THE SHORT-RUN VERSUS LONG-RUN TRADEOFF IN MACROECONOMIC POLICY

THE SHORT-RUN VERSUS LONG-RUN TRADEOFF IN

MACROECONIMIC POLICY: AN OPTIMAL CONTROL

ANALYSIS IN THE CANADIAN CONTEXT

by

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TO NAZHA

with love

ABSTRACT

The intertemporal dimension of macroeconomic policy, which has been largely neglected, constitutes the main focus of this study. Two questions are posed: What are the long-run implications of pursuing a short-run myopic policy, and what are the short-run consequences of long-sighted policy? Is there a tradeoff relationship between the performance of the economy in the short run and in the long run?

To answer these questions, an optimal control approach is used. The two principal elements of this approach are: a macroeconomic model describing the functioning of the system under control, and an objective function specifying explicitly the targets pursued by the policy maker. A macroeconomic model for Canada is formulated and estimated for the period 1962-1982. Some short-run and long-run features characterize the structure of the model. In particular, a government budget constraint, a capital accumulation identity and a production-function that defines the level of potential output in the long run are integrated with a conventional demand-oriented short-run Keynesian model. The objective function specified is quadratic and penalizes the squared deviations of its arguments from their desired values.

The optimal control experiments consist of minimizing the objective function subject to the constraints of the macro model. Two approaches are used. The first consists of varying the relative

structure of the weights over time by changing the rate of time preference in the objective function. The second approach aims at measuring and comparing the costs implied by different time horizons used by policy makers. In both cases, tradeoff relationships are derived between the performance of the economy in the short run and the long run, expressed in terms of the weighted squared deviations in the objective function.

The major conclusions drawn from this study are that a short-run/long-run tradeoff does exist within the given structure of the economy. In addition, this tradeoff implies that as more emphasis is placed on the near-term, the higher will be the cost incurred in the longer term relative to the gain in the near-term, and vice versa. Other conclusions relate to the existence of inflation-unemployment and inflation-balance-of-payments tradeoffs, and the assignment of particular instruments to particular targets.

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CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

1.1 ECONOMIC POLICY AND ECONOMIC THEORY

Economic policy in its broadest sense is a well specified course of actions undertaken to achieve some predetermined economic targets. However, the decision-making process involved in any policy formulation necessitates the need for knowledge concerning the economic system or entity subject to inquiry. This knowledge is provided by economic theory through the process of model building, the main purpose of which is to capture the most relevant aspects of the real world and its evolution. Thus economic theory and economic policy are intimately related, and economic models remain simply descriptive unless they are integrated with the theory of quantitative economic policy. After all, "knowledge is useful if it helps us to make the best decision"[1]. This chapter begins with a brief review of the theory of economic policy.

Tinbergen [1952], whose pioneering work has underlined and shaped most of the existing literature on quantitative economic policy, opened the exposition of his theory by classifying the types of variables that exist in a structural macroeconometric model. These are the target variables which represent the policy objectives and the instrument variables which are the policy parameters under the command of the policy maker. There are two other types of variables, which, though indispensable for a complete picture of the economy, are not of particular interest in a policy context. These are the data variables external to the problem considered, and the irrelevant variables endogenous to the model but not included in the targets. If the latter are eliminated from the structural model, we obtain the policy model that Tinbergen has developed. This model would have as many equations as targets and thus would be "operational". Given such a model, the policy maker will choose a set of instruments that satisfy his objective represented by the target variables without violating the boundary conditions or constraints imposed on the targets and the use of instruments. Tinbergen has emphasized the fact that while conventional economic theory treats the instruments as given and the targets as unknown, economic policy considers the targets to be known while the instruments are to be determined in order to attain these targets. This is the so-called inverse relation between instrument and target variables in the theory of economic policy. However, the difference between the two theories lies in their analytical approach rather than in their substance.

The simulation techniques used nowadays permit the policy maker to determine exactly the values of the endogenous or target variables corresponding to each set of policy instruments. This does not necessarily mean that the use of simulations in the analysis of economic policy does not involve any preference function. Although it may not be explicitly specified, the objective function could be implicit and known to the decision maker, but its maximum may never be attained by means of repeated simulations because of the cost and efforts associated with this procedure, especially in the case of large nonlinear macromodels. This problem can be overcome by fixing a set of targets and asking the question: What values should the policy variables take in order to reach this target? The answer to this question provides a direct and efficient way of choosing an optimal policy with respect to a specified set of objectives. This is the Tinbergen approach.

The Tinbergen approach to economic policy, however, has various limitations. In a linear model it implies that the number of instruments should be equal to the number of targets, for otherwise, no unique optimal solution can be found. Tinbergen himself has mentioned that there are no a priori reasons to believe that this situation will hold. In the case when the number of instruments exceeds the number of targets, there will be an infinity of solutions. By contrast, a solution does not exist if the number of targets surpasses the number of instruments. The only way out of this impasse is to increase the number of instruments or reduce the number of targets until they are equally numerous. Even if the number of instruments equals the number of targets, the solution values for the instruments may not be economically sensible; for example, negative government spending or interest rates. Furthermore, in nonlinear models there may be no solution for the instruments even though the number of instruments equals the number of targets. Finally, the basic Tinbergen approach takes no account of uncertainty.

Some of the above shortcomings have been corrected by Theil's [1964] contribution to the theory of quantitative economic policy. Still within the context of a linear model, the fixed targets have been replaced by a quadratic objective function and he showed that a solution exists even with unequal numbers of targets and instruments. The arguments of the objective function are the target and instrument variables specified in terms of deviations from their desired values. An optimal solution can be obtained by simply maximizing this quadratic function subject to the constraints of the macro model. In addition, it has been shown that this solution remains the same in the first period even after the introduction of uncertainty into the model by means of additive random errors. This is known as the "first period certainty equivalence theorem" which can be stated as follows: Under certain conditions, the policy adopted under uncertainty is exactly the same as the one that would be obtained if the random errors were replaced by their expectations. In short, the two significant steps that Theil has taken beyond Tinbergen's theory are: (1) the introduction of uncertainty into the policy model, and (2) the substitution of flexible for fixed targets.

1.2 WHY OPTIMAL CONTROL?

In recent years, there has been a growing interest in optimal control theory as an efficient tool for a systematic search for good economic policies. This was due to the explosion of computer technology which makes the solution of the control problem more manageable, the success that control applications have encountered in the engineering field, and more importantly, the proven superiority of the optimal control approach over the Tinbergen-Theil approach, especially in the case of uncertainty.

As mentioned earlier, the simulation technique can shed some light on the effects of alternative policies and the dynamic behaviour of the system, but it is highly improbable, no matter how hard we try, that the desired targets will be exactly met or reached. Optimal control theory is very useful in this context. It is capable of providing the timing and magnitude of the policy instruments, and achieving a desirable time path for the economic variables that represent the policy objective.

The three basic ingredients in any macroeconomic optimal control problem are:

a) A dynamic structural macro model that is considered to be an acceptable description of the economy. This is the "law of motion" describing the evolution of the system subject to investigation. This model can be represented by a set of difference (or differential) equations relating the state variables (i.e., targets) to the control variables (i.e., instruments), as well as to data variables.

b) A set of constraints and boundary conditions on the variables of the model.

c) An objective function representing the preferences of the policy maker and including target and perhaps instrument variables. Given this structure, then, the task of the "controller" is to steer the state variables towards their desired destination by manipulating the instruments under this control. In other words, the policy maker maximizes (c) subject to (a) and (b) in order to derive the optimal decision rules.

In the 1970s, numerous studies applied optimal control theory to the formulation of macroeconomic policies. These studies can be categorized according to various criteria (type of objective function, size of macro model, etc.), but we have chosen to categorize these studies according to their deterministic or stochastic nature, a classification which coincides, more or less, with their historical evolution. A number of these studies are surveyed in Sections 1.3 and 1.4[2].

1.3 DETERMINISTIC OPTIMAL CONTROL STUDIES

Pindyck [1973] applied a deterministic control approach to a simple quarterly linear model of the United States economy in order to derive the optimal time paths of the policy variables for short-term stabilization purposes. The model contains 28 state variables and three control variables, namely, government expenditures, the tax rate and the money supply. Among the targets are consumption, investment, interest rates, and disposable income. But the most important objectives were the price and employment levels. The loss function used is quadratic and given by

$$L = \sum_{t=0}^{T} \{(y_t - y_t^*)' Q(y_t - y_t^*) + (x_t - x_t^*)' R(x_t - x_t^*)\}$$

where y_t is a vector of state variables at time t and x_t is the control vector at time t; y_t^* and x_t^* are the optimal or desired values of state and instrument variables, respectively. Q and R are two diagonal matrices, the elements of which are the relative costs or penalties for deviating from the optimal paths of y and x. The inclusion of control variables in the objective function indicates that manipulating policy instruments cannot be done freely but should obey some constraints and boundary conditions.

The optimal control problem specified by Pindyck is therefore to minimize the loss function L subject to a linear macro model given by:

$Y_{t+1} - Y_t = AY_t + BX_t + CZ_t$ and some initial condition: Y_0 = constant.

Z is a vector of exogenous noncontrollable variables (data variables in Tinbergen terminology), and A, B, C are coefficient matrices. By varying the weights in the two diagonal matrices Q and R, Pindyck provided an empirical measure of the tradeoff between inflation and unemployment over the 21-quarter period beginning with the first quarter of 1957 and ending with the first quarter of 1962. The Phillips curve turned out to be dynamic in nature, i.e., to take different shapes over time and also to depend on other policy objectives. The several policy "experiments" also permitted the comparison between the actual historical policies pursued during the five-year period and what we might have expected, had the policy instruments been chosen optimally. Pindyck has also emphasized the idea that, although optimal control theory was originally intended to derive optimal decision rules, it was shown to be very useful in analyzing and better understanding the dynamic behaviour of an econometric model. This latter point was stressed by Oudet [1976] who considered the primary benefit of using optimal control to be its relevance as a tool for studying the dynamic behaviour of macroeconomic models rather than deriving economic policies because of the inaccuracy of these models and the arbitrariness in choosing an objective function. He calculated deterministic control rules based on a linearized version of a model of the French economy using a quadratic loss function and these rules in turn were applied to the original nonlinear model over the period 1972-76. The major conclusion drawn from this exercise was that controlled simulations are better tools than conventional or trial and error simulations for understanding the dynamic behaviour of large non-linear models, and that the results given by a nonlinear model and its linear approximation are fairly close.

Craine, Havenner, and Tinsley [1976] have derived the optimal paths of policy variables that minimize a fourteen-quarter (1971:I-1974:II) quadratic loss function subject to the constraint of a medium size (21 behavioural equations, 40 identities) nonlinear, deterministic model of the U.S. economy. The deterministic control solutions were analyzed in order to see whether or not historical policies could have been improved and to examine the possibility of a gain from coordinating monetary and fiscal policies and what advantage -- if any -- this coordination would have had over the use of monetary policy alone. The loss function includes four targets: the primary ones are the unemployment and inflation rates; the secondary ones are the rate of change of inflation and the rate of change of short-term interest rate. Only two instruments were considered: the money supply (M1) and government expenditures. The loss function is given by

$$L = \sum_{t=1}^{T} \{(Y_t^{'}QY_t) + (x_t - x_t^{*})^{'}R(x_t - x_t^{*})\}$$

where Y_t is a (4×1) vector of state variables, and x_t is a (2×1) vector of control variables. Y_t^{*} does not appear in L since the desired values of the four targets were set equal to zero. By weighting the unemployment rate twice as heavily as inflation, the authors discovered that optimal unemployment and inflation paths lie below and above their respective historical levels, which suggest that their objective function was significantly different from that of the policy maker. However, the main conclusion of the study is that economic policy, over the relevant period, could have been improved by being more expansionary and that monetary and fiscal policies appeared to be substitutes, in the sense that less use of one instrument could be compensated for by more use of the other.

Still in the context of deterministic optimal control studies, Holbrook [1973] applied an optimization method that he had developed to a deterministic version of the RDX2 model of the Canadian economy during the period 1969:I-1970:IV. The loss function used is quadratic and includes two targets, inflation and unemployment rates, and four instruments, government expenditures, income tax rate, interest rate and secondary reserve requirements. His function takes the following form:

$$L = \sum_{t=1}^{T} \{(Y_t' Q Y_t) + (x_t - x^*)' R (x_t - x^*)\}$$

Again, the desired targets, Y^*, were set equal to zero. Several experiments were performed involving different instruments, different cost or penalty coefficients and different time horizons. The choice of optimal policy for the first quarter of 1969 was described in detail and it was shown that the welfare gain was not great, and that the gain obtained was due solely to reduction of unemployment, since inflation was increasing at the same time. However, the main purpose of the paper was "not to prescribe economic policy", but rather to develop an optimization technique capable of handling large nonlinear econometric models such as RDX2 in order to reduce the computational and other costs associated with real world optimal control policy formulation. To test the simplicity and efficiency of his method, Holbrook [1974] applied it to the Michigan model of the U.S. economy which is a medium-size (61 equations) nonlinear model, and extended the control problem to a stochastic framework. But once more the purpose of the exercise was to illustrate the ease and limitations of the technique developed since the instruments, targets and cost coefficients were chosen arbitrarily.

1.4 STOCHASTIC OPTIMAL CONTROL STUDIES

Of course, macroeconometric models are not really deterministic, but the complexity and the costs involved in solving a stochastic optimal control problem, particularly at the beginning of optimal control applications, induced many authors to justify the use of deterministic models by invoking some simplifying assumptions. Pindyck [1973], for example, relied on the principle of "certainty equivalence" mentioned earlier, which is valid in the first period when uncertainty is represented by uncorrelated error terms added to each equation of the model[3]. On the other hand, many authors have questioned the desirability of introducing uncertainty into the optimal control problem. Holbrook [1974] has raised the question of whether the gain attributed to the inclusion of uncertainty outweighs the costs of excluding it, especially in the case of large nonlinear models. His response seems to suggest that for practical policy-making purposes the deterministic solution provides sufficient useful information regarding the signs and magnitudes of the policy instruments.

Chow [1972, 1975, 1976] was one of the first advocates of the use of stochastic optimal control in macroeconomic policy analysis. He has shown that the benefits of incorporating random disturbances into an econometric model are much larger than those that one could get by ignoring them. He argued that in the case of deterministic control, welfare gain or loss which depends on future policies cannot be measured since these policies are not provided in an optimal way, except for the first period using the "certainty equivalence" theorem. Relying on this theorem in a multiperiod control problem, with say n periods, requires the planner to solve n first period certainty equivalence problems [Chow, 1976]. This would be a costly procedure to use and that is why we have noted earlier that optimal control theory is more efficient than the Theil approach, particularly in the case of uncertainty. The model that Chow constructed in order to demonstrate the above results was a simple annual model of the U.S. for the period 1931-40 and 1948-63. The loss function was quadratic with a ten-period time horizon. The problem then was that of minimizing

$$L = E \sum_{t=1}^{T} \{(Y_t - Y_t^*)' Q (Y_t - Y_t^*)\}$$

subject to a linear econometric model in its reduced form:

$$Y_t = A_{1t}Y_{t-1} + \ldots + A_{mt}Y_{t-m} + C_{0t}X_t + \ldots + C_{nt}X_{t-n} + b_t + U_t$$

The exogenous variables not subject to control are considered to be either a part of the intercept b_t, or a part of U_t, the random disturbance vector. To simplify the analysis, the above model was redefined in a state-variable form; that is it was rewritten as a first order system of difference equations[4].

$$Y_t = A_t Y_{t-1} + C_t X_t + b_t + U_t$$

In this case, Y_t could include current and lagged state (dependent) variables as well as current and lagged control variables. That is why the above objective function does not include explicitly the instrument variables in its arguments, since these are embodied in the vector Y_t.

Apart from being efficient, the stochastic control approach taken by Chow is more realistic, in the sense that it yields a solution in the form of feedback control equations or reaction functions, i.e., the future values of the policy variables will be revised automatically in the light of new information. This is known in the control literature as closed loop policy. By contrast, the open loop policy generated by a deterministic control problem consists of a set of preassigned values determined in advance in period 0 -- and not subject to change during the planning period, since uncertainty is assumed away.

The usefulness of feedback control equations has been questioned by Craine, Havenner and Berry [1978], who have examined the "issue of fixed rules vs. 'activism' in the conduct of monetary policy" over the two-year period starting 1973:III and ending in 1975:II. Their loss function is quadratic and includes three targets, the inflation rate, the unemployment rate, and the change in the treasury bill rate, and one instrument, the rate of growth of M1. This function can be represented by the general form:

$$L = \sum_{t=1}^{T} \{(\tilde{Y}_t)' Q(\tilde{Y}_t) + (X_t - X^*)' R(X_t - X_t^*)\}$$

where $\tilde{Y}_t = Y_t - Y_t^*$ if $Y_t > Y_t^*$

$= 0$ otherwise.

The choice of Y_t^* is based on the announced objectives of the administration at the given time. Six policies were constructed and compared in terms of the losses that they would have incurred. It came as a surprise that "policies that used a minimal amount of information gave the best performance".

The above result, however, has been contradicted by the finding of Buiter and Owen [1979]. They have compared the results obtained with fixed-rule policies and those generated by an optimal feedback control approach when a quadratic loss function, including four targets and four instruments, was minimized subject to the constraint of a nonlinear small (12 behavioral equations) open economy model of the Netherlands over the period 1961-1976. The relative performance of optimal control solutions was impressive since the variances of unemployment, inflation and GNP growth were substantially lower than in the case of non-discretionary policy. On the other hand, the fixed rules were shown to be satisfactory only if the economy is stable and free of external and internal shocks.

Ando and Palash [1977] have attempted to isolate the contribution of external exogenous non-controllable shocks and that of fiscal and monetary policies to the stagflation in the United States during the period 1973-1975, using the MPS model and a quadratic objective function. They have shown rather successfully that because of the misspecification of the objective function which failed to take into consideration the imported component of inflation which was not under the control of the policy maker, the recession was much worse than it would have been otherwise. Their argument is fairly simple and straightforward: at least two-thirds of the inflation in 1974 was due to factors not related to domestic excess demand. Thus, the attempt to curb inflation by restricting final demand resulted in a sharp reduction in output and employment without much moderation of inflation. An appropriate policy could have been obtained by treating the exogenous noncontrollable inflation as one of the constraints imposed on the optimization problem. This would have yielded a much lower unemployment rate without worsening inflation.

A similar question has been raised by Klein and Su [1980]. That is: was there a set of policies that could have been pursued by the government over the period 1971:I-1975:I such that the inflationary recession could have been avoided, or at least moderated? If such policies had existed and been implemented, then how large is the improvement that could have been achieved over the actual policies?

To investigate this, they applied a stochastic optimal control approach to a large-scale (450 equations and identities) nonlinear quarterly model of the U.S. economy, using a quadratic objective function that included four targets and three instruments. The optimal control solution suggested, as in the case of Ando and Palash [1977], that even though inflation could not have been much lower, the unemployment rate could have been reduced significantly. In other words, the severity of the recession could have been moderated and the welfare of the economy could have been improved by approximately 40 percent had the optimal policies been implemented.

The choice of the objective function has always been a controversial matter in the theory of optimal control. Bray [1975] has suggested that a series of questions should be put to the policy maker in order to represent his priorities in the welfare function in terms of partial derivatives, which are the marginal improvements in each target and instrument. He applied a stochastic optimal control approach to a small (15 behavioural equations and 10 identities) linearized model of the U.K. economy over the period 1973-1977. The loss function included 6 targets and 6 instruments, and sensitivity tests were carried out by varying the weights attached to them. Moreover, he made a comparison between "fixed" and "flexible" policies, and between the simulated control model and the level and variations of variables that actually occurred. He found, for example, that the variations in unemployment and balance of payments could be reduced at the cost of an increase in variation of some policy instruments. However, the conclusion that he has drawn from his experiments is that despite the limits on the quality of the optimal control solution, especially in the case of a highly aggregated model, optimal control is very useful for stabilization purposes.

The use of optimal control theory can illustrate another aspect of economic policy, namely, the choice of instruments and target variables. Pindyck and Roberts [1974] have shown that following an interest rate target might be better for monetary policy than targetting the money stock (M1) when both of them are not under the direct control of the federal reserve system. This is so because attaining a given target for M1 can be accomplished only at the cost of large fluctuations in some other variables, whereas this problem does not arise when the interest rate is considered to be the policy target. Along the same lines, Litterman [1982] has investigated the question of instrument instability in the conduct of monetary policy. The interest rate may fluctuate considerably in order to achieve some money supply target. By specifying a quadratic loss function which penalized both money supply deviations from target and interest rate volatility, he was able to prove that interest rate fluctuation can be reduced considerably without damaging the degree of monetary control, and hence to argue that the optimal control approach would improve the federal reserve operations procedure.

Garbade [1975] has examined, among other things, the relative merits of the treasury bill rate and the money supply as alternative policy instruments. His conclusion suggests, in a stochastic context, that when discretion is allowed, there is little difference between the two instruments, while controlling the money supply yields a lower expected welfare loss in the case of "fixed" policy rules. Using a nonlinear intermediate size macro-model of the U.S. economy which he constructed, and a quadratic welfare criterion, he has shown that optimal control theory is a very helpful tool in the theory of quantitative economic policy. Furthermore, it can contribute significantly to economic stability and performance by reducing the expected welfare loss by about 50 percent from that incurred by a nondiscretionary policy.

The optimal control approach can be used also to evaluate the relative effectiveness of monetary versus fiscal policy. Mathieu [1976], for example, has tested the principle of "effective market classification" in the context of the Canadian economy. He used an open economy model and a quadratic loss function to show that the above principle holds; that is, that fiscal policy should be directed towards domestic targets and monetary policy towards external equilibrium. At the same time, it was clear from the analysis, that external and internal objectives cannot be satisfied simultaneously, therefore suggesting a tradeoff facing the policy maker.

1.5 OBJECTIVES AND ORGANIZATION OF THE STUDY

It is evident from the above mentioned studies that optimal control theory is a useful tool in most aspects of macroeconomic policies. It can be used as a rational device in the formulation of policies by providing the optimal time paths of the control variables that are under the direct control of the policy-maker. Moreover, optimal control theory can be helpful in the choice of targets and instruments and in the evaluation of prospective monetary and fiscal policies, as well as historical policies. Recently, Fair [1978] and Chow [1978] have developed two methods, based on the theory of optimal control, which can be used to measure economic performance in any historical period and have applied these methods to those four-year periods associated with the American political administration[5].

However, most of these control applications have dealt with the issue of short-run stabilization policies which are concerned with a myopic time horizon, ranging from a few quarters to a few years. On the other hand, there were few studies that dealt with long-term planning as a problem in optimal control[6]. The integration of short run and long run in the theory of economic policy has been neglected in the control literature and in any other policy framework for that matter. In fact, one cannot treat short-run policies in isolation from their long-run effects, and by the same logic, long-run objectives cannot be pursued without regard to their short-run implications. In other words, the optimal planning of short-run stabilization policies should be made in such a way that their long-run consequences do not contradict or offset their immediate results, since all short-term objectives have long-term aspects as well. For instance, reducing the unemployment rate is a short-run as well as a long-run objective for any government. At the same time, pursuit of a long-term objective should not be initiated at the cost of huge sacrifices in the present period. There must be some balance between the Keynesian statement that "in the long-run we are all dead" and the extreme neoclassical assertion that only the long run matters.

Having insisted on the importance of both short-run and long-run aspects of economic policy, it should be noted, however, that it may be impossible to satisfy both types of objectives simultaneously, and a tradeoff may exist between them which the policy maker should be aware of. The use of optimal control in order to investigate this tradeoff and the linkage between short run and long run in the theory of economic policy is the purpose of this study.

The limitations of the use of optimal control theory in macro-economic policy should be mentioned at the outset. It simplifies substantially the complex nature of the decision process, which is due to political, administrative and institutional factors, by reducing it to a possible oversimplified problem. Furthermore, the application of optimal control in macro planning is predicated on the premise that the economy follows well defined "laws of motion". While in physical sciences the existence of such laws is quite conceivable, few people would accept this premise without qualifications as a realistic description of economic systems. Many economists would go even further and argue that change in policies will not leave the structure of the model in question unchanged, since economic agents adjust their behaviour to the policies they perceive or anticipate. This is the main argument advanced by the rational expectations theorists, and it rules out very often the effect of any policy on the economy. In this study we are not concerned with the discussion of such arguments, but rather we apply optimal control theory in the belief that it can improve the quality of the decisions which are made.

Since the application of optimal control requires the existence of a macro model, this is the subject matter of Chapter II, wherein a small and simple open economy model of the Canadian economy is specified and estimated. In Chapter III some historical and policy simulation experiments are carried out in order to evaluate the model's ability to replicate the historical paths of the endogenous variables and to test the short-run and long-run responses of the model to different policy shocks. These experiments shed some light on the convergence and stability properties of the model, which are important when long-run aspects of the model are considered. Chapter IV deals with the specification of the objective function and its arguments, namely the targets and instruments, and with their weights, and includes also a discussion of the algorithm used to solve the optimization problem. Chapter V presents the results of the optimal control experiments and their interpretation, as well as a discussion of the tradeoff and coordination between short- and long-run economic policies. Finally, Chapter VI summarizes the major conclusions of the study, notes some caveats, and provides some suggestions for future research.

FOOTNOTES

Chapter 1

1. Marshak, J.,[1953].

2. The following studies do not constitute an exhaustive list. See Kendrick [1976] for a survey of over sixty such studies.

3. In the context of the macroeconomic optimal control problem, uncertainty can be introduced in three different ways:

   a) System noise can be incorporated in the form of random disturbances U_t added to each equation of the model. For example, $Y_{t+1} = AY_t + BX_t + U_t$.

   b) The coefficients of the model can be specified as functions of some unknown parameters, i.e., $Y_{t+1} = A(\theta_t)Y_t + B(\theta_t)X$.

   c) Uncertainty can be introduced in the form of measurement error. For example, $Y_t^* = Y_t + \eta$; where Y_t^* = true vector; Y_t = observation vector, and η = measurement noise.
   Needless to say, the above three forms of uncertainty are not mutually exclusive and can be incorporated simultaneously.

4. The state variable form is the result of reducing a high order system of difference or differential equations to a first order system. See Pindyck [1973], Appendix A, for details.

5. Fair [1978] has examined the performance of five U.S. administrations: Eisenhower I and II; Johnson, Kennedy and Nixon I. See Fair [1978] for details.

6. One of these is by Martens and Pindyck [1972]. It deals mainly with the allocation of investment resources among different sectors over time in Tunisia. This is cited in Rufatt [1981].

CHAPTER 2

A SMALL MACROECONOMIC MODEL FOR CANADA

2.1 INTRODUCTION

In this chapter, a small macroeconomic model for Canada will be constructed and estimated over the period 1962-82. The design and construction of any model depends to a great extent on the purpose for which the model is built. Since the major concern of this study is related to policy formulation, the model should be a 'control' or decision model. In fact, the interpretation and understanding of the control solutions will be easier if the model attempts to specify as closely as possible the cause effect mechanism at work and the channels through which the control variables exercise their influence on the targets in question. Hence, the major policy targets and instruments (such as unemployment rate, inflation rate, government expenditure, tax rate, etc.) should be included explicitly in the model. Moreover, the present model should highlight the distinction between the short-run and long-run relationships, since this constitutes the major point of investigation. However, when constructing a macroeconomic model, one is confronted by different alternatives and various possibilities, and the final choice must involve some compromises.

The model used in the present context is highly aggregated, simple and small, relative to some existing models[1]. Also an attempt has been made to keep nonlinearities to a minimum and thereby to facilitate the computation of optimal control solutions. Hence, it should be mentioned at the outset that the present model is intended to be illustrative rather than a complete and disaggregated description of the Canadian economy. But for the model to be useful and for the results to have any relevance at all, it should incorporate the basic macro relationships that capture the distinctive characteristics of the Canadian economic system.

One final aspect of the methodology used in this chapter is worth noting before discussing the structure of the model. In the last decade or so, there has been a growing body of literature, associated mainly with the neo-classicists, criticizing the way in which conventional macro models were built and used in policy simulations, namely their lack of microfoundations. Such criticism is relevant to the present model, the equations of which are specified directly in their structural form without any explicit attempt to justify them in the light of microeconomic theory.

In the remainder of this chapter, the structure of the model will be specified and discussed first. The estimation procedures and results will then be presented and explained.

2.2 THE STRUCTURE OF THE MODEL

2.2.1 General Properties of the Model

The model described in this chapter does not belong to either the "text book" Keynesian nor neo-classical schools of economic analysis. It incorporates some features commonly associated with both. For example, although effective demand is the proximate determinant of the level of output, the production function in the model constrains the level of output in the long run. Any increase in one of the components of GNP will induce a rise in wages and prices, and so limit the growth of demand for output. An increase in exports, for example, will raise actual output relative to capacity or potential output, thus inducing a price increase, and therefore limiting the growth of foreign demand for home output. Such a price increase can also result from an increase in consumer expenditure or investment expenditures, which in turn leads to an increase in the interest rates and, consequently, to a decline in these expenditures. However, total output is not constrained by the aggregate production function at every point in time but can fluctuate in the short run. More output can be produced by more intensive use of the inputs [Knight and Wymer, 1978].

In terms of channels of transmission of economic policy in the model, one can notice that while the effect of fiscal policy on output is direct, monetary policy exerts its influence via its effect on the interest rates; and that is there is no real balance effect on effective demand. The model assumes that Canada is a small open economy; that is, that Canada is a price taker on the international market and thus the prices of our exports and imports do not influence the prices in the rest of the world. Another important assumption in this regard is that the Canadian exchange rate is taken as an exogenous policy instrument. This assumption becomes controversial for the period starting in 1971, when the authorities ceased to declare 'explicitly' a par value for the Canadian dollar. However, that does not imply that foreign exchange market intervention has been abandoned, but rather that the Canadian authorities have taken a flexible approach to management of the exchange rate. Having made such an assumption, the exchange rate becomes a major link in the transmission of foreign shocks[2].

The dynamics of the model come from many sources. The price and wage equations, together with the unemployment rate equation, represent a block characterized by simultaneity. Nominal wages tend to rise whenever there is excess demand for labour and/or a price increase in the economy, and prices tend to rise whenever wages and/or the capacity utilization rate rise, which also leads to a decline in unemployment. In addition, the exchange rate determines the level of import prices which affects the domestic price level with a lag. Other dynamic elements of the model come from capital accumulation, from technical progress, and from the public sector financing requirements. One distinguishing feature of the Canadian economy which has not been incorporated into the model at this stage is the external debt and interest payments abroad and their domestic implications.

2.2.2 Description of the Model

The model consists of 21 equations, of which twelve are behavioural and nine are identities. The model can be organized conceptually around six interrelated blocks: a domestic components of gross national expenditure block; a wage-price-unemployment block; a monetary sector block; a balance of payments block; a potential GNP block; and a government sector block. A considerable degree of interrelationship exists among the endogenous variables as can be easily seen from the block diagram presented in Figure 2.1. The block structure of the model is shown in Figure 2.2.

DOMESTIC EXPENDITURES AND OUTPUT

This block involves the determination of consumption (CON), gross fixed investment (GFI), and the change in non-farm inventories (NFINV). These variables, together with the change in farm inventories (FINV), government expenditures (G), and the net trade balance (X-M), determine

FIGURE 2.1 BLOCK DIAGRAM OF THE MACROECONOMIC MODEL

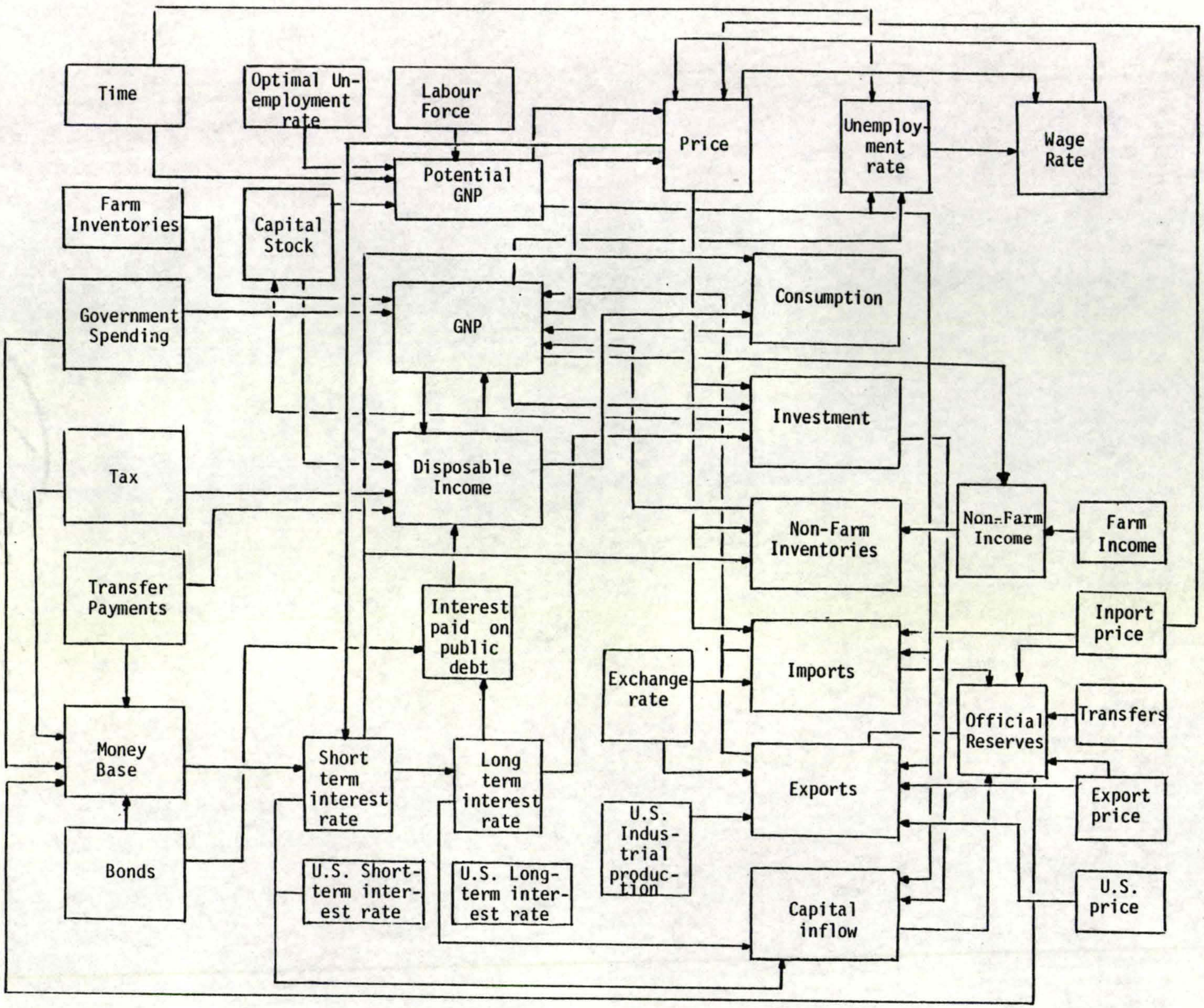
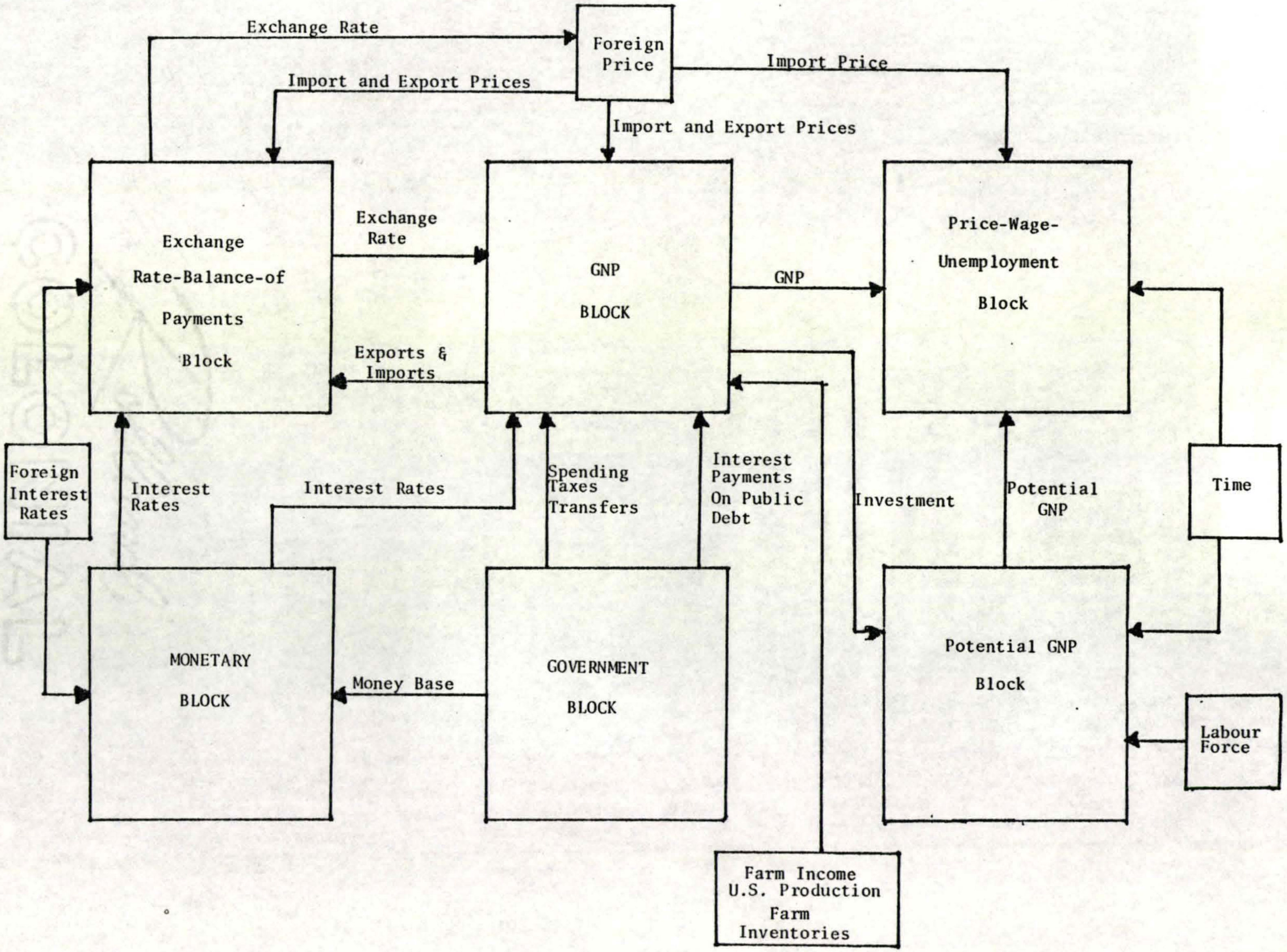


FIGURE 2.2 BLOCK STRUCTURE OF THE MACROECONOMIC MODEL



the gross national product (Q), as indicated by the identity

$$Q = CON + GFI + NFINV + FINV + G + X\text{-}M \tag{2.1}$$

where X is exports and M imports. The GNP and its components are defined in real terms[3].

The consumption function relates consumer expenditures on goods and services to disposable income (YD), the short-term interest rate (RS), and consumption in the previous period. Disposable income is defined as net national product (NNP), minus total taxes (T), plus transfer payments to persons (TRP), and interest payments on the public debt (IPPD):

$$YD = NNP - T + TRP + IPPD \tag{2.2}$$

where NNP is defined as the GNP minus capital consumption allowances.

The inclusion of the short-term interest rate (RS), as an explanatory variable captures the dependence of consumer spending, especially on durable goods, on the cost of borrowing. The short-term interest rate, which serves as a measure of credit cost, constitutes a direct link between the monetary and real sectors. This link seems to suggest that monetary policy would have a larger impact on GNP than fiscal policy, since increasing government expenditures would raise the interest rates, and thus both investment and consumption expenditures would be "crowded out". On the other hand, monetary policy reduces the level of the short-term interest rate, and the effect on GNP would be larger because of the additional increase in consumption. However, the above supposition is true only in the impact period and when the government financing constraint is not taken into consideration as we will see later in this chapter. The lagged consumption effect might be attributed to either or both of the following factors: habit formation and thus the dependence of current consumption on previous levels of disposable income, or expectations about future income and its effect on the current consumption level. Thus the consumption function takes the following form:

$$CON = f[YD, RS, CON_{-1}] + U_1 \tag{2.3}$$

Total investment is broken into two categories. The first is gross fixed investment (GFI), which is a function of gross national product (Q), and the expected real long-run interest rate (RRL). The latter is defined as the long-term interest rate (RL), minus the expected inflation rate, which is equal, by assumption, to the previous period inflation rate $(\hat{P})$. The expectation hypothesis is simple and static, in the sense that the last period rate is expected to prevail in the next period, with no adaptive or other adjustment processes being allowed. This assumption is questionable especially in the case of an annual model. However, the assumption is made in order to keep the model manageable, for the purpose of control experiments. The investment function is then of the form

$$GFI = f[Q, RRL_{-1}] + U_2 \tag{2.4}$$

Investment plays a dual role in the model: in the short run it has an expenditure impact on GNP, as represented by the above equation; and in the long run it leads to the accumulation of capital stock which in turn influences the long-run production level. The capital accumulation process is represented by the identity

$$K = (1 - \delta)K_{-1} + GFI_{-1} \tag{2.5}$$

where δ is the depreciation rate and K is the capital stock at the beginning of the year.

The second category of investment, change in non-farm inventories (NFINV), is considered to be a function of its own lagged value, the change in non-farm income (NFQ), and the change in the expected short-term real rate of interest (RRS), the latter representing the opportunity cost of holding inventories. RRS is defined in the same manner as the expected long-term rate, i.e., $RRS = RS - \hat{P}_{-1}$. Thus

$$NFINV = f[\Delta NFQ, \Delta RRS, NFINV_{-1}] + U_3 \tag{2.6}$$

Non-farm income is given by the identity

$$NFQ = Q - FQ \tag{2.7}$$

where FQ is farm-income.

WAGE, PRICE, UNEMPLOYMENT AND POTENTIAL OUTPUT

The rate of change of the money wage rate ($\hat{W}$), is assumed to be a decreasing function of the unemployment rate (UR), an increasing function of the previous period rate of change of wages (representing a sluggish wage adjustment), and the rate of change of the GNP deflator ($\hat{P}$).

$$\hat{W} = f[UR, \hat{P}, \hat{W}_{-1}] + U_4 \tag{2.8}$$

The above equation implies a Phillips curve relating the rate of change in wages to the unemployment rate. The higher the unemployment rate is, the less is the ability of workers to bargain for higher wages. $\hat{P}$ can can be viewed as an expectation term, for reasons similar to those explained earlier, and we would then have an expectation-augmented Phillips curve. The GNP deflator may be appropriate to the demand side of the labour market however, the consumer price index might be more appropriate to the supply side, since employees are interested in the purchasing power of their wages rather than the overall price index. However, we have opted for a single price index in the model in order to keep the size of the model manageable in the optimization part of this study.

The rate of change of the GNP deflator ($\hat{P}$) is related on the supply side to the rate of change of labour cost represented by the wage rate, and on the demand side to the capacity utilization rate, expressed as the ratio of actual to potential output (Q/QP). An external factor, which is the rate of change of import prices ($\hat{PM}$), has been added to the equation since these prices are assumed given to Canada, and because imports constitute a substantial proportion of total goods and services consumed by Canada residents. Thus the price equation is of the form

$$\hat{P} = [\hat{W}, Q/QP, \hat{PM}_{-1}] + U_5 \tag{2.9}$$

It is useful at this stage to explain how the potential output series was obtained. A constant returns to scale Cobb-Douglas production function that captures both short-term fluctuations and the long-term growth was estimated by ordinary least squares. It has the following form

$$Q_t = aK_t^{\alpha}((1 - UR_t^*)LF_t)^{\beta}(1-UR_t/1 - UR_t^*)^{\delta}e^{\gamma t}$$

where K is the capital stock, UR^* and UR are the "full employment" and actual unemployment rates, respectively, t is a time trend reflecting technological change, and LF is the total labour force. Since some difficulties were encountered in estimating the above equation, restrictions were imposed on the share of capital and labour: α was set equal to 0.35, and β to 0.65[4]. The "full employment" level of the unemployment rate (UR^*) was generated by linking the troughs in the unemployment rate series which occurred in 1966 and 1974 and then extrapolating for the rest of the period. Potential or peak output was generated by setting actual employment equal to full employment, so that $[1-UR/1-UR^*] = 1$. Thus

$$QP_t = aK^{\alpha}((1-UR_t^*)LF_t)^{\beta}e^{\gamma t} \tag{2.10}$$

Notice that the ratio Q_t/QP_t equals $(1-UR_t/1-UR_t^*)^{\delta}$, which implies that the deviation of actual output from its potential level is due solely to the deviation of the unemployment rate from its "full employment" level.

