A Stopping Rule for Facilities Location Algorithms

BY

ROBERT F. LOVE
McMaster University

and

WEE Y. YEONG
University of Singapore

FACULTY OF BUSINESS
McMASTER UNIVERSITY
HAMILTON, ONTARIO

Working Paper No. 170
September, 1980

A STOPPING RULE FOR FACILITIES LOCATION ALGORITHMS

Robert F. Love
McMaster University

Wee Y. Yeong
University of Waterloo

Abstract. The single facility location model with Euclidean distances and its multi-facility and ℓ_p distance generalizations are considered. With present algorithms a user is unable to decide how close to optimal any given feasible solution is. This article describes a procedure for calculating a lower bound on the optimal objective function when a proposed solution is given.

Location problems concern themselves with finding the best location for new facilities (one or more warehouses for example) with respect to a number of existing facilities (retailers to be supplied from the warehouses). In continuous location problems the cost elements are the transportation costs between the new facility and each of the existing facilities. The transportation costs are usually assumed to be proportional to some measure of the distance between the facilities, with the constant of proportionality incorporating the annual volume transported and the transportation cost per unit volume per unit distance. A practical application of a continuous location model involved the location of two new machines in an existing plant layout [11].

This article proposes a rational stopping criterion for use with iterative algorithms used to solve continuous facility location problems. The method is developed for both single and multi-facility location problems with generalized ℓ_p distance measures. In practice, the advantages of computing a lower bound on the optimal cost of a location problem is the computer time savings that may be achieved and the confidence gained by the user that his solution is as close to optimality as he desires while using an iterative solution procedure. Computation time savings are achieved by knowing exactly when to stop the iterative procedure rather than carrying on needless calculations attempting to satisfy an arbitrary stopping rule. Computational savings may be especially significant when very large numbers of individual location problems must be solved during a single computer run. This occurs, for example, when one utilizes a location-allocation algorithm to solve warehouse and plant location problems [8].

The single-facility location problem is to

$$\text{minimize } W(x) = \sum_{j=1}^{n} w_j \, d(x,a_j). \tag{1}$$

where

> n is the number of existing facilities, w_j converts the distance between the new facility and existing facility j into cost,
>
> $x = (x_1,x_2)$ is the location of the new facility on the plane,
>
> $a_j = (a_{j1},a_{j2})$ is the location of existing facility j, and
>
> $d(x,a_j)$ is the Euclidean distance between the new facility and existing facility j.

It is generally conceded that the first iterative procedure for solving (1) was proposed by Weiszfeld [15]. The stopping rule given here is not restricted to be used only with Weiszfeld's procedure. However, since the Weiszfeld procedure is so well known, we shall use it and a new generalized version of it to demonstrate the method.

At the k^{th} iteration of Weiszfeld's procedure, a point $x_k = (x_1^k,x_2^k)$ is generated as follows:

$$x_i^{k+1} = \sum_{j=1}^{n} \frac{w_j a_{ji}}{d(x_k, a_j)} \Big/ \sum_{j=1}^{n} \frac{w_j}{d(x_k, a_j)}, \text{ for } i=1,2. \tag{2}$$

The convergence properties of the sequence (2) have been discussed by Weiszfeld [15], Katz [4,5], Kuhn [6] and Ostresh [14], among others. From a practical computing aspect, however, the important issue of a useable stopping criterion has been ignored. It has generally been the practice to stop the sequence when an iteration produces an improvement in the objective function which is less than a pre-specified value, or when the partial derivatives of W(x) become "small". However, these types of criteria are arbitrary and give no assurance that the present solution is close to optimal, either in decision or value space. In this article, we develop a computational procedure which enables the user to compute at each step of (2) and a generalized version of (2) a lower bound on the optimal objective function value. The iterative procedure given by (2) can then be automatically terminated when the difference between the current solution value and the lower bound is within a prespecified tolerance set by the user.

Development of the Stopping Criterion

The points of interest for the sequence generated by (2) do not include the a_j, j=1,...,n. These points are generally tested for optimality using the following criterion given by Kuhn [7].

