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The Cost of Equity Capital with Personal Income Taxes and Flotation Costs

M.J. Gordon and L.I. Gould*

This paper establishes the cost of retention-financed capital and the cost of stock-financed capital in the presence of personal income taxes and flotation costs under alternative theories of share valuation that are considered plausible.

In the absence of the tax and flotation costs (and information content to the dividend), the cost of equity capital is independent of its source. It then depends on whether a share's yield is independent of (SYI) or is dependent on (SYD) the firm's investment decision. It also depends on whether the return on investment functions in future periods are independent of (RIFI) or are dependent on (RIFD) the firm's current investment decision. Consequently, under the conditions stated there are four possible answers to the question, what is a corporation's cost of equity capital? Each of the pairs (1) SYI-RIFI, (2) SYD-RIFI, (3) SYI-RIFD, and (4) SYD-RIFD results in a different answer.

Part I reviews these four solutions to the cost of equity capital. Part II extends the four solutions to establish the costs of retention-financed and stock-financed capital in the presence of flotation costs and the differential tax treatment of dividends and capital gains. Part III compares the results obtained with the existing literature.

All that a search of the literature could find is the Lewellen [6] solution to the problem. That solution was confined to the SYI-RIFI case, and it assumed the pre-tax share yield is independent of the firm's investment decision.

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In fact, when the no-tax share yield is taken to be independent of the firm's investment decison, it is the after-tax share yield that takes on this property. The pre-tax share yield becomes a decreasing function of the dividend's rate of growth, and the Lewellen solution materially understates the costs of retention-financed and stock-financed capital under the SYI-RIFI assumptions.

I.

The cost of equity capital depends on whether the return on investment functions in future periods are dependent on (RIFD) or independent of (RIFI) the firm's current investment decisions. We will first examine the cost of equity capital models under each of these assumptions in the absence of personal taxes and flotation costs.

A. The RIFD Cost of Equity Capital Models

The implications for the cost of equity capital of the constant expected growth rate stock value model,

$$P = (1-b)Y/(k-g), \tag{1}$$

were examined in the early sixties by Gordon [3][4] and Lintner [7][8], and by Gordon and Gould [5] in a recent paper. In this equation:

P = Present value of the firm's stock;

Y = Expected value of the firm's earnings per share in the coming year;

b = Expected value of the firm's retention rate, expressed as a fraction of earnings;

g = Expected rate of growth in the dividend to a current share;

k = Before-personal-tax required return or yield at which the stock is selling.

Miller and Modigliani [10,p. 423] have shown that in the absence of personal taxes and transaction costs,

$$g = [qr(1-b)-sk]/(1-q), \tag{2}$$

where the additional variables are defined as:

s = Expected value of the firm's stock financing rate, expressed as a fraction of earnings;

q = Expected value of the firm's investment rate, expressed as a fraction of earnings = b+s;

r = Expected value of the return on investment, with investment the fraction q of earnings.

Substituting this expression for g in Eq. (1), and simplifying, results in

$$P = (1-q)Y/(k-qr). \tag{3}$$

If r and k are both functions of q, the partial derivative with respect to q is

$$\frac{\partial P}{\partial q} = \frac{Y}{(k-qr)^2}\{-k+qr+(1-q)[r+q(\frac{\partial r}{\partial q})-\frac{\partial k}{\partial q}]\}. \tag{4}$$

In the above $r' = r+q\ (\partial r/\partial q)$ is the marginal rate of return on investment when investment is at the rate q. Setting Eq. (4) equal to zero and solving for r' we find that the value of P is maximized when q is set to satisfy

$$r' = (k-qr)/(1-q) + \partial k/\partial q. \tag{5}$$

The left hand side of Eq. (5), the marginal rate of return on investment, is a decreasing function of q, the firm's investment rate.

The right hand side of Eq. (5) is also a function of q, and it may be called the firm's cost of equity capital, since the value of q that satisfies Eq. (5) maximizes share price. In other words, if the value of r' at some investment rate is above the right hand side of Eq. (5), undertaking the next most profitable investment will raise the value of the firm's stock and vice versa.

