Stoic propositional logic: a new reconstruction

David Hitchcock

McMaster University

hitchckd@mcmaster.ca

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Abstract: I reconstruct Stoic propositional logic, from the ancient testimonies, in a way somewhat different than the 10 reconstructions published before 2002, building especially on the work of Michael Frede (1974) and Suzanne Bobzien (1996, 1999). In the course of reconstructing the system, I draw attention to several of its features that are rarely remarked about, such as its punctuation-free notation, the status of the premisses of an argument as something intermediate between a set and a sequence of propositions, the incorrectness of the almost universal translation of the Greek label for the primitives of the system as *indemonstrable arguments*, the probable existence of an extended set of primitives which accommodates conjunctions with more than two conjuncts and disjunctions with more than two disjuncts, the basis for the system's exclusion of redundant premisses, and the reason why the hypothetical syllogisms of Theophrastus are not derivable in the system. I argue that, though sound according to its originator's (Chrysippus's) conception of validity, the system as reconstructed is not complete according to that conception. It is an open problem what one needs to add to the system in order to make it Chrysippean-complete, or even whether it is possible to do so without making it Chrysippean-unsound.

Key words: Stoicism, logic, history of logic, Stoic logic, Chrysippus, reconstruction, propositional logic, soundness, completeness

0. Introduction

In *Aristotle's Earlier Logic* (Woods 2001), John Woods finds in Aristotle's earliest logical writings considerable grist for his ongoing sophisticated defence of classical validity against contemporary relevantist objections. Woods' approach is to keep the account of validity in his reconstruction of Aristotle's early theory of syllogisms as simple as is consistent with Aristotle's

own assertions and other demands on the theory. This methodological principle allows him to interpret the concept of following of necessity in Aristotle's definition of a syllogism, a concept which Aristotle himself nowhere defines, as the concept of following according to the classical concept of validity, with the proviso that the premisses and conclusion of the argument belong to the same discipline. The properties of Aristotelian syllogisms which contemporary advocates of relevance logic and connexionist logic find so congenial–exclusion of redundant premisses, non-identity of the conclusion with any premiss, multiplicity of premisses–turn out to be constraints over and above the constraint imposed by the requirement that a syllogism be a valid argument. In fact, in order to preserve the possibility of demonstrations of logical truths, Woods revises Aristotle's definition. The basic conception of a syllogism underlying what Aristotle has to say in the *Topics* and *Sophistical Refutations*, Woods claims, is sixfold: (1) A syllogism is an argument, in the sense of a discourse consisting of a set of premisses (which may be the null set) and a conclusion. (2) A syllogism is valid, in the sense that it has no counter-model. (3) Each premiss of a syllogism is a proposition, in Aristotle's sense of a statement which says one thing about one thing; and the conclusion of a syllogism is also a proposition in this sense. (4) Each term in the conclusion of a syllogism occurs in at least one premiss. (5) Each premiss has a term which occurs in the conclusion. (6) The conclusion repeats no premiss. The last of these conditions echoes the condition in Aristotle's own definition of a syllogism that something other than the premisses follows from them. The condition in Aristotle's own definition that a syllogism have more than one premiss falls out of Woods' reconstruction as a derived consequence. The condition that there be no redundant premisses, implicit in Aristotle's requirement that the conclusion follow because of the premisses, is qualified so as to allow for demonstrations of

logical truths; apart from this qualification, syllogisity is not only non-monotonic but counter-monotonic, in the sense that any addition of a premiss to a syllogism produces something which is not a syllogism.

Woods' reconstruction, which is confessedly anachronistic, shows how sophisticated positions in philosophical logic can be extracted from reflection on ancient logical texts when the resources of contemporary logical theory are brought to bear on them. Similar sophistication has emerged from contemporary scholarship and logical reflection on the system of propositional logic invented by Chrysippus (c. 280-207 BCE), the third head of the Stoic school.

At the time of preparing this paper, there were at least ten reconstructions of Stoic propositional logic: those by Mates (1953), Becker (1957), Kneale and Kneale (1962), Frede (1974), Hitchcock (1982), Egli (1982), Long and Sedley (1987), Ierodiakonou (1990), Milne (1995) and Bobzien (1996, 1999). Despite the steadily increasing adequacy of these reconstructions, I propose to add an eleventh, which I believe to be more adequate than any of them. It stands on the shoulders of previous reconstructions, in particular those of Frede and Bobzien. I do not share the pessimism of Mignucci (1993), who concludes that "if we keep to the texts two of the Stoic *themata* can be identified in a more or less sure way, and we must simply confess that we have no clue for detecting the others." (238) I aim to show, on the contrary, that a faithful but judicious adherence to the textual evidence gives us a sound basis for inferring the content of the Stoic *themata* for which we have no direct testimony. Along the way, I shall draw attention to some little noticed features of the system invented by Chrysippus: its punctuation-free notation, the status of the premisses of an argument as something intermediate between a set and a sequence of propositions, the incorrectness of the almost universal translation of the Greek

label for the primitives of the system as *indemonstrable arguments*, the great generality of the general descriptions of these primitives in our sources (as compared to the moods and examples), the probable existence of an extended set of primitives which accommodates conjunctions with more than two conjuncts and disjunctions with more than two disjuncts, how Chrysippus avoided both invalidity and fudging in the latter case, the absence of a contraction rule, the basis for the system's exclusion of redundant premisses, the reason why the hypothetical syllogisms of Theophrastus are not derivable in the system, the equivalence in both applicability and deductive power of the two extant ancient versions of the third *thema*. I shall also make comparisons and contrasts to John Woods' reconstruction of Aristotle's early theory of the syllogism. Finally, I shall argue that, even if we use the criterion of completeness which Chrysippus himself would have accepted (and could have formulated), the system is not complete; a counter-example proposed by Peter Milne (1995) still applies. It is an open problem, I shall conclude, what one needs to add to the system of Chrysippus in order to make the system Chrysippean-complete, or even whether it is possible to do so without making it Chrysippean-unsound.

Because of the fragmentary character of the ancient testimonies, their frequent polemical intentions, the logical obtuseness of some of their authors, and the paucity of direct quotations, the reconstruction of Stoic propositional logic is a difficult hermeneutic enterprise. In what follows I shall endeavour to be judiciously faithful to all the ancient textual evidence, rejecting or modifying testimonies only if there are clear grounds for doing so.

1. Axiômata

1.1 Definition

For the Stoics, a proposition (*axiôma*) is a kind of sayable (*lekton*), i.e. an incorporeal (*asômaton*) item capable of being expressed in speech. As incorporeal (i.e. not possessing both three-fold extension and resistance (D.L. 7.135)), it is non-existent (*mê on*) and incapable of affecting anything or being affected. We have Chrysippus' own definition of a proposition:

> A proposition is that which is either true or false, or a thing[1] complete in itself which is assertible[2] insofar as concerns itself, as Chrysippus says in his *Dialectical Definitions*: 'A proposition is that which is assertible or affirmable insofar as concerns itself, for example *It is day*, *Dion is walking*.' ... A proposition is what we assert when speaking, which is either true or false.[3] (Diocles 7.65-66; cf. A.L. 2.73-74, Gellius 16.8.4 = FDS 877)

Thus a proposition is an assertible complete sayable. Propositions include the illocutionary force of an assertion; the sentence 'Is it day?' for example does not signify the proposition *It is day* but the question *Is it day?* (Diocles 7.66) Despite their abstract status, some propositions change truth-value over time; for example, the proposition *It is day* is true when it is day and false when

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[1]The word "thing" (*pragma*) in such contexts refers to a sayable, as Mates (1953, 28) points out. The parallel definitions in Sextus Empiricus (P.H. 2.104) and Gellius (16.8.4 = FDS 877) are verbally identical, except for the use of "sayable" (*lekton*) instead of "thing".

[2]Bobzien (1996, 1999) translates *axiôma* as *assertible*. This translation has the attractive feature of marking the fact that Stoic *axiômata*, unlike Fregean *Gedanken*, include the illocutionary force of asserting. But the Stoics had a word for *assertible*, namely, the word *apophanton*, which Diocles uses here. Bobzien's translation of *axiôma* would render the Stoics' standard definition as 'An assertible is a sayable complete in itself which is assertible insofar as concerns itself', which is hardly the illuminating account intended. Hence I stick to the usual translation of *axiôma* as *proposition*.

[3]Here and elsewhere, translations are my own.

it is not day (Diocles 7.65, A.L. 2.103).[4] Some propositions even come into being and pass away. In all three respects (assertoric force, possible changing truth-value, possible generability and destructibility), Stoic propositions are unlike Frege's *Gedanken*, which they otherwise resemble. They should be sharply distinguished from the "propositions" (*protaseis*) of Aristotle, which are sentences in which a speaker requests an interlocutor to grant a statement in which one thing is affirmed or denied of one thing; An Aristotelian *protasis* is a certain kind of simple linguistic item in which one thing is affirmed or denied of one thing; a Stoic *axiôma* in contrast is a non-existent incorporeal which need not be simple. Much confusion has resulted in the western logical tradition from the use of the same word *proposition* for both Aristotle's *protasis* and the Stoics' *axiôma*.

Every Stoic proposition is either true or false (Diocles 7.65, Cicero *On Fate* 38); the principle of bivalence was central to Stoic philosophy, being the basis for their belief in universal determinism or 'fate'. Although propositions are not linguistic items, the Stoics maintained that assertions in a regimented form of Greek exactly correspond in their structure to the propositions which they signify. Thus they were able to produce in Greek a logic for a class of items which are not linguistic. The appeal to a regimented form of Greek is the source of the formalism for which they were notorious in antiquity.

1.2 Simple propositions

Simple propositions (*hapla axiômata*) are "those composed of a proposition which is not

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[4]The truth-value of this proposition is also relative to place, since there are times when it is day at one place on the earth but it is not day at another place on the earth. Although the Stoics would have known this, there is no evidence that they recognized this additional complication in using unqualified occasion sentences as surrogates for propositions.

duplicated" (Diocles 7.68), non-simple propositions (*ouch hapla axiômata*) "those put together out of a duplicated proposition [e.g. *If it is day, it is day*–DH] or of propositions [e.g. *If it is day, it is light*–DH]" (Diocles 7.68). Thus some negations (*apophatika*) are simple propositions, e.g. the proposition *Not it is day*[5], but others are not, e.g. the proposition *Not both it is day and it is night*. The Stoics had a taxonomy of simple propositions (*hapla axiômata*), for which they provided rules of formation and truth-conditions. Except for simple negations, which are considered in the next section, this taxonomy has no bearing on their system of propositional logic, which abstracts from the internal structure of simple propositions.

1.3 Non-simple propositions

Four types of non-simple propositions are noticed in the primitives of Stoic propositional logic: negations (*apophatika*), conjunctions (*sumpeplegmena*), conditionals (*sunêmmena*) and disjunctions (*diezeugmena*). The recursive grammar of the system's formal language permitted in its base clause not only simple propositions but also propositions in which a subject has some propositional attitude towards a proposition (e.g. *You know that you are dead*: Origen, *Contra Celsum* 7.15, p. 167 = FDS 1181) and perhaps causal (*aitiôdê*) propositions (e.g. *Because it is day it is light*: Diocles 7.72), propositions indicating what is more likely (e.g. *More likely it is day than it is night*: Diocles 7.72) and propositions indicating what is less likely (e.g. *Less likely it is night than it is day*: Diocles 7.73).

1.3.1 Negations

The negation (*apophatikon*) of a proposition is the proposition formed by prefixing a negative,

[5]The solecism, which I will repeat throughout for propositions which are simple negations, reflects the Stoic requirement that the negative in a negation precede the negated proposition.

apophasis (*ouk* or *ouchi*, English *not*) to the proposition (Diocles 7.69, A.L. 2.88-90). This formation rule reflects a clear understanding that the scope of the negative *not* is an entire proposition and not, for example, the predicate in a simple proposition. Negation is classically truth-functional: the negation of a true proposition is false, and of a false proposition true (A.L. 2.103).

The concept of negation differs from that of a contradictory (*antikeimenon*). A proposition and its negation are said to be "contradictories" (*antikeimena*), meaning that one of the pair is true and the other false (Diocles 7.73, A.L. 2.89). Thus the term *contradictory* is more general than the term *negation*. The negation of a proposition always exceeds the negated proposition by a negative (*apophasis*) which is prefixed to it, but the contradictory of a negated proposition may fall short by a negative. For example, the negation of *Not it is day* is the double negation (*hyperapohatikon*, Diocles 7.69) *Not not it is day*, but the contradictory of *Not it is day* may be either this double negation or the affirmative proposition *It is day*. The distinction between the negation and the contradictory of a proposition is important in getting an accurate understanding of Stoic propositional logic and appreciating its power.

1.3.2 Conjunctions

A conjunction (*sumpeplegmenon*, *sumplokê*) is "a proposition which is conjoined by some conjunctive connectives, for example, *Both it is day and it is light*" (Diocles 7.72). The formation rule allows more than two conjuncts, as other examples in our sources attest (cf. e.g. Gellius 16.8.10 = FDS 967). Diocles' example indicates that an initial conjunctive connective was required, as is necessary to avoid syntactic ambiguity when a conjunction is negated. The conjunctive connective is classically truth-functional: a conjunction is true if all its conjuncts are

true and false if a conjunct is false (A.L. 2.125, Gellius 16.8.11 = FDS 967).

