A COMMENT ON "OPTIMAL AND SYSTEM MYOPIC POLICIES FOR MULTI-ECHELON PRODUCTION/INVENTORY ASSEMBLY SYSTEMS"

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A COMMENT ON "OPTIMAL AND SYSTEM MYOPIC POLICIES FOR MULTI-ECHELON PRODUCTION/INVENTORY ASSEMBLY SYSTEMS"

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Leroy B. Schwarz and Linus Schrage [1] suggest a cost model for multi-echelon production/inventory systems in their equations (8) and (4):

$$(1)\quad \underset{\text{w.r.t.}\,Q_1}{\text{Min}} \quad C = \textstyle\sum_{j=1}^{n} (DK_j/m_j)/Q_1 + (Q_1/2)\sum_{j=1}^{n} m_j h_j$$

where: $h_j = (1+D/p_j)h'_j + 2(D/p_j)\sum_{k\varepsilon A(j)} h'_k$, and

$j=n$ denotes the first stage; $j=1$ is the last stage.

Symbols are defined on page 1287 in [1]. Among these symbols, h'_j is the "holding cost per unit time charged against the echelon stock of stage j." They define "the echelon stock of stage j as the number of units in the system which are or have passed through stage j but have as yet not been sold." This definition of h'_j may "permit some very convenient mathematical simplifications" as the authors claim, but is certainly too complex, if not confusing, for anyone who wants to apply the model in practice.

In a multi-stage production/inventory system, the unit holding cost of the product in a certain stage j is the most one could reasonably expect to determine. We call this stage unit holding cost, c_j. For instance, if the holding cost is expressed as 20 percent of the value (cost) of the product after stage j is completed, and this value is \$10.0, then the stage unit holding cost is: c_j=\$2.

The purpose of this note is to show that the cost model (1) is only true if the unit holding cost of the echelon stock is interpreted as the increment in stage unit holding costs between two adjacent stages j and (j+1).

When the cost model (1) is applied to a serial production case where $Q_1=Q_2=\ldots=Q_n=Q$, the terms $m_1=m_2=\ldots=m_n=1$; and the cost function is as follows:

$$(2)\quad C = (D/Q)\sum_{j=1}^{n}K_j+(Q/2)\sum_{j=1}^{n}(D/p_j+1)h'_j+Q\sum_{j=1}^{n}((D/p_j)\sum_{i=j+1}^{n}h'_i)$$

When the stage unit holding cost (c_j) is known for all stages, a corresponding cost function can easily be determined. For each production order cycle of duration Q/D, the inventory at stage j builds up to Q during Q/p_j period, then depletes to zero during Q/p_{j-1} period. Therefore, the average inventory holding cost at stage j is:

$$c_j((Q^2/2)/p_j+(Q^2/2)/p_{j-1})/(Q/D) = (Q/2)(D/p_j+D/p_{j-1})c_j.$$

Summing this expression over all stages, the average inventory holding cost of the system is:

$$(Q/2)\sum_{j=1}^{n}(D/p_j+D/p_{j-1})c_j, \text{ where } p_0=D.$$

Consequently, the total of the average set-up costs and the average inventory holding costs can be expressed as follows:

$$(3)\quad C = (D/Q)\sum_{j=1}^{n}K_j+(Q/2)(D/p_1+1)c_1+(Q/2)\sum_{j=2}^{n}(D/p_j+D/p_{j-1})c_j$$

Rearranging (2), the following expression can be obtained:

$$(4)\quad C = (D/Q)\sum_{j=1}^{n}K_j+(Q/2)(D/p_1+1)\sum_{j=1}^{n}h'_j+(Q/2)\sum_{j=2}^{n}((D/p_j+D/p_{j-1})\sum_{i=j}^{n}h'_i)$$

Comparison between (3) and (4) clearly shows that

$c_j = \sum_{i=j}^{n}h'_i$, and $h'_j = c_j-c_{j+1}$; where $c_{n+1}=0$.

This relationship between the h'_j-s and c_j-s also applies to the general model in (1). Unfortunately, the definition for the unit holding cost of the echelon stock given by Schwarz and Schrage does not express this relationship; therefore, it is rather obscure and misleading for practical application.