The term (k-qr)/(1-q) reflects the assumption that the return on investment functions in future periods are dependent on the investment decision in a particular way. Hence, Eq. (5) with $\partial k/\partial q=0$ is the SYI-RIFD cost of equity capital, and with $\partial k/\partial q\neq 0$, we have the SYD-RIFD cost of equity capital.

On the rationale for the RIFD function reflected in (k-qr)/(1-q), a recent paper by Elton and Gruber [1] investigated a wide range of assumptions with regard to a firm's future rate of return on investment functions, and derived the value of the firm and its cost of capital for each of these functions. For our purposes, the results of that paper may be summarized as follows. First, if the return on investment function in each future period is independent of the firm's current investment decision, regardless of how these return on investment functions shift over time, a firm's cost of capital is k. That is, the firm's investment decision should equate the marginal rate of return on investment with the firm's share yield, assuming that $\partial k/\partial q=0$.

On the other hand, if the return on investment function in period j depends in some way on the investment decision in period $t<j$, then regardless of what this relation is, a firm's cost of capital depends on its investment decision and is not equal to k, even if k is independent of that investment decision. The expression (k-qr)/(1-q) incorporates the

particular assumption that the return on investment expressed as a function of q in every future period is the same and independent of q, which means the return as a function of the level of investment shifts upward at a constant rate that increases with the value of qr. Arguments for and against this assumption may be found in Gordon and Gould [5] and Miller [9].

The theoretical basis for believing that the yield investors require on a share is an increasing function of the firm's investment rate ($\partial k/\partial q>0$) is contained in Fewings [2]. An expression for share yield that incorporates this assumption in an empirically feasible manner is provided in Gordon [3] and Gordon and Gould [5].

B. The RIFI Cost of Equity Capital Models

In order to arrive at the cost of equity capital when the return on investment functions in future periods are independent of the current investment decisions, we define Δ to equal the rate of growth in price that is independent of the investment decision in the current period. The appropriate stock value model is

$$P=[Y(1-b)+P+P\Delta+Y(b+s)r/k-Ys]/(1+k). \qquad (6)$$

The first term in the numerator is the dividend in the coming period. The next two terms are the recovery of the investment in the stock, plus the appreciation in the stock due to investments the firm will make in subsequent periods, as well as other events that are independent of the firm's current decisions.

The next term is the price appreciation due to the current investment of Y(b+s) to earn r. The last term is the value of the shares issued

to cover the stock financed portion of the investment.

Rearranging Eq. (6) gives

$$P=[Y(1-q+qr/k)]/(k-\Delta). \tag{7}$$

Taking the partial derivative with respect to q, setting the result equal to zero, and solving for the marginal rate of return on investment results in

$$\frac{\partial qr}{\partial q} = r' = k + \frac{\partial k}{\partial q}\left[\frac{k(1-q)+qr}{k-\Delta} + \frac{qr}{k}\right]. \tag{8}$$

If $\partial k/\partial q=0$ we have the well known SYI-RIFI solution. The cost of equity capital is k. Without the assumption that $\partial k/\partial q=0$, Eq. (8) is the SYD-RIFI solution to the cost of equity capital.

II.

This section will extend the previous models to recognize the presence of transaction costs and personal income taxes. With the debt ratio fixed, the corporate income tax may be ignored, and all that will concern us is the differential tax treatment of dividends and realized capital gains. The transaction costs of buying and selling outstanding shares is relatively small, and the only transaction costs that will be incorporated in the analysis are the flotation and under-pricing costs of a new issue.

The notation that will be employed beyond the terms introduced previously is:

t_d = Personal income tax rate on dividends;

t_g = Personal income tax rate on realized capital gains;

k^τ = Discount rate which equates the expected after-tax cash flows from a share with its price;

w = Flotation costs on a new issue expressed as a fraction of the funds raised from the issue, after adjusting for any corporate tax deductions;

c_s = Cost of stock-financed capital;

c_b = Cost of retention-financed capital.

