A MODEL FOR CONFIDENCE JUDGMENTS

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IN
CHOICE TASKS

By

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ABSTRACT

It is proposed that the basis of confidence judgments in choice reaction time tasks is the distance between the initial and terminating values of a random walk decision process. Several qualitative predictions and one important quantitative prediction about confidence are derived from the model and tested in a "same-different" line length discrimination reaction time task with two levels of response time deadline. In the task, S simultaneouly indicates whether two sequentially presented line lengths are the same length or different lengths and a level of confidence associated with the choice response. The model predicts all the major qualitative features of the data. In addition, a predicted linear relationship between [P(correct)- P(error)] and the average "signed" confidence fits the data very well.

A second test of the model , using two deadlines and an "accuracy" condition in which response time was unimportant, replicates both the results of earlier experimenters and the results of the first experiment. Despite a concensus in the literature to the contrary, confidence and response time are shown to be positively related.

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TABLE OF CONTENTS

Quod enim mavult homo verum esse, id potius credit.

(For whatever a man would prefer to be true,
he more readily believes.)

Francis Bacon, 1561-1626

LIST OF FIGURES

LIST OF TABLES

TABLE XII- Estimates of mean response time, movement time and release time for three Ss (NN,NW,RB) pooled.

TABLE XIII-Predicted and observed values of average "signed" confidence.

An almost universal feature of human decisions is the feeling of some degree of certainty that the correct choice has been made. These feelings, commonly referred to as "confidence", are usually considered to lie on a scale ranging from "complete confidence", associated with unquestionably correct decisions, to "lack of confidence", associated with "wild guesses". Beginning with Peirce and Jastrow (1884) attempts have been made to relate confidence judgments to skill, task difficulty, various personalogical variables, judgment time and accuracy. This thesis is concerned with the development of a theoretical framework for the relationships between confidence, accuracy and judgment time.

Since confidence is an introspective variable, one would expect it to be well defined in the psychological literature. Unfortunately, the meaning of confidence was considered too self evident for anyone to bother to define it. Typically, subjects in the experimental psychology literature were simply instructed to rate their confidence on some arbitrary scale, the endpoints of which were called "complete confidence" and "no confidence". The magnitude of the response was taken as a measure of the magnitude of S's belief that the response chosen was correct. Confidence will be defined, for purposes of this thesis, as a response which indicates the intensity of an individual's belief in some state of affairs.

The confidence response is, in a sense, S's estimate of the probability that the choice response is correct. Data in the confidence literature show that these estimates are positively correlated with the "objective" probability that the choice response is correct. The problem of establishing a basis for these confidence responses will be the focus of this study.

In the past, confidence has often been investigated in the context of psychophysical discrimination. The discrimination paradigm allows presentation within a single session of a wide range of stimuli which differ along one simple physical dimension. The relationship between stimuli of various magnitudes on the physical dimension and the confidence with which responses to the stimuli are made is of general interest and easily obtained. This thesis will examine confidence within the framework of the psychophysical discrimination experiment.

Bayes (1743) developed the first normative model for confidence associated with a choice judgment. "Everyone sees in general that there is reason to expect an event with more or less confidence according to the greater or less number of times in which ... it has happened without failing, but we here see exactly what this reason is, on what principles it is founded, and how we ought to regulate our expectations." Bayes' model, roughly stated, proposed that if a rational individual had no reason to believe that either of two possible mutually exclusive causes of an event, e, is more likely than the other to be the actual cause of e prior to the occurence of e, then following the occurence of e, he should adjust his belief dependent upon the actual event observed.

Intensity of belief that any particular cause, θ_i (where i indexes the set of possible causes,) is responsible for the value of e observed should be related to the proportion of the total likelihood of the observed event which is associated with the cause θ_i. Given only possible causes, θ_1 and θ_2, if there is a large likelihood of e when θ_1 is operating and a low likelihood of e when θ_2 is operating, the occurence of e should intensify the belief that θ_1 was operating and decrease the intensity of the belief that θ_2 was operating. This may be stated as

$$P(\theta_i|e) = \frac{P(e|\theta_i)\ P(\theta_i)}{P(e|\theta_i)P(\theta_i) + P(e|\theta_j)P(\theta_j)} \tag{1}$$

where $P(\theta_i|e)$ is related to the intensity of belief that θ_i caused the observed event e, $P(e|\theta_i)$ is the likelihood of the event e when cause θ_i is in effect and $P(\theta_i)$ is the intensity of belief in the operation of θ_i prior to the occurence of e. If equal likelihoods are associated with the occurence of e when either θ_1 or θ_2 is operating then the occurence of e doesn't affect the intensity of belief in the operation of either cause.

Bayes' approach can be applied to a discrimination or choice experiment to yield predictions about confidence. Let there be two stimuli, s_1 and s_2. On each trial, s_1 is presented with probability $P(s_1)$; otherwise s_2 is presented. Let each stimulus s_i have associated with it a sensory random variable with a probability density function $f_i(x)$, $(i=1,2)$. Prior to each trial the intensity of belief in the occurence of s_i on the trial should be $P(s_i)$. If the sensory value X occurs on a trial, the intensity of S's belief that s_i was presented should be given by

$$P(s_i|X) = \frac{P(X|s_i)\ P(s_i)}{P(X|s_i)P(s_i) + P(X|s_j)P(s_j)}\ ;\ j \neq i\ . \tag{2}$$

If equal likelihoods are associated with the two stimulus values then (2) reduces to $P(s_i|X) = P(s_i)$ and the occurence of the sensory value X has no effect on belief. As the ratio of the likelihoods, $P(X|s_j)/P(X|s_i)$, increases, the intensity of belief that s_i was the stimulus presented decreases toward zero. As the ratio of the likelihoods decreases, the intensity of belief in s_i being the stimulus increases toward one. If the stimuli s_1 and s_2 have similar sensory distributions associated with them, (i.e., $f_1(x) \approx f_2(x)$ for most x) then $E[P(s_i|X)]$ should be approximately $P(s_i)$. However, if the sensory distributions associated with s_1 and s_2 are very different, then $E[P(s_i|X)] > P(s_i)$ for correct responses and $E[P(s_i|X)] < P(s_i)$ for errors.

C. S. Peirce (1878), (mentor of Wm. James and John Dewey,) appears to be the first to have considered the experimental study of confidence. He reasoned that a) intensity of belief should vary with the odds of an event (where odds are the ratio of the likelihood of the event to the likelihood of its complement); b) an event which occurs with absolute certainty should lead to an infinite belief; c) when there is an even chance (i.e., the odds equal one) intensity of belief should be equal to zero; d) as the odds of an event decrease below 1, belief in its complement should strengthen to infinity; and e) only the logarithm (log) of the odds has the required characteristics of intensity of belief. Therefore, intensity of belief (or confidence) can be equated to a $\log(p/(1-p))$, where p is the likelihood of the event with which the belief is associated. Peirce noted that Fechner's psycho-physical law was a similar relationship in that the "intensity of any sensation is proportional to the logarithm of the external force which produces it " (Peirce, 1878). In addition, the log of the odds has the property that the belief resulting from the occurence of both of two independent events is the sum of the beliefs resulting from each event separately.

The relationship between Peirce's Law and Bayes' Theorem can be seen in the following argument. The inverse of Peirce's Law is

$$p = (1 + e^{-c/a})^{-1} . \tag{3}$$

$P(\theta_1|e)$ in Bayes' Theorem, (1), has been called the "subjective probability" of θ_1 (DeKlerk and Oppe, 1970). Replacing p in Peirce's Law with $P(\theta_1|e)$ yields for the case of two possible causes,

$$c = a \log[\frac{P(\theta_1|e)}{P(\theta_2|e)}] = a \log [\frac{P(e|\theta_1)P(\theta_1)}{P(e|\theta_2)P(\theta_2)}] . \tag{4}$$

Substituting the value of c from (4) into (3) yields

$$p = [(\frac{P(e|\theta_2)\ P(\theta_2)}{P(e|\theta_1)\ P(\theta_1)}) + 1)^{-1}$$

$$= \frac{P(e|\theta_1)\ P(\theta_1)}{P(e|\theta_1)\ P(\theta_1) + P(e|\theta_2)\ P(\theta_2)}$$

which is identical to Bayes' theorem.

Peirce's and Bayes' formulations of confidence agreed that confidence and the likelihood of an event are related. Peirce's Law however, was based upon the idea that confidence was unbounded (an "odds" statement) while Bayes felt that bounds were appropriate for a measure of confidence and chose zero and one (for a "probability" statement). If the $P(\theta_1|e)$ are "subjective probabilities" in Bayes' model, then Peirce's c can be interpreted as the "subjective odds".

Peirce designed an experiment to demonstrate that the sensations produced by a stimulus are normally distributed random variables (Peirce and Jastrow, 1884). Anticipating Thurstone (1927) he supposed that if two stimuli are presented and the subject required to indicate whether the second stimulus is greater or less in magnitude than the first stimulus, the subject should simply respond on the basis of the sign of the difference between the two sensations that were associated with the two stimulus values. The distribution of the sensation differences is normal if the distributions of sensations are normal. The mean difference between sensations should be the difference between the means of the sensations associated with each of the stimuli. The proportion of "greater than" responses should be the integral of the normal distribution from zero to infinity (all possible positive values of the difference,) while the proportion of "less than" responses should be the integral of the normal distribution from zero to minus infinity. Opposed to his view of the perceptual-decision process was the Fechnerian differential threshold theory which claimed that for sensory differences below some "threshold" the probability of a correct response would simply be one-half. In order to determine if stimulus differences were above or below the threshold, Ss' confidence responses were recorded in addition to their choice responses. According to the Fechnerians, little or no confidence indicated that the threshold had not been exceeded. The proportion of errors made with various levels of confidence were, therefore of primary interest. Peirce proposed the relationship $c = a \log(\frac{p}{1-p})$ as an alternative to the threshold interpretation of the meaning of the confidence response.

In order to test Peirce's Law, Peirce and Jastrow performed as subjects in a tactile pressure discrimination task. A sequence of three stimuli was presented on each trial. On half the trials the stimulus was a 250 g pressure applied to a finger followed by an increase in pressure followed by a return to 250 g. On the other half of the trials, the stimulus was a pressure greater than 250 g followed by a decrease in pressure to 250 g followed by an increase to the original pressure. The 250 g stimulus was called the "standard". The "comparison" was fixed for a block of 25 trials at one of six pressures - 250.125, 250.250, 250.750, 251.000, 251.500, and 252.000 g. Ss task was to determine which type of trial (increase-decrease or decrease-increase) had been presented on the trial and to rate his confidence in his response on a four point scale (0, 1, 2, 3). Data were presented which showed a direct relationship between confidence levels and the probability of a correct response. Predictions of confidence based upon the probabilities of a correct response were presented and showed close agreement with the data. However, Peirce's methods of parameter estimation are unclear. Since the complete data were not presented, independent parameter estimation needed to test the adequacy of the model is impossible.

Later, Fullerton and Cattell (1892) collected confidence ratings on a four point (A, B, C, D) scale following each trial in a series of intensity discrimination experiments. Successively presented stimuli (brightness of lights in one experiment and lifted weights in another,) were used to determine the just noticeable difference (jnd) and examine its relationship to stimulus difference. Following Peirce and Jastrow, they used the confidence response as a way to combine two methods of determining the jnd. In the first of these methods, S was required to state whether a comparison stimulus was greater than or less than some standard stimulus. If the difference was too small to tell, a response of "doubtful" was recorded. Fullerton and Cattell proposed that the confidence response D was equivalent to a "doubtful" response in this method. It therefore indicated a "sub-threshold" difference between the sensations on that trial. In the other method, S was simply required to state whether a particular stimulus difference was "just noticeable" or not. Since the meaning of "just noticeable" was somewhat imprecise, Fullerton and Cattell proposed that a confidence judgment of B was equivalent to the just noticeable response if a just noticeable difference was taken to mean that S "feels some confidence" that there is a difference, while if a just noticeable difference meant that S was "sure" of a difference, then A corresponded to a just noticeable difference.

The sensitivities of Ss as determined by the two methods were compared and found to be unrelated. That is, a subject reporting a high proportion of A or B responses (indicating the occurrence of many supra-threshold trials,) did not necessarily have a small jnd as determined by the proportion of right and wrong responses. They emphasized that the meanings of category responses like "quite confident" (A), "less confident" (B), "uncertain" (C), and "doubtful" (D) were variable from one subject to another (as were the judgments of "just noticeable" with which Fechner and Weber had determined thresholds). Such responses, they stressed, should not be relied upon to provide information about a subject's sensitivity. The proportion of correct responses associated with each of the confidence responses, however, was directly related to the degree of confidence and the proportion of high confidence (A or B) responses was directly related to the stimulus difference.

Henmon (1911) studied the problem of the relationship between judgment time and accuracy in a forced choice line length discrimination experiment. He hypothesized that there are four different types of judgment processes. Two of these processes generate "quick" judgments with high confidence and accuracy while the other two processes generate longer judgments with "little confidence and reliability".

Thus, Henmon expected that when judgments were timed and Ss reported confidence judgments, confidence and accuracy would be associted with fast responses. Confidence and accuracy were expected to decrease in magnitude as judgment time increased. In order to test this hypothesis, he collected, after each judgment of line length, confidence judgments on a four point (a,b,c,d) scale. Two horizontal lines were presented simultaneously, 50 cm from S, horizontally separated by 10 mm, until approximately 300 ms after S made a judgment. In one half of the sessions, S was required to press a key with the hand corresponding to the longer line, while in the remaining sessions S was required to indicate the shorter of the two lines. Ss were aware that their judgment times were being recorded, but there were no special instructions to discourage them from taking more time than was necessary to be correct.

Henmon found that judgment times decreased as the confidence decreased, and that the average time for errors was longer than the average time for correct responses. Data were presented for 10 Ss, 3 of whom were run for 1000 trials, while the remaining seven were run for 500 trials. Five subjects had judgment times which decreased with increasing confidence while the other five subjects' judgment times were either increasing or U-shaped functions of confidence. Five subjects had faster average judgment times for their correct responses while the other five subjects had faster average judgment times for their errors. Nonetheless, it was Henmon's summary of his results (that as confidence decreased, judgment time increased and that errors take longer on the average than correct responses) rather than his amply presented data which formed the basis for subsequent ideas and investigations of confidence.

Henmon also found a direct relationship between confidence and accuracy and a lack of reliability between Ss in the use of confidence judgments. These relationships had been reported earlier by Fullerton and Cattell (1892) and Peirce and Jastrow (1888), but Henmon was the first investigator to examine these relationships within a single value of stimulus difference.

Kellogg (1931) argued that determinations of the relationship between accuracy , confidence and judgment time were marred by the fact that the subjects knew that their judgments were being timed and may have adopted some "unnatural" strategies of response. He felt that judgment times should be recorded under "normal" circumstances where the "judgment itself and not the time factor is the primary consideration". Subjects were required to judge which half of a lighted disc (6° visual angle, presented for one second) was brighter. Judgments were indicated by pressing keys and the time taken to do this after the onset of illuminationof the disc was recorded without the S's knowledge. On 3/7 of the trials the left half of the disc was brighter, on 3/7 of the trials the right half was brighter and on the remaining 1/7 of the trials both halves were equally illuminated. On two days Ss were required to judge which half of the disc was brighter while on the remaining two days the judgment "equally bright" was permitted. After 42 trials Ss estimated the proportion of trials in the block on which they "felt sure of " their judgment. This proportion was recorded as the confidence.

Kellogg presented the confidence averaged over blocks for each day (118 trials in 4 blocks of 42 trials/block) for each S in both the two-choice and three-choice conditions. The two-choice day with the higher mean judgment time had the lower average confidence for only 2 of the 5 Ss but the three-choice day with the longer mean judgment time had the higher average confidence for all 5 Ss. One could hesitate to infer any kind of relationship between confidence and judgment time on the basis of this somewhat meager evidence, but Kellogg concluded that "the time of judgment is a better index of the confidence of Ss when their judgments are permitted to vary among three categories than when they are restricted to two".

The next major contribution to the confidence literature was made by Volkmann (1934). He proposed a relationship between confidence and judgment time of the form $T = \frac{A}{(2c-1)}$. T was judgment time, c was some measure of confidence and A was a constant. As c approaches .5, T approaches infinity. Confidence was measured on an 11-point scale ranging from 0 to 1 where .5 indicated no confidence. Three observers made "comparative judgments upon the inclinations of lines visually presented using the categories greater or less ". The data showed that the "greater the difference in certainty between the two categories, the shorter the time of judgment".

Stimulated by Volkmann, Johnson (1939) chose to describe the by now "well known" inverse relationship between confidence and judgment time by means of an equation of the form $T = A\,e^{B(1-C)}$. This equation has the advantage over Volkmann's of predicting finite judgment times for responses made with "no confidence" while still predicting a decelerating decrease in judgement times as confidence increased. He used comparative judgments of line lengths to examine the relationships between a) confidence and magnitude of stimulus difference b) judgment time and magnitude of stimulus difference and c) judgment time and confidence. In order to control for attitudinal variables, which he suggested might influence the subjects' set for speed or accuracy, "attitude instructions" were used. In one condition ("usual") attitude was left uncontrolled, in a second condition ("speed") speed was emphasized over accuracy, and in a third condition ("accuracy") accuracy was emphasized while speed was unimportant. The stimulus was a horizontal line with short vertical uprights at each end and 50 mm from the left end. The line was thus divided into two segments, the left segment ("standard") always 50 mm long and the right segment ("comparison") was one of 15 randomly determined lengths - 50 mm $\pm$ 0,2,3,4,6,8 or 10 mm. Judgments of the comparison line as either shorter or longer than the standard were made by pressing one of two keys. Judgment time was recorded and then S made a confidence judgment on a 0 to 1.00 scale. Each of three Ss was run in blocks of 75 trials for 2 blocks per day for

3 days. Within each block the attitude instructions were constant. For all three Ss the sequence of attitude conditions was usual, usual, accuracy, speed, speed, accuracy. A total of 10 judgments were made by each S at each level of stimulus difference in each attitude condition.

