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A FUZZY DECISION MAKING MODEL

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Abstract

A fuzzy decision making model is proposed to support decision making under uncertainty. This model incorporates three theories and methodologies: classical decision making theory under conflict, as suggested by Luce and Raiffa (1957), the fuzzy set theory of Zadeh (1965, 1984), and a modified version of the back propagation neural network algorithm originated by Rumelhart et al. (1986). An algorithm which implements the model is also described.

Key words: Fuzzy sets, games and decisions, neural networks.

A FUZZY DECISION MAKING MODEL

Introduction

Research in artificial intelligence has shown that the use of Bayes probability functions for describing subjective judgments is unjustified and may lead to erroneous results (Wierzchon 1982; Shortliffe 1976; Szolovits & Pauker 1978). In recent years, fuzzy set theory (Zadeh 1965) has regarded as a useful and systematic theory that can be more applicable when dealing with uncertainty and vagueness in human-originated information.

Research into fuzzy decision making is still at an early phase, and faces two major challenges. First, fuzzy decision making models need the supporting frameworks of general theories (Dubois and Prade 1980). Since fuzzy set descriptions make sense in human information processes, fuzzy decision making should be more closely associated with general human decision behavior and linguistic preferences. Secondly, on a practical level, the question of fuzzy membership function elicitation which has a lack of simple and convincing techniques has raised many criticisms (Dubois & Prade 1989; French 1984). Sophisticated mathematical formulations are useful for theoretical discussion, but the associated assumptions are often too restrictive for practical applications. Recently, there has been a rapid advance in the theory and application of neural networks and fuzzy reasoning

(Kosko 1992). Interpolation plays a central role in both neural networks and fuzzy reasoning (Zadeh 1992). In fuzzy systems, the input-output pairs have the structure of IF-THEN rules that relate fuzzy variables whose values are inexact. On the other hand, neural networks are able to create an approximation framework that can be used to generalize the IF-THEN rules through learning from examples, due to the adaptive nature of the neural network learning process.

In the light of this, this paper incorporates three theories and methodologies into the fuzzy decision making paradigm.

(1) The classical decision making theory, suggested by Luce and Raiffa (1957), provides a theoretical framework for dealing with conflict compromise in decision making under uncertainty.

(2) The fuzzy set theory, created by Zadeh (1965, 1984), presents uncertainty in a more natural form in dealing with human subjective judgements.

(3) A modified version of back propagation neural networks (Rumelhart et al. 1986) serves as a powerful tool in implementing fuzzy membership functions and fuzzy decision models.

Fuzzy Decision Making Models

The Simple Fuzzy Decision Making Model

Zadeh's (1965) original basis for fuzzy sets was to consider a membership function $\mu_Y(\mathbf{X})$ which associates an observation $\mathbf{X}$ (a vector variable, $\mathbf{X}=(x_1 \ldots x_m)$) with a real number y in the interval [0,1] (the likelihood of the observation belonging to Y) (See Figure 1). In the fuzzy decision making context, **X** is a decision input or an object, and y is a decision output or response.

**** Insert Figure 1 about here ****

A crucial issue in the practical applications of fuzzy sets is to find a fuzzy membership function $\mu_Y(\mathbf{X})$. Because fuzzy membership functions describe subjective judgements for particular problems, the only way of finding a fuzzy membership function is to obtain sample judgements from decision makers. A general form in representing human knowledge is the IF-THEN production rule:

$$\text{IF} \quad \mathbf{X} \quad \text{is} \quad \mathbf{A}_i \qquad \text{THEN} \quad y \quad \text{is} \quad c_i \qquad i=1 \ldots n$$

where n is the number of observations. Since fuzzy membership functions are monotonic (Zadeh 1984), a monotonic interpolation curve (in the wide sense of a curve, surface or hypersurface) which passes through all the data points representing the IF-THEN relation in syllogism (1) can be used for approximate reasoning in decision making. As will be shown in this article, neural networks with a certain learning algorithm can be employed for modeling these fuzzy membership functions.

In the context of fuzzy decision making, the problem of inference in approximate reasoning can generally be stated as the following syllogism.

$$\begin{array}{llll} \text{IF } \mathbf{X} \text{ is } \mathbf{A}_i & \text{THEN } y \text{ is } c_i & & i=1 \ldots n \\ \underline{\quad \mathbf{X} \text{ is } \mathbf{A}' \quad} & & & \\ & y \text{ is } c' & & (1) \end{array}$$

where **A′** is the constraint on independent variable **X**, and c′ is the deduced constraint on dependent variable y. This means that, for practical purposes, each of the n production rules associates **X**, specified by a non-fuzzy vector **A**, with y, specified by a non-fuzzy number c. c is usually called the certainty factor (CF) value in production rule systems. In this model, the fuzzy membership is crisp. The generalization of the set of rules and the decision making syllogism are illustrated in Figure 2(a).

