ADAPTIVE CONTROL OF MULTIVARIABLE SYSTEMS VIA POLE ASSIGNMENT

by

FRANCIS K. OMANI, B.Sc., M.Sc.(Eng.), D.I.C.

A Thesis
Submitted to the School of Graduate Studies
in Partial Fulfilment of the Requirements
for the Degree
Doctor of Philosophy

McMaster University
 December 1985

ADAPTIVE CONTROL OF MULTIVARIABLE SYSTEMS VIA POLE ASSIGNMENT

TO AGNES AND YVONNE

DOCTOR OF PHILOSOPHY (1985)
(Electrical Engineering)

McMASTER UNIVERSITY
Hamilton, Ontario

TITLE: Adaptive Control of Multivariable Systems via Pole Assignment

AUTHOR: Francis K. Omani B.Sc. (University of Wales)
M.Sc. Eng. (University of London)
D.I.C. (Imperial College)

SUPERVISOR: N.K. Sinha
Professor of Electrical Engineering
B.Sc. Eng. (Banaras Hindu University)
Ph.D. (University of Manchester)

NUMBER OF PAGES: xi, 160

ABSTRACT

The adaptive control of linear discrete time multivariable systems is considered. A unifying survey of a number of adaptive control strategies is presented. The various algorithms are shown to be special cases of a more general algorithm. The state space design of self-tuning controllers is considered in detail. Two new algorithms for state space pole assignment self-tuning control are proposed. The first algorithm follows an explicit approach, thus a modification of the bootstrap estimator was used for joint parameter and state estimation of an innovations model. The resulting self-tuning controller is more efficient computationally than the methods based on block canonical forms since a minimal realization can be adopted. The second algorithm may be regarded as an implicit pole assignment controller. The recursive prediction error algorithm is used for joint parameter and state estimation in the controller canonical form. The main contribution of this approach is that on-line computation of transformation matrices is avoided. The subsequent computation of controller parameters is trivial, and the resulting self-tuning controller is robust to over-parameterization. To demonstrate a practical application, the second algorithm was used to design a robust autopilot for a simulated nonlinear model

of a Royal Navy frigate subjected to sea disturbances. The autopilot was found to perform well for both the course keeping and course changing modes.

ACKNOWLEDGEMENTS

The author wishes to express his sincere appreciation of the expert guidance and supervision given by Dr. N.K. Sinha. He also thanks Dr. P.A. Taylor and Dr. S. Haykin members of his supervisory committee for their useful suggestions.

Thanks are also due to Dr. J.F. MacGregor and Dr. O. Vidal for helpful disscusions on the topic of adaptive control.

The author would also like to acknowledge the stimulating academic interaction with his coleagues S. Puthenpura, A. Karnik, J. Raol, L. Liu., and last but not the least the moral support of A. Jimoh.

Financial support provided by the Department of Electrical and Computer Engineering at McMaster University is gratefully acknowledged.

TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER 1

1. INTRODUCTION

1.1 Background

The design of efficient controllers for physical systems requires an accurate a priori knowledge of the system dynamics. In practice however this information is often not available since many systems are too complex to analyze. Also the system may be nonlinear and time varying, in which case the linearized system parameters will alter with time and set point changes. Such problems may be tackled by designing controllers which can adapt to any detectable changes in the system dynamics.

The subject of adaptive control has received a great deal of attention since the 1950's. In a recent survey paper Astrom [10], noted that over 1500 papers have been published on the topic.

Early work on adaptive control concentrated on model reference controllers (MRCs). Such controllers were originaly designed for continous time models, based on deterministic theory [29,57,70,74,81,92]. Later papers [28,58,60,68], have applied model reference techniques to discrete time systems. The techniques have been extended to improve their performance in the stochastic environment in [25,26,58,59,61].

Since the early 1970's however, the availability of efficient micro-computers has made self-tuning control the more popular method for adaptive control. The two methods are by no means unrelated. Several unification papers [2,3,4,26,28,29,35,36,58,59,60,61,67,68], have derived general adaptive control algorithms for which the various discrete time model reference and self-tuning control algorithms are special cases.

Self-tuning control may be divided into two distinct steps. Firstly, a recursive estimation algorithm is used to estimate the unknown system parameters. At each sampling interval the current estimates are used to calculate the controller parameters. The assumption that the estimated parameters are equal to the system parameters leads to 'certainty equivalence' control [9,16].

The term 'self-tuning' applies if the resulting controller parameters would converge to those values that would have been obtained if the system parameters were known exactly.

The minimum variance controller of Astrom [9,13] would lead to unbounded control for inverse unstable (non-minimum phase) systems. This is a severe restriction, since such systems often occur in sampled data systems even when the underlying continous time system is inverse stable

(Astrom et al. [12]).

Clarke and Gawthrope [22] extended the minimum variance controller by weighting the control variable, thus making the self-tuning controller (STC) more widely applicable. Their technique however is not foolproof. The closed loop system stability would depend on a diophantine equation relating the weighting polynomials to the unknown system parameters. Since the parameter adaptation is done implicitly it is not known a priori how to choose the weighting polynomials to give satisfactory closed loop perfomance.

Other techniques for dealing with non-minimum phase systems include linear quadratic gaussian (LQG) control and pole assignment. LQG control involves the solution of a matrix Riccati equation or alternatively a spectral factorization algorithm, both of which require iterative solutions hence are unsuitable for on-line control, due to the limitation on computing time imposed by the sampling intervals. Furthermore, the choice of weighting matrices is problem dependent.

Recent work on adaptive control has thus been focused on pole assignment objectives [14,30,31,32,41,100,101]. This approach has two main attractions. Firstly, it gives a simple solution to the control of non-minimum phase systems. Secondly, desirable closed loop performance characteristics are more easily specfied via pole configuration than by the selection of weighting polynomials such as proposed in [22], or of weighting matrices for LQG control.

Early work on pole assignment self-tuning control concentrated on systems represented by auto-regressive moving average with auxilary input (ARMAX) models. The techniques involve on-line solution of polynomial equations. The method was extended to multi-input/multi-output (MIMO) systems by Prager and Wellstead [84]. However, the computational complexity of on-line solution of polynomial equations has recently led to interest in state space based methods.

State space self-tuning pole assignment controllers for single-input/single-output (SISO) systems have been proposed by Tsay and Shieh [97] and Warwick [98,99]. The attraction of state space methods is that the SISO case can be easily extended to MIMO systems. Also the main computational load in controller design is in the inversion of a controllability matrix which has a much smaller dimension than the Sylvester matrix [50], associated with the solution of polynomial equations.

The techniques have been extended to MIMO systems by Benzanson and Harris [18] and Shieh et al. [88]. Hesketh [43], has proposed a MIMO pole assignment STC which uses input/output data for feedback instead of state estimates.

1.2 Contents and contribution of this thesis.

The main effort of the research reported in this thesis is the investigation of the relative advantages to be gained in adaptive control, by the careful selection of model representation, identification algorithm and control law.

It was found that two joint state and parameter estimation algorithms when combined with pole assignment control result in more efficient algorithms for state space STCs than those reported in [18,43,84,88,97,98,99].

The first method employs a modification of the original bootstrap estimator (BSE) proposed in [33,34,85]. The original algorithm would not converge in the presence of significant disturbance noise [1,93,94]. A modification was suggested, sufficient conditions for convergence were derived and the technique extended to MIMO systems. The main contribution of the state space STC based on the modified BSE is that the canonical form adopted is a minimal realization hence results in smaller dimensional system matrices than the block canonical forms adopted in [18,43,88,89,90], thus reducing the computational effort in identification and subsequent controller design.

The need for on-line computation of transformation matrices is the major drawback the state space STCs from two points of view. Firstly, the techniques involve on-line matrix inversion which is undesirable from a computational point of view. Secondly, since the inverse of a controllability matrix is required, the method cannot be applied to over-parameterized systems. This is a severe restriction on the practical usefulness of adaptive pole placement, since very often in adaptive control, the exact model order is unknown or may be time varying.

To overcome the problem of on-line computation of transformation matrices, the flexibility on model representation offered by recursive prediction error (RPE) methods was employed to derive a joint state and parameter estimation algorithm in the controller canonical form. Since in this canonical form the computation of the feedback gains becomes trivial the resulting algorithm may be regarded with some justification as an implicit adaptive (direct) pole placement algorithm. The overall algorithm is robust to over-parameterization and more efficient computationally when applied to MIMO systems than the algorithms proposed in [18,43,84,88].

To demonstrate a practical application of the state space STC, an adaptive autopilot was designed for a simulated nonlinear model of a Royal Navy frigate, subjected to sea disturbances. The autopilot consisted of an RPE algorithm for joint state and parameter estimation followed by nonlinear state feedback for pole assignment and nonlinear compensation. The proposed autopilot thus overcomes some of the problems encountered in designing adaptive controllers for ship manoevering, such as encountered in [5,6,7,8,11,51,72,73,86].

Chapter 2 is of a tutorial nature hence in that chapter the general theory of adaptive control is discussed from a unifying point of view. Thus the discrete time model reference controller and the various self-tuning controllers are shown to be special cases of a more general algorithm. The polynomial approach to pole shifting and its extension to the MIMO case are discussed in this section.

Chapter 3 introduces state space self-tuning pole assignment controllers, and the various proposed extensions to the MIMO case. An improved algorithm based on the modified BSE is introduced and simulation results are presented.

In chapter 4 the general theory of recursive prediction error estimation is discussed. This technique was used to design a state space STC based on pole assignment for SISO systems. Simulation results are presented to show the effectiveness of the proposed algorithm on non-minimum phase and over-parameterized systems. The techniques developed in chapter 4 are extended to the MIMO case in chapter 5 and simulation results presented.

Chapter 6 discusses a practical application to the design of a ship autopilot with simulation results. Conclusions and suggestions for further work in the general area of adaptive pole placement are discussed in chapter 7.

CHAPTER 2

2. UNIFICATION OF MODEL REFERENCE AND STC ALGORITHMS

2.1 Introduction

The pioneering works on STCs were done by Peterka [82] and Astrom and Wittenmark [13]. The methods combined a recursive least squares (RLS) algorithm with a minimum variance controller. Since then STCs based on other design criteria such as 'weighted control action' (Clarke and Gawthrope [22]) and pole placement techniques (Wellstead et. al. [100,101]) have been reported.

Simultaneously, much work has been done to improve the performance of discrete time MRCs in the stochastic environment [25,26,58,59]. Thus the resulting algorithms for stochastic MRCs may infact be interpreted as pole-zero placement self-tuning controllers. This has led to several attempts to unify the two approaches by deriving general algorithms for which the various STC and MRC algorithms are special cases.

2.2 A Generalized Algorithm For Controller Design.

Consider a SISO system represented by the ARMAX model

$$A'(z^{-1})y_t = z^{-kd}B'(z^{-1})u_t + C'(z^{-1})e_t \qquad (2.1a)$$

where y_t, u_t are the output and input variables respectively, and t is the discrete time variable. e_t is a Gaussian white noise sequence with mean $E[e_t]=0$ and variance $E[e_t^2]=\gamma$. Also $E[.]$ denotes the expected value. $kd \geq 1$, denotes the integral periods of delay in the system. $A'(z^{-1}), B'(z^{-1})$ and $C'(z^{-1})$ are polynomials in the backward shift operator z^{-1}, and are given by

$$A'(z^{-1}) = 1 + a_1' z^{-1} + \ldots a_{na}' z^{-na} \qquad (2.1b)$$

$$B'(z^{-1}) = b_0' + b_1' z^{-1} + \ldots b_{nb}' z^{-nb} \qquad (2.1c)$$

$$C'(z^{-1}) = 1 + c_1' z^{-1} + \ldots c_{nc}' z^{-nc} \qquad (2.1d)$$

Then a general controller may be designed for 2.1 by considering the optimization problem

$$\min_u \left\{ y_{t+kd} - \frac{Bm(z^{-1})}{Am(z^{-1})} u_{r,t} + \frac{Qm(z^{-1})}{Am(z^{-1})} u_t \right\}^2 \qquad (2.2)$$

Subject to (2.1)

Where $Bm(z^{-1})$, $Am(z^{-1})$ are polynomials defining the dynamics of a desired reference model. $u_{r,t}$ is a given input sequence and $Qm(z^{-1})$ is a weighting polynomial.

2.2.1 <u>Model reference control.</u>

An algorithm for discrete model following control may be regarded as a special case of (2.2) with $Qm(z^{-1}) = 0$. The optimisation problem may then be solved in two steps.

(i) Predict the value of y_{t+kd} at time t

$$\hat{y}_{t+kd|t} = \frac{G'(z^{-1})}{C'(z^{-1})} y_t + \frac{B'(z^{-1})F'(z^{-1})}{C'(z^{-1})} u_t \qquad (2.3)$$

where $G'(z^{-1})$ and $F'(z^{-1})$ are given by the equation

$$C'(z^{-1}) = A'(z^{-1})F'(z^{-1}) + z^{-kd}G'(z^{-1}) \qquad (2.4)$$

The prediction error is then given by

$$\varepsilon_{t+kd} = F'(z^{-1})e_{t+kd} \qquad (2.5)$$

(ii) Set $$\hat{y}_{t+kd} = \frac{Bm(z^{-1})}{Am(z^{-1})} u_{r,t} \qquad (2.6)$$

Thus the closed loop system is given by

$$y_{t+kd} = \frac{Bm(z^{-1})}{Am(z^{-1})} u_{r,t} + F'(z^{-1})e_{t+kd} \qquad (2.7)$$

the control law to achieve this is given by

$$u_t = \frac{Bm(z^{-1})C'(z^{-1})}{Am(z^{-1})B'(z^{-1})F'(z^{-1})} u_{r,t} - \frac{G'(z^{-1})}{B'(z^{-1})F'(z^{-1})} y_t . \qquad (2.8)$$

2.2.2 Minimum variance control.

Minimum variace control is achieved by selecting

$$Bm(z^{-1}) = Am(z^{-1}) = 1$$

The closed loop system then reduces to

$$y_{t+kd} = u_{r,t} + F'(z^{-1})e_{t+kd} \qquad (2.9)$$

and the resulting control law is given by

$$u_t = -\frac{G'(z^{-1})}{B'(z^{-1})F'(z^{-1})} y_t + \frac{C'(z^{-1})}{B'(z^{-1})F'(z^{-1})} u_{r,t} \qquad (2.10)$$

Thus a condition for bounded control is that $B'(z)$ must have all roots inside the unit circle. The same restriction applies to the model reference control in view of (2.8).

2.3 Pole-zero placement control.

To overcome the restriction to minimum phase systems, it is essential to prevent the cancellation of unstable zeros. Astrom and Wittenmark [14] proposed a pole-zero placement controller which retains the unstable zeros. The technique is a special case of (2.2) with $Qm(z^{-1})=0$, $C'(z^{-1})=1$, and $e_t \equiv 0$.

The reference model is replaced by

$$y'_{r,t} = \frac{B^-(z^{-1})Bm(z^{-1})}{Am(z^{-1})}u_{r,t} \qquad (2.11)$$

where $B^-(z^{-1})$ is obtained by factorizing $B'(z^{-1})$ as

$$B'(z^{-1}) = B^+(z^{-1})B^-(z^{-1})$$

The polynomial $B^-(z)$ has zeros outside the restricted stability area hence may not be cancelled.

The above algorithm is essentially a servo controller and would not perform well for regulation when applied to a non-minimum phase system, since for $e_t \neq 0$ the transfer function relating u_t to e_t is

$$u_t = -\frac{G'(z^{-1})}{B'(z^{-1})}e_t \qquad (2.12)$$

which would give ubounded control if $B'(z)$ has zeros outside the unit circle.

The extended minimum variance controller proposed by Clarke and Gawthrope [21] considers the entire cost function (2.2). Using the definitions

$$\frac{Am(z^{-1})C'(z^{-1})}{A'(z^{-1})} = F(z^{-1}) + \frac{z^{-kd}G(z^{-1})}{A'(z^{-1})} \tag{2.13}$$

$$H(z^{-1}) = B'(z^{-1})F(z^{-1}) + Qm(z^{-1})C'(z^{-1}) \tag{2.14}$$

$$E'(z^{-1}) = -C'(z^{-1})B'(z^{-1}) \tag{2.15}$$

the control law to minimize (2.2) is then given by

$$H(z^{-1})u_t + G(z^{-1})y_t + E'(z^{-1})u_{r,t} = 0 \tag{2.16}$$

giving the closed system

$$y_t = \frac{z^{-kd}B'(z^{-1})Bm(z^{-1})}{B'(z^{-1})Am(z^{-1}) + A'(z^{-1})Qm(z^{-1})}u_{r,t} + \frac{H(z^{-1})}{B'(z^{-1})Am(z^{-1}) + A(z^{-1})Qm(z^{-1})}e_t \tag{2.17}$$

The closed loop stability thus depends on the zeros of

$$B'(z^{-1})Am(z^{-1}) + A'(z^{-1})Qm(z^{-1}) = T(z^{-1}) \tag{2.18}$$

thus all the open loop zeros are retained provided that $T(z)$ and $B'(z)$ have no common factors.

The technique is applicable to both regulatory and servo control. The method is however not foolproof. Since adaptation is done implicitly it is not known how to choose the weighting polynomials $Am(z^{-1})$ and $Qm(z^{-1})$ to give satisfactory performance. Alternatively one could use some approximate (estimated) values of $A'(z^{-1})$ and $B'(z^{-1})$ to update $Am(z^{-1})$ and $Qm(z^{-1})$ for a given $T(z^{-1})$. Such an algorithm was proposed in [3,4].

The pole placement controller of Wellstead et al. [97,98] overcomes the above problem in that $T(z^{-1})$ is specified by the designer and pole placement is done explicitly. The pole shifting control law is given by

$$G^*(z^{-1})y_t + F^*(z^{-1})u_t + Bm(z^{-1})u_{r,t} = 0 \qquad (2.19)$$

where $F^*(z^{-1})$ and $G^*(z^{-1})$ satisfy the Diophantine relationship

$$A'(z^{-1})F^*(z^{-1}) + B'(z^{-1})G^*(z^{-1}) = C'(z^{-1})T(z^{-1}) \qquad (2.20)$$

$$nt \leqslant na + nb + kd - nc$$

$$ng = na - 1$$

$$nf = nb + kd - 1$$

where nt, nf and ng are the orders of $T(z^{-1})$, $F^*(z^{-1})$ and $G^*(z^{-1})$ respectively. The resulting closed loop system is then given by

$$y_t = \frac{F^*(z^{-1})}{T(z^{-1})}e_t + \frac{B'(z^{-1})}{C'(z^{-1})T(z^{-1})}u_{r,t} \qquad (2.21)$$

2.4 Multivariable pole placement control.

The controller design algorithms described in sections 2.2 and 2.3 have been extended to multivariable systems. Borrison [20] and Keviczky et al. [52] have extended the minimum variance techniques of Astrom to multivariable systems. Koivo [53] and Favier [36] have extended the techniques of Clarke and Gawthrope [22] to multivariable systems. Prager and Wellstead [84] have extended their pole placement controller to multivariable systems.

General algorithms similar to the SISO cases of 2.2 may be derived for the MIMO cases (Favier et. al. [36]). Of immediate interest however is the multivariable pole placement controller.

Consider the MIMO system represented by the ARMAX model

$$[A(z^{-1})]Y_t = [B(z^{-1})]U_t + [C(z^{-1})]\ e_t \tag{2.22a}$$

where

$$[A(z^{-1})] = I + A_1 z^{-1} + \ldots A_{na} z^{-na} \tag{2.22b}$$

$$[B(z^{-1})] = z^{-kd}(B_0 + B_1 z^{-1} + \ldots B_{nb} z^{-nb}) \tag{2.22c}$$

$$[C(z^{-1})] = I + C_1 z^{-1} + \ldots C_{nc} z^{-nc} \tag{2.22d}$$

also I is an mxm unit matrix and Y_t, U_t are m dimesional output and input vectors respectively. A_i, B_i and C_i are mxm dimensional matrices which determine the dynamics of the system. For simplicity it is assumed that the no. of inputs = the no. of outputs = m, although the various algorithms may be extended to the more general case. e_t is an m dimensional vector gaussian white noise with mean $E[e_t]=0$, with covariance $E[e_t e_t]=\Lambda$, where Λ is an mxm matrix.

The explicit pole placement regulator may be extended to MIMO systems ([81,102]) as follows. Select the control law

$$U_t = -[G(z^{-1})][F(z^{-1})]^{-1}Y_t \tag{2.23}$$

the controller parameters are obtained by solving

$$[A(z^{-1})][F(z^{-1})] + [B(z^{-1})][G(z^{-1})] = [C(z^{-1})][T(z^{-1})] \tag{2.24}$$

where $[T(z^{-1})]$ is the desired closed loop mode polynomial matrix. The closed loop system is thus

$$Y_t = [F(z^{-1})][T(z^{-1})]^{-1}e_t \tag{2.25}$$

The main computational load of the method is the solution of (2.24) which may be written in matrix form as

$$\left[\begin{array}{cccccc}
I & & & 0 & & \\
A_1 & \ddots & & \vdots\,\}kd & & \\
\vdots & & \ddots & 0 & & \\
 & & & -B_0 & & \\
\vdots & & I & \vdots & \ddots & \\
 & & A_1 & \vdots & & \\
A_{na} & & \vdots & & & -B_0 \\
 & \ddots & & -B_{nb} & & \vdots \\
 & & \vdots & & \ddots & \vdots \\
 & & A_{na} & & & -B_{nb}
\end{array}\right]
\left[\begin{array}{c}
F_1 \\ \vdots \\ F_{nf} \\ G_0 \\ \vdots \\ G_{ng}
\end{array}\right]
=
\left[\begin{array}{cc}
S_1 & -\ A_1 \\
\vdots & \vdots \\
\cdot & -\ A_{na} \\
 & 0 \\
\vdots & \vdots \\
S_{ns} & 0
\end{array}\right] \tag{2.26a}$$

$(nb+kd) \times m \qquad na \times m$

where S_i are the coefficients of $[S(z^{-1})]$ given by

$$[S(z^{-1})] = [C(z^{-1})][T(z^{-1})] \tag{2.26b}$$

$$ns = \deg\{\ [S(z^{-1})]\ \}$$

As shown by Prager and Wellstead [81] the implementation of the control law (2.23) would require the inversion of the polynomial matrix $[F(z^{-1})]$. The alternative to this would be to compute the pseudo-commutative polynomial matrices

$$[F(z^{-1})][G(z^{-1})] = [G(z^{-1})][\tilde{F}(z^{-1})] \qquad (2.26c)$$

The control law may then be implemented as

$$[F(z^{-1})]U_t = [G(z^{-1})]Y_t \qquad (2.26d)$$

The main computational load in the controller design thus involves the inversion of an $[m(na+nb+kd)]x[m(na+nb+kd)]$ matrix for the solution of (2.26a) and also the inversion of an $[m(nf+ng)]x[m(nf+ng)]$ matrix in the solution of (2.26c). The solution of (2.26c) may be side stepped by the the technique proposed by Gong Wei Bo [35,36] which implements (2.23) by introducing an intermediate vector variable.

Apart from the large computational load, the explicit pole placement algorithm requires knowledge of the exact model order since over-parameterization would mean that $[A(z^{-1})]$ and $[B(z^{-1})]$ have common factors hence the Sylvester matrix in (2.26a) would be singular. However in self-tuning control the estimated parameters $[\hat{A}(z^{-1})]$ and $[\hat{B}(z^{-1})]$ are stochastic, thus the probability of pole-zero cancellation is on the set of measure zero. However near pole-zero cancellation can lead to severe ill-conditioning, thus result in $[F(z^{-1})]$ and $[G(z^{-1})]$ having large coefficients [38].

2.5 Parameter adaptation.

Self-tuning control employs the certainty equivalence principle of [9,16], which allows for the separation of the adaptive control problem into two independent problems. The first step involves the estimation of the system parameters. The second step involves the mapping of the estimated system parameters into the controller parameters by any of the various proposed controller design techniques. The overall structure of the adaptative controller is as shown in figure 2.1.

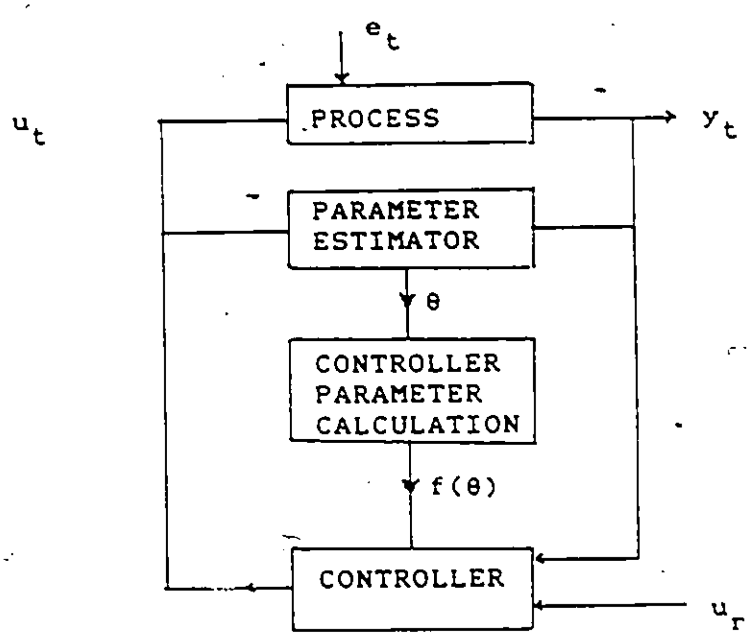


Fig. 2.1 Schematic diagram of a parameter adaptive controller.

The choice of model structure and the identification algorithm adopted are important. Careful choice of model structure could make subsequent controller design very simple. For the special case where the mapping from the estimated system parameters to the controller parameters is the trivial mapping, the algorithm is referred to as "implicit" or "direct" adaptive control. Otherwise the term "explicit" or "indirect" adaptive control applies.

2.5.1 Explicit adaptive control.

For explicit adaptive control the recursive least squares algorithm is often employed for parameter adaptation. Thus considering the model structure of (2.1a), we assume that

$$C'(z^{-1}) = 1$$

then defining

$$\theta = [a_1', \dots, a_{na}', b_1', \dots, b_{nb}']^T$$
$$\phi_t = [-y_{t-1}, \dots, -y_{t-na}, u_{t-kd}, \dots, u_{t-nb-kd}]^T$$

the elements of θ are the estimated parameters of (2.2a) which may be biased due to assuming the nosie term to be white. The RLS algorithm may be implemented as follows.

$$\varepsilon_t = y_t - \phi_t^T \theta_{t-1} \tag{2.27}$$

$$L_t = \frac{P_{t-1}\phi_t}{\lambda_t + \phi_t^T P_{t-1} \phi_t} \tag{2.28}$$

$$\theta_t = \theta_{t-1} + L_t \varepsilon_t \tag{2.29}$$

$$P_t = (P_{t-1} - \frac{P_{t-1}\phi_t\phi_t^T P_{t-1}}{\lambda_t + \phi_t^T P_{t-1}\phi_t})/\lambda_t \qquad (2.30)$$

where P_t is an (na+nb)x(na+nb) covariance matrix with $P_0 = \sigma I$, $\sigma >> 0$. θ_t is the current estimate of the parameter vector θ, also λ_t is a variable forgetting factor given by

$$\lambda_t = \frac{g\varepsilon_t^2}{\overline{Y}}$$

$$g = \frac{1}{1000}$$

$\overline{Y}$ = the mean value of ε_t^2 over a given period.

The variable forgetting factor allows the tracking of time varying parameters yet prevents the exponential growth of the coavriance matrix [24,37,42,101]. The numerical properties of the basic RLS algorithm may be greatly improved by using various factorization techniques to update the covariahce matrix ([19,66]).

At each sampling interval the estimated (possibly biased) parameters are used to calculate the controller parameters as if they were exact. The self-tuning principle, if it holds for the controller design selected, would guarantee the convergence of the controller parameters to their correct values.

The self-tuning principle has been proved for various controller designs. Self-tuning has been shown to occur (Astrom [9]) for the STC based on minimum variance control.

Clarke and Gawthrope [22] have proved the self-tuning principle for their extended minimum variance controller. Prager and Wellstead have also shown self-tuning to occur for the STC based on explict pole placement control [84].

2.5.2 Implicit adaptive pole placement.

Several implicit adaptive pole placement techniques have been proposed [2,10,13,36,39,46,47,48,49,101]. However from a unifying point of view, the basic idea behind many of these methods is the use of the RLS algorithm to solve the Diophantine equation (2.20). To do this (2.20) is multiplied by an auxilary signal. The use of different auxilary signals give rise to the various proposed algoritms, i.e. by the partial state [10,47,48], by y_t [2,49] or by u_t [49].

Consider the relationship

$$T(z^{-1}) = F^*(z^{-1})A(z^{-1}) + G^*(z^{-1})B(z^{-1}) \qquad (2.31)$$

where the coefficients of $A(z^{-1})$ and $B(z^{-1})$ have been previously estimated by RLS. Multiplying (2.31) by y_t gives the following implicit algorithm.

$$T(z^{-1})y_t = \theta_c^T\phi_t^f \qquad (2.32)$$

where

$$\theta_c^T = [f_1,\ldots,f_{nf},g_0,\ldots,g_{ng}]$$

$$\phi_t^f = [A(z^{-1})y_t,\ldots,A(z^{-1})y_{t-nf},B(z^{-1})y_t,\ldots,B(z^{-1})y_{t-ng}]$$

also f_i and g_i are the coefficients of $F^*(z^{-1})$ and $G^*(z^{-1})$ respectively.

Thus the controller parameters may be estimated from (2.32) using the RLS algorithm. The above implicit pole placement technique results in a billinear estimation problem since the data vector ϕ_t^f involves the estimated system parameters. The billinear estimation problem may however be converted into a linear estimation problem with twice the number of parameters [10,30,46].

Convergence problems have been reported with implicit adaptive pole placement controllers, especially in connection with their reduced order behaviour [47].

2.6 Concluding remarks.

Self-tuning controllers were traditionally designed for systems represented by ARMAX models. STC s based on pole assignment offer a feasible solution to the adaptive control of non-minimum phase systems, where model reference type controllers would give unstable control. However explicit pole placement involves on-line solution of polynomial equations which is computationaly undesirable for adaptive control.

Alternatively the use of implicit algorithms results in the estimation of a large number of parameters which can be equally unattractive. The above problems become even more acute when the methods are extended to MIMO systems.

One way to get round some of the above problems is to approach adaptive pole placement from the more natural state space approach. Techniques based on such an approach will be the main focus of this thesis.

CHAPTER 3

3. STATE SPACE SELF-TUNING CONTROL via POLE ASSIGNMENT.

3.1 Background

Self-tuning pole assignment control based on polynomial modeling was discussed in chapter 2. In the SISO case these techniques have proved to be very successful. For the MIMO cases however, the computational complexity of on-line solution of polynomial matrix equations has recently led to interest in state space based methods.

Warwick [98,99], Tsay and Shieh [97] have proposed state space pole assignment STCs for SISO systems. The techniques have been extended to MIMO systems by Benzanson and Harris [18], Shieh et. al. [88]. Hesketh [43] has proposed a MIMO state space STC based on input/output data.

The MIMO techniques proposed in [18,43,88,89], adopted state space models based on block canonical forms. Such "pseudo-canonical" forms are useful in the absence of a priori knowledge of the Kronecker structural indices [21,83] of the system. This however results in system matrices with larger than minimal dimensions. Furthermore all the techniques proposed above require the system model to be transformed on-line to the controllable canonical form. This in view of the large dimensions of the matrices involved, results in excessive computation. In some cases [18,43,88] this can lead to a similar computational load, as the methods based on polynomial matrices.

The MIMO state space pole assignment STCs proposed in [18,43,88] may be summed up as follows. Assume the system to be represented by the MIMO ARMAX model (2.22). At each sampling interval the parameters of (2.22) are estimated by the ELS algorithm. The estimated parameters are then used to obtain the one step prediction estimates of the states via the state space innovations model thus

$$\varepsilon_t = Y_t - \hat{Y}_t \tag{3.1a}$$

$$\hat{X}_{t+1} = A_o\hat{X}_t + B_oU_t + D_o\varepsilon_t \tag{3.1b}$$

$$\hat{Y}_{t+1} = C_o\hat{X}_{t+1} \tag{3.1c}$$

$$A_o = \begin{bmatrix} 0 & \cdot & & & A_n \\ I & & \cdot & & A_{n-1} \\ & & & \cdot & \cdot \\ & & & & \cdot \\ & & & & \cdot \\ 0 & & & I & A_1 \end{bmatrix} \tag{3.1d}$$

$$B_o = \begin{bmatrix} B_{nb} \\ B_{nb-1} \\ \cdot \\ \cdot \\ \cdot \\ B_o \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad D_o = \begin{bmatrix} C_n + A_n \\ C_{n-1} + A_{n-1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ C_1 + A_1 \end{bmatrix} \tag{3.1e}$$

where

$$A_i, C_i = 0, \text{ for } na, nc < i < n = \max(na, nb+kd, nc)$$

The computation of the pole assignment feed back law then involves the following steps.

(i) Define the matrix A_m as

$$A_m = \begin{bmatrix} 0 & I & 0 & & 0 \\ & & \cdot & & \\ & & & \cdot & 0 \\ 0 & & & & I \\ T_n & T_{n-1} & \cdot & \cdot & T_1 \end{bmatrix} \qquad (3.2)$$

where the mxm dimensional matrices T_i are the coefficients of $[T(z^{-1})]$ the desired closed loop mode matrix polynomial.

(ii) Transform the estimated system characteristic matrix A_o to the block controller canonical form [87] by the similarity transformation

$$A_c = PA_oP^{-1} \qquad (3.3a)$$

where

$$A_c = \begin{bmatrix} 0 & I & 0 & & 0 \\ & & \cdot & & \\ & & & \cdot & \\ 0 & & & & I \\ A_{c,n} & A_{c,n-1} & \cdot & \cdot & A_{c,1} \end{bmatrix} \qquad (3.3b)$$

The transformation (3.3a) must be physicaly carried out since unlike the SISO case $A_o^T \neq A_c$. The state feed back law to assign the closed loop poles to the roots of $[T(z^{-1})]$

is then given by

$$U_t = G_f \hat{X}_t \tag{3.4a}$$

where

$$G_f = [0,\ldots,0,I](A_m - A_c)P \tag{3.4b}$$

The transformation matrix P is unique and may be obtained [18,88,90] as follows Define the controllability test matrix

$$\Gamma = [B_o, A_o B, \ldots, A_o^{n-1} B_o] \tag{3.5a}$$

$$P = \begin{bmatrix} P_1 \\ P_1 A_o \\ \cdot \\ \cdot \\ \cdot \\ P_1 A_o^{n-1} \end{bmatrix} \tag{3.5b}$$

where

$$P_1 = [0,\ldots,0,I]\Gamma^{-1} \tag{3.5c}$$

It is evident from equations (3.5c) and (3.3a) that the computation of the feed back gain matrix requires the inversion of two nm x nm matrices. Thus the computational effort is similar to that of the methods proposed in [43,84].

The high dimensionality associated with block canonical forms may be avoided by adopting a modification of the bootstrap estimator (BSE) proposed in [85,34] for joint parameter and state estimation in the observable canonical form. The resulting adaptive pole placement algorithm has the following advantages over the methods proposed in [18,43,88].

(a) Fewer parameters are estimated due to the minimal realization of the canonical form adopted.

(b) The computation of the transformation matrix involves the inversion of an n x n controllability matrix which has much smaller dimensions than the 'controllability test' matrix [90] which has dimensions nm x nm.

(c) Although a transformation matrix is computed, the actual transformation of the system characteristic matrix to the controllable canonical form is not required. This is a significant saving over the methods in [18,43,88], since this avoids the second matrix inversion to determine the inverse of the transformation matrix.

3.2 The Modified BSE.

The use of bootstrap algorithms for joint state and parameter estimation has been reported in [33,34,85]. The main advantage of the method is that the canonical forms adopted, results in equation error type estimation algorithms. The method thus results in a pseudo linear regression (PLR) type algorithm which is much simpler to implement than the extended Kalman filter (EKF). This advantage becomes even more advantageous when the method is applied to MIMO systems [34].

The main drawback of the bootstrap algorithms proposed in [34,85] is that it was derived in an ad-hoc manner. Thus conditions for asymptotic convergence of the overall algorithm were not available. Recently it has been pointed out in [1,94] that the algorithm in its present form would not converge in the presence of significant observation and driving noise.