1.3.3 Conditionals

A conditional (*sunêmmenon*) is:

> as Chrysippus says in his *Dialectical Definitions* ..., that which is put together by the conditional connective *if*. This connective declares that the second follows from the first, for example *If it is day, it is light*. (Diocles 7.71; cf. A.L. 2.109-111, Gellius 16.8.9 = FDS 953)

By "the first", as Sextus Empiricus explains (A.L. 2.110), the Stoics meant the proposition after the connective *if* (Greek *ei* or *eiper*), even if it is uttered second, as in the sentence "It is light if it is day." They used technical terms for the antecedent (*hêgemoun*, literally "leader") and consequent (*lêgon*, literally "ceaser" or "terminater"). The comma which our printed sources insert between the antecedent and consequent, although it does not occur in the manuscripts on which they are based (which are without punctuation marks), will turn out to be necessary for syntactical disambiguation

Stoic logicians took the consequent of a conditional to follow from its antecedent if and only if the contradictory of the consequent "conflicts with" (*machetai*, literally "battles with") its antecedent:

> A conditional is true in which the contradictory of the consequent conflicts with the antecedent, for example *If it is day, it is light*. This is true, for *Not it is light*, the contradictory of the consequent, conflicts with *It is day*. A conditional is false in which the contradictory of the consequent does not conflict with the antecedent, for example *If it is day, Dion is walking*. For *Not Dion is walking* does not conflict

with *It is day*. (Diocles 7.73; cf. P.H. 2.111)

The example shows that conflict is not simply logical incompatibility, since *Not it is light* and *It is day* are at most incompatible in meaning, and perhaps just physically incompatible. Conflicting propositions cannot be simultaneously true (I.L. 4.2, Apollonius Dyscolus 218 = FDS 926, Gellius 16.8.14 = FDS 976), but in some cases can be simultaneously false, as in the example, e.g. at night by lamplight. In his report on the ancient dispute about the truth-conditions of a conditional proposition, Sextus Empiricus (P.H. 2.110-112) attributes the criterion of conflict to "those who introduce connection (*sunartêsis*)" (P.H. 2.111), i.e. those who maintain that in a true conditional there is a connection between antecedent and consequent. The contrast is to the criteria of Philo and Diodorus Cronus, predecessors of the Stoics in the so-called "dialectical school"; a conditional is true for Philo whenever it does not have a true antecedent and a false consequent, and for Diodorus if it never has a true antecedent and a false consequent (P.H. 2.110-11). Philo's criterion is met at any time when a conditional has either a false antecedent or a true consequent, e.g. *If it is day, Dion is walking* whenever *It is day* is false or *Dion is walking* is true. Diodorus' criterion is met if a conditional has either an always false antecedent or an always true consequent, e.g. *If not there are partless elements of existents, Dion is walking* or *If Dion is walking, there are partless elements of existents*. Thus there are Diodorean-true (and *a fortiori* Philonian-true) conditionals in which there is no connection between antecedent and consequent. The criterion of conflict between the antecedent and the contradictory of the consequent is an attempt to add to the criterion of Diodorus a requirement of such a connection. Sextus' example of a conditional which is Diodorean-true but Alexandrian false, *If not there are partless elements of existents, there are partless elements of existents*, shows that a proposition does not conflict

with itself, even if it is necessarily false. It is a reasonable extrapolation from this example that the mere fact that a proposition is always false, or even necessarily false, is not sufficient for it to conflict with any arbitrarily chosen proposition; thus *not there are partless elements of things* does not conflict with *Dion is walking*. In other words, the Stoics reject the medieval principles *ex falso quodlibet* (from what is necessarily false, anything follows) and *e quolibet verum* (from anything, what is necessarily true follows). Although not all conflicting propositions are contradictories of one another, any proposition conflicts with its contradictory (Apollonius Dyscolus 218 = FDS 926). Hence it cannot be a requirement for conflict that each proposition is at some time true, nor can it be a requirement that neither proposition is necessarily false; either requirement would make it impossible for an always or necessarily true proposition to conflict with its contradictory. To sum up, one proposition conflicts with another only if (1) they are distinct, (2) they cannot both be simultaneously true, and (3) this impossibility is not due to the necessary falsity of one of them. The ancient testimonies do not give us any basis for identifying further necessary conditions for conflict. The difficulty is to specify the conditions under which an always false, or necessarily false, proposition conflicts with another proposition.

1.3.4 Disjunctions

A disjunction (*diezeugmenon*) is "that which is disjoined by the disjunctive connective *or*, for example *Either it is day or it is night*." (Diocles 7.72). As with the conjunction, the initial *either* (Greek *êtoi*) prevents syntactic ambiguity, e.g. when a disjunction is negated. The definition, and examples elsewhere, indicate that there are disjunctions with more than two disjuncts, e.g. *Either pleasure is evil or pleasure is good or both not pleasure is good and not pleasure is bad* (Gellius 16.8,12 = FDS 976).

Our sources convey a confused message about the truth conditions for a disjunction.

According to Diocles, the disjunctive connective "declares that one or the other of the <disjoined> propositions is false" (7.72). This is truth-functional exclusive disjunction, according to which a proposition like *Either it is day or Dion is walking* would be true whenever *It is day* is false and *Dion is walking* is true (and also whenever *It is day* is true and *Dion is walking* is false). As stated, it applies only to disjunctions with two disjuncts, but it can easily be extended, as the condition that one disjunct is true and the remaining one or remaining ones are false.

Sextus Empiricus adds the requirement of conflict: "The sound[6] disjunction declares that one of its disjuncts is sound and the remaining one or remaining ones are false with conflict." (P.H. 2.191) This seems to mean that one disjunct is true and each disjunct conflicts with each other disjunct.[7] This is quasi-connexionist exclusive disjunction, quasi-connexionist rather than fully connexionist because it is not necessary that one disjunct be true; on this account, *Either Dion is walking or Dion is sitting or Dion is standing* would be true when one of its disjuncts is true but false when Dion is lying down. The proposition *Either it is day or Dion is walking* would however always be false, because its disjuncts do not conflict with each other.

Gellius (16.8.13 = FDS 976) reports that in a disjunction (1) the disjuncts ought to conflict with one another and (2) their contradictories ought to be opposed among themselves; he adds that (3) it is necessary that one of the disjuncts is true and the others false. Condition (1)

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[6]Here and elsewhere in our sources, the adjective *sound* (Greek *hugies*, literally "healthy") when applied to propositions is a synonym of *true* (Greek *alêthes*).

[7]It cannot be meant merely that each false disjunct conflicts with each other false disjunct, because "the remaining one ... <is> false with conflict" would then make no sense.

implies that no two disjuncts can be true, along with the other conditions involved in the concept of conflict. Condition (2) must be interpreted to mean that not all the contradictories of the disjuncts can be true, i.e. that at least one disjunct must be true; the alternative interpretation, pairwise opposition among contradictories, would imply along with condition (1) that no disjunction with more than two disjuncts is true, but Chrysippus clearly regarded some such disjunctions as true (P.H. 1.69, 2.150). Condition (3) is a consequence of conditions (1) and (2), but not equivalent to it, since it can be met by disjunctions in which there is no connection among the disjuncts, e.g. *Either not there are partless elements of existents or if it is day it is light*. Gellius' account thus amounts to fully connexive exclusive disjunction. According to it, the disjunction *Either wealth is good or wealth is bad or wealth is indifferent* (P.H. 2.150) would be true, since each disjunct conflicts with each other disjunct but it is not possible for all their contradictories (*not wealth is good*, *not wealth is bad*, *not wealth is indifferent*) to be true. But the disjunction *Either Dion is walking or Dion is sitting or Dion is standing* would be false even when one of its disjuncts was true, because it is possible for all the contradictories of its disjuncts to be true, e.g. when Dion is lying down.

On either the truth-functional or the quasi-connexive version of exclusive disjunction, an argument from a disjunction with multiple disjuncts and the contradictory of one disjunct to the disjunction of the remaining disjuncts is valid, for example, *Either wealth is good or wealth is bad or wealth is indifferent; but not wealth is good; therefore either wealth is bad or wealth is indifferent*. But on the fully connexive version this argument is invalid: its premisses are true but its conclusion is false, because it is possible for all the contradictories of the disjuncts in the conclusion to be true. On the connexive account, the only valid argument from a disjunction with

more than two disjuncts and a negation is one whose added premiss (*proslêgon*) is the negation of the disjunction of all the disjuncts but one and whose conclusion is the remaining disjunct. Our only source does in fact report this form of argument as primitive in Stoic propositional logic: "There arise then ... two other hypothetical moods, the fourth ..., and the fifth which from a disjunction by the denial of one or of the remaining ones introduces the one left behind." (Philoponus 245,32-35 = FDS 1133) Unfortunately, the Stoics' first *thema*, a contraposition rule for arguments, allows one to transform an argument of this form into an argument from the denial of one disjunct to the disjunction of the remaining ones. And this argument is invalid on the fully connexive interpretation of disjunction. To preserve the soundness of the system, we must suppose that Chrysippus had at most the quasi-connexive account of disjunction reported by Sextus.[8] In what follows I will assume the quasi-connexive acccount, i.e. that a disjunction is true if and only if one disjunct is true and each disjunct conflicts with each other disjunct.

1.4 Punctuation-free notation

Apart from the comma between the antecedent and consequent of a conditional, the formation rules prevent syntactical ambiguity without the need for punctuation devices. This fact is evidence of the Stoics' well-known fussiness about how to construct the sentences which they used as surrogates for propositions in their propositional logic. It is a consequence of the fact that each type of non-simple proposition begins with the connective for that type–the same feature

[8]There are independent reasons for discounting Gellius' testimony. He is an amateur, the author of a charming account of books he has read. He devotes just three pages to a report of what he found in the first section of an *Introduction to Logic*, the section on propositions. He has little logical acumen: his truth-conditions if taken literally would imply that no disjunction with more than two disjuncts is true, he does not notice that the third condition is a consequence of the first two, and he asserts falsely that every false disjunction is called a quasi-disjunction (i.e. an inclusive disjunction). He is clearly confused.

which makes it possible for Polish notation to do without brackets. It is quite possible that the inventor of Polish notation, Jan Łukasiewicz, got the idea from the syntax of Stoic propositional logic, which he knew. If we put Stoic punctuation-free notation into the garb of contemporary symbolism, we would have the following formation rules:

1. If p is a proposition, then so is $\neg p$.

2. If p and q are propositions, then so is $\rightarrow p \rightarrow q$. (Read: *if p then q*.)

3. If $p_1, ..., p_n$ (n > 1), then so are $\& p_1 \& ... \& p_n$ (read: *both* p_1 *and .. and* p_n) and $\veebar p_1 \veebar ... \veebar p_n$ (read: *either* p_1 *or .. or* p_n).

In what follows, I shall use this notation.

2 Anapodeiktoi

2.1 Preliminaries

2.1.1 Arguments

Stoic propositional logic took as primitives arguments (*logoi*) of five types (Diocles 7.79-81). The Stoics defined an argument as "a system <put together> out of premisses (*lêmmata*) and a conclusion (*epiphora*)" (D.L. 7.45). The plural form of "premisses" is quite intentional: Chrysippus denied that there are one-premissed arguments (A.L. 2.443), and a later head of the Stoic school, Antipater of Tarsus, was recognized as making an innovation when he allowed one-premissed arguments such as *It is day; therefore it is light*, perhaps in response to sceptical challenges (A.L. 2.443, Alexander *In Top*. 8,16-19 = FDS 1052). Thus Stoic orthodoxy was even more hostile than Aristotle to one-premissed arguments; whereas Aristotle nowhere denies that there are one-premissed arguments but makes a minimum of two premisses a necessary condition of syllogisms in the broad sense, Chrysippus explicitly excluded even one-premissed arguments

which were not (Stoic) syllogisms. *A fortiori*, no Stoic syllogism has one premiss; that is, the system of propositional logic developed by Chrysippus does not permit the proof of any argument with one premiss.

Why did Chrysippus insist that an argument must have at least two premisses? Not because he took apparently one-premissed arguments to be elliptical, with an implicit unstated additional premiss needed for validity. For he apparently took the proposition *It is light* to follow from the simple proposition *It is day*[9], so that the system *It is day; therefore it is light* would be valid without supplementation if it were an argument. For Chrysippus, then, the discourse "It is day, so it is light" does not signify an argument; we have no testimony concerning what sort of item he thought it *did* signify. Nor do we know why he rejected one-premissed arguments.

The rejection of one-premissed arguments shows that the premisses of a Stoic argument did not constitute a set, contrary to Peter Milne's suggestion (1995, 41). If they did, Stoic propositional logic would easily generate one-premissed arguments (in fact, one-premissed syllogisms), a fact Chrysippus could hardly fail to have noticed. To get ahead of our story for a moment, the application of the first *thema* to the first *anapodeiktos* argument *If not it is day, it is day; not it is day; therefore it is day* produces the argument *Not it is day; not it is day; therefore not if not it is day, it is day*. If the two premisses of the latter argument are a set, it is a set with one member, so that the argument would have one premiss. Thus repetitions of premisses can occur.

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[9]I assume that Chrysippus' standard example of a conditional, *If it is day, it is light*, was chosen partly because it is obviously true. Since the conditional connective declares that the consequent follows from the antecedent, the truth of this conditional means that *It is light* follows from *It is day*. If the assumption is incorrect, choose another true conditional composed of two simple propositions.

On the other hand, the Stoics' premisses appear not to be a sequence either. There is no evidence that Stoic logic had a permutation rule which allows one to change the order of premisses. If order of premisses were important, some proofs would not go through without such a rule. Given the silence of our sources, it is a reasonable inference that order of premisses was not important.

Thus the premisses of a Stoic argument are something intermediate between a set and a sequence. Repetitions of premisses counts, but order of mentioning does not.

2.1.2 The translation of "anapodeiktoi"

A primitive argument in Stoic propositional logic was called *anapodeiktos*. The suffix *-tos* is ambiguous, corresponding either to the English suffix *-ed* or to the English suffix *-able*/*-ible*. Thus we can translate *anapodeiktos* either as *undemonstrated* or as *indemonstrable*, just as we can translate *agenêtos* either as *ungenerated* or as *ungenerable*, *aphthartos* either as *undestroyed* or as *indestructible*, and *lektos* either as *thing said* or as *sayable*. The usual translation of *anapodeiktos* as *indemonstrable* implies that the validity of arguments of these types cannot be demonstrated. But in some cases it can: a first *anapodeiktos* argument can be analysed by the first *thema* into a corresponding second *anapodeiktos* argument, and vice versa. Further, we are told that the Stoics' arguments are *anapodeiktos* because they do not need demonstration (Diocles 7.79; cf. A.L. 2.223), an explanation which makes far more sense if we translate *anapodeiktos* as *undemonstrated* than if we translate it as *indemonstratable*. I propose therefore to translate *anapodeiktos* as *undemonstrated*, intending thereby to indicate that no demonstration of these arguments is given, because it is thought not to be required; such arguments, as Sextus Empiricus reports, "have no need of demonstration because of its being at once conspicuous in their case

that they are valid" (A.L. 2.223).

2.1.3 Validity

For the Stoics, the syllogisms of their system are a species of valid (*perantikos*, *perainôn*, *sunaktikos*, *sunagôn*) argument (Diocles 7.78); in such an argument the conclusion "follows from" (*sunagetai ek*, *akolouthei*, *hepetai*) the premisses and is "that which follows" (*sunagoumenon*), and the premisses "yield" (*sunagousi*) the conclusion.[10] An argument is valid if and only if the contradictory of its conclusion conflicts with the conjunction of its premisses (Diocles 7.77). Thus validity is the same as truth of the argument's associated conditional, the conditional whose antecedent is the conjunction of the argument's premisses and whose consequent is the argument's conclusion (P.H. 2.137, A.L. 2.415-417; cf. A.L. 2.112). Stoic validity is in some respects narrower than classical validity, since it requires a connection between premisses and conclusion; for example, the argument *Dion is walking; but not Dion is walking; therefore, it is light* is classically valid but not Stoically valid. In other respects Stoic validity is wider than classical validity, since the conflict of the contradictory of a valid argument's conclusion with the conjunction of its premisses may be partly a function of the meaning of its extra-logical terms or even of natural necessities; for example, the argument *The first is greater than the second; but the second is greater than the third; therefore the first is greater than the third* is Stoically valid (Alexander *In An. pr.* 21,28-22,1) but not classically valid.

