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DISCRETE LOT STREAMING IN TWO-MACHINE FLOW SHOPS

By

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Working Paper # 405

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Abstract

Lot streaming is the process of splitting a job or lot to allow overlapping between successive operations in a multistage production system. This use of transfer lots usually results in a substantially shorter makespan for the corresponding schedule. In this paper, we study the discrete version of the two-machine flowshop problem to minimize the makespan. We present new insights into the structure of optimal schedules, which lead to a strongly polynomial solution for the problem.

1 Introduction

Lot streaming is the process of using transfer batches to move the processed portion of a production lot to downstream machines so that the makespan of the schedule can be shortened and the work-in-process inventory levels can be lowered. The term was introduced by Reiter [19], but the idea has been considered many times under different names. The increased interest in its applications over the last few years is probably due to the fact that it is consistent with the Just-In-Time (JIT) philosophy of making small or single unit sublots and it also agrees with the basic idea of the commercially successful OPT scheduling package [8], [10].

Szendrovits [20] analyzes the lot streaming problem in a flow shop for a single job with equal sublot sizes. Goyal [11] finds the optimal sublot sizes in Szendrovits' model. Moily [15], Jacobs and Bragg [13], Kulonda [14] and Graves and Kostreva

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[12] also demonstrate reductions in production time and cost by using transfer lots. Steiner and Truscott [21] find the optimal lot streaming schedules in an open shop with equal size transfer lots and no idling on the machines.

Many papers on lot streaming consider the objective of minimizing the makespan in an m-machine flow shop where each item is processed on the m machines in the order $1, \ldots, m$. Trietsch, in [22] and [23], and Baker [1] independently develop a conceptual framework for the problem. They present a classification scheme and review the most important results in [24]. Most papers study the *continuous version* of the problem where it is assumed that the sublot sizes can be fractional. The sublots are *consistent* if their size does not change between machines, otherwise they are *variable*. Baker [1] shows that linear programming can be used to find the consistent sublot sizes which minimize the makespan. When items become available for processing at the next machine after the current machine finished processing the last item in their sublot (batch), we have *batch availability*. In this case, Potts and Baker [17] show that it is sufficient to consider consistent sublot sizes on the first two machines, and on the last two machines, for a single job. An efficient direct solution — which does not use linear programming — is presented for the two-machine case in [17] and in [22]. Glass et. al. [9] develop directly the continuous solution to minimize the makespan for a single job in a three-stage production process, by analyzing the structure of the optimal solution. These results are extended by Chen and Steiner, in [6] and [7], to the case with detached and attached job setup times. Cetinkaya and Gupta [4] solve the continuous lot streaming problem for a single job in a flow shop with the total flow time criterion.

Another lot streaming model uses the assumption of *item availability* when individual items become available for processing at the next machine as soon as they are finished on the current machine (unit size transfer lots). As it is pointed out in [24], there is no difference between item and batch availability in the two-machine case. Vickson and Alfredsson [26] solve the makespan minimization problem in the two-machine flow shop. The same problem is solved with detached setups in [5] and with attached setups in [2]. Warrillow [27] studies the three-machine case with multiple jobs and setups.

Much less is known about the *discrete version* of lot streaming problems where it is required that each sublot size must be integer. Simple examples [24] show that the solution of the discrete version of a problem could be substantially different from the best continuous solution. Of course one could use the linear programming formulation of Baker [1] for minimizing the makespan in a flow shop with consistent sublots, but this would require finding the best integer solution for the linear program, for which no efficient algorithm is available.

In this paper, we consider the problem of minimizing the makespan by splitting a single job into s *integer sized* sublots in a two-machine flow shop. The job has U items in it, and each item has positive processing times p_1 and p_2 on machine 1 and 2, respectively. It is known, [17] and [24], that it is sufficient to consider only consistent sublot sizes even for this discrete version of the problem. Trietsch and Baker [24] gave a dynamic programming algorithm, which solves the two- and three-machine problem in $O(s^2U)$ time. Cetinkaya [3] extends this to include job setup and removal times. Vickson [25] and Cetinkaya [3] independently show that the lot streaming problem for multiple jobs in a two-machine flow shop decomposes into an easily idenfiable sequence of single job problems, even with job setup times and sublot transfer times. Vickson also presents a new linear programming formulation for the single job problem, and proves that, using bisection search, one can find an integer valued solution to the model in $O(s \log U)$ time. We present a *strongly polynomial* algorithm which finds an optimal solution in $O(s)$ time if $p_1 \neq p_2$, and in constant time when $p_1 = p_2$. What is perhaps more important, we provide new insights into the structure of the optimal solutions for the discrete version of the problem. Among these, we show that the shortest makespan realizable with integer valued sublots is *always very close* to the continuous optimum for the problem. In other words, the integrality requirement does *not* significantly reduce the savings achievable in the makespan by lot streaming.

The paper is organized as follows. Section 2 gives a network representation for the problem. Section 3 contains the detailed analysis of the structure of the optimal solutions. The next section proves worst-case bounds for the approximations by "equal" sublots and the adjusted continuous solution. Section 5 presents the polynomial time solution for the problem. A summary and conclusions are presented in Section 6.

2 Network representation

In the two-machine case there is only one set of sublots (transfers from machine 1 to machine 2), so we can assume without loss of generality that the sublots are consistent. Let x_j represent the jth sublot size and let $C_{i,j}$ denote the completion time of sublot j on machine i ($i = 1, 2, \quad j = 1, 2, ..., s$). The following constraints must be satisfied by any feasible solution.