The unemployment rate (UR) is explained as a function of the capacity utilization rate (Q/QP), the unemployment lagged one period, and a time trend t, representing the change over time in the composition of the labour force, and possible increasing structural unemployment. Thus

$$UR = f[Q/QP,\ UR_{-1},\ t] + U_6 \tag{2.11}$$

THE MONETARY SECTOR

The monetary sector consists of two equations. The first relates the nominal short-term interest rate (RS) to the rate of change of prices ($\hat{P}$) and the ratio of high powered money to nominal GNP($H/P \cdot Q$). That is

$$RS = f[H/P \cdot Q,\ \hat{P}] + U_7 \tag{2.12}$$

The second relates the long-term interest rate to the level of the short-term rate as well as to changes in RS which can be considered to be an expectation term, and to the long-term rate lagged one period. A monetary policy will therefore affect RS directly and RL indirectly (through RS). This equation, used by Pindyck [1973], takes the following form:

$$RL = f[RS,\ \Delta RS,\ RL_{-1}] + U_8 \tag{2.13}$$

THE BALANCE OF PAYMENTS

The foreign sector involves imports (M), exports (X), a capital inflow equation, and a balance of payments identity. Imports of goods and services are a function of an activity variable represented by total output (Q), and a relative price term which is the ratio of the GNP deflator (P) to the import price deflator. The import price deflator is equal to the foreign price (PF) multiplied by the exchange rate (e)[5]. Thus

$$M = f[Q, P/PM] + U_9 \quad (2.14)$$

and

$$PM = e \cdot PF \quad (2.15)$$

Exports of goods and services are a function of a foreign activity level, which is the industrial production index in the United States (IPU), the ratio of the export price to the U.S. price, expressed in Canadian dollars $(PX/PU \cdot \overset{e}{\cancel{E}})$, and the capacity utilization ratio (Q/QP). The impact of the latter is expected to be negative. This negative effect can be explained by the fact that output becomes less available for exports whenever actual output approaches its potential level[6]. The export equation then, is

$$X = f[IPU, PX/PU \cdot e, Q/QP] + U_{10} \quad (2.16)$$

Net capital inflow as a fraction of nominal GNP$(CF/P \cdot Q)$ is a function of the changes in short-term and long-term interest rate differentials between Canada and the United States. These interest rate differentials exert a great influence on the movement of capital between the two countries, since the capital account of the Canadian balance of payments is dominated by transactions with the United States. Also, capital inflow is a function of the change in the investment output ratio, which captures the importance of foreign funds in domestic investment projects. Thus the capital flow equation is

$$CF/P \cdot Q = f[\Delta RSD, \Delta RLD, \Delta(GFI/Q)] + U_{11} \quad (2.17)$$

where RSD and RLD are the short-term and long-term interest rate differentials between Canada and the United States. These three equations are related in the balance of payments identity as follows:

$$\Delta R = PX \cdot X - PM \cdot M + CF + TRAN \quad (2.18)$$

where TRAN is transfers (assumed exogenous) and R is official international reserves. Thus, the fourth variable defined by the foreign sectors' four equations is either the exchange rate (e), in the case of a flexible exchange rate regime or the change in reserves (ΔR), in the case of fixed rate regime. As noted earlier, either the exchange rate or the change in official reserves could be taken as the policy instrument associated with the foreign exchange market. In this study the exchange rate is going to be considered as a policy instrument.

THE GOVERNMENT SECTOR

The government finance constraint stipulates that government must balance the sources and uses of its funds. Moreover, this identity implies an interrelationship which has an important bearing on conventional fiscal and monetary policy multipliers and the stability properties associated with them[7].

The difference between the government's expenditures and its tax receipts, in nominal terms, may be financed either by issuing government debt (B) or by creating high-powered money (H). Thus, the change in the monetary base equals spending, less taxes (T), plus the difference between the accumulation of international reserves (ΔR) and public sector borrowing requirements (ΔB). This is the amount that the Canadian government must finance by creating high-powered money, as described in the following equation:

$$\Delta H = P \cdot G + TRP + IPPD + \Delta R - T - \Delta B \tag{2.19}$$

Public spending is broken down into government expenditures on goods and services, in nominal terms ($P \cdot G$), transfer payments (TRP), and interest payments on outstanding debt (IPPD), which is defined by

$$IPPD = RL \cdot B \tag{2.20}$$

It should be pointed out that including the interest paid on bonds as an expenditure item in the government budget constraint, has many implications for the model. Not only are the dynamics of the system modified, but also the steady-state or long-run multipliers will have different impacts, depending on whether the deficit is financed by money or bonds. As shown by Blinder and Solow [1973], in the case of bond-financing the budgetary gap is harder to close, and this may lead to instability in the system. This is certainly true in the present model, since increasing outstanding debt (B), will also lead to an increase in the interest rate (RL), and therefore increase IPPD even further. The model is stable only if there is an accompanying increase in income to generate tax receipts sufficient to close the budgetary gap. However, this condition requires a very high tax rate. In our model the tax rate is estimated by fitting a simple tax function, that is

$$T = f[P \cdot Q] + U_{12} \tag{2.21}$$

If the stability condition is satisfied, then it may well be the case that the bond-financed government expenditure multiplier is higher than the money-financed multiplier because of the positive effect of the increase in interest payments on disposable income and subsequently on consumption and GNP. Thus our earlier contention that, given our consumption function, fiscal policy is less effective than monetary policy, may be true only in the impact period. In the long run, however, bond-financed fiscal policy has two opposing effects. First, the interest rate tends to be higher and this would reduce consumption and investment. Second, a higher disposable income, due to the increase in debt servicing, will increase consumption and investment. The final result depends on the magnitudes of these two effects.

2.3 ESTIMATION OF THE MODEL

2.3.1 Estimation Techniques

The model was estimated using annual time-series data. Depending on the equation being estimated, the sample period ranged between 1962-1982 and 1964-1982, since the largest lag period in the model is two years. Initially, the model was estimated by Ordinary Least Squares (OLS). However, in simultaneous-equation models, such as the present one, inconsistency of the OLS estimator of the structural parameters may arise because the assumption of zero correlation between the right-hand-side variables and error terms is often violated. The model was therefore estimated by the method of two-stage least squares (2SLS). As is well known, this method can be thought of as consisting of two separate stages. The first stage involves regressing each right-hand-side endogenous variable on the predetermined variables of the model. Then, in the second stage, an OLS regression is performed with the endogenous right-hand-side variables replaced by their first stage estimated values. In the present model, however, the number of predetermined variables exceeds the sample size, so that the first stage of the 2SLS method breaks down. Even if this were not the case, the number of degrees of freedom in the first stage would be small if there were a large number of predetermined variables, and the estimation would be unsatisfactory [Johnston, 1970]. A solution to this difficulty is to reduce the number of predetermined variables through the technique of principal components. Kloek and Mennes [1960] suggested different methods of selecting principal components of the predetermined variables of the model. There are essentially two methods. The first is to choose a set of principal components (PCS) of those predetermined variables that do not appear in the equation considered. This would avoid the possibility of perfect correlation between the PCS and the right-hand-side variables. The second method reduces the computational work involved in the first procedure and consists of selecting a single subset of PCS from the complete set extracted from all of the predetermined variables in the system. Both methods were tried but the final form of the estimated model, reported here is based on the second one, not just because it is simple, but also because it produced better results, overall. A set of six principal components was computed amounting to 95 percent of the total variation among the predetermined variables. This set was used in the first stage as instrumental variables or regressors for all endogenous variables appearing on the right-hand-side of the simultaneous equations. The estimated values of the endogenous variables were then used in the second stage of the two-stage least squares principal component method (2SLSPC). Those equations with significant autocorrelation were re-estimated using the method proposed by Fair [1970] which consists of augmenting the list of instruments by adding to it the lagged dependent and independent variables in order to obtain consistent estimates. The test for serial correlation was based on the standard Durbin-Watson statistic DW. Since this test loses its power somewhat in equations involving lagged dependent variables, an h-statistic as suggested by Durbin [1970], was also employed. The h-statistic is defined as follows:

$$h = \hat{\rho}\sqrt{\frac{N}{1 - NV}}$$

where $\hat{\rho}$ is equal to (1 - 1/2 DW), N is the sample size, and V is the sampling variance of the coefficient of the lagged dependent variable. For all values of h greater than 1.645, one would reject the hypothesis of zero autocorrelation at the 5 percent level of significance[8].

2.3.2 Estimation Results

This section presents the estimated form of the Canadian model. The variables of the model are defined in Table 2.1 and the estimated equations, along with the identities, are presented in Table 2.2. All the behavioural equations were estimated by the 2SLSPC method except the capital inflow equation, which was estimated by OLS. The t statistic is given in brackets under each coefficient and beneath each equation appear the adjusted R-squared statistic ($\bar{R}^2$), the standard error of regression (SER), and the Durbin-Watson Statistic DW. These are followed by an estimate of the first-order autocorrelation parameter ρ (if used), obtained by using Fair's procedure, as mentioned earlier, and the value of the h statistic, in cases in which a lagged dependent variable appears on the right-hand-side of the equation. Also, the period of estimation is stated for each equation.

TABLE 2.1 VARIABLES OF THE MODEL

ENDOGENOUS

CF	= Net Capital Inflow
CON	= Real Consumption
GFI	= Real Investment
H	= High Powered Money[9]
IPPD	= Interest on the Public Debt
K	= Real Capital Stock
M	= Real Imports
NFINV	= Real Non-Farm Inventories
NFQ	= Real Non-Farm GNP
P	= General Price Level (GNP Implicit Deflator)
PM	= Import Price
Q	= Real GNP
QP	= Real Potential GNP
R	= Official International Reserves
RL	= Long-Term Interest Rate
RS	= Short-Term Interest Rate
T	= Total Taxes
UR	= Unemployment Rate
W	= Wage Rate
X	= Real Exports
YD	= Real Disposable Income

Table 2.1 Variables of the Model (continued)

EXOGENOUS

B	=	Total Bonds
e	=	Exchange Rate
FINV	=	Real Farm Inventories
FQ	=	Real Farm GNP
G	=	Real Government Expenditures on Goods and Services
IPU	=	US Industrial Production Index
LF	=	Labour Force
PF	=	Foreign Price Level
PX	=	Price of Exports
PU	=	US Price Level
RLU	=	US Long-Term Interest Rate
RSU	=	US Short-Term Interest Rate
t	=	Time Trend
TRP	=	Transfer Payments to Persons
TRAN	=	Current Account Net Transfer
UR*	=	'Full-Employment' Unemployment Rate

δ depreciation rate

Δ denotes backward difference operator; $X = X - X_{-1}$

$\hat{}$ denotes a proportional rate of change: $\hat{X} = \Delta X / X_{-1}$

TABLE 2.2 EQUATIONS OF THE MODEL

ESTIMATED EQUATIONS

1) $$CON = \underset{(2.67)}{4087.02} + \underset{(2.14)}{.571973}\ YD - \underset{(-3.52)}{57060.8}\ RS + \underset{(1.16)}{.375024}\ CON_{-1}$$

$\bar{R}^2 = .9956$; SER = 1079.47; DW = 1.92; h = 1.1; 1963-82

2) $$GFI = \underset{(.60)}{631.537} - \underset{(-2.55)}{18046}\ (RL - \hat{P})_{-1} + \underset{(23.0)}{.2222}\ Q$$

$\bar{R}^2 = .988$; SER = 573.39; DW = 1.63; $\rho = .43$; 1964-82

3) $$NFINV = \underset{(-3.35)}{-1184.25} + \underset{(5.23)}{.2715}\ NFQ - \underset{(-.44)}{3548.75}\ \Delta(RS - \hat{P}) + \underset{(2.39)}{.5382}\ NFINV_{-1}$$

$\bar{R}^2 = .6277$; SER = 748.0; DW = 2.02; $\rho = -.51$; 1964-82

4) $$\hat{W} = \underset{(2.59)}{.02695} - \underset{(-1.27)}{.4044}\ UR + \underset{(3.02)}{.5012}\ \hat{P} + \underset{(3.07)}{.6217}\ \hat{W}_{-1}$$

$\bar{R}^2 = .8627$; SER = .0116; DW = 1.69; h = 1.15; 1964-82

Table 2.2 Equations of the Model (continued)

5) $$\hat{P} = -.0576 + \underset{(-.27)}{}\ .3043\ \hat{W} + .3924\ \hat{PM}_{-1} + .0756\ (Q/QP)_{-1}$$
$$(-.27)\quad(-.59)\quad(1.71)\quad(.33)$$

$\bar{R}^2 = .5654$; SER = .0241; DW = 1.99; 1964-82

6) $$UR = .2325 - .2510\ (Q/QP) + .0018\ t + .3506\ UR_{-1}$$
$$(3.93)\quad(-3.73)\quad(2.54)\quad(1.43)$$

$\bar{R}^2 = .8566$; SER = .0070; DW = 1.41; $\rho = .65$; 1963-82

7) $$RS = .3102 - 4.6660\ (H/(Q \cdot P)) + .5918\ \hat{P}$$
$$(-2.65)\quad(-2.54)\quad(2.15)$$

$\bar{R}^2 = .7307$; SER = .0202; DW = 1.85; $\rho = .51$; 1963-82

8) $$RL = .0071 + .2749\ RS + .1230\ \Delta RS + .7031\ RL_{-1}$$
$$(1.51)\quad(2.90)\quad(1.27)\quad(5.21)$$

$\bar{R}^2 = .9819$; SER = .0038; DW = 1.98; h = .05; 1963-82

9) $$M = -17678.6 + .3529\ Q + 7774.01\ (P/PM)$$
$$(-2.78)\quad(42.65)\quad(1.16)$$

$\bar{R}^2 = .9895$; SER = 923.91; DW = 1.13; 1962-82

10) $$X = 35725.3 + 282.83\ IPU - 1098.95\ (PX/PU \cdot e) - 47068.8\ (Q/QP)$$
$$(2.91)\quad(8.03)\quad(-.21)\quad(3.46)$$

$\bar{R}^2 = .9801$; SER = 1092.75; DW = 1.77; 1962-82

11) $$CF/Q \cdot P = .0172 + .5961\ \Delta(RS - RSU) + .8009\ \Delta(RL-RLU) + .1759\ \Delta(GFI/Q)$$
$$(6.98)\quad(2.20)\quad(.85)\quad(.56)$$

$\bar{R}^2 = .3377$; SER = .0108; DW = 1.78; 1963-82

12) $$T = 0.3295\ (Q \cdot P)$$

$\bar{R}^2 = .99634$; SER = 2013.22; DW = 1.74; $\rho = .68$; 1962-82

IDENTITIES*

13) $$Q = CON + GFI + NFINV + FINV + G + X - M$$

14) $$YD = \frac{Q \cdot P - \delta(K \cdot P) - T + TRP + IPPD}{P}$$

15) $$K = (1 - \delta)K_{-1} + GFI_{-1}$$

16) $$\Delta R = PX \cdot X - PM \cdot M + CF + TRAN$$

* A residual term has been added to each identity (except (17) and (20)) in order that the identity hold exactly in the data.

Table 2.2 Equations of the Model (continued)

17)	$QP = e^{1.21289}K^{.35}((1-UR^{*})LF)^{.65}e^{.00724682\cdot t}$
18)	$IPPD = RL\cdot B$
19)	$\Delta H = P\cdot G + TRP + IPPD + \Delta R - T - \Delta B$
20)	$PM = e\cdot PF$
21)	$NFQ = Q - FQ$

On the whole, the empirical results for the model are quite satisfactory. All estimated parameters have the expected signs, although their significance varies over a wide range. The goodness-of-fit of each equation was not of particular concern in the present context. Our major aim was not the testing and refining of alternative hypotheses, but rather the specification of a theoretically sound and empirically reasonable model. Whenever an estimation difficulty was encountered, an attempt was made to correct for it within the framework of the hypothesis in question, instead of changing the structural equation itself.

In the estimated consumption function (1), the cost of borrowing represented by the short-term interest rate is significant and the marginal propensity to consume out of disposable income falls within the conventional range. Gross fixed investment (2) depends very significantly on the value of gross national product, and with less significance on the expected real rate of interest. The inventory equation (3) performed rather well, given its nature and the difficulty usually associated with estimating this type of equation. According to the above hypothesis, the opportunity cost represented by the short-term interest rate has not been a significant factor in determining the change in inventory holdings. On the other hand, the previous change in inventories, as well as the change in income, are more important in determining the present change of these holdings.

The wage and price equations (4) and (5), are estimated in terms of rates of change. All the explanatory variables in the wage equation are statistically significant. The large coefficient of the lagged rate of change of wages indicates some rigidity in wage change. In the price equation, however, the explanatory variables are statistically insignificant, especially the excess demand variable represented by the ratio of actual to potential output (Q/QP). The relative importance of imported inflation ($\hat{PM}$) and its positive influence on domestic prices are as expected, due to the degree of openness of the Canadian economy. Demand considerations exert a greater influence on the rate of unemployment in equation (6) along with the time trend, which reflects the change in composition of the labour force. The lagged unemployment rate would slow down the adjustment of the present rate whenever changes occurred in the other variables that affect employment conditions in the economy. This seems to be quite reasonable in the short run, when firms are expected to react cautiously to changing market conditions. One would expect them to modify the intensity of use of their work force, rather than adjusting its size, and thereby avoiding the possibility of high costs associated with hiring, firing and training. Thus, the positive coefficient of the lagged rate of unemployment is expected.

The equations for short and long-term interest rates, equations (7) and (8), have fairly good fits. The high significance of the lagged dependent variable in (8) indicates a slow adjustment of the long-term rate. But in a long-run situation, where changes in both rates are set equal to zero, the long-term rate of interest would be almost identical to the short-term rate, a desirable steady-state property.

A common feature of the equations for imports (9) and exports (10) is the insignificance of the relative price term, although this is particularly evident in the case of the export equation. The price elasticities of foreign demand for Canadian exports and of Canada's demand for imports are about -0.006 and -0.3, respectively. These results contradict previous Canadian estimates published in the past two decades, using similar structural equations[10]. However, the activity variables in both equations proved to be highly significant. In addition, the excess demand variable in the export equation has a negative sign which, as mentioned earlier, can be attributed to the negative relationship between the capacity utilization rate and the availability of output for export purposes.

The overall fit of the capital inflow equation (11) is poor, which is understandable because of the wide fluctuations and the rapid movement of capital between Canada and the United States.

Finally, total taxes are expressed as a simple function of nominal GNP in equation (12). The average rate was found by forcing the regression line through the origin, and was estimated to be 0.32. This rate, as well as the intercept in the equation (referred to as surtax), are assumed to be policy parameters, and either one of them can be considered to be a control variable in a policy framework.

2.4 CONCLUDING REMARKS

In this chapter, a small annual macroeconomic model for Canada was constructed and estimated over the period 1962-1982. The aim was to construct the simplest model that could be used as a vehicle for the examination of short-run and long-run economic policies in the Canadian context. This model has not been wedded to either the Keynesian or the neoclassical school of economic analysis, but contains elements that traditionally have been attributed to both schools. Moreover, the model should not be compared to the much more detailed and disaggregated models already in existence for the simple reason that we lack both the resources and time needed to construct a large scale comprehensive model. It is nonetheless of importance to note that 'small' does not necessarily imply 'bad', especially when the purpose of the model does not require a high degree of disaggregation. Furthermore, in the case of a small model, it is easier to highlight the major macro relationships and to interpret the results of simulations and optimization.

The problem of simultaneity involved in the model has been dealt with in estimation by using a two-stage least squares principal components method. The computed set of principal components that were used in the first stage explained 95 percent of the total variation among the predetermined variables. The estimated results were, on the whole, satisfactory with only a few exceptions. As we shall see in the next chapter, those equations that are characterized by relatively poor statistical fit turn out to perform rather well as components of the complete system in the simulation experiments.

FOOTNOTES

Chapter 2

1. These include such models as RDX2, CANDIDE, QFM and TRACE. See DeBever et. al. [1979].

2. Alternatively, and equivalently, the change in foreign exchange reserves could be taken as the policy instrument associated with the foreign exchange market. Given all the other variables which impinge on the balance of payments, the choice of a level for the exchange rate implies a value for the change in foreign exchange reserves, and the choice of a value for the change in foreign exchange reserves implies a level for the exchange rate.

3. All output and expenditure variables are defined in real terms, while variables that represent financial holdings are valued in money terms.

4. A priori one might expect the value of α to be between 0.2 and 0.4. Thus a search procedure that minimized the sum of squared residuals was applied over this range and the values of α and $\beta = 1 - \alpha$ were chosen accordingly.

5. Since the national accounts measures of imports and exports include interest and dividend payments abroad, interest rates could be included in both the import and export equations. In future work, this component of the balance of payments will be treated separately.

6. See Artus [1970] for the short term effect of domestic demand pressure on British exports.

7. See Scarth [1973] for details in the Canadian context.

8. Durbin [1970] has shown that the h statistic is approximately normally distributed with unit variance, and the test for first-order serial correlation can be done directly by using the normal distribution table which shows a critical value of 1.645 for a 5 percent level of significance.

9. H and B cannot both be endogenous or exogenous at the same time. The endogeneity of one implies the exogeneity of the other, by virtue of the government constraint.

10. Rhomberg [1964] obtained export and import price elasticities of approximately -2 and -1, while in Houthhaker and Magee (1969) these elasticities were equal to -0.59 and -1.46. More recently, Mathieu (1976) estimated an export price elasticity of -0.65 and an import price elasticity of -1.25. The import price elasticity was also estimated by Yadav [1977] and was equal to -1.37. The much lower values implied by our estimates are almost certainly due to the different estimation period.

APPENDIX 2.A

DATA

Sources of Data:

BCR Bank of Canada Review

EEH Employment, Earnings and Hours (Statistics Canada, Cat. 72-002).

GFC Gross Fixed Capital Stocks and Flows (Statistics Canada, Cat. 13-568).

LF The Labour Force (Statistics Canada, Cat. 71-001).

NIEA National Income and Expenditure Accounts (Statistics Canada, Cat. 13-201).

SCB Survey of Current Business (U.S. Department of Commerce/Bureau of Economic Analysis).

DEFINITION OF SERIES

B Total Bonds.
Government of Canada direct and guaranteed securities and loans.
Source: BCR.

CF Net Capital Inflow.
This series is the sum of long-term and short-term capital inflows. Source: BCR.

CON Real Consumption.
Personal expenditures on goods and services in 1971 constant prices. Source: NIEA.

e Exchange Rate.
This is the price of the US dollar in terms of Canadian dollars.
Source: BCR.

FINV Real Farm Inventories.
Value of physical change in farm inventories in 1971 constant prices. Source: NIEA.

G Real Government Expenditures.
Government current expenditure on goods and services in 1971 constant prices. Source: NIEA.

GFI Real Investment.
Total gross fixed capital formation in 1971 constant prices.
Source: NIEA.

H High Powered Money.
This series is the sum of currency and chartered bank deposits at the central bank. Source: BCR.

IPPD Interest on the Public Debt.
Source: NIEA.

IPU United States Industrial Production Index. 1967 = 100.0.
Source: SCB.

K Real Capital Stock.
Total capital stock at the beginning of the year in 1971 constant prices. Source: See Appendix 2.B.

LF Labour Force.
Total labour force. Source: LF.

M Real Imports.
Imports of goods and services in 1971 constant prices. Source: NIEA.

NFINV Real Non-Farm Inventories.
Value of physical change in non-farm inventories in 1971 constant prices. Source: NIEA.

NFQ Real Non-Farm GNP.
Gross national product less farm income in 1971 constant prices. Source: NIEA.

P Price Level.
Gross national product implicit price deflator, 1971 = 100.00. Source: NIEA.

PF Foreign Price Level.
This is derived by dividing the foreign price index by the exchange rate. Source: NIEA.

PM Price of Imports.
Implicit price index for imports of goods and services. 1971 = 100.00. Source: NIEA.

PU U.S. Price Level.
The United States gross national product implicit price index. 1972 = 100.00. Source: SCB.

Q Real GNP.
Gross national product in 1971 constant prices. Source: NIEA.

QP Real Potential Output.
Potential gross national product in 1971 constant prices. Source: See Chapter 2.

R	Official International Reserves. Canada's official international reserves in Canadian dollars. Source: BCR.
RL	Long-Term Interest Rate. Average yield on long-term Canadian bonds. Source: BCR.
RS	Short-Term Interest Rate. Average yield on 91-day treasury bills. Source: BCR.
RLU	U.S. Long-Term Interest Rate. Aaa rate on domestic corporate bonds (Moody's). Source: SCB.
RSU	U.S. Short-Term Interest Rate. Rate paid on 3-month government securities. Source: SCB.
t	Time Trend. Time; 1962 is 1, and 1963 is 2, etc. Source: SCB.
T	Total Taxes. Source: NIEA.
TRP	Transfer Payments to Persons. Source: NIEA.
TRAN	Current Account Net Transfers. Source: BCR.
UR	Unemployment Rate. Annual average unemployment rate. Source: LF.
UR*	'Full-Employment' Unemployment Rate. Source: This was generated by connecting the two trough points which occurred in 1966 and 1974, and extrapolating for the rest of the period.
W	Wage Rate. This is an hourly wage rate in the manufacturing sector. Source: EEH.
X	Real Exports. Exports of goods and services in 1971 constant prices. Source: NIEA.
YD	Real Disposable Income. Disposable income at current prices deflated by the GNP price deflator, P. Source: NIEA.
NOTE:	Stock series are measured as averages over the year, except for the capital stock, which is measured at the beginning of the year.

APPENDIX 2.B

CAPTIAL STOCK

The available capital stock time series, which is published in "Gross Capital Stocks and Flows" (Statistics Canada, Cat. 13-568) excludes the housing sector. This would have created an inconsistency with the output measure had we not created our own capital stock series, because GNP includes imputed residential rent and other rents. In order to generate a capital stock for housing, a benchmark was created for the year 1926 by assuming that the ratio of gross investment in residential construction to that of nonresidential construction over the period 1926-1930 was equal to the corresponding ratio of their capital stocks at the beginning of 1926. That is,

$$\left(\frac{\overline{IR}}{IB}\right)_{1926-30} = \left(\frac{KR}{KB}\right)_{1926}$$

where IR, KR and IB, KB are gross investment and capital stock for residential and nonresidential construction, respectively. Given this assumption, we could solve for KR(1926), and then accumulate the series on housing investment over time, assuming a 3 percent depreciation rate. The capital stock for the rest of the economy was computed by taking a benchmark from the published stock data and then accumulating over time, assuming a 4.7 percent depreciation rate, which was found to be consistent with the published data. Having obtained both series, we combined them and an average depreciation rate was found by regressing (K - GFI(-1)) on K(-1), since K = (1 - S(K-1) + GFI(-1)). The estimated overall depreciation rate is 0.042675.

CHAPTER 3

SIMULATIONS WITH THE MODEL

3.1 INTRODUCTION

In the preceding chapter, the structural macro model was estimated by a single equation method, and the individual performance of each equation was evaluated by referring to various statistics ($\bar{R}^2$,t-ratios, etc.), and by examining the signs and magnitudes of the estimated coefficients. This, however, does not necessarily imply that the model as a whole has the same properties as the individual equations of which it is composed. In other words, there is no one-to-one correspondence between the performance of the model as a whole and of its individual components, and it may well be the case that an equation with very poor statistical fit performs quite well as a part of a complete system. The converse may be also true; the individual equations may have a good statistical fit, but the model as a whole may not closely replicate the historical data over the period for which the model was estimated. An obvious reason is that the dynamic structure of the model is more complex with a high degree of simultaneity, and this can render the characteristics of the whole different from those of its parts. It seems that the above argument can be inuitively stretched to the extent where an analogy can be drawn between this question and the controversial question of the microfoundation of macroeconomics. That is, there may exist a case in which each equation of a given macroeconomic model had been carefully derived from a well defined maximization problem, but once these equations are treated as a system the model might not make sense on the macro level. This may be nothing more than the materialization of the principle which says that the whole is more than the simple sum of its parts.

Thus, however well single equations may explain the behaviour of individual components of aggregate economic activity, the ultimate test of a structural model remains the fit of all equations, taken together in simulation. There are various types of simulations which can be performed depending on the objective for which these simulation experiments are conducted.[1] Historical simulations, for example, are usually carried out in order to test the validity and realism of the model by comparing the original data series with the simulated series of the endogenous variables. This type of ex-post simulation has also proven to be useful in policy analysis. The values of certain parameters or policy instruments can be changed for the historical period or a part of it, in order to examine the response that might have taken place, of some key endogenous variables. Forecasting, on the other hand, involves simulating the model forward in time beyond the historical period in order to study the predictive power of the model conditional on a particular set of assumptions about the exogenous variables. Since the primary concern of this study is related to the analysis of short and long-run economic policies, the type of simulation that is most appropiate in this context is policy simulation. First, it can shed some light on the stability properties of the model, especially when the long run is involved, and secondly it allows us to examine the short-run response of the model to some exogenous shocks. But before doing that, one should test the validity and realism of the model and its capability of reproducing the actual data. In other words, the model is supposed to be a plausible description of the economy subject to investigation in order for any result to be useful and relevant. In the next section, historical or ex-post simulation is conducted to evaluate and test the performance of the model over the period for which the model was estimated. In the subsequent section, some policy simulation experiments are carried out in order to examine the short-run and long-run responses of endogenous variables to exogenous shocks as well as the stability properties of the model around its steady-state.

3.2 HISTORICAL SIMULATION

Two types of ex-post simulation were performed in order to evaluate the model's ability to replicate the actual data. The first is static simulation, which is equivalent to simulating endogenous variables for each observation in the sample period, with all predetermined variables taking on their actual values. The second simulation is dynamic, in the sense that the values of lagged endogenous variables for the first period are the observed values but for all subsequent periods the lagged endogenous variables have the values simulated by the model for the previous period[2]. In this simulation process, errors tend to be larger than those of the single period (static) case and this may lead to poorer tracking in later years. But at the same time, a dynamic simulation provides a reasonable test for the dynamic structure of the model and of the feedbacks within the system. Static simulation, however, is useful in examining the ability of the model to match the behaviour of the endogenous variables in the short run or on a year-to-year basis when annual data is used, as is the case here. In other words, the major difference between long-run (dynamic) and short-run (static) simulation is in the treatment of initial conditions. The short-run extrapolation is re-initialized every year and lagged values of endogenous variable are set at observed levels. For the long-run simulation, lagged values are set initially at levels prevailing before the start of simulation and are generated by the model thereafter. The long-run simulation (dynamic) was conducted over two time-periods. One coincides with the estimation period (1962-1982), and the other starts in 1973 and ends in 1982. The choice of the second time interval was more or less arbitrary, its purpose being to test the sensitivity of the model to the choice of starting point for simulation. If the model accurately represents the underlying structure of the economy, it should not matter much in what year the simulation is begun; the results should not be drastically different. A second purpose for carrying out the simulations over two different time periods is to compare the ability of the model to track the response to different types and different severities of shocks. In particular, the 1973-1982 period was characterized by more severe shocks of a different nature than those during the earlier sub-period.

The results of these historical simulation experiments for the two-time periods are summarized in Tables 3.1 and 3.2. Since we are interested in examining how closely each endogenous variable tracks its corresponding historical time series, two oftenly used measures, the RMS (Root-Mean-Square) and RMSP (Root-Mean-Square-Proportionate) simulation errors are reported in the two tables for the 18 endogenous variables of the model along with the mean value of each variable. The RMS and RMSP simulation errors for a variable Y are defined as

$$\text{RMS} = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(Y_t^s - Y_t^a)^2}$$

$$RSMP = \sqrt{\frac{1}{N} \sum_{t=1}^{N} \left(\frac{Y_t^s - Y_t^a}{Y_t^a}\right)^2}$$

where Y_t^s = Simulated value of Y_t

Y_t^a = Actual value of Y_t

N = Sample size

RMS can be interpreted only by comparing it with the average size of the variable in question. The smaller the values of RMSP and RMS are, the better is the fit of individual variables, and hence the tracking ability of the model. In addition, actual and simulated values for the selected endogenous variables are plotted in Figures 3.1 to 3.13, for the simulation that covers the entire estimation period.

The tracking record of major endogenous variables in the dynamic simulation is remarkably good, given the relative simplicity and high degree of aggregation that characterize the model. Of the 18 endogenous variables reported in Table 3.1, only 6 have RMSP simulation errors of more than 20%. For half the varibles (9 out of 18) these errors are less than 10%. As expected, the RMSP simulation errors are somewhat smaller in the static simulation. They are more than 20% for 5 variables and less than 10% for 10 variables. The differences however

TABLE 3.1 RESULTS OF HISTORICAL SIMULATIONS (1964-1982)

Variable	Mean	DYNAMIC RMS	DYNAMIC RMSP	STATIC RMS	STATIC RMSP
Q	99199.2	2429.6	0.02	2274.5	0.02
CON	60475.9	2751.3	0.05	1588.3	0.03
GFI	22294.0	803.6	0.03	680.5	0.03
NFINV	531.1	833.47	2.11	1336.87	3.29
M	24640.4	1061.5	0.04	749.6	0.04
X	22401.3	1867.6	0.09	1615.8	0.08
RS	0.0727	0.01	0.17	0.01	0.13
RL	0.0824	0.01	0.11	0.01	0.06
UR	0.0618	0.01	0.17	0.01	0.11
$\hat{P}$	0.0652	0.02	0.30	0.02	0.30
$\hat{W}$	0.0847	0.02	0.25	0.01	0.17
T	46023.3	2613.5	0.09	2885.4	0.09
CF	2870.3	4286.3	5.4	3592.9	2.76
ΔR	88.0	2204.9	30.9	2627.67	29.5
YD	67737.5	3379.6	0.06	3229.5	0.06
QP	102625.0	2234.5	0.02	1573.4	0.01
K	245889.0	4784.9	0.02	0.06	0.001
ΔH	666.4	637.72	1.25	789.3	1.05

TABLE 3.2 RESULTS OF HISTORICAL SIMULATIONS (1973-1982)

Variable	Mean	DYNAMIC RMS	DYNAMIC RMSP	STATIC RMS	STATIC RMSP
Q	122408	1320.5	0.01	2115.4	0.02
CON	76177.4	1789.6	0.02	987.3	0.02
GFI	27781.7	607.8	0.02	615.5	0.02
NFINV	343.7	757.0	1.68	1560.3	3.79
M	32992.2	1038.6	0.03	1350.6	0.04
X	29105.6	1889.5	0.07	1567.9	0.05
RS	0.101	0.01	0.15	0.01	0.15
RL	0.104	0.01	0.10	0.01	0.07
UR	0.075	0.01	0.15	0.01	0.11
$\hat{P}$	0.100	0.03	0.32	0.02	0.31
$\hat{W}$	0.112	0.03	0.32	0.01	0.21
T	74320.9	2813.0	0.05	2885.4	0.05
CF	4908.7	1930.5	0.84	3592.9	3.75
ΔR	-131.12	1568.8	13.8	2627.6	14.4
YD	87652.9	1511.0	0.02	3836.5	0.03
QP	126818	1664.9	0.01	1573.1	0.01
K	312043	1726.3	0.01	0.06	0.001
ΔH	1048.2	517.2	0.39	789.3	1.31

are not pronounced, and this may be attributed to the simple structure of the model. Also, a comparison between Tables 3.1 and 3.2 proves that the model is not very sensitive to the period of simulation since the results do not differ drastically when the period is changed. Generally, the results show that small simulation errors are associated with the real variables, whereas large errors are associated with nominal, financial and price variables. Furthermore, a number of variables have simulation errors that are very large, most notably, inventories, capital flow, change in international reserves and high-powered money. This, however, was not surprising given the volatile nature of these variables.

The historical simulation results can be investigated more thoroughly by examining the plots of some selected variables provided in Figures 3.1-3.13. As mentioned above, the expenditure side of the model performs very well and the simulated gross national product and its components follow closely their historical values. Nonetheless, one can notice some difference in the performance of each variable. For example, GNP and consumption are below their historical values up to 1974 but are almost identical thereafter. On the other hand, investment reproduces closely its historical trend and the change in inventories reproduces most of the turning points which is unexpected in view of the difficulty surrounding the specification and estimation of an equation for this variable. In the case of the external sector,

