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COST-VOLUME-PROFIT ANALYSIS AND THE VALUE OF INFORMATION

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ABSTRACT

COST-VOLUME-PROFIT ANALYSIS AND THE VALUE OF INFORMATION

In this paper, a new view point on cost-volume-profit analysis is presented. In contrast to the usual concern with the actual accept/reject decision, it is pointed out that, first, a decision must be made on the adequacy of the input data for use with the decision model. The use of the concept of the expected value of perfect information in this decision is described. In addition, the sensitivity of the decision variables to uncertainty in the input variables is investigated. Equations to be used for these two purposes are derived and illustrated for two types of probability distributions of the input data, the normal and lognormal distributions.

COST-VOLUME-PROFIT ANALYSIS AND THE VALUE OF INFORMATION

INTRODUCTION

Cost-volume-profit (CVP) analysis, in both its full form and the restricted form of breakeven analysis, has been extensively taught and researched because it is a valuable business decision tool. While the basic CVP model has been modified in many directions - extensions to sales mix, uncertainty about input variables, multiple time periods, etc., have been presented - previous work has generally focussed on the basic decision to accept or reject a project. In this paper, a new view point on CVP analysis leads to examination of the following two points : (i) the decision on the adequacy of the input data for the model, involving the economics of seeking additional information before making the accept-reject decision, and (ii) the effects on the decision variables of uncertainty in the input variables, involving analysis of the sensitivity of the decision variables to the various input variables.

In order to minimize the length of the presentation, only the single product single time period case will be considered. After a brief summary of previous work, the analysis using the normal distribution will be presented because early work used that distribution and it is a familiar and often useful distribution. Then, the analysis for the lognormal distribution will be considered, because it has certain properties which, some feel, make it more realistic for CVP analysis.

An important point to be noted is that the extensions which are considered here are all based upon the same inputs required for CVP analysis under uncertainty. With the aid of standard tables and a modern calculator, all calculations can easily be done manually. The ease with which this additional information for decision making can be calculated enhances the practical utility of the methods discussed in the paper.

BACKGROUND

In CVP analysis, the model variables generally are:

P the unit selling price

V the total unit variable cost

M = P−V the contribution margin per unit

F the total fixed costs

Q the quantity to be sold

Z the profit

Q_b the breakeven quantity

In basic CVP analysis, the fundamental assumption is that all variables are deterministic and the two following equations are the basis for decision making:

$$Z = Q\,(P-V) - F = QM - F \qquad (1)$$

$$Q_b = \frac{F}{P-V} = \frac{F}{M} \qquad (2)$$

In principle, if values for Q, P, V and F are available, then the decision to accept or reject a project can be made on the basis of equations 1 and 2.

In order to improve the usefulness of CVP analysis, Jaedicke and Robichek (1964) incorporated explicitly the uncertainty of knowledge of the variables of the decision model. Their particular assumptions were:

i) $\tilde{Q}$, $\tilde{P}$, $\tilde{V}$ and $\tilde{F}$ are normally distributed random variables (random variables are indicated by a tilde)

ii) $\tilde{Q}$, $\tilde{P}$, $\tilde{V}$ and $\tilde{F}$ are statistically independent

iii) $\tilde{Z}$ is treated as a normally distributed random variable.[1]

Each random variable $\tilde{X}$ is thus characterized by its mean μ_X and its variance σ_X^2 and its distribution is given by

$$f(\tilde{X}) = \frac{1}{\sigma_X \sqrt{2\pi}} \exp\left[- \frac{(\tilde{X} - \mu_X)^2}{2\sigma_X^2} \right] \tag{3}$$

If the expected value and variance for each of the input variables ($\tilde{Q}$, $\tilde{P}$, $\tilde{V}$ and $\tilde{F}$) can be specified, then the distribution parameters for the profit $\tilde{Z}$ can be readily calculated from the equations

$$\mu_M = \mu_P - \mu_V \tag{4}$$

$$\mu_Z = \mu_Q \mu_M - \mu_F \tag{5}$$

$$\sigma_M^2 = \sigma_P^2 + \sigma_V^2 \tag{6}$$

$$\sigma_Z^2 = \sigma_Q^2 \sigma_M^2 + \mu_Q^2 \sigma_M^2 + \mu_M^2 \sigma_Q^2 + \sigma_F^2 \tag{7}$$

The increased usefulness of this model results from the fact that it allows calculation of the probability of profit being in any particular range. For example, a quantity of fundamental interest is the probability of profit exceeding breakeven which is given by

$$\Pr(\tilde{Z} \geqslant o) = \int_o^\infty f(\tilde{Z}) d\tilde{Z} \tag{8}$$

Once μ_Z and σ_Z are calculated from equations 5 and 7, this probability is obtained from standard normal probability tables.

Subsequent work of this type in CVP analysis seems to have been concentrated in two main areas. Some investigators (Ferrara, Hayya and Nachman, 1972) have been concerned with the limitations of assumptions (ii) and (iii) above, while others (Buzby 1974, Hilliard and Leitch 1975, Liao 1975) have been concerned with assumption (i) and have discussed using alternative probability distributions, etc.

DECISION ANALYSIS

In the Jaedicke and Robichek model of CVP analysis, the sequence of events in deciding to accept or reject a project is essentially as shown in Figure 1. From the estimates of μ and σ for $\tilde{Q}$, $\tilde{P}$, $\tilde{V}$ and $\tilde{F}$, the calculations of equations 4 to 8 are made and the decision to accept or reject is made on the basis of whether μ_Z > or < 0 and the magnitude of Pr $(\tilde{Z} \geqslant 0)$. In actual fact, the decision process follows the sequence in Figure 2, which shows that, first, a decision is made - explicitly or by default - on the sufficiency and/or adequacy of the information about the input variables before the accept/reject decision itself is made.