The confidence judgments were averaged within each stimulus and attitude condition. Confidence judgments associated with the judgment "longer" were assigned positive values and those associated with the judgment "shorter" were assigned negative values. When plotted as a function of stimulus difference, the obtained average confidences were found to approximate a normal ogive. (For a small number of trials the average confidence can take on more different values than can the estimated probability of a correct response upon which estimates of sensitivity were based. Johnson probably saw this "confidence function" as the basis for a less time consuming and more convenient method of measuring sensitivity.)

Johnson examined the effects of the attitude conditions on response time, the confidence functions and the average confidence (when all confidence responses were given a positive sign). He found that the attitude conditions affected response times in the expected direction-average response times in the "accuracy" condition were longest followed by those in the "usual" and "speed" conditions. There was, however, no significant difference between the parameters of the normal ogives fitted to the confidence functions determined separately for each of the attitude conditions. In addition, although the average levels of confidence in the "speed" and "accuracy " conditions differed significantly from each other, the effects of both the special conditions on the average confidence were in the same direction. For each of the five Ss the average confidence in the two special conditions were either both higher or both lower than the average confidence in the usual condition. Furthermore, although Johnson did not examine the effect of attitude instructions on accuracy, the psychometric functions he presented did not appear to differ between conditions.

Together, Johnson's results suggest that Ss in his experiment delayed emitting responses for a period of time which they chose according to the requirements of the attitude condition. Average response times ranged from 390 ms to 8,500 ms - quite a bit longer than would be expected in a discrimination task of this nature. They provide further evidence for the hypothesis that the attitude conditions simply influenced the length of a delay period which preceeded S's emitting a response rather than influencing S's attitude or the relative importance S attached to speed and accuracy .

The fact that the attitude conditions were varied between blocks is probably partly responsible for the lack of any effect of the attitude conditions on the confidence levels. The criteria for confidence responses appear to be unstable and dependent upon the context in which they are produced. A subject who responds with low levels of accuracy may have maximum confidence associated with a stimulus judged correctly only 70% of the time while the same subject in an easier task, where his performance is more accurate, might have a maximum confidence response associated with a stimulus judged correctly 90% of the time.

Finally, Johnson demonstrated the relationship

$$T = A\, e^{B(1-C)} \qquad (7)$$

by plotting the average judgment time for responses whose confidence (regardless of the direction of judgment) fell into each of five 20-unit categories (i.e., 0-20,21-40,etc.) against the average confidence of the responses falling into each category. The data were well described by the equation (7) when separate parameters were estimated for each of the attitude conditions. Johnson didn't reveal why the data had to be analysed in this particular fashion to demonstrate the relationship nor were the complete data presented. Consequently it is not possible to determine if a more detailed analysis (e.g., separate analysis of correct and error responses) would have revealed departures from the proposed confidence-decision time relationship.

Cartwright and Festinger (1943) proposed a topological model of choice behavior having implications for confidence. The response choice was determined by the value of the resultant of a number of randomly fluctuating vectors with lengths which depended upon the stimulus presented and S's tendency to react or not react. Although the model was originally presented in topological terms, Vickers (1972) has recently constructed a simplified version of the theory (in terms of a strategy of sequential sampling,) which will be presented here.

The sensory value associated with a presentation of a particular stimulus was assumed to be a normally distributed random variable, x, which fluctuated in magnitude over time. The mean sensory value was directly related to the stimulus value presented. In addition, a criterion value,k, was a normally distributed random variable also fluctuating in magnitude over time. The expected value of k is related to S's motivations and instructions. S sampled values of x as long as they continued to fall in the interval $-k>x>k$. When the value of x first fell outside of the interval, sampling stopped and a response was generated. The response R_1 was generated if the final value of $x>k$, while the response R_2 was generated if the final value of $x<-k$.

The expected value of k was the mean "restraining force" in the original formulation of the model. It was negatively correlated to S's "tendency to respond". The larger was the value of E(k), the less likely it was that a given x value would meet the criteria for a response to be produced. Decision time was shown to be a positive linear function of the probability that a given x did not produce a decision. Therefore, the higher the value of E(k), the longer were the decision times.

Confidence was related to the expected values of x based on the sampled values prior to the decision criteria being met. A large positive stimulus difference, therefore, generated sensory values which had a large mean value and which were likely to exceed the criterion for an R_1 response. A majority of fast high confidence responses were expected to be associated with such a stimulus. Similarly, a small stimulus difference would produce slow, low confidence responses and more errors.

Festinger (1943) tested the model by requiring five Ss to make comparative judgments of line lengths. Two vertical lines separated horizontally by 9in., 130in. in front of S were presented simultaneously for 500 ms. The "standard" (7 1/2 in.) line always appeared to S's left while a "comparison" stimulus, always on S's right, was randomly selected from 15 lengths (7 1/2 in. ± 0,1,2,4,6,8,12,or 16 sixteenths of an inch). Six hundred judgments were made in each of four separate attitude conditions, 10 at each stimulus difference in each attitude condition. Responses made verbally, were recorded via a voice key and timed from the simultaneous onset of the two lines. After each response, a confidence judgment was reported on a 100 point scale. Using instructions similar to Johnson's, all Ss were run with the usual instructions first, followed by the accuracy and then speed conditions.

The confidence functions for each attitude condition were approximately normal ogives. As in Johnson's study, it was expected that accuracy instructions would increase the precision of the ogive while the speed instructions were expected to decrease the precision of the confidence function. However, the parameters of the ogives were not affected in any systematic fashion by the attitude instructions. The precision of the relationship between the probability of a "longer" response and stimulus difference was also not systematically affected by the attitude instructions. Since the speed condition followed the accuracy condition for all 5 Ss, Festinger attributed the failure of the instructions to influence the ogives in the predicted fashion to a strong practice effect. A plausible alternative is that (as in Johnson's study) Ss delayed emitting their responses after their decisions had already been determined. Performance in all three attitude conditions would be based upon the same decision process but some varying amount of non-processing time would be added on to the "true" judgment time to satisfy the demands of the experimenter for speed in the speed condition and more "considered" responses in the accuracy condition. The generally high values of response time (600 ms -2000 ms) also tend to support this hypothesis.

With the exception of Hemmon's experiment, all of the studies reviewed confounded stimulus difference and confidence. It is clear that there is a positive relationship between average confidence and stimulus difference while at the same time there is a negative relationship between average response time and stimulus difference. The common approach to the problem of the confidence- response time relationship which was based on average "signed" confidences and average response time results obtained when stimulus difference was varied, necessarily resulted in a negative relationship. Hemmon's alternative of examining the relationship within a single level of stimulus difference allows differentiation of the confidence- response time relationship and the effects of stimulus difference on the form of that relationship.

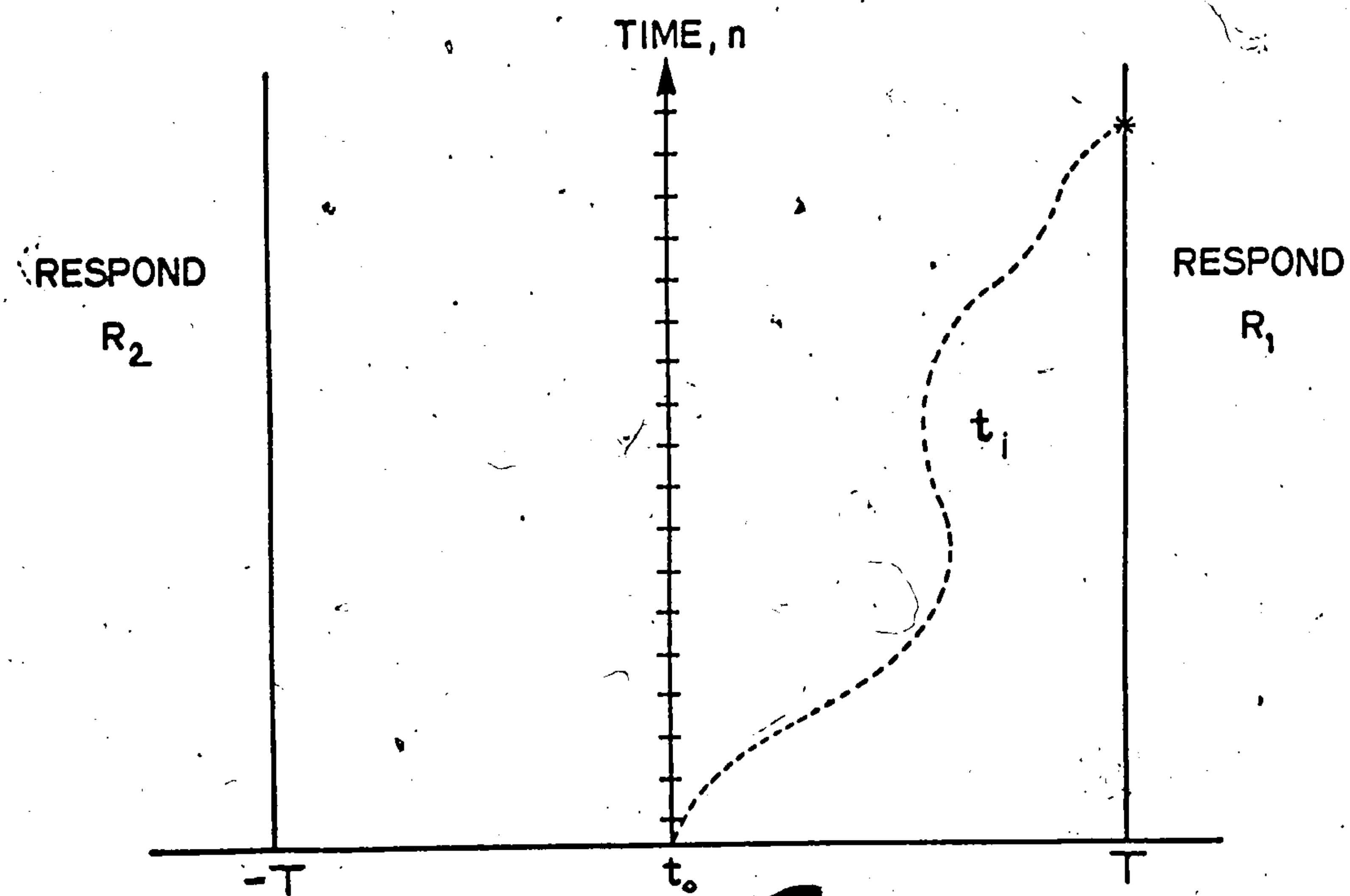
In addition, none of the previous investigations included any attempt to control the time at which the confidence judgments were made. The confidence judgments were simply reported after the choice response. The time at which the confidence response ocurred must, therefore, have varied considerably. Confidence responses are generally regarded as "introspective" responses; S is assumed to be reporting on the basis of some feature of his judgment process. Since judgment processes are probably time dependent, a method of collecting confidence responses which controls the time of the confidence response as well as the time of the choice response is desirable for any investigation of confidence, choice and judgment time.

A THEORY OF CONFIDENCE

A class of decision-making models based upon Wald's (1947) sequential sampling procedures have been suggested as a basis for human performance in choice and discrimination tasks (Stone, 1960; Edwards, 1965; Laming, 1968; and Link & Heath, 1974). It is assumed by all of these models that associated with each stimulus value, s_k is a unique distribution of sensory values, $f_k(x)$. Subjects are assumed in each moment of time, i, ($i = 1, 2, \ldots, n$) to sample sensory values, x_i, which fluctuate in magnitude. The sensory values may undergo a transformation to $y_i = g(x_i)$ which is added to a running total of the transformed sensory values $t_i = t_o + \sum_{j=1}^{i} y_j$, where t_o is the initial value of the sum. A decision is generated by the process when the value of t_i falls outside the interval $-T < 0 < T$. If $t_i \leq -T$ then a response R_2 is generated; if $t_i \geq T$ then a response R_1 is generated. Otherwise, an additional sensory value is sampled and the process continues with the decision rule being reapplied.

The decision process is conveniently treated as a random walk performed by t_i in the interval $-T < t_i < T$. The value of t_i fluctuates as additional sensory values are sampled, increasing with some samples and decreasing with others until the process ends in a decision. After the i^{th} sensory value has been sampled, the value t_i is determined by adding a randomly distributed increment, y_i to t_{i-1}. The random walk continues until t_i hits or exceeds one of two absorbing boundaries at $-T$ and at T, as is illustrated in Figure 1. For example, assume that the function y_i takes the value 1 if x_i exceeds some criterion value, x_c and the value -1

Figure 1- A realization of the random walk process.


TIME, n
RESPOND
R2
RESPOND
R1
t
i
-T
t0
T

otherwise. The probability that any x_i exceeds x_c is determined by $f_k(x)$ and will be labelled p_k. The probability that x_i does not exceed x_c is then $1-p_k = q_k$. After each sensory value is sampled, with probability p_k, the random walk takes a unit step in the positive direction, toward the boundary value T and decision R_1. With probability q_k a unit step in the negative direction, toward boundary value -T and decision R_2, occurs. When the number of positive steps first exceeds the number of negative steps by $T-t_o$ or when the number of negative steps first exceeds the number of positive steps by $T+t_o$, then the process terminates.

Confidence is related to the probability that once a decision has been generated by the process it is in fact the correct decision. Applying the approach of Bayes, the larger the likelihood that the decision reached by the process would have been generated if s_1 was presented relative to the likelihood that the same decision would have been generated if s_2 was presented, the higher should be the confidence that s_1 was presented and the lower the confidence that s_2 was presented. Thus confidence should be directly related to the <u>a posteriori</u> probability of the stimulus for which the decision that was made would be correct.

In the class of random walk models, the <u>a posteriori</u> probability of either stimulus is, at any moment in time, a function of the distance of the random walk from its starting position. A sample value that drives the random walk toward the positive boundary makes the belief that s_1 has been presented more tenable, while a sample that moves the walk toward the negative boundary increases the belief that an s_2 was presented. In the case of the simple binomial random walk discussed above, (assuming

$t_o = 0$), the current value of the process, t_i consists of u positive and w negative unit increments. The total number of steps taken equals i while the position of the random walk equals the number of positive steps, u, minus the number of negative steps, w. Thus

$$u + w = i \qquad \text{and} \qquad u - w = t_i \quad .$$

Solving these equations yields $u = \frac{(i + t_i)}{2}$ and $w = \frac{(i - t_i)}{2}$. Since each increment is determined independently,

$$P(t_i | s_k) = p_k^{\,u} q_k^{\,w} = (p_k q_k)^{i/2} \left(\frac{p_k}{q_k}\right)^{t_i/2}$$

When there are only two stimuli, s_1 and s_2, Bayes' Theorem permits a simple calculation of the probability of the stimulus given the current position of the random walk. For example, if $P(s_1) = P(s_2) = 1/2$, then

$$P(s_1 | t_i) = \frac{(p_1 q_1)^{i/2} \left(\frac{p_1}{q_1}\right)^{t_i/2}}{(p_1 q_1)^{i/2} \left(\frac{p_1}{q_1}\right)^{t_i/2} + (p_2 q_2)^{i/2} \left(\frac{p_2}{q_2}\right)^{t_i/2}}$$

$$= \left(1 + \left(\frac{p_2 q_2}{p_1 q_1}\right)^{i/2} \left(\frac{p_2 q_1}{p_1 q_2}\right)^{t_i/2}\right)^{-1}.$$

Thus, for any given number of samples, i, $P(s_1 | t_i)$ is determined by t_i. When $p_2 > p_1$ then $P(s_1 | t_i)$ reaches its maximum at the smallest possible value of t_i, that is at $-T$; when $p_1 > p_2$ then $P(s_1 | t_i)$ reaches its maximum at the largest possible value of t_i, i.e., T.

It is an assumption of this thesis that the value t_o reflects S's expectation about which stimulus will be presented. Large values of t_o are chosen when S expects s_1 on the trial and wishes to both reduce R_1 response time and increase the probability of an R_1 response. Small values of t_o reflect a similar expectation of s_2. Over a series of trials S's expectations may vary from trial to trial based upon the preceding sequence of stimuli, responses, and reinforcements, giving rise to a distribution of values of t_o. For simplicity it will be assumed that t_o is distributed symmetrically about its expected value $E(t_o)$.

The major proposal of this thesis is that confidence is a linear function of the difference between the final and initial values of the random walk process. For example a response which is generated when the process terminates at a boundary close to the starting point of the process should have little confidence while if the boundary at which the process stops is far from the initial value the response will be made with greater confidence.