In developing an after-tax model of stock value, we cannot assume as with Eq. (1) that the investor buys a stream of dividends for the infinite future. In that event, the investor would ignore the capital gains tax in putting a value on the share, and our tax problem would disappear. We assume that the investor plans on selling the share at the end of one period. As noted below in Part III, the extension of the model to the case where the investor plans on selling the share two or more periods hence only requires a reduction in the value of t_g. We also make the simplifying assumptions that all investors are subject to the same tax rate on dividends and the same tax rate on capital gains, and these two tax rates are not expected to change over time.

A. The RIFD Assumption

We first arrive at a model of stock valuation and cost of capital under the assumption that the firm's return on investment function in future periods is dependent on the current investment decision. As derived in the appendix, we can express the price of the firm's stock as

$$P = \frac{Y[(1-b)(1-t_d) - (1-t_g)s/(1-w)]}{k^\tau - (1-t_g)(b+s)r} \tag{9}$$

subject to the usual convergence criterion, $k^\tau > (1-t_g)(b+s)r$, and a positive numerator.

If we set $t_d = t_g = w = o$, Eq. (9) reduces to Eq. (3). However, with $t_d \neq t_g$ the one-period and infinite horizon models are different. To determine the cost of retention capital with no stock financing we set s=o and take the derivative with respect to b.

$$\frac{\partial P}{\partial b} = \frac{Y(1-t_d)}{[k^\tau - br(1-t_g)]^2} \{\frac{\partial br}{\partial b}(1-b)(1-t_g) - \frac{\partial k^\tau}{\partial b}(1-b) - k^\tau + br(1-t_g)\}. \tag{10}$$

Solving for the before-tax marginal rate of return on investment, we find that P is maximized when b is set to satisfy

$$\frac{\partial br}{\partial b} = r' = c_b = \frac{k^\tau - (1-t_g)br}{(1-b)(1-t_g)} + \frac{\partial k^\tau/\partial b}{1-t_g} . \tag{11}$$

To obtain the cost of stock-financed capital with no retention financing, we set b=o and take the derivative of P with respect to s.

$$\frac{\partial P}{\partial s} = \frac{Y}{[k^\tau - sr(1-t_g)]^2} \{- \frac{1-t_g}{1-w}[k^\tau - (1-t_g)sr] - [\frac{\partial k^\tau}{\partial s} - \frac{\partial sr}{\partial s}(1-t_g)]$$

$$[1-t_d - s\frac{(1-t_g)}{(1-w)}]\} . \tag{12}$$

Solving for r' once again we find that P is maximized by the stock financing rate that satisfies

$$\frac{\partial sr}{\partial s} = r' = c_s = \frac{k^\tau - (1-t_g)sr}{(1-t_d)(1-w) - s(1-t_g)} + \frac{\partial k^\tau/\partial s}{1-t_g} . \tag{13}$$

A comparison of Eqs. (11) and (13) reveals the difference between the costs of retention and stock-financed capital when the other is set equal to zero, for a given investment rate q equal to b or s. On the reasonable assumption that $\partial k^{\tau}/\partial b = \partial k^{\tau}/\partial s$, the two equations differ only in the denominators of the first term.[1] The denominator of Eq. (11) is larger by $t_d - t_g + w(1-t_d)$. It follows that the cost of stock-financed capital is larger due both to flotation costs and the difference between the tax rates on dividends and capital gains.

The conclusion that $c_s > c_b$ holds when the alternative source of funds is non-zero as well as when the alternative source of funds is zero. It follows that, with the investment rate constrained so that $b+s<1$, a firm will never engage in stock financing. This conclusion is contrary to fact. Some firms do on occasion engage in stock financing. Two possible explanations are our assumption with regard to the dependence of the firm's investment function on prior investment decisions, and that the firm's dividend has no informational content.

B. The RIFI Assumption

We will see shortly that, with the firm's return on investment function independent of prior investment decisions, a firm may engage in stock financing. To establish the values of c_b and c_s for the RIFI case, we introduce personal income taxes and flotation costs to Eq. (6) to obtain

$$P = \{Y(1-b)(1-t_d) + P + P\Delta(1-t_g) + (1-t_g)[Ybr(1-t_d)/k^{\tau}] + (1-t_g)Ys\frac{[r(1-t_d) - k^{\tau}/(1-w)]}{k^{\tau}}\}/(1+k^{\tau}). \quad (14)$$

[1]This represents the SYD-RIFD case, but in the SYI-RIFD case $\partial k^{\tau}/\partial b = \partial k^{\tau}/\partial s = 0$, and the subsequent argument is unchanged.