Presumably, the criterion for deciding whether or not to seek further information is based on benefit-cost analysis, a criterion which is conceptually straight forward, but difficult to implement. Generally, the cost of obtaining information can be determined reasonably accurately, once it has been decided what type of information is required and how it will be obtained. The basic problem is how to determine the benefit which will result from obtaining information. Ideally, the benefit of the particular information gathering process should be obtained. But, because this would depend upon particular circumstances, a general first approximation is desirable. The approximation which is particularly useful is the expected value of perfect information (EVPI), since this provides an upper limit to the benefit which could result from any possible information gathering process.[2] Comparison of the EVPI with the estimated cost of obtaining the information will at least provide a first stage in the decision on gathering further information; in cases where the cost of information exceeds, or is considerably less than, the EVPI, no further information will be required to make the information-seeking decision.

EXPECTED VALUE OF PERFECT INFORMATION

From the Jaedicke and Robichek model, the expected profit can be calculated from equation 5. If the usual expected value criterion is used, then the decision on the project will be to accept if $\mu_Z > 0$ and to reject if $\mu_Z < 0$.

If it is assumed that $\mu_Z > 0$ and the project is accepted, then one still must consider what the actual outcome of the project is. Because the profit $\tilde{Z}$ is assumed to be a normally distributed random variable, it is possible that the actual profit may be greater than or less than the expected profit.

For the moment, it is assumed that the project was accepted. If it turns out that a profit actually results, then the decision was correct. But it is possible that a profit does not result, that the decision was incorrect in this case and gave rise to an opportunity loss (OL), which can be summarized as

$$OL = -\tilde{Z} \qquad -\infty < \tilde{Z} < 0 \qquad (9)$$

$$OL = 0 \qquad 0 \leqslant \tilde{Z} \qquad (10)$$

Before actually undertaking the project, the result is not known. However, by multiplying each possible opportunity loss calculated from equations (9) and (10) by the probability of its occurring, which is given by the probability distribution of profit $f(\tilde{Z})$ calculated by the Jaedicke and Robichek model, the expected opportunity loss (EOL) can be determined and this is equal to the expected value of perfect information (Winkler, 1972). This procedure, which is illustrated in Figure 3, is represented analytically by the equation

$$\begin{aligned} EVPI = EOL &= \int_{-\infty}^{0} \tilde{Z}\, f(\tilde{Z})\, d\tilde{Z} \\ &= \sigma_Z\, L_N\left(\frac{\mu_Z}{\sigma_Z}\right) \qquad (11) \end{aligned}$$

where $L_N\ (\mu_Z/\sigma_Z)$ is the normal loss function which can be found in standard tables (Winkler, 1972, p. 516-8).

In the case of the reject decision having been made because $\mu_Z < 0$, the expression for the EVPI is

$$\text{EVPI} = \sigma_Z\ L_N \left(\frac{|\mu_Z|}{\sigma_Z}\right) \tag{12}$$

The similarity to equation 11 results because the function f $(\tilde{Z})$ is symmetrical.

The EVPI as calculated from equations 11 or 12 provides an upper limit to the benefit which can result from any information gathering procedure. Comparison of this to the estimated cost of gathering the information may, in many cases, be sufficient to allow a decision on whether or not to seek more information to be made.

SENSITIVITY ANALYSIS

A significant factor which has not yet been incorporated into CVP analysis is the potential effect of errors in the estimates made of the input variables. The results calculated above depend upon the values of μ_Z and σ_Z which, in turn, are calculated from the μ and σ for the input variables $\tilde{Q}$, $\tilde{P}$, $\tilde{V}$ and $\tilde{F}$. However determined, the latter are only estimates of future values. Thus, the question naturally arises as to how sensitive the model output variables are to errors in the estimates of input variables. If they are highly sensitive, then the decision model may lose some of its utility.

In the Jaedicke and Robichek model, the output variables for decision making are μ_Z and probabilities, say for convenience, the probability of profit exceeding breakeven, Pr $(\tilde{Z} \geq 0)$. The expected profit μ_Z depends upon

μ_Q, μ_M and μ_F (equation 5) and the sensitivity of μ_Z to errors in those variables is readily determinable because the function is simple; it will not be considered further. Because the determination of the sensitivity analysis of Pr ($\tilde{Z} \geq 0$) is somewhat complex, the method of calculation is outlined and the results given.

The probability of profit exceeding breakeven is given by

$$\Pr(\tilde{Z} \geq 0) = \frac{1}{\sigma_Z \sqrt{2\pi}} \int_0^\infty \exp\left[-\frac{(\tilde{Z} - \mu_Z)^2}{2\sigma_Z^2}\right] d\tilde{Z} \tag{13}$$

and to determine its sensitivity to errors in the μ and σ for the input variables, the best procedure is to use the equations

$$\frac{\partial Pr}{\partial \mu_i} = \frac{\partial Pr}{\partial \mu_Z}\frac{\partial \mu_Z}{\partial \mu_i} + \frac{\partial Pr}{\partial \sigma_Z}\frac{\partial \sigma_Z}{\partial \mu_i} \tag{14}$$

$$\frac{\partial Pr}{\partial \sigma_i} = \frac{\partial Pr}{\partial \mu_Z}\frac{\partial \mu_Z}{\partial \sigma_i} + \frac{\partial Pr}{\partial \sigma_Z}\frac{\partial \sigma_Z}{\partial \sigma_i} \tag{15}$$

where μ_i and σ_i are the mean and standard deviation of input variable i. After some mathematical manipulation, this can be reduced to the equations in Table 1.