W(x) is minimum at (a_{r1}, a_{r2}) if and only if

$$\left[\left(\sum_{\substack{j=1\\ \neq r}}^{n} \frac{w_j(a_{r1}-a_{j1})}{d(a_r,a_j)}\right)^2 + \left(\sum_{\substack{j=1\\ \neq r}}^{n} \frac{w_j(a_{r2}-a_{j2})}{d(a_r,a_j)}\right)^2\right]^{1/2} \leq w_r. \tag{3}$$

The a_j are tested using (3) before the computation of the sequence (2) is started, since it is only in the event that none of the a_j are optimal that we are interested in generating the sequence. For our purposes, we ignore the a_j and assume that none of them will become part of the sequence generated by (2).

Note that W(x) is a convex function, and except for the points a_j, j=1,...,n, W(x) is differentiable. Thus

$$W(y) \geq W(x) + \nabla W(x)'(y-x) \text{ for all } x, y \in E^2, \tag{4}$$

where $\nabla W(x)$ is the gradient of W(x) and the prime denotes transpose. Let Ω be the convex hull generated by the fixed points a_j, j=1,...,n, and for $x \in E^2$, let

$$\bar{\sigma}(x) = \max_{y \in \Omega} \{d(x,y)\}.$$

Since the maximum of a convex function defined on a compact convex set occurs at an extreme point, it follows that

$$\bar{\sigma}(x) = \max_{j=1,\ldots,n} \{d(x,a_j)\} \tag{5}$$

Proposition

For the single facility location problem, let x_k be the solution point given by the Weiszfeld procedure at the k^{th} iteration. Then a bound on the improvement in the objective function value in succeeding iterations is given by $\bar{\sigma}(x_k) \, \|\nabla W(x_k)\|$.

Proof

The optimum solution x* for the single facility location problem is in Ω, and the Weiszfeld procedure generates the sequence $\{x_k\}_{k=1}^{\infty}$ such that $x_k \to x^*$. Let $y \in \Omega$ such that $y \neq x_k$. Then $y = x_k + \sigma r$, where $\sigma > 0$, and r is the unit directional vector from x_k to y.

Since $W(x)$ is convex,

$$\begin{aligned} W(y) &\geq W(x_k) + \nabla W(x_k)' \, (y-x_k) \\ &= W(x_k) + \sigma \nabla W(x_k)' r \\ &\geq W(x_k) + \sigma \nabla W(x_k)' \left(- \frac{\nabla W(x_k)}{\|\nabla W(x_k)\|} \right) \\ &= W(x_k) - \sigma \, \|\nabla W(x_k)\| \\ &\geq W(x_k) - \bar{\sigma}(x_k) \, \|\nabla W(x_k)\| \end{aligned}$$

Since the above inequality holds for all $y \varepsilon \Omega$,

$$W(x_k) - W(x^*) \leq \bar{\sigma}(x_k) \, \|\nabla W(x_k)\| \, .$$

This result suggests the following stopping criterion for the Weiszfeld procedure. At iteration k, let x_k be the current solution point given by the Weiszfeld procedure, with current objective function value $W(x_k)$. Compute $\bar{\sigma}(x_k)$ and $\|\nabla W(x_k)\|$. Then, if $\frac{\bar{\sigma}(x_k) \, \|\nabla W(x_k)\|}{W(x_k)} \times 100\% \leq \alpha\%$, stop and accept x_k as an adequate solution for problem (1), where $\alpha > 0$ is a prespecified tolerance.

Extension to the Multi-Facility ℓ_p Distance Problem

The multi-facility ℓ_p distance location problem is to

$$\text{minimize } WM_p(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} w_{1ij} \left[|x_{i1} - a_{j1}|^p + |x_{i2} - a_{j2}|^p \right]^{1/p}$$

$$+ \sum_{i<r} w_{2ir} \left[|x_{i1} - x_{r1}|^p + |x_{i2} - x_{r2}|^p \right]^{1/p}, \qquad (6)$$

where

- m is the number of new facilities,
- n is the number of existing facilities,
- w_{1ij} is the parameter which converts the distance between new facility i and existing facility j into cost,
- w_{2ir} is the parameter which converts the distance between the ith and rth new facilities into cost ($i \neq r$), and
- $x_i = (x_{i1}, x_{i2})$ are the location coordinates of new facility i,
- $a_j = (a_{j1}, a_{j2})$ are the location coordinates of existing facility j, and
- p is the ℓ_p distance parameter. (Note the notational difference between x_i, $i=1,2$, in this section as compared to that used in discussing the single facility model.)