In this thesis and also in [79] the author has proposed some modifications to the basic algorithm in [85] to ensure asymptotic convergence of the parameter estimates to their unbiased values. Thus the innovations model is used to obtain the one step prediction state estimates, and an ELS type algorithm is used for parameter adaptation. Following Ljung et al. [63,64,65,66] the 'ordinary differential equation' (ODE) approach may be used to derive the sufficient conditions for asymptotic convergence of the estimated parameters to their unbiased values, see the APPENDIX and also reference [79].

Consider the MIMO system represented by the state space innovations model

$$X_{t+1} = \mathbf{F}_o X_t + \mathbf{G}_o U_t + \mathbf{K}_o e_t \qquad (3.6a)$$

$$Y_t = \mathbf{H}_o X_t + e_t \qquad (3.6b)$$

where

$$\mathbf{F}_o = [\mathbf{F}_{o,ij}] \quad , \qquad \begin{array}{l} i = 1, \ldots, m \\ j = 1, \ldots, m \end{array}$$

$$F_{o,ii} = \begin{bmatrix} a_{ii}(1) & \\ \cdot & I_{n_i-1} \\ \cdot & \\ \cdot & \\ a_{ii}(n_i) & \end{bmatrix} \tag{3.6c}$$

$$F_{o,ij} = \begin{bmatrix} a_{ij}(1) & \\ \cdot & 0 \\ \cdot\cdot & \\ \cdot & \\ a_{ij}(n_{ij}) & \end{bmatrix} \tag{3.6d}$$

$$G_o^T = [g_{o,1} \cdots g_{o,i} \cdots g_{o,n}] \tag{3.6e}$$

$$K_o^T = [k_{o,1} \cdots k_{o,i} \cdots k_{o,n}] \tag{3.6f}$$

$$H_o = \begin{bmatrix} u_{\ell_1}^T \\ \vdots \\ u_{\ell_i}^T \\ \vdots \\ u_{\ell_m}^T \end{bmatrix} \tag{3.6g}$$

$$\ell_i = n_1 + n_2 + \dots n_{i-1} + 1$$

$$n_1 + n_2 + \dots n_m = n \tag{3.6f}$$

n_i are the Kroneker invariant indices [83]. and are assumed to be known a priori. A survey of different methods for estimating the structural indices of transfer function matrices may be found in [91]. u_{ℓ_i} = a unit column vector with a 1 in the ℓ_i'th row, taking $\ell_1 = 1$, i.e, $n_0 = 0$. $g_{o,i}$ are m dimensional column vectors corresponding to the transpose of the rows of the input matrix G_o. $k_{o,i}$ are m dimensional column vectors corresponding to the transpose of the rows of the kalman gain matrix K_o.

The modified BSE algorithm may be implemented as follows.

(i) Define the parameter vectors

$$\theta_{i,t} = [a^T_{i,1}, \ldots, a^T_{i,m}, g^T_{o,\ell_i}, \ldots, g^T_{o,\ell_i+n_i-1}, k^T_{o,\ell_i}, \ldots, k^T_{o,\ell_i+n_i-1}] \qquad (3.7a)$$

$$i = 1, \ldots, m$$

where the $a_{i,j}$ are the non-trivial columns of of $[F_{i,j}]$ and

(ii) Define the data vectors

$$\phi_{i,t} = [X_{1,t-1}, \ldots, X_{1,t-n_i}, \ldots, X_{\ell_m,t-1}, \ldots, X_{\ell_m,t-n_i}, U^T_{t-1}, \ldots, U^T_{t-n_i}, e^T_{t-1}, \ldots, e^T_{t-n_i}] \qquad (3.7c)$$

Let $\theta_{i,t}$ be the current estimate of the parameter vector θ_i. Then the parameter identifcation involves m decoupled RLS type adaptation algorithms given by

1: Estimate prediction error

$$\varepsilon_{i,t} = Y_{i,t} - \hat{Y}_{i,t} \qquad (3.8a)$$

2: Compute adaptation gain

$$L_{i,t} = \frac{P_{i,t-1}\phi_{i,t}}{\lambda_t + \phi^T_{i,t}P_{i,t-1}\phi_{i,t}} \tag{3.8b}$$

3: Update parameter vector

$$\theta_{i,t} = \theta_{i,t-1} + L_{i,t}\varepsilon_{i,t} \tag{3.8c}$$

4: Update covariance matrix

$$P_{i,t} = \frac{1}{\lambda_t}[P_{i,t-1} - \frac{P_{i,t-1}\phi_{i,t}\phi^T_{i,t}P_{i,t-1}}{\lambda_t + \phi^T_{i,t}P_{i,t-1}\phi_{i,t}}] \tag{3.8d}$$

The estimates of $\hat{Y}_t$ required to construct the $\phi_{i,t}$ are obtained from the innovations model

$$\hat{X}_{t+1} = \mathbf{F}_{o,t}\hat{X}_t + \mathbf{G}_{o,t}U_t + \mathbf{K}_{o,t}\varepsilon_t \tag{3.9a}$$

$$\hat{Y}_{t+1} = \mathbf{H}_{o,t}\hat{X}_{t+1} \tag{3.9b}$$

where $\mathbf{F}_{o,t}$, $\mathbf{G}_{o,t}$ and $\mathbf{K}_{o,t}$ are the current estimates of $\mathbf{F}_o$ $\mathbf{G}_o$, and $\mathbf{K}_o$ respectively.

$P_{i,t}$ = a $\dim(\theta_i) \times \dim(\theta_i)$ matrix.

$L_{i,t}$ = a $\dim(\theta_i)$ vector.

For the special case where $n_1 = .. = n_i = .. = n_m$, $P_1 = .. = P_i = .. = P_m$, only one covariance matrix needs to be updated.

3.3 Pole Assignment by State Feedback.

In view of the separation theorem for stochastic control [9,16], a pole assignment STC may be designed by using the parameter estimates from the BSE algorithm to calculate the controller gain parameters in place of the exact system parameters.

The feedback gain to arbitrarily assign the closed loop poles of (3.6) may be computed [95] as follows.

(i) Define the controllability matrix

$$\Gamma = [g_1, \ldots, F_o^{n_1-1} g_1, g_2, \ldots, F_o^{n_m-1} g_m] \qquad (3.10a)$$

g_i is the i'th column of G_o. The n_i are assumed to be known a-priori as in (3.6f).

Since Γ is nonsingular, we can compute

$$\Gamma^{-1} = \begin{bmatrix} \eta_{1,1}^T \\ \vdots \\ \eta_{1,n_1}^T \\ \vdots \\ \eta_{r,n_m}^T \end{bmatrix} \qquad (3.10b)$$

Denoting the last row of each chain as

$$\eta_i^T = \eta_{i,n_i}^T \quad , \qquad i = 1, \ldots, m$$

the required control law to arbitrarily place the closed loop poles is then given by

$$U_t = K_f \hat{X}_t \qquad (3.11a)$$

where the feedback gain matrix K_f may be computed from

$$K_f = Q^{-1} \begin{bmatrix} \eta_1^T \pi_1(F_{o,t}) \\ \cdot \\ \cdot \\ \cdot \\ \eta_m^T \pi_m(F_{o,t}) \end{bmatrix} \qquad (3.11b)$$

$$\pi_i(z^{-1}) = 1 - \alpha_{ii}(1) z^{-1} - \ldots - \alpha_{ii}(n_i) z^{-n_i} \qquad (3.11c)$$

The coefficients $\alpha_{ii}(\cdot)$ determine the characteristic polynomial of the closed loop system matrix

$$F_c = [F_o + G_o K_f] \tag{3.11d}$$

thus the polynomials $\pi_i(z^{-1})$ are such that

$$\Pi_c(z^{-1}) = \prod_{i=1}^{m} \pi_i(z^{-1}) \tag{3.11e}$$

where

$$z^n \Pi_c(z^{-1}) = \det(zI - F_c) \tag{3.11f}$$

The matrix Q is an m x m input transformation matrix defined by

$$Q = \begin{bmatrix} 1 & q_{1,2} & \cdot\cdot & q_{1,j} & \cdot\cdot & q_{1,m} \\ 0 & \cdot & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & 0 & 1 \end{bmatrix} \tag{3.11g}$$

where the elements $q_{i,j}$ may be determined as follows

$$q_{i,j} = n_i^T [F_o^{n_i - 1} g_j] \tag{3.11h}$$
$$i = 1, \ldots, m-1$$
$$j = i+1, \ldots, m$$

The inversion of the m x m matrix Q in (3.11g) is computationaly inexpensive since Q is an upper triangular matrix with 1's on the leading diagonal. Thus the inversion process is simply a set of backward substitutions. Also since usually m << n this inverse is relatively insignificant.

3.4 Simulation Results

To illustrate the STC proposed in this chapter, the following system was simulated.

$$X_{t+1} = \begin{bmatrix} 1.4 & 1 & 0.2 \\ -0.48 & 0 & 0 \\ 0.22 & 0 & 0.75 \end{bmatrix} X_t + \begin{bmatrix} 1.0 & 0.0 \\ 1.3 & 0.2 \\ 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} U_{1,t} \\ U_{2,t} \end{bmatrix}$$

$$+ \begin{bmatrix} 1.8 & 0.2 \\ -0.48 & 0.0 \\ 0.22 & 1.25 \end{bmatrix} \begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix} \qquad (3.12a)$$

$$\begin{bmatrix} Y_{1,t} \\ Y_{2,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} X_t + \begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix} \qquad (3.12b)$$

$$E[ee^T] = \begin{bmatrix} 0.1 & 0.0 \\ 0.0 & 0.1 \end{bmatrix}$$

The open loop characteristic equation of $\mathbf{F}_o$ is is given by

$$\Pi_o(z) = \det(zI - \mathbf{F}_o) = z^3 - 2.15z^2 + 1.486z - 0.36$$

giving open loop poles at

$$z = 0.53 \pm j\,0.22,\; 1.01$$

thus the simulated system is open loop unstable. it is also non-minimum phase due the open loop zero at $z = -1.3$. The above system satisfies the sufficient conditions for the asymptotic convergence of the modified bootstrap estimator derived in the APPENDIX, since

$$\text{Real} \left(\frac{1}{1 + 0.5z^{-1}} - \frac{1}{2} \right) > 0$$

and

$$\text{Real} \left(\frac{1}{1 + 0.4z^{-1}} - \frac{1}{2} \right) > 0$$

The closed loop characteristic polynomial was selected to place all the closed loop poles at the origin to give minimum time (dead beat) control, thus

$$\Pi_c(z^{-1}) = 1$$

The following initial conditions were assumed.

$$P_{1,o} = 10^4 I, \quad P_{2,o} = 10^4 I$$

$$\theta_{1,o} = [0,0,0,0,0.1,0,0,0]$$

$$\theta_{2,o} = [0,0,0,0.1,0,0]$$

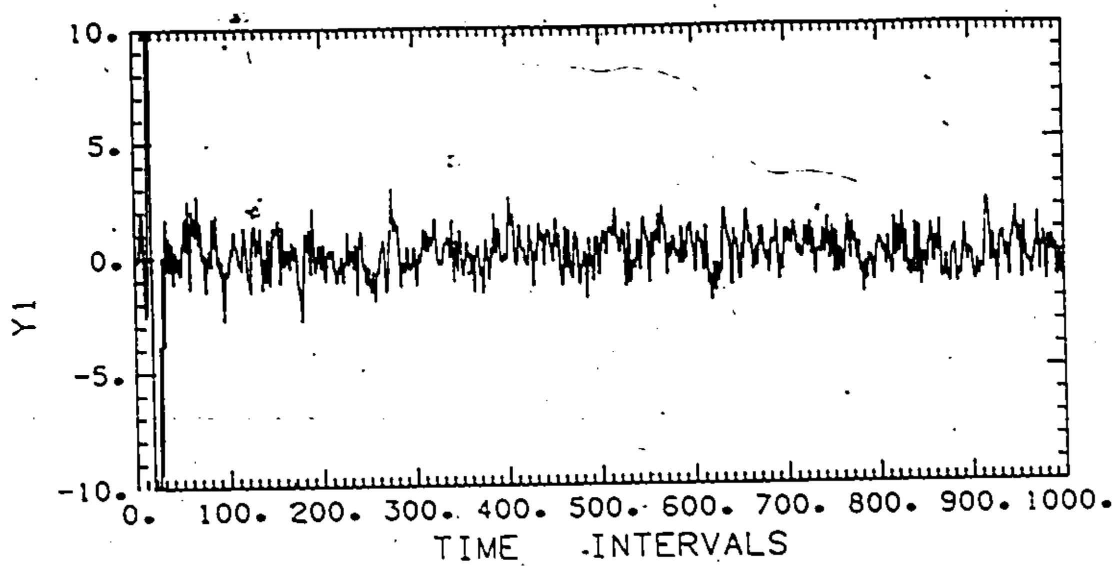
$$\lambda_o = 0.9$$

Figure 3.1 shows plots of the output variables $Y_{1,t}$ and $Y_{2,t}$ for the closed loop system. The closed loop variances of the output variables after settling down were

$$\mathrm{Var}(Y_{1,t}) = 0.67$$

$$\mathrm{Var}(Y_{2,t}) = 0.23$$

Figure 3.2 shows plots of the control variables $U_{1,t}$ and $U_{2,t}$. The variances of the control variables were observed



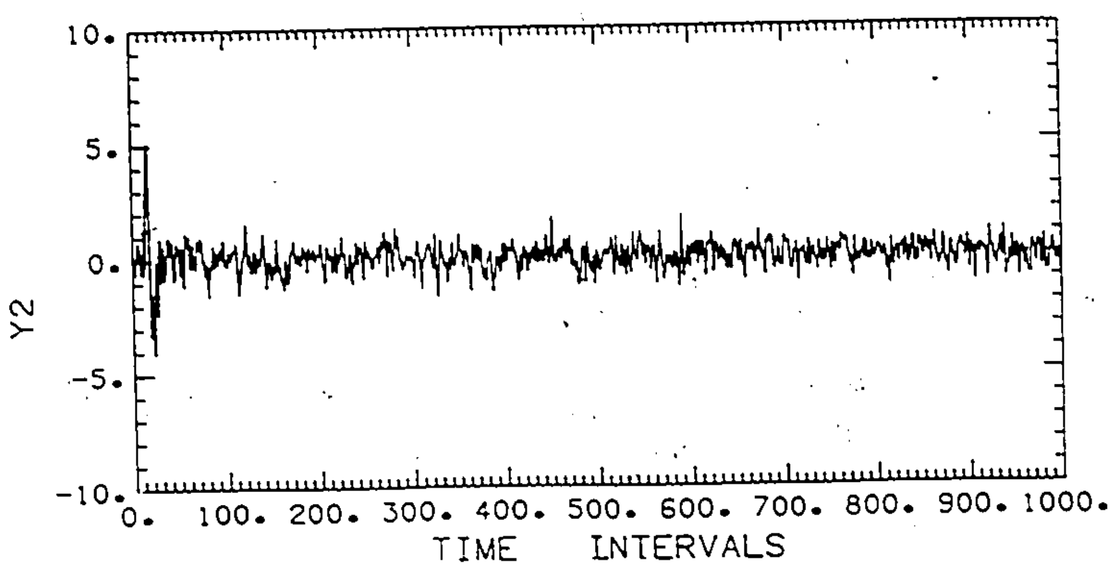


Fig. 3.1 Plots of the output variables for the closed loop system (example 1)

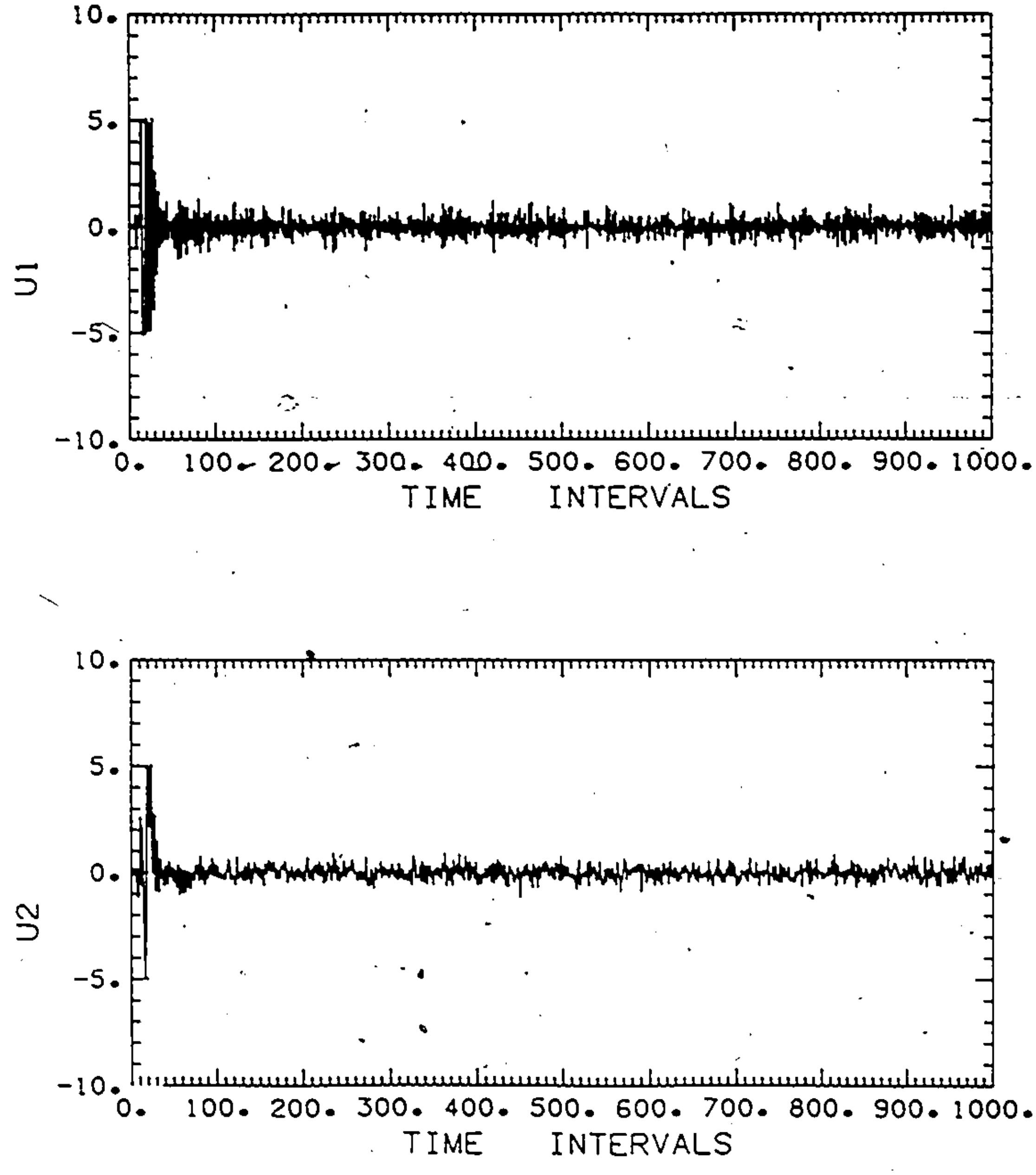


Fig. 3.2 Plots of the control action variables (example 1)

to be

$$Var(U_{1,t}) = 0.127$$

$$Var(U_{2,t}) = 0.048$$

thus the simulated open loop unstable and non-minimum phase system has been adaptively stablized by the algorithm with reasonable regulatory performance and moderate control action.

The estimation algorithm would converge to the exact system parameters with probabilty one (Appendix), if the input variables are sufficiently exciting and the sufficient conditions on the noise model are satisfied. Thus self-tuning would occur for the above simulation subject to persistency of the excitations, which is not guaranteed under closed loop control. However this is only a problem when the estimated parameters have converged, thus methods to keep the estimator open to incoming information such as the variable forgetting factor and covariance resseting could be used to overcome this problem. Otherwise dither signals should be injected to ensure the persistency of the input signals.

The exact system parameters are

$$\theta_1 = [1.4,-0.48,0.2,0.0,1.0,0.0,1.3,0.2,1.8,0.2,-0.48,0.0]$$

$$\theta_2 = [0.22,0.75,0.0,1.0,0.22,1.25]$$

The estimated parameters at t = 1000 were

$$\theta_{1,1000} = [1.42,-0.56,0.26,-0.1,0.99,0.00,1.22,0.07,1.72]$$

$$\theta_{2,1000} = [0.23,0.74,0.0,0.99,0.1,1.08]$$

The above parameter estimates are sufficiently accurate for the purposes of adaptive control since we assume the system to be time varying. Figure 3.3 shows plots of the rates of convergence of the estimated parameters as measured by the variables EN1 and EN2, where

$$EN1 = \frac{\|\theta_{1,t} - \theta_1\|^2}{\|\theta_1\|^2}$$

$$EN2 = \frac{\|\theta_{2,t} - \theta_1\|^2}{\|\theta_2\|^2}$$

thus from figure 3.3, it is evident that the estimated parameters have converged after about 150 sampling intervals.

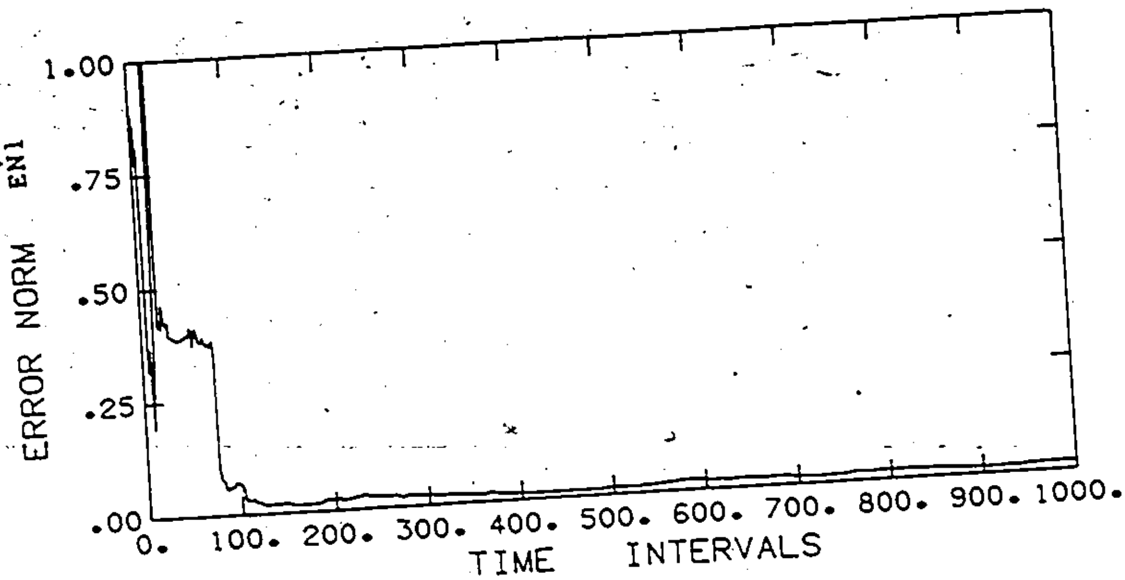
Figure 3.4 shows plots of the residual sequences. The variances of the residuals after settling down were found to be

$$Var(\varepsilon_{1,t}) = 0.12$$

$$Var(\varepsilon_{2,t}) = 0.11$$

Both sequences passed whiteness tests, based on the auto-correlation of the residuals. Thus the estimated parameters are unbiased, and hence self-tuning may be assumed to have occured.

It was observed in chapter 2, that adaptive pole placement control based on explicit parameter identification would give unsatisfactory performance when applied to over-parameterized systems, since pole-zero cancellation would cause the controllability (state space approach) or Sylvester (polynomial approach) matrix to be ill-conditoned.



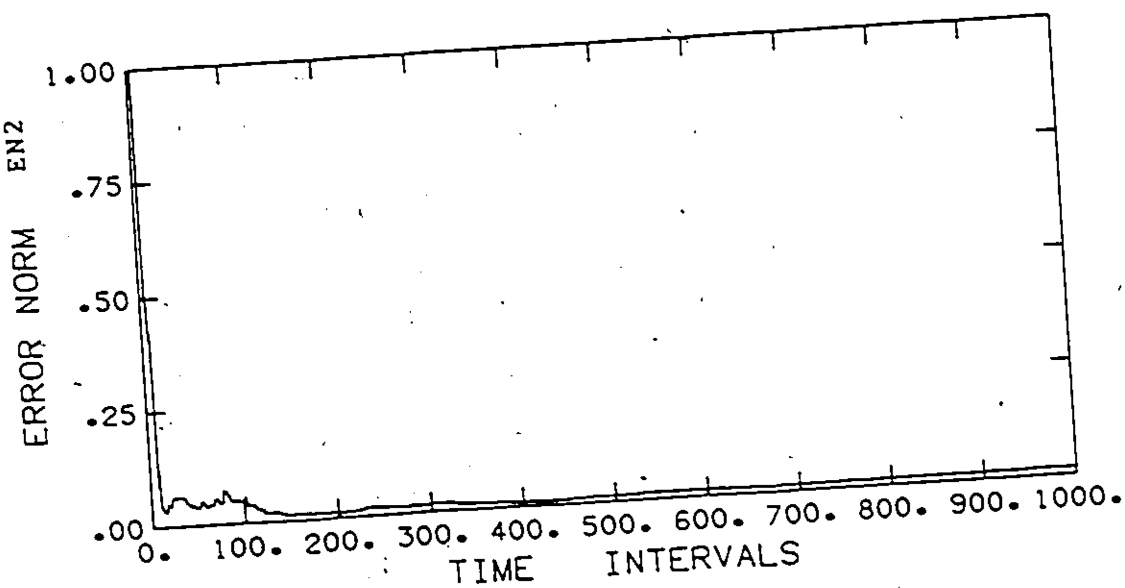
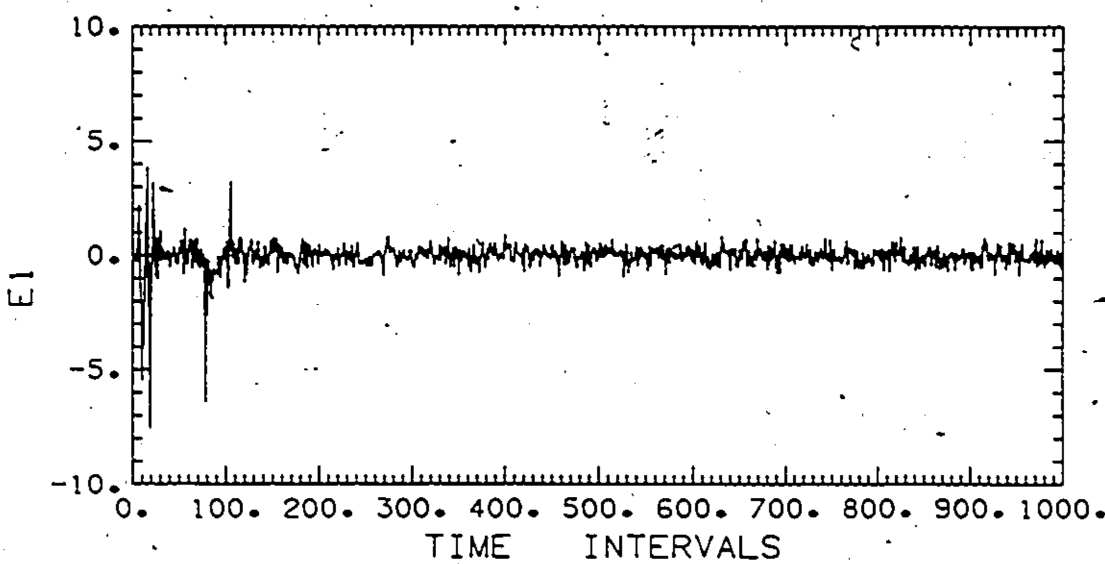


Fig. 3.3 Rates of convergence of the estimated parameter vectors (example 1)



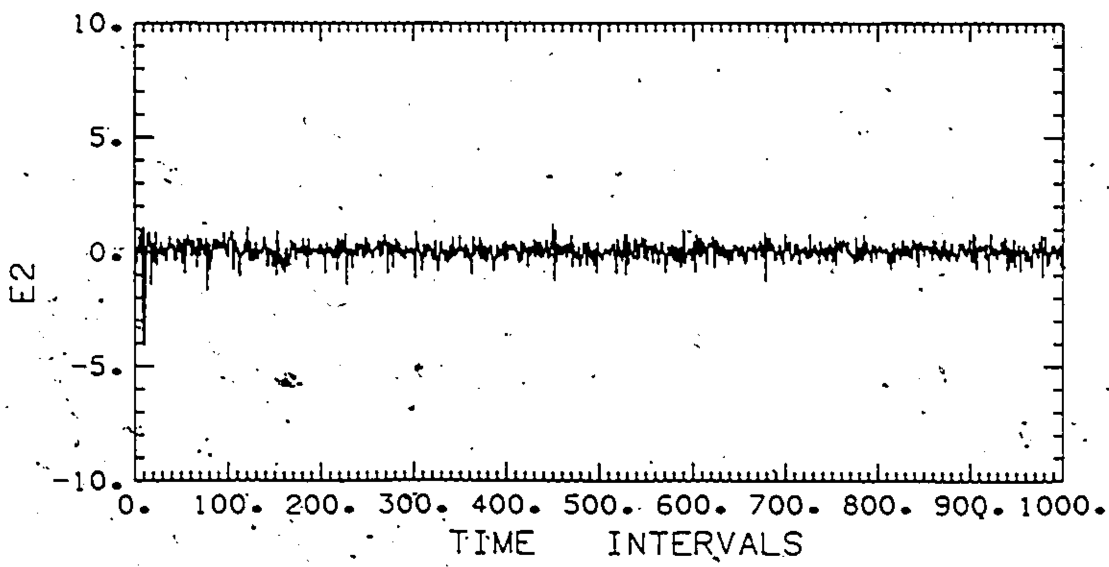


Fig. 3.4 Plots of the residual sequences (example 1)

Thus although the proposed algorithm and other explicit pole placement algorithms have been observed to work satisfactorily for several simulated models, such algorithms would give poor performance when applied to uncontrollable but stabilizable systems. To demonstrate this problem, the proposed algorithm was applied to the following model.

$$X_{t+1} = \begin{bmatrix} 1.4 & 1 & 0.0 \\ -0.48 & 0 & 0.0 \\ 0.22 & 0 & 0.75 \end{bmatrix} X_t + \begin{bmatrix} 1.0 & 0.0 \\ 0.8 & 0.2 \\ 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} U_{1,t} \\ U_{2,t} \end{bmatrix}$$

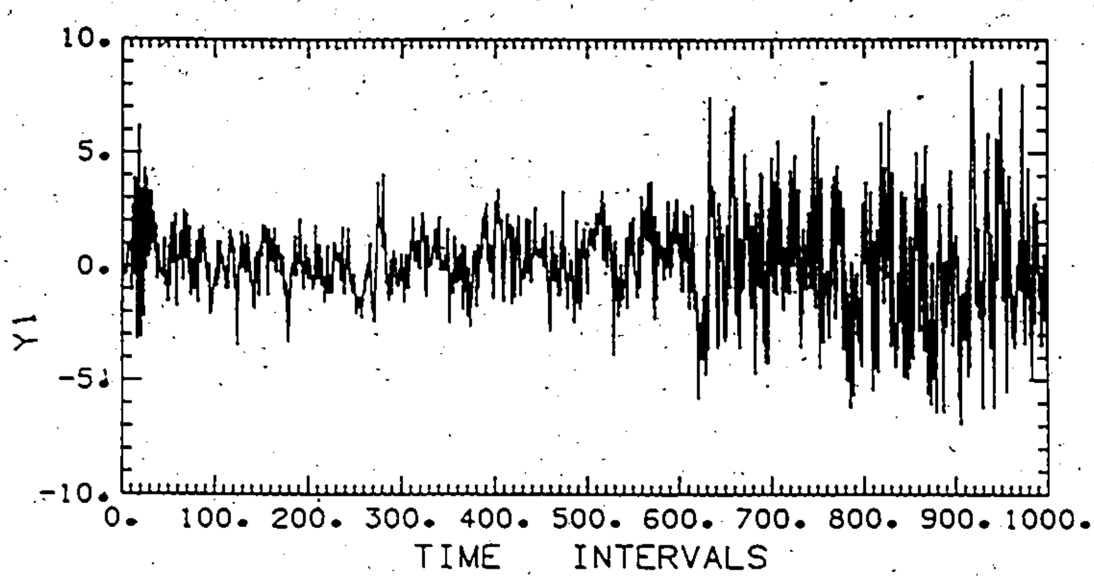
$$+ \begin{bmatrix} 1.8 & 0 \\ -0.48 & 0 \\ 0.22 & 1.25 \end{bmatrix} \begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix}$$

$$\begin{bmatrix} Y_{1,t} \\ Y_{2,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} X_t + \begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix}$$

$$E[ee^T] = \begin{bmatrix} 0.1 & 0.0 \\ 0.0 & 0.1 \end{bmatrix}$$

The aove system has a pole-zero cancellation at $z = 0.8$. For parameter adaptation the initial conditions were selected as

$$P_{1,0} = 10^4 I, \ P_{2,0} = 10^4 I$$



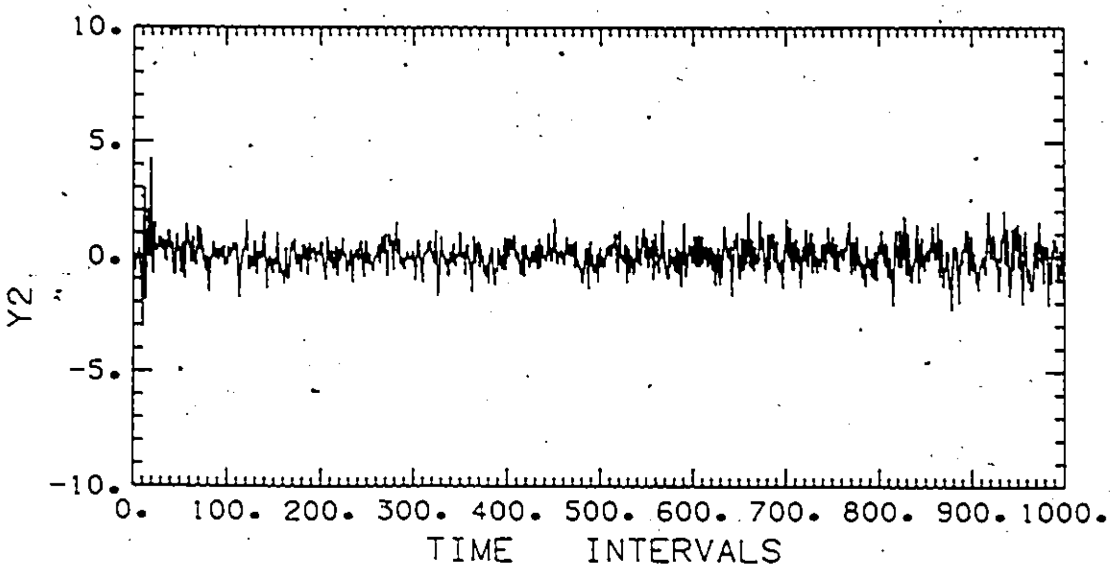


Fig. 3.5 Plots of the output variables for the closed loop system (example 2), showing the onset of instability due to pole-zero cancellation

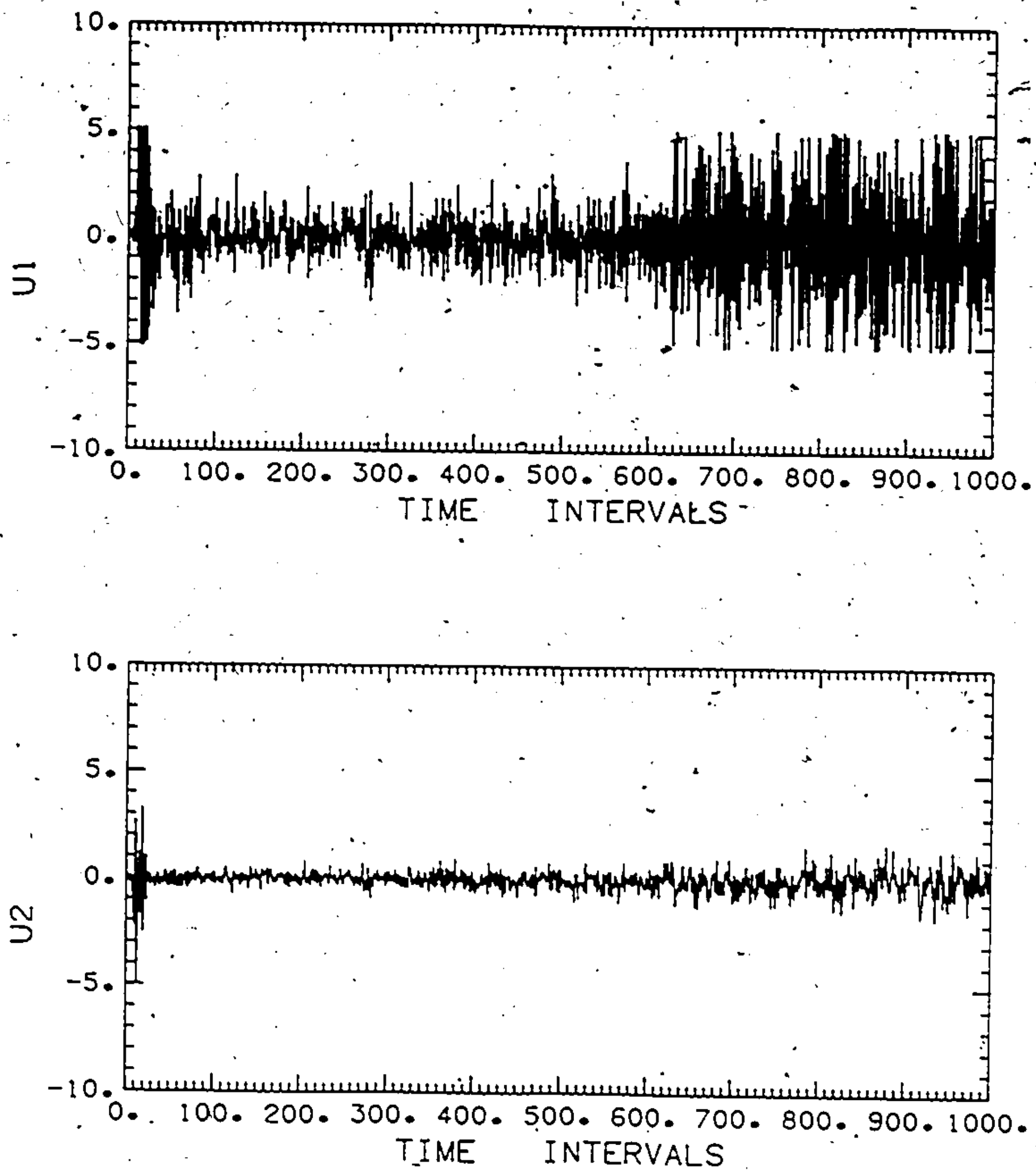


Fig. 3.6 Plot of the control action variables showing saturation of the control action due to ill conditioning (example 2)

$$\theta_{1,0} = [0,0,0,0,0.1,0,0,0,0]$$
$$\theta_{2,0} = [0,0,0,1,0,0]$$
$$\lambda_o = 0.9$$

Figure 3.5 shows plots of the closed loop output variables $Y_{1,t}$, and $Y_{2,t}$. It can be observed from these plots that the variances of the output variables are reasonably small in the early sampling periods, but increases with time. This may be explained by the fact that in the initial periods the controllability matrix is nonsingular since the estimated parameters are far from their exact values. However as the parameters converge, the controllablity matrix becomes more and more ill conditioned due to near pole-zero cancellation.

