SUPPLY SIDE EFFECTS OF MONETARY POLICY

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A Thesis

Submitted to the School of Graduate Studies

in Partial Fulfilment of the Requirements

for the Degree

Doctor of Philosphy

January 1986

SUPPLY SIDE EFFECTS OF MONETARY POLICY

DOCTOR OF PHILOSOPHY (1986)
[ECONOMICS]

McMASTER UNIVERSITY
Hamilton, Ontario

TITLE: Supply Side Effects of Monetary Policy

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NUMBER OF PAGES: xiii, 156

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TO THE MEMORY OF

MY LATE MOTHER

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ABSTRACT

The primary purpose of this thesis is to combine two branches of empirical work concerning firms' demands for factors. One set of studies involves the simultaneous estimation of money, labour and capital demand functions, where the focus is on the role of money as a factor production, and adjustment costs for capital are ignored. The other set of studies involves estimation of an investment function, together with other factor demand functions. Writers in this area typically exclude money as factor in the production process, though we wish to overcome that omission in this thesis.

We also study the possible supply side effects of monetary policy that arise because of the role of money as a factor of production and deduce macroeconomic policy implications.

Our empirical work is divided into three stages. In each stage we assume that firms minimize the cost of producing a given level of output subject to a production function that includes real money balances as a factor input. In the first stage we estimate a full equilibrium

model in which firms can adjust their capital stocks without any lags and there are no costs associated with the adjustment of these capital stocks. In the second stage of estimation we introduce the fixity of capital stocks in the short-run into the firms' optimization problem. In this temporary equilibrium model, costs of adjustment of capital are not assumed. Finally, we estimate a dynamic model of the firm, based on the assumption of non-linear internal costs of adjustment for capital. The three empirical models are estimated at the aggregate level for non-financial corporations in the United States.

On statistical grounds the full equilibrium model did not fit the data well. The own price elasticity of real money balances was not significantly different from zero. In the other two models, where capital is fixed in the short-run, all the own price elasticities are significantly different from zero and have negative signs. Furthermore, in the full equilibrium model, autocorrelation seemed to be present even after making a first order correction for the errors.

The temporary equilibrium model was also statistically rejected, conditional on the particular functional form of the cost function employed in the dynamic model. We conclude that the dynamic specification is most appropriate in this context.

The price elasticities varied substantially across the different models. The cost minimizing interest rate elasticity of labour demand was significantly different from zero and negative in sign in the dynamic model. However, the implied profit maximizing interest rate elasticity of labour demand was not statistically significant. This suggests that earlier estimates of money's role in the production process were contaminated by the restriction that there were no costs of adjustment.

ACKNOWLEDGEMENTS

Allahumman Laka al-hamd Wa laka al-shukr

It is a great pleasure in acknowledging the help and guidance I have received from Professor William M. Scarth, my supervisor. He has provided constructive comments on the entire study and gave encouragement at each and every stage of it. The time I have spent with him has left a pleasant and invaluable impression on me.

I would also like to express my gratitude to the other members of my committee, Professors A. Leslie Robb and Lonnie J. Magee, who gave numerous useful suggestions. Professor A. Leslie Robb made a number of useful comments on the model specification and gave many helpful suggestions on the empirical issues.

I have also had some useful discussions with Mr. Eataz Ahmed on the econometric issues.

I also sincerely appreciate the help of Dr. Z. Alavi in providing the word processing facilities.

I am deeply obliged for the patience and support of my wife, Naeem, during the long period of this study.

Finally, for any errors, of either omission or commission, I alone am responsible.

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CHAPTER 1

INTRODUCTION

The importance of money in aggregate economic activity is universally recognized. In recent years, several studies have been undertaken to increase our understanding of the differences in motives for holding cash balances between individuals and business firms[1]. The results of these empirical studies indicate that real money balances do play a significant role in firms' production processes and they should be considered as a factor input in the production function. These studies typically estimate money, labour and capital demand functions simultaneously and assume that capital stocks can be adjusted costlessly and instantaneously. However, recent studies on the role of energy in production have questioned the assumption of instantaneous adjustment of capital stock and have successfully developed empirical models that explicitly incorporate the notion of the cost of adjustment of capital in the optimization framework of firms[2]. These studies have made clear that the assumption of fully adjustable capital stocks does contaminate the results. Writers in this area, however, exclude money as a factor in the production process.

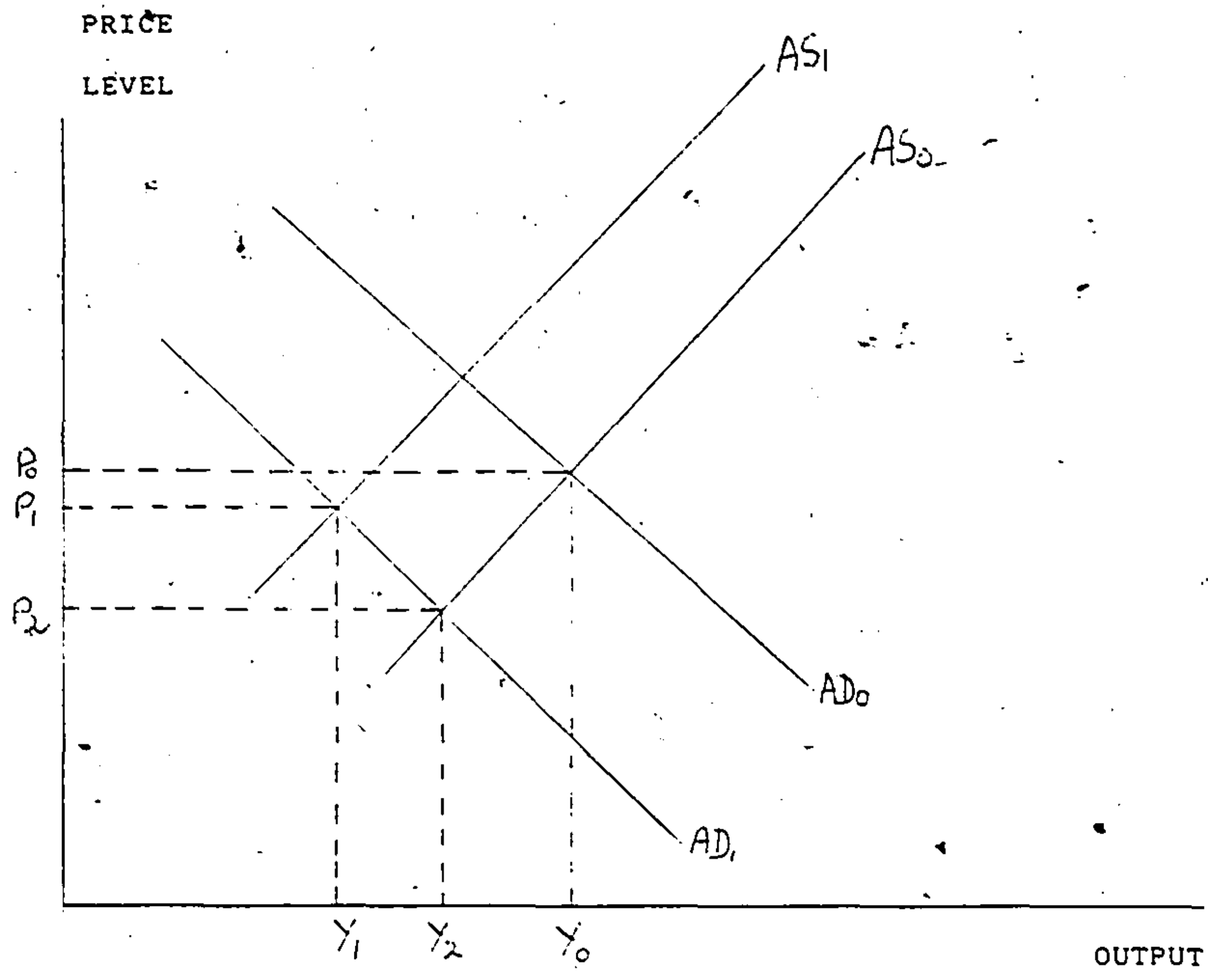
The primary aim of the empirical work in this thesis is to re-examine the role of real money balances in a production framework that does not assume instantaneous adjustment of capital stocks.

The limitations of the existing empirical work are not the only motivations for our study. A secondary issue, on which we like to focus on, is the macroeconomic implications of introducing real money balances into the production function. Standard macroeconomics ignores the role of real money balances in the production function. Therefore, the aggregate demand function for labour is independent of the rate of interest in the short-run and there are no supply side effects of changes in the rate of interest on the level of employment. Inclusion of real money balances in the production function as a factor input, on the other hand, provides potential for these supply side effects on the employment levels.

Consider the standard aggregate supply (AS) and demand (AD) functions in Figure 1. In this model we also assume that real money balances enter into the aggregate production function as a factor input. A contractionary monetary policy, aiming to reduce inflation in the short-run, leads to an increase in the rate of interest. The increase in the interest rate shifts the aggregate

FIGURE 1

AGGREGATE DEMAND AND AGGREGATE SUPPLY ANALYSIS



demand function from AD_o to AD_1. However, what is generally unrecognized is that the aggregate supply function may also shift from AS_o to AS_1, if labour and real money balances are complements in the production process. With supply side effects, as shown, the price level goes down from P_0 to P_1 rather than to P_2 if the aggregate supply function did not shift. Not only is the decrease in price smaller in the face of this sort of supply shift, the decrease in the level of output and therefore the level of employment is more with the shift in the aggregate supply function. Clearly the shift in the aggregate supply function offsets the aim of the policy. In fact, if the shift in the aggregate supply function outweighs the shift in the aggregate demand function then we would find that the policy actually resulted both in an increase in the price level and unemployment. It is these possibilities that further motivate the need for the work that follows.

There are two main tasks that we undertake in this chapter. **First**, we estimate the size of the supply side effects of changes in the rate of interest on employment. These macroeconomic implications are based on estimates of production function parameters that follow from the existing empirical studies of the demand for money based on the production function approach.

Second, we argue that there is a need to re-examine the role of real money balances in the firms' optimization problem because the empirical studies that have been conducted so far in this area assume instantaneous adjustment of capital stocks.

1.1 MACROECONOMIC IMPLICATIONS

In this section we introduce a supply-determined macro model with real wage rigidity[3]. We demonstrate the significance of the supply-side effects to changes in the rate of interest on the level of employment by comparing the interest rate elasticities of employment in two models: one in which output is supply-determined and one in which output is demand determined.

Without allowing any role for money in the production process, the interest elasticity of employment is zero, when real wage rigidity is assumed. The aggregate supply function is vertical and therefore any shift in the aggregate demand function (such as that caused by a change in the interest rate) does not change the level of output. However, when real money balances are introduced into the production function, the demand for labour becomes a function of the opportunity cost of money, i.e., the rate of interest, and the aggregate supply function shifts with changes in the rate of interest. The first order conditions for profit maximization (with static expections for the firm) together with the production function are

[1.1] $F_n(N,m,K) = \bar{w}$ [Labour Demand]

[1.2] $F_m(N,m,K) = r$ [Money Demand]

[1.3] $I/K = \beta\{F_k/(r+\delta) - 1\}$ [Investment Function]

[1.4] $Y = F(N,K,m) = AN^{\alpha}K^{\theta}m^{\gamma}$ [Production Function]

where F_n and F_m are the marginal products of labour(N) and real money balances(m) respectively. $\bar{w}$ and r are real wages and the rate of interest respectively. δ is the rate depreciation. [1.3] is an investment(I) function based on the assumption of costs of adjustment for capital(K), where β is the reciprocal of the coefficient of costs of adjustment[4].

Since capital is fixed in the short-run, and real wages are fixed by the assumed labour supply behaviour, and since we take the interest rate as the exogenous instrument of monetary policy for this demonstration of policy relevance, [1.1] and [1.2] can be solved for the level of employment and money supply. The level of investment in [1.3] can be solved recursively. Using the particular functional form of the production function specified in [1.4], we can calculate the interest elasticity of employment by differentiating [1.1] and [1.2] simultaneously[5],

[1.5] $(dN/N)/(dr/r) = -\gamma/(1-\alpha-\gamma)$.

Several empirical studies attempt to measure the productivity of cash balances as a factor input in the production function. Sinai and Stokes[1972], using a Cobb–Douglas function, were the first to report direct estimates on this productivity. Their model was later extended by Short[1979] and Simos[1981] to a translog production function. Short also provides estimates of the parameters of a Cobb–Douglas production function as a special case. Dennis and Smith [1978] utilized a cost minimization framework constrained by a production function that includes real money balances as a factor input.

We use the production function parameter estimates of Short[1979], for the Cobb–Douglas function, to get a preliminary estimate of the interest elasticity of employment in [1.5], as

[1.6] $(dN/N)/(dr/r) = -0.143.$

The interest elasticity of employment in [1.6] is zero if money did not enter the production function in [1.4], i.e., if $\gamma=0$. It is important to establish whether the difference between 0 and –0.143 is economically significant. A base for comparison can be had by calculating the magnitude of the interest elasticity of employment in the standard textbook model where output is demand–determined.

Consider the following aggregate demand model,

[1.7] $Y = C\{(1-t)Y\} + I + G$ [Aggregate Demand]

[1.8] $I/K = \beta\{(F_k/(r+\delta)) - 1\}$ [Investment function]

[1.9] $Y = F(N,K) = AN^{\alpha}K^{\theta}$ [Production Function]

where G and C are the government expenditure and consumption respectively and t is the tax rate(the rest of variables and parameters are defined on page 6).

Substituting [1.8] in [1.7] and taking the total differential, we have,

[1.10] $\lambda dY = dG-[\beta KF_k/(r+\delta)^2]dr$

where $\lambda=[1-C_y(1-t)-(\beta F_{kn}/F_n)/(r+\delta)]$.

and C_y is the marginal propensity to consume, F_{kn} is the partial differential of the marginal product of capital with respect to labour and F_n is the marginal product of labour.

Rewriting [1.10] for the particular functional form of the production function in [1.9], we have

[1.11] $\lambda dY= dG-[\beta\theta Y/(r+\delta)^2]dr$

where $\lambda=[1-C_y(1-t)-(\beta\theta/(r+\delta))]$

The interest elasticity of aggregate demand can be derived from [1.11] directly[6],

[1.12] $(dY/Y)/(dr/r) = -[(1/\lambda)(\beta\theta Y)/(r+\delta)^2][r/Y]$

$= -(1/\lambda)\beta\theta r/(r+\delta)^2$ (Using [1.3])

We get a representative estimate of the interest elasticity of aggregate demand in [1.12] by using the following parameter values: $C_y=0.8$ and $t=0.4$ (taken from Blinder and Solow [1973]), $\beta=0.1$ and $r=0.025$(taken from Tobin and Brainard [1977]), $\theta=0.163$ (taken from Short[1979]) and a value of 0.05 has been assumed for δ.

Substituting these values we get,

[1.13] $(dY/Y)(dr/r) = -0.2398$

The aggregate demand elasticity in [1.13] can be translated into the interest elasticity of employment by noting that $dY/Y = \alpha dN/N$, from the production function in [1.9]. Setting $\alpha = 0.8$, where the value of α has been taken from Short[1979],

[1.14] $(dN/N)/(dr/r) = -0.2997$.

We find that the interest elasticity in the supply-determined model, given in [1.6], is about half the absolute size of this elasticity of the demand-determined model. Since the profession has given great stress to the effects of interest rates on aggregate demand, our comparison suggests that supply side effects of monetary policy should receive more emphasis than it currently does. Although, the basic purpose of this thesis is to do a better job of estimating the production function parameters than exists in the literature, more accurate estimates of the supply side effects of monetary policy may be obtained as a byproduct.

1.2 OUTLINE OF THE EMPIRICAL WORK

One major shortcoming of all the empirical studies on the demand for real money balances by firms based on the production function approach is the assumption of instantaneous adjustment of capital stocks. In general, investment cannot take place immediately in response to some external shock. Additions to plants require some planning on the part of firms and can not be affected instantaneously. Previous studies of the demand for money by firms have tended to ignore the disequilibrium process by which firms adjust to external shocks. Analysis of this disequilibrium process is important for the analysis of any short-run macro policy implications. For example, the response to external shocks by a firm in the short-run, when the firm is in disequilibrium with respect to its holdings of capital stocks, may be very different from its response in the long-run when it can fully adjust its capital stock. Recent studies on the role of energy in production have questioned the assumption of instantaneous adjustment of the capital stock and have developed quite successful empirical models that explicitly incorporate the notion of the cost of adjustment of capital in the optimization framework. The factor demand functions in these studies not only depend on the factor prices but also on the level of capital stocks and net investment. These studies have also made clear that the results of the static models are contaminated by imposing unreasonable cross-equation restrictions on the factor demand system by ignoring the fixity of capital stocks in the short-run.

The basic aim of the empirical work in this thesis is to re-examine the role of real money balances in a production framework that does not assume instantaneous adjustment of capital stocks. Our empirical analysis of the demand for money by firms, based on the production function approach is divided into three stages. In the **first stage**, we estimate a full equilibrium model, where firms are assumed to minimize cost and can instantaneously adjust the capital stock. We make use of the translog function, a more generalized functional form for the cost function[7]. Unlike the Cobb-Douglas function, this function does not impose any restrictions on the substitution possibilities between factor inputs.

In the **second stage**, we estimate a temporary equilibrium model in which capital is a quasi-fixed factor. However, no assumption about the costs of adjustment of capital has been explicitly incorporated into firms' optimization. This model also employs the translog cost function. Therefore, it also enables us to get a direct estimate of short-run elasticites in which capital stocks are held constant.

Finally, a full dynamic model is estimated in which non-linear costs of adjustment have been incorporated into the firms' optimization framework. The model assumes that these costs of adjustment are internal. In this model, investment uses some of the resources of the firm, including labour, and so the demand for each of the variable factor inputs is also a function of investment. Since investment is inversely related to the rate of interest, changes in the interest rate may affect indirectly the labour demand. Therefore any policy that affects the level of investment in the short-run, also influences the labour demand. This provides an additional route for interest rate effects on the demand for labour through changes in investment demand. These supply side effects of monetary policy on the level of employment are independent of whether money enters into the production function or not. We estimate both direct and indirect supply side effects of monetary policy on labour demand[8].

1.3 STRUCTURE OF THE DESSERTATION

Following this introduction, we review the literature concerning the demand for money by firms. in Chapter 2. In Section 2.1, three alternative approaches to examine the role of money balances in the firms' optimization problem has been discussed. It has been argued that the production approach is general one of these and provides a convenient framework to study the substitution possibilities between money and other factors of production. Section 2.2 reviews the empirical work that has been done to-date using the production function approach. In Section 2.3 the implications of partial adjustment models are discussed.

In Chapter 3, an empirical model based on the assumption of instantaneous adjustment of capital stocks is developed. Section 3.1 considers a translog cost function, a member of the group of flexible functional forms which allows the elasticities of substitution to take any values. This functional form is shown to be desirable for our study where we would like to allow the elasticities of substitution between money and other factors to be determined by the data and to allow them to assume any value, including of zero. In Section 3.2 the issues related to production are discussed. In Section 3.3, we discuss the sources of data employed in this thesis and the results of the full equilibrium model.

In Chapter 4, an empirical model based on fixity of capital in the short-run is analysed. In Section 4.1 we examine the theoretical foundations of a temporary equilibrium model and develop an empirical model in which capital is held fixed in the short-run. In Section 4.2 we discuss the issues related to production structure. In Section 4.3 the results of the model are discussed.

In Chapter 5, we develop an empirical model for the firms' dynamic demand for real balances based on the notion of internal costs of adjustment for capital. Section 5.1 reviews the literature on the costs of adjustment of capital. Section 5.2 discusses the theoretical model based on internal costs of adjustment and develops the empirical model in which adjustment costs are incorporated into the firms optimization problem. In Section 5.3 the issues related to production structure are discussed. In Section 5.4 we discuss the econometric technique used to estimate the dynamic model and also discuss the results of the model.

Chapter 6 presents conclusions and gives suggestions on ways in which the research could be extended.

FOOTNOTES:

Chapter 1

[1]See Coates[1976] for a survey of these studies.

[2]These models are based on the internal costs of adjustment for capital (see for example Morrison and Berndt[1981]). In chapter 5 of this thesis these models are discussed in detail.

[3]We use the example of a supply-determined model to simplify our derivation of interest elasticity of employment, since fewer parameters are involved. Theoretically, the argument is also valid for a positively sloped aggregate supply function, as we discussed above.

[4]See Scarth[1984].

[5]Taking the total differential of these equations and setting $dw=dK=0$ gives the formula for elasticity in [1.5] for the Cobb-Douglas production function in [1.4].

[6]The rate of interest has been taken as an instrument of monetary policy, so the LM curve is horizontal and we do not need the parameters associated with LM curve in order to derive the elasticity. Furthermore, if there were no reason for aggregate demand to depend directly on the price level, the aggregate demand function would be vertical. As long as we assume non-indexed taxes, this will shift the IS curve with the changes in the price level and keep the aggregate demand function downward sloping.

[7]The empirical models in this thesis are based on the assumption that firms minimize cost subject to given level of output. This framework has been chosen by most authors because, apparently, cost minimizing models fit better to data. In order to deduce the macro implications we indirectly calculate the profit maximizing interest price elasticities of labour demand and compare them with the elasticity in [1.6].

[8] There is some inconsistancy in the specification of the macro models discussed in Section 1.1. The assumption of costs of adjustment implies that some of the output, Y, is used up in the installation of new capital stocks. Therefore, the output available for consumption is less than Y (see Scarth[1984] for more details). We did not make this correction in our illustration of the macroeconomic importance of money entering the production function (in Section 1.1), in order to keep the calculations simplified. The empirical work reported in the later chapters is not limited in this way, since we allow for both routes for supply-side effects there.

CHAPTER 2

REVIEW OF THE LITERATURE ON THE DEMAND FOR MONEY BY FIRMS

There have been both theoretical and empirical discussions in the literature about the behaviour of business firms concerning their holdings of cash balances. The literature on this subject is vast and diverse. The purpose of this chapter is to provide a survey of this work, with an emphasis on the issues that are relevant to the subsequent development of the empirical models in this thesis.