The initial value t_o, therefore, simultaneously sets the level of confidence that will be associated with each of the two responses. Increasing t_o simultaneously decreases confidence in R_1 responses and increases confidence in R_2 responses. Thus there is a tradeoff between the confidence associated with an R_1 response and the confidence associated with an R_2 response as t_o varies.

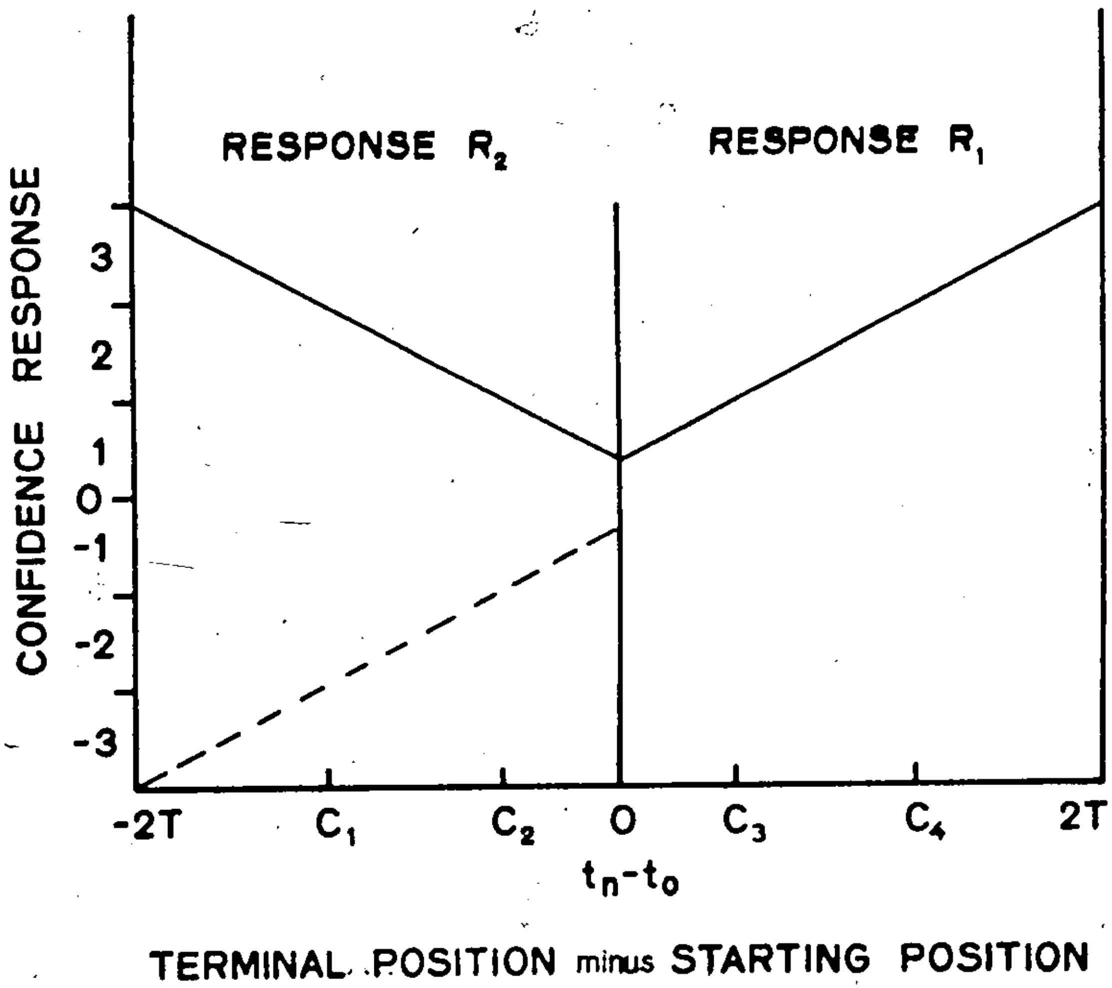
Traditionally confidence responses have been treated in two ways. The confidence response considered without regard to the choice response has been called the "absolute" confidence, while if a negative sign is assigned to those confidence responses associated with a particular choice response the resulting values have been called "signed" confidence.

The proposal that confidence is a linear function of the distance $|t_n - t_o|$ means that absolute confidence, $|c| = m|t_n - t_o| + b$, where m and b are positive constants primarily determined by the scale on which the confidence responses are expressed. Figure 2 illustrates how $|c|$ changes as a function of $t_n - t_o$. Note that the function is symmetrical about the point $t_n - t_o = 0$ which is the value which separates the choice responses, R_1 and R_2. Equal distances between initial and terminating values correspond to equal confidence levels.

When S is allowed only a finite number of confidence responses, the continuum of $t_n - t_o$ values may be divided by criterion values c_i into discrete categories as shown in Figure 2. In the example shown, any trial on which $t_n - t_o$ falls into the interval $-2T < t_n - t_o < C_1$ generates a confidence response 1. When $C_1 < t_n - t_o < C_2$ the confidence response 2 is generated and so on. The values of C_1 and C_2 are assumed to be chosen such that the expected values of $t_n - t_o$ for adjacent categories within each choice response are separated by a constant, 1/m. Thus, results of calculations based upon observed confidence responses may differ only by a constant from the values that would be obtained if "true" c were observed.

Confidence responses have the property that the higher the confidence, the higher is the probability that an associated choice response is correct. The a posteriori $P(S_2|R_2)$, which is a correct choice, is shown in Appendix I to be an increasing function of t_o, the initial value of the process. Thus, as t_o increases, $|-T - t_o| = T + t_o$ increases, confidence associated with an R_2 response increases and the a posteriori probability that given the occurrence of an R_2 response, the response is correct, increases. It is also shown in Appendix I that a posteriori $P(S_1|R_1)$ is a decreasing function of t_o. Therefore, as t_o increases, $T - t_o$ decreases, confidence in an R_1 response decreases and the a posteriori probability that, given an R_1 response has occurred it is

Figure 2- The relationship between distance travelled and confidence.


RESPONSE R2
RESPONSE R1
CONFIDENCE RESPONSE
3
2
1
0
-1
-2
-3
-2T
C1
C2
0
C3
C4
2T
tn-to
TERMINAL POSITION minus STARTING POSITION

correct, decreases.

A series of discrimination trials can be conceived of as a series of random walk decisions where t_o is a random variable with probability density function $f_o(t)$ symmetric about $K_o(t)$ and fixed boundary values T and -T. Several predictions have been generated (in Appendix I) about $E(|t_n - t_o|)$ for a series of decisions with these properties. These predictions can be interpreted by using the linear function $C = m|t_n - t_o| + b$ in terms of absolute confidence.

1) The expected distance travelled to the -T boundary, $E(T+t_o|R_2)$, is a decreasing function of the expected value of the increments, E(y), while the expected distance travelled to the boundary T, $E(T - t_o|R_1)$, is an increasing function of E(y). Since |c| is linearly related to distance travelled, average absolute confidence must be a decreasing function of E(y) for R_2 responses and an increasing function of E(y) for R_1 responses. $E(y) = \int f_k(x) * g(x)\, dx$ and is determined by the stimulus value s_k. If the distributions of sensory values, $f_k(x)$ are considered in terms of Thurstonian or Theory of Signal Detection conceptions of sensory representations (i.e., sensory value is positively related to stimulus value plus random noise) and if g(x) is a monotonic increasing function, then E(y) is positively related to stimulus magnitude. Thus, an increase in stimulus magnitude should lead to an increase in the average absolute confidence associated with R_1 responses and a decrease in the average absolute confidence associated with R_2 responses.

2) Average distance travelled, $E(|t_n - t_o|)$ increases as T increases. If S can be induced to systematically increase T, then the average absolute confidence should increase for all stimulus values. This increase

in confidence should be accompanied by an increase in $P(R_1|s_1)$ and, if $P(R_1|s_1) > 1/2$, an increase in $E(n|s_1)$, where n is the number of observations sampled prior to the termination of the process at $t_1 = t_n$. Therefore, average absolute confidence, accuracy and mean decision times should all increase for all stimulus values.

The signed confidence is equal to $|c|$ when the response R_1 is chosen and $-|c|$ when the response R_2 is chosen. This is illustrated in Figure 2 by the dotted line which shows the relationship between values of $t_n - t_o$ and confidence.

For any particular stimulus value and t_o value, the average "signed' confidence is given by

$$E(\text{confidence}|s_1 \cap t_o) = P(R_1|s_1 \cap t_o)(m(T-t_o)+b) + P(R_2|s_1 \cap t_o)(m(-T-t_o)+b).$$

Since $P(R_2|s_1 \cap t_o) = 1 - P(R_1|s_1 \cap t_o)$ then,

$$E(\text{confidence}|s_1 \cap t_o) = [2P(R_1|s_1 \cap t_o) - 1]\ (mT+b) - mt_o\ .$$

To calculate $E(\text{confidence}|s_1)$ the last expression has to be multiplied by $f(t_o)$ and then integrated with respect to t_o. This yields

$$E(\text{confidence}|s_1) = [2P(R_1|s_1) - 1\ ]\ (\ mT + b) - mE(t_o)\ .$$

Thus average "signed" confidence is a linear function of $[2P(R_1|s_1) - 1]$. The slope of the linear function, $mT + b$, and the intercept, $-mE(t_o)$, have values which are linearly realted to T and $E(t_o)$, respectively, and which are independent of the stimulus value.

A TEST OF THE MODEL

In order for the model to be tested confidence responses must be based solely upon information at the time that the choice decision is determined. Otherwise, confidence might be based upon an independent process or by continuation of the random walk process to a second set of boundary values T', $-T'$, for example. An experiment was proposed in which two features were designed to attempt to influence S to base both the confidence response and choice response upon the result of a single decision process. The predictions of the model about the effects of changes in the boundary value T, the initial value t_o and E(y) were examined within this experiment.

The first feature of the experimental design was the requirement that S indicate simultaneously, by the depression of a single key, both the choice response and the level of confidence associated with the response. In order to maximize the compatability between the response keys and the corresponding theoretical events, the keys indicating each response choice were grouped together. Within each group of keys the confidence assigned each key increased as the key was further removed from the alternative choice. Thus, from left to right the values of $t_n - t_o$ corresponding to each key (i) either increased monotonically or decreased monotonically, (depending upon the assignment of R_1 responses to the group of keys to the right or to the left, respectively). The keys i = 1, 6 corresponded to values of average confidence for key i equal to -3, -2, -1, 1, 2, 3, when R_1 responses were to the right, and 3, 2, 1, -1, -2, -3, when R_1 was on the left.

The other way in which S was encouraged to adopt an appropriate response strategy for testing the model was by limiting the amount of time allowed for each decision. This was accomplished by the imposition of response time deadlines similar to those employed by Pachella and Pew (1968) and Link (1971). In the deadline procedure S is instructed to respond within an experimenter imposed time criterion. Deadlines were chosen such that S had to extract as much information as possible within a relatively short time. S was thereby discouraged from using secondary processes to generate the confidence response.

There are three features which completely determine a random walk process. These are the boundary values T , - T , the starting value, t_o, and the distribution of increments to the walk F(y). The experiment was designed to manipulate all three characteristics of the process in order to provide a basis for comparison of the obtained results with the predictions made from the model.

An attempt was made to control the boundary value, T, by using two time criteria, one of which was chosen at random to be applied on each trial. Link (1971) has demonstrated the ability of practiced Ss to alter discrimination reaction time performance to conform to deadline conditions imposed on successive trials without significant sequential effects. Deadlines were randomly determined from trial to trial in order to avoid the practice and order effects which marred earlier studies (Johnson, 1939 and Festinger, 1943a, b) using "attitude instructions". A longer deadline was employed to influence S to use a larger value of T. The predictions of increased absolute confidence, increased response times and more accurate performance were, therefore, compared with the results of the deadline manipulation.

The starting point of the process t_o is difficult to regulate. At best S can be influenced, through payoffs and sequences of reinforcement and/or stimuli, to increase or decrease the value of t_o from trial to trial. The response keys, however, indicate for a fixed T, that a particular range of t_o was used on the trial. High confidence R_1 responses indicate small values of t_o while high confidence R_2 responses indicate large values of t_o. The use of multiple response keys thus allowed the effect of changes in t_o to be studied.

Finally, the expected values of the distributions of increments $E_k(y)$ were manipulated through the use of four levels of stimulus difference in a "same-different" discrimination reaction time task. The E(y) for a set of increasing stimulus differences can be considered at least to be ordered in the same way as the stimuli. For example, consider the binomial distributions discussed in the example above. In the Theory of Signal Detection conception of distributions of sensory values, increased stimulus differences generate higher sensory values. Thus, $p_k = P(x_i > x_c)$ increases with increased stimulus difference. E(y) then which is equal to $p_k - q_k$, increases with increased stimulus difference.

On each trial, the comparison line length was preceded by a presentation of a standard line length. In this way the criterion S used for the discrimination was kept stable over trials. The sequential presentation also fixed the amount of time for which S could attend to the standard as opposed to S sharing decision time between attending to the standard and attending to the comparison, as could occur with the simultaneous presentation of a standard and comparison stimulus.

One "same" (the same length as the standard,) and three levels of "different" stimuli were chosen to cover a range from near chance to near perfect discrimination performance. The results of this manipulation provided data with which to test the predictions of the model that average "signed" confidence, average confidence in an R_1 response and $P(R_1 \backslash s_1)$ should all increase and that average confidence in an R_2 response whould decrease as stimulus difference was increased.

METHOD

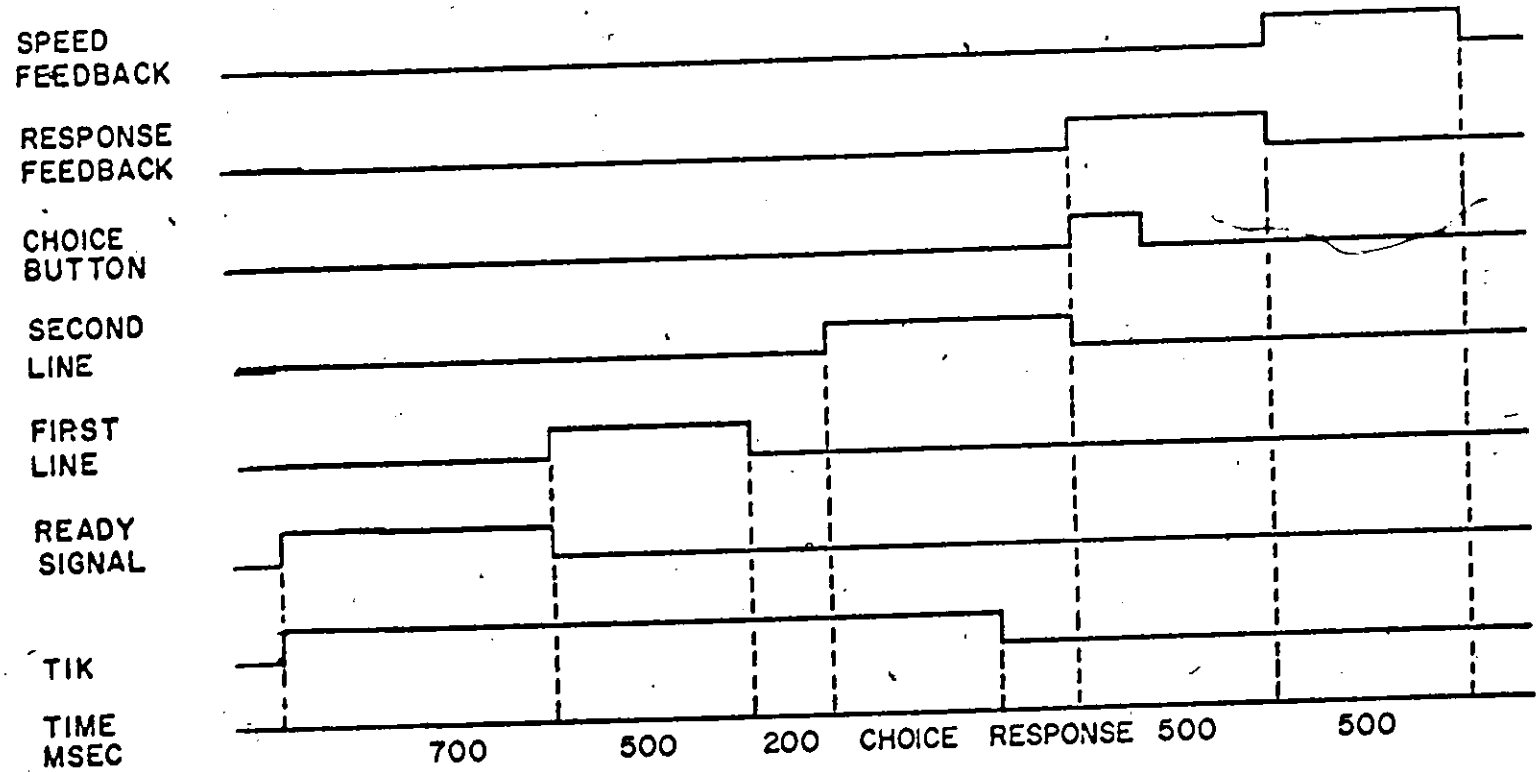
Two right handed university students with 20/20 vision (as determined on a Snellen eye chart) were each paid $26.00 to serve as subjects in five practice and eight experimental sessions. A session lasted approximately 40 min and consisted of two blocks of 310 trials each, with a rest break of 5 min between blocks. Ss were instructed to make their responses within E-determined time criteria (indicated to S at the start of each trial), and encouraged to "beat the deadline" while being as accurate as possible.