We shall now illustrate how one could generalize the stopping rule to any iterative algorithm used to solve problem (6). Define $\bar{\Omega} \subseteq E^{2m}$ as $\bar{\Omega} = \{x = (x_{11}, x_{12}, \ldots, x_{m1}, x_{m2}) \mid (x_{i1}, x_{i2}) \in \Omega,\ i=1,\ldots,m\}$. For any $x \in E^{2m}$, let $\bar{\bar{\sigma}}(x) = \max \{d(x,y) \mid y \in \bar{\Omega}\}$. Since for any $y \in \bar{\Omega}$,

$d(x,y) = \|x-y\| = (\sum_{i=1}^{m} [d(x_i,y_i)]^2)^{1/2} \leq (\sum_{i=1}^{m} (\bar{\sigma}(x_i))^2)^{1/2}$. Therefore,

$\bar{\bar{\sigma}}(x) \leq (\sum_{i=1}^{m} (\bar{\sigma}(x_i))^2)^{1/2}$. Let $x_k = (x_{11}^k, \ldots, x_{m1}^k, x_{m2}^k)$ be a point generated by any iterative procedure at the kth iteration and let $WM_p(\bar{x}) = \min WM_p(x)$. Using a result by Juel [3], the optimal solution $\bar{x}$ is in $\bar{\Omega}$, and we have, by analogous argument to the single facility case

$$WM_p(x_k) - WM_p(\bar{x}) \leq \bar{\bar{\sigma}}(x_k) \, \|\nabla WM_p(x_k)\| \, . \tag{7}$$

The first order derivatives of $WM_p(x)$ are not defined at the existing facility locations [9]. In an effort to overcome this problem, Wesolowsky and Love [16] and Eyster, White and Wierwille [1] proposed the the following hyperbolic approximating function to replace $WM_p(x)$:

$$WM_{ph}(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} w_{1ij}[((x_{i1}-a_{j1})^2 + \varepsilon)^{p/2} + ((x_{i2}-a_{j2})^2 + \varepsilon)^{p/2}]^{1/p}$$

$$+ \sum_{i<r} w_{2ir}[((x_{i1}-a_{r1})^2 + \varepsilon)^{p/2} + ((x_{i2}-x_{r2})^2 + \varepsilon)^{p/2}]^{1/p}, \tag{8}$$

where $\varepsilon > 0$. The function $WM_{ph}(x)$ is strictly convex and is differentiable to any order everywhere. It can easily be shown that $WM_{ph}(x)$ is uniformly convergent to $WM_p(x)$ as $\varepsilon \to 0$ since

$$\max\ (WM_{ph}(x)-WM_p(x)) = 2^{1/p}\ \varepsilon^{1/2}\left(\sum_{i=1}^{m}\sum_{j=1}^{n} w_{1ij} + \sum_{i<r} w_{2ir}\right). \tag{9}$$

By setting $\partial WM_{ph}(x)/\partial x_{rs}$ to zero, the iterative sequence given by (2) generalizes to the multi-facility case for $r=1,2,\ldots,m$; and $s=1,2$ as:

$$x_{rs}^{k+1} = (A+B)/(C+D), \tag{10}$$

where

$$A = \sum_{i=1}^{m} \frac{w_{2ri}x_{is}^k}{\{[(x_{r1}^k-x_{i1}^k)^2+\varepsilon]^{p/2} + [(x_{r2}^k-x_{i2}^k)^2+\varepsilon]^{p/2}\}^{(p-1)/p}[(x_{rs}^k-x_{is}^k)^2+\varepsilon]^{(2-p)/2}},$$