Friedman[1956, p.60] rejected the possibility that the business demand for money may depend on different variables than consumer demand,

> *"the demand for money on the part of business enterprises can be regarded as expressed by a function of the same kind as equation (7) [developed for consumers], with the same variables on the right hand side."*

In the late 1960's, however, several studies analyzed the differences between household and business holdings of cash balances and concluded that businesses and individuals use different criteria for determining their cash holdings[1]. Moreover, in an appraisal of the shortcomings of aggregate demand models, Goldfeld[1976] suggested that the motives for holding cash balances are different across firms and individuals and the use of a single aggregate model for this behaviour may represent too much of a compromise of economic theory.

It is possible to identify four different theoretical approaches to modelling a firm's demand for money. <u>**First**</u>, the production function approach treats real money balances as a factor of production in the production process. <u>**Second**</u>, the Austrian approach treats money as a catalyst in the production process. This approach argues that money should not be included in the production function as a factor. <u>**Third**</u>, the transaction demand model has the firm keep the cash to meet its transaction requirements and try to minimize the costs of holding the cash balances necessary for transactions. The <u>**fourth**</u> approach is the portfolio model which introduces risk and uncertainty motives for holding cash balances[2].

In this chapter we limit ourselves to the first three theoretical approaches. These theoretical approaches are discussed in Section 2.1. In Section 2.2, we cover some of the related empirical studies on the firm's demand for money. Section 2.3 discusses the limitations of the existing empirical work on the demand for money by firms and builds up the foundation for our empirical work in this thesis.

2.1 MAJOR THEORETICAL APPROACHES TO THE ANALYSIS OF DEMAND FOR MONEY BY FIRMS

2.1.1 MODEL A: PRODUCTION FUNCTION MODEL

The inclusion of money in the production function model is based upon the assumption that holding of money results in smaller capital and/or labour inputs than would otherwise be required for a given level of output. In other words, the output of a firm can be described by a production function,

$$Y = F(K, N, m) \qquad [2.1]$$

where K, N, and $m=M/P$ are the capital stocks, labour, and real cash balances respectively. P is the general price level.

On theoretical grounds, several authors have argued that real money balances are a factor of production[3]. Baily[1962, p.59] states that,

> *"...cash balances held by business firms are obviously a productive service similar to any other.....they reduce the other resources required for a given level of production by facilitating payments."*

Nadiri [1969, pp.17] also takes a similar view.

According to Nadiri,

> *"Real cash balances serve as productive inputs..They are part of the working capital of the firm facilitating its productive process, often by indirect means, such as hedging against changes in the prices of capital and labor and the interest rate".*

The standard neoclassical production function relates real output to real inputs. Yet, firms must adopt some kind of exchange arrangement which can combine and organize inputs in order to produce output. In a barter economy, considerable labour and capital may be expended in order to ensure trade.

> *"Labor must be hired, capital services rented and factors combined at various physical locations so that production can occur. Laborers, owners of capital goods and entrepreneurs must go to markets and exchange physical goods and services in return for services and goods. Time, effort and capital may be utilized in the process. Thus, labor and capital may be diverted from production to distribution in order to achieve the "double coincidence " required in a barter economy....In a monetary economy, productive efficiency may increase as labor and capital services, released from the special tasks required in a barter economy, are used in production".*[4]

Inclusion of real money balances in the production function is somewhat analogous to its inclusion in the utility function in theories of consumer demand for money. The fact that firms hold real cash balances at a positive opportunity cost suggests that they yield some ill-defined flow of services to the firms. Stein[1970] has suggested that the expected return on real balances can be decomposed into three components- a marginal product, opportunity cost of having an asset which fluctuates less than some other assets in real value. The marginal product of real money balances reflects the increase in output as a result of increasing the money balances which may release labour and capital services for utilization in production instead of distribution.

One may develop a logical basis for treating real money balances as a factor input in the production process by examining the theoretical underpinnings of the production function itself. Shephard[1970, p.4] defines a production function as a mathematical summary of output that is implied by the use of factor inputs within alternative feasible arrangements of technical processes (Dennis and Smith p.796). Therefore, a production function summarizes both the engineering activities involved in transforming inputs into outputs, and the productive function performed by money in facilitating the process of combining and acquiring the factor inputs over the time interval of the production process[5]. Fischer[1974, p.529] has argued along these lines as follows,

> *"the first reason money is different from the other factors of production, is that we are used to thinking of a production function as an expression of hard technological relations, valid independently of market arrangements. ..... We have shown that even if there is a separate physical production function, a production function including real cash balances can be defined-though, to be sure, it depends on the particular exchange arrangements which the firm faces."*

In a micro-analytic framework we may assume that firms minimize costs subject to a production function such as in [2.1]. The firm chooses the levels of factor inputs by equating the ratios of any pair of the marginal products to the corresponding ratio of the pair of rental prices in competitive factor markets, the conditions for factor demands would depend on the the level of output and the ratios of factor prices[6]. Thus we expect,

$$
[2.2] \qquad
\begin{aligned}
N &= N\{Y,\ V/W,\ r_m/W\} \\
K &= K\{Y,\ W/V,\ r_m/V\} \\
m &= m\{Y,\ W/r_m,\ V/r_m\}
\end{aligned}
$$

where W, V and r_m represent the rental prices of labour, capital, and real money balances M/P respectivly.

One of the interesting features of the derived factor demand functions in [2.2] is the dependence of both labour and capital demands on the opportunity cost of money, the rate of interest. Furthermore, we note that the production function approach does not restrict the substitution possibilities between money and other factors of production. We expand on this point in the later part of this chapter.

2.1.2 MODEL B: A MONEY REQUIREMENTS FUNCTION

An alternative approach to Model A is taken by Gabor and Pearce[1958] , Vickers[1968] and others. They adopt an Austrian approach to the demand for money. According to this approach, money is considered as a capital fund from which advances are made to meet the rental payments to other factors. However, a distinction between real capital and money capital is made. Real capital is viewed as specific machines and equipment while money is "conceived as a Knight-like fund of value or general purchasing power", (Moroney [1972], p.339). As Gabor and Pearce[1958, pp.540–41) put it:

> *"....there is an essential difference between money and any other factor. Production could conceivably be carried on without money if other inputs were made available by some dictatorial hand; but the removal of a unit of any other factor would immediately reduce the product. Money capital is a catalyst".*

The model therefore separates the pure technical relationships between output and factors, such as labour and capital, from the relationships between money and output. This is done by specifying a "money requirements function" in addition to a neoclassical production function. Turnovsky [1970], for example, specifies a money requirements function that depends on the level of input usage. This dependence is rationalized by assuming that the payments to the factors are made at intervals over the period and, therefore, the average working capital depends on the level of input usage. These functions may formally be written as:

[2.3] $$Y = F(N, K) \qquad F_1, F_2 > 0$$

[2.4] $$M = M(WN, VK) \qquad M_1, M_2 > 0$$

where F_i and M_i represent the partial derivatives with respect to the ith argument in the functions and WN and VK are the rental cost of labour and capital serivices.

Cost minimization results are obtained when the ratio of marginal products of labour and capital are equated to the ratio of their marginal costs. These marginal costs include both the rental cost and any costs arising out of an increased requirement of money balances(the effects being given by M_1 and M_2 in [2.4]). Thus we have

[2.5] $$F_1/F_2 = \{W + r_m M_1\}/\{V + r_m M_2\}.$$

The resulting labour and capital demands depend on the level of output and the ratio of marginal costs, i.e,

[2.6a] $$N = N\{Y, (V + r_m M_2)/(W + r_m M_1)\},$$

[2.6b] $$K = K\{Y, (W + r_m M_1)/(V + r_m M_2)\}.$$

One essential difference in the resulting demand functions for labour and capital in Model A and Model B is the nature of their response to changes in the interest rate. If we differentiate [2.6] with respect to r_m, we have,

[2.7a] $$dN/dr_m = N'\{ (WM_2 - VM_1)/(W+r_m M_1)^2 \}$$

[2.7b] $$dK/dr_m = K'\{ (VM_1 - WM_2)/(V+r_m M_2)^2 \}$$

where N' and K' are the partial derivatives of N and K with respect to the appropriate ratios of the marginal costs. The cost minimizing condition in [2.5] implies that these partial derivatives are positive. The numerators (in the bracketed expression) in [2.7] are opposite in sign and denomonators are positive. Therefore, either m is a complement to N and a substitute for K or m is a complement to K and a substitute for N. On the other hand, Model A is less restrictive in this respect since money can be substituted for one or both of the inputs.

Saving[1972] extended the money requirements approach by replacing the money requirements function in Model B by a generalized transactions cost function. The transactions cost function gives the minimum transactions cost associated with nominal sales, expenditure on labour and capital goods, average real money balances and the average levels of output, labour and capital, i.e.,

[2.8] $$T = T\{PY,\ WN+VK,\ Y,\ N,\ K,\ M/P\}$$

where Y, N, and K represent average inventory levels and T is the transaction cost.

[2.8] could be interpreted as representing the minimization of transactions cost over alternative time paths of transactions requiring the same level of inventories and money balances. The resulting first order conditions from [2.8] and [2.3] imply that the N and K demand functions are independent of the rate of interest. This result depends crucially on the assumption that the transaction costs in [2.8] depend on the total expenditure on factors, i.e, WN+VK. If we specify a more general transaction cost function as,

[2.9] $T = T\{PY, WN, VK, Y, N, K, M/P\}$

then labour and capital demand functions again depend on the rate of interest[7].

In summary, the factor demand functions that follow from the money requirements approach are more restrictive than those derived from the production function approach and typically can be thought of as a special case of the production function approach.

2.1.3 MODEL C: THE TRANSACTION DEMAND MODEL

The basic inventory or transaction demand model was developed by Baumol[1952] and Tobin[1956]. In comparison to Model A and Model B, where money is treated as being directly or indirectly involved in production, in the transaction demand model the demand for money is determined by modelling only the firm's transactions. The Baumol–Tobin inventory model, as most other inventory models, is deterministic though Miller and Orr[1968] extended the inventory model to a stochastic framework.

Application of inventory theory to the theory of demand has been justified by Baumol[1952, p.545] on the basis that,

> *"A stock of cash is its holder's inventory of the medium of exchange, and like an inventory of a commodity, cash is held because it can be given up at the appropriate moment, serving then as its possessor's part of the bargain in an exchange."*

In Model C we assume that labour is the only factor input which is used in the production process. This assumption is made in order to simplify the analysis of the model. A firm's production function can be written as follows:

[2.9] $Y(t) = F\{N(t)\}, \quad F_n > 0 \; ; \; F_{nn} < 0$

where Y(t) and N(t) represent the flow of output and the flow of labour input respectively. F_n and F_{nn} are the first and second partial derivatives of the production function with respect to labour.

The model assumes that there are no transactions cost involved in selling the product or hiring the labour. Goods produced are sold at a fixed price P and payments to the labourers are made at the time they are used. The resources that are generated by selling the product after payments to the labourers, can be held either in the form of cash, which earns a rate r_m, or transferred at a fixed cost b per transaction into bonds, which earn interest at a rate r_b ($r_m < r_b$). Furthermore, all the bonds are transferred into money at the end of the period, again giving rise to a fixed cost b, and the cash is distributed among the owners of the firm.

The first order profit maximization condition is given by:

[2.10] $PF_n = W$ for all t.

Therefore, for given output and input prices, a corresponding cash flow is generated which is given by:

[2.11] $$R = PF(N^*) - WN^* = R\{P,W\}$$

where N^* is the profit maximizing level of labour demand.

With this flow of cash, for given prices, a firm's problem is to hold an amount of cash which minimizes the cost of transactions relating to the transfers between money and bonds, and which minimizes interest earnings that are lost by holding money rather than bonds. The model also assumes that firms divide the period into sub-periods of uniform length. At the beginning of each sub-period, the firm cashes some of its bonds, (represented by C), and makes factor payments in that period. The cash balance of the firm takes on a saw tooth shape during the period, i.e., at the beginning of each sub-period the firm cashes C and uses it until the cash holdings are zero at which time it cashes another amount C, and so on. Therefore, the average cash holdings of the firm is C/2. The minimization problem can be formulated in two ways. First, the firm can determine the size of each cash withdrawal, C, and therefore the number of withdrawals, n, can be determined by dividing the total cash flow R by C. Alternatively, the firm can choose its number of withdrawals and then determine the size of cash holdings in each period by dividing R by n. Both approaches result in a single type of demand for money function. By latter approach, the total cost of holding cash balances can be divided into two parts, the net interest lost on the average cash holdings, and the fixed transaction cost paid for each cash withdrawal. The total cost can be written as,

[2.12] $G = bn + (r_b - r_m)R/2n.$

Minimization of [2.12] with respect to n results in,

[2.13] $n = \{R(r_b - r_m)/2b\}^{1/2}.$

The average cash holdings are given by,

[2.14] $M = C/2 = R/2n.$

Combining [2.13] and [2.14] give[8],

[2.15] $M = \{bR/2(r_b - r_m)\}^{1/2}.$

The fact that in this model there is no interaction between production and financial decisions, leads to a demand for labour that is independent of the interest rate.

Fischer[1974] discusses a simple reinterpretation of this Baumol-Tobin inventory model by involving labour in financial transactions as well. Specifically, he assumed that one transaction uses one unit of labour services, i.e., b=W. Total labour services hired by the firm are divided into two types. N_1 represents labour that is used for production and N_2 is the labour used for transactions, so that,

[2.16] $N = N_1 + N_2.$

The production function in [2.9] and [2.16] gives,

[2.16] $$Y = F(N-N_2)$$

and the money demand function in [2.13] can be written as,

[2.17] $$M/R = 1/2n = 1/2N_2.$$

Substituting [2.17] in [2.16] for N_2,

[2.18] $$Y = F(N - R/2M)$$

[2.19] or $$Y = G(N,M/R) = G(N,m) \text{ where } m = M/R$$

The derived function [2.19] can not be distinguished from a production function. Therefore an alternative motivation for a production function with real money balances can be found in a transaction demand model in which labour is needed in financial as well as other transactions.

2.2 EMPIRICAL STUDIES OF THE FIRMS' DEMAND FOR MONEY

There are a number of empirical studies of firms' demand for money that have appeared in the literature[9]. In this section, however, we focus only on studies that are based on the production function approach.

Nadiri[1969] first utilized the neoclassical framework to estimate a firms' demand for money. He hypothesized that a firm's total cost is:

[2.21] $$C = WN + VK + rm$$

where W, V and r are the cost of labour, N, capital, K, and real money balances, m. Firms are then assumed to minimize the expected cost subject to a production function which is assumed to be twice differentiable:

[2.22] $Y^* = f(N, K, m)$

where Y^* is the expected level of output.

Using [2.21] and [2.22], Nadiri derived a demand function for real money balances as:

[2.23] $m^* = f(V/W, r, Y^*)$.

Nadiri also introduced distributed lags in real cash balances by using a Koyck distributed lag mechanism:

[2.24] $m_t/m_{t-1} = \{m_t{}^*/m_{t-1}\}^b$

and a forecasting mechanism for output:

[2.25] $Y_t^* = [Y_t]^{k1} [Y_{t-1}]^{k2}$.

Finally, he derived the money demand function using [2.24] and [2.25] in log form:

[2.26] $\ln m_t = a_o + a_1 \ln r + a_2 \ln(V/W) + a_3 \ln Y + a_4 \ln Y_{-1} + a_5 \ln m_{-1}$

Nadiri estimated equation [2.26], employing quarterly data for U.S. manufacturing(1947–1963), using ordinary least squares. Nadiri's conclusions are: first, real money balances appear to be a factor of production. Second, the long term interest rate performed somewhat better than the short term interest rate. Third, the long-run elasticity of real money balances with respect to its own price is -0.18, and with respect to (V/W) it is 0.27. The latter result was interpreted by Nadiri as indicating substitution between investment in real assets and investment in real money balances.

There are several limitations of Nadiri's approach. **First**, his model limits the substitution possibilities between real money balances and other inputs. In particular, in [2.26], if a_2 is positive, money is complementary with labour and substitutable for capital, while if a_2 is negative, the converse is true. Real money balances can be substituted either for capital services or for labour but not for both. **Second**, he estimated the money demand function in isolation. The efficiency of the estimates could have been increased by incorporating the cost function and other demand functions. **Third**, the link between the long-run and short-run is quite arbitrary and **finally**, the model specified a partial adjustment process for the real money balances but ignored the adjustment lags associated with capital stocks. According to Coates[1976, p.41], cash balances are more fungible than the real goods, and costs of changing these balances are smaller on average when the rate of investment or disinvestment is greater.

A few years later Sinai and Stokes[1972] were the first to make use of a Cobb-Douglas production function. They found significant and positive correlation between real money balances and real output.

One of the major problems with the Sinai and

Stokes study is the inconsistency in the data set. The data used on output, capital and labour are for the private domestic sector, while the data on money balances are for the total economy (including the money holdings of consumers). The other major problem is the use of the restrictive Cobb-Douglas form, which imposes the restriction that the elasticity of substitution is unity. In addition, one can note that the production function is estimated rather than a reduced form and no recognition of capital adjustment is made.

Dennis and Smith[1978] later extended the Nadiri cost minimization approach. They used the four factor translog cost function for 11 two-digit industries of the U.S.A. over the period 1952-1973. Utilizing the first-order conditions for cost minimization, Dennis and Smith derived four cost share equations for real money balances, capital, non-production labour and production labour. Their results indicate that the interest elasticity of demand for cash balances ranged from -0.22 to -0.41 across industries. From their results, they also concluded that the neoclassical model provides considerable promise for modelling the firm's demand for money. In particular they note [p.807],

> *"Real cash balances are not separable from the remaining factor inputs. Thus omitting them in previous analyses of production technology for these industries offers, in principle, the potential for serious specification error."*

In their study they did not estimate the cost function with the subset of cost share equations. The parameters in the cost share equations are a subset of the parameters in the cost function. Therefore the efficiency of estimated parameters could have been increased by estimating the cost function together with the share equations. Furthermore, they also assume instantaneous and costless adjustment of capital stocks.

Short[1979] also adopted the production function approach to real money balances. She derived and estimated the revenue share equations, using the first order conditions for profit maximization[10]. She employed both Cobb-Douglas and the more flexible functional form of the translog and found a positive and significant role of real money balances in the production process.

Short has used a stacked regression technique to estimate the set of share equations. The problem with this technique is that it attaches the same weight to the errors of the production function and the share equations. It has been shown, in other contexts, that a more appropriate technique to estimate a system of share equations and a cost or production function is to use the iterative Zellner efficient estimator[11]. Her model also ignores the fixity of capital stocks in the short-run.

Simos[1981] recently estimated the aggregate production function and the resulting revenue share equations for the private domestic economy of the U.S.A.(1929–1972). Simos used the same source of data as employed by Short, for output, labour, and capital. He used the iterative Zellner efficient estimator to estimate the system of revenue share equations.

Simos's primary conclusion was to reject the hypothesis that the relationship between capital and labour is independent of the level of real money balances. According to Simos, [pp.223–224],

> *"The statistical results of the translog production function clarify the debate which has occurred on whether real money balances are an original factor input or a catalyst with a role similar to technological innovation in the production process. Furthermore we find that real money balances are substitutes for capital but complements with labor, implying that the derived demand for labor is inversely related to the cost of using money.......omission of real money balances from the production function overlooks the relative complementarity between labor and capital and creates a serious specification error that has important implications for the estimates of primary factor substitution."*

A common problem with the studies of Short[1979] and Simos[1981] (and, as mentioned earlier, with Sinai and Stokes' work) relates to the inconsistency of the data base they have employed. In both studies, the total money holdings of the private sector of the United States has been employed. On the other hand, the data on labour and capital relate to the manufacturing industries of the United States (1929–67).

Although all the studies reviewed in this section do indicate that the use of a production function model of real money balances may yield interesting results, <u>they also assume that capital stocks can be costlessly and instantaneously adjusted to the desired level.</u> Therefore the results of these studies may be contaminated by imposing the cross equation restrictions that exist by assuming costless adjustment of capital stocks. In the next section of this chapter we discuss the implications of partial adjustment models and costs of adjustment in modelling the production role of money.

2.3 CONCLUDING REMARKS

In the preceding sections, we have discussed both the theoretical and empirical models that deal with the demand for money by firms. At the theoretical level we discussed three models: **first**, the production function model that treats real money balances as any other input; **second** , the money requirement function approach and an extension of this model in terms of a transaction cost model; and **third**, the transaction demand model in which money is held in order to meet the day to day expenses of the factors employed.

The choice of a theoretical framework to develop an empirical model for studying the firm's demand for real cash balances depends on the issues that are intended to be addressed by the model. Therefore we give a summary of these models with reference to the issues that we intend to examine.

The **first** issue we wish to examine is whether real money balances can be substituted for other inputs. Model A provides a framework that allows for substitution possibilities between real money balances and other factor inputs. In this framework, we can also test whether real money balances enter in a more general way into the production function as a factor input or whether they are separable from other factors. We test the separability of real cash balances from other factors in the full equilibrium model in Chapter 3.

The **second** interesting issue relates to the interest sensitivity of both real money balances and other factor inputs in the production process. Model A provides a convenient framework to evaluate the cross price elasticities of both labour and capital with respect to the rate of interest. For example if the cross price elasticity of labour demand with respect rate of interest is negative then it may imply that any contractionary monetary policy aiming to reduce inflation, may not be very effective. An increase in the interest rate shifts both aggregate demand and aggregate supply functions(as shown in Chapter 1).

A **final** question of interest is to evaluate these elasticities in a model which introduces fixity of capital in the short-run. The fixity of capital can be introduced arbitrarily or by introducing costs of adjustment for capital into the firm' optimization problem. These modifications of the production model will be discussed later in this chapter and in Chapters 4 and 5.

Consequently, we choose the production function model as the basic framework for our empirical work. This framework can most conveniently incorporate the range of input substitution possibilities, interest rate sensitivities and can be generalized to a dynamic framework that incorporates the costs of adjustment of capital.

Both the theoretical and the empirical examinations of the production function approach to real money balances reveal that, in general, no explicit assumption has been made about the cost of adjusting factors and possible lags in the adjustment of factors of production. In particular, all the studies reviewed assume no lags associated with adjusting capital stocks. The empirical models discussed earlier are static or equilibrium models which imply that firms are able to adjust instantaneously to exogeneous shocks. The one exception is Nadiri's model where partial adjustment has been assumed for cash balances. The assumption of instantaneous adjustment of capital stocks is open to examination. Recent empirical studies on energy demand have explicitly introduced costs of adjustment associated with capital stocks into the optimization framework of the firm. These models open up several interesting issues in the modelling of resource substitution and have proven to be quite sucessful.