1) Machine capacity constraints :

$$C_{i,j} \geq C_{i,j-1} + p_i\, x_j \qquad (\; i = 1, 2, \quad j = 2, ...s \;);$$

2) Production constraints :

$$C_{2,j} \geq C_{1,j} + p_2\, x_j \qquad (\; j = 1, 2, ..., s \;);$$

3) Initialization constraint :

$C_{1,1} \geq p_1 x_1$

4) $x_j \geq 0$ is integer. $\quad\quad\quad\quad$ $(j = 1, 2, ..., s)$.

Following the approach in [16], such a solution can be represented by a network $N(x)$ which contains a vertex for each sublot on every machine (see Fig. 1). Machine i is denoted by M_i for $i = 1, 2$. The directed arc from vertex $(1, j)$ to vertex $(2, j)$ represents the production constraint that sublot j can be processed on M_2 only after it is completed on M_1. The directed arc from vertex (i, j) to vertex $(i, j+1)$ $(1 \leq j < s)$ represents the machine capacity constraint that sublot $(j+1)$ can start on M_i only when the jth sublot is completed on it. The vertex (i, j) has weight $p_i x_j$ $(1 \leq i \leq 2,\ 1 < j \leq s)$, where x_j is an integer.

Using the network representation, the objective becomes to determine the sublot sizes which minimize the length of the longest path in the network, where the length of any path is the sum of the weights of the vertices on it. Any path from vertex $(1, 1)$ to $(2, s)$ is called *maximal* and any longest path is referred to as a *critical path*. A subpath of a (critical) path is called a *(critical) segment*. We call sublot j *critical* in $N(x)$ if there is a critical segment containing the arc from $(1, j)$ to $(2, j)$. We use $M(x)$ to denote the length of a critical path, and $M_i(x) = p_1 \sum_{l=1}^{i} x_l + p_2 \sum_{l=i}^{s} x_l$ to denote the length of the path $(1,1) - ... - (1,i) - (2,i) - ... - (2,s)$ $(i = 1, 2, ..., s)$ for sublot sizes x.

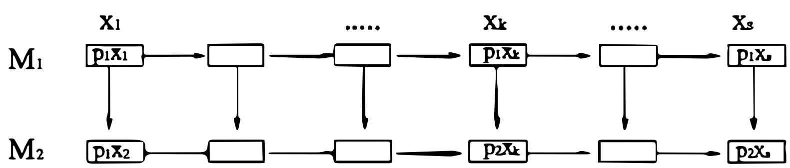

Figure 1: The network for a solution

Lemma 1 $M(x) \geq \min\{p_1, p_2\} + U \max\{p_1, p_2\}$.

Proof. First we assume $p_1 \leq p_2$, and let i be the first sublot with positive sublot size. By looking at the length of the path $(1,1) - ... - (1,i) - (2,i) - ... - (2,s)$, we see that

$$M(x) \geq p_1 x_i + p_2 \sum_{l=i}^{s} x_l = p_1 x_i + p_2 U \geq p_1 + p_2 U.$$

If $p_1 > p_2$, we can similarly show that $p_2 + p_1 U$ is a lower bound by looking at the path $(1,1) - ... - (1,j) - (2,j) - ... - (2,s)$, where j is the last positive sublot. □

Lemma 2 *If $U \leq s$, the optimal sublot sizes are $x_i = 1$ for $1 \leq i \leq U$ and $x_j = 0$ for $U < j \leq s$.*

Proof. If $p_1 \leq p_2$, then it is clear that the path $(1,1) - (2,1) - ... - (2,s)$ is critical in $N(x)$, with length

$$M(x) = p_1 x_1 + p_2 \sum_{l=1}^{s} x_l = p_1 + p_2 U,$$

which is a lower bound by Lemma 1. Therefore, x is optimal.

If $p_1 > p_2$, then it is clear that the path $(1,1) - ... - (1,U) - (2,U) - ... - (2,s)$ is critical in $N(x)$, with length

$$M(x) = p_2 x_U + p_1 \sum_{l=1}^{U} x_l = p_2 + p_1 U,$$

which is a lower bound by Lemma 1. Therefore, x is optimal indeed. □

In view of Lemma 2, we assume $U > s$ for the remainder of the paper.

3 Structure of optimal solutions

We must distinguish three cases, depending on whether $p_1 < p_2$, $p_1 = p_2$ or $p_1 > p_2$.

Case 1 $p_1 < p_2$

If sublot k is critical, then the path $(1,1) - (1,2) - ... - (1,k) - (2,k) - ... - (2,s)$ is a critical path, and

$$M(x) = M_k(x) = p_1 \sum_{i=1}^{k} x_i + p_2 \sum_{i=k}^{s} x_i.$$

When the optimal makespan is equal to the lower bound of Lemma 1, then the first sublot is critical and has only one item in it. We will refer to this situation by saying that the *extreme solution* is optimal. Potts and Baker [17] have proved that every sublot must have a positive size in the optimal solution for the continuous version of the two-machine problem. In the following theorem, we show that the same property holds for the discrete version of the problem, if the extreme solution is not optimal.

Theorem 3 *If $p_1 < p_2$ and the first sublot is not critical or it is critical but $x_1 > 1$, then all sublot sizes are positive, i.e., $x_i > 0$ $(1 \leq i \leq s)$ in* every *optimal solution.*

Proof. Suppose there is an optimal solution in which some sublot sizes are zero. W.l.o.g. it can be written as $x = (x_1, ..., x_r, 0, ..., 0)$, where $x_i > 0$ for $1 \leq i \leq r$. We construct alternative sublot sizes $x' = (x'_1, ..., x'_s)$ with a shorter critical path, which yields a contradiction with x being optimal.