$$B = \sum_{j=1}^{n} \frac{w_{1rj}a_{js}}{\{[(x_{r1}^k-a_{j1})^2+\varepsilon]^{p/2} + [(x_{r2}^k-a_{j2})^2+\varepsilon]^{p/2}\}^{(p-1)/p}[(x_{rs}^k-a_{js})^2+\varepsilon]^{(2-p)/2}},$$

$$C = \sum_{i=1}^{m} \frac{w_{2ri}}{\{[(x_{r1}^k-x_{i1}^k)^2+\varepsilon]^{p/2} + [(x_{r2}^k-x_{i2}^k)^2+\varepsilon]^{p/2}\}^{(p-1)/p}[(x_{rs}^k-x_{is}^k)^2+\varepsilon]^{(2-p)/2}},$$

and

$$D = \sum_{j=1}^{n} \frac{w_{1rj}}{\{[(x_{r1}^k-a_{j1})^2+\varepsilon]^{p/2} + [(x_{r2}^k-a_{j2})^2+\varepsilon]^{p/2}\}^{(p-1)/p}[(x_{rs}^k-a_{js})^2+\varepsilon]^{(2-p)/2}}$$

Ostresh [13] has proved that for $p=2$ this generalized Weiszfeld sequence is strictly decreasing and Morris [12] has given a convergence proof for $1<p\leq 2$. In numerical tests run by the authors and others, the sequence has always come within any specified tolerance to an optimum solution [10].

Let x^* be the optimal solution of $WM_{ph}(x)$ and x_k be the point generated by (10) at the kth iteration. Then

$$\begin{aligned} WM_{ph}(x_k) - WM_p(\bar{x}) &= WM_{ph}(x_k) - WM_{ph}(x^*) + WM_{ph}(x^*) - WM_p(\bar{x}) \\ &\leq WM_{ph}(x_k) - WM_{ph}(x^*) + WM_{ph}(\bar{x}) - WM_p(\bar{x}) \\ &\leq \bar{\bar{\sigma}}(x_k) \,\|\nabla WM_{ph}(x_k)\| \\ &\quad + 2^{1/p} \varepsilon^{1/2} \left(\sum_{i=1}^{m} \sum_{j=1}^{n} w_{1ij} + \sum_{i<r} w_{2ir} \right). \end{aligned} \tag{11}$$

Hence, by rearranging (11) a lower bound for $WM_p(\bar{x})$ can be obtained at each iteration of the generalized Weiszfeld procedure (10).

Computational Experience

The stopping criterion has been tested on several single and multi-facility test problems. The added computational cost is slight since the criterion uses values which have been computed in the course of the Weiszfeld procedure anyway. In all examples that the authors have run, the total cost function was strictly decreasing as is proved theoretically by Weiszfeld [15], Katz [4,5], and Morris [12]. The following example has 5 existing facilities and 3 new facilities.

j	1	2	3	4	5
a_j	(2, 3)	(4, 2)	(5, 4)	(3, 5)	(6, 7)

Table 1

Existing Facility Locations

w_{1ij}

i \ j	1	2	3	4	5
1	1	1	10	1	6
2	4	1	1	1	1
3	1	1	1	1	1

Table 2

New-to-Existing Facility Weights

w_{2ir}

i \ r	1	2	3
1	--	1	1
2	--	--	1

Table 3

Weights Between New Facilities

With $p = 1.8$, and $\varepsilon = 10^{-6}$, the following table contains computational results for the problem using the generalized Weiszfeld procedure. The initial solution points taken were:

$$(x_{11},x_{12}) = (x_{21},x_{22}) = (x_{31},x_{32}) = (0,\ 0).$$

After 20 iterations the objective function has come within .87% of the optimal solution; after 40 iterations it is within .127% of the optimal solution.