Another area in which sensitivity analysis is clearly in order is in the calculation of the EVPI described earlier. Proceeding from the equation

$$EVPI = \frac{\sigma_Z}{\sqrt{2\pi}} \int_{\mu_Z/\sigma_Z}^\infty \left[\theta - \frac{\mu_Z}{\sigma_Z}\right] \exp\left[-\frac{\theta^2}{2}\right] d\theta \tag{16}$$

and the equations

$$\frac{\partial EVPI}{\partial \mu_i} = \frac{\partial EVPI}{\partial \mu_Z}\frac{\partial \mu_Z}{\partial \mu_i} + \frac{\partial EVPI}{\partial \sigma_Z}\frac{\partial \sigma_Z}{\partial \mu_i} \tag{17}$$

$$\frac{\partial EVPI}{\partial \sigma_i} = \frac{\partial EVPI}{\partial \mu_Z}\frac{\partial \mu_Z}{\partial \sigma_i} + \frac{\partial EVPI}{\partial \sigma_Z}\frac{\partial \sigma_Z}{\partial \sigma_i} \tag{18}$$

leads, again after some mathematical manipulation, to the equations given in Table 2.

THE LOGNORMAL DISTRIBUTION

The use of the normal distribution in CVP analysis has been objected to by some on the grounds that this distribution (i) is symmetrical and (ii) allows negative values of the variables, both properties being unrealistic in some cases. A distribution which does not have these shortcomings and has been proposed for use in CVP analysis is the lognormal distribution (Hilliard and Leitch 1975). This distribution can be used with a prespecified lower limit and amount of skewness. Under the assumptions that $\tilde{Q}$ and $\tilde{M}$ are lognormally distributed with the range 0 to ∞ and F is fixed at a value μ_F, the probability density function for the profit $\tilde{Z}$ is given by the equation (Hilliard and Leitch 1975)

$$f(\tilde{Z}) = \frac{1}{(\tilde{Z} + \mu_F)\sigma_* \sqrt{2\pi}} \exp\left[-\frac{(\log(\tilde{Z}+\mu_F) - \mu_*)^2}{2\sigma_*^2}\right] \tag{19}$$

where

$$\mu_* = \log\left[\frac{\mu_Q^2}{\sqrt{\sigma_Q^2 + \mu_Q^2}}\right] + \log\left[\frac{\mu_M^2}{\sqrt{\sigma_M^2 + \mu_M^2}}\right] \tag{20}$$

$$\sigma_*^2 = \log\left[\left(\frac{\sigma_Q}{\mu_Q}\right)^2 + 1\right] + \log\left[\left(\frac{\sigma_M}{\mu_M}\right)^2 + 1\right] + \log\left[\frac{\sigma_Q\,(\rho_{QP}\sigma_P - \rho_{QV}\sigma_V)}{\mu_Q\mu_M} + 1\right] \tag{21}$$

The appearance of this function depends upon the specific values of the parameters; one possibility is shown in Figure 4.

The discussion for the lognormal distribution will parallel that for the normal distribution, the EVPI calculation being followed by an analysis of the sensitivity of Pr $(\tilde{Z} \geqslant 0)$ and EVPI to the input variables. Because the principles of the calculations are basically the same as before, the calculations will be outlined only briefly and the results presented.

<u>EVPI - LOGNORMAL DISTRIBUTION</u>

With the lognormal distribution, the expected value of perfect informatio can be calculated to provide an upper limit to the benefit which will resul from seeking further information. However, because the distribution is no symmetric, the calculation is different for the cases of $\mu_Z > 0$ and < 0.

For $\mu_Z > 0$, the method of calculation is illustrated in Figure 5 and the result is given by

$$EVPI_1 = EOL = -\int_{-\mu_F}^{o} \tilde{Z}\, f(\tilde{Z})\, d\tilde{Z}$$

$$= -\exp\left[\frac{\sigma_*^{\,2} + 2\mu_*}{2}\right] F_N\left[\frac{\log \mu_F - \mu_*}{\sigma_*} - \sigma_*\right] + \mu_F\, F_N\left[\frac{\log \mu_F - \mu_*}{\sigma_*}\right] \qquad ..2)$$

where $$F_N(a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a} \exp\left[-\frac{\theta^2}{2}\right] d\theta \qquad)$$

and the value of $F_N(a)$ is readily calculated using standard normal probability tables.

For $\mu_Z < 0$, the method of calculation is illustrated in figure 6 and the result is given by the following equation.

$$EVPI_2 = EOL = \int_{o}^{\infty} \tilde{Z}\, f(\tilde{Z})\, d\tilde{Z} \qquad (24)$$

However, it is also know that the following is true:

$$\mu_Z = E(\tilde{Z}) = \int_{-\mu_F}^{\infty} \tilde{Z} f(\tilde{Z}) d\tilde{Z} = \int_{-\mu_F}^{0} \tilde{Z} f(\tilde{Z}) d\tilde{Z} + \int_{0}^{\infty} \tilde{Z} f(\tilde{Z}) d\tilde{Z}$$

$$= -EVPI_1 + EVPI_2 \tag{25}$$

Thus, equation 24 can be rewritten in the form

$$EVPI_2 = EVPI_1 + \mu_Z \tag{26}$$

Or, in a more general form, the two equations for the EVPI using the lognormal distribution can be combined into the single equation

$$EVPI = EVPI_1 + \min\ (\mu_Z, 0) \tag{27}$$

Equation 27 reduces to equation 22 for $\mu_Z > 0$ and to equation 26 for $\mu_Z < 0$.

SENSITIVITY ANALYSIS - LOGNORMAL DISTRIBUTION

As in the case of the normal distribution, the caclulations and decisions are based upon the estimated values of the input μ and σ for the variables $\tilde{Q}$, $\tilde{M}$ and F, and the question as to the sensitivity of the decision variables to the input variables arises. Here, the two decision variables examined for the normal distribution - Pr $(\tilde{Z} \geqslant 0)$ and EVPI - will be considered.