Figure 3.6 shows plots of the resulting control action. Note that the closed-loop system does not become unstable due to imperfect pole-zero cancellation (the estimated parameters are stochastic) and the saturation of the control variables.

3.5 Concluding Remarks

A state space self-tuning pole placement controller was proposed which identifies the parameters of a MIMO state space innovations model directly. The proposed method has the following advantages over previously proposed methods.

(i) The method sidesteps the need to firstly identify the parameters of a polynomial model and its use to construct a block canonical model. Thus a minimal realisation can be adopted which reduces the number of parameters to be estimated, if the structural indices are known.

(ii) The method of controller design adopted requires one matrix inversion of order n x n whereas the methods adopted in [18,43,88] would require two matrix inversions of order nm x nm.

The main drawback of the explicit adaptive pole placement algorithms is the need to know the correct model order in order to prevent pole-zero cancellations. This drawback was demonstrated in the simulations section. Implicit pole placement is one way to overcome this problem, however it was shown in chapter 2 that this could lead to a billinear estimation problem, the convergence properties of such estimation algorithms are not well established. Alternatively the billinear estimation problem may be be converted into an equivalent linear estimation problem, but this would result in the estimation of a large number of parameters as shown in chapter 2. Some of the disadvantages of explicit adaptive pole placement mentioned above may be avoided by the algorithms proposed in chapter 4.

CHAPTER 4

4. State Space Self-tuning STC via Recursive Prediction Error Estimation.

4.1 Introduction

Adaptive control via pole placement has proved to be a very attractive method, due to the applicability of the techniques to a very large class of systems. However the various algorithms proposed suffer from one main drawback. The computation of the controller gains require the inversion of a Sylvester matrix (2.26a) or alternatively the inversion of a controllability matrix (3.10a), for the state space approach. The existence of the inverses of these matrices requires the absence of pole zero cancellations. This is thus a severe restriction on the applicability of pole placement adaptive controllers, since very often in adaptive control the correct model orders are unknown a priori.

The design of pole placement controllers can be easily carried out, if the system is modeled in the controller canonical form [69]. However 'equation error' type identification algorithms are more easily associated with observable canonical forms [31,77].

The adoption of a system model in the controller canonical form would lead to a nonlinear estimation problem. The resulting nonlinear estimation problem may be tackled by the extended Kalman filter [EKF], by treating the unknown system parameters as additional states, and using the standard Kalman filter algorithms. The EKF however often produces biased parameter estimates and may sometimes diverge if the initial parameter values are not close to their correct values.

The recursive prediction error (RPE) algorithm for joint state and parameter estimation, was originaly proposed by Ljung [63] and Moore et al. [71] as variations of the EKF to guarantee asymptotic convergence of the estimated parameters to their unbiased values. It has been shown [66] that when the techniques are applied to the state space innovations model, the Kalman gain matrix becomes explicitly parameterized, thus may be estimated on-line together with the system parameters.

The RPE algorithm is equivalent to the off-line maximum likelihood algorithm since they both minimize the same cost function (4.5a). Thus provided the input signal is bounded and sufficiently exciting, the estimated parameters should converge to a local minimum of (4.5a). If the correct model order is known the estimated parameters would converge to the global minimum.

An adaptive pole placement controller based on the RPE estimator thus overcomes the main disadvantages of the previous indirect adaptive pole placement controllers. The resulting algorithm would be less sensitive to over-parameterization, since the ill-conditioning associated with inverting a nearly singular controllability or Sylvester matrix is avoided. Furthermore the computation of the feedback gains becomes trivial in this canonical form. Thus for large systems the computational savings of avoiding on-line state transformation which involves matrix inversions, more than offsets the extra recursive steps required by the RPE method.

4.2 Structure Of The Innovations Model.

Consider an SISO system represented by the polynomial model (2.1), in order to facilitate identification in cases where the integral time delay kd is either unknown or time varying, (2) may be represented by a more general model

$$A(z^{-1})y_t = B(z^{-1})u_{t-kd+1} + C(z^{-1})e_t \qquad (4.1a)$$

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + \ldots a_n z^{-n} \qquad (4.1b)$$

$$B(z^{-1}) = b_1 z^{-1} + \ldots . b_n z^{-n} \qquad (4.1c)$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + \ldots c_n z^{-n} \qquad (4.1d)$$

$$n = \max(na, nb + km - 1, nc)$$

km = maximum number of integral periods of delay expected, i.e. $km \geq kd \geq 1$.

Following [97,98] (4.1) can be represented in the state space innovations form as

$$Z_{t+1} = F_o Z_t + G_o u_t + K_o e_t \tag{4.2a}$$

$$y_t = H_o Z_t + e_t \tag{4.2b}$$

where

$$F_o = \begin{bmatrix} -a_1 & 1 & 0 & 0 \\ -a_2 & 0 & 1 & 0 \\ \vdots & \vdots & & \\ -a_{n-1} & 0 & 0 & 1 \\ -a_n & 0 & 0 & 0 \end{bmatrix} \tag{4.2c}$$

$$F_o = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix} \tag{4.2d}$$

$$H_o = [1, 0, \;..\;, 0] \tag{4.2e}$$

$$K_o = \begin{bmatrix} c_1 - a_1 \\ c_2 - a_2 \\ \vdots \\ c_{n-1} - a_{n-1} \\ c_n - a_n \end{bmatrix} \tag{4.2e}$$

Z_t = an n dimensional state vector.

The above model is, however, in the observable canonical form. Assuming the noise-free portion of (4.2) is controllable, (4.2) may be transformed to the controllable canonical for using the similarity transformation

$$Z_t = JX_t \tag{4.3a}$$

where

$$J = \Gamma_c(\Gamma_o^T)^{-1} \tag{4.3b}$$

$$\Gamma_c = [G_o, F_oG_o, \ldots, F_o^{n-2}G_o, F_o^{n-1}G_o] \tag{4.3c}$$

$$\Gamma_o^T = [H_o^T, F_o^TH_o^T, \ldots, (F_o^T)^{n-2}H_o^T, (F_o^T)^{n-1}H_o^T]$$

Using (4.3) the controller canonical form of (4.3) is thus

$$X_{t+1} = FX_t + Gu_t + Ke_t \tag{4.4a}$$

$$y_t = HX_t + e_t \tag{4.4b}$$

where

$$F = \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_{n-1} & -a_n \\ 1 & 0 & \cdots & 0 & 0 \\ & \ddots & & & \vdots \\ 0 & & & 1 & 0 \end{bmatrix} = J^{-1}F_oJ \tag{4.4c}$$

$$G = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = J^{-1}G_o \tag{4.4d}$$

$$K = \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{bmatrix} = J^{-1}K_o \tag{4.4e}$$

$$H = [b_1, b_2, \ldots, b_n] = H_oJ \tag{4.4f}$$

X_t = the state vector of the controllable canonical model.

4.3 Joint Parameter and State Estimation RPE Methods

Following [66] the RPE method may be used for joint parameter and state estimation of (4.4). Define the cost function

$$V(\theta,\gamma) = E\{ \frac{1}{2}\frac{\varepsilon_t^2}{\gamma} + \log\gamma \} \tag{4.5a}$$

where ε_t is the prediction or innovations sequence generated by

$$\varepsilon_t = y_t - \hat{y}_t \tag{4.6b}$$

$$\hat{X}_{t+1} = F(\theta_t)\hat{X}_t + G(\theta_t)u_t + K(\theta_t)\varepsilon_t \tag{4.6c}$$

$$\hat{y}_{t+1} = H(\theta_t)\hat{X}_{t+1} \tag{4.6d}$$

and

θ = a 3n parameter vector of the unknown elements of (4.4)

θ_t = the estimated parameter vector at time t

$\hat{y}_t$ = the predicted output at time t

$\hat{X}_t$ = the estimated state vector at time t.

$V(\theta,\gamma)$ is the negative log-likelihood function, if we assume the error sequence ε_t to be Gaussian distributed.

Consider

$$\min_{\theta,\gamma} V(\theta,\gamma) \tag{4.7a}$$

Subject to (4.6b),(4.6c), and (4.6d)

Also define

$$\Psi_t = -\left(\frac{d\varepsilon_t}{d\theta}\right)^T = \left(\frac{d\hat{y}_t}{d\theta}\right)^T \quad \text{(a 3n vector)} \tag{4.7b}$$

Ψ_t is the negative gradient of the prediction error, hence provides a descent direction for the recursive minimization of (4.7a). In order to compute Ψ_t, the following quantities need to be determined.

$$W_t = \frac{d}{d\theta}[\hat{X}_t(\theta)] \quad \text{(an n x 3n matrix)} \tag{4.7c}$$

$$D_t = \frac{\partial}{\partial\theta}[H(\theta)\hat{X}_t]|_{\theta = \theta_t} \quad \text{(a 3n row vector)} \tag{4.7d}$$

$$M_t = \frac{\partial}{\partial\theta}[F(\theta)\hat{X}_t + G(\theta)u_t + K(\theta)\varepsilon_t]|_{\theta = \theta_t} \quad \text{(an n x 3n matrix)} \tag{4.7e}$$

Then Ψ_t may be computed from

$$\Psi_t = W_t^T H(\theta)^T + D_t^T \tag{4.7f}$$

where W_t satisfies the dynamics

$$W_{t+1} = [F(\theta) - K(\theta)H(\theta)]W_t + M_t - K(\theta)D_t \tag{4.7g}$$

The RPE algorithm for the recursive minimization of (4.7a) is the given by:

$$\theta = [a_1, \ldots, a_n; b_1, \ldots, b_n, k_1, \ldots, k_n] \tag{4.8a}$$

$$D_t = [0, \ldots, 0, \hat{x}_{1,t}, \ldots, \hat{x}_{n,t}, 0, \ldots, 0] \tag{4.8b}$$

$$M_t = \left[\begin{array}{ccc|ccc|ccc} -\hat{x}_{1,t} & \ldots, & -\hat{x}_{n,t} & 0, & \ldots, & 0 & \varepsilon_t, & 0, \ldots, & 0 \\ 0, & \ldots, & 0, & \cdot & & \cdot & 0 & \varepsilon_t, \ldots, & 0 \\ \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot & & \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot & & \cdot & \cdot & & \\ 0, & \ldots, & 0, & 0, & & 0 & 0, & \ldots\ldots, & \varepsilon_t \end{array}\right]$$

1: Compute prediction error

$$\varepsilon_t = y_t - \hat{y}_t \tag{4.9a}$$

2: Update var(ε_t)

$$\gamma_t = \gamma_{t-1} + \frac{1}{t}[\varepsilon_t^2 - \gamma_{t-1}] \tag{4.9b}$$

3: Compute adaptation gain

$$L_t = \frac{P_{t-1}\Psi_t}{\lambda_t\gamma_t + \Psi_t^T P_{t-1}\Psi_t} \tag{4.9c}$$

4: Update parameter estimates

$$[\theta_t = \theta_{t-1} + L_t\varepsilon_t]|_{\theta_t} \quad D_s \tag{4.9d}$$

5: Update covariance matrix

$$P_t = \frac{1}{\lambda_t}\left[P_{t-1} - \frac{P_{t-1}\Psi_t\Psi_t^T P_{t-1}}{\lambda_t\gamma_t + \Psi_t^T P_{t-1}\Psi_t} \right] \tag{4.9e}$$

6: Predict next state estimates

$$\hat{X}_{t+1} = F_t\hat{X}_t + G_t u_t + K_t\varepsilon_t \tag{4.9f}$$

7: Predict next output

$$\hat{y}_{t+1} = H_t\hat{X}_{t+1} \tag{4.9g}$$

8: Compute gradient of ($\hat{X}_{t+1}$)

$$W_{t+1} = [F_t - K_t H_t]W_t + M_t - K_t D_t] \tag{4.9h}$$

9: Compute gradient of ($\hat{y}_{t+1}$)

$$\Psi_{t+1} = W_{t+1}{}^T H_t^T + D_{t+1}^T \tag{4.9i}$$

where we have used the notation

$F_t = F(\theta_t)$

$G_t = G(\theta_t)$

$H_t = H(\theta_t)$

$K_t = K(\theta_t)$

D_s = the stability region of the predictor:

$$D_s\{\theta | (F_t - K_t H_t) \text{ is strictly stable}\} \quad (4.9j)$$

L_t = an n dimensional vector of adaptation gains

P_t = a 3n x 3n covariance matrix.

The time varying forgetting factor λ_t is generated by the first order difference equation

$$\lambda_t = \lambda_o \lambda_{t-1} + (1 - \lambda_o) \quad (4.10)$$

$0 < \lambda_o$, usually $\lambda_o = 0.95$

The above forgetting factor ensures that the covariance matrix is open to incoming data, but asymptotically sets λ_t to unity. For cases where the input/output data is not persistently exciting, the use of the above forgetting factor could lead to exponential growth of the covariance matrix. For such cases the data dependent variable forgetting factor described in section 2.5.1 should be used.

From theorem 4.3 of [66] a necessary condition for the convergence of the estimated parameters θ_t to the set

$$D_c = \{\theta | \frac{d}{d\theta} V(\theta, \gamma) = 0\}$$

or to the boundary of D_s, is that θ_t D_s (at least infinitely often). Thus to ensure the stability of the RPE algorithm,

θ_t must be monitored and if necessary projected into the stability region of the predictor (4.9j).

The projection facility may be implemented as follows. Define

$$\bar{F} = F_t - K_t H_t \qquad (4.11a)$$

then the characteristic polynomial of F is given by

$$\Pi(z) = \det(zI - \bar{F}) \qquad (4.11b)$$

$$= (z^n + p_1 z^{n-1} + \dots p_{n-1} z + p_n) \qquad (4.11c)$$

The adaptation law (4.9d) may be implemented as :

1: update parameters $\theta_t = \theta_{t-1} + L_t \varepsilon_t$

2: compute $\bar{F} = F_t - K_t H_t$

3: determine $\Pi(z) = \det(zI - \bar{F})$

4: if $\Pi(z) = 0$, for $|z| \geq 1$; set $\theta_t = \theta_{t-1}$

Thus if a particular measurement would take θ_t outside the stability region it is simply ignored. A more refined algorithm would be to successively halve the correction term $L_t \varepsilon_t$, till θ_t is inside the stability region. Step 4 above involves testing to ensure that all roots of $\Pi(z)$ lie inside the unit circle. A suitable algorithm for doing this based on the Jury stability criterion, is given in [55].

The coefficients of $\Pi(z)$ may be generated recursively by using Leverier's algorithm A(23) of [50]. Unfortunately, Leverier's algorithm is computationally inefficient for large matrices. However, by exploiting the special structures of F,K, and H, $\Pi(z)$ may be determined as follows:

$$\Pi(z) = \det(zI - F)\det(zI + (zI - F)^{-1}KH) \qquad (4.12a)$$

$$= a(z) + H\,\mathrm{adj}(zI - F)K \tag{4.12b}$$

using (A33) of [50]

$$H\,\mathrm{adj}(zI - F)K = (\beta(z)\delta(z)) \bmod a(z) \tag{4.12c}$$

where

$$a(z) = z^n A(z^{-1}) \tag{4.12d}$$

$$\beta(z) = b_1 z^{n-1} + \ldots b_n \tag{4.12e}$$

$$\delta(z) = \delta_n z^{n-1} + \ldots \delta_1 \tag{4.12f}$$

$$\begin{bmatrix} \delta_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \delta_n \end{bmatrix} = \begin{bmatrix} 1, & a_1 & \cdot & \cdot & \cdot & a_{n-1} \\ 0, & 1, & a_1 & \cdot & \cdot & a_{n-2} \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ 0, & & & & & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ k_n \end{bmatrix} \tag{4.12g}$$

also $(\beta(z)\delta(z)) \bmod a(z)$ stands for the remainder after the division of $(\beta(z)\delta(z))$ by $a(z)$.

4.4 Pole Assignment Self-tuning Control by State Feedback

In view of the separation theorem for stochastic control [9], the STC algorithm may be implemented as follows. Each iterartion of the joint parameter and state estimation algorithm is followed by the state feedback law

$$u_t = K_f \hat{X}_t + K_r y_r \tag{4.13a}$$

$$K_f = [a_1, \ldots, a_n] - [\alpha_1, \ldots, \alpha_n] \tag{4.13b}$$

where $\alpha_1, \ldots, \alpha_n$ are the coefficients of the desired closed loop mode polynomial $A_m(z^{-1})$ and

$$A_m(z^{-1}) = 1 + \alpha_1 z^{-1} + \ldots \alpha_n z^{-n} \tag{4.13c}$$

Also y_r is a reference input sequence and K_r is a feed forward gain. To ensure steady state error to a step input, K_r is selected

$$K_r = \frac{1 + \sum_{i=1}^{n} a_i}{\sum_{i=1}^{n} b_i}, \quad \sum_{i=1}^{n} b_i \neq 0$$

The closed loop output subject to the above control law is then

$$y_t = H(zI-F-GK_f)^{-1}GK_r y_r + H(zI-F-GK_f)^{-1}Ke_t + e_t \qquad (4.14a)$$

which reduces to

$$y_t = \frac{B(z^{-1})}{A_m(z^{-1})}K_r y_r + \frac{D(z^{-1})}{A_m(z^{-1})}e_t \qquad (4.14b)$$

where

$$D(z^{-1}) = z^{-n}\{(\beta(z)\delta'(z)) \bmod \alpha(z)\} + A_m(z^{-1}) \qquad (4.14c)$$

$$= 1 + d_1 z^{-1} + \dots\dots d_n z^{-n} \qquad (4.14d)$$

$$\alpha(z) = z^n A_m(z^{-1}) \qquad (4.14e)$$

$$\delta'(z) = \delta_n' z^{n-1} + \dots\dots\delta_1' \qquad (4.14f)$$

$$\begin{bmatrix} \delta_1' \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \delta_n' \end{bmatrix} = \begin{bmatrix} 1, & a_1 & \cdot & \cdot & \cdot & \cdot & a_{n-1} \\ 0, & 1, & a_1 & \cdot & \cdot & \cdot & a_{n-2} \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ 0, & 0, & \cdot & \cdot & \cdot & 0, & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ k_n \end{bmatrix} \qquad (4.14g)$$

For pure regulation, i.e $y_r = 0$, the stationary variance of the output y_t can be evaluated from (4.14b) thus

$$\sigma_y^2 = \frac{\gamma}{2\pi j}\oint \frac{D(z)D(z^{-1})}{A_m(z)A_m(z^{-1})}\frac{dz}{z} \tag{4.14h}$$

A recursive algorithm to compute the above integral is given in [55]. For dead beat control, $A_m(z^{-1}) = 1$, i.e. all closed loop poles are placed at the origin. Equation (4.14h) then reduces to

$$\sigma_y^2 = (1 + d_1^2 + \ldots . d_n^2)\gamma \tag{4.14i}$$

The control action for the regulatory mode can be related to e_t by

$$u_t = \frac{r(z^{-1})}{A_m(z^{-1})}e_{t-1} \tag{4.14j}$$

where

$$r(z^{-1}) = z^{-2}\{(\omega(z)\delta'(z))\text{mod}\alpha(z)\} \tag{4.14k}$$

$$\omega(z) = (a_1-\alpha_1)z^{n-1} + \ldots\ldots (a_n-\alpha_n) \tag{4.14l}$$

and hence the control action variance may be computed from

$$\sigma_u^2 = \frac{\gamma}{2\pi j}\oint \frac{r(z)r(z^{-1})}{A_m(z)A_m(z^{-1})}\frac{dz}{z} \tag{4.14m}$$

4.5 Simulation Results

To demonstrate the effectiveness of the proposed algorithm, the following system was simulated.

$$y_t-1.6y_{t-1}+0.63y_{t-2} = u_{t-1}+1.5u_{t-2}+e_t+0.5e_{t-1} \tag{4.15a}$$

$$e_t \approx (0,0.1)$$

The above system has poles at z=0.9, and z=0.7 and is non-minimum phase. Using (4.3) the above model may be put in the observable canonical form.

$$Z_{t+1} = \begin{bmatrix} 1.6 & 1 \\ -0.63 & 0 \end{bmatrix} Z_t + \begin{bmatrix} 1.0 \\ 1.5 \end{bmatrix} u_t + \begin{bmatrix} 2.1 \\ -0.63 \end{bmatrix} e_t \qquad (4.15b)$$

$$y_t = [1,0]Z_t + e_t \qquad (4.15c)$$

On transformation to the controllable canonical form we get

$$X_{t+1} = \begin{bmatrix} 1.6 & -0.63 \\ 1 & 0 \end{bmatrix} X_t + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_t + \begin{bmatrix} 1.026 \\ 0.716 \end{bmatrix} e_t \qquad (4.15d)$$

$$y_t = [1.0, 1.5]X_t + e_t \qquad (4.15e)$$

thus from (4.8a)

$$\theta = [-1.6, 0.63, 1.0, 1.5, 1.026, 0.716]^T$$

Figure 4.1 shows the open loop response of the system with $u_t \equiv 0$, and $y_r \equiv 0$. The uncontrolled system gave an output variance

$$\mathrm{Var}(y_t) = 9.68$$

Both the desired closed loop poles were placed at the origin to obtain dead beat control; thus

$$A_m(z^{-1}) = 1$$

The starting values were chosen as

$$\theta_o = [0.01, 0.01, 0.1, 0.01, 0.01, 0.01]^T$$

$$P_o = 10^4 I$$

$$\lambda_o = 0.96$$

The input sequence y_r was selected as a square wave sequence with mean of 2.5, amplitude of ±2.5 and a period of

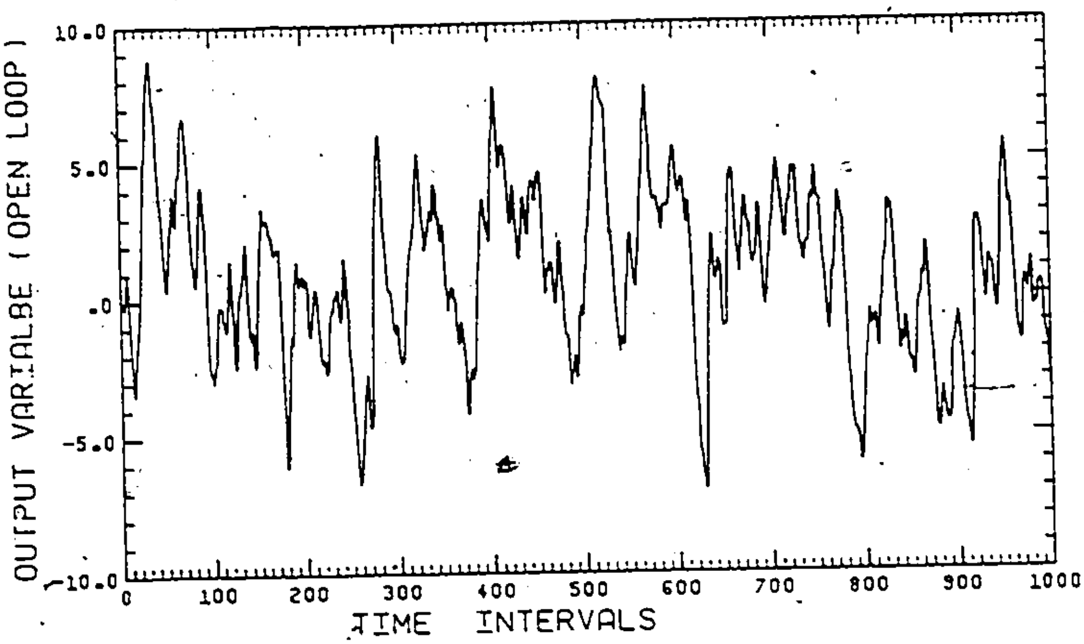


Fig. 4.1 Open loop response, u ≡ 0, (example 3)

100 sampling intervals. Figure 4.2 shows the closed-loop response, superposed on the input sequence y_r. After settling down the closed loop output variance was

$$Var(y_t - y_r) = 1.43$$

The theoretical value obtained via (4.14i) is $\sigma_y^2 = 0.78$. The discrepancy may be attributed to the combined effects of inexact parameter estimates, and the transient response to y_r.

The projection algorithm implemented in section 4.4, ensures the stability of the predictor matrix $[F_t - K_t H_t]$. Under closed loop control however, the stability of the overall algorithm would also depend on the dynamics of $[F + GK_f]$ which is uncertain, even if $[F_t + GK_f]$ is always stable. Thus to prevent the algorithm from blowing up in the initial stages when the estimated parameters are far from their correct values, it was found necessary to hard limit the control variable, thus

$$-5 < u_t > 5$$

Figure 4.3 shows a plot of the control action. After settling down the control action variance was

$$Var(u_t) = 0.27$$

The estimated parameters at t = 500 were

$$\theta_{500} = [-1.67, 0.68, 0.91; 1.48, 1.02, 0.81]$$

Figures 4.4 and 4.5 show plots of the estimated system and Kalman gain parameters respectively. It may be noticed from these plots that the estimated parameters never really

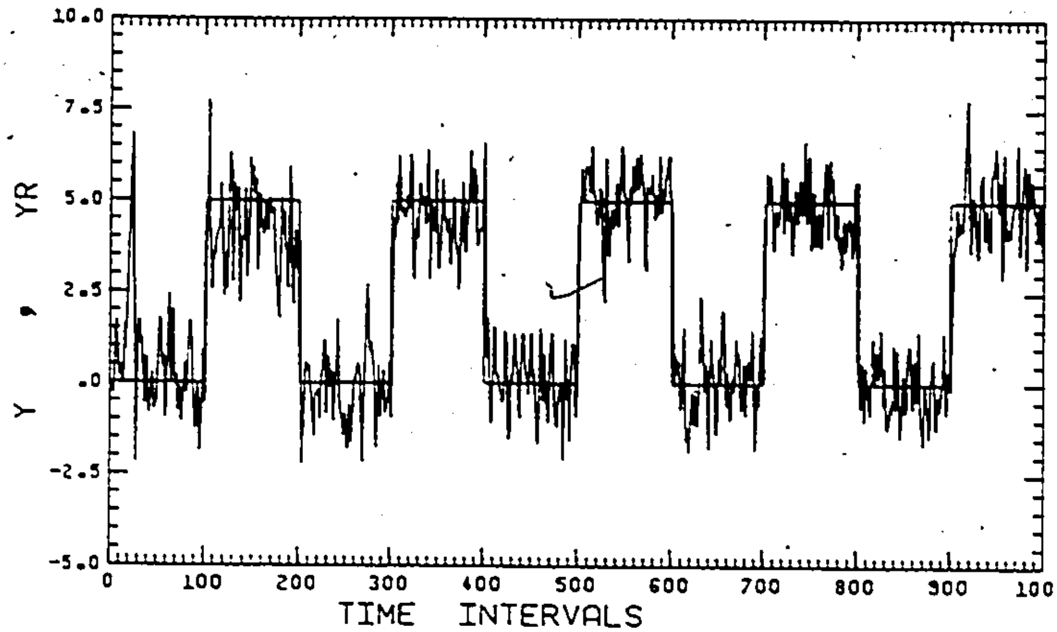


Fig. 4.2 Closed loop response (example 3)

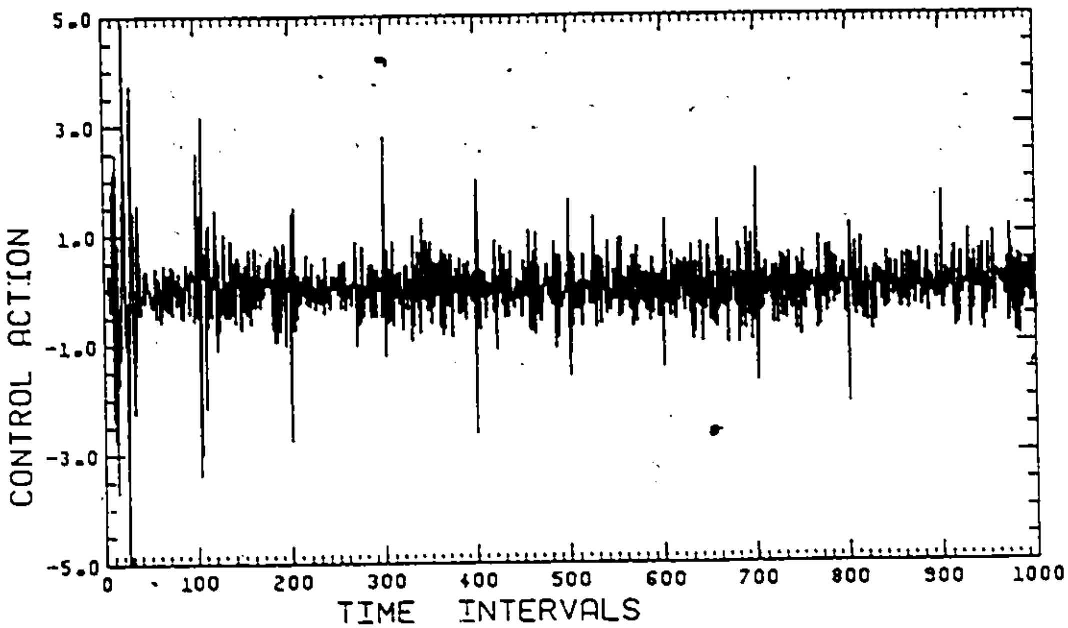


Fig. 4.3 Plot of control variable (example 3)

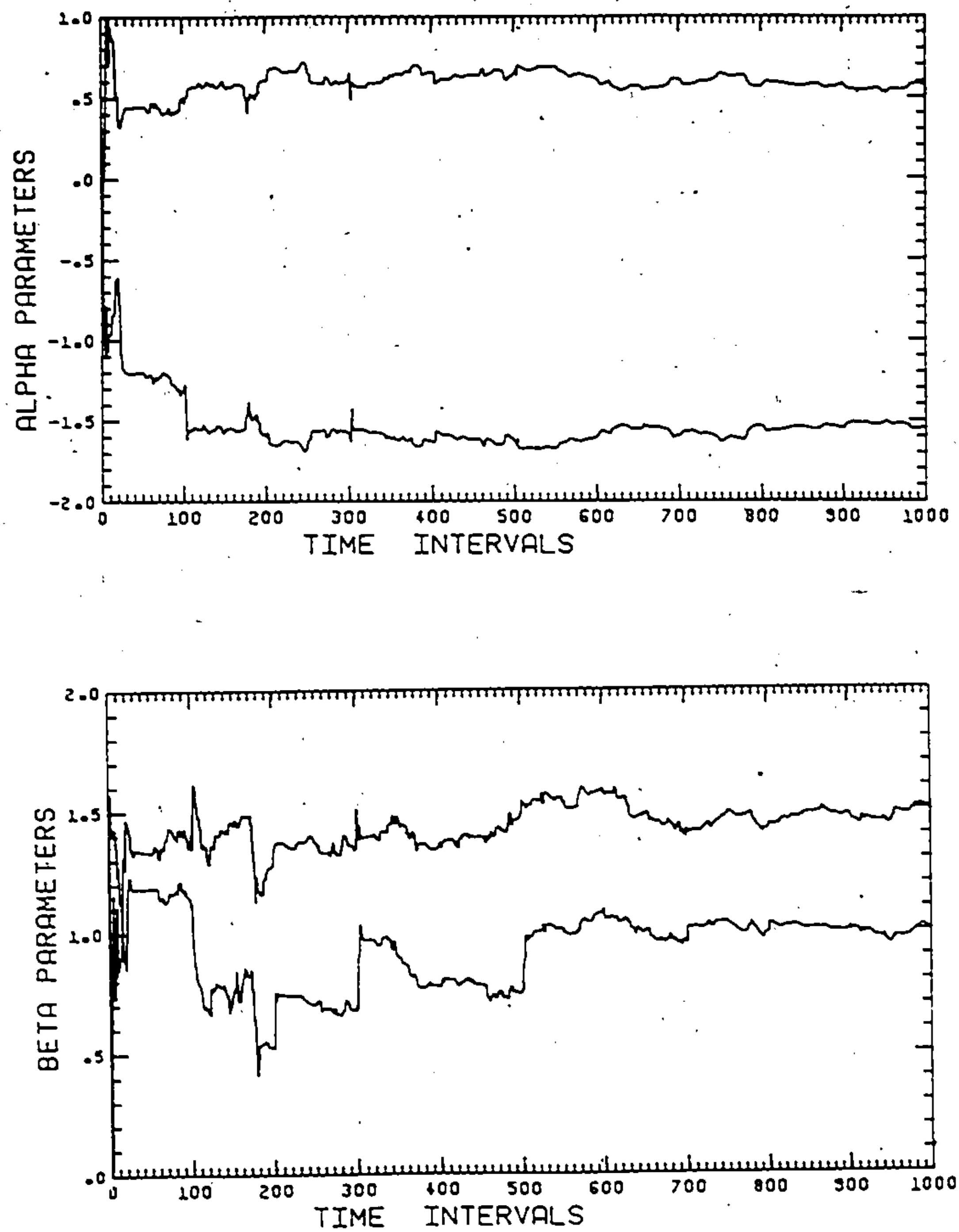


Fig. 4.4 Plot of the estimated system parameters (example 3)

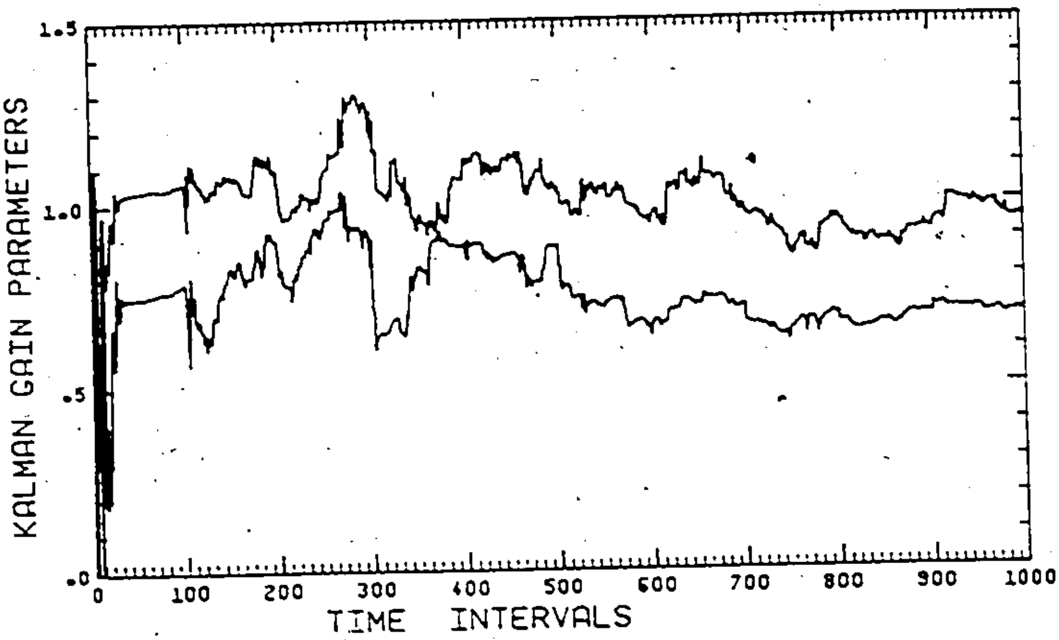


Fig. 4.5 Plot of the estimated Kalman gain parameters (example 3)

settle down. This is due to closed loop identifiability problems, which cause the estimated parameters to be scaled by b_1. If the input signal is made persistently exciting the estimated parameters converge more rapidly. Alternatively if a good estimate of b_1 is used and the corresponding diagonal element of P_o is set to a low value, the estimated parameters again converge rapidly.