Multi-facility problems are usually much slower to converge than single facility problems on average (based on our experiences with running about 20 multi-facility problems and about 50 single facility problems). A single facility problem will usually converge to within .1% of the optimal solution in less than 5 iterations. There are, of course, exceptions. R. L. Francis has

TABLE 4: COMPUTATIONAL RESULTS FOR EXAMPLE

Iteration No.	x_1	x_2	x_3	$WM_p(x)$	$\nabla WM_{ph}(x_k)$	Lower Bound On $WM_p(\bar{x})$ (L.B.)	$[WM_p(x)-L.B.] \div (L.B.)$
0	(.0000, .0000)	(.0000, .0000)	(.0000, .0000)	210.7866	21.8458	.0000	–
1	(.0030, .0132)	(.0023, .0102)	(.0183, .0445)	210.1930	22.3861	.0000	–
2	(.0347, .1906)	(.1544, .4791)	(.3065, .8064)	201.2360	23.2518	.0000	–
3	(1.8084, 3.2586)	(1.3233, 2.5660)	(2.6273, 3.2464)	108.6370	20.3162	.0000	–
4	(3.3494, 3.9221)	(2.4800, 3.2035)	(2.7707, 3.4146)	75.0533	12.4520	.0000	–
5	(4.3133, 4.1231)	(2.5473, 3.2736)	(2.9978, 3.5704)	63.7436	9.6475	.0000	–
6	(4.8041, 4.0844)	(2.6383, 3.3290)	(3.1973, 3.6766)	59.0732	7.8966	1.2401	46.6367
7	(4.9541, 4.0412)	(2.7380, 3.3770)	(3.4117, 3.7518)	57.6079	6.6123	10.433	4.5216
8	(4.9882, 4.0187)	(2.8389, 3.4184)	(3.5803, 3.8012)	57.1308	5.8829	15.989	2.5735
9	(4.9961, 4.0084)	(2.9326, 3.4546)	(3.7054, 3.8326)	56.9143	5.6086	18.337	2.1037
10	(4.9985, 4.0037)	(3.0129, 3.4852)	(3.7963, 3.8528)	56.798	5.4583	19.748	1.8762
11	(4.9993, 4.0017)	(3.0732, 3.5088)	(3.8623, 3.8664)	56.7365	5.1638	22.026	1.5759
12	(4.9997, 4.0008)	(3.1259, 3.5280)	(3.9101, 3.8756)	56.7001	4.2147	28.587	.9834
13	(4.9998, 4.0004)	(3.1694, 3.5435)	(3.9447, 3.8819)	56.6787	2.3391	41.165	.3769
14	(4.9999, 4.0002)	(3.2050, 3.5560)	(3.9696, 3.8864)	56.6661	.8027	51.359	.1033
15	(4.9999, 4.0002)	(3.2338, 3.5661)	(3.9873, 3.8895)	56.6585	.2584	54.947	.0311
16	(4.9999, 4.0002)	(3.2570, 3.5743)	(3.9994, 3.8916)	56.654	.1499	55.658	.0179
17	(4.9999, 4.0002)	(3.2757, 3.5808)	(4.0063, 3.8930)	56.6514	.1141	55.892	.0135
18	(4.9999, 4.0002)	(3.2906, 3.5862)	(4.0117, 3.8938)	56.6497	.0905	56.046	.0108
19	(4.9999, 4.0002)	(3.3026, 3.5905)	(4.0157, 3.8944)	56.6486	.0723	56.164	.0086
20	(4.9999, 4.0002)	(3.3122, 3.5939)	(4.0186, 3.8948)	56.6479	.0073	56.161	.0087
21	(4.9999, 4.0002)	(3.3199) 3.5967)	(4.0208, 3.8951)	56.6475	.0470	56.328	.0057
22	(4.9999, 4.0002)	(3.3261, 3.5989)	(4.0224, 3.8954)	56.6472	.0456	56.337	.0055
23	(4.9999, 4.0002)	(3.3309, 3.6007)	(4.0236, 3.8955)	56.6471	.0305	56.436	.0037
24	(4.9999, 4.0002)	(3.3349, 3.6021)	(4.0244, 3.8957)	56.6469	.0258	56.466	.0032
25	(4.9999, 4.0002)	(3.3381, 3.6032)	(4.0251, 3.8957)	56.6469	.0207	56.499	.0026
30	(4.9999, 4.0002)	(3.3467, 3.6063)	(4.0265, 3.8959)	56.6468	.0108	56.563	.0015
35	(4.9999, 4.0002)	(3.3495, 3.6074)	(4.0269, 3.8959)	56.6468	.0092	56.574	.00129
40	(4.9999, 4.0002)	(3.3505, 3.6077)	(4.0270, 3.8959)	56.6467	.0090	56.575	.00127