The probability of having a profit is given by the equation

$$\Pr\ (\tilde{Z} \geqslant 0) = \frac{1}{\sigma_* \sqrt{2\pi}} \int_0^{\infty} \frac{1}{\tilde{Z} + \mu_F} \exp\left[-\frac{(\log\ (\tilde{Z} + \mu_F) - \mu_*)^2}{2\sigma_*^{\,2}}\right] d\tilde{Z} \tag{28}$$

and its sensitivity to the input variables μ_Q, σ_Q, μ_M, σ_M and μ_F is determined in the same way as for the normal distribution and the results are given by the equations in Table 3.

For the EVPI, the sensitivity to the input variables in again determined as for the normal distribution and the results are given by the equations of Table 4 both for $\mu_Z > 0$ ($\delta = 0$ in the equations) and for $\mu_Z < 0$ ($\delta = 1$ for the equations).

CONCLUSION

Previous work on CVP analysis has focused on the decision to accept or reject a proposal. In this paper, it has been pointed out that (i) a preliminary decision must be made to seek or not to seek further information; this decision is based on the "quality" of the information available (ii) the sensitivity of the decision variables to errors in the input data should be considered.

The decision on information seeking is based on a benefit-cost criterion and it has been shown that the expected value of perfect information can be calculated for both the normal and lognormal distributions and provides an upper limit on the magnitude of the usually difficult to measure benefit. Because cost can normally be determined reasonably accurately, this provides a method for deciding whether or not to seek further information before making an accept-reject decision.

In both the information seeking and accept-reject decisions, the decision is based on the value of a decision variable calculated from the model input variables. Because the model input variables are normally just estimates of future values, knowledge of the sensitivity of the decision variable to errors in the input variables is useful. The equations given provide a means for estimating the sensitivity of Pr $(\tilde{Z} \geq 0)$ and EVPI for both the normal and lognormal distributions.

The extensions of CVP analysis described here require no inputs beyond those required for the usual calculation under uncertainty. Consequently, only a few calculations need be carried out to obtain further information useful for decision-making.

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FOOTNOTES

1. Z is treated as normally distributed even though it strictly is not. The conditions for validity of this approximation has been examined by Ferrara, Hayya and Nachman (1972).

