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A STABLE RESIDENCE EXCHANGE PROBLEM

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ABSTRACT

This paper introduces a new matching problem, the Stable Residence Exchange Problem, which originates from the needs for residence exchange in China. The problem involves n families wishing to exchange their residences voluntarily on the basis of their own preferences. Residence exchange can be arranged through exchange rings where each family in a ring moves to the residence of the next family in the ring. A residence exchange assignment is stable if under the assignment, there does not exist any unassigned ring in which at least one family is better off and none is worse off. For any instance of the problem, a stable solution is unique, always exists and can be found by using a forward chaining algorithm. A family cannot be better off by misrepresenting its true preferences and cannot be worse off by submitting more choices as long as they are desirable. Finally, computer simulation is used to assess the effect of pool size and number of choices on the result of residence exchange.

Keywords:

Matching, Multi-person game, Assignment, Public service

A Stable Residence Exchange Problem

I. INTRODUCTION

A variety of problems can be classified as matching problems where pairing is made between participants based on their preferences (see Gusfield and Irving [1989] for a general discussion). Three related matching problems are introduced by Gale and Shapley [1962]. Among them, the simplest and most well known one is the stable marriage problem. This problem involves pairing men and women for marriage according to their preferences towards the marriage. The unique feature which makes the problem different from the classic personnel job assignment problem is the requirement of stability for the assignment. A matching assignment is stable if there do not exist a man and a woman who prefer each other to their assigned partners. Gale and Shapley prove that a stable solution always exists and can be found through a "deferred-acceptance" procedure.

A polygamous version of the stable marriage problem is the stable college admission problem where the matching is made between the students and college programs with a targeted number of enrollments. The stable assignment algorithm for the problem has been successfully applied to the National Resident Matching Program (NRMP) for matching medical graduates to hospital internship positions in the United States, as well as in Canada (see Graettinger and Perason [1981], Roth [1984]).

The third matching problem is the stable roommates problem where students are paired to be roommates based on their preferences and the stability is defined in a similar way. Since the roommate pairing is formed for the members within one group instead of from two disjoint gender groups, the roommates problem is a generalization of the stable marriage problem.

Irving (1985) shows that a stable solution for the roommates problem does not always exist. Irving develops an algorithm to verify the potential existence of a stable solution and to find it when it does exist.

In this paper a new matching problem called the stable residence exchange problem is introduced. The problem involves a voluntary exchange of residence based on families' preferences for change. Since the exchange is not necessarily pair-wise, the problem can be considered as an extension of the stable roommates problem.

The stable residence exchange problem originates from the needs for residence exchange in China. Most urban people in China live in public housing. The rent subsidized by the government is very low but there is a serious shortage in housing. It is very difficult for a family to be assigned to better accommodation, and it often takes many years of waiting. In this situation people have to exchange their residences (actually, exchange the right of renting, not the ownership of, the residence) to meet their specific needs. For instance, they exchange residences in order for the location to be closer to the working place or to their parents, to trade space for private bathroom or kitchen, to trade one living place for two separate living places because of a child's marriage, etc.

The demand for residence exchange is very high. It was unofficially estimated by Mr. Cai Yutian, Deputy Director of Shanghai Municipal Housing Administration Bureau, during a meeting with the author in 1991, that in Shanghai more than one hundred thousand families are actively searching for residence exchange. To be able to find a willing and satisfactory exchange partner, however, is very difficult. Less than 5 percent of the families actually made the exchange. One major problem is that there is no efficient information channel available to facilitate the search. Most people search for exchange either through personal contact or by attending residence exchange fairs. This searching method seriously limits the chance for people to find a suitable exchange partner. Another problem is that mutually preferred exchange pairs seldom exist. It is often the case that one likes another's residence but not vise versa. In this situation, no pair-wise exchange can be arranged. However, a ring exchange, if it can be formed, may lead to a better result. For instance, it may happen that A likes B's residence but not vise versa, B likes C's but not vise versa, and C likes A's but not vise versa. A circle, or a ring exchange, can be arranged to let A change to B, B change to C, and C change to A so that everyone is satisfied. The problem is that such a ring exchange is very difficult to explore through individual contact. Furthermore, even a ring exchange is possible, the parties involved may still hesitate to make the final decision due to the fear of losing a potentially better opportunity later on. The question therefore is to define and to find out what is the best residence exchange.

It should be mentioned that some types of residence exchange problem have already been studied in the literature. A housing reallocation problem is formed by Wright [1975] where tenants are allowed to change housing from one category to another category. The objective of reallocation, from a housing administrator's point of view, is to minimize the length of the exchange circuits. A network best path algorithm is applied to solve the problem. The model however does not allow tenants to have more than one choice and the reallocation criterion is different from the one studied here. In this paper, the context of residence exchange is viewed as a market or a multi-player game. The stability is introduced as a criterion for residence exchange. The residence exchange is formalized as a stable residence exchange problem and an algorithm is developed to find the solution. The rest of the paper is organized as follows. In section II, the stable residence exchange problem is defined. In section III, a forward chaining algorithm is developed to find a stable solution. In section IV, properties of the solution are discussed. In section V, computer simulation is used to investigate factors which may affect the results of residence exchange. The final section is the discussion and the conclusion.

II. THE STABLE RESIDENCE EXCHANGE PROBLEM

Assume that there are n families in a city who wish to exchange their residences for a variety of reasons. Each family is allowed to submit a preference list consisting of up to n choices with the last choice being its own residence with no exchange. A residence exchange may be arranged to reallocate n residences to n families in accordance to families' preferences.

If the residence exchange is restricted to be pair-wise only, the problem can be formalized as the stable roommates problem where a pair of families for residence exchange corresponds to a pair of students who are assigned to be roommates and their preference of being paired can also be interpreted accordingly. Since a family may choose not to exchange its residence if no desirable exchange partner is available, pairing with itself therefore should be allowed. A solution of the problem is stable if with the assignment there does not exist a pair of families who prefer exchange their residences with each other than with their assigned one's. However, a stable solution for the roommates problem does not always exist, and even if it does, it may not necessarily be the best for residence exchange.

Without the restriction of pair-wise exchange, the residence exchange can be made through any feasible reallocation as long as each residence is occupied by only one family and each family moves only to one of the residences on its preference list. In fact, this kind of reallocation can always be implemented through disjoint exchange rings where each family moves to the next one's residence in a cycle or a ring (see Lemma 1 later in the paper). A question therefore is to define what is the desirable or the best residence exchange.

One approach assumes the existence of a single authority or decision maker who takes care of all families' interests. For each residence reallocation, an utility or a cost is specified according to families' preferences. The objective therefore is to maximize the total utility or to minimize the total cost for the residence reallocation assignment. The problem can be formed as an integer programming problem with multiplicative utility converted from families' preferences (see Mehrez et al. [1988], Keeney and Raiffa [1976]).

Another approach, that is taken in this paper, regards the residence exchange as a market or multi-player game where each family acts as an independent decision maker to compete with each other. Obviously, every family wishes to move to a more favorable residence. It is quite likely that two or more families may wish to move into the same residence. However, at most only one family may be able to move in when the family that currently live in this residence is willing and able to move out. But the question is: how to decide who can move into whose residence? Due to the conflict of the interests among families, a residence exchange must resulting from a fair competition. As a desired criterion, the stability of a residence exchange can be defined as follows.

Definition 1. A residence exchange is stable if there does not exist any family subset consisting of more than one family in which the rearrangement of residence exchange makes at least one family better off but none worse off.

The stability indicates that no collusion among families is capable to further improve their situation. To illustrate the concept let us look at a simple example shown in Table 1. There are four families wishing to exchange their residences. Assume that residence i is currently occupied by family i. The families' residence preference list P is shown in Table 1 a) where each row represents a family's preference. For instance, row 1 indicates that family 1's first choice is family 2's residence, second choice is family 3's residence, and the last choice is its own residence, i.e., no exchange.

<Table 1 here>

One possible family-residence assignment (the residence exchange), say M', is shown in Table 1 b) where family 1 changes to residence 3, family 2 changes to residence 4, family 3 changes to resident 1, and family 4 changes to resident 2. This assignment is unstable since there exist a subset consisting of family 1 and family 2, among them a better residence exchange can be made to make both better off (family 1 changes to residence 2 which is more favorable than the assigned residence 3 and family 2 changes to residence 1 which is more favorable than the assigned residence 4).

The solution M show in Table 1 c) is stable. To verify the stability of this solution we should check all possible subsets of families to verify if any exchange within a subset can break the stability. To simplify the stability check, we introduce the following lemma.

Lemma 1. Any family-residence assignment (residence exchange) can be implemented through a set of disjoint exchange rings.

Proof. With any assignment (represented by a permutation of the residence numbers assigned to families) the corresponding exchange subsets can be determined by sequentially linking families by moving them to the assigned residences until a cycle (exchange ring) is formed. Since each family can only move to one residence and the number of families is equal to the number of residences, at least one cycle must be formed and all cycles are disjoint. It should be mentioned that with an arbitrarily specified assignment, the families' preferences or acceptances of exchange have not been considered.

Lemma 1 leads to an equivalent definition of the stability.

Definition 2. A residence exchange assignment is <u>stable</u> if there does not exist any other assignment under which there is a exchange ring consists of more than one family in which at least one is better off and none is worse off.

Using the new definition to check the stability of assignment M in the above example, we select, for instance, an alternative assignment M" shown in Table 1 d). The corresponding disjoint exchange subsets for the assignment M" are {1,2,4} and {3}. In subset {1,2,4}, family 1 is assigned to residence 2, the same as in M, family 2 is assigned to residence 4, which is worse than residence 3 assigned to it in M, and family 4 is assigned to residence 1, which is better than residence 4 assigned to it in M. Subset

{3} consists only one family, definitely cannot be better off, so it does not need to be checked. Since in each subset at least one is worse off or none is better off, the alternative assignment M" cannot break the stability of the assignment M. Notice that the number of all possible assignments will be n!, the permutation of all the n residences (in this case it is 4! = 24), hence we need to check the stability of M with rest of 22 assignments.

It is interesting to see that in this simple example no stable pair-wise exchange exists. Notice there are 9 possible pair-wise exchange assignments and none is stable. When the assignment consists of the pair-wise exchange {1,2} and/or {3,4}, families 2 and 3 will prefer each other to their assigned partners or without exchange (paired with a dummy family). When the assignment consists of the pair-wise exchange {1,3} and/or {2,4}, families 1 and 2 will prefer each other to their assigned partners or without exchange. When the assignment consists of the pair-wise exchange {1,4} and/or {2,3}, families 1 and 3 will prefer each other to their assigned partners or without exchange.

III. THE FORWARD CHAINING ALGORITHM

An algorithm called the Forward Chaining Algorithm is developed to find a stable solution for residence change. Assume that there are n families in a city who wish to participate the residence exchange. Each family is allowed to submit a preference list consisting of up to n choices with the last choice being its own residence with no exchange. To simplify the notation we assume family i currently occupies residence i and do not distinguish between the family and the family's residence if it does not create any ambiguity. For instance, i prefers j to k means that family i prefers family j's residence j to family k's residence k, etc.

Specific terms are introduced to describe the procedure used by the algorithm. The term "i *proposes* to j" means that family i proposes to move into family j's residence; "j *holds* i" means family j is seeking to move out so that family i may be able to move into residence j; "j *rejects* i" means that family j cannot or does not want to move out so that it rejects family i's proposal to move into its residence; and "j *accepts* i" means that family i is able to move into family j's residence because family j is able to go to somewhere else.

A *chain* is used to represent the sequence of proposal; for example, the chain i->j->k->h indicates that i proposes to j, j proposes to k, k proposes to h, and h is going to propose to someone else who has not been determined yet. A *ring* is used to represent a cyclical proposal chain. For example, in the above chain, if h proposes to j, then a ring <j, k, h> is formalized, which indicates that j proposes to k, k proposes to h and h proposes to j in a cycle. Once a ring is formed, the actual residence exchange can be arranged by accepting each proposal in the ring. A *waiting list* is used to represent all the families whose move-in residences have not been arranged yet, and an *arranged list* represents all the families whose "move-in" residences have been finally arranged.

The forward chaining works as follows. At the beginning, all families are put into the waiting list and the arranged list is empty. Starting from one family in the waiting list, the algorithm develops a proposal chain where each preceding family in the chain proposes to its best choice on its preference list. The family who receives the proposal became the succeeding family in the chain and continue to propose to its best choice on its preference list. The chain will continue to grow until one of the following situations happens:

1) The last family in the chain proposes to one already in the chain. In this case a ring is formed. That ring will be the final exchange arrangement for the members of the ring. These families will be removed from the waiting list and put into the arranged list; their residence will be removed from the residence preference list of all other families in the waiting list. The ring then will also be removed from the chain. If no family is left in the chain, a new chain should start with a family in the waiting list. If the rest of the chain is not empty, the last family in the remaining chain, who has been rejected by the first family of the ring, will continue to propose to the best choice in its remaining preference list.

2) The last family in the chain has no other residence to choose in its remaining preference list except its own residence. In this case a ring with single family is formed. It will also be removed from the chain and that family will be put into the arranged list. The family's residence will be removed from other families' preference lists, in a way similar to that in the above step. If no family is left in the chain, a new chain should start with a family in the waiting list. If the rest of the chain is not empty, the last family in the remaining chain, who has been rejected by the first family of the ring, will continue to propose to the best choice in its remaining preference list.

Since the number of families and the number of choices are limited when the choices are exhausted, all families will eventually be assigned to a residence, either someone else's residence or their own residence. The forward chaining algorithm is summarized in appendix 1.

We illustrate the algorithm through a simple example which involves 6 families for residence exchange. The preference list of the families and the forward chaining process is shown in Table 2.

<Table 2 here>

In Table 2 a), starting from family 1, we have family 1 proposes to family 6, family 6 proposes to family 4, and the proposal chain is formed as 1->6->4. Since family 4 proposes to family 6, the ring <6, 4> is formed. We remove the ring <6, 4> from the chain and save it in the arranged list. families 6 and 4 are removed from the waiting list and their residences are marked off from the residence preference list (indicated by *). The remaining chain needs to grow again, which is shown in Table 2 b). In this case family 1 proposes to family 5, family 5 proposes to family 3, and family 3 proposes to family 1. A ring <1, 5, 3> is formed. After removing families 1, 5, and 3 (marked by #) from the waiting list and marking off the corresponding residences in the preference list, we have only family 2 left which has to stay in its own residence without exchange. The corresponding ring then is <2>, indicated in Table 2 c). The residence exchange is complete. The arranged set now consists of three rings: <6, 4>, <1, 5, 3>, and <2>.

It can be verified from the example that the solution does not depend on the sequence in which a family is picked for forward chaining. For instance, starting from family 2, the forward chaining is 2->6->4->6, a ring <6, 4> is formed and family 2's proposal is rejected by family 6. Then, continuing with the forward chaining 2->5->3->1->5, another ring <5, 3, 1> is formed. Family 2's proposal is rejected by family 5. The last ring is <2>. All the rings generated are exactly the same. We introduce the following Lemma.

Lemma 2. The sequence of picking a family from the waiting list to initiate forward chaining does not affect the result.

Proof. Using graph presentation each family can be represented as a node and each proposal creates a arrowhead link from one node to another. The direction of the forward link from a node does not depend on which node has a link pointing to it. Suppose from an initial node, say i, the first ring R is formed. Within the ring R, every node must be linked to its available first choice node. If other node, say j, is picked as the initial node, there are two possible cases. Case 1 is that node j belongs to the ring R. Starting from node j, the same ring will be formed. Case 2 is that node j does not belong to ring R. Starting from node j, the chain may first reach one node, say k, which belongs to the ring R. Continue chaining from k, the route will be the same and the ring R will be formed. Another possibility is that from node j the chain may never be able to reach any node in ring R. The ring R is not formed at this time, and is waiting to be formed later when one of its member is picked as a starting node or is reached by other node. In any case, the ring R will be the same. Removing the members of ring R from other nodes' preference list will not affect the chaining of other nodes since any node outside of ring R will be rejected when that node proposes to any member of ring R. The same argument can be applied to the rings formed after the removal of ring R. Finally all the same rings will be formed regardless of the order that the initial nodes are picked.

IV. PROPERTIES OF THE SOLUTION

We now study the properties of the solution generated from the Forward Chaining Algorithm.

Theorem 1. The solution generated from the Forward Chaining Algorithm is stable.

Proof. Assume that the solution generated from the Forward Chaining Algorithm is not stable. According to the definition of stability, there must exist a family subset consisting of more than one family in which the reallocation of the residences within the subset makes at least one better off and none worse off than the currently assigned one. Since any residence reallocation can be represented as a ring, or a set of partitioned rings. With the assumptions we have made, there must exist a ring which is different from the rings generated from the Forward Chaining Algorithm and every family in the ring is better than or at least the same as the assigned one. If a family can move into a better residence, it should be able to propose to that residence with success. However, based on our algorithm, the family must have proposed to that residence before its current assigned residence and must have been rejected. This contradicts with our assumption.

Theorem 1 leads to the following corollary:

Corollary 1.1. For any instance of the residence exchange problem, a stable solution always exists.

Theorem 2. For any instance of the residence exchange problem, the stable solution is unique.

Proof. Assume that there are two different stable solutions. We first remove all the exchange rings which are the same for both solutions. All the remaining families must have been assigned to the residences of the remaining families from both solutions. Since the two solutions are not the same, the set of remaining families is not empty. There must exists a family, say i, assigned to residence j in one solution and residence k in another solution. Start from family i, each remaining family proposes to the best residence between the two residences assigned to that family in two solutions. Continue the proposal chain, a ring must be formed and every member of the ring will be better than o at least the same as the existing two solutions. Since the ring must be different from one of the two solutions, the stability of that solution will be violated.

Theorem 3. No family will be better off by misrepresenting its true preferences.

Proof. Assume that all families except family i, represent their true preferences. According to lemma 2, the sequence of picking a family from the waiting list to initiate a exchange chain does not affect the resulting stable solution. We start with any family other than family i, say family h, to do a forward chaining. The growth of the chain from h will not depend on family i's preference until it reaches family i or until it is discarded completely by forming rings with other families. In the first case family i will be accepted by h if it proposes to h and in the second case family i will be rejected by h if it proposes to h . All the families can therefore be classified into two groups when each is proposed by family i: the "achievable group" and the "unachievable group". The classification does not depend on whether family i misrepresents its preferences as long as all other families keep their preferences unchanged. The stable solution therefore depends on which family from the achievable group is first proposed by family i.

Assume that with its true preference P, family i is assigned to residence j and with its misrepresented preference P′, family i is assigned to residence k which is better than residence j. Since both family j and k should belong to family i's achievable group. If family truly prefers residence k to residence j, it will propose to family k first with its true preferences and be accepted by k. This contradicts our assumption.

Theorem 4. A family will not be worse off by listing more choices.

Proof. Assume that family i adds one more choice, say family j, on its preference list, while the rest of families keep their preferences unchanged. By the same argument used in the proof of Theorem 3, family j can be classified into either the achievable group or the unachievable group of family i. If family j belongs to the achievable group and is ranked higher than the assigned one, say k, in the original solution, then according to the forward chaining algorithm, i will be assigned to j in the new solution. If family j belongs to the unachievable group or is ranked lower than family k, the solution will not change. In either case family i cannot be worse off.

Theorem 3 shows that there is no incentive of misrepresenting true preferences. Theorem 4 shows that there is no risk of submitting more choices. It is clear that the optimal strategy for each family therefore is to submit its true preferences and to list more choices as long as they are desirable.

V. FACTORS THAT AFFECT EXCHANGE RESULTS

In practice it is often desirable to know what factors will affect the exchange result. These factors could be the number of families that participate the residence exchange, the number of choices listed by each family, etc. The performance of resulting exchange can be measured by the percentage of families being arranged for exchange and the choice preference obtained. Computer simulation is conducted to investigate possible relation between these variables. It is assumed that each family's preferences is independent of other families' preferences and it is also assumed that every residence has equal chance to be ranked by any family at any order. Simulation is done in two different settings. In first setting, the number of families that participate the residence exchange is fixed to 50. The number of choices that each family can submit is varied from 2 to 50 with the last choice of no exchange. The simulation runs for 1000 times and the mean and standard deviation of success rate are shown in Table 3. Here the success rate is the ratio of the number of families that have been assigned for residence exchange to the total number of families that participate the exchange.

<Table 3 here>

It is clear from Table 3 that the more choices listed, the higher success rate. When each family lists only 2 choices (including the second choice of no change), the success rate is only 0.173. By adding 2 choices, the success rate jump to 0.630. When more and more choices are added, the gain of success rate diminishes. To list more choices costs more and requires more time and effort in searching and evaluating alternatives. The larger number of choices also indicates the lower achievable standard for residence exchange.

Table 4 shows the choice preference received when different number of choices are listed.

<Table 4 here>

Although as indicated in Theorem 4 a family will not be worse off by listing more choices, the side effect on other families cannot be determined. Adding a choice by one family may help other families to form a ring therefore they are all better off. The newly formed ring however may force other families outside the ring to reform rings therefore to be worse off. For instance, in the example shown in Table 2, the stable rings are <1,5,3>, <6,4>, and <2>. If family 1 adds residence 2 as its new second choice before residence 5, the resulting rings will be <1,2,5,3> and <6,4>. Family 1 gets its new but still second choice and family 2 gets its second choice, residence 5 instead of no exchange before. If family 1 adds residence 3 as its second choice, the resulting ring will be <1,3>, <5,2>, and <6,4>. Family 2 is better off and family 5 is worse off. Because of the mixture of both positive and negative effects, it is difficult to predict the overall result even when only one family has added one choice. It might be expected that when more choices are listed by many families, the overall choice preference received will be worse due to the intensified competition. The simulation, however, shows the opposite direction. When the number of choices increases, the percentage of families that receive the first choice increases most rapidly, followed by groups with less preferred choices. Since the success rate increases significantly, it seems that the dominate effect of adding choices is to make those families to form exchange rings that otherwise may not be able to form. The positive effect seems to offset the negative effect on those families that already are able to form exchange rings before.

The ring length distribution with different numbers of choices listed by each family is shown in Table 5 where the percentages of families that made residence exchange for different ring length ranges are listed. Ring length 1 means no exchange. The data in Table 5 show that when the number of choices listed by each family increases, the largest portion of families are still making residence exchanges with short rings ranging from 2 to 5 families. Considering the cost and benefit involved in deciding the number of choices listed, about 5 to 6 choices in our case seem reasonable and good enough.

<Table 5 here>

To investigate the effect of pool size, i.e. the number of families participating in a residence exchange, simulation is carried out with pool size that increases from 6 to 1000. In all situations, each family lists 6 choices with the last choice of no exchange. The simulation runs for 1000 times. Table 6 shows that the success rate decreases when the pool size increases. It seems somehow unexpected. Notice that the number of choices is fixed to 6 in our simulation. For small pool sizes 6 and 10, the high success rate is somehow artificial because families in general will not be able to list 6 choices with such a small pool size. When the pool size increases from 50 to 1000, more options will be available for families to chose but the choices can also be diverse and widely spread. With the fixed number of choices, the success rate only decreases slightly with large pool sizes.

<Table 6 here>

The choice preference distribution for different pool size is shown in Table 7. The percentage of families that receive the first choice for residence exchange decreases most significantly when the pool size increases from 6 to 50. The percentage decrease diminishes when the pool size further increases from 50 to 1000. However, it does not necessarily mean the quality of the exchange will be worse since the rank order only represents the relative preferences. Choices with the same rank selected from a large pool usually should be better than that from a small pool when more intensive searching is possible.

<Table 7 here>

In summary, the simulation shows that listing more choices will improve both the success rate and the overall satisfaction level of residence exchange. Although increasing pool size does not directly raise the success rate and the rank of received choices, it increases the chance for families to have more and better choices and therefore indirectly improve the quantity and the quality of residence exchange.

VI. CONCLUSION

In this paper we have introduced a new matching problem -- the stable residence exchange problem and have developed a forward chaining algorithm to find a stable solution. For any instance of the problem, the stable solution is unique and always exists. The best strategy for families to improve their residence exchange is to represent their true preferences and list all desirable choices.

To make the stable residence exchange possible, a centralized residence exchange center needs to be established. A computerized information retrieval system with a large database can be used to register the requests for residence exchange and to facilitate the search for desirable residences. Due to the cost and time limitation, families may not be able to make intensive searching therefore may miss some good choices especially when the pool size is large. To improve exchange results, their preference lists can be extended with the help of exchange center. Based on submitted preferences, achievable group can be calculated for each family. Compared with their submitted preference list, a list of additional achievable residences can be provided to each family for further consideration. Preference adjustment therefore can be made when it is desirable. After several runs of preference adjustment, each family needs to confirm its final preferences and make a commitment of accepting the assigned exchange. A computerized forward chaining algorithm then can be used to generate the stable solution for residence exchange. The exchange assignment can be run periodically, say, for instance, monthly or quarterly, to arrange exchanges officially for participated families. With this approach, the quality of residence exchange can be significantly improved.

The residence exchange problem can be applied to other settings such as the room exchange in a student dormitory, the rotation of people among different jobs or working places within the military, governments, or any large organization.

In this paper, we did not consider the situation when some families may come from or move to other cities so that they need to be assigned to new accommodations or vacate their current residences. If a city housing administration office is responsible in both assisting residence exchange and assigning new residences, the residence exchange should be considered jointly with residence assignment for optimization. Further research is needed to define and solve the problem.

APPENDIX: The Forward Chaining Algorithm

```
put all families in a waiting list W;
while the waiting list W is not empty do
begin {the beginning of a forward chaining}
     clean chain C;
     S := a family from the waiting list W;
     put S as the first member of the chain C;
     while chain C is not empty do
     begin {the beginning of one forward step}
     T := the last member of the chain C;
     U := the first choice from T's preference list P to whom T has not
          yet proposed;
     if U = any member V of chain C then {form a ring}
          begin
               clean ring R;
               put chain members start from V to the end into ring R;
               put ring R into arranged list M;
               remove members of ring R from the waiting list W;
               remove members of ring R from the chain C;
               remove members of ring R from the preference list P of
               all families in the waiting list W;
          end;
     else {extend the chain}
          put U as the last member of chain C;
     end; {the end of one forward step}
end; {the end of a forward chaining}
output the stable exchange solution stored in the arranged list M
```

REFERENCES

Gale, D., and Shapley, L.S. (1962), "College Admission and the Stability of Marriage," American Mathematical Monthly, 69: 9-15.

Graettinger J. S. and Perason E. (1981), "The Matching Program," The New England Journal of Medicine, 304: 1163-1165.

Gusfield, D. and Irving, R.W. (1989), The stable marriage problem: structure and algorithms, The MIT Press: Cambridge, Massachusetts.

Irving, R. W. (1985), "An Efficient Algorithm for the 'Stable roommates' Problem," Journal of Algorithms, 6: 577-595.

Keeney, R. L. and Raiffa, A. (1976), Decisions with Multiple Objectives: Preferences and Value Tradeoffs, Wiley: New York.

Mehrez, A., Yuan, Y., and Gafni, A. (1988), "Stable Solutions vs. Multiplicative Utility Solutions for the Assignment Problem." Operations Research Letters, 7: 153-162.

Roth, A. (1984), "The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory," Journal of Political Economy, 92: 991-1016.

Wright, J. W. (1975), "Reallocation of Housing by Use of Network Analysis," Operational Research Quarterly, 26: 253-258.

Table 1. An Example

a) A sample preference list

Family	Residence preference			
1	2	3	1	
2	3	1	4	2
3	1	2	3	
4	1	2	3	4

b) An unstable solution M′

Family	Residence assigned
1	3
2	4
3	1
4	2

c) The stable solution M

Family	Residence assigned
1	2
2	3
3	1
4	4

d) An alternative assignment M" for stability check

Family	Residence assigned
1	2
2	4
3	3
4	1

Table 2. An Illustration of Forward Chaining Process

a) Chain starts from family 1 and the first ring <6,4> is formed.

Family	Residence preference						Chaining
1	6*	5	4*	1			1->6
2	6*	5	4*	1	2		
3	1	2	4*	5	6*	3	
4*	6	2	3	4			4->6
5	3	2	1	6*	5		
6*	4	3	6				6->4

b) Chain continues from family 1 and the second ring <1,5,3> is formed.

Family	Residence preference				Chaining
1#	5	1			1->5
2	5#	1#	2		
3#	1	2	5	3	3->1
4*	6				
5#	3	2	1	5	5->3
6*	4				

c) Chain starts from family 2 and the third ring <2> is formed.

Family	Residence preference	Chaining
1#	5	
2	2	2->2
3#	1	
4*	6	
5#	3	
6*	4	

Table 3. Simulation Studies: Exchange Success Rate with Different Number of Choices

Success Rate	2	3	4	5	6	10	20	50
	Number of Choices Listed by Each Family							
Mean	0.173	0.483	0.630	0.722	0.775	0.878	0.951	0.990
STD	0.083	0.086	0.077	0.062	0.060	0.043	0.028	0.010

Note: For each run, 50 families involved in residence exchange. Simulation runs for 1000 times

Table 4. Simulation Studies: Received Choice Preference Distribution with Different Number of Choices Listed

Choice Preference Received	Number of Choices Listed by Each Family							
	2	3	4	5	6	10	20	50
1st	17.3%	37.6%	44.2%	47.4%	48.7%	51.0%	51.5%	52.1%
2nd		10.7%	12.9%	14.1%	14.7%	15.8%	16.5%	16.4%
3rd			6.2%	6.6%	7.0%	7.9%	8.2%	8.2%
4th				4.0%	4.1%	4.6%	4.8%	5.0%
5th					2.8%	3.0%	3.2%	3.3%
6-10th						5.7%	6.3%	6.5%
>10th						0.0%	4.5%	7.4%

Note: For each run, 50 families involved in residence exchange. Simulation runs for 1000 times.

Table 5. Simulation Studies: Ring Length Distribution with Different Number of Choices Listed by Each Family

Ring Length	Number of Choices Listed by Each Family							
	2	3	4	5	6	10	20	50
1	82.7%	51.7%	37.0%	27.8%	22.5%	12.2%	4.9%	1.0%
2-5	7.7%	20.2%	26.7%	31.1%	34.0%	40.8%	48.0%	51.2%
6-10	6.8%	19.7%	26.8%	30.4%	32.1%	35.1%	35.1%	36.2%
10-15	1.9%	7.3%	8.5%	9.4%	9.5%	10.1%	10.6%	10.0%
>15	0.9%	1.1%	1.0%	1.3%	1.9%	1.8%	1.4%	1.6%

Note: For each run, 50 families involved in residence exchange. Simulation runs for 1000 times.

Table 6. Simulation Studies: Exchange Success Rate with Different Pool Size

Success Rate	Pool Size 6	10	50	200	1000
Mean	0.932	0.863	0.776	0.741	0.722
STD	0.082	0.097	0.059	0.034	0.015

Note: For each run, different number of families involved in residence exchange. Each family has 6 choices. Simulation runs for 1000 times.

Table 7. Simulation Studies: Choice Preference Distribution with Different Pool Size

Choice Preference Received	Pool Size 6	10	50	200	1000
1st	64.2%	58.0%	49.2%	46.7%	45.8%
2nd	13.4%	14.3%	14.5%	14.2%	13.9%
3rd	7.8%	7.4%	7.1%	6.8%	6.5%
4th	4.6%	4.0%	4.2%	3.9%	3.7%
5th	3.2%	2.5%	2.7%	2.5%	2.3%

Note: For each run, different number of families involved in residence exchange. Each family has 6 choices. Simulation runs for 1000 times

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