The first term is the after-tax dividend in the coming period. The next two terms are the tax-free recovery of the investment in the stock, plus the after-tax appreciation in the stock due to the investments the firm will make in subsequent periods, as well as other events that are independent of the firm's current decisions.

The next term is the after-tax capital gain due to the retention financed investment in the current period. This term assumes that the entire return on the investment will be paid out in dividends. In that event, the retention-financed investment will increase subsequent dividends by Ybr. Their after-tax periodic amount is $Ybr(1-t_d)$. Dividing by k^τ produces their end-of-period present value, and multiplying by $(1-t_g)$ results in the after-tax capital gain. The last term is the end-of-period after-tax capital gain due to the stock-financed investment during the period. The after-tax periodic income that accrues to the start-of-period shareholders is $Ysr(1-t_d)$, less the outflow to new shareholders of $Y[s/(1-w)]k^\tau$. Here, also, the capital gain reflects the assumption that the return on the investment will be paid in dividends.

Valuing a firm's stock on the assumption that the future earnings on the current investment will all be paid in dividends may not be considered correct on the grounds that some part of these earnings will be retained and invested. Under this reasoning, Eq. (14) understates P, and the use of Eq. (14) to arrive at the cost of capital from each source would bias the figure upwards. However, the subsequent decision to retain any part of these earnings will depend on the after-tax profitability that these earning will finance. In other words, the tax advantage from retaining these earnings accrues to the future investments that will make their retention possible. It would then follow that with the no-tax return on investment

function independent of prior investment decisions, the Eq. (14) treatment of the future earnings generated by current investment as being paid in dividends is correct.[1]

Solving Eq. (14) for P and taking the partial derivative with respect to b gives:

$$\frac{\partial P}{\partial b} = \frac{Y(1-t_d)}{[k^\tau-\Delta(1-t_g)]^2} \left[\begin{array}{l} [k^\tau-\Delta(1-t_g)][-1+\{[k^\tau(1-t_g)\frac{\partial br}{\partial b}]-[(1-t_g)br]\frac{\partial k^\tau}{\partial b}\} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad k^{\tau 2} \\ - [(1-b) + \frac{(1-t_g)br}{k^\tau}]\frac{\partial k^\tau}{\partial b} \end{array} \right] . \quad (15)$$

Solving for the marginal rate of return on investment, we find that the value of b which satisfies

$$\frac{\partial br}{\partial b} = r' = c_b = \frac{k^\tau}{1-t_g} + \frac{\partial k^\tau}{\partial b}\{\frac{k^\tau(1-b) + (1-t_g)br}{[k^\tau-\Delta(1-t_g)](1-t_g)} + \frac{br}{k^\tau}\} \quad (16)$$

maximizes the stock price.

Setting b=0, and carrying out the analogous operations for the stock financing decision, we find that

$$\frac{\partial sr}{\partial s} = r' = c_s = \frac{k^\tau}{(1-t_d)(1-w)} + \frac{\partial k^\tau}{\partial s}\{\frac{k^\tau[1-\frac{s(1-t_g)}{(1-t_d)(1-w)}] + (1-t_g)sr}{[k^\tau - \Delta(1-t_g)](1-t_g)} + \frac{sr}{k^\tau}\}. \quad (17)$$

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[1]On the other hand, the level of future investment that can enjoy the favorable tax treatment of retention financing depends on the level of future earnings, which in turn depends on the level of current investment, retention or stock-financed. Hence, even when the no-tax return on investment function is independent of prior investment decisions, the differential tax treatment of dividends and capital gains creates dependence. We assume that this dependence is small and can be ignored.