2. It is assumed here that the decision maker decides on the basis of expected values.

FIGURE 1

Representation of the Decision Process Usually Assumed in CVP Analysis

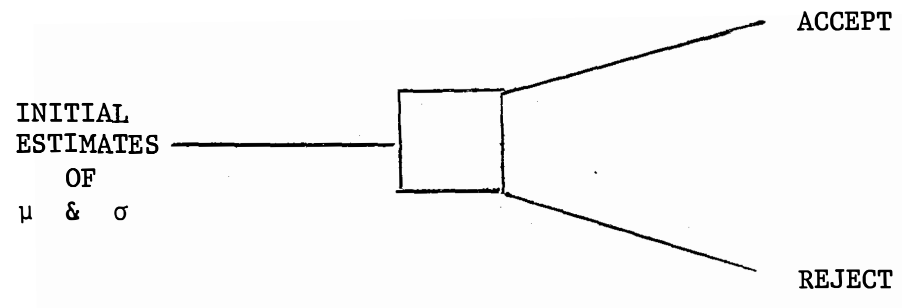

FIGURE 2

Representation of the Decision Process Actually Occuring in CVP Analysis

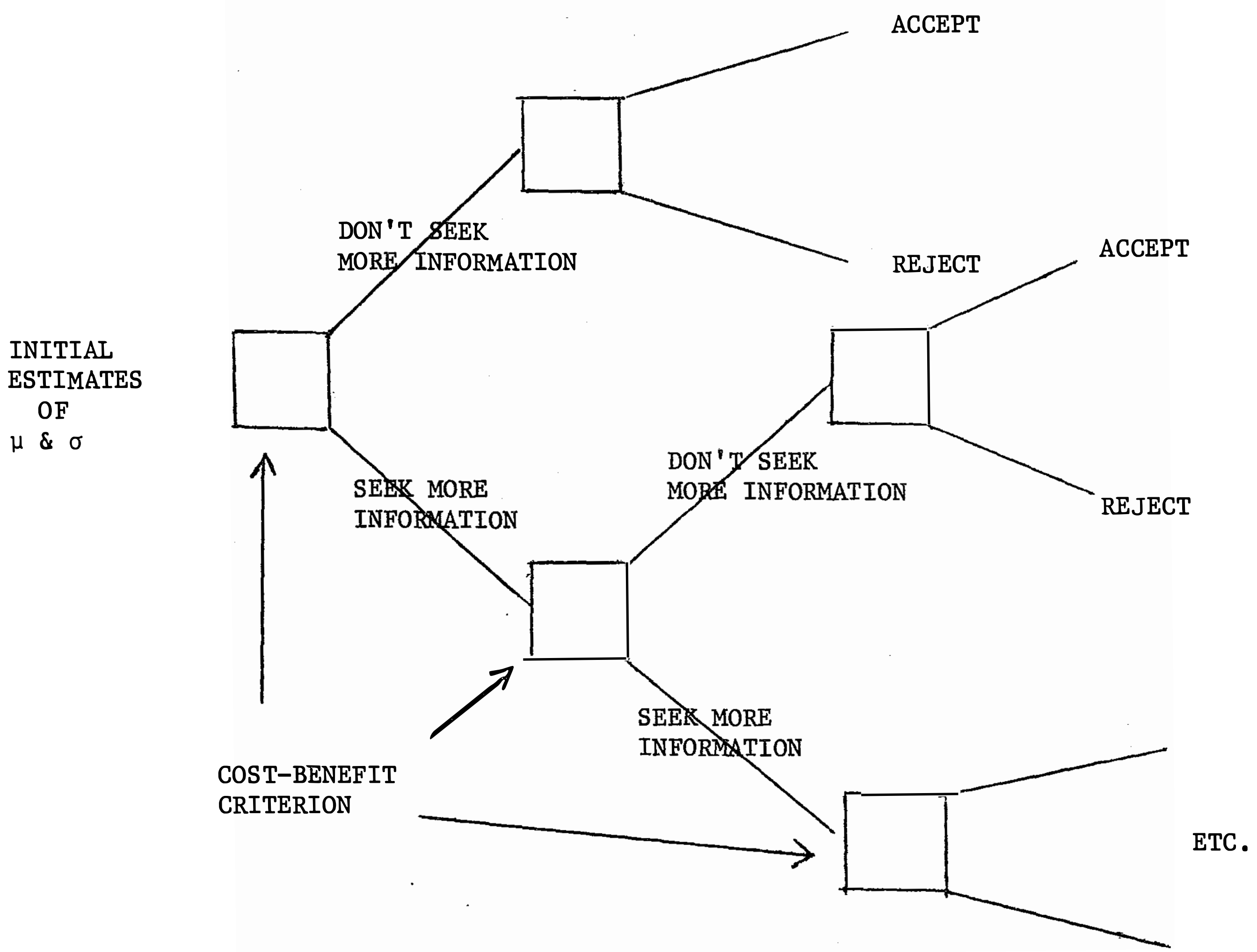

FIGURE 3

Schematic Representation of the Calculation of the EVPI when $\mu_Z > 0$ And The Profit $\tilde{Z}$ is Normally Distributed

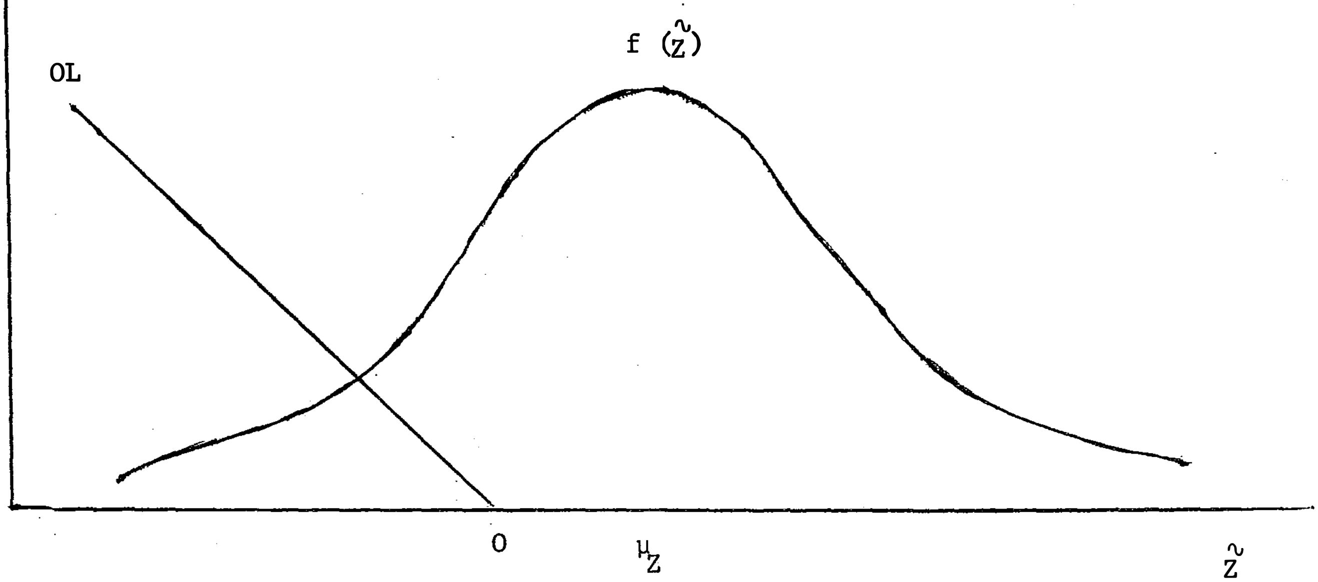

FIGURE 4

LOGNORMAL DISTRIBUTION

A possible Appearance of the Probability Distribution For Profit $\tilde{Z}$ Following A Lognormal Distribution

$$f(\tilde{Z}) = \frac{1}{(\tilde{Z} + \mu_F)\, \sigma_* \sqrt{2\pi}} \operatorname{Exp}\left[- \frac{(\text{LOG } (\tilde{Z} + \mu_F) - \mu_*)^2}{2\sigma_*^{\,2}} \right]$$

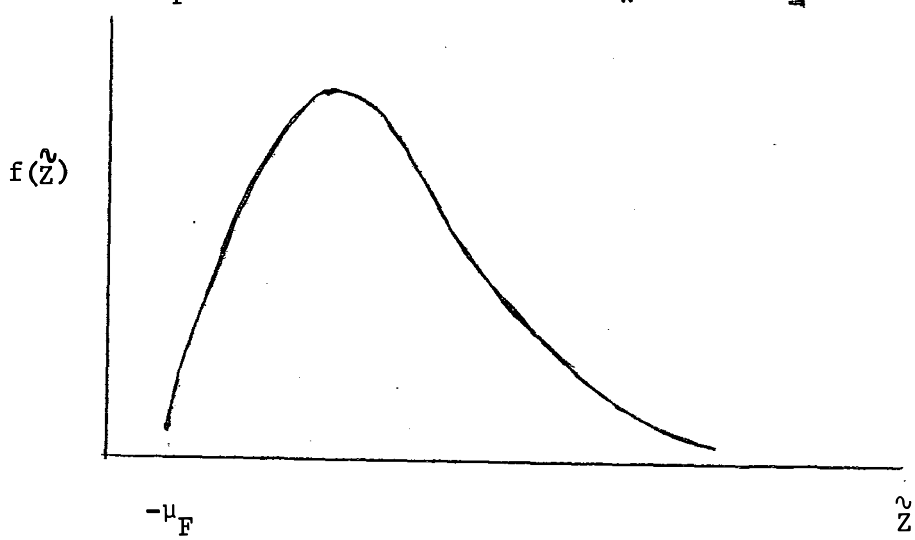

FIGURE 5

Schematic Representation of the Calculation of the EVPI When $\mu_Z > 0$ and the Profit $\tilde{Z}$ is Lognormally Distributed