FIGURE 3.1 HISTORICAL SIMULATION OF GNP

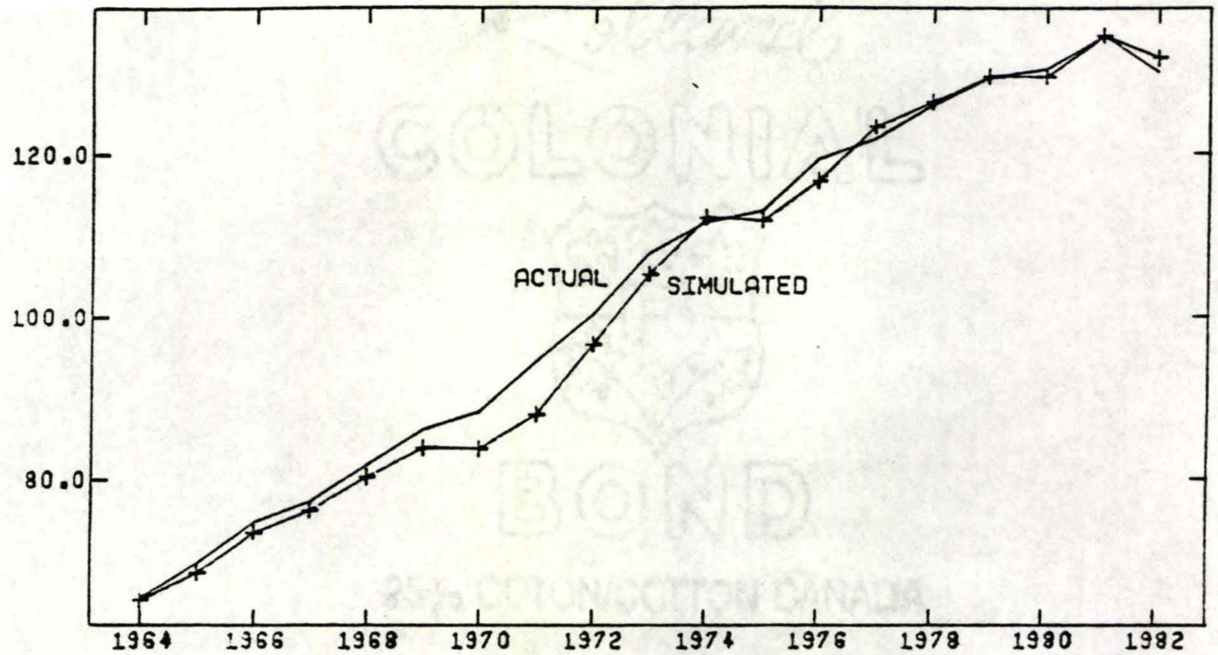


FIGURE 3.2 HISTORICAL SIMULATION OF CONSUMPTION

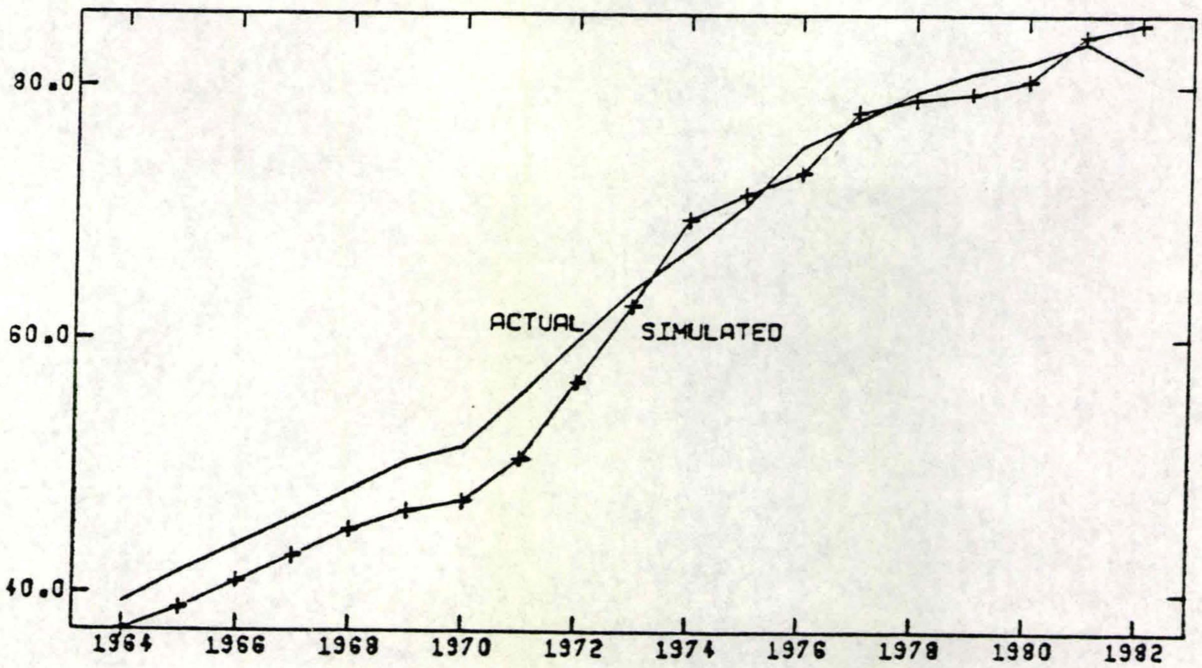


FIGURE 3.3 HISTORICAL SIMULATION OF INVESTMENT

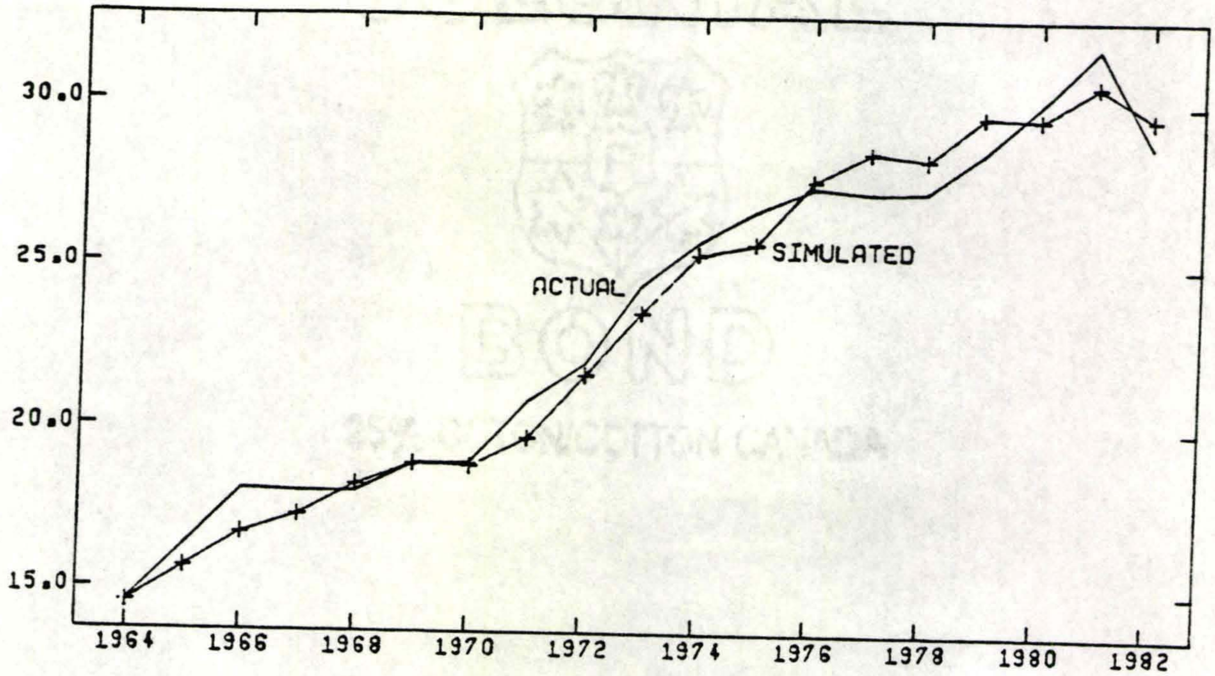


FIGURE 3.4 HISTORICAL SIMULATION OF NON-FARM INVENTORIES

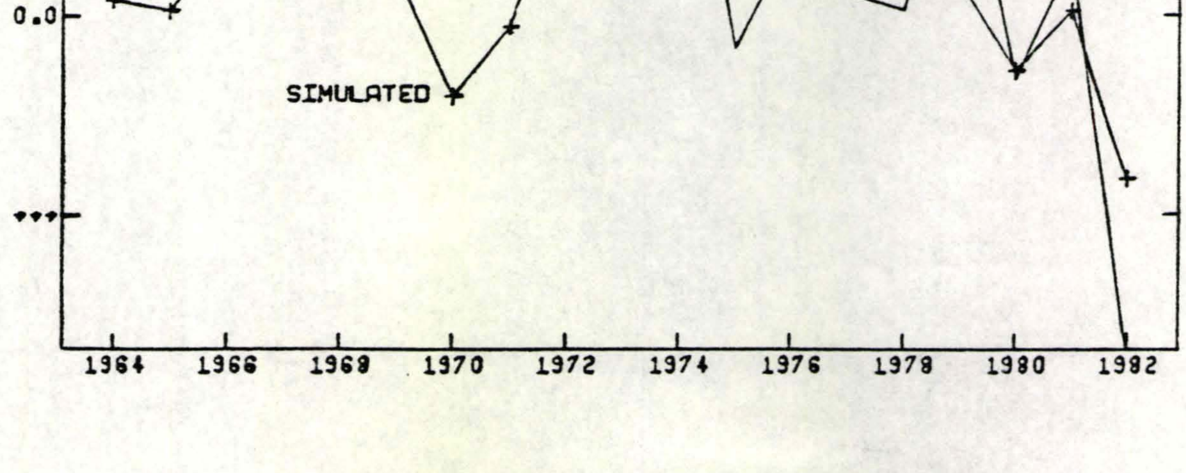


FIGURE 3.5 HISTORICAL SIMULATION OF IMPORTS

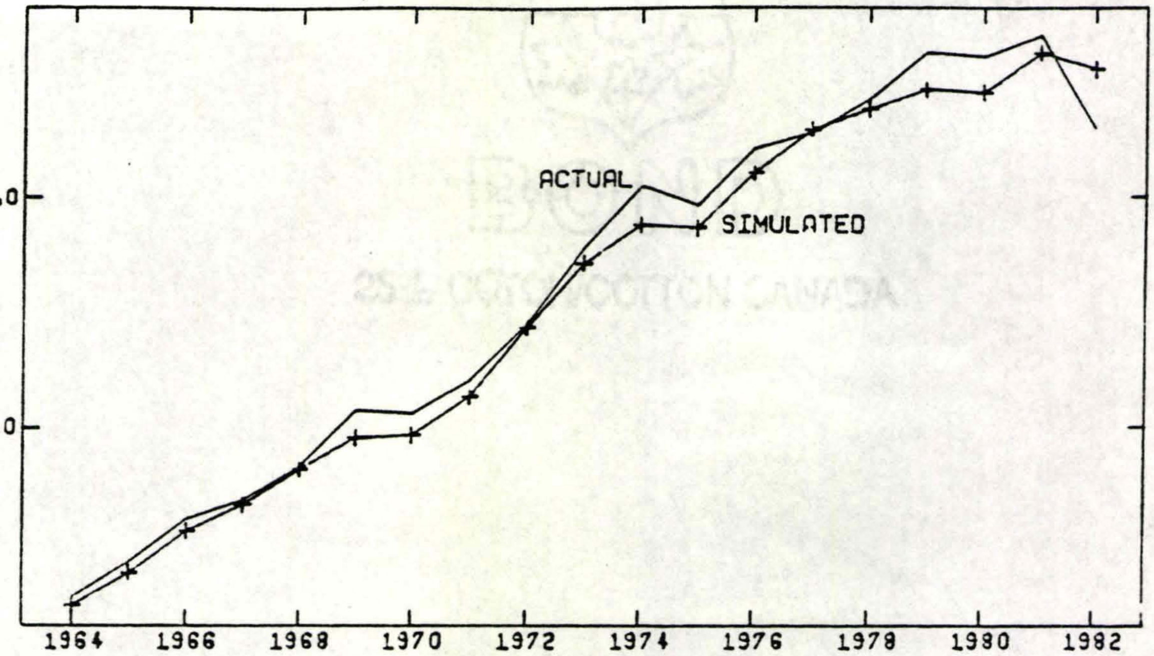


FIGURE 3.6 HISTORICAL SIMULATION OF EXPORTS

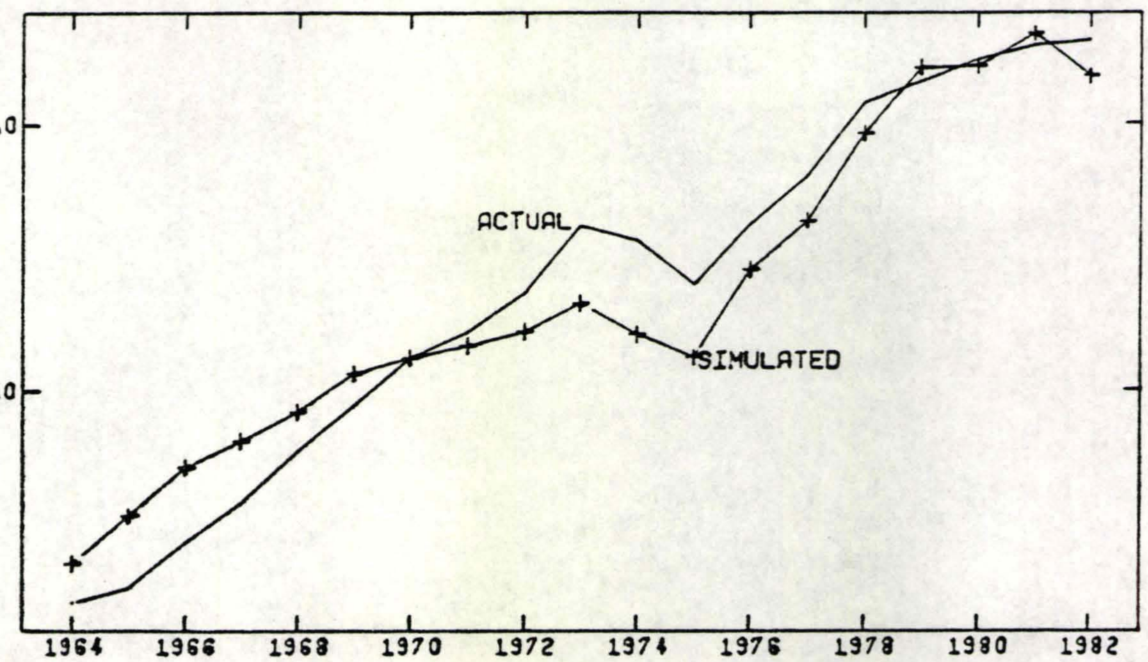


FIGURE 3.7 HISTORICAL SIMULATION OF SHORT-TERM INTEREST RATE

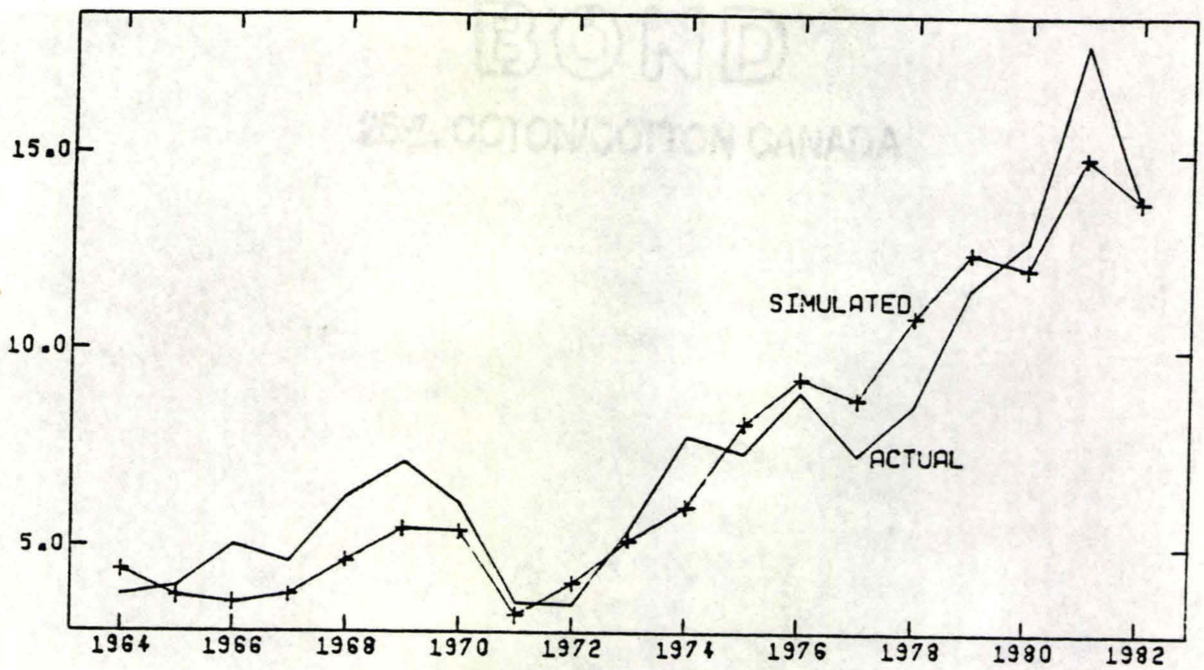


FIGURE 3.8 HISTORICAL SIMULATION OF LONG-TERM INTEREST RATE

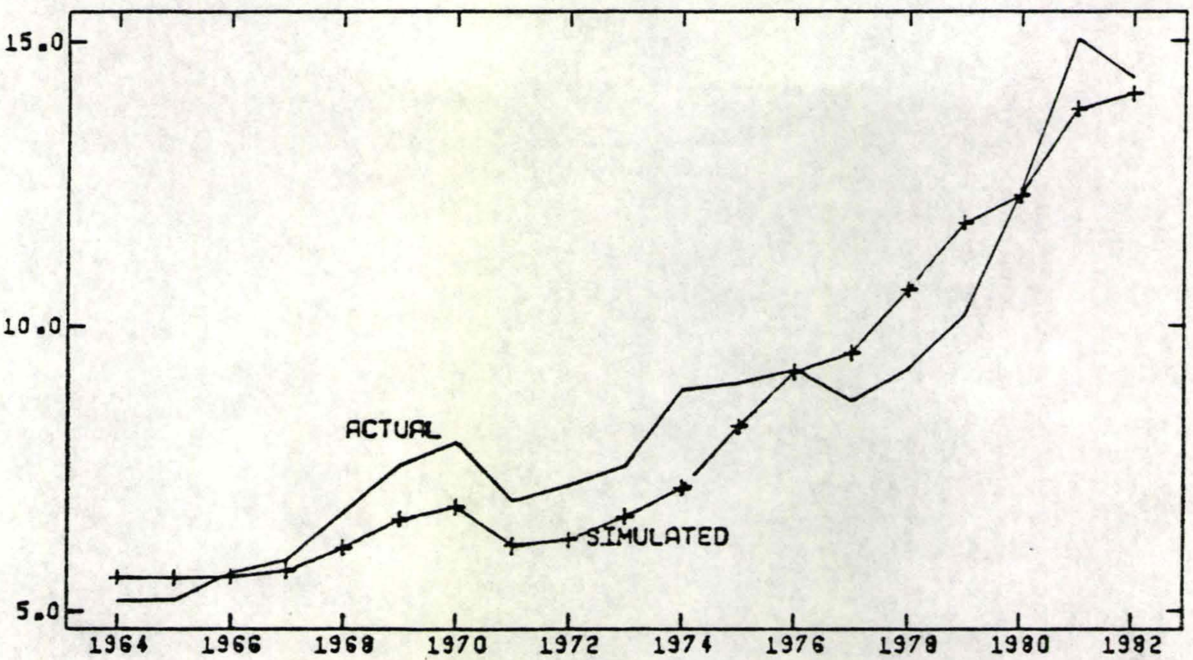


FIGURE 3.9 HISTORICAL SIMULATION OF UNEMPLOYMENT RATE

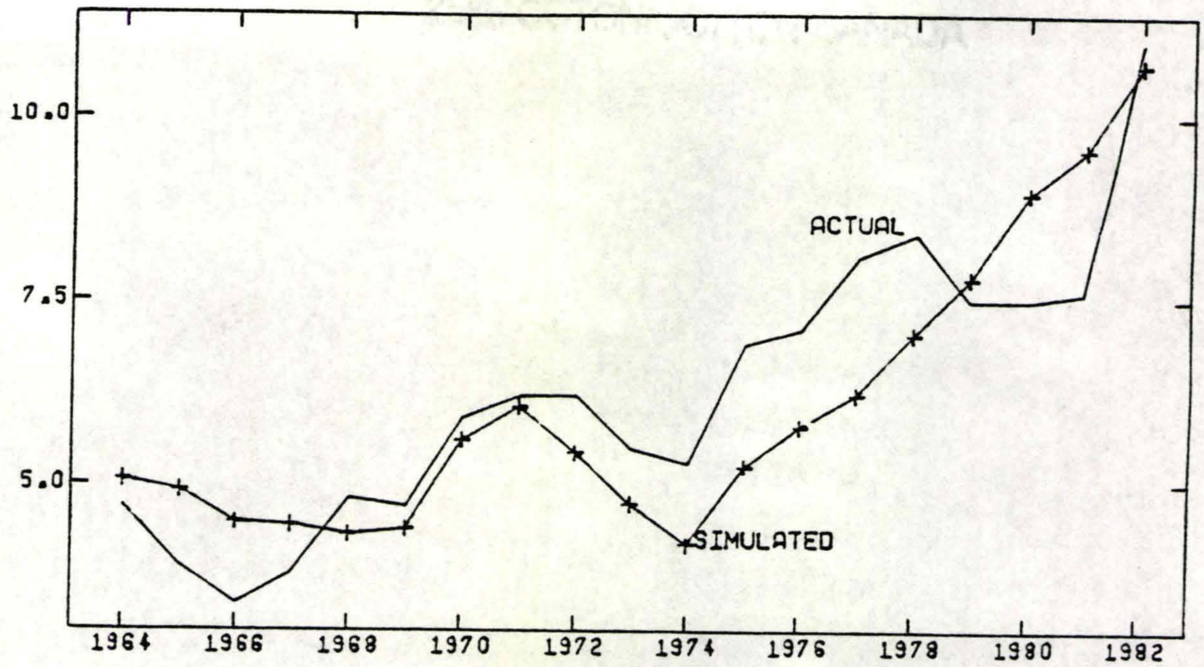


FIGURE 3.10 HISTORICAL SIMULATION OF ANNUAL RATE OF GROWTH OF PRICE LEVEL

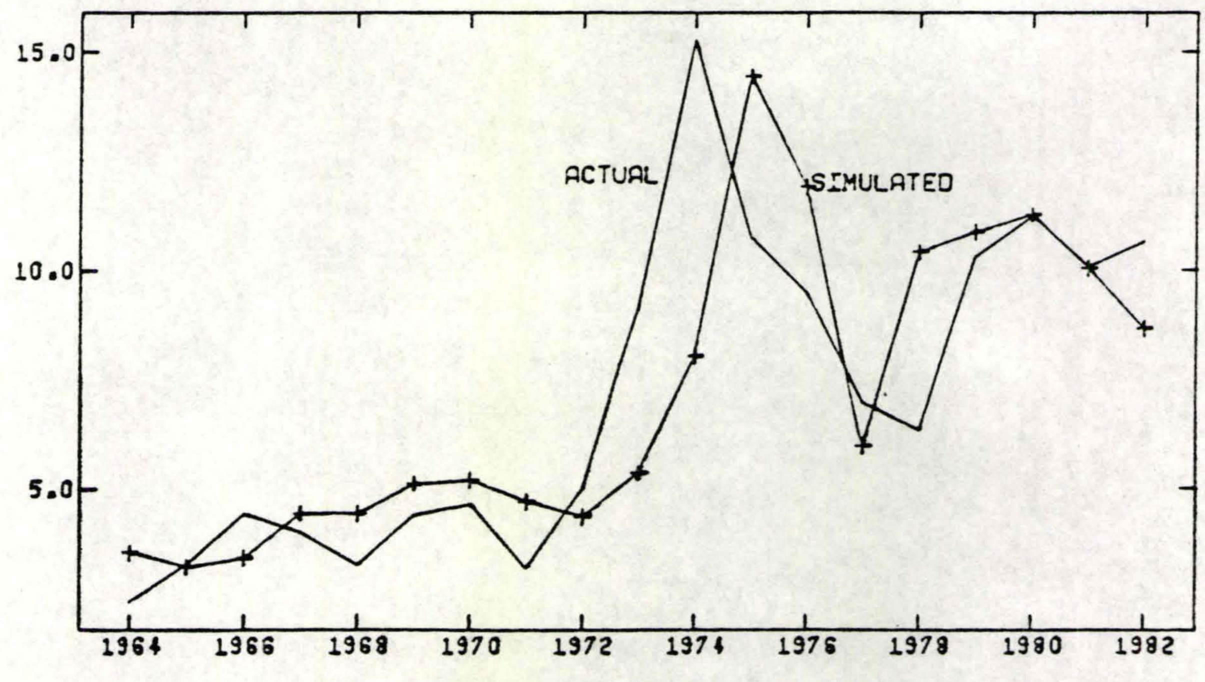


FIGURE 3.11 HISTORICAL SIMULATION OF ANNUAL RATE OF GROWTH OF WAGE RATE

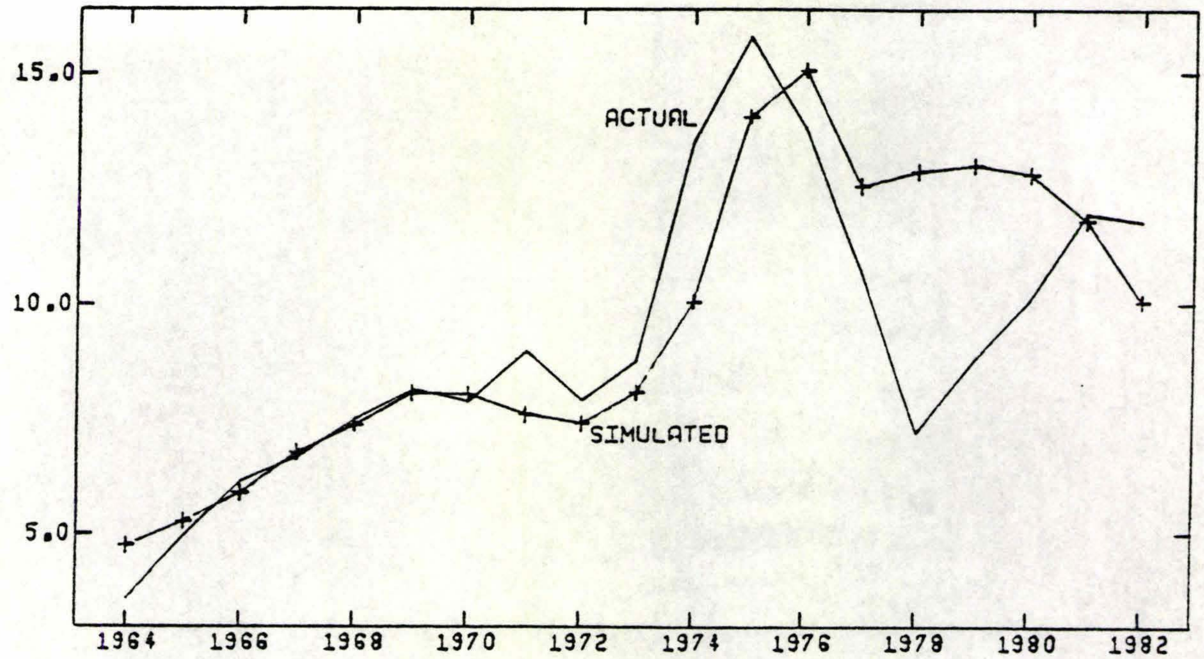


FIGURE 3.12 HISTORICAL SIMULATION OF TAX REVENUE

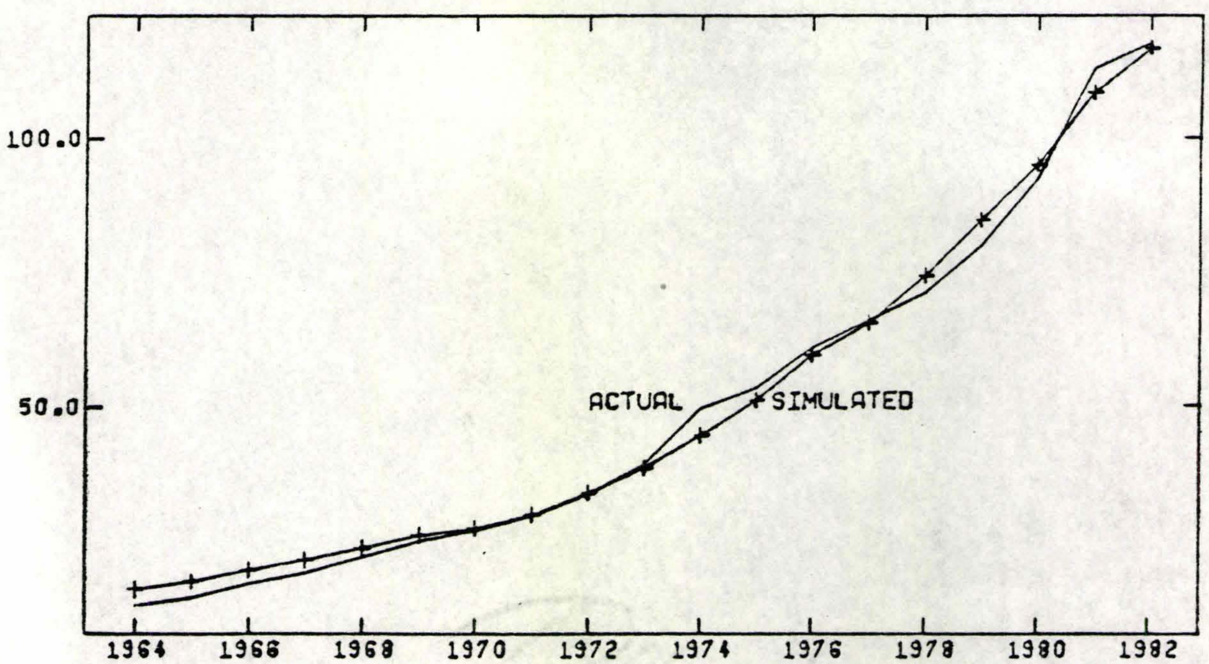
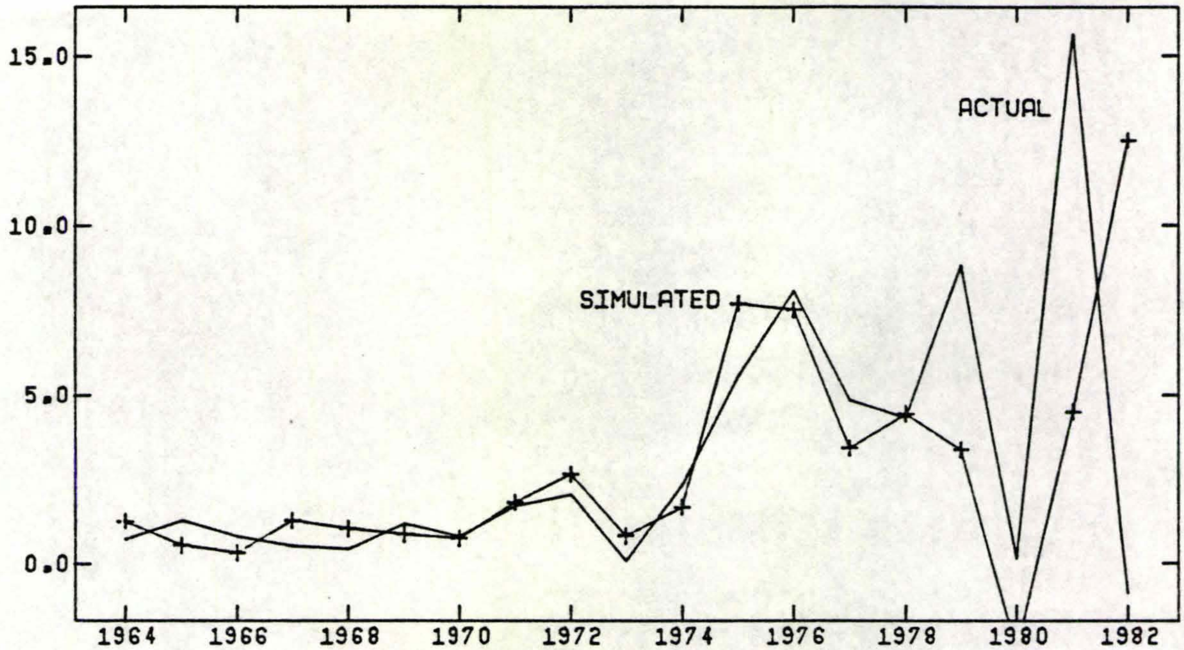


FIGURE 3.13 HISTORICAL SIMULATION OF CAPITAL INFLOW



the simulation of imports tend to underestimate the actual time series but does capture the turning points over the simulation period. The simulation of exports is less satisfactory but does reproduce the major cycle that occurred in 1973-1976. Our simulation of capital inflow succeeds in picking up most of the turning points with a short lag, which is quite satisfactory, given the extreme volatility of this variable over the 1975-1982 sub-period. Further, this shows up -- since the fit of this equation was poor -- that the performance of an equation as a part of a complete system may not be the same as its performance as an individual equation.

The monetary sector, represented by the short and long-term interest rates, reproduces the general historical trend of these variables, though it does not follow the year-to-year movements very well. In the case of the wage-price-unemployment sector, the simulated unemployment rate smoothes its historical series while the simulation of the rate of change of prices captures the major swing that occurred in the 1972-77 period, with a one-period lag. The rate of change of the wage rate follows closely the historical path at the beginning of the simulation but diverges considerably from it during the last five years.

In concluding this section, it can be asserted that the performance of the model in historical simulations was satisfactory since we were able to replicate the historical behaviour of the key endogenous variables fairly closely. Thus the model can be used for policy analysis with some confidence.

3.3 THE DYNAMICS OF THE MODEL

In the previous section, the macromodel proved to be able to reproduce closely the general trend of the actual performance of the Canadian economy during the 1962-1982 period. Having established this, we can now proceed to study the dynamic behaviour of the model. As indicated earlier, the model was designed for policy analysis in the short and the long term. For such analysis to be valid, the model must exhibit plausible short-run and long-run behaviour. In the long run, it has been argued, economic variables bear fairly stable relationships to one another. The broad movements of these variables and their regularities (stylized facts) suggest that a steady-state situation exists toward which economies tend in the long run. If this is the case, then a properly specified model should have a plausible steady-state solution, even if the model is built for short term analysis[3]. In what follows, a steady-state solution of the model will be derived and then the stability of the model around this steady-state will be tested by simulation.

3.3.1 THE STEADY-STATE SOLUTION

It should be noted at the outset that the state we call "steady state" is a device that has been often used for analytical convenience and the assumptions on which it is based are, most of the time, unrealistic. But no matter how unrealistic this state is, it can be extremely useful when used as a point of reference when analyzing the dynamic properties of a system.

The model expressed in the previous chapter can be represented by $DY(t) = f[Y(t), X(t), \theta]$, where D is a difference operator, $Y(t)$ is the vector of endogenous variables, $X(t)$ the vector of exogenous variables and θ is the set of structural parameters. This system of difference equations is said to have a steady-state solution if a particular solution $Y_i(t) = Y_i^* e^{\rho_i t}$ exists for all variables i, when each exogenous variable grows at a constant rate, that is, $X_i(t) = X_i^* e^{\lambda_i t}$. X^* and Y^* are the steady-state levels of the exogenous and endogenous variables, respectively, at

$t = 0$, and λ_i and ρ_i are the corresponding growth rates [Knight and Wymer, 1978].

The steady-state solution that we are going to derive is a special case of the above particular solution. It is a zero-growth steady state where ρ_i and λ_i are set equal to zero, that is, every variable of the model stays constant (a stationary state). This simplistic assumption may not be plausible when the focus is mainly on the steady-state properties of the model and the values assumed by the variables of the model at this state. But for the purpose of stability analysis, which is of major concern here, (especially when dealing with long-term policies), the above assumption is quite appropriate. This is so because, as shown in the next section, by displacing the model from the steady state it will tend to return to this state only if it is stable.

Thus the assumptions underlying the present steady-state solution are: $Y_i(t) = Y_i(t-n)$ and $X_i(t) = X_i(t-n)$ for every variable i of the model and for every time period n. In addition, the steady-state values of the unemployment rate UR^* and that of the capacity utilization ratio $(Q/QP)^*$ are those values at which wages and prices remain constant. This implies, from equations (4) and (5) in Table 2.2, that $UR^* = 0.066$ and $(Q/QP)^* = 0.76$. The existence of zero-growth steady-state solution may call for the imposition of restrictions on the structural coefficients in order to avoid any inconsistency. In the present case, the steady-state values of UR and (Q/QP), along with the assumption of no technological change, imply a slight change in the intercept of the unemployment rate equation (from 0.2325 to 0.2634).

Based on the above assumptions, the empirical model presented in Chapter 2 (Table 2.2) is expressed in its steady-state form as in Table 3.3 below.

The steady-state values of all endogenous and exogenous variables, when the above system is solved for a particular year (1973) by simulation, are presented in the appendix. However, it should be stressed that to justify the reasonableness of these values is not an issue since they are not used for policy analysis but rather only as a point of reference against which the stability of the model can be tested[4].

3.3.2 POLICY SIMULATIONS AND STABILITY ANALYSIS

The stability properties of the model can be analyzed by using the steady-state solution of the previous section as initial conditions. Since the model is nonlinear, it could be argued that these properties should be evaluated by simulation[5]. This method consists of displacing the model from its steady-state by means of shocks, i.e., changing the values of some exogenous variables or parameters for one period (or more), and then examining the path of the endogenous variables to see whether or not they exhibit stable or explosive behaviour. If, after the shock, these variables have a tendency to go back to their initial steady-state values, then one can conclude that the model is stable. Generally this is an indication

TABLE 3.3: THE STEADY-STATE EQUATIONS OF THE MODEL

Estimated Equations

(1) $CON = 6539.48 + 0.9152\ YD - 91300.78\ RS$

(2) $GFI = 631.53 - 18046\ RL + 0.2222\ Q$

(3) $RS = 0.3102 - 4.666\ (H/(Q.P))$

(4) $RL = 0.0240 + 0.9259\ RS$

(5) $M = -17678.6 + 0.3529\ Q + 7774.01\ (P/PM)$

(6) $X = -104.88 + 282.83\ IPU - 1098.95\ (PX/PU.e))$

(7) $CF = 0.01716\ (Q.P)$

(8) $T = 0.32957\ (Q.P)$

Identities

(9) $Q = CON + GFI + FINV - 2564.45 + G + X - M$

(10) $$YD = \frac{(Q.P - 0.042675\ (K.P) - T + TRP + IPPD)}{P}$$

(11) $K = 23.43\ GFI$

(12) $$PM = \left(\frac{PX.X + CF + TRAN}{M}\right)$$

(13) $QP = 1.3136\ Q$

(14) $$P = \left(\frac{T - TRP - IPPD}{G}\right)$$

(15) $IPPD = RL.B$

(16) $$e = \frac{PM}{PF}$$

of local stability, in the sense that the dynamic behaviour of the model is stable in the neighbourhood of the initial condition determined by the steady-state solution[6]. Nonetheless, this can be still informative about the dynamic behaviour of the model. The shocks used to disturb the model are mostly policy instrument shocks. This choice was made in order to shed some light on the effects of these instruments on selected target variables, which is the subject matter of later chapters. Many simulation experiments have been carried out in order to assess the stability properties of the model. However, we have opted to discuss the results of only three particular shocks, as shown in Figures 3.14-3.55. The first is a money-financed government expenditure shock, the second is a bond-financed government expenditure shock, and the third is an exchange rate shock.

A MONEY-FINANCED GOVERNMENT EXPENDITURE SHOCK. In this experiment, government spending on goods and services was increased by 20 percent in the first year and then reduced to its original level thereafter. The results are shown graphically in Figures 3.14 to 3.27, where the straight line indicates the steady-state value of the variable in question and the line of X's shows the response to the shock for a 48-year time period.

The increase in government spending on goods and services has, as expected, an expansionary impact or first-year effect on GNP. The increase in GNP that results reduces the unemployment rate (Figure 3.22). This boosts the rate of increase of the wage rate and the rate of inflation, which is even higher in the second year due to the lagged effect of GNP

