous, as well as when this assumption is relaxed to allow for consumption-leisure choice with either a Stone-Geary or a Cobb-Douglas utility function.

The models developed in this study are based on the assumption that individuals leave zero bequests and receive no inheritances or gifts. Furthermore, the capital market is assumed to be perfect, and there are no expectational lags or uncertainties. What would happen to the interest elasticity of saving if these assumptions were relaxed is left for future investigation?

My analysis of the interest elasticity of saving is based on a partial equilibrium framework and the interest rate is assumed to be exogenous. An obvious extension of the present study would incorporate a general equilibrium analysis in which the interest rate was allowed to change. Savings would then be affected, and hence the capital stock, and this in turn would have a feed-back effect on the interest rate. Using such an approach it would be possible to address various issues in addition to those with which this thesis has been concerned.

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