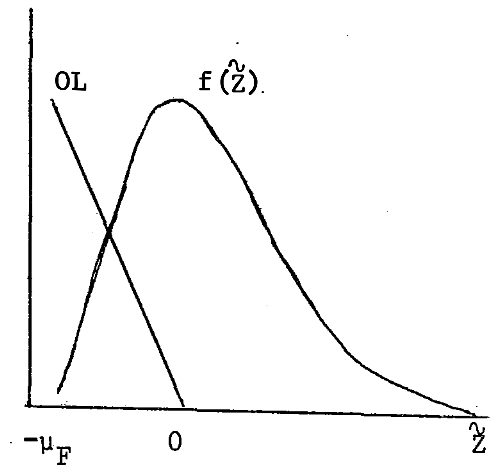

FIGURE 6

Schematic Representation of the Calculation of the EVPI When $\mu_Z < 0$ and the Profit $\tilde{Z}$ is Lognormally Distributed

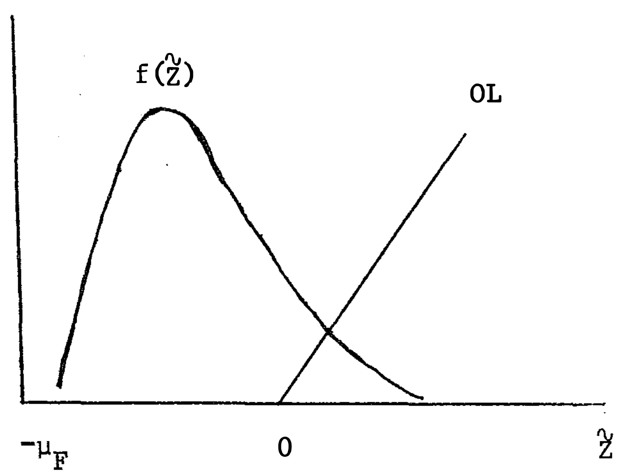

TABLE 1

Equations giving the sensitivity of the probability of profit exceeding zero ($\Pr(\tilde{Z} \geqslant 0)$) to the input variables for the case when the profit $\tilde{Z}$ is normally distributed.

$$\frac{\partial \Pr(\tilde{Z} \geqslant 0)}{\partial \mu_Q} = \left[\mu_M - \frac{\mu_Q \mu_Z \sigma_M^2}{\sigma_Z^2} \right] R$$

$$\frac{\partial \Pr(\tilde{Z} \geqslant 0)}{\partial \sigma_Q} = - \left[\frac{\mu_Z \sigma_Q (\sigma_M^2 + \mu_M^2)}{\sigma_Z^2} \right] R$$

$$\frac{\partial \Pr(\tilde{Z} \geqslant 0)}{\partial \mu_M} = \left[\mu_Q - \frac{\mu_M \mu_Z \sigma_Q^2}{\sigma_Z^2} \right] R$$

$$\frac{\partial \Pr(\tilde{Z} \geqslant 0)}{\partial \sigma_M} = - \left[\frac{\mu_Z \sigma_M (\sigma_Q^2 + \mu_Q^2)}{\sigma_Z^2} \right] R$$

$$\frac{\partial \Pr(\tilde{Z} \geqslant 0)}{\partial \mu_F} = -R$$

$$\frac{\partial \Pr(\tilde{Z} \geqslant 0)}{\partial \sigma_F} = - \frac{\mu_Z \sigma_F}{\sigma_Z^2} R$$

$$R = \frac{1}{\sigma_Z \sqrt{2\pi}} \exp \left[-\tfrac{1}{2} \left(\frac{\mu_Z}{\sigma_Z} \right)^2 \right]$$

Table 2

Equations giving the sensitivity of the EVPI to the input variables for the case when profit $\tilde{Z}$ is normally distributed.

$$\frac{\partial EVPI}{\partial \mu_Q} = \mu_M H + \frac{\mu_Q \sigma_M^2}{\sigma_Z} I$$

$$\frac{\partial EVPI}{\partial \sigma_Q} = \frac{\sigma_Q(\sigma_M^2 + \mu_M^2)}{\sigma_Z} I$$

$$\frac{\partial EVPI}{\partial \mu_M} = \mu_Q H + \frac{\mu_M \sigma_Q^2}{\sigma_Z} I$$

$$\frac{\partial EVPI}{\partial \sigma_M} = \frac{\sigma_M(\sigma_Q^2 + \mu_Q^2)}{\sigma_Z} I$$

$$\frac{\partial EVPI}{\partial \mu_F} = -H$$

$$\frac{\partial EVPI}{\partial \sigma_F} = \frac{\sigma_F}{\sigma_Z} I$$

$$H = \frac{-1}{\sqrt{2\pi}} \int_{\frac{\mu_Z}{\sigma_Z}}^{\infty} \exp\,[-1/2\theta^2]d\theta$$

$$I = \frac{1}{\sqrt{2\pi}} \exp\,[-1/2(\frac{\mu_Z^2}{\sigma_Z})]$$

TABLE 3

Sensitivity of the probability of profit exceeding zero (Pr ($\tilde{Z} \geq 0$)) to the input variables for the case when the profit $\tilde{Z}$ is lognormally distributed.