Figures 4.6 and 4.7 show plots of the residual sequence ε_t and its auto-correlation function respectively. The confidence bounds on the auto-correlation sequence represent the 95% confidence limits. Thus the residual sequence may be passed as white noise and hence the resulting estimated parameters as unbiased.

To demonstrate the applicability of the algorithm to over-parameterized systems, the following model was simulated.

$$(1 - 0.9z^{-1})(1 - 0.7z^{-1})y_t = (1 - 0.7z^{-1})u_{t-1} + (1 + 0.5z^{-1})e_t$$

the above system has a pole-zero cancelation at z = 0.7, thus the traditional indirect pole-placement controllers cannot be applied sucessfully due to the ill-coditioning in the controller design algorithms.

The above system is equivalent to the state space model

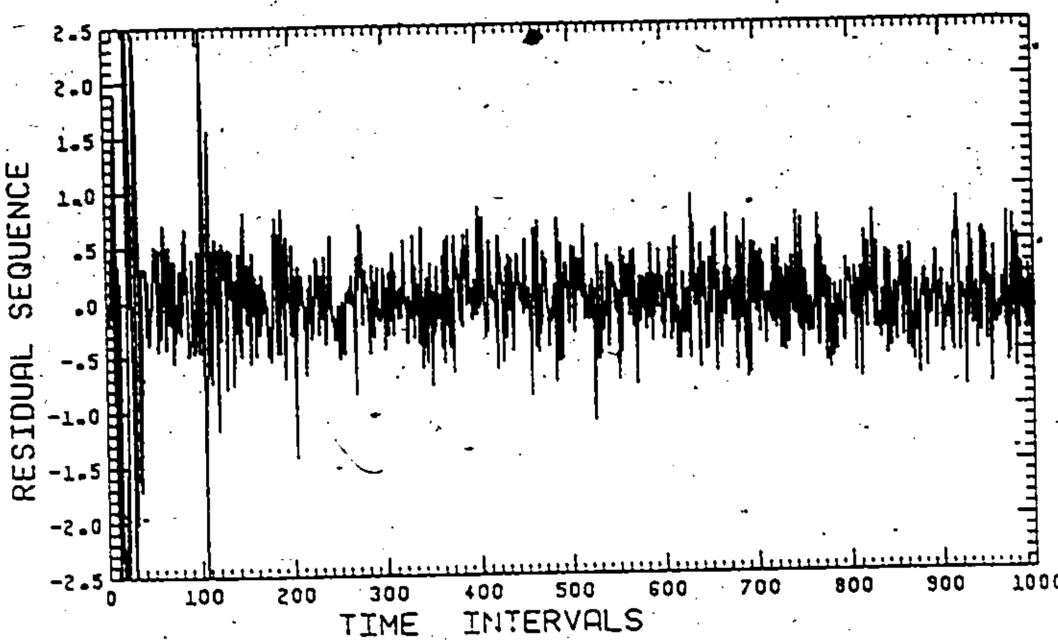


Fig. 4.6 Plot of the residual sequence (example 3)

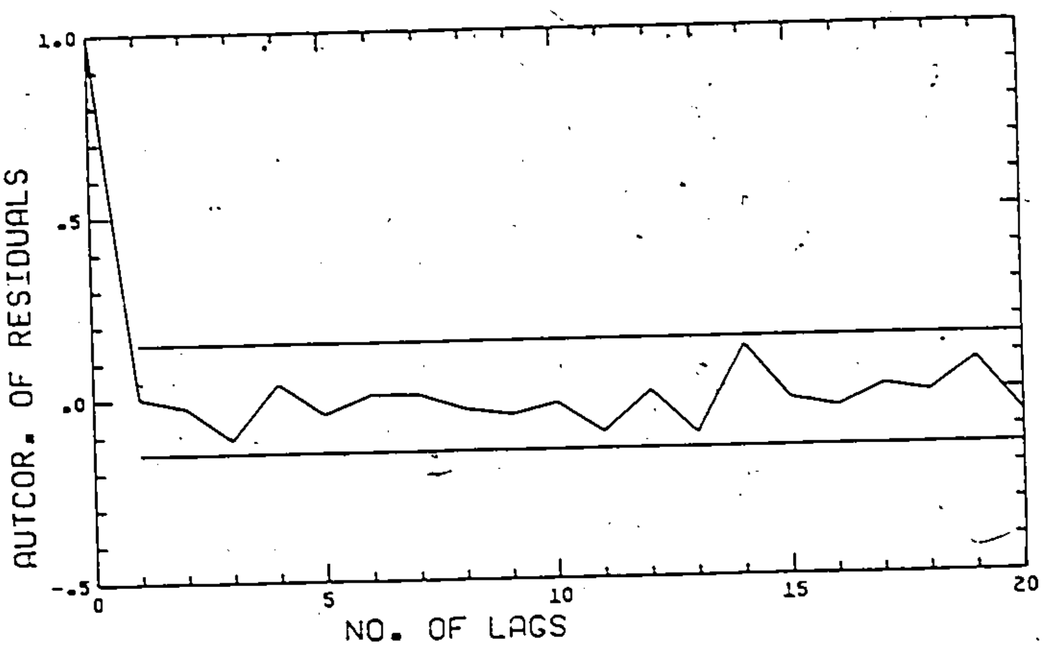


Fig. 4.7 Plot of the auto-correlation of the residuals (example 3)

$$Z_{t+1} = \begin{bmatrix} 1.6 & 1 \\ -0.63 & 0 \end{bmatrix} Z_t + \begin{bmatrix} 1 \\ -0.7 \end{bmatrix} u_t + \begin{bmatrix} 2.1 \\ -0.63 \end{bmatrix} e_t$$

$$y_t = [1 \; 0] Z_t + e_t$$

$$e_t \simeq (0, 0.1)$$

The simulated system is uncontrollable, since the pole at $z = 0.7$ cannot be shifted. This pole is however well damped. It was required that the pole at $z = 0.9$ be shifted to the origin to improve the transient response.

The starting values for the adaptation process were chosen as:

$$P_o = 10^4 I$$

$$\theta_o = [0.01, 0.01, 0.1, 0.01, 0.01, 0.01]$$

Figures 4.8 and 4.9 show plots of the output and control variables respectively. It is evident from these plots that satisfactory regulatory action has been achieved without the saturation of the controller variables experienced in section 3.4. Thus the state space STC based on the RPE gives a dramatic improvement over other indirect adaptive pole placement algorithms for systems with unknown or time varying model orders. For such systems an upper bound on the model order could be used and the algorithm proposed in this chapter may be used to get round the resulting ill-conditioning of the controller and estimation algorithms.

4.6 Concluding Remarks

The state space STC is an effective way to adaptively

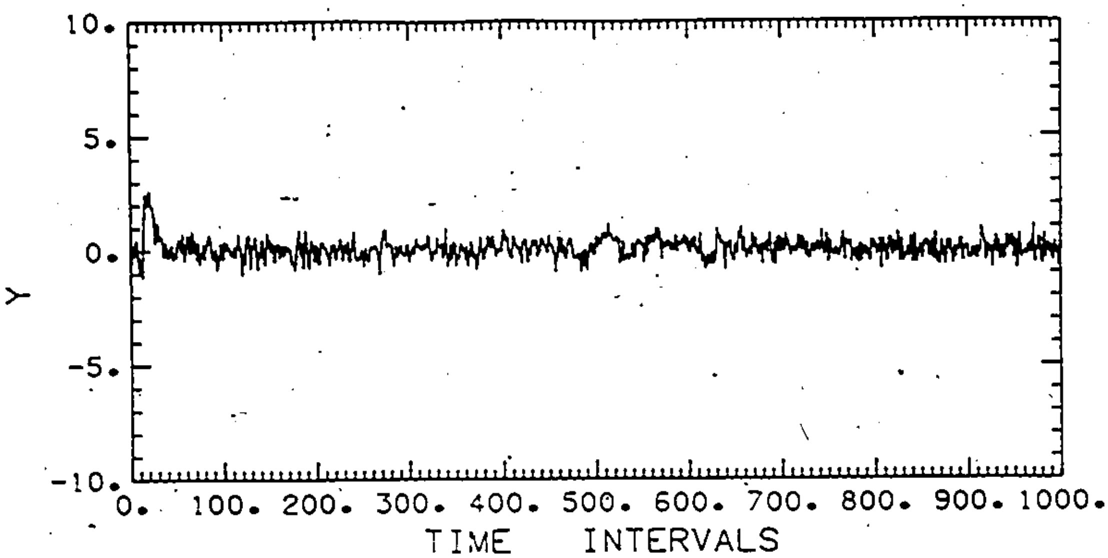


Fig. 4.8 Closed loop response for the overparameterized system (example 4)

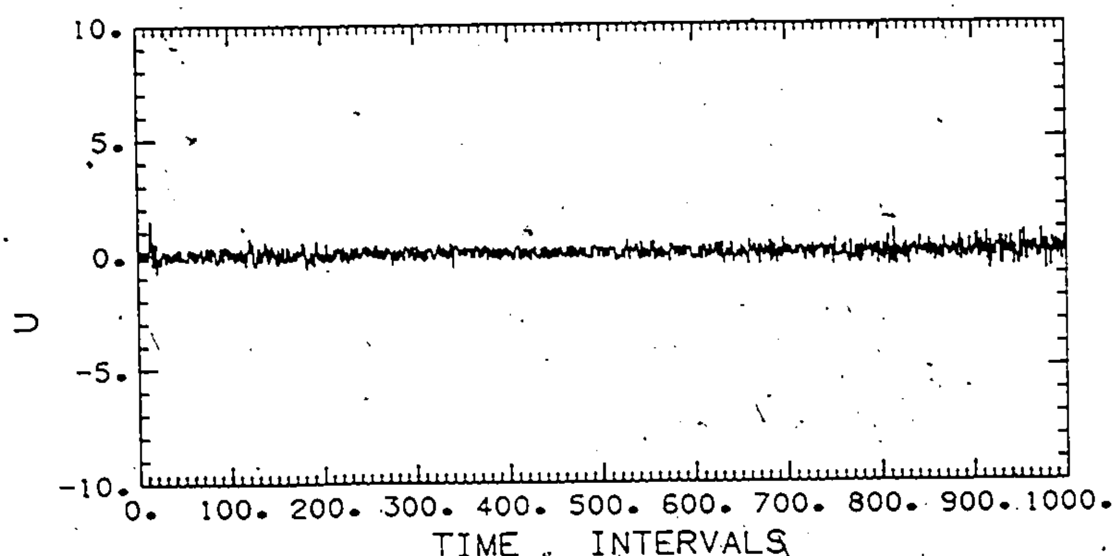


Fig. 4.9 Plot of the control variable (example 4), showing stable control

control non-minimum phase systems. The main disadvantage of the previously reported methods, and the algorithm proposed in chapter 3, is the on-line computation of transformation matrices. This was due to the fact that the then existing algorithms for joint state and parameter estimation required special canonical forms. In this chapter we have demonstrated that recent algorithms for joint state and parameter estimation such as proposed by [66], may be used for joint state and parameter estimation in the controllable canonical form. This overcomes the main drawback of the previous state space STC's.

Intuitively the modified algorithm should be less sensitive to over-parameterization, since the ill-conditioning associated with the inversion of a nearly singular controllablity matrix is avoided. Simulation results presented in this chapter show that the proposed algorithm worked well when it was applied to a system with a stable pole-zero cancellation.

The RPE method is equivalent to the off-line maximum likelihood method, since they both minimize the same objective function (4.6), Thus the parameter estimates are asymptoticaly unbiased. This suggests that self-tuning would occur if the model order is known a-priori. A further advantage of the proposed method is that the matrices of partial derivatives are sparse and have very simple structures hence are easily computed on-line. The method can be easily extended to MIMO systems. This extension is the main focus of chapter 5.

CHAPTER 5

5. MULTIVARIABLE STATE SPACE STC VIA RPE ESTIMATION

5.1 Introduction

In section 3.3 an explicit adaptive STC based on the state space model and BSE estimation was proposed. The limitation of such algorithms to controllable systems was demonstrated by simulations. Using the estimation techniques discussed in chapter 4 however, a modified version of the MIMO state sapce STC of 3.3 may be developed to side step the on-line transformation of the state vector, thus making the MIMO state space STC more widely applicable.

Using the BSE estimator the observability indices are assumed to be known, the estimation is done in the observable canonical form. The controllability indices can then be determined and the pole assignment control law implemented by transforming the system to the controllable canonical form. Unlike the SISO case the controllable canonical realization of the system characteristic matrix is not the transpose of its observable canonical realisation. Thus the computational savings in avoiding this transformation is of great significance.

Using the RPE estimator the controllablity indices are assumed to be known, the estimation is done in the controllable canonical form. Pole assignment control can then be implemented quite easily. The modified algorithm has a further advantage in that it is easy to select by inspection, a feed back matrix to decouple the system dynamics as seen by the inputs. However some cross coupling terms may be unavoidable if cross coupling terms are present in the system output matrix.

The algorithm proposed in this chapter and also in [78] is applicable to unstable but stabilizable and or non-minimum phase systems. There is however a restriction that the number of inputs have to be equal to the number of outputs.

5.2 MIMO Parameter and State Estimation by RPE Method

Consider the MIMO system, modeled in the state space innovations form.

$$X_{t+1} = F_c X_t + G_c U_t + K_c e_t \tag{5.1a}$$

$$Y_t = H_c X_t + e_t \tag{5.1b}$$

where

$$F_c = [F_{c_{i,j}}] \qquad i = 1,\ldots,m \qquad j = 1,\ldots,m \tag{5.1c}$$

$$F_{c_{i,j}} = \begin{bmatrix} 0 & & & \\ \cdot & & I_{n_i-1} & \\ \cdot & & & \\ \cdot & & & \\ 0 & & & \\ a_{ii}(1) & \cdots & & a_{ii}(n_i) \end{bmatrix}, \quad \underset{i \neq j}{F_{c_{i,j}}} = \begin{bmatrix} 0 & & & 0 \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ 0 & & & 0 \\ a_{ij}(1) & \cdots & & a_{ij}(n_{ij}) \end{bmatrix}$$

$$G_c = [\mu_{\ell_1}, \ldots, \mu_{\ell_m}] \; , \; K_c = [k_1, \ldots, k_m]$$

$$H_c = \begin{bmatrix} b_1^T \\ \cdot \\ \cdot \\ b_m^T \end{bmatrix}$$

$a_{ij}(.)$ are scalar parameters which determine the dynamics of F_c.

b_i^T are n dimensional row vectors corresponding to the rows of H_c.

k_i are n dimensional vectors corresponding to the columns of the Kalman gain matrix K_c.

μ_{ℓ_i} is a unit column vector of dimension n, with a 1 in the ℓ_i'th row, where

$$\ell_i = n_1 + \ldots n_i$$

$$n_1 + \ldots n_i + \ldots n_m = n$$

n_i are the controllability indices of the system which must be estimated a-proiri. However due to the robustness of this algorithm to over-parameterization, over-estimation of these indices would not cause unstable control as would be the case for the algorithms presented in chapter 3.

The RPE estimation algorithm may then be generalized to the multivariable case as follows.

Consider

$$\min_{\theta,\Lambda} V(\theta,\Lambda) = \min_{\theta,\Lambda}\{\varepsilon_t^T \Lambda^{-1} \varepsilon_t + \log|\Lambda|\} \qquad (5.2a)$$

ε_t is a vector prediction error sequence given by

$$\varepsilon_t = Y_t - H_c(\theta)X_t \tag{5.2b}$$

$$\hat{X}_{t+1} = F_c(\theta)\hat{X}_t + G_c(\theta)U_t + K_c(\theta)\varepsilon_t \tag{5.2c}$$

where θ is a parameter vector containing the unknown elements of (5.1).

Define

$$\Psi_t = -\left(\frac{d\varepsilon_t}{d\theta}\right)^T = \left(\frac{d}{d\theta}\,[H_c(\theta)X_t(\theta)]\right)^T \quad \text{(a } \dim(\theta) \times m \text{ matrix)} \tag{5.3a}$$

The columns of the matrix Ψ_t represent the negative gradients of the elements of the innovations vector ε_t with respect to the parameter vector θ.

$$W_t = \frac{d}{d\theta}[\hat{X}_t(\theta)] \quad \text{(an } n \times \dim(\theta) \text{ matrix} \tag{5.3b}$$

$$D_t = \frac{\partial}{\partial\theta}[H_c(\theta)\hat{X}_t]|_{\theta = \theta_t} \quad \text{(an } n \times \dim(\theta) \text{ matrix)} \tag{5.3c}$$

$$M_t = \frac{\partial}{\partial\theta}[F_c(\theta)\hat{X}_t + G_c(\theta)U_t + K_c(\theta)\varepsilon_t]|_{\theta = \theta_t} \quad \text{(an } n \times \dim(\theta) \text{ matrix)} \tag{5.3d}$$

Let

$$\theta = [a_1^T \cdot\cdot a_i^T \cdot\cdot a_m^T, b_1^T \cdots b_m^T, k_1^T \cdots k_m^T]$$

where

a_i^T corresponds to the unknown rows of the F_c matrix.

Then an algorithm for the recursive minimization of (5.2) is given by:

1: Compute the innovations vector

$$\varepsilon_t = Y_t - \hat{Y}_t \tag{5.4a}$$

2: Update $\mathrm{cov}(\varepsilon_t)$

$$\Lambda_t = \Lambda_{t-1} + \frac{1}{t}(\varepsilon_t \varepsilon_t^T - \Lambda_{t-1}) \tag{5.4b}$$

3: Compute adaptation gain matrix

$$L_t = P_{t-1}\Psi_t(\lambda_t\Lambda_t + \Psi_t^T P_{t-1}\Psi_t)^{-1} \tag{5.4c}$$

4: Update parameter estimates

$$[\theta_t = \theta_{t-1} + L_t\varepsilon_t]|_{\theta_t \in D_s} \tag{5.4d}$$

5: Update covariance matrix

$$P_t = \frac{1}{\lambda_t}[P_{t-1} - P_{t-1}\Psi_t\Psi_t^T P_{t-1}(\lambda_t\Lambda_t + \Psi_t^T P_{t-1}\Psi_t)^{-1}]$$

6: Predict next state estimates

$$\hat{X}_{t+1} = F_{c,t}\hat{X}_t + G_{c,t}U_t + K_{c,t}\varepsilon_t \tag{5.4f}$$

7: Predict next output

$$\hat{Y}_{t+1} = H_{c,t}\hat{X}_{t+1} \tag{5.4g}$$

8: Compute gradient of $(\hat{X}_{t+1})$

$$W_{t+1} = [F_{c,t} - K_{c,t}H_{c,t}]W_t + M_t - K_{c,t}D_t \tag{5.4h}$$

9: Compute gradient of $(\hat{Y}_{t+1})$

$$\Psi_{t+1} = W_{t+1}^T H_{c,t}^T + D_{t+1}^T \tag{5.4i}$$

Where we have used the notation

$F_{c,t} = F_c(\theta_t)$

$G_{c,t} = G_c(\theta_t)$

$H_{c,t} = H_c(\theta_t)$

$K_{c,t} = K_c(\theta_t)$

L_t = an adaptation gain matrix of dimensions $\dim(\theta) \times m$

P_t = the covariance matrix, of dimensions

$$\dim(\theta) \times \dim(\theta) \text{ with } P_o = \sigma I,\ \sigma >> 0.$$

D_s is the stability region of the predictor:

$$D_s\{\theta | (F_c - K_c H_c) \text{ is strictly stable}\} \tag{5.4j}$$

The projection of the estimated parameters into the stability region is required for the asymptotic convergence of the RPE estimator, and may be implemented via the projection algorithm described in section 4.4. The characteristic polynomial of the predictor matrix $[F_c - K_c H_c]$ cannot be determined via equations (4.12) for the multivariable case. It may however be determined numericaly by using Leverier's algorithm.

Define

$$\begin{aligned} \Pi_c(z) &= \det(zI - F_c + K_c H_c) \\ &= \det(zI - F_c) \\ &= z^n + p_1 z^{n-1} + \dots p_{n-1} z + p_n \\ T_i &= \text{trace}(F_c^i) \end{aligned}$$

then the coefficients p_i can be determined recursively from

$$p_i = -\frac{1}{i}(p_{i-1}T_1 + p_{i-2}T_2 + \dots p_1 T_{i-1} + T_i)$$

where

$$i \quad \{1, \dots, n\}$$

5.3 <u>Pole Assignment Control Law</u>

Using the certainty equivalence principle [9,16] the STC algorithm may be implemented by following each iteration of the joint state and parameter estimation algorithm by the state feedback law

$$U_t = -K_F\hat{X}_t + K_R Y_R \tag{5.5a}$$

where

$$K_F = [K_{F_{ij}}] \tag{5.5b}$$

$$i = 1,\dots m$$

$$j = 1,\dots m$$

$$K_{F_{ii}} = [a_{ii}(1)-\alpha_{ii}(1)\dots a_{ii}(n_i)-\alpha_{ii}(n_i)] \tag{5.5c}$$

$$\underset{i\neq j}{K_{F_{ij}}} = [a_{ij}(1)\dots a_{ij}(n_{ij})] \tag{5.5d}$$

a_{ij} are the estimated system parameters.

α_{ij} are the desired closed loop dynamic parameters.

With the above control law the closed loop system matrix asymptotically becomes

$$F_c = F_c - G_c K_F \tag{5.6a}$$

where

$$F_c = [F_{c,ij}] \tag{5.6b}$$

$$F_{c,ii} = \begin{bmatrix} 0 & & \\ \cdot & & \\ \cdot & I_{n_i-1} & \\ \cdot & & \\ 0 & & \\ \alpha_{ii}(1) & \cdots & \alpha_{ii}(n_i) \end{bmatrix} \tag{5.6c}$$

$$\underset{i\neq j}{F_{c,ij}} = [0] \tag{5.6d}$$

Thus the closed loop characteristic polynomial is given by

$$\Pi(z) = \prod_{i=1}^{m} \pi_i(z) \tag{5.7a}$$

where

$$\pi_i(z) = (-\alpha_{ii}(1) - \alpha_{ii}(2)z \ldots - \alpha_{ii}(n_i)z^{n_i-1} + z^{n_i}) \tag{5.7b}$$

Y_R is a reference input vector, K_R is selected to ensure that for a constant Y_R, $E\{Y_t\} = Y_R$, thus

$$K_R = [H_{c,t}(I - F_c)^{-1}G_c]^{-1} \tag{5.8a}$$

$$\det[H_{c,t}(I - F_c)^{-1}G_c] \neq 0 \tag{5.8b}$$

Alternatively, if the cross-coupling terms in the output matrix may be neglected, (5.8a) reduces to

$$K_R = \mathrm{diag}\{K_R(ii)\} \tag{5.8c}$$

where

$$K_R(ii) = \frac{1 - \sum_{j=1}^{n_i} \alpha_{ii}(j)}{\sum_{j=n_1+\ldots n_{i-1}+1}^{n_1+\ldots n_i} H_{c,ij}}, \quad \sum_{j=n_1+\ldots n_{i-1}+1}^{n_1+\ldots n_i} H_{c,i,j} \neq 0 \tag{5.8d}$$

$H_{c,ij}$ represents the ij'th element of the H_c matrix.

The overall closed loop system is then given by

$$Y_t = H_c(zI - F_c)^{-1}G_cK_RY_R + [H_c(zI - F_c)^{-1}K_c + I]e_t \tag{5.9a}$$

For regulatory action the output covariance matrix may be computed [9] from

$$\mathrm{Cov}_Y = (I + \sum_{i=0}^{\infty}[H_cF_c^iK_c][H_cF_c^iK_c]^T)\Lambda \tag{5.9b}$$

for dead beat control

$$F_c^i = 0 \quad , \quad i \geq n \tag{5.9c}$$

thus (9b) reduces to

$$Cov_Y = (I + \sum_{i=0}^{n-1}[H_c F_c^i K_c][H_c F_c^i K_c]^T)\Lambda \tag{5.9d}$$

For a multivariable system the required feedback gain matrix to arbitrarily place the closed-loop poles is non-unique. Thus the remaining degrees of freedom after closed-loop pole assignment, may be used to satisfy other design criteria such as decoupling, or the reduction of the sensitivity of the closed-loop system to parameter variation. In this algorithm this extra design freedom was used to decouple the system dynamics as seen by the inputs. However cross-coupling zeros may be unavoidable if there are corresponding terms in the system output matrix. Attempts to cancel such zeros may lead to unstable control in non-minimum phase systems.

5.4 Simulation Results

To illustrate the method, the following system was simulated.

$$X_{t+1} = \begin{bmatrix} 0 & 1 & 0 \\ -0.63 & 1.6 & 0 \\ 0 & -0.2 & 0.8 \end{bmatrix} X_t + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} U_{1,t} \\ U_{2,t} \end{bmatrix} + \begin{bmatrix} 0.716 & 0.3 \\ 1.026 & -0.3 \\ -0.2 & 1.3 \end{bmatrix}\begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix} \tag{5.10a}$$

$$Y_t = \begin{bmatrix} 1.5 & 1.0 & 0 \\ 0 & 0 & 1.0 \end{bmatrix} X_t + \begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix} \tag{5.10b}$$

$$\Lambda = E\{ee^T\} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

The above model has poles at z=0.9, z=0.7 and z=0.8, and is non-minimum phase. The exact parameter vector is thus:

$$\theta = [-0.63,1.6,0,0,-0.2,0.8,1.5,1.0,0,0,0,1.0,0.716,1.026,-0.2,0.3,-0.3,1.3]^T$$

The matrices of partial derivatives are then:

$$M_t = \left[\begin{array}{c|c|ccc|c|c} \bigcirc & \bigcirc & \bigcirc & \bigcirc & & & \\ \hat{X}_t^T & \bigcirc & \bigcirc & \bigcirc & & I\varepsilon_{1,t} & I\varepsilon_{2,t} \\ \bigcirc & \hat{X}_t^T & \bigcirc & \bigcirc & & & \end{array}\right]$$

$$D_t = \left[\begin{array}{cc|c|c|cc} \bigcirc & \bigcirc & \hat{X}_t^T & \bigcirc & \bigcirc & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc & \hat{X}_t^T & \bigcirc & \bigcirc \end{array}\right]$$

where

$$\bigcirc = [0\ 0\ 0]$$

The desired closed loop system characteristic matrix was selected as

$$F_c = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

to obtain dead beat control. The closed loop system is also effectively decoupled since there are no cross-coupling terms in the system output matrix H_c.

To maintain stability during the initial stages of the adaptation, it was found necessary to limit the control signals to

$$\begin{bmatrix} -5 \\ -5 \end{bmatrix} \leqslant U_t \leqslant \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

However as pointed out in [18], this could lead to control signals which are not persistently exciting. This was noticed in the simulations, but was overcome by adding a dither signal vector to the limited control signal. Thus

$$U_T = -K_F\hat{X}_t + K_R Y_R$$

$$\begin{bmatrix} -5 \\ -5 \end{bmatrix} \leqslant U_T \leqslant \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$U_t = U_T + DTH$$

where DTH is a vector of dither signals each of variance 0.1.

If a fairly accurate guess of the open loop gain parameters is available and the corresponding elements of P_o are set to a low value, the initial values of the control variables are not so excessive. The limiters may then be removed, thus making the injection of the dither signals redundant.

The uncontrolled system i.e with $U_t \equiv 0$, gave the output variances:

$$Var(Y_1) = 18.58$$

$$Var(Y_2) = 3.74$$

Figure 5.1 shows plots of the output variables for the open loop system.

The initial values were

$$\theta_o = [0.01,\ldots,0.01]$$

$$P_o = 10^5 I$$

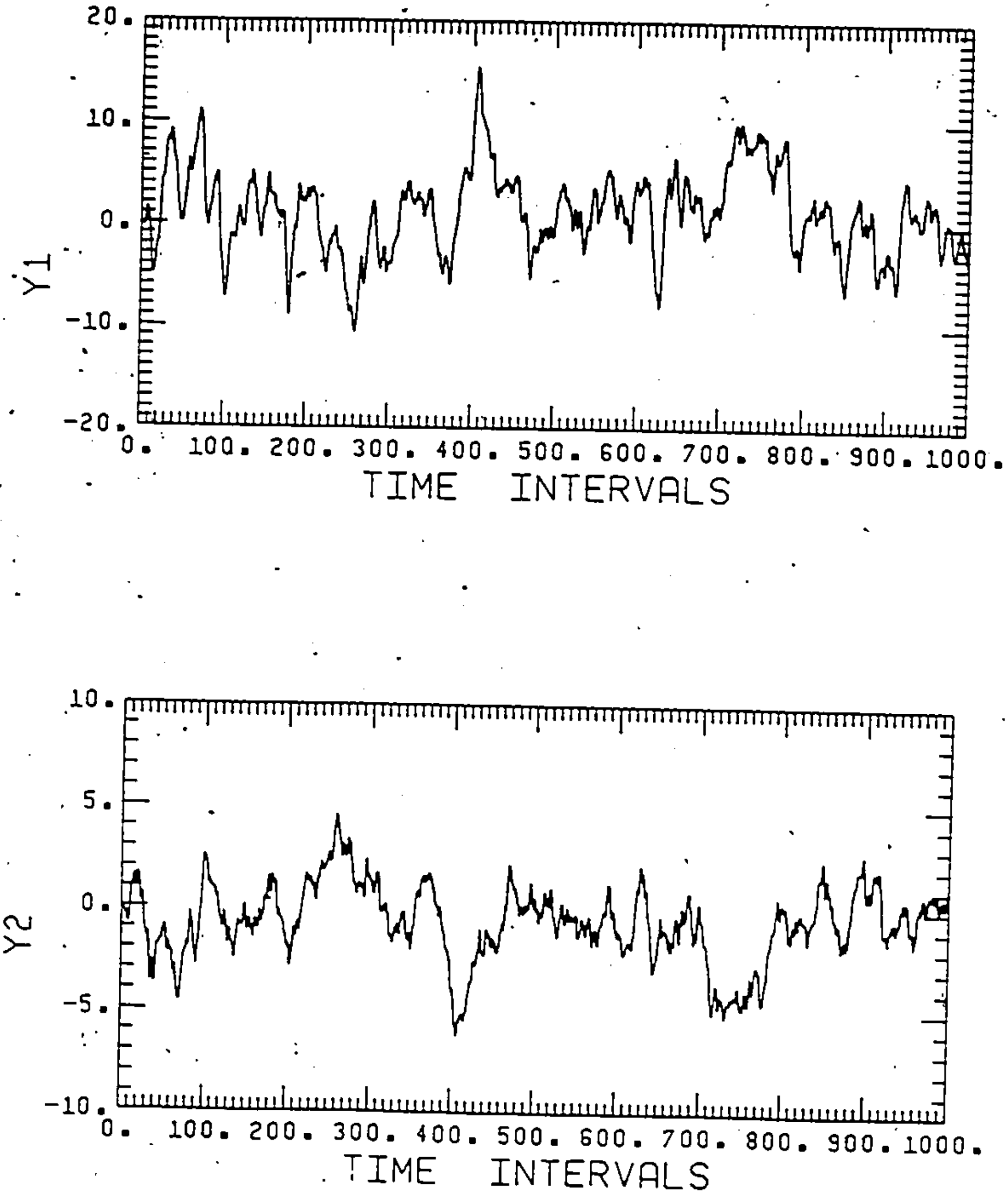


Fig. 5.1 Open loop response (example 5)

$$\lambda_o = 0.95$$

The estimated values after 500 sampling intervals were

$$\theta_{500} = [-0.613, 1/589, 0.004, 0.017, -0.227, 0.798, 1.497, 0.964, -0.002, 0.059, 0.0034, 1.00, 0.718, 0.826, -0.212, 0.272, -0.387, 1.212]^T$$

$$\Lambda_{500} = \begin{bmatrix} 0.17 & 0.01 \\ 0.01 & 0.1 \end{bmatrix}$$

and after settling down, the closed loop output variances were

$$Var(Y_1 - Y_{R,1}) = 3.97$$

$$Var(Y_2 - Y_{R,2}) = 1.56$$

The theoretical values computed from (5.9d) are given by

$$Cov_y = \begin{bmatrix} 0.81 & -0.02 \\ -0.02 & 0.27 \end{bmatrix}$$

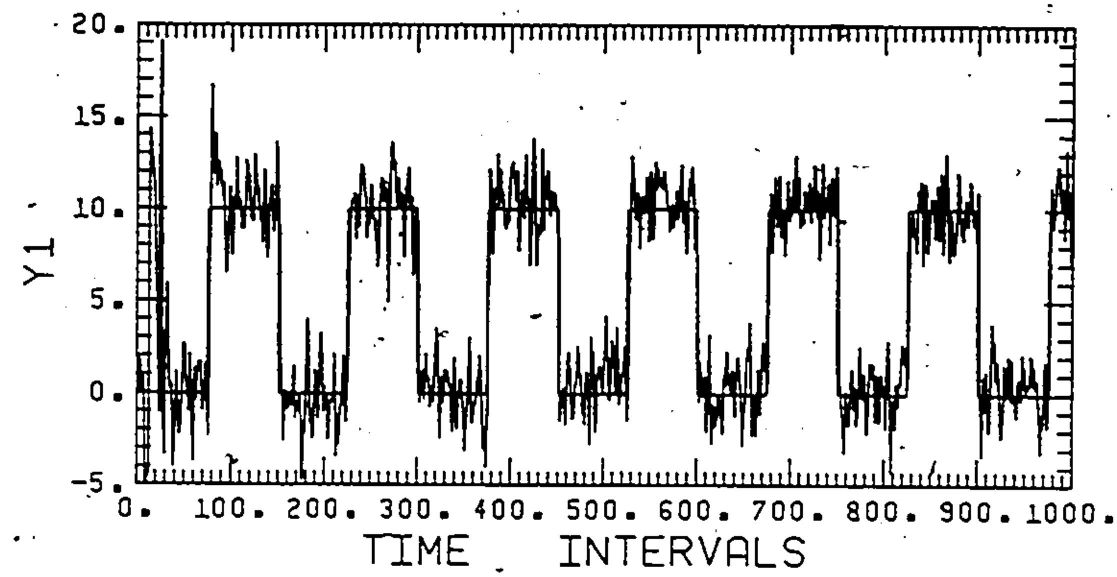
The large discrepancies are obviously due to the effects of the transient response to the set point changes, and the injected dither signals.