noted that if one cluster of fixed facilities is near the origin and another fixed facility or cluster of fixed facilities is relatively far away from the origin, given certain weight structures convergence may be very slow [2]. In cases of this type the stopping criterion is especially useful.

Acknowledgements

This research was supported by grants from the Office of Research of the University of Waterloo, the Office of Research at McMaster University, the Natural Scineces and Engineering Research Council, Canada, and the Graduate School of the University of Wisconsin, Madison, Wisconsin.

References

[1] Eyster, J. W., White, J. A., and Wierwille, W. W., "On Solving Multifacility Location Problems Using a Hyperboloid Approximation", AIIE Transactions, 5 1(1963).

[2] Francis, R. L., Verbal communication (1980).

[3] Juel, H., "Properties of Location Models", Operations Research Technical Report No. 3, Graduate School of Business, University of Wisconsin, Madison, 1975.

[4] Katz, I. N., "On the Convergence of A Numerical Scheme for Solving Some Locational Equilibrium Problems", Siam Journal of Applied Mathematics, Vol. 17, No. 6, 1969, pp. 1224-1231.

[5] ____________, "Local Convergence in Fermat's Problem", Mathematical Programming, Vol. 6, 1974, pp. 89-104.

[6] Kuhn, H.W., "A Note on Fermat's Problem", Mathematical Programming, Vol. 4, 1973, pp. 98-107.

[7] Kuhn, H.W., "On A Pair of Dual Nonlinear Problems", Chapter 3 in T. Abadie (ed.), Nonlinear Programming, New York: John Wiley and Sons, 1967.

[8] Love, R. F., and Juel, H., "Properties and Solution Methods for Large Location-Allocation Problems", MRC Technical Report No. 1634; University of Wisconsin Mathematics Research Center, June 1976.

[9] Love, R.F., and Morris, J.G., "Modelling Inter-City Road Distances by Mathematical Functions", Operational Research Quarterly, Vol. 23, No. 1, 1972, pp. 61-71.

[10] Love, R.F. and Wesolowsky, G.O., "Subroutine for the Generalized Weiszfeld Procedure", Wisconsin Working Paper, Graduate School of Business, University of Wisconsin-Madison, August, 1976.

[11] Love, R. F., and Yerex, L., "An Application of a Facilities Location Model In The Prestressed Concrete Industry", Interfaces, 6 1(1976).

[12] Morris, J.G., "Analysis of a Generalized Empirical Distance Function For Use in Location Problems", paper presented at the International Symposium on Locational Decisions, Banff, Alberta, April, 1978.

[13] Ostresh, L.M., "The Multifacility Location Problem: Applications and Descent Theorems", Journal of Regional Science, Vol. 17, No. 3, 1977, pp. 409-419.

[14] Ostresh, L.M., "On the Convergence of a Class of Iterative Methods for Solving the Weber Location Problem", Operations Research, Vol. 26, No. 4, 1978, pp. 597-609.

[15] Weiszfeld, E., "Sur le point lequel la somme des distances de n points donnés est minimum", _Tôhoku Mathematics Journal_, Vol. 43, 1937, pp. 355-386.

[16] Wesolowsky, G.O. and Love, R.F., "A Nonlinear Approximation Method For Solving a Generalized Rectangular Distance Weber Problem", _Management Science_, Vol. 18, No. 11, 1972, pp. 656-663.

Faculty of Business
McMaster University

WORKING PAPER SERIES

101. Torrance, George W., "A Generalized Cost-effectiveness Model for the Evaluation of Health Programs," November, 1970.