Let $x'_1 = 1$, $x'_i = x_{i-1}$ for $2 \leq i \leq k$ and $x'_{k+1} = x_k - 1$, where k is the last critical sublot in the network $N(x)$, and $x'_j = x_{j-1}$ for $k+1 < j \leq s$.

First we show that $x_k > 1$ if $k \geq 2$ (if $k = 1$, then $x_k > 1$ by assumption). Suppose it was not, i.e., $x_k = 1$. Segment $(1, k-1) - (1, k) - (2, k)$ should be at least as long as the segment $(1, k-1) - (2, k-1) - (2, k)$, as k is a critical sublot. Therefore, $p_2 x_{k-1} \leq p_1 x_k = p_1$, which obviously violates the assumption $p_1 < p_2$.

Now we look at the length of maximal paths in $N(x')$. For the path containing segment $(1,1) - (2,1)$, we have

$$M_1(x') = p_1 x'_1 + p_2 \sum_{l=1}^{s} x'_l = p_1 + p_2 U.$$

If lot 1 is not critical in $N(x)$, then

$$M_1(x') = p_1 + p_2 U \leq p_1 x_1 + p_2 U = M_1(x) < p_1 \sum_{l=1}^{k} x_l + p_2 \sum_{l=k}^{s} x_l = M(x). \qquad (1)$$

If lot 1 is critical, then $x_1 > 1$ by assumption, so we have

$$M_1(x') = p_1 + p_2 U < p_1 x_1 + p_2 U = M(x). \qquad (2)$$

For the path containing segment $(1, i) - (2, i)$, where $1 < i < k+1$, we have

$$\begin{aligned} M_i(x') &= p_1 \sum_{l=1}^{i} x'_l + p_2 \sum_{l=i}^{s} x'_l \\ &= p_1 \sum_{l=1}^{i-1} x_l + p_2 \sum_{l=i-1}^{s} x_l + p_1 - p_2 \\ &< M(x), \end{aligned} \qquad (3)$$

where the last inequality is strict because $p_1 < p_2$.

For the path containing segment $(1, k+1) - (2, k+1)$, we have

$$\begin{aligned} M_{k+1}(x') &= p_1 \sum_{l=1}^{k+1} x_l' + p_2 \sum_{l=k+1}^{s} x_l' \\ &= p_1 \sum_{l=1}^{k} x_l + p_2 \sum_{l=k}^{s} x_l - p_2 \\ &< M(x). \end{aligned} \quad (4)$$

For the path containing segment $(1, i) - (2, i)$, where $k+1 < i \leq s$, we have

$$\begin{aligned} M_i(x') &= p_1 \sum_{l=1}^{i} x_l' + p_2 \sum_{l=i}^{s} x_l' \\ &= p_1 \sum_{l=1}^{i-1} x_l + p_2 \sum_{l=i-1}^{s} x_l \\ &< M(x), \end{aligned} \quad (5)$$

where the last inequality is strict because of the definition of k.

Combining (1), (3), (4) and (5), we get $M(x') < M(x)$, yielding a contradiction with the optimality of x. □

Potts and Baker [17] have shown that each maximal path has the same length and is critical in the optimal network for the continuous problem. The following theorem shows that this is almost true for an integer optimal solution too, unless we are dealing with the extreme case.

Theorem 4 *Let x be an optimal solution. If $p_1 < p_2$ and the first sublot is not critical in $N(x)$ or it is critical but $x_1 > 1$, then there is an optimal solution x' for which* every *maximal path has a length which is within p_2 of the longest path, i.e., $M(x') < M_i(x') + p_2$ for $1 \leq i \leq s$.*

Proof. For the maximal path containing $(1, s) - (2, s)$, if $M(x) \geq M_s(x) + p_2$, then we construct an alternative solution $x' = (x_1', ..., x_s')$, such that x' is still optimal and $M(x') - M_s(x') < p_2$.

Let $x_j' = x_j$ for $j = 1, ..., s-2$, $x_{s-1}' = x_{s-1} - 1$ and $x_s' = x_s + 1$. We note that $x_{s-1}' \geq 0$ by Theorem 3.

Consider the lengths of the maximal paths from $(1, 1)$ to $(2, s)$ in $N(x')$.

For the path containing $(1,j)-(2,j)$, where $j < s-1$, we have

$$\begin{aligned} M_j(x') &= p_1 \sum_{l=1}^{j} x_l' + p_2 \sum_{l=j}^{s} x_l' \\ &= p_1 \sum_{l=1}^{j} x_l + p_2 \sum_{l=j}^{s-2} x_l + p_2(x_{s-1}-1) + p_2(x_s+1) \\ &= M_j(x). \end{aligned} \tag{6}$$

For the path containing $(1,s-1)-(2,s-1)$, we have

$$\begin{aligned} M_{s-1}(x') &= p_1 \sum_{l=1}^{s-1} x_l' + p_2 \sum_{l=s-1}^{s} x_l' \\ &= p_1 \sum_{l=1}^{s-1} x_l + p_2 \sum_{l=s-1}^{s} x_l - p_1 \\ &< M_{s-1}(x). \end{aligned} \tag{7}$$

For the path containing $(1,s)-(2,s)$, we have

$$\begin{aligned} M_s(x') &= p_1 \sum_{l=1}^{s} x_l' + p_2 x_s' \\ &= p_1 \sum_{l=1}^{s} x_l + p_2 x_s + p_2 \\ &\leq M(x), \end{aligned} \tag{8}$$

where the inequality follows from the assumption $M_s(x) + p_2 \leq M(x)$.