Sextus Empiricus reports a fourfold taxonomy, attributed to an anonymous "they", of

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[10]Unfortunately, no single English root will do the quadruple duty of the Greek root *sunag-* in these contexts.

ways in which an argument can become invalid (*aperantos*): disconnection (*diartêsis*), redundancy, bad form, deficiency (A.L. 2.429-432). This taxonomy is consistent with the basic concept of validity; it indicates different ways in which the contradictory of an argument's conclusion can fail to conflict with the conjunction of its premisses.

Disconnection is the failure of the premisses to have any association and connection (*sunartêsis*) with each other and with the conclusion, as in the argument *If it is day, it is light; but wheat is being sold in the market; therefore it is light*. Since the criterion of conflict is designed to assure a connection between premisses and conclusion, disconnection is a direct failure to conflict.

Redundancy is the introduction of a superfluous premiss into an argument which would otherwise be valid[11], as in the argument *If it is day, it is light; but it is day; and virtue benefits; therefore it is day*. It is not immediately obvious why the contradictory of this argument's conclusion fails to conflict with the conjunction of its premisses, especially since it is impossible

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[11]As Bobzien points out (1996, 180), this definition needs to be interpreted so as to accommodate some arguments recognized as syllogisms in Stoic propositional logic. One way to do so is to create an exception where the argument has the same form as a valid argument without a redundant premiss. An example is *If there exists a sign, there exists a sign; if not there exists a sign, there exists a sign; but either not there exists a sign or there exists a sign; therefore, there exists a sign* (A.L. 2.281), where the third-mentioned premiss is redundant. But this argument has the same form as the argument *If it is day, it is light; if it is night by lamplight, it is light; either it is day or it is night by lamplight; therefore it is light*, which is valid and has no redundant premiss. This way of creating an exception seems preferable to that proposed by Bobzien (1996, 180), since it counts as exceptions not only arguments whose validity is provable in the Stoics' system; but also intuitively valid arguments which are not so provable, e.g. the hyposyllogism *That there exists a sign follows from that there exists a sign; that there exists a sign follows from that not there exists a sign; but either not there exists a sign or there exists a sign; therefore, there exists a sign*. (For hyposyllogisms, so-called "specifically valid" (Diocles 7.8) non-syllogistic arguments which become syllogisms if a premiss is replaced with an equivalent proposition, see Alexander *In an. pr*. 84,12-25 and 373,29-35 = FDS 1084 and 1085, as well as I.L. 19.6 = FDS 1086.)

for both these propositions to be simultaneously true. The explanation is perhaps that, according to the Stoics' first *thema*, if this argument is valid, then so is its contrapositive[12]: *If it is day, it is light; but it is day; and not it is day; therefore not virtue benefits*. But this argument is clearly invalid on the connexive criterion of validity; the contradictory of the conclusion has nothing to do with the conjunction of the premisses.

Bad form is any form different from the sound forms, exemplified by the argument *If 3 is 4, 6 is 8; but not 3 is 4; therefore not 6 is 8*. Although this argument has true premisses and a true conclusion, its form *If the first, the second; but not the first; therefore not the second* is bad because there are bad arguments in this form, e.g. *If it is day, it is light; but not it is day; therefore not it is light*, which sometimes has true premisses and a false conclusion. The definition of this type of invalidity of course needs qualification for the exceptions where an argument is valid in virtue of some feature other than the bad form which it has.

Deficiency Sextus defines as a deficiency in a validating premiss, as in the argument *Either wealth is bad or wealth is good; but not wealth is bad; therefore wealth is good*. The disjunctive premiss, Sextus explains, is not true unless one adds the disjunct *wealth is indifferent*. But, we will immediately object, this is a problem with the truth of a premiss, not with the argument's validity. Sextus has distorted his source, which perhaps spoke of deficiency *of* (as opposed to *in*) a validating premiss. We can get an example of such a deficiency by only slightly modifying Sextus' example: *Either wealth is bad or wealth is good or wealth is indifferent; but*

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[12]Woods (2001) follows Aristotle in calling the operation of transforming an argument in this way *argument conversion*. Since conversion of sentences does not involve changing their quality from affirmative to negative or vice versa, but this operation (like the contraposition of sentences) does involve such a change in quality, it seems less misleading to call the operation *argument contraposition*. Our sources do not tell us what name the Stoics used for it.

not wealth is bad; therefore wealth is good. The problem here is that the argument is not valid unless one adds the premiss *not wealth is indifferent*. And this *is* a problem with the argument's validity.

Thus Stoic (i.e. Alexandrian) validity not only necessarily excludes true premisses and a false conclusion but also requires a connection between premisses and conclusion and excludes redundant premisses. It is a more complex concept than the simple one which Woods uses in his reconstruction of Aristotle's early logic.

2.2 Undemonstrateds

We have many testimonies about each of the five types of undemonstrated arguments recognized by Chrysippus, always in the same order with the same numbering, and largely consistent with one another. For each type we find in one or more sources a description, a mood (*tropos*) or schema (*schêma*), and one or more examples. It is important for capturing the full power of Stoic propositional logic to recognize that the descriptions, which are more general than the moods, are more authoritative, as we can infer from the fact that some examples fit the description but not the mood, and also from the fact that our only surviving examples of analyses of arguments within the system appeal to the descriptions rather than the moods to support the claim that a given argument used in a reduction is an undemonstrated argument (A.L. 2.232-238). In what follows, I shall first cite a description and standard example of each type. I shall then comment in turn on the significance of the use of the term *contradictory* rather than *negation* in the descriptions; the indeterminacy in the descriptions about which conjunct or disjunct occurs in the added premiss of a third, fourth or fifth undemonstrated argument; and the probably existence of extended descriptions of third, fourth and fifth undemonstrated arguments for conjunctions and

disjunctions with more than two conjuncts or disjuncts. Having thus clarified the power of the descriptions of the undemonstrated arguments, I shall then give in contemporary punctuation-free notation of the Stoic type a complete list of the moods of their undemonstrated arguments.

2.2.1 Basic description

A *first undemonstrated argument* is one "in which an entire[13] argument is constructed out of a conditional and the antecedent from which the conditional begins and concludes to the consequent" (Diocles 7.80; cf. A.L. 2.224, P.H. 2.157, I.L. 6.6). The standard example is: *If it is day, it is light; but it is day; therefore it is light* (A.L. 2.224, P.H. 2.157).

A *second undemonstrated argument* is one "which through a conditional and the contradictory of the consequent has as conclusion the contradictory of the antecedent" (Diocles 7.80; cf. A.L. 2.225, P.H. 2.157, I.L. 6.6). The standard example is: *If it is day, it is light; but not it is light; therefore not it is day* (Diocles 7.80, A.L. 2.225, P.H. 2.157).

A *third undemonstrated argument* is one "which through a negation of a conjunction and one of those in the conjunction concludes to the contradictory of the remaining one of those in the conjunction" (A.L. 2.226; cf. Diocles 7.80, P.H. 2.158, I.L. 14.4). A standard example is: *Not both it is day and it is night; but it is day; therefore not it is night* (A.L. 2.226, P.H. 2.158).

A *fourth undemonstrated argument* is one "which through a disjunction and one in the disjunction has as conclusion the contradictory of the remaining one" (Diocles 7.81; cf. P.H. 2.158, I.L. 6.6). An example is: *Either it is day or it is night; but it is day; therefore not it is night* (P.H. 2.158).

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[13]The qualifier "entire" (*pas*) perhaps emphasizes the absence of a redundant premiss, in accordance with the Stoics' concept of validity as excluding redundancy of premisses.

A *fifth undemonstrated argument* is one "in which an entire argument is constructed out of a disjunction and the contradictory of one of those in the disjunction and concludes to the remaining one" (Diocles 7.81; cf. P.H. 2.158). The standard example is: *Either it is day or it is night; but not it is night; therefore it is day* (D.L. 7.81, P.H. 2.158, ps.-Galen 15, 608,1-2 = FDS 1129).

2.2.2 Contradictories vs. negations

The descriptions systematically refer to the contradictory of a proposition rather than to its negation, except in the case of the conjunction in the leading premiss of a third undemonstrated argument, where the contradictory must be the negation of the conjunction. Thus the descriptions accommodate examples in which the contradictory falls short by a negative, rather than exceeding by a negative. One source, Martianus Capella occasionally gives moods or examples for such arguments: an example of a second undemonstrated argument with a negative antecedent and consequent and affirmative added premiss and conclusion (4.415 = FDS 1139), both a mood and an example of a third undemonstrated argument with a leading premiss whose second conjunct is negative and an affirmative conclusion (4.416, 420 = FDS 1139). A complete statement in a formal language of the moods of the Stoics' undemonstrated arguments should accommodate such examples.

2.2.3 Indeterminacy in choice of conjunct or disjunct

The description of a third undemonstrated argument leaves unspecified which conjunct in the negated conjunction of the leading premiss occurs as the added premiss. Galen confirms that the indeterminacy is intended by providing two examples with the same leading premiss (*Not both Dion is walking and Theon is conversing*) but different conjuncts as added premiss (I.L. 14.7-8).

Similarly the description of a fourth undemonstrated argument leaves indefinite which disjunct is the added premiss, and the description of a fifth undemonstrated argument leaves unspecified which disjunct is contradicted in the added premiss. In the latter case, our sources confirm that the indefiniteness is intended: the added premiss is in a report of the mood (Cicero, *Topics* 56 = FDS 1138) the contradictory of the first disjunct, but in a report of an example (Diocles 7.81, P.H. 2.158) the contradictory of the second disjunct. A complete statement in a formal language of the moods of the Stoics' undemonstrated arguments should reflect this indeterminacy.

2.2.4 Extended description

The above descriptions of undemonstrated arguments presuppose that a conjunction has two conjuncts and a disjunction has two disjuncts. But the formation rules allow for conjunctions with more than two conjuncts and for disjunctions with more than two disjuncts. Probably the above descriptions were basic ones for the two-conjunct or two-disjunct case; Diogenes Laertius, for example, quotes them as part of an excerpt from a first century BCE survey of philosophers designed to state as much as comes within the scope of a Stoic introductory handbook (D.L. 7.48).[14] Evidence that there was an extended description to cover conjunctions with more than two conjuncts and disjunctions with more than two disjuncts comes from scattered more general descriptions, albeit in unreliable sources.

We have two accounts of third undemonstrated arguments whose negated conjunction has more than two conjuncts. According to Cicero, the added premiss in such an argument asserts all

[14]Reconstructing the whole system from reports which are at best this incomplete is like trying to reconstruct *Principia Mathematica* from an eight-page summary of the relevant portions of Irving Copi's *Introduction to Logic* in a history of western philosophy written two centuries from now.

but one of the conjuncts and the conclusion denies (i.e. is the contradictory of) the remaining one: "when you negate some conjuncts and assume one or more of them, so that what remains may be denied, that is called a third mode of inference" (*Topics* 54 = FDS 1138).[15] According to Philoponus, on the other hand, the added premiss asserts just one conjunct and the conclusion denies the remaining ones: "the third mood of hypothetical <syllogisms> .. out of a negation of a conjunction by the affirmation of one denies the remaining ones" (Philoponus *In an. pr.* 245, 23-24 = FDS 1133).[16] The Ciceronian and Philoponian versions are equivalent, as can be shown using the Stoics' first *thema*, which permits argument contraposition. Since the Philoponian version requires less information in the added premiss, it seems more natural to choose it as the basis. I shall assume that Chrysippus did so; a *third undemonstrated argument* would thus be one "which through a negation of a conjunction and one of those in the conjunction concludes to the contradictory of the remaining one or remaining ones in the conjunction", where the contradictory of the remaining ones is the negation of their conjunction.

Philoponus also describes the conclusion of a fourth undemonstrated argument as denying the remaining one *or ones*, and gives an example in which the disjunction has three disjuncts and the conclusion denies the remaining two of them (Philoponus *In an. pr.* 245, 33-34, 36-37 = FDS

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[15]Cicero's *Topics* is a highly dubious source. Cicero tells its addressee in the preface that he wrote this brief work from memory while on a sea-voyage to Greece, with no books at his disposal. He lists as a separate sixth mood a mood which fits his description of the third mood for the case where one disjunct is affirmed. And he lists a seventh mood which is patently invalid. And, as an orator writing a book for a fellow orator, he treats the leading premiss as a *topos* or *locus* for finding arguments, rather than as part of the argument itself.

[16]Philoponus too is unreliable. A late (c. 490-570s) commentator on Aristotle, he works with a confused mixture of Peripatetic and Stoic logic, nicely disentangled by Speca (2001, 64-65).

1133). The example formulates the conclusion as a *neither-nor* statement, which can be put into standard Stoic form as either a *both not ...and not* statement or a *not either ... or* statement. Neither formulation permits a further inference in the Stoic system to the contradictory of an arbitrarily chosen disjunct other than the one assumed in the added premiss, even though an argument to any such single disjunct is valid by Stoic criteria. But a formulation that permitted inference to the contradictory of an arbitrarily chosen disjunct other than the one assumed in the added premiss would permit a further inference in the Stoic system to the conclusion as formulated by Philoponus (whether interpreted as a *both not ...and not* statement or a *not either ... or* statement), as can be demonstrated using the *themata*. Since Philoponus is our only source for an extended description of a fourth undemonstrated argument, and he is an unreliable witness, I shall assume that Chrysippus had enough logical acumen to have formulated an extended description in the logically more powerful way, as follows: A *fourth undemonstrated argument* is one "which through a disjunction and one in the disjunction has as conclusion the contradictory of the remaining one or of one of the remaining ones".

Philoponus is again our only source for an extended description of a fifth undemonstrated argument. A fifth "hypothetical mood", he says, is one "which from a disjunction by the denial of one or of the remaining ones introduces the one left behind" (Philoponus *In an. pr.* 245, 34-35 = FDS 1133). This description means that, if the leading premiss has more than two disjuncts, the added premiss is the contradictory (i.e. the negation) of a disjunction formed by omitting one of the original disjuncts. Philoponus' example of a fifth undemonstrated argument whose disjunction has more than two premisses, although it is not in Stoic canonical notation, fits his description in the relevant respect: *The side is either equal to or greater than or less than the*

side; but it is neither greater nor less; therefore it is equal (Philoponus *In an. pr.* 246, 4-5 = FDS 1133). I shall assume that the extended description provided by Chrysippus had as its conclusion a single disjunct and as its added premiss the negation of the disjunction of the remaining disjuncts. Such a negation of a disjunction follows from premisses which separately contradict each disjunct, as can be demonstrated using the *themata*. Hence it would not be difficult in practice to use an argument of the type envisaged. On the extended description, then, a *fifth undemonstrated argument* is one "in which an entire argument is constructed out of a disjunction and the contradictory of one or of the remaining ones of those in the disjunction and concludes to the one left behind".