102. Isbester, A. Fraser and Sandra C. Castle, "Teachers and Collective Bargaining in Ontario: A Means to What End?" November, 1971.

103. Thomas, Arthur L., "Transfer Prices of the Multinational Firm: When Will They be Arbitrary?" (Reprinted from: Abacus, Vol. 7, No. 1, June, 1971).

104. Szendrovits, Andrew Z., "An Economic Production Quantity Model with Holding Time and Costs of Work-in-process Inventory," March, 1974.

111. Basu, S., "Investment Performance of Common Stocks in Relation to their Price-earnings Ratios: A Text of the Efficient Market Hypothesis," March, 1975.

112. Truscott, William G., "Some Dynamic Extensions of a Discrete Location-Allocation Problem," March, 1976.

113. Basu, S. and J.R. Hanna, "Accounting for Changes in the General Purchasing Power of Money: The Impact on Financial Statements of Canadian Corporations for the Period 1967-74," April, 1976. (Reprinted from Cost and Management, January-February, 1976).

114. Deal, K.R., "Verification of the Theoretical Consistency of a Differential Game in Advertising," March, 1976.

114a. Deal, K.R. "Optimizing Advertising Expenditures in a Dynamic Duopoly," March, 1976.

115. Adams, Roy J., "The Canada-United States Labour Link Under Stress," [1976].

116. Thomas, Arthur L., "The Extended Approach to Joint-Cost Allocation: Relaxation of Simplifying Assumptions," June, 1976.

117. Adams, Roy J. and C.H. Rummel, "Worker's Participation in Management in West Germany: Impact on the Work, the Enterprise and the Trade Unions," September, 1976.

118. Szendrovits, Andrew Z., "A Comment on 'Optimal and System Myopic Policies for Multi-echelon Production/Inventory Assembly Systems'," [1976].

119. Meadows, Ian S.G., "Organic Structure and Innovation in Small Work Groups," October, 1976.

120. Basu, S., "The Effect of Earnings Yield on Assessments of the Association Between Annual Accounting Income Numbers and Security Prices," October, 1976.

121. Agarwal, Naresh C., "Labour Supply Behaviour of Married Women - A Model with Permanent and Transitory Variables," October, 1976.

122. Meadows, Ian S.G., "Organic Structure, Satisfaction and Personality," October, 1976.

123. Banting, Peter M., "Customer Service in Industrial Marketing: A Comparative Study," October, 1976. (Reprinted from: European Journal of Marketing, Vol. 10, No. 3, Summer, 1976).

124. Aivazian, V., "On the Comparative-Statics of Asset Demand," August, 1976.

125. Aivazian, V., "Contamination by Risk Reconsidered," October, 1976.

126. Szendrovits, Andrew Z. and George O. Wesolowsky, "Variation in Optimizing Serial Multi-Stage Production/Inventory Systems, March 1977.

127. Agarwal, Naresh C., "Size-Structure Relationship: A Further Elaboration," March 1977.

128. Jain, Harish C., "Minority Workers, the Structure of Labour Markets and Anti-Discrimination Legislation," March, 1977.

129. Adams, Roy J., "Employer Solidarity," March, 1977.

130. Gould, Lawrence I. and Stanley N. Laiken, "The Effect of Income Taxation and Investment Priorities: The RRSP," March 1977.

131. Callen, Jeffrey L., "Financial Cost Allocations: A Game-Theoretic Approach," March 1977.

132. Jain, Harish C., "Race and Sex Discrimination Legislation in North America and Britain: Some Lessons for Canada," May, 1977.

133. Hayashi, Kichiro. "Corporate Planning Practices in Japanese Multinationals." Accepted for publication in the Academy of Management Journal in 1978.

134. Jain, Harish C., Neil Hood and Steve Young, "Cross-Cultural Aspects of Personnel Policies in Multi-Nationals: A Case Study of Chrysler UK", June, 1977.

134a Jain, H. et al.

135. Aivazian, V. and J. L. Callen, "Investment, Market Structure and the Cost of Capital", July, 1977.

136. Adams, R. J., "Canadian Industrial Relations and the German Example", October, 1977.

137. Callen, J. L., "Production, Efficiency and Welfare in the U.S. Natural Gas Transmission Industry", October, 1977.

138. Richardson, A. W. and Wesolowsky, G.O., "Cost-Volume-Profit Analysis and the Value of Information", November, 1977.

139. Jain, Harish C., "Labour Market Problems of Native People in Ontario", December, 1977.

140. Gordon, M.J. and L.I. Gould, "The Cost of Equity Capital: A Reconsideration", January, 1978.

141. Gordon, M.J. and L.I. Gould, "The Cost of Equity Capital with Personal Income Taxes and Flotation Costs", January 1978.

142. Adams, R. J., "Dunlop After Two Decades : Systems Theory as a Framework For Organizing the Field of Industrial Relations", January, 1978.

143. Agarwal, N.C. and Jain, H.C., "Pay Discrimination Against Women in Canada: Issues and Policies", February, 1978.

144. Jain, H. C. and Sloane, P.J., "Race, Sex and Minority Group Discrimination Legislation in North America and Britain", March, 1978.

145. Agarwal, N.C., "A Labor Market Analysis of Executive Earnings", June, 1978.

146. Jain, H. C. and Young, A., "Racial Discrimination in the U.K. Labour Market : Theory and Evidence", June, 1978.

147. Yagil, J., "On Alternative Methods of Treating Risk," September 1978.

148. Jain, H. C., "Attitudes toward Communication System: A Comparison of Anglophone and Francophone Hospital Employees," September, 1978

149. Ross, R., "Marketing Through the Japanese Distribution System", November, 1978.

150. Gould, Lawrence I. and Stanley N. Laiken, "Dividends vs. Capital Gains Under Share Redemptions," December, 1978.

151. Gould, Lawrence I. and Stanley N. Laiken, "The Impact of General Averaging on Income Realization Decisions: A Caveat on Tax Deferral," December, 1978.

152. Jain, Harish C., Jacques Normand and Rabindra N. Kanungo, "Job Motivation of Canadian Anglophone and Francophone Hospital Employees", April, 1979.

153. Stidsen, Bent, "Communications Relations", April, 1979.

154. Szendrovits, A. Z. and Drezner, Zvi, "Optimizing N-Stage Production/Inventory Systems by Transporting Different Numbers of Equal-Sized Batches at Various Stages", April, 1979.

155. Truscott, W. G., "Allocation Analysis of a Dynamic Distribution Problem", June, 1979.

156. Hanna, J. R., "Measuring Capital and Income", November, 1979.

157. Deal, K. R., "Numerical Solution and Multiple Scenario Investigation of Linear Quadratic Differential Games", November, 1979.

158. Hanna, J. R., "Professional Accounting Education in Canada : Problems and Prospects", November, 1979.

159. Adams, R. J., "Towards a More Competent Labor Force : A Training Levy Scheme for Canada", December, 1979.

160. Jain, H. C., "Management of Human Resources and Productivity", February, 1980.

161. Wensley, A., "The Efficiency of Canadian Foreign Exchange Markets", February, 1980.

162. Tihanyi, E., "The Market Valuation of Deferred Taxes", March, 1980.

163. Meadows, I. S., "Quality of Working Life : Progress, Problems and Prospects", March, 1980.

164. Szendrovits, A. Z., "The Effect of Numbers of Stages on Multi-Stage Production/Inventory Models - An Empirical Study", April, 1980.

165. Laiken, S. N., "Current Action to Lower Future Taxes : General Averaging and Anticipated Income Models", April, 1980.

166. Love, R. F., "Hull Properties in Location Problems", April, 1980.

167. Jain, H. C., "Disadvantaged Groups on the Labor Market", May, 1980.

168. Adams, R. J., "Training in Canadian Industry : Research Theory and Policy Implications", June, 1980.

169. Joyner, R. C., "Application of Process Theories to Teaching Unstructured Managerial Decision Making", August, 1980.