**** Insert Figure 2 about here ****

It is also possible to consider a fuzzy conclusion c~ given a fuzzy constraint **A~**. Intuitively, one may define a fuzzy number **A~**=[**A-**, **A**, **A+**] as shown in Figure 2(b), and model the decision making based on information about **A-** and **A+** as well as the associated possibility distributions (Tanaka et al. 1989; Liang & Wang 1992). However, the possibility distribution of **A~** is often very difficult to justify. Using an assumption based on model developers' opinions such as "a fuzzy number **A~** has a triangular distribution" may or may not pertinent to a particular situation.

The Ultrafuzzy Decision Making Model

Frequently, the membership function takes on fuzzy values itself. This type of fuzzy set is called ultrafuzzy (Zadeh 1984). Ultrafuzzy sets are causally connected to the real world due mainly to imprecise knowledge obtained from domain experts (Turksen 1989). There has been a great deal of work concerning the techniques of approximate reasoning relating to ultrafuzzy sets (see, for instance, Hirota 1977, Zadeh 1979, Bandler and Kohout 1984, Turksen 1989, Martin-Clouaire 1989). Most of these techniques address the issue of deducing conclusions under uncertainty. Considering that the n rules representing the judgments of the decision maker constitute an ultrafuzzy set, the relationship between **X** and y can be expressed in the form:

IF $\mathbf{X}$ is $\mathbf{A}_i$ THEN y is $[c-, c+]_i$ $i=1...n$

where [c-, c+] specifies the ultrafuzzy interval, as shown in Figure 3.

**** Insert Figure 3 about here ****

Without loss of generality, we do not differentiate between situations where the rules are elicited from different experts, or from a single decision maker, due to the fuzzy nature of decision making. Any decision maker - a single human being or an organization - which can be thought of as having a unitary interest motivating its decisions, can be treated as an individual when the utility of a decision is being considered (Luce & Raiffa 1957, p13).

In practice, the knowledge available for deriving a ultrafuzzy interval is usually very limited. We often do not have enough information to justify an assumed distribution of y. Estimating the fuzziness of c- and c+ is even more controversial. More importantly, from our point of view, the "actual" fuzzy decision which is made is subjective, based on the particular decision environment. The modeling of fuzzy decision making should based on considerations of how to incorporate information provided by both historical data and the current decision situation. From this point of view, decision making models based on distribution functions often have drawbacks due to their after-the-fact nature (Archer & Wang 1991). In our approach we try to model decision making based on a fundamental consideration that decision making behavior is rational instead of random.

We now investigate ultrafuzzy functions in more detail. In Figure 3, the lower fuzzy function provides information that the membership y value for a given **X** should not be below the specified value according to available evidences. The upper fuzzy function provides information that the membership y value for a given **X** should not be above the specified value according to available evidence. These two functions can be treated as a pair of belief and plausibility functions respectively (Shafer 1976; Zadeh 1978). Note the following relationships represented by the two fuzzy functions (Shafer 1976; Zadeh 1978):

$$Pl_Y(\mathbf{X}) + Pl_{-Y}(\mathbf{X}) \geq 1 \tag{2.1}$$

$$Bel_Y(\mathbf{X}) + Bel_{-Y}(\mathbf{X}) \leq 1 \tag{2.2}$$

$$Pl_Y(\mathbf{X}) + Bel_{-Y}(\mathbf{X}) = 1 \tag{2.3}$$

$$Bel_Y(\mathbf{X}) + Pl_{-Y}(\mathbf{X}) = 1 \tag{2.4}$$

where $Pl_Y(\mathbf{X})$, $Pl_{-Y}(\mathbf{X})$, $Bel_Y(\mathbf{X})$ and $Bel_{-Y}(\mathbf{X})$ are the plausibility function that an observation belongs to Y, the plausibility function that an observation does not belong to Y, the belief function that an observation belongs to Y, and the belief function that an observation does not belong to Y, respectively. If the equality holds in equations (2.1) and (2.2), the two fuzzy functions merge into one crisp fuzzy function.

We now define two fuzzy functions based on these plausibility and belief functions for ultrafuzzy.

Definition 1: A participation function is defined as follows:

$$Par_Y(\mathbf{X}) = (Pl_Y(\mathbf{X}) + Bel_Y(\mathbf{X})) / 2 \tag{3}$$

A participation function measures the extent to which the set Y "participates" in **X** (Tsichritzis 1971). It represents the average of the y values for the two functions at a particular realization of **X** (Figure 4(a)). Given an **X**, the fuzzy decision making result should be in the region of Par(**X**), but should be fuzzy. Participation functions possess the property

$$Par_Y(\mathbf{X}) + Par_{-Y}(\mathbf{X}) = 1 \tag{4}$$

Definition 2: A moderation function is defined as follows:

$$\mathrm{Mod}_Y(\mathbf{X}) = \mathrm{Pl}_Y(\mathbf{X}-\Delta\mathbf{X}) = \mathrm{Bel}_Y(\mathbf{X}+\Delta\mathbf{X}) \tag{5}$$