By combining (6), (7) and (8), it follows that x' is still an optimal solution. By Theorem 3, we must have $x_i' > 0$ for $i = 1, 2, ..., s$. Therefore, we can repeat the preceding procedure until the theorem holds for the path containing $(1,s)-(2,s)$, i.e., $M(x') < M_s(x') + p_2$.

We can similarly prove that the theorem holds for $i = s-1, ..., 2$. So suppose x' is an optimal solution for which $M(x') < M_i(x') + p_2$, $2 \leq i \leq s$. By Theorem 3, $x_i' > 0$ for $2 \leq i \leq s$.

Consider now the path $(1,1)-(2,1)-...-(2,s)$. Suppose we had $M(x') \geq M_1(x') + p_2$. We can construct alternative sublot sizes $x'' = (x_1'', ..., x_s'')$ yielding a shorter schedule, which contradicts the assumption that x' is optimal.

Let $x_1'' = x_1' + 1$, $x_i'' = x_i'$ for $2 \leq i < s$ and $x_s'' = x_s' - 1$. We have

$$M_1(x'') = p_1 x_1'' + p_2 \sum_{l=1}^{s} x_l''$$

$$
\begin{aligned}
&= p_1 x_1^{'} + p_1 + p_2 \sum_{l=1}^{s} x_l^{'} \\
&= M_1(x^{'}) + p_1 \\
&< M(x^{'}),
\end{aligned} \tag{9}
$$

where the inequality holds since $p_1 < p_2$ and we assumed $M(x^{'}) \geq M_1(x^{'}) + p_2$.

For the other paths containing $(1, i) - (2, i)$, where $2 \leq i < s$, we have

$$
\begin{aligned}
M_i(x^{"}) &= p_1 \sum_{l=1}^{i} x_l^{"} + p_2 \sum_{l=i}^{s} x_l^{"} \\
&= p_1 \sum_{l=1}^{i} x_l^{'} + p_2 \sum_{l=i}^{s} x_l^{'} + p_1 - p_2 \\
&= M_i(x^{'}) + p_1 - p_2 \\
&< M(x^{'}).
\end{aligned} \tag{10}
$$

For the path containing $(1, s) - (2, s)$, we have

$$
\begin{aligned}
M_s(x^{"}) &= p_1 \sum_{l=1}^{s} x_l^{"} + p_2 x_s^{"} \\
&= p_1 \sum_{l=1}^{s} x_l^{'} + p_2 x_s^{'} - p_2 \\
&= M_s(x^{'}) - p_2 \\
&< M(x^{'}).
\end{aligned} \tag{11}
$$

Combining (9), (10) and (11), we obtain a contradiction with the optimality of $x^{'}$, so $x^{'}$ must satisfy all the conditions of the theorem. □

Theorem 4 states that there is an optimal solution in which *all* maximal paths have length close to the length of a critical path, i.e., the network is "balanced" in this sense. This very strong property will be exploited in our algorithm for finding an optimal solution.

The formula for the optimal makespan in the continuous problem shows that the makespan strictly decreases if we increase the number of sublots. The following theorem shows that this is also true in the discrete case, unless the extreme solution is optimal.

Theorem 5 *If $p_1 < p_2$ and the first sublot is not critical in the optimal $N(x)$ or it is critical but $x_1 > 1$, then the addition of one more sublot strictly decreases the makespan.*

Proof. Let k be the last critical sublot in $N(x)$. We construct alternative sublot sizes $x^{'} = (x^{'}_1, ..., x^{'}_{s+1})$ with a shorter critical path.

Let $x^{'}_1 = 1$, $x^{'}_{k+1} = x_k - 1$ and $x^{'}_{i+1} = x_i$, $i \in \{1, ..., s\} - \{k\}$.

For the maximal path containing segment $(1,1) - (2,1)$ in $N(x^{'})$, we have

$$M_1(x^{'}) = p_1 x^{'}_1 + p_2 \sum_{l=1}^{s+1} x^{'}_l = p_1 + p_2 U.$$

If the first sublot is not critical in $N(x)$, we have

$$\begin{aligned} M_1(x^{'}) &\leq p_1 x_1 + p_2 \sum_{l=1}^{s} x_l \\ &< M(x). \end{aligned} \tag{12}$$

If the first sublot is critical, then $x_1 > 1$ by assumption, so we have

$$\begin{aligned} M_1(x^{'}) &< p_1 x_1 + p_2 \sum_{l=1}^{s} x_l \\ &= M(x). \end{aligned} \tag{13}$$

For the path containing segment $(1,i) - (2,i)$ in $N(x^{'})$, for $1 < i \leq k$, we have

$$\begin{aligned} M_i(x^{'}) &= p_1 \sum_{l=1}^{i} x^{'}_l + p_2 \sum_{l=i}^{s+1} x^{'}_l \\ &= M_{i-1}(x) + p_1 - p_2 \\ &< M(x). \end{aligned} \tag{14}$$

For the path containing segment $(1, k+1) - (2, k+1)$ in $N(x^{'})$, we have

$$\begin{aligned} M_{k+1}(x^{'}) &= p_1 \sum_{l=1}^{k+1} x^{'}_l + p_2 \sum_{l=k+1}^{s+1} x^{'}_l \\ &= M_k(x) - p_2 \\ &< M(x). \end{aligned} \tag{15}$$

For the path containing segment $(1,j) - (2,j)$ in $N(x^{'})$, for $k+1 < j \leq s+1$, we have

$$\begin{aligned} M_j(x^{'}) &= p_1 \sum_{l=1}^{j} x^{'}_l + p_2 \sum_{l=j}^{s+1} x^{'}_l \\ &= M_{j-1}(x) \\ &< M(x), \end{aligned} \tag{16}$$

where the last inequality is strict by the definition of k.