Comparison of the two cost of capital expressions supports the conclusion that $c_s > c_b$. The denominator of the first term for c_b, which is $1-t_g$, exceeds the corresponding term in c_s by $t_d-t_g+w(1-t_d)$. This is the same difference that we had in Eqs. (11) and (13). The second term on the right hand side of Eq. (17) is greater than the corresponding term in Eq. (16) since $b<s(1-t_g)/(1-t_d)(1-w)$. However, the contribution of these terms to the excess of c_s over c_b is likely to be small, because we are only changing the levels of the current retention and stock financing rates, and k^τ varies with the long run rate of growth. Furthermore, these terms disappear in the SYI-RIFI case where $\partial k^\tau/\partial b = \partial k^\tau/\partial s = 0$.

It does not follow that the firm will never engage in stock financing, for now the investment decision is not constrained so that $b+s<1$. With a firm's return on investment function dependent on prior investment decisions, $q=b+s$ is the firm's long run investment rate, and a firm with $q>1$ under these conditions is a sink. However, with the return on investment function independent of prior investment decisions, q may vary in any way from one period to the next, and $q>1$ in any period does not imply that the firm is a net drain on society.

With $c_b<c_s$ the firm would use retention financing as long as r' remained greater than the right hand side of Eq. (16). However, the use of retention financing for investment is subject to a constraint. At a maximum $b=1$, and if the dividend contains information with regard to future dividends and earnings, a firm may well constrain $b<1$. If the constrained value of b does not satisfy Eq. (16), then stock financing is not precluded. We can take the partial derivative of Eq. (14) with respect to s, treating the constrained value of b as a constant, and solve for the marginal return on investment to obtain

$$\frac{\partial qr}{\partial q} = \frac{\partial qr}{\partial s} = r' = c_s = \frac{k^\tau}{(1-t_d)(1-w)} + \frac{\partial k^\tau}{\partial s}\left\{\frac{k^\tau\left[1-b-\frac{s(1-t_g)}{(1-t_d)(1-w)}\right] + (1-t_g)qr}{[k^\tau - \Delta(1-t_g)](1-t_g)} + \frac{qr}{k^\tau}\right\}. \quad (1$$

Finally, in comparing Eqs. (11) and (13) with Eqs. (16) and (17), it is not possible to make any simple statements with regard to how the cost of each source of funds differs depending on the properties of the firm's return on investment function. Nonetheless, it would seem that the optimal investment is higher with the return on investment function independent of prior investment decisions than under the alternative assumption.

III.

As stated earlier Lewellen [6] contains the only previous solution to the problem that was turned up by our search of the literature.[1] He posed the following question for a firm that is initially paying out all of its earnings in dividends: if an investment of bY is made, and all

[1] See [6], Ch.5, pp. 53-74. Pye [11] was concerned primarily with the dynamic choice between retention and stock financing, that is, the extent to which a firm should retain earnings in order to avoid stock financing in subsequent periods. He arrived at the same conclusions with regard to the cost of capital as Lewellen.

Solomon [12] argued that the cost of equity capital with the differential tax treatment of dividends and capital gains is simply k on the reasoning that firms may invest in the shares of other firms to earn k without limit and thereby avoid paying any dividends. There is some merit in Solomon's solution insofar as firms are free to repurchase their own shares. If the tax authorities allow firm's to label dividends as share repurchases, the cost of equity capital from each source is as presented below with t_d=0.

subsequent earnings from this investment are paid out in dividends, what rate of return will leave the stockholder indifferent to receiving bY in dividends? If bY is paid in dividends the investor will receive $bY(1-t_d)$ after taxes. Alternatively, he argued, the retention of bY will provide a perpetual stream of earnings of bYr', which when discounted at k results in an after-tax capital gain of $(bYr'/k)(1-t_g)$. Equating the two wealth levels gives

$$bY(1-t_d)=(bYr'/k)(1-t_g). \tag{18}$$

Solving for the required return on investment, Lewellen found that the cost of retention capital is

$$r'=c_b=k(1-t_d)/(1-t_g). \tag{19}$$

In reaching this solution Lewellen implicitly assumed that both k and the return on investment functions in future periods are independent of the current investment decision. Hence, our SYI-RIFI model, which incorporates the same assumptions, is comparable to the Lewellen model. Our solution in that case was Eq. (16) with $\partial k/\partial b=0$ or