On each trial S initiated the sequence of trial events by depressing a trial initiation key (TIK). Immediately upon depression of TIK the characters R and 1 or R and 2 were presented on a computer controlled calligraphic display (Tektronix 602 P4 phosphor) for 700 msec. When the digit 1 appeared in the ready display, a 350 msec RT deadline was in effect on the trial. The digit 2 meant a 475 msec RT deadline was in effect. Immediately following the ready signal, the first of two horizontal line segments was presented and displayed for 500 msec. A 200 msec interstimulus interval followed during which the display screen was blank. The second line segment was then presented until S made a response by releasing TIK and the depressing of one of six (80 g) microswitches arranged in an arc with 3 1/2 in separating the center of TIK from the center of each microswitch, and 1 1/2 in separating the centers of adjacent microswitches. The six microswitches corresponded to (from left to right,) high confidence in an R_1 response, medium confidence in an R_1 response, low confidence in an R_1 response, low confidence in an R_2 response, medium confidence in an R_2 response and high confidence in an R_2 response. RTs were measured from the onset of the second line segment to the depression of one of the response microswitches. After responding, S was informed via the calligraphic display as to whether the response (R_1 or R_2) was correct (YES or NO) and whether the time deadline for the trial had been exceeded (SPEED OK or TOO SLOW). Each of the feedback displays lasted 500 msec and was followed by a new trial when S next depressed TIK. If TIK was released before the presentation of the second line segment the trial was terminated immediately and was restarted by the next TIK depression. Figure 3 shows the sequence of trial events.

Four horizontal line segments, 2.0, 1.9, 1.8, and 1.6 cm, (0Δs, 1Δs, 2Δs, and 4Δs, respectively) presented approximately 1 meter from S, were used throughout the experiment. The 2.0 cm line segment was always used as a standard and presented as the first of the two line segments on each trial. The deadline on each trial was selected randomly from two deadlines - 350 and 475 msec. The second line segment or comparison stimulus on each trial was also randomly chosen so that on one half of the trials the 0Δs stimulus was presented and the remaining stimuli (1Δs, 2Δs, and 4Δs) were each presented on one third of the remaining trials. One S was required to respond R_1 when the comparison stimulus stimulus was 0Δs while the other was required to respond R_2. At the end of each block S was informed of the number of trials on which he had exceeded the time criterion, and was encouraged to keep this number low.

The first ten trials of each block were discarded in the analysis

Figure 3- The sequence of trial events.


SPEED FEEDBACK
RESPONSE FEEDBACK
CHOICE BUTTON
SECOND LINE
FIRST LINE
READY SIGNAL
TIK
TIME MSEC
700
500
200
CHOICE
RESPONSE
500
500

so that each day's session yielded 150 0Δs trials at each of the two deadlines, and 50 kΔs trials (k=1,2,4), at each of the two deadlines. Since two Ss were run for 8 days, the total number of the trials was 2x8x600=9,600. 2400 trials were run in each 0Δs and deadline combination and 800 trials in each kΔs (k=1,2,4) and deadline combination.

RESULTS

Before the data from this experiment can be considered in terms of the confidence literature it must first be established that the responses Ss generated in this experimental situation have the properties generally attributed to confidence responses collected by previous investigators. The two major characteristics reported have been a positive relationship between the probability that a choice response is correct and the confidence associated with the response, and a "confidence function" which has the form of a normal ogive.

Table I shows the estimates of the probability that a choice response was correct as a function of the confidence-choice response for each of the two Ss. Probability correct is an increasing function of the confidence associated with the response for all response and deadline combinations.

Table II shows the average "signed" confidence for each S at each deadline as a function of stimulus difference (where the numbers -3, -2, -1 were assigned to high medium and low confidence "same" responses, respectively, and 3, 2, 1 were assigned to high medium and low confidence "different" responses respectively). The results in this table correspond to what Johnson (1939) and Festinger (1943a) have called the "confidence function" and which they supposed to have the form of a normal ogive. Ogives were, therefore, fitted to the data for each deadline by finding the best fitting straight line relating stimulus difference and the value of a dummy variable z for which the integral of a normal distribution from minus ∞ to z equals the confidence value transformed to a zero to one scale. The estimated ogives fit the data about as well as Festinger's data fit ogives.

Table I - Probability of a response being correct as a function of the confidence.

subject	response	confidence	475 ms P(c)	350 ms P(c)
JW	same	high	.840 (990)	.763 (520)
	same	medium	.843 (89)	.684 (215)
	same	low	.679 (81)	.604 (235)
	diff	low	.566 (196)	.503 (530)
	diff	medium	.818 (610)	.641 (736)
	diff	high	.903 (434)	.823 (164)
EL	same	high	.706 (947)	.656 (503)
	same	medium	.571 (247)	.634 (503)
	same	low	.519 (108)	.478 (341)
	diff	low	.563 (327)	.551 (768)
	diff	medium	.697 (218)	.643 (367)
	diff	high	.774 (553)	.782 (78)

Table II - Average "signed" confidence for each stimulus difference-deadline combination.

Subject	deadline	475 ms				350 ms			
	stim difference	0Δ	1Δ	2Δ	4Δ	0Δ	1Δ	2Δ	4Δ
JW	observed	-1.89	0.09	1.91	2.31	-0.66	0.19	0.86	1.60
	predicted	-1.32	-.17	1.02	2.57	-0.50	0.09	0.67	1.69
		μ = 1.14		σ = 1.96		μ = 0.85		σ = 4.05	
EL	observed	-1.41	-0.47	0.64	1.51	-0.68	-0.34	0.04	0.75
	predicted	-1.25	-0.52	0.25	1.66	-0.60	-0.26	0.09	0.76
		μ = 1.67		σ = 3.05		μ = 1.75		σ = 6.94	

The predicted average confidence values and the parameters of the ogives are also shown in Table II. It can be seen that there is a clear effect of the speed condition upon the estimated standard deviations associated with the ogives. For both Ss the estimated standard deviation is considerably smaller in the 475 ms condition than the 350 ms condition.

Since confidence responses in this study do possess the fundamental attributes of confidence responses collected in the traditional manner of earlier studies, comparisons with the confidence results of earlier investigators, as well as analyses of previously unreported features of confidences responses, will be of general interest. Among the latter are analyses of the distributions of confidence responses as a function of stimulus, deadline, and choice response.

The distributions of confidence responses are shown in Table III conditionalized upon presentation of either a "same" (i.e. 0Δs) or "different" (non-0Δs) stimulus difference in each deadline condition. It should be noted that these distributions are not of the form that is associated with rating responses. The 350 ms 0Δs column, for example, contains estimates of the probability of each confidence-choice response which (looking from high confidence "same" to high confidence "difference") decrease within the "same" response category, increase abruptly to a second peak at the low confidence "different" response and then decrease within the "different" response category. All four distributions show a similar discontinuity in the response distribution at the "same"-"different" response boundary which would not be expected in rating response data.

Table III also shows the effect of the deadline condition upon the forms of the distribution of responses. The probability density for

Table III - Probability of each confidence response combination for "same" and for "different" stimuli

	Stimulus	0Δs		± kΔs (k ≠ 0)	
	Deadline	475	350	475	350
Response	Conf				
Same	high	.625	.259	.182	.100
Same	medium	.090	.194	.050	.105
Same	low	.046	.127	.033	.113
Diff	low	.095	.236	.123	.305
Diff	medium	.074	.165	.271	.295
Diff	high	.070	.019	.342	.082

both stimulus conditions shifts away from the higher confidence responses as the deadline is decreased. This shift is further exhibited in the results of Table IV in which the average "absolute" confidence (where the numbers 1, 2 and 3 were assigned to the low, medium and high confidnece responses, respectively) associated with each stimulus is shown to be lower for the shorter deadline.

Table IV shows that the average confidence associated with "same" responses decreased and the average confidence associated with "different" responses increased as stimulus difference was increased. The average "absolute" confidence shows relatively little effect of changes in stimulus difference.

The observed increases in average confidence in the longer deadline condition are an indicator of S's increased accuracy in that condition. Table V shows that the longer deadline condition is associated with a higher proportion of correct responses for both Ss. This change in accuracy and the observed shifts in average confidence both stand in marked contrast to the failures of Festinger (1943a,b,) and Johnson (1939) to influence either variable with the attitude instructions used in their experiments.

The relationship between confidence and response time is shown in Figure 4 . Generally, mean response times increased as the confidence in the response increased. There was also an average difference of approximately 75 ms between mean RTs for each confidence response as a result of the deadline instructions. Figure 4 also shows the mean

Table IV - Average confidence as a function of stimulus difference for each deadline and response.

Subject	deadline	475			350		
	stim difference	$\bar{c}$	$\bar{c}$ "same"	$\bar{c}$ "diff"	$\bar{c}$	$\bar{c}$ "same"	$\bar{c}$ "diff"
JW	0Δ	2.61	2.81	1.82	2.05	2.37	1.62
	1Δ	2.38	2.73	2.14	1.88	2.28	1.63
	2Δ	2.29	2.34	2.38	1.89	2.01	1.85
	4Δ	2.37	2.60*	2.37	1.88	1.65	1.90
EL	0Δ	2.50	2.71	1.95	1.78	2.09	1.33
	1Δ	2.42	2.68	2.13	1.68	2.00	1.35
	2Δ	2.40	2.51	2.34	1.65	1.84	1.49
	4Δ	2.35	2.10	2.41	1.62	1.68	1.60

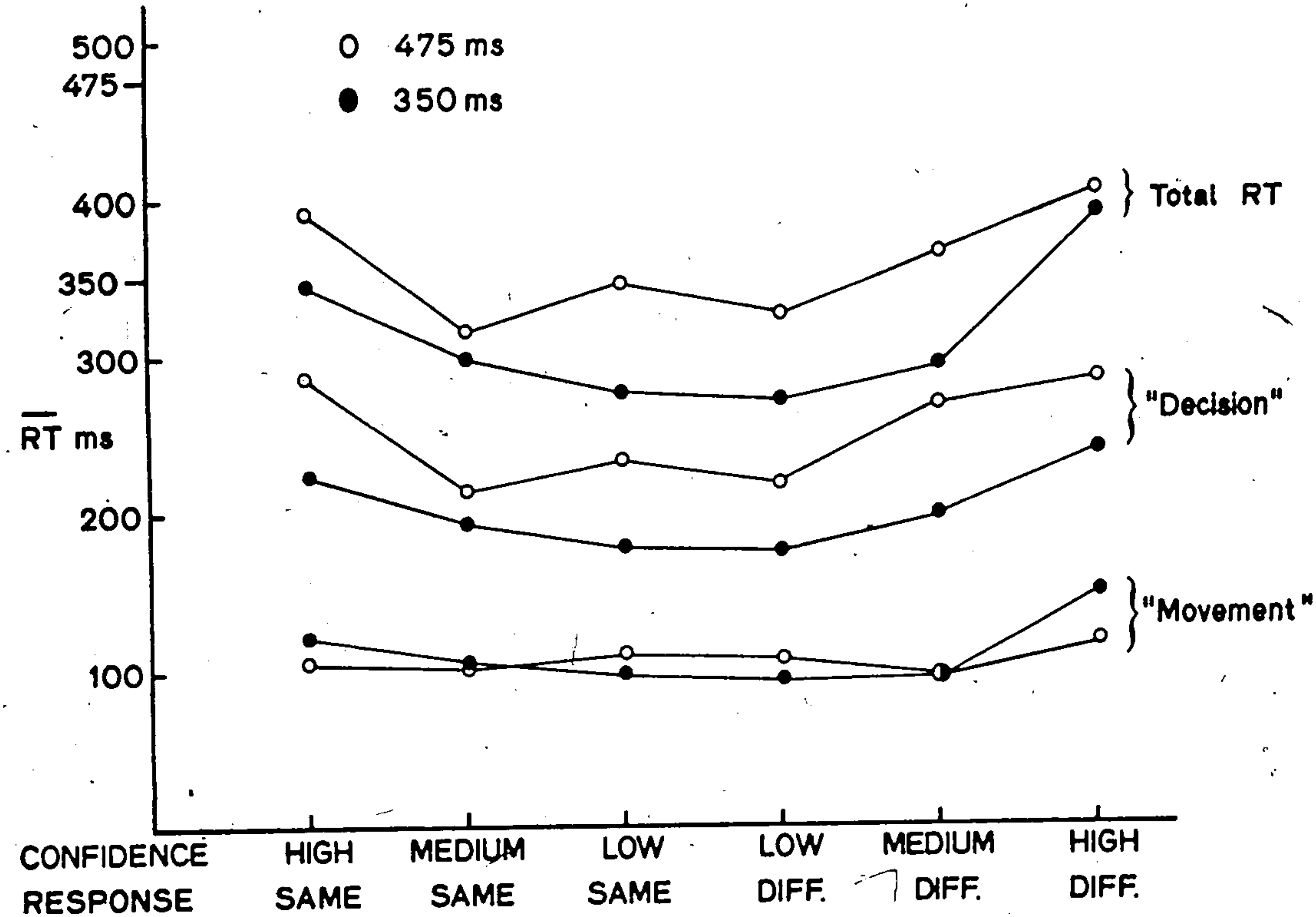
* based upon 5 responses.

Table V - Probability of a correct response as a function of stimulus difference for each deadline.

Subject	deadline / stim. difference	475	350
JW	0Δ	.80	.57
	1Δ	.60	.63
	2Δ	.92	.75
	4Δ	.99	.92
EL	0Δ	.72	.59
	1Δ	.46	.50
	2Δ	.65	.57
	4Δ	.80	.74

Figure 4- Estimates of mean total response times, "movement" times, and "decision" times as a function of confidence response for each choice response. (msecs)

*Note- Estimates of standard errors for all mean RTs may be found in Appendix II.



500
475
400
350
300
200
100
RT ms
O 475 ms
● 350 ms
Total RT
"Decision"
"Movement"
CONFIDENCE RESPONSE
HIGH SAME
MEDIUM SAME
LOW SAME
LOW DIFF.
MEDIUM DIFF.
HIGH DIFF.

RTs separated into two components, the "decision time" (,or "release time"), which is the time taken by S to release TIK following presentation of the comparison stimulus, and the "movement time", which is the time it took from the release of TIK to the depression of the response key. The decision time functions retain the form of the total RT functions while the movement time functions are relatively flat. In addition, the mean difference between the decision times in each of the two deadline conditions is approximately 72 ms while the mean difference for the movement times is only 2 ms. These data suggest that movement time factors played a relatively minor role in determining the pattern of RT results.

The basic result, that within each of the deadline conditions high confidence was associated with long RTs, is contrary to the conclusions of Henmon (1911), Johnson (1939) and Festinger (1943a,b), although it is not clear from the data they each presented that their conclusions were justified. The results shown in Figure 4 are also the first reported instance of any effect of speed instructions upon the level of confidence. In the present study, the condition with the longer RTs also had higher levels of confidence.

Table VI provides more evidence for the relatively minor contribution of movement time factors to the RT results. On the average, deadline had no effect upon the movement times as a function of stimulus difference, although there is a substantial effect of deadline on the decision time component of the RT. This table also shows that there was little if any effect of stimulus difference on mean RTs when pooled over all responses. The effects of stimulus difference on the mean

Table VI- Estimates of mean response time, movement time and release time.

		Total $\overline{RT}$		Movement Time		Release Time	
	stim diff/deadline	475	350	475	350	475	350
Subject							
JW	0Δ	405	317	113	120	292	197
	1Δ	419	309	125	117	294	192
	2Δ	401	307	108	114	293	193
	4Δ	371	304	91	108	280	196
EL	0Δ	352	285	100	91	251	194
	1Δ	367	302	109	95	258	207
	2Δ	347	289	110	92	237	197
	4Δ	352	309	106	102	246	208

RTs are relatively small, varying considerably between Ss and deadline conditions. Johnson (1939) and Festinger(1943b) have reported a decrease in mean RT as a function of stimulus difference.

Figure 5 shows the mean RT for each stimulus difference when conditioned upon a "same" or a "different" response. There is a general increase in mean RTs for the "same" responses accompanied by a general decrease in mean RTs for "different" responses as stimulus difference increases. This trade-off occurs in both of the deadline conditions.

In an earlier argument it was suggested that the effects of speed instructions in the experiments of Johnson (1939) and Festinger (1943a,b) were primarily to add non-processing time to a basic level of processing time. The argument was based in part on the lack of any improvement in performance for the conditions requiring more emphasis on accuracy. In the case of the data reported herein, however, evidence has been presented for improved performance as a result of relaxed speed requirements. There was also a difference in the effect of the speed instructions between those parts of the response time that are primarily "non-decision making" time and those parts during which "decision making" or stimulus processing is going on. Further evidence of the control over processing time exerted by the deadline procedure used here is presented in Figure 6. If the effect of the longer deadline was to add a constant deadtime to RT then the difference between the mean RTs for any two conditions within one deadline condition would be the same as the difference between the RTs for the same two conditions in the other deadline condition. In Figure 6, the difference between the RTs for a "same" and a "different" response is shown for each stimulus difference in each deadline condition. Since these differences are

Figure 5– Estimates of mean response time as a function of stimulus difference for "same" and "different" responses in each deadline condition.