From (12), (13), (14), (15) and (16), we see that $x^{'}$ yields a shorter critical path than x. □

Case 2 $p_1 = p_2$

Theorem 6 *If* $p_1 = p_2$ *, then there is an optimal solution* x *and sublot* k *such that*

i) k *and every sublot before* k *is critical, i.e.,* $M_i(x) = M(x)$ *for* $1 \leq i \leq k$*;*

ii) every sublot after k *is non-critical, i.e.,* $M_j(x) < M(x)$ *for* $k < j \leq s$*.*

Proof. Let $x = (x_1, ..., x_s)$ be the optimal solution in which the last critical sublot has the smallest index k among all optimal solutions. Suppose there was an $i < k$, such that sublot i is not critical. Then segment $(1,i) - ... - (1,k) - (2,k)$ should be longer than segment $(1,i) - (2,i) - ... - (2,k)$, so we have $p_1 \sum_{l=i+1}^{k} x_l > p_2 \sum_{l=i}^{k-1} x_l$. This, in view of $p_1 = p_2$, implies

$$x_k > x_i. \tag{17}$$

We can construct alternative sublot sizes $x^{'} = (x_1^{'}, ..., x_s^{'})$ such that $x^{'}$ is also optimal and the index of the last critical sublot in $N(x^{'})$ is smaller than k, which yields a contradiction with the definition of x. Let $x_j^{'} = x_j$ for $j \in \{1, ..., s\} - \{i, k\}$, $x_i^{'} = x_i + 1$ and $x_k^{'} = x_k - 1$. We note that $x_k^{'} \geq 0$ by (17).

For the path containing $(1,j) - (2,j)$, where $1 \leq j < i$, we have

$$\begin{aligned} M_j(x^{'}) &= p_1 \sum_{l=1}^{j} x_l^{'} + p_2 \sum_{l=j}^{s} x_l^{'} \\ &\leq M(x). \end{aligned} \tag{18}$$

For the path containing $(1,i) - (2,i)$, we have

$$\begin{aligned} M_i(x^{'}) &= p_1 \sum_{l=1}^{i} x_l^{'} + p_2 \sum_{l=i}^{s} x_l^{'} \\ &= p_1 \sum_{l=1}^{i} x_l + p_2 \sum_{l=i}^{s} x_l + p_1 \\ &\leq p_1 \sum_{l=1}^{k-1} x_l + p_2 \sum_{l=k}^{s} x_l + p_1 x_i + p_1 \\ &\leq M_k(x) \\ &\leq M(x), \end{aligned} \tag{19}$$

where the second inequality holds by (17).

For the path containing $(1,j)-(2,j)$, $i<j<k$, we have

$$\begin{aligned} M_j(x') &= p_1 \sum_{l=1}^{j} x'_l + p_2 \sum_{l=j}^{s} x'_l \\ &= M_j(x) + p_1 - p_2 \\ &\leq M(x). \end{aligned} \tag{20}$$

For the path containing $(1,k)-(2,k)$, we have

$$\begin{aligned} M_k(x') &= p_1 \sum_{l=1}^{k} x'_l + p_2 \sum_{l=k}^{s} x'_l \\ &= M_k(x) - p_2 \\ &< M(x). \end{aligned} \tag{21}$$

For the path containing $(1,j)-(2,j)$, $k<j\leq s$, we have

$$\begin{aligned} M_j(x') &= p_1 \sum_{l=1}^{j} x'_l + p_2 \sum_{l=j}^{s} x'_l \\ &= M_j(x) \\ &< M(x). \end{aligned} \tag{22}$$

It follows from (18), (19), (20), (21) and (22) that x' is still optimal, but the index of the last critical sublot is less than k in $N(x')$. This yields a contradiction with the definition of x. □

The heavy lines in Fig.2 show the critical activities in the optimal network.

Case 3 $p_1 > p_2$

It is well known [17] that this problem is equivalent to its inverse, in which p_2 is the unit processing time on M_1 and p_1 is the unit processing time on M_2. The inverse problem falls under Case 1.

4 Approximations

In certain operating environments, it may be desirable to use an equal sublot policy, because of its operational simplicity. In the following, we define a discrete version

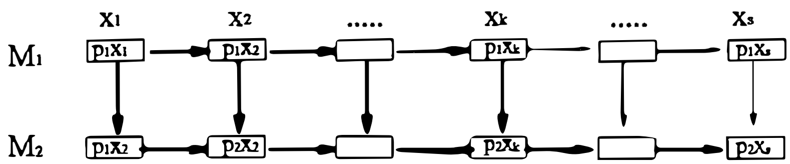

Figure 2: Optimal network structure when $p_1 = p_2$

of the "equal sublot policy" — for all cases — and provide an upper bound for its worst-case performance.

Rounded Equal Sublot Policy:

If U is divisible by s, then $x_i = U/s$ for $1 \leq i \leq s$.

If U is not divisible by s, then $\begin{cases} x_i = \lceil U/s \rceil & \text{for } 1 \leq i \leq U - s\lfloor U/s \rfloor \\ x_j = \lfloor U/s \rfloor & \text{for } U - s\lfloor U/s \rfloor < j \leq s, \end{cases}$

where $\lfloor y \rfloor$ and $\lceil y \rceil$ denote the largest integer not greater than y and the smallest integer not smaller than y, respectively.