where $\Delta\mathbf{X}$ is an increment of $\mathbf{X}$ along the direction of $x_1=x_2=...=x_m$. A moderation function measures the extent to which the fuzzy set Y is "moderated" by the plausibility and belief functions. It represents an average of the $\mathbf{X}$ values for the two functions at a particular realization of a fuzzy membership y value (Figure 4(b)). It can be verified readily that the moderation function possesses the property

$$\mathrm{Mod}_Y(\mathbf{X}) + \mathrm{Mod}_{-Y}(\mathbf{X}) = 1 \tag{6}$$

It can easily be proved that $\mathrm{Par}_Y(\mathbf{X})$ and $\mathrm{Mod}_Y(\mathbf{X})$ intersect at least one in the range of $\mathbf{X}$. They may also entirely overlap, where $\mathrm{Par}_Y(\mathbf{X})=\mathrm{Mod}_Y(\mathbf{X})$. The intersection set of $\mathrm{Par}_Y(\mathbf{X})$ and $\mathrm{Mod}_Y(\mathbf{X})$ is called the consensus set $\mathrm{Con}_Y(\mathbf{X})$ (Figure 4(c)).

**** Insert Figure 4 about here ****

According to classical decision making theory (Luce & Raiffa 1957), there are interactions between human perceptions of independent variables and the responses humans make in decision making. Decision making can be regarded as a kind of game to maximize utility and minimize risk. In the ultrafuzzy context, the plausibility and belief functions can be regarded as an analogy of the two decision functions used by two cooperative persons who are playing competitive games in decision making. The ultrafuzzy interval is a measure of conflict in decision making. We will use the participation and moderation functions to solve conflicts in the ultrafuzzy context.

Suppose a decision maker (DM) is given an object which has attribute values represented by $\mathbf{X}$. If a decision based on $\mathbf{X}$ is to be made in a fuzzy environment, the DM does not perceive $\mathbf{X}$ as a non-fuzzy value. In this case, the DM should make a decision $Par_Y(\mathbf{X})$ in order to minimize the biases (the Utility conflict), caused by the inconsistency between the plausibility and belief functions (Figure 5(a)). On the other hand, using the moderation function $Mod_Y(\mathbf{X})$, the DM may act to reduce the differences in the perceived $\mathbf{X}$ given a decision y (Figure 5(a)). The subjective inconsistency between the plausibility and belief functions, given a decision y, is called the Risk conflict. The final decision is a consequence of the compromise between the two Utility and Risk conflicts (see (Luce & Raiffa 1957)). A rational DM would make a decision which falls within the interval between $Par_Y(\mathbf{X})$ and $Mod_Y(\mathbf{X})$ where both the plausibility and belief fuzzy aspects are "satisfied" (Figure 5(a)) and utility difference and risk difference are minimized simultaneously. The interval between $Par_Y(\mathbf{X})$ and $Mod_Y(\mathbf{X})$ is called the equilibrium fuzzy set $Equ_Y(\mathbf{X})$. This decision region is a "no-winner-no-loser" interval. We then conclude that, given an ultrafuzzy decision region and a non-fuzzy decision object $\mathbf{X}$, the fuzzy decision making result is $[Par_Y(\mathbf{X}), Mod_Y(\mathbf{X})]$ (Figure 5(a)); that is

$$c\sim \in \mathrm{Equ}_Y(\mathbf{X} \mid \mathbf{X}{=}\mathbf{A}) \tag{7}$$

**** Insert Figure 5 about here ****

In cases where c~ is an interval instead of a single value, the decision could be fuzzy. In other words, the DM can make any decision within the interval. If the DM is risk averse, the decision would be more close to $\mathrm{Par}_Y(\mathbf{X})$, and vice versa. Unlike statistical models, this rational fuzzy decision making model does not result in a random distribution of outcomes.

We now consider cases where the **X** attributes are fuzzy. Fuzzy inputs for decision making may be due to one of two reasons. In some cases, a fuzzy decision input is explicitly defined by the environment, and the fuzzy interval is given to the DM. More often, a decision is a sequence of reasoning processes, where the decision output at a decision stage is in turn an input of the next decision stage. Assume **X** has a fuzzy interval [**A-**, **A+**]. In high dimensional cases, [**A-**, **A+**] is a hyper-cube if we assume that there is no interaction between the decision variables, and **A-** and **A+** represent the lowest and highest vertexes. We model decision making by recognizing the ultrafuzzy interval as a whole. By maximizing utility and minimizing risk, the final decision should be in a fuzzy interval such that

$$c\sim \in \mathrm{Equ}_Y(\mathbf{X} \mid \mathbf{X}{=}\mathbf{A-}) \cap \mathrm{Equ}_Y(\mathbf{X} \mid \mathbf{X}{=}\mathbf{A+}) \tag{8}$$