FIGURE 3.14 EFFECT OF A MONEY-FINANCED GOVERNMENT EXPENDITURE SHOCK ON GNP

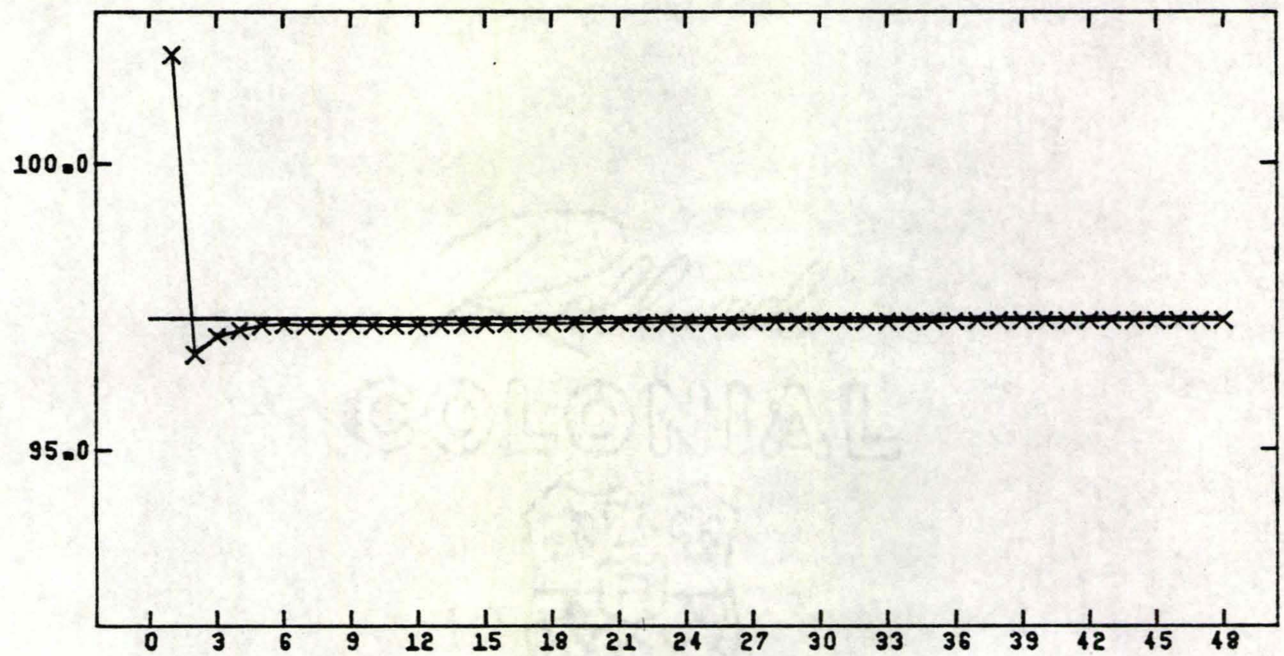


FIGURE 3.15 EFFECT OF A MONEY-FINANCED GOVERNMENT EXPENDITURE SHOCK ON CONSUMPTION

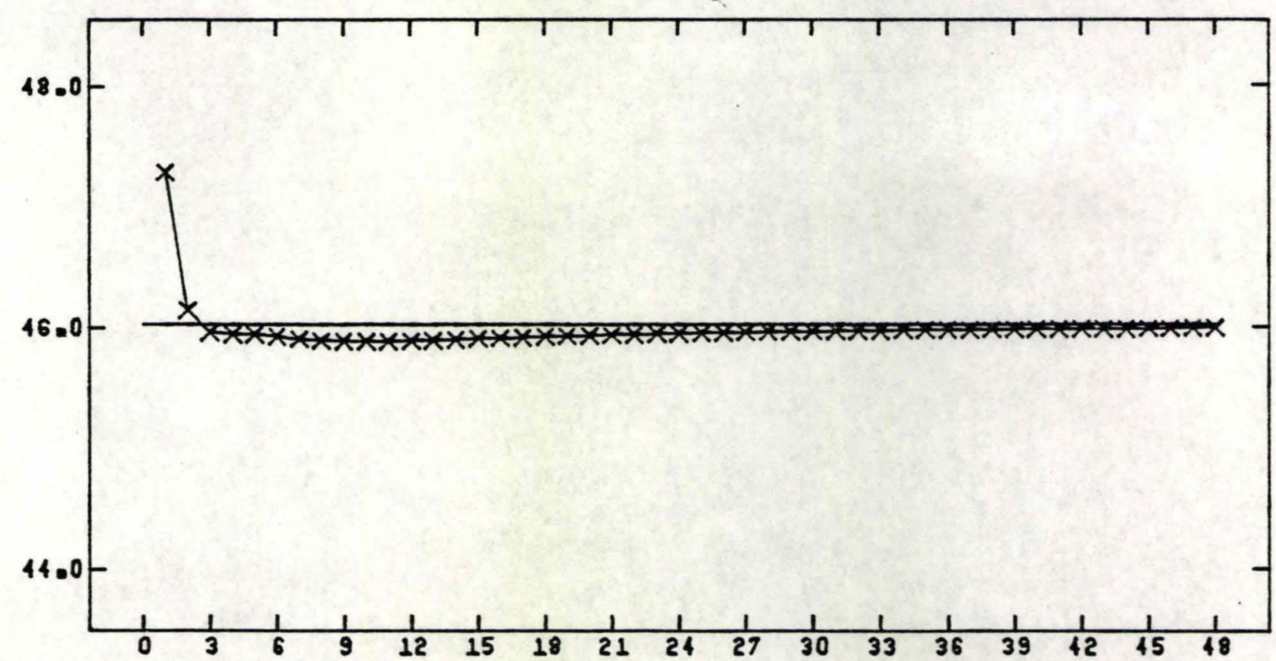


FIGURE 3.16 EFFECT OF A MONEY-FINANCED GOVERNMENT EXPENDITURE SHOCK ON INVESTMENT

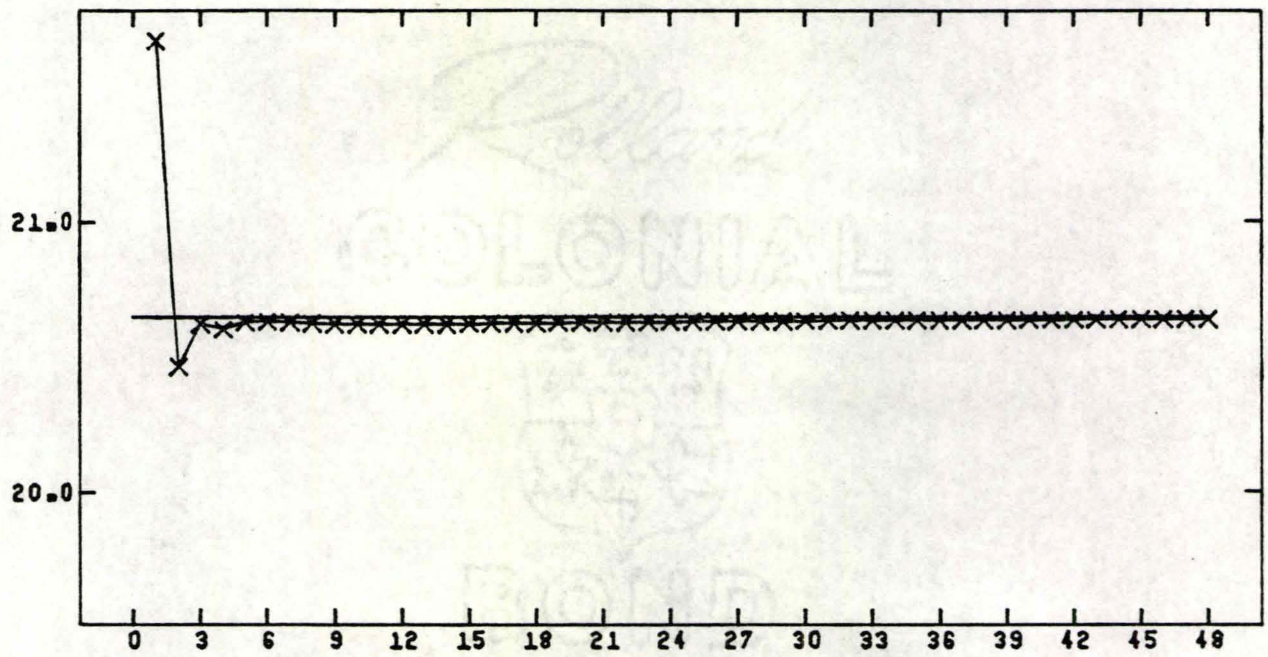


FIGURE 3.17 EFFECT OF A MONEY-FINANCED GOVERNMENT EXPENDITURE SHOCK ON NON-FARM INVENTORIES

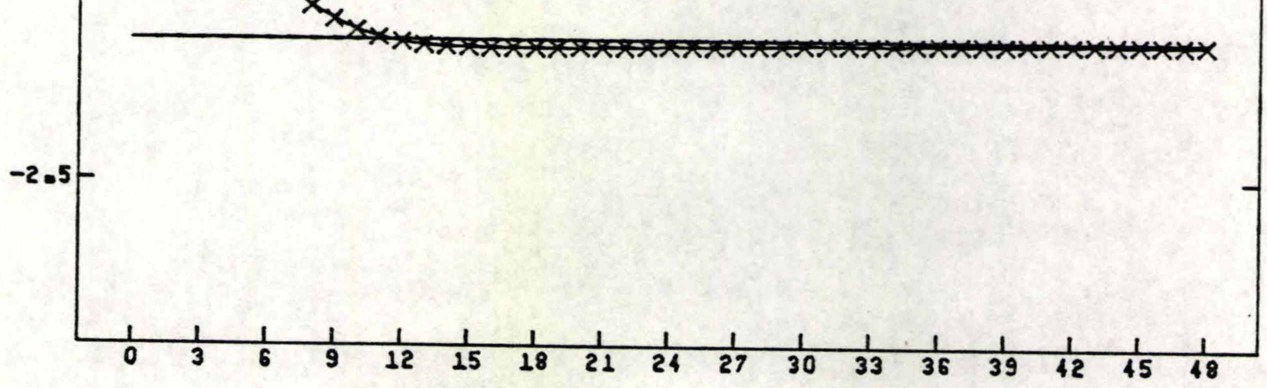


FIGURE 3.18 EFFECT OF A MONEY-FINANCED GOVERNMENT EXPENDITURE SHOCK ON IMPORTS

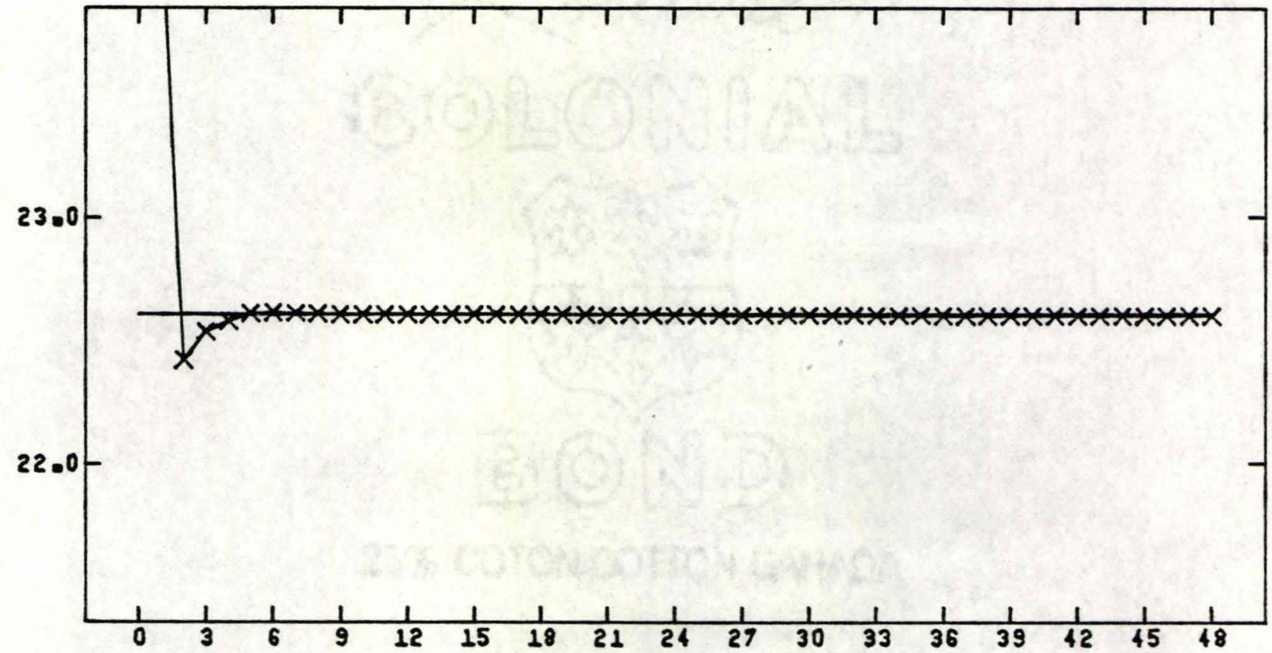


FIGURE 3.19 EFFECT OF A MONEY-FINANCED GOVERNMENT EXPENDITURE SHOCK ON EXPORTS

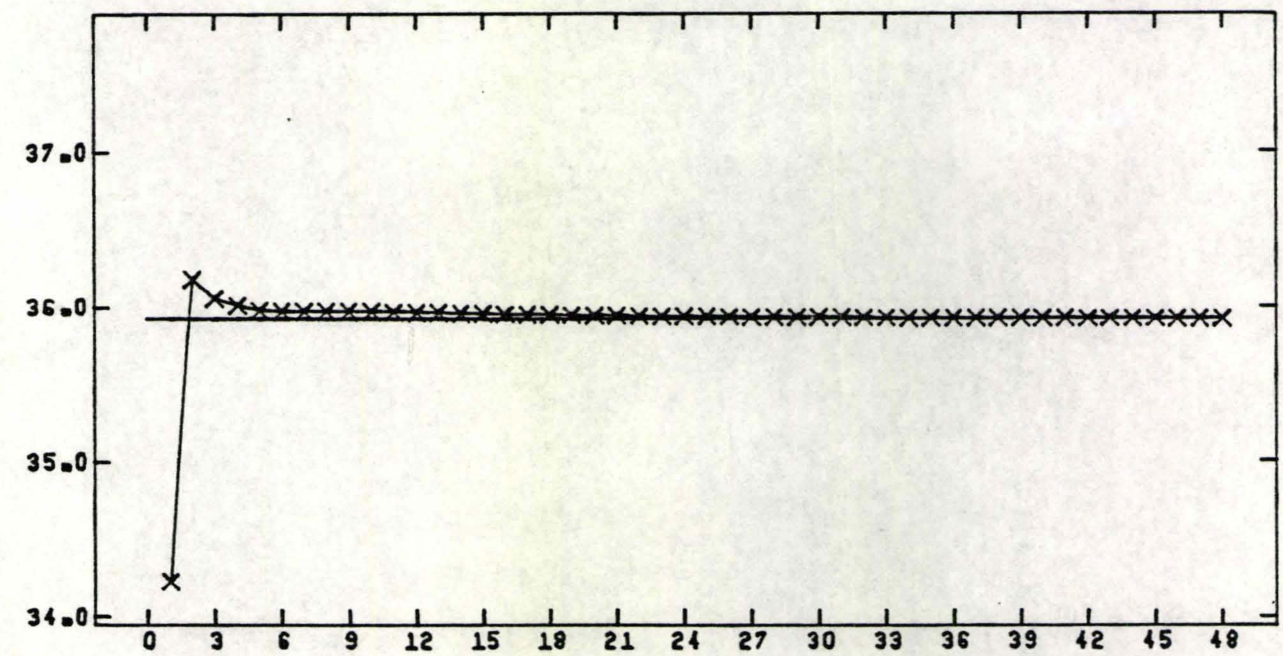


FIGURE 3.20 EFFECT OF A MONEY-FINANCED GOVERNMENT EXPENDITURE SHOCK ON SHORT-TERM INTEREST RATE

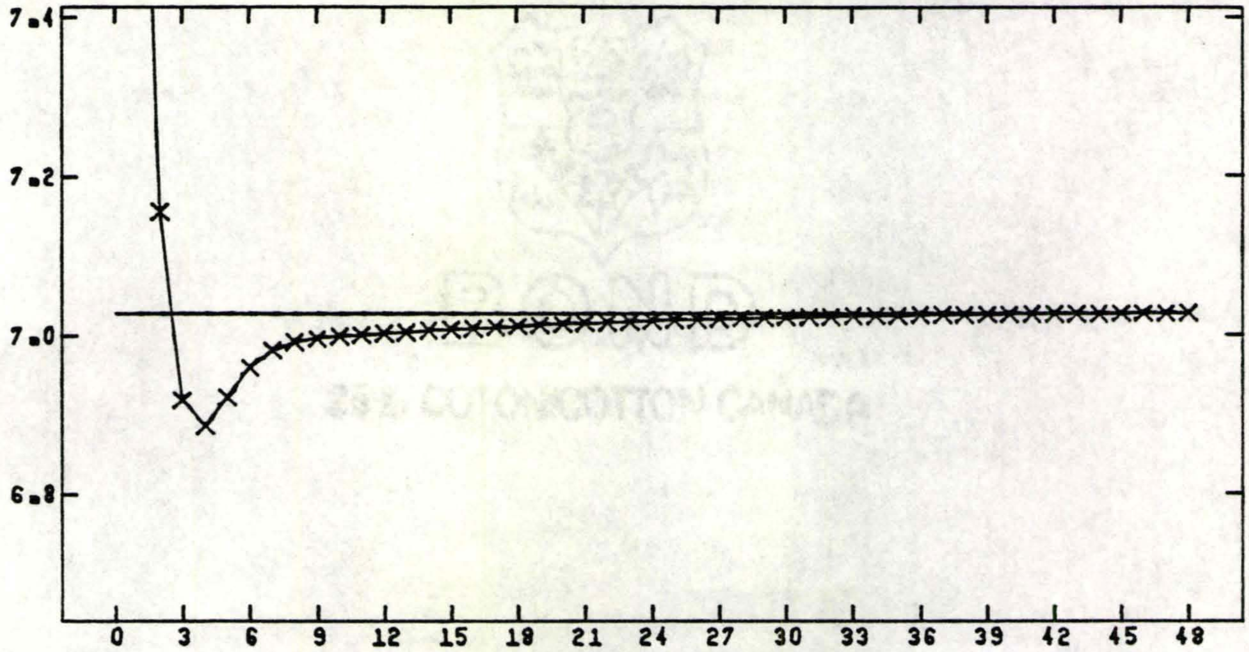


FIGURE 3.21 EFFECT OF A MONEY-FINANCED GOVERNMENT EXPENDITURE SHOCK ON LONG-TERM INTEREST RATE

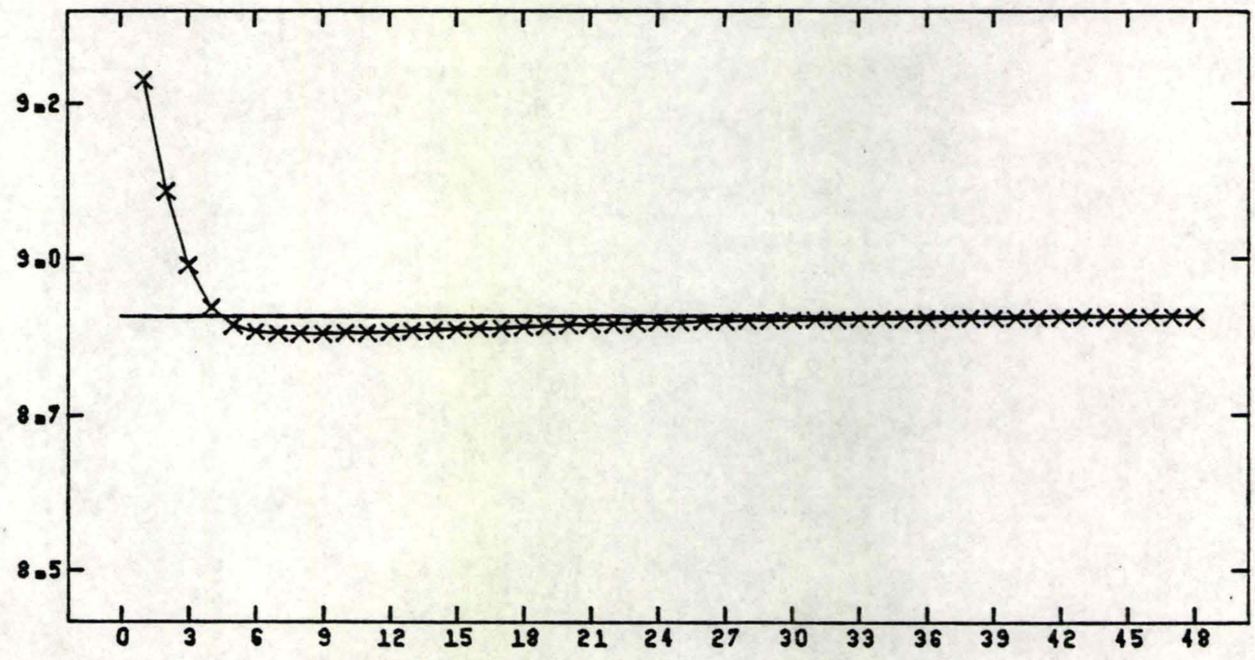


FIGURE 3.22 EFFECT OF A MONEY-FINANCED GOVERNMENT EXPENDITURE SHOCK ON UNEMPLOYMENT RATE

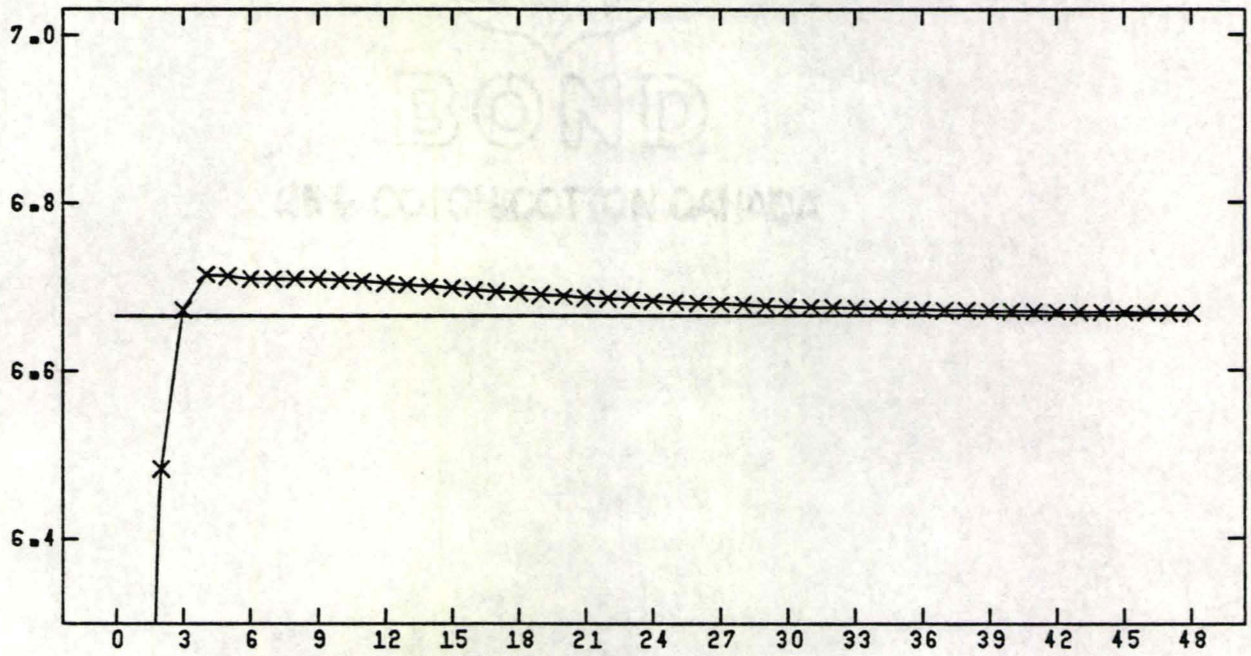


FIGURE 3.23 EFFECT OF A MONEY-FINANCED GOVERNMENT EXPENDITURE SHOCK ON GROWTH RATE OF PRICE LEVEL

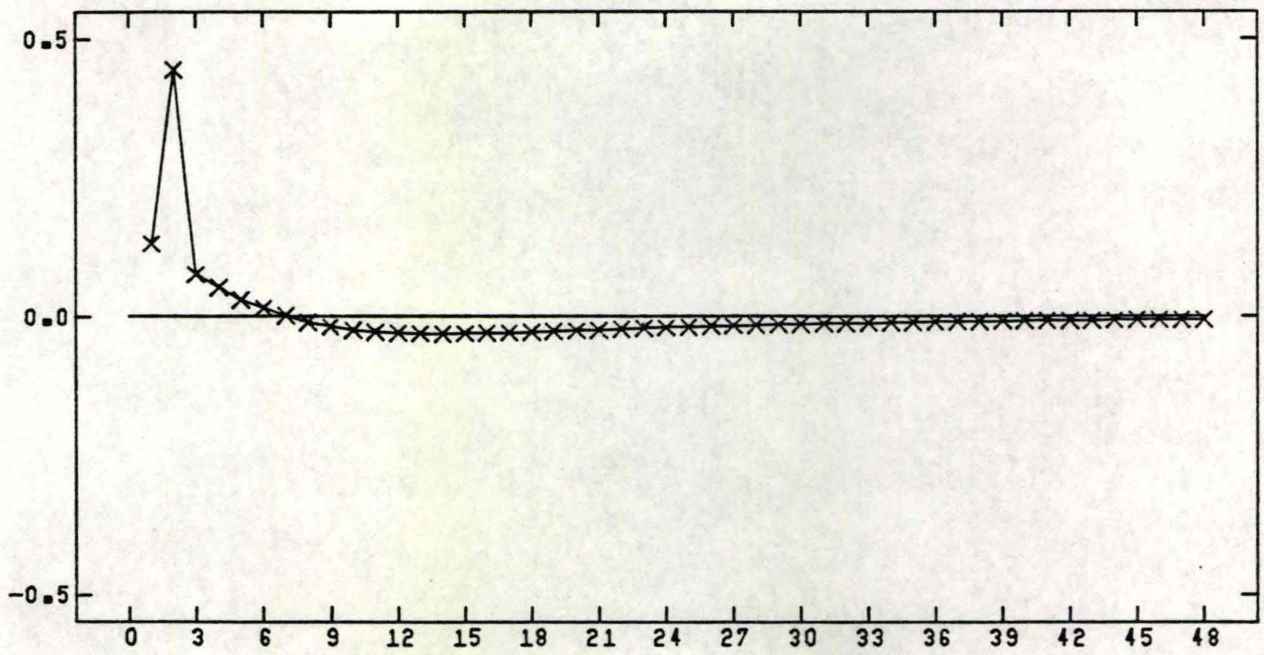


FIGURE 3.24 EFFECT OF A MONEY-FINANCED GOVERNMENT EXPENDITURE SHOCK ON GROWTH RATE OF WAGE RATE

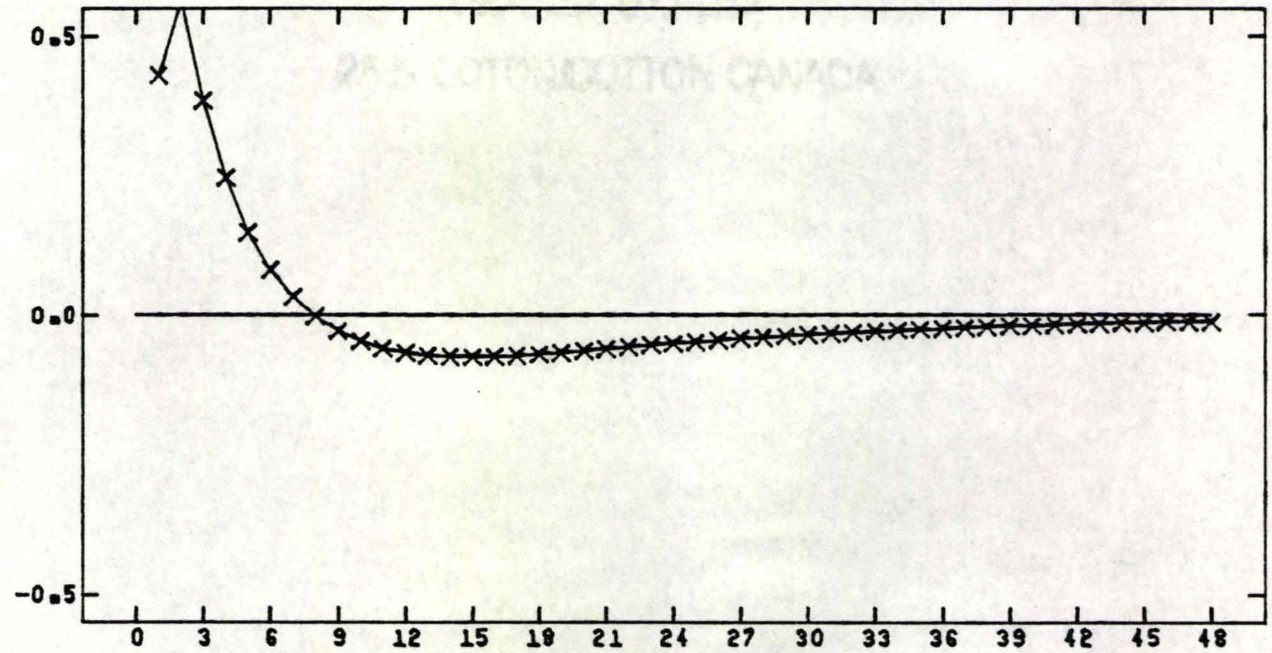


FIGURE 3.25 EFFECT OF A MONEY-FINANCED GOVERNMENT EXPENDITURE SHOCK ON TAX REVENUE

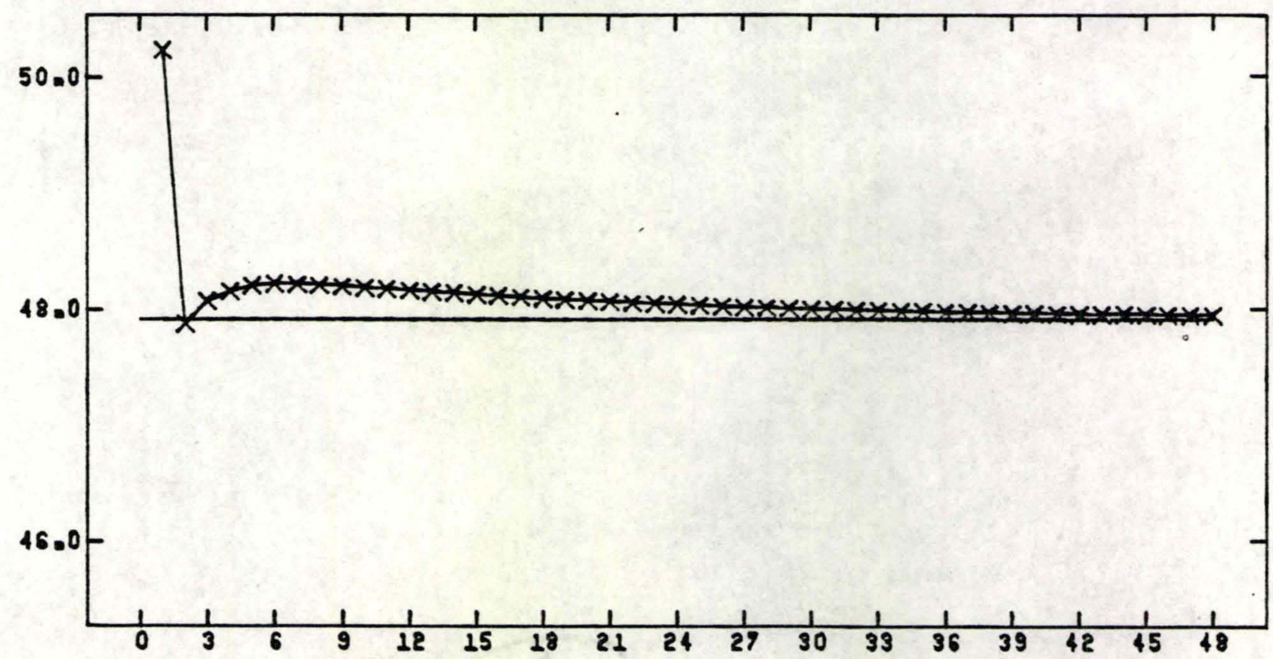


FIGURE 3.26 EFFECT OF A MONEY-FINANCED GOVERNMENT EXPENDITURE SHOCK ON CAPITAL INFLOW

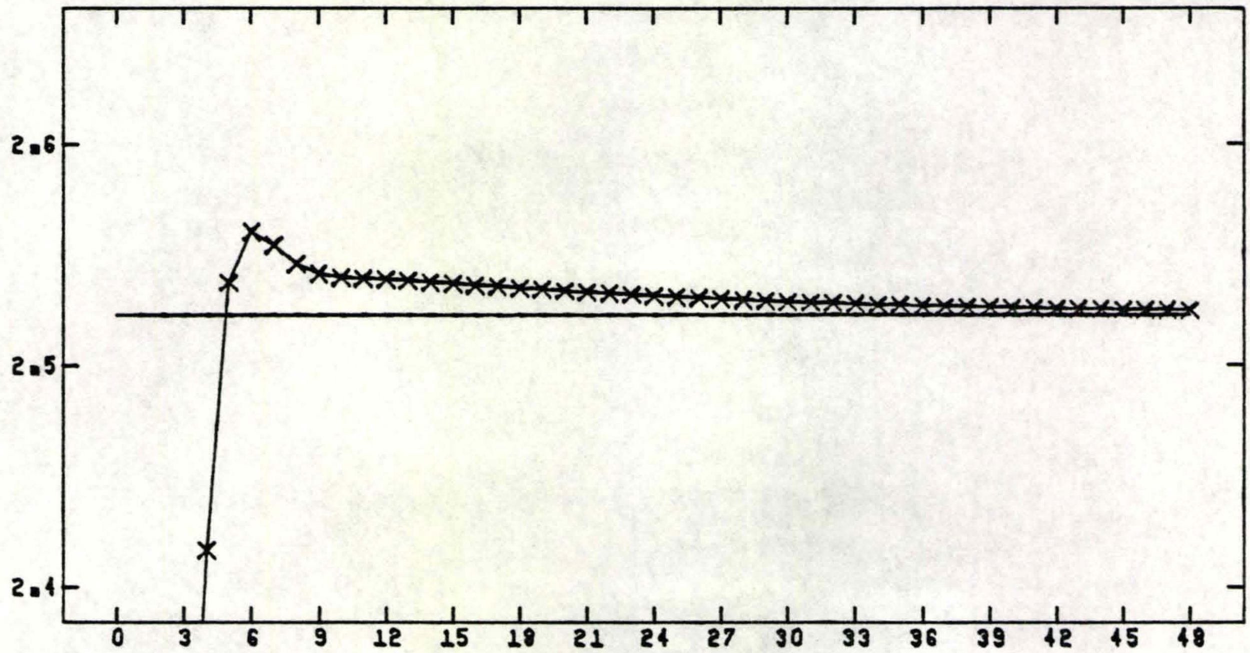
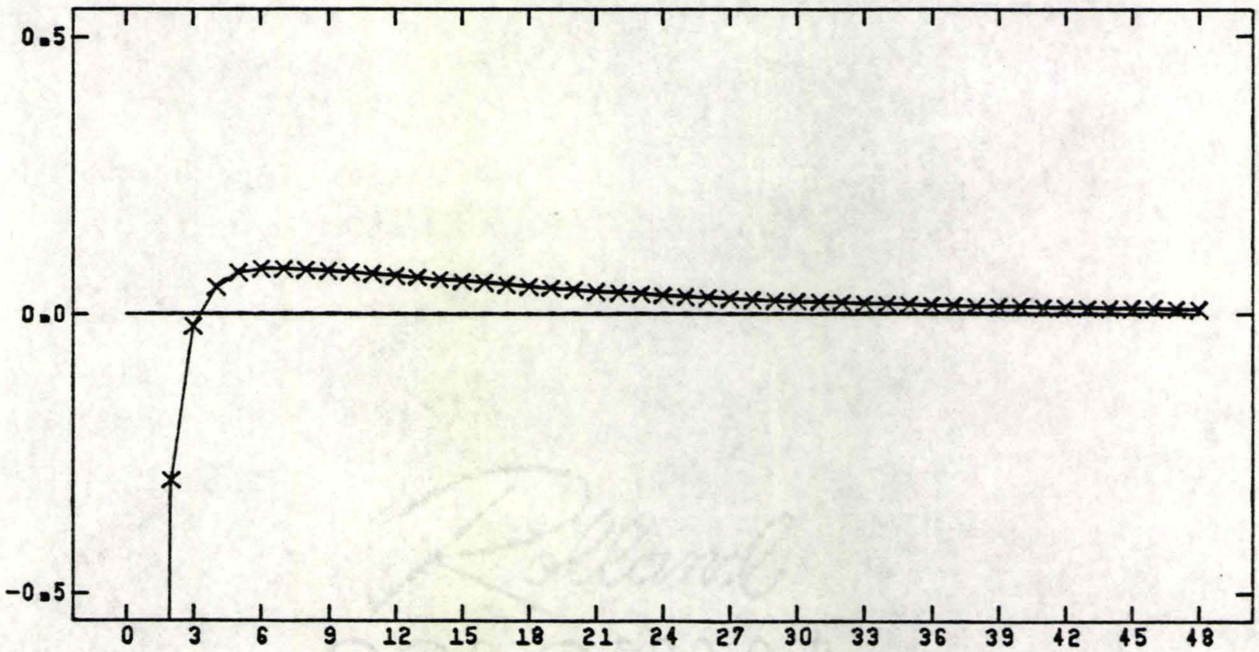


FIGURE 3.27 EFFECT OF A MONEY-FINANCED GOVERNMENT EXPENDITURE SHOCK ON BALANCE OF PAYMENTS



(Figures 3.23 and 3.24). The final impact of this shock on the short-term interest rate is the result of three effects: higher inflation rate, higher GNP and an increase in high-powered money resulting from the government financing constraint. The positive effects of the higher GNP level and inflation rate offset the negative effects of a larger money base and therefore the short-term interest rate responds positively to the shock. As for the long-term interest rate, the impact is also positive since it is positively related to the short-term rate. On the other hand, consumption, investment and non-farm inventories have all risen in the initial year following the shock. This is due mainly to the direct effect of GNP, which is larger in magnitude than the negative effect of interest rates on these variables. It should be noted that real interest rates (short and long-term) have risen, since the increase in the nominal rates outweighs the increase in price level. For the external sector, the increase in GNP stimulates imports and decreases exports, leading to a current account balance deterioriation. This more than offsets the increased inflow of capital, thus leading to an overall balance of payments deficit[7].

This foregoing is a brief description of the workings of the model that led to the results shown in the plots. But what is of more importance to us in the present context is not the first period response of the model but rather its behaviour in the long run. It is clear from the plots that the model is quite stable. Most of the variables respond as expected in the impact period. But as soon as the shock works its way through the model, these variables tend to reverse their initial reaction and start returning to their steady-state values. The convergence of the model to its steady-state situation was relatively quick and without much oscillation. This remained true when the magnitude of the shock was increased.

A BOND-FINANCED GOVERNMENT EXPENDITURE SHOCK. We have argued, in Chapter 2, that the incorporation of the government budget constraint in the model has many implications for the dynamics of the system and its stability. In order to investigate this point, a bond-financed government expenditure shock experiment was carried out over a period of 26 years[8]. It consisted, as in the previous case, of increasing government expenditure by 20 percent in the first year, and then reducing it to its steady-state level in subsequent years. There is one difference from the previous experiment, however; this shock and consequent changes in the government budget deficit are financed endogenously by issuing bonds. The results are shown in Figures 3.28 to 3.41.

The impact effect of this shock on gross national product exceeds that of the money-financed case. This is due to the additional effect that interest payments on the public debt have on disposable income, and ultimately on GNP. The increase in GNP lowers the unemployment rate, which in turn leads to an increase in the growth rates of wages and prices. The short-term interest rate is pushed