Theorem 7 *If $p_1 < p_2$ and $M(x^*)$ and $M_{eq}(x)$ denote the optimal makespan and the makespan under the Rounded Equal Sublot Policy, respectively, then*

$$\frac{M_{eq}(x) - M(x^*)}{M(x^*)} < \frac{p_1}{p_2}\frac{1}{s} < \frac{1}{s}.$$

Proof. It is clear that $(1,1) - (2,1) - \ldots - (2,s)$ is a critical path under the Rounded Equal Sublot Policy and

$$\begin{aligned} M_{eq}(x) &= p_1 x_1 + p_2 \sum_{i=1}^{s} x_i \\ &= p_1 \lceil U/s \rceil + p_2 U. \end{aligned}$$

For the worst-case ratio, we have

$$\frac{M_{eq}(x) - M(x^*)}{M(x^*)} \leq \frac{p_1 \lceil U/s \rceil + p_2 U}{p_1 + p_2 U} - 1$$

$$
\begin{aligned}
&< \frac{p_1 + p_1 U/s + p_2 U}{p_1 + p_2 U} - 1 \\
&< \frac{p_1 U/s + p_2 U}{p_2 U} - 1 \\
&= \frac{p_1}{p_2}\frac{1}{s} \\
&< 1/s,
\end{aligned}
$$

where we have used the lower bound of Lemma 1 in the first inequality and $p_1 < p_2$ in the last one. □

Theorem 8 *For the case $p_1 < p_2$, if $x^c = (x_1^c, ..., x_s^c)$ is the optimal solution for the continuous version of the problem, with makespan M_c, then $M(x^*) < M_c + p_1$.*

Proof. Given x^c, we construct an integer solution $x' = (x_1', ..., x_s')$ such that $M(x') < M_c + p_1$: Let $x_i' = x_i^c$ if x_i^c is integer, and $x_i' = \lceil x_i^c \rceil$ for the first u sublots which are not integer in $N(x^c)$, and $x_i' = \lfloor x_i^c \rfloor$ for the rest of the sublots, where $u = U - \sum_{i=1}^{s} \lfloor x_i^c \rfloor$. Let $n(u)$ be the last index for which $x_i' = \lceil x_i^c \rceil$, i.e. the last index where we rounded up to get x_i'. Then

$$\sum_{l=1}^{n(u)} x_i' + \sum_{l=n(u)+1}^{s} x_i' = \sum_{l=1}^{n(u)} \lfloor x_i^c \rfloor + u + \sum_{l=n(u)+1}^{s} \lfloor x_i^c \rfloor = U, \tag{23}$$

so x' is an (integer valued) solution indeed. Let $\delta_i = x_i' - x_i^c$ and note that $|\delta_i| < 1$ *for* $i = 1, 2, ..., s$, and $\sum_{i=1}^{s} \delta_i = 0$, by (23). Consider the length of the maximal paths, $M_i(x')$ *for* $i = 1, 2, ..., s$, in $N(x')$:

$$
\begin{aligned}
M_i(x') &= p_1 \sum_{l=1}^{i} x_l' + p_2 \sum_{l=i}^{s} x_l' \\
&= M_i(x^c) + p_1 \sum_{l=1}^{i-1} \delta_l + p_1 \delta_i + p_2 \sum_{l=i}^{s} \delta_l & (24) \\
&= M_i(x^c) + p_1 \sum_{l=1}^{i-1} \delta_l + p_1 \delta_i - p_2 \sum_{l=1}^{i-1} \delta_l \\
&\leq M_c + p_1, & (25)
\end{aligned}
$$

where the last equality holds because $\sum_{i=1}^{s} \delta_i = 0$. □

5 Problem solution

Case 1 $p_1 < p_2$

Our approach is based on trying to find the best solution satisfying the conditions of Theorem 4. There is one case, however, when this solution may not be optimal. First we deal with this, in the next theorem.

Theorem 9 *We can determine in $O(s)$ time whether there exist lot sizes $x = (x_1, ..., x_s)$ satisfying the following additional conditions:*

i) the first sublot is critical,

ii) $x_1 = 1$.

If such a solution exists, then it is optimal and we can find it in $O(s)$ time.

Proof. Assume that the first sublot is critical and $x_1 = 1$, then segment $(1,1) - ... - (1,i) - (2,i)$ should be no longer than segment $(1,1) - (2,1) - ... - (2,i)$ for $1 < i \leq s$, i.e.,

$$p_2 \sum_{l=1}^{i-1} x_l \geq p_1 \sum_{l=2}^{i-1} x_l + p_1 x_i \text{ for } 1 < i \leq s. \tag{26}$$

Let x_i $(1 < i \leq s)$ be the largest nonnegative integer value satisfying (26), subject to the additional constraints $\sum_{j=1}^{i} x_j \leq U$. These unique values can be determined recursively for $i = 2, ..., s$, requiring $O(s)$ time in total.

If $\sum_{l=1}^{s} x_l = U$, then x is an optimal solution, since $M(x) = M_1(x) = p_1 + p_2 U$ realizes the lower bound of Lemma 1 for the optimal makespan.

If $\sum_{l=1}^{s} x_l < U$, then we show that no solution exists which would satisfy conditions i) and ii) of the theorem. Assume there were (optimal) sublot sizes $x^* = (x_1^*, ..., x_s^*)$, for which the first sublot is critical in $N(x^*)$ and $x_1^* = 1$. Then segment $(1,1) - ... - (1,i) - (2,i)$ should be no longer than segment $(1,1) - (2,1) - ... - (2,i)$ for $1 < i \leq s$, i.e., x^* must also satisfy (26). This implies, however, that $x_i^* \leq x_i$ for $2 \leq i \leq s$. Thus $\sum_{l=1}^{s} x_l^* \leq \sum_{l=1}^{s} x_l < U$, which yields a contradiction with the feasibility of x^*. □

The following result shows that when the extreme solution is optimal, it is also "nearly" balanced.