as shown in Figure 5(b). The decision shown in Figure 5(b) is called a strong solution since each of the "players" is satisfied with one decision aspect, either more utility or less risk. However, there may be cases where the uncertainty or fuzziness in decision input is too great, and c~ as represented in equation (8) is empty. In this case, there is no strong solution. Nevertheless, the DM may make a decision by adjusting the perception of the fuzzy input to minimize utility difference and risk difference simultaneously. The resulting decision when there is no strong solution is called a weak solution, and

$$c\sim \in [\ \inf(Equ_Y(\mathbf{X} \mid \mathbf{X=A+})),\ \sup(Equ_Y(\mathbf{X} \mid \mathbf{X=A-}))\ ] \qquad (9)$$

as shown in Figure 5(c)(d).

In the above discussion, the concept of plausibility and belief functions was used to construct conflict measures. In our opinion, a conflict is connected with fuzzy inferential evidence, and is appropriate to model human strategies in decision making. Whether in the individual decision making context or in the group decision making context (e.g. (Poole et al. 1991)), the nature of conflict measured by the plausibility and belief functions helps to explain the compromise phenomena observed in decision making.

The Back-Propagation Neural Networks and Fuzzy Membership Functions

A serious problem in application of the fuzzy theory is in implementing it without strict assumptions or sophisticated construction techniques, because fuzzy membership functions are all based on subjective judgements in a particular problem domain. Fortunately, our fuzzy decision making model can be readily implemented by using neural network interpolation techniques.

In the last few years, interest in neural networks has grown dramatically. Research into neural networks is still in its infancy, but it is expected that neural network models will be useful both as models of real brain functions and as computational devices. More complete overviews of artificial neural networks for the latter purpose may be found in Lippmann (1987) and Carpenter (1989). One of the most popular neural networks is the layered neural network, implemented with the back-propagation least mean square error (BPLMS) learning algorithm (Rumelhart et al. 1986). A back-propagation (BP) neural network with a single output node (see Figure 6) will perform a transformation from an input vector **X** to a scalar output y; that is

$$y = \Psi(\mathbf{X}) \tag{10}$$

**** Insert Figure 6 about here ****

Note that the presence of a single output node has the special meaning in our discussion context, that y is a single conclusion in a production rule. If there are t output nodes representing the t functions G_g (g=1,...t) in multiple conclusion cases, then each of these t functions G_g (g=1,...t) could be represented by a single output node neural network. This paper assumes the neural network with a single output node for our purposes.

A training data set is usually a set of samples from some functional mapping, which neural networks are able to learn through the BPLMS algorithm. Neural network training can be described as an interpolation problem. When the neural network has learned the training set, it implements a function that passes through the points defined by the training set. A full explanation of the BPLMS learning mechanism is given in Rumelhart (1986). Cybenko (1989) proved that, using the BPLMS algorithm, layered neural networks with only one hidden layer and sigmoid function nodes can closely approximate any continuous function. That is, in principle there is no need for more than one hidden layer in order to generate an arbitrary function.

As discussed earlier, each production rule to be generalized represents a pair (**X**, y) describing a certain relationship between the conditions and conclusion. If we present these data to the neural network, the neural network with the BPLMS learning algorithm will be able to learn these data and generate a "perfect fitting" function, given that the neural network has enough hidden nodes. However, our objective is to generalize knowledge represented by the individual production rules, rather than to simply recite these rules. In fact, we are more interested in the effectiveness of the neural networks in interpolation. The standard BPLMS neural networks learning algorithm, however, have difficulties in generating an effective interpolation. That is, given a set of training data for the standard BPLMS neural network, the final fitted result could be unpredictable (Kawabata 1991). This is because the linkage of a limited number of training data points may be implemented by the neural network in unnatural ways. To provide regular curve fitting based on a limited number of training points, one may use additional information, or heuristics. One possible solution is to use a form of local linear fitting, called interpolation training or k-neighbour interpolation training (Kawabata 1991). Instead of using local information to regulate the curve fitting, in which the smoothness of the curve depends on the training point sample density (Kawabata 1991), our approach uses a global smoothing training strategy suggested by (Wang 1990) and applied in (Archer & Wang 1991, 1993). In this training method, the BPLMS neural network is limited by monotonicity constraints during the training process so that it will generate a monotonic function. Monotonicity is an important characteristic of fuzzy membership functions (Zadeh 1984) (see Figure 1) which are commonly used in production rule-based expert systems and represented in CF values (Buchanan and Shortliffe 1985). This is a basic principle we have used in applying neural network techniques to generate fuzzy membership functions (Wang 1994).