In Chapter 4, we develop an empirical model which introduces fixity of capital stocks in the short-run, without making any explicit assumption about costs of adjustment for capital. In Chapter 5, however, we introduce the assumption of costs of adjustment of capital into firms' optimization framework. If one argues that cash balances are needed to facilitate the adjustment of other assets then the production function in [2.1] can be rewritten to incorporate the assumption of internal costs of adjustment associated with capital stocks as[12]:

[2.26] $Y = F\{K, N, m, I_g\}$ $\quad \partial F/\partial I_g < 0$

where K, N and m are the stocks of capital, labour, and real money balances and I_g is gross physical investment. [2.26] implies that a change in the capital stocks results in a reduction of the output produced. Money in [2.26] can be viewed as reducing the cost of adjustment if $\partial^2 F/\partial I_g \partial m < 0$.

Based on the type of production function as specified in [2.26], an empirical model of the demand for money has been estimated in Chapter 5.

FOOTNOTES:

Chapter 2

[1]See Meltzer[1963], Vogel and Maddala[1967], Whelen [1965] and McCall[1960]. A review of these studies can be found in Coates[1976].

[2]Turnovsky[1970] has developed a model for the demand for money by firms by providing a choice of financing their purchases of producer goods by borrowing money or issuing equity. Risk is introduced in the net income stream of firms. The firms are then assumed to maximize net present value and determine their cash requirements. For a review of the studies of demand for money by the firms that make use of financial theory, see Butterfield[1970].

[3]Baily [1971], Friedman(1959), Johnson(1967) and Nadiri[1969] argue that money should enter the production function.

[4]Sinai and Stokes[1972]

[5]Gabor and Pearce[1958] point out that the need for money arises because in practice the flow of production cannot be instantaneously created. There has to be a build up period which is important in determining equilibrium.

[6]Butterfield[1971]

[7]Butterfield[1971]

[8]Brunner and Meltzer[1967] have criticized Baumol's square root rule in [2.15]. According to them the square root rule is only applicable to the firm which has steady state stream of expenditures.

[9]A review of cross-section and time-series studies can be found in Coates[1976].

[10]See Christensen, Jorgenson and Lau[1973] for the derivation of revenue shares for a translog production function.

[11]Barten[1969].

[12]The concepts of internal and external cost of adjustment of capital are discussed in Chapter 5.

CHAPTER 3

EMPIRICAL MODELS BASED ON COST MINIMIZING BEHAVIOUR WITH NO COST OF ADJUSTMENT

The purpose of building an optimizing model of a firm is to deduce the technological and behavioral relationships which are implicit in the firm's market actions. In this way, one can predict a firm's response to changes in the prices of factors and other relevant economic variables. In this chapter, we assume that firms act to minimize costs in the context of a static or equilibrium model. The model is quite general in that it imposes no a priori restrictions on the factor substitution possibilities.

Such a static or equilibrium model assumes that firms are able to adjust fully to exogenous shocks within one period of time. Like most studies of the demand for money by firms undertaken in the 1970s, except Nadiri's[1969] model where partial adjustment for cash balances has been assumed, this model assumes that all inputs adjust to their long-run equilibrium levels within the period.

In Section 3.1, of this chapter, we summarize the features of static or equilibrium cost models. In Section 3.2 we discuss issues relating to the production structure. In Section 3.3, we discuss the estimation procedures used for the equilibrium model, the data, and the empirical results of the static model.

3.1 A COST MINIMIZATION MODEL WITH NO ADJUSTMENT COSTS

In order to estimate the underlying technology, one can either examine the production function of a firm or the associated cost function. The application of duality theory to this optimization problem has established that, under rather weak regularity conditions, there is a unique correspondence between the production function and the cost function. Furthermore, all the information about the underlying technology is contained in each function (Shephard[1970]).

Suppose a firm uses n inputs $X_1, \ldots\ldots, X_n$ to produce output Q. The production function can be written as

$$Q = F\{X_1, \ldots\ldots\ldots\ldots, X_n\}. \qquad [3.1]$$

Assuming that [3.1] is concave, twice differentiable and that the firm minimizes cost, then there exists a cost function that is dual to [3.1] and relates minimum cost to the output level and factor prices:

[3.2] $$C = G(Q, r_1, r_2, \dots\dots\dots, r_n)$$

where r_i is the exogenous price of factor i. Moreover, a cost function of the form [3.2] that satisfies a certain set of properties (to be outlined below), is an alternative representation of a technology such as [3.1]. The basis for this duality is the Shephard Duality Theorem (Shephard[1970]), which states that a cost function (a function which describes the dependence of minimum total cost on a given level of output and factor prices) and a production function are equivalent representations of technology , under the assumption that firms minimize cost and face parametric factor prices. Accordingly, a cost function, C(Q,r), satisfying the following properties[1]:

(a) **Domain.** C(Q;r) is a positive real valued function, defined and finite for all $Q > 0$ and $r>0$,

(b) **Monotonicity.** C(Q;r) is a non-decreasing left continuous function in Q and tends to plus infinity as Q tends to plus infinity for every

[3.3] $r>0$ and is also non-decreasing in prices,

(c) **Homogeneity.** C(Q;r) is (positive) linear homogeneous in r for every $Q > 0$,

(d) **Concavity.** C(Q;r) is a concave function in r for every $Q>0$,

(e) **Continuity.** $C(Q;r)$ is continous from below in Q and continuous in r;

is equivalent to a production function, $F(X)$ satisfying,

(a) **Domain.** F is a real valued function of X defined for every $X > 0$, and $F(X)$ is finite if X is finite,

(b) **Monotonicity.** $F(0)=0$, and $F(X)$ is non-decreasing function in X,

[3.4] (c) **Continuity.** $F(X^N)$ is continuous from above, and $F(X^N)$ tends to plus infinity for at least one non-negative sequence of vectors$\{X^N\}$.

(d) **Concavity.** F is a quasi concave function over Ω^n, the non-negative ortchant R^n .

Consequently, if we define any arbitrary cost function which satisfies the properties [3.3], then there exists a corresponding production function which satisfies conditions [3.4] (and these conditions are the ones we usually want the production function to satisfy).

The choice between employing a cost function or a production function is a matter of statistical convenience and analytical purpose. We have chosen to use the cost function rather than the production function for the following two reasons. <u>First</u>, it is easier to derive reduced form factor demands in the case of the cost function approach and one can estimate directly various

Allen-Uzawa elasticities of substitution. These parameters are important for describing the pattern and degree of substitutability and complementarity among the factors of production.

The **second** advantage of using a cost function is that it is easier to develop empirical models which introduce fixity of capital stocks in the short-run. We discuss two such models in Chapters 4 and 5.

We have chosen the transcendental logarithmic (translog) specification of the cost function for our empirical work in this Chapter. This specification can be viewed as a second-order logarithmic approximation to an arbitrary twice-differentiable production function (Christensen, Jorgenson and Lau[1973]). A translog cost function imposes no *a priori* restrictions on the substitution possibilities among the factors of production. This is especially desirable in our study where we would like to allow the elasticities of substitution between money and other inputs to be able to assume any value (in particular, zero)[2]. The translog cost function can be written as:

$$[3.5] \qquad \ln C = \alpha_o + \alpha_q \ln Q + 1/2\gamma_{qq}(\ln Q)^2 + \Sigma_i \alpha_i \ln r_i + 1/2\Sigma_i\Sigma_j\gamma_{ij}(\ln r_i)(\ln r_j) + \Sigma_i \gamma_{qi}(\ln Q)(\ln r_i) \qquad i,j=1,\ldots,n$$

where C is the total nominal cost, Q is the output, and r_i represents the exogenously given price of the ith input.

In order to correspond to a firm's budget constraint, the cost function must be homogeneous of degree one in factor prices. This imposes the following adding up constraints on the cost function [3.5]:

[3.6a] $\Sigma_i \alpha_i = 1$

[3.6b] $\Sigma_i \gamma_{qi} = 0$

[3.6c] $\Sigma_i \gamma_{ij} = \Sigma_j \gamma_{ij} = \Sigma_i \Sigma_j \gamma_{ij} = 0.$ $i,j=1,\ldots,n$

In addition to the adding up constraints, symmetry on the γ_{ij} matrix is also imposed as a matter of necessity (since γ_{ij} can not be distinguished from γ_{ji} empirically).

Factor demand equations are derived by partially differentiating the cost function with respect to the factor prices and applying Shephard's lemma:

[3.7] $\partial C/\partial r_i = X_i$.

This result may be conveniently expressed in logarithmic form in the case of the translog cost function; [3.5]:

[3.8] $\partial \ln C/\partial \ln r_i = \partial C/\partial r_i \; r_i/C = r_i X_i/C = S_i$

where S_i indicates the cost share of the ith factor input. The translog cost function [3.5] yields the following share equations:

[3.9] $S_i = \alpha_i + \gamma_{qi} \ln Q + \Sigma_j \gamma_{ij} \ln r_j.$

The cost equation [3.5] and three share equations for labour, capital and money of the form of [3.9] constitute our first model (Model A) with no adjustment costs for capital.

In discussing the estimates of the cost equation parameters, we focus on the following issues:

(a) What is the nature of the derived demand for real cash balances?

(b) Are real money balances substitutes or complements in production for either capital or labour?

(c) What is the nature of the variable output price elasticities, derived from the cost function parameters?

(d) Are the primary inputs, capital and labour, separable from real money balances in the production process?

In the following sections, the above issues are dealt with in detail. (a) and (b) are discussed in Section 3.2.1. Issues (c) and (d) are discussed in Sections 3.2.2 and 3.2.3 respectively.

3.2 THE STRUCTURE OF PRODUCTION

3.2.1 FACTOR DEMAND ELASTICITIES

Elasticities of substitution provide us with a tool for investigating the degree of substitution and/or complementarity of the factor inputs. For a production function, $Q= f(X_1,X_2)$, where Q is the output and X_1 and X_2 are the inputs, with prices r_1 and r_2, respectively, the elasticity of substitution, ES, between input 1 and input 2 is calculated as:

$$[3.10] \quad ES_{12} = d\ln(X_1/X_2)/d\ln(r_1/r_2).$$

The ES becomes larger as substitution becomes easier between the two inputs.

When there are only two inputs, there is only one relative price between the two inputs and there is no ambiguity in measuring the price change. When more than two variable factors are involved in a production process, however, the degree of substitutability between n factors, measured by the elasticity of substitution, may be defined in a variety of ways. These definitions depend on the *ceteris paribus* conditions under which partial derivatives are obtained. Allen's definition of the elasticity of substitution, which is widely used in this field, will be the measure we focus on. It measures the response of relative demands to an input price change, holding output and all other input prices constant. Other elasticities of substitution hold constant the quantities of other inputs rather than their prices.

When output Q is related to inputs $X_1,\ldots,X_n$, by a positive, twice differentiable, strictly quasi-concave production function, [3.1], Allen's partial elasticity of substitution can be computed by:

$$[3.11] \quad \sigma_{rk} = \{\Sigma_i X_i f_i / X_k X_r\}(f^{-1})_{rk}$$

where $(f^{-1})_{rk}$ is the rkth element of the inverse of the bordered Hessian matrix of the production function and $f_i = \partial Q/\partial X_i$ (Binswanger [1974, p.378]).

Uzawa[1962] has shown that Allen's partial elasticities of substitution can be computed directly from the cost function [3.2] by the formula:

[3.12] $\sigma_{rk} = \{CC_{rk}\}/C_r C_k$

where subscripts on C indicate partial differentiation with respect to factor prices. For the translog cost function specified in [3.5], Allen's substitution elasticities can be calculated as[3]:

[3.13] $\sigma_{rk} = \{\gamma_{rk} + S_r S_k\}/S_r S_k$ and

[3.14] $\sigma_{rr} = \{\gamma_{rr} + S_r(S_r - 1)\}/S_r^2$.

The own price elasticities of factor demand (η_{ii}) can be obtained from,

[3.15] $\eta_{rr} = \sigma_{rr} S_r$

and, similarly, the cross price elasticities of factor demand can be found as:

[3.16] $\eta_{rk} = \sigma_{rk} S_k$

where η_{rk} is interpreted as the percentage change in the demand for the rth factor as a result of a one percent change in the price of factor k.

3.2.2 VARIABLE OUTPUT OR PROFIT MAXIMIZING ELASTICITIES

The cross price elasticities, discussed in the last section, were derived by holding the level of output and other-factor prices constant. For matters of public policy, we should also take into account both input substitution along the isoquants and the impact of changes in output on input demand.

Field and Allen [1981] have derived both the general expression for the profit maximizing price elasticities, Π_{ij}, and a more specific expression for the translog cost function. In this section we follow their derivation for Π_{ij}. In deriving the profit maximizing elasticity, we make use of the general expressions for the production function and cost function in [3.1] and [3.2] respectively. We also specify a demand function for output as:

[3.17] $\quad Q = \phi(P)$

where P is the output price.

The first order profit maximizing condition for the market clearing requires:

[3.18] $\quad C_q(r,Q) = P.$

C_q denotes the partial derivative of the cost function with respect to output, i.e. the marginal cost. Totally differentiating [3.18] and setting dr=0 except for the jth factor price, we have:

[3.19] $\quad C_{qq}dQ + C_{qj}dr_j = dP$

In a perfectly competitive market, each firm faces a horizontal demand curve for its output. For this market, dP=0, [3.19] can be rewritten as:

[3.20] $\partial Q/\partial r_j = -(C_{qj}/C_{qq})$.

Differentiating the cost minimizing demand function in [3.7] and incorporating the output effects on factor demand, we get:

[3.21] $\partial X_i/\partial r_j = C_{ij} + C_{iq}\partial Q/\partial r_j$.

Using [3.21], the profit maximizing cross price elasticity can be written as:

[3.22] $\Pi_{ij} = (r_j/X_j)\{C_{ij} - (C_{iq}C_{jq})/C_{qq}\}$

Making use of [3.12] and the fact that $S_i = r_iX_i/C$, we can rewrite [3.22] as:

[3.23] $\Pi_{ij} = S_jA_{ij}\{1-(C_{iq}C_{jq})(C_{ij}C_{qq})\}$

where A_{ij} is Allen's[6] partial elasticity of substitution and Π_{ij} is the profit maximizing response of the ith factor to changes in the price of the jth factor.

For the translog cost function in [3.5], we can solve for the partial derivatives in [3.23] and derive the expression for Π_{ij} as:

[3.24] $$\Pi_{ij} = S_jA_{ij}[1- \{(\gamma_{iq}+S_iS_q)(\gamma_{jq}+ S_jS_q)\}/ \{\gamma_{ij}+S_iS_j)(\gamma_{qq}+S_q{}^2)\}]$$

where S_i and S_j are the cost shares of ith and jth factors respectively and $S_q = \alpha_q + \Sigma_j\gamma_{qj}\ \ln r_j + \gamma_{qq}\ln Q$.

In this chapter and in Chapter 4, we use the profit maximizing cross price elasticity formula in [3.24] to evaluate the supply side effects of changes in rate of interest on the demand for labour and relate it to the motivation discussed in Chapter 1.

3.2.3 SEPARABILITY OF FACTORS IN PRODUCTION

In this section we discuss the testing of separability of one factor from others in the production process. Dennis and Smith[1978] and Simos[1981] have tested for the separability of real money balances in the production function.

The mathematical condition for two factor inputs X_i and X_j to be (weakly) functionally separable from a third input X_k is that the ratio of the marginal products of X_i and X_j (or the marginal rate of technical substitution between X_i and X_j) be independent of the level of X_k. Berndt and Christensen [1973] provide a complete demonstration of the conditions necessary for weak (as well as strong or additive) input separability and the relationship of these conditions to Allen-Uzawa partial elasticities of substitution in the context of the translog cost function[4]. They also propose that the separability of X_k from X_i and X_j can be tested by testing the equality of σ_{ik} and σ_{jk}, i.e.,

[3.25] $\quad \sigma_{ik} = \sigma_{jk}.$

Substituting [3.13] in [3.25], we have,

[3.26] $\quad \{\gamma_{ik} + S_i S_k\}/S_i S_k = \{\gamma_{jk} + S_j S_k\}/S_j S_k,$

or

[3.27] $\quad S_j \gamma_{ik} - S_i \gamma_{jk} = 0.$

For the three input case, i.e., labour, capital, and real money balances, we can write [3.27] as follows:

$$\begin{array}{lll} & \text{N \& K separable from m} \Leftrightarrow & S_k\gamma_{nm} - S_n\gamma_{km} = 0, \\ [3.28] & \text{K \& m separable from N} \Leftrightarrow & S_k\gamma_{nm} - S_m\gamma_{nk} = 0, \\ & \text{N \& m separable from K} \Leftrightarrow & S_n\gamma_{km} - S_m\gamma_{nk} = 0. \end{array}$$

All three conditions of separability in [3.28] can be mutually satisfied only if $\gamma_{nm}=\gamma_{nk}=\gamma_{km}=0$. This would reduce the underlying production structure to a Cobb-Douglas technology, where all the cross elasticities of substitution are unity. Following Berndt and Christensen[1973] we call this condition complete separability.

We first test the hypothesis of complete separabilty by utilizing the likelihood ratio test. Only if this hypothesis is rejected need one test each individual separability condition in turn.

The separability condition in [3.27] can hold in two possible ways. **First**, if $\gamma_{ik}=\gamma_{jk}=0$ in [3.27], which implies that $\sigma_{ik}=\sigma_{jk}=1$, we refer to these as linear conditions. **Second**, if $\gamma_{ik}\neq0$ and $\gamma_{jk}\neq1$. In this case, non-linear conditions can be derived by substituting for S_i and S_j in [3.27] and rearranging(Berndt and Chritensen [1973,p.91]). Table 3.1 summarizes these conditions for the three factor case.

TABLE 3.1

Conditions for Separabilty

Separability Type	Linear Conditions	Non-Linear Conditions
NK from m	$\gamma_{nm}=\gamma_{km}=0$ $(\sigma_{nm}=\sigma_{km}=1)$	$\gamma_{mm}=\gamma_{nm}^2/\gamma_{nn}$ $\alpha_m=(1+\alpha_n\gamma_{nm}/\gamma_{nn})$
Km from N	$\gamma_{nm}=\gamma_{nk}=0$ $(\sigma_{nm}=\sigma_{nk}=1)$	$\gamma_{mm}=\gamma_{nm}^2/\gamma_{nn}$ $\alpha_m=(\alpha_n\gamma_{nm}/\gamma_{nn})$
Nm from K	$\gamma_{km}=\gamma_{nk}=0$ $(\sigma_{km}=\sigma_{nk}=1)$	$\gamma_{mm}=\gamma_{nm}^2/\gamma_{nn}$ $\alpha_m=(\alpha_n-1)\gamma_{nm}/\gamma_{nn}$

3.3 DATA AND ESTIMATION OF MODEL

3.3.1 DATA

We estimate the parameters of the cost function in [3.5] for three factors, labour, money, and capital, using aggregate annual data for the United States non-financial corporations over the time period 1948–71[5]. The particular data series have been been compiled by J.Gorman and published in the Survey of Current Business [March, 1972, pp.22–52], for the years 1948–71 only. We need data on the following variables: real output of non-financial corporations, Q, with corresponding price, P, nominal money balance holdings by the non-financial corporations, M; the amount of labour services used by the non-financial corporations, N, and the corresponding wage, W; the users' cost of capital, V; and the market rate of interest, r. In addition to these variable we need data on capital stocks and investment for the temporary equilibrium and dynamic models, discussed in Chapters 4 and 5 where capital is fixed in the short-run.

The data on real output, Q, was derived by deflating current dollar gross product of non-financial corporations by their output price index. These series were published in the Survey of Current Business [March, 1972, Table 1, p.22]. The data on price of output was also taken from the Survey of Current Business [March, 1972, Table 1, p.22].

The measure of labour input, taken from the Survey of Current Business [March 1972, Table 2, p.24], is an index of total man-hours worked by the employees of non-financial corporations. The wage rate has been derived by dividing total wages and salaries paid to these employees by their total man-hours. The data on total wages and salaries were taken from the Survey of Current Business [March 1972, p.22].

The data for the holdings of money balances by non-financial corporations is based on the Flow of Funds accounts published by the Board of Governors of the Federal Reserve System. Cash plus demand deposits have been taken as the measure of money balances. The data was obtained from various issues of the Statistical Abstract of the United States.

The user cost of capital is calculated by the method described by Hall and Jorgenson[1967]. It can be considered as a function of the price of new capital goods and the discounted value of all future services derived from capital. In addition it incorporates the effects of tax credits for investment expenditures and a proportional tax on business income. Assuming static expectations, the nominal user cost of capital can be calculated by the following formula:

$$[3.29] \qquad V = \frac{(1-k)(1-uz)\{P_k(r+\delta)\}}{(1-u)}$$

where V = user cost of capital,

k = tax credit allowed on investment goods,

u = tax rate,

P_k = price of capital goods,

r = discount rate (nominal),

δ = rate of depreciation,

z = present value of depreciation deductions for tax purposes, $z = (1/rt)(1-e^{-rt})$.

Following Hall and Jorgenson, the tax credit is assumed to be zero, except in 1962 and 1963, when it is given a value of 0.06. The tax rate u has been assumed to be 0.52 for all years (Nadiri and Rosen [1973]). The rate of depreciation, δ, is estimated from data on investment and capital stocks for non-financial corporations, using the equation for the perpetual inventory method. The market rate of interest on four to six month commercial paper, r, is used for the discount rate. The data on r has been taken from various issues of the Statistical Abstract of the United States. The main source of this data is the series published by the Board of Governors of the Federal Reserve System in the Federal Reserve Bulletin.

The data on the current net capital stocks have been taken from data published in the Survey of Current Business[April 1976, pp.48–52]. These stock estimates are derived by the perpetual inventory method which starts with investment flows and calculates gross capital stock at any given year-end by accumulating past investment flows and deducting discards. The net capital stocks are derived by depreciating the gross stocks. The data on constant dollar net capital stocks for non-financial corporations is deduced by deflating the current dollar net capital stocks by the implicit price deflator for fixed private domestic investment. The series on the price deflator for fixed investment is published in the National Income and Product Account of the United States, 1929–74(Table 7.1, p.264). In Appendix A, we present the data used for estimation.