Corollary 10 *If the extreme solution x described in Theorem 9 is optimal and r is the index of the last nonzero sublot in x, then*

$$M(x) = p_1 + p_2 U < M_i(x) + p_1 \text{ for } 1 \leq i < r.$$

Proof. Each x_i $(1 \leq i < r)$ is chosen to be largest possible while satisfying (26). Therefore,

$$p_1 \sum_{l=2}^{i-1} x_l + p_1 x_i + p_1 \geq p_2 \sum_{l=1}^{i-1} x_l \text{ for } 1 < i < r. \tag{27}$$

Adding $p_2 \sum_{l=i}^{s} x_l$ to both sides of (27) proves the corollary. □

Theorem 11 *If $p_1 < p_2$ and no optimal solution satisfying the conditions of Theorem 9 exists, then the optimal sublot sizes can be obtained in $O(s)$ time.*

Proof. Let M_0 denote a trial value for the length of a critical path in the optimal network. If M_0 is the optimal value for the makespan, then there is also an optimal solution $x^0 = (x_1^0, ..., x_s^0)$ which satisfies Theorem 4.

From Theorem 4, we know that

$$M_s(x^0) > M_0 - p_2. \tag{28}$$

From the network, we get $M_s(x^0) = p_1 \sum_{i=1}^{s} x_i^0 + p_2 x_s^0 = p_1 U + p_2 x_s^0$. Therefore, $M_0 \geq p_1 U + p_2 x_s^0 > M_0 - p_2$, implying

$$\frac{M_0 - p_1 U}{p_2} \geq x_s^0 > \frac{M_0 - p_1 U}{p_2} - 1. \tag{29}$$

Note that there is a *unique* integer value for x_s^0 which satisfies (29). By Theorem 4,

$$M_0 \geq M_i(x^0) = p_1 \sum_{l=1}^{i} x_l^0 + p_2 \sum_{l=i}^{s} x_l^0 > M_0 - p_2 \text{ for } 1 \leq i < s, \tag{30}$$

which is equivalent to

$$\frac{M_0 - p_1(U - \sum_{l=i+1}^{s} x_l^0)}{p_2} \geq \sum_{l=i}^{s} x_l^0 > \frac{M_0 - p_1(U - \sum_{l=i+1}^{s} x_l^0)}{p_2} - 1 \text{ for } i = s-1, ..., 1. \tag{31}$$

The recursive solution of the inequalities (31) requires the calculation of the *unique* integer values $x_{s-1}^0, x_{s-2}^0, ..., x_1^0$ — in this order — which satisfy (31).

Therefore, we can obtain x_i^0 for $i = s-1, ..., 1$ in $O(s)$ time.

If $\sum_{i=1}^{s} x_i^0 > U$, then we show that $M(x^*) \leq M_0$. Assume there were optimal sublot sizes $x^* = (x_1^*, ..., x_s^*)$ with $M(x^*) > M_0$. We can assume without the loss of generality that x^* is the balanced solution described in Theorem 4. Thus, similarly to (31), we have

$$\frac{M(x^*) - p_1(U - \sum_{l=i+1}^{s} x_l^*)}{p_2} \geq \sum_{l=i}^{s} x_l^* > \frac{M(x^*) - p_1(U - \sum_{l=i+1}^{s} x_l^*)}{p_2} - 1 \text{ for } 1 \leq i \leq s. \tag{32}$$

If $M(x^*) > M_0$, then each of the upper and lower bounds is at least as large in (32) as the corresponding bound in (31). Thus, we have $\sum_{i=1}^{s} x_l^* \geq \sum_{i=1}^{s} x_i^0 > U$, which is in a contradiction with the feasibility of x^*. So we must have $M(x^*) \leq M_0$ indeed in this case.

If $\sum_{i=1}^{s} x_i^0 < U$, then we show that $M(x^*) > M_0$ for the balanced optimal solution x^* described in Theorem 4. Suppose it was not, i.e., $M(x^*) < M_0$. From Theorem 4, we know that

$$M(x^*) \geq M_s(x^*) = p_1 U + p_2 x_s^* > M(x^*) - p_2. \tag{33}$$

Comparing (29) with (33), we obtain

$$x_s^* \leq x_s^0. \tag{34}$$

Since x^* has to satisfy (32) again, we must have $\sum_{j=i+1}^{s} x_j^* \leq \sum_{j=i+1}^{s} x_j^0$ for $1 \leq i \leq s-1$. Therefore, $\sum_{i=1}^{s} x_i^* \leq \sum_{i=1}^{s} x_i^0 < U$, which yields the contradiction with the feasibility of x^*.

If $\sum_{i=1}^{s} x_i^0 = U$, then we show that $M(x^*) \leq M_0$. Since x^0 satisfies all the inequalities in (31), (30) also holds. Together with (29), these imply that $M_i(x^0) \leq M_0$ for $1 \leq i \leq s$. $M(x^0) = \max_{1 \leq i \leq s} M_i(x^0)$, however, so we also have $M(x^*) \leq M(x^0) \leq M_0$.