According to Wang (1994), a crisp fuzzy function can be generated using a single monotonic neural network. In the case of crisp fuzzy functions, there is no conflict involved in decision making, as discussed earlier. However, in cases where human knowledge does not conform to a crisp fuzzy membership function, a pair of fuzzy functions $Pl_Y(\mathbf{X})$ and $Bel_Y(\mathbf{X})$ are to be found to form an ultrafuzzy set for a specific problem. Accordingly, two elemental neural networks, NN_{Pl} and NN_{Bel} are needed. It is worth noting that, from the point of view of pure fuzzy set theory, $Pl_Y(\mathbf{X})$ and $Bel_Y(\mathbf{X})$ themselves are fuzzy and do not really exist, or could be arbitrarily defined. Our purpose in using $Pl_Y(\mathbf{X})$ and $Bel_Y(\mathbf{X})$ is simply to define a region for providing uncertainty information to decision makers. In the current context, the development of the functions $Pl_Y(\mathbf{X})$ and $Bel_Y(\mathbf{X})$ depends on available human knowledge. In cases where the available data are collected through a carefully designed knowledge acquisition procedure and carry little noise, one may use the actual frontiers of a data set (cf.(Keeney & Raiffa 1976)) to define $Pl_Y(\mathbf{X})$ and $Bel_Y(\mathbf{X})$. However, when the data carry much noise, only one outlier could distort the entire region, resulting in inadequate uncertainty information. In order to find $Pl_Y(\mathbf{X})$ and $Bel_Y(\mathbf{X})$ for which the ultrafuzzy region effectively and efficiently covers the data points, we developed an algorithm (Wang & Archer 1994) to find a pair of fuzzy functions which are less influenced by outliers, but without making any statistical assumptions. The basic idea of the method is as follows. Suppose that we train the neural networks with an initial data set. If the ultrafuzzy region is too loose due to a outlier which carries much noise for decision making, then we reduce the region between the pair $Pl_Y(\mathbf{X})$ and $Bel_Y(\mathbf{X})$ by excluding an extreme point from the training data set. A compromise can be found by employing an iterative procedure. If reducing the region does not improve the compactness of the ultrafuzzy region, then the iterative procedure stops (Wang & Archer 1994). This paper places the emphasis on the fuzzy decision process, and simplifies the issue of finding $Pl_Y(\mathbf{X})$ and $Bel_Y(\mathbf{X})$ by assuming that the available knowledge for fuzzy decision making is true, and that the data representing the expert knowledge carries little noise. Thus, each of the data points representing an expert's opinion has equal power. After finding $Pl_Y(\mathbf{X})$ and $Bel_Y(\mathbf{X})$ by training the two neural networks NN_{Pl} and NN_{Bel}, $Par_Y(\mathbf{X})$ is readily determined since the value of $Par_Y(\mathbf{X})$ is simply the average of the output of neural networks NN_{Pl} and NN_{Bel}. Thus, there is no need to construct a neural network for function $Par_Y(\mathbf{X})$. However, we must employ a neural network to build the function $Mod_Y(\mathbf{X})$. Based on the values of the sample data points on the generated functions $Pl_Y(\mathbf{X})$ and $Bel_Y(\mathbf{X})$, we can find a data set for generating the function $Mod_Y(\mathbf{X})$, according to Definition 2 and equation (5). Using these data, we can then train the neural network NN_{Mod} under the monotonicity constraints and obtain function $Mod_Y(\mathbf{X})$. After generating the three fuzzy functions, the decision maker will be able to make a decision, by using the neural networks in response to a given decision input according to the fuzzy decision making model discussed earlier in this paper.

A Fuzzy Decision Making Algorithm

Suppose there is an available data set which represents decision preferences of decision maker(s) in a fuzzy environment, based on historical data, surveys, or subjective estimations. According to the foregoing discussion, the DM may wish to use an algorithm that helps to make a decision which will maximize utility and minimize risk. We summarize such an algorithm as follows. The algorithm consists of two major parts. Part (1) generates the fuzzy participation and moderation functions for the ultrafuzzy analysis, based on available human knowledge which is used to train the neural networks. Part (2) produces decision support information according to the fuzzy decision making models which were generated from the prior data, in response to a non-fuzzy or fuzzy decision input.

Part (1):

Step 1. Verify the monotonic relationship between decision input $\mathbf{X}$ and fuzzy decision response y, based on common knowledge of the relationship. If it is not satisfied, then transform or decompose $\mathbf{X}$ such that the monotonic relationship is verified (cf.(Zadeh 1984)).

Step 2. Collect a data set S representing decision making knowledge, based on subjective estimates, historical records of decision preferences, or surveys. Each observation is a data point $s(\mathbf{X}, y) \in S$ in the decision space.

Normalize $\mathbf{X}$ on [0,1] and y on [y_{min}, y_{max}] for calculation purposes in neural network training, where [y_{min}, y_{max}] is a desired range within (0,1); e.g.,[0.2,0.8] (see (Wang and Archer 1994)).

Step 3. Find sets S_{Pl} and S_{Bel} such that: $s \in S_{Pl}$ if decision s is a superior-frontier point of S, and $s \in S_{Bel}$ if s is an inferior-frontier point of S (cf.(Keeney & Raiffa 1976)).