$$r'=c_b=k^{\tau}/(1-t_g). \tag{20}$$

Comparison of Eqs. (19) and (20) raises two questions. Is $k^{\tau}=k(1-t_d)$? In that case our solution is the same as Lewellen's. If $k^{\tau}\neq k(1-t_d)$ which solution is right? The relation between k and k^{τ} can be established by noting that the price of a share can be expressed in terms of either before-tax or after-tax cash flows, as long as the appropriate discount rate is used. That is, with D the next period's expected dividend, and g the expected growth in price, we have

$$P = [D + P(1+g)]/(1+k) = D/(k-g) \tag{21}$$

or

$$P = \{D(1-t_d) + P(1+g) - t_g[P(1+g)-P]\}/(1+k^\tau)$$

$$= D\ (1-t_d)/[k^\tau-g(1-t_g)]. \tag{22}$$

With P the same in both equations, we may equate the two expressions for P and solve for k^τ. The result is

$$k^\tau = k(1-t_d) + g(t_d-t_g). \tag{23}$$

Solving for k, we obtain

$$k = \frac{k^\tau}{1-t_d} - \frac{g(t_d-t_g)}{1-t_d}. \tag{24}$$

It is clear then that $k^\tau=k(1-t_d)$ only in the uninteresting special case where g=0. Furthermore, as g and t_d-t_g increase, the difference between k and k^τ decreases.

The determination as to whether Eq. (19) or (20) is the correct expression for the cost of retention capital under the SYI-RIFI assumptions is facilitated by deriving our Eq. (20) under the Lewellen mode of analysis. The investment of bY generates a periodic future dividend of bYr', but the future dividends that an investor receives is net of taxes, or $(1-t_d)bYr'$. Hence, the price appreciation that results is $(1-t_d)bYr'/k^\tau$ and not bYr'/k. It is as simple as that. Consequently, the correct equality between the wealth levels represented by paying bY in dividends and investing bY is

$$bY(1-t_d)=(1-t_d)(bYr'/k^\tau)(1-t_g) \tag{25}$$

Solving this expression for $r'=c_b$ results in our Eq. (20).

Comparison of what had been considered and what we now see is the correct solution for the SYI-RIFI cost of retention capital is facilitated by substituting Eq. (23) for k^{τ} in Eq. (20). The result is

$$r' = c_b = \frac{k(1-t_d)}{1-t_g} + \frac{g(t_d-t_g)}{1-t_g} . \tag{26}$$

We see that c_b exceeds the Lewellen solution by an amount that varies with the share's expected rate of growth and with t_d-t_g. A simple numerical illustration provides some indication of the quantitative importance of the correction. Assume a share with $k=.13$, $g=.07$, $t_d=.50$ and $t_g=.25$. The Lewellen solution is $c_b=.0867$ and our solution is $c_b=.1100$.

For the cost of stock-financed capital Lewellen recommended $c_s=k/(1-w)$. Our solution under the SYI-RIFI assumptions is Eq. (17) with $\partial k^{\tau}/\partial s=0$ or

$$r'=c_s=k^{\tau}/(1-t_d)(1-w). \tag{27}$$

Substituting for k^{τ} results in

$$r'=c_s= \frac{k}{1-w} + \frac{g(t_d-t_g)}{(1-t_d)(1-w)}. \tag{28}$$

Again we see that Lewellen's result is obtained only with $g=0$, and the error increases with g and with t_d-t_g.

The following observations may be of some help in interpreting and using our SYI-RIFI models for the costs of retention and stock-financed capital. We start with the generally accepted proposition that investors use after-tax cash flows in arriving at an asset's value. Hence, the SYI assumption in the absence of taxes implies that with a personal income tax it is k^{τ} and not k that takes on this independence property. Accordingly, Eq. (20) and not Eq. (19) is the correct measure of the cost of retention capital.

However, we cannot observe k^{τ} directly, while we can observe k. Accordingly, given g, t_d and t_g as well as k, we may make the substitution that results in Eqs. (26) and (28) for c_b and c_s. Assume now that a change in g is contemplated for a firm. We cannot use the old k and the new g to arrive at a new c_b in Eq. (26). From Eq. (20) we see that c_b does not change with g. The old k and g may be used with Eq. (23) to arrive at k^{τ}. Eq. (24) may then be used to arrive at the change in k with g.