The price of holding one nominal dollar is measured by the interest rate, r. In the model, we assume that the services derived from the nominal money to the firms are directly proportional to the the level of real stocks of money, i.e., nominal balances divided by the price level. The price of holding one real dollar is the interest rate multiplied by the price level. Thus an increase in the price level will increase the price of a given real level of money services (Butterfield[1971]). The opportunity cost of real money balances is given by:

[3.30] $r_m = p^*r$,

where p is the price of output.

3.3.2 ESTIMATION PROCEDURES

The parameters of the translog cost function can be estimated in one of three ways. One can use ordinary least squares to estimate the cost function only. This technique is attractive from the point of view of simplicity. However, it neglects the additional information contained in the cost share equations (Christensan and Greene[1976]). Alternatively, we can estimate the set of share equations in a simultaneous equation framework, excluding the cost equation. For example, Berndt and Wood[1975] have estimated share equations as a multivariate regression system. Finally, Christensan and Greene[1976] have estimated cost function together with the share equations. We follow this latter approach to estimate the parameters of the cost function in [3.9].

Following conventional practice, we specify additive disturbances for the cost function [3.5] and each of the share equations in [3.9]. We also assume that these disturbances have a joint normal distribution and allow for contemporaneous correlation across equations.

The cost shares in [3.9] must add to unity by definition, and the right hand sides of the cost share equations must also add to unity. Hence, the errors in the share equations must add to zero for each observation.

This implies that one of the share equations must be dropped, as their covariance structure is not of full rank (Barten [1969]).

Following Zellner[1962], the system can be estimated by using the seemingly unrelated regression technique, and the estimates so obtained are invariant to the choice of the equation to be dropped[6]. Barten [1969] has shown that a maximum likelihood estimate of the share equations guarantee such invariance. Kmenta and Gilbert [1968] and Dhrymes[1970] have shown that continuing iteration of Zellner's method, until the covariance matrix converges, yields the maximum likelihood estimates. This method is computationally equivalent to maximum likelihood and ensures invariance of parameter estimates to the choice of the share equation dropped.

Since there are three cost share equations in our model, we used the property of invariance to check our estimates by estimating three alternative specifications of the model. In each case one of the share equations was dropped by using the linear homogeneity conditions in [3.6]. For example, solving the homogeneity conditions for the parameters of the capital equation (α_k, γ_{kk}, γ_{nk}, γ_{km} and γ_{qk}) and substituting them into the cost function gives the following set of equations:

$$[3.31a] \quad \ln C = \alpha_o + a_q \ln Q + 1/2\gamma_{qq} (\ln Q)^2 + \alpha_n \ln(W/V) + \alpha_m \ln(r_m/V) + \ln V + 1/2\gamma_{nn} \ln(W/V)^2 + 1/2\gamma_{mm} \ln(r_m/V)^2 + \gamma_{nm} \ln(W/V) \ln(r_m/V) + \gamma_{qn} \ln Q \ln(W/V) + \gamma_{qm} \ln Q \ln(r_m/V) + v_{ct},$$

$$[3.31b] \quad S_n = \{WN\}/\{WN+VK+r_m m\} = \alpha_n + \gamma_{nn} \ln(W/V) + \gamma_{nm} \ln(r_m/V) + \gamma_{qn} \ln Q + v_{nt},$$

$$[3.31c] \quad S_m = \{r_m m\}/\{WN+VK+r_m m\} = \alpha_m + \gamma_{nm} \ln(W/V) + \gamma_{mm} \ln(r_m/V) + \gamma_{qm} \ln Q + v_{mt}.$$

where W, V and r_m are the prices of labour(N), capital(K), and real money balances(m) respectively. v_{ct}, v_{nt} and v_{mt} are the additive disturbances in the cost, labour share, and money share equations respectively.

There are five parameters which do not appear in [3.31], namely, $\alpha_k, \gamma_{kk}, \gamma_{nk}, \gamma_{km}$, and γ_{qk}. However these are linear combinations of the consistent and asymptotically efficient estimates of $\alpha_n, \alpha_m, \gamma_{mm}, \gamma_{nm}, \gamma_{qn}$ and γ_{qm} in [3.6][7].

Since we employ Zellner's iterative efficient estimation technique, we obtain maximum likelihood estimates and can therefore use likelihood ratio tests to test different hypotheses, outlined in Table 3.1. Writing the likelihood ratio as,

$$[3.32] \quad \lambda = \Omega(R) - \Omega(U).$$

where $\Omega(R)$ and $\Omega(U)$ are the log likelihood values under the restricted and unrestricted versions respectively. -2λ is then asymptotically distributed as Chi-square with degrees of freedom equal to the number of independent restrictions imposed.

3.3.3 DISCUSSION OF RESULTS

The full equilibrium model discussed in Section 3.1.4 was estimated using the Zellner technique discussed earlier. In the first round of estimation, convergence was achieved. The estimated parameters are presented in Table 3.2. Most of the parameters are significantly different from zero at the 95% level of confidence. However in all the estimated equations the Durbin-Watson statistics are low (0.78, 0.83, and 0.804 in the cost equation, labour share equation and money share equation respectively)[8]. We followed the method of correcting for serial correlation discussed by Berndt and Savin [1975], for the singular equation system. It is assumed that v_t, in the stochastic version of the share equations of the full equilibrium model [3.9], follows a stationary vector stochastic process and satisfies the following difference equation:

$$[3.33] \quad v_t = Rv_{t-1} + \varepsilon_t$$

where

TABLE 3.2

Results of Full Equilibrium Model
(No correction for serial correlation)

Parameter	(Sk dropped)
α_o	3.9966 (2.946)
α_q	−0.18923 (−0.418)
γ_{qq}	0.17292 (2.2724)
α_n	0.94408 (9.3664)
α_k	0.0392
α_m	0.01674 (5.1875)
γ_{nn}	0.15527 (6.9895)
γ_{kk}	0.14811
γ_{mm}	0.00356 (6.2166)
γ_{nk}	−0.14991
γ_{nm}	−0.00536 (−9.6551)
γ_{km}	0.00180
γ_{qn}	−0.11188 (−9.655)
γ_{qk}	0.10987
γ_{qm}	0.00201 (4.2026)
Ω	295.456

Note: t-ratios(asymptotic) are reported below the estimates.

$$R = \begin{bmatrix} R_{nn} & R_{nk} & R_{nm} \\ R_{kn} & R_{kk} & R_{km} \\ R_{mn} & R_{mk} & R_{mm} \end{bmatrix} \text{ and } \xi_t = \begin{bmatrix} \xi_{nt} \\ \xi_{kt} \\ \xi_{mt} \end{bmatrix}$$

and where ξ_t is assumed to be independently and normally distributed with mean vector zero and singular covariance matrix.

We can write the stochastic process in [3.33] for the three shares of our model in [3.9] as:

[3.34a] $v_{nt} = R_{nn}v_{nt-1}+R_{nk}v_{kt-1}+R_{nm}v_{mt-1}+\xi_{nt}$,

[3.34b] $v_{mt} = R_{mn}v_{nt-1}+R_{mk}v_{kt-1}+R_{mm}v_{mt-1}+\xi_{mt}$,

[3.34c] $v_{kt} = R_{kn}v_{nt-1}+R_{kk}v_{kt-1}+R_{km}v_{mt-1}+\xi_{kt}$.

It has been argued in Section 3.1.4 that one of the share equations must be dropped before estimating the full equilibrium model in [3.9]. In [3.31], we have dropped the capital share equation. The error in the capital share equation, v_{kt}, is the negative sum of the errors in the labour and money share equation.

Now we can rewrite [3.34a] and [3.34b] by substituting $v_{kt}=-(v_{nt}+v_{mt})$,

[3.35a] $v_{nt} = (R_{nn}-R_{nk})v_{nt-1}+(R_{nm}-R_{nk})v_{mt-1}+\xi_{nt}$

[3.35b] $v_{mt} = (R_{mn}-R_{mk})v_{nt-1}+(R_{mm}-R_{mk})v_{mt-1}+\xi_{mt}$

or, substituting ρ_{ij} for $(R_{ij}-R_{ik})$, [3.36a] $v_{nt} = \rho_{nn}v_{nt-1}+\rho_{nm}v_{mt-1}+\varepsilon_{nt}$,

[3.36b] $v_{mt} = \rho_{mn}v_{nt-1}+\rho_{mm}v_{mt-1}+\varepsilon_{mt}$.

We use the stochastic process, in [3.36a] and [3.36b], for the errors of the labour and money share equation and rewrite the share equations in [3.29] as follows:

[3.37a] $$S_{nt} = [\alpha_n(1-\rho_{nn})-\alpha_m\rho_{nm}]+\gamma_{nn}\ln W/V+\gamma_{nm}\ln r_m/V+\gamma_{qn}\ln Q+ \rho_{nn}S_{n-1}+\rho_{nm}S_{mt-1}-(\gamma_{nn}\rho_{nn}+\gamma_{nm}\rho_{nm})\ln(W/V)_{t-1}- (\gamma_{nm}\rho_{nn}+\gamma_{mm}\rho_{nm})\ln(r_m/V)_{t-1}- (\gamma_{qn}\rho_{nn}+\gamma_{qm}\rho_{nm})\ln(Q)_{t-1}+ (v_{nt}-\rho_{nn}v_{nt-1}-\rho_{nm}v_{mt-1});$$

[3.37b] $$S_{mt} = [\alpha_m(1-\rho_{mm})-\alpha_n\rho_{mn}]+\gamma_{nm}\ln W/V+\gamma_{mm}\ln r_m/V+\gamma_{qm}\ln Q+ \rho_{mm}S_{mt-1}+\rho_{mn}S_{nt-1}-(\gamma_{nm}\rho_{mm}+\gamma_{nn}\rho_{mn})\ln(W/V)_{t-1}- (\gamma_{mm}\rho_{mm}+\gamma_{nm}\rho_{mn})\ln(r_m/V)_{t-1}- (\gamma_{qm}\rho_{mm}+\gamma_{qn}\rho_{mn})\ln(Q)_{t-1}+ (v_{mt}-\rho_{mn}v_{nt-1}-\rho_{mm}v_{mt-1}).$$

The error in the cost equation v_{ct} is assumed to follow a separate first order autoregressive scheme:

[3.38] $v_{ct} = \rho_c v_{ct-1} +\varepsilon_{ct}$.

The two share equations, [3.37a] and [3.37b], and the cost function, following the first order autoregressive scheme in [3.38], now constitute one out of three alternative transformed specifications which we estimate. In the other two models, the labour and money share equations are dropped alternatively.

All three models are estimated as a check on the invariance of the estimated parameters to the share equation dropped and to provide some assurance that the global maximum has been attained. The parameters of the transformed model are now non-linear and the estimated results depend on the starting values of the parameters. Different starting values of parameters are tried and the values of the log likelihood function are compared. Comparison of different likelihood values reveals that the one with the highest value also gives invariance of the estimated parameters across three sets of estimates. The results of the estimated parameters, after making this first order correction for serial correlation, are presented in Table 3.3.

We test the null hypothesis that the coefficients of R matrix are jointly zero. In the transformed model, we have estimated five additional parameters of the R matrix (see [3.37]). The computed likelihood ratio test statistic is 71.44 compared to computed Chi-square of 15.09 with five independent restrictions at 99% confidence level. Therefore we reject the restrictions imposed by the untransformed model for serial correlation.

The elements of the R matrix in [3.33] are underidentified and therefore we can not solve for their values. However, we present the estimated ρ's in Table 3.4, which also includes the values of ρ_c across the three models. The ρ's based on the "own-autocorrelation" are

TABLE 3.3

Results of Full Equilibrium Model
(After Correcting for Serial Correlation)

Parameter	Model 1 (S_k dropped)	Model 2 (S_n dropped)	Model 3 (S_m dropped)
α_o	4.01104 (1.155)	4.00899 (1.154)	4.00612 (1.154)
α_q	0.49297 (0.513)	0.49309 (0.513)	0.49314 (0.514)
γ_{qq}	−0.04952 (−0.325)	−0.04962 (−0.325)	−0.04979 (−0.326)
α_n	−0.32095 (−0.822)	0.32091	−0.32099 (−0.822)
α_k	1.29286	1.29289 (3.449)	1.29295 (3.449)
α_m	0.02803 (1.3289)	0.02802 (1.329)	0.02804
γ_{nn}	0.16052 (7.415)	0.16042	0.16053 (7.421)
γ_{kk}	0.15564	0.15556 (6.621)	0.15565 (6.624)
γ_{mm}	0.00388 (2.890)	0.00388 (2.893)	0.00388
γ_{nk}	−0.15614	−0.15605	−0.15615 (−7.081)
γ_{nm}	−0.00438 (−4.758)	−0.00437	−0.00438
γ_{km}	0.00050	0.00049 (0.236)	0.00050

Contd./...

TABLE 3.3 (Continued)

Parameter	Model 1 (S_k dropped)	Model 2 (S_n dropped)	Model 3 (S_m dropped)
γ_{qn}	0.08365 (1.556)	0.08364	0.08365 (1.556)
γ_{qk}	−0.08306	−0.08305 (−1.637)	−0.08305 (−1.637)
γ_{qm}	−0.00059 (−0.157)	−0.00059 (−0.157)	−0.00060
Ω	331.178	331.178	331.178

Note: t-ratios are in parentheses. These ratios are not distributed exactly as Student's t.

TABLE 3.4

Estimated ρ's

$^{*}\rho_{nn} = R_{nn}-R_{nk}$	0.96 (13.43)	$^{x}\rho_{nk} = R_{nk}-R_{km}$	3.16 (0.46)
$^{*}\sigma_{mm} = R_{mm}-R_{mn}$	0.98 (9.32)	$^{x}\rho_{kn} = R_{kn}-R_{km}$	−3.15 (−0.46)
$^{+}\rho_{kk} = R_{kk}-R_{kn}$	0.96 (13.40)	$^{+}\rho_{km} = R_{km}-R_{kn}$	3.16 (0.47)
$^{*}\rho_{nm} = R_{nm}-R_{nk}$	−3.18 (−0.47)	$^{+}\rho_{mk} = R_{mk}-R_{kn}$	0.001 (−0.71)
$^{*}\rho_{mn} = R_{mn}-R_{nk}$	0.001 (0.71)	$^{*}\rho_{c}$	0.49 (1.94)

Note: In the parentheses are t-ratios(asymptotic).
* the estimates have been taken from Model 1 in Table 3.3.
\+ the estimates have been taken from Model 2 in Table 3.3.
x the estimates have been taken from Model 3 in Table 3.3.

significantly different from zero. These ρ's are the combination of the elements of the R matrix and they are not interpreted as correlation coefficients.

Even after making a correction for serial correlation the Durbin-Watson statistics are low (1.36, 1.78 and 1.48 in the cost equation, labour share equation and money share equation respectively). Low Durbin-Watson values also indicate that a more fundamental solution of the autocorrelation problem may be appropriate (see footnote 8). For example, a model that allows for costs of adjustment of capital in the firms optimization problem may fit the data better. We estimate such a model in Chapter 5.

Six out of the sixteen parameters in Table 3.3 that we have estimated are significantly different from zero at the 95% level of confidence. Several parameters that were significant before we corrected for serial correlation are now insignificant. Again, this suggests that a model which presumes full adjustment of capital holdings within each period may be inadequate.

The hypotheses of separabilty of factor inputs have been tested next. First, the hypothesis of global separabilty, discussed in Section 3.2.3, is tested. Complete global separability is rejected with 99% confidence(see Table 3.5). Since this hypothesis also tests for Cobb-Douglas technology, with all the partial

TABLE 3.5

Separabilty of Factor Inputs

	# of independent restrictions	$\Omega(U)$	$\Omega(R)$	Tabulated Chi-Square $\alpha=0.01$	Tabulated Chi-Square $\alpha=0.05$	Calculated Chi-Square
Complete Global Separability ($\sigma_{nm}=\sigma_{nk}=\sigma_{km}=1$)	3	331.178	291.128	11.34	7.81	80.10
Non-Linear Separabilty						
a) NK-m ($\sigma_{km}=\sigma_{nm}\neq 1$)	2	331.178	308.237	9.21	5.99	45.88
b) Nm-K ($\sigma_{nk}=\sigma_{km}\neq 1$)	2	331.178	322.256	9.21	5.99	17.84
c) Km-N ($\sigma_{nk}=\sigma_{nm}\neq 1$)	2	331.178	327.170	9.21	5.99	8.02

elasticities equal to unity, Cobb–Douglas technology is also rejected. **Second**, we test each of the non-linear separabilty hypotheses presented in Table 3.1. All three types of non-linear separability hypotheses are also rejected at 95% confidence level. Since our results reject equality between the elasticities of substitutions, including equality of the elasticities to unity, we do not test for the linear separabilty conditions.

The own and cross price elasticities are also presented in Table 3.6. The own price elasticities of labour and capital indicate fairly inelastic demands for these two factors. The own price elasticity of real money balances is close to the estimates of Dennis and Smith[1978]. Their estimates of the own price elasticity of money ranged between –0.22 to –0.40.

We have also generated confidence intervals for the own price elasticities[9]. These intervals are presented in Table 3.7. The own price elasticities of labour and capital are significantly different from zero and have a negative sign. On the other hand the own price elasticity of real money balances is not significantly different from zero. The result is unsatisfactory as we expect own price elasticities to be negative on theoretical grounds.

The cross price elasticities are also presented in Table 3.6, using [3.16]. All the cross price elasticities are positive in sign, indicating that all

TABLE 3.6

Factor Price Elasticities

η_{nn}	-0.08246	η_{nm}	0.00094
η_{kk}	-0.17996	η_{km}	0.01112
η_{mm}	-0.18357	η_{nk}	0.01953
η_{nr}	+0.00094	Π_{nr}	-0.1475

Note: η_{nn}, η_{kk} and η_{mm} are the own price elasticities of labour, capital and real money balances respectively. η_{nm} and η_{nk} are the cross price elasticity of labour demand with respect to the price of money and rental price of capital and η_{km} is the cross price elasticity of capital demand with respect to the price of money. η_{nr} and Π_{nr} are cost minimizing and profit maximizing interest rate elasticities of labour demand.

TABLE 3.7

Confidence Intervals for Own Price Elasticities (Full Equilibrium Model)

	MEAN ELASTICITY FROM 900 DRAWS	ELASTICITIES: LOWER TAIL 2.5%	LOWER TAIL 5%	UPPER TAIL 5%	UPPER TAIL 2.5%
LABOUR	-0.08246	-0.14596	-0.13342	-0.031034	-0.0215
MONEY	-0.18357	-0.5412	-0.43192	+0.02154	+0.0503
CAPITAL	-0.17996	-0.30011	-0.28315	-0.077556	-0.0661

three factors of production are substitutes for each other. Dennis and Smith[1978] have estimated these elasticities in the cost-minimization framework. In their study, the price elasticity between production labour and money was positive in nine out of eleven industries. Similarly, the cross price elasticity of money and capital was also positive in ten industries. Simos[1981], on the other hand, found a complimentary relationship between labour and money. However, his study was based on a profit maximization approach.

The profit maximizing interest rate elasticity of labour demand, Π_{nr}, is calculated using [3.24]. Capital is treated as a variable factor while evaluating this elasticity. In Chapter 4 we estimate this elasticity for the translog cost function, while holding capital fixed in the short-run. We can compare this elasticity directly with the Short[1979] and Simos[1981] results for the full equilibrium model. Simos's [1981] estimate of Π_{nr} ranged between −0.038 in 1929 to −0.099 in 1972. On the other hand Short's estimates are much higher, −4.80 in the Cobb–Douglas case. Our estimates of Π_{nr} are close to Simos' result and they indicate the potential for strong supply side effects of changes in the interest rate on both labour demand and supply of output (Table 3.6). As noted earliar these supply side effects are comparable to the one discussed in Chapter 1, because capital is not fixed in the short-run.

3.4 CONCLUDING REMARKS

In this chapter we estimated a full equilibrium model in which firms can adjust their capital stocks instantaneously. The results of the price elasticities were consistant with the results of the existing studies on demand for money by firms. We also rejected the hypothesis of separabilty of real money balances in the production function. This result is also consistant with the findings of Simos[1981] and Dennis and Smith[1978].

In the first round of estimation the Durbin-Watson statistics were low, indicating the presence of serial correlation in the errors of the equations of the model. We followed the method suggested by Berndt and Savin[1975] to correct the problem of serial correlation. Even after making the correction for serial correlation, Durbin-Watson statistics were still low. The significance of the estimated parameters of the cost function was considerably decreased in the transformed model. Furthermore, the own price elasticity of real money balances was not significantly different from zero.

These results suggest that the full equilibrium model did not fit the data well and a more fundamental solution may be appropriate. For example one could introduce the costs of adjustment of capital into firms optimization problem. We estimate such a model in Chapter 5.

FOOTNOTES

CHAPTER 3

[1]Diewert[1971] discusses and proves these properties. Also see McFadden[1978].

[2]The translog functional form has been employed in Chapters 3 and 4 only. In Chapter 5 we introduce the quadratic cost function.

[3]For a detailed derivation of own and cross price elasticities , see Binswanger[1974].

[4]In these texts we follow the common practice of treating the translog cost function as exact. Slightly more complex tests are required if the function is to be treated as an approximation(see Denny and Fuss[1977]).

[5]In preliminary work we used the aggregated US manufacturing data(1947–1971) to estimate the model. However, some of the estimated own price elasticities were positive. Denny, Fuss and Waverman([1980], p.42) report similar problems with this data. Consequently we decided to use data on the non-financial corporations of the United States.

[6]See Christensen and Greene[1976].

[7]These five parameters are determined residually, using the constraints in [3.6].

[8]The critical range of the Durbin–Watson statistic is appropriate for single equation estimation with homoscedastic errors, and so are not applicable here. However, the statistics can still be used informally as a measure of the extent of autocorrelation in disturbances.

[9]The elasticities are non–linear functions of the parameters and the exact confidence intervals are not available. We report instead confidence intervals generated by a Monte Carlo method assuming a normal distribution for the estimated parameters, reported in Table [3.3]. The method involves taking drawings on the parameters according to the estimated variance–covariance matrix. We use the IMSL routine GGNSM to make 900 drawings. The distributions of these drawings is then used in generating the confidence intervals of the elasticities using the values of the estimated parameters as means (for more details see Krinsky and Robb[1985]). This is not the same as bootstrapping, which would have involved drawings of the dependent variables and using them to get estimates of parameters for each drawing. This would have been too expensive in terms of computer costs.

