



AN ANALYSIS OF MULTI-PERIOD AFTER-TAX RATES OF RETURN ON INVESTMENT

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McMASTER UNIVERSITY

HAMILTON, ONTARIO, CANADA

Research and Working Paper Series No. 195
November, 1982

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no.195

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July 1982

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An Analysis of Multi-period After-Tax Rates of Return on Investment

Lawrence I. Gould and Stanley N. Laiken*

This paper presents a framework for the systematic analysis of after-tax returns from a wide variety of investments. It deals with the situation of an individual who has exhausted the \$1,000 investment income deduction and faces the problem of maximizing after-tax returns on further investments. Examples of the magnitude of tax effects on investment returns under the November 12, 1981 Budget proposals which were included in the June 1982 draft legislation are presented and examined.

The effects of income taxation on investment returns and, therefore, their effects on the investment priorities of the individual investor must be considered if appropriate investment decisions are to be made. Commonly quoted pre-tax rates of return must be converted to after-tax returns which are dependent on the individual investor's tax position and investment holding period, in addition to the different ways that various types of investment returns are taxed. Once expected investment returns have been reduced to an after-tax amount for a given investor, a proper comparison of all investments available can be made and adjustments for the risk preferences of the individual can be considered.

In a previous paper¹, the authors addressed the question of which of the three basic returns of Canadian-source interest, taxable dividends or taxable capital gains from the disposition of Canadian securities an

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¹Lawrence I. Gould and Stanley N. Laiken, "Effects of the Investment Income Deduction on the Comparison of Investment Returns", (March-April 1982), 30 Canadian Tax Journal, 228-239.

individual investor should favour if the \$1,000 investment income deduction provided in section 110.1 of the Income Tax Act has not been fully utilized and the returns are taxed annually. That paper developed and illustrated an approach to computing the pre-tax return for a given investment producing one form of return that would provide an equivalent after-tax rate of return for an investment producing another form of return. The approach was based primarily on a comparison of pre-tax returns and it was appropriate in that case because the \$1,000 investment income deduction is provided as a pre-tax dollar amount. Furthermore, a one-year holding period was used in that analysis because the \$1,000 investment income deduction is available as an annual deduction. Thus, the analysis did not consider the effects of potential tax deferrals beyond one year.

This paper extends the analysis and comparison of investment returns beyond those eligible for the \$1,000 investment income deduction. It will present a framework for comparison of investment returns from virtually all investments on an after-tax basis. In doing so, the effects of the investor's planning horizon or holding period will be examined. First, the areas in which such a framework can and should be used will be identified and discussed. Then, sample tables of after-tax return equivalents for both single-return and combination-return investments computed using the rules proposed in the November 1981 Budget and included in the June 1982 draft legislation will be presented and examined. From these sample data, it will be possible to demonstrate how tax effects provide the investor with required premiums for risk where these are considered necessary. It will also be possible to examine some of the effects of changes proposed in the November 12, 1981 Budget and contained in the June 1982 draft legislation on after-tax investment returns. Finally, a brief demonstration of sample pre-tax equivalent investment returns will be given.

Need for After-Tax Investment Return Comparisons

A comparison of after-tax rates of return on investment is appropriate for many aspects of personal investment decision-making. Yet the effects of taxation are not typically analyzed beyond the single-period comparison of interest and dividends. Thus, a framework for analyzing investments on a multi-period after-tax basis would be important in the following areas.

1. Security Analysis. In comparisons of the variety of securities available to an individual investor, each security's return should be adjusted for the differential tax treatment of interest, dividends and capital gains. If different returns are not adjusted for their tax effects risk differentials will be obscured and cannot be considered properly. For example, a comparison of a pre-tax dividend yield on preferred shares of 12% with a pre-tax interest yield on corporate bonds of 17% does not show any premium in return which must be provided to the preferred shareholder whose investment position is somewhat riskier than that of the bondholder. However, when after-tax returns from these possible investments are computed, a risk premium for the stock investment can be observed and evaluated.