In using Eq. (23) to arrive at k^{τ} or in using Eq. (26) to arrive at c_b directly, we require t_d and t_g as well as k and g. A resonable figure for t_d is a weighted average of the marginal tax rates on ordinary income with the weights based on the taxpayers relative holdings of shares. If the capital gains tax rate is 25% or half the ordinary tax rate, these figures should be used if investors hold shares for one period. The longer the average holding period for shares, the lower the effective tax rate on capital gains. In the limit if all investors hold all shares forever, t_g goes to zero.[1/]

[1]See Whaley [13] for the determination of how the effective capital gains tax rate declines with the holding period and an estimate of the effective rate.

Appendix

The additional symbols which will be used in this appendix are defined below:

V_t = Value of the firm at the end of period t;

X_t = Expected value of the firm's earnings in period t;

n = Number of shares currently outstanding at t=0;

We can express the current value of the firm, V_o, as

$$V_o = \frac{X_1(1-b)(1-t_d)}{1+k^\tau} + \frac{[V_1 - \frac{s}{1-w} X_1]}{1+k^\tau} - \frac{t_g\{[V_1 - \frac{s}{1-w} X_1] - V_o\}}{1+k^\tau}. \tag{A1}$$

The first term on the right hand side is the discounted value of the after-tax dividend. The second term is the discounted value of the investors' holdings after any retention or new stock financing, which are assumed to occur at the end of the period. Notice that the firm must sell $(\frac{s}{1-w})X_1$ stock in order to provide new equity capital of sX_1 after flotation costs. The third term is the discounted capital gains tax.

Now, letting $\emptyset = [(1-b)(1-t_d) - \frac{s}{1-w}(1-t_g)]$, we can rearrange Eq. (A1) to give

$$V_o = \frac{X_1\emptyset}{1+k^\tau-t_g} + (\frac{(1-t_g)}{1+k^\tau-t_g})(V_1). \tag{A2}$$

In the following period, earnings will grow due to retention financing by bX_1r and due to stock financing by sX_1r, and we have

$$X_2 = X_1 + bX_1r + sX_1r = X_1(1+br+sr)$$

and generalizing:

$$X_{t+1} = X_1(1+br+sr)^t.$$

We can therefore express V_1 as

$$V_1 = \frac{X_1(1+br+sr)\emptyset}{1+k^\tau-t_g} + \frac{(1-t_g)}{1+k^\tau-t_g} V_2, \tag{A3}$$

and generalizing:

$$V_t = \frac{X_1(1+br+sr)^t\emptyset}{1+k^\tau-t_g} + \frac{(1-t_g)}{1+k^\tau-t_g} V_{t+1}. \tag{A4}$$

Substituting Eq. (A4) into Eq. (A2) gives

$$V_o = \frac{X_1\emptyset}{1+k^\tau-t_g} + \left[\frac{1-t_g}{1+k^\tau-t_g}\right]\left[\frac{X_1(1+br+sr)\emptyset}{1+k^\tau-t_g}\right] + \left[\frac{1-t_g}{1+k^\tau-t_g}\right]^2\left[\frac{X_1(1+br+sr)^2\emptyset}{1+k^\tau-t_g}\right] + \ldots$$

Therefore,

$$V_o = \left(\frac{X_1\emptyset}{1+k^\tau-t_g}\right)\left(\sum_{t=o}^{\infty} \frac{[(1+br+sr)(1-t_g)]^t}{[1+k^\tau-t_g]^t}\right) \tag{A5}$$

Evaluating the infinite sum and substituting the definition of $\emptyset$ results in

$$V_o = \frac{X_1[(1-b)(1-t_d) - (\frac{s}{1-w})(1-t_g)]}{k^\tau-(b+s)r(1-t_g)} \tag{A6}$$

if $k^\tau > (b+s)r(1-t_g)$.

Since $V_o = nP_o$, and $X_1 = nY$, we can also express Eq. (A6) on a per-share basis. Dividing both sides by n results in Eq. (9) in the text.

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