Train the neural networks NN_{Pl} and NN_{Bel} with S_{Pl} and S_{Bel}, respectively, under monotonicity constraints, and find the fuzzy functions $Pl_Y(\mathbf{X})$ and $Bel_Y(\mathbf{X})$.

$Par_Y(\mathbf{X})$ is then determined by averaging the NN_{Pl} and NN_{Bel} outputs.

Step 4. Find an artificial data set S_{Mod} by the following three sub-steps:

Step 4.1. For each observation $s \in S_{Pl}$ or $s \in S_{Bel}$ find its $Pl_Y(\mathbf{X}_s)$ or $Bel_Y(\mathbf{X}_s)$ value.

Step 4.2. Find its symmetrical points s' according to Definition 2 and equations (5) such that

$Bel_Y(\mathbf{X}_{s'}) = Pl_Y(\mathbf{X}_s)$ if $s \in S_{Pl}$,

$Pl_Y(\mathbf{X}_{s'}) = Bel_Y(\mathbf{X}_s)$ if $s \in S_{Bel}$.

Step 4.3. Find the middle point s" of each segment [s, s']. These points s" constitute S_{Mod}.

Step 5. Train neural network NN_{Mod} with S_{Mod} under the monotonic constraints. $Mod_Y(\mathbf{X})$ is then represented by the trained

NN_{Mod}.

Part (2):

Step 6. Present a new decision input to the neural networks. The decision conclusion is derived as follows.

Case 1: When the decision input **A** is non-fuzzy, the fuzzy decision conclusion is an interval:

$c\sim = [\min\{Par_Y(\mathbf{A}), Mod_Y(\mathbf{A})\}, \max\{Par_Y(\mathbf{A}), Mod_Y(\mathbf{A})\}]$.

Case 2: When the decision input **A~** is fuzzy, with its interval [**A-**, **A+**],

Case 2.1: When

$\min\{Par_Y(\mathbf{A+}), Mod_Y(\mathbf{A+})\} \le \max\{Par_Y(\mathbf{A-}), Mod_Y(\mathbf{A-})\}$,

$c\sim = [\min\{Par_Y(\mathbf{A+}), Mod_Y(\mathbf{A+})\}, \max\{Par_Y(\mathbf{A-}), Mod_Y(\mathbf{A-})\}]$

is a strong decision making solution.

Case 2.2: When

$\min\{Par_Y(\mathbf{A+}), Mod_Y(\mathbf{A+})\} > \max\{Par_Y(\mathbf{A-}), Mod_Y(\mathbf{A-})\}$,

$c\sim = [\max\{Par_Y(\mathbf{A-}), Mod_Y(\mathbf{A-})\}, \min\{Par_Y(\mathbf{A+}), Mod_Y(\mathbf{A+})\}]$

is a weak decision making solution.

Step 7. It may be desirable to transform the neural network output values to the initial scale [0,1] by de-normalizing the neural network output values according to (Wang and Archer 1994).

Conclusion

In this paper, a fuzzy decision making model is developed, and an algorithm in implementing the model using neural networks is suggested. According to fuzzy set theory, it is asserted that an ultrafuzzy set which is built based on historical data, subjective judgements, and expert knowledge can serve as a foundation in fuzzy decision making. Instead of using probability or possibility distributions, the model applies the classical decision making theory of maximum utility and minimum risk. There are two advantages in using classical decision making theory over distribution-based methods. First, on a practical level, probability or possibility distributions are often difficult to verify in a fuzzy decision making environment, while the classical decision model does not require such strict assumptions. Second, decision making models based on probability of possibility distributions may result in divergent decision behavior. The fuzzy decision model developed in this paper does not have this pitfall.

We use fuzzy neural network models to implement our fuzzy decision making model. The algorithm suggested in this paper can be applied in supporting decision making in a fuzzy environment, where only limited decision data are available, and also to approximate reasoning in expert systems.

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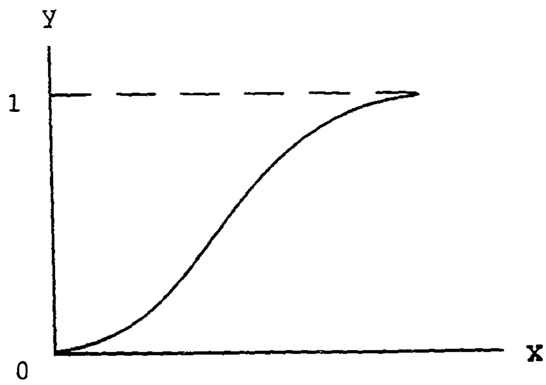
Figure Captions