2. Portfolio Evaluation. The comparison of returns from portfolios of different types of investments, including mutual funds and investment companies, must be made on an after-tax basis in the hands of the individual investor. Therefore, the comparison of pre-tax returns in the financial press for such different investments will not be particularly meaningful to the individual investor. Just as the pre-tax return on bonds will be higher than the pre-tax return on preferred shares, the pre-tax return on bond funds will be higher than the pre-tax return on funds

concentrating their investments in preferred stock. This does not mean that the after-tax return to an individual investor from the bond fund will necessarily be higher.

3. Risk Premium Adjustments. The analysis of a premium for risk or the component of investment return that compensates the investor for taking risk on certain securities should be done on an after-tax basis. This is particularly important in the face of major changes to the tax legislation in the area of taxation of investment returns as in the case of the November 12, 1981 Budget. Generally, risk premiums are estimated from historical pre-tax yield spreads between types of investments available in the market. Such an historical risk premium estimate will be inappropriate if there have been changes in the tax legislation affecting the returns analyzed. Furthermore, in some situations, such as the regulation of returns for public utilities, a risk premium is estimated for some stock market index and then adjusted to arrive at the risk premium for a particular company's securities. This pre-tax adjustment is inappropriate if the return on the particular company's securities is composed of a different proportion of dividend yield and expected capital gain than that generally available in the market index, since after-tax returns will vary with this yield mix.

4. The Decision to Invest or Reduce Personal Debt. The decision of whether to invest in securities or to reduce debt which bears non-deductible interest such as a mortgage on a personal home or other forms of personal debt should be based on the after-tax rate of return available on the alternative investments. In this case the interest cost of the personal debt can be regarded as an after-tax return, since the amount of debt reduced by funds available to invest elsewhere saves the individual the non-deductible rate of interest on the debt. In making this

comparison, it must be recognized that the effects of taxation of investment returns for individuals in relatively low tax brackets will not be nearly as great as for individuals in relatively high tax brackets. This might make the reduction of debt a higher priority for those in the higher tax brackets than, perhaps, an investment in preferred shares.

5. The Decision to Invest or Consume. The decision of whether to invest in securities, thereby deferring consumption, or to consume immediately, again, should be based on the after-tax rate of return available on the alternative investments relative to the expected increase in the cost of consumption in the future. For example, an individual may be faced with the decision to take an expensive vacation trip immediately or to defer it for a year when the cost of that trip is expected to be 10% higher. If the decision is made to defer the trip, the funds that would have been spent could be invested for the year. A pre-tax interest return of 17% during the year for a person in the top tax bracket would mean an after-tax return of less than the 10% increase in cost of the trip, making the decision to defer consumption unwise for that individual. However, for a person in a lower tax bracket, the after-tax return from investment might be higher than 10%, making the deferral of consumption the better alternative, all other things being equal.

After-Tax Equivalence Tables for Single-Return Investments

For investments that produce only one type of return, computing an after-tax rate of return equivalent to a given pre-tax return is relatively straightforward. The results can be presented in tabular form showing the differing marginal tax rates and various holding periods for an investment. As an example of such a tabular presentation, consider Table 1 which shows a selection of after-tax equivalents in the Province

TABLE 1

Sample After-Tax Equivalents to Different 15% Pre-Tax Returns in Ontario

Holding Period (Years)	Selected Federal Marginal Tax Rates				
	16%	18%	23%	25%	34%
	<u>Annual Dividend Return^a</u>				
1	17.22%	16.55%	14.89%	14.22%	11.23%
3	17.22	16.55	14.89	14.22	11.23
5	17.22	16.55	14.89	14.22	11.23
10	17.22	16.55	14.89	14.22	11.23
15	17.22	16.55	14.89	14.22	11.23
	<u>Compounding Capital Gain Return^b</u>				
1	13.22%	13.00%	12.45%	12.23%	11.23%
3	13.42	13.22	12.72	12.52	11.60
5	13.60	13.42	12.96	12.78	11.93
10	13.93	13.79	13.43	13.29	12.61
15	14.16	14.05	13.77	13.65	13.10
	<u>Maximum Compounding Interest Return^c</u>				
1	11.45%	11.00%	9.89%	9.45%	7.45%
2	11.63	11.20	10.12	9.69	7.71
3	11.80	11.39	10.35	9.92	7.97
4	11.71	11.29	10.23	9.80	7.84
5	11.73	11.31	10.26	9.83	7.87
6	11.80	11.39	10.35	9.92	7.97
7	11.75	11.34	10.28	9.85	7.90
8	11.76	11.34	10.29	9.86	7.91
9	11.80	11.39	10.35	9.92	7.97
10	11.77	11.35	10.30	9.87	7.92
15	11.80	11.39	10.35	9.92	7.97

^aThe basis of these calculations is provided in Equation B-1 of the Appendix.