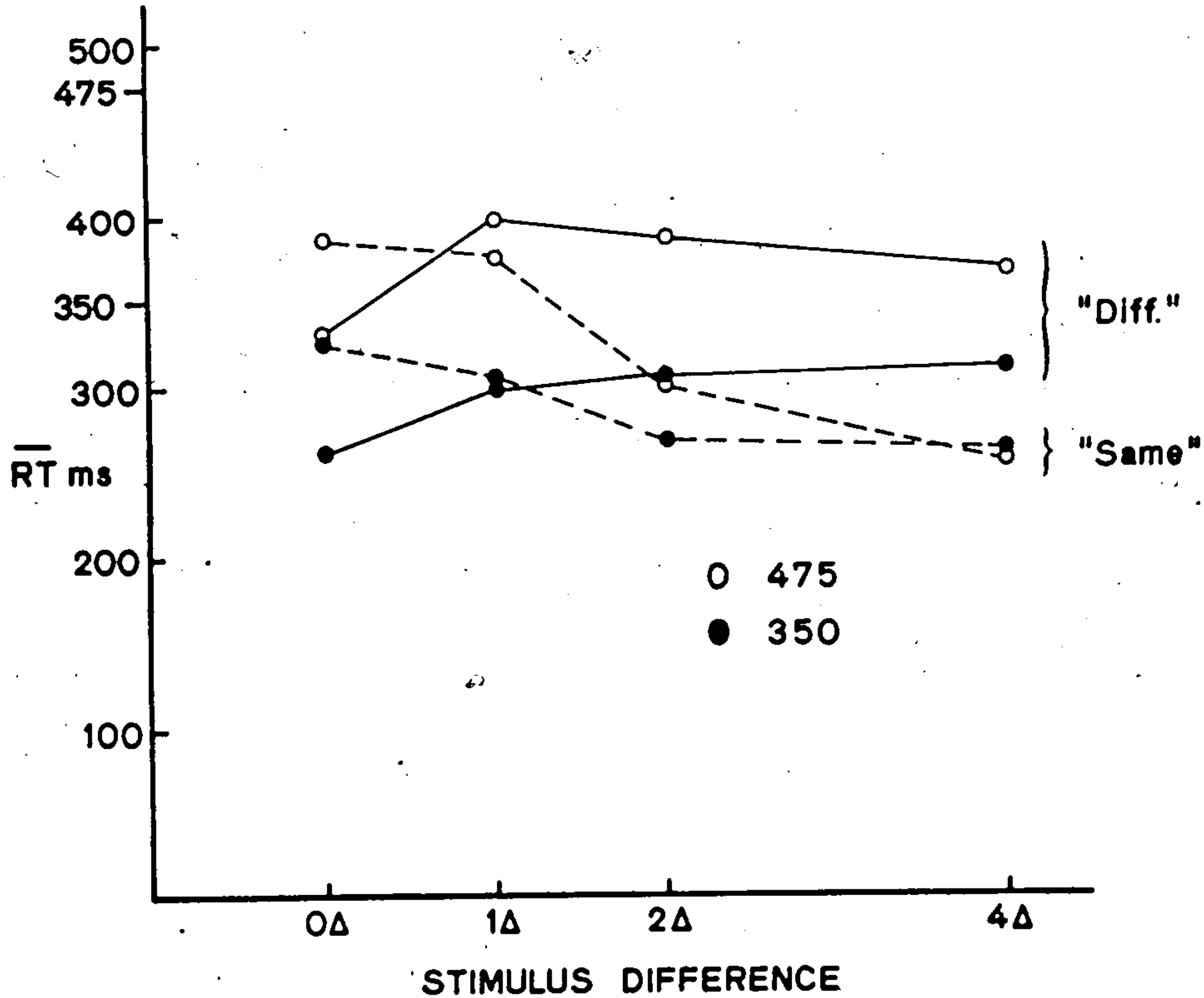
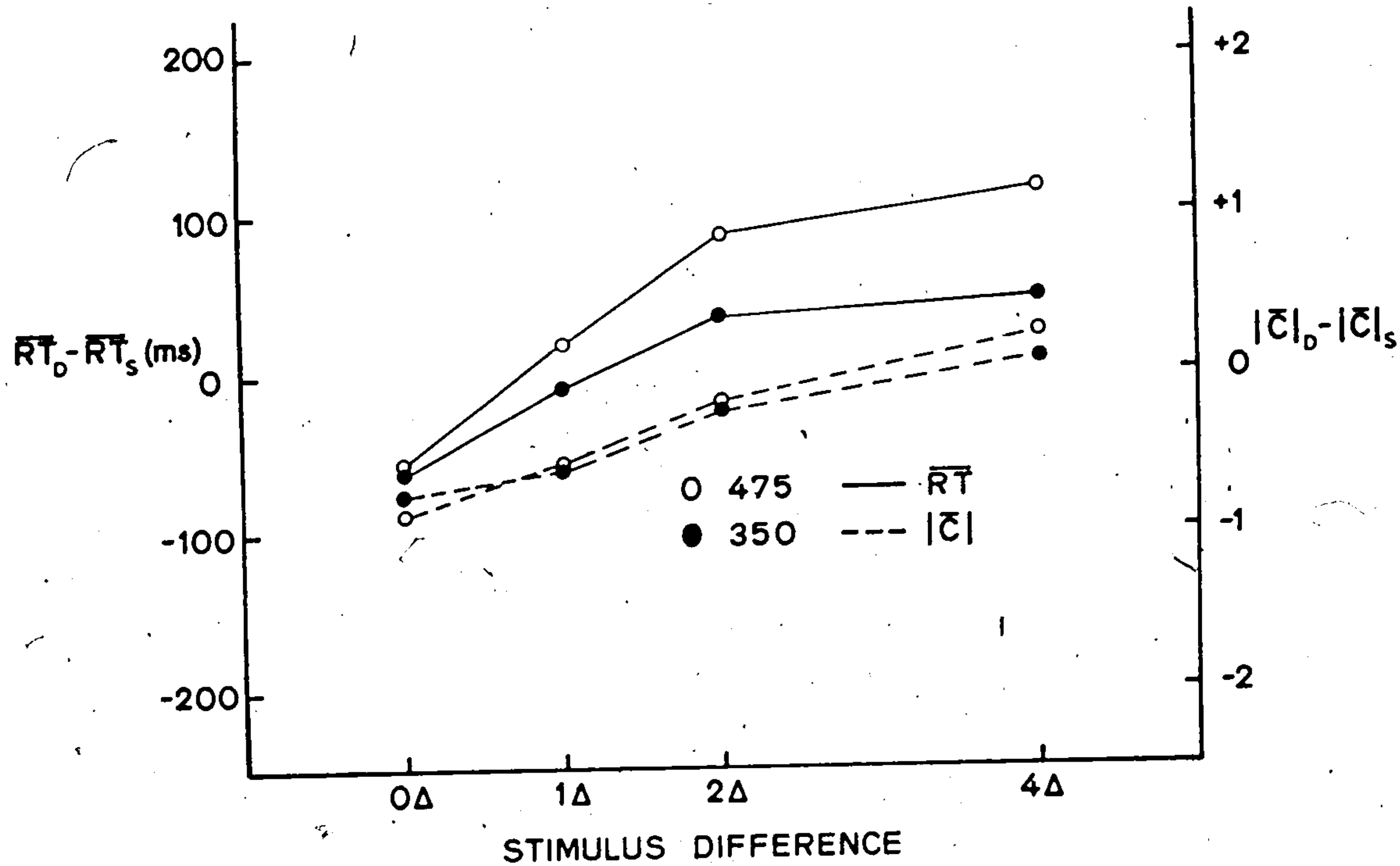

500
475
400
350
300
RT ms
200
100
0Δ
1Δ
2Δ
4Δ
STIMULUS DIFFERENCE
"Diff."
"Same"
0 475
● 350

Figure 6– Differences between estimates of mean RT for "same" and "different" responses and between average absolute confidences as a function of stimulus difference in each deadline condition.



200
100
0
-100
-200
RT_D - RT_S (ms)
+2
+1
0
-1
-2
|C|_D - |C|_S
O 475
● 350
—— RT
---- |C|
0Δ
1Δ
2Δ
4Δ
STIMULUS DIFFERENCE

different for the two deadlines, the hypothesis of a constant additive effect of deadline on the RT is not supported.

Figure 6 also shows the difference between the average absolute confidence for the "same" and "different" responses for each stimulus difference in each deadline condition. There is no appreciable difference between the differences for the two deadline conditions despite an effect of stimulus difference. Thus, the hypothesis of a constant additive effect of deadline on the average absolute confidence is supported.

The results of this experiment showed that confidence is positively related to RT both between and within deadline conditions. Long RTs were associated with high confidence and short RTs were associated with low confidence. The average confidence with which a "different" response was a made increased and the average confidence with which a "same" response was made decreased as stimulus difference increased.

EVALUATION OF THE MODEL

One of the primary assumptions upon which any evaluation of the model rests is that the average increment of the random walk process, E(y), was positively related to stimulus difference. Specifically, it was assumed that the average increment was negative when the 0Δs stimulus was presented and as stimulus difference increased from 1Δs to 2Δs to 4Δs, the average increment became increasingly positive. Evidence in support of these assumptions is provided by the results shown in Table IV. The majority of 0Δs stimulus presentations resulted in "same" responses while the proportion of kΔs (k=1, 2, 4) stimulus presentations which resulted in "different" responses increased as stimulus difference increased.

The confidence results shown in Table IV show that the average confidence increased for "same" responses and decreased for "different" responses as stimulus difference increased. In addition, as the results shown in Figure 6 demonstrate, there is an almost constant effect of deadline on the average confidence levels. These two sets of results agree with the predictions of the model for the effects of changes in the size of the average increment of the random walk and of a change in the boundary value T, respectively.

A quantitative test of the model is provided by the prediction of a linear relationship between [2P("different"$|s_i$) − 1] and the average signed confidence. The slope of the function is itself a linear function of T, the boundary value of the process, while the intercept is linearly related to the average initial position of the random walk,

$E(t_o)$. Table VII shows that the values of average signed confidence predicted by the model are in very close agreement with the observed values. In addition, the estimated slopes for the predicting functions are larger for the longer deadline condition for both Ss. This result confirms that the effect of the deadlines, as was suggested by the results of Table IV, was to induce changes in the boundary value T in the random walk process. The average starting position of the process, on the other hand, appears to have remained unaffected by the deadlines.

The response time data shown in Figure 4 are also consistent with the predictions of the random walk model. Average RT for each response is positively related to the distance between the initial value and final values of the process. This means that low confidence responses (, which are associated with small differences between t_n and t_o,) should be faster than high confidence responses (, which are associated with large values of $|t_n - t_o|$). The results in Figure 4 show that RT increased as confidence increased within both "same" and "different" responses and both deadline conditions. In addition, average RT was longer for every response and confidence combination in the 475 ms deadline condition than for the same response and confidence combination in the 350 ms condition. This is again consistent with the interpretation of deadlines as controllers of the value of T, the boundary value.

Although most of the results that have been examined here are analyses of the data that were not reported by previous investigators, there are some differences between the results obtained in this experiment and those that have been reported by other experimenters. In particular, the deadline procedure altered the form of the confidence

Table VII - Predicted and observed values of average "signed" confidence.

Subject	stim diff	0Δs		1Δs		2Δs		4Δs	
	deadline	obs	pred	obs	pred	obs	pred	obs	pred
JW	475 ms	-1.89	-1.94	0.09	0.19	1.91	1.90	2.21	2.26
		slope = 2.67			intercept = -0.34				
JW	350 ms	-0.66	-0.68	0.19	0.26	0.86	0.80	1.60	1.61
		slope = 2.36			intercept = -0.35				
EL	475 ms	-1.41	-1.44	-0.47	-0.42	0.64	0.65	1.51	1.49
		slope = 2.81			intercept = -0.20				
EL	350 ms	-0.68	-0.68	-0.34	-0.31	-0.04	-.01	0.75	0.76
		slope = 2.18			intercept = - 0.29				

functions in this study while the attitude instructions used by Festinger (1943a) and Johnson (1939) had no systematic effect on the confidence functions. Furthermore, while Johnson showed average response times were a negative function of average absolute confidence, the results of this study showed confidence and response time to be positively related.

PERFORMANCE UNDER "ACCURACY" CONDITIONS

It was suggested earlier in this thesis that the Ss in Johnson's and Festinger's studies were always working with accuracy as a primary consideration, simply adding additional time to their "true" response times as they thought they were expected to do by the experimental instructions. Speed instructions required little or no additional time, usual instructions required some additional time and accuracy instructions implied that still more time was expected to be taken before a response was made.

In order to investigate the possibility of a difference between Ss' performances under deadline conditions and in an "accuracy" condition, a second group of Ss was required to perform a task similar to that in Experiment I, but containing two deadline conditions and an accuracy condition. The relationship between RT and confidence was expected to show the difference if any existed, between the bases of confidence judgments in deadline and accuracy conditions. If there was no appreciable change in this relationship, then the accuracy condition was to be a source of additional information about the effect of changes in the boundary value, T, on performance.

METHOD II

Four right handed university educated Ss with 20/20 vision (as determined by a Snellen eye chart) were each paid $48.00 to participate in eleven practice and thirteen experimental sessions. The procedure was identical to that used in Experiment I except that on each trial one of three equally likely instruction conditions was in effect. During the ready display, R1 indicated that a 350 ms deadline was in effect on the trial, R2 indicated a 475 ms deadline was in effect and R3 indicated that the accuracy condition was in effect. In the accuracy condition Ss were instructed to take as long as necessary for their responses to be correct. No response time feedback was given on those trials on which the accuracy condition was in effect.

Two Ss (NN and NW) were assigned to indicate "same" responses on the left side of the response panel while the other two Ss (JS and RB) were to indicate "different" responses on the left side of the response panel.

298 trials were presented in each of two blocks in each session. The first ten trials of each block were discarded in the analysis, leaving 192 trials in each instruction condition in each session. Each session contributed 96 0Δs trials in each instruction condition and 32 kΔs (k=1,2,4) in each instruction condition. For each S the 13 experimental days yielded a total of 7,488 trials- 1248 0Δs trials and 416 kΔs trials in each of the three instruction conditions.

RESULTS II

It is nececessary, as in Experiment I, to ascertain that the confidence responses which have been collected have the basic properties of confidence responses collected in the past. Table VIII shows that the confidence responses of three of the four Ss (excluding JS) are positively related to the probability that the response was correct. The data of the subject JS was anomalous in a number of respects and will not be included in further analyses.

Table IX shows the confidence functions and estimated parameters of the best fitting normal ogives for each deadline for the confidence data pooled over the three Ss NN, NW, and RB. The estimates of standard deviation increase as the time allowed for a response (by the instruction condition) increased. These results again demonstrate that the speed instructions controlled the confidence with which Ss responded. The result is consistent with the results of Experiment I but again contrary to the findings of Johnson (1939) and Festinger (1943a,b).

Table X also shows that the instruction conditions had a clear effect on the average confidence associated with each of the responses "same" and "different", as well as the average absolute confidence associated with each level of stimulus difference. Stimulus difference, however, had little effect on the average absolute confidence. Furthermore, Table X shows that, as in Experiment I, the average confidence associated with "different" responses increased and the average confidence associated with "same" responses decreased as stimulus difference was increased.

Table VIII- Probability of a response being correct as a function of the confidence.

Subject		N N			N W		
Deadline		Acc	475	350	Acc	475	350
Response	Conf						
Same	high	.859 (1148)	.784 (827)	.779 (801)	.969 (1226)	.883 (282)	.810 (79)
Same	medium	.827 (133)	.786 (364)	.731 (427)	1.000 (18)	.856 (854)	.722 (126)
Same	low	.750 (8)	.508 (59)	.400 (100)	1.000 (2)	.833 (6)	.700 (989)
Diff	low	.755 (102)	.652 (454)	.302 (504)	0.000 (1)	.600 (5)	.668 (871)
Diff	medium	.849 (318)	.822 (625)	.799 (542)	.859 (71)	.787 (1177)	.615 (179)
Diff	high	.907 (787)	.910 (167)	.910 (122)	.975 (1178)	.942 (172)	.829 (252)

Subject		R B			J S		
Deadline		Acc	475	350	Acc	475	350
Response	Conf						
Same	high	.859 (1267)	.600 (15)	.778 (9)	.791 (445)	.659 (91)	.575 (630)
Same	medium	.400 (30)	.638 (357)	.565 (267)	.780 (341)	.726 (317)	.726 (175)
Same	low	.609 (87)	.618 (1137)	.554 (1144)	.728 (302)	.689 (660)	.660 (174)
Diff	low	.700 (20)	.633 (504)	.548 (693)	.602 (176)	.602 (430)	.505 (206)
Diff	medium	.771 (35)	.742 (469)	.619 (372)	.668 (563)	.674 (886)	.580 (578)
Diff	high	.923 (1057)	.857 (14)	.909 (11)	.771 (669)	.795 (112)	.592 (733)

Table IX - Average "signed" confidence as a function of stimulus difference for three Ss pooled (NN, NW, RB).

deadline	accuracy				475 ms				350 ms			
stim diff	0Δ	1Δ	2Δ	4Δ	0Δ	1Δ	2Δ	4Δ	0Δ	1Δ	2Δ	4Δ
observed	-2.51	1.04	2.65	2.81	-1.22	-0.06	1.16	1.66	-0.80	-0.15	0.60	1.05
predicted	-1.57	0.12	1.75	2.94	-0.89	-0.15	0.61	1.89	-1.01	-0.32	0.39	1.65
	$\mu = 0.93$				$\mu = 1.91$				$\mu = 1.46$			
	$\sigma = 1.31$				$\sigma = 3.13$				$\sigma = 3.56$			

Table X - Average confidence as a function of stimulus difference for each deadline and response.