Figure 5.2 shows plots of the output variables for the closed loop system, superposed on the command signals. The observed control action variances were

$$Var(U_1) = 1.85$$

$$Var(U_2) = 4.64$$

The observed variances of the control action includes the



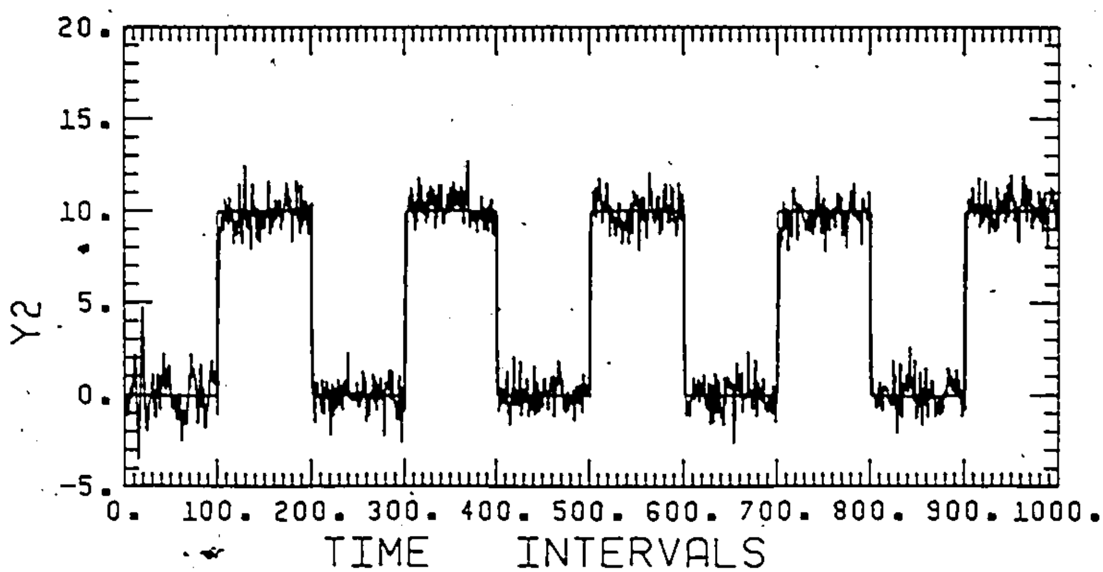
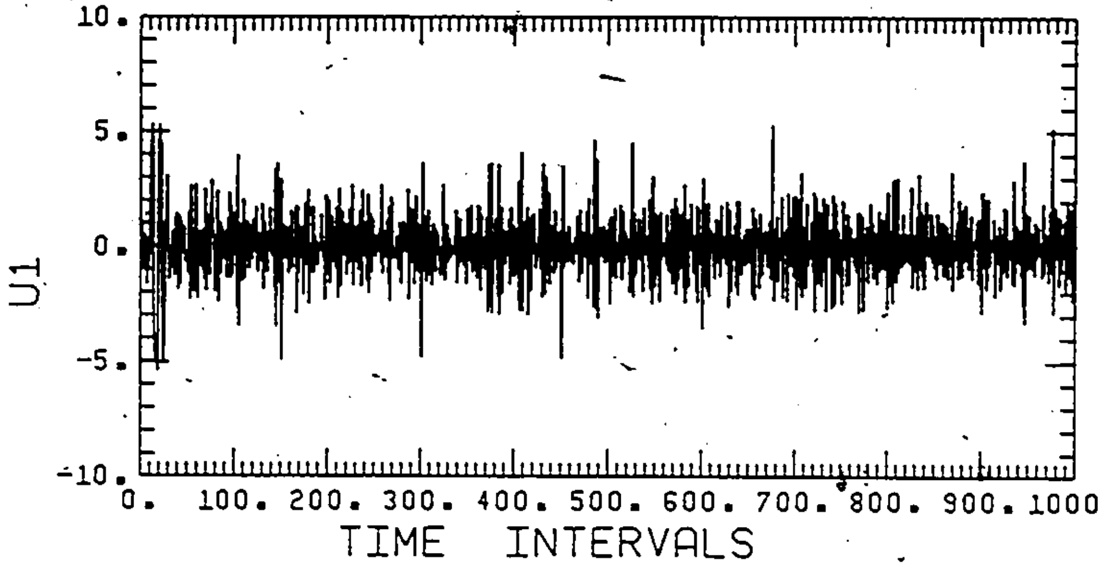


Fig. 5.2 Closed loop response (example 5)



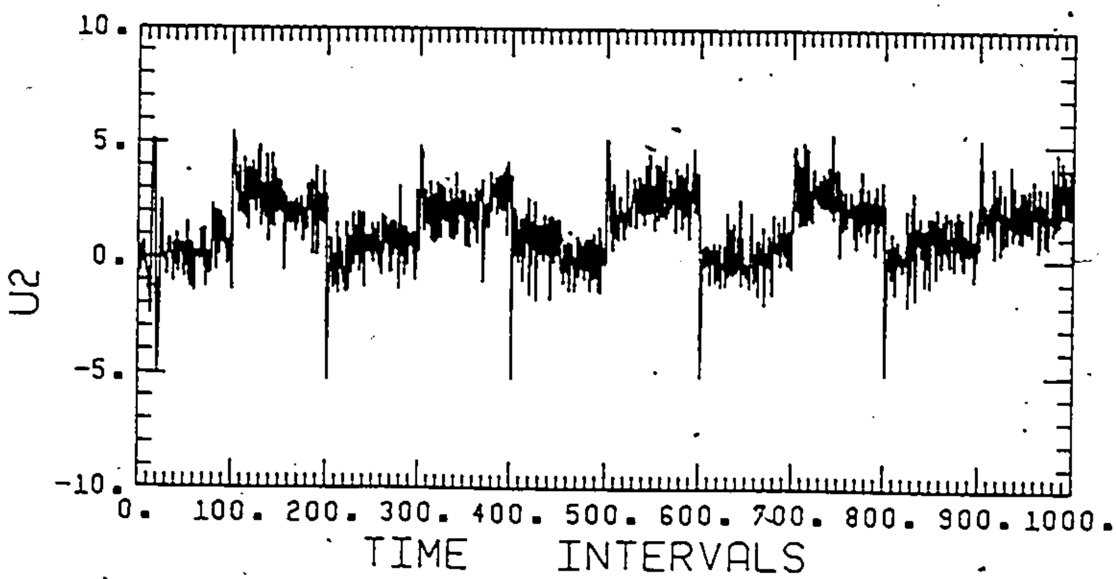


Fig. 5.3 Plot of the control action variables (example 5)

effects of both regulatory and servo action, hence are appreciably higher than would be expected for pure regulatory action. Figure 5.3 shows plots of the control variables.

To study the rate of convergence of the estimated parameters and states, the following quantities were plotted. Figure 5.4 shows a plot of

$$\text{GAMA} = \frac{\|\theta-\theta_t\|^2}{\|\theta\|^2}$$

Figure 5.5 shows plots of

$$\text{GAMA X} = \frac{\|X_t-\hat{X}_t\|^2}{\|X_t\|^2}$$

From figures 5.4 and 5.5, it is evident that the norms of both the state and parameter vectors have reasonably converged to zero after after 500 sampling intervals.

Figures 5.6,5.7 and 5.8 show plots of the estimated system and Kalman gain parameters whilst Figure 5.9 shows plots of the residual sequences. To check the adequacy of the estimated parameters, the auto-correlation functions of the residuals were computed. Figure 5.10 shows plots of the auto-correlation functions of the residuals. From the plots it is evident that the residual sequences may be accepted as white with 95% confidence, hence the estimated model may be accepted as adequate and thus implying that self-tuning has occured.

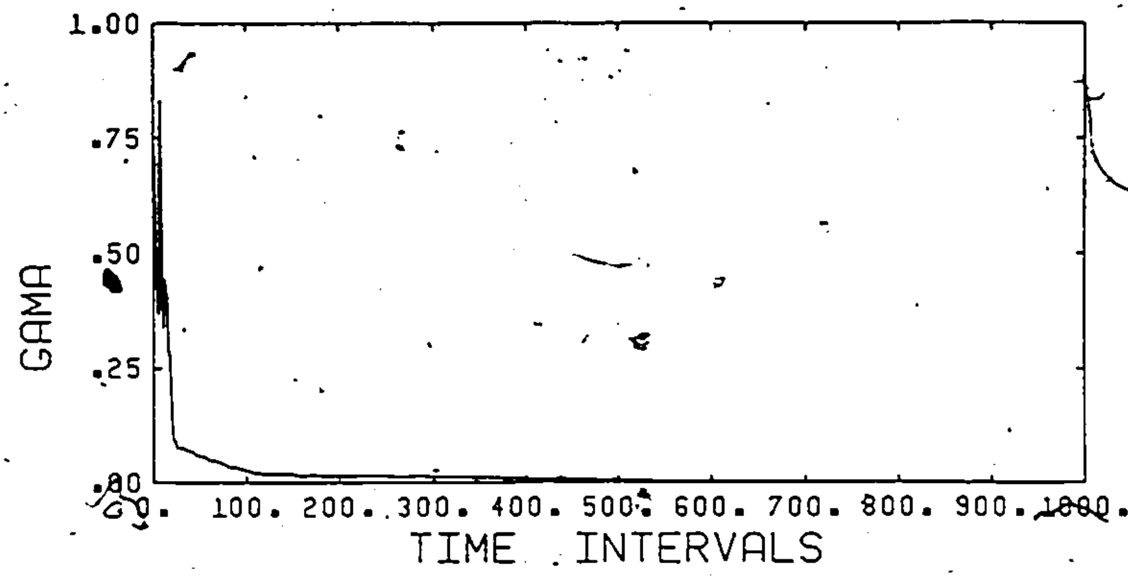


Fig. 5.4 Rate of convergence of the parameter error (example 5)

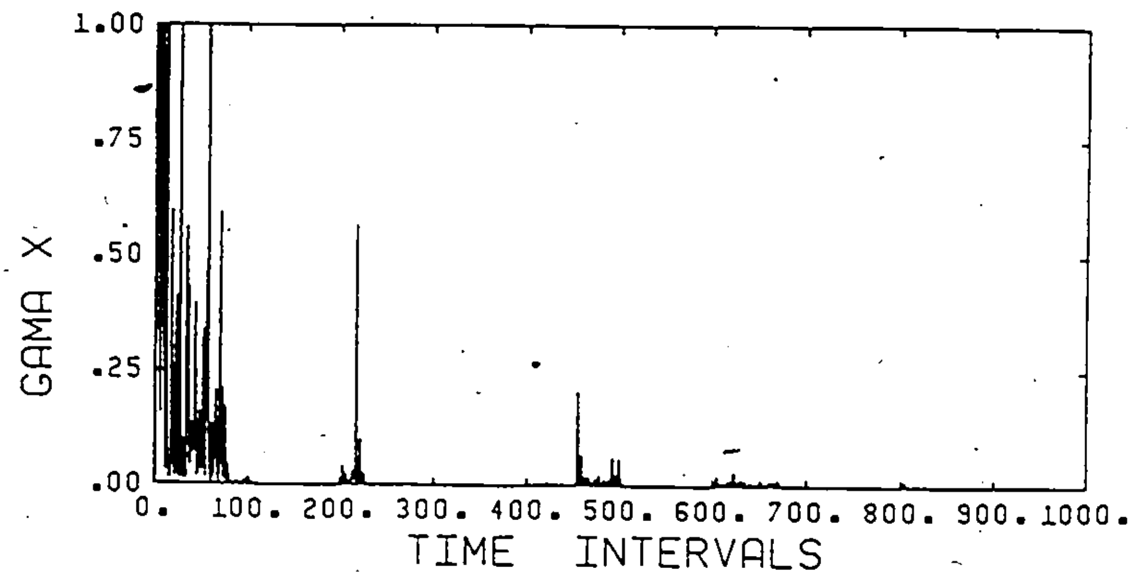
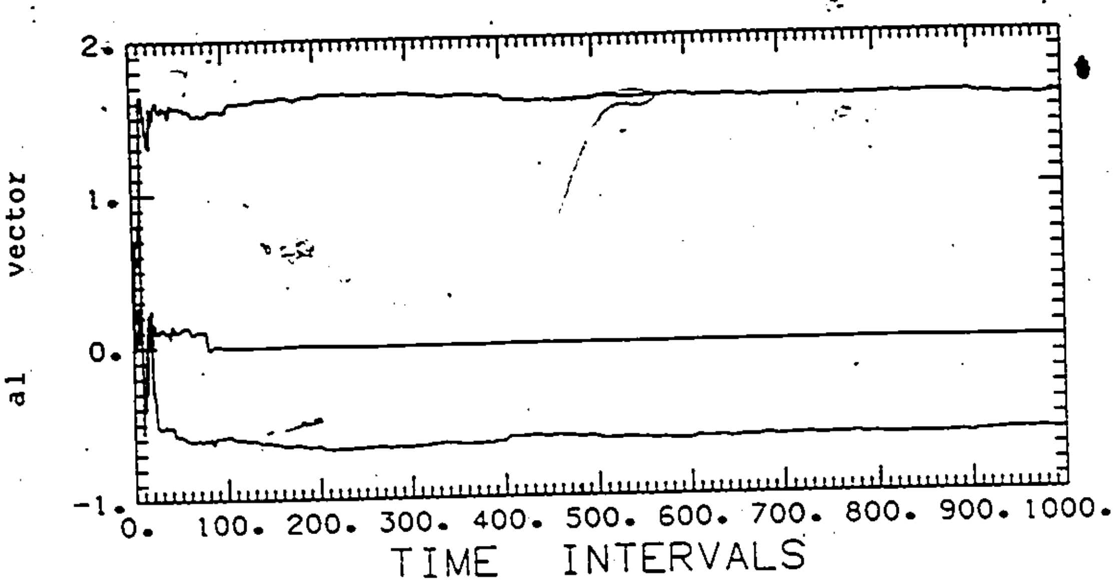


Fig. 5.5 Rate of convergence of the state error (example 5)



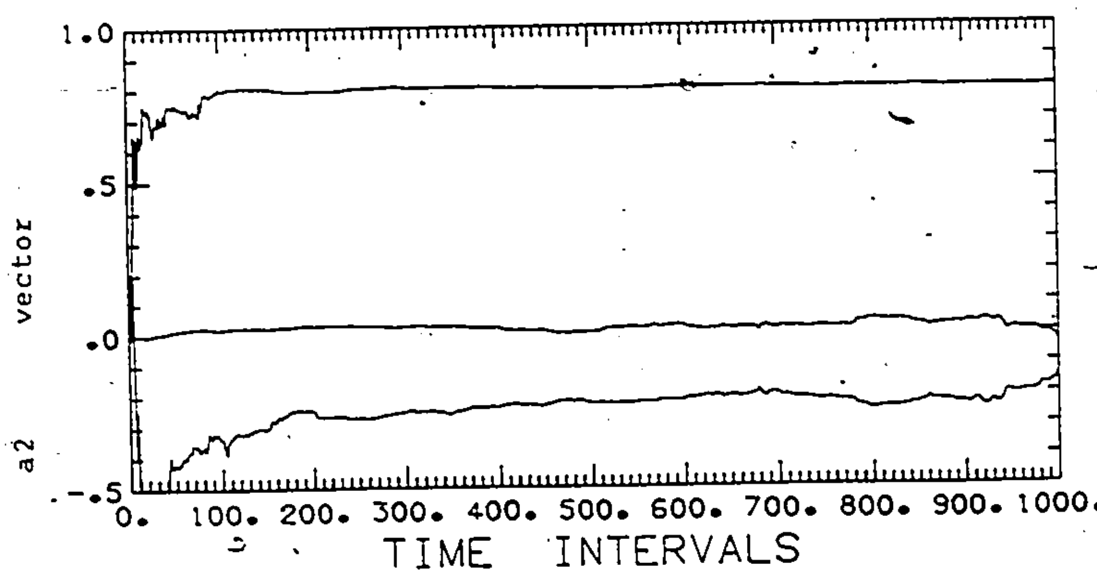
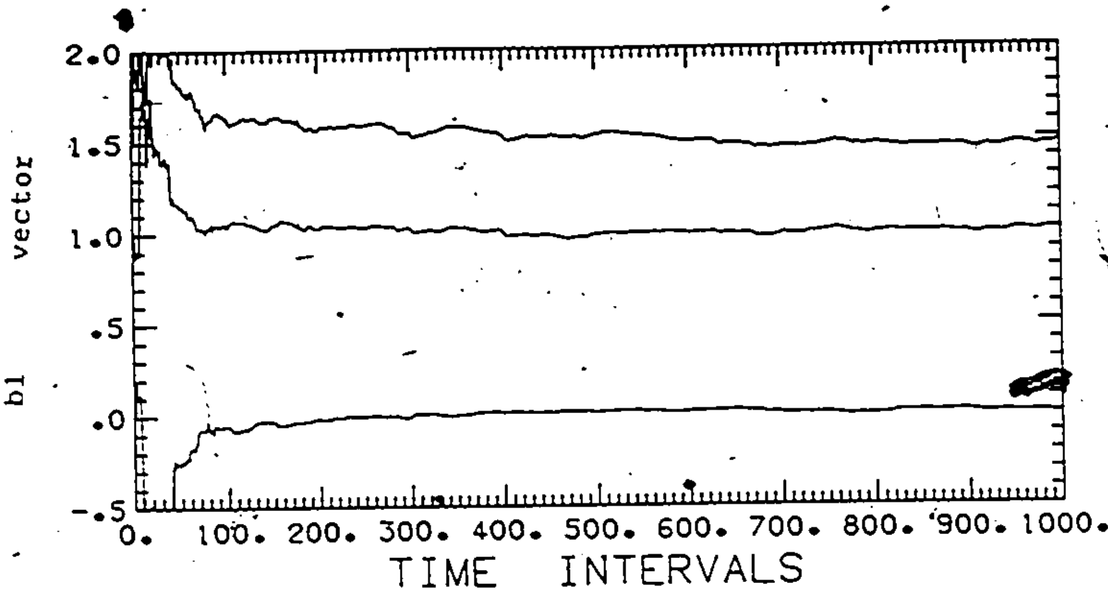


Fig. 5.6 Plot of the estimated F-matrix parameters (example 5)



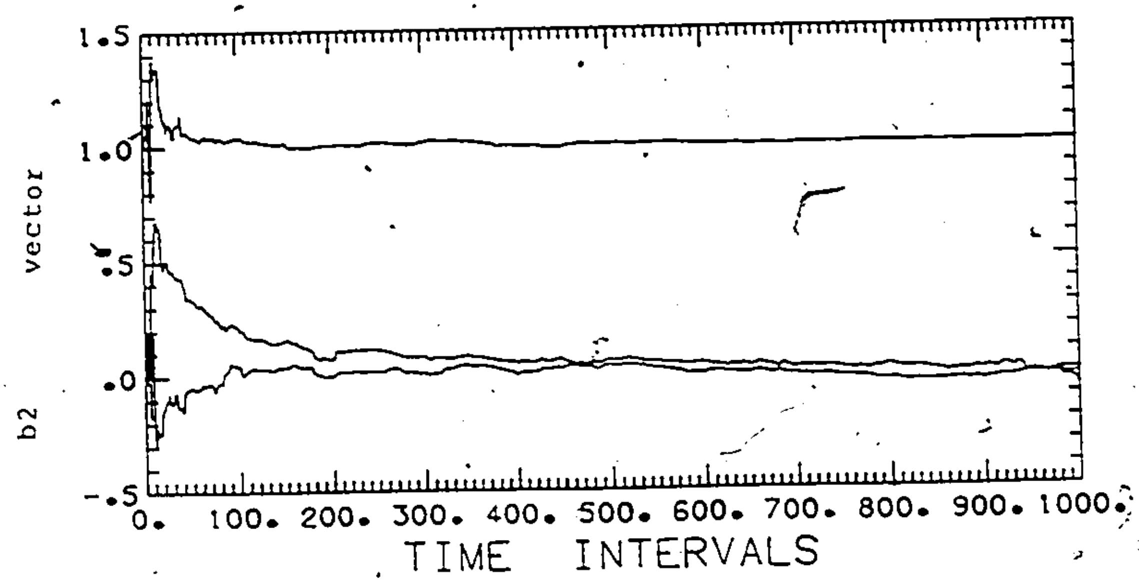


Fig. 5.7 Plot of the estimated H-matrix parameters (example 5)

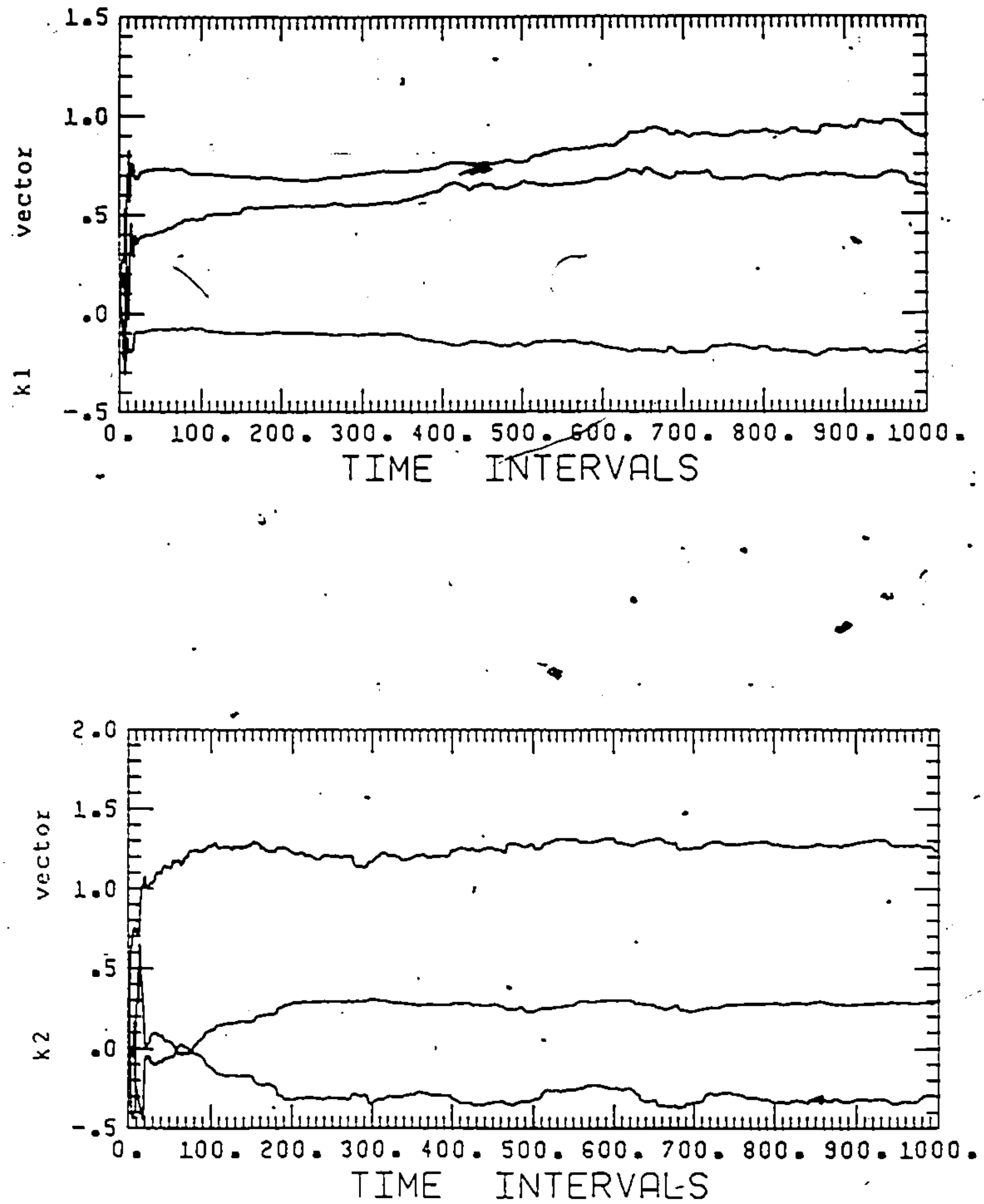
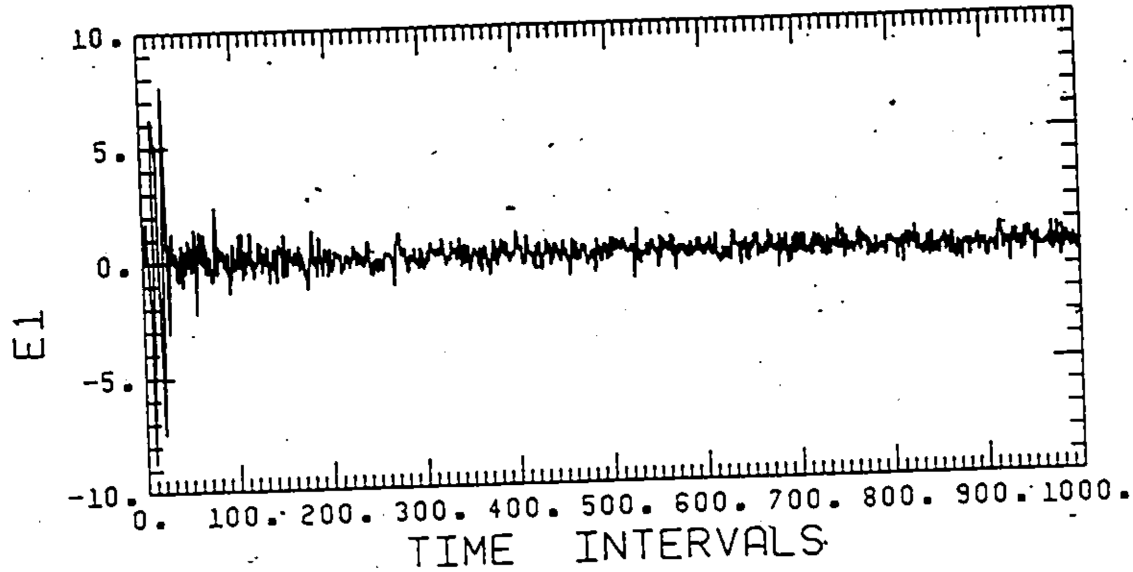


Fig. 5.8 Plot of the estimated K-matrix parameters (example 5)



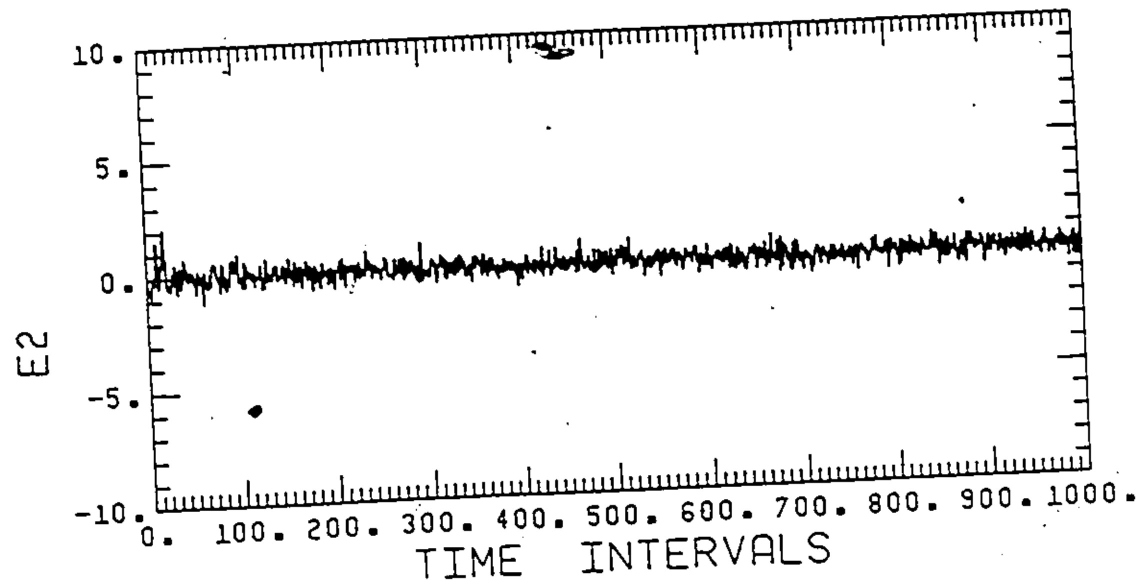
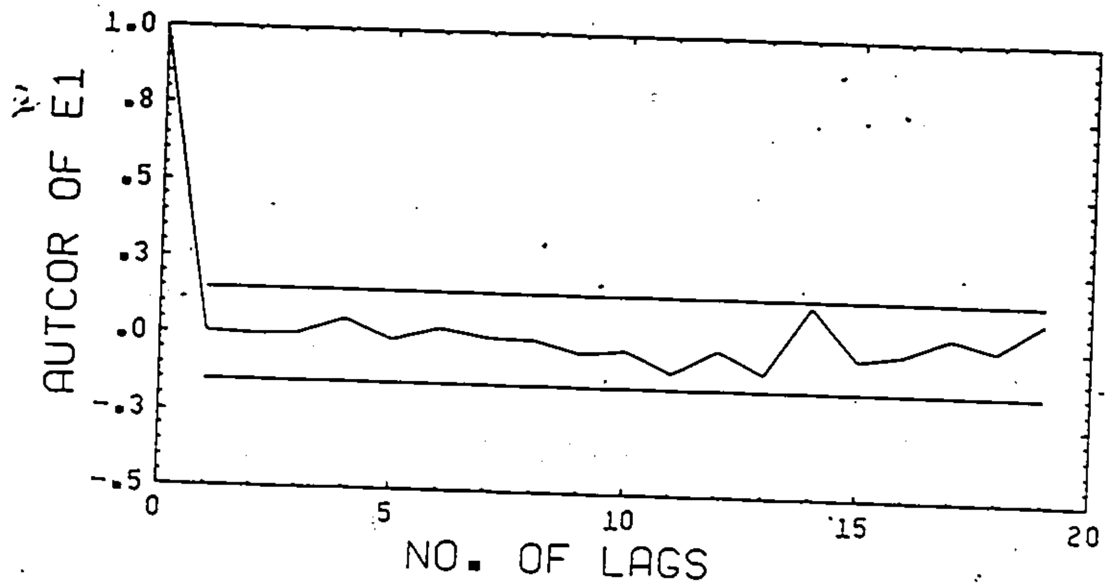


Fig. 5.9 Plot of the residual sequences (example 5)



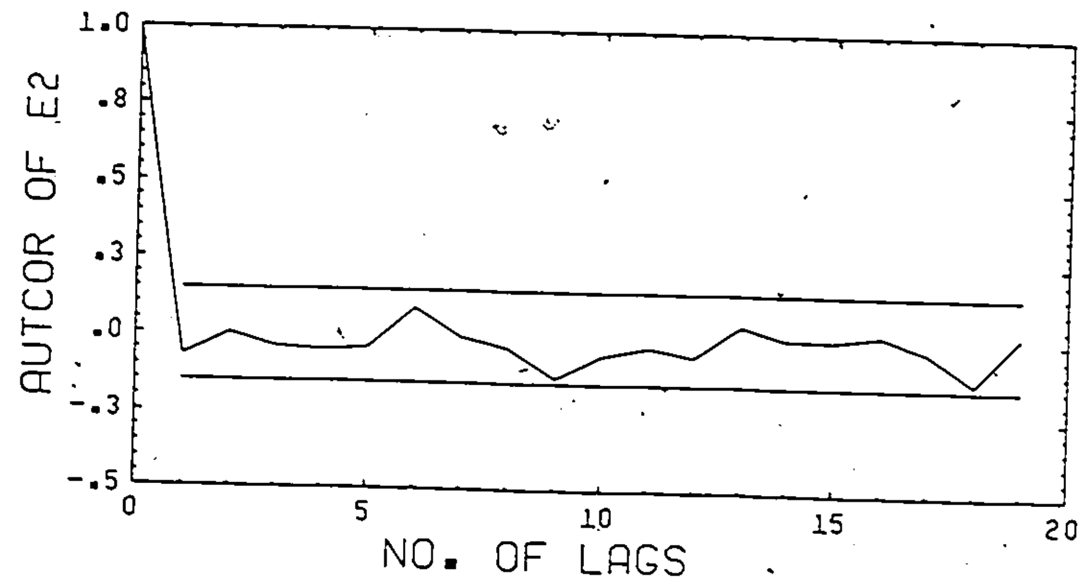


Fig. 5.10 Plot of the auto-correlation of the residuals

5.5 Concluding Remarks

In this chapter we developed an efficient algorithm for the suboptimal control of MIMO systems with unknown parameters and subject to unknown stochastic disturbances. The algoritm proposed is more efficient computationally than either the polynomial matrix approach or the previously reported state space based algorithms, due to the large computational savings in avoiding on-line transformation of the system matrices. Although the estimation algorithm requires more steps, this extra computation, mainly to determine the gradient matrix Ψ need not cause excessive computation since the matrices of partial derivatives are sparse and have very simple structures.

Since the inversion of a controllability matrix is avoided, the proposed algorithm is more applicable to uncontrollable but stablizable systems, as shown by the simulation results for the SISO case in section 4. This robustness to over-parameterization is further reinforced by the fact that for an over-parameterized system it is known [66] that provided the control and output signals are bounded and suitably exciting, the parameter estimates would converge to a local minimum of (5.2a).

A noticeable drawback of the proposed algorithm is the need to project the estimated parameters into the stability region of the predictor. This involves finding the characteristic polynomial of an nxn matrix and the monitoring of its stability via the Routh–Schur or Jury stability tests. However in many applications less refined projection algorithms such as keeping the parameter estimates within some a priori selected limits are often enough to prevent divergence in the initial stages.

CHAPTER 6

6. AN ADAPTIVE AUTOPILOT FOR NONLINEAR SURFACE SHIPS

6.1 Background

In recent years, a large number of publications have appeared on the application of adaptive control techniques to ship steering. The reason for interest in adaptive autopilots is that the ship and disturbance system is inherently nonlinear and time varying. Such characteristics make it difficult for bridge officers to optimally adjust the settings of the PID type controllers, traditionally employed in ship autopilots.

Several authors, Astrom [11], Kallstrom et al. [51], Lim et al. [62], Mort et al. [72,73] and Quevedo et al. [86] represented the nonlinear and time varying ship and disturbance system by linear stochastic models. Various self-tuning controllers were then designed for these models. The philosophy behind such an approach being that, on-line identification should be able to track the effects of time varying system parameters, thus confer a high degree of adaptability on the autopilot.

The various STC autopilots proposed were found to perform well during course keeping, when the ship is essentially operating in the linear region. However, for ship manouvering, which requires large course alterations with corresponding large rudder angles, the STC was found to perform unsatisfactorily. Ashworth and Towill [8] and Ashworth [7] have shown that the main reason for this poor performace, is that the main mode of nonlinear behaviour was essentially complete, before the identification algorithms could sufficiently adapt to it. This resulted in large overshoots to step changes in ship's heading.

In a recent paper Ashworth and Towill [8] proposed an inverse nonlinear compensation scheme to approximately cancel out the effects of the nonlinearity. The resulting system could thus be approximated by a linear time varying system. Thus this would suggest that an ideal adaptive autopilot should consist of a two tier control scheme. A nonlinear compensator to reduce the effects of the nonlinearity, followed by a self-tuning controller to handle the resulting linear time varying system. This should allow the controller to perform optimally during course keeping, course changing, speed alterations, or when in the proximity of canal walls and passing ships.

In this chapter and also in [80] the author proposes an adaptive autopilot based on state space self-tuning control with pole assignment, and nonlinear compensation. The novelty of the proposed method is that the structure of the state space model adopted allows the separation of the system model into a linear dynamic system and a nonlinear control input. A nonlinear state feedback scheme could thus be designed which cancels out the effects of the nonlinearity, and simultaneously, arbitrarily places the closed loop poles for the resulting linear time varying system.

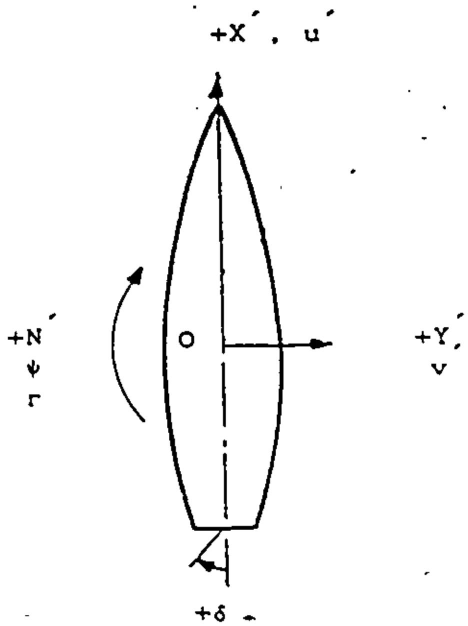
6.2 Mathematical Modelling of Surface Ship Dynamics

The analytical treatment of surface ship equations of motion in six degrees of freedom has been extensively discussed (Comstock [23], Nicholson [75]). The standard nomenclature and sign convention for manoevering in the horizontal plane, neglecting the cross-coupling effects of heave, pitch and roll is shown in figure 6.1.