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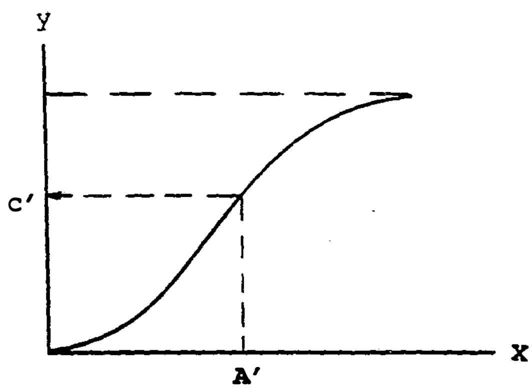

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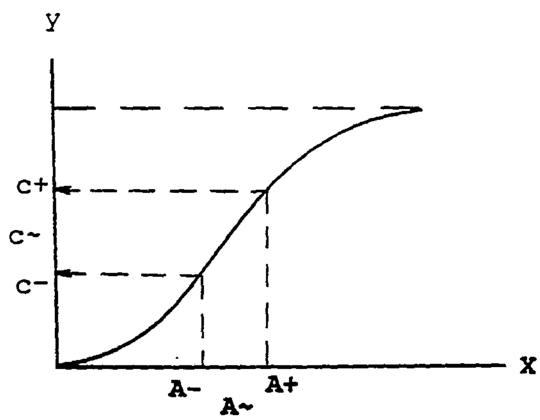

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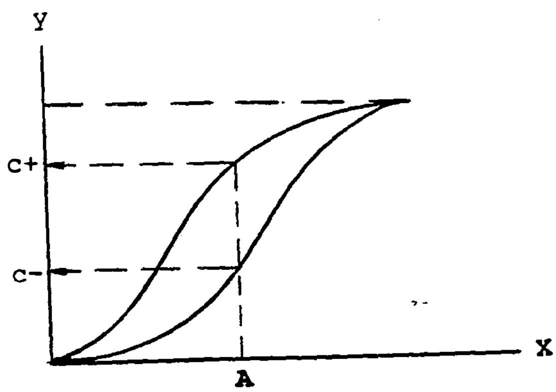

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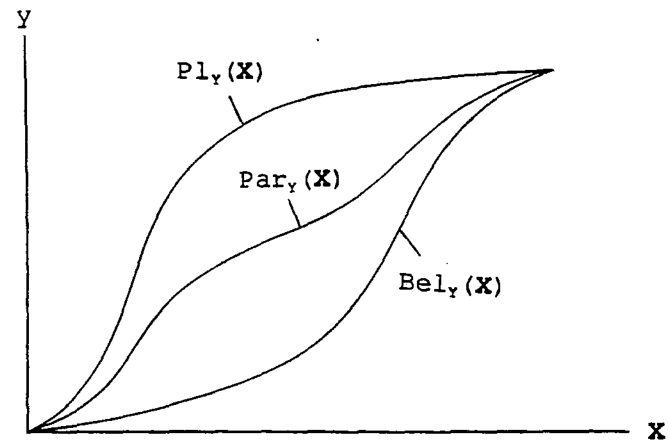

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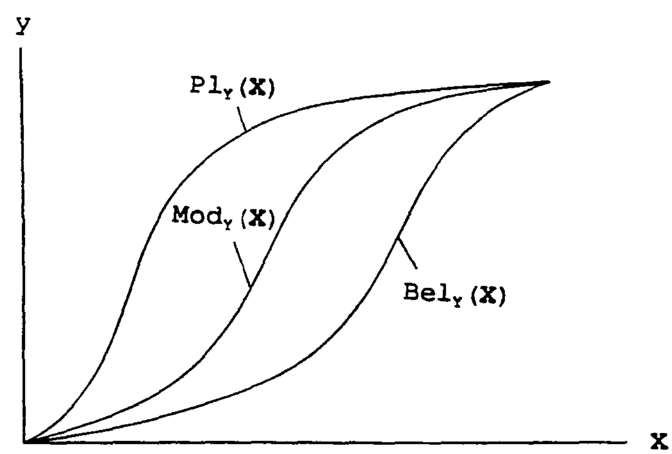

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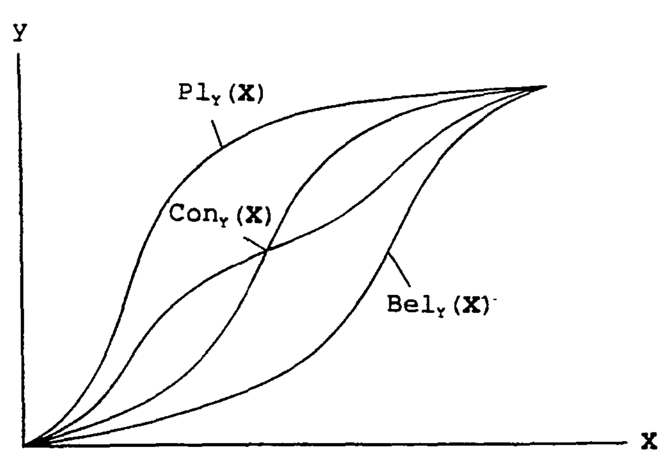