CHAPTER 4

EMPIRICAL MODELS BASED ON COST MINIMIZATION WITH CAPITAL FIXED IN THE SHORT-RUN

In Chapter 3, the analysis of a firm's factor demands was based on a number of assumptions including one which allowed the firm to purchase and install new capital both costlessly and within one period (year). Most empirical studies on firms' money demand made similar assumptions, either implicitly or explicitly. But the assumption of costless and instantaneous installation of capital seems inappropriate for several reasons. **First**, as noted by Nickell([1978],p.25), "the consequence of this particular assumption was that the firm responded to changes in relevant parameters with instantaneous changes in its capital stock, a most unappealing result, given the nature of capital goods and one which is in little accord with casual observation". **Second**, this view ignores the role of future expectations implied by the durability of capital (Abel[1978]). In other words, the firms' optimization problem, in the static framework, does not incorporate any future returns to capital and the firm is assumed to buy and sell fixed capital solely for the purpose of making capital gains. **Third**, factors other than capital may respond differently in the short-run when capital is fixed, as compared to the long-run when all factors are variable[1].

In many instances firms are unable to adjust their factor demands instantaneously. There are two main approaches that relax the assumption of instantaneous adjustment of factor inputs in the full equilibrium model. **First**, costs of adjustment can be incorporated into the firms' optimization problem explicitly. In this type of model, firms are continuously in dynamic disequilibrium instead of being in full static equilibrium. We discuss this approach in some detail in Chapter 5. **Second**, firms can be assumed to be in static equilibrium for some of the factors, conditional on the levels of some other factors. These models are referred to as partial static equilibrium models[2]. In this thesis, however, we use the term temporary equilibrium model instead of partial equilibrium model as partial equilibrium is also used to distinguish the analysis from "general equilibrium". Variable factors are assumed to be in temporary static equilibrium while the remaining factors are treated as quasi-fixed factors. The theoretical basis for temporary equilibrium models can be found in Samuelson[1953] and McFadden[1978].

In Section 4.1, we discuss the theoretical foundations of temporary equilibrium models and derive its estimating equations. In Section 4.2 we discuss the relationship between the elasticities of temporary and full equilibrium models. In Section 4.3 we report on the estimation procedure and the results.

4.1 A COST MINIMIZATION MODEL WITH TEMPORARY EQUILIBRIUM

In Chapter 3, we utilized a cost function in which all factors were adjusted instantaneously, to represent the full equilibrium model in analyzing the role of real money balances in the production process. The important assumption of this model was that firms have the ability to fully adjust to changes in exogeneous variables instantaneously. An alternative way of modelling the production process is to introduce the notion of a restricted variable cost function. In this model, firms are assumed to be in temporary equilibrium in the short-run. Some of the factors in this model are not fully adjusted (i.e. are restricted) to their long-run equilibrium values within any period. Hence, we assume that it is the capital stock that is a quasi-fixed factor, while labour and real money balances are assumed to be variable factors of production. The firms' behaviour can be described by a short-run restricted variable cost function:

[4.1] $CV = \phi\{W, r_m, Q, K\}$

where W and r_m are prices of the variable factors labour and real money balances, respectively, Q is the level of output and K represents the level of capital stocks. CV is the minimum variable cost of producing the output Q, conditional on the level of capital K.

The substitution possibilities between the variable factors of production can be evaluated by estimating the elasticities of substitution. The method of estimating these elasticities is much the same as in the case of the full equilibrium model. The only difference in the short-run is that capital is fixed in the temporary equilibrium model. The temporary static equilibrium elasticities of substitution can be computed from the variable cost function as:

[4.2] $\sigma_{ij} = (CV)(CV_{ij})/(CV_i CV_j)$

where $CV_i = \partial CV/\partial P_i$ and $CV_{ij} = \partial CV_i/\partial P_j$.

The elasticities in [4.2] are evaluated conditionally for given levels of the fixed factor. Therefore these are like / direct elasticities of substitution. These elasticities provide information about the short-run adjustments by firms to exogenous shocks. They do not, however, provide information about substitution possibilities between the variable and the quasi-fixed factor or about a long-run in which capital is allowed to adjust.

The total cost, when all the variable factors are in equilibrium, can be defined as follows:

[4.3]. $$CT = \Phi(W, r_m, Q, K) + VK$$

where V is the rental price of capital.

Next we consider the long-run relationship between the three factors of production. Samuelson [1953, pp.19-20] has shown that the derivative of the restricted variable cost function, [4.1], with respect to the quasi-fixed factor K must equal the negative of the rental price of the quasi-fixed factor <u>in the long-run</u>[3], i.e.,

[4.4] $$-V = \partial CV/\partial K = \psi(Q, W, r_m, K).$$

For given Q, W, r_m, V, and a particular functional form for [4.3], [4.4] can be solved for the cost minimizing demand for capital, K,

[4.5] $$K^* = \zeta(Q, W, r_m, V).$$

By substituting [4.5] into [4.3], we can derive the long-run cost function for a given functional form for the variable cost function. The long-run cost function can then be used to derive substitution relationships between capital and the two variable factors, labour and money balances.

We specify a translog function for the variable cost function. In Chapter 3 we have discussed the advantages of employing this particular functional form and these advantages are again relevant here. The variable cost function with an additive error term can be written as

$$[4.6] \quad \ln CV = \alpha_o + \alpha_Q \ln Q + 1/2\alpha_{QQ}(\ln Q)^2 + \alpha_n \ln W + \alpha_m \ln r_m + \alpha_k \ln K + 1/2\{\gamma_{nn}(\ln W)^2 + \gamma_{mm}(\ln r_m)^2 + \gamma_{kk}(\ln K)^2\} + \gamma_{nm}\ln r_m \ln W + \gamma_{nk}\ln K \ln W + \gamma_{mk}\ln K \ln r_m + \gamma_{Qn}\ln Q \ln W + \gamma_{Qm}\ln Q \ln r_m + \xi_c.$$

Using Shepherd's lemma[1953], we can derive the demand functions for the variable factors that minimize variable cost. The derivatives $\partial \ln CV/\partial \ln P_i$, where Pi represent the variable factor prices W and r_m, are equal to the shares of variable factor cost to total variable cost, i.e.,

$$[4.7] \quad \partial \ln CV/\partial \ln W = (W/CV)(\partial CV/\partial W) = S_n = \alpha_n + \gamma_{nn}\ln W + \gamma_{nm}\ln r_m + \gamma_{Kn}\ln K + \gamma_{Qn}\ln Q + \xi_n,$$

$$[4.8] \quad \partial \ln CV/\partial \ln r_m = (r_m/CV)(\partial CV/\partial r_m) = S_m = \alpha_m + \gamma_{nm}\ln W + \gamma_{mm}\ln r_m + \gamma_{Km}\ln K + \gamma_{Qm}\ln Q + \xi_m.$$

[4.7] and [4.8] have been derived by assuming symmetry, $\gamma_{nm}=\gamma_{mn}$. The set of [4.6], [4.7] and [4.8] constitute the system of equations used to estimate the parameters of the variable cost function.

On theoretical grounds the variable cost function [4.6] is restricted to be homogeneous of degree one in input prices. This homogeneity imposes the following linear restrictions on the parameters:

[4.9a] $\alpha_n + \alpha_m = 1,$

[4.9b] $\gamma_{nn} + \gamma_{nm} = 0,$

[4.9c] $\gamma_{nm} + \gamma_{mm} = 0,$

[4.9d] $\gamma_{Qn} + \gamma_{Qm} = 0,$

[4.9e] $\gamma_{Kn} + \gamma_{Km} = 0.$

The cost shares of money and labour add up to unity by definition as must the right hand sides of these share equations. The errors in the share equations must, therefore, sum to zero. The resulting covariance matrix is singular, and one of the share equation must be deleted before estimating the system as in the previous Chapter.

The restrictions in [4.9] are solved and imposed on the cost function and the factor demand equations to be estimated. In the first instance we drop the money share equation, while in the second case we drop the labour share equation. The first specification of the model can be written as follows:

[4.10] $\ln CV = \alpha_o + \alpha_Q \ln Q + 1/2\alpha_{QQ}(\ln Q)^2 + \alpha_n \ln(W/r_m) + \ln r_m + \alpha k \ln K +$
$1/2\{\gamma_{nn}(\ln W/r_m)^2 + \gamma_{kk}(\ln K)^2\} + \gamma_{Kn} \ln K \ln(W/r_m) +$
$\gamma_{Qn} \ln Q \ln(W/r_m) + \gamma_{KQ} \ln Q \ln K + \xi_c;$

[4.11] $S_n = \alpha_n + \gamma_{nn} \ln(W/r_m) + \gamma_{Kn} \ln K + \gamma_{Qn} \ln Q + \xi_n.$

Similarly the other set of equations, in which we drop the labour cost share equation, can be written as,

$$[4.12]\quad \ln CV = \alpha_o + \alpha_Q \ln Q + 1/2\alpha_{QQ}(\ln Q)^2 + \alpha_m \ln(r_m/W) + \ln W + \alpha_k \ln K + 1/2\gamma_{mm}(\ln r_m/W)^2 + \gamma_{kk}(\ln K)^2) + \gamma_{Km}\ln K \ln(r_m/W) + \gamma_{Qm}\ln Q \ln(r_m/W) + \gamma_{KQ}\ln Q \ln K + \xi_c;$$

$$[4.13]\quad S_m = \alpha_m + \gamma_{mm}\ln(r_m/W) + \gamma_{Km}\ln K + \gamma_{Qm}\ln Q + \xi_m.$$

Equations [4.10] and [4.11], and equations [4.12] and [4.13] constitute the two alternative specifications of the temporary equilibrium model.

In discussing the estimates of the variable cost function, we focus on the following issues:

(a) What is the nature of the derived demand for real cash balances and how do the results compare with those of the full equilibrium model?

(b) Are real money balances substitutes or complements in production for labour in the short-run and for labour and capital in the long-run? Again, how do the results in the long-run compare with those of the full equilibrium model?

(c) How does the implied profit maximizing interest rate elasticity of labour demand, in the short-run, compare with the corresponding estimates which we discussed in Chapter 1?

In the following section we discuss these three issues.

4.2 THE STRUCTURE OF PRODUCTION

4.2.1 FACTOR DEMAND ELASTICITIES

The methodology for estimating the short-run own and cross price elasticites is the same as that discussed in section 3.1.1 of Chapter 3, except now capital stocks are also held constant and there are only two variable factors of production. In the long-run the own price elasticity of demand of a variable factor of production, for example labour, can be evaluated as follows:

[4.14] $\quad d\ln N/d\ln W = (d\ln N/d\ln W)_{\bar{Q},\bar{K}} + (d\ln N/d\ln K)(d\ln N/d\ln W)$

where the first term on the right hand side of [4.14] is the short-run own price elasticity of labour and Q and K indicate that this elasticity has been evaluated while holding output and capital constant. For the long-run response we need to evaluate dlnK/dlnW and dlnN/dlnK also.

In order to evaluate dlnK/dlnW, we first differentiate [4.6] and use the condition in [4.4],

[4.15] $\quad d\ln CV/d\ln K = (K/CV)(\delta CV/\delta K) = (K/CV)(-V)$

Therefore,

[4.16] $\quad (K/CV)(-V) = S_k$

$$= \alpha_K + \gamma_{KK}\ln K + \gamma_{KQ}\ln Q + \gamma_{Ki}\ln(W/r_m)$$

Taking the log of [4.16] and differentiating with respect to $\ln W^4$,

[4.17] $\quad d\ln K/d\ln W = \{1/(1-S_k)\}\{S_n - (1/S_k)\gamma_{Kn}\}$

where $S_K=\alpha_K+\gamma_{KK}\ln K+\gamma_{KQ}\ln Q+\gamma_{Kn}\ln(W/r_m)$.

$d\ln N/d\ln K$ can be determined by first taking the log of variable cost share equation, [4.11], and differentiating it with respect to $\ln K$ and rearranging to give

[4.18] $$d\ln N/d\ln K = (1/S_n)dS_n/d\ln K + d\ln CV/d\ln K$$

Substituting for $dS_n/d\ln K$ and $d\ln CV/d\ln K$, by differentiating [4.11] and [4.6] with respect $\ln K$, into [4.18], we have

[4.19] $$d\ln N/d\ln K = (1/S_n)\gamma_{Kn}+(\alpha_K+\gamma_{KK}\ln K+\gamma_{KQ}\ln Q+ + \gamma_{Kn}\ln(W/r_m).$$

[4.17] and [4.19] can be evaluated by using mean values of output Q, capital stocks K and factor price ratio W/r_m.

In a similar way we can estimate the own price elasticity of real money balances.

In order to derive the cross price elasticies of the variable factors, for example the cross price elasticity of labour demand with respect to opportunity cost of real money balances, we have,

[4.20] $$d\ln N/d\ln r_m=(d\ln N/d\ln r_m)_{\bar{Q},\bar{K}}+(d\ln N/d\ln K)(d\ln K/d\ln r_m)$$

4.3 ESTIMATION OF MODEL

4.3.1 ESTIMATION PROCEDURES

It was argued in Chapter 3 that the iterative

Zellner technique gives estimates of parameters that are invariant to which equation is dropped and, in the limit yields maximum likelihood estimates. Equations [4.10] and [4.11] are estimated using the iterative Zellner technique. As a check we also estimated equations [4.12] and [4.13].

Following conventional practice, we specify additive disturbances for the cost function [4.10] and the share equation [4.11]. Similar assumptions have been made for the disturbances in the alternative specification in [4.12] and [4.13].

4.3.2 DISCUSSION OF RESULTS

The results of the two alternative specifications of the temporary equilibrium model in [4.10] and [4.11] (alternatively [4.12] and [4.13]) are presented in Table 4.1. The results of the estimated parameters show that there is essentially no difference across the two models. The Durbin-Watson statistic for the cost equation and cost share equations were considerably low[5]. We corrected for serial correlation, as in Chapter 3 using the Berndt and Savin [1975] method for a singular equation system. We discussed this method in chapter 3 (p.59, [3.28]). The variable cost equation was also corrected for serial correlation and the error was assumed to follow a first order autoregressive scheme (p.62, [3.33]). We compute the likelihood ratio to test the null hypothesis that the coefficients of serial correlation (ρ_c and ρ_n) are zero. The computed likelihood ratio test statistic is 19.94 compared to a tabulated Chi-square of 9.21 with two independent restrictions at a 99% level of confidence. Therefore the null hypothesis has been rejected with 99% confidence.

The estimated parameters of the cost function, with the correction for serial corelation, are presented in Table 4.2. The first order correction for serial correlation has not fully removed autocorrelation from the residuals. The t-ratios in Table 4.2, which are distributed as Student's t (in fact, normally) asymptotically, indicate that there are seven out of ten directly estimated parameters of the temporary equilibrium model that are significantly different from zero at the 90% (asymptotic) level of confidence. Recall that in the full equilibrium model of Chapter 3, there were only three out of ten directly estimated parameters that were significantly different from zero at the 90% level of significance (Table 3.3). This gives some indication of an overall better fit of the temporary equilibrium model over the full equilibrium model.

The own and cross price elasticities, both in the short-run and in the long-run, are presented in Tables 4.3

TABLE 4.1

Estimated Parameters of the Temporary Equilibrium Model (Without Serial Correlation Correction)

	A	B		A	B
α_0	2.4608 (1.720)	2.4276 (1.704)	γ_{Qn}	−0.00358 (−1.510)	−0.00359
α_Q	4.89609 (2.907)	4.92764 (2.928)	γ_{Qm}	0.00358	0.00359 (1.513)
α_{QQ}	−2.0618 (−0.755)	−2.0402 (−0.750)	α_K	−4.5763 (−2.76)	−4.5963 (−2.77)
α_n	0.9916 (153.88)	0.9917	γ_{KK}	−0.5920 (−0.204)	−0.5630
α_m	0.0084	0.00827 (0.463)	γ_{Kn}	−0.0004 (−0.175)	−0.0004
γ_{nn}	0.00506 (8.701)	0.00505	γ_{Km}	0.0004	0.0004 (0.174)
γ_{mm}	0.00506	0.00505 (8.695)	γ_{QK}	1.3250 (0.472)	1.2985
γ_{nm}	−0.00506	−0.00505	Ω	213.261	213.261
			DW_{cv}	0.8730	0.8730
			DW_n	1.0758	*
			DW_m	*	1.0758

Note: In A and B, the money share equation and the labour share equation have been dropped respectively. The t-ratios are in parentheses. The parameters without any t-ratio are derived residually using [4.9]. DW_{cv}, DW_n and DW_m are the Durbin-Watson statistic for the cost, labour and money share equations respectively. Ω is the computed value of the log likelihood function.
* Since the errors are additive we can compute the DW for the dropped cost share equation, but we have not computed it here.

TABLE 4.2

Estimated Parameters of the Temporary Equilibrium Model
(After Correcting for Serial Correlation)

α_o	2.746 (1.1449)	γ_{nm}	−0.00635
α_Q	2.247 (3.8604)	γ_{Km}	0.00583
α_{QQ}	−0.6719 (−2.164)	γ_{Kn}	−0.00583 (−1.3851)
α_n	0.9219 (32.625)	γ_{Qm}	−0.00496
α_m	0.0781	γ_{Qn}	0.00496 (1.8584)
γ_{QK}	0.3700 (1.9596)	α_K	−1.9910 (−1.845)
γ_{mm}	0.00635	γ_{KK}	0.3785 (1.5078)
γ_{nn}	0.00635 (13.522)	Ω	223.231
DW_{cv}	1.610	ρ_c	0.6194 (3.860)
DW_n	1.257	ρ_n	0.990 (21.89)
DW_m	*		

Note: In parentheses are the t-ratios. The parameters without t-ratios are derived residually using [4.9].
* See the note in Table 4.1.

and 4.5. We also include in this table the results of the full equilibrium model for comparison. We discuss the results in the latter part of this section.

We have also generated confidence intervals for the own price elasticities in the short-run. These confidence intervals have been generated by the method discussed in Chapter 3 (see footnote 9). The results are presented in Table 4.4. Unlike the results of the full equilibrium model, both own price elasticities are significantly different from zero and are negative in sign. Therefore the model with fixity of capital stocks seems to give more reasonable results and suggests the restrictions imposed by the full equilibrium model are inappropriate.

The own price elasticity of labour is considerably smaller in the short-run than in the long-run. This result can be explained by the fact that in the short-run both output and capital stocks are fixed. Therefore the demand for labour is quite insensitive to changes in the wage rate. However, in the long-run, when capital stocks are adjusted to the desired equilibrium, there is a large increase in the absolute size of the own price elasticity of labour. The long-run elasticity is smaller in magnitude than that of the full equilibrium model.

The own price elasticity of real money balances indicates that demand for real money balances is more

TABLE 4.3

Own Price Elasticities

	TEMPORARY EQUILIBRIUM MODEL		FULL EQUILIBRIUM MODEL [Chapter 3]
	SHORT-RUN	LONG-RUN	
LABOUR	-0.002611	-0.0590	-0.08246
MONEY	-0.21966	-0.2821	-0.17996
CAPITAL	-	-0.4540	-0.05720

TABLE 4.4

Confidence Intervals for Own Price Elasticities (Temporary Equilibrium Model, Short-Run)

	MEAN ELASTICITY FROM 900 DRAWS	ELASTICITIES			
		LOWER TAIL		UPPER TAIL	
		2.5%	5%	5%	2.5%
LABOUR	-0.00261	-0.00356	-0.00341	-0.00259	-0.00173
MONEY	-0.21966	-0.3350	-0.31593	-0.21832	-0.11311

elastic both in the short-run and in the long-run than the own price elasticity for labour demand. The long-run estimate of η_{mm} is about 36% more than the size of the elasticity in the full equilibrium model.

The cross price elasticity of labour with respect to the opportunity cost of real money balances, r_m, shows that labour and real money balances are substitutes in the short-run and in the long-run(Table 4.5). The size of the long-run cross elasticity is quite different from the full equilibrium result.

The cross price elasticity of demand for labour with respect to the rental price of capital indicates that labour and capital are substitutes. On the other hand, capital and money are long-run complements. The latter result is in contrast to the full equilibrium result, where capital and money are substitutes.

Finally we calculated the profit maximizing interest rate cross elasticity, Π_{nr}, in the short-run, using the formula discussed in Chapter 3 ([3.19])[6]. The elasticity is much closer to zero than Short's [1979] estimate of the profit maximizing elasticity, which was discussed in Chapter 1. Therefore the results of temporary equilibrium model indicate that the supply side effects, discussed in Chapter 1, are economically insignificant.

TABLE 4.5

Cross Price Elasticities

	TEMPORARY EQUILIBRIUM MODEL		FULL EQUILIBRIUM MODEL [Chapter 3]
	SHORT-RUN	LONG-RUN	
η_{nm}	+0.00158	+0.00790	+0.00094
η_{nk}	–	+0.0213	+0.01953
η_{mk}	–	–0.2828	+0.01112
η_{nr}	+0.00158	+0.0221	–0.00616
Π_{nr}	–0.0010	–	–0.14750

Note: η_{nm} is the cross price elaticity of labour with respect to the price of money and η_{nk} and η_{mk} are the cross price elasticities of labour and money with respect to the rental price of capital respectively. η_{nr} and Π_{nr} are the cost minimizing and profit maximizing interest price elasticities of labour demand repectively.

4.4 CONCLUDING REMARKS

In the first round of estimation of the temporary equilibrium model, the Durbin-Watson statistics were considerably low. The model was corrected for serial correlation, using the method proposed in Chapter 3. The first order correction for serial correalation did not fully remove the serial correlation. However, the significance of the estimated parameters of the cost function was considerably increased in the transformed model.

The own price elasticities of labour and money in the short-run were significantly different from zero and negative in sign. This result shows an improvement over the full equilibrium model where the own price elasticity of real money balances was not significantly different from zero.

We also found substantial differences in the short-run and long-run price elasticities. Furthermore, the profit maximizing interest rate cross elasticity of labour demand was almost zero in the short-run, compared to -0.14 calculated in Chapter 1. The differences in the size of these interest rate elasticities clearly demonstrates the contamination in the estimates of the full equilibrium model, that follow from ignoring the quasi-fixed nature of capital in the short-run. Similar conclusions can be drawn for the own and other cross price elasticities.

FOOTNOTES:

CHAPTER 4

[1]Therefore two kinds of responses to external shocks can be analysed, short-run and long-run.

[2]See for example McFadden[1978].