The lateral equations of motion (sway and yaw) for surface ships, when expressed in nondimensional terms with forces and moments expanded as a Taylor's series to include first order derivatives may be expressed as

$$\begin{bmatrix} (m' - Y_{\dot{v}}') & Y_{\dot{r}}' \\ N_{\dot{v}}' & (I_{zz}' - N_{\dot{r}}') \end{bmatrix} \begin{bmatrix} \dot{v}' \\ \dot{r}' \end{bmatrix} = \begin{bmatrix} Y_v' & (Y_r' - m'u') \\ N_v' & N_r' \end{bmatrix} \begin{bmatrix} v' \\ r' \end{bmatrix} + \begin{bmatrix} Y_\delta' \\ N_\delta' \end{bmatrix} \delta + \begin{bmatrix} Y^w \\ N^w \end{bmatrix} \tag{6.1}$$

where $Y_{\dot{v}}'$, Y_v', $Y_{\dot{r}}'$, Y_r', Y_δ', $N_{\dot{r}}'$, N_r', $N_{\dot{v}}'$, N_v' and N_δ' are the first order hydrodynamic derivatives of the particular force or moment component with respect to the subscripted degree of motion. Y^w and N^w represent the effects of



m' = mass of ship

I'_{zz} = moment of inertia about the vertical axis

N' = moment component about the vertical axis

ψ = yaw angle

r = yaw rate

δ' = rudder angle

u' = foward velocity in x-axis

v' = drift velocity along y-axis

X', Y' = force components along x and y-axes

Fig. 6.1 Nomenclature and sign convention for lateral ship manoevering (the ′ after a variable denotes a nondimensionalized variable).

environmental disturbances such as wind and waves.

Eliminating v' from (6.1) the differential equation relating yaw to rudder deflection may be written as

$$\dddot{\psi} + \left(\frac{1}{T_1} + \frac{1}{T_2}\right)\ddot{\psi} + \frac{1}{T_1T_2}\dot{\psi} = \frac{K_g}{T_1T_2}(T_3\dot{\delta} + \delta) \tag{6.2}$$

where T_1, T_2, T_3 and K_g are related to the hydrodynamic derivatives by

$$\frac{K_g}{T_1T_2} = \frac{N_v'Y_\delta' - Y_v'N_\delta'}{(Y_{\dot{v}}' - m')(N_{\dot{r}}' - I_{zz}') - (Y_{\dot{r}}' - m'x_G)(N_{\dot{v}}' - m'x_G)}$$

$$T_3 = \frac{(N_{\dot{v}}' - m'x_G)Y_\delta' - (Y_{\dot{v}}' - m')N_\delta'}{N_v'Y_\delta' - Y_v'N_\delta'}$$

$$\frac{1}{T_1T_2} = \frac{Y_v'(N_r' - m'x_Gu') - N_v'(Y_r' - m'u')}{(Y_{\dot{v}}' - m')(N_{\dot{r}}' - I_{zz}') - (Y_{\dot{r}}' - m'x_G)(N_{\dot{v}}' - m'x_G)}$$

$$\left(\frac{1}{T_1} + \frac{1}{T_2}\right) = \frac{(Y_{\dot{v}}' - m')(N_r' - m'x_Gu') + (N_{\dot{r}}' - I_{zz}')Y_v' - (Y_{\dot{r}}' - m'x_G)N_v' - (N_{\dot{v}}' - m'x_G)(Y_r' - m'u')}{(Y_{\dot{v}}' - m')(N_{\dot{r}}' - I_{zz}') - (Y_{\dot{r}}' - m'x_G)(N_{\dot{v}}' - m'x_G)}$$

Bech [17] noted that during a manoevre the coefficients $\left[\frac{1}{T_1} + \frac{1}{T_2}\right]$, $\frac{K_g}{T_1T_2}$ and T_3 all remain constant for a given ship's speed. The coefficient $\frac{1}{T_1T_2}$ however was found to vary rapidly.

It is well known from ship trials that the steady state yaw rate is related to the rudder deflection by

$$\delta = H(\dot{\psi}) \tag{6.3a}$$

For a slender ship $H(\dot{\psi})$ may be approximated by the cubic polynomial

$$H(\dot{\psi}) = \nu_1\dot{\psi} + \nu_2\dot{\psi}^3 \qquad (6.3b)$$

Bech [17] suggested that the steering equation may be more accurately represented by the nonlinear equation

$$\dddot{\psi} + \left(\frac{1}{T_1} + \frac{1}{T_2}\right)\ddot{\psi} + \frac{K_g}{T_1T_2}H(\dot{\psi}) + \frac{K_g}{T_1T_2}(T_3\dot{\delta} + \delta) \qquad (6.4)$$

$H(\dot{\psi})$ is thus assumed to account for the net effect of all the nonlinearities in the steering model. The coefficients ν_1 and ν_2 in (6.3b) may be obtained from ship trials by the Dieudonné spiral test for directionaly stable ships, or the reversed spiral test for directionaly unstable ships (Bech [17]). The parameter ν_2 is known to remain fairly constant, but ν_1 is time varying due to the effects of changes in operating conditions, particularly caused by water depth variations, (Nicholson [75]).

For the purposes of controller design, $\dot{\delta}$ may be neglected (Mort and Linkens [72,73]), thus (6.4) may be approximated by

$$\dddot{\psi} + a_T\ddot{\psi} + K_g'H(\dot{\psi}) = K_g'\delta \qquad (6.5)$$

where

$$a_T = \frac{1}{T_1} + \frac{1}{T_2}, \qquad K_g' = \frac{K_g}{T_1T_2}$$

Using (6.3b) an essentially linear state space model with a nonlinear feedback term may be obtained for (6.5).

$$\begin{bmatrix} \dddot{\psi} \\ \ddot{\psi} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} -a_T & -K_g'\nu_1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \ddot{\psi} \\ \dot{\psi} \\ \psi \end{bmatrix} + K_g' (\delta - \nu_2 \dot{\psi}^3) \tag{6.6a}$$

$$\begin{bmatrix} \psi \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \ddot{\psi} \\ \dot{\psi} \\ \psi \end{bmatrix} \tag{6.6b}$$

or more compactly

$$Z = AZ + b(\delta - \nu_2 \dot{y}^3) \tag{6.7a}$$

$$y = C_1 Z \tag{6.7b}$$

$$\dot{y} = C_2 Z \tag{6.7c}$$

where

$$y = \psi \qquad , \; Z = \begin{bmatrix} \ddot{\psi} \\ \dot{\psi} \\ \psi \end{bmatrix}$$
$$\dot{y} = \dot{\psi}$$

also (6.7b) and (6.7c) are obtained by appropriately partitioning (6.6b).

For the purposes of digital control, the sampled data equivalent of (6.7), assuming zero order sample and hold is given by

$$Z_t = FZ_{t-1} + GU_{t-1} \tag{6.8a}$$

$$y_t = C_1 Z_t \tag{6.8b}$$

$$\dot{y}_t = C_2 Z_t \tag{6.8c}$$

where

$$Z_t = \begin{bmatrix} \ddot{\psi}_t \\ \dot{\psi}_t \\ \psi_t \end{bmatrix}, \qquad y_t = \psi_t, \quad \dot{y}_t = \dot{\psi}_t$$

$$F = e^{AT_s}, \qquad G = [\int_0^{T_s} e^{A(T_s-\tau)} d\tau]b \tag{6.8d}$$

$$U_t = \delta_t - \nu_2 \dot{\psi}_t^3 \tag{6.8e}$$

T_s = the sampling interval.

The effects of wind and wave disturbances have been modelled extensively by various impirically derived sea spectra (Comstock [23]). However for the purposes of adaptive control Astrom [11] has demonstrated that the effects of the sea disturbances may be adequately represented by the moving average noise term of an ARMAX model. Thus the net effects of all the external disturbances (wind and waves) may be incorporated into the state space model by adopting the innovations representation (4.4), thus

$$Z_t = FZ_{t-1} + GU_{t-1} + Ke_{t-1} \tag{6.9a}$$

$$y_t = C_1Z_t + e_t \tag{6.9b}$$

$$\dot{y}_t = C_2Z_t \tag{6.9c}$$

The above model incorporates the effects of steady state wind disturbances. since F has a pole on the unit circle.

The discretized state space models (6.8) and (6.9) are not in any special canonical form. Thus in order to obtain a parsimonious model for the purposes of identification and control (6.9) may transformed to the controller canonical form, using the nonsingular transformation

$$Z_t = T_x X_t \tag{6.10a}$$

where

$$T_x = \Gamma_z \Gamma_x^{-1} \tag{6.10b}$$

$$\Gamma_z = [G, FG, F^2G, \ldots, F^{n-1}G] \tag{6.10c}$$

$$\Gamma_x^{-1} = \begin{bmatrix} 1 & a_1 & \cdots & a_{n-1} \\ 0 & 1 & a_1 \cdots & a_{n-2} \\ \vdots & & & \vdots \\ 0 & & \cdots & 1 \end{bmatrix} \tag{6.10d}$$

The transformed system may thus be written as

$$X_t = F_c X_{t-1} + G_c U_{t-1} + K_c e_{t-1} \tag{6.11a}$$

$$y_t = H_{c_1} X_t + e_t \tag{6.11b}$$

$$\dot{y}_t = H_{c_2} X_t \tag{6.11c}$$

where F_c, G_c, K_c, and H_{c_1} have the general structure of (4.5). H_{c_2} has no special form.

Pole placement STCs have been successfully applied to ship auto-pilot design (Quevedo et al. [86]). However in that paper the polynomial approach to pole placement was employed. The state space STC developed in chapter 4 may be used to adaptively place the poles of (6.11), and simultaneously cancel out the effects of the nonlinearity, this may be done easily as follows.

(i) Compute U_t from (4.13a) to assign the desired closed loop poles.

(ii) The equivalent rudder deflection to achieve U_t and hence ensure the linearity of (6.9), is then given by

$$\delta_t = U_t + \nu_2 \dot{\psi}_t^3 \tag{6.12}$$

$\dot{\psi}_t$ is assumed to be available from rate gyros, with a tolerable amount of measurement noise. Alternatively, if the nominal values of H_{c_2} are known $\dot{\psi}_t$ may be estimated via (6.9c). The parameter ν_2 is assumed to be known a priori from spiral tests, hence not estimated on-line.

The cancellation of the nonlinear dynamics need only be approximate. Ashworth [7] has demonstrated that very significant improvements to the transient response may be obtained, even when the nonlinear compensation is only approximate. The closed loop system subject to the control laws (4.13) and (6.12) with imperfect cancellation of the nonlinearity may be represented by the Lure′ type nonlinear sampled data system [102]

$$X_t = F_c X_{t-1} + G_c f(\dot{\psi}_{t-1}) + K_c e_{t-1} \tag{6.13a}$$

$$\dot{y}_t = H_{c_2} X_t \tag{6.13b}$$

F_c is the desired closed loop characteristic matrix. $f(\psi_t)$ represents the remaining nonlinearity due to imperfect cancellation.

Using Tsypkin′s criterion [102], the sufficient conditions for the stability of (6.13) may be stated as follows.

Define

$$G_{tf}(z) = H_{c_2}(zI - F_c)^{-1} G_c = \frac{B^c(z)}{A_m(z)} \tag{6.14a}$$

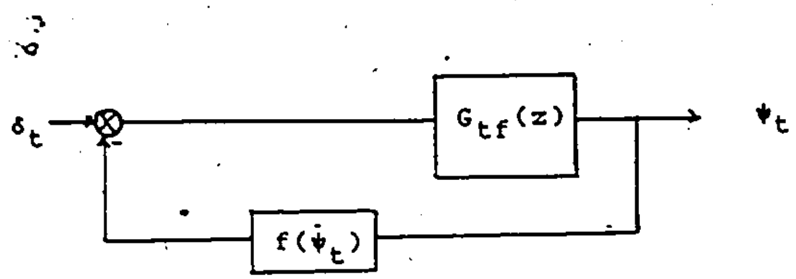


Fig. 6.2 Lure' representation of the closed loop system with imperfect cancellation fo the nonlinearity.

then if the zeros $A_m(z)$ lie inside the unit disc, the origin of (6.13) is asymptotically stable in the large, if

$$0 \leq \frac{f(\dot{\psi}_t)}{\dot{\psi}_t} \leq K_{stab}, \qquad \frac{d\, f(\dot{\psi}_t)}{d\dot{\psi}_t} \geq 0 \tag{6.14b}$$

and

$$\text{Real}\,[1 + \alpha_{stab}(1 - z^{-1})G_{tf}(z)] + \frac{1}{K_{stab}} > 0 \tag{6.14c}$$

for

$$|z| = 1 \text{ and } \alpha_{stab} \geq 0.$$

thus (6.14) shows that even an approximate cancellation of the nonlinearity such that $|f(\dot{\psi}_t)| << |H(\dot{\psi}_t)|$, considerably increases the stability margin of the closed loop system.

6.3 Simulation of ship and sea disturbance model

To test the effectiveness of the proposed autopilot, a discretized a discretized version of the frigate model used by Mort et al. [72], was simulated as shown in figure 6.3.

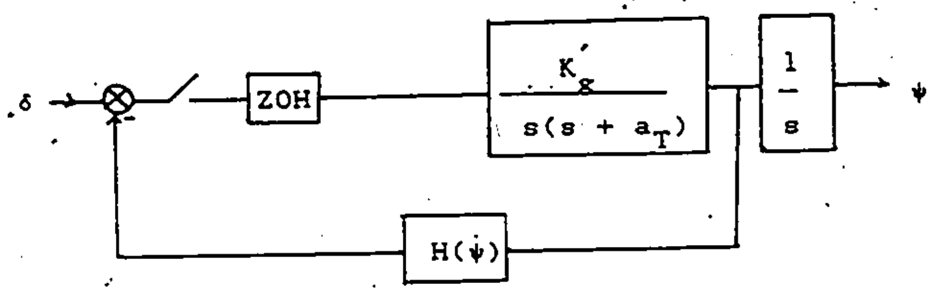


Fig. 6.3 Discretized nonlinear frigate model.

$$K_g' = 0.107$$

$$a_T = 0.11$$

$$H(\dot{\psi}) = 9.42\dot{\psi} + 2.24\dot{\psi}^3,$$

Figure 6.4 shows the δ-$\dot{\psi}$ characteristics of the frigate model. Using a sampling interval of T_s = 2.0 secs., the above model corresponds to the sampled data state space model

$$X_{t+1} = \begin{bmatrix} 0.246 & -0.049 & 0.803 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} X_t + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u_t \tag{6.15a}$$

$$y_t = [0.111, 0.333, 0.098]X_t \tag{6.15b}$$

$$\dot{y}_t = [0.141, -0.015, -0.13]X_t \tag{6.15c}$$

To introduce the effects of sea disturbance, we first consider the equivalent ARMAX representation of (6.15) thus

$$A(z^{-1})y_t = B(z^{-1})U_{t-1} + C(z^{-1})e_t \tag{6.16a}$$

where

$$A(z^{-1}) = 1 - 0.246z^{-1} + 0.049z^{-2} - 0.803z^{-3} \tag{6.16b}$$

$$B(z^{-1}) = 0.11 + 0.333z^{-1} + 0.098z^{-2} \tag{6.16c}$$

the disturbance term could then be simulated by selecting an

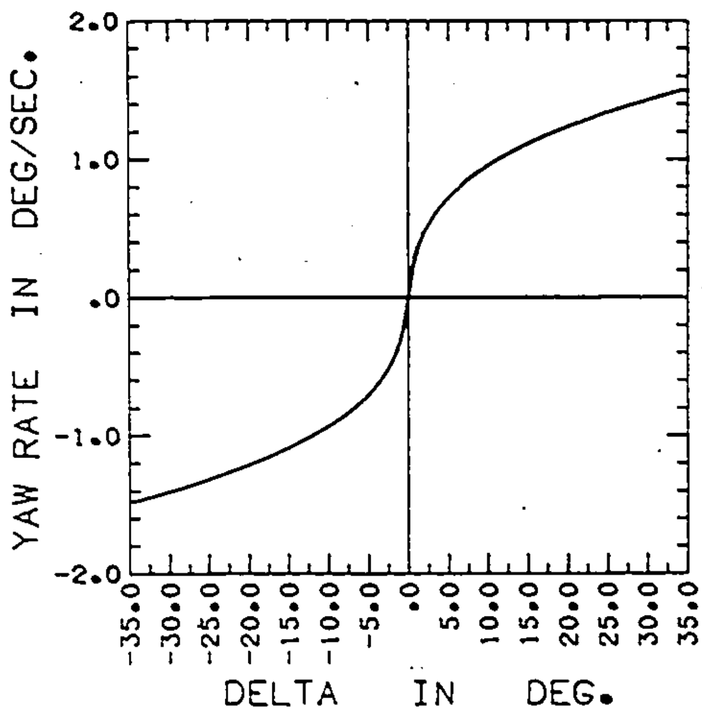


Fig. 6.4 The $\delta - \dot{\psi}$ characteristics of the frigate model

an appropriate noise polynomial. For this simulation the noise polynomial was arbitarily selected as

$$C(z^{-1}) = 1 - 0.7z^{-1} + 0.1z^{-2} \tag{6.16d}$$

the innovations representation of (6.16) is then

$$X_t = \begin{bmatrix} 0.246 & -0.049 & 0.803 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} X_{t-1} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} U_{t-1} + \begin{bmatrix} -0.484 \\ -2.281 \\ 3.663 \end{bmatrix} e_{t-1} \tag{6.17a}$$

$$y_t = [0.111, 0.333, 0.098]X_t + e_t \tag{6.17b}$$

$$U_{t-1} = \delta_{t-1} - 2.24\dot{\psi}^3_{t-1} \tag{6.17c}$$

$$\dot{\psi}_{t-1} = [0.141, -0.015, -0.013]X_{t-1} \tag{6.17d}$$

6.4 Selection of closed loop poles to meet autopilot performance criteria.

An autopilot has two basic functions.

(i) Course Keeping: this is regulatory action to minimize the heading errors from a given reference course. However tight control could lead to excessive rudder action, which increases rudder induced drag. Thus for course keeping a suitable performance index which minimizes the propulsion losses due to steering, (Norrbin [76]) may be written as

$$J_1 = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} [(y_t - \psi_r)^2 + \sigma_w \delta_t^2] \tag{6.18}$$

ψ_r is a desired reference course, and σ_w weights the rudder action relative to the course error. The optimal choice of σ_w would depend on the particular ship and on the operational and environmental conditions.

(ii) Course Alteration: This is essentially servo action, to transfer the ship from one heading to another. During course alteration, especially in congested waters and canals, safety requirements are the most important factors. Thus the ship should be transferred to the new course in minimum time, with no overshoots or undershoots. This suggests a performance criterion of the form

$$J_2 = \frac{1}{N_f} \sum_{t=1}^{N_f} [y_t - \psi_r]^2 \qquad (6.19)$$

i.e. the rudder deflection is not penalized. N_f is a a suitable transition period. The optimal choice of N_f depends on the dynamics of the particular ship. Tiano et al. [96] have suggested that N_f should be chosen in such a way that the course alteration is executed at a constant rate of turn.

For an autopilot based on pole assignment control, the closed loop pole configuration, should be selected to minimize the cost functions (6.18) and (6.19) for the appropriate mode of operation. Quevedo et al. [86] suggested that all the closed loop poles be placed at a

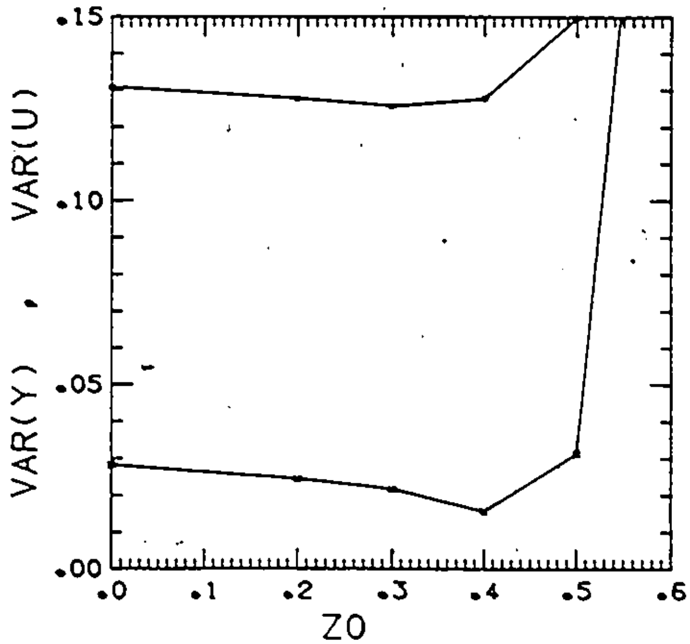


Fig. 6.5 Plot of input and output variances for different values of z_o

single-point on the real axis, thus

$$A_m(z^{-1}) = (1 - z_0 z^{-1})^3 \quad (6.20a)$$

$$0 \leq z_0 < 1 \quad (6.20b)$$

This should ensure that there are no overshoots or undershoots to a step change in ships heading. Placing z_0 closer to the origin should ensure fast response, thus optimizing (6.19) as closely as possible. Obviously the ship may not achieve deadbeat control, due to limiting rate of turn and saturating rudder deflections.

6.5 Closed loop simulation results

For closed loop simulation studies the model (6.17) was simulated, subject to the STC described in chapter 4. The STC was implemented in two modes. For course keeping, the closed loop poles were all placed at $z_0 = 0.4$. This corresponds approximately to the minimum values of both σ_y^2 and σ_u^2 from figure 6.5. For course changing, the closed loop poles were placed at $z_0 = 0.2$, to ensure faster response.

The results of two simulation runs at different noise levels are presented in this thesis. For run no. 1, the disturbance was selected as e_t=(0,0.01), ψ_r was selected as a square wave sequence representing 40° alterations of course. Such large course alterations would obviously take the system outside the linear region. Figure 6.6 shows a plot of the ship's heading superposed on the demand

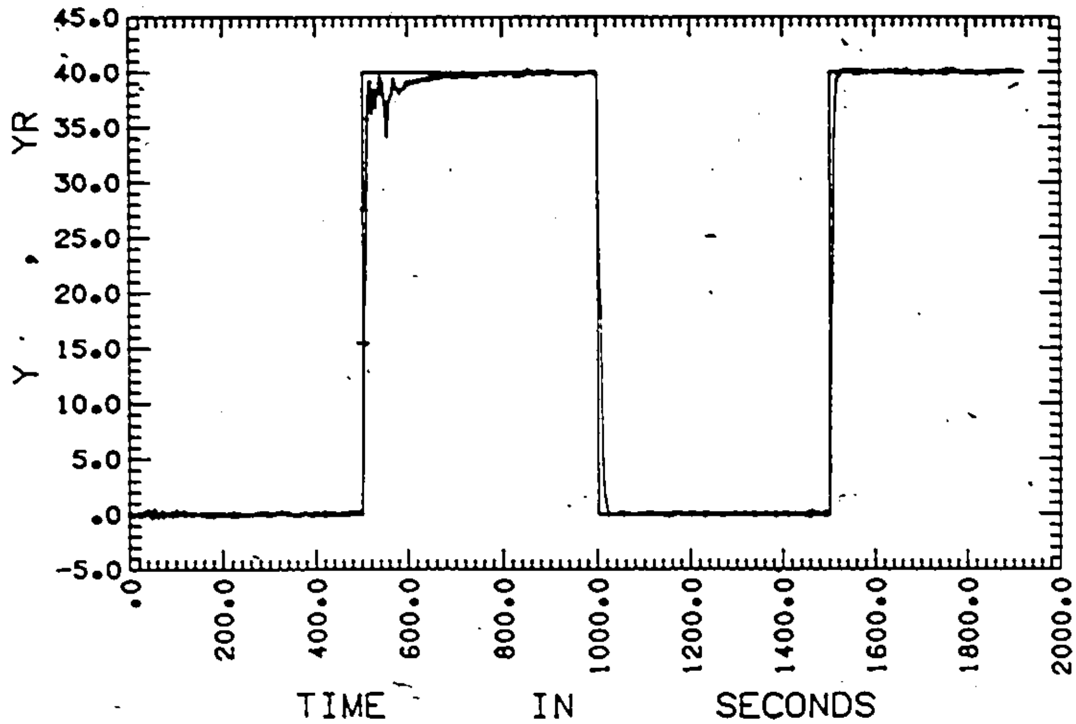


Fig. 6.6 Closed loop response (simulation run no. 1)

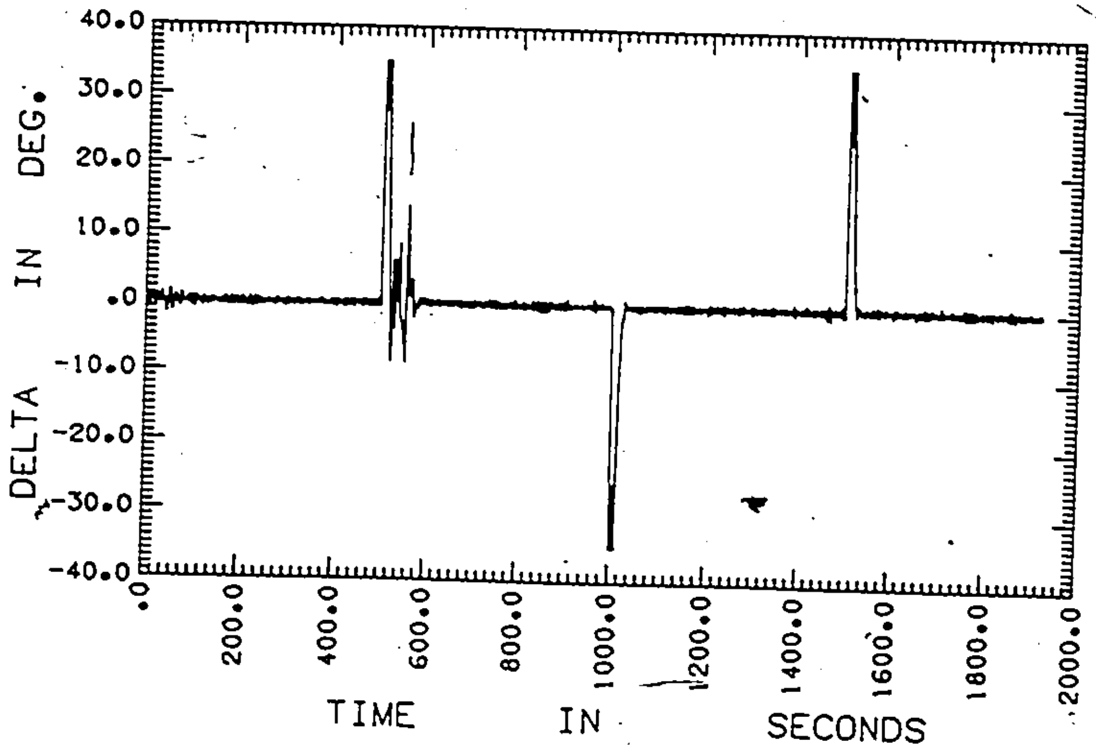


Fig. 6.7 Plot of rudder deflections (simulation run no. 1)

reference course for run no. 1. A significant undershoot was observed at the first course alteration. This was due to inaccurate parameter estimates at t = 500 secs. No significant overshoots or undershoots were apparent at subsequent alterations of course.

Figure 6.7 shows the corresponding rudder deflections for both regulation and servo action. For practical purposes the rudder deflection was limited to ±35°. Thus with $z_0 = 0.2$ the rudder saturates for a few seconds during a course alteration of 40°, thus limiting the speed of response.

Figure 6.8 shows a plot of the associated yaw rate response. The ship was assumed to have a maximum rate of turn of 1.75°/sec. Hence from figure 6.8, we see that for a significant portion of the turning manouvre, the ship was turning at its maximum rate of turn.

The simulated values of the estimated parameters were

$$\theta^T = [-0.246, 0.049, -0.803, 0.111, 0.333, 0.098, -0.484, -2.281, 3.663]$$

The starting (assumed nominal values) values were chosen as

$$\theta_o^T = [-0.2, 0, -1, 0.2, 0.2, -0.2, -2, 4]$$

the initial value of the covariance matrix was

$$P_o = 50.0I$$

the estimated values after 500 were

$$\theta_{1000}^T = [-0.322, 0.1558, -0.831, 0.0756, 0.3534, 0.0767, -0.114, -1.254, 2.716]$$

From figure 6.9 we see that the actual system parameters

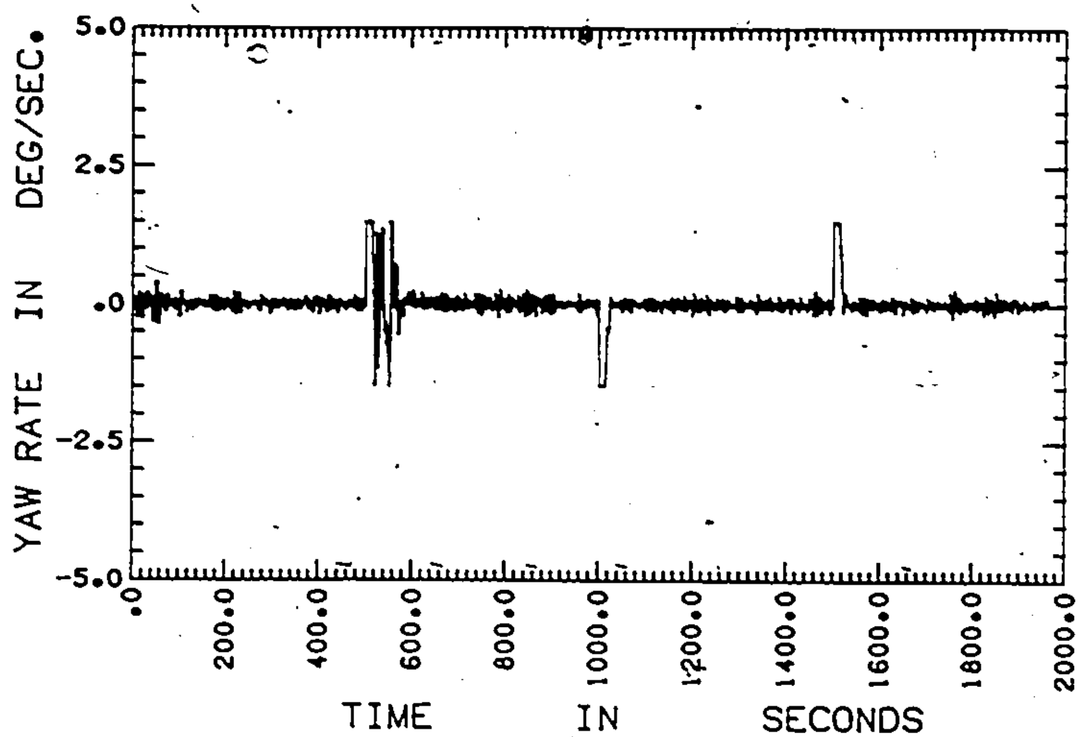


Fig. 6.8 Plot of yaw rate response (simulation run no. 1)

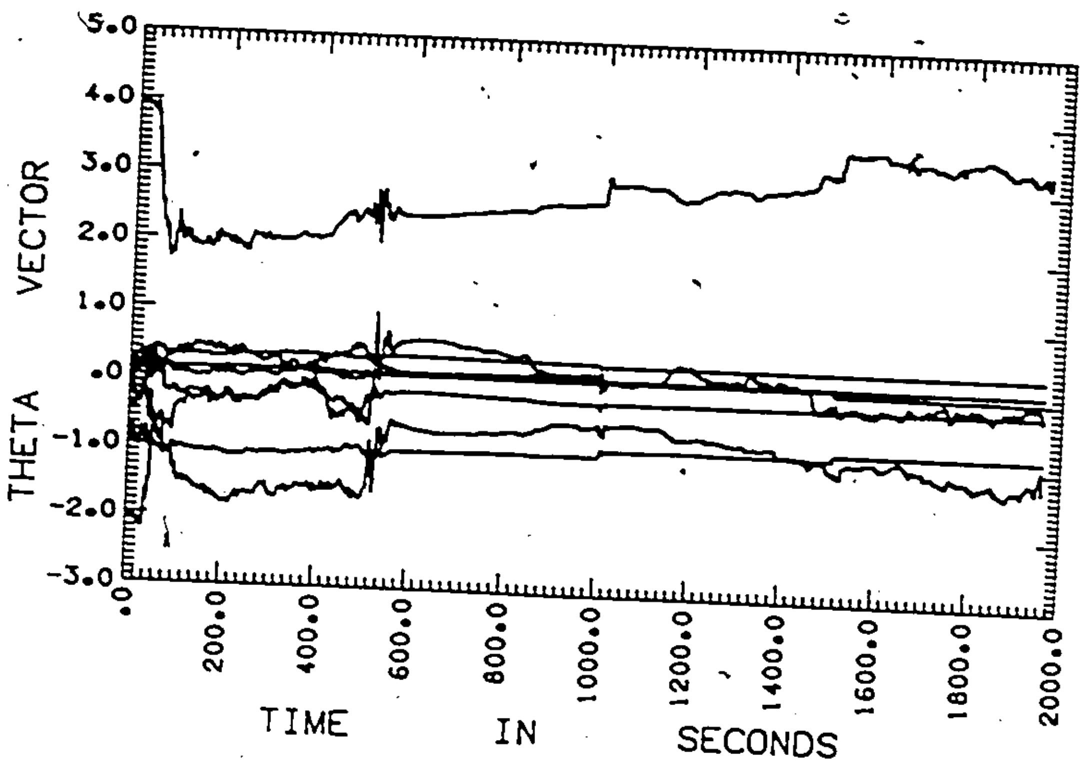


Fig. 6.9 Plot of the estimated system parameters (simulation run no. 1)

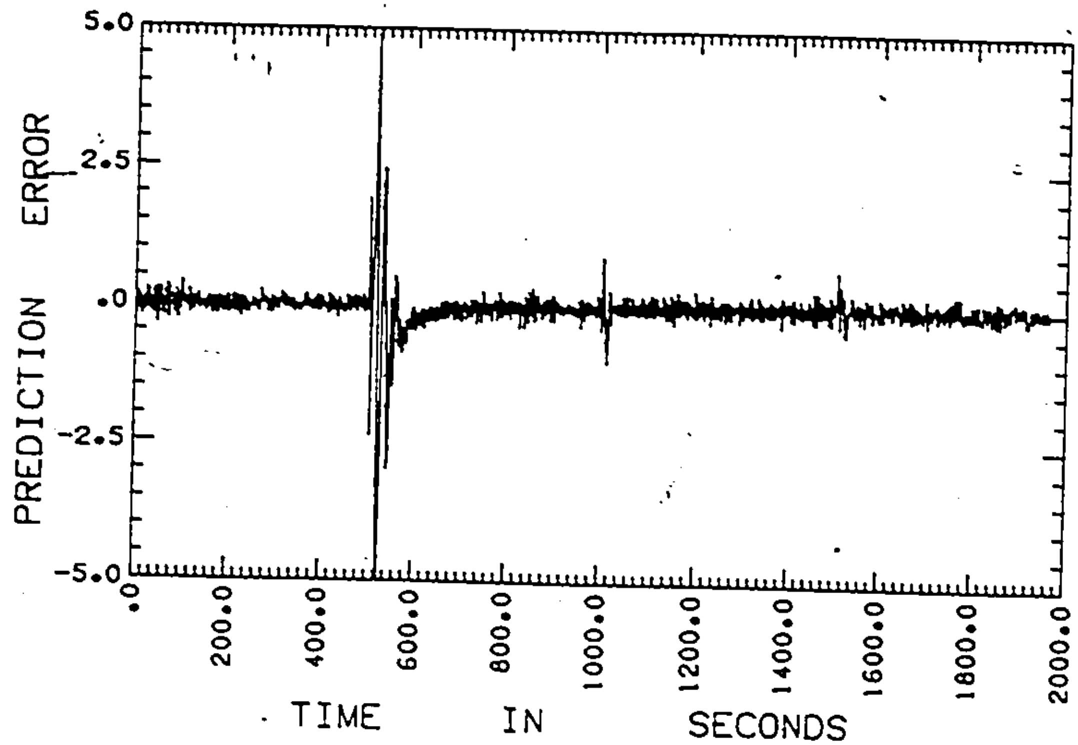


Fig. 6.10 Plot of the residual sequences (simulation run no. 1)

could be estimated with reasonable accuracy. The Kalman gain parameters were however estimated less accurately. This may be attributed to the effects of imperfect cancellation of the nonlinear terms. Thus additional (numerical noise) noise is introduced, due to the effects of deviations from the linearized model.