$$\frac{\partial Pr}{\partial \mu_Q} = \frac{2\sigma_Q^2 + \mu_Q^2}{\mu_Q(\sigma_Q^2 + \mu_Q^2)} S - \frac{1}{2\mu_Q\sigma_*}\left[\frac{2\sigma_Q^2}{\sigma_Q^2 + \mu_Q^2} + E\right] T$$

$$\frac{\partial Pr}{\partial \mu_M} = \frac{2\sigma_M^2 + \mu_M^2}{\mu_M(\sigma_M^2 + \mu_M^2)} S - \frac{1}{2\mu_M\sigma_*}\left[\frac{2\sigma_M^2}{\sigma_M^2 + \mu_M^2} + E\right] T$$

$$\frac{\partial Pr}{\partial \sigma Q} = \frac{\sigma_Q}{\sigma_Q^2 + \mu_Q^2} S + \frac{1}{2\sigma_*}\left[\frac{2\sigma_Q}{\sigma_Q^2 + \mu_Q^2} + E\right] T$$

$$\frac{\partial Pr}{\partial \sigma_M} = -\frac{\sigma_M}{\sigma_M^2 + \mu_M^2} S + \frac{\sigma_M}{\sigma_*(\sigma_M^2 + \mu_M^2)} T$$

$$\frac{\partial Pr}{\partial \mu_F} = \frac{-1}{\mu_F\sigma_*\sqrt{2\pi}} \text{EXP}\left[-\tfrac{1}{2}\left(\frac{\mu_* - \text{LOG}\mu_F}{\sigma_*}\right)^2\right]$$

$$D = \frac{\text{LOG}\mu_F - \mu_*}{\sigma_*} + \sigma_*$$

$$E = \frac{\rho_{QP}\sigma_P - \rho_{QV}\sigma_V}{\sigma_Q(\rho_{QP}\sigma_P - \rho_{QV}\sigma_V) + \mu_Q\mu_M}$$

$$S = \frac{1}{\sigma_*\sqrt{2\pi}} \text{EXP}\left[-\frac{(\text{LOG}\mu_F - \mu_*)^2}{2\sigma_*^2}\right]$$

$$T = \frac{\text{LOG}\,\mu_F - \mu_*}{\sqrt{2\pi}\sigma_*^2} \text{EXP}\left[-\tfrac{1}{2}\left(\frac{\text{LOG}\mu_F - \mu_*}{\sigma_*}\right)^2\right]$$

TABLE 4

Sensitivity of the EVPI to the Input Variables for the Case When the Profit $\tilde{Z}$ is Lognormally Distributed

$$\frac{\partial EVPI}{\partial \mu_Q} = \frac{2\sigma_Q^2 + \mu_Q^2}{\mu_Q(\sigma_Q^2 + \mu_Q^2)} B - \frac{1}{2\mu_Q\sigma_*}\left[\frac{2\sigma_Q^2}{\sigma_Q^2 + \mu_Q^2} + \sigma_Q E\right] C + \delta\mu_M$$

$$\frac{\partial EVPI}{\partial \mu_M} = \frac{2\sigma_M^2 + \mu_M^2}{\mu_M(\sigma_M^2 + \mu_M^2)} B - \frac{1}{2\mu_M\sigma_*}\left[\frac{2\sigma_M^2}{\sigma_M^2 + \mu_M^2} + \sigma_Q E\right] C + \delta\mu_Q$$

$$\frac{\partial EVPI}{\partial \sigma_Q} = -\frac{\sigma_Q}{\sigma_Q^2 + \mu_Q^2} B + \frac{1}{2\sigma_*}\left[\frac{2\sigma_Q}{\sigma_Q^2 + \mu_Q^2} + E\right] C$$

$$\frac{\partial EVPI}{\partial \sigma_M} = -\frac{\sigma_M}{\sigma_M^2 + \mu_M^2} B + \frac{\sigma_M}{\sigma_*(\sigma_M^2 + \mu_M^2)} C$$

$$\frac{\partial EVPI}{\partial \mu_F} = F_N\left[\frac{LOG\mu_F - \mu_*}{\sigma_*}\right] - \delta$$

$$B = -EXP\left[\frac{\sigma_*^2 + 2\mu_*}{2}\right] F_N\left(\frac{LOG\mu_F - \mu_*}{\sigma_*} - \sigma_*\right)$$

$$C = EXP\left[\frac{\sigma_*^2 + 2\mu_*}{2}\right]\left\{\frac{1}{\sqrt{2\pi}} EXP\left[-\frac{1}{2}\left(\frac{LOG\mu_F - \mu_*}{\sigma_*} - \sigma_*\right)^2\right] - \sigma_* F_N\left(\frac{LOG\mu_F - \mu_*}{\sigma_*} - \sigma_*\right)\right\}$$

$$E = \frac{\rho_{QP}\sigma_P - \rho_{QV}\sigma_V}{\sigma_Q(\rho_{QP}\sigma_P - \rho_{QV}\sigma_V) + \mu_Q\mu_M}$$

$$F_N(a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a} EXP\left(-\frac{1}{2}\theta^2\right) d\theta$$

$$\delta = \begin{cases} 0 & \text{for } \mu_Z > 0 \\ 1 & \text{for } \mu_Z < 0 \end{cases}$$

APPENDIX

To illustrate the calculations for normally distributed variables, consider the data given in Table A-1. From these data, equations 4 - 7 can be used to calculate

$$\mu_Z = (980)(5.40) - 5000 = \$292 \qquad \text{(A-1)}$$

$$\sigma_Z = \$617.50 \qquad \text{(A-2)}$$

and from standard normal probability tables, Pr $(\tilde{Z} \geqslant 0) = 0.682$.