[3]In calculating V we used a short-term rate in [3.29] in Chapter 3. Studies with short-run fixity of capital have used the long-term rate. We have used short-term rate throughout the thesis, since most of the money studies have used the short-term rate.

[4][4.16] is totally differentiated, allowing K to vary as follows:

$$\partial lnK/\partial lnW - \partial lnCV/\partial lnW - (\partial lnCV/\partial lnK)(\partial lnK/\partial lnW) = \partial lnS_k/\partial lnW$$

[5]The critical values for the Durbin-Watson test are only appropriate for single equations with homoscedastic disturbances, and cannot be used here, as noted in Chapter 3; footnote 8.

[6]Taking the log of the labour share equation in [4.7], we have,

$$lnW+lnN-lnCV = lnS_n$$

$$= ln(\alpha_n+\gamma_{nn}lnW+\gamma_{nm}lnr_m+\gamma_{kn}lnK+\gamma_{Qn}lnQ);$$

differentiating both sides,

$$\partial lnN/\partial lnr = \partial lnCV/\partial lnr+(1/S_n)(\partial Sn/\partial lnr)$$

or

$$\partial lnN/\partial lnr = S_m+(1/S_n)\gamma_{nm}$$

The partial derivatives $\partial Sn/\partial lnr$ and $\partial lnSn/\partial lnr_m$, are the same because r_m=rp. Therefore in the short-run there is no difference between the cross price elasticities of labour with respect to r_m and r.

CHAPTER 5

EMPIRICAL MODELS BASED ON COST MINIMIZING BEHAVIOUR WITH COSTS OF ADJUSTMENT FOR CAPITAL

In Chapter 4 we estimated a cost minimizing model that incorporated fixity of capital stocks in the short-run. The model was promising as it explicitly recognized the interrelatedness of the disequilibrium process and also generated short-run and long-run demand equations for variable and fixed factors. However, the model was incomplete. There was no explicit constraint on the firms' optimization that justified the fixity of capital stocks in the short-run. Several alternative ways have been discussed in the economic literature to introduce fixity of capital stocks explicitly into a firm's optimization problem. One of these alternatives is to introduce the notion of non-linear costs of adjustment. These are costs associated with changes in the level of investment in the short-run which can either be associated with a monopolistic capital goods market in which the price of capital rises with the level of investment (external costs of adjustment) or with rising costs per unit of investment associated with using factors that are internal to the firm (internal costs of adjustment). The latter are measured in terms of output foregone by the firms because an increase in the investment level in the short-run may divert some of the resources of firms from production to investment[1].

In this chapter we examine the role of real money balances in the production process by developing a dynamic model which incorporates internal costs of adjustment of capital into firms' optimization problems.

Section 5.1 introduces the costs of adjustment and reviews some of relevant theoretical developments. In Section 5.2, we discuss both theoretical and empirical models that incorporate costs of adjustment of capital in the optimization framework. Section 5.3 discusses issues related to production structure, such as price elasticities and the role of costs of adjustment in the model. In Section 5.4, estimation procedures and empirical results are explained.

5.1 REVIEW OF THE ADJUSTMENT COST LITERATURE

There are a number of ways to introduce the cost of adjusting capital into the firm's maximization problem. In earlier versions of the neo-classical models of investment, originated by Jorgensen[1963], the firm's optimization problem starts with an objective of maximizing the present discounted value of net cash flows constrained by the production function. Typically these models also assume constant returns to scale and derive the optimal stock of capital for an exogenously given output. In order to make investment determinate, the model is completed by a distributed lag function for net investment. One of the major criticisms of this approach is that there is no mechanism which determines the rate of investment (see Abel[1978]). The rate of investment is determined in an *ad hoc* manner, which is often stated in its continuous form as:

[5.1] $$\dot{K} = \beta(K^{*} - K)$$

where $\dot{K}$ is the continuous time derivative of the capital stock, K^{*} is the desired level of the capital stock and β is the partial adjustment coefficient.

As noted by Lucas[1967], Gould[1968] and Treadaway[1969], there are several other shortcomings in this earlier version of the neo-classical model. In particular, K^* in [5.1] is determined without taking into account the adjustment mechanism even though the adjustment mechanism in [5.1] constrains capital accumulation. Therefore many variables, such as sales or profits, needed to define K^* are in fact affected by [5.1]. In other words, the actual investment level is itself a decision variable which may affect profits and therefore should be in the criterion function (Gould[1968]). Furthermore, if one major input is out of equilibrium, for example capital in [5.1], the other more rapidly adjustable factors will be adjusted.

Nadiri and Rosen[1973] attempted to rectify some of the problems of the single equation distributed lag models by introducing the notion of general disequilibrium. They defined a vector of n inputs, at time t, as $X_t = (x_{1t}, x_{2t}, \ldots, x_{nt})$ and the long-run equilibrium levels of these inputs as X_t^*. The adjustment mechanism, in its discrete time version, was then generalized to:

[5.2] $$X_t - X_{t-1} = B(X_t^* - X_{t-1})$$

where B is an (n x n) partial adjustment matrix.

The representation in [5.2] has some important features. First, it allows disequilibrium with respect to one factor to affect the demand for the other factors. Second, a typical equation of [5.2] is of the following form:

[5.3] $$x_{it} = \Sigma_{j \neq i}\, b_{ij}(x_{jt}^* - x_{j,t-1}) + (1 - b_{ii})x_{i,t-1} + b_{ii}x_{it}^*,$$

The factor demand elasticities, obtained from [5.2], depend not only on b_{ii} but also on all the b_{ij}'s. Nadiri and Rosen did not consider their representation in [5.2] as a generalized ad hoc adjustment mechanism, but rather viewed it as an approximation to some underlying differential equations. The basis for their argument was the Lucas [1967] paper, in which it was shown that when all the inputs are decomposed into variable and quasi fixed inputs then the flexible accelerator equation,

[5.4] $$X_t - X_{t-1} = M_t^*(X_t^* - X_{t-1})$$

could be considered as an approximate solution to a differential equation system, derived explicitly from a dynamic economic optimization problem. However, the Nadiri and Rosen specification differs from that of Lucas in two important respects. First, in the Lucas framework M_t^* in [5.4] is a function of the exogenous prices and therefore not necessarily constant over time. In the Nadiri and Rosen model, B in [5.2] is a matrix of constant parameters. Secondly, M_t^* in [5.4] is an adjustment matrix for quasi-fixed variables only. On the other hand, in the Nadiri and Rosen model the adjustment matrix is also extended to variable inputs. More generally, since the Nadiri and Rosen model does not explicitly specify the objective function and the constraints involved in the dynamic optimization, it is not easy to defend the particular equations in, in say [5.3].

The theoretical foundations of dynamic equilibrium, with explicit costs of adjustment, were provided by Eisner and Strotz[1963], Lucas[1967], Gould[1968] and Treadway[1969]. The distinguishing feature of these models is that they are based explicitly on dynamic economic optimization, incorporating costs of adjustment for the quasi-fixed factors.

Internal costs of adjustment models have been used mainly by Lucas[1967] and Treadway[1969]. Under this approach the production function [3.1] is replaced by,

[5.5] $Q(t) = \{X(t),\ K(t),\ I_n(t)\}$

where $X(t)$ is the vector of all the variable factors, $I_n(t)$ is the gross investment and $K(t)$ is the capital stock. The marginal product of $I_n(t)$ is assumed to be negative, so that (ceteris paribus) increases in gross investment reduce the level of output.

These dynamic models, with an explicit treatment of the cost of adjustment, have been extended theoretically and tested empirically in the literature. At a general level, Lucas[1967b] extended his one factor quasi-fixed factor model to n factors of production subject to adjustment costs. Mortenson[1973] and Treadway [1969,1970] discussed some interesting implications of internal adjustment costs while Brechling[1975] extended the approach to a cost minimization framework and provided some empirical results.

More recently, Berndt, Fuss and Waverman [1977, 1979, 1980] and Morrison and Berndt [1981] have utilized the dual cost minimization approach and developed a dynamic model in which capital is a quasi-fixed factor. Denny, Fuss and Waverman[1980] have also used a similar approach for manufacturing industries in the United States and Canada in the context of substitution possibilities for energy. With a quadratic cost of adjustment for capital, their model provides the explicit solution to the optimal investment plan in a static expectations framework of factor and output prices. There have been several variations and applications of their model in the resource literature. Pindyck and Rotemberg[1983] have extended the approach to the rational expectations framework. They assume external costs of adjustment for capital as opposed to the internal costs of adjustment employed in the Berndt, Fuss and Waverman model.

In the next section of this chapter we develop a dynamic model for real money balances based on the notion of internal costs of adjustment of capital. The following section discusses such a model which draws heavily on the Morrison and Berndt[1981] and Denny et al ([1980]) papers.

5.2 A COST MINIMIZATION MODEL WITH COSTS OF ADJUSTMENT FOR CAPITAL

In developing a dynamic model that allows explicitly for the cost of adjustment to enter into a firm's optimization problem, we assume that factor markets for variable factors are perfectly competitive and the prices of the factors are known with certainty and expected to remain constant over time. We define the production function, based on the notion of internal costs of adjustment for one quasi-fixed factor, capital, K, as

$$[5.6] \quad Q(t) = F\{X(t), K(t), I_n(t)\}$$

where X(t) is the vector of variable factors and K(t) is capital, a quasi-fixed factor. $I_n(t)$ is net investment, the rate of change in K(t) over time, and Q(t) is the level of output. We also assume that for an increase in $I_n(t)$ ($I_n(t)>0$) output falls for given levels of capital stocks and variable factors because some of the resources are now devoted to changing the stocks instead of being used in production. These changes in output resulting from accumulation or depletion of capital stocks constitute the basis for internal costs of adjustment.

In the short-run, firms are assumed to maximize variable profits for given input prices W, output price P, level of quasi fixed factor K and net investment I_n. We can alternatively view the firms' optimization problem as minimizing the normalized variable cost G, which is conditional on the prices of the variable factors W, output Q, capital stock K and net investment I_n, i.e,

[5.7] $G = \Sigma_j w_j X_j$, $w_j = W_j / W_1$, $j=2,\ldots,m$

The dynamic economic problem of the firm can be stated as minimizing the present value of the stream of variable costs. The objective function of the firm, under cost minimization can be written as follows,

[5.8] $L(o) = \int_o^\infty e^{-rt}\{\Sigma_j W_j X_j(t) + P_k I_g(t)\}\, dt$

where r is the discount rate, $I_g(t)$ is gross investment in units of capital ($I_g = I_n + \delta K$), where δ is the rate of depreciation and P_k is the purchase price of capital goods.

The objective of the firm is then to minimize L subject to the production function in [5.6]. For a given level of output and technology, the problem is to choose the time paths for the control variables X(t) and $I_n(t)$ and state variable K(t) that minimizes L for a given level of initial capital stocks and variable factors.

We now relate this primal dynamic optimization problem to the normalized restricted variable cost funtion. Solving the production function in [5.6] for one of the variable factors gives:

[5.9] $X_1(t) = f[X_2(t),\ldots,X_m(t), K(t), I_n(t), Q(t)]$.

Substituting [5.9] into [5.8], we have

$$[5.10] \quad L(o) = \int_o^{\infty} e^{-rt}[W_1 f(X_2(t)..X_m(t), K(t), I_n(t), Q(t)) + \Sigma_j W_j X_j(t) + P_k I_g(t)], \qquad j=2...,m$$

<u>In the first stage</u> the demands for the variable factors can be determined. The necessary conditions for optimization are given by,

$$[5.11] \quad \partial L(o)/\partial X_j = W_1 f_j + W_j = 0 \qquad j=2,...,m$$

$$\text{or} \quad f_j = -w_j, \quad w_j = W_j/W_1$$

where f_j is the partial derivative of $f[.]$ with respect to the jth variable factor of production. The partial derivative f_j is negative since for a given level of output and other factors, an increase in the use of the jth factor reduces the use of the first factor X_1.

[5.11] can then be solved for cost minimizing short-run factor demands, given the strict concavity of the production function in [5.6]. The demand for the jth variable factor is represented by,

$$[5.12] \quad X_j^*(t) = \theta\{w_2(t), ...w_m(t), K(t), I_n(t), Q(t)\}.$$

Substituting [5.12] in [5.7], we have,

$$[5.13] \quad G(t) = \Sigma_j w_j X_j^*(t) = G\{w(t), K(t), I_n(t), Q(t)\} \qquad j=2,...,m$$

G(t) in [5.13] is the normalized variable cost which is conditional on the level of capital stocks, net investment and output in period t. G(t) is conceptually the same function that we discussed in Chapter 4, with the exception that now the notion of internal costs of

adjustment of capital are also introduced into the firms' optimization problem and investment as well as capital is a conditioning argument. Under reasonable regularity conditions on the production function, the normalized variable cost function can be shown to be increasing and concave in w, increasing and convex in K(t), increasing in Q(t) and decreasing and convex in $I_n(t)$ (Lau[1976]). In the subsequent paragraphs of this section, we drop the t subscripts for simplification.

In the second stage, one can analyze the long-run problem of the firm which is to determine its optimal capital stocks and to achieve its long-run objectives over time (Morrison and Berndt(p. 343, [1981]). Substituting [5.13] into [5.8], we have,

$$[5.14] \quad L(o) = \int_o^{\infty} W_1 e^{-rt}\{G(w,K,I_n,Q,t)+ p_k I_g\}dt$$

where $p_k=P_k/W_1$.

Next we substitute for gross investment, $I_g=I_n+\delta K$, in [5.14], yielding,

$$[5.15] \quad L(o)/W_1 = \int_o^{\infty} e^{-rt}\{G(w,K,I_n,Q)+ \delta p_k K\}dt+$$

$$\int_o^{\infty} e^{-rt}(p_k I_n)dt$$

The last term in [5.15] can then be integrated by parts, as,

$$[5.16] \quad \int_o^\infty e^{-rt}(p_k I_n)dt = p_k K e^{-rt}\Big]_o^\infty - \int_o^\infty p_k K(-re^{-rt})dt$$

$$= -p_k K(0) + \int_o^\infty e^{-rt}\, rp_k K dt$$

Substituting [5.16] into [5.15], we get,

$$[5.17] \quad L(o)/W_1 + p_k K(0) = \int_o^\infty e^{-rt}[G(w,K,I_n,Q) + vK]dt$$

where $v = (r+\delta)p_k$, the normalized rental price of capital.

The solution to the minimization problem in [5.17], with respect to the state variable K and control variable I_n, is obtained by applying Pontryagin's maximum principle. In constructing the Hamiltonian function we assume static expectations with respect to the normalized factor prices. The Hamiltonian function for [5.17] can be written as,

$$[5.18] \quad H(K,I_n,J,t) = \int_o^\infty e^{-rt}\,[G(w,\ K,I_n,Q) + vK] + JI_n$$

where J is a co-state variable, which is a dynamic equivalent of the Langrange multiplier in a static optimization problem subject to constraints. The necessary conditions for a minimum are,

$$[5.19] \quad dH/dI_n = e^{-rt}G_I + J = 0$$

and

$$[5.20] \quad -dH/dK = -(e^{-rt}(G_k + G_{kI}I+v) = \dot{J}$$

Differentiating [5.19] with respect to time,

$$[5.21] \quad e^{-rt}G_I I_{nt} + re^{-rt}G_I + \dot{J} = 0$$

where I_{nt} denote the time derivative of I_n.

Eliminating $\dot{J}$ from [5.21] by substituting [5.20], we have,

[5.22] $-G_k - rG_I - v + G_{II}I_{nt} + G_{kI}I_n = 0$.

where the subscripts k and I represent partial derivatives and I_{nt} denotes the time derivative of I_n.

A steady state solution K^* is given by,

[5.23] $-G_k(w,K^*,I_n) - rG_I(w,K^*,I_n) - v = 0$

where * indicates that it is evaluated at $K=K^*$ and $I_n=0$.

The solution K^* is shown to be unique as long as,

[5.24] $-G_{kk} - rG_{kI} = 0$.

[5.23] can be rewritten as

[5.25] $-G_k(w,K^*,0) = v + rG_I(w,K^*,0)$.

The left hand side of [5.25] measures the marginal benefit to the firm of changing the quasi-fixed factor and the right hand side is composed of the rental price and the marginal adjustment cost of changing the quasi-fixed factor by one unit at $I_n = 0$.

Treadway[1971, 1974] has shown that the solution of the demand for quasi-fixed factors K, generated from [5.23] or [5.25], can be linked to the flexible accelerator model by approximating the solution of K to the differential equation,

[5.26] $I_n = \beta^*(K^* - K)$

where β^* satisfies the condition,

[5.27] $-(G_{II})(\beta^*)^2 - rG_{II}(\beta^*)+(G_{kk} + rG_{kI}) = 0$

and [5.27] can be solved for β^*(taking the stable roots),

[5.28] $\beta^* = -1/2\ [r-\{(r^2 + 4(G_{kk} + rG_{kI})/G_{II}\}^{1/2}]$.

Two properties of the normalized restricted cost function in [5.13], also noted by Berndt and Morrison (p.353, [1981]), relate to a firm's demand for variable and quasi-fixed factors, which can be derived directly from the optimization conditions discussed earlier in this section. **First**, the partial derivative of the cost function G with respect to the jth normalized factor price equals the demand for the jth factor. This property follows directly from [5.13] (Shephard's Lemma), i.e.,

[5.29] $\partial G/\partial w_j = X_j^*$.

Second, in the steady state, a firm's demand for the quasi-fixed factor K is determined by solving [5.23] for K^* when $G_I=0$, i.e.,

[5.30] $\partial G/\partial K = -v$.

In the following section we utilize this theory to build an empirical model for real money balances, with capital as a quasi-fixed factor. The demand for the variable factors, including real money balances, differs in its arguments from the factor demands derived in the full and temporary equilibrium models in several respects. In the full equilibrium model the factor demands depended only on the factor prices and the level of output. Capital in that model was assumed to be adjusted costlessly and instantaneously, along with the firm's holdings of labour and money. In the temporary equilibrium model, some account for the fixity of capital stocks was introduced. Factor demand functions in that model not only depended on output and factor prices but also on the level of stocks of fixed factors. In the dynamic model, the demand for factors not only depends on factor prices, output, and the level of capital stocks, but it also depends on net investment. The adjustment process of the capital stocks is now endogenous in the system and results directly from the application of economic theory.

In the following section, we first discuss the empirical model in the dynamic framework based on the particular functional form for the variable cost function employed by Denny, Fuss and Waverman [1980]. Then we discuss some of the shortcomings of the functional form used by Denny et al [1980]. In the latter part of the section we derive our alternative estimating equations for the dynamic model based on a different functional form for the variable cost function.

5.2.1 AN EMPIRICAL MODEL FOR REAL MONEY BALANCES WITH INTERNAL COSTS OF ADJUSTMENT FOR CAPITAL

In this section we describe the structure that is imposed on entrepreneurial decisions: First, we assume that the firm sells only one product Q(t) at a price p per unit. __Second__, firms are cost minimizers who treat their output and input prices as exogeneous. __Third__, there are four factors of production, labour N, real money balances m, capital K, and I_n a negative factor representing internal costs of adjustment, in the production process. Capital is assumed to be a quasi-fixed factor, while the others are treated as variable. __Fourth__, we assume that firms expect all current prices to prevail in the future. __Fifth__, the continuous changes in capital, I_n, can be represented by discrete changes, $\Delta K=K_t - K_{t-1}$.

We assume that there are non-linear internal costs of adjustment associated with changes in net investment. These costs are represented by an increase in variable costs, G, due to changes in the level of net investment. Berndt and Morrison[1981] approximate the normalized restricted cost function by a quadratic function. This specification has been chosen for the following reasons (Denny et al [1980]). First, the Hessian of second order partial derivatives is a matrix of constants which facilitates linking the short and long-run responses [Lau,1976]. Second, the quadratic approximation of the underlying differential equations are linear and therefore the optimal path for the quasi-fixed factor is globally as well as locally valid.

Several studies (see for example, Berndt and Morrison [1981], Berndt, Fuss and Waverman [1977,1980] and

Denny, Fuss and Waverman [1980]), have chosen the following quadratic specification for the normalized variable cost function:

$$[5.31] \quad G = X_1 + \Sigma_j w_j X_j$$
$$= \alpha_o + \Sigma_j \alpha_j w_j + \alpha_Q Q + \alpha_K K + 1/2 \Sigma_j \Sigma_i \gamma_{ji} (w_j)^2$$
$$+ 1/2\{\gamma_{QQ} Q^2 + \gamma_{KK} K^2 + \gamma_{\Delta\Delta}(\Delta K)^2\}$$
$$+ \Sigma_j \gamma_{jK} w_j K + \Sigma_j \gamma_{Qj}(w_j/W_1)Q + \gamma_{QK} KQ$$
$$+ \Sigma_j \gamma_{j\Delta} w_j \Delta K + \gamma_{Q\Delta} Q(\Delta K)$$
$$+ \gamma_{K\Delta} K(\Delta K) + \alpha_\Delta(\Delta K) \qquad i,j=2,\ldots,m$$

where w_j is the normalized factor price, $w_j = W_j/W_1$.