The danger is that the ill-defined h'_j in [1] would probably be mistaken by any practitioner for c_j, the consequences of which become evident if (2) is written in terms of $h'_j = c_j-c_{j+1}$:

$$(5)\quad C = (D/Q)\sum_{j=1}^{n}K_j+(Q/2)\sum_{j=1}^{n}(D/p_j+1)c_j+Q\sum_{j=1}^{n}((D/p_j)\sum_{i=j+1}^{n}c_i)$$
$$-(Q/2)\sum_{j=1}^{n}(D/p_j+1)c_{j+1}-Q\sum_{j=1}^{n}((D/p_j)\sum_{i=j+1}^{n}c_{i+1}).$$

Differentiating this cost function with respect to Q yields the optimal lot size (Q^*) which consists of a non-optimal lot size (Q^{no}) and a correction factor (β):

$$(6)\quad Q^* = Q^{no}\beta,\text{ where } \beta = (1-B)^{-\frac{1}{2}},\text{ and}$$

$$(7)\quad Q^{no} = ((2D\sum_{j=1}^{n}K_j)/\sum_{j=1}^{n}[(D/p_j+1)c_j+2(D/p_j)\sum_{i=j+1}^{n}c_i])^{\frac{1}{2}},$$

(8) $B = \sum_{j=1}^{n}[(D/p_j+1)c_{j+1}+2(D/p_j)\sum_{i=j+1}^{n}c_{i+1}]/\sum_{j=1}^{n}[(D/p_j+1)c_j+2(D/p_j)\sum_{i=j+1}^{n}c_i]$.

The following conclusions can be drawn from these expressions:

(a) If $h'_j=c_j$ is used instead of $h'_j=c_j-c_{j+1}$ in (2), there is a double counting of inventory costs because the negative expressions in (5) are omitted from the cost function.

(b) Consequently, the lot size computed with $h'_j=c_j$ in (7) is non-optimal (Q^{no}) and, since $0<B<1$, is smaller than the optimal lot size (Q^*) computed in (6) with $h'_j=c_j-c_{j+1}$.

(c) The excess cost ratio generated by using Q^{no} in lieu of Q^* can be expressed as follows:

(9) $C^{no}/C^* = (1+\beta^2)/2\beta$.

Let us illustrate these consequences with an example:

$n=5$, $D=200$

$K_1=280$, $K_2=130$, $K_3=150$, $K_4=40$, $K_5=25$

$p_1=1000$, $p_2=1600$, $p_3=400$, $p_4=2500$, $p_5=800$

$c_1=6.30$, $c_2=4.30$, $c_3=2.60$, $c_4=1.30$, $c_5=0.50$

A closer examination of this example shows that applying the solution method described in [1] yields $Q_1=Q_2=\ldots=Q_5=Q$ and, therefore, $m_1=m_2=\ldots=m_5=1$.

If $h'_j=c_j$ is used in (2), Q^{no} is computed as shown in (7):

$$Q^{no} = [(2)(200)(625)/24.787]^{\frac{1}{2}} = 100.43.$$

However, the optimal lot size Q* according to (8) and (6) is:

$B = 13.285/24.787 = 0.536$, hence

$$\beta = (1-0.536)^{-\frac{1}{2}} = 1.468, \text{ and}$$

$$Q^* = 100.43(1.468) = 147.43.$$

The excess cost ratio in (9) generated by using Q^{no} instead of Q^* is:

$$C^{no}/C^* = (1+1.468^2)/(2(1.468)) = 1.075.$$

This can be easily verified by computing the cost in (5) for Q^{no} and Q^* which yields:

$C^{no} = 1822.20$, $C^* = 1695.70$, therefore, $C^{no}/C^* = 1.075$.

Consequently, for the given example, there is a 7.5 percent cost increase over the optimal cost, if $h_j' = c_j$ is used instead of the correct $h_j' = c_j - c_{j+1}$ for computing the lot size according to the cost function suggested by Schwarz and Schrage. Further, by examining (8) one can see that the excess cost ratio becomes larger as the number of stages (n) increases.

References:

1. Leroy B. Schwarz and Linus Schrage, "Optimal and System Myopic Policies for Multi-Echelon Production/Inventory Assembly Systems", Management Science, Vol. 21, No. 11, July 1975, pp. 1285-1294.