FIGURE 3.28 EFFECT OF A BOND-FINANCED GOVERNMENT EXPENDITURE SHOCK ON GNP

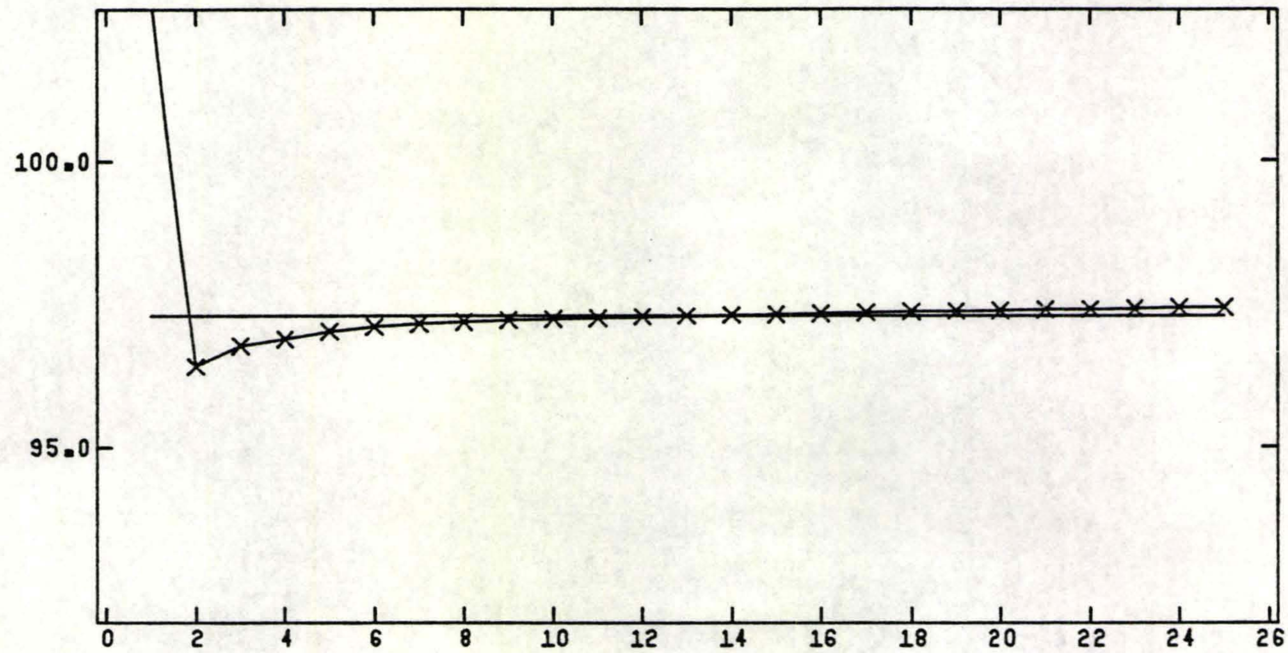


FIGURE 3.29 EFFECT OF A BOND-FINANCED GOVERNMENT EXPENDITURE SHOCK ON CONSUMPTION

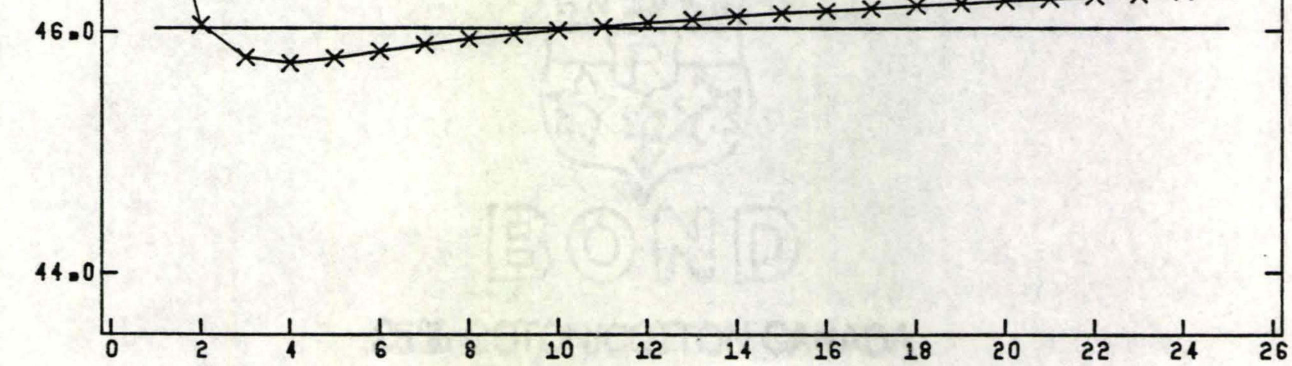


FIGURE 3.30 EFFECT OF A BOND-FINANCED GOVERNMENT EXPENDITURE SHOCK ON INVESTMENT

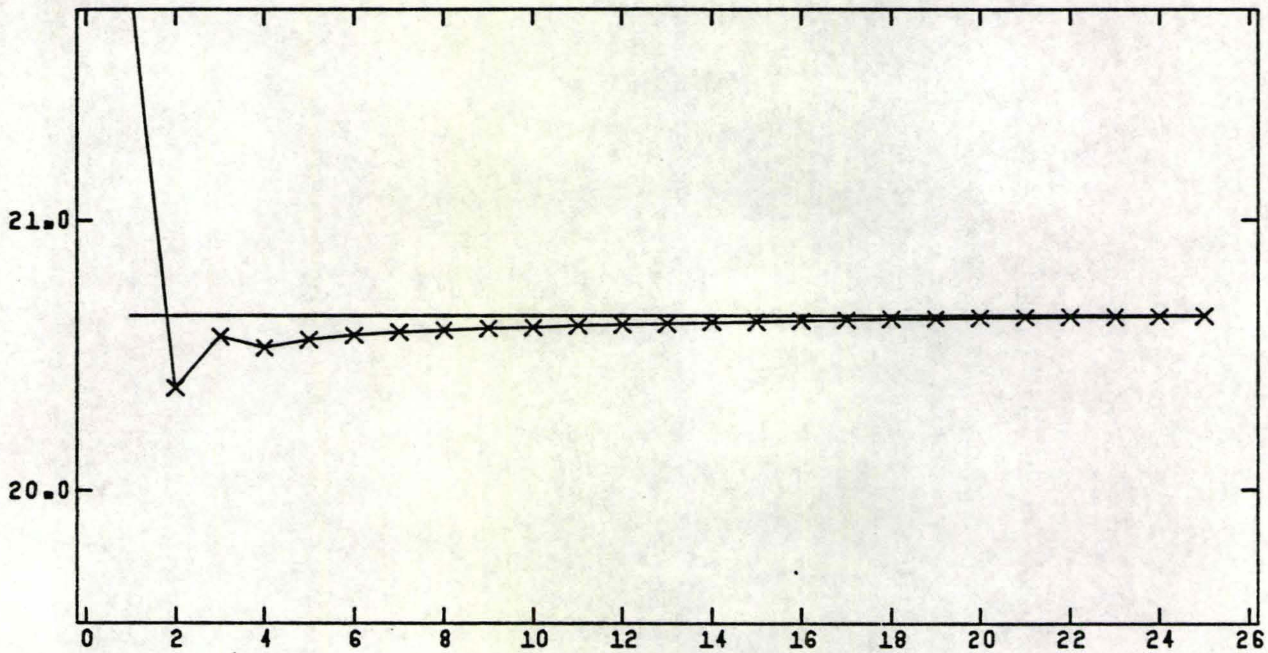


FIGURE 3.31 EFFECT OF A BOND-FINANCED GOVERNMENT EXPENDITURE SHOCK ON NON-FARM INVENTORIES

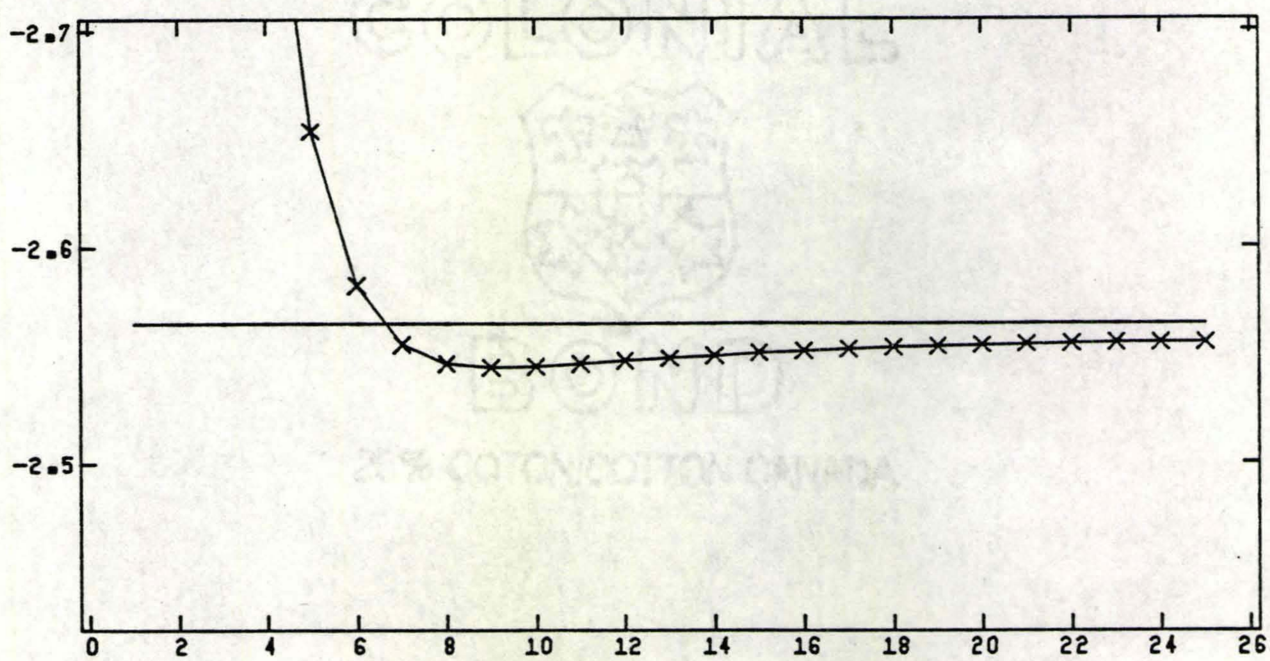


FIGURE 3.32 EFFECT OF A BOND-FINANCED GOVERNMENT EXPENDITURE SHOCK ON IMPORTS

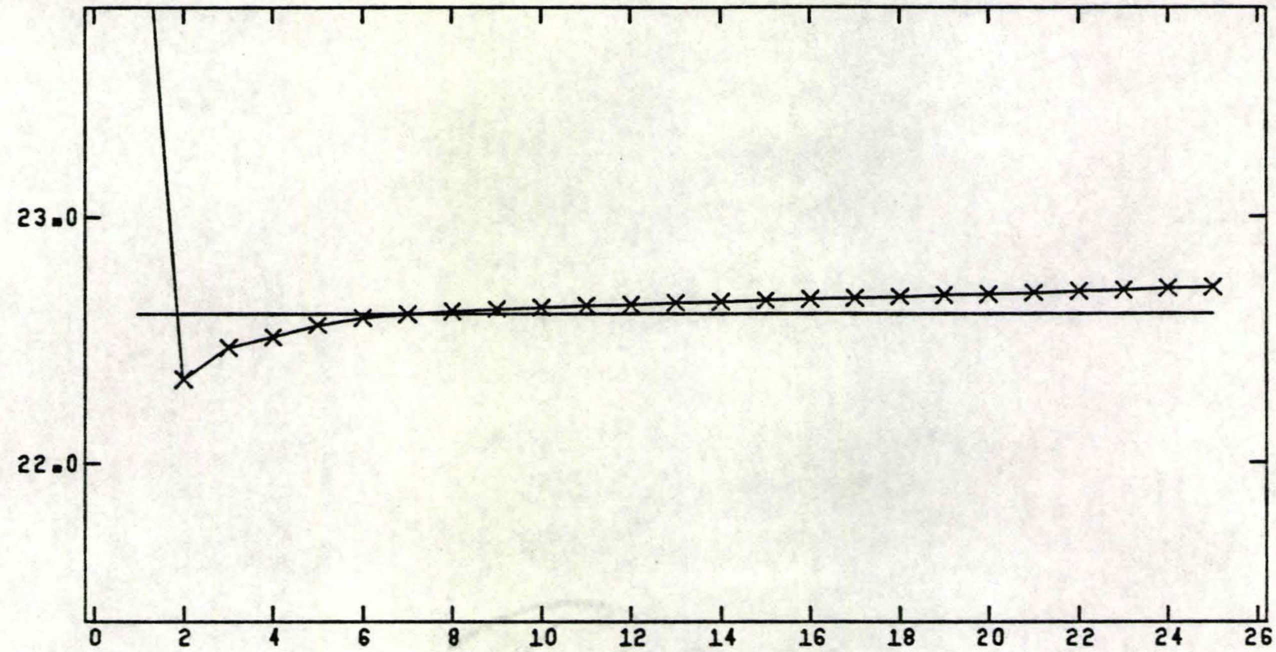


FIGURE 3.33 EFFECT OF A BOND-FINANCED GOVERNMENT EXPENDITURE SHOCK ON EXPORTS

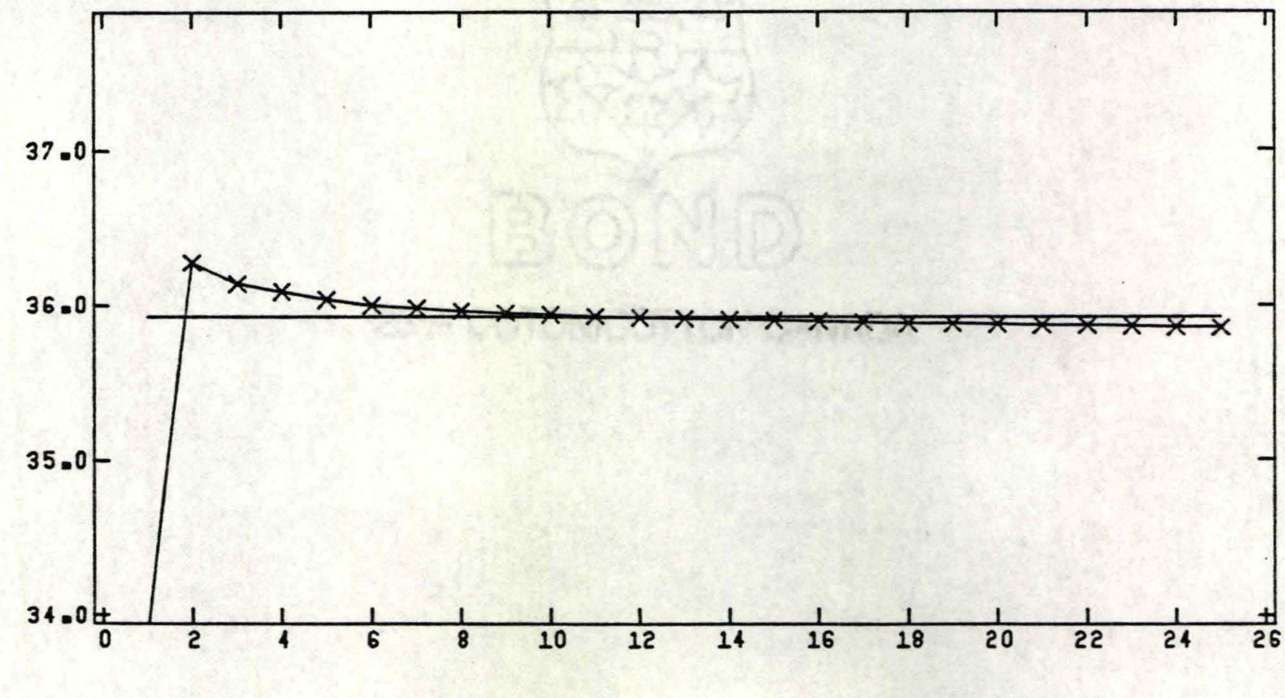


FIGURE 3.34 EFFECT OF A BOND-FINANCED GOVERNMENT EXPENDITURE SHOCK ON SHORT-TERM INTEREST RATE

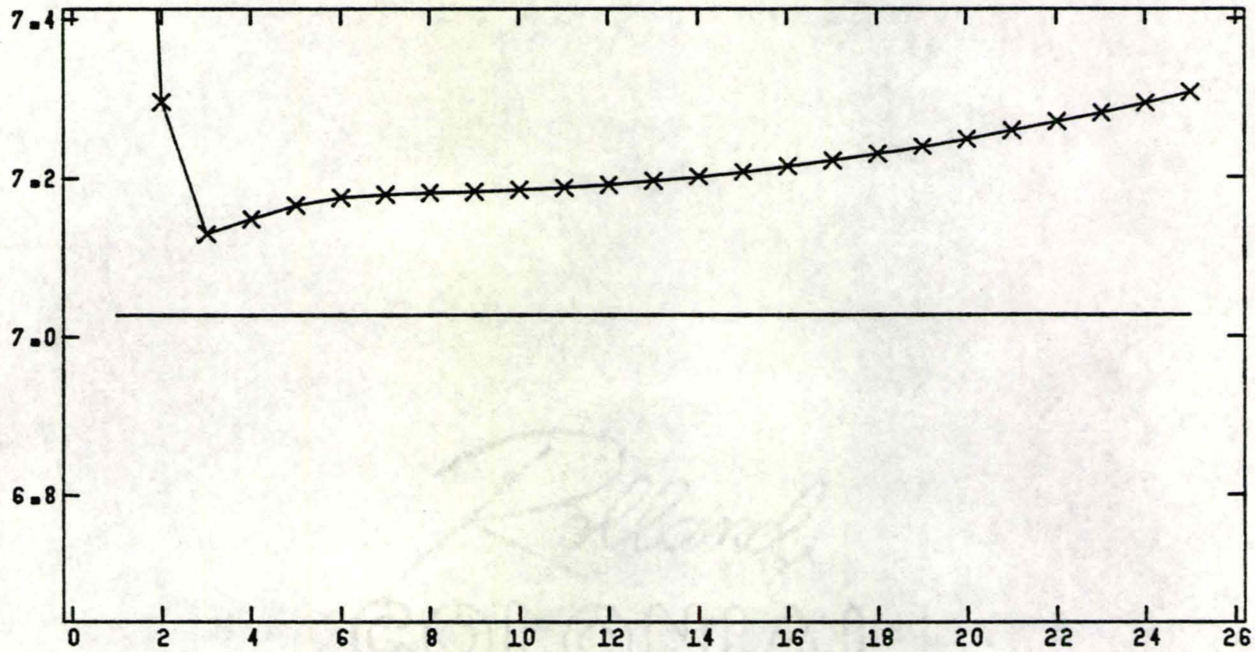


FIGURE 3.35 EFFECT OF A BOND-FINANCED GOVERNMENT EXPENDITURE SHOCK ON LONG-TERM INTEREST RATE

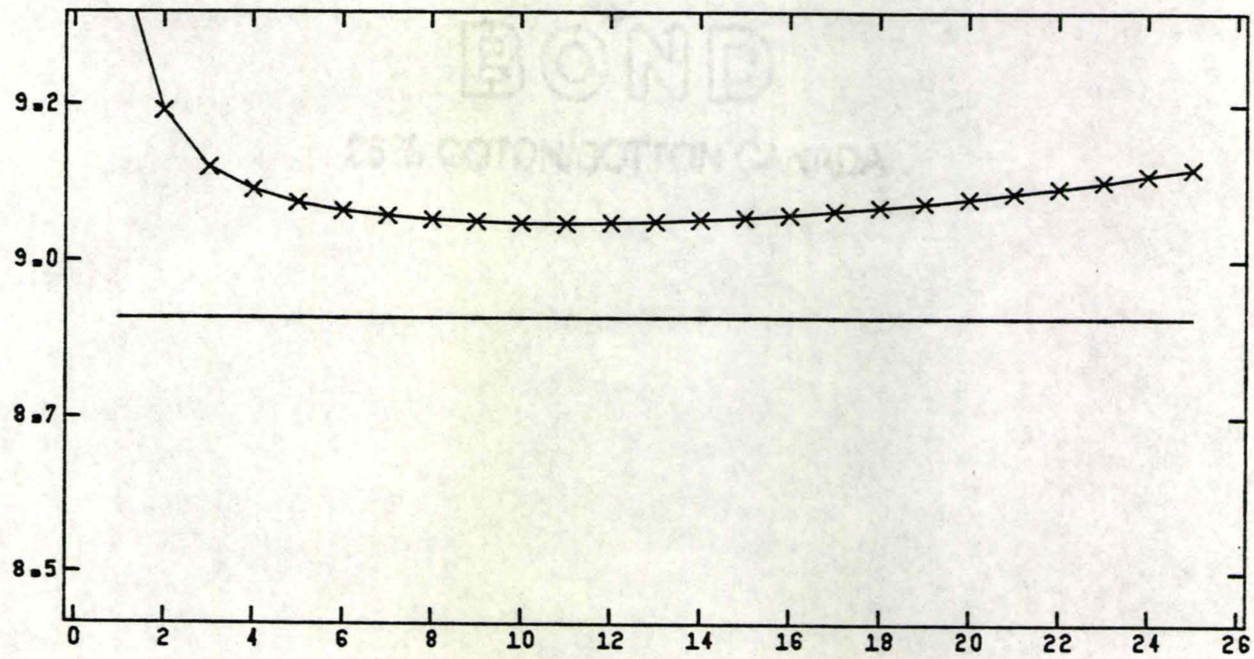


FIGURE 3.36 EFFECT OF A BOND-FINANCED GOVERNMENT EXPENDITURE SHOCK ON UNEMPLOYMENT RATE

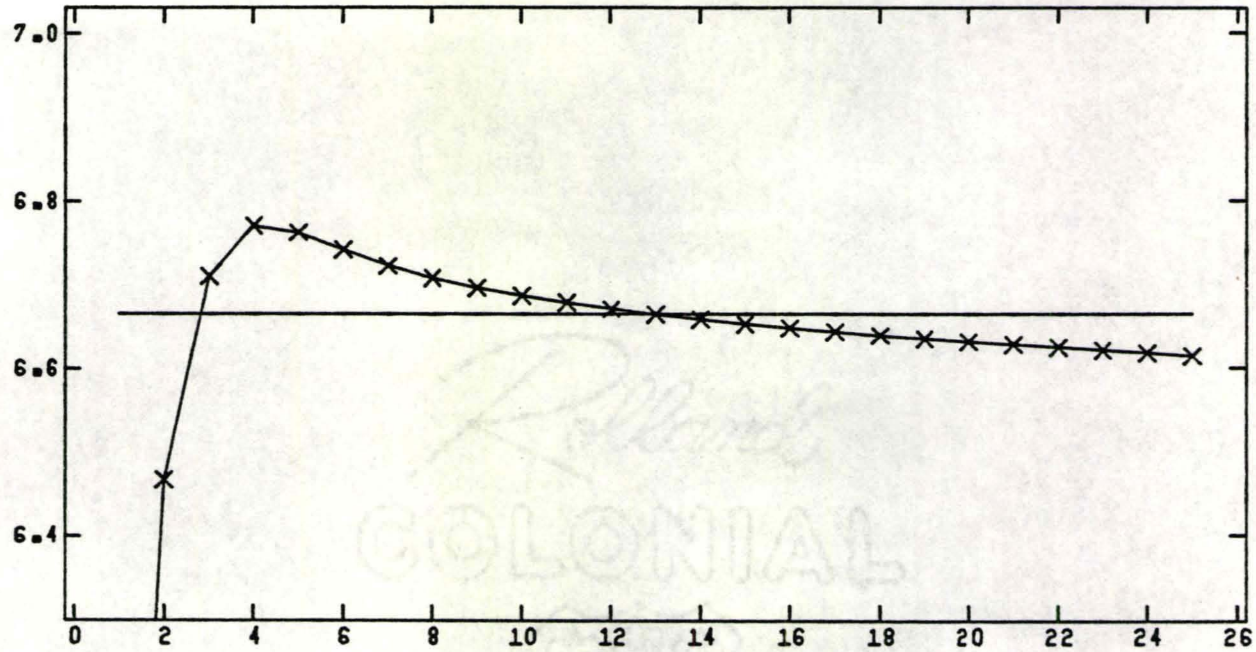


FIGURE 3.37 EFFECT OF A BOND-FINANCED GOVERNMENT EXPENDITURE SHOCK ON GROWTH RATE OF PRICE LEVEL

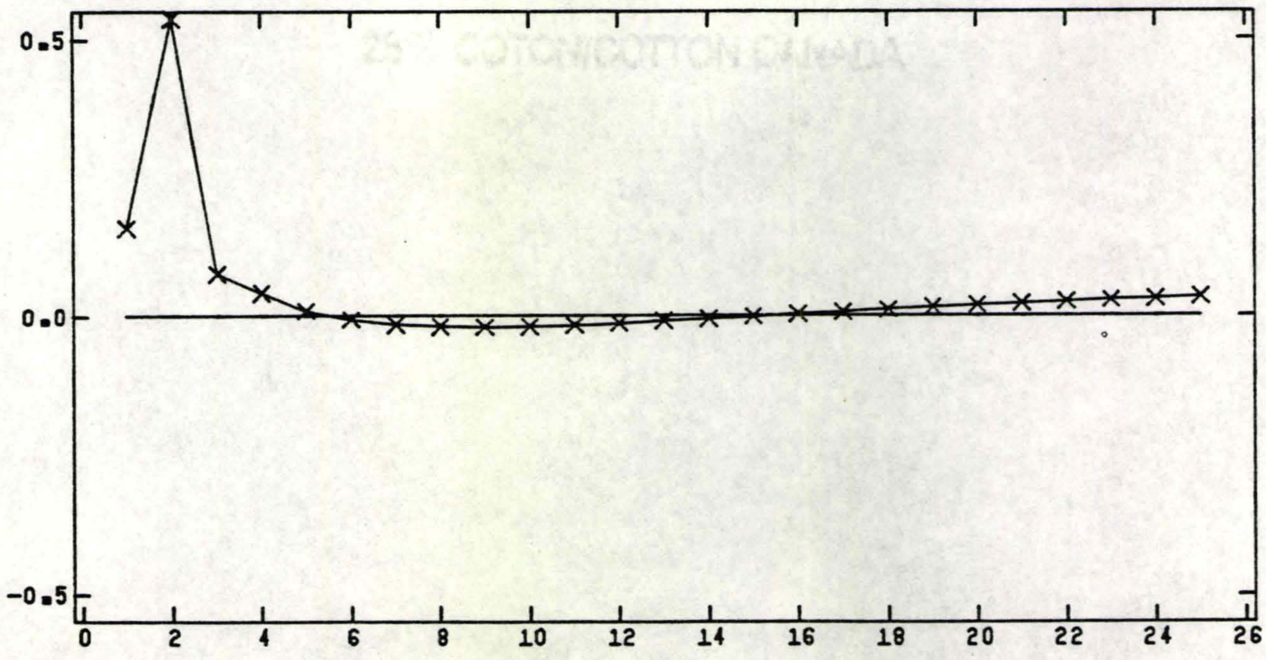


FIGURE 3.38 EFFECT OF A BOND-FINANCED GOVERNMENT EXPENDITURE SHOCK ON GROWTH RATE OF WAGE RATE

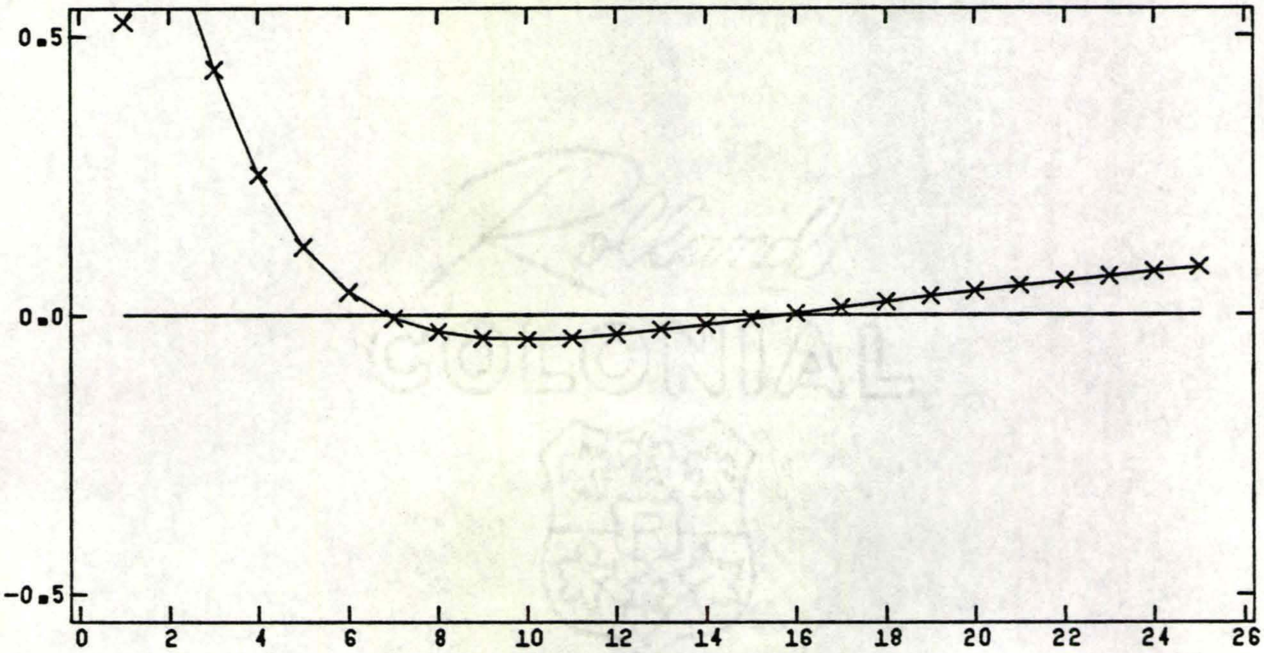


FIGURE 3.39 EFFECT OF A BOND-FINANCED GOVERNMENT EXPENDITURE SHOCK ON TAX REVENUE

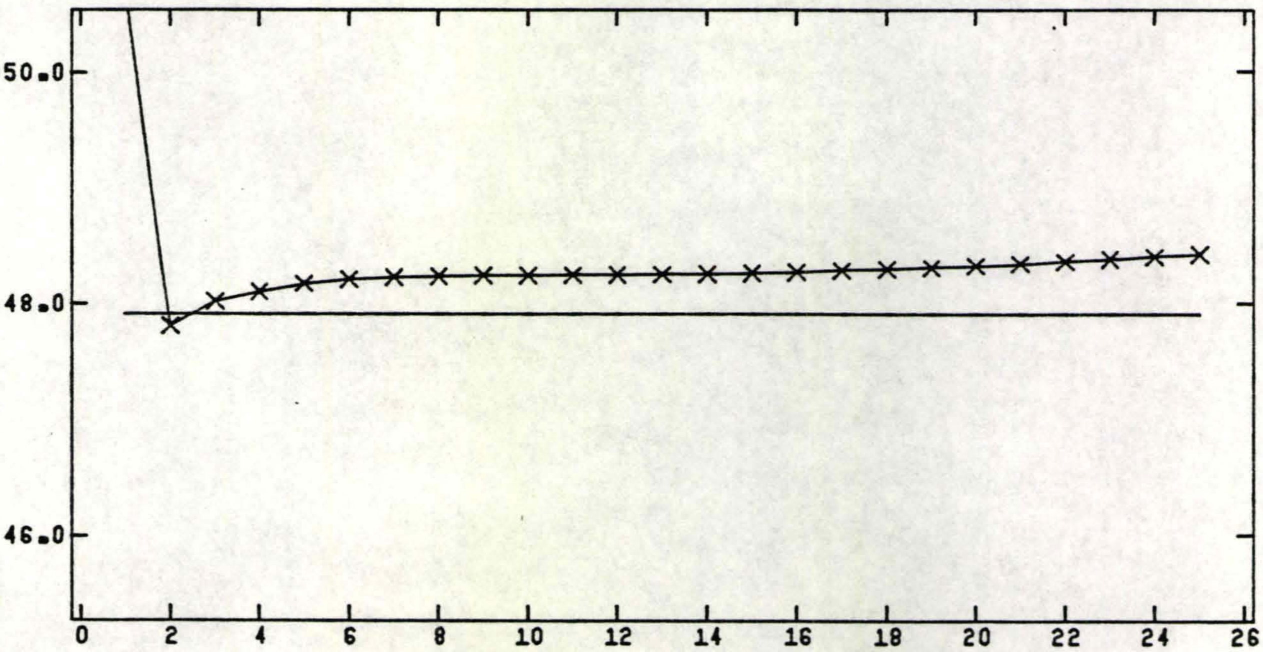


FIGURE 3.40 EFFECT OF A BOND-FINANCED GOVERNMENT EXPENDITURE SHOCK ON CAPITAL INFLOW

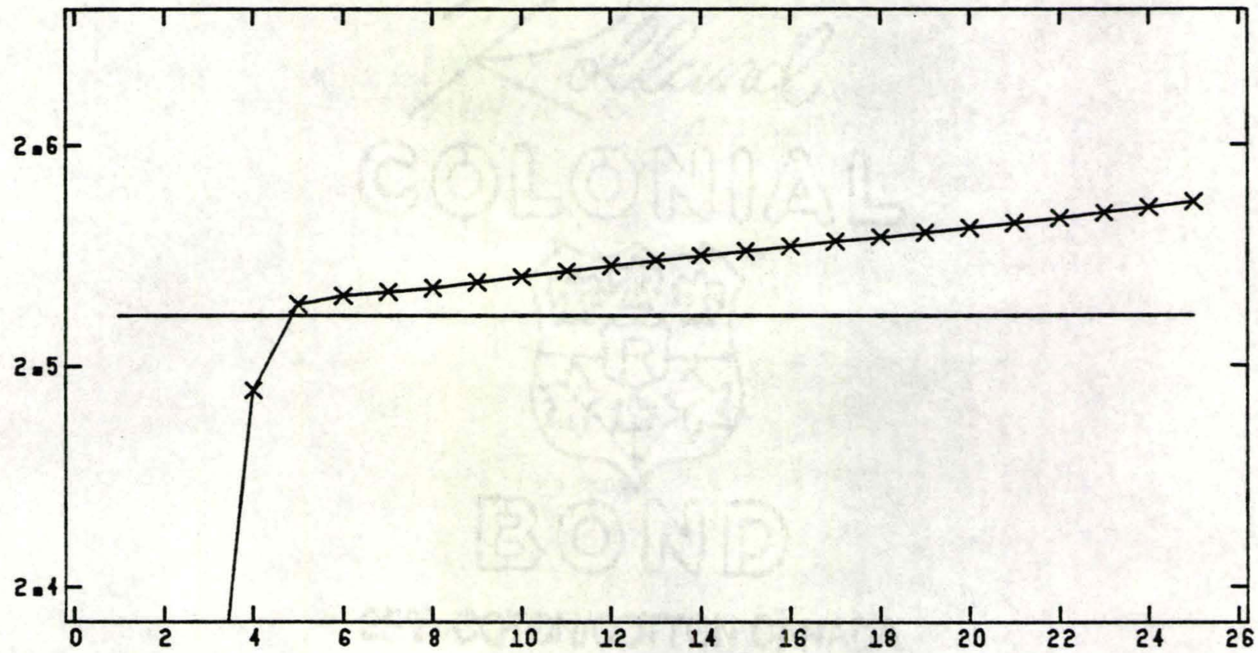
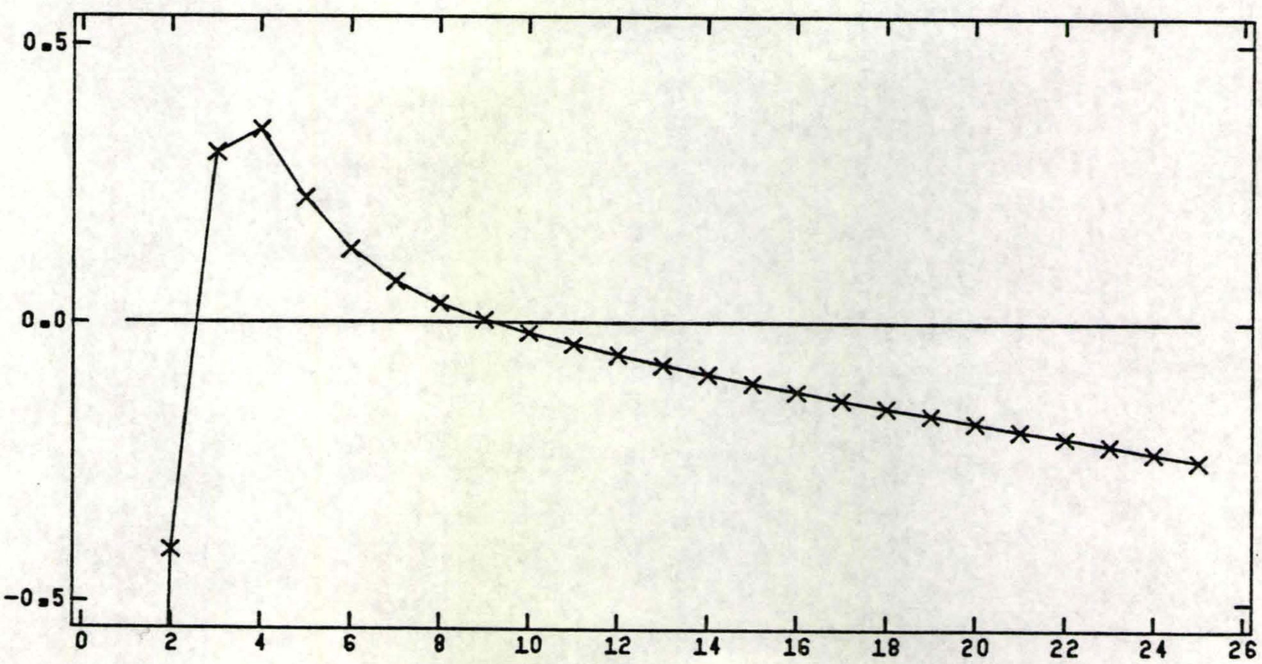


FIGURE 3.41 EFFECT OF A BOND-FINANCED GOVERNMENT EXPENDITURE SHOCK ON BALANCE OF PAYMENTS



upward because of higher inflation and higher GNP, and the long-term-rate increases as well. Real interest rates also tend to increase, and this lessens the effect of the higher GNP on investment and non-farm inventories, after the initial shock. Similarly, the effect of the nominal short-term rate on consumption tends to reduce consumption after the impact period. As for the external sector, the shock initially stimulates imports and reduces exports and capital inflow, leading to a balance of payments deficit.

Thus far we have briefly discussed the impact effect of a bond-financed government expenditure shock on the model, which is very similar to that of the money-financed case. However, with the passage of time, the difference between the two responses becomes increasingly clear. The fact that taxes do not rise enough to offset the increase in interest payments on the public debt raises disposable income, consumption and GNP. As a consequence of the government budget constraint and as long as the budgetary gap is not filled, new bonds have to be issued, interest rates will rise, interest payments on the debt will increase as a result, and the whole process will continue indefinitely.

Thus, as shown in the plots, most variables diverge from their steady-state values as time progresses. One can draw the conclusion therefore, that the model is unstable in the long-run when government expenditure is financed by the issuing of bonds. This seems to be in agreement with most macro models which explicitly incorporate the government budget constraint and the influence of interest payments on the expenditure side of the model.

AN EXCHANGE RATE SHOCK. In this experiment, the exchange rate was increased by 10% which amounts to the devaluation of the Canadian dollar by that proportion. The results are shown in Figures 3.42 to 3.55. In our model, the exchange rate affects the economy by influencing the imports and exports of goods and services, and also the domestic price level. A decrease in the value of domestic currency, reduces real imports since they become relatively more expensive and increases real exports as they become cheaper. This, however, does not translate, as one might have expected, into a current account surplus. This is so because whether the money value of imports rises or falls depends on whether the decrease in real imports outweighs the increase in price -- that is, on the elasticity of demand for imports. This demand was estimated to be highly inelastic, and thus a balance of payments deficit arises since the increase in the nominal value of imports offsets not only the higher money value of Canadian exports but also the inflow of capital which results from high domestic interest rates. The latter was brought about by a decrease in the quantity of money, which was necessary for the government budget constraint to hold. The increase in interest rates caused a lowering of consumption and investment and consequently of the GNP. Unemployment was higher and