Figure 6.10 shows a plot of the residual sequence. The marked increase in the prediction error at t = 500 secs. was due to inaccurate parameter estimates at the first alteration of course. However the resulting large innovations caused the system to adapt rapidly, hence improve the parameter estimates.

For simulation run no. 2, the disturbance noise was selected as e_t = (0,0.1) i.e., ten times the variance of run no. 1. Figures 6.11 and 6.12 show plots of the heading response and rudder deflections respectively for run no. 2. From those plots it is evident that the proposed algorithm performs well in the presence of significant sea disturbances. However it was noticed from simulations that the rudder variance increases significanly if the yaw rate required to implement (6.12) is measured with significant noise levels, but the performance is reasonably acceptable even when $\dot{\psi}_t$ is estimated from X_t via (6.11c) using nominal parameter values.

To demonstrate its relative efficiency, the proposed autopilot was compared with a fixed gain PID controller.

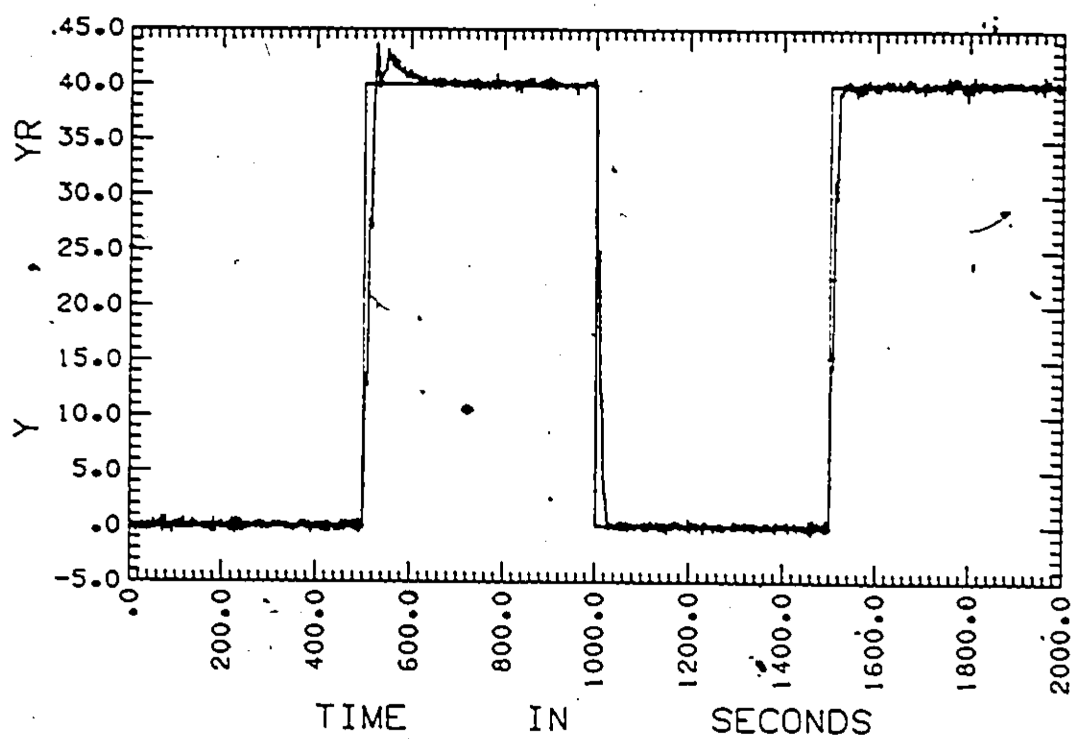


Fig. 6.11 Closed loop response for simulation run no. 2, with increased sea disturbance

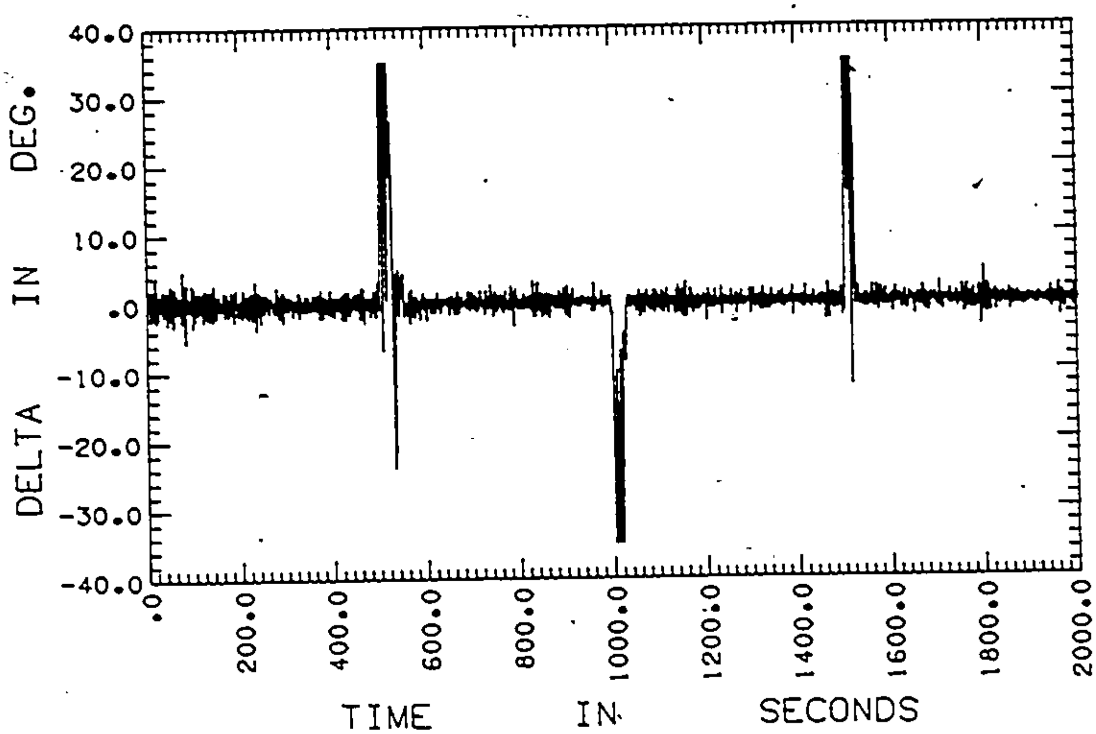


Fig. 6.12 Plot of rudder deflections for simulation run no. 2

Noting that the open loop characteristic equation already contains an integrator i.e. $A(z^{-1}) = A^*(z^{-1})(1-z^{-1})$, a PID controller can be designed as follows.

$$u_{t+1} = -(p_o + p_1 z^{-1} + p_2 z^{-2})(y_t - \psi_r) \qquad (6.21a)$$

the controller gains may be determined rapidly [52,44,45] as follows

$$p_o + p_1 z^{-1} + p_2 z^{-2} = p_o(1 + a_1^* z^{-1} + a_2^* z^{-2}) \qquad (6.21b)$$

The closed loop system subject to the control law (6.21) is thus

$$A^*(z^{-1})[(1 - z^{-1}) + p_o B(z^{-1})]y_t = p_o B(z^{-1}) A^*(z^{-1}) \psi_{r,t} + C(z^{-1}) e_t \qquad (6.22)$$

p_o can then be chosen such that

$$[(1 - z^{-1}) + p_o B(z^{-1})] = A_m(z^{-1})$$

has suitably well damped poles. Thus (6.22) reduces to

$$y_t = \frac{p_o B(z^{-1})}{A_m(z^{-1})} \psi_{r,t} + \frac{C(z^{-1})}{A_m(z^{-1}) A^*(z^{-1})} e_t \qquad (6.23)$$

From equation (6.23) it is evident that while the PID controller is adequate for course changing, its regulatory (course keeping) performance would depend on the poles of $A^*(z^{-1})$ in addition to those of $A_m(z^{-1})$. Thus for some parameter values this would not be a satisfactory controller in the stochastic environment. Figures 6.13 and 6.14 show plots of the ship's heading and correspondig rudder action respectively.

The ship model was controlled with by the 3 term controller (6.21) with

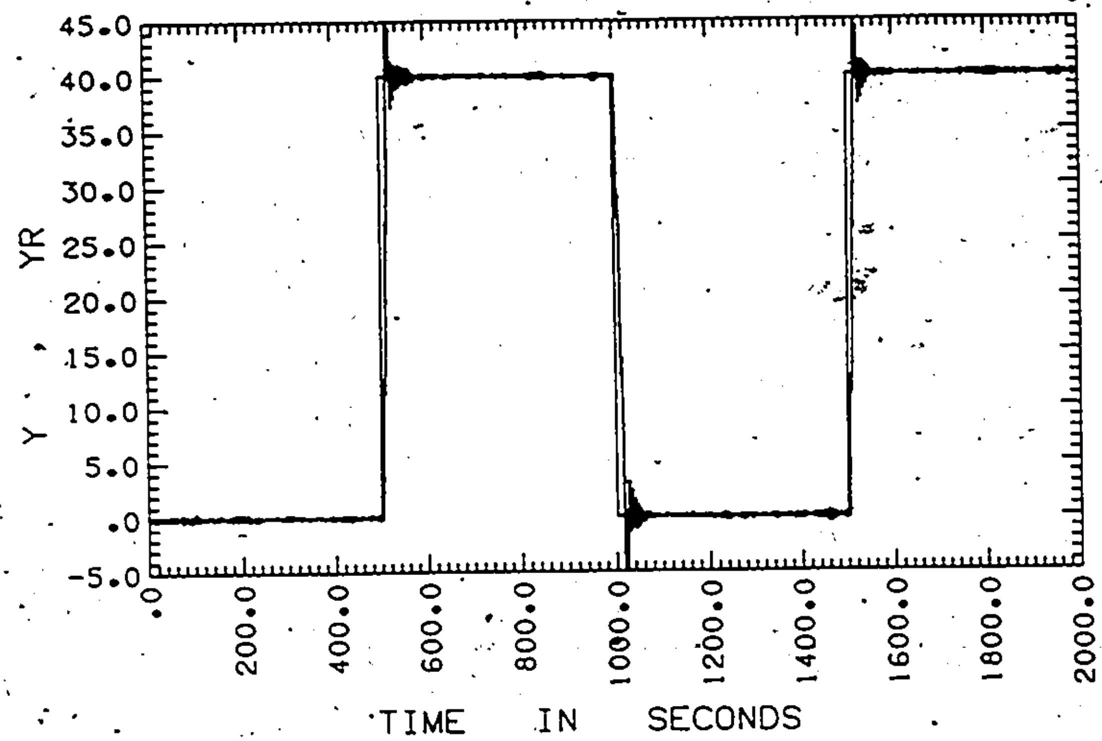


Fig. 6.13 Closed loop response under PID control, $P_o = 0.8$

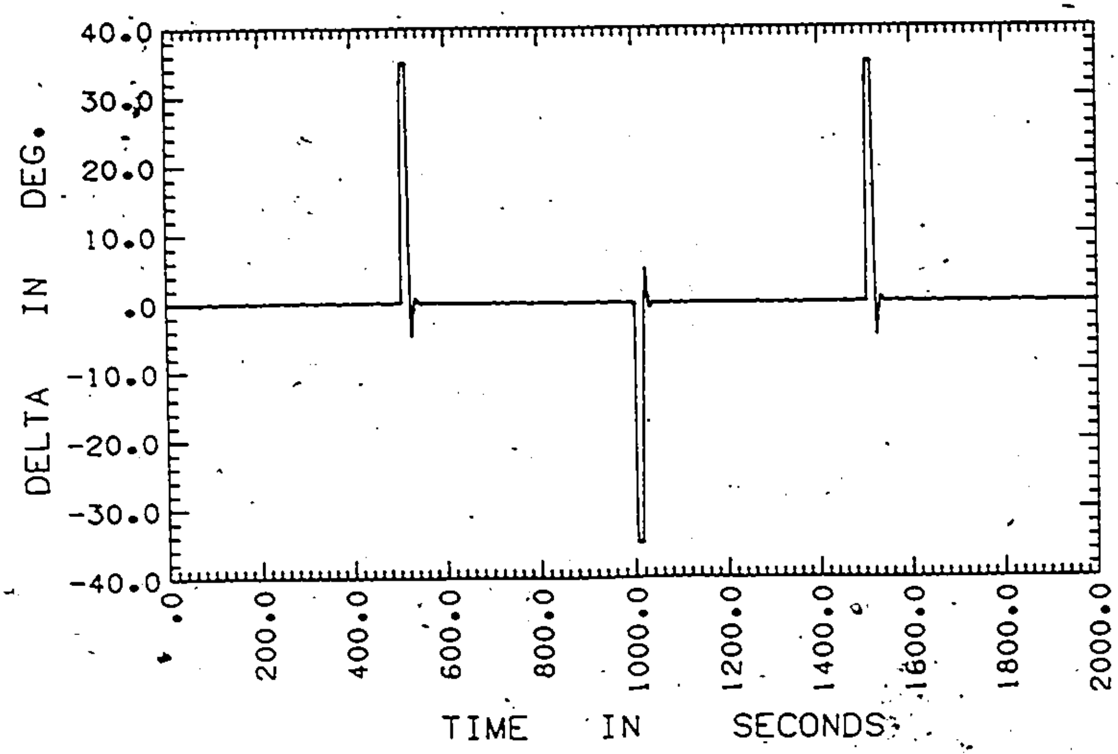


Fig. 6.14 Rudder deflections under PID control

$$p_o = 0.8$$

and

$$A^*(z^{-1}) = 1 + 0.754z^{-1} + 0.803z^{-2}$$

which is assumed to be known. Thus

$$A_m(z) = (z - 0.542 + j0.4)(z - 0.542 - j0.4)(z + 0.172)$$

the marked increase in overshoot to course alteration was attributed to a contribution of the under damped poles of $A^*(z^{-1})$ and the effects of ignoring the nonlinearity.

The contribution of the nonlinear cancellation term (6.12) in reducing the overshoot to a step change in course direction is demonstrated in figures 6.15 and 6.16, for this simulation a pole assignment STC was used to place the poles of the linearized system, ignoring the effects of the nonlinearity. The plot of the ship's heading figure 6.16 shows that ignoring the nonlinear rate feedback terms leads to significant overshoots to step course changes, which are persistent even after the system has settled down.

6.6 Concluding Remarks

Several adaptive control strategies have been adopted for the design of ship autopilots. Of these the model reference type controllers (Amerongen [5,6]), perform well during course changing, but have poor performance in the stochastic environment.

Self-tuning controllers were found to confer a high degree of adaptability on autopilots with respect to slow variations in the system dynamics, such as due to the

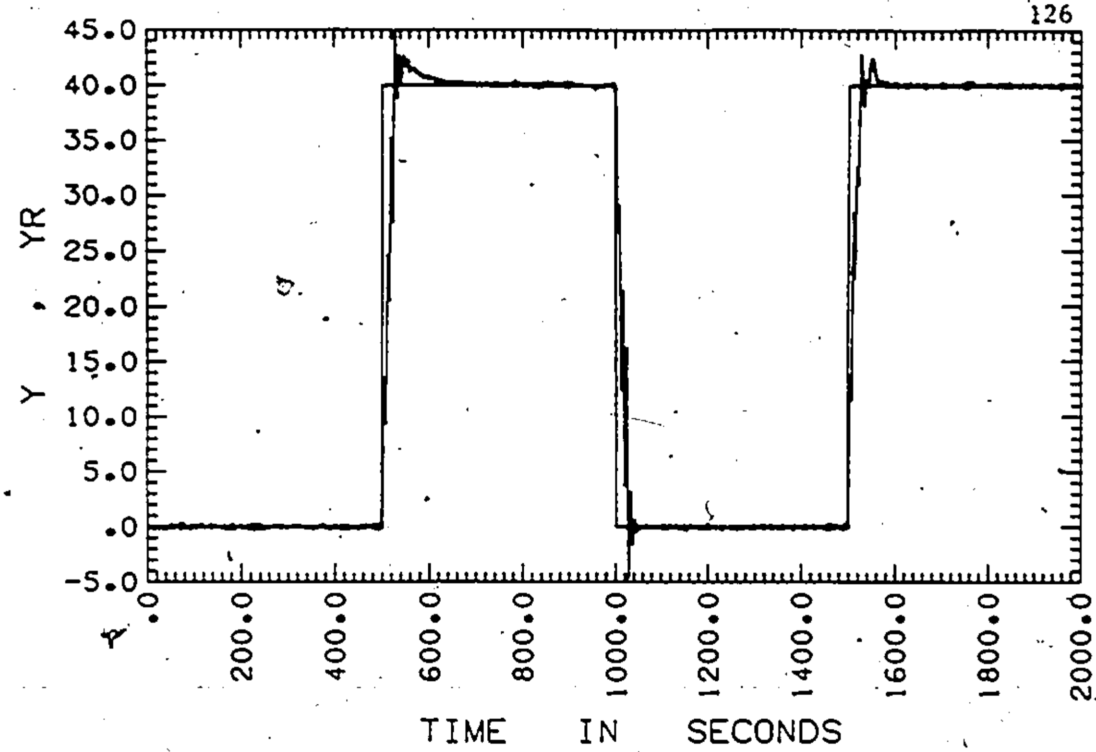


Fig. 6.15 Closed loop response under pole placement STC, with z_o = 0.4, without nonlinear compensation

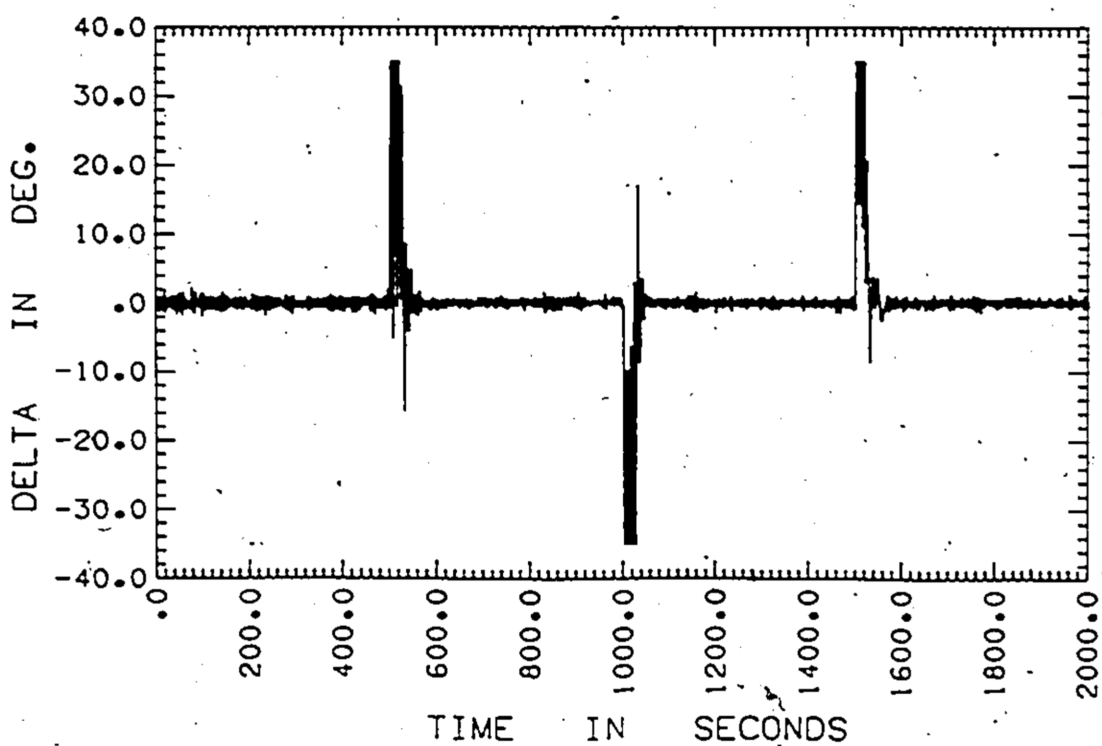


Fig. 6.16 Rudder deflections under pole placement STC without nonlinear compensation

effects of changes in speed, weather conditions or the interaction of proximate objects such as shallow water or canal walls. The STC's have been used succesfully as course keeping controllers. For course alteration however, nonlinear terms predominate. For such cases it is well known that the STC does not perform well, since the identification algorithms cannot track the nonlinear effects fast enough.

In this chapter the author has presented a suboptimal but robust self-tuning autopilot, which uses a combination of linear state feedback to arbitarily place the closed loop poles, and a nonlinear feedback term to compensate for the nonlinearity. The cancellation of the nonlinearity need only be approximate. Ashworth [7] has shown that even an approximate cancellation of the nonlinearity, can reduce significantly the range of variation attributed to the hypothetically linear time varying ship. Furthermore, the effects of stabilizing feedback is to reduce the closed loop sensitivity to parameter variations. The effects of the remnant nonlinearity appears as increased residuals and can be handled by the adaptation algorithm.

A notable disadvantage of the proposed scheme is the requirement for an accurate yaw rate sensor. However it was observed from simulations that even an estimated yaw rate using nominal parameter values gave satisfactory performance.

In section 6.4 techniques were presented to aid the selection of the closed loop poles to satisfy the quadratic performance indices traditionally used in the design of autopilots.

The choice of the simulated noise model was rather simple. This was for illustration purposes. However even if a more elaborate filter is used to simulate the sea disturbance, the structure of the innovations model would remain the same. Hence the estimation and control algorithms would not be altered.

Further work remains to be done mainly on practical implementation on an ocean going vessel. Related to this is the investigation of micro-computer implementation, and the effects of finite word lengths. However since large sampling intervals are envisaged, computation times should not be a problem and hence double precision could be used on smaller word length computers.

CHAPTER 7

CONCLUSIONS

The problem of the adaptive control of multivariable systems using pole assignment techniques has been investigated in some detail. Adaptive control algorithms can be classified as 'explicit' or 'implicit'. Explicit adaptive control is the more natural solution to the adaptive control problem. In such an algorithm a recursive estimation algorithm is used to estimate the system parameters on-line. At each sampling interval these estimated parameters are used to implement a control law. The computation of the controller gains could lead to a number of problems depending on the control law adopted.

(i) For non-minimum phase systems zero cancellation can lead to unbounded control.

(ii) Pole placement control avoids the problem in (i) but has the disadvantage that lack of knowledge of the exact model order can lead to over-parameterization and subsequently to ill-conditioning and unstable control.

Alternatively careful selection of model representation and identification algorithm, could lead to simplification of subsequent controller design. The particular case where the mapping from the estimated parameters to the controller parameters is the trivial mapping may be termed "implicit" adaptive control.

Implicit pole placement algorithms have been proposed by various authors, however most of these algorithms were derived in an ad-hoc manner. Since the resulting estimation algorithms are nonlinear, various convergence problems have been encountered with the implicit algorithms, particularly in the presence of strongly correlated disturbance noise. When applied to non-minimum phase systems these algorithms become even more complex due to the need to factorize out the unstable zeros.

This thesis concentrates on the design of a class of pole assignment self-tuning control algorithms based on state space models. Although such an approach has been adopted for the design of STCs by a number of authors, all the previously reported approaches estimate the parameters of an ARMAX model. The estimated parameters are then mapped into a state space model for subsequent controller design. For the MIMO case the need to keep this mapping simple, leads to nonminimal realizations such as 'block canonical' forms.

In chapter 3 an explicit adaptive pole placement algorithm was derived for MIMO systems. A state space model in the observable canonical form was assumed and a modification of the BSE algorithm was used for joint state and parameter estimation of this model. The proposed algorithm involves the estimation of fewer parameters, if the observability indices are known, than the algorithms proposed in [18,43,88]. Subsequent controller design is also more efficient computationally, mainly due to the reduction in the number of matrix inversions required to implement the control law.

In chapter 4 a modified algorithm was proposed for state space self-tuning control which retains some of the advantages of both explicit and implicit adaptive control. The state space model was assumed to be in the controller canonical form, thus the computation of the controller parameters for pole assignment becomes trivial. The proposed algorithm may thus be interpreted as an implicit controller.

The RPE algorithm was adopted for joint state and parameter estimation. Thus unlike the previously reported implicit pole placement controllers, sufficient conditions for asymptotic convergence of the estimated parameters are available. These conditions require that the estimated parameters be projected into the stability region of the predictor. Thus the key to the success of the proposed modified STC is the inclusion of a projection facility, such a projection facility can be implemented quite easily as shown in chapter 4. In chapter 5 the techniques developed in chapter 4 was extended to the MIMO case, and simulation results demonstrating the effectiveness of the algorithm were presented.

To demonstrate a practical application, the modified state space STC proposed in chapter 4 was used to design an adaptive autopilot for a nonlinear model of a Royal Navy frigate. The novelty of the proposed autopilot is that the model structure adopted allows the ship model to be separated into a linear dynamic system and a nonlinear control input. A 'moment allocation' type controller could thus be designed which uses nonlinear state feedback to arbitarily place the closed loop poles and simultaneously cancel out the nonlinear dynamics.

The cancellation of the nonlinearity need only be approximate. Analytical treatment shows that even an approximate cancellation of the nonlinearity improves the stability margin considerably. Furthermore it is known that the effect of stablizing feedback is to reduce the closed loop sensitivity to parameter variation. Thus the residual nonlinearity has only marginal effects on the closed loop system, and can be handled very effectively by the adaptation algorithm.

The proposed autopilot thus overcomes many of the problems encountered in the design of an adaptive autopilot which can perform efficiently, the conflicting tasks of course keeping and course changing. The simulation results presented show that the proposed algorithm performs satisfactorily in the two operating modes in the presence of significant sea disturbance.

7.1 Suggestions For Further Research

Sufficient conditions for global convergence of explicit STCs in a stochastic environment have yet to be established. Some encouraging work done in this area for continous time systems in [27,54] and Goodwin and Sin [40,42] have suggested that global stability can be ensured by constraining the estimated parameters to within some parametric distance of a nominal parameter vector. Thus some analytical investigation of the effects of various innitial conditions and projection algorithms would be useful in this area.

The algorithms proposed in chapters 4 and 5 were found to perform well when applied to overparameterized systems. This is mainly due to the elimination of on-line matrix inversion in the controller design. The reduced order behaviour however, cannot be so easily analyzed. The reduced order performance of the proposed algorithm would also depend on the significance of the neglected dynamics of the controlled system. Thus an analytical and experimental investigation in this area, especially in comparison with the observations made by Johnson et al. [46] for other proposed implicit pole placement algorithms would be important.

A wide area of practical applications remains to be investigated, especially the application to systems where the estimated states have physical meanings would make the state space STC a more useful tool than the polynomial approach, since extra information would then be available about the internal states of the controlled system.

The adaptive autopilot proposed in chapter 6 could be extended to the MIMO case. Thus an adaptive control algorithm could be designed for ship control which can combine roll stabilization, speed and steering controls in an optimal manner, incorporating some of the MIMO features developed in chapter 5 such as partial decoupling.

A more elaborate simulation of the ship manouvering equations, incorporating a more realistic wave filter could be implemented. This would be a good test for the reduced order behaviour of the proposed autopilot.

Finally the practical implementation of the proposed autopilot, and its trial evaluation at sea, is an area for further research. Closely associated with this is the hardware implementation, the choice of a suitable micro-computer, and the investigation of computing times, sampling intervals and the effects of finite word lengths. However for ship steering long sampling times are envisaged. Thus double precision can be used, hence non of the above considerations are crucial to the practical usefulness of the proposed autopilot.

REFERENCES

[1] M.S. Ahmed, "On bootstrap estimation of systems and states", IEEE Trans. Aut. Control, Vol. AC-28, No.7, pp. 805-806, 1983.

[2] F. Alix, J.M. Dion, L. Dugand and I.D. Landau "Adaptive control of non-minimum phase systems, comparisms of several algorithms, and improvements" Ricerche Di Automatica. Vol.13, No.1, 1982, ITALY.

[3] A.Y. Allidina and F.M. Hughes, "A general adaptive scheme", Int. J. Control. No.6, pp. 1115-1120, 1983.

[4] A.Y. Allidina and F.M. Hughes, "Self-tuning controllers for deterministic systems", Int. J. Control. Vol.37, No.4, pp. 831-841, 1983.

[5] J. Amerongen and L.H.R. Nauta, "Optimum steering of ships with an adaptive autopilot", Proc. 5th Ship Control Systems Symposium, Maryland, 1978.

[6] J. Amerongen, "Adaptive steering of ships - a model reference approach", Automatica, Vol.20, pp. 1984.

[7] M.J. Ashworth, "Feedback design of systems with significant uncertainty", Research Studies Press Herts, England, 1982.

[8] M.J. Ashworth and D.R. Towill, "An approach to autopilot design based on nonlinear ship model with uncertain parameters", Proc. 6th Ship Control Systems Symposium, Ottawa, 1981.

[9] K.J. Astrom, "Introduction to stochastic control theory". Academic Press, New York, 1970.

[10] K.J. Astrom, "Theory and application of adaptive control - a survey", Automatica, Vol. 19, No.5 , pp. 471-486, 1983.

[11] K.J. Astrom, "Why use adaptive techniques for steering large tankers?", Int. J. Control Vol. 32, pp. 689- ,1980.

[12] K.J. Astrom, P. Hagander and J. Sternby, "Zeros of sampled systems", Automatica, Vol.20, No.1 , pp. 31-38, 1984.

[13] K.J. Astrom and B. Wittenmark, "On self-tuning regulators", Automatica, Vol.19, pp. 185-, 1973.

[14] K.J. Astrom and B. Wittenmark, "Self-tuning controllers based on pole-zero placement", IEE Proc. Vol.127, Part D , No. 3, pp. 120-129 , 1980.

[15] Cs. Banyasz and I. Keviczky, "Direct methods for self-tuning PID regulators", 6th. IFAC symp. on Identification and Parameter Estimation, pp. 1249-1254, 1982.

[16] Y. Bar-Shalom and E. Tse, "Dual effect, certainty equivalence and separation in stochastic control" ,IEEE Trans. Aut. Control, Vol. AC-19, pp. 494, 1974.

[17] M.I. Bech, "Some aspects of the stability of automatic control of ships", J. Mech. Sci. (supplementary issue) vol. 14, pp. 123-131, 1972.

[18] L.W. Bezanson and S.L. Harris, "State space design of multivariable self-tuning regulators", Int. J. Control, Vol. 39, pp. 395-411, 1984.

[19] G.J. Bierman, "Factorization methods for for discrete sequential estimation". Academic Press, New York, 1977.

[20] U. Borisson, "Self-tuning regulators for a class of multivariable systems", Automatica, Vol. 15, pp. 209-215, 1979.

[21] P. Brunovsky, "A classification of linear controllable systems". SIAM J. Control Vol. 10, pp. 252-377, 1971.

[22] D.W. Clarke and P.J. Gawthrop, "Self-tuning controller", IEE Proc., Vol. 122, No. 9, 1975.

[23] Comstock, "Principles of naval architecture", SNAME, New York, 1967.

[24] A. O. Cordero and D.Q. Mayne, "Deterministic convergence of a self-tuning regulator with variable forgetting factor", IEE Proc. Part D, Vol. 128, No.1, pp.19- 1980.

[25] L. Dugand and I.D. Landau, "Recursive output error identification algorithms theory and evaluation", Automatica, Vol. 16, pp. 443-462, 1980.

[26] L. Dugand and I.D. Landau, "Stochastic model

reference adaptive controllers", Proc. 19th IEEE CDC Conference, Alburquerque, USA, 1980.

[27] P. De Larimnat, "General stability conditions for continous time adaptive control systems", Ricerche Di Automatica, Vol. 13, No.1, 1982, Italy.

[28] B. Egardt, "Unification of some discrete-time adaptive control schemes", IEEE Trans. Aut. Control, Vol. AC-25, pp. 693 697, 1980.

[29] B. Egardt, "Unification of some continous-time adaptive control schemes", IEEE Trans. Aut. Control, Vol. AC-24, pp. 588- , 1979.

[30] D.H. Elliot, "Direct adaptive pole placement with application to nonminimum phase systems", IEEE Trans. Aut. Control, Vol. AC-27, pp. 720- ,1982.

[31] H. Elliot and W.A. Wolovich, "Multivariable adaptive pole placement", Proc. 20th IEEE Conf. Dec. Contr. San Diego, CA, pp. 537-538, 1981.

[32] H. Elliot and W.A. Wolovich, "a parameter adaptive control structure for linear multivariable systems", IEEE Trans. Aut. Control, Vol. AC-27, No.2, pp. 340-351, 1982.

[33] H. El-Sherief and N.K. Sinha, "Suboptimal control of linear stochastic multivariable systems with unknown parameters", Automatica, Vol. 18, No. 1, pp. 101-105, 1979.

[34] H. El-Sherief and N.K. Sinha, "Bootstrap estimation of parameters and states of linear multivariable systems", IEEE Trans. Aut. Control, Vol. AC-24, No.2, pp. 340-343 ,1979.

[35] G. Favier and M. Hassani, "Multivariable self-tuning controllers based on generalized minimum variance strategy", Proc. 21st IEEE Conf. Dec. Contr. Vol. 2, pp. 770-777, 1982.

[36] G. Favier, "Adaptive control algorithms for MIMO plants , ISIS Workshop on Adaptive Control, Firenzo, Italy, 1982.

[37] T.R. Fortescue, L.S. Kershenbaum, and B.E. Ydstie, "Implementation of self-tuning regulators with variable forgetting factors", Automatica, Vol. 17, pp. 831-835.

[38] W.B. Gong, "Correspondence on multivariable pole

assignment self-tuning regulators", IEE Proc. D, CTA, Vol. 129, No. 4, pp. 142- ,1982.

[39] W.B. Gong, "Correspondence on multivariable assignment self-tuning regulators", IEE Proc. D, CTA, Vol. 130, No. 4, pp. 200- ,1983.

[40] G.C. Goodwin, P.J. Ramadge and P.E. Caines, "Discrete time stochastic adaptive control", SIAM J. Control & Optimization,

[41] G.C. Goodwin and K.S. Sin, "Adaptive control of nonminimum phase systems", IEEE Trans. Aut. Contr., Vol. AC-26, No. 9, pp. 478-483, 1981.

[42] G.C. Goodwin and K.S. Sin, "Adaptive filtering prediction and control", Prentice Hall, 1984.

[43] T. Hesketh, "State space pole placing self-tuning regulator using input-output values", IEE Proc. D, CTA, Vol. 129, pp. 123-128, 1982.

[44] R. Iserman, "Parameter adaptive control algorithms - a tutorial", Proc. 6th IFAC/IFIP Conf. on Digital Computer Applications to Process Control, 1980.

[45] R. Iserman, "Digital control systems", Springer-Verlag, New York, 1981.

[46] C.R. Johnson, Jr. and H. Elliott "On three similar (but different) approaches to direct adaptive pole placement", Proc. 15th Conf. Info. Sci. Syst., Baltimore, MD, 1981.

[47] C.R. Johnson, Jr., D.A. Lawrence and J.P. Lyons, Jr., "A flaw in the reduced order behaviour of a direct adaptive pole placer", IEEE Trans. Aut. Control, Vol. AC-28, no. 9, 1983, pp. 922-924.

[48] C.R. Johnson, Jr., "On direct adaptive control of nonminimum-phase plants", Information Sciences, Vol. 20, pp. 165-179, 1980.

[49] C.R. Johnson, Jr., "Input matching, error augmentation, self-tuning and output error identification: Algorithmic similarities in discrete adaptive model following ", IEEE Trans. Aut. Control, Vol. AC-25, No. 4 ,pp. 697-703, 1980.

[50] T. Kailath, "Linear Systems", Prentice Hall, Inc., 1980.

[51] C.G. Kallstrom, K.J. Aatrom, N.E. Thorell, J.

Erikson and L. Stern, "Adaptive autopilots for tankers", Automatica, Vol. 15, pp. 241- ,1979.

[52] L. Keviczky and J. Hethessy, "Self-tuning minimum variance control of MIMO discrete time systems", Automatic Control Theory Appl., Vol. 5, pp. 11-17.

[53] H.N. Koivo, "A multivariable self-tuning controller", Automatica, Vol. 16, pp. 351-366, 1980.

[54] G. Kreisselmeier, "On adaptive state regulation", IEEE Trans. Aut. Control, Vol. AC-27, No. 1, pp. 3-17, 1982.

[55] V. Kucera, "Discrete linear control. The polynomial equation approach", Chichester, 1979.

[56] P.R. Kumar, "A survey of some results in stochastic adaptive control", SIAM J. Ccontrol and Optimization, Vol. 23, No. 3, 1985.

[57] I.D. Landau, "Adaptive control; the model reference approach", Marcel Dekker Inc. New York, 1979.

[58] I.D. Landau, "Adaptive controllers with explicit and implicit reference models and stochastic self-tuning regulators - equivalence and duality aspects", Proc. 17th IEEE - CDC conference, San Diego, USA, 1981.

[59] I.D. Landau, "Model reference adaptive control and stochastic self-tuning regulators - towards a cross fertilization", Proc. AFOSR Workshop: Adaptive control, Champaign, USA.

[60] I.D. Landau and R. Lozano, "Unification and evaluation of discrete time explicit model reference adaptive control designs", Automatica, Vol. 17, pp. 593- ,1981

[61] I.D. Landau, "Combining model reference adaptive controllers and stochastic self-tuning regulators" Automatica, Vol. 18, No. 1, pp. 77-84, 1982.

[62] C.C. Lim and W. Forsythe, "Autopilots for ship control", Proc. IEE Part D, CTA, Vol. 130, pp. 281- ,1983.

[63] L. Ljung, "Asymptotic behaviour of the extended Kalman filter as a parameter estimator for linear systems", IEEE Trans. Aut. Control, Vol. AC-24, No. 1, pp. 36-50, 1979.

[64] L. Ljung, "Analysis of recursive stochastic

algorithms", IEEE, Trans. Aut. Control, Vol. AC-22, No. 4, pp. 551-1977.

[65] L. Ljung, "On positive real transfer functions and the convergence of some recursive schemes", IEEE Trans. Aut. Control, Vol. AC-22, No. 4, pp. 539-551, 1977.

[66] L. Ljung and Soderstrom, "Theory and practice of recursive identification", MIT Press, 1983.

[67] L. Ljung and I.D. Landau, "Model reference adaptive systems and self-tuning regulators - some connections", Proc. 7th IFAC Congress, Vol. 3, pp. 1973-1980, 1978.

[68] R. Lozano and I.D. Landau, "Redesign of explicit and implicit discrete time model reference adaptive control schemes", Int. J. Control, Vol. 33, pp. 1981.

[69] D.G. Luneberger, "An introduction to observers", IEEE Trans. Aut. Control, Vol. AC-16, pp. 596-602, 1971.

[70] R.V. Monopoli, "Model reference adaptive control with an augmented error signal", IEEE Trans. Aut. Control, Vol. AC-25, pp. 433- ,1974.

[71] J.B. Moore and H. Weiss, "Recursive prediction error methods for adaptive estimation", IEEE Trans. on Systems Man and Cybernetics, Vol.SMC-9, No. 4, pp. 197-205, 1979.

[72] M. Mort and D.A. Linkens, "Identification and self-tuning control of linear and nonlinear ship models", Proc. 6th ship control systems symposium, Ottawa, 1981.

[73] M. Mort and D.A. Linkens, "Self-tuning controllers for surface ship manoeuvering", IMC Symposium, Dynamic Analysis of Vehicle Ride and Manoeuvering Characteristics", London, 1978.

[74] K.S. Narendra and R.P. Singh, "A golbaly stable adaptive controller for multivariable systems", Proc. 20th IEEE Conf. Dec. Contr., San Diego, CA, 1981.

[75] H. Nicholson, "Modeling of dynamical systems", IEE, Vol. 1, London, 1980.

[76] N.H. Norrbin, "On the added resistance due to steering on a straight course", 13th International Towing Tank Conf. Berlin, 1972.

[77] F.K. Omani and N.K. Sinha, "A modified approach to state space self-tuning control with pole assignment", Accepted for publication, IEE Proc. Part D, CTA, 1985.

[78] F.K. Omani and N.K. Sinha, "A modified state space approach to multivariable self-tuning control with pole assignment", Accepted for publication, IEE Proc. Part D CTA, 1985.

[79] F.K. Omani and N.K. Sinha, "Modification of the bootstrap algorithm to ensure asymptotic convergence of the estimated parameters", Submitted for publication, IEEE Trans. Aut. Control, 1984.

[80] F.K. Omani and N.K. Sinha, "An adaptive autopilot for nonlinear surface ships", Submitted for publication, Automatica, 1985.

[81] P.C. Parks, "Liapunov redesign of model reference adaptive control systems", IEEE Trans. Aut. Control, Vol. AC-11, pp. 362-367, 1966.

[82] V. Peterka, "Adaptive digital regulation of noisy systems", Proc. 2nd IFAC symposium on identification and process parameter estimation, Prague, 1970.

[83] V.M. Popov, "Invariant description of linear time invariant controllable systems", Kybernetika Vol. 3, pp. 173-187, 1970.

[84] D.L. Prager and P.E. Wellstead, "Multivariable pole assignment self-tuning regulators", IEE Proc. D, CTA Vol. 128, No. 1 , pp. 9-18, 1981.

[85] R.M. Prasad and A.K. Sinha, "On bootstrap identification using stochastic approximation", IEEE Trans. Aut. Control Vol. AC-29, pp. 576- , 1984.

[86] J. Quevedo and L. Basanez, "Adaptive autopilot based on pole assignment", Proc. 6th Ship Control Systems Symposium, Ottawa, 1981.

[87] C. Samson and J.J. Fuchs, "Discrete adaptive regulation of not necessarily minimum-phase systems", IEE Proc. D, CTA, Vol. 128, No. 3, pp. 102-108, 1981.

[88] L.S. Shieh, C.T. Wang and Y.T. Tsay, "Multivariable state feedback self-tuning controllers", Stochastic Analysis and Applications, Vol. 3, No.2, pp. 189-212, 1985.

[89] L.S. Shieh, C.T. Wang and Y.T. Tsay, "Fast suboptimal state space self-tuner for linear

stochastic multivariable systems", IEE Pro. D, CTA, Vol. 130, No. 4 pp. 143-154, 1983.

[90] L.S. Shieh and Y.T. Tsay, "Transformations of a class of multivariable control systems to block companion forms" IEEE Trans. Aut. Control, Vol. AC-27, pp. 199- ,1982.

[91] N.K. Sinha, "Minimal realization of transfer function matrices a comparative study of different methods", Int. J. Control, Vol. 22, pp. 627-639, 1975.

[92] N.K. Sinha, M.M. Gupta and P.N. Nikiforuk, "Recent advances in adaptive control", Journal of Cybernetics, Vol. 6, pp. 79-100, 1976.

[93] A.K. Sinha, "Reply on bootstrap estimation of system parameter and states", IEEE Trans. Aut. Control, Vol. AC-29, pp. 576 , 1984.

[94] D. Sprevak and M.M. Newman, "Comments on bootstrap estimation of parameters and states of linear multivariable systems", IEEE Trans. Aut. Control, Vol. AC-25, pp. 138-139, 1980.

[95] V. Strejc, "State space theory of discrete linear control", John Wiley & Sons, 1981.

[96] A. Tiano and E. Volta, "Comparison of various adaptive control techniques applied to autopilot design", Proc. 6th Ship Control Systems Symposium, Ottawa, 1981.

[97] Y.T. Tsay and L.S. Shieh, "State-space approach for self-tuning feedback control with pole assignment", IEE Proc. D, CTA, Vol. 128, No.3, pp. 93-101, 1981.

[98] K. Warwick, "Self-tuning regulators: a state space approach", Int. J. Control, Vol. 33, No. 4, pp. 839-858, 1981.

[99] K. Warwick and J.H. Westcott, "Arbitary linear output feedback self-tuners", Proc. 6th IFAC Symposium on Identification and System Parameter Estimation, Washington, DC, pp. 1040-1045, 1982.

[100] P.E. Wellstead, D. Prager and P. Zanker, "Pole assignment self-tuning regulators", Proc. IEE, Vol. 126, No. 8, pp. 781- ,1979.

[101] P.E. Wellstead and S.P. Sanoff, "Extended self-tuning algorithm", Int. J. Control, Vol. 34, No. 3, pp. 433-455, 1982.

[102] M.I. Younis, W.G. Vogt and M.S. Mahmoud, "Linear estimation of nonlinear sampled-data systems", Automatica, Vol. 13, pp. 519-531, 1977.

[103] H. Chen, "Recursive estimation and control for stochastic systems", John Wiley & Sons, 1985.

[104] J. Neveu, "Discrete parameter martingales", North-Holland, New York, 1975.

[105] V. Solo, "The convergence of AML", IEEE. Trans. Aut. control, Vol. AC-24, No. 6, pp. 958-962.

APPENDIX

MODIFICATION OF THE BOOTSTRAP ESTIMATOR TO ENSURE ASYMPTOTIC CONVERGENCE OF THE ESTIMATED PARAMETERS

A1. The Bootstrap Estimator

The original bootstrap estimator proposed in [32,82] may be described as follows. Consider the system modeled in the observability canonical form

$$X_{t+1} = \left[\begin{array}{c|c} 0 & I \\ \hline \multicolumn{2}{c}{a_b^T} \end{array}\right] X_t + \begin{bmatrix} b_b \end{bmatrix} u_t + W_t \qquad (A1.a)$$

$$y_t = [1,0..,0]X_t + e_t \qquad (A1.b)$$

or

$$X_{t+1} = A_b X_t + B_b u_t + W_t \qquad (A1.c)$$

$$y_t = C_b X_t + e_t \qquad (A1.d)$$

where e_t and W_t are zero mean gaussian noise sequences of the appropriate dimensions. The vectors a_b and b_b represent the non-trivial elements of A_b and B_b respectively. The special canonical structure of A_b allows us to rewrite (A1)

as

$$y_t = [a_b^T, b_b^T] \begin{bmatrix} X_{t-n} \\ u_{t-1} \\ \vdots \\ u_{t-n} \end{bmatrix} + \xi_t \qquad (A2.a)$$

or more compactly

$$y_t = \theta_b^T \phi_{b,t} + \xi_t \qquad \text{(A2.b)}$$

where

$$\xi_t = \sum_{i=1}^{n} w_{i,t-i} + e_t \qquad \text{(A2.c)}$$

ξ_t is uncorrelated with $\phi_{B,t}$, hence the RLS algorithm was used to estimate θ_b from (A2.a). However, X_t is not available, hence a stochastic approximation algorithm was used to update the state estimates thus:

$$\hat{X}_{t|t} = \hat{X}_{t|t-1} + \Omega_t C_b^T (y_t - C_b \hat{X}_{t|t-1}) \qquad \text{(A3)}$$

However as pointed out in [1,90,91], $\Omega_t \to 0$, as $t \to \infty$. Hence it becomes impossible to update $\hat{X}_t$. The resulting large error bounds on $\hat{X}_t$ prevents the convergence of the parameter estimates to their unbiased values [1,94].

A2. The Modfied Algorithm

Any system represented by (A1) may after applying the standard Kalman filtering algorithm and suitable transformation be also represented by

$$X_{t+1} = \left[a_c \,\middle|\, \begin{matrix} I \\ \hline 0 \end{matrix} \right] X_t + \begin{bmatrix} b_c \end{bmatrix} u_t + \begin{bmatrix} k_c \end{bmatrix} e_t \qquad \text{(A4.a)}$$

$$y_t = [1, 0, \ldots, 0] X_t + e_t \qquad \text{(A4.b)}$$

or more compactly

$$X_{t+1} = A_c X_t + B_c u_t + K_c e_t \qquad \text{(A4.c)}$$

$$y_t = C_c X_t + e_t \qquad \text{(A4.d)}$$

Due to the canonical structure of A_o, (A4a-d) can be written as

$$y_t = [a_c^T, b_c^T, k_c^T] \cdot \begin{bmatrix} X_{1,t-1} \\ \vdots \\ X_{1,t-n} \\ u_{t-1} \\ \vdots \\ u_{t-n} \\ e_{t-1} \\ \vdots \\ e_{t-n} \end{bmatrix} + e_t \qquad \text{(A4.e)}$$

which can be written in compact form as

$$y_t = \theta_c^T \bar{\phi}_{c,t} + e_t \qquad \text{(A4.f)}$$

Since the innovations sequence is white one may use an ELS type algorithm to estimate the parameters from (A4.e). The estimates of the state variable $X_{1,t}$ required in (A4.e) is obtained as the one step prediction states of the innovations model (A4.c).

The modified algorithm is thus:

$$\phi_{c,t}^T = [\hat{X}_{1,t-1}, \ldots, \hat{X}_{1,t-n}, u_{t-1}, \ldots, u_{t-n}, \bar{\varepsilon}_{t-1}, \ldots, \bar{\varepsilon}_{t-n}] \qquad \text{(A5.a)}$$

$$P_o = \sigma I \ , \ \sigma >> 0$$

$$\lambda_t = \lambda_{oo}\lambda_{t-1} + (1-\lambda_{oo}) \qquad \text{(A5.b)}$$

$$\lambda_{oo} = 0.99, \ 0 < \lambda_o < 1$$

$$\varepsilon_t = y_t - \hat{y}_t \qquad \text{(A5.c)}$$

$$L_t = \frac{P_{t-1}\phi_{c,t}}{\lambda_t + \phi_{c,t}^T P_{t-1}\phi_{c,t}} \qquad \text{(A5.d)}$$

$$\theta_{c,t} = \theta_{c,t-1} + L_t\varepsilon_t \qquad \text{(A5.e)}$$

$$\bar{\varepsilon}_t = y_t - \theta_{c,t}^T\phi_{c,t} \qquad \text{(A5.f)}$$

$$P_t = \frac{1}{\lambda_t}\left[P_{t-1} - \frac{P_{t-1}\phi_{c,t}\phi_{c,t}^T P_{t-1}}{\lambda_t + \phi_{c,t}^T P_{t-1}\phi_{c,t}}\right] \qquad \text{(A5.g)}$$

$$\hat{X}_{t+1} = A_c(\theta_{c,t})\hat{X}_t + B_c(\theta_{c,t})u_t + K_c(\theta_{c,t})\varepsilon_t \tag{A5.h}$$

$$\hat{y}_{t+1} = \hat{X}_{1,t+1} \tag{A5.i}$$

where $\theta_{c,t}$ is the current estimate of θ_c.

A3. Comparison With The RPE Estimator

Consider

$$\operatorname*{Min}_{\theta_c} \left[\frac{1}{2} E\{\varepsilon_t^2\} \right] \tag{A6.a}$$

Subject to (A4.a) and (A4.b)

In view of (A4.e),(A5.a) and (A5.b), the above problem is equivalent to

$$\operatorname*{Min}_{\theta_c} \left[\frac{1}{2} E\{\varepsilon_t^2\} \right]$$

Subject to

$$\varepsilon_t = y_t - \theta_{c,t}^T \phi_{c,t} \tag{A7.a}$$

Define

$$\psi_t = -\frac{d\varepsilon_t}{d\theta_{c,t}} = \phi_{c,t} + \frac{d\phi_{c,t}^T}{d\theta_{c,t}}\theta_{c,t} \tag{A7.b}$$

The RPE algorithm described in chapter 4 uses ψ_t as a stochastic descent direction for the recursive minimization of (A6.a). Using the ordinary differential equation (ODE) approach of Ljung [60,61;63] it is easy to show that $\theta_{c,t} \rightarrow \theta_c$ with probability 1, where θ_o is a local minimum of (A6). However if we ignore the implicit dependence of $\phi_{c,t}$ on $\theta_{c,t}$, i.e. the second term on the right hand side of

(A7.b) we get a PLR type algorithm, i.e

$$\psi_t \cong \phi_{c,t}$$

For the modified bootstrap algorithm the exact relationship between ψ_t and $\phi_{c,t}$ may be obtained by differentiation; thus

$$\psi_t = \frac{d\hat{y}_t^T}{d\theta_c} \tag{A8.a}$$

Define

$$W_t = \frac{d\hat{X}_t}{d\theta_t} \tag{A8.b}$$

$$M_t = \frac{\partial}{\partial\theta_t}[A_c\hat{X}_t + B_c u_t + K_c\bar{\varepsilon}_t] \tag{A8.c}$$

then

$$W_{t+1} = [A_c - K_cC_c]W_t + M_t + [0] \tag{A8.d}$$

introducing the forward shift operator z

$$W_t = (zI - A_c + K_cC_c)^{-1}M_t \tag{A8.e}$$

$$\psi_t = C_c(zI - A_c + K_cC_c)^{-1}M_t \tag{A8.f}$$

$$= \frac{C_c[\mathrm{adj}(zI - A_c + K_cC_c)M_t}{\det(zI - A_c + K_cC_c)} \tag{A8.g}$$

$$= \frac{[z^{-1},\ldots,z^{-n}]}{c(z^{-1})}\,[IX_{1,t} \quad Iu_t \quad I\bar{\varepsilon}_t] \tag{A8.h}$$

where

$$c(z^{-1}) = 1 + \sum_{i=1}^{n}(a_i - k_i)z^{-i} \tag{A8.i}$$

a_i, k_i are the elements of a_c and k_c respectively

$i = 1,\ldots,n$

which reduces to

$$\frac{1}{c(z^{-1})}\begin{bmatrix} x_{1,t-1} \\ \vdots \\ x_{1,t-n} \\ u_{t-1} \\ \vdots \\ u_{t-n} \\ \bar{\varepsilon}_{t-1} \\ \vdots \\ \bar{\varepsilon}_{t-1} \end{bmatrix} \tag{A8.j}$$

i.e

$$\Psi_t = \frac{1}{c(z^{-1})}\phi_{c,t} \tag{A8.k}$$

Thus inspite of using the estimated states in the data vector the asymptotic properties of the modified bootstrap algorithm is the same as that of the ELS estimator. The usefulness of the modified BSE however, is in its application to the joint parameter and state estimation of a parsimonious MIMO state space model as demonstrated in chapter 3.

A4. Convergence Analysis of The Modified BSE Estimator

Theorem A1:-

Consider the algorithm (A5), with the system modeled by

$$y_t = \theta_c^T \bar{\phi}_t + e_t \tag{A9.a}$$

assume that the following conditions are verified:

(a) The correct model order has been selected.

(b) $$c(z^{-1})(\bar{\varepsilon}_t - e_t) = (\theta_c - \theta_{c,t})^T \phi_{c,t} \tag{A9.b}$$

(c) e_t is a white noise sequence

(d) $E\{\phi_{c,t} e_t\} = 0$, for all θ

Then using the now standard ODE approach of Ljung [61,62,63] or alternatively the martingale approach [40,105], the sufficient conditions for the asymptotic convergence of (A5) is given by

$$\text{Prob}\{ \lim_{t \to \infty} \theta_{c,t} \to \theta_c \} = 1 \quad \text{globally} \tag{A10.a}$$

if

$$\text{Real} \left\{ \frac{1}{c(z^{-1})} - \frac{1}{2} \right\} > 0 \tag{A10.b}$$

Condition (d) above is easily verified since e_t is white noise and $\phi_{c,t}$ does not contain e_t. Condition (b) may be verified as follows.

$$\bar{\varepsilon}_t = y_t - \hat{y}_t \tag{A12.a}$$

$$\bar{\varepsilon}_t = \theta_o^T \bar{\phi}_t - \theta_{c,t}^T \phi_{c,t} + e_t \tag{A12.b}$$

$$= \theta_c^T (\bar{\phi}_t - \phi_{c,t}) + (\bar{\phi}_{c,t} - \phi_{c,t})^T \theta_{c,t} + e_t \tag{A12.c}$$

$$= [a_c^T, b_c^T, k_c^T] \begin{bmatrix} -e_{t-1} + \bar{\varepsilon}_{t-1} \\ \vdots \\ -e_{t-n} + \bar{\varepsilon}_{t-n} \\ 0 \quad\quad 0 \\ \vdots \\ 0 \quad\quad 0 \\ e_{t-1} - \bar{\varepsilon}_{t-1} \\ \vdots \\ e_{t-n} - \bar{\varepsilon}_{t-n} \end{bmatrix} + (\theta_c - \theta_{c,t})^T \phi_{c,t} + e_t \tag{A12.d}$$

and using (A8.i), (A12.d) reduces to

$$c(z^{-1})(\bar{\varepsilon}_t - e_t) = (\theta_c - \theta_{c,t})^T \phi_{c,t} + e_t$$

This verifies condition (b). Thus provided the correct model order is known condition (a) is verified and (A10) holds.

A5. The Modified Algorithm For The Observability Canonical Form

Consider the system modeled by

$$X_{t+1} = \begin{bmatrix} 0 & \vdots & I \\ \hline & a_o^T & \end{bmatrix} X_t + \begin{bmatrix} b_o \end{bmatrix} u_t + \begin{bmatrix} k_o \end{bmatrix} e_t \qquad \text{(A13.a)}$$

$$y_t = [1,0,\ldots,0]X_t + e_t \qquad \text{(A13.b)}$$

or more compactly

$$X_{t+1} = A_o X_t + B_o u_t + K_o e_t \qquad \text{(A13.c)}$$

$$y_t = CX_t + e_t \qquad \text{(A13.d)}$$

Due to the canonical structure of A_o (A12) can be written as

$$y_t = [a_o^T, b_o^T, k_o^T] \begin{bmatrix} X_{t-n} \\ u_{t-1} \\ \vdots \\ u_{t-n} \\ e_{t-1} \\ \vdots \\ e_{t-n} \end{bmatrix} + e_t \qquad \text{(A14.a)}$$

which can be written in compact form as

$$y_t = \theta_o^T \phi_{o,t} + e_t \qquad \text{(A14.b)}$$

Define

$$\phi_{o,t} = [\hat{X}_{t-n}^T, u_{t-1}, \ldots, u_{t-n}, \bar{\varepsilon}_t, \ldots, \bar{\varepsilon}_{t-n}]^T \qquad \text{(A14.c)}$$

then algorithm (A5) may be used to estimate the parameters of (A14.a), by replacing θ_c and $\phi_{c,t}$ with θ_o and $\phi_{o,t}$.

The exact gradient vector Ψ_t for this canonical form may be obtained by differentiation thus:

$$\Psi_t = \frac{d}{d\theta_{o,t}}(\theta_{o,t}^T\phi_{o,t}) \qquad \text{(A15.a)}$$

Using (A8) Ψ_t can be evaluated from

$$\Psi_t = \frac{[z^{-1}\alpha_1(z^{-1}),\ldots,z^{-i}\alpha_i(z^{-1}),\ldots,z^{-n}]}{c(z^{-1})}\left[\begin{array}{c|c|c} 0 & & \\ 0 & Iu_t & I\bar{\varepsilon}_t \\ \hat{X}_t^T & & \end{array}\right] \qquad \text{(A15.b)}$$

where

$$\alpha_i(z^{-1}) = 1 - \sum_{1}^{n-i} a_{o,j}z^{-j}, \qquad i = 1, \ldots, n-1$$

$$\Psi_t = \frac{1}{c(z^{-1})}\left[\begin{array}{c|c|c} I & 0 & 0 \\ \hline 0 & D & 0 \\ \hline 0 & 0 & D \end{array}\right]\phi_{o,t} \qquad \text{(A15.c)}$$

where $\quad D = \mathrm{diag}(\alpha_1(z^{-1}),\ldots,\alpha_i(z^{-1}),\ldots,\alpha_{n-1}(z^{-1}),1)$

(A15.c) thus reduces to

$$\Psi_t = [H(z^{-1})]\phi_{o,t} \qquad \text{(A15.d)}$$

In section A6. we shall establish that a sufficient condition for the asymptotic convergence of the estimated parameters, i,e .

$$\operatorname*{prob}_{\lim t \to \infty}\{\theta_{o,t} \to \theta_o\} = 1$$

is that

$$\text{Real } \{ [H(z^{-1}) - \frac{1}{2}[I]\} > 0 \qquad \text{(A16.a)}$$

Since $[H(z^{-1})]$ is diagonal the above condition reduces to

$$\text{Real } \{ \frac{1}{c(z^{-1})} - \frac{1}{2} \} > 0 \qquad \text{(A16.b)}$$

and

$$\text{Real } \{ \frac{a_i(z^{-1})}{c(z^{-1})} - \frac{1}{2} \} > 0 \qquad \text{(A16.c)}$$

$$i = 1, \ldots, n-1$$

A6. Convergence Analysis For The Observability Canonical form

Define

$$U_{t-1} = (u_{t-1}, \ldots, u_{t-n})^T$$
$$E_{t-1} = (e_{t-1}, \ldots, e_{t-n})^T$$
$$\hat{E}_{t-1} = (\bar{\varepsilon}_{t-1}, \ldots, \bar{\varepsilon}_{t-n})^T$$

Now

$$\bar{\varepsilon}_t = y_t - \hat{y}_t$$
$$= \theta_o^T \bar{\phi}_{o,t} - \theta_{o,t}^T \phi_{o,t} + e_t \qquad \text{(A17.a)}$$
$$= \theta_o^T [\bar{\phi}_{o,t} - \phi_{o,t}] + [\theta_o - \theta_{o,t}]^T \phi_{o,t} + e_t \qquad \text{(A17.b)}$$
$$= (a_o - a_{o,t})^T \hat{X}_{t-n} + (b_o - b_{o,t})^T U_{t-1} + (k_o - k_{o,t})^T \hat{E}_{t-1}$$
$$+ a_o^T (X_{t-n} - \hat{X}_{t-n}) + k_{o,t}^T (E_{t-1} - \hat{E}_{t-1}) + e_t \qquad \text{(A17.c)}$$

Using (A13) and performing recursive forward substitutions

$$X_{t-n} = \begin{bmatrix} y_{t-n} - e_{t-n} \\ y_{t-n+1} - e_{t-n+1} - b_{o,1}u_{t-n} - k_{o,1}e_{t-n} \\ \cdot \\ \cdot \\ \cdot \\ y_{t-n+i-1} - e_{t-n+i-1} - \sum_{j=1}^{i-1} b_{o,j}u_{t-n+i-j-1} \\ \qquad - \sum_{j=1}^{i-1} k_{o,j}e_{t-n+i-j-1} \\ \cdot \\ \cdot \\ \cdot \\ y_{t-1} - e_{t-1} - \sum_{j=1}^{n-1} b_{o,j}u_{t-j-1} - \sum_{j=1}^{n-1} k_{o,j}e_{t-j-1} \end{bmatrix}$$

(A17.d)

similarly

$$\hat{X}_{t-n} = \begin{bmatrix} \hat{y}_{t-n} \\ \hat{y}_{t-n+1} - b_{o,t,1}u_{t-n} - k_{o,t,1}\bar{\varepsilon}_{t-n} \\ \cdot \\ \cdot \\ \cdot \\ \hat{y}_{t-n+i-1} - \sum_{j=1}^{i-1} b_{o,t,j}u_{t-n+i-j-1} - \sum_{j=1}^{i-1} k_{o,t,j}\bar{\varepsilon}_{t-n+i-j-1} \\ \cdot \\ \cdot \\ \cdot \\ \hat{y}_{t-1} - \sum_{j=1}^{n-1} b_{o,t,j}u_{t-j-1} - \sum_{j=1}^{n-1} k_{o,t,j}\bar{\varepsilon}_{t-j-1} \end{bmatrix}$$

(A17.e)

therefore

$$a_o^T(X_{t-n} - \hat{X}_{t-n}) =$$

$$a_o^T \begin{bmatrix} \bar{\varepsilon}_{t-n} - e_{t-n} \\ \bar{\varepsilon}_{t-n+1} - e_{t-n+1} - (b_{o,1} - b_{o,t,1})u_{t-n} - (k_{o,1} - \\ k_{o,t,1})\bar{\varepsilon}_{t-n} + k_{o,1}(\bar{\varepsilon}_{t-n} - e_{t-n}) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \bar{\varepsilon}_{t-n+i-1} - e_{t-n+i-1} - \sum_{j=1}^{i-1} (b_{o,j} - b_{o,t,j})u_{t-n+i-j-1} \\ - \sum_{j=1}^{i-1}(k_{o,j} - k_{o,t,j})\bar{\varepsilon}_{t-n+i-j-1} + \sum_{j=1}^{i-1} k_{o,j}(\bar{\varepsilon}_{t-n+i-j-1} - e_{t-n+i-j-1}) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \bar{\varepsilon}_{t-1} - e_{t-1} - \sum_{j=1}^{n=1}(b_{o,j} - b_{o,t,j})u_{t-j-1} - \sum_{j=1}^{n=1}(k_{o,j} - k_{o,t,j})\bar{\varepsilon}_{t-j-1} \\ + \sum_{j=1}^{n-1} k_{o,j}(\bar{\varepsilon}_{t-j-1} - e_{t-j-1}) \end{bmatrix}$$

(A17.f)

sub for (A17.f) in (A17.c), rearranging and using (A8.i):

$$c(z^{-1})(\bar{\varepsilon}_t - e_t) = \begin{aligned}[t] &[\, (a_o - a_{o,t})^T \hat{X}_{t-n} + \\ &(b_{o,1} - b_{o,t,1})\alpha_1(z^{-1})u_{t-1} + \dots \\ &+ (b_{o,i} - b_{o,t,i})\alpha_i(z^{-1})u_{t-i} + \dots \\ &+ (b_{o,n} - b_{o,t,n})u_{t-n} + \\ &(k_{o,1} - k_{o,t,1})\alpha_1(z^{-1})\bar{\varepsilon}_{t-1} + \dots \\ &+ (k_{o,i} - k_{o,t,i})\alpha_i(z^{-1})\bar{\varepsilon}_{t-i} + \dots \\ &+ (k_{o,n} - k_{o,t,n})\bar{\varepsilon}_{t-n} \,] \end{aligned} \qquad \text{(A17.g)}$$

Thus using the definition for $[H(z^{-1})]$ from (A15), (A17.g) can be written as

$$\bar{\varepsilon}_t = (\theta_o - \theta_{o,t})^T [H(z^{-1})] \phi_{o,t} + e_t$$

This verifies assumption (b) of theorem A1. Now e_t is white noise and is not contained in $\phi_{o,t}$ hence is independent of $\phi_{o,t}$ for all θ, this verifies assumptions (c) and (d). Thus if the correct model order has been selected, by theorem A1:

$$\text{prob}\{ \lim_{t \to \infty} \theta_{o,t} \to \theta_o \} = 1$$

if $\{[H(z^{-1})] - \frac{1}{2}[I]\}$ is strictly positive real. This establishes conditions (A16) as required.

Comparing equations (A16) and (A10b) it is obvious that the sufficient conditions for the asymptotic convergence of the bootstrap algorithm is more restrictive when it is applied to the observability canonical form, than the case for the observer canonical form. Hence it is the latter canonical form that should be adopted for joint parameter and state estimation by the BSE estimator.

A7. Proof Of Theorem A1

Theorem A1 may be proved by the ODE approach of Ljung [64], although a useful method, such approach is based to some extent on heuristic arguments (Goodwin and Sin [42]). The approach adopted here follows the methods proposed by Solo [105], based on the asymptotic properties of martingale stochastic processes. In a recent work, Chen [103] has also proposed a more rigorous version of the ODE approach using the asymptotic properties of martingale sequences.

Define

$$\tilde{\theta}_t = \theta_{c,t} - \theta_c \tag{A18.a}$$

then subtracting θ_c from both sides of (A5.e) and using (A5.f) and (A5.g)

$$\tilde{\theta}_t = \tilde{\theta}_{t-1} + P_{t-1}\phi_{c,t}\bar{\varepsilon}_t \tag{A18.b}$$

Select the stochastic Lyapunov function

$$V_t = \tilde{\theta}_t^T P_t^{-1} \tilde{\theta}_t \tag{A18.c}$$

then multiplying (A18.b) by $\tilde{\theta}_t^T P_t^{-1}$ and using (A5), V_t satisfies the recursive relationship

$$V_t = V_{t-1} + b(t)^2 - 2b(t)\bar{\varepsilon}_t - \phi_{c,t}P_{t-1}\phi_{c,t}\bar{\varepsilon}_t^2 \tag{A18.d}$$

$$= V_{t-1} + b(t)^2 - 2b(t)z(t) - 2b(t)e_t - \phi_{c,t}P_{t-1}\phi_{c,t}\bar{\varepsilon}_t^2 \tag{A18.e}$$

where

$$b(t) = -\phi_{c,t}^T\tilde{\theta}_t \tag{A18.f}$$

$$z(t) = \bar{\varepsilon}_t - e_t \tag{A18.g}$$

Let F_t be the set of increasing σ-algebra's generated by

$$\{e_0, \ldots, e_t, \phi_{c,0}, \ldots, \phi_{c,t}\}$$

also let

$$E\{\, e_t^2 | F_{t-1}\} = \sigma^2$$

then

$$E\{V_t | F_{t-1}\} = V_{t-1} + E\{(b(t)^2 - 2b(t)z(t)) | F_{t-1}\}$$
$$-2\phi^T_{c,t} P_{t-1} \phi_{c,t} \sigma^2 - E\{(\phi^T_{c,t} P_{t-1} \phi_{c,t}) \bar{\varepsilon}_t^2 | F_{t-1}\} \tag{A18.h}$$

Define

$$g(t) = z(t) - \frac{1}{2} b(t) \tag{A19.a}$$

thus

$$E\{V_t | F_{t-1}\} = V_{t-1} - 2E\{b(t)g(t) | F_{t-1}\}$$
$$-E\{\phi_{c,t}^T P_{t-1} \phi_{c,t} \bar{\varepsilon}_t^2 | F_{t-1}\} + 2\phi^T_{c,t} P_t \phi_{c,t} \sigma^2 \tag{A19.b}$$

from (A9) and (A18.a)

$$g(t) = \left(\frac{1}{c(z^{-1})} - \frac{1}{2}\right) b(t)$$

but from (A10.b)

$$\text{Real}\left(\frac{1}{c(z^{-1})} - \frac{1}{2}\right) > 0$$

thus $E\{g(t)b(t)\} > 0$, (see Ljung et. al [66]).

Thus the following inequality can be established

$$E\{(V_t + 2b(t)g(t)) | F_{t-1}\} \leq V_{t-1} + 2\phi_{c,t}^T P_t \phi_{c,t} \sigma^2 \tag{A19.c}$$

<u>Theorem A2 Martingale Stochastic Convergence Theorem (MGCT)</u>

Let T_t, ν_{t+1} and β_{t+1} be non-negative sequences of random variables and F_t a sequence of increasing σ-algebras.

Then if

$$E\{T_t | F_{t-1}\} \leq T_{t-1} + \nu_t - \beta_t \qquad \text{(A20.a)}$$

and

$$\sum_1^{\infty} \nu_t < \infty \quad \text{a.s} \qquad \text{(A20.b)}$$

then

$$T_t \to T < \infty \quad \text{w.p.1} \qquad \text{(A20.c)}$$

and

$$\sum_1^{\infty} \beta_t < \infty \quad \text{w.p.1} \qquad \text{(A20.d)}$$

The proof of theorem A2 due to Neveu is given in [104].

Now define

$$T_t = V_t + 2b(t)g(t) \qquad \text{(A20.e)}$$

then (A19.c) reduces to

$$E\{T_t | F_{t-1}\} \leq T_{t-1} + 2\phi_{c,t}^T P_t \phi_{c,t} \sigma^2 \qquad \text{(A20.f)}$$

dividing (A20.f) by t

$$E\{\frac{T_t}{t} | F_{t-1}\} \leq \frac{T_{t-1}}{t-1} - \frac{T_{t-1}}{t(t-1)} + \frac{2\phi_{c,t}^T P_t \phi_{c,t} \sigma^2}{t} \qquad \text{(A20.g)}$$

Now

$$2\sigma^2 \sum_1^{\infty} \frac{\phi_{c,t}^T P_t \phi_{c,t}}{t} < \infty \text{ , ([66,105]).}$$

thus from (A20c) (A20d)

(a) $\dfrac{T_t}{t} < \infty$ w.p.1

(b) $\sum_{1}^{\infty} \frac{1}{t} \frac{T_{t-1}}{t-1} < \infty$ w.p.1

((a) and (b)) imply $T_t \to 0$ w.p.1 (to avoid contradiction).

Thus from (A20.e)

$$\tilde{\theta}_t^T P_t^{-1} \tilde{\theta}_t + 2b(t)g(t) \to 0 \qquad \text{(A21.a)}$$

and by positivity

$$\tilde{\theta}_t P_t^{-1} \tilde{\theta}_t \to 0 \quad \text{w.p.1}$$

$$\theta_{c,t} \to \theta_c \quad \text{w.p.1}$$

and

$$2b(t)g(t) \to 0 \quad \text{w.p.1} \qquad \text{(A21.b)}$$

hence since $\left(\frac{1}{c(z^{-1})} - \frac{1}{2} \right)$ is a stable filter

$$\Rightarrow b(t) \to 0 \qquad \text{(A21.c)}$$

but

$$z(t) = \bar{\varepsilon}_t - e_t = \frac{1}{c(z^{-1})} b(t)$$

thus (A21.c) implies that

$$\bar{\varepsilon}_t - e_t \to 0$$

i.e the residual sequence converges to the driving noise.

This completes the proof.