The Effect of Numbers of Stages on Multi-Stage Production/Inventory Models — An Empirical Study

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THE EFFECT OF NUMBERS OF STAGES ON MULTI-STAGE PRODUCTION/INVENTORY MODELS -- AN EMPIRICAL STUDY[1]

Multi-stage manufacturing usually involves work-in-process inventory which increases progressively with the number of stages. Models dealing with this phenomenon are intended for a variety of production/inventory situations. Two typical models, each of which represents a particular organization of the manufacturing process, are examined in this paper. An empirical study draws attention to the underlying assumptions and significant characteristics of these two models, and reveals a remarkable difference with regard to the merit of the process organizations as the number of stages increases.

1. Characteristics of Multi-Stage Inventory Models

1.1 Applicability of Lot-size Models

The efficiency of large production runs is a long-standing and widely-accepted doctrine in production management. The reason for this may be that setting up machines for a production run is time consuming and the related costs are far more conspicuous than those of carrying inventory. The need to control inventory levels sets a natural limit on the application of this doctrine. Early inventory-control research focused on the common EPQ (Economic Production Quantity) model. One must realize, however, that this model assumes a single manufacturing operation and only accounts for the finished product inventory. It ignores the work-in-process (later called process inventory) which is inherent in any process whenever a lot is manufactured through several operations. The larger the production lot-size and the number of manufacturing stages, the longer the manufacturing cycle time which, in turn, increases the process inventory. This relationship is recognized, to varying degrees, in multi-stage production/inventory models.

Since the single-stage EPQ model overstates the optimal lot size, dramatic savings could be generated by replacing it with a multi-stage model when appropriate. Any multi-stage production/inventory model is intended for some particular organization of the manufacturing process. The same models are usually advocated for both "serial" systems, in which each stage has only one predecessor stage, and for "assembly" systems, in which various stages precede the final assembly stage with an arborescent configuration. Some reservation is in order, however, in accepting this claim. It is assumed that, in recurring cycles, the quantity produced at each stage feeds the final assembly stage for a certain length of time. This single cycle heuristic may not be optimal because certain components might be more economically produced in quantities that would supply more than a single cycle. However, a serial

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system always entails a single cycle process organization.

Serial systems are very common in practice. Parts for a complex product often involve a series of operations (stages) being performed in production lots. Specifically, an overwhelming number of such parts are produced in the automobile industry and generally in the machine industry. It is also notable that the number of manufacturing stages is usually rather large (5-15 stages). Although a single part may not be (and usually is not) sold directly, the assembly of the product incorporating this part could be continous over time. Since part-manufacturing is seldom synchronized with the rhythm of the assembly line, a single part may be manufactured intermittently. When this happens the net effect is that of having a continuous demand fed by a part produced in lots. This is the reason for the applicability of lot-size models.

Obviously, scheduling precedence must be given to products whose production must follow the process organization implied by the multi-stage model. When facilities are shared by several intermittently-manufactured product lots, the scheduling of EPQ's for the entire spectrum of products is rarely feasible. However, experience has shown often that a relatively small portion of all products constitute a very large part of the process inventory. Scheduling priorities in manufacturing a small number of products, as the multi-stage model requires, do not present an obstacle in practice. The reason is that these products do not tie-up a prohibitive proportion of the manufacturing capacity; rather, they leave ample room to manipulate the manufacturing schedule of the remaining product line. This practical consideration justifies the limited application of EPQ models even in very tightly scheduled MRP (Material Requirements Planning) systems. The key problem is to select the right products for production according to an EPQ model.

1.2 Selective Review of the Literature

Inventory models are forerunners of the birth of operations research. Raymond (1931), in his early study of the subject, describes a host of variations in order quantity models. Whitin (1954) presents a comprehensive survey of the development of inventory control research. Niland (1970) has shown that the capital tied up by process inventory in certain industries can be as large as 60% of that for the total inventory. Nevertheless, the control of this particular type of inventory has received relatively little attention. Eilon (1962) introduces a variation of the conventional EPQ models that optimizes the return on the whole production cycle by determining the length of the cycle and the size of the buffer stock for the required consumption of finished products. However, his model treats the multi-stage case in aggregate, as if executed by a single facility.

An extensive survey of multi-echelon (stage) models by Clark (1972) summarizes the state-of-the-art until 1972. One class of these models is distinguishable on the basis of allowing different lot sizes across stages. Crowston, Wagner and Williams (1973) present a model for a multi-stage assembly system and compute a set of optimal lot sizes, but the model assumes instantaneous production at each stage, an improbable characteristic in practice. Jensen and Khan (1972) allow non-instantaneous production and assume that each stage will be periodically shut down and restarted so that the average production rate at that stage is equal to the demand rate. Their model ignores the cost of transporting batches and permits any number of shipments between stages. Taha and Skeith (1970) assume non-instantaneous production with variable lot-sizes at different stages. In their model, they calculate the optimum lot size of the finished product and allow over-production at those stages where a large set-up cost warrants a holdover of process inventory for subsequent manufacturing cycles. Their model, however, is based on backlogging of unfulfilled demand of the finished product; therefore, it presents considerable difficulties in practice when the product is required for a further echelon (e.g., assembly) of the production process. Schwarz and Schrage (1975) in a similar model, exclude backlogging and allow decreasing variable lot-sizes at the various stages so that only complete lots are transported to the next stage. Their model incorporates an "echelon inventory-holding cost" which may be mistaken for the more familiar and explicit stage inventory-holding cost. If this happens, a seriously distorted non-optimal result may be obtained as shown by Szendrovits (April, 1978). Nevertheless, their model is typical of variable lot-size models.

Another class of multi-stage models has uniform lot sizes at all stages, but allows portions of a lot to be transported to the next stage in batches. Szendrovits (1975) explores the functional relationship between the manufacturing cycle time and the process inventory and, based on his findings, offers a model in which equal-sized batches can be transported between stages. However, this model assumes an average process-inventory holding cost and "sunk" costs of transporting batches. A refinement of his model (Szendrovits, 1976) incorporates stage inventory-holding costs and transportation costs of batches over all stages. Goyal (1977) advocates that unequal batch-sizes within each stage could be chosen so as to decrease the process inventory in a two-stage system. However, Szendrovits (October, 1978) has shown that the additional transportation cost of unequal batch-sizes within a stage usually eliminates the savings in inventory costs. Moreover, unequal batch sizes within a stage become technically infeasible for more than two stages. Consequently, Szendrovits' (1976) model for a uniform lot size with equal-sized batch shipments at all stages is representative of this class of models.

The objective of this paper is to examine which of the process organizations represented by two typical multi-stage production/inventory models yields a lower cost, especially when the number of stages increases.

2. Focusing on Two Typical Process Organizations

2.1 Basic Assumptions in the Models

Models in the literature reflect some of the many possible variations in process organization. No model can be exhaustive and numerous variations could be added to any model; but, the problem would soon become unmanageable and the solution method analytically intractable.

In the models discussed in this paper, and particularly in the two typical ones on which we will focus our empirical analysis, some conventional assumptions can be noted. A "lot" denotes the quantity produced with one set-up at a stage and a "batch" denotes the portion of a lot transported to the next stage. The lot- or batch-size does not have to be an integer, the implication of which is that units of the product are "infinitely divisible".

This, of course, is not restrictive if optimal lot- and batch-sizes are found to be sufficiently large. The production system consists of a fixed sequence of operations, with constant production rates at each stage. The "one time" output quantity (lot size) is not constrained by limited production or storage capacity. Each lot involves a fixed set-up cost and the transportation cost of each batch to the next stage is also fixed. Note that only completed lots or batches are transported between stages. Set-up times and transportation times are not considered to be significant and, hence, are ignored. The inventory system is based on deterministic (and constant) demand rates and linear inventory-holding costs over an infinite time horizon. The stage inventory holding-cost is interpreted as the cost, per unit time, that is incurred by carrying one unit of physical inventory of a product on which the stage is completed.

The following assumptions to simplify the analysis are also common in most of the existing lot size models. The unit inventory-holding cost for a stage is assumed to be never lower than that for the preceding stage. This is not unreasonable in practice, since the inventory-holding cost is often proportional to the increasing value of the product. However, exceptions to this are possible when special handling costs are implemented at a particular stage and are not required at later stages. Also, it is assumed that the lot size of a stage is an integer multiple of the lot size that follows it. One could show that this decreasing lot size requirement is never optimal in any of the existing variable lot-size models (except for the unrealistic case of instantaneous production). However, one must admit that this is a necessity for analytical tractability in the variable lot-size models. In the uniform lot-size models the assumption of an integer number of batches helps analytical tractability and the resulting equal-sized batches facilitate the accounting for transportation costs.

Following frequent practice in the literature, we denote the stages in the production process by $i = 1,2,...,n$; the final stage, the one which meets the demand for the finished product, is stage 1. Other notation is as follows:

D = demand (consumption) rate of the final product (at stage 1);

P_i = production rate at stage i (note that $P_i > D$);

S_i = set-up (fixed) cost of one lot at stage i;

T_i = transportation (fixed) cost of one batch from stage i to stage i-1;

c_i = unit inventory-holding cost, per unit time, at stage i;

Q_i = the lot size at stage i;

$m_i = Q_i/Q_1$ (note that m_i is required to be an integer);

Q = a uniform lot size at all stages;

b = the number of batches in lot size Q at all stages (note that b is an integer);

$x = Q/b$, the size of batches in lot size Q at all stages (note that the sizes of batches are equal).

All parameters above are greater than zero. For the convenience of our cost equations we define: $c_{n+1} = 0$, $P_0 = D$, $m_0 = 1$ and $Q_0 = Q_1$.

For convenience in the following discussion, we will refer to a lot at stage i as a Q_i lot.

Let us now focus our attention on two basic models that we intend to compare.

2.2 Variable Lot-sizes with Full-lot Shipments - Model 1

A typical model in this class is described by Schwarz and Schrage (1975). Figure 1 illustrates the time-weighted stage inventories resulting from the underlying process organization.

Figure 1

Variable Lot Size Model When n=3 And $m_1=1$, $m_2=2$, $m_3=6$

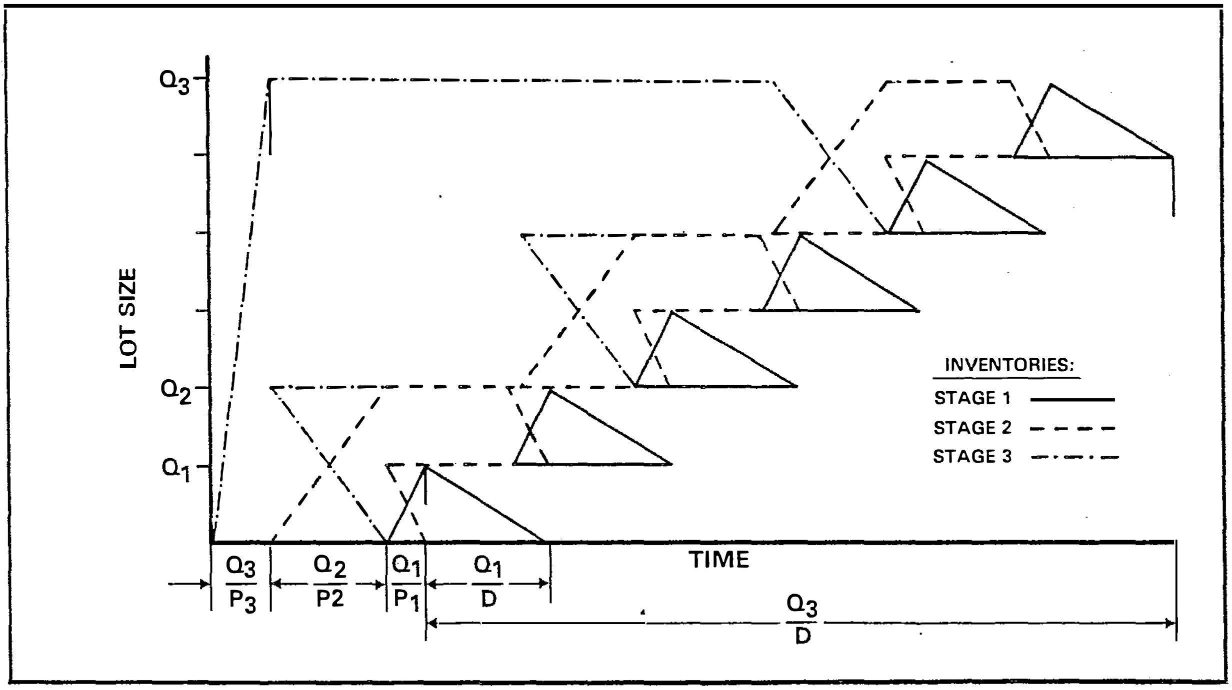


Figure 1 shows that lot-size Q_3 is produced at stage 3 during the time Q_3/P_3. Only after the full lot is completed can units be transported to stage 2. Here the first Q_2 lot (i.e., lot at stage 2), the size of which is $Q_3m_2/m_3 = Q_3/3$, is produced during the time Q_2/P_2 while some portions of the Q_3 lot are carried with a unit inventory-holding cost of c_3 until the second and third Q_2 lots absorb them. When the first Q_2 lot is completed the production of the first Q_1 lot takes place during the time Q_1/P_1 and the inventory is carried at c_1 holding-cost to satisfy the demand for a period of Q_1/D. Again some portion of the Q_2 is stored at c_2 holding-cost until it is used up for the next Q_1 lot. The same phenomenon occurs for the second and third Q_2 lots. The entire inventory cycle is repeated for each subsequent period Q_3/D over an infinite time horizon.

The cost function expressed in terms of stage holding-costs as shown by Szendrovits (April, 1978) in equation (3) is:

$$C1 = \frac{D}{Q_1} \sum_{i=1}^{n} \frac{(S_i + T_i)}{m_i} + \frac{Q_1}{2} \sum_{i=1}^{n} c_i \left[\left(\frac{D}{P_i} + 1\right) m_i + \left(\frac{D}{P_{i-1}} - 1\right) m_{i-1} \right] \quad (1)$$

subject to Q_i/Q_{i-1} and m_i must be integers; $m_i \geq m_{i-1}$; $c_{i+1} \leq c_i$ for $i = 1,2,\ldots,n$.

The first term is the average fixed cost (set-up and transportation cost); the second term is the cost of holding the average inventory per unit time. Note that in our particular example the fixed cost is required once for Q_3 lot, three times for Q_2 lots and six times for Q_1 lots.

However, the largest proportion of the inventory is carried at the lowest unit holding-cost and the smallest inventory involves the highest unit holding-cost. As a point of interest, one must remember that the model and its cost function are valid only if the unit holding-costs of the stage inventories are monotonically increasing, i.e., $c_{i+1} < c_i$ for $i = 1,2,\ldots,n$. The optimization procedure for the cost function (1) is described by Szendrovits and Wesolowsky (1979).

2.3 Uniform Lot-size With Equal-size Batch Shipments -- Model 2

A typical model in this class is described by Szendrovits (1976). Figure 2 illustrates the time-weighted stage inventories resulting from the underlying process organization.

Figure 2

Uniform Lot-size Model With Batch Shipments When b=6

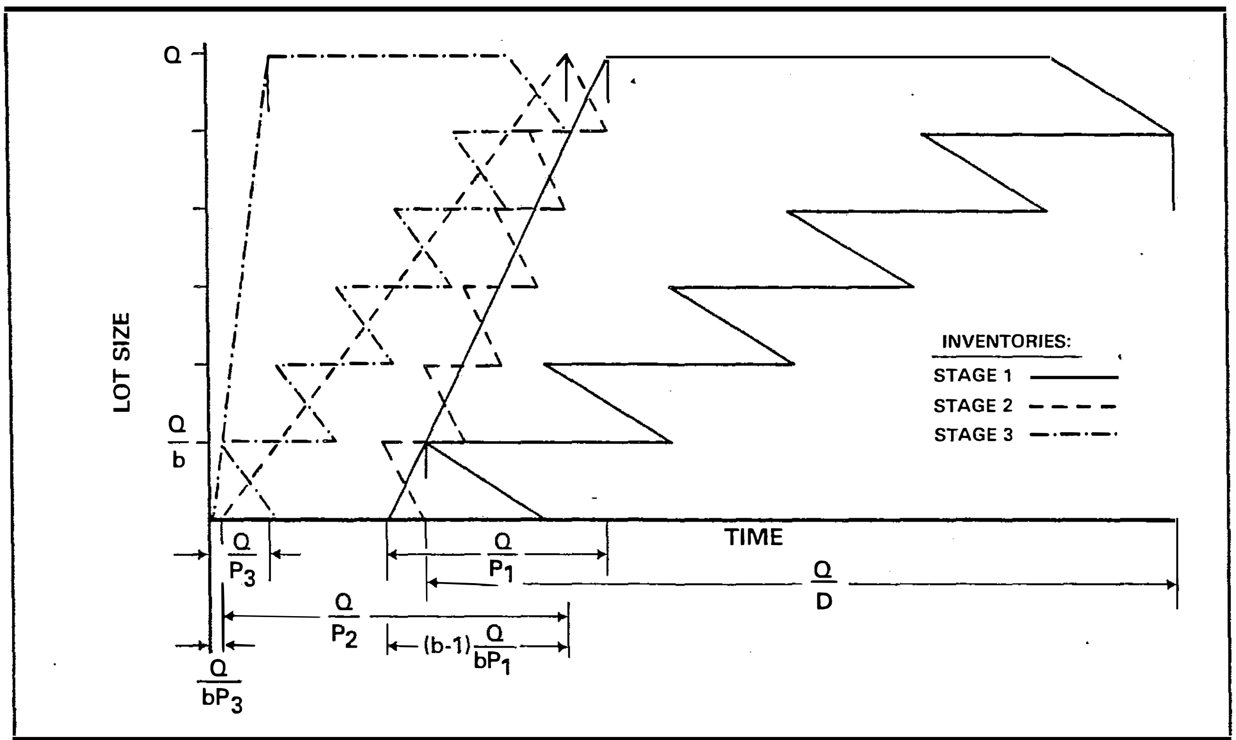


Figure 2 shows that a uniform lot size Q is produced with a single set-up at each stage. At stage 3 the production time of the lot is Q/P_3. Since batch shipments of Q/b units are allowed, we do not wait until the full Q_3 lot is completed; rather, we start producing at stage 2 after $Q/(bP_3)$ time. We can see that there is always a sufficient supply of units from stage 3 to produce the Q lot without interruption at stage 2. A similar schedule is always possible whenever a shorter operation time is followed by a longer one, or when adjacent operation times are equal; i.e., $1/P_i \leq 1/P_{i-1}$. It is clear, however, that we must delay the start of stage 1 to ensure uninterrupted production of Q lot at this stage. Nevertheless, we can start producing at stage 1 after $Q/P_2 - (b-1)\, Q/(bP_1)$ time; that is before the full Q lot is completed at stage 2. Such a delay is always necessary whenever a longer operation time is followed by a shorter one; i.e., $1/P_i > 1/P_{i-1}$. Since all production rates are greater than the demand rate ($P_i > D$), units are available to satisfy continuous demand for Q/D time as soon as the first batch is finished at stage 1 in $Q/(bP_1)$ time. Stage inventories are carried at the corresponding unit stage-inventory holding-costs, c_3, c_2 and c_1.

The cost function and its optimization method are given by Szendrovits and Wesolowsky (1979). The cost function is:

$$C2 = \frac{D}{Q} \sum_{i=1}^{n} (S_i + bT_i) + \frac{QD}{2b} \sum_{i=1}^{n} c_i \left[\left(\frac{1}{P_i} + \frac{1}{P_{i-1}} \right) + \left| \frac{1}{P_i} - \frac{1}{P_{i-1}} \right| (b-1) \right] \quad (2)$$

subject to: b = integer; $1 \leq b \leq Q$.

The first term contains the average fixed cost (set-up costs and transportation costs of b batches), the second term is the holding cost of the average inventory per unit time. Note that only one set-up cost is incurred at each stage, but transportation costs must be included for each of the b batches at all stages. In contrast to Model 1, if the unit stage holding-costs were increasing, a relatively larger proportion of the inventory would be carried at higher unit holding-costs. On the other hand, monotonically increasing unit holding-costs are not required in this model for its optimization.

3. Comparison of the Process Organizations for Different Numbers of Stages

3.1 The Experiment and the Problem Parameters

Whether one or the other process organization yields a lower cost depends on problem parameters. Therefore we tested the two models with different numbers of stages (n = 5, 10, 15 and 20), using 400 problems for each. The problem parameters were randomly generated within the following ranges: Range (D) = [5000, 50000], Range (P_i) = [60000, 625000], Range (c_i) = [0.1, 2.5], Range (S_i) = [1 , 500] and Range (T_i) = [0, 50]. The values for the parameters in each test problem were generated from a uniform probability distribution within each range. Since c_i in Model 1 must increase with decreasing stage numbers ($c_{i+1} \leq c_i$), the c_i values were ordered accordingly. (Note that this condition is not required for Model 2 but was necessary to compare the two models.)

Empirical results were obtained using the 400 randomly generated problems. These were studied to observe how the number of stages influences the performance of the two models under different assumptions. Examining the models in the light of different assumptions helps us to understand their behaviour and to draw conclusions as to the merit of the process organization. While the selected ranges of the parameters are considered to be reasonably realistic, the results obtained must be regarded as illustrative rather than as a statistical justification for the conclusions drawn. Recognizing the shortcoming of such an empirical study, we have checked the effect of exaggerated ranges for certain parameters on 100 contrived problems similar to those in the tests. We will illustrate the test results for various basic assumptions graphically, interpret their meaning and explain the effect of contrived parameters.

3.2 Magnitude of the Average Inventory

A cursory examination of Figure 1 and Figure 2 is sufficient to realize that, if Model 1 is applied with optimal Q_n^* and m_n^* quantities, Model 2 could be an alternate solution if we choose $Q = Q_n^*$ and $b = m_n^*$. In our figures Q_n, m_i and P_i for $i = 1,2,\dots,n$ were construed so that the inventory areas are the same. This is the exception rather than the rule when $m_n > 1$, but is always so when $m_n = 1$. We can see that in both cases the time-weighted inventory takes the shape of a trapezoid, the area of which depends on its base. The larger the base for one of the models, the larger is the average inventory for that model. We can easily express the base for Model 1:

$$BA1 = \sum_{i=1}^{n} \frac{Q_i}{P_i} . \tag{3}$$

For Model 2 when $Q = Q_n^*$ and $b = m_n^*$ the base is:

$$BA2 = \frac{Q}{b} \sum_{i=1}^{n} \left[\frac{1}{P_i} + (b - 1) \left(\frac{1}{P_i} - \frac{1}{P_{i-1}}\right) I_i \right], \tag{4}$$

$$\text{where } I_i = \begin{cases} 1 \text{ if } P_i < P_{i-1}; \\ 0 \text{ if } P_i \geq P_{i-1}. \end{cases}$$

Using the test problems described earlier we computed optimal Q_i and m_i values for Model 1 and established BA1 and BA2 for 400 random cases. The results for various numbers of stages are summarized in Table 1.

Table 1

Relative Magnitude Of Average Inventory For The Two Models

No. of Stages	Percent of 400 cases		
n	BA1>BA2	BA1=BA2	BA1<BA2
5	50	33	17
10	55	19	26
15	51	16	33
20	58	9	33

A larger base for the particular model indicates a larger average inventory. It is obvious from (3) and (4) that the bases and, therefore, the inventories are the same for both models when $m_n = 1$. As Table 1 shows, Model 1 yields a larger inventory than Model 2 in at least 50 percent of the cases. Equal average inventory sizes decrease from 33 percent for 5-stage cases with increasing numbers of stages. Model 2 involves a larger average inventory than Model 1 in only 17 percent of 5-stage cases; this increases to 33 percent, while the percent of equal inventory sizes decreases, with increasing numbers of stages.

Considering that Model 2 never involves more than one set-up at each stage, these data suggest that Model 2 (with $Q = Q_n^*$ and $b = m_n^*$) is generally better than Model 1 because it yields a smaller or equal average inventory in at least one third of the cases. This is certainly true if an average unit holding cost (uniform for all stages) is applied to the process inventory and if the transportation cost of batches is not taken into account (i.e., "sunk" transportation costs). One should note that these conditions are found quite frequently in practice.

Contrived problems with expanded and contracted ranges for set-up costs and unit inventory holding-costs did not indicate appreciable changes in the results shown in Table 1. This suggests that the magnitude of average inventories, under the given circumstances, is rather insensitive to the ranges of these parameters.

3.3 Stage Holding Costs and Sunk Transportation Costs of Batches

The transportation system in a plant usually handles a host of products and it is difficult to allocate transportation costs to particular product lots. Due to this difficulty, these costs are often handled as sunk costs. This is equivalent to setting all T_i values to zero in the cost functions. On the other hand, we may be able to establish separate unit inventory-holding cost, c_i, for each stage. To examine whether Model 2 would still be a favourable alternative to Model 1 under these assumptions, we computed the following costs for each problem:

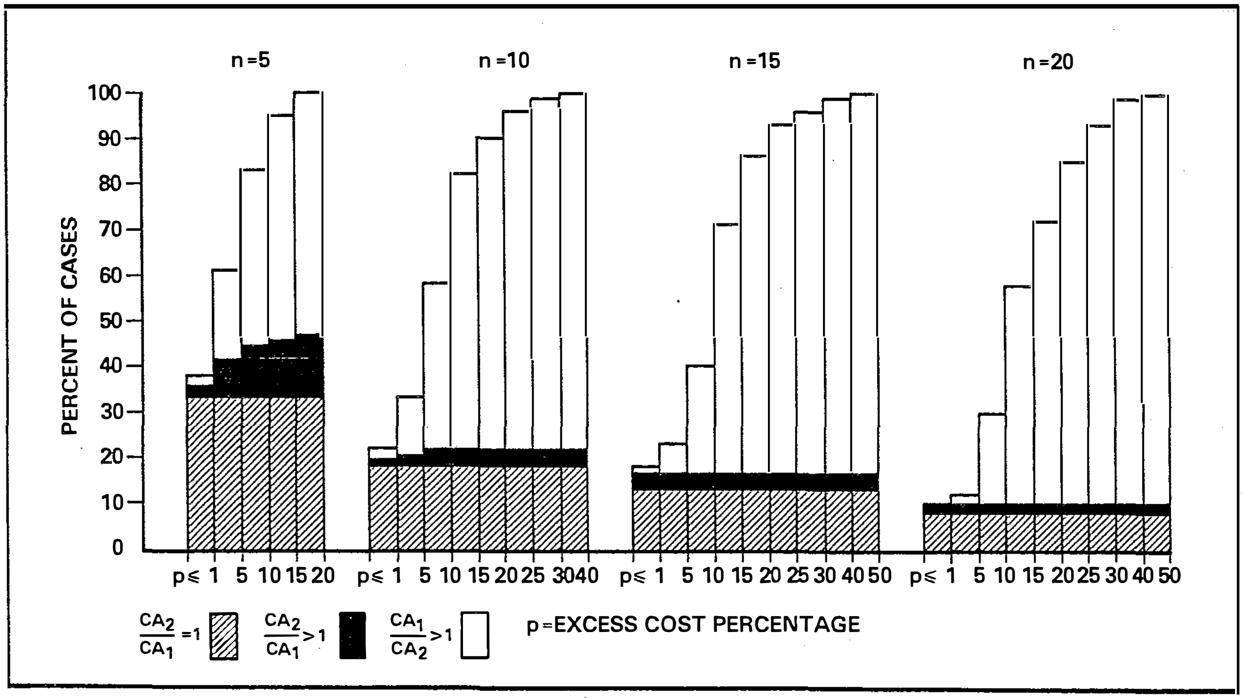
CA1 = the optimal cost for Model 1 in cost function (1);
CA2 = the cost for Model 2 at $Q = Q_n^*$ and $b = m_n^*$ in cost function (2).

Using these costs we examined which of these costs was higher and computed the percentage by which one cost exceeded the other: $p = (CA1/CA2 - 1)100$ or $p = (CA2/CA1 - 1)100$. The results for various numbers of stages are illustrated in Figure 3.

Let us first interpret the histogram for $n = 5$ in Figure 3. The numbers on the horizontal axis indicate, in less than or equal form, the percent by which one cost exceeds the other. The vertical bars indicate the cumulative frequencies of all cases (in percent). The cross-hatched bar shows that in 33 percent of all cases the costs were equal, CA1 = CA2. At $p \leq 1\%$ the white bar indicates that in 38-36 = 2 percent of all cases, the cost of Model 1 exceeded that of Model 2, CA1 > CA2, by 1% or less. The black bar shows that Model 2 involved a higher cost than Model 1, CA2 > CA1, in 36-33 = 3 percent of all cases. We can see that the excess cost percentage for Model 2 grows less rapidly than for Model 1, and at $p \leq 20\%$ we find CA1 > CA2 in 100-47 = 53

Figure 3

Comparison Of Models With Stage Holding Costs And Sunk Transportation Costs



percent, CA2 > CA1 in only 47-33 = 14 percent of all cases. The histograms for n = 10, n = 15 and n = 20 show the same phenomenon even more dramatically.

We can conclude that, for the assumptions specified, the process organization represented by Model 2 is preferable to that of Model 1, and it is apparent that it is increasingly so as the number of stages increases. The check problems (i.e., with exaggerated ranges) showed a similar pattern and almost identical results when the ranges of parameters for the set-up costs and for the stage inventory unit holding-costs were exaggerated.

3.4 Including Transportation Costs of Batches

If both the unit inventory-holding cost, c_i, and the transportation cost of a batch, T_i, can be established for each stage, we can compute the optimal cost for the two models:

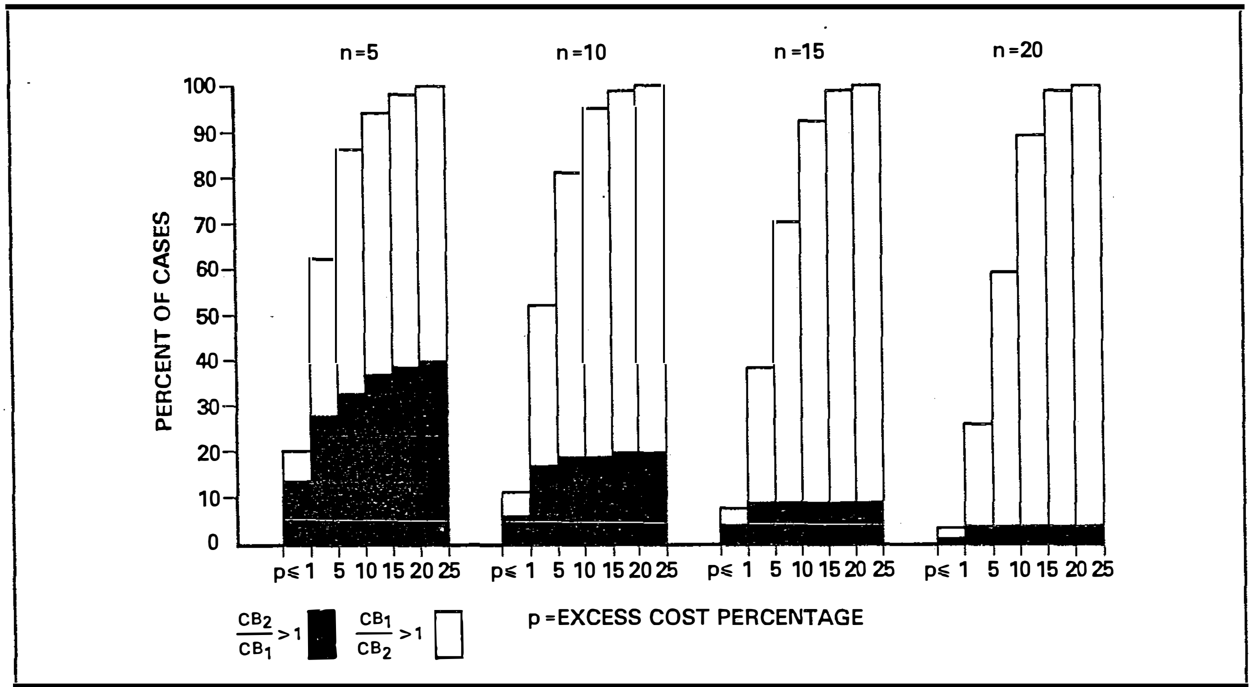
CB1 = the optimal cost for Model 1 in cost function (1);
CB2 = the optimal cost for Model 2 in cost function (2), at optimal values of Q^* and b^*.

Figure 4, which is constructed in a manner similar to Figure 3, demonstrates the cost comparison between Models 1 and 2.

The interpretation of the histograms in Figure 4 is similar to that described in connection with Figure 3. We can see that, for the chosen

Figure 4

Comparison Of Models With Stage Holding-costs And
Transportation Costs



problem parameters, Model 2 performs considerably better than Model 1, and becomes overwhelmingly better when the number of stages becomes large. We can see that, when n = 20 the optimal cost for Model 2 exceeds that of Model 1 only in 4 percent of all cases and the excess cost is always below 5%. Excess cost for Model 1 occurs in 96 percent of all cases and it could be as high as 25%; in fact, it is between 5% and 25% in 96-22 = 74 percent of all cases.

Check problems have shown that the optimal costs of the models were somewhat sensitive to exaggerated ranges, especially for set-up costs and/or transportation costs, and to a lesser extent for unit inventory holding-costs. Therefore, some caution is in order with respect to the better performance of Model 2. Under the given assumptions reliable discrimination could only be attained by determining the optimal cost of both models and choosing the better one.

3.5 The Effect of Limited Transport Capacity

Examining the cost function (1) for Model 1, we can see that only a single transportation cost is accounted for each Q_i lot regardless of the lot size. On the other hand, the transportation cost for each of the batches is included in the cost function (2) for Model 2. Therefore, if we assume that the transportation cost is related to a limited transport capacity which is the batch-size x, the modified cost function for Model 1 is as follows:

$$CC1 = C1 + \frac{D}{Q_n} \sum_{i=1}^{n} T_i \left[\left(\frac{Q_i}{x} \right)_{\uparrow} - 1 \right] \tag{5}$$

where $(Q_i/x)_{\uparrow}$ indicates a non-integer Q/x value rounded up to the nearest integer and C1 is the optimal cost of Model 1 from equation (1).

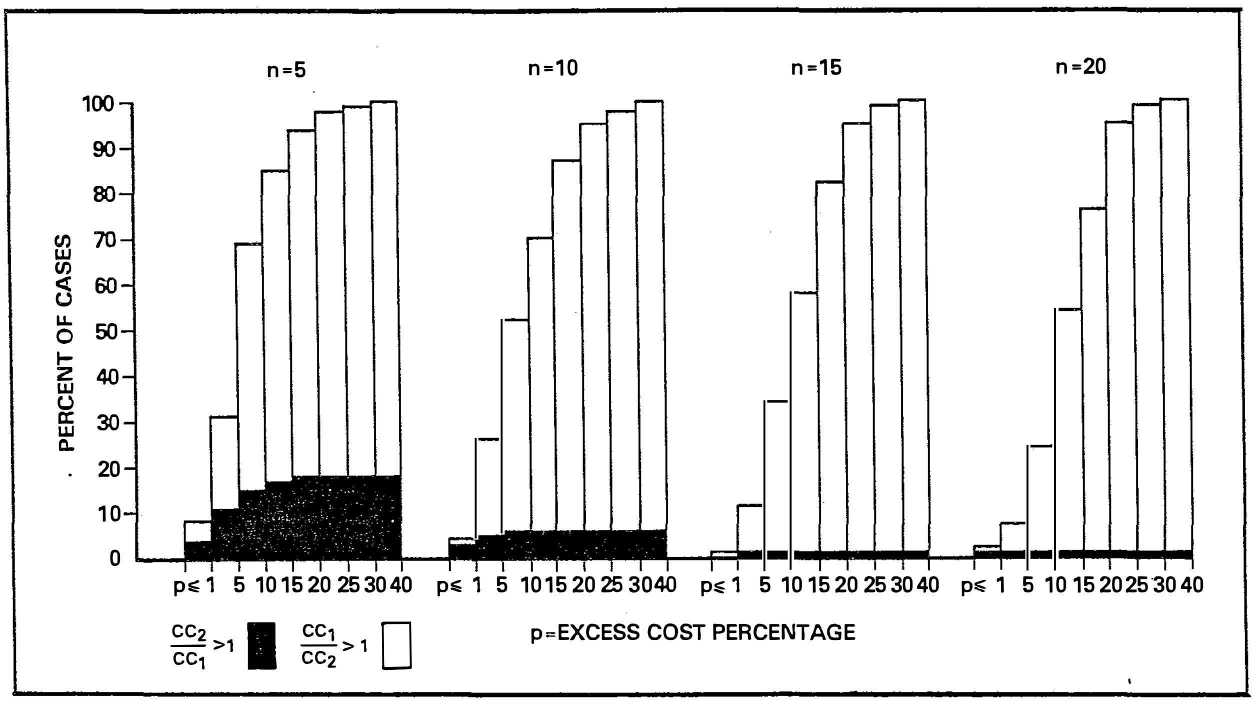
The second term of this cost function accounts for the additional average transportation cost at those stages where $Q_i > x$. Now we can compare more realistically the modified optimal cost of Model 1 in (5) with that of Model 2 which is:

CC2 = the optimal cost of Model 2 in cost function (2).

To simplify the examples used for comparing the above costs, we assumed that the transportation cost, T_i, is related to the transport equipment, the load capacity of which is equal to the optimal batch size (x = Q/b) computed for Model 2. The comparison of costs CC1 and CC2 is illustrated in Figure 5.

Figure 5

Comparison With Transportation Costs Included In Both Models



The histograms in Figure 5 illustrate that, considering the additional transportation cost in (5) for Model 1, the performance of Model 2 is better than in the cases illustrated in Figure 4. For 5 stages, the optimal cost of Model 2 only exceeds that of Model 1 in 18 percent of all cases and the excess cost is always below 20%. For 10 stages, the excess cost is always below 10% and only occurs with 6 percent frequency; for 15 and 20 stages, the excess cost is less than 1% and occurs only in one percent of all cases. This suggests that, for 10 stages or more, Model 2 is superior to Model 1 if the given assumptions prevail.

Interestingly most check problems did not indicate appreciable sensitivity to exaggerated ranges of set-up and/or transportation costs and inventory unit holding costs. Intuitively this can be explained with the correction for the cost of multiple transport loads in Model 1.

4. Conclusion

4.1 Generalization From the Empirical Results

We must emphasize again that inferences drawn from experiments based on a chosen range of parameters must be accepted with due caution and scrutiny. Nevertheless, the results strongly support some general conclusions concerning the merits of the process organizations represented by Models 1 and 2.

(a) First we examined the case where transportation costs are "sunk" and unit holding-costs are not known for the stage inventories, i.e., only an average unit holding-cost can be established for the process inventory. We found that Model 2, with the same lot-size and with numbers of batches identical to the numbers of smallest lots in Model 1, usually yields a smaller (or at least equal) average inventory quantity and cost than Model 1. The risk to the contrary (i.e., the occurrence of the opposite case) increases with the number of stages but appears to be within 33 percent. This risk does not show appreciable sensitivity to contrived parameters.

(b) When transportation costs are "sunk" and unit holding costs are established for the stage inventories, the optimal cost of Model 2, based on numbers of batches identical to the optimal numbers of smallest lots in Model 1, is usually smaller than (or at least equal to) that of Model 1. The risk to the contrary appears to be within 15 percent for 5 stages and it declines considerably with increasing numbers of stages to as low as 2 percent for 20 stages. The sensitivity of this risk to contrived parameters is minimal.

(c) When both transportation costs and unit holding costs are known for the stages Model 2 frequently yields a lower cost than Model 1. The risk to contrary appears to be about 40 percent for 5 stages, diminishing with increasing numbers of stages to about 5 percent for 20 stages. However, when the set-up costs are very low relative to the unit holding-costs at all stages, or when the transportation costs of batches are quite large relative to the set-up costs, Model 1 is expected to perform better than Model 2. Reliable discrimination can only be made by comparing the optimal costs of the two models.

(d) When the load capacity of the transport equipment is close to the optimal batch size, Model 2 usually yields a lower optimal cost than Model 1 after adjusting for multiple transportation costs for lot-sizes larger than the load capacity. The risk to the contrary appears to be within 18 percent for 5 stages, rapidly diminishing to 1 percent for 20 stages. There is a low risk sensitivity to contrived parameters.

(e) The performance of Model 2, under varying circumstances, seems to be generally better than that of Model 1. One must also remember that the optimal cost of Model 1 can be established only if the unit holding cost of stages is monotonically increasing towards the stage that satisfies demand, a requirement not needed for determining the optimal cost of Model 2.

4.2 Need for Improvement

The process organization represented by Model 2 required equal batch sizes over all stages. A significant improvement on this model can be achieved be relaxing the constraint requiring that the number of equal-sized batches be the same at each stage. Obviously a low transportation cost at a certain stage would allow a larger number of batches than a larger transportation cost at another stage. Depending on the transportation cost, varying the number of batches across stages could reduce the process inventory, and therefore the cost, in a uniform lot-size model similar to Model 2.

A further development is a model that incorporates the advantages of both models discussed in this paper. Combining variable lot-sizes with different numbers of batch shipments at various stages allows even more flexible inventory policies which would further reduce costs.

The author is working on articles discussing these models and their optimization methods. For the varying batch-size model, an optimization procedure is possible. A model, which unifies variable lot-sizes and different numbers of batch shipments at various stages, presents a considerably more complex problem and for this only a heuristic solution appears to be possible.

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