FIGURE 3.42 EFFECT OF EXCHANGE RATE SHOCK ON GNP

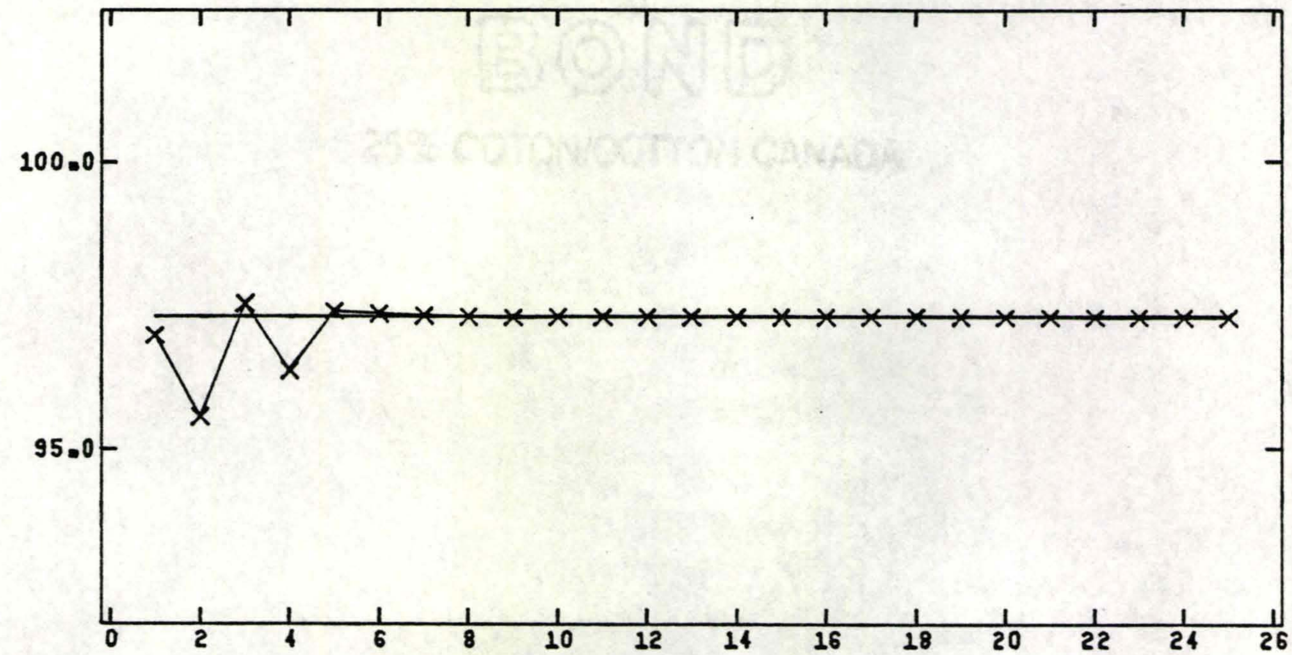


FIGURE 3.43 EFFECT OF EXCHANGE RATE SHOCK ON CONSUMPTION

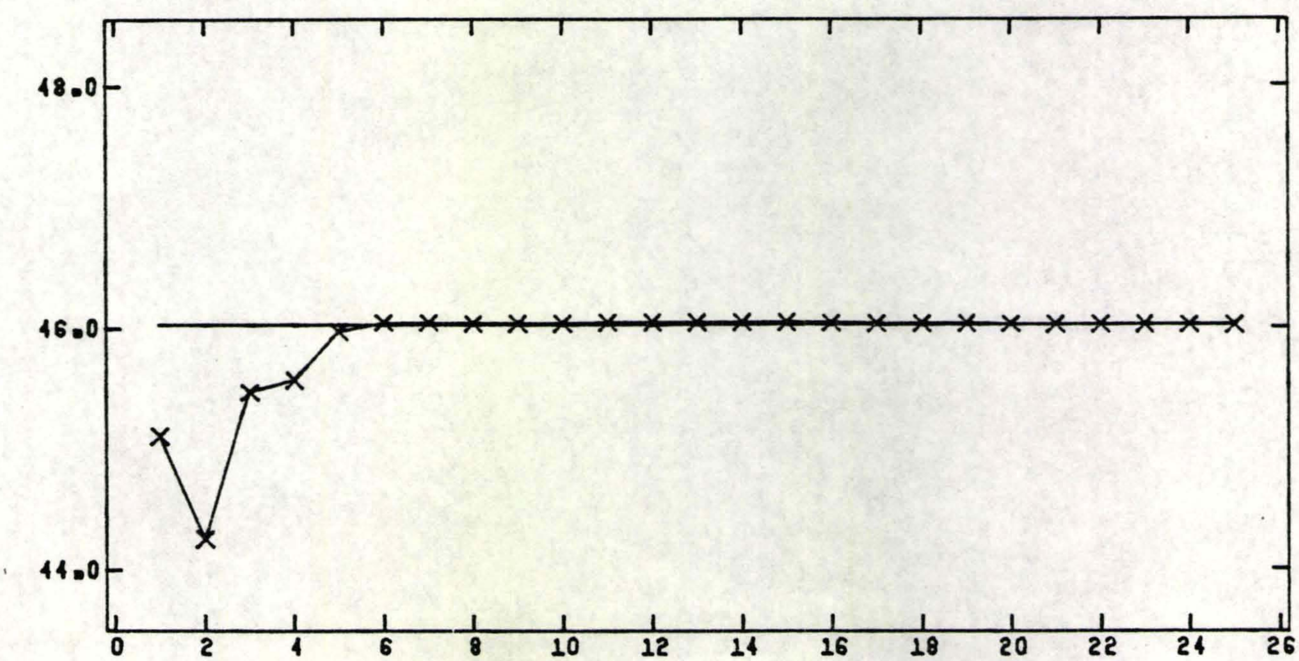


FIGURE 3.44 EFFECT OF EXCHANGE RATE SHOCK ON INVESTMENT

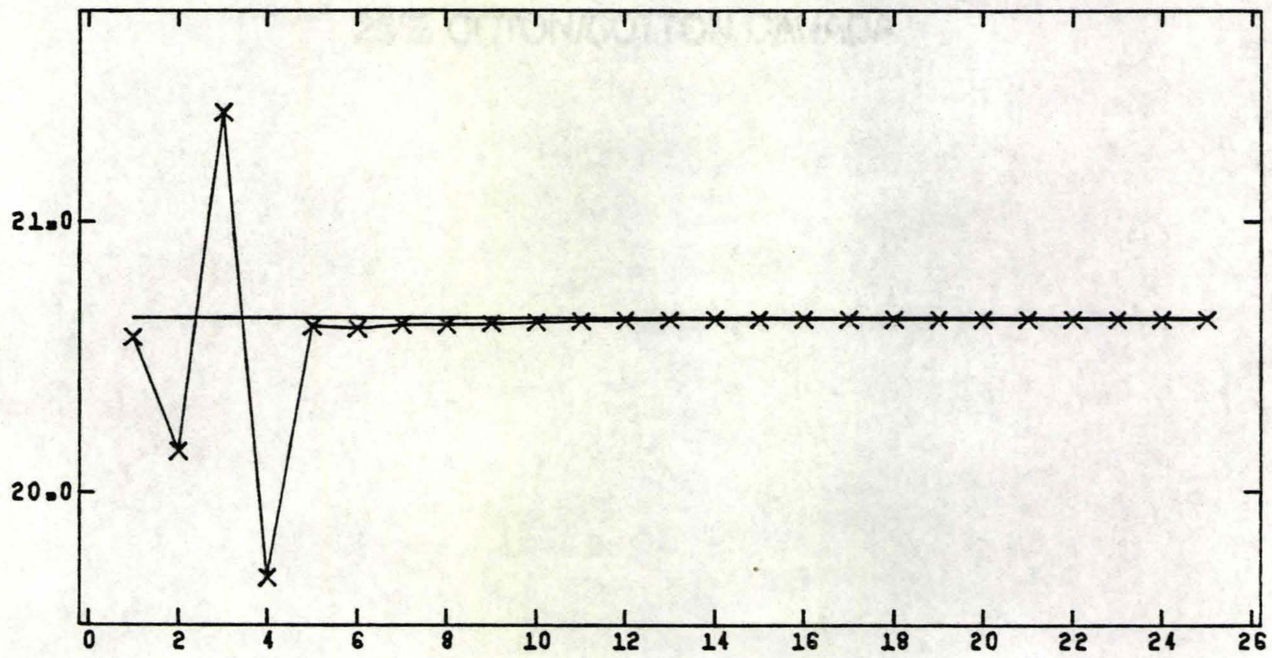


FIGURE 3.45 EFFECT OF EXCHANGE RATE SHOCK ON NON-FARM INVENTORIES

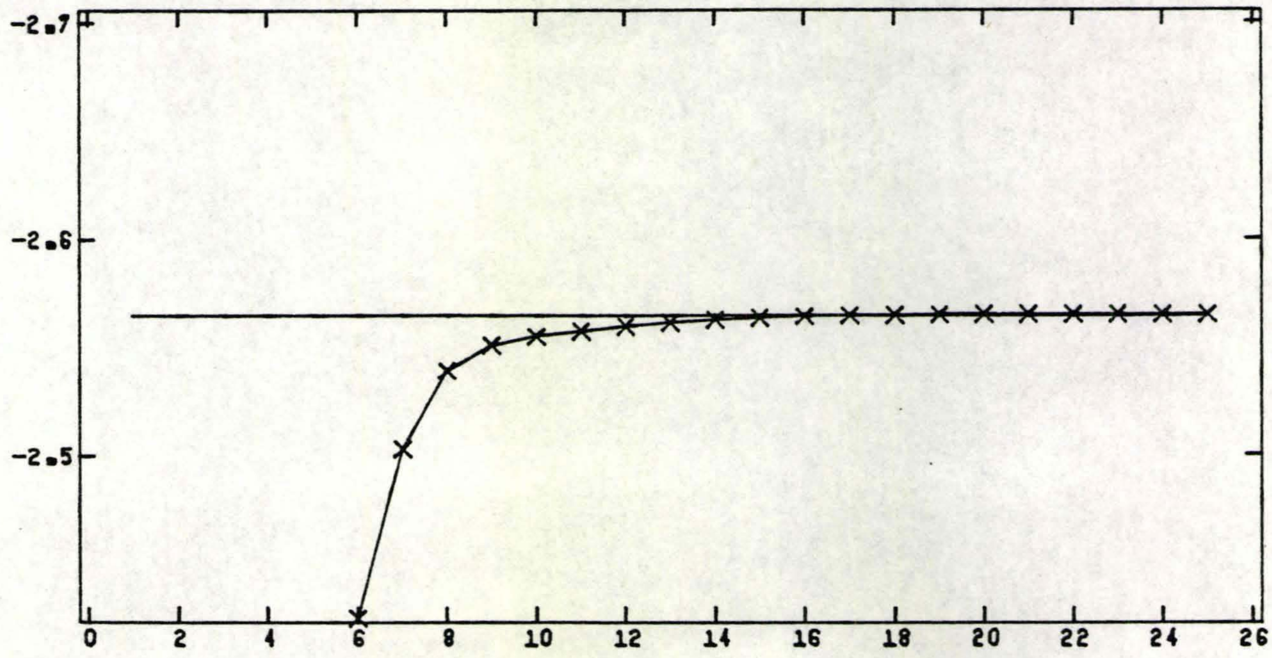


FIGURE 3.46 EFFECT OF EXCHANGE RATE SHOCK ON IMPORTS

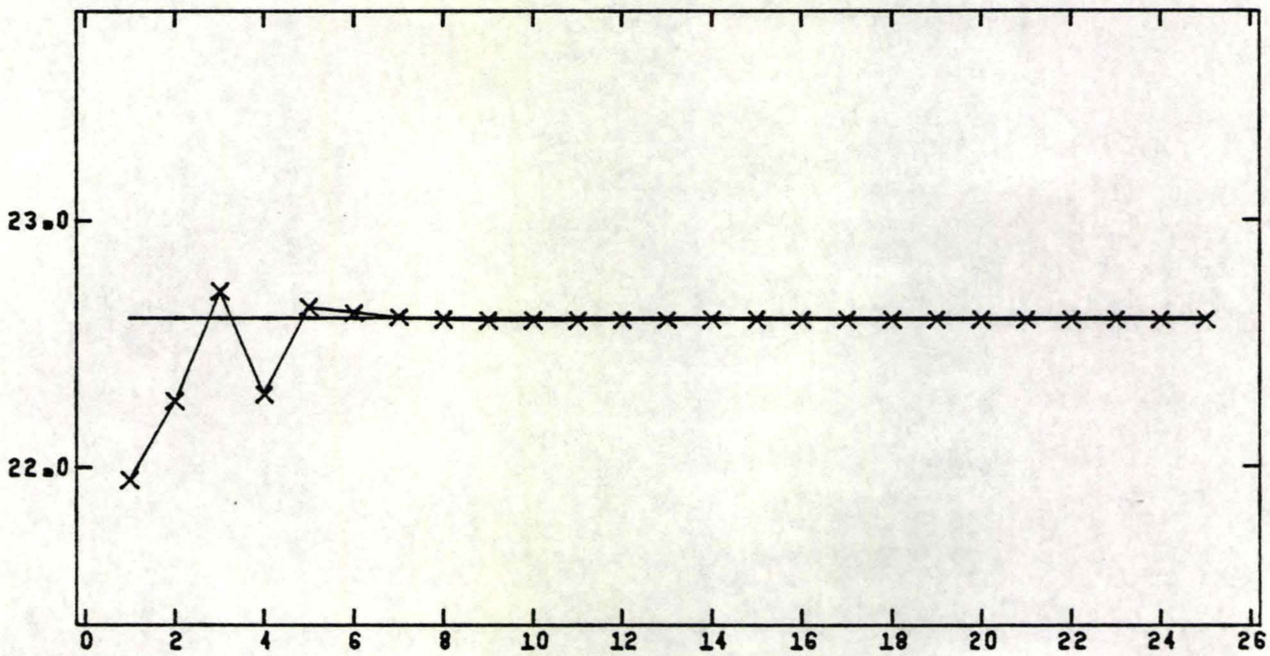


FIGURE 3.47 EFFECT OF EXCHANGE RATE SHOCK ON EXPORTS

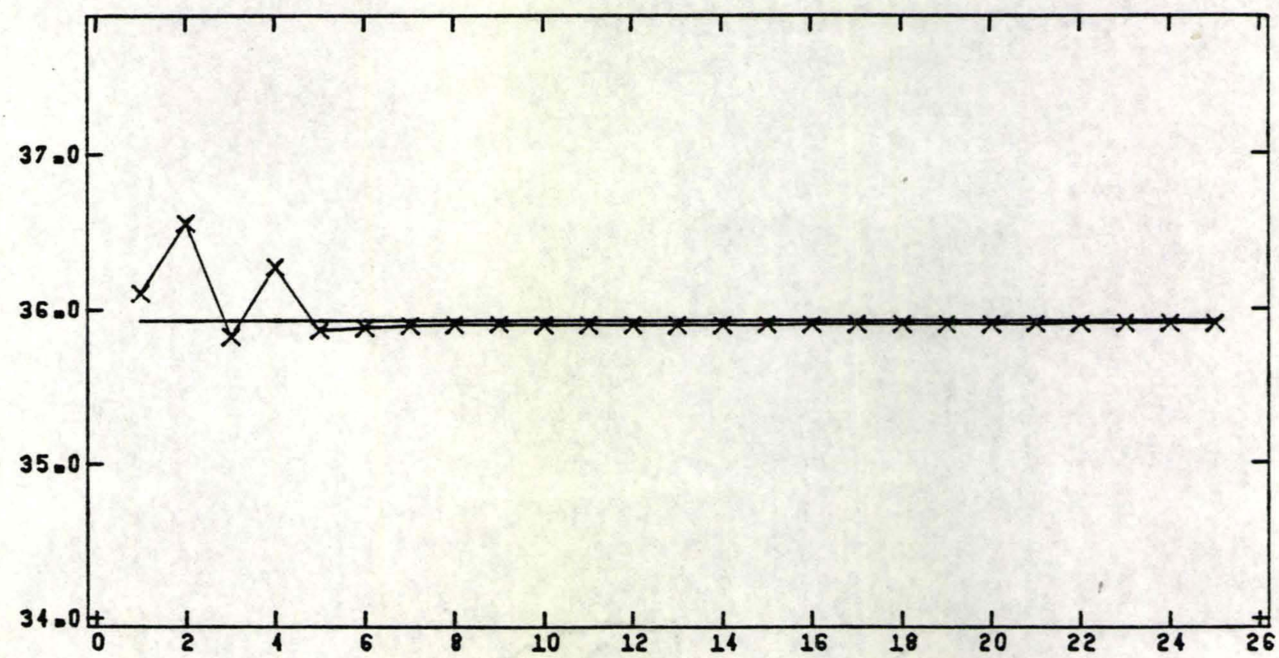


FIGURE 3.48 EFFECT OF EXCHANGE RATE SHOCK ON SHORT-TERM INTEREST RATE

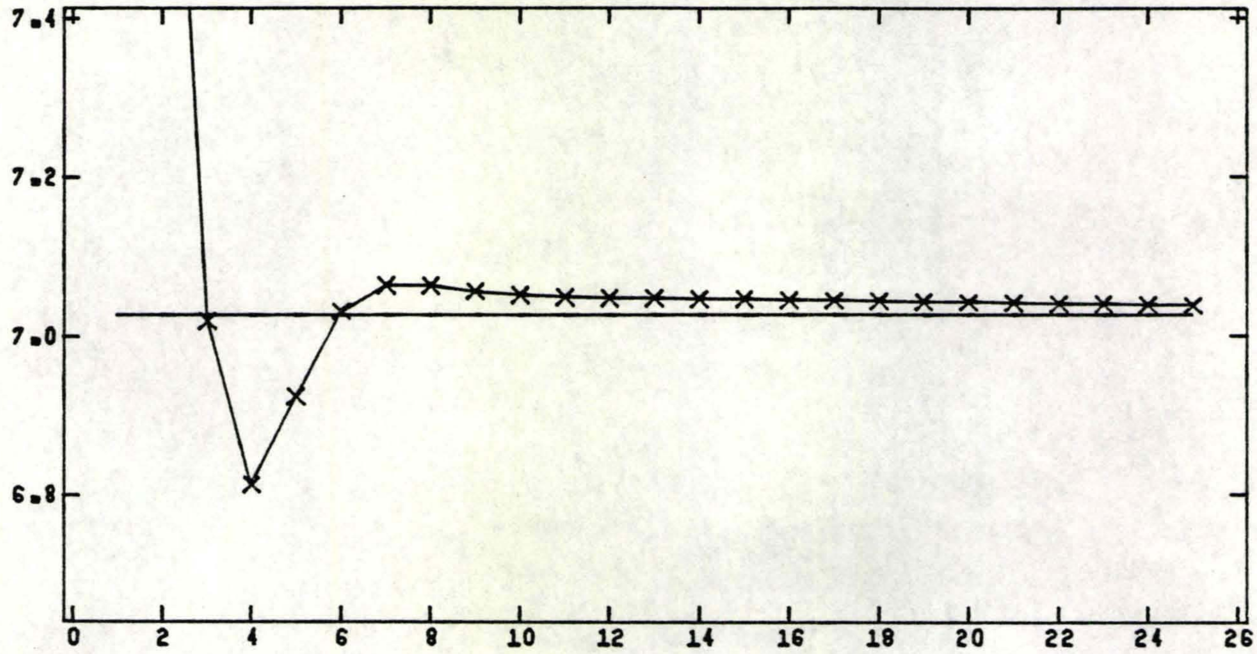


FIGURE 3.49 EFFECT OF EXCHANGE RATE SHOCK ON LONG-TERM INTEREST RATE

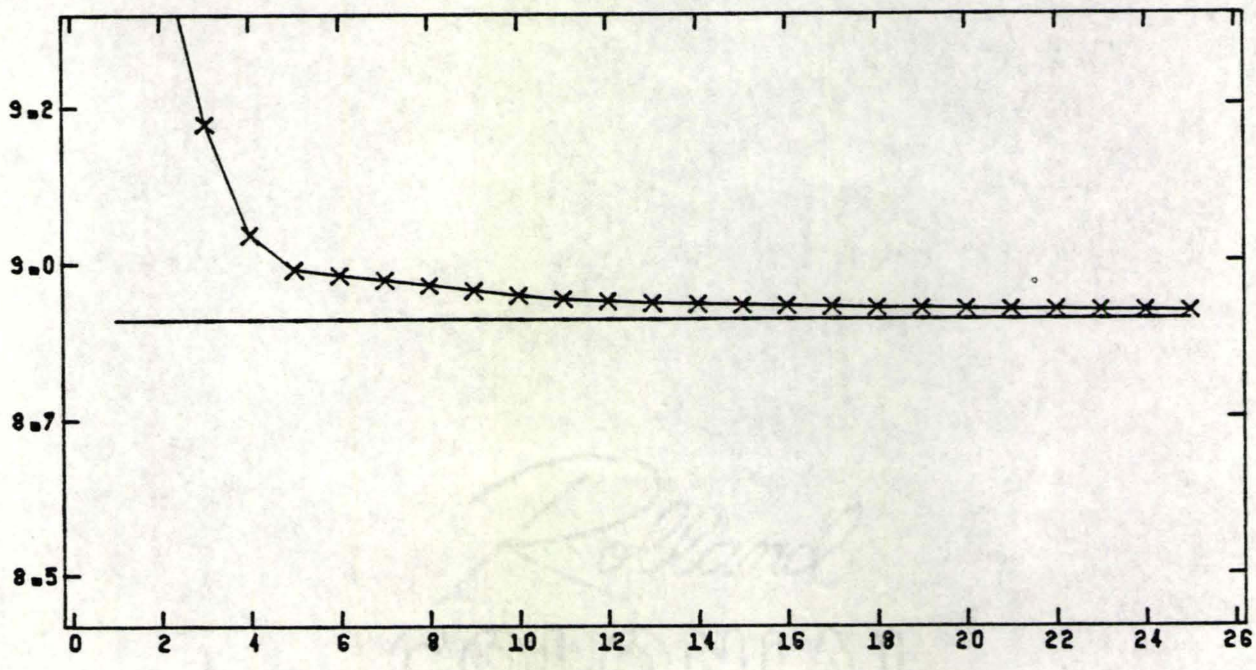


FIGURE 3.50 EFFECT OF EXCHANGE RATE SHOCK ON UNEMPLOYMENT RATE

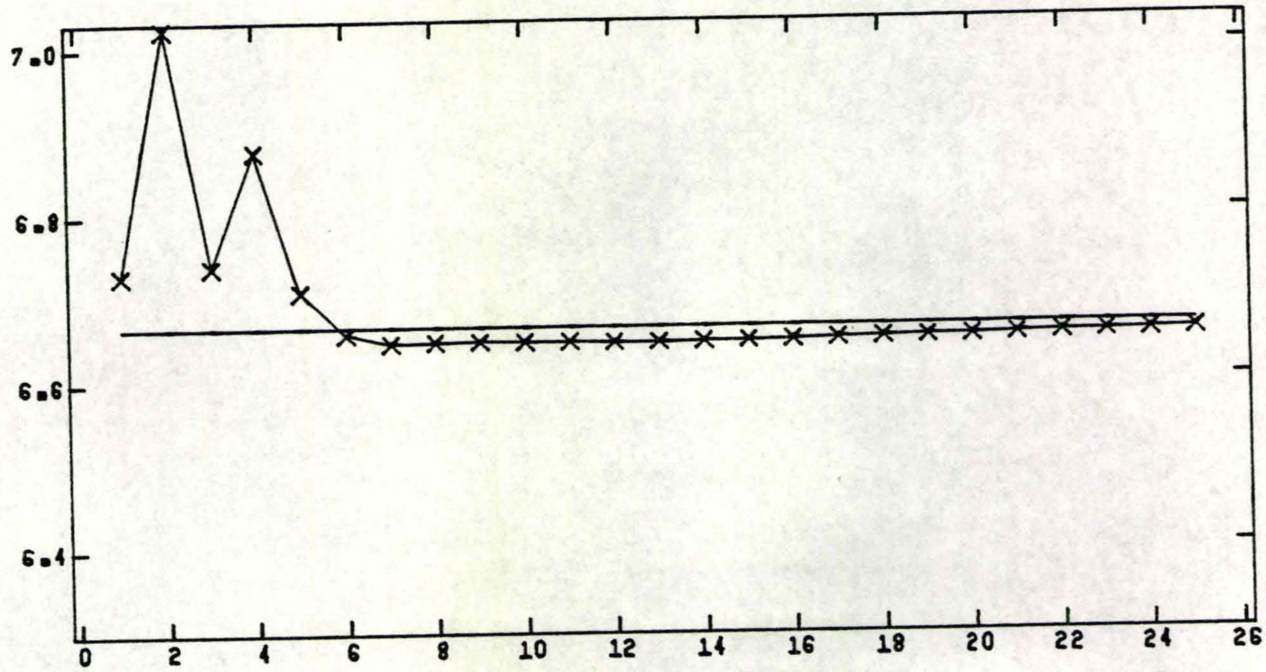


FIGURE 3.51 EFFECT OF EXCHANGE RATE SHOCK ON GROWTH RATE OF PRICE LEVEL

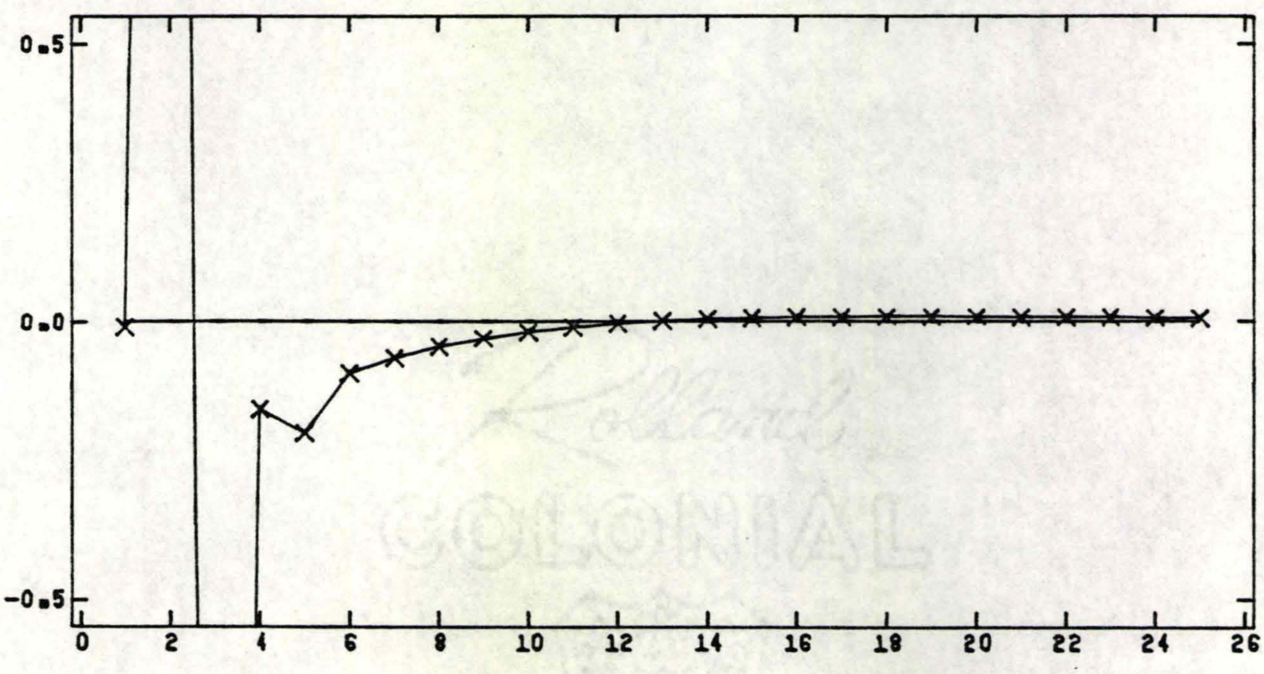


FIGURE 3.52 EFFECT OF EXCHANGE RATE SHOCK ON GROWTH RATE OF WAGE RATE

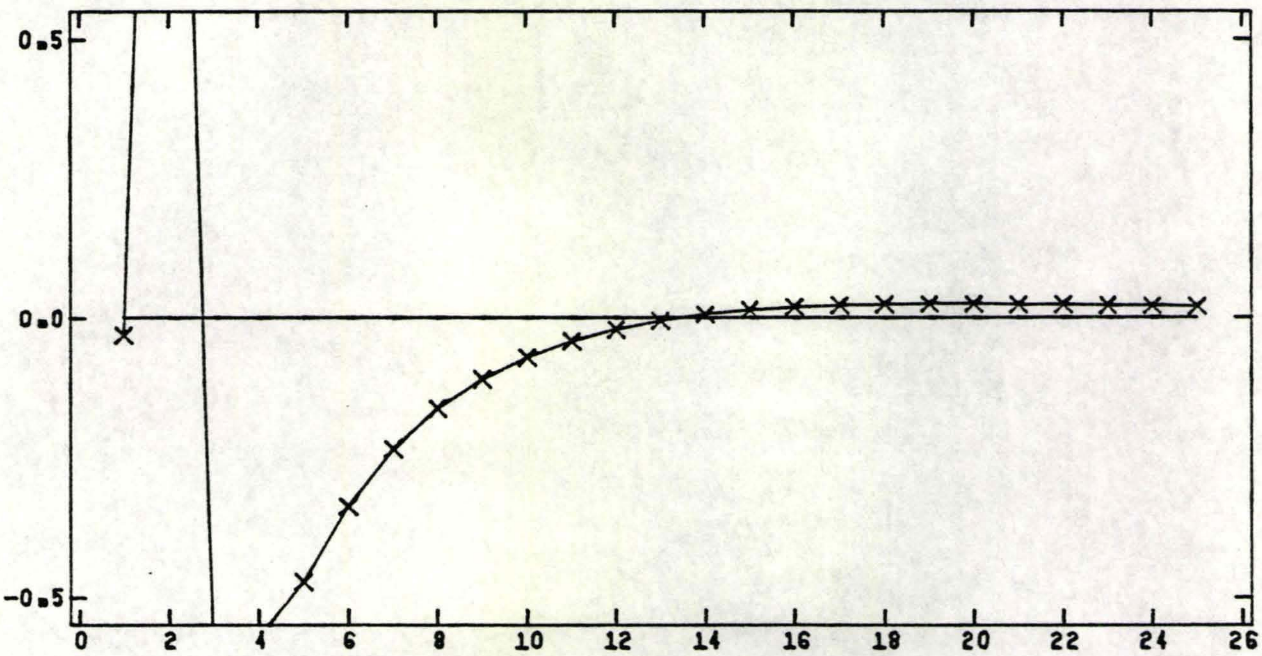


FIGURE 3.53 EFFECT OF EXCHANGE RATE SHOCK ON TAX REVENUE

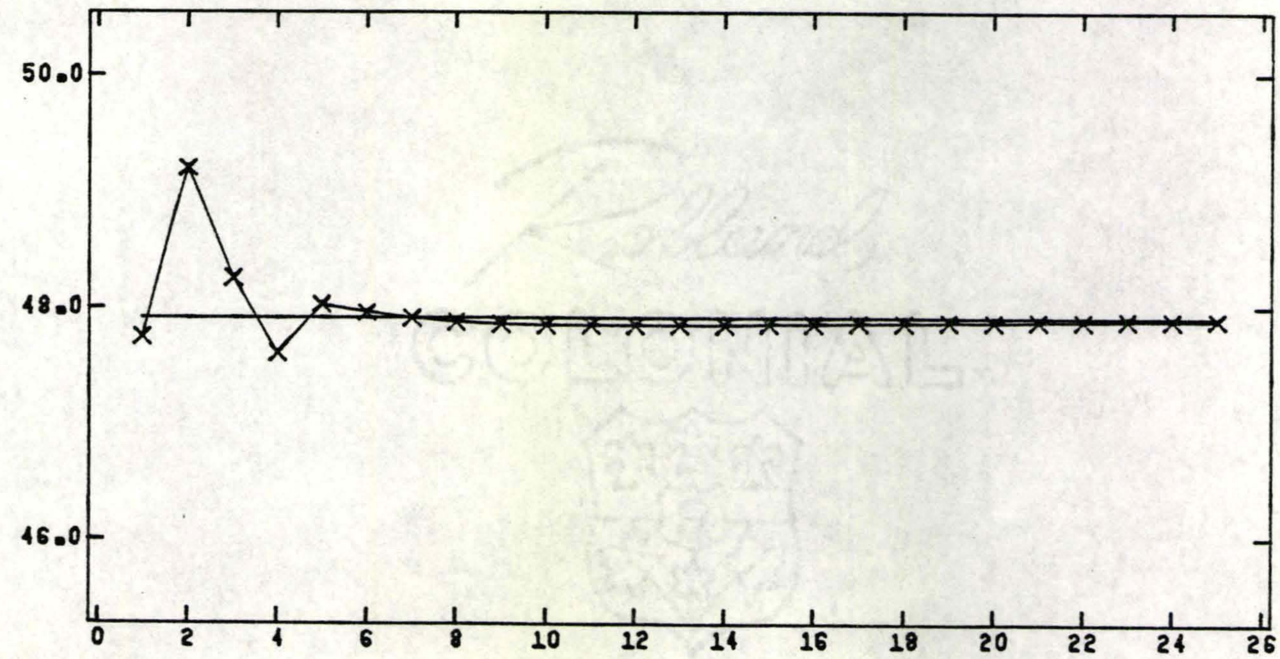


FIGURE 3.54 EFFECT OF EXCHANGE RATE SHOCK ON CAPITAL INFLOW

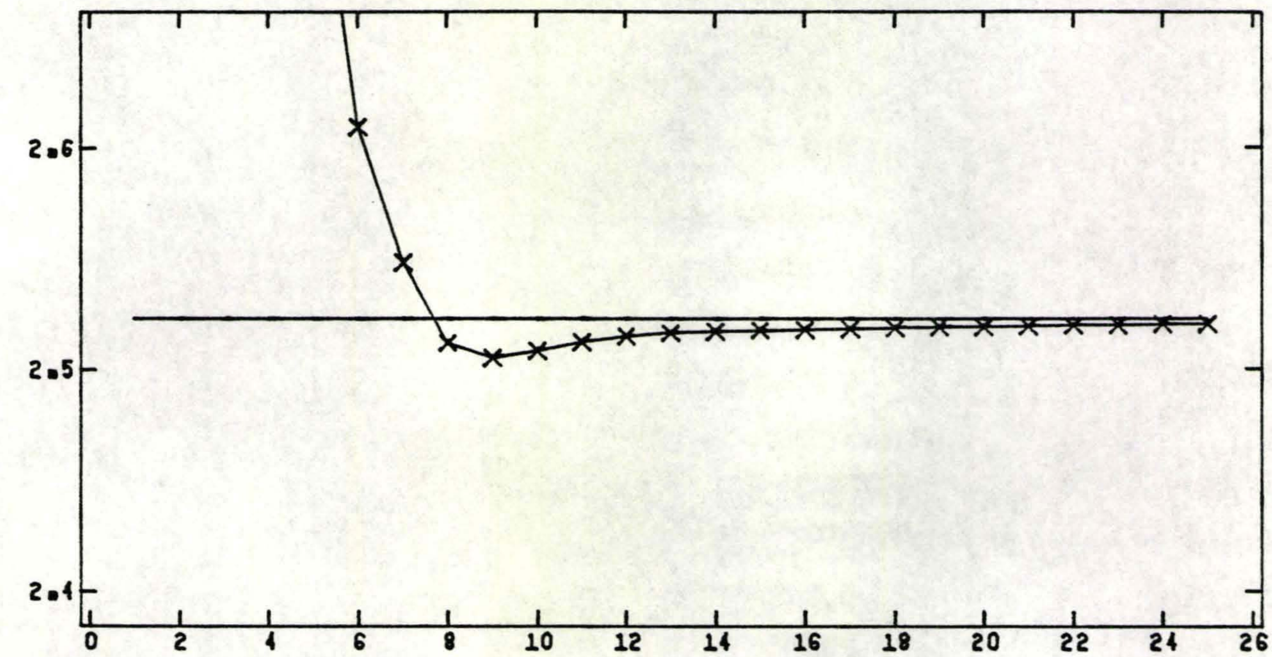
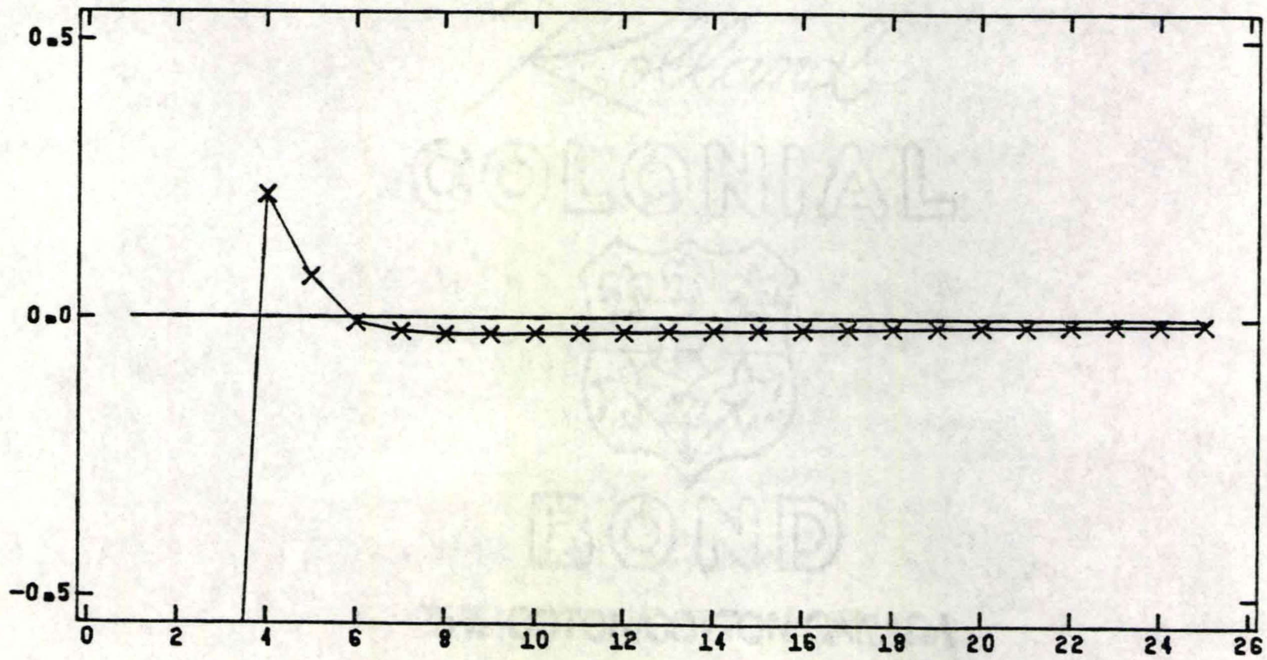


FIGURE 3.55 EFFECT OF EXCHANGE RATE SHOCK ON BALANCE OF PAYMENTS



the rates of increase of the wage rate and prices were lower. All of this occurred in the initial year, but in the long run all variables converged to their steady-state values. Although stability was demonstrated in this experiment, the model showed some oscillations before settling down to its original state. Moreover, when the magnitude of the shock was amplified (by doubling the exchange rate) the response was very sharp and oscillatory, and the model required a far greater amount of time before showing signs of convergence to a steady-state.

OTHER SHOCKS. The dynamics and stability of the model were also tested by studying its response to shocks coming from other sources. The effects of higher taxes, for example, brought about by increasing either the tax rate or the surtax, which is the intercept in the tax function, had almost the opposite effect from an increase in public spending, and posed no stability problem whatsoever. Another experiment was carried out in which the U.S. short-term interest rate was doubled. This led to outflow of capital and an increased balance of payments deficit which, in turn caused a decrease in the money base to satisfy the government budget constraint. The decrease in the quantity of money pushed domestic interest rates up and consumption and investment down, and therefore, reduced the value of GNP. This caused a higher unemployment rate, a lower rate of change in the wage rate and lower inflation, and aggravated the balance of payments deficit by decreasing exports and increasing imports. However, after about 6 periods all variables started to return to their original steady-state values. The model proved to be stable also in response to a U.S. price shock, which led to a balance of payments surplus and higher GNP in the impact period.

3.4 CONCLUDING REMARKS

In this chapter, the ability of the model to describe the actual performance of the Canadian economy over the period 1962-1982 was evaluated by conducting ex-post historical simulations. This has shown that the model's ability to replicate the actual historical paths of most endogenous variables is, on the whole, satisfactory. Having established this, it can be argued that the conclusions that will be drawn when the model is used for policy analysis are meaningful and useful in the Canadian context. Since this analysis will be carried over the short and long term, the stability of the model is of major importance. The model was solved for a zero-growth steady state, and a series of shocks were then administered to examine the stability and dynamic properties of the model in relation to the steady state. In all the experiments carried out, only the bond-financed government spending shock indicated instability of the model which was not surprising, since it is consistent with existing theoretical and empirical work.

Having established that the model is representative of the Canadian economy and that it is stable permits us now to move to the next step. There, an objective function will be specified and optimized, subject to the constraint of the macro model, in order to investigate the tradeoff relationship between the short-run and long-run effects of economic policies.

FOOTNOTES

Chapter 3

1. The discussion of types of simulation with econometric models is based, in part, on Chapters 11-14 in Pindyck and Rubinfeld [1981].

2. All simulation experiments were conducted using the TSP (Time Series Processor) programme. Two simulation methods are available in this program: SOLVE and SIML. The latter, which is based on Newton's Method, was used since it proved to be more powerful in the case of nonlinear models.

3. This point has been emphasized by Knight and Wymer [1978].

4. By and large, the values do seem to be reasonable except for the short and long-term interest rates, which are high, especially for a case with zero inflation and no technological change.

5. Another method of doing this consists of linearizing the model about the steady state and determining its stability properties using the eigenvalues of the linear model. If these eigenvalues are less than one, then the linearized model is stable. However, there is no guarantee that the full nonlinear model will exhibit the same stability properties.

6. The steady-state we have derived is not unique and need not be. Thus, the model could have not shown the same stability properties, had we started from another set of initial conditions.

7. The inflow of capital in the first three periods is not shown in Figure 3.26 since it is outside the dimensions of the plot.

8. The choice of the time period in this experiment and the others is more or less arbitrary. In this case, the 26 years were sufficient to prove the point concerning the stability properties of the model.

APPENDIX

THE STEADY-STATE SOLUTION VALUES OF THE MODEL

B	CF	CON	e	FINV
29131.0	2522.53	46018.4	1.7748	3.0
G	H	IPPD	IPU	K
19795.0	7558.0	5183.45	129.8	4833.710
M	NFINV	P	Pf	PM
22602.2	-2564.45	1.5105	1.1123	1.9742
PX	PU	Q	QP	RL
1.1744	1.058	97288.0	127804	0.0891
RS	T	T1	TRP	TRAN
0.0703	47905.9	0.3295	11198.0	344.0
UR*	X	YD		
0.066	35919.6	50146.8		

CHAPTER 4

FORMULATION AND SOLUTION OF THE CONTROL PROBLEM

4.1 INTRODUCTION

The control problem consists, as mentioned earlier, of maximizing an objective function subject to the constraint of a macroeconometric model. Having specified, estimated and tested the macro model in the preceding two chapters, the logical step to be taken in this chapter is the specification of the objective function and its arguments. Also, a brief discussion of the algorithm used to compute the optimal solution is in order.

The specification of a suitable objective function represents a major problem in the process of optimal decision making. This is particularly true in the case where the prescription of new policy and/or the evaluation of historical policy is the object of analysis. Here the objective function is the main criterion for decision and evaluation, and should therefore be representative of the goals pursued by the government, and/or a good approximation to its historical objectives. This problem is somewhat less important in the present study because our main purpose is neither to prescribe nor to evaluate economic policies, but rather to investigate the intertemporal tradeoff implicitly imbedded in policy formulation. This can be accomplished by using an acceptable and computationally convenient objective function, as discussed in the next section.

4.2 THE OBJECTIVE FUNCTION

This study uses a quadratic loss function penalising the weighted sum of squared deviations of the target variables from their desired values over a chosen finite time horizon. It is of the form

$$L = \sum_{t=1}^{T} (y_t - y_t^*)' (Q \cdot \lambda^t)(y_t - y_t^*)$$

where y_t and y_t^* are vectors of computed and desired values of the targets, respectively, Q is a known diagonal positive semi-definite matrix, and λ is a discount factor that can be given different values reflecting different time preferences. Note that, as formulated by Chow [1975], the vector y_t may include both state and control variables that are targeted in the objective function[1].

The inclusion of policy instruments in the objective function has been justified on the ground that policy makers are not indifferent to the use of these instruments [Holbrook 1973, 1974]. The cost associated with the use of some control variables may be excessive. This reflects the social, administrative, and political constraints on changing public policy[2]. But when it comes to solving a control problem, the above argument seems to be peripheral. From a practical point of view, the main reason for including control variables in the objective function is to avoid instrument instability. This is a significant problem in control theory applications, especially when the desired values of the targets are set at very ambitious levels relative to their historical values. An idealistic approach to target setting may prevent the computation of a solution at all by requiring the intensive use of instruments, which in turn leads to an explosive search for a solution and instability in the computational algorithm. Conversely, if the use of instruments is severely constrained, then we would lose much on target performance. Thus, as long as a solution exists, a trade-off can be traced between the weights attached to the instruments, on one hand, and the achievement of targets, on the other. Within a feasible region, the penalties imposed on the use of instruments can be chosen in such a way as to reflect the institutional factors mentioned above.

A quadratic objective function is widely used in most optimal control studies because it is simple to implement and gives a simple measure of the welfare cost in terms of weighted squared deviations of the variables from their desired values. Furthermore, the mathematical properties it possesses, such as convexity and differentiability, that are required to compute an optimal solution, and the characteristics of the solution, are highly desirable and have been extensively researched. However, two shortcomings of the quadratic objective function, have been traditionally cited in the literature. First, it is not a satisfactory representation of preferences. In any given period of time, exact desired values for policy targets pursued by the policy maker may never exist [Friedman 1975]. These preferences can be best represented by a range, rather than a fixed value for each target. Secondly, a more serious defect of a quadratic objective function is the implication that deviations of a target or instrument from its desired value are equally costly regardless of the direction of the deviation. In the present context, however, this symmetry problem may not be serious because of the way the desired values for the target variables have been chosen, as shown in the next section.

4.2.1 TARGETS AND INSTRUMENTS

The targets chosen to be included in the objective function are the rate of growth of output[3], the unemployment rate, the inflation rate and the balance of payments. The instruments to be used to steer these targets to their desired values are government expenditures, the change in the money base, a surtax (the intercept in the tax function), and the exchange rate. All the policy instruments are weighted in the objective function in order to avoid the instrument instability problems mentioned above, and to represent institutional constraints on the extent to which these instruments can be used. The fact that the change in the money base is penalised in the objective function, while that of bonds is not, is expected (due to government budget constraint) to create instability, as experienced in the previous chapter. If, in the course of the analysis reported in the next chapter, instability does arise, then a penalty will be imposed on government borrowing in order to avoid this problem.

The desired trajectories for the four target variables were chosen as follows: First, the means of the four variables were computed over the estimation period and used as the desired values in the objective function in the first experiment. The optimal solution obtained from this run showed that the computed values overshoot favourably their targets in many instances, thus leading to a high welfare 'cost', which is not a true cost, arising form the symmetry problem characterizing a quadratic function. In order to avoid this, the desired trajectories of the targets were then shifted slowly away from their historical averages, and in each case a new set of optimal values was computed. This continues until a solution was reached in which the welfare cost represented only unfavourable deviations of the targeted variables from their desired values. These values were then chosen to be included in the objective function in its final form. The desired trajectories of the instruments were set equal to their historical averages. These values are presented in Table 4.1.

The above procedure may seem somewhat ad hoc in nature, and to have subordinated the choice of the desired targets to the problem of symmetry in the objective function. However, the procedure is appropiate for two reasons. First, as pointed out earlier, we are not passing judgement on the merits of the policies pursued by the government over the historical period. The chosen desired values for the targets were the same in all of the optimal control experiments and have little effect, if any, on the results and conclusions to be drawn from this study. Secondly, the effect of a symmetric quadratic objective function that penalizes deviations, whatever their direction, is quite serious in the present context. This is so because the changes in those deviations and the associated welfare costs are the very basis on which the results as to the tradeoff between short and long-run economic policies are based. If these costs were misleading, so too would be the results obtained from the optimal control experiments.

4.2.2 LOSS FUNCTION WEIGHTS

Another important set of arguments to be included in the objective function are the weights attached to the target

TABLE 4.1 HISTORICAL AVERAGES AND DESIRED VALUES FOR TARGET AND INSTRUMENT VARIABLES

Target variables	Historical Average	Desired Value
% Rate of Growth (GNP)	4.3	5.0
% Unemployment Rate	6.2	4.0
% Rate of Change of Prices	7.1	4.0
Balance of Payments Surplus (million $)	98.0	0.0
Instrument Variables		
% Rate of Growth of Government (%) Expenditures	4.1	4.1
Exchange Rate (Can $/US $)	1.0774	1.0774
Change in Money Base (million $)	729.3	729.3
Surtax (Tax Function Intercept) (million $)	-2849.4	-2849.4

and instrument variables.

The role that the weights play in the optimization process is, of course, crucial. They represent the preference ordering of the policy maker, provide a useful way (especially for the instruments) of avoiding instability and incorporating institutional rigidities in the problem at hand. More importantly, a change in the relative structure of these weights permits the investigation of the tradeoff between different targets at any particular point in time or the tradeoff between the same set of targets at different points in time. This can be accomplished in the first case by changing the weights in the matrix Q, keeping the discount factor λ the same; and in the second case by changing λ, keeping Q the same.

In all respects, the choice of desired trajectories and the assignment of weights are closely related. If a target variable is to be effectively steered towards its desired value, a high weight or penalty can be attached to it, or the desired value can be set at a more ambitious level. On the other hand, if an instrument is to be widely used, then a low penalty can be attached to it, or the constraint inposed on its use may be partly relaxed by adjusting its desired value upward or downward, depending on the instrument in question. In the contrary case, the effective use of an instrument may be curtailed by imposing a high penalty on it or by adjusting its desired value accordingly.

Important as they are, the choice of the weights is a difficult task and has been always subject to a great deal of arbitrariness. However, the absolute values of the weights, per se, are of no great importance when a quadratic function is involved. Rather, their relative structure is the relevant element, especially in the case where the tradeoff between targets is the focal point of analysis.

There are several approaches to the choice of the weight coefficients. One approach consists of choosing the weights relative to a particular variable whose weight is normalized, taking into consideration the differences in units in which the variables are measured [Pindyck 1972, Matthieu 1976]. Another approach is to use an interactive procedure suggested by Rustem et al.[1979]. After an initial run has been completed, the policy-maker is asked to reveal his preferences and reservations about the 'optimal' paths shown to him. These reservations are then translated back into the weights of the objective function. The process continues until the results become satisfactory in the view of the policy maker. A third approach, which is used here, is to derive the weights from some measures of dispersion around the target variables. The measures chosen in this study are the variances of the target and instrument variables around their historical means. The inverse of the historical variance of each target variable scaled upward by a factor of 10^6, is used as a weight for that particular variable[4].

$$W_i = \{1 / \frac{1}{T} \sum_{t=1}^{T} (y_{it} - \bar{y}_i)^2\} \cdot 10^6$$

where W_i is weight for the i^{th} variable, y_{it} is the value of the i^{th} variable in year t, and $\bar{y}_i$ is its historical average.

This technique takes care of the scaling problem.

Moreover, this approach implies that the greater the variability of a variable historically, the smaller is the weight attached to it. If, for example, the variance of a policy instrument were small, indicating its moderate use and fluctuation, perhaps because of institutional constraints, then its coefficient would be large, and its use would thereby be restricted, in effect. Thus instead of making our own judgement about the intensity with which a particular instrument should be used, the weights, as computed, reflect the historical pattern, which is assumed to prevail also during the period of experimentation. The same argument applies with regard to the target variables. If a target variable has a small variance, then its weight coefficient will be high reflecting the fact that policy makers attach great importance to its stability.

One severe limitation of applying the same criteria to the choice of the instrument and target variable weights is that the use of instruments may be effectively reduced, and this may severely constrain the optimization problem. For this reason, the weights on the control variables were scaled down by a factor of 10 relative to those of the targets, allowing more freedom in their use. The structure of the weighting system is presented in Table 4.2.

4.3 SOLUTION OF THE CONTROL PROBLEM

There are basically two general approaches to solving an optimal control problem. The first applies the Lagrange-multiplier

TABLE 4.2 THE STRUCTURE OF WEIGHTS IN THE OBJECTIVE FUNCTION

Target Variable Weights		
W_Q	GNP	0.02
W_U	Unemployment Rate	2.93×10^9
W_P	Inflation Rate	7.88×10^6
W_R	Balance of Payments	35.32
Instrument Variable Weights		
W_G	Government Expenditures	0.0687
W_e	Exchange Rate	1.87×10^7
W_H	Change in the Money Base	3.427
W_T	Surtax	108.16

technique, which consists of absorbing the constraints into the objective function, and then optimizing [Chow 1975; Pindyck 1973]. The second requires the elimination of constraints by substituting them into the objective function and finding the unconstrained optimum [Holbrook 1973; Fair 1974].

The algorithm used in the next chapter is due to Chow [1975]. Formally, the problem is as follows:

$$\text{Minimize } E(L) = \{\sum_{t=1}^{T}(y_t - y_t^*)'(Q\cdot\lambda^t)(y_t - y_t^*)\} \tag{1}$$

subject to the nonlinear constraints represented by the model of Chapter 2, which can be represented as

$$y_t = \phi(y_t, y_{t-1}, \ldots, y_{t-m}, x_t \ldots x_{t-n}, z_t) + u_t \tag{2}$$

where

y_t is a vector of endogenous variables (state variables)

x_t is a vector of exogenous variables (control variables)

z_t is a vector of exogenous variables not subject to control

u_t is a vector of random errors with zero means and variance-covariance matrix V. It is serially uncorrelated.

ϕ is a (nonlinear) functional operator.

The algorithm involves the following steps:

(1) The model (2) is converted into state space form. That is, for a discrete time model, the dynamics of the system are described by a set of first-order difference equations. All the lagged endogenous variables dated prior to t-1 are eliminated by introducing new identities of the form $y_{kt} = y_{i,t-1}$[5]. Then (2) can be rewritten as

$$y_t = \phi(y_t, y_{t-1}, x_t, x_{t-1}, z_t) + u_t \tag{3}$$

It is of interest to digress at this point and note that most optimal control studies have used discrete time models for two main reasons. First, they are more appropriate for economic systems, since measurements are taken at discrete time intervals and instruments are used discretely. Second, a discrete time model is easier to implement in the case of dynamic programming. In continuous time deterministic models, when the constraints are represented in differential equation form, the maximum or minimum principle of Pontryagin proves to be more useful but more complicated in its application[6].

(2) Initially the model must be linearized. Solving the deterministic part (setting $u_t = 0$) of the model by the Gauss-Seidel interative method gives y_t^0 as an initial deterministic solution vector. Then, assuming a trial solution x_t^0 for the control variables, the model can be linearized around y_t^0, x_t^0 and y_{t-1}^0, as given. This gives

$$y_t = y_t^0 + B_{1t}(y_t - y_t^0) + B_{2t}(y_{t-1} - y_{t-1}^0) + B_{3t}(x_t - x_t^0) + \varepsilon_t$$

where B_{1t}, B_{2t} and B_{3t} are the partial derivatives of the vector function ϕ with respect to y_t, y_{t-1} and x_t, respectively. Thus (3) can be written as

$$y_t = A_t y_{t-1} + C_t x_t + b_t + \varepsilon_t \tag{4}$$

where

$$A_t = (I - B_{1t})^{-1} B_{2t}$$

$$C_t = (I - B_{1t})^{-1} B_{3t}$$

$$\varepsilon_t = (I - B_{1t})^{-1} u_t$$

$$b_t = y_t^0 - A_t y_{t-1}^0 - C_t x_t^0$$

(3) The solution then proceeds by minimizing the objective function (1) subject to the linearized model as expressed in (4). This problem can be decomposed into two problems: deterministic and stochastic.

a) The deterministic problem consists of minimizing

$$\frac{1}{2} \sum_{t=1}^{T} (\bar{y}_t - y_t^*)' Q_t (\bar{y}_t - y_t^*) \tag{5}$$

subject to

$$\bar{y}_t = A_t \bar{y}_{t-1} + C_t \bar{x}_t + b_t \tag{6}$$

by differentiating the following expression:

$$\frac{1}{2} \sum_{t=1}^{T} (\bar{y}_t - y_t^*)' Q_t (\bar{y}_t - y_t^*)$$

$$- \sum_{t=1}^{T} \mu_t' (\bar{y}_t - A_t \bar{y}_{t-1} - C_t x_t - b_t) \tag{7}$$

with respect to $\bar{y}$, $\bar{x}_t$ and the vector of Lagrange multipliers μ_t and where $Q_t = Q \cdot \lambda^t$.

Following the principle of dynamic programming the solution is first found for the last period and then sequentially for each preceding period. The solution of this problem for the control variables for period T[7] is

$$\bar{x}_T = G_T \bar{y}_{T-1} + g_T \tag{8}$$

where

$$G_T = -(C_T' Q_T C_T)^{-1} C_T' Q_T A_T$$

$$g_T = -(C_T' Q_T C_T)^{-1} C_T' (Q_T b_T - Q_T y_T^*)$$

This procedure is repeated until $G_T, G_{T-1}, \ldots, G_1$ and $g_T, g_{T-1}, \ldots, g_1$ are calculated. Then $\bar{x}_t$ and $\bar{y}_t$ are calculated for $t = 1,2,\ldots,T$ using (6) and (8).

The resulting solution x_t is now used to replace the initial guess x_t^0 in step 2 and the minimization problem is repeated until convergence occurs in $\bar{x}_t$.

b) The existence of an error term makes the actual y_t deviate from $\bar{y}_t$ and, therefore, the optimal path $\bar{x}_t$ has to be adjusted in order to minimize these deviations. That is, the stochastic problem consists of minimizing

$$E \sum_{t=1} \hat{y}_t' Q_t \hat{y}_t \tag{9}$$

subject to

$$\hat{y}_t = A_t \hat{y}_{t-1} + C_t \hat{x}_t + \varepsilon_t \tag{10}$$

obtained by subtracting (6) from (4) and defining $\hat{y}_t = y_t - \bar{y}_t$ and $\hat{x}_t = x_t - \bar{x}_t$. It was shown by Chow [1975] that the matrix G_t in the decision rule $\hat{x}_t = G_t \hat{y}_{t-1}$ is identical with G_t in the deterministic problem. In other words, minimizing the variance of the vector $\hat{y}_t$ is compatible with steering the deterministic vectors $\bar{y}_t$ towards their desired targets.

(4) The optimal control equation x_t is obtained by combining the stochastic and deterministic optimal feedback equations. That is:

$$x_t = \bar{x}_t + \hat{x}_t = G_t\bar{y}_{t-1} + g_t + G_t\hat{y}_{t-1}$$

$$= G_t y_{t-1} + g_t \qquad (11)$$

Once (11) is reached, then the welfare cost associated with the given optimal control problem is obtained by adding its two components. The deterministic cost corresponds to the weighted squared values summed over time of $(\bar{y}_t - y_t^*)$, and the stochastic cost representing the weighted variances of $(y_t - \bar{y}_t)$ summed over time. These welfare costs can be used to compare alternative policies and assess the importance of stochastic disturbances in the optimization process.

In summary, the proposed method begins with a trial path x^0 for the control variables and y^0 as a solution to the nonlinear model obtained by the Gauss-Seidel iterative procedure. A linear approximation is computed for each period around these tentative paths. A linear feedback equation is then obtained by minimizing the given objective function subject to the linearized version of the model. The numerical values of the solution are used to generate a second tentative path (x^0, y^0), around which a second linear approximation is performed, and the process continues until convergence is reached[8].

4.4 SUMMARY

In this chapter, the objective function and its arguments were specified. We have argued that the choice of a quadratic function is justifiable mainly because of its simplicity in implementation and interpretation. The targets to be included in the objective function are: the rate of growth of output, the rate of change of prices, the unemployment rate and balance of payments equilibrium. The instruments that steer these targets toward their desired values are government expenditures, the surtax, the exchange rate and the change in the money base. The desired values of the instruments were set equal to their historical averages and those of the targets were chosen so as to avoid the problem of symmetry in a quadratic function. The weight of each target variable in the objective function was set equal to the inverse of its variance over the historical period. Finally, the algorithm developed by Chow [1975] to solve optimal control problems was briefly discussed.

FOOTNOTES

Chapter 4

1. From a mathematical point of view the distinction between target and instrument variables is no longer clear. Both have positive weights and enter in the same format in the objective function. It is needless to say, however, that the distinction between the two types of variables is still meaningful from the point of view of economic policy.

2. This argument has been advanced in virtually all studies using an optimal control approach in macroeconomic policy. See the bibliography for references.

3. Actually both GNP and G enter as levels in the objective function, with those levels chosen to represent a 5% rate of growth from the base year.

4. An exception occurs in the case of the surtax, for which the variance was based on the residuals in the estimated tax equation.

5. If, for example, the model includes a variable such as C_{t-3} then two identities should be introduced: $C_{1,t} = C_{t-1}$ and $C_{2,t} = C_{1,t-1}$. This permits the user to write C_{t-3} as $C_{2,t-1}$ and thus represent the model by a set of first-order difference equations. For details, see Pindyck [1973, pp. 155-59], or Chow [1975, pp. 153-54].

6. The Pontryagin principle can be put in discrete form, but according to Chow [1975], it does not accomplish any more than the method of Lagrange multipliers in this case.

7. A complete and detailed derivation of the optimal feedback equation can be found in Chow [1975, Chapter 7].

8. A computer program is available to execute all these steps. This program was developed at Princeton University by Chow and Butters [1977]. It is called 'optimal control of nonlinear systems' (OPTNL).

CHAPTER 5

SHORT-RUN VERSUS LONG-RUN IN MACROECONOMIC POLICY

5.1 INTRODUCTION

In this chapter, we shall describe and discuss several optimal control experiments that are carried out in order to determine whether or not a tradeoff exists between the performance of the economy in the short run and the long run. That is, the long-run implications of pursuing a myopic or short-sighted policy, and the short-run consequences of long-sightedness in policy making are explored. The optimal control approach is appropriate in this context, as explained earlier, since we are dealing with multiperiod optimization and because it enables us to trace out the "best" intertemporal tradeoff, a tradeoff that cannot be dominated by any other possibility [Chow, 1981].

Two general approaches are used to derive this optimal tradeoff: the first consists of giving different degrees of importance to the present vis à vis the future by changing the discount factor in the objective function while keeping the time horizon the same (Section 5.2). The second approach (Section 5.3) consists of carrying out the optimization over different time horizons when the discount factor is constant. This would give some indication as to the long-run implications of different planning horizons. In both cases an analysis of the results will be provided and a short-run/long-run tradeoff will be derived.

5.2 TIME PREFERENCE IN POLICY-MAKING

In this section, the impact of varying the weights attached to the set of policy targets and instruments overtime is investigated and a tradeoff relationship between short and long-run performance is derived. Here, we distinguish between three types of policy programs. First is a "Base" program, in which the discount factor λ in the objective function is set equal to 1 and kept the same over the whole time horizon. This implies that the policy maker is indifferent as to the performance of the economy between the present and the future and the acheivement of the targets is equally desirable in any period. Secondly, a "Short-run" program is specified in which the policy maker is interested more in the present than in the future. This assumption of short-sightedness in policy-making might be regarded as realistic given that governments are apt to be interested more in their re-election and the immediate effects of their actions than in costs that may be incurred in the long run. The third policy program is "Long-run" in the sense that the long run has more importance to the policy maker than the short run. This seems to be unrealistic, especially when the long run is in the distant future. Nevertheless, one may argue that governments resort in some circumstances to policies that are beneficial in the long run but costly in the short run in order to redirect some targets toward their long-run paths.

Moreover, this long-sighted program is useful for comparison of its outcome to those of the Base and Short-run programs.

Thus, given the objective function of Chapter 4,

$$L = \sum_{t=1}^{T} (y_t - y_t^*)'(Q\cdot\lambda^t)(y_t - y_t^*)$$

the value assigned to the discount factor λ indicates the type of program that is being implemented. The specifications are as follows:

Base program	$\lambda = 1$
Short-run program	$\lambda < 1$
Long-run program	$\lambda > 1$

A discount factor less than 1 implies that, as time goes by, less and less weight will be assigned to the targets. On the other hand, when $\lambda > 1$ the future has more weight than the present. However, the relative structure of the weights assigned to the variables in the objective function remains the same at each point in time. This facilitates the comparison of the results, since any differences can be attributed entirely to the differences in the intertemporal structure of the weights.

5.2.1. ANALYSIS OF THE RESULTS

Several experiments were performed for the purpose of analysing the implications of different rates of time preference.

It is useful, however, to analyse in detail the results of the Base program when λ = 1, which can be used as a point of reference for further analysis.

Base Program Experiment

In this experiment, the algorithm described in Chapter 4 is applied to the objective function of that Chapter, and the econometric model of Chapter 2, over the 1964-1982 period. Thus, the problem is to

$$\text{minimize } L = \sum_{t=1}^{19} \{W_Q(\hat{Q}_t - 5.0)^2 + W_U(UR_t - 4.0)^2 + W_p(\hat{P}_t - 4.0)^2$$

$$+ W_R(\Delta R - 0.0)^2\} + \sum_{t=1}^{19} W_G(G_t - 4.2)^2 + W_e(e_t - 1.0774)^2$$

$$+ W_H(\Delta H_t - 729.3)^2 + W_T(T_{0t} - 2849.4)^2\}^1$$

subject to $y_t = \phi(y_t, y_{t-1}, x_t, x_{t-1}, z_t) + u_t$

where $\hat{Q}$ = rate of growth of GNP

UR = unemployment rate

$\hat{P}$ = inflation rate

ΔR = balance of payments

$\hat{G}$ = rate of change of government expenditure

e = exchange rate, i.e., price of U.S. dollar in terms of Canadian dollar

ΔH = change in the money base

T_0 = surtax

and where W_{Y_i} is equal to $\frac{1}{var(Y_i)}$, as defined in the previous chapter.

The results of this experiment showed that the major factors in shaping the behaviour of the target and instrument variables are the existence of the government budget constraint and the interest paid on the public debt. The solution calls for the predominant use of fiscal measures and the exchange rate while monetary policy is contractionary and generally, the change in the money base is below its target. The reason that monetary policy was not used in a stimulative way in this experiment is due partly to the existence of the government budget constraint in the model. The fact that no penalty has been imposed on the extent to which government can issue bonds makes it easier to resort to borrowing than to increasing the money base, especially when a penalty is imposed on the use of the latter. The increase of interest payments on the debt, which comes about as a result of the increase in the stock of bonds as well as a higher interest rate, has to be met by increasing the debt more and more, and the cycle continues. This is the stability problem of bond-financing discussed in Chapter 3, which can be avoided only if taxes rise enough to offset the servicing of the debt. The two targets of high GNP growth and a low unemployment rate, call for a larger public debt. This is so because disposable income is positively related to the increase in interest payments on the debt, which then leads to a higher level of consumption spending, and therefore a higher level of GNP. Furthermore, the increase in GNP calls indirectly for a rise in the rate of interest, the positive effects of which on consumption spending -- through the interest payments on the debt -- outweigh the negative effects. Even a higher level of investment spending is compatible with a high rate of interest, since it is sensitive to the level of output as well as to the cost of borrowing. Having established these facts, it becomes easier to understand why the optimal Base Program shows high levels of short- and long-term nominal interest rates. The increase in the demand for money due to the increase in income, accompanied by a restrictive monetary policy pushes the short-term nominal rate to a staggering level of 22% at the end of the planning period.

In light of the above results, it is desirable to remove the undesirable and unrealistic behaviour of some of the endogenous variables. Thus, a penalty was imposed on government borrowing in the objective function for the purpose of limiting the explosive behaviour of the public debt and the accompanying high interest rates. Another term was therefore added to the objective function, and the experiment is repeated. A rate of growth of government bonds of 15% was chosen in such a way that the ratio of government debt to GNP follows its historical time path. The weight for this new variable was chosen in a manner similar to the other variables, i.e.,

$$W_B = \frac{1}{\mathrm{var}(B)} \; .$$

In terms of achievement of targets, the optimal solution to the revised problem was successful overall. The computed optimal value for GNP overshoots its targets in the first 4 years by a relatively small amount but on average it was lower that the target by about 2 billion dollars. As for the unemployment rate, the average value generated by the solution was around 5.5%, 1.5% higher than its desired value and ranging between a minimum value of 4 and a maximum value of 6.2 percentage points. The same kind of behaviour applies to the rate of change of prices, which fluctuated between 3.3 and 6.5%, averaging about 5.5% for the whole period. For the balance of payments equilibrium target, the solution shows that on average the balance of payments experienced a small deficit of 16 million dollars.

The question now is: How did these results come about? How did the endogenous variables of the system behave? More importantly, how did the four instruments react in order to steer the target variables toward their desired destinations?

Roughly speaking, the time paths of the policy instruments are identical to those of the previous run in which no weight was attached to government borrowing. However, in the present case, the use of monetary policy was mildly expansionary and the change in the money base taken as an average exceeded its target by just under a billion dollars. Government spending, on the other hand, was used more intensively and on average it was higher than its desired value by about 3.5 billion dollars. The exchange rate was above its target for only the first 4 years and then showed strong appreciation of the Canadian relative to the U.S. dollar. The tax surcharge was cut by about 900 million dollars on average, which is considered to be expansionary, given that it affects the model in the same way as government expenditures.

The 4.5% increase in GNP was brought about by an increase in its individual components moderated by a current account deficit. The average rate of growth of imports (6%) was much higher than that of exports (4%). The improvement in the terms of trade and the increase in income encouraged imports and discouraged exports. The rate of growth of potential GNP was about 4%, thus increasing the ratio of capacity utilization ($\frac{Q}{QP}$), which exerts a negative effect on the quantity of exports. A question arises in this case: if the GNP target called for an improvement in the current account and this required the devaluation of Canadian currency, why then does the exchange rate appreciate over the planning period? This question has to be answered by examining the other targets in the objective function. The rate of change of price target is sensitive to the price of imports and, therefore, requires the appreciation of the exchange rate. Although a current account deficit occurred, capital inflow increased due to high domestic interest rates; and, therefore, the balance of payments target was met. It appears from the results that the exchange rate instrument is directed mainly towards steering the inflation rate to its desired value. Thus, the need for appreciation outweighs the need for depreciation required for the growth of GNP, which can be achieved by other means. This growth target is consistent with the unemployment target which was close to its desired trajectory. As indicated above, the ratio of actual GNP to potential GNP was growing over time allowing the unemployment rate to remain relatively low, offsetting, in part, the exogenous effect of increases in the Canadian labour force over the simulation period.

Short-Run and Long-Run Experiments

We have so far described briefly the result of the Base program and the channels through which the policy instruments were able to steer the targets toward their desired paths. In what follows, a description of the short-run and long-run programs that correspond to λ = 0.8541 and λ =1.1708 will be provided. These particular values of λ were chosen in such a way that the short-sighted planner attaches 20 times more importance to the first period than the last period, while the long-run planner gives 20 times more weight to the last period than the first.

These results, along with those of the Base program are summarized graphically in Figures 5.1 to 5.8. These plots allow easy comparison of the three programs.

In the case of GNP (Figure 5.1), the time paths of the three solutions are very close. However, the short-run program yields a somewhat higher level of GNP than the long-run program, but almost coincides with the Base program path. This behaviour may be explained by the fact that putting emphasis on the present, relative to the future, will achieve an overall better performance of GNP. Thus, if we start with a high level of GNP at period t, then it is more likely that a higher level of GNP will be attained in the subsequent years, given the achievable growth rate of GNP.

Since the unemployment rate is negatively correlated with the level of GNP in the model, then one would expect the kind of behaviour shown in Figure 5.2. The unemployment rate is consistently higher in the long-run program than in the base program and consistently is higher in the latter than in the Short-run program. Another factor which may have rendered these results even more pronounced than they would otherwise have been, is the positive linkage of unemployment rate in the model have been, is the positive linkage of unemployment rate in the model to its lagged value. Therefore, a low rate in the present period would lead to a lower rate in the future. However, exceptions to this pattern occurred at both ends of the planning horizon. At the beginning, we have the additional constraints of the initial values, and more time is needed for the optimization to run its course, because of the lag structure of the model. On the other hand, strange behaviour may occur at the end of the period since we are penalizing the deviations from the desired targets only for a finite time horizon and in the last period the planner is not concerned with what happens in subsequent years[2].