In summary, the unique integer solution to (29) and (31) can be obtained in $O(s)$ time. By Theorem 9, $M_c \leq M(x) < M_c + p_1$. Therefore, the range of M_0 should be $[M_c,\ M_c + p_1)$. From (29), there are *at most two* different integer x_s^0 values which could satisfy (29), if we use any M_0 value from this range. Therefore, the optimal solution can be obtained in $O(s)$ time, by repeating the calculations from (29) and (31) at most twice. □

Case 2 $p_1 = p_2$

Theorem 12 *If* $p_1 = p_2$*, then* $p_2U + p_1 \lceil U/s \rceil$ *is a lower bound for the optimal makespan.*

Proof. Let k be the index of the last critical sublot as defined in Theorem 6. We show first that $x_k \geq x_i$ for $i \in \{1, ..., s\} - \{k\}$.

From Theorem 6, we know that both segments $(1,i) - (1,i+1) - (2,i+1)$ and $(1,i) - (2,i) - (2,i+1)$ are critical for $1 \leq i < k$. Therefore, they should have the same length, i.e., $p_1 x_{i+1} = p_2 x_i$, implying

$$x_i = x_{i+1} \text{ for } 1 \leq i < k. \tag{35}$$

Suppose there was a sublot with $x_j > x_k$, where $j > k$. By comparing the length of $(1,1) - ... - (1,j) - (2,j) - ... - (2,s)$ with $(1,1) - ... - (1,k) - (2,k) - ... - (2,s)$, we get $M(x) = M_k(x) < M_j(x)$, a contradiction. Therefore, $\sum_{i=1}^{s} x_i \leq s x_k$. Since $\sum_{i=1}^{s} x_i = U$, we obtain

$$x_k \geq U/s. \tag{36}$$

The length of critical path $(1,1) - ... - (1,k) - (2,k) - ... - (2,s)$ is

$$\begin{aligned} M(x) &= p_1 \sum_{l=1}^{k} x_l + p_2 \sum_{l=k}^{s} x_l \\ &= p_2 \sum_{l=1}^{k-1} x_l + p_2 \sum_{l=k}^{s} x_l + p_1 x_k \\ &= p_2 U + p_1 x_k \\ &\geq p_2 U + p_1 \lceil U/s \rceil . \end{aligned}$$

Therefore, $p_2U + p_1 \lceil U/s \rceil$ is a lower bound indeed. □

Theorem 13 *For* $p_1 = p_2$*,*

i) if U *is divisible by* s*, then the optimal sublot sizes are* $x_i = U/s$ *for* $1 \leq i \leq s$;

ii) if U *is not divisible by* s*, then the optimal sublot sizes are*

$$\begin{array}{ll} x_i = \lceil U/s \rceil & \text{for } 1 \leq i \leq k \\ x_j = \lfloor U/s \rfloor & \text{for } k < j \leq s \end{array}, \quad \text{where } k = U - s \lfloor U/s \rfloor .$$

Proof. If U is divisible by s, it is clear that all paths have the same length in the network when $x_i = U/s$ for $1 \leq i \leq s$, and

$$\begin{aligned} M(x) &= p_1 x_1 + p_2 \sum_{l=1}^{s} x_l \\ &= p_1 U/s + p_2 U. \end{aligned}$$

Thus $M(x)$ equals the lower bound of Theorem 12, so x is optimal.

If U is not divisible by s and x is as defined in ii), then it is clear that the first k sublots are critical and the last $s - k$ sublots are non-critical. Thus,

$$\begin{aligned} M(x) &= p_1 x_1 + p_2 \sum_{l=1}^{s} x_l \\ &= p_1 \lceil U/s \rceil + p_2 U. \end{aligned}$$

Thus $M(x)$ equals the lower bound again, so x is optimal. □

Corollary 14 *If $p_1 = p_2$, then the optimal lot streaming schedule can be identified in constant time.*

Proof. If we do not require outputting the actual sublots, the corollary follows from the preceding theorem. □

6 Summary and concluding remarks

We have analyzed the structural properties of discrete lot streaming schedules which minimize the makespan for a single job in a two-machine flow shop. We have shown that, unless an easily recognizable extreme solution is optimal, there is always a balanced optimal schedule in which the length of every maximal path is within $\max\{p_1, p_2\}$of the optimal makespan. We have also proved that the optimal integer solution's makespan is within $\max\{p_1, p_2\}$ of the best makespan achievable with fractional sublots allowed. This is important from the practical point of view, as the use of integer size transfer lots is clearly desirable in applications. So the savings achievable in the makespan through lot streaming are *essentially the same* when we allow only integer valued sublots. We have also shown that the optimal solution can be obtained in $O(s)$ time, by exploiting its balanced nature. Since it was shown in

[25] and [3] that the problem with multiple jobs decomposes into a sequence of independently schedulable single jobs, all of these structural results are *true for the case of multiple jobs too.*

Sometimes *no-idling* may be required, i.e., each machine should be kept working without any idle time once it starts. (This was called the *contiguity of work* assumption in [20] and in [21].) It is straightforward to transform the optimal schedules into a no-idling schedule in the two-machine case. In certain situations it is desirable to have *no-wait* schedules, i.e., to be able to start the processing of each sublot on each machine immediately after it is finished on the preceding machine. It is known [17] that the optimal schedule for the continuous problem is totally balanced, resulting in a schedule which satisfies the no-wait requirement. Since we have proved that $M(x) < M_i(x) + \max\{p_1, p_2\}$ for $i = 1, 2, ..., s$, this means that the time a transfer lot may have to wait between being finished on M_1 and starting to be processed on M_2 is also minimal, i.e., less than $\max\{p_1, p_2\}$.

There are many related topics for future investigation. It seems to be natural to try to extend the results in this paper to the case including job setup times, and there is reason to believe that most of the structural insights would remain true in this case too. Another possible extension is to the case of more than two machines. Some of these problems will be studied in the future.

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