^bThe basis of these calculations is provided in Equation C-2 of the Appendix.

^cThe basis of these calculations is provided in Equation A-2 of the Appendix.

of Ontario to a 15% pre-tax rate of return in dividends from Canadian corporations, capital gains or interest using the tax bracket rates and the dividend tax credit proposed for 1982 in the November 12, 1981 Budget and included in the June 1982 draft legislation. With the aid of a computer, similar tables can be produced for any imaginable rate of return in all provinces and territories for all tax brackets and any set of holding periods.

The assumptions underlying the data in Table 1 are based on realistic examples which can be varied to accommodate other situations. The assumption common to all parts of Table 1 is that the individual investor has exhausted the \$1,000 investment income deduction in all years of the investment holding period. The after-tax dividend equivalents, which might apply to a preferred stock with no expectation of a capital gain, assume that dividends are received and taxed annually such that, even if the after-tax dividend is reinvested annually, the compounding effect over the holding period is on an after-tax basis. As a result, holding period has no effect on after-tax returns in this situation, as can be seen from the dividend segment of Table 1 which shows the after-tax return at the same rate for all holding periods for an individual in a given tax bracket.

On the other hand, the after-tax capital gain equivalents, which might apply to a non-dividend-paying common stock, assume that capital gains compound through the holding period at the specified pre-tax rate until they are realized and taxed at the end of that period. This, of course, provides an opportunity to defer taxes on such returns during the investment holding period. As a result, the after-tax return on such investments can be seen to increase with the length of holding period for an individual in a given tax bracket. For an individual in the top tax bracket, a 15-year holding period would add 187 basis points to the one-

year return for the case shown in Table 1. The longer the holding period and, therefore, the tax deferral, the less significant is the tax in present value terms.

The after-tax interest equivalents, which in the case shown in Table 1 might apply to a long-term compounding guaranteed investment certificate, Canada Savings Bond or similar type of compound-interest-bearing security, assume that interest is compounded to a maximum period of every three years throughout the holding period. This assumption shows the effects of the November 12, 1981 Budget which proposes to tax such accrued investment income in this manner. This proposal, of course, limits the benefits of deferring tax on this form of investment return, thereby reducing significantly the beneficial effects of a long holding period as can be seen in Table 1.

In addition to the effects already noted from Table 1, some further effects can be observed. While it should be no surprise that after-tax returns decrease with increasing tax rates, the magnitude of decrease may be of interest. The biggest such decrease in after-tax return is on dividends where the difference in after-tax return between the lowest and highest tax brackets shown is almost 600 basis points for the example used. Note that for tax brackets under 23% (federal), the after-tax return is higher than the pre-tax return of 15% because the dividend tax credit exceeds the tax on the grossed-up dividend. Of course, there must be a source of other income subject to tax to absorb this excess dividend tax credit and achieve the indicated after-tax return. The magnitude of the decrease is not nearly as great on compounding capital gain returns, because only one-half of the capital gain is subject to tax. The difference shown in Table 1 is only about 200 basis points from the lowest bracket shown to the highest with a one-year holding period and this

difference decreases with longer holding periods. Finally, the magnitude of the decrease in after-tax return for compounding interest returns is about 400 basis points for a one-year holding period between the extremes of tax rates shown and this difference decreases very slightly for multiples of three-year holding periods when accrued interest must be subjected to tax under the Budget proposal and draft legislation.

The ranking of after-tax returns from the three types shown in Table 1 is quite clear. For most taxpayers, the ranking for a given pre-tax rate of return will be dividends first, capital gains second and interest or other similar forms of income from property third. The exception to this ranking would appear to be for individuals in the higher tax brackets who contemplate a longer holding period for their investments. In these cases, a given pre-tax rate of capital gain may rank above the same pre-tax rate of dividends on an after-tax basis because the longer holding period means a deferral of the tax on the capital gain.