The optimal time path of the inflation rate (Figure 5.3) indicates that the lowest rate is associated with the long-run experiment for most of the planning horizon. This kind of behaviour may be

FIG 5.1 OPTIMAL GNP IN THE THREE PROGRAMS

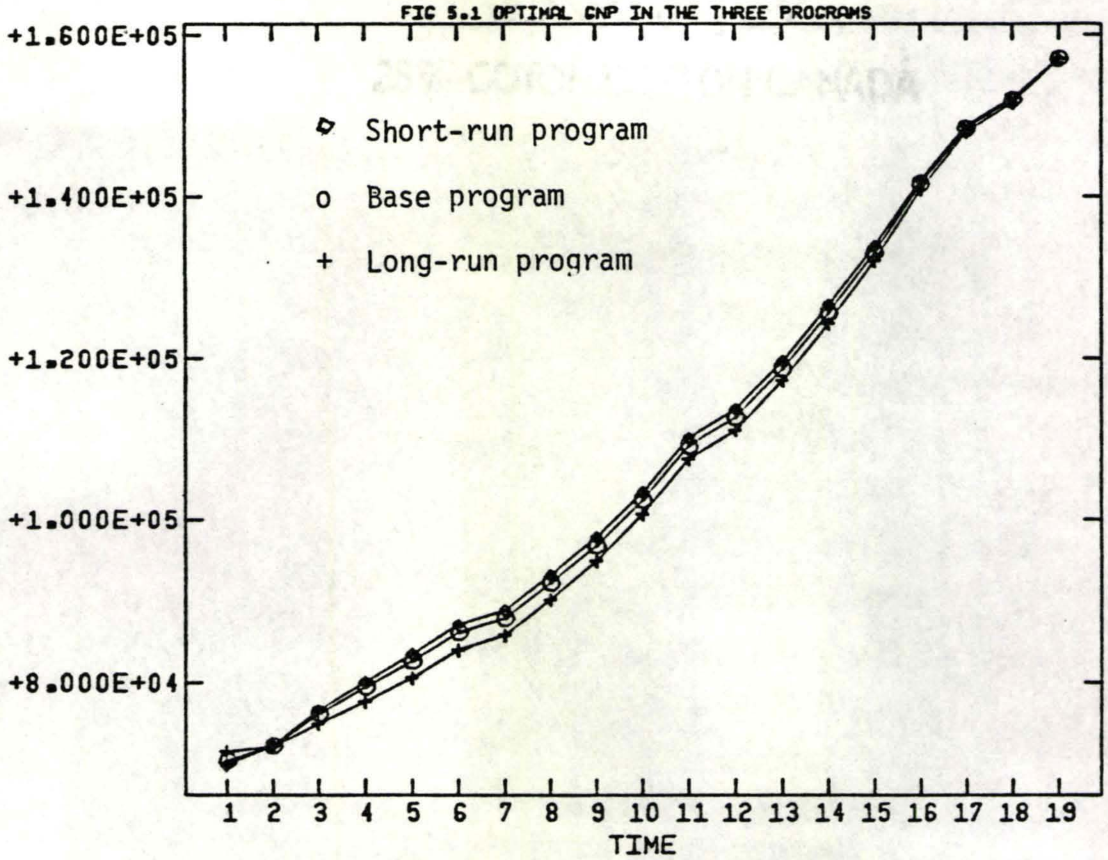
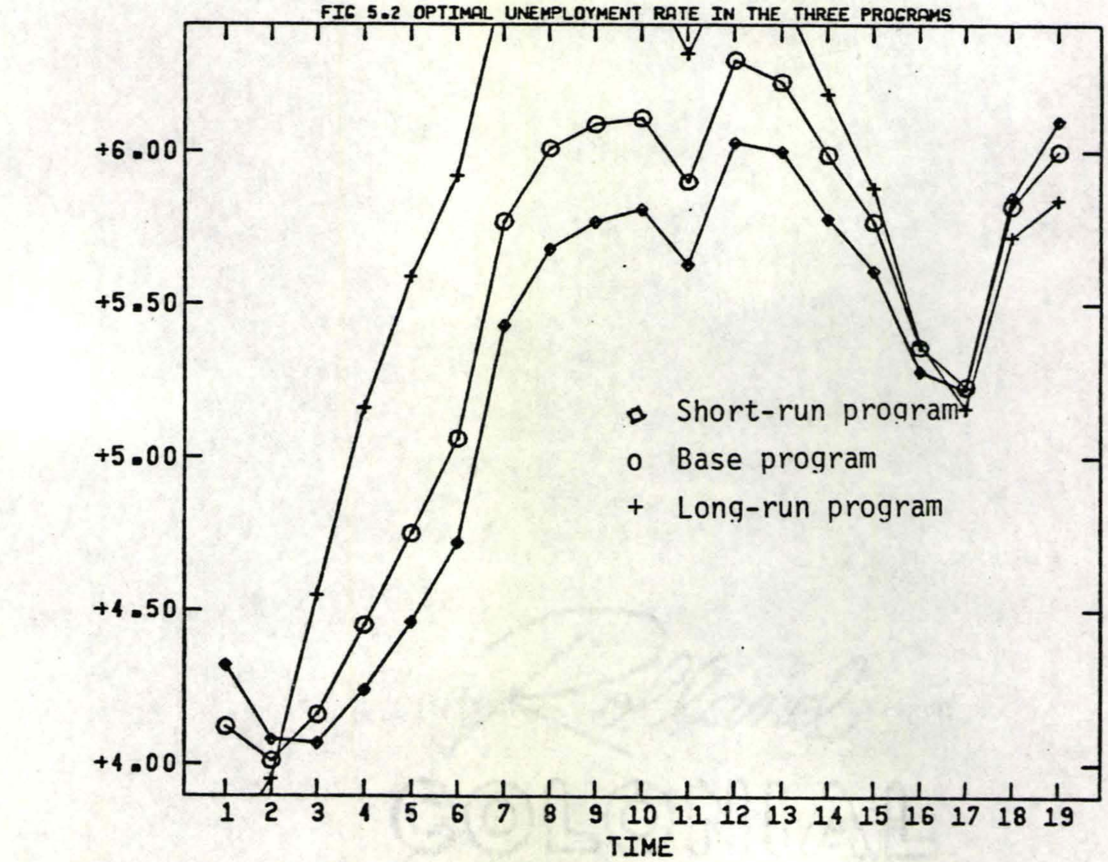


FIG 5.2 OPTIMAL UNEMPLOYMENT RATE IN THE THREE PROGRAMS



explained by referring to the optimal trajectories of the unemployment rate, the balance of payments and the exchange rate. As shown in Figure 5.4, the balance of payments experienced a surplus at the beginning and a deficit at the end of the period. However, the low weights in the early years of the optimization process in the Long-run program attached to the target variables imply that the need to control the balance of payments surplus is not pressing and the exchange rate need not be appreciating as fast as in the other two programs. That is why the inflation rate is above that in the other two programs in the first 4 years. But starting from the fifth year, this rate becomes lower and closer to the target until the end of the optimization period. This may be explained by the influence of the unemployment rate and GNP. As we have seen earlier, the lower GNP in the Long-run program generated a higher unemployment rate, and therefore a lower inflation rate. The tradeoff between unemployment and inflation embedded in the model may have helped the inflation rate to remain close to its target for the rest of the period in the long-sighted program, thus removing the need to appreciate the currency.

Another tradeoff between the balance of payments and inflation came out of these experiments. In the first few years, there was no conflict between achieving a low price level and equilibrium in the balance of payments (which was in surplus), since both targets require the appreciation of the exchange rate. But once a deficit in the balance of payments is developed, than a conflict

FIG 5.3 OPTIMAL INFLATION RATE IN THE THREE PROGRAMS

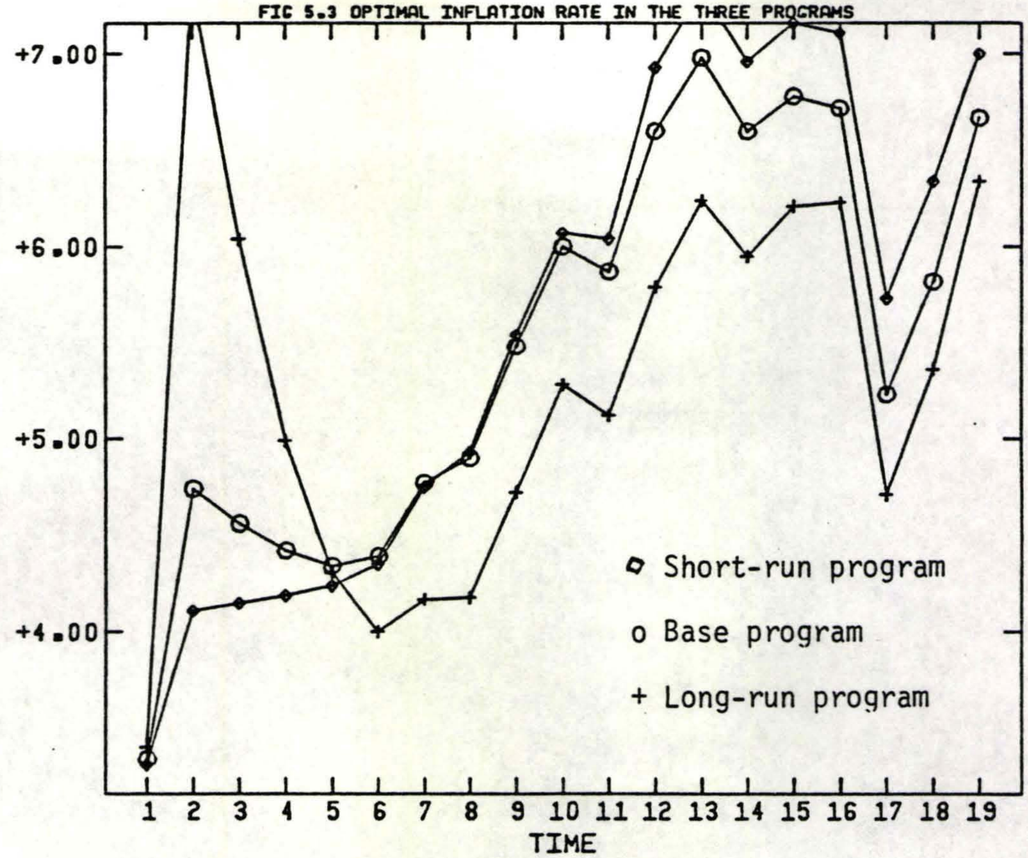
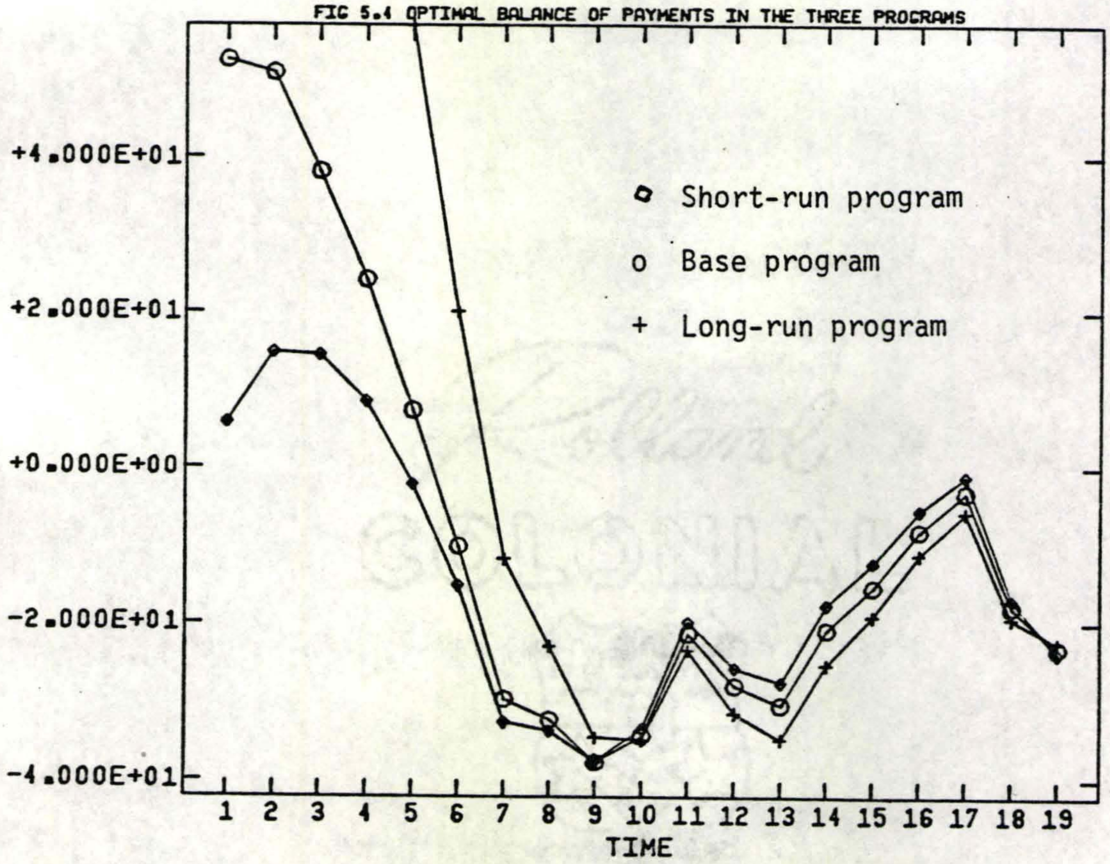


FIG 5.4 OPTIMAL BALANCE OF PAYMENTS IN THE THREE PROGRAMS



arises. To eliminate this deficit, the exchange rate should depreciate. But to keep the inflation rate close to its target, it should appreciate. As shown in Figures 5.3 and 5.4, while the inflation rate is closer to its target in the Long-run program, the balance of payments is further away from its target, relative to the other two programs. The opposite is true for the Short-run program.

The behaviour of the control variables depicted in Figures 5.5 to 5.8 show that the Short-run program is the most stimulative. Government expenditures are higher for most of the period since they are directed primarily to the growth and unemployment targets, which were closer to their values in the Short-run plan. The same remark can be made about the surtax instrument, which was cut more vigorously, and the exchange rate, which depreciated more, in the Short-run program. As for the change in the money base, there is little difference between the three cases except toward the end of the period. It is obvious, however, that monetary policy was used less intensively in the three programs, simply because it is less effective in moving the target variables in the model toward their target values.

5.2.2 SHORT-RUN/LONG-RUN TRADEOFF

The optimal control solutions for the three programs discussed above are based on the minimization of the sum of the squared deviations of the variables in the objective function from

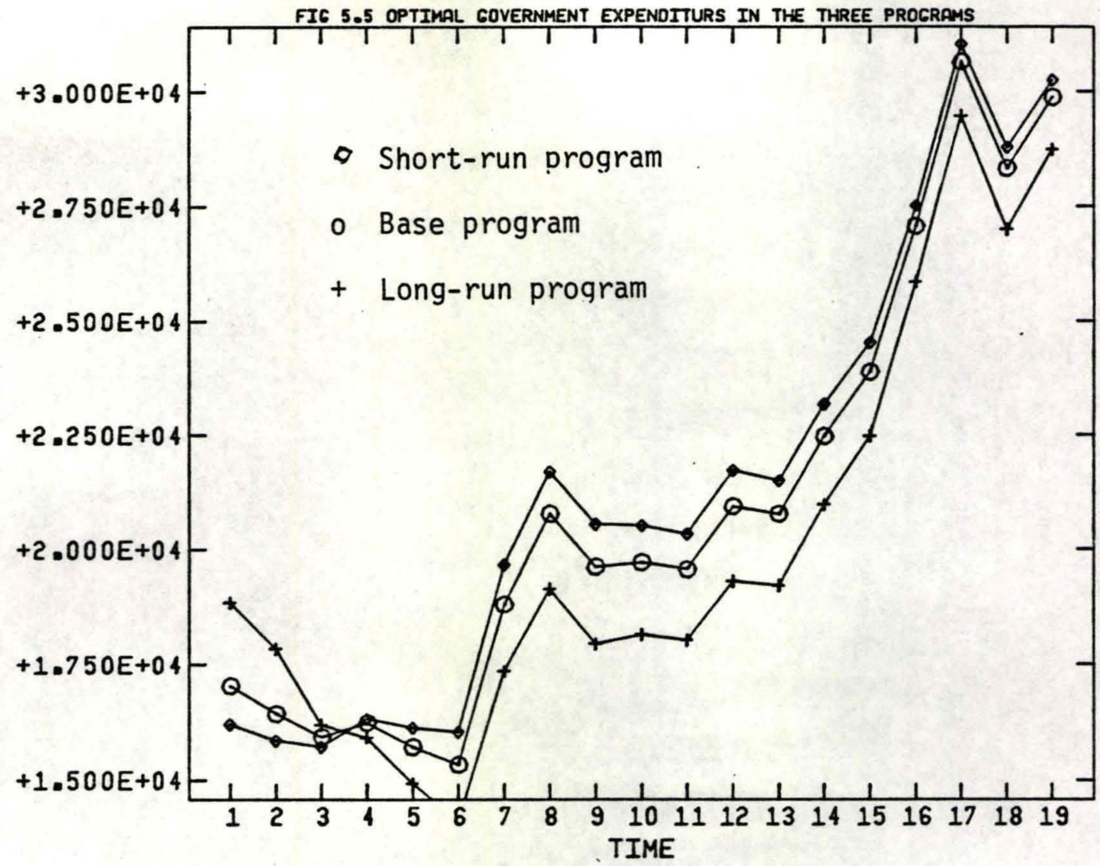


FIG 5.5 OPTIMAL GOVERNMENT EXPENDITURS IN THE THREE PROGRAMS
+3.000E+04
+2.750E+04
+2.500E+04
+2.250E+04
+2.000E+04
+1.750E+04
+1.500E+04
Short-run program
o Base program
+ Long-run program
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
TIME

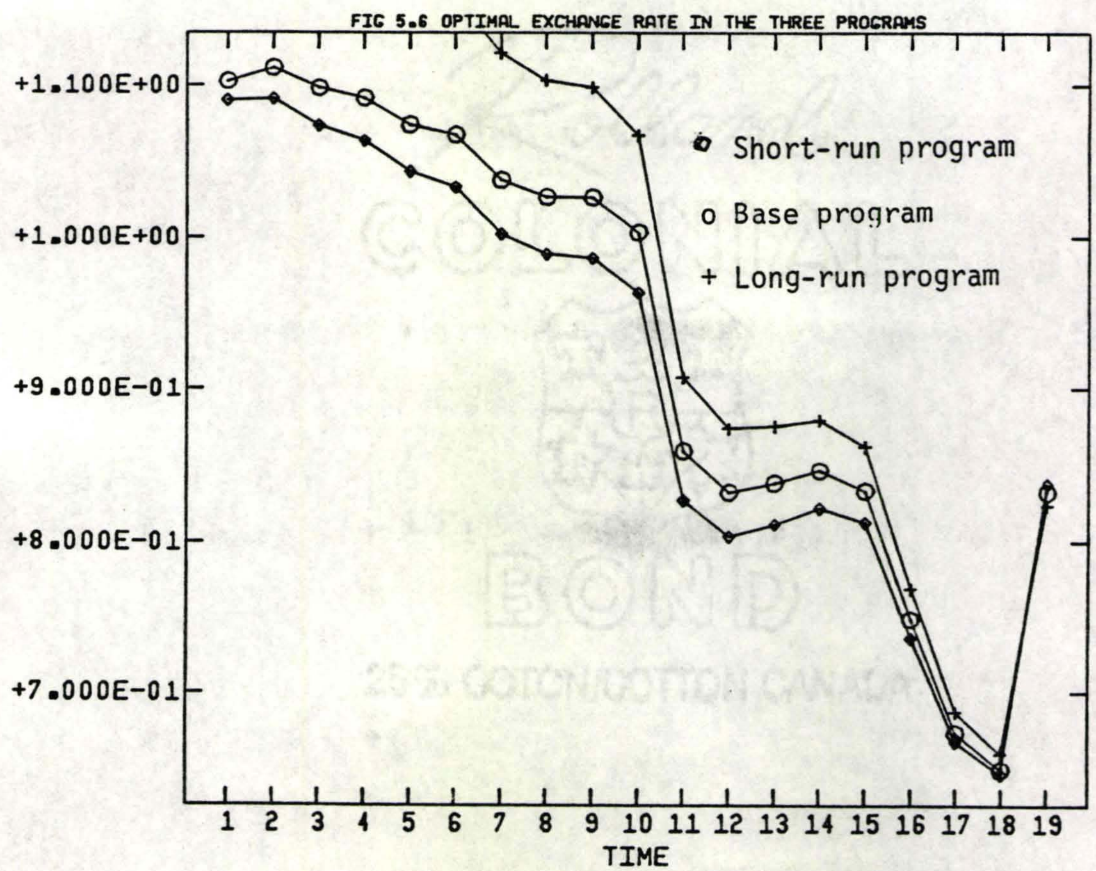


FIG 5.6 OPTIMAL EXCHANGE RATE IN THE THREE PROGRAMS
+1.100E+00
+1.000E+00
+9.000E-01
+8.000E-01
+7.000E-01
Short-run program
o Base program
+ Long-run program
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
TIME

FIG 5.7 OPTIMAL MONEY BASE IN THE THREE PROGRAMS

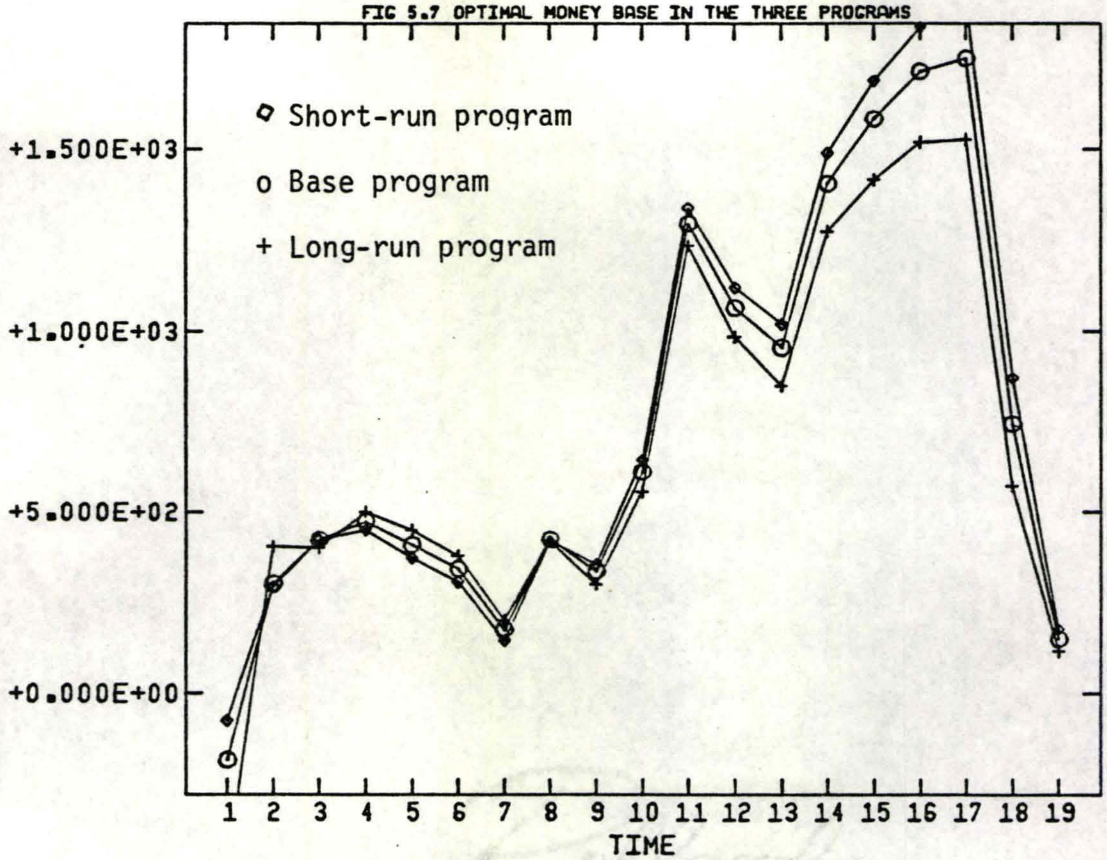
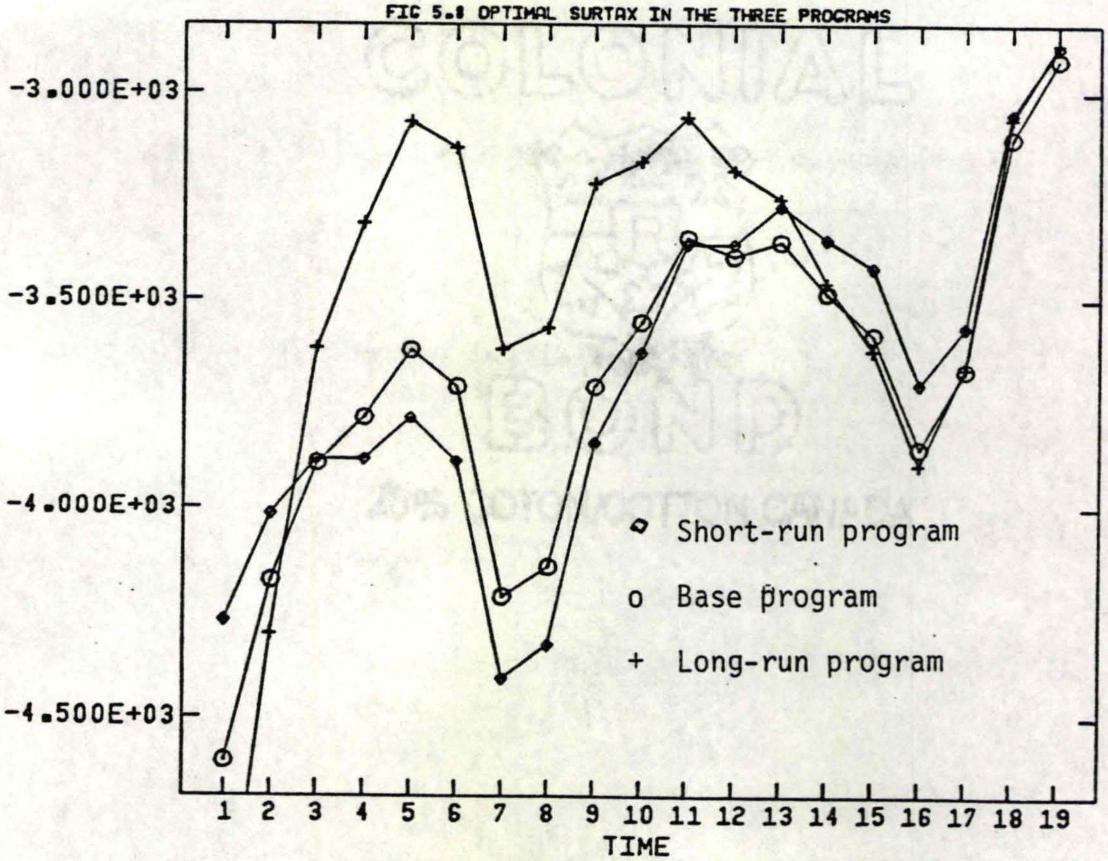


FIG 5.8 OPTIMAL SURTAX IN THE THREE PROGRAMS



their desired values, weighted by the weight matrix Q_t. While in the base program case Q_t is constant, in the other two cases it is changing from year to year by the discount factor λ, i.e., $Q_t = Q \cdot \lambda^t$. Thus, each variable is weighted differently between one period and another, but the relative weights among different variables at any point in time are the same. By varying the weights over time, one can trace out the tradeoff possibilities in terms of the squared deviations summed over all variables for each time period, and adjusting by the original weights Q. That is,

$$L_t = (Y_t - Y_t^*)' Q (Y_t - Y_t^*) \text{ for } t = 1,2,\ldots 19$$

The results of these computations are shown in Figure 5.9 where time runs along the horizontal axis and the welfare cost L_t along the vertical axis. Observe that the Long-sighted program has the highest cost in the first 10 years and the lowest cost in the second 10 years of the planning period. As expected, the Base program has generated a welfare loss time-path that lies between the other two, and the crossing point occurs at the middle of the planning period.

In figures 5.10 to 5.11 the welfare cost is shown for only the target and control variables, respectively. It is interesting to note that the gain realized in terms of target achievement in the short-sighted program is obtained by incurring more cost in the use of instrument variables. On the other hand, the Long-run planner uses his instruments less intensively but in return he is not as

FIG 5.9 TOTAL WELFARE COST

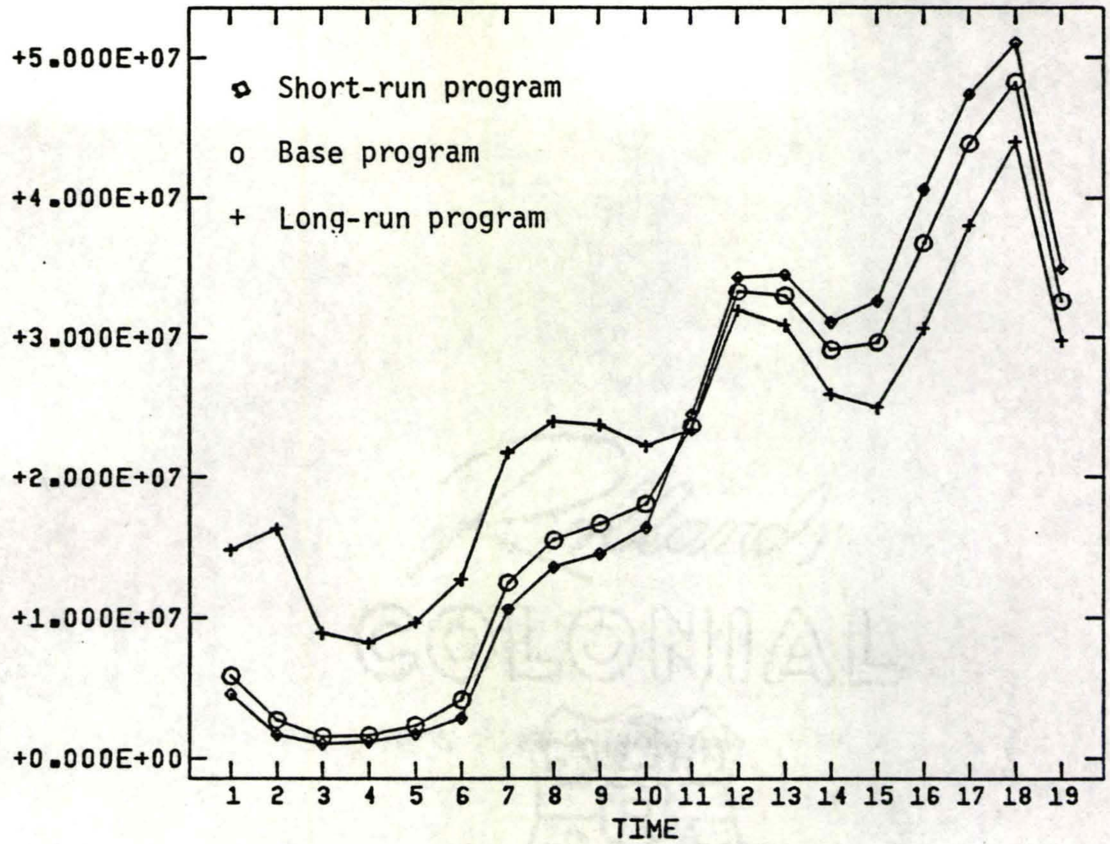
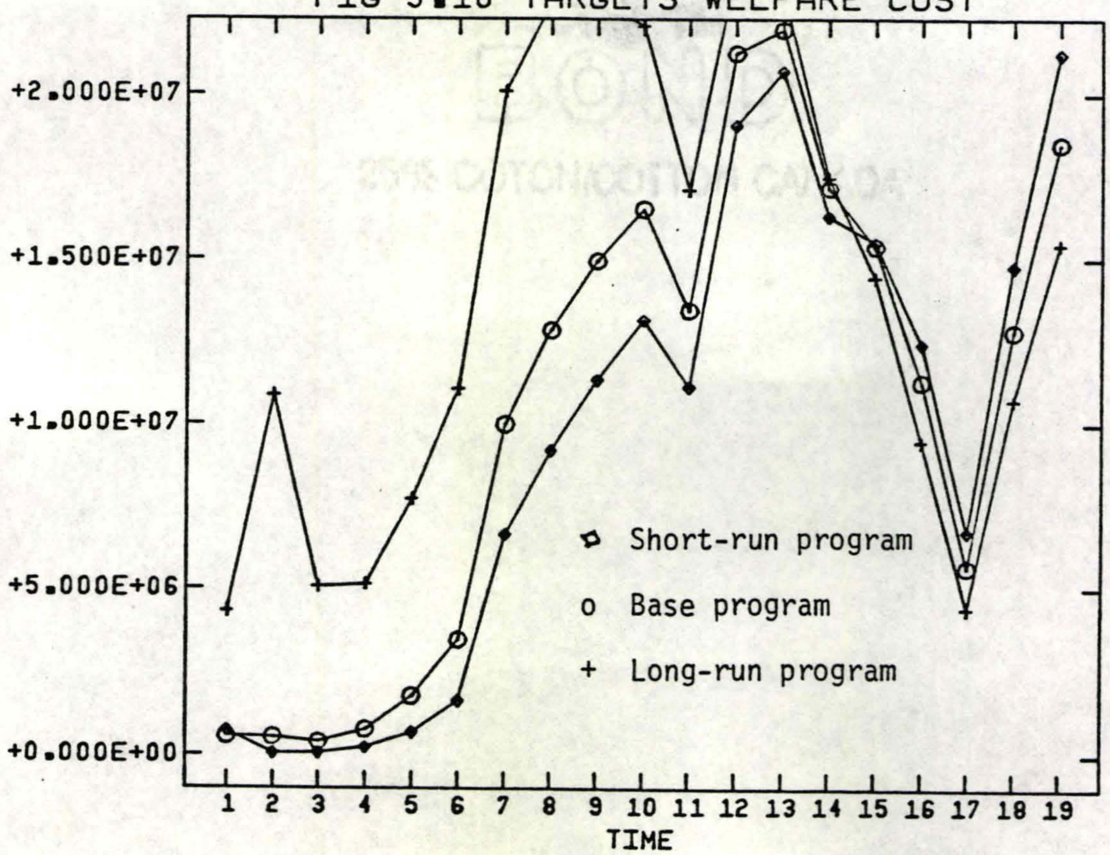


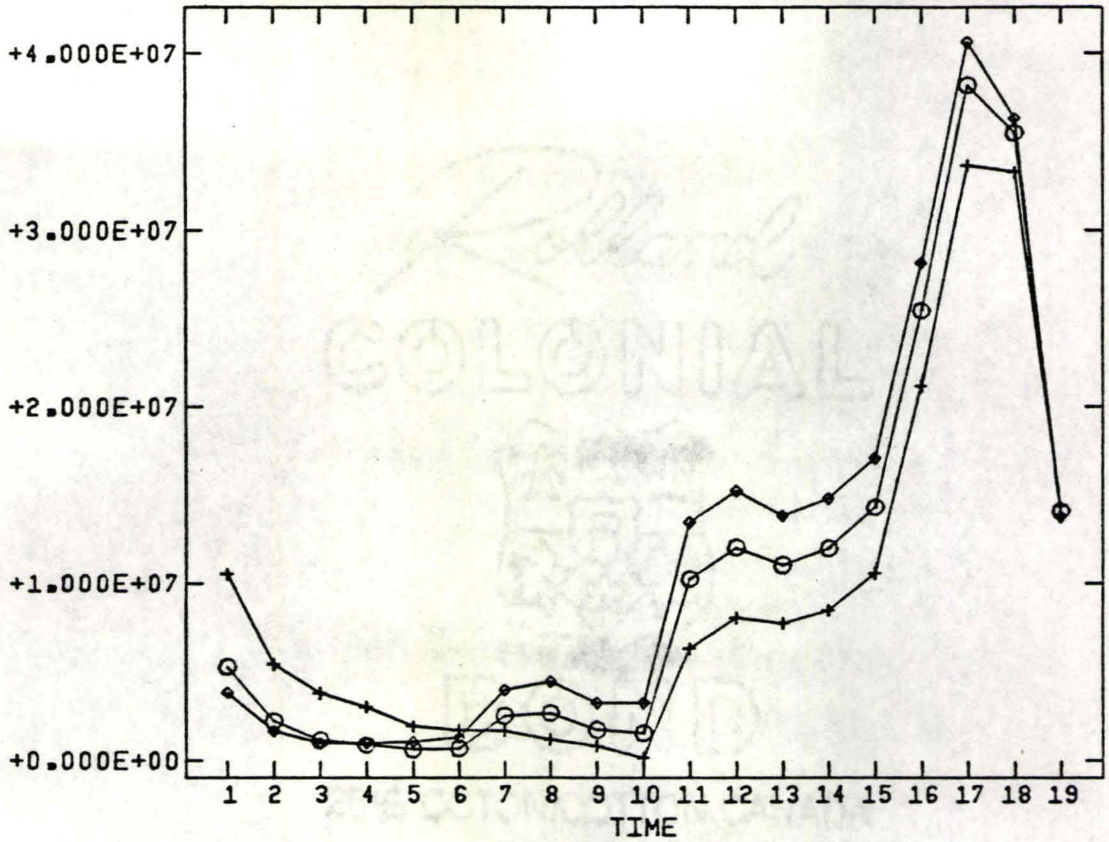
FIG 5.10 TARGETS-WELFARE COST



successful as the Short-run planner in achieving his targets, except in the last 5 years. Some insight into this result may be gained by recalling the simulation experiments reported in Chapter 3. Many of the instrument variables have strong impacts in the first period followed by offsetting reactions in later periods. For the short-run planner these offsetting reactions are of relatively little importance, but for the long-run planner they may be equally or more important than the initial effects. Thus, the long-run planner may not value the active use of instruments as highly as the short-run planner.

The Short- and Long-run programs described so far correspond to two particular values of the discount factor λ. These values imply that the first period (last period) in the planning horizon is 20 times more important than the last period (first period). Five more experiments were carried out, each based on a different degree of emphasis on the short- or long-run. In all these experiments, the time profiles of welfare loss of the three programs, cross at the same point, namely, the 10th period. Thus the welfare loss before and after this period can be accumulated for each program, and can be translated into a single point in a two-dimensional graph, in which the horizontal axis represents the sum of the losses in the first 10 years, and the vertical axis the sum of the losses in the second 10-year period. The first sum represents the short-run cost and the second sum the long-run cost.

FIG 5.11 INSTRUMENTS WELFARE COST



In other words, for each program, we compute two values:

$$L_1 = \sum_{t=1}^{10} (Y_t - Y_t^*)' Q(Y_t - Y_t^*); \text{ and } L_2 = \sum_{t=10}^{19} (y_t - Y_t^*)' Q(Y_t - Y_t^*)$$

These two values generate a single point on the tradeoff curve.

The results of the eight experiments, expressed in terms of Root Mean Squared weighted deviations, i.e., $[\sum_t (Y_t - Y_t^*)' Q(Y_t - Y_t^*)/T]^{1/2}$, are presented in Table 5.1, in which the degree of emphasis refers to the first period vis a vis the last period, or vice versa.

The eight points corresponding to the eight experiments are plotted in Figure 5.12 and joined by straight lines to form a tradeoff

TABLE 5.1

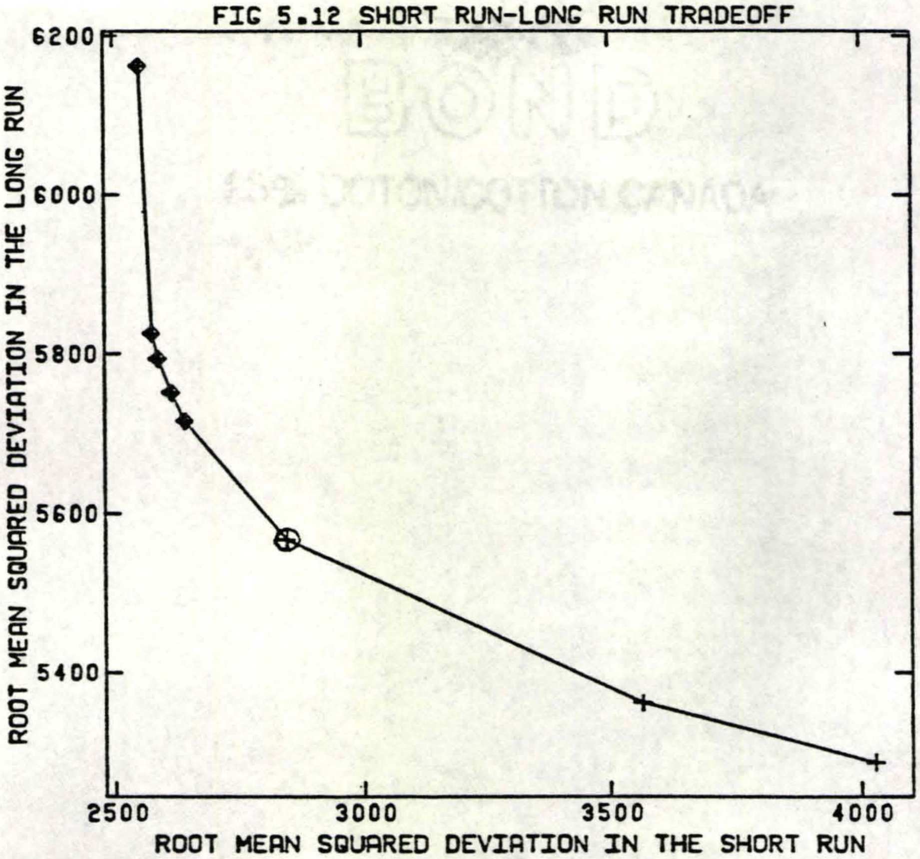
Root Mean Squared Deviations of the Optimal Levels from Their Targets in the Base, Short- and Long-Run Programs

Discount Factor (λ)	Degree of Emphasis	Short-Run (SR)	Long-Run (LR)
0.6813	1000	2554	6162
0.7742	100	2578	5825
0.8046	50	2591	5793
0.8466	20	2617	5751
0.8799	10	2645	5715
1.0000	--	2848	5567
1.1364	10	3565	5361
1.1810	20	4030	5286

curve between the short run and the long run. The five points that correspond to the Short-run program experiments are marked by small squares. The Long-run experiments are marked by crosses, and the base program result, by circles. The tradeoff curve appears to have a smooth and continuous shape and tends to approach the two axes asymptotically. Note that this curve becomes almost parallel to the vertical axis. The movement from the second highest to the highest point on the graph represents a shift of the emphasis on the short run from 100 to 1000 times for the first period relative to the last. This big shift, however, generated only a small gain in the short run but a large increase in cost in the long run. From Table 5.1 the gain in terms of the Root Mean Square Deviation is less than 1 percent (0.93 percent) while the cost is around 6 percent. Thus an extremely myopic view in formulating economic policy may be able to realize a small gain in the present, but only at the cost of large sacrifice in the future. The same conclusion can be drawn if the emphasis is shifted to the long run. As shown in Figure 5.12, the shift from the second lowest point to the lowest point represents doubling the emphasis on the last period, but the gain in the long run is smaller compared to the gain obtained from moving from the base to the first point downward. In other words, the slope of the line becomes smaller as we approach the horizontal axis and larger as we approach the vertical axis.

The lessons that can be learned from analyzing the above results are that a tradeoff between the performance of the economy in the short run and the long run does exist, and that as more and more emphasis is placed on the short run versus the long run in policy formulation, the less is the gain in the present compared to the cost in the future, and vice versa. That is, there are limits that one cannot surpass, and emphasizing the performance of the economy in any particular period is going to restrain the economy in other periods, given the structural constraints of the economy represented by the macro model.

FIG 5.12 SHORT RUN-LONG RUN TRADEOFF



5.3 THE TIME HORIZON EFFECTS

In this section a different approach will be taken. In order to evaluate and compare the performance of the economy, the same objective function will be used but different time horizons will be assumed. In this way, an alternative short-run/long-run tradeoff relationship will be derived.

It is a frequent criticism of economic policy-making that an insufficiently long view is taken of the consequences of certain policies that focus attention on the outcome for the economy in the near future. In order to investigate this point in the present context three experiments, each corresponding to a different time horizon, are carried out. A four-year time horizon that coincides more or less with the duration of most Canadian governments in office, and a two-year time horizon, which is even more short-sighted, are the alternatives for the short-run case. A 10-year time horizon represents the case of long-run policy. The three optimizations are conducted in the following way. First, we start the process in the first year by optimizing over the specific time horizon -- say a 4-year time horizon -- using the actual historical values as initial conditions. Second, we assume that in the second year the policy maker is free to change his strategy and to choose a new policy, given the optimal solution of the first period. Thus, the values of the endogenous variable (targets and instruments) are re-initialized and the optimization is repeated for a second 4-year time horizon. That is, if the planning period is 1964-1967 in the first case, it becomes 1965-1968 in the second case. This implies that decisions are revised sequentially year by year, rather than being made once and for all years of the planning horizon. It is more realistic to assume that policy makers do not adhere to a decision they have made, but rather that they revise their decisions frequently in light of the new information that is available to them each year. The above process is started in 1964 and ends in 1973, i.e., the final run was made over the period 1973-1976 for the 4-year program. The same procedure is followed in the case of the other two programs. By having 10 runs for each program we can derive the corresponding three sequences of optimal solutions for the period 1964-1973.

Assume that a new administration came to power in 1973. It is believed that this administration does not concern itself only with the short run but it takes account of the longer-term future as well. It sets out a long-run plan for the next 10 years (1973-1982)[3], taking as an initial condition the situation inherited from the previous administration. Now, the question is what implications the three previous programs have on the present long-run plan. To answer this question, three more experiments are carried out, each one taking the values of all variables in the 1973 optimal solution that corresponds to the 2-, 4- and 10-year time horizon, as initial conditions. The differences in the solution of these three runs are attributable to the difference in the initial conditions, which are in turn due to the effects of the planning horizon used by the policy makers in the first ten years of the optimization process. In other words, these differences measure the damage done to the long-run planner by the economic conditions he inherited from previous administrations. If we compute these differences in terms of weighted squared deviations and plot the results, as we did in the previous section, then we can measure the long-run costs of pursuing more or less myopic economic policies. On the other hand, the weighted squared deviations from the first ten periods represent the short-run costs of pursuing more or less far-sighted policies. (Because the ten-year planner must take into account events in the future, the economy is expected to perform more poorly in the short run).