2.2.5 Moods

Taking into account the observations of the last three sections, we can say that, if p, q and p_1, ..., p_n are propositions, then an argument of any of the following forms is a primitive (an undemonstrated argument) in Stoic propositional logic:

1. $\neg p \rightarrow q, p \vdash q$

2a. $\neg p \rightarrow q, \neg q \vdash \neg p$

2b. $\neg p \rightarrow \neg q, q \vdash \neg p$

2c. $\neg \neg p \rightarrow q, \neg q \vdash p$

2d. $\neg \neg p \rightarrow \neg q, q \vdash p$

3a. $\neg \& p_1 \& \ldots \& p_n, p_i \vdash \neg \& p_1 \& \ldots \& p_{i-1} \& p_{i+1} \& \ldots \& p_n$ ($n > 1$, $1 \leq i \leq n$)

3b. $\neg \& p \& \neg q, p \vdash q$

3c. $\neg \& \neg p \& q, q \vdash p$

4a. $\veebar p_1 \veebar \ldots \veebar p_n, p_i \vdash \neg p_j$ ($n > 1$, $1 \leq i,j \leq n$, $i \neq j$)

4b. $\veebar p_1 \veebar \ldots \veebar \neg p_j \veebar \ldots \veebar p_n, p_i \vdash p_j$ ($n > 1$, $1 \leq i,j \leq n$, $i \neq j$)

5a. $\veebar p_1 \veebar \ldots \veebar p_n, \neg\veebar p_1 \veebar \ldots \veebar p_{i-1} \veebar p_{i+1} \veebar \ldots \veebar p_n \vdash p_i$ ($n > 1$, $1 \leq i \leq n$)

5b. $\veebar \neg p \veebar q, p \vdash q$

5c. $\veebar p \veebar \neg q, q \vdash p$

No other arguments are primitives in the system.

3. Themata

3.1 Reduction

The Stoics called arguments *syllogistic* "which either are undemonstrated or are reduced[17] to undemonstrated ones by one or more of the *themata*" (Diocles 7.78). This definition implies that the latter type of argument is not a syllogism until it has actually been reduced; before reduction, Sextus Empiricus tells us, arguments which have not been demonstrated are rather confusingly called *undemonstrated arguments*, in a different sense of *undemonstrated* from that used of the primitives of the system (A.L. 2.223). Thus, for Stoic logic just as much as for Aristotle, a *sullogismos* is what we mean by a *deduction* (cf. Corcoran 1974, 91-92). The process of reducing an argument to undemonstrated arguments was also described as an analysis (*analusis*, literally "breaking up") of the syllogism into those arguments (Galen, *On the Doctrines of Hippocrates and Plato* 2 3.18-19 = FDS 1160; A.L. 2.231; Simplicius *In de caelo* 236,33-237,4 = FDS 1168). Both the term *analysis* and the use of the plural "undemonstrated ones" in Diocles' report imply that every reduction is to two or more undemonstrated arguments. But in fact some reductions are

[17] *Anagomenoi*, literally "led up". Cf. Aristotle *An. pr.* 29b1, 41b4, 50b5-51b2.

to a single undemonstrated argument.[18] So the word *reduction* is more appropriate, and Diocles' definition should be modified to read "reduced to one or more undemonstrated ones".

3.2 The attested themata

As used in Stoic propositional logic, the word *thema* has no exact English equivalent; I follow others in leaving the word untranslated. A *thema* is a rule by which one can reduce an argument to one or more other arguments. One can most easily appreciate the type of rule by considering the only statements we have of the Stoics' *themata*:

First thema: An unknown author of a late (3rd or 4th century) short compilation of Aristotelian logic says that the Stoics define the first *thema* as follows:

> If from two a third follows, from either one of them together with the contradictory of the conclusion there follows the contradictory of the remaining one. (Ps.-Apuleius, *De Interpretatione* 191,5-11 = FDS 1161)

Third thema (first citation): Alexander of Aphrodisias (fl. c. 200) describes himself as having created a "synthesis" of two two-premissed syllogisms according to what is called "by the moderns"[19] the third *thema*, whose summary statement he quotes:

> Whenever from two a third follows, and external assumptions[20] syllogize one of

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[18]For example, an argument of the form *p; q; therefore both p and q* is reduced by the first *thema* to the corresponding third undemonstrated argument of the form *not both p and q; but p; therefore, not q*.

[19]*hupo tôn neôterôn*, literally "by the newer ones", a phrase Alexander uses of Stoic logicians, in contrast to "us" or "Aristotle and his circle", a euphemism for Aristotle. At *In an. pr*. 284,13 he explicitly attributes to the Stoa the third *thema* which he quotes here.

[20]Bobzien (1999, 143) translates: "two external assumptions". But nothing in the Greek restricts the external assumptions to two, and the system would lose its power considerably if there were such a restriction. In her previous translation of the passage (Bobzien 1996, 145), the

> them, from the remaining one and from the external ones which syllogize the other one the same will follow. (Alexander, *In an. pr.* 278,12-14)

Third thema (second citation): Simplicius (writing after 532) says that Alexander's analysis of an argument by Aristotle (*De Caelo* 275b6 ff.) as consisting of two second-figure syllogisms is valid according to what is called by the Stoics the third *thema*, whose definition "according to the ancients"[21] is as follows:

> If from two a third follows, and that which follows along with another from outside yields something, then also from the first two and the one assumed in addition externally there will follow the same one. (Simplicius, *In de caelo* 237,2-4 = FDS 1168)

All three citations are general conditional statements, whose antecedent postulates one or two valid arguments and whose consequent states that another argument will be valid. Such conditional statements would license a transition in a demonstration from an argument or arguments meeting the conditions in the antecedent to an argument related to it or them as the consequent describes. In other words, they are just the sorts of general principles which could be used as the *themata* are reported to have been used, for reducing arguments to the

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word "two" does not occur before "external assumptions" and Bobzien notes specifically (145-146) that the second-mentioned argument can have more than two premisses, so that the argument to be analysed can have more than three premisses. She gives no explanation of why she added the word "two" in her (1999).

[21]*kata tous palaious*. It is not clear who Simplicius is referring to. Bobzien (1995, n. 30) and Ierodiakonou (1990, 64) assume that he is referring to some Peripatetics; in late antiquity, it was usual to describe Peripatetic logicians as "the ancients" and Stoics logicians as "the newer ones". As Bobzien states, the use of a Peripatetic source does not mean that Simplicius is quoting a Peripatetic modification of the Stoics' third *thema*, only that he got this definition of the Stoics' third *thema* from a Peripatetic work.

undemonstrated arguments. The reduction would consist in a sequence of arguments, each of which either was an undemonstrated argument or followed from one or more previous arguments in the sequence in accordance with a *thema*. The terminus of the sequence would be the reduced argument, and the undemonstrated arguments appearing along the way would be those to which the reduced argument was reduced.

The antecedents of all three statements assume an input valid argument with two premisses. This condition fits the fact that all undemonstrated arguments have two premisses.

Except for the second clause in Alexander's version of the third *thema*, the antecedents are phrased in terms of the validity of the input arguments rather than their syllogisticity. Thus they are applicable outside the Stoics' syllogistic, to their so-called "specifically valid" arguments (Diocles 7.78; Alexander *in An. pr.* 21,30-23,2 = FDS 1087, 1118; 373,29-35 = FDS 1085). For example, one can apply the first *thema* to the "unmethodically valid" argument: *The first is greater than the second; and the second is greater than the third; therefore the first is greater than the third* to show that the following argument is valid: *The first is greater than the second; but not the first is greater than the third; therefore not the second is greater than the third*.

3.3 The first thema

The first *thema*, quoted above in the version cited by pseudo-Apuleius, is a contraposition rule for arguments, of the same sort as we find already in Aristotle's *Topics* (8.14.163a32-36) and *Sophistical Refutations* (33.182b37-183a2); there is also an argument contraposition rule in Aristotle's *Prior Analytics* (2.8.59b1-8), but formulated in terms of contraries as well as contradictories.

The first *thema* makes it possible to eliminate many undemonstrated arguments without loss from the basis of the system. Thus, its application to any first undemonstrated argument will produce a corresponding second undemonstrated argument, and vice versa. Further, as Milne (1995) points out, since the first *thema* is stated in terms of contradictories rather than negations, it could be used to analyse oblique second through fifth undemonstrated arguments (of moods 2b, 2c, 2d, 3b, 3c, 4b, 5b or 5c above) into direct arguments where the contradictory is a negation. Also, it can be used to change which conjunct appears as added premiss in a third undemonstrated argument whose leading premiss has two conjuncts; similarly with fourth or fifth undemonstrated arguments with two-disjunct disjunctions. Given these possibilities, we can reduce the moods of undemonstrated arguments to the following, without loss of demonstrative power:

1. $\neg p \rightarrow q, p \vdash q$

3. $\neg \& p_1 \& \ldots \& p_n, p_i \vdash \neg \& p_1 \& \ldots \& p_{i-1} \& p_{i+1} \& \ldots \& p_n$ $(n > 1, 1 \leq i \leq n)$

4. $\veebar p_1 \veebar \ldots \veebar p_n, p_i \vdash \neg p_j$ $(n > 1, 1 \leq i,j \leq n, i \neq j)$

5. $\veebar p_1 \veebar \ldots \veebar p_n, \neg \veebar p_1 \veebar \ldots \veebar p_{i-1} \veebar p_{i+1} \veebar \ldots \veebar p_n \vdash p_i$ $(n > 1, 1 \leq i \leq n)$

Further, the first *thema* makes the following variant formulation of a fifth undemonstrated argument equivalent to the above formulation:

5′. $\veebar p_1 \veebar \ldots \veebar p_n, \neg p_i \vdash \veebar p_1 \veebar \ldots \veebar p_{i-1} \veebar p_{i+1} \veebar \ldots \veebar p_n$ $(n > 1, 1 \leq i \leq n)$

Galen mentions a variant of the principle of argument contraposition which applies to syllogisms with more than two premisses:

> Those two-premissed syllogisms convert with one another in which one premiss is common and the remaining premiss in each contradicts the conclusion of the

> other; for multi-premissed syllogisms we will say not simply "one premiss" but will add "or more", making the whole statement as follows: "An argument is the converse of an argument if in them one or more premisses are common and the remaining premiss in each contradicts the conclusion of the other." (I.L. 6.5)[22]

Although Galen does not specifically refer to the Stoics' first *thema* in this passage, it is a reasonable conjecture that there was an extended version of the first *thema* to handle syllogisms with more than two premisses, since the third *thema* generates such syllogisms:

> *Extended first thema*: If from two or more a conclusion follows, the contradictory of any one of them follows from the remaining one or remaining ones together with the contradictory of the conclusion.

3.4 Reconstruction of the remaining themata: constraints and desiderata

The third *thema* turns out to be closely related to the second and fourth *themata*, as Alexander indicates by equating them jointly to a so-called "synthetic theorem" which he attributes to Aristotle (*In an. pr.* 284,12-17 = FDS 1165). It is therefore more helpful to think first about constraints on the reconstruction of the missing second and fourth *themata*[23] and then to discuss the content of the extant third *thema* along with that of the other two.

1. Each *thema* should be a generalized conditional statement which licenses a transition from one or more valid arguments to a new argument which will also be valid. This constraint,

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[22]Alexander (*In an. pr.* 29,7-13 = FDS 1163) has a similar description of what he calls "conversion of syllogisms", but formulated exclusively in terms of a two-premissed syllogism.

[23]Frede (1974, 172-185) first adopted this approach of setting out requirements for an acceptable reconstruction. Bobzien (1996, 143-144) extends the list. The following list incorporates and supplements their proposals.

based on Diocles' report of how the Stoics defined a syllogism (Diocles 7.78), immediately rules out three candidates which have been proposed for a missing *thema*. First, the version of the deduction theorem whose validity the Stoics recognized[24] cannot be a *thema*, for this principle would license a transition to a proposition rather than an argument; the Stoics' system showed the validity of arguments, not the truth of propositions. Second, it cannot be a repetition "rule", that any argument is valid whose conclusion is the same as a premiss; such a principle would entitle one to introduce an argument into a reduction rather than to generate one from previous argument(s), and besides no such discourse is a valid argument by Chrysippus' criteria.[25] Third, it cannot be the "dialectical theorem" mentioned by Sextus Empiricus at A.L. 2.231 and applied at A.L. 2.232-241; the analyses given by Sextus using the dialectical theorem are a kind of natural deduction whose steps are propositions, not arguments.

2. Each *thema* should preserve validity. That is, in the circumstances described in its antecedent, the discourse described in its consequent will be a valid argument by the criterion of Chrysippus. For it is unlikely that the inventor of this system would have failed to notice an unsoundness in the system, given how carefully it is constructed. This criterion of validity preservation rules out one candidate for a missing *thema*, Michael Frede's proposal (1974, 185-190) that the second *thema* allows one to drop a repeated premiss. Such a rule would immediately license the derivation of one-premissed "arguments", by application to the argument

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[24]If an argument is valid, then that conditional proposition is true whose antecedent is the conjunction of the argument's premisses and whose consequent is the argument's conclusion (A.L. 2.415-417 and 2.112, cited in section 2.1.3 above).

[25]If there is only one premiss, identical to the conclusion, the discourse is not an argument. If there is more than one premiss, then the argument is invalid by redundancy, unless it falls under the exception mentioned below.

It is day; and it is day; therefore not if it is day it is not day, which can be derived by the first *thema* from the first undemonstrated argument *If it is day, it is not day; it is day; therefore not it is day*. But Chrysippus insisted that there were no one-premissed arguments, so the discourse *It is day; therefore not if it is day it is not day* can hardly be a valid argument.

3. The *themata* should not overlap with one another. This desideratum is a matter of elegance rather than soundness, and might be violated at the margin. But it is unlikely to be violated in so egregious a manner as would result if one took as a *thema* the so-called "dialectical theorem" which Sextus Empiricus describes as "handed down for the analyses of syllogisms":

> Whenever we have premisses which yield a conclusion, we also have that conclusion potentially among these premisses, even if it is not expressly stated. (A.L. 2.231)

If the dialectical theorem were one of Chrysippus' *themata*, there would be no need for the third *thema*, in either of the two forms in which it is cited. Non-overlap, then, provides an additional reason for rejecting the suggestion that the dialectical theorem was one of Chrysippus' *themata*.

4. The second, third and fourth *themata* should have, or at least have plausibly seemed to Alexander to have, the same demonstrative power as the "synthetic theorem", which Alexander claims (*In an. pr.* 284,13-15 = FDS 1165) the Stoics took from Aristotle,[26] divided up, and

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[26]This claim of historical provenance should be treated sceptically. The synthetic theorem does not occur in Aristotle's surviving corpus, and there was an interval of more than 500 years between Aristotle and Alexander during which the Peripatetics could have articulated it. As holder of a chair in Peripatetic philosophy and a defender of Aristotle, Alexander is grinding an axe, trying to show that whatever is valuable in Stoic logic Aristotle came up with first, and whatever the Stoics originated is worthless. Nevertheless, the claim of Aristotle's historical provenance would make no sense unless the three *themata* were similar to the synthetic theorem. And we know by direct inspection that the third *thema* was.

transformed into what they called the second, third and fourth *thema*. Alexander quotes it twice:

> Whenever from some something follows, and that which follows along with one or more yields something, then those from which it follows, along with the one or more with which it yields that something, will themselves also yield the same thing. (Alexander *In an. pr.* 274,21-24 = FDS 1166; 278,8-11 = FDS 1167; cf. 283,15-17 = FDS 1165)

Symbolically, if $p_1, ..., p_n \vdash q$ (n > 1) and $q, r_1, ..., r_m \vdash s$ (m > 0), then $p_1, ..., p_n, r_1, ..., r_m \vdash s$. The synthetic theorem is thus a rule for chaining arguments together, or in contemporary language a cut rule (Gentzen 1969). As stated by Alexander, it requires that each input argument have at least two premisses, which is a requirement for an Aristotelian syllogism; we can expect a similar restriction in the Stoics' cut rules, since for Chrysippus all arguments (and *a fortiori* all syllogisms) must have at least two premisses. The synthetic theorem imposes no other restrictions; both the subordinate argument (i.e. the one whose conclusion is a premiss of the other argument) and the superordinate argument (i.e. the one whose premiss is a conclusion of the other argument)[27] can have any number of premisses greater than one, and repetition of

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[27]The term *subordinate* is in common use in contemporary informal logic texts for an argument in a chain of reasoning whose conclusion is a premiss of another argument. It seems natural to use the term *superordinate* for the other argument in the relation. The Stoics themselves, as Alexander tells us (*In an. pr.* 283,13-14 = FDS 1165), used the terms *epiballôn* (*throwing itself upon*, *falling upon*) and *epiballoumenos* (*fallen upon*) for two arguments chained together. But our sources differ on which argument is which. For Alexander (*In an. pr.* 283, 19-28 = FDS 1165) the subordinate argument whose conclusion is left out of the composite is first in order and is *epiballoumenos* (fallen upon), while the superordinate argument whose premiss is demonstrated is second in order and is *epiballôn* (falling upon). For Sextus (A.L. 2.216, 240), on the other hand, the subordinate argument *epiballei* (i.e. falls upon) the superordinate argument. Alexander supports his claim that the subordinate argument is first in order by analysing a three-premiss Aristotelian syllogism first into the subordinate argument and second into the superordinate argument of which it is composed. Sextus' analyses using the "dialectical theorem"

premisses is not excluded. We should expect the Stoics' second, third and fourth *themata* to cover jointly all the cases covered by the synthetic theorem.

5. The second and fourth *themata* should be useless from a Peripatetic perspective. Alexander finds the third *thema* useful: he cites it (*In an. pr.* 278,12-14) and says that it can be used instead of Aristotle's synthetic theorem to link a pair of two-premissed syllogisms. But he describes the Stoics, in their division of the synthetic theorem into three *themata*, as "being careless about usefulness but discussing fully and striving for everything which in any way can be said in such a theorem, even if it is useless" (*In an. pr.* 284, 15-17 = FDS 1166). The comment implies that, from Alexander's Aristotelian perspective, the Stoics' second and fourth *themata* are useless, although they are encompassed by the synthetic theorem; that is, they are useless for chaining together Aristotelian two-premiss syllogisms. Elsewhere (*In an. pr.* 164,27-31 = FDS 1169), Alexander explicitly stigmatizes the second *thema* in particular as useless.

6. The third *thema* should correspond to either Simplicius' version (possibly extended to allow one argument to have more than two premisses) or Alexander's.

7. Each of the *themata* should be applicable for the reduction of syllogisms, without fudging by mentally using other ways of proof. That is, it should be possible, given an argument which can be reduced to one or more undemonstrated arguments, to use the *themata* to find out how to reduce it, without mentally using other rules. There should be at least reasonable heuristics for the system. The *themata* may or may not provide a decision procedure; that is, we

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also begin with the subordinate argument, but he says that this argument *epiballei* the superordinate argument, thus exactly reversing Alexander's application of the terms. It is unclear why he does so. In any case, we cannot use the Stoics' terms to identify unambiguously the subordinate and superordinate argument in a pair of arguments chained together.

do not know in advance whether they provide a method for showing, for an arbitrary argument, whether it is reducible to one or more undemonstrated arguments.

8. The *themata* should not allow one to generate one-premissed (or zero-premissed) syllogisms.

9. All known Stoic syllogisms should be provable in the system.

10. They should not permit the reduction of arguments with redundant premisses, except in cases where application of the first *thema* to the redundant premiss preserves conflict between the contradictory of the conclusion and the conjunction of the premisses. These are cases where the argument has a mood which has instances without redundant premisses; for an example, see note 11 in section 2.1.3 above.

11. Syllogisms from two moodals should be provable using the first or second *thema* (or both), in accordance with Galen's comment that such syllogisms were analysed using those two *themata* (*On the Doctrines of Hippocrates and Plato* 2 3.18 = FDS 1160). Syllogisms from two moodals are syllogisms whose mood is *If the first, the second; if the first, not the second; therefore not the first* (Origen, *Contra Celsum* 7.15 = FDS 1181). A "moodal" (*tropikon*)[28] is a type of non-simple proposition which can be the leading premiss of a mood (*tropos*) of undemonstrated argument, i.e. either a conditional or a negated conjunction or a disjunction.

12. Non-differently concluding syllogisms should be provable using the first or second *thema* (or both), in accordance with Galen's comment that such syllogisms were analysed using

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[28]The Greek word was as much a neologism as the English is, to judge by the rather fanciful etymologies proposed by Galen (I.L. 7.1) and ps.-Ammonius (*In an. pr.* 68,6-7). Both etymologies presuppose that a *tropikon* is a proposition which can function as the leading premiss of an undemonstrated argument.

those two *themata* (*On the Doctrines of Hippocrates and Plato* 2 3.18 = FDS 1160), perhaps the second *thema* alone, to judge by Alexander's lumping them in with the second *thema* as useless (*In an. pr.* 164,27-31 = FDS 1169). A non-differently concluding argument is an argument whose conclusion is the same as one of the premisses (Alexander *In Top.* 10,10-11). Most arguments with a conclusion identical to a premiss are invalid through redundancy, as pointed out in note 25 above. But some are both valid and provable within the system; the examples we are given are the argument *Either it is day or it is light; but it is day; therefore it is day* (Alexander *In Top.* 10,11-12) and the argument *If it is day, it is light; but it is day; therefore it is day* (Alexander *In An. pr.* 18,15-16).[29] It is a mistake to interpret Alexander as claiming that the Stoics proved all non-differently concluding arguments in their system, just as it would be a mistake to interpret Alexander as claiming that the Stoics proved all arguments from two moodals in their system.[30]

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[29]In the absence of a rule of repetition, it is hard to imagine how the second of these arguments could be proved within the system. But the rather similar argument *If it is day, it is light; and if it is light it is day; but it is day; therefore it is day* will turn out to be provable in the system as reconstructed below. I conjecture that, here as in many of our other ancient sources of information about Stoic propositional logic, part of the argument in Alexander's text has dropped out through a scribal error. Such errors are understandable when there is so much repetition and the scribe does not understand what he is copying. And in fact the manuscripts collated in Wallies' edition differ at this point: B has the above reading, A has *If it is day; but it is day; therefore it is day*, and L and M have *If it is day, it is light; but it is day; therefore it is light*. (Wallies mistakenly chooses the readings of L and M, even though Alexander defines a non-differently concluding argument as one whose conclusion is the same as a premiss.) An alternative explanation is that the example at *In an. pr.* 18,15-16 was regarded by the Stoics as a non-differently concluding argument which could not be shown to be valid in their system; as it stands, it is arguably invalid. This explanation is implausible, however, because in the context Alexander is defending Aristotle's claim that such arguments are not syllogisms on the ground that what is not useful for proof is not a syllogism, and such a defence would be otiose if the Stoic targets of his polemic did not themselves claim that they were syllogisms.

[30]Note the distinction between non-differently concluding *arguments* and non-differently concluding *syllogisms*. A syllogism is a valid argument whose validity is proved in the system. Galen talks about how non-differently concluding syllogisms were analysed, Alexander about the

13. Each thema should begin with the clause *If from two a third follows* or *If from two or more a third follows*, by analogy to the three formulations which are extant.

14. The whole system should be described completely formally, with no fudging. That is, at no point should it be necessary to rely on implicit principles of transforming one argument into another or one proposition into another. This constraint reflects the thoroughgoing formalism of the extant parts of the Stoic system. The analyses given by Sextus Empirics at A.L. 2.232-238 indicate the care with which proofs were set out.

15. It should be possible to formulate the *themata* in ordinary Greek.

16. It would be confirmatory, but is not necessary, for so-called *duplicated arguments* and *indefinite content* to be demonstrable using the second *thema*. For Alexander (*In an. pr.* 164, 28-31) lumps these together with non-differently concluding arguments and the second *thema* as useless innovations of the Stoics, and Galen tells us that the analysis of non-differently concluding arguments used the second *thema*. Alexander gives as an example of a duplicated argument: *If it is day, it is day; it is day; therefore it is day* (*In an. pr*. 18,17-18; 20,10-12 = FDS 1171; *In Top*. 10,8-10 = FDS 1170); evidently these were arguments with a duplicated proposition (a proposition in which the same proposition occurs twice–cf. Diocles 7.69, A.L. 2.109) as a premiss. There is no explanation anywhere in our ancient sources of which arguments the Stoics meant by *indefinite content*; perhaps they were arguments with an iterated conditional as a major premiss, such as Sextus (A.L. 2.230-233) analyses: *If it is day, if it is day, it is light;*

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uselessness for (Aristotelian) demonstration of non-differently concluding arguments. Alexander attributes to the Stoics only the admission of non-differently concluding arguments, not the claim that all such arguments could be analysed as syllogisms. By definition, only those non-differently concluding arguments which were valid and whose validity could be proved within the system could become non-differently concluding syllogisms.

but it is day; therefore it is light, since the content of the inmost antecedent could be iterated indefinitely many times.

17. There should be a class of arguments which can be analysed using the third and fourth *themata*, since Galen (*On the Doctrines of Hippocrates and Plato* 2.3.18 = FDS 1160) refers to "such others [i.e. other arguments–DH] as they [those who are expertly trained–DH] analyse as syllogisms by the third or fourth *thema*".[31]

18. The method of reduction using the *themata* should be replaceable by a more concise method such as Galen credits Antipater with introducing (*On the Doctrines of Hippocrates and Plato* 2.3.19 = FDS 1160).

19. There should be four *themata* altogether, in accordance with Galen's reference to the first, second, third and fourth *themata* (*On the Doctrines of Hippocrates and Plato* 2.3.18-19 = FDS 1160). Although Galen does not explicitly state that Stoic propositional logic had only four *themata* for analysing syllogisms, the reference to the four *themata* occurs in a polemic against the waste of effort by Chrysippus and his school on such a useless task as analysing syllogisms, when they wrote nothing worth mentioning on how to distinguish scientific premisses from

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[31]Reading: *hôsper amelei kai ep' allois hosous dia tou tritou thematos ê tetartou sullogismous analuousi*, following the mss. De Lacy, followed by Hülser, gratuitously transposes *ep'* and *allois* so as to make the clause say that other people are trained to analyse syllogisms using the third or fourth *thema*. It is a bizarre idea, not supported by the manuscripts, that some people would be trained in analyses using the first and second *thema*, and others trained in analyses using the third or fourth *thema*. Mignucci, who discusses the passage at length (1993, 233-234), likewise finds the idea bizarre, but rejects the reading of the manuscripts on the ground that it says absurdly that there are syllogisms whose syllogisms are analysed by the third and fourth *themata*. But *analuontai sullogismoi* earlier in the sentence can be translated as *are analysed as syllogisms* and *sullogismous analuousi* here as *they analyse as syllogisms*, so that the present clause would read "just as of course also with respect to such others as they analyse as syllogisms by the third or fourth *thema*" and "such others" would refer to arguments other than those mentioned at the beginning of the sentence. Bobzien (1996, 143) assumes such a reading.

dialectical, rhetorical and sophistical ones. For the purposes of such a polemic, Galen would be expected to mention all the *themata* of the Stoic system. Further, we have references elsewhere to the first (ps.-Apuleius, *De interpretatione* 191,5-10 = FDS 11621), second (Alexander, *In an. pr.* 164,30-31; 284,15), third (Alexander, *In an. pr.* 278,6-7,11-14; 284,15; Simplicius, *In de caelo* 236,33-237,4) and fourth (Alexander, *In an. pr.* 284,15) *themata*, but to no other.

20. There should be a natural explanation of why Chrysippus enumerated the second, third and fourth *themata* in that order. In other words, there should be a principle of listing the components of a cut rule which implies that one would mention the second *thema* first, then the third *thema*, and finally the fourth *thema*.

21. Whichever extant version of the third *thema* is taken to be the one invented by Chrysippus, there should be a plausible explanation of how and why someone invented the other version as a replacement.

3.5 The two extant versions of the third *thema*

Armed with the foregoing requirements and desiderata, we can now consider the difference between Alexander's and Simplicius' versions of the third *thema*

> Whenever from two a third follows, and external assumptions syllogize one of them, from the remaining one and from the external ones which syllogize the other one the same will follow. (Alexander, *In an. pr.* 278,12-14)

Symbolically, if $q_1, q_2, \vdash r$ and $p_1, ..., p_n \vdash q_i$, then $q_j, p_1, ..., p_n \vdash r$ (i, j = 1, 2; i ≠ j; n > 1; $p_k \neq q_m$ for $1 \leq k \leq n$ and m = 1 or 2).

> If from two a third follows, and that which follows along with another from outside yields something, then also from the first two and the one assumed in

addition externally there will follow the same one. (Simplicius, *In de caelo* 237,2-4 = FDS 1168)

Symbolically, if $p_1, p_2 \vdash q_1$ and $q_1, q_2, \vdash r$, then $p_1, p_2, q_2 \vdash r$ ($p_1 \neq q_2, p_2 \neq q_2$).

Both versions assume a chaining together of two arguments, one of which has as conclusion a premiss of the other. Both assume that each argument chained together has at least two premisses. Both mention first a two-premiss argument, presumably because any ultimate input in any chain of arguments in the system will be an undemonstrated argument with two premisses. Both versions assume that the superordinate argument has no premiss in common with the subordinate argument; the word *external* (*exôthen*) means *not identical with a premiss of the other argument*. Both versions license the production of a new valid argument which has one premiss less than the two arguments chained together. Both versions produce a new valid argument with more than two premisses, so that neither can be used to analyse a two-premiss argument.

The versions differ in two respects. Alexander's allows the subordinate argument to have more than two premisses, whereas Simplicius restricts both arguments to two premisses. This means that Alexander's version can be used for the analysis of arguments with four or more premisses, as well as for those with three premisses, whereas Simplicius' can be used only for the analysis of arguments with three premisses. Simplicius may however be quoting a basic version, just as ps.-Apuleius appears to quote a basic version of the first *thema*. A natural extension of Simplicius' version would permit more than two premisses in the second-mentioned argument, i.e. the superordinate argument, and would read as follows:

If from two a third follows, and that which follows along with one or more from

> outside yields something, then from the first two and that or those assumed in addition externally there will follow the same one.

Symbolically, if $p_1, p_2 \vdash q_1$ and $q_1, ..., q_n \vdash r$, then $p_1, p_2, q_2, ..., q_n \vdash r$ ($p_1 \neq q_i, p_2 \neq q_i$, $1 < n$, $1 < i \leq n$).

The more consequential difference, as Bozien (1996, 145) notes, is that Alexander's version mentions first the superordinate argument, whereas Simplicius' version mentions first the subordinate argument. This difference implies a difference in the method of analysis. The third *thema* is a tool for analysis, not for synthesis. That is, it is to be applied by someone working backwards from a given argument which is to be reduced to undemonstrated arguments, not by somebody working forwards from undemonstrated arguments to see what follows when one chains them together. The order in which the third *thema* mentions the components is, it may be reasonably assumed, the order in which a backwards-working analyst will generate them. For the two-premissed argument mentioned first will be either an undemonstrated argument or one which can be readily reduced to an undemonstrated argument, and that is what one would be looking for first. Using Alexander's version, one looks first for a single premiss of the argument to be analysed from which the analysandum's conclusion follows if another unstated premiss is added. For example, if one premiss of the analysandum is a negated conjunction (e.g. *not both it is light and it is night*) and the conclusion of the analysandum is the contradictory of one of its two conjuncts (e.g. *not it is night*), then adding the other conjunct as an unstated premiss (e.g. *it is light*) will produce a valid superordinate argument; the subordinate argument will then have this unstated premiss as conclusion and the remaining premiss or premisses of the analysandum as premisses. Using Simplicius' version, one looks first for two premisses of the analysandum

from which something follows which is not stated as a premiss.[32] For example, if one premiss is a conditional (e.g. *if it is day, it is light*) and another is the antecedent of that conditional (e.g. *it is day*), and the consequent of the conditional (e.g. *it is light*) is not a premiss of the analysandum, then the argument with those two premisses and the unstated proposition as a conclusion will be a valid subordinate argument; the superordinate argument will then have the conclusion of the subordinate argument and the remaining premiss of the analysandum as premisses. The difference in the two methods of analysis can be made clear by exhibiting them at work side by side on a single three-premissed argument:

Not both it is light and it is night; and if it is day it is light; but it is day; therefore not it is night

"Alexandrian" analysis	**"Simplician" analysis**
Superordinate argument: *Not both it is light and it is night; but it is light; therefore not it is night* (a third undemonstrated argument)	Subordinate argument: *If it is day, it is light; but it is day; therefore it is light* (a first undemonstrated argument)
Resulting subordinate argument: *If it is day, it is light; but it is day; therefore it is light* (a first undemonstrated argument)	Resulting superordinate argument: *Not both it is light and it is night; but it is light; therefore not it is night* (a third undemonstrated argument)

[32]Bobzien (1996, 159-160) implies that the component argument so identified will either be an undemonstrated argument or be reducible to an undemonstrated by one application of the first *thema*. But an argument with a conditional as its leading premiss and the double negation of the conditional's antecedent as its added premiss will need two applications of the first *thema* before it is reduced to an undemonstrated argument. More complicated cases can be constructed.

Both analyses reduce the analysandum to a first undemonstrated and a third undemonstrated, but the order of discovery is different. The Simplician analysis seems, to me at least, easier and more natural; it is easier to think of a conclusion which follows from two of a number of given premisses than to think of a premiss which can be added to one of a number of given premisses to yield a given conclusion. Further, the Simplician analysis corresponds to the method which Sextus Empiricus uses to analyse two arguments into undemonstrated arguments (A.L. 2.232-238) using the "dialectical theorem":

> Whenever we have premisses which yield a conclusion, we also have that conclusion potentially among these premisses, even if it is not expressly stated. (A.L. 2.231; for analyses using the theorem, cf. 2.232-238)

Although the dialectical theorem is a vague piece of advice for breaking up composite arguments, the method it licenses is very similar to that of the Simplician third *thema*; find two premisses of the analysandum which yield a conclusion, then put that conclusion as a premiss together with some other premiss in the analysandum.

Most reconstructions of Stoic propositional logic have assumed that the Alexandrian version was the original one. Bobzien (1996), however, has argued for the Simplician version, on the ground that none of the arguments given for the Alexandrian version (by Frede 1974 and Ierodiakonou 1990) is compelling and that the Simplician version leads to a family of reconstructions which are much more easily workable than those which have been proposed on the basis of the Alexandrian version. She explains Alexander's version as perhaps a modification by Alexander or some earlier Peripatetic to adjust it to the Aristotelian case which Alexander is describing.

One can multiply considerations for and against each version, perhaps without end. The more salient considerations appear to be the following. Both Alexander and Simplicius introduce their versions as quoted statements of the Stoics' third *thema*, and both are careful and logically astute commentators in the midst of writing a commentary on one of Aristotle's works; these facts point to both versions being genuinely Stoic. Alexander in particular contrasts his version of the Stoics' third *thema* which he quotes with the Peripatetic synthetic theorem, which makes it unlikely that he is quoting some Peripatetic modification of the third *thema*,[33] whereas Simplicius gives a definition of the third *thema* "according to the ancients", the Peripatetics, which may (but need not) mean that he is quoting a Peripatetic variant. Both are writing many centuries after Chrysippus invented the system, Alexander more than 400 years later, Simplicius about 750 years later; thus, either or both could be quoting a version of the third *thema* which originated in the Stoic (or Peripatetic) tradition after Chrysippus. Alexander quotes the *thema* in the context of an extended justification of Aristotle's claim in *Prior Analytics* I.25 that every syllogism has two premisses, a justification in which the chaining together of two-premiss syllogisms is a central focus; Simplicius quotes the *thema* rather incidentally as a basis for analysing into two two-premiss syllogisms an argument of Aristotle's in his *On the Heavens*. The differences between the two versions cannot easily be explained as the result of a scribal error in copying a manuscript.

The following seem to be less salient considerations. Alexander's formulation allows for

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[33]Bobzien's suggestion (1996, 150-151) that Alexander might have altered the Stoics' third *thema* to fit the example he was discussing is strained, since Simplicius' version also fits; Alexander would only need to alter the order in which he mentions the two syllogisms he is chaining together.

subordinate arguments with more than two premisses, of just the kind which show up when one does complex proofs in the system, whereas Simplicius' formulation requires both the subordinate and the superordinate argument to have two premisses; as Bobzien (1995, 147-149) points out, Simplicius' version could be a basic version of an extended third *thema*, as the version of ps.-Apuleius is a basic version of an extended first *thema*, or alternatively the chaining of superordinate arguments with more than two premisses could have been handled by the second or fourth *thema*. Analysis is more difficult with Alexander's version than with Simplicius', but that fact can be used in favour of Alexander's version as well as against it; if the more difficult method of analysis was the original one, we have a natural explanation for the emergence of the other version: to make analysis easier.

It seems unlikely that any combination of considerations such as those mentioned in the last two paragraphs will make a compelling case for identifying one of the two versions with the version invented by Chrysippus. Bobzien chooses the Simplician one mainly because it makes it possible to generate a family of easily applicable reconstructions of the second and fourth *themata*, whereas the most defensible reconstructions which use the Alexandrian version (Frede 1974, Ierodiakonou 1990) are extremely difficult to apply. The difficulty of application, however, comes from Frede's choice, imitated by Ierodiakonou, to postulate as the fourth *thema* a rule whose application requires one to come up with two unstated premisses from which the conclusion follows; this procedure opens up an enormous number of possibilities, most of which will be blind alleys in generating a reduction: even when one takes into account fairly obvious heuristics (e.g. include in the premisses only simple propositions which already occur in the stated premisses or conclusion), producing an analysis is extremely difficult, even impossible. An

alternative approach to an Alexandrian reconstruction, however, is to imitate Bobzien's Simplician reconstruction, which construes the second *thema* as covering cuts in which the superordinate argument has only an internal added premiss (i.e. a premiss of the subordinate argument),[34] and the fourth *thema* as covering cuts where the superordinate argument has both an internal premiss and an external premiss. The Alexandrian parallel would construe the second *thema* as covering cuts in which the subordinate argument has an internal added premiss (i.e. a premiss of the superordinate argument), and the fourth *thema* as covering cuts where the subordinate argument has both an internal premiss and an external premiss. Unlike Frede's reconstruction, the method of analysis dictated by such a combination of chain rules does not permit the introduction of propositions which do not already occur as components of the analysandum, since each premiss in a two-premiss syllogism is composed of propositions which occur in the other premiss and the conclusion.[35]

In fact, as pointed out above, the method suggested by Simplicius' version of the third *thema* is very similar to the "dialectical theorem handed down for the analysis of syllogisms" which Sextus Empiricus mentions:

> Whenever we have premisses which yield a conclusion, we also have that conclusion potentially among these premisses, even if it is not expressly stated.

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[34]This proposal for the second *thema* was first advanced by Hitchcock (1982) and apparently independently by Ierodiakonou (1990).

[35]The proof is by induction. In each undemonstrated argument, each premiss is composed of one or more propositions which each occur once in the other premiss or the conclusion; similarly, the conclusion is composed of propositions which each occur once in the two premisses. Application of the first *thema* to an argument with this property produces an argument with this property. So, apparently, does application of the second *thema* to two two-premissed arguments with this property. No other *thema* produces an argument with two premisses.

(A.L. 2.231)

It is a short step from saying that, if from two premisses a conclusion follows, and that conclusion along with one or more others yields something as conclusion, then the first two along with the other(s) will yield the same thing as conclusion, to saying that if two premisses yield a conclusion, then you have that conclusion potentially among the premisses. If, as seems plausible, the dialectical theorem is the simplification introduced by Antipater which Galen mentions (*On the Doctrines of Hippocrates and Plato* 2.3.19 = FDS 1160), it seems hardly credible that it would have taken Stoic logicians more than 50 years (Chrysippus' death 204 BCE, Antipater's accession to headship of the Stoa 152 BCE) to have gotten from a Simplicius-type set of chain rules to Antipater's dialectical theorem. As Bobzien points out, however, (1996, 165-166) the dialectical theorem is less precise than either the two versions of the third *thema* or the synthetic theorem, in that it leaves open how many premisses there are to be in a second component argument and how many applications are required for some arguments. It is therefore quite possible that Antipater's simplification was the transformation of Chrysippus' rules, parallelling the one transmitted by Alexander, into rules which parallel the one transmitted by Simplicius. The dialectical theorem would then be an informal summary of the second, third and fourth *themata*, as transformed by Antipater.

3.6 Alexandrian and Simplician reconstructions

Here is a statement of the four *themata* as reconstructed using as a starting point Apuleius' statement of the first *thema* and Alexander's statement of the third *thema*. For comparative purposes, I put the proposed reconstruction side by side with Bobzien's

reconstruction based on Simplicius' statement of the third *thema* (1996, 151)[36]:

Proposed "Alexandrian" reconstruction	**Bobzien's "Simplician" reconstruction**
First thema (basic version): If from two a third follows, either one of them together with the contradictory of the conclusion yields the contradictory of the remaining one.	*First thema* (basic version): When from two propositions a third follows, then from either one of them together with the contradictory of the conclusion the contradictory of the remaining one follows.
First thema (extended version): If from two or more another follows, all but one of them together with the contradictory of the conclusion yield the contradictory of the remaining one.	*First thema* (extended version): When from two or more propositions another follows, then from all but one of them together with the contradictory of the conclusion the contradictory of the remaining one follows.
Second thema: Whenever from two a third follows, and one of them and an external assumption syllogize the remaining one, from that one and the external assumption the same will follow.	*Second thema*: When from two propositions a third follows and from the third and one (or both) of the two another follows, then this other follows from the first two.

[36]I choose this version rather than the one in Bobzien (1999, 138 and 145), because the 1999 reconstruction omits the extended version of the first *thema* and interpolates the word *assertible* instead of *proposition*. Otherwise Bobzien's two reconstructions are identical.

Third thema: Whenever from two a third follows, and external assumptions syllogize one of them, from the remaining one and from the external ones which syllogize the other the same will follow.	*Third thema*: When from two propositions a third follows and from the third and one (or more) external proposition(s) another follows, then this other follows from the first two and the external(s).
Fourth thema: Whenever from two a third follows, and one of them and external assumptions syllogize the remaining one, from that one and the external assumptions the same will follow.	*Fourth thema*: When from two propositions a third follows and from the third and one (or both) of the two and one (or more) external propositions another follows, then this other follows from the first two and the external(s).

The two reconstructions appear to be equivalent.

(1) The first *thema* is identical in each reconstruction, being an extension of the basic *thema* quoted by ps.-Apuleius.

(2) As for the second *thema*, the Alexandrian reconstruction cannot directly accommodate an application of the Simplician second *thema* in which both premisses of the subordinate argument occur in the superordinate argument. It seems however that it can accommodate such an application indirectly. For the superordinate argument in such a case will have three premisses, and so will require reduction by the third (or fourth) *thema*. And it appears that the whole reduction can in that case be accomplished with the use of only the basic second *thema*.[37]

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[37]For example, one can reduce the argument $\neg\&\ F\ \&\ G, F \vdash \neg\veebar\&\ F\ \&\ G \veebar F \veebar G$ to a third, a fourth, and a fifth undemonstrated argument by applying successively the Simplician extended second *thema*, the first *thema*, and the Simplician third *thema*. But one can reduce the same argument to the same three undemonstrated arguments by applying successively the first

To transform an application of the basic Simplician second *thema* into an application of the Alexandrian second *thema*, just switch the position of the subordinate argument and superordinate argument; similarly for the reverse transformation. Consider for example the Alexandrian and the Simplician reduction of the argument from two moodals: *If you know that you are dead, you are dead; if you know that you are dead, you are not dead; therefore, you do not know that you are dead* (Origen, *Contra Celsum* 7.15 = FDS 1181). Let F = *you know that you are dead* and G = *you are dead*.[38] Then in symbols the reductions go as follows:

Alexandrian reduction	**Simplician reduction**
$\rightarrow F \rightarrow G, \rightarrow F \rightarrow \neg G \vdash \neg F$	$\rightarrow F \rightarrow G, \rightarrow F \rightarrow \neg G \vdash \neg F$
$\uparrow$ (T1)	$\uparrow$ (T1)
$\rightarrow F \rightarrow G, F \vdash \neg \rightarrow F \rightarrow \neg G$	$\rightarrow F \rightarrow G, F \vdash \neg \rightarrow F \rightarrow \neg G$
$\uparrow$(T2$_A$)	$\uparrow$(T2$_S$)

$F, G \vdash \neg \rightarrow F \rightarrow \neg G$ $\quad$ $\underline{\rightarrow F \rightarrow G, F \vdash G}$ (A1) $\quad$ $\underline{\rightarrow F \rightarrow G, F \vdash G}$ (A1) $\quad$ $F, G \vdash \neg \rightarrow F \rightarrow \neg G$

$\uparrow$ (T1) $\qquad\qquad\qquad\qquad\qquad\qquad$ $\uparrow$ (T1)

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thema, the Simplician basic second *thema*, the first *thema*, the Simplican basic second *thema* again, and the first *thema*. If the strategy used in this example is perfectly generalizable, then any use of the Simplician extended second *thema* is eliminable. A possible counter-example to the eliminability of the Simplician extended second *thema* is the argument $\veebar F \veebar G \veebar H, F \vdash \& \neg G \& \neg H$, which can be reduced to a third and two fourth undemonstrated arguments by successive applications of the Simplician extended second *thema*, the Simplician third *thema* and the first *thema*; I have been unable to discover a way of reducing it without using the Simplician extended second *thema*.

[38]I use capital letters as abbreviations of propositions. In section 2.2.5 above, in symbolizing the moods of undemonstrated arguments, I used small letters as abbreviations of meta-linguistic propositional variables. The Stoics rather confusingly used ordinals (*the first*, *the second*, etc.) for both purposes, i.e. both in stating moods of undemonstrated arguments and as abbreviations in analyses.

$\underline{\rightarrow F \rightarrow \neg G,\ F \vdash \neg G}$ (A1) $\underline{\rightarrow F \rightarrow \neg G,\ F \vdash \neg G}$ (A1)

Reductions are written from top to bottom in the order in which their components are generated from the argument to be reduced, which is at the top. The reduction is complete when each branch of the descending tree terminates in an underlined undemonstrated argument, whose number is indicated by "(A<number>)". Applications of a *thema* are indicated by "(T<number of *thema*>)", with an arrow pointing upwards to show the direction of proof. The reduction could be rewritten in the form of deduction in a Gentzen-type sequent calculus, with the undemonstrated arguments initiating the deduction. The two parallel reductions above both require application of the first *thema* before the second *thema* can be applied. Application of the Alexandrian second *thema* requires that one first generate the superordinate argument on the left by finding a suitable premiss to go with one of the premisses of the argument being reduced; generation of the subordinate argument is then mechanical. Application of the Simplician second *thema* requires that one first generate the subordinate argument on the left by finding a conclusion which follows from the two given premisses; generation of the superordinate argument requires that one make a suitable choice of which premiss of the analysandum is to be a premiss. The two arguments generated are exactly the same; one of them requires further reduction by the first *thema* before the reduction is complete. In both cases, the argument from two moodals is analysed using the first and second *themata*, as Galen stated (*On the Doctrines of Hippocrates and Plato* 2 3.18 = FDS 1160).

(3) Any application of either version of the third *thema* in which both subordinate and superordinate argument have two premisses can be transformed into an application of the other version of the third *thema* by switching the position of the two arguments, as in the case of the

second *thema*. More generally, any application at all of the Alexandrian third *thema* can be transformed into successive applications of the first *thema*, the Simplician third *thema*, and the first *thema* (twice). Similarly, Any application of the Simplician third *thema* can be transformed into successive applications of the first *thema*, the Alexandrian third *thema*, and the first *thema* (twice). Consider for example the argument: $\rightarrow\&\ F\ \&\ G\rightarrow\veebar H \veebar J, F, G, H \vdash \neg J$.

Alexandrian reduction

$\rightarrow\&\ F\ \&\ G\rightarrow\veebar H \veebar J, F, G, H \vdash \neg J$

$\uparrow(T3_A)$

$\underline{\veebar H \veebar J, H \vdash \neg J}$ (A4)

$\rightarrow\&\ F\ \&\ G\rightarrow\veebar H \veebar J,$
$F, G \vdash \veebar H \veebar J$

$\uparrow(T3_A)$

$\underline{\rightarrow\&\ F\ \&\ G\rightarrow\veebar H \veebar J,}$
$\underline{\&\ F\ \&\ G \vdash \veebar H \veebar J}$
(A1)

$F, G \vdash \&\ F\ \&\ G$
$\uparrow(T1)$
$\underline{\neg\&\ F\ \&\ G, F \vdash \neg G}$
(A3)

Simplician transformation

$\rightarrow\&\ F\ \&\ G\rightarrow\veebar H \veebar J, F, G, H \vdash \neg J$

$\uparrow(T1)$

$\rightarrow\&\ F\ \&\ G\rightarrow\veebar H \veebar J, F, H, J \vdash \neg G$

$\uparrow(T3_S)$

$H, J \vdash \neg\veebar H \veebar J$
$\uparrow(T1)$

$\underline{\veebar H \veebar J, H \vdash \neg J}$ (A4)

$\rightarrow\&\ F\ \&\ G\rightarrow\veebar H \veebar J,$
$F, \neg\veebar H \veebar J \vdash \neg G$

$\uparrow(T1)$
$\rightarrow\&\ F\ \&\ G\rightarrow\veebar H \veebar J,$
$F, G \vdash \veebar H \veebar J$
etc.

The Simplician transformation of the one-step application of the Alexandrian third *thema* begins from the same argument and ends with the same two arguments, but requires four steps. Similarly, the Alexandrian transformation of the one-step application of the Simplician third *thema* would begin from the same argument and end with the same two arguments, but require four steps.

(4) Transformation of an application of the fourth *thema* proceeds in the same way as for the third. Any application of one version of the fourth *thema* can be transformed into a sequence with the same input and output but with a sequence of applications of the first *thema*, the other version of the fourth *thema*, and the first *thema* (twice). Consider for example the first step in the

Simplician reduction of the argument: →F→→& F & G→⊻ H ⊻ J, F, G, H ⊢ ¬J.

Simplician reduction (in part)

→F→→& F & G→⊻ H ⊻ J, F, G, H ⊢ ¬J

↑($T4_S$)

<u>→F→→& F & G→⊻ H ⊻ J, F ⊢→& F & G→⊻ H ⊻ J</u> (A1)	→& F & G→⊻ H ⊻ J, F, G, H ⊢ ¬J etc.

Alexandrian transformation

→F→→& F & G→⊻ H ⊻ J, F, G, H ⊢ ¬J

↑(T1)

F, G, H, J ⊢¬→F→→& F & G→⊻ H ⊻ J

↑($T4_A$)

¬→& F & G→⊻ H ⊻ J, F ⊢¬→F→→& F & G→⊻ H ⊻ J	F, G, H, J ⊢¬→& F & G→⊻ H ⊻ J
↑(T1)	↑(T1)
<u>→F→→& F & G→⊻ H ⊻ J, F ⊢→& F & G→⊻ H ⊻ J</u> (A1)	→& F & G→⊻ H ⊻ J, F, G, H ⊢ ¬J etc.

The most economical application of the two methods of proof produces in some cases a shorter Simplician proof and in other cases a shorter Alexandrian proof. Other things being equal, the Simplician proof is shorter if the original analysandum has two premisses which yield an unexpressed conclusion but does not have a premiss which with another premiss yields the expressed conclusion. And, also other things being equal, the Alexandrian proof is shorter in the converse situation.

3.7 Syllogistic moods

I list below some moods of arguments whose validity can be proved within the system. The parenthetical remark following each mood indicates in code how the reduction goes in the Alexandrian reconstruction above. The code is explained in the note to the first mood.

S1. (through two moodals) $\rightarrow p \rightarrow q, \rightarrow p \rightarrow \neg q \vdash \neg p$ (T1, $T2_A$ [T1, A1; A1])[39]

S2. (non-differently concluding #1) $\veebar p \veebar q, p \vdash p$ ($T2_A$ [A5; A4])

S3. (non-differently concluding #2) $\rightarrow p \rightarrow q, \rightarrow q \rightarrow p, p \vdash p$ ($T3_A$ [A1; A1)

S4. (indefinite content) $\rightarrow p \rightarrow \rightarrow p \rightarrow q, p \vdash q$ ($T2_A$ [A1; A1])

S5. (conjunction introduction) $p, q \vdash \& p \& q$ (T1, A3)

S6. (through three moodals) $\rightarrow p \rightarrow r, \rightarrow q \rightarrow r, \veebar p \veebar q \vdash r$ (T1, $T4_A$ [T1, A1; $T3_A$ (A5; A2)])[40]

S7. $p, \neg q \vdash \neg \rightarrow p \rightarrow q$ (T1, A1)

S8. $p, q \vdash \neg \veebar p \veebar q$ (T1, A4)

S9. $\neg p, \neg q \vdash \neg \veebar p \veebar q$ (T1, A5)

S10. $\veebar p \veebar q \veebar r, \neg p, \neg q \vdash r$[41] ($T3_A$ [A5; T1, A5])

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[39]The sequence in parentheses encodes the reduction displayed in the previous section, in the discussion of the equivalence of the two versions of the second *thema*. Start with the argument to be reduced. Apply the first *thema* (T1); note that the encoding does not say which premiss is to be contradicted and made into the conclusion of the new argument. Apply the Alexandrian version of the second *thema* ($T2_A$); the sequence in square brackets indicates what is to be done to the two outputs from this application, first the superordinate argument (before the semi-colon), then the subordinate argument (after the semi-colon). Apply the first *thema* (T1) to the superordinate argument, then note that the result is a first *anapodeiktos* (A1). Note that the subordinate argument is a first *anapodeiktos* (A1).

[40]An argument of this mood was used against indeterminism: *If you will mow, not both perhaps you will mow and perhaps you will not mow; if not you mow, not both perhaps you will mow and perhaps not you will mow; but either you will mow or not you will mow; therefore not perhaps you will mow and perhaps you will not mow*. (Ammonius, *In de int.* 131,28)

[41]Chrysippus is reported to have said that a dog makes use of this syllogism when, arriving at a spot where three ways meet, after smelling two ways and finding no scent of its quarry, he rushes off down the third way without stopping to smell. The dog reasons: *The beast went either this way or this way or this way; but neither this way nor this way; therefore this way*. (P.H. 1.69)

S11. $\veebar p \veebar q \veebar r, p \vdash \& \neg q \& \neg r$ ($T2_{S\text{-}ext}$ [A4; $T3_S$ (A4; T1, A3)])[42]

S12. $\veebar p \veebar q \veebar r, p \vdash \neg \veebar q \veebar r$ ($T2_{S\text{-}ext}$ [A4; $T3_S$ (A4; T1, A5)])[43]

S13. $\rightarrow \& p \& q \rightarrow r, \neg r, p \vdash \neg q$ (T1, $T3_A$ [A1; T1, A3])[44]

S14. $\rightarrow p \rightarrow q, \rightarrow \& \rightarrow r \rightarrow s \& t \rightarrow p, \neg q, t \vdash \neg \rightarrow r \rightarrow s$ (T1, $T3_A$ [A1; $T3_A$ (A1; T1, A3)])[45]

S15. $\neg \& p \& q, \rightarrow \neg r \rightarrow q, \veebar s \veebar p, \rightarrow r \rightarrow s \vdash \neg p$ (T1, $T4_A$ [T1, A4; $T3_A$ (A1; $T3_A$ [A2;A3])])[46]

S16. $\rightarrow \& p \& \neg q \rightarrow \veebar \neg r \veebar s \veebar t \veebar u \veebar \neg v, \neg\neg r, \neg s, \neg t, \neg u, \neg\neg v, p \vdash q$ ($T3_A$ [A3; $T3_A$ (A2; T1, $T3_A$ [A5; T1, $T3_A$ (A5; T1, $T3_A$ [A5; T1, A5])])])[47]

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[42]It seems impossible to reduce arguments of this mood in the Alexandrian or basic Simplician system.

[43]Arguments of this mood also seem irreducible in the Alexandrian or the basic Simplician system.

[44]Sextus Empiricus states an argument of this mood by the Pyrrhonian sceptic Aenesidemus (A.L. 2.215-216) and analyses it into a first and third undemonstrated argument using the so-called "dialectical theorem" (A.L. 2.234-238).

[45]Bobzien (1996, 161 n. 54) offers an argument of this mood as a challenge to any reconstruction based on Alexander's version of the third *thema*. She comments that anyone using such a reconstruction would almost certainly come up with the analysis only after working out the complete hidden structure by another method, which as she says is a kind of cheating. But her comment applies only to Frede's (1974) Alexandrian reconstruction, not to the one proposed here.

[46]Bobzien (1996, 161 n. 54) offers an argument of this mood as a challenge to any reconstruction based on Alexander's version of the third *thema*. I worked the proof out, without cheating, in the Alexandrian reconstruction. The heuristic key is to see that you need an even number of occurrences of each of *p*, *q*, *r* and *s*. But there are an odd number of occurrences of *p*, so you need to add another one by using the second or fourth *thema*. To do so without at the same time creating an odd number of occurrences of *q*, *r* or *s*, you have to contrapose the argument, putting the contradictory of $\veebar s \veebar p$ in the conclusion of the new argument, because that contradictory follows from *p* along with a simple unstated premiss *s*.

[47]This is the mood of an argument which Cicero (*De Divinatione* I 38,82-39,84 = Hülser 466) reports the Stoics as having used to prove that there is divination.

S17. $\neg\&\, p\, \&\, \neg\veebar\, q \veebar r, \rightarrow q \rightarrow s, p, \neg s \vdash r$ (T1, $T3_A$ [A1; $T3_A$ (A5, A4)])

S18. $\rightarrow\&\, p\, \&\, q \rightarrow r, \veebar\, p \veebar s, \neg r, \neg s \vdash \neg q$ (T1, $T3_A$ [A5 ; $T3_A$ (A3, A2)])

3.8 Rejected invalid moods

The following moods are not syllogistic, and furthermore are invalid. The parenthetical remark following each mood indicates why it is invalid.

RI1. (hypothetical syllogism) $\rightarrow p \rightarrow q, \rightarrow q \rightarrow r \nvdash \rightarrow p \rightarrow r$ (counter-example: *you know that you are dead* for *p*, *you are dead* for *q*, *not you know that you are dead* for *r*)

RI2. $p, \neg q \nvdash \veebar\, p \veebar q$ (counter-example: *grass is green* for *p* and *snow is pink* for *q*)

RI3. $\neg\veebar\, p \veebar q, p \nvdash q$[48] (counter-example: *grass is green* for *p* and *snow is pink* for *q*)

RI4. (dilemma) $\rightarrow p \rightarrow r, \rightarrow q \rightarrow s, \veebar\, p \veebar q \nvdash \veebar\, r \veebar s$ (counter-example: *it is day* for *p*, *it is light* for *r*, *it is night* for *q*, *not the sun is shining* for *s*)

RI5. $\neg p, \neg q \nvdash \neg\&\, p\, \&\, q$ (invalid by redundancy: the conclusion follows from either premiss)[49]

RI6. Any mood with one premiss (Instances of such moods are not arguments, since an argument must have at least two premisses.)

RI7. (ex false quodlibet) $p, \neg p \nvdash q$ (invalid because of disconnection)

3.9 Rejected valid moods

The following moods are not syllogistic, even though they are valid in virtue of the defined meanings of the logical operators of the system. That they are not syllogistic can be shown by construing the logical operators as truth-functions with more than one distinguished value, in

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[48]Cf. Milne (1995, 52-53).

[49]Milne (1995, 47) uses the indemonstrability of arguments of this form as evidence of the incompleteness of what he calls Basic Symbolic Logic. But he does not use Chrysippus' concept of validity as his gold standard against which to measure what is demonstrable.

such a way that the undemonstrated arguments preserve distinguished values and the application of a *thema* to a distinguished-value-preserving argument or arguments produces a distinguished-value-preserving argument, but some arguments of the mood do not preserve a distinguished value.

RV1. $\neg p \rightarrow q, \neg p \rightarrow \neg q \nvdash \veebar p \veebar \neg q$ (Milne 1995)

RV2. $\veebar p \veebar q, p \nvdash \neg p \rightarrow \neg q$[50]

3.10 Adequacy of the reconstruction

The Alexandrian reconstruction meets most of the 21 constraints and desiderata listed in section 3.4 above. (1) Each *thema* is a generalized conditional statement which licenses a transition from one or more valid arguments to a new argument which will also be valid. (2) Each *thema* preserves validity, as can be shown by applying Chrysippus' definition of validity as incompatibility of the contradictory of the conclusion with the conjunction of the premisses. (3) The *themata* do not overlap with one another. (4) The second, third and fourth *themata* have almost as much deductive power as the Peripatetic synthetic theorem, and could well have seemed to Alexander to have the same deductive power. The exception is in the inability of the Alexandrian reconstruction to demonstrate S11 and S12. It is possible that the transition from the Alexandrian version to the Simplician version was motivated partly by noticing this deficiency. (5) The second and fourth *themata* are indeed useless from a Peripatetic perspective, because they require the two arguments chained together with their use to have a common premiss as well as a conclusion in one which is a premiss of the other. Such a double overlap never occurs when

[50]Erik Krabbe pointed out in November 2012 that this mood is invalid through redundancy. I have left the text as it was written in 2002.

one is chaining Aristotelian syllogisms together. The Stoics had to provide for it because it occurs and because repetition counted among their premisses. (6) In the Alexandrian reconstruction, the third *thema* is exactly as Alexander quotes it, with no need for extension. (7) As evidenced by the list of syllogistic moods, and the encoding of how arguments in those moods are reduced to undemonstrated arguments, the reconstructed *themata* are applicable in practice. In particular, it is possible to prove in the Alexandrian reconstruction proposed here the validity of arguments which Bobzien produced as a challenge, without mentally using heuristics of a Simplician sort. And there are useful heuristics, for example, that one will need the second *thema* or fourth *thema* if any simple proposition occurs an odd number of times.[51] (8) No *thema* licenses a transition to an "argument" with one premiss, or with no premisses. (9) The syllogistic moods in section 3.7 include the moods of all known Stoic syllogisms. (10) The *themata* do not permit the reduction of arguments with redundant premisses, unless the argument is an instance of a mood which has other instances without redundant premisses. (11) Syllogisms from two moodals, are demonstrated using the first and second *thema*, as Galen claimed, as shown in section 3.6. (12) Of the attested non-differently concluding syllogisms, one (S2) is demonstrated using the second *thema*, as Galen claimed, but the other (S3) using the third *thema*, contrary to Galen's claim. Two explanations of this discrepancy immediately come to mind: perhaps Galen over-generalized, or perhaps the conjectured reconstruction of the second non-differently concluding argument is incorrect. (13) All four *themata* begin with the clause *If from two (or more) a third follows*. (14) The whole system is described completely formally, without appeal to

[51]Note that, if the simple proposition occurs only once, the argument is not demonstrable in the system. For each simple proposition in an undemonstrated argument occurs at least twice in it, and the *themata* preserve this property.

implicit principles of transformation. (15) The wording of the reconstructed *themata* corresponds closely to that of the extant *themata*, and would go into Greek easily. (16) Indefinite content syllogisms (S4) are demonstrated using the second *thema*, as suggested by Alexander's lumping them together. Duplicated arguments are first undemonstrated arguments, and so not in need of demonstration; this fact is consistent with Alexander's lumping them in the same category of useless Stoic innovations as the second *thema*. (17) Arguments of moods S6, S11-12 and S14-18 are analysed using the third or fourth *thema*, sometimes in combination with the first. The distinction Galen makes between arguments analysed using the first and second *themata* and arguments analysed using the third and fourth *themata* is the distinction between two-premissed arguments analysable in the system and multi-premissed arguments analysable in the system. (18) The Alexandrian system is more difficult to use than the Simplician system (as well as being in one respect less powerful). It is a plausible conjecture that some later Stoic changed the method of search for component arguments from one which focused first on how the conclusion could be proved to one which focused first on what could be inferred from some pair of premisses. This later Stoic could be Antipater, head of the Stoic school from 152 BCE to 129 BCE, whom Galen credits with introducing a simpler method for the analysis of syllogisms. The dialectical theorem used by Sextus Empiricus to analyse two syllogisms is an informal statement of the method licensed by the Simplician version of the *themata*. (19) There are four *themata*, as implied by Galen. (20) There is a natural transition from the second to the third to the fourth *thema*. The second *thema* deals with the case where the subordinate argument shares a premiss with the superordinate argument and has just one external premiss. The third *thema* deals with the case where the subordinate argument shares no premiss with the superordinate argument and

has more than one external premiss. The fourth *thema* deals with the combined case where the subordinate argument both shares a premiss with the superordinate argument and has more than one external premiss. (21) There is a plausible explanation of why someone would have invented the Simplician version if Chrysippus produced the Alexandrian one: the Simplician version makes discovery of demonstrations easier, and it is more powerful.

3.11 Adequacy of the system as reconstructed

In assessing the adequacy of Stoic propositional logic, it is only fair to assess it with respect to its inventor's conception of a valid argument: a system composed of at least two premisses and a conclusion, the contradictory of which conflicts with the conjunction of the premisses, meaning that at least one of the conflicting propositions must be false, the contradictory of the conclusion does not conflict with a proposition which is a proper part of the conjunction of the premisses, and there is some connection between the two conflicting propositions.[52]

Soundness: Every undemonstrated argument is Chrysippean-valid. The proof is straightforward. In a first undemonstrated argument, the contradictory of the conclusion is the contradictory of the consequent of the argument's leading premiss, and the conjunction of the premisses is the conjunction of a conditional and the antecedent of that conditional, which is true if and only if both the antecedent is true and the contradictory of the consequent is incompatible with the antecedent. Thus the contradictory of the conclusion cannot be true at the same time as the conjunction of the premisses; given the truth of the antecedent and the incompatibility of the

[52]These three conditions may not be sufficient for conflict, whose definition by the Stoics is not preserved. But they are individually necessary. In the proofs which follow, I shall take the three necessary conditions as jointly sufficient. If the three individually necessary conditions are not jointly sufficient, the proofs which follow need to be expanded to take account of whatever additional necessary conditions are required for joint sufficiency.

contradictory of the consequent with the antecedent, the contradictory of the consequent is not true. Further, in general, the contradictory of the conclusion does not conflict with a proper part of the conjunction of the premisses.[53] And there is a connection between the two propositions. Similarly for the other undemonstrated arguments.

Further, each *thema* generates a Chrysippean-valid argument when it is applied to one or two Chrysippean-valid arguments. In the case of the basic first *thema*, for example, suppose that $p, q \vdash r$ is Chrysippean-valid. Then $\neg r$ conflicts with $\& p \& q$. Hence it cannot be that both propositions are true, i.e. that r is false, p is true, and q is true. Further, there is a connection between $\& p \& q$ on the one hand and r on the other. Further, $\neg r$ does not conflict with p and $\neg r$ does not conflict with q. From the first consequence, we can infer that it cannot be that p is true, q is true, and $\neg r$ is true, i.e. that it cannot be that both the contradictory of p is false and $\& q \& \neg r$ is true. From the second consequence, along with the fact that each simple proposition in every syllogistic argument occurs in different components of the argument, it follows that there is a connection between $\& q \& \neg r$ on the one hand and $\neg p$ on the other.[54] From the third consequence, it follows that p does not conflict with q[55] and p does not conflict with $\neg r$. Similarly for the other *themata*.

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[53]There is however such a conflict when the conditional is true. Sextus used this fact to argue quite cogently that on Chrysippus' own assumptions a first undemonstrated argument either is invalid through redundancy or has a false leading premiss (A.L. 2.438-442). Antipater's admission of one-premissed arguments (A.L. 2.443) may have been a response to this criticism.

[54]The additional assumption is necessary to accommodate the case in which the connection is between q and r, which both end up in the premisses of the contraposed argument so that theoretically there might be no connection between q and $\neg r$ on the one hand and $\neg p$ on the other.

[55]Erik Krabbe questioned this inference in his comments in November 2012 on this paper.

Completeness: A Chrysippean-complete system would allow one to analyse into one or more of the five Stoic indemonstrables every argument which is an instance of a mood whose component premisses and conclusion contain only propositional variables and one or more of the four logical operators of Chrysippus' system and whose instances are all Chrysippean-valid arguments. That is, the mood should have at least two premisses. No instance of the mood should be able to have true premisses and a false conclusion. Some instance of the mood should have true premisses. Some instance of the mood should have a false conclusion. In at least some instances of the mood no premiss should be redundant; that is, the contradictory of the conclusion should not conflict with any proposition which is a proper part of the conjunction of the premisses.

The requirements for completeness outlined in the preceding paragraph require a much less powerful system than the requirement imposed by some interpreters (e.g. Becker, Kneale and Kneale) that all classically valid arguments using only conjunction and negation which are expressible in the system be provable in it. Such anachronistic requirements have led to historically ungrounded reconstructions of Stoic propositional logic which are in fact unsound on Chrysippus' conception of a valid argument; they allow one to prove things which are not valid arguments by Chrysippus' criteria.

On the other hand, they require in some respects a more powerful system than the requirements laid down by Bobzien (1996, 192; 1999, 151 n. 158). She proposes that Stoic propositional logic would be complete if in it any argument with two or more premisses is either undemonstrated or reducible to undemonstrated arguments which:

(1) because of its form can never have true premisses and a false conclusion, and

(2) contains as logical constants relevant to their form only the four recognized in Stoic propositional logic, and

(3) contains no premiss doublets, and

(4) contains no redundant premisses, and

(5) is not wholly hypothetical, and

(6) is self-evidently valid, or is composed of nothing but self-evidently valid arguments.

The first four conditions are accommodated in my proposal in the preceding paragraph. The fifth and sixth of these requirements seem unnecessarily restrictive. They seem motivated by the fact that no mood of an undemonstrated argument is wholly hypothetical and that each undemonstrated argument is self-evidently valid. But nothing in Chrysippus' conception of a valid argument licenses their inclusion. Thus, they function more as excuses for the incompleteness of the system than as necessary conditions for completeness. On the other hand, Bobzien does not take into account the requirement of a connection between premisses and conclusion which is central to Chrysippus' concept of validity. Thus, she seems to impose on Chrysippus' system a requirement to demonstrate, for example, ex falso quodlibet ($p, \neg p \vdash q$), unless this argument is not self-evidently valid or composed of self-evidently valid arguments.

Milne's counter-example (RV1) shows that Chrysippus' system of propositional logic is not complete, as does RV2. It is however surprisingly difficult to come up with Chrysippean-valid moods with only propositional variables and one or more of the four logical operators which cannot be demonstrated within the system. The difficulty is surprising because the system at first glance has glaring deficiencies. You can prove valid arguments from a conditional but not arguments to a conditional, arguments to a conjunction but not from a conjunction, and

arguments from a disjunction but not to a disjunction. The reasons for these deficiencies are various. Valid "arguments" from a conjunction need no other premiss, so either are not arguments by Chrysippus' criterion or are invalid because of redundancy. Valid arguments to conditionals or disjunctions require an assumption of some conflict between two propositions, an assumption which cannot be expressed simply in terms of the propositions and the logical operators of the system. Thus, Chrysippus' system of propositional logic came closer to being Chrysippean-complete than we might imagine, because of the restrictions imposed in his concept of a valid argument. It is nevertheless noteworthy that his system allows one to prove the validity of those arguments with formally valid moods expressible in the system which we are inclined to use in real reasoning and argument. From this point of view, the Chrysippean restrictions on valid arguments may not be as unmotivated as we might suppose.

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Abbreviations

A.L. = Sextus Empiricus, *Against the Logicians* (= *Against the Professors* 7-8)

Alexander *In An. pr.* = Alexander of Aphrodisias, *In Aristotelis Analyticorum Priorum Librum I*

Commentarium

Alexander *In Top*. = Alexander of Aphrodisias, *In Aristotelis Topicorum Libros Octo Commentaria*

Ammonius *In de int.* = Ammonius, *In Aristotelis De Interpretatione Commentarius*

Aristotle *An. pr.* = Aristotle, *Analytica priora*

D.L. = Diogenes Laertius

FDS = Hülser, *Die Fragmente zur Dialektik der Stoiker*

I.L. = Galen, *Institutio Logica* (original Greek title *Eisagôgê Dialektikê*)

LSJ = Liddell-Scott-Jones, *A Greek-English Lexicon*

P.H. = Sextus Empiricus, *Outlines of Pyrrhonism* (*Pyrrhôneiôn Hupotupôseôn*

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