In [5.31], the variable cost has been normalized by the price of the factor X_1. The internal costs of adjustment of capital in [5.31] are represented by,

$$[5.32] \quad C(\Delta K) = \alpha_\Delta(\Delta K) + 1/2\gamma_{\Delta\Delta}(\Delta K)^2 +$$
$$\Sigma_j \gamma_{j\Delta} w_j \Delta K + \gamma_{Q\Delta}(\Delta K)Q + \gamma_{K\Delta}(\Delta K)K.$$

These costs of adjustment are required to be non-negative (for both positive and negative values of ΔK) and the marginal costs of adjustment are required to be zero when $\Delta K=0$. Differentiating [5.32] with respect to ΔK, we have,

$$[5.33] \quad \partial G/\partial(\Delta K) = \alpha_\Delta + \gamma_{\Delta\Delta}(\Delta K) + \Sigma_j \gamma_{j\Delta} w_j$$
$$+ \gamma_{Q\Delta} Q + \gamma_{K\Delta} K.$$

The marginal adjustment cost in [5.33] is zero at $\Delta K=0$ if and only if we impose the following restrictions on the parameters of [5.31],

$$[5.34] \quad \alpha_\Delta = \gamma_{j\Delta} = \gamma_{Q\Delta} = \gamma_{K\Delta} = 0, \qquad j=2,\ldots,m$$

After imposing the restrictions in [5.34] on the cost function in [5.31], the cost function can be re-written as follows,

$$[5.35]\quad G = X_1 + \Sigma_j w_j X_j$$

$$= \alpha_o + \Sigma_j \alpha_j w_j + \alpha_Q Q + \alpha_K K + 1/2\Sigma_j \Sigma_i \gamma_{ji} w_j{}^2$$

$$+ 1/2\{\gamma_{QQ}Q^2 + \gamma_{KK}K^2 + \gamma_{\Delta\Delta}(\Delta K)^2\}$$

$$+ \Sigma_j \gamma_{jK} w_j K + \Sigma_j \gamma_{Qj} w_j Q + \gamma_{QK}KQ$$

$$+ \gamma_{Q\Delta}Q(\Delta K) \qquad i,j=2,\ldots,m$$

Short-run demand functions for the variable inputs are derived by using Shephard's Lemma as in [5.29]. The demand equation of the normalized factor can be derived residually since the function G embodies the optimizing behaviour of the firms (Denny et al, p.13, [1980]). Therefore we can substitute for X_j in $G = X_1 + \Sigma_j w_j X_j$ and solve residually for X_1. These demand functions are given by[2]

$$[5.36]\quad X_j = \alpha_j + \Sigma_j \gamma_{ji} w_j + \gamma_{Qj} Q + \gamma_{jK} K$$

and

$$[5.37]\quad X_1 = G - \Sigma_i w_i X_i$$

$$= \alpha_o - 1/2\Sigma_j \Sigma_i \gamma_{ji} w_j w_i + \alpha_{QQ} + \alpha_{KK} + \gamma_{QK}KQ +$$

$$1/2\{\gamma_{KK}K^2 + \gamma_{QQ}Q^2 + \gamma_{\Delta\Delta}\Delta K^2\}$$

$$j,i=2,\ldots,m$$

In addition to the demand for variable factors, the demand equation for net investment can be obtained by using [5.26], [5.28] and [5.35]:

$$[5.38]\quad \Delta K = -1/2\{r-(r^2 + 4\gamma_{KK}/\gamma_{\Delta\Delta})^{1/2}\}\{K^* - K_{t-1}\}.$$

The expression for the steady state K^* is obtained by using [5.23], with $G_I=0$:

[5.39] $K^* = \{(-1/\gamma_{KK})\ (\alpha_K + \Sigma_j \gamma_{jK} w_j + \gamma_{QK}\ Q + v)\}$

where v is the normalized rental or user cost of capital.

Substituting [5.39] in [5.38], we get

[5.40] $$\begin{aligned}\Delta K &= \beta^*(K^* - K_{t-1}) \\ &= -1/2[r - \{r^2 + 4\gamma_{KK}/\gamma_{\Delta\Delta}\}^{1/2}\ ] \\ &\qquad [(-1/\gamma_{KK})\ (\alpha_K + \Sigma_j \gamma_{jK} w_j + \gamma_{QK}\ Q + v) - K_{t-1}].\end{aligned}$$

The demand functions for the freely variable factors in [5.36] and [5.37] and the net investment demand function in [5.40] constitute the system of equations which is estimated in the studies referred to earlier.

The problem with this particular specification of the cost function in [5.35] is that the resulting demand function for the normalized factor, X_1, depends on different variables than the demand for all other variable factors. In particular, the demand for X_1 has additional arguments Q^2, K^2 and ΔK^2 that do not appear in the other demand function. Furthermore, the particular functional form of G also limits the model to have only one factor of production, the normalized factor X_1, adjust when investment takes place. The rest of the variable factors do not depend on ΔK, the argument that represents the costs of adjustment in the production process, and hence are not varied in the adjustment process. This is clearly unsatisfactory; the arguments of a factor demand function should not depend on the researcher's normalization rule. We therefore specify an alternative parameterization of G. The alternative proposed here can be thought of as homogeneous of order one in the factor prices (a Diewert type cost function) and a quadratic approximation in Q, K and ΔK.

[5.41] $$VC = \Sigma_i W_i X_i$$
$$= \Sigma_i \Sigma_j \gamma_{ij} (W_i W_j)^{1/2} + \Sigma_i \gamma_{qi} W_i Q + \Sigma_i \gamma_{ki} W_i K + \Sigma_i \gamma_{\Delta i} W_i \Delta K + 1/2\{\Sigma_i \theta_{qi} W_i Q^2 + \Sigma_i \theta_{ki} W_i K^2 + \Sigma_i \theta_{\Delta i} W_i \Delta K^2\} + \Sigma_i \psi_i W_i KQ + \Sigma_i \phi_i W_i K(\Delta K) + \Sigma_i \zeta_i W_i Q(\Delta K).$$

The variable cost in [5.41] is measured in nominal terms as opposed to G in [5.35] where the variable cost was normalized by one of the factor prices. The cost function in [5.41] is homogeneous of degree one in the factor prices by construction and is symmetric in all factors. Therefore, it gives exactly the same variable factor demand equation whether we start from a non-normalized cost function or a normalized one. We choose the former to derive the factor demand equations.

It was argued earlier that the marginal cost of adjustment should be required to be zero when net investment is zero. Differentiating VC in [5.41] and equating it to zero, we have

[5.42] $$\partial VC/\partial \Delta K = \Sigma_i \theta_{\Delta i} \Delta K + \Sigma \phi_i W_i K + \Sigma_i \gamma_{\Delta i} W_i + \Sigma_i \zeta_i W_i Q = 0.$$

[5.42] therefore imposes the following zero restrictions on the parameters of the cost function,

[5.43] $\phi_i = \zeta_i = \gamma_{\Delta i} = 0,$ $i=1,\ldots,m$

Imposing the restrictions in [5.43] on the cost function and rewriting it, yields

[5.44]
$$\begin{aligned} VC &= \Sigma_i W_i X_i \\ &= \Sigma_i \Sigma_j \gamma_{ij} (W_i W_j)^{1/2} + \Sigma_i \gamma_{qi} W_i Q + \Sigma_i \gamma_{ki} W_i K + \\ &\quad 1/2\{\Sigma_i \theta_{qi} W_i Q^2 + \Sigma_i \theta_{ki} W_i K^2 + \Sigma_i \theta_{\Delta i} W_i \Delta K^2\} + \\ &\quad \Sigma_i \psi_i W_i (KQ). \end{aligned}$$

The demand for the variable factors in the short-run can be derived by using the Shephard's lemma,

[5.45]
$$\begin{aligned} \partial VC/\partial W_i &= X_i^* \\ &= \gamma_{ii} + \Sigma_j \gamma_{ij} (W_j/W_i)^{1/2} + \gamma_{qi} Q + \gamma_{ki} K + \\ &\quad 1/2\{\theta_{qi} Q^2 + \theta_{ki} K^2 + \theta_{\Delta i} \Delta K^2\} + \psi_i (KQ). \end{aligned}$$

The demand for net investment can be derived by using [5.26], [5.28] and [5.44],

[5.46]
$$\Delta K = -0.5\{r - (r^2 + 4(\Sigma_i \theta_{ki} W_i)/(\Sigma_i \theta_{\Delta i} W_i))^{1/2}\} \{K^* - K_{-1}\}.$$

The expression for the steady state capital stocks is obtained by using [5.23] and [5.44],

[5.47]
$$K^* = -[(\Sigma_i \gamma_{ki} W_i + \Sigma_i \psi_i W_i Q + V)/(\Sigma_i \theta_{ki} W_i)].$$

Finally, substituting [5.47] into [5.46], yields the investment equation,

[5.48]
$$\begin{aligned} \Delta K = &-0.5\{r - [r^2 + 4(\Sigma_i \theta_{ki} W_i)/(\Sigma_i \theta_{\Delta i} W_i)]^{1/2}\} \\ &\{[-(\Sigma_i \gamma_{ki} W_i + \Sigma_i \psi_i W_i Q + V)/(\Sigma_i \theta_{ki} W_i)] - K_{-1}\}. \end{aligned}$$

The demand functions for the variable factors in [5.45] and the net investment demand function in [5.48] comprise the system of equations that we estimate. We also estimate the variable factors in [5.36] and [5.37] and the investment demand function in [5.38], based on the Denny et al [1980] specification of the cost function. This will enable us to compare the price elasticities based on two different specifications of the cost function. In our empirical specification, we have two variable factors of production: labour and real money balances, while capital is a quasi-fixed factor. The variable demand functions have identical arguments and the same functional form. We now present the empirical specification of the dynamic model:

[5.49a] $$N^{*} = \gamma_{nn} + \gamma_{nm}(r_m/W)^{1/2} + \gamma_{qn}Q + \gamma_{kn}K + 1/2\{\theta_{qn}Q^2 + rknK2 + \theta_{\Delta n}\Delta K^2\} + \psi_n(KQ) + \xi_n$$

[5.49b] $$m^{*} = \gamma_{mm} + \gamma_{nm}(W/r_m)^{1/2} + \gamma_{qm}Q + \gamma_{km}K + 1/2\{\theta_{qm}Q^2 + \theta_{km}K^2 + \theta_{\Delta m}\Delta K^2\} + \psi_m(KQ) + \xi_m$$

[5.49c] $$\Delta K = -0.5\{r - [r^2 + 4(\theta_{kn}W + \theta_{km}r_m)/(\theta_{\Delta n}W + \theta_{\Delta m}r_m)]^{1/2}\} \{[-(\gamma_{kn}W + \gamma_{km}r_m + \psi_n WQ + \psi_m r_m Q + V)/ (\theta_{kn}W + \theta_{km}r_m)] - K_{-1}\} + \xi_{\Delta}$$

where W and r_m represent the wage rate and opportunity cost of real money balances and V is the user price of capital. ξ_n, ξ_m and ξ_{Δ} are the additive disturbances in the three equations respectively.

In discussing the estimates of the dynamic model, we deal mainly with the following issues:

(a) Does the cost of adjustment of capital matter ?

(b) What is the nature of the short-run adjustment process ?

(c) How do the own and cross price elasticities compare in the short-run and in the long-run?

(d) What are the effects of changes in the interest rate on the demand for labour?

(a) and (b) are dealt with in Section 5.3.1. (c) is discussed in Section 5.3.2. (d) is discussed in Section 5.3.3.

5.3 THE STRUCTURE OF PRODUCTION

5.3.1 ROLE OF COSTS OF ADJUSTMENT OF CAPITAL

In Section 5.1.3, we solved the expression for steady state capital K^* in [5.47]. Desired levels of capital stocks can be obtained by using [5.47], and can be compared with the actual levels. The discrepancies between actual and desired capital stocks can indicate the potential for disequilibrium in the short-run. In the full equilibrium model, discussed in Chapter 3, there was no such distinction. Firms could always have the desired levels of capital stocks. Furthermore, the partial adjustment coefficient β^* in [5.39] indicates the degree to which the firm reduces the gap between desired and actual capital stocks in response to exogenous shocks. A value of β^* closer to zero indicates slower adjustment in capital stocks and a value closer to unity would indicate a quicker adjustment. Theoretically, this coefficient could take any value between 0 and unity.

The variable factor demands, in the cost minimizing dynamic framework, are conditionally derived for given levels of capital stocks and investment in [5.49a] and [5.49b]. However, changes in the factor prices have both direct and indirect effects on the factor demands in the short-run (Denny et al [1979]). For example, an increase in the level of the rate of interest will reduce or increase employment directly, depending on the complementarity or substitutability of labour demand with real money balances. In this model, labour demand is also a function of investment which in turn is a function of the interest rate. Therefore, the total effects of changes in the rate of interest also include the indirect effects, including the changes in investment behaviour, in the short-run. These total cross price effects are discussed in more detail in the following section on price elasticities.

5.3.2 FACTOR DEMAND ELASTICITIES

Both the own and cross price elasticities in the dynamic framework are classified into three groups. First, there are the restricted short-run elasticites, RS. These elasticities are evaluated while keeping all of the following constant: the level of capital, the level of output and net investment. These elasticities are calculated in order to check the sign consistency of own and cross price elasticities with theoretical priors. Second, there are the estimates of the short-run elasticities, SR. These elasticities include both the direct effects of changes in factor prices on input demand and the indirect effects through the changes in the demand for net investment in the short-run. Finally, the long-run estimates, LR, are evaluated when firms have acquired the desired level of capital stocks and net investment is zero.

The own price elasticity of the ith factor in [5.45] in the restricted short-run is given by:

[5.50] $$(\eta_{ii})^{RS} = (W_i/X_i)\{\partial X_i/\partial W_i\}$$
$$= -(W_i/X_i)\{0.5\gamma_{nm}(W_j/W_i)^{1/2}(1/W_i).$$

For positive factor prices, the own price elasticity in [5.50] is negative if and only if $\gamma_{nm}>0$. The short-run estimate of the own price elasticity can be obtained by adding the indirect effect on factor demand of changes in net investment to [5.50], i.e.

$$[5.51] \quad (\eta_{ii})^{SR} = (\eta_{ii})^{RS} + W_i/X_i(\partial X_i/\partial \Delta K)(\partial \Delta K/\partial W_i)$$
$$= (\eta_{ii})^{RS} + W_i/X_i[\{\theta_{Wi}(\Delta K)\}\{(\partial \beta^*/\partial W_i)(K^*-K_{-1}) + \beta^*(\partial K^*/\partial W_i)\}]$$

The long-run own price elasticites are evaluated as follows,

$$[5.52] \quad (\eta_{ii})^{LR} = (\eta_{ii})^{RS} + W_i/X_i(\partial X_i/\partial K^*)(\partial K^*/\partial W_i)$$

Similarly, the cross price elasticites are calculated for the three different cases as follows[3],

$$[5.53] \quad (\eta_{ij})^{RS} = W_j/X_i\{\partial X_i/\partial W_j\}$$
$$= W_j/X_i\{0.5\gamma_{nm}(W_j/W_i)^{-1/2}(1/W_i)$$

$$[5.54] \quad (\eta_{ij})^{SR} = (\eta_{ij})^{RS} + W_j/X_i\{(\partial X_i/\partial \Delta K)(\partial \Delta K/\partial W_j)$$
$$= (\eta_{ij})^{RS} + W_j/X_i[\{\theta_{\Delta i}(\Delta K)\}\{(\partial \beta^*/\partial W_j)(K^*-K_{-1}) + \beta^*(\partial K^*/\partial W_j)\}]$$

and

$$[5.55] \quad (\eta_{ij})^{LR} = (\eta_{ij})^{RS} + W_j/X_i\{(\partial X_i/\partial K^*)(\partial K^*/\partial W_j)\}$$

The own price elasticity of capital can be estimated by using the equation for the desired capital stocks in [5.47] as,

$$[5.56] \quad \eta_{kk} = (V/K)(\partial K^*/\partial V)$$
$$= (V/K)\{-1/(\theta_{kn}W+\theta_{km}r_m)\}$$

5.3.3 VARIABLE OUTPUT OR PROFIT MAXIMIZING ELASTICITIES

We can obtain the the marginal cost function from the variable cost function in [5.45] by differentiating variable cost with respect to output,

$$[5.57] \quad MC = \partial G/\partial Q$$

$$= \Sigma_i \gamma_{qi} W_i + \Sigma_i \theta_{qi} W_i Q + \Sigma_i \psi_i W_i K.$$

There are increasing, decreasing or constant returns to scale depending on whether the derivative of MC with respect to output is negative, positive or zero respectively,

$$[5.58] \quad \partial MC/\partial Q = \Sigma_i \theta_{qi} W_i.$$

We can derive the supply of output function by equating the MC in [5.57] to the price of output and solving for Q, i.e.,

$$[5.59] \quad Q^* = \{1/(\Sigma_i \theta_{qi} W_i)\}\{P - \Sigma_i \gamma_{qi} W_i - \Sigma_i \psi_i W_i K\}$$

where P is the output price, normalized by the wage rate.

We may use [5.59] in order to derive the output variable or profit maximizing price elasticities.

We determine the profit maximizing cross price elasticity of labour with respect to rate of interest as,

$$[5.60] \quad (\Pi_{ij})^{SR} = (\eta_{ij})^{SR} + (W_j/X_i)(\partial X_i/\partial Q)(\partial Q/\partial W_i)$$

$$= (\eta_{ij})^{SR} + (W_j/X_i)(\gamma_{qi} + \theta_{qi} Q + \psi_i K)(\partial Q/\partial W_i)$$

where $\partial Q/\partial W_i = (\Sigma_i \theta_{qi} W_i)^{-1}(-\gamma_{qi} - \psi_i K) + (P - \Sigma_i \gamma_{qi} W_i - \Sigma_i W_i K)(\theta_{qi})$.

5.4 ESTIMATION OF MODEL

5.4.1 ESTIMATION PROCEDURES

In this section we outline the statistical techniques that have been used to estimate the system of equations [5.49] developed in Section 5.1.3.

There are several problems associated with the estimation of equations of the dynamic model, discussed in Section 5.1.3. The first set of problems relates to the fact that the errors of the equations may not satisfy the conditions of Gauss-Markov. The individual equations have different scales, therefore we do not expect the variances of the error terms from different equations to be the same. We also expect that the contemporaneous errors are correlated with each other. The second set of estimation problems arises due to the non-linearity of the net investment equation [5.49c] in the parameters. Furthermore, ΔK, an endogeneous variable, appears on the right hand side in the variable factor demand equations in [5.49a] and [5.49b]. Therefore, it appears that the estimation of the dynamic model involves non-linear simultaneity in variables and parameters. However, as noted by Morrison and Berndt[1981], the system is recursive. ΔK in [5.49c] depends only on exogenous variables. Therefore, the Jacobian matrix is unit triangular and the likelihood function of the simultaneous equations reduces to a likelihood function of a non-linear 'seemingly unrelated' regression problem. We therefore use the TSP routine for the seemingly unrelated regression technique. The method is computationally equivalent to maximum likelihood.

Due to the non-linearity of the equations, the estimated parameters depend on the starting values. Numerous starting values were tried and the values of the log likelihood function were compared at different points of convergence. In Table 5.1, we present the estimated parameters of the variable cost function which has the highest value of the log likelihood function.

We make use of the likelihood ratio to test the zero restrictions imposed on the serial correlation coefficients by model A (Table 5.1). The computed likelihood ratio test statistic is 17.08 compared to a tabulated Chi-square of 13.28 with 4 independent restrictions at 99% confidence level. Therefore we reject the null hypothesis at the 99% confidence level. The result of this test clearly indicates that the transformed model has corrected some of the serial correlation from the residuals of the dynamic model. The estimated autoregression coefficients and Durbin-Watson statistic are presented in Table 5.2[4].

The results of the corrected version of the dynamic model for serial correlation are presented in the column B of Table 5.1. The transformed equations of labour demand and money demand together with the net investment equation were simultaneously estimated, using the iterative Zellner technique for seemingly unrelated equations.

TABLE 5.1

Results of the Dynamic Model

	A	B		A	B
γ_{nn}	40.95 (8.63)	54.196 (6.974)	θ_{kn}	0.0001 (1.24)	0.00021 (2.17)
γ_{mm}	−9.694 (−1.19)	−1.072 (−0.176)	θ_{km}	−0.0014 (−1.12)	0.00004 (0.213)
γ_{nm}	3.393 (5.505)	3.311 (6.47)	$\theta_{\Delta n}$	0.0016 (1.30)	0.0037 (3.27)
γ_{qn}	0.062 (2.480)	−0.0034 (−0.085)	$\theta_{\Delta m}$	0.0019 (1.24)	0.0117 (7.43)
γ_{qm}	0.156 (1.924)	0.0746 (1.096)	ψ_n	−0.0001 (−1.26)	−0.00027 (−2.43)
γ_{kn}	0.001 (0.247)	0.0023 (0.336)	ψ_m	0.0014 (1.10)	−0.00013 (−0.89)
γ_{km}	−0.0001 (−0.001)	0.0367 (0.70)	Ω	−161.28	−152.74
θ_{qn}	0.00012 (0.933)	0.00046 (2.561)			
θ_{qm}	−0.00016 (−1.14)	0.00013 (0.59)			

Note: In the parentheses are the t-ratios. The estimated parameters in column A are not corrected for serial correlation. In column B, the results of the model after correcting for serial correlation are presented.

Table 5.2

Estimated ρ's and Durbin-Watson Statistics

	ρ		A	B
ρ_{nn}	0.4114 (3.87)	DW_n	0.97	1.793
ρ_{mm}	−0.4674 (−3.68)	DW_m	2.017	2.075
ρ_{nm}	−0.2213 (−1.96)	DW_Δ	1.920	1.864
ρ_{mn}	3.311 (6.47)			

Note: DW's in column A belongs to the uncorrected model for serial correlation and the DW's in column B are for the model corrected for serial correlation. The net investment equation in [5.49c] was not adjusted for first order serial correlation. Therefore there is no estimate of p for the investment equation.

5.4.2 DISCUSSION OF RESULTS

In discussing the results of the dynamic model, we first test the importance of the costs of adjustment in a firm's optimization framework. There are two key parameters in the variable cost function [5.45] that signify the internal cost of adjustment, $\theta_{\Delta n}$ and $\theta_{\Delta m}$. These two parameters also appear in the two variable factor demand functions in [5.49a] and [5.49b]. The role of net investment in the factor demand functions is the key difference between the factor demand functions of the temporary equilibrium model and the dynamic model. We therefore tested the null hypothesis that these two parameters are jointly zero. The computed likelihood ratio test statistic is 60.58 compared to a tabulated Chi-square of 10.6 with two independent restrictions at the 99% confidence level. The null hypothesis is therefore strongly rejected at the 99% confidence level. With the two parametric restrictions discussed above, the dynamic model reduces to temporary equilibrium model with a quadratic functional form for the variable cost function. Therefore, based on the above result, we reject the temporary equilibrium model conditional on the functional form of the variable cost function.

The coefficient β^* in the net investment equation was estimated for each year, using the expression for β^*

in [5.49c]. The series of estimates of β^* is presented in Table 5.3. The coefficient showed little variation over time. Its average value is 0.21407, indicating that about 21% of adjustment of the capital stocks takes place in one year. This result is consistent with other estimates. For example, Morrison and Berndt[1981] estimated β^* at 0.266 for the U.S. manufacturing, although in their model energy replaces money as the other factor of production.

The level of desired capital stocks, K^*, was also estimated over the years (Table 5.3), using equation [5.47]. The results indicate that the average difference betweeen desired and actual capital stocks has been about 81.237 billion 1972 dollars.