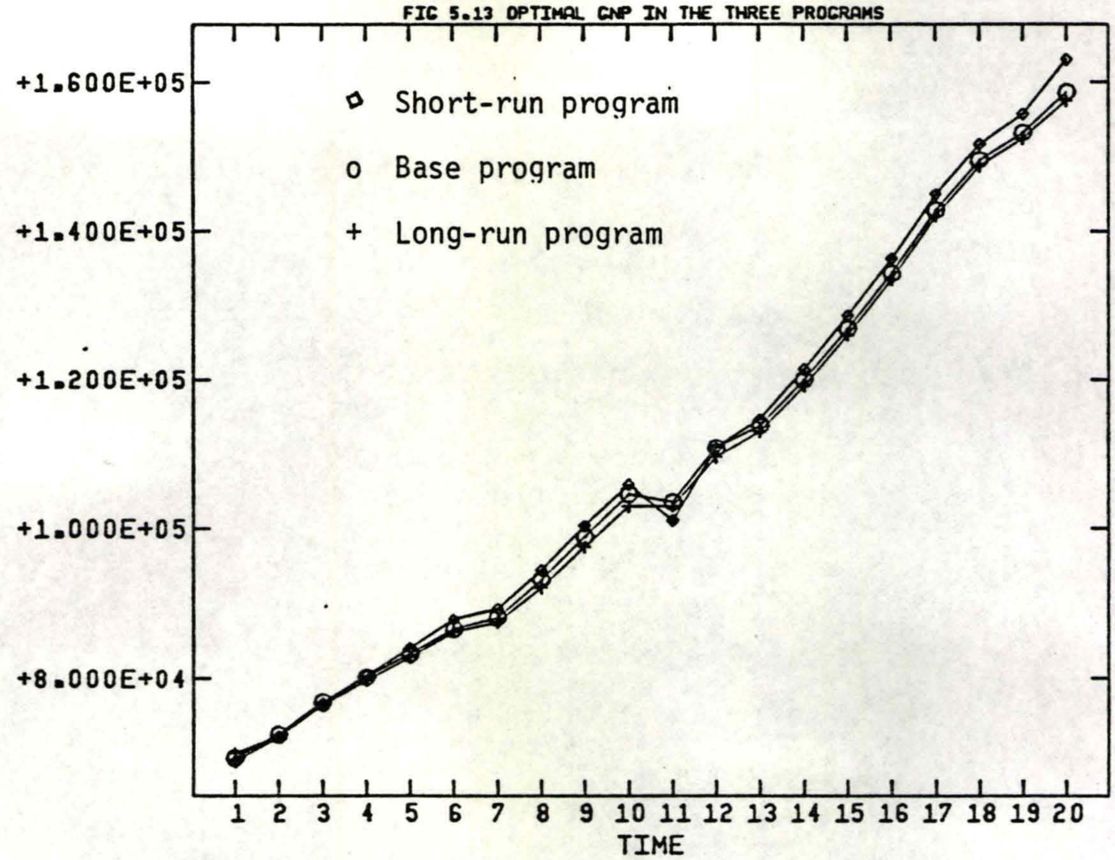
However, before doing the calculation of this kind, let us look at the behaviour of the targets and instruments in the three programs.

5.3.1 ANALYSIS OF THE RESULTS

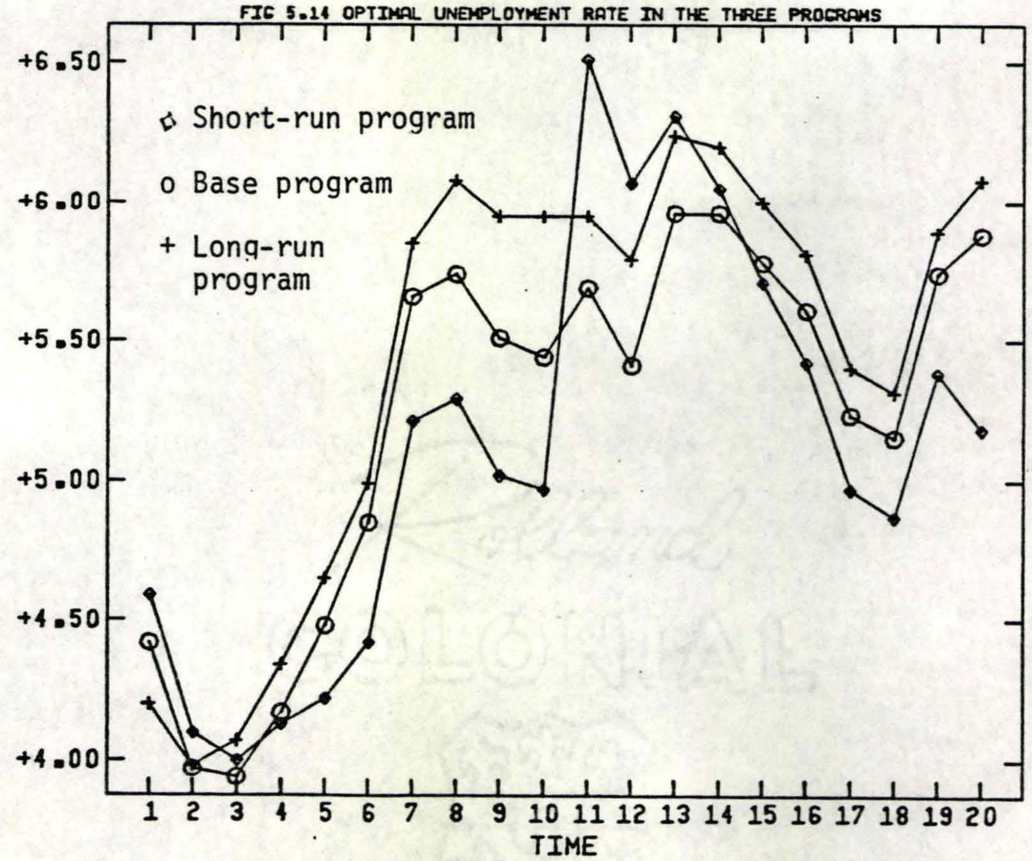
The same mechanism explained in connection with the first approach applies here, the only difference being that the behaviour of the endogenous variables is affected by the change in the duration of the period over which the optimization takes place. The results are shown graphically in Figures 5.13 to 5.20.

The three time paths of the GNP in Figure 5.13 are almost identical. The 2-year program generates a slightly higher level than the 4-year program, which in turn generates a slightly higher one than the 10-year program. A jump occurs in the 11th year when all three programs adapt a 10-year planning horizon with no further revisions thereafter. However, the change in direction is only temporary, and GNP in the 2-year plan keeps increasing, and is greater than in the other two plans over most of the last ten years of the 20-year period. The behaviour of the GNP in the 2-year plan in this approach is similar to its behaviour in the previous approach. That is, the short-run program was able to achieve a higher production level in both the short run and the long run. The myopic (sequential two-period) optimization results in better global (multiperiod) performance when the period-by-period results are positively correlated, as is the case with the GNP and employment variables. This result is consistent

FIC 5.13 OPTIMAL GNP IN THE THREE PROGRAMS



FIC 5.14 OPTIMAL UNEMPLOYMENT RATE IN THE THREE PROGRAMS



with the result obtained, theortically, by Tesfatsion [1980].

As in the previous experiment, the unemployment rate is influenced mainly by its own lagged value and by the level of output. This is reflected in Figure 5.14 where the unemployment rate jumps to 6.5% in the 11th year, corresponding to the decline of GNP in this year but then decreases thereafter. The behaviour of the inflation rate, (Figure 5.15) is more complicated and more difficult to understand because of its close interdependence with other target and instrument variables. Up until the 10th year the rate of change of prices seems to be determined exclusively by the behaviour of the exchange rate, Figure (5.18). In the first 4 years the exchange rate trajectory is identical in the three programs, and so is that of the inflation rate. A divergence starts to develop in the 5th year. For the next four years, the exchange rate in the 4-year program is lower than in the other two programs, thus generating a lower inflation rate. In the second half of the planning period, however, the behaviour of the exchange rate and inflation rate is not consistent with this explanation in the case of the 4- and 10-year plans; the 4-year one has the highest inflation rate but not the lowest exchange rate. The Philips curve relationship effect may dominate that of the exchange rate and give rise to a higher inflation rate for the 4-year plan than for the 10-year plan. This kind of tradeoff has been mentioned earlier in analyzing the results of the first approach.

With respect to the balance of payments equilibrium target, there are essentially two factors that influence its behaviour: the

FIG 5.15 OPTIMAL INFLATION RATE IN THE THREE PROGRAMS

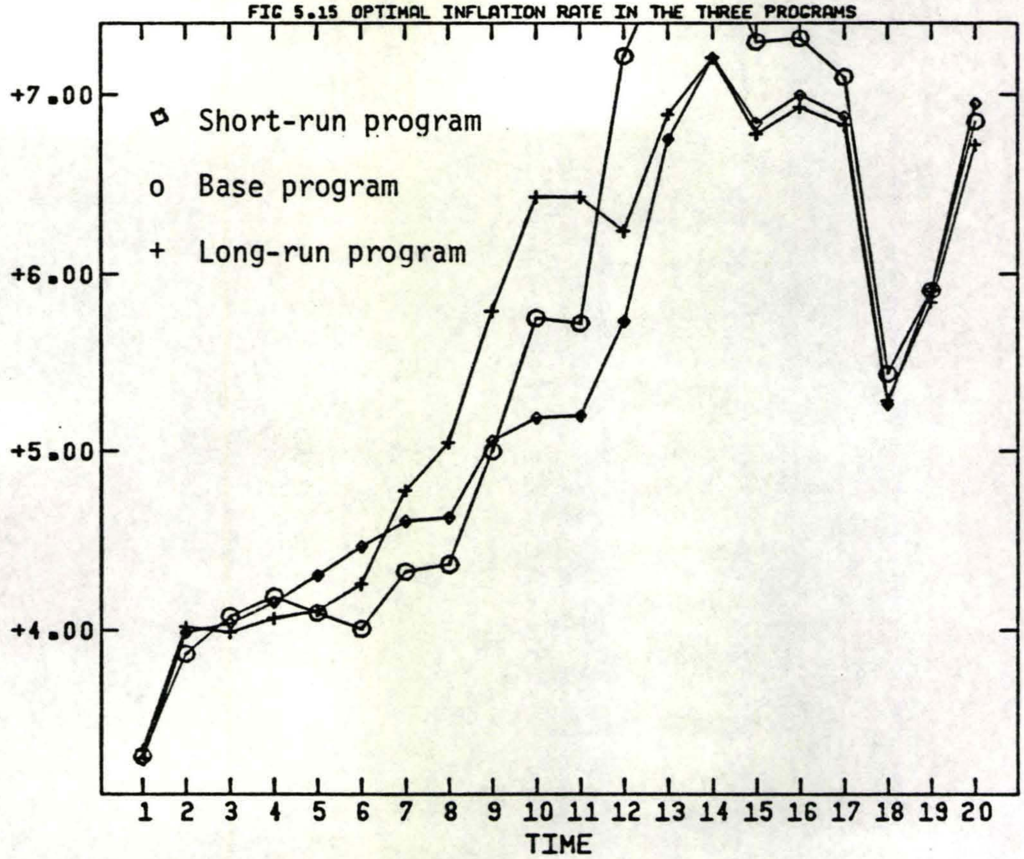
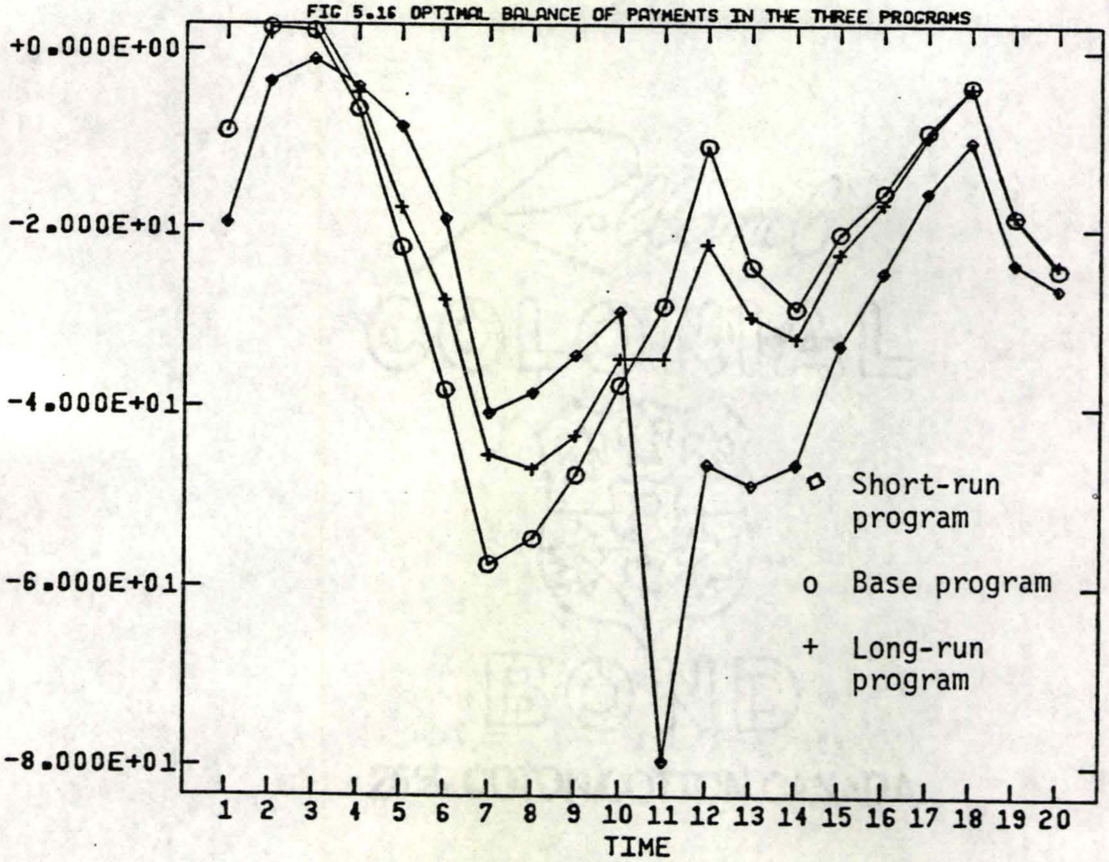


FIG 5.16 OPTIMAL BALANCE OF PAYMENTS IN THE THREE PROGRAMS



exchange rate and the level of output. It is clear from Figure 5.16 that in the first few years, the GNP level achieved is almost the same in the three plans and the exchange rate determines the balance of payments. In the second half of the planning horizon, a higher GNP and a lower exchange rate in the 2-year plan generate a greater deficit in the balance of payments. However, the performance of the balance of payments in the other two plans is generally similar.

The optimal trajectories of the control variables are depicted in Figures 5.17 to 5.20. The level of government expenditures varies inversely with the length of the time horizon, except in the first few years. This proves once more that short-sightedness in policy making requires more extensive use of policy instruments. As shown in Figure 5.17, government spending is much higher in the 2-year plan but is brought down in the 11th year when the second phase 10-year plan is put in place. The exchange rate (Figure 5.18) follows the same path in the three programs but it appreciates more in the myopic program. There is no noticeable difference in the change in the money base (Figure 5.19), and overall monetary policy is contractionary in the first half but expansionary in the second half of the optimization process. The surtax (Figure 5.19) associated with the 2-year plan displays more contractionary behaviour (higher taxes) relative to the other two programs. The reason that taxes are not used in a stimulative way may lie in the fact that the increase in government borrowing in the three program is large, especially in the myopic horizon case, therefore preventing a large decrease in taxes while still satisfying the government budget constraint.

FIG 5.17 OPTIMAL GOVERNMENT EXPENDITURES IN THE THREE PROGRAMS

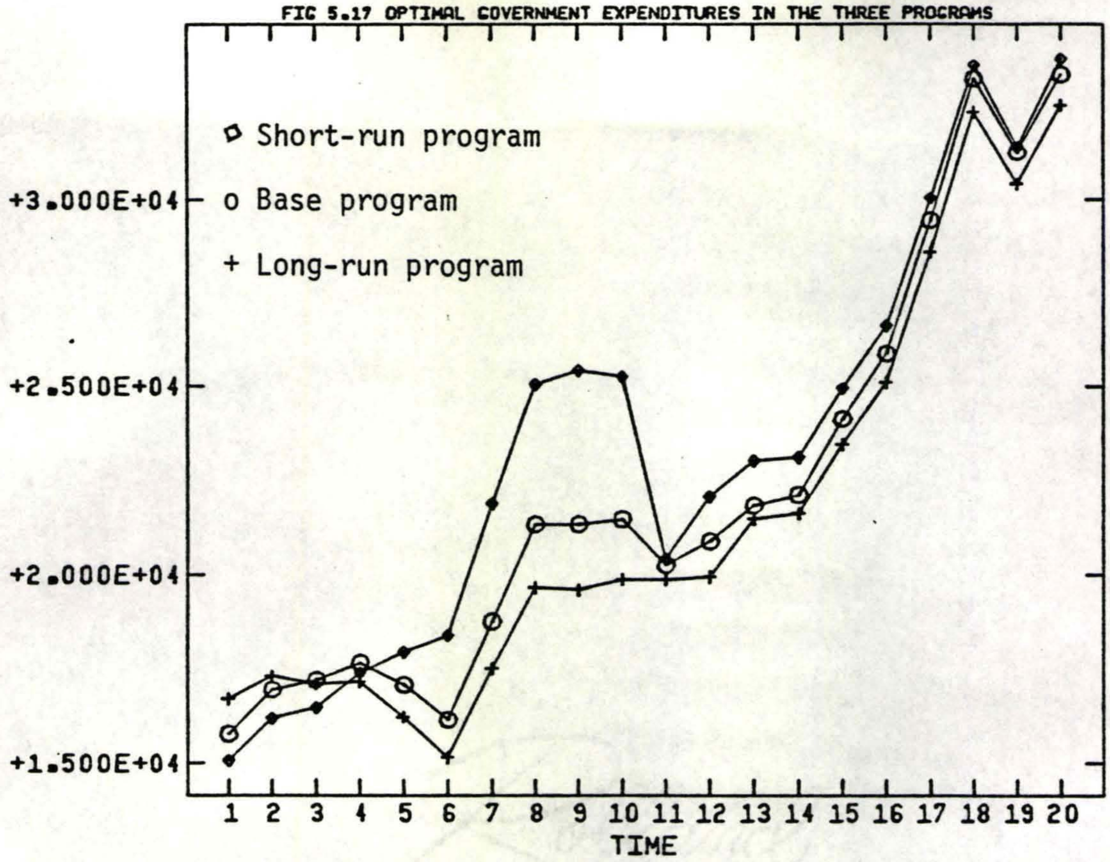


FIG 5.18 OPTIMAL EXCHANGE RATE IN THE THREE PROGRAMS

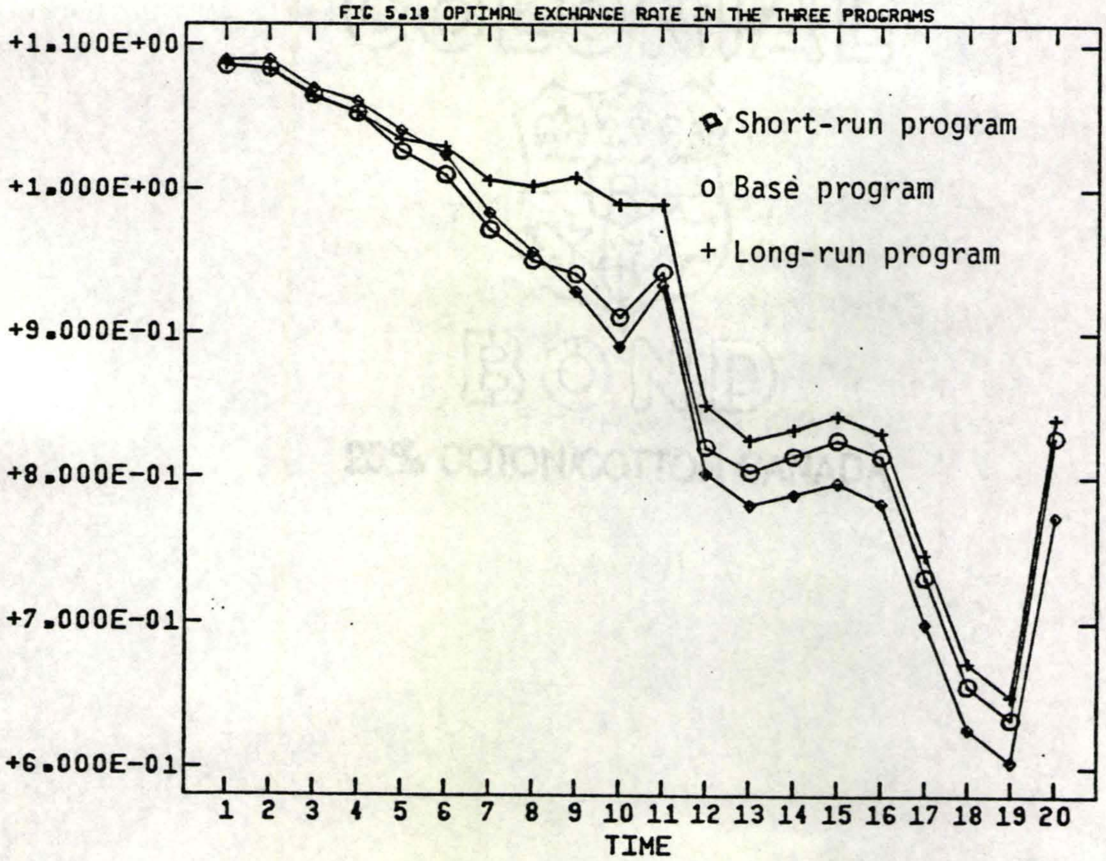


FIG 5.19 OPTIMAL MONEY BASE IN THE THREE PROGRAMS

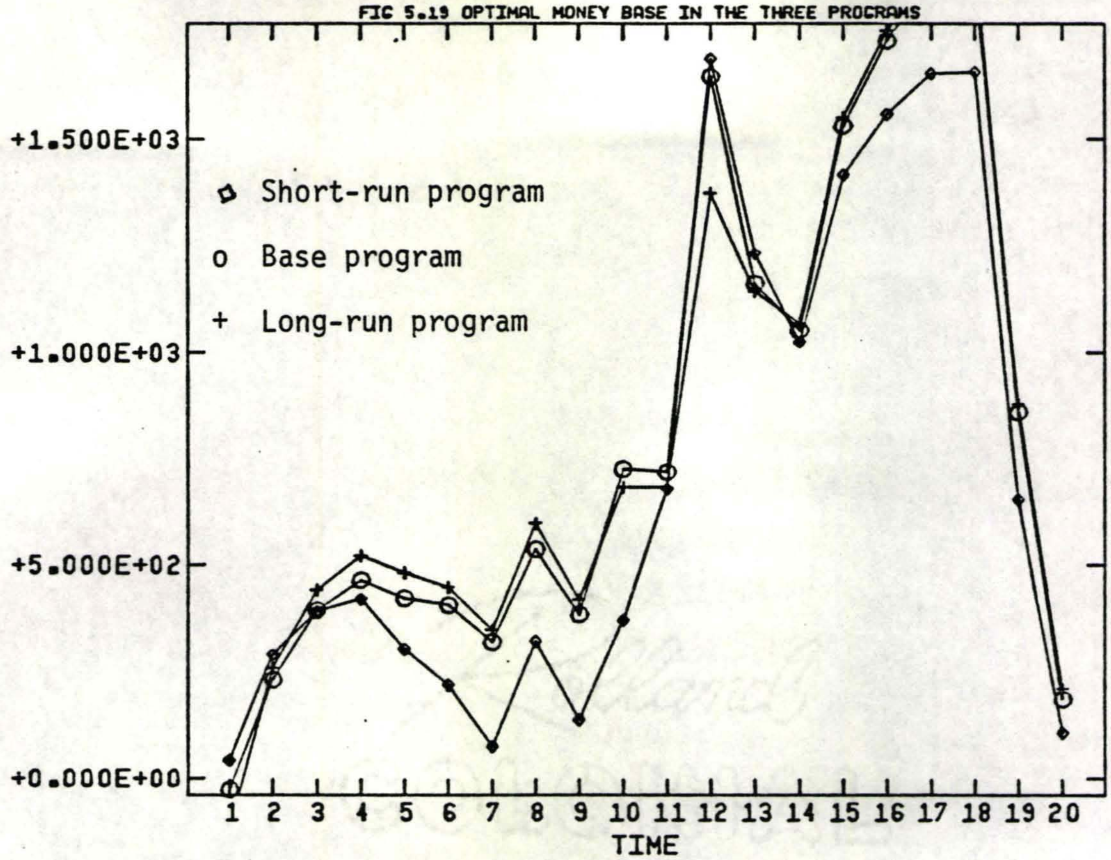
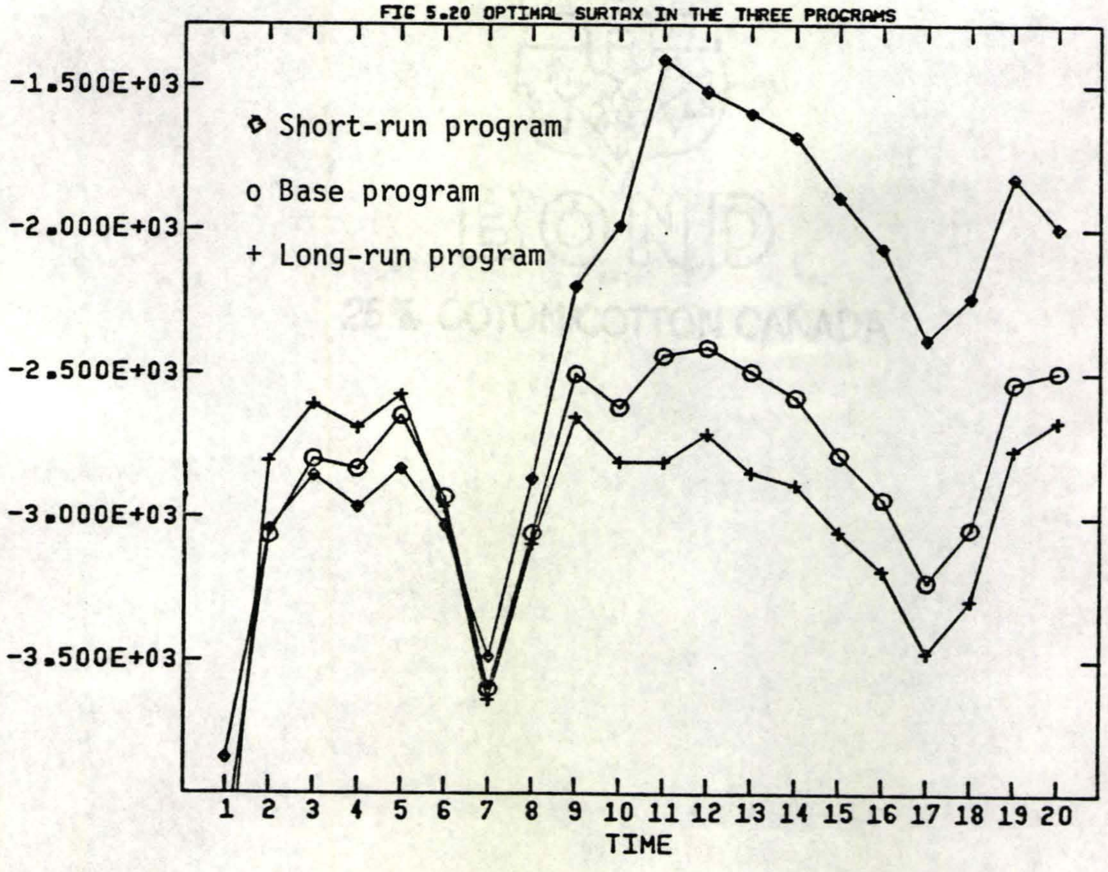


FIG 5.20 OPTIMAL SURTAX IN THE THREE PROGRAMS



5.3.2 SHORT-RUN/LONG-RUN TRADEOFF

Following the same procedure as in the previous section, the weighted squared deviations of the three programs are computed and their time paths are plotted in Figure 5.21. The 2-year program, denoted by small squares, has the lowest cost in the first 10 years and the highest cost in the last 10 years. The opposite is true for the 10-year program, denoted by crosses, while the cost path of the 4-year plan, denoted by circles, lies between the two.

These results accord with intuition. If the planner did not take into consideration the future he would be less constrained, and therefore more successful in achieving his desired short-run targets. On the other hand, if the planner had a sufficiently long view of the future, then he would have to make allowance for some anticipated shocks which would constrain him in the present, and therefore make his policies less successful in the short run. In other words, if the planner suffers from myopia, he will not be concerned with the consequences of his present actions in the years beyond his short time horizon, and may thus achieve a good performance in the near future, but only at the cost of bad performance further down the road.

As shown in Figure 5.21, the Root Mean Square weighted deviations are computed for the first 10 years and for the last 10 years for each of the three programs. Thus we have 6 values which

FIG 5.21 TOTAL WELFARE COST

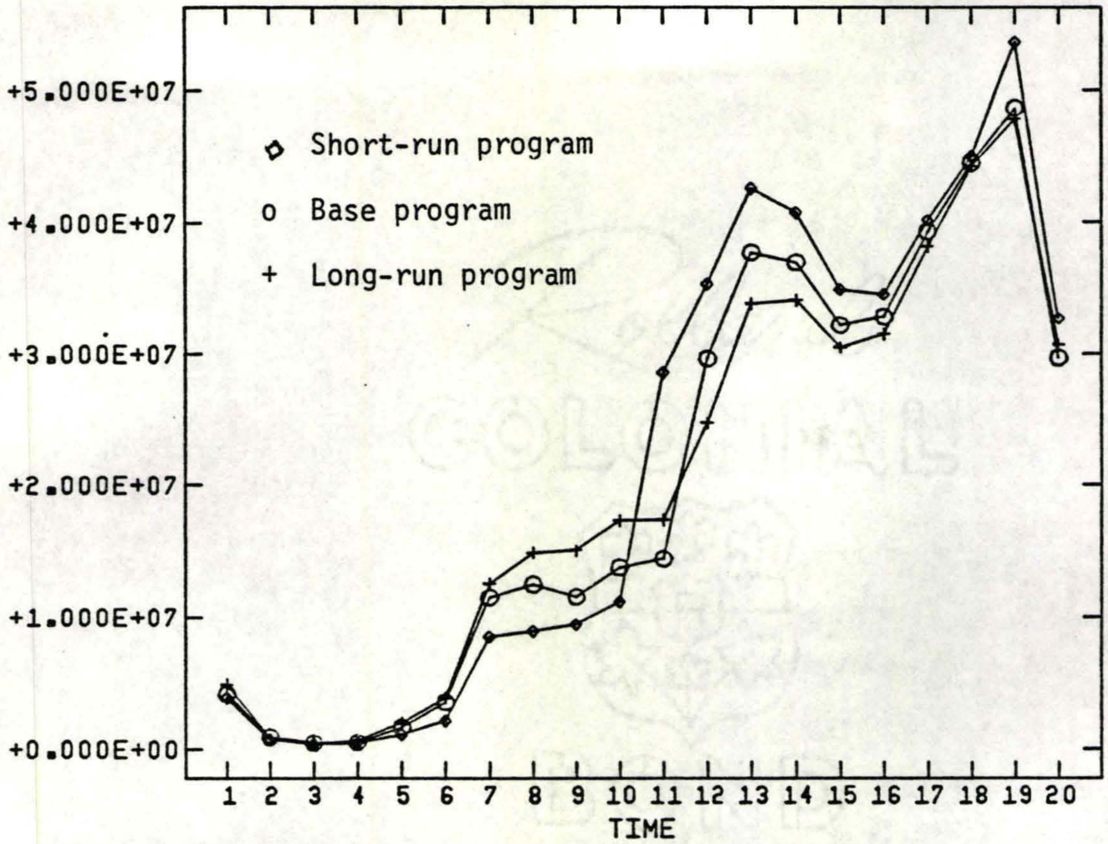
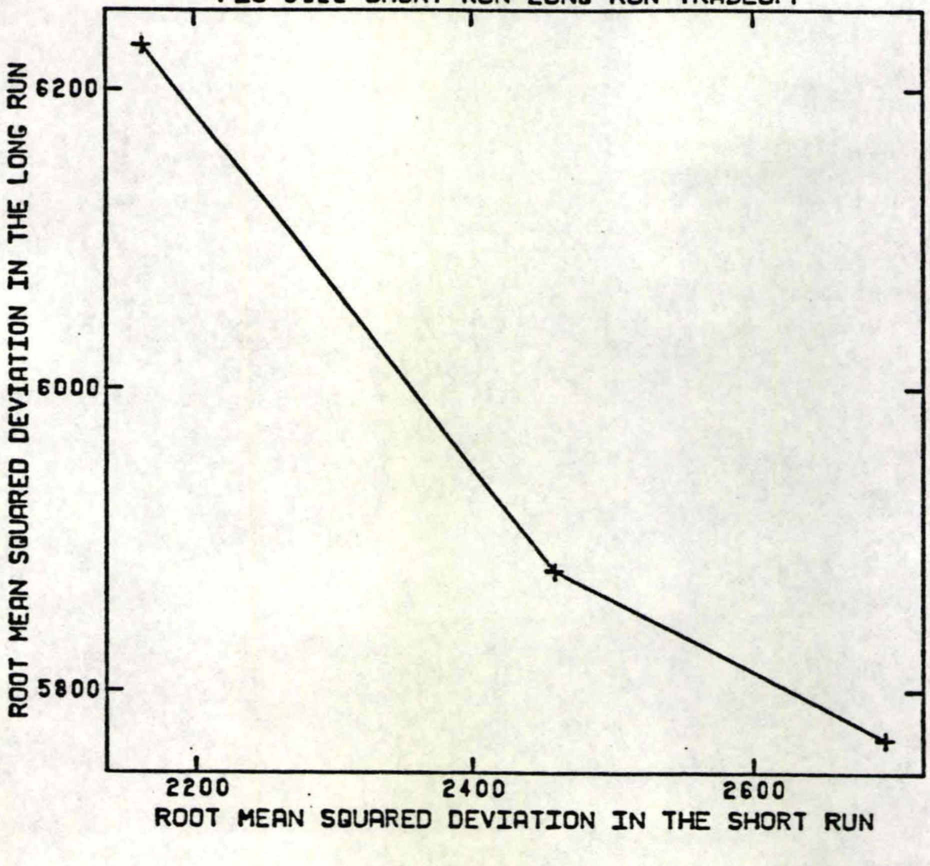


FIG 5.22 SHORT RUN-LONG RUN TRADEOFF



are represented by 3 points on a two-dimensional graph as in Figure 5.22. By connecting these three points we obtain a tradeoff curve between optimal policies in the short run and in the long run. The highest point on the plot represents the 2-year plan, the middle point the 4-year plan, and the lowest point the 10-year plan. The movement from the 2- to the 4-year time horizon engenders a greater benefit in the long run than the movement from the 4-year to the 10-year time horizon. If the planning horizon is extended indefinitely, the cost in the short run will increase considerably while the benefit in the long run will be limited. Put differently, the tradeoff curve will approach the axes asymptotically, as in the first approach[4].

5.4 SUMMARY

In this chapter, the objective function of Chapter 4 was minimized subject to the constraints of the macro model of Chapter 2, in order to derive a tradeoff relationship between short-run and long-run performance in macroeconomic policy. Two approaches were used for this purpose. The first consists of changing the value of the discount factor in the objective function and tracing the responses of the endogenous variables. In the second approach three experiments were carried out, each corresponding to a different time horizon: 2-year and 4-year time horizons, representing a short-run view, and a 10-year horizon, representing a long-run view. These experiments were carried out sequentially, the policy maker being allowed to review his decision every year in light of new information. Ten runs were done for each program. Then a 10-year time horizon optimization was carried out for each case, taking the results of the previous sequence of runs as initial conditions. This enabled us to determine the long-run cost of pursuing short-run policies.

In both approaches the time paths of the welfare costs associated with each program were derived and plotted, and tradeoff curves between the long-run cost and the short-run cost were calculated. The shape of the two tradeoff curves suggested that the heavier the emphasis is on the present, the greater will be the cost in the future. However, there is a limit to the improvement one can make in any one period, given the structure of the economy represented by the macro model. Therefore, if the planner keeps emphasizing more and more the present (future) the benefit will be small compared to the cost in the future (present). Having identified this intertemporal tradeoff, it is up to the policy maker to decide at what point on the tradeoff curve he wants to be.

FOOTNOTES

Chapter 5

1. As noted in footnote 3 in Chapter 4, GNP and G enter as levels in the objective function, with those levels chosen to represent a 5% and 4% rate of growth, respectively, per year.

2. The problem of a finite time horizon was raised by Pindyck [1973]. He suggested either increasing the time horizon beyond that of interest to the policy maker or neglecting the behaviour of the target and instrument variables in the last few periods. However, the neglect of the last 2 years in the present case did not alter the results obtained regarding the short-run/long-run tradeoff.

3. The experiments were carried out over the historical period only. Any attempt to extend the planning period beyond 1982 would require generating forecasts for the exogenous variables, and that has not been attempted here.

4. All the results related to the short-run/long-run tradeoff are based on deterministic welfare cost, as measured by the squared deviations of the mean path of the endogenous variables from their desired values weighted by the matrix Q. However, as explained in Chapter 4, there is also a stochastic welfare cost representing the weighted deviations of the solution around its mean path, i.e., $(Y_t - \bar{Y})$, summed over time. The stochastic cost was larger in magnitude than the deterministic cost in all experiments, and this seems to be consistent with the results obtained by many authors [Chow 1975, Mathiew 1976]. Furthermore the deterministic and stochastic costs tend to be proportional in these studies. On this basis, if the deterministic cost in experiment i is greater than it is in experiment j, then the stochastic cost in i will also be greater than it is in j. Hence the inclusion of the stochastic cost will not alter the main conclusion of the analysis, although it will shift the tradeoff curve outward.

CHAPTER 6

CONCLUSION

This chapter offers a review of the major findings of the study and discusses some possibilities for further research.

The intertemporal dimension of macroeconomic policy, which has been largely neglected, constitutes the main focus of this study. Two principal questions were posed at the beginning: What are the long-run implications of pursuing short-run myopic policy, and what are the short-run consequences of long-sighted policy? Is there a tradeoff between the performance of the economy in the short run and in the long run?

To answer these questions, an optimal control approach was used. This approach lies in the area of quantitative economic policy, as developed by Tinbergen It enables us to derive the best possible policies in the short run and the long run in the case of multiperiod optimization, and is more efficient than a series of repeated simulations which would be required to derive and identify policy sets consistent with some predetermined targets. The two principal elements of the approach are a macroeconomic model describing the functioning of the system under control, and an objective function which specifies explicitly the targets pursued by the policy maker, and the welfare losses associated with failures to achieve these targets.

The first step consisted then of formulating a macroeconomic model for Canada. The model described in Chapter 2 is a small and highly aggregated model that combines some features associated with Keynesian and Classical schools of macroeconomics. In particular, some long-run characteristics, such as the existence of a production function determining the long-run level of potential output in the absence of short-run fluctuations, and a capital accumulation identity were added to the conventional demand-oriented Keynesian model. In addition, the present model goes beyond traditional short-run analysis by allowing for the accumulation of financial assets via the government budget constraint. The model was estimated for the period 1962-1982 by a principal components variant of the two-stage least squares procedure, principal components being used because the number of predetermined variables exceeded the number of observations. The estimated model was then subjected to a series of ex-post historical simulations and to simulations involving various shocks, in order to examine its realism and its stability properties. On the whole, the model was able to reproduce the time paths of most of the endogenous variables, and its stability was confirmed, except when government spending was financed by issuing bonds.

The second step, as described in Chapter 4, was to specify an objective function. The function specified was quadratic, penalizing the weighted squared deviations of its arguments from their desired values. These arguments included both the target variables which were the rate of growth of GNP, the rate of change of prices, the unemployment rate, and the balance of payments, and the instrument variables namely government expenditures, taxes, the change in the money base and the exchange rate.

Optimal control experiments were carried out, as described in Chapter 5, in order to investigate the short-run/long-run policy tradeoffs. Two approaches were used. The first consisted of varying the relative structure of the weights over time by means of changing the rate of time preference in the objective function. A tradeoff curve, with points corresponding to different discount factors, was then derived between the welfare losses in the short run and the long run. The second approach aimed at measuring and comparing the costs implied by different time horizons used by policy-makers. Tradeoff relationships were then derived between the performance of the economy in the short run and the long run, expressed in terms of the weighted deviations in the objective function. The results of these experiments can be summarized as follows:

- A short-run/long-run tradeoff does exist within the given structure of the economy represented by the constraints of the macro-economic model.

- As more and more emphasis is placed on the present (future), the higher is the cost incurred in the future (present) and the less is the additional gain in the period being emphasized.

- An inflation/unemployment and an inflation/balance of payments tradeoff was identified from the responses of those variables in the experiments conducted.

- A myopic approach in policy-making is likely to lead to better results in the short and long run when a target variable is positively-correlated over time, such as GNP and employment.

- A short-sighted policy requires the extensive and frequent use of policy instruments. If the change in these instruments is subject to social, administrative and political constraints, then the cost associated with the use of these instruments is the price of achieving a short-run objective.

- As far as the present model is concerned, fiscal policy was directed mainly to the growth and unemployment targets while the exchange rate proved to be effective in controlling the rate of inflation, implying that Canada is vulnerable to inflation rates prevailing abroad. As for monetary policy, it was found to have only a limited effect on the targets.

- A byproduct of the optimal control experiments is the confirmation that a potential stability problem exists when government spending is financed by increasing the debt.

Several qualifications should be kept in mind when interpreting the results. These qualifications relate to the use of optimal control theory in policy-making in general, and to the particular model and objective function, as well as to some of the assumptions that were made.

First, all the results mentioned above are based on the implicit assumption of structural stability of the economy through the period of experimentation. Thus, we are dealing with conjectural policy which pursues a more efficient performance conditional on a given structure [Dagum, 1981]. This assumption is questionable, especially when dealing with the distant future. However, it is expected that this assumption would be more likely to prevail with a shorter time horizon.

Second, the symmetry property of a quadratic objective function is difficult to justify unless the stability of the targets around their desired values is the only welfare criterion of interest.

The third, and most important general qualification, is the limitation of the use of optimal control theory in the formulation of macroeconomic policy. As pointed out in the introduction, this approach reduces the very difficult problem of decision making to a simple mechanistic optimization problem. The general reluctance to endorse this approach in policy making stems from the unreliability of macroeconomic models and the inadequacy of the objective function in reflecting the preferences of the policy maker. These difficulties, however, do not threaten the viability of our results, since our objective was neither to evaluate historical policies nor to prescribe new policies. Nevertheless, two main arguments against the use of optimal control theory as a policy tool cast some doubt on any conclusion drawn from control experiments. One is that the assumption of a single policy maker who can control the economy is obviously unrealistic, since in economics there is always a game-theory element in most decisions [Buiter, 1983]. The government attempts to control a system, the response of which depends to a great extent on the expected payoff of each player, and this in turn, depends on the strategy of other players. Thus, modelling economic systems as a dynamic game leads to decision rules that are different from those obtained in the special case where the number of players is equal to one, as in most optimal control studies.

The second argument relates to the existence and formation of expectations which differentiate social systems from physical systems. If changes in public policy alter the expectations of economic agents, then a model that does not specify this link will lead to misleading conclusions [Buiter, 1983]. The rational expectations hypothesis and the Lucas critique, which say that the private sector knows the true structure and the parameters of the model, and that private sector behaviour varies with any perceived change in policy, is a strong and special case of expectations formation. Some authors [Kydland and Prescott, 1977] have gone so far as to argue that in the presence of rational expectations, optimal control theory should not be used in economic planning, since "current decisions of economic agents depend in part upon their expectations of future policy actions ....Only if these expectations were invariant to the future policy plan selected would optimal control theory be appropriate".

These strong criticisms have drawn some sharp responses "to dispel the mistaken notion that policy evaluations and optimization are impossible if economic agents form rational expectations" [Chow, 1980]. Chow proposed a method in which he allowed for the effects of future policy on current actions and he then derived a variable coefficients feedback rule which became time invariant as the time horizon increased. Also some work has been done to embody the game-theory aspect in optimal control studies applied to economic policy [Buiter 1983, Chow 1981]. This would transform the game of the policy maker against nature into a game with many players, which might be viewed as more representative of the nature of real economic systems.

Future research should be directed at those areas in which optimal control theory is still controversial. The use of a time-varying macroeconomic model would allow for structural change over the period in which the policy maker is interested [Chow, 1975]. The problem of symmetry in a quadratic objective function should be avoided if the welfare costs deviations are not symmetric in the view of the policy maker, and the use of asymetric objective functions is a research priority in this area.

As for specific future research relating to the question of the short-run/long-run policy relationship, the optimal control approach can be modified in many respects to serve this purpose. A modified algorithm that permits the imposition of terminal conditions is very useful in this context. The use of terminal conditions can be used as a proxy for the explicit treatment of long-run considerations and the terminal conditions imposed on a short-run program can be chosen so as to allow the return of the economy to its long-run time path.

An approach similar to the one used in this study would permit the policy maker to identify the optimal time over which a given target should be achieved. Suppose that a government wishes to achieve a specific target -- say a balanced budget. Suppose, too, that the government has the choice between two programs: a short-period (2-year) program and a long-period (5-year) program. The question is what are the necessary reductions in government expenditures and/or the necessary tax increase (control variables) needed to achieve this target in the two programs. Which of the two programs has the least undesirable effects on the other targets, such as output and unemployment? Given a "plausible" discount rate, the optimal control approach would allow us to determine the optimal time horizon, that is the program associated with the minimum welfare loss, required to achieve the target of balancing the budget. This would also provide an answer to the question of "gradualism" versus "shock treatment" in the theory of economic policy.

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