The sensitivity of Pr $(\tilde{Z} \geqslant 0)$ can be calculated using the equations of Table 2, which yield the results given in Table A-2. If, for purposes of illustration, it is assumed that a 1% error in input variables is likely, and a linear variation is assumed for simplicity, then the approximate changes in Pr $(\tilde{Z} \geqslant 0)$ are calculated from the results in Table A-2 to give the results in Table A-3. In this case, the results are clearly more sensitive to errors in the expected values than to errors in the standard deviations of the input variables.

With the values of μ_Z and σ_Z calculated in equations A-1 and A-2, it is easy to calculate the expected value of perfect information

$$\text{EVPI} = 617.50\ L_N \left(\frac{292.00}{617.50}\right) = \$127.38 \qquad \text{(A-3)}$$

If the cost of gathering information exceeds \$127.38, then it is clearly not worth it.

The sensitivity of the previously calculated value of EVPI = \$127 to the input variables is summarized in Table A-4 and Table A-5 gives the effect on the EVPI of a 1% error in the input variables assuming a linear variation of EVPI with changes in individual input variables. The results here are also more sensitive to errors in the expected values than to errors in the standard deviations.

To illustrate the calculations for the lognormal distribution, the data of Table A-6 will be used; these are the same as those in Table A-1 with the changes that $\tilde{Q}$ and $\tilde{M}$ are lognormally distributed, $\tilde{Q}$ and $\tilde{M}$ have a minimum value of 0, and F is deterministic. For these data, μ_Z is calculated to be \$292 and so the decision would be to accept the project. Because $\mu_Z > 0$, the EVPI is calculated from equation 22 and is found to be \$109.54; this provides an upper limit to the amount which should be spent to obtain further information.

Calculation of the sensitivity of Pr $(\tilde{Z} \geqslant 0)$ using the equations of Table 3 gives the results listed in Table A-7; if one assumes a 1% change in the input variables and a linear variation for illustration, the numerical values of Table A-8 can be calculated. Again, one can see that the sensitivity to the expected values is greater than to the standard deviations.

The sensitivity of the EVPI value to the input variables is calculated using the equations of Table 4 ($\delta = 0$ because μ_Z = \$292 i.e. > 0) to give the sensitivity data of Table A-9. The same assumptions of a 1% change in the input variables and linear variation lead to the numerical results in Table A-10. As before, the sensitivity to uncertainty in the expected values is greater than to uncertainty in the standard deviations.

Table A-1

Example Data for the Case Where All Variables are Normally Distributed

VARIABLE	μ	σ
Q	980 units	80 units
P	\$10 per unit	\$0.346 per unit
V	\$4.60 per unit	\$0.20 per unit
M	\$5.40 per unit	\$0.40 per unit
F	\$5,000	\$200

Table A-2

Calculated Sensitivity of $Pr(\tilde{Z} \geqslant 0)$ to the Input Variables for the Normal Distribution Example

	$\frac{\partial Pr}{\partial \mu_i}$	$\frac{\partial Pr}{\partial \sigma_i}$
Q	3.050×10^{-3}	-1.038×10^{-3}
M	5.509×10^{-1}	-1.711×10^{-1}
F	-5.777×10^{-4}	-8.848×10^{-5}

Table A-3

Approximate Changes in $\Pr(\tilde{Z} \geq 0)$ for a 1% Change in Individual Input Variables - Assuming a Linear Variation

	μ	σ
Q	0.030	0.00083
M	0.030	0.00068
F	0.029	0.00018

Table A-4

Calculated Sensitivity of EVPI to The Input Variables for the Normal Distribution Example

	$\frac{\partial EVPI}{\partial \mu_i}$	$\frac{\partial EVPI}{\partial \sigma_i}$
Q	-1.6274	1.3551
M	-291.8224	223.4155
F	0.3182	0.1155

Table A-5

Approximate Change in EVPI for a 1% Change in Individual Input Variables - Assuming a Linear Variation

	μ	σ
Q	\$15.95	\$1.08
M	15.76	0.89
F	15.91	0.23

Table A-6

Example Data for Lognormal Distribution Calculation. Q and M are lognormally Distributed with Range 0 - ∞ and F is Deterministic

VARIABLE	μ	σ
Q	980 units	80 units
M	\$5.40 per unit	\$0.40 per unit
F	\$5,000	-

Table A-7

Calculated Sensitivity of $Pr(\tilde{Z} \geqslant 0)$ to the Input Variables for the Lognormal Distribution Example

$Pr(\tilde{Z} \geqslant 0) = 0.677$

	$\frac{\partial Pr}{\partial \mu_i}$	$\frac{\partial Pr}{\partial \sigma_i}$
Q	3.441×10^{-3}	-1.399×10^{-3}
M	6.208×10^{-1}	-2.307×10^{-1}
F	-6.520×10^{-4}	-

Table A-8

Approximate Changes in $Pr(Z \geqslant 0)$ for a 1% Change in Individual Input Variables - Assuming a Linear Variation

	μ	σ
Q	0.034	0.0011
M	0.034	0.0009
F	0.033	-

Table A-9

Calculated Sensitivity of EVPI to the Input Variables for the Lognormal Distribution Example

	$\frac{\partial EVPI}{\partial \mu_i}$	$\frac{\partial EVPI}{\partial \sigma_i}$
Q	-1.6442	1.3487
M	-294.8891	222.3602
F	0.3225	-

Table A-10

Approximate Changes in EVPI for a 1% Change in Individual Input Variables - Assuming A Linear Variation

	μ	σ
Q	$16.11	$1.08
M	15.92	0.89
F	16.13	-

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