In examining the price elasticities of factors, we first discuss the own price elasticities of the three factors of production. Table 5.4 presents the cost minimizing own price elasticities of the dynamic model. These elasticities are classified into three groups. First are the restricted short-run elasticities [5.50], RS. Second are the short-run elasticities [5.51], SR. Finally, the long-run elasticities [5.52], LR. The own price elasticities in the restricted short-run indicate movement along the isoquant. Theoretically we expect this substitution effect to be negative. In the short-run, however, the effect of changes in the net investment are also added.

TABLE 5.3

Estimated Desired Capital Stocks and β^*

YEAR	DESIRED K	(K^*-K)	β^*	ΔK
1949	109.3250	−133.9589	0.2247	−30.1120
1950	183.6210	−68.3859	0.2249	−15.3862
1951	187.7650	−81.9699	0.2214	−18.1553
1952	205.4545	−71.8484	0.2206	−15.8545
1953	253.6347	−34.2882	0.2197	−7.5350
1954	304.2621	4.4211	0.2243	0.9919
1955	345.0876	33.8886	0.2214	7.5037
1956	298.5690	−33.3749	0.2159	−7.2082
1957	284.8242	−64.8097	0.2136	−13.8446
1958	349.6291	−12.7118	0.2200	−2.7975
1959	347.1630	−27.2719	0.2128	−5.8058
1960	389.3083	7.1853	0.2134	1.5337
1961	464.9969	73.4809	0.2176	15.9923
1962	561.2080	159.2530	0.2162	34.4409
1963	608.0737	163.7457	0.2149	35.1895
1964	643.9567	187.9747	0.2129	40.0279
1965	716.3701	269.6671	0.2110	56.9122
1966	747.0443	270.0793	0.2057	55.5612
1967	797.4016	288.3466	0.2077	59.9091
1968	840.8060	301.4160	0.2041	61.5450
1969	784.1845	211.5905	0.1957	41.4292
1970	776.6283	171.5653	0.1962	33.6748
1971	941.9804	308.6104	0.2077	64.1187
AVG.	484.4041	83.5915	0.2140	17.0491

Note: Capital stocks and net investment are measured in billions of 1972 U.S dollars.

Table 5.4 also reports the earlier estimates of own price elasticities for the full equilibrium model and temporary equilibrium model, discussed in Chapters 3 and 4 respectively. The three models can be compared in terms of long-run elasticities, while for the temporary equilibrium model and the dynamic model, short-run elasticities can also be compared. We discuss these comparisons in the latter part of this section.

We have also generated confidence intervals for the restricted short-run and the short-run price elasticities of the dynamic model (see footnote 9 of Chapter 3).

The signs of the own price elasticities for labour and real money balances, in the restricted short-run, are consistent with the theory. The confidence limits for both own price elasticities are presented in Table 5.5. In the restricted short-run, RS, the own price elasticites are significantly different from zero at 0.05 level of significance.

The own price elasticity of labour in the short-run, with investment effects included, is positive in sign. The hypothesis for zero own price elasticity of labour is rejected in favour of positive own price elasticity. The positive effect on labour of changes in net investment outweighs the negative own substitution effect. Intuitively, the result indicates that in the short-run, when the wage rate goes up, firms start increasing investment in order to substitute capital for labour in the long-run. However, in making more investment, the demand for labour services for investment purposes goes up and outweighs the decrease in the demand for labour for production purposes. Finally, the own price elasticity of demand for labour in the long-run is negative and inelastic as expected. Morrison and Berndt[1981] have found similar results[5]. Their estimates of labour's own price elasticity are −0.283 and −0.347 in the short and in the long-run respectively for the U.S. manufacturing industries(1952–71). The estimate from the dynamic model of the own price elasticity of labour in the long-run is greater than the estimates from the full equilibrium model.

The own price elasticity of real money balances has a negative sign both in the short-run and in the long-run. The elasticity is significantly different from zero both in the restricted short-run and short-run (see Table 5.5). The estimates of η_{mm} are different in the short-run and in the long-run. The long-run elasticity is not that different from those of Dennis and Smith[1978] and our mean estimate of the full equilibrium model. The estimate of Dennis and Smith of η_{mm} ranged between −0.224 to −0.409 for the 11 two digit SIC code industries of United States (1952–73).

TABLE 5.4

Own Price Elasticities
(Cost Minimization)

	FULL EQUILIBRIUM MODEL	TEMPORARY EQUILIBRIUM MODEL		DYNAMIC DISEQUILIBRIUM MODEL		
	LR	SR	LR	RS	SR	LR
LABOUR (η_{nn})	−0.08246	−0.00261	−0.0590	−0.0014	0.0532	−0.1326
MONEY (η_{mm})	−0.17996	−0.21966	−0.2821	−0.1463	−0.1508	−0.1791
CAPITAL (η_{kk})	−0.05720	−	−0.4540	−	−	−0.3840

Note: RS is the short-run, when capital is fixed, SR is the intermediate-run when capital has started adjusting partially as determined by β^* and LR is the long-run when capital has fully adjusted to the desired level.

TABLE 5.5

Confidence Intervals for Own Price Elasticities
(Dynamic Model, RS and SR)

	ELASTICITIES				
	MEAN ELASTICITY FROM 900 DRAWS	LOWER TAIL		UPPER TAIL	
		2.5%	5%	5%	2.5%
LABOUR (RS)	−0.001373	−0.01800	−0.001735	−0.001005	−0.00095
MONEY (RS)	−0.146320	−0.1919	−0.18496	−0.107100	−0.10153
LABOUR (SR)	+0.053245	+0.01006	+0.014862	+0.113840	+0.18841
MONEY (SR)	−0.150810	−0.19845	−0.19073	−0.111180	−0.10274

The estimated own price elasticity of capital is -0.38. The result of the dynamic model is not very different from the temporary equilibrium model. But there is a substantial difference, when it is compared to the full equilibrium model estimate (Table 5.4). Morrison and Berndt[1981] estimate this elasticity at -0.207.

Cross price elasticities are listed in Table 5.6. Money and labour are substitutes both in the short-run and in the long-run. The elasticity is significantly different from zero in the restricted short-run. However, it is not different from zero when investment effects are added in the short-run. The confidence limits for this elasticity are presented in Table 5.8. Table 5.6 also indicates that labour and capital are substitutes and money and capital are substitutes.

The results of the dynamic model are different from the two previous models. For example, capital and money were found to be complement in the temporary equilibrium model while they are substitutes in the dynamic model. Labour and money are subtitutes in the three models, both in the short-run and in the long-run. However, the magnitude of the long-run cross price elasticity of labour demand with respect to opportunity cost of money in the dynamic model is subtantially larger than the full equilibrium model result. Therefore, ignoring the role of cost of adjustment in Chapters 3 and 4 has contaminated estimates of the price elasticities.

TABLE 5.6

Cross Price Elasticities
(Cost Minimization)

	FULL EQUILIBRIUM MODEL	TEMPORARY EQUILIBRIUM MODEL		DYNAMIC DISEQUILIBRIUM MODEL		
	LR	SR	LR	RS	SR	LR
LABOUR/MONEY	0.00094	0.00158	0.0079	0.0012	0.0006	0.01280
LABOUR/CAPITAL	0.01953	–	0.0213	–	–	0.0340
CAPITAL/MONEY	0.00111	–	–0.2828	–	–	0.0624

We also estimated the earlier version of the dynamic model, [5.36]-[5.38], employing the Morrison and Berndt specification of the variable cost function. In general the estimated elasticites are different at their mean levels. For example, the own price elasticity of money demand is -0.186, for our specification, in the restricted short-run compared to -0.146. Our estimate of own price elasticity of labour in the long-run is -0.1326 in comparision to -0.1899 for Berndt and Morrison specification. The own price of elasticity for capital is -0.1948 compared to -0.384 in our model. Finally, our result indicate that money and labour are substitute in the long-run while they are complement in the Berndt and Morrison case. These large differences suggest that the arbitrary normalization rule involved in the Berndt and Morrison specification are critical. It is hoped that there will be quite general interest in our suggested alternative functional form.

We now turn to the discussion of the supply side effects of changes in the rate of interest on the demand for labour. The cost minimizing interest elasticity of labour demand is significantly different from zero (row #4, Table 5.8). In the restricted short-run, when these supply side effects arise only due to the fact that money enters into the production function, the effect is positive. However, when the indirect effects of changes in the rate of interest on investment are included, the elasticity is negative and significantly different from zero. Therefore, the role of costs of adjustment is statistically significant in determining these supply side effects in the cost minimizing framework.

In order to deduce the macroeconomic implications we calculate profit maximizing interest price elasticity of labour demand, Π_{nr}, using [5.60]. In Chapter 1, we estimated the implied profit maximizing interest price elasticity of labour demand from the estimates of the Short[1980] study and it was -0.14. The mean result of the dynamic model, -0.0105, is much lower than Short's estimates and the profit maximizing interest elasticity is, in fact, not significantly different from zero (row # 6, Table 5.8). Therefore, the profit maximizing elasticity indicates that the supply side effects are economically insignificant. However, the cost minimizing results do indicate that the interest price elasticity of labour demand is statistically different from zero.

5.5 CONCLUDING REMARKS

In summarizing the results of the dynamic model we note: first, the estimates of price elasticities of the dynamic model are different compared to the full

TABLE 5.7

Effects of Changes in Interest Rate on Labour Demand

	FULL EQUILIBRIUM MODEL	TEMORARY EQUILIBRIUM MODEL		DYNAMIC DISEQUILIBRIUM MODEL		
	LR	SR	LR	RS	SR	LR
η_{nr}	0.00094	0.00158	0.0221	0.00130	−0.0158	0.0157
Π_{nr}	−0.1475	−0.0010	--	−0.0105	−0.0105	−0.0033

Note: η_{nr} is cost minimizing cross price elasticity of labour demand with respect to rate of interest. Π_{nr} is the profit maximizing cross price elasticities.

TABLE 5.8

Confidence Intervals for Cross Price Elasticities (Dynamic Model, RS and SR)

	ELASTICITIES				
	MEAN ELASTICITY FROM 900 DRAWS	LOWER TAIL		UPPER TAIL	
		2.5%	5%	5%	2.5%
$(\eta_{nrm})^{RS}$	+0.001372	+0.000949	+0.0009998	+0.001733	+0.00179
$(\eta_{nr})^{RS}$	+0.001307	+0.000905	+0.000952	+0.001651	+0.0090
$(\eta_{nrm})^{SR}$	+0.000672	−0.002221	−0.001205	+0.002139	+0.00288
$(\eta_{nr})^{SR}$	−0.01581	−0.059190	−0.034410	−0.004617	−0.00334
$(\Pi_{nr})^{RS}$	−0.002792	−0.02615	−0.017756	+0.012564	+0.01501
$(\Pi_{nr})^{SR}$	−0.01050	−0.055490	−0.04395	+0.006501	+0.07289

Note: η_{nrm} represent the cost minimizing cross price elasticity of labour with respect to the opportunity cost of money, r_m

equilibrium model and the temporary equilibrium model. Second, we rejected the temporary equilibrium model based on the particular quadratic functional form of the cost function. Third, we found that the role of costs of adjustment is statistically significant in firms' optimization problem. Fourth, the result of the dynamic model indicates that the cost minimizing interest price elasticity of labour demand is statistically significant. However, the implied profit maximizing interest price elasticity of labour demand is statistically insignificant. Finally, in the dynamic model Durbin-Watson statistics increased substantially, after making the adjustment for serial correlation, relative to the other models in Chapters 3 and 4.

FOOTNOTES:

Chapter 5

[1] Lucas[1967, p.324] gives two examples to justify the reduction in output as a result of a change in capital stock. One of the example includes the case where workers are trained for the newly installed capital and it takes some time before they can be fully productive.

[2] Compare to equations [25] and [26] on page 348 in Berndt and Morrison[1981], and [27], [28] and [29] on page 13 in Denny et al [1980].

[3] In evaluating the interest cross price elasticities, adjustment has been made for the fact that rental price of capital, V, is also a function of rate of interest.

[4] The critical values of the Durbin-Statistic are not appropriate for the system of equations in general. In the dynamic model, the applicability of these critical values is further restricted because in the net investment equation [5.49c] K_{-1} appears on the right-hand side of the equation and $\Delta K=K-K_{-1}$ (see Johnston[1972], p.262).

[5] In most of the models estimated for the US economy, the demand for labour was inelastic(see for example, Denny, Fuss and Waverman [1980], Simos [1981], Pindyck and Rotemberg [1983]).

CHAPTER 6

SUMMARY AND CONCLUSIONS

In the introduction (Chapter 1) we emphasized both the need for re-examining the role of money in firms' optimization problem by introducing the fixity of capital in the short-run, and the importance of supply side effects of changes in the rate of interest on the level of employment that arise due to the role of money in the firms' production process. Our empirical work has covered new ground in two principal respects. First, we estimated firms' demand for money in a model in which capital is exogenous in the short-run and costs of adjustment of capital are ignored. The existing empirical studies in this area are restricted as they ignore the fixity of capital.

Second, we estimated a dynamic model in which non-linear costs of adjustment of capital are incorporated explicitly into the firms' optimization. In this model, changes in the rate of interest can affect firms' demand for labour through two channels. In the first case, changes in the rate of interest directly affect demand for labour since real money balances enter the production function as a factor of production. Secondly, there are indirect effects of changes in the net investment, as the rate of interest changes, on the demand for labour. These indirect effects arise because in the dynamic model, with internal costs of adjustment, the demand for labour is also a function of the rate of investment.

In Chapter 3, we ignored the fixity of capital and estimated a full equilibrium model, employing a translog cost function. Dennis and Smith [1978] have also estimated a similar model. However, our empirical model differs from the Dennis and Smith in two ways. Firstly, we have included the cost function along with the cost share equations while estimating the parameters of the cost function. Secondly, we employed a different data base for our empirical analysis.

The calculated elasticity of the demand for money with respect to its opportunity cost is -0.1777, which is consistent with other estimates in the context of full equilibrium models. Following the empirical work in this area, we also tested for the separability of real money balances from other factors of production. We rejected the hypothesis of separability with 99% confidence. This result is also consistent with the findings of others.

In Chapter 4, we estimated a temporary equilibrium model in which capital is assumed to be constant in the short-run. We estimated a translog variable cost function together with the cost share equations. We estimated both short-run and long-run price elasticities. The comparison of short-run and long-run elasticities indicates that there is substantial difference between them. For example, the own price elasticity of labour is −0.0026 in the short-run compared to −0.059 in the long-run. Therefore, the assumption of instantaneous adjustment of capital, which is not supported by data, significantly changes the estimated elasticities.

The implied profit maximizing interest price elasticity of labour demand, Π_{nr}, in the short-run is −0.001. In Chapter 1 we calculated Π_{nr} using Short's estimates of Cobb-Douglas production function (Π_{nr}=−0.14). The result here indicates that the interest sensitivity of the demand for labour, when estimated directly by assuming fixity of capital stocks, is almost zero.

Finally, in Chapter 5, we estimated a dynamic model in which not only is capital exogenous in the short-run but also the model assumes that there are non-linear costs of adjustment associated with the changes in the level of capital stocks in the short-run. Following the empirical studies on dynamic models, we employed a quadratic functional form for the variable cost function. However, we noted that there were some problems with this functional form. For example, the derived demand for the normalized factor depends on different variables than the demand for all other variable factors[1]. Therefore we specified an alternative parameterization of the cost function where all of the factor demand functions have an identical functional form.

In the dynamic model we calculated three types of elasticities. In the first case we estimated price elasticities that are evaluated while keeping capital and net investment constant (restricted short-run). Secondly, we estimated price elasticities in which indirect effects of changes in the net investment on the variable factors are also added in the short-run. Finally, we estimated the long-run price elasticities. These elasticities are evaluated at the point where capital has fully adjusted to its desired level and net investment is zero.

We have also generated confidence intervals for the short-run price elasticities, both in the restricted and unrestricted short-run. The own price elasticities for the demand for labour and real money balances are negative and significantly different from zero at the 95% confidence level in the restricted short-run. The long-run own price elasticity for labour and real money balances are also negative. The own price elasticities in the short-run are significantly different from the long-run responses. Furthermore the short-run elasticities are substantially different than the responses of the full equilibrium model in Chapter 3.

The cost minimizing interest rate elasticities of labour demand were significantly different from zero both in the restricted and unrestricted short-run. In the restricted short-run when net investment is held constant the elasticity is positive and significantly different from zero. However, in the short-run, with investment effects included, the cross price elasticity is negative and significantly different from zero at the 95% confidence level. The mean value of the cross price elasticity, η_{nr}, is −0.0158. On the other hand the implied profit maximizing elasticity, Π_{nr}, is not significantly different from zero at 95%. The 95% confidence interval ranged between −0.055 to +0.0028.

In Chapter 1 we also motivated our work in this thesis by noting the importance of supply side effects of monetary policy on employment. Re-examination of these effects, in the dynamic framework, indicates that these effects are not as important from the macroeconomic policy point of view as the existing parameter values suggested. However, from the statistical point of view the results clearly reveal that money belongs in the production function, and that the full equilibrium model is clearly contaminated by ignoring the fixity of capital and the costs of adjustment of capital.

Comparison of the three empirical models, on statistical grounds, indicates that the full equilibrium model did not fit the data well. The Durbin-Watson statistics were relatively low even after making first order serial correlation adjustments in the errors. The significance of the estimated parameters was decreased considerably after adjusting for the serial correlation. Furthermore, the own price elasticity of real money balances was not significantly different from zero. The results of the temporary equilibrium model were relatively better in these respects. However, we statistically tested the restrictions imposed by the temporary equilibrium model for the quadratic functional form of the variable cost function in Chapter 5[2]. These restrictions were rejected at the 99% confidence level.

In the dynamic model Durbin-Watson statistics increased considerably, after making adjustments for serial correlation. However, we must note here that the critical ranges of the Durbin-Watson statistics are not valid in simultaneous equations models.

Having summarized our principle findings, we now discuss some of the ways in which the empirical work of this thesis could be improved and extended. First, the dynamic model in Chapter 5 has been estimated by assuming static expectations for the output and factor prices. The model can be extended to incorporate rational expectations. Pindyck and Rotemberg[1983] have estimated such a dynamic model for energy. This framework can be employed for a model which also includes real money balances as factor of production.

Another way to extend the model is to apply it to disaggregated data for the manufacturing industries of the United States. It would be interesting to see how the demand for labour responds to changes in the rate of interest across different manufacturing industries.

The model can also be extended by including energy as a factor input in the production function. The studies on resource substitution ignore the mole of real money balances in the production process. It will be interesting to analyse different substitution possibilities between factors of production in this extended framework.

FOOTNOTES:

CHAPTER 6

[1] See for example Morrison and Berndt[1981].

[2] In Chapter 4 we employed a translog cost function in order to compare results directly with the full equilibrium model in Chapter 3. However, a functional form with quadratic adjustment cost was needed in order to use the Treadway's approximation to derive the net investment equation. Therefore a translog cost function is not employed in Chapter 5.

APPENDIX A

In this appendix we list the data used in this study. For sources and construction of the data see Chapter 3 , pp.55–58.

TABLE A.1

DATA ON PRICES
(Non-Financial Corporations, United States)

YEAR	W	V	r_m	r	p
1948	1.50	0.063170	0.00861	0.0144	0.5980
1949	1.56	0.065137	0.00904	0.0149	0.6070
1950	1.63	0.066708	0.00890	0.0145	0.6140
1951	1.75	0.080457	0.01395	0.0216	0.6460
1952	1.86	0.084324	0.01544	0.0233	0.6630
1953	1.97	0.087852	0.01683	0.0252	0.6680
1954	2.04	0.076496	0.01069	0.0158	0.6770
1955	2.10	0.086014	0.01499	0.0218	0.6880
1956	2.23	0.106538	0.02366	0.0331	0.7150
1957	2.35	0.118009	0.02811	0.0381	0.7380
1958	2.44	0.098016	0.01854	0.0246	0.7540
1959	2.53	0.121583	0.03037	0.0397	0.7650
1960	2.62	0.120277	0.02976	0.0385	0.7730
1961	2.69	0.106634	0.02304	0.0297	0.7760
1962	2.78	0.103813	0.02536	0.0326	0.7780
1963	2.87	0.108993	0.02761	0.0355	0.7780
1964	2.99	0.123621	0.03120	0.0397	0.7860
1965	3.08	0.131712	0.03473	0.0438	0.7930
1966	3.22	0.155025	0.04484	0.0555	0.8080
1967	3.39	0.152520	0.04233	0.0510	0.8300
1968	3.63	0.173212	0.05056	0.0590	0.8570
1969	3.86	0.219739	0.06984	0.0783	0.8920
1970	4.12	0.228173	0.07202	0.0772	0.9330
1971	4.39	0.186059	0.04972	0.0511	0.9730

Note: W is the wage rate; V is the rental price of capital; r_m is the opportunity cost of real money balances; r is the interest rate; p is the output price index in 1972 dollars.

TABLE A.2

Data on Factor Inputs and Output
(Non-Financial Corporations, United States)

YEAR	N	m	K	Q
1948	55.6	42.3077	243.284	229.097
1949	51.9	43.6573	252.007	219.605
1950	54.7	45.7655	269.735	247.068
1951	58.9	46.4396	277.303	269.814
1952	59.6	46.4555	287.923	274.510
1953	61.2	46.5569	299.841	291.467
1954	57.8	49.3353	311.199	283.013
1955	61.2	50.2907	331.944	314.390
1956	63.0	48.6713	349.634	323.357
1957	62.3	47.2900	362.341	327.778
1958	58.8	49.6021	374.435	312.997
1959	61.9	47.4510	382.123	344.706
1960	62.4	48.1242	391.516	353.299
1961	61.4	52.9639	401.955	358.763
1962	63.5	56.1697	444.328	389.203
1963	64.4	59.7686	455.982	411.311
1964	66.1	60.1781	446.703	440.204
1965	69.1	62.9256	476.965	476.166
1966	72.5	61.0149	509.055	511.139
1967	72.7	65.1807	539.390	519.036
1968	74.1	67.6779	572.594	548.308
1969	76.0	61.5471	605.063	562.780
1970	74.1	60.9861	633.370	553.269
1971	72.7	64.7482	649.635	568.345

Note: N is the billions of labour hours; m are the real money balances in billions of 1972 dollars; K is the capital stock in billions of 1972 dollars; Q is the output in billions of 1972 dollars.

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