S	deadline	Accuracy			475			350		
	stim difference	$\bar{c}$	$\bar{c}$|"same"	$\bar{c}$|"diff"	$\bar{c}$	$\bar{c}$|"same"	$\bar{c}$|"diff"	$\bar{c}$	$\bar{c}$|"same"	$\bar{c}$|"diff"
NW	0Δ	2.97	2.98	2.70	2.20	2.25	2.03	1.20	1.26	1.39
	1Δ	2.90	3.00	2.89	2.13	2.23	2.08	1.41	1.23	1.45
	2Δ	2.97	-	2.97	2.12	2.00	2.12	1.51	1.10	1.63
	4Δ	2.98	-	2.98	2.21	-	2.21	1.51	1.23	1.56
RB	0Δ	2.89	2.90	2.79	1.30	1.26	1.41	1.25	1.21	1.32
	1Δ	2.87	2.78	2.94	1.30	1.26	1.51	1.28	1.17	1.44
	2Δ	2.91	2.25	2.95	1.41	1.21	1.56	1.29	1.23	1.36
	4Δ	2.88	1.53	2.93	1.48	1.27	1.56	1.32	1.18	1.41
NN	0Δ	2.82	2.89	2.33	2.38	2.64	1.50	2.35	2.60	1.48
	1Δ	2.61	2.87	2.43	2.13	2.62	1.65	2.04	2.42	1.61
	2Δ	2.61	2.75	2.61	1.85	2.35	1.83	1.85	2.26	1.72
	4Δ	2.70	2.0	2.70	1.96	1.82	1.97	1.82	1.97	1.81
pooled	0Δ	2.89	2.92	2.54	1.96	2.06	1.63	1.63	1.74	1.38
	1Δ	2.77	2.77	2.77	1.87	1.93	1.81	1.57	1.60	1.54
	2Δ	2.84	2.46	2.85	1.82	1.55	1.89	1.55	1.45	1.60
	4Δ	2.85	1.56	2.87	1.89	1.32	1.95	1.55	1.28	1.62

Table XI demonstrates that the increase in confidence associated with the instruction conditions which allowed more time for the response is associated with an increase in accuracy. Again, this finding is consistent with those of Experiment I.

Estimates of mean RT for each stimulus difference are shown in Table XII. In the accuracy condition the effect of the stimulus difference is to increase mean RT as the "category boundary" (i.e., 1Δs) is approached. This is the result reported by Johnson and Festinger and upon which many of their conclusions about the confidence-RT relationship were based. The mean RTs in the 475 ms and 350 ms deadline conditions, however, show little or no effect of stimulus difference. (Note that the mean RT in the 475 ms condition was 414 ms and in the 350 ms condition 342 ms. This again demonstrates the ability of Ss to closely control their RTs under deadline conditions.) The movement times reveal another anomalous characteristic of the accuracy condition results. The movement times in the two deadline conditions show little or no effect of the deadline difference but those in the accuracy condition are considerably longer than those obtained from the two deadline conditions.

In Figure 7 the mean RTs for "same" and "different" responses are plotted separately as a function of stimulus difference. The results for the two deadline conditions are very similar to those of Experiment I. Mean RTs for "different" responses tended to increase while those for "same" responses decrease as stimulus difference was increased. In the accuracy condition, however, the RT functions for the "same" responses are below and parallel to that for "different" responses. Both functions

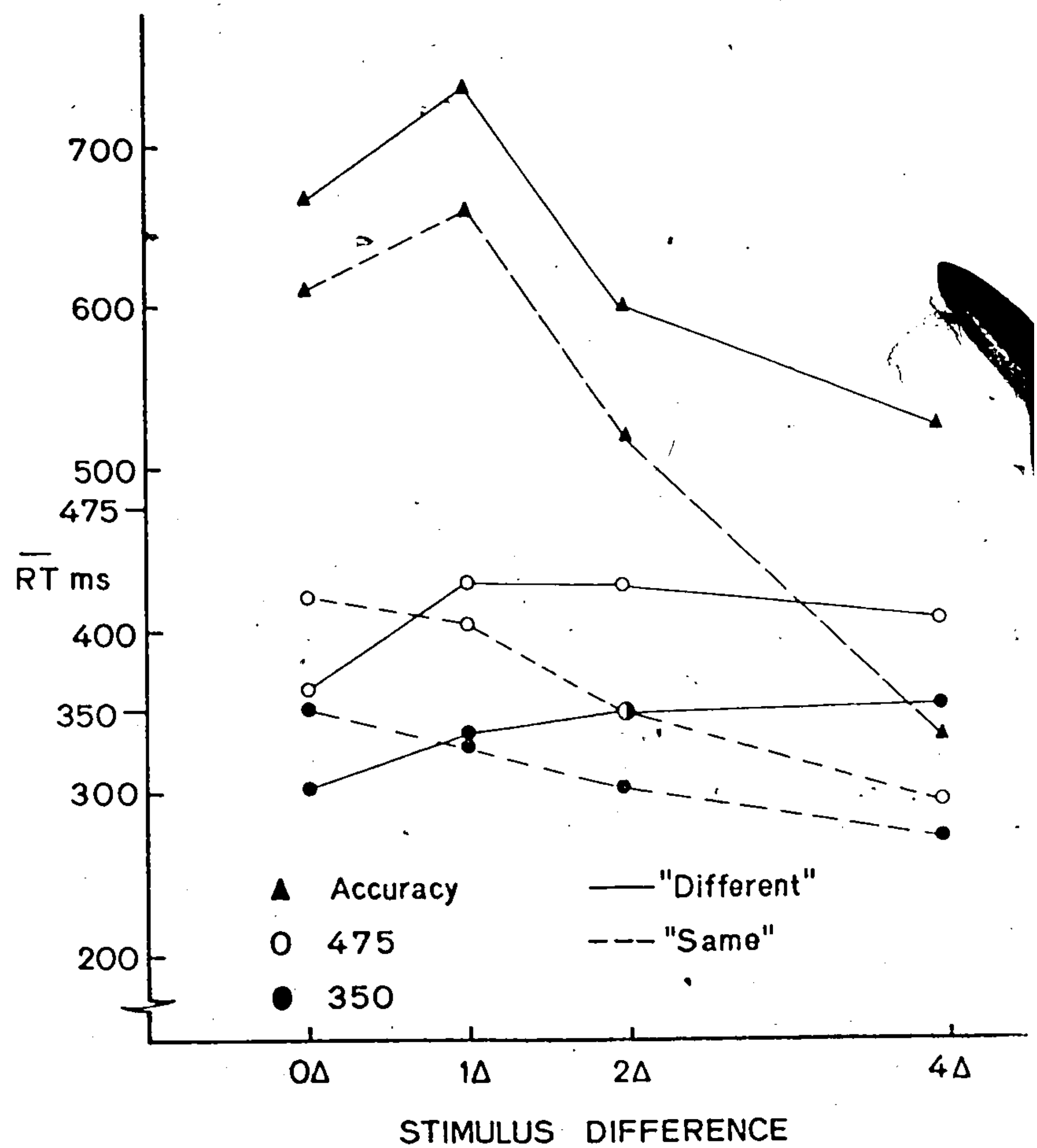
Table XI - Estimates of the probability of a correct response as a function of stimulus difference.

Subject	deadline stim diff	accuracy	475 ms	350 ms
NW	0 Δ	0.968	.789	.679
	1 Δ	.909	.659	.516
	2 Δ	1.000	.964	.769
	4 Δ	1.000	1.000	.880
RB	0 Δ	.924	.753	.635
	1 Δ	.553	.334	.399
	2 Δ	.933	.563	.488
	4 Δ	.959	.736	.603
NN	0 Δ	.883	.772	.782
	1 Δ	.601	.505	.469
	2 Δ	.952	.834	.762
	4 Δ	.998	.974	.923

Table XII - Estimates of mean response time, movement time and release times pooled for the three Ss NN, NW, RB.

	Total $\overline{RT}$			Movement Time			Release Time		
deadline	Acc	475	350	Acc	475	350	Acc	475	350
stim diff									
0Δ	619	413	342	192	135	132	427	278	210
1Δ	722	423	340	258	143	131	464	280	209
2Δ	606	418	341	188	142	133	418	276	208
4Δ	536	405	344	156	129	132	380	276	212

Figure 7- Estimates of mean response time as a function of stimulus difference for "same" and "different" responses in each speed condition.



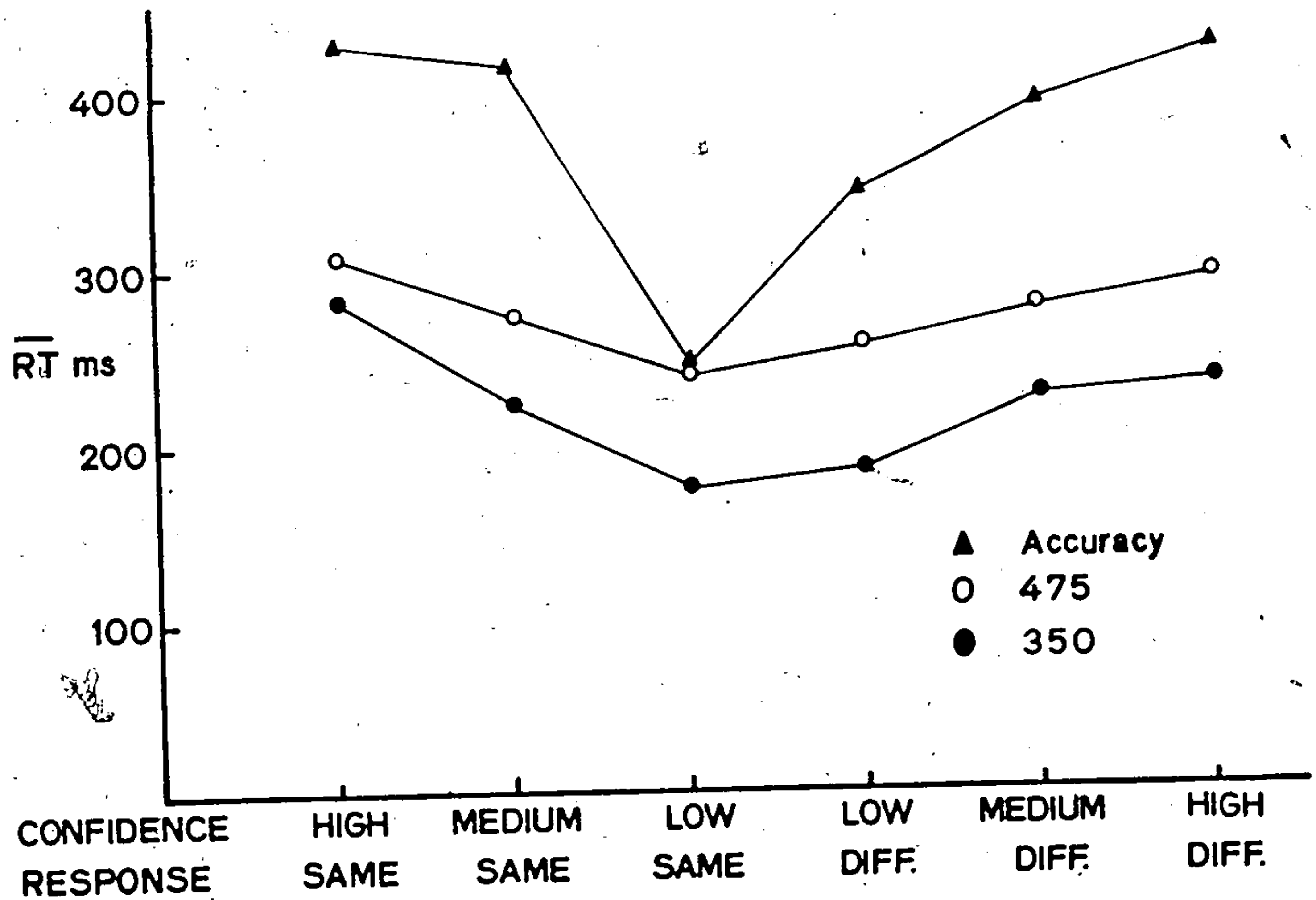
700
600
500
475
RT ms
400
350
300
200
Accuracy
475
350
"Different"
"Same"
0Δ
1Δ
2Δ
4Δ
STIMULUS DIFFERENCE

decreased as stimulus difference changed from the "category boundary" (1Δs). This means that in the accuracy condition errors were faster than correct responses for the kΔs (k=1,2,4) stimuli but errors were slower than correct responses for the 0Δs stimulus difference. In the deadline conditions, on the other hand, errors were always faster than the correct responses on the average.

Figure 8 shows that estimates of mean "release time" tended to increase as the confidence associated with a response increased. The functions which represent the confidence-RT relationship for the two deadline conditions are similar to those obtained in Experiment I. They are separated by approximately 66 ms on the average as compared with a separation of 72 ms in Experiment I. The function obtained from the accuracy condition data, however, is rather odd looking, due, in part, to the paucity of data at the low confidence responses in this condition. Generally, however, the positive relationship between level of confidence and mean RT is retained.

The results for the two deadline conditions replicated those of Experiment I while the results from the accuracy condition were similar in several respects to those reported by Festinger and Johnson. The speed instructions were shown to influence not only RT but accuracy and confidence as well.

Figure 8- Estimates of mean "decision" time as a function of confidence response for each choice response.


400
300
200
100
RT ms
CONFIDENCE
RESPONSE
HIGH SAME
MEDIUM SAME
LOW SAME
LOW DIFF.
MEDIUM DIFF.
HIGH DIFF.
Accuracy
475
350

DISCUSSION AND CONCLUSIONS

The major features of the data from the two deadline conditions in Experiment II replicated those found in Experiment I. A U-shaped function again related mean response time and the confidence-choice responses and the 475 ms deadline condition again produced mean RTs roughly 65-70 ms longer than the mean RTs in the 350 ms condition. Stimulus difference had a relatively small effect on average absolute confidence and mean RTs. However, when separated into "same" and "different" responses, average confidence and mean RT both increased for "different" responses while average confidence and mean RT both decreased for "same" responses as stimulus difference was increased.

The results from the accuracy condition appear to have some features in common with the deadline conditions and other features that are unique. The relatively U-shaped relationship between mean RT and confidence-choice responses was rather irregular in form. It lacked the symmetry associated with the functions obtained in the deadline conditions and was not parallel to the other two functions (which were parallel to each other). Also in contrast to the results from the deadline conditions, mean RT for "same" responses and the mean RT for "different" responses both decreased as stimulus difference increased, while the average confidence results, on the other hand, were consistent with the results from the deadline conditions. In addition, marginal mean RTs were greatly affected by the stimulus difference in only the accuracy condition.

Table XIII shows the observed values of average signed confidence compared to values predicted from a linear function of $[2P(\text{"different"}|s_i)-1]$.

Table XIII- Predicted and observed values of average "signed" confidence

Subject	stim diff	0Δs		1Δs		2Δs		4Δs	
	deadline	obs	pred	obs	pred	obs	pred	obs	pred
RB	Accuracy	-2.47	-2.46	0.38	0.36	2.61	2.61	2.75	2.76
		slope = 2.96				intercept = 0.05			
RB	475 ms	-0.60	-.59	-0.34	-0.33	0.35	0.32	0.81	0.83
		slope = 1.45				intercept = 0.14			
RB	350 ms	-0.29	-0.27	-0.13	-0.18	0.03	0.06	0.38	0.37
		slope = 1.34				intercept = 0.09			
NN	Accuracy	-2.28	-2.32	0.32	0.40	2.35	2.37	2.69	2.63
		slope = 2.81				intercept = -0.17			
NN	475 ms	-1.70	-1.74	-0.47	-0.41	1.14	1.16	1.87	1.83
		slope = 2.39				intercept = -0.44			
NN	350 ms	-1.70	-1.69	-0.53	-0.55	0.77	0.78	1.52	1.52
		slope = 2.27				intercept = -0.41			
NW	Accuracy	-2.80	-2.86	1.44	2.11	2.87	2.62	2.98	2.62
		slope = 2.83				intercept = -0.21			
NW	475 ms	-1.35	-1.37	0.61	0.64	1.97	2.00	2.21	2.16
		slope = 2.24				intercept = -0.07			
NW	350 ms	-0.41	-0.41	0.16	0.18	1.00	0.94	1.23	1.27
		slope = 1.50				intercept = -0.13			

Not only was there close agreement between observed and predicted values of average confidence, but the slope of the functions obtained increased as the time criterion was relaxed. This again indicates that the effect of the speed criteria was to adjust the values of the boundary value T, in the random walk process.

The data from this experiment generally support the proposed model in the deadline conditions. The results from the accuracy condition appear to indicate that a somewhat different mode of decision making might come into play when the control of response times is unimportant to S. The Ss in the present study probably exhibited a mixture of decision processes under accuracy instructions due to the general "beat the deadline" set of the experiment. Ss in Johnson's (1939) and Festinger's (1943a,b) studies were probably using decision processes of the "accuracy" type in all instruction conditions.