These results are consistent with those reported in the previous paper² where an analytical approach to ranking equal pre-tax returns was taken. However, these results show the magnitude of the differences in ranking after-tax returns rather than simply a breakeven tax rate. Note that these differences between dividends and capital gains are greater at lower tax rates because of the effect of the excess dividend tax credit. Since one-half of capital gains are excluded from taxation, differences between capital gains and interest increase with higher tax rates. It must be recognized that these comparisons only cover

²Ibid., at 231.

cases in which equal pre-tax returns are available as in the choice between cash dividends and stock dividends as explained in the previous paper. More realistic comparisons can be made by using a more complete set of other similar tables which convert other pre-tax returns to their after-tax equivalents based on tax bracket and holding period.

After-Tax Equivalence Tables for Combination-Return Investments

Many investments provide a combination of the basic forms of return. For example, the return on a common stock may consist of dividends through the investment holding period plus a capital gain at the end of that period. Similarly, the return on a corporate bond may consist of interest through the investment holding period plus a capital gain at the end of the period. Since the previously described tabular presentation handles only a single type of investment return common to many, but not all investments, the analysis and presentation must be expanded to accommodate combination-return investments.

Consider Table 2 as an example of such a presentation for a stock. The top half of the table illustrates the after-tax return from a 10% dividend taxed annually throughout the holding period plus a 5% capital gain compounding through the holding period and taxed at the end of that period. This combination might be typical of a utility stock. Note that the return increases with increases in the holding period. However, the magnitude of this increase for the holding periods shown is not very large, particularly at the lower tax rates. This is due to the relatively low capital gain component in the return combination. On the other hand, the high dividend component in the return is of particular benefit to the lower tax brackets.

The bottom half of the table illustrates the after-tax return when

TABLE 2

Sample After-Tax Equivalents to 15% Pre-Tax Returns on Stock in Ontario^a

Holding Period (Years)	Selected Federal Marginal Tax Rates				
	16%	18%	23%	25%	34%
	<u>10% Dividend + 5% Capital Gain</u>				
1	15.89%	15.37%	14.08%	13.56%	11.23%
3	15.97	15.45	14.18	13.66	11.35
5	16.06	15.53	14.26	13.75	11.46
10	16.15	15.66	14.42	13.93	11.67
15	16.23	15.74	14.53	14.05	11.83
	<u>5% Dividend + 10% Capital Gain</u>				
1	14.56%	14.19%	13.26%	12.89%	11.23%
3	14.70	14.35	13.45	13.09	11.47
5	14.82	14.48	13.62	13.27	11.69
10	15.05	14.74	13.94	13.62	12.14
15	15.21	14.91	14.16	13.86	12.46

^aThe basis of these calculations is provided in Equation D-1 of the Appendix.

the rates associated with the components of the total return are reversed. This combination might be typical of an industrial stock. Note how the higher capital gain component results in a greater effect from the length of the holding period, particularly in the higher tax brackets. On the other hand, the lower dividend component results in lower after-tax returns for individuals in the lower tax brackets.

Of course, many other numerical combinations of dividend yields and capital gain rates are possible for stocks. Table 3 presents a selection of dividend yields from 0 to 15% in combination with possible capital gain rates from 0 to 25% in terms of their after-tax equivalent returns in Ontario. Only two tax rates and three holding periods are shown in this table to reduce the number of after-tax equivalent returns presented. Note that observations made previously in the less extensive table hold for the results shown in Table 3 in terms of the effects of increasing tax rates, increasing holding period and the relative proportion of dividends and capital gains in the combination of pre-tax returns.

Some specific observations can be made from Table 3. Within a given tax bracket, as holding periods are increased, combinations containing relatively high capital gains produce larger increments to after-tax returns because of the deferral of tax with compounding capital gains. In the 16% federal marginal tax bracket, differences range up to about 150 basis points from a one-year holding period to a ten-year holding period. This effect is magnified at higher tax rates such as the 34% federal marginal tax bracket where differences range up to over 300 basis points from the one-year holding period to the ten-year holding period. Within a given holding period, as the tax rate increases, combinations containing relatively high dividend yields produce a larger decrease to after-tax returns because they lose the much more favourable tax treatment of dividends relative to capital gains which was obtained at the lower tax rates. With a one-year holding period such

TABLE 3

Sample After-Tax Equivalentents to Various Pre