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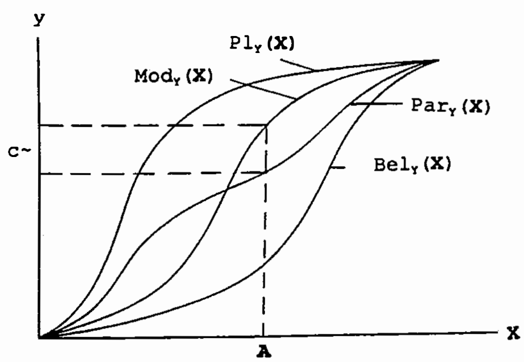

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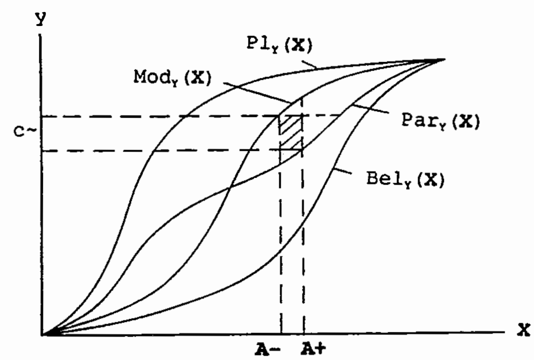

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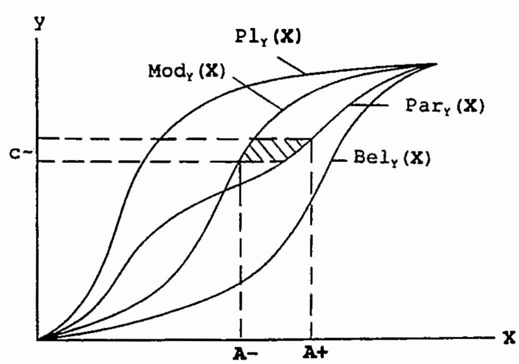

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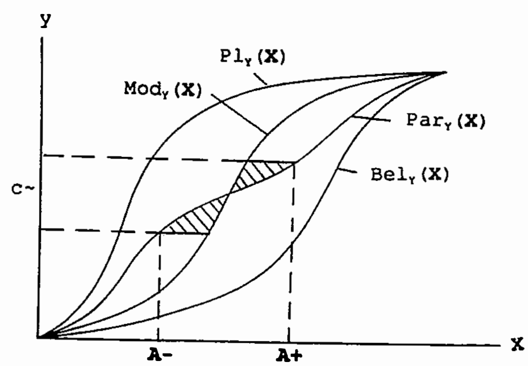

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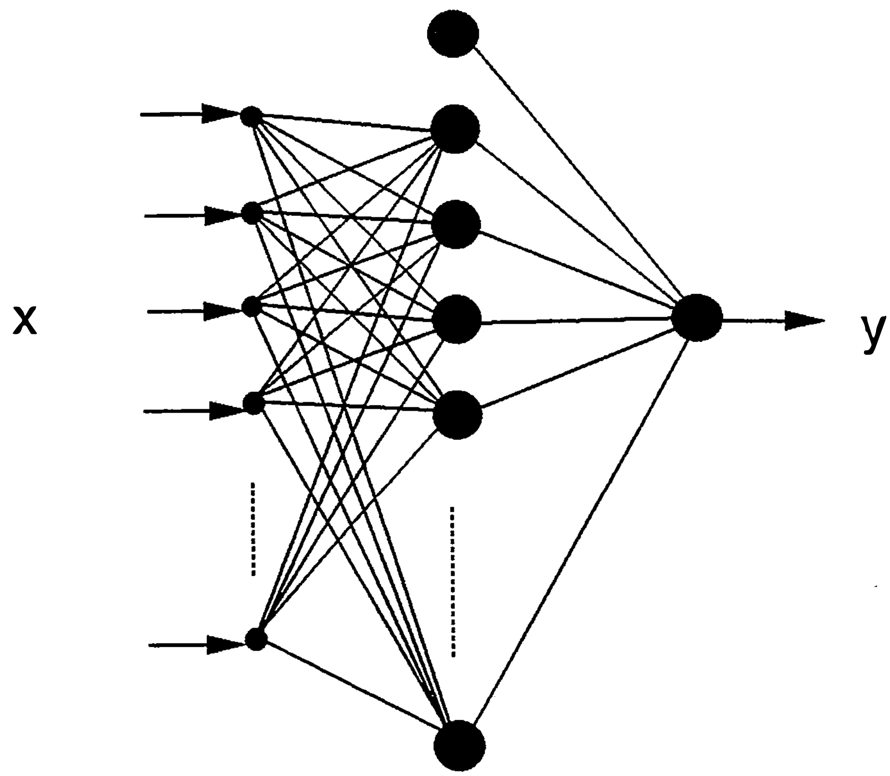

Fig.6

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