Johnson reported that the average response time associated with high confidence responses was approximately one-half second while for low confidence responses, average response times of over eight seconds were reported! These Ss were clearly not concerned about response time in the way we are inclined to think of response time today. The kinds of decision making activities that Johnson's Ss probably engaged in must be quite different from those employed by Ss in other discrimination experiments (including the present study) where the maximum RT rarely exceeds two seconds for a single response.

Johnson and Festinger both depended heavily upon the average signed confidence in formulating their conclusions about the basis of the confidence response and its relationship to other features of the decision process. However, conclusions about confidence based upon the average signed confidence are misleading. A stimulus which generates high confidence "same" and high confidence "different" responses has the same value of average signed confidence associated with it as does a stimulus which generates low confidence "same" and low confidence "different" responses if the proportion of responses of each type is equal for the two stimuli. The effects of experimental manipulations upon the confidence with which responses are actually made are buried within the average confidence value. Separate computation of average confidence judgments for "same" and "different" responses removed this distortion and revealed that the average confidence associated with each stimulus difference within each speed condition resulted from a trade-off between the confidence associated with "same" responses (which decreased as stimulus increased) and the confidence associated with "different" responses (which increased as stimulus difference increased). These analyses also revealed the constant effect of the deadline conditions on confidence which could never have been inferred from average signed confidence results.

The results support the idea that confidence is primarily determined by non-stimulus factors which may have set the confidence levels even before the stimulus was presented. An amount of "confidence stuff" or <u>gewissheitstoff</u> (as might have been preferred by Johnson and Festinger) might be made available at the beginning of a trial based upon past performance in the condition in effect on the trial or based on the amount of time allowed for a response. If the condition is one in which S has performed well in the past more gewissheitstoff might be allotted to the trial than if the condition has proven to be difficult. The gewissheitstoff is divided between the possible responses, possibly on the basis of the preceding sequence of stimuli and/or responses and/or feedback, so that if the trial results in one response one level of confidence judgment is made, while if the trial produces the alternative response a confidence judgment of a different level is made. Trial to trial variations in the proportions of gewissheitstoff allocated to each response combined with changes in the probability of a particular response being made as a result of the stimulus difference presented, lead to the patterns of confidence results obtained. In the model, the boundary values of the random walk process determine how much gewissheitstoff available on the trial. The proportion of confidence stuff which is allocated to each response is determined by the initial position of the random walk; the nearer the initial value is to a particular boundary the less gewissheitstoff is available to responses made at that boundary and the more gewissheitstoff is available to response made at the alternative boundary.

Once the initial position and boundary values have been chosen, the confidence is determined completely by the response that the process generates. The probability of each of the responses is determined by the value of the stimulus difference presented on the trial. The initial value of the random walk is assumed to be a random variable. Therefore, the average confidence for any particular stimulus difference and response is determined by the likelihoods of the response being generated by the process starting at each of the possible initial values and by the likelihoods of the various initial values. The effect of stimulus difference on the average confidence associated with a given response is an effect based upon changes in the likelihoods of the response being generated from each of the initial values. On any particular trial, S is equally confident in a particular response regardless of stimulus difference. It is the probability of observing any particular level of confidence with each of the responses that differs from stimulus difference to stimulus difference and which causes the changes in average confidence that have been reported.

Further tests of the model should include more direct attempts to control the initial value of the random walk process. Possible approaches to controlling the initial value include trial to trial variations in the stimulus presentation probabilities, where S is informed of the presentation probabilities prior to each trial, and prediction of the stimulus by S before each trial.

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Appendix I- Theoretical development of the model.

Laming (1968), Edwards (1965) and Stone (1968) have presented models of choice based upon Wald's (1947) sequential probability ratio test. The models assume that S makes a response on the basis of the sum, t_n, of a stationary sequence of random variables, y_i $(i = 1, 2, \ldots, n)$ whose distribution function $F_k(y)$ is determined by the particular stimulus presented, s_k. The sequences $\{y_i\}$ and $\{t_i\}$ are assumed to continue until either $t_n \geq T$ or $t_n \leq -T$ where $0 < T$. When t_n meets one of these conditions the process generates a response—R_1 if $t_n \geq T$ and R_2 if $t_n \leq -T$.

Laming (1968) suggested that the initial value of the process, t_o, might be a random variable with distribution function $F_o(t)$, the form of which could be determined by the processing of random noise in the system prior to the onset of the sequence $\{y_i\}$. Alternatively, $F_o(t)$ could be based upon Ss attempts to maximize performance by shifting toward the value T or -T depending on which stimulus he expects to receive next in the sequence of stimulus presentations. A positive value of t_o will decrease the time it takes to make a response R_1 so an S expecting the stimulus to be one which is paired with the R_1 response could increase t_o in order to achieve this.

Increasing the value of t_o will also have the effect of decreasing the a posteriori probability that a response R_1 was actually generated by a stimulus S_1 and increase the a posteriori probability that a response R_2 was actually generated by a stimulus S_2. The likelihood that an R_1 response is correct (if it occurs) decreases while the likelihood that an R_2 response is correct (if it occurs) increases. If t_o is decreased then the likelihood that an R_1 response is correct if it occurs increases, while the likelihood that an R_2 response is correct if it occurs decreases.

Since confidence is associated with the probability that a response is correct it would be expected that the confidence associated with R_1 responses would be an inverse function of t_o while the confidence associated with R_2 responses would be an increasing function of t_o.

Theorem Given monotonic distributions functions $F_k(y)$ with moment generating functions $E_k(e^{-\theta y})$ $(k = 1, 2)$ and $E_2(y) < 0 < E_1(y)$ determining the increments, y_i of the decision process, t_i and fixed values T and -T for the boundary values of the process, t_n, then the a posteriori probability $P(S_1|R_1)$ is a decreasing function of t_o and $P(S_2|R_2)$ is an increasing function of t_o.

Note

The starting point and boundary values of the random walk process can be changed by the addition of any constant value of c to define another random walk space with starting value t and boundaries at A and -B, where $A = T + c$, $-B = -T + c$, and $t = t_o + c$. A random walk occurring in the transformed space has exactly the same characteristics as a random walk in the original space with the same value of θ_k where θ_k is the non-zero root of the moment generating function $E_k(e^{-\theta y})$. For several of the following proofs it is convenient to deal with a transformed random walk space such that $c = -E(t_o)$. Thus $A = T - E(t_o)$, $-B = -T - E(t_o)$, $t = t_o - E(t_o)$.

Proof: 1)

$$P(s_1|R_1) = \left(1 + \frac{P(R_1|s_2)}{P(R_1|s_1)}\right)^{-1}$$

$$\frac{d}{dt}[P(s_1|R_1)] = \frac{d}{dt}\left[\frac{P(R_1|s_1)}{P(R_1|s_2)}\right]\left[P(s_1|R_1)\right]^2 .$$ From

Cox and Miller(1965)

$$\frac{d}{dt}\left[\frac{P(R_1|s_1)}{P(R_1|s_2)}\right] = \frac{d}{dt}\left[\frac{e^{\theta_1 B} - e^{-\theta_1 t}}{e^{\theta_2 B} - e^{-\theta_2 t}} \cdot \frac{e^{\theta_2 B} - e^{-\theta_2 A}}{e^{\theta_1 B} - e^{-\theta_1 A}}\right]$$

$$= \frac{d}{dt}\left[\frac{e^{\theta_1 B}-e^{-\theta_1 t}}{e^{\theta_2 B}-e^{-\theta_2 t}}\right] \text{x (negative constant)}$$

Since $\theta_2 < 0$ and $e^{\theta_2 B}-e^{-\theta_2 A} < 0$.

Then, letting $\theta_1 = \theta_2 + k$,

$$Q = \frac{e^{\theta_1 B}-e^{-\theta_1 t}}{e^{\theta_2 B}-e^{-\theta_2 t}} = \frac{e^{(\theta_2+k)B} - e^{-(\theta_2+k)t}}{e^{\theta_2 B} - e^{-\theta_2 t}} \quad \text{and}$$

$$\frac{dQ}{dt} = \frac{(e^{\theta_2 B}-e^{-\theta_2 t})\ (\theta_2+k)e^{-(\theta_2+k)t} - (e^{(\theta_2+k)B}-e^{-(\theta_2+k)t})\ (\theta_2 e^{-\theta_2 t})}{(e^{\theta_2 B} - e^{-\theta_2 t})^2}$$

$$= \frac{e^{-\theta_2 t}}{(e^{\theta_2 B}-e^{-\theta_2 t})^2}\left[\theta_2 e^{\theta_2 B}(e^{-kt}-e^{kB}) + ke^{-kt}(e^{\theta_2 B}-e^{-\theta_2 t})\right]$$

Since the terms outside of the brackets are always positive they will be ignored. The terms within the brackets sum to zero when $t = -B$. The derivative of these terms with respect to t will then tell us the sign of $\frac{dQ}{dt}$.

$$\frac{d}{dt}\,[\theta_2 e^{\theta_2 B}(e^{-kt}-e^{kB}) + ke^{-kt}(e^{\theta_2 B}-e^{-\theta_2 t})]$$

$$= -\theta_2 k e^{\theta_2 B-kt} - k^2 e^{\theta_2 B-kt} + k(\theta_2+k)e^{-(k+\theta_2)t}$$

$$= ke^{-kt}(\theta_2+k)\ (e^{-\theta_2 t}-e^{\theta_2 B})$$

$$= \theta_1 k e^{-kt}(e^{-\theta_2 t}-e^{\theta_2 B})$$

$$\geq 0 \text{ for all } \theta_2 < 0$$

$$\therefore \frac{dQ}{dt} \geq 0 \text{ and } \frac{d\,P(s_1|R_1)}{dt} \leq 0 \qquad \text{so } \frac{d\,P(s_1|R_1)}{dK} \leq 0$$

2) $$P(s_2|R_2) = \left(1 + \frac{P(R_2|s_1)}{P(R_2|s_2)}\right)^{-1}$$

$$\frac{d}{dt}\,(P(s_2|R_2)) = \frac{d}{dt}\left[\frac{P(R_2|s_2)}{P(R_2|s_1)}\right] \times \;[P(s_2|R_2)]^2$$

$$\frac{d}{dt}\left[\frac{P(R_2|s_2)}{P(R_2|s_1)}\right] = \frac{d}{dt}\left[\frac{e^{-\theta_2 A} - e^{-\theta_2 t}}{e^{-\theta_1 A} - e^{-\theta_1 t}}\right] \times \text{(negative constant)}$$

Since $\theta_1 > 0$ and $e^{-\theta 1 A} - e^{\theta 1 B} < 0$

let $\theta_1 = \theta_2 + K$ and then

$$\frac{dQ}{dt} = \frac{d}{dt}\left[\frac{e^{-\theta_2 A} - e^{-\theta_2 t}}{e^{-(\theta_2 + K)A} - e^{-(\theta_2 + K)t}}\right] = \frac{e^{-\theta_2 t}}{e^{-(\theta_2+K)A} - e^{-(\theta_2+K)t}}\left[\theta_2 e^{-\theta_2 A}(e^{-KA} - e^{-Kt}) + Ke^{-Kt}(e^{-\theta_2 A} - e^{-\theta_2 t})\right]$$

Again the terms in brackets sum to zero when $t = A$ and the derivative of the sum will give us the sign of $\frac{dQ}{dt}$.

$$\frac{d}{dt}\,[\theta_2 e^{-\theta_2 A}(e^{-Ka} - e^{-Kt}) + Ke^{-Kt}(e^{-\theta_2 A} - e^{-\theta_2 t})]$$

$$= K(\theta_1)e^{-Kt}(e^{-\theta_2 t} - e^{-\theta_2 A})$$

$$\leq 0 \text{ for } \theta_2 < 0$$

$$\therefore \frac{dQ}{dt} \leq 0 \text{ and } \frac{d\,P(s_2|R_2)}{dt} \geq 0 \text{ and } \frac{d\,P(s_2|R_2)}{dK} > 0$$

A given value of t_o thus corresponds to one value of confidence if the response R_1 occurs on the trial and a different value of confidence if the response R_2 occurs. If we assume that S's expectations about the stimulus presentations are such that the distribution of t_o is symmetrical about $E(t_o)$ [i.e., $f_o(t_o) = \frac{dF_o(t)}{dt} = f_o(2E(t_o)-t_o)$] then, $E(t) = 0$

and $f_o(t) = f_o(-t)$

$$1)\quad E((t+B)|R_2) = \begin{cases} B - \dfrac{E(t\,e^{\theta_i t})}{E(e^{\theta_i t}) - e^{-\theta_i A}} & ; \quad i = 1, 2 \quad \theta_i \neq 0 \\[2ex] B - \dfrac{E(t^2)}{A} & ; \quad \theta = 0 \end{cases}$$

$$2)\quad E((A-t)|R_1) = \begin{cases} A - \dfrac{E(t\,e^{\theta_i t})}{e^{\theta_i B} - E(e^{\theta_i t})} & ; \quad i = 1, 2 \quad \theta_i \neq 0 \\[2ex] A - \dfrac{E(t^2)}{B} & ; \quad \theta_i = 0 \end{cases}$$

$$3)\quad E(|t_n - t|) = \begin{cases} A\,P(R_1|s_1) + B\,P(R_2|s_1) - \dfrac{2E(te^{\theta_i t})}{e^{\theta_i B} - e^{\theta_i A}} & ; \quad i = 1, 2; \quad \theta \neq 0 \\[2ex] A\,P(R_1|s_1) + B(P\,R_2|s_1) - \dfrac{2E(t^2)}{A+B} & ; \quad \theta = 0 \end{cases}$$

Proof

$$1)\quad E(t+B|R_2) = \frac{\int_{-B}^{A} f_o(t)\ (B+t)P(R_2|S_1)\ dt}{\int_{-B}^{A} f_o(t)P(R_2|S_1)\ dt}$$

$$= B - \frac{\int t\ f_o(t)e^{-\theta A}\ dt - \int t\ f_o(t)e^{-\theta t}\ dt}{e^{-\theta A}\int f_o(t)\ dt - \int f_o(t)e^{-\theta t}\ dt} \quad ; \quad \theta \neq 0$$

But since $f_o(t) = f_o(-t)$, then $E(t) = \int_{-B}^{A} t\ f_o(t)\ dt = 0$

$$E(t+B|R_2) = B - \frac{E(te^{-\theta t})}{e^{-\theta A} - E(e^{-\theta t})}$$

$$= B - \frac{E(te^{\theta t})}{E(e^{\theta t}) - e^{-\theta A}} \qquad \text{Since } E(e^{\theta t}) = E(e^{-\theta t}) \text{ and } E(te^{\theta t}) = -E(te^{-\theta t})$$

In the special case when $\theta = 0$, then

$$E(t+B|R_2) = \frac{\int_{-B}^{A} (B+t) \left(\frac{A-t}{A+B}\right) f_o(t)\, dt}{\int_{-B}^{A} \frac{A-t}{A+B} f_o(t)\, dt}$$

$$= B - \frac{E(t^2)}{A}$$

2) $$E(A-t|R_1) = \frac{\int_{-B}^{A} (A-t)\,(e^{\theta B} - e^{-\theta t})\, f_o(t)dt}{\int_{-B}^{A} (e^{\theta B} - e^{-\theta t})\, f_o(t)dt} \qquad \theta \neq 0$$

$$= A - \left[\frac{\int t\, f_o(t)e^{-\theta t}dt - \int t\, f_o(t)e^{\theta B}dt}{\int f_o(t)e^{-\theta t}dt - \int e^{\theta B} f_o(t)\, dt}\right]$$

$$= A - \frac{E(te^{-\theta t})}{E(e^{-\theta t}) - e^{\theta B}}$$

$$= A - \frac{E(te^{\theta t})}{e^{\theta B} - E(e^{\theta t})} \quad ; \quad \theta \neq 0$$

$$E(A-t|R_1) = \frac{\int (A-t)\left(\frac{B+t}{A+B}\right) f_o(t)dt}{\int \frac{B+t}{A+B} f_o(t)\,dt} \qquad \text{when } \theta = 0$$

$$= A - \frac{E(t^2)}{B}$$

3) $$E(|t_n-t|) = E(A-t|R_1)\,P(R_1|s_1) + E(t+B|R_2)\,P(R_2|s_1)$$

$$= A\,P(R_1|s_1) + B\,P(R_2|s_1) - \frac{2\,E(te^{\theta t})}{e^{\theta B}-e^{-\theta A}} \qquad \theta \neq 0$$

$$= \frac{2[A-B - E(t^2)]}{A+B} \quad ; \qquad \theta = 0$$

<u>Theorem</u> $E(A-t|R_1)$ is an increasing function of θ while $E(B+t)$ is a decreasing function of θ.

<u>Proof</u> Consider a binomial distribution

$$g_o(t) = \begin{cases} t \text{ with probability} = 1/2 \\ -t \text{ with probability} = 1/2 \end{cases}$$

$t > 0.$

$$E^*(A-t|R_1) = A - \frac{t\left(\frac{e^{\theta t}-e^{-\theta t}}{2}\right)}{e^{\theta B} - \left(\frac{e^{-\theta t}+e^{\theta t}}{2}\right)}$$

and $$\frac{d\,E^*(A-t|R_1)}{d\theta} = \frac{2C}{[2e^{\theta B}-(e^{\theta t}+e^{-\theta t})]^2}$$

where $C = 2\,t^2 + t[(B-t)\,e^{\theta(B+t)} - (B+t)\,e^{\theta(B-t)}]$

$$\frac{dC}{d\theta} = (B^2-t^2)e^{\theta B}(e^{\theta t}-e^{-\theta t}) \quad \begin{cases} < 0 & \text{if } \theta < 0 \\ > 0 & \text{if } \theta > 0 \\ = 0 & \text{if } \theta = 0 \end{cases}$$

When $\theta = 0$, C reaches its minimum value = 0 and therefore

$$\frac{dE^*(A-t|R_1)}{d\theta} \geq 0 \quad \text{for all } \theta.$$

Now let us consider any symmetric distribution $f_o(t)$ as a mixture of a set of $g_o(t)$ $(0 < t < Q;\ Q = \min(A,B))$.

Then $$E(A-t|R_1) = \int_0^Q E^*(A-t|R_1 \cap t) f_o(t)dt$$

and $$\frac{d\,E(A-t|R_1)}{d\theta} = \int_0^Q \frac{d\,E^*(A-t|R_1 \cap t)}{d\theta} f_o(t)dt > 0$$

Since $\dfrac{E^*(A-t|R_1)}{d\theta} > 0$ for all θ and all t.

Similarly if we consider $E^*(B+t|R_2)$ and $g_o(t)$, then

$$E^*(B+t|R_2) = B - t\,\frac{\left(\frac{e^{\theta t}-e^{-\theta t}}{2}\right)}{\frac{d^{\theta t}+e^{-\theta t}}{2} - e^{-\theta A}} \quad ; \quad t \geq 0$$

$$d\,E^*(B+t|R_2) = \frac{2C}{[2e^{-\theta A}-(E^{\theta t}+e^{-\theta t})]^2} \quad \text{where}$$

$$C = e^{-\theta A}t(e^{\theta t}(A+t) - e^{-\theta t}(A-t))-2t)$$

$$\frac{dC}{d\theta} = (A^2-t^2)te^{-\theta A}(e^{-\theta t}-e^{\theta t}) \quad \begin{matrix} < 0 \text{ if } \theta > 0 \\ > 0 \text{ if } \theta < 0 \\ = 0 \text{ if } \theta = 0 \end{matrix}$$

and $C = 0$ when $\theta = 0$; therefore the maximum value of C is 0

and $$\frac{d\,E^*(B+t|R_2)}{d\theta} \leq 0 \text{ for all } \theta.$$

Again $f_o(t)$ is a mixture of $g_o(t)$ $(0 \le t \le Q)$ and

$$\frac{d\,E(B+t|R_2)}{d\theta} = \int_0^Q \frac{d\,E^*(B+t|R_2 \cap t)}{d\theta}\, f_o(t)\, dt \le 0$$

Since $\dfrac{d\,E^*(B+t|R_2)}{d\theta} \le 0$ for all θ and all t.

<u>Theorem</u> $E[(|t_n - t_o|)|s_i]$ is positively related to the boundary value, T.

<u>Proof</u>

$$E((|t_n - t_o|)|t_o \cap s_i) = (T+t_o)\, P(R_2|s_i \cap t_o) + (T-t_o)\, P(R_1|s_i \cap t_o)$$

$$= T - t_o[2\, P(R_1|s_i \cap t_o) - 1]$$

Integrating over t_o yields

$$E((|t_n - t_o|)|s_i) = T - [2\, P(R_1|s_i) - 1]E(t_o)$$

$$\frac{d\,E((|t_n - t_o|)|s_i)}{dT} = 1$$

An increase in the boundary values T, -T should lead to increased accuracy.

<u>Theorem</u>

$$\frac{d\,P(R_1|s_i)}{dT} \begin{cases} < 0 \text{ if } E(y|s_i) < 0 \\ > 0 \text{ if } E(y|s_i) > 0 \end{cases}$$

and

$$\frac{d\,P(R_2|S_i)}{dT} \begin{cases} < 0 \text{ if } E(y|s_i) > 0 \\ > 0 \text{ if } E(y|s_i) < 0 \end{cases}$$

<u>Proof</u> θ is always of the same sign as $E(y|s_i)$

$$P(R_1|s_1) = \frac{e^{\theta T} - e^{-\theta t_o}}{e^{\theta T1} - e^{-\theta T}}$$

$$\frac{d\ P(R_1|s_1)}{dT} = \theta \left[\frac{e^{\theta t_o}(e^{\theta T}+e^{-\theta T})}{(e^{\theta T}-e^{-\theta T})^2}\right]$$

The quantity in parentheses is always positive. Therefore the sign of the product is the same as the sign of θ. Since θ is of the same sign as $E(y|s_1)$, then $\frac{d\ P(R_1|s_1)}{dT}$ is also of the same sign as $E(y|s_1)$.

$$P(R_2|s_1) = 1 - P(R_1|s_1)$$

$$\frac{d\ P(R_2|s_1)}{dT} = \frac{-d\ P(R_1|s_1)}{dT}.$$

Therefore, $\frac{d\ P(R_2|s_1)}{dT}$ is of the opposite sign to $E(y|s_1)$

The average duration of the process should also generally increase as T increases.

Theorem $\frac{d\ E(n|s_1)}{dT} \geq 0$ for all s_1 when P(Correct decision) > P(error) where t_n is the last value of the random walk.

Proof It has been shown that

$$E(n|s_1) = [2T\ P(R_1|s_1) - (T'+t_o)]\ \frac{1}{E(y|s_1)}$$

$$\frac{d\ E(n|s_1)}{dT} = \left[(2T)\frac{d\ P(R_1|s_1)}{dT} + 2P(R_1|s_1) - 1\right]\left(\frac{1}{E(y|s_1)}\right)$$

$$= \left[2T\left(\frac{d\,P(R_1|s_i)}{dT}\right) + (P(R_1|s_i) - P(R_2|s_i))\right] \frac{1}{E(y|S_i)}$$

In the case where $E(y|s_i) > 0$, $\frac{d\,P(R_1|s_i)}{dT} > 0$ and

$P(R_1|s_i) > P(R_2|s_i)$ by assumption. The product of the positive sum inside the parentheses and the positive reciprocal of $E(y|s_i)$ is positive as well.

In the case where $E(y|s_i) < 0$, $\frac{d\,P(R_1|s_i)}{dT} < 0$ and

$P(R_1|s_i) < P(R_2|s_i)$ by assumption. The sum of terms in parentheses is negative. The product of the negative sum and the negative reciprocal term is positive.

Appendix II - The raw data.

response		different						same					
confidence		HIGH		MEDIUM		LOW		LOW		MEDIUM		HIGH	
deadline	stim diff/	N	RT	N	RT	N	RT	N	RT	N	RT	N	RT
475		42	410	111	352	85	357	55	426	75	420	832	414
350	0Δs	29	331	264	259	221	262	142	229	147	363	397	376
475		74	443	124	401	41	396	16	368	12	456	133	432
350	1Δs	23	440	115	278	114	280	37	282	33	345	78	357
475		147	437	176	483	45	374	9	[illegible]	2	370	21	357
350	2Δs	51	445	150	288	97	287	37	[illegible]	27	281	38	305
475		171	387	199	361	25	341	[illegible]	199	---	---	4	310
350	4Δs	61	402	207	304	98	265	19	232	8	251	7	276

Table XIV - Data for subject JW

response		different						same					
confidence		HIGH		MEDIUM		LOW		LOW		MEDIUM		HIGH	
deadline	stim diff/	N	RT	N	RT	N	RT	N	RT	N	RT	N	RT
475		125	359	66	289	143	285	56	313	144	299	669	385
350	0Δs	17	320	131	267	345	261	163	283	319	286	225	329
475		82	429	43	325	59	381	13	293	44	284	159	370
350	1Δs	11	399	48	341	139	298	59	246	84	303	59	316
475		149	421	51	354	60	285	15	286	38	241	87	314
350	2Δs	19	348	73	353	134	274	66	272	69	252	39	282
475		197	395	58	402	65	292	24	288	24	223	32	260
350	4Δs	31	350	115	353	150	291	53	285	31	241	20	298

Table XV - Data from subject EL

response		DIFFERENT									SAME								
confidence		HIGH			MEDIUM			LOW			LOW			MEDIUM			HIGH		
deadline	stim diff/	N	RT	T1	N	RT	T1	N	RT	T1	N	RT	T1	N	RT	T1	N	RT	T1
475	0Δs	167	372	260	177	328	233	228	312	211	111	369	252	216	341	236	1501	401	294
350		46	327	218	395	262	176	566	261	170	305	290	187	466	310	199	622	359	230
475	1Δs	156	436	304	167	382	275	100	387	257	29	334	232	56	321	208	292	398	285
350		34	427	265	163	297	207	253	290	184	96	260	166	117	315	207	137	339	218
475	2Δs	296	429	296	227	377	283	105	323	215	24	327	213	40	247	164	108	323	237
350		70	419	247	223	309	209	231	280	180	103	269	174	96	260	175	77	293	204
475	4Δs	368	392	285	257	371	280	90	306	206	25	285	190	24	223	136	36	265	183
350		92	384	237	322	322	215	248	281	187	72	271	176	39	243	160	27	292	191

Table XVI - Data for subjects JW and EL pooled - RT is total response time measured, T1 is "release time".

response		different						same					
confidence		High		Medium		Low		Low		Medium		High	
deadline	stim diff/	N	RT	N	RT	N	RT	N	RT	N	RT	N	RT
Acc	0Δs	29	981	10	1257	1	305	2	428	18	911	1188	592
475		10	394	251	394	2	376	5	416	731	455	249	491
350		43	284	69	373	289	281	692	351	91	379	64	419
Acc	1Δs	337	689	41	885	---	---	---	---	---	---	38	590
475		21	482	253	463	---	---	---	---	110	438	32	451
350		48	368	26	401	141	331	167	317	21	324	13	371
Acc	2Δs	403	554	13	620	---	---	---	---	---	---	---	---
475		52	458	347	418	2	372	1	460	13	363	1	342
350		83	353	46	378	196	318	87	280	8	311	1	218
Acc	4Δs	409	511	7	518	---	---	---	---	---	---	---	---
475		89	415	326	382	1	339	---	---	---	---	---	---
350		83	364	38	375	245	312	43	256	6	297	1	265

Table XVII - Data from subject NW

response		different						same					
confidence		HIGH		MEDIUM		LOW		LOW		MEDIUM		HIGH	
deadline	stim diff/	N	RT	N	RT	N	RT	N	RT	N	RT	N	RT
Acc	0Δs	73	427	48	456	25	540	6	1644	110	797	986	532
475		15	430	111	361	158	318	30	385	286	382	648	396
350		11	313	109	309	152	289	40	322	312	354	624	373
Acc	1Δs	141	616	75	660	34	856	2	444	17	1069	147	537
475		17	403	103	400	90	384	13	429	52	385	141	400
350		15	306	88	344	92	332	27	360	75	327	119	370
Acc	2Δs	270	513	96	480	30	439	---	---	5	954	15	375
475		50	412	189	445	108	380	11	366	23	354	35	347
350		37	395	153	374	127	340	22	345	29	273	48	340
Acc	4Δs	303	435	99	417	13	360	---	---	1	193	---	---
475		85	392	222	407	98	375	5	326	3	258	3	192
350		59	388	192	366	133	329	11	218	11	220	10	284

Table XVIII - Data from subject NN

response		DIFFERENT						SAME					
confidence		HIGH		MEDIUM		LOW		LOW		MEDIUM		HIGH	
deadline	stim diff	N	$\overline{RT}$	N	$\overline{RT}$	N	$\overline{RT}$	N	$\overline{RT}$	N	$\overline{RT}$	N	$\overline{RT}$
Acc	0Δs	153	433	187	387	70	359	220	432	266	480	352	529
475		23	403	289	365	191	370	455	415	230	416	60	392
350		299	293	243	307	102	313	115	362	127	1 27	388	383
Acc	1Δs	104	508	102	409	44	433	46	401	52	458	68	514
475		14	462	129	400	68	373	130	401	63	397	12	363
350		121	321	100	316	30	304	28	344	26	333	111	344
Acc	2Δs	182	481	136	416	32	392	24	328	20	385	22	408
475		29	417	198	388	98	401	52	391	21	398	18	359
350		131	363	108	298	36	324	24	319	18	302	99	335
Acc	4Δs	230	461	138	398	30	375	12	323	3	357	3	296
475		46	448	270	402	73	383	23	320	3	366	1	277
350		182	369	127	354	38	363	7	274	4	378	58	276

Table XIX- Data from subject JS

response		different						same					
	confidence	HIGH		MEDIUM		LOW		LOW		MEDIUM		HIGH	
deadline	stim diff/	N	RT	N	RT	N	RT	N	RT	N	RT	N	RT
Acc		81	872	8	1022	6	416	53	433	12	753	1088	711
475	0Δs	2	567	121	379	185	371	703	408	228	443	9	694
350		1	259	142	328	313	318	634	338	151	376	7	504
Acc		219	882	8	1251	3	1044	15	337	10	1251	161	762
475	1Δs	2	530	67	454	70	448	207	405	67	395	3	455
350		3	459	67	345	96	349	209	331	39	321	2	768
Acc		378	776	5	980	5	632	8	444	75	446	75	610
475	2Δs	4	668	123	488	107	452	145	367	36	316	1	213
350		2	222	69	359	132	386	165	325	48	299	---	---
Acc		379	683	14	730	6	554	11	294	3	282	3	637
475	4Δs	6	597	158	495	142	456	82	305	26	289	2	388
350		5	492	94	428	152	391	136	285	29	323	---	---

Table XX- Data from subject RB

response		DIFFERENT									SAME								
confidence		HIGH			MEDIUM			LOW			LOW			MEDIUM			HIGH		
deadline	stim diff/N	N	RT	T1	N	RT	T1	N	RT	T1	N	RT	T1	N	RT	T1	N	RT	T1
Acc		183	712	459	66	623	384	32	510	305	61	430	269	140	729	364	3262	614	432
475	0Δs	27	426	281	483	383	248	345	347	235	738	408	252	1245	436	284	906	425	313
350		55	290	184	320	331	204	754	298	171	1366	344	184	554	364	239	695	379	285
Acc		697	735	486	124	763	486	37	818	427	17	349	190	27	1023	474	346	643	424
475	1Δs	40	451	324	423	446	286	160	412	277	220	406	250	229	414	269	176	410	308
350		66	358	224	181	352	225	329	336	200	403	327	179	135	325	216	134	376	285
Acc		1051	623	431	114	518	373	35	466	329	8	444	175	10	559	330	30	493	324
475	2Δs	106	444	292	659	439	294	217	416	275	157	368	220	72	336	222	37	344	269
350		117	364	241	268	371	249	455	344	198	274	312	170	85	291	180	49	338	264
Acc		1091	550	390	120	459	341	19	421	268	11	294	150	4	260	143	3	637	410
475	4Δs	180	411	297	706	415	289	241	422	272	87	307	171	29	286	164	5	270	162
350		147	378	264	324	385	264	530	339	196	190	275	139	46	295	167	11	282	216

Table XXI - Data for subjects NN,NW,RB pooled - RT is total response time measured, T1 is "release time"

response		different						same					
confidence		HIGH		MEDIUM		LOW		HIGH		MEDIUM		LOW	
deadline	stim diff/	T1	T2	T1	T2	T1	T2	T1	T2	T1	T2	T1	T2
475	0Δs	9	5	7	3	7	4	10	8	6	4	2	3
350		14	18	3	1	4	3	5	4	4	5	4	6
475	1Δs	8	9	6	6	17	17	20	11	11	16	6	7
350		26	25	9	3	8	7	8	5	8	8	7	13
475	2Δs	5	8	6	3	10	5	16	13	10	4	9	3
350		11	23	7	4	6	4	10	3	6	4	9	5
475	4Δs	4	4	6	3	11	4	13	7	11	4	10	4
350		8	15	5	4	6	3	11	7	7	3	17	16

Table XXII - Estimated standard errors of mean "release times"(T1) and mean "movement times" (T2) in msec for both Ss (EL and JW) pooled.

response		different						same					
confidence		HIGH		MEDIUM		LOW		HIGH		MEDIUM		LOW	
deadline	stim diff/	T1	T2	T1	T2	T1	T2	T1	T2	T1	T2	T1	T2
Acc		13	17	30	40	28	57	17	9	17	32	2	3
475	0Δs	14	36	4	4	5	4	4	3	3	3	3	3
350		10	5	5	5	3	2	2	2	5	4	4	3
Acc		6	8	20	27	55	24	23	10	56	87	9	10
475	1Δs	16	16	7	5	9	10	7	7	6	8	7	7
350		11	10	7	6	5	5	4	3	8	5	7	6
Acc		3	5	11	16	20	29	26	125	48	154	21	22
475	2Δs	11	16	3	5	7	8	8	4	9	4	11	3
350		9	7	6	6	4	5	3	3	9	4	11	7
Acc		2	3	7	13	17	32	16	8	28	13	126	34
475	4Δs	5	8	4	3	6	7	7	3	12	5	29	30
350		7	7	5	6	4	4	5	2	12	13	21	7

Table XXIII-Estimates of standard errors of mean "release times" (T1) and of mean "movement times" (T2) in msecs for the three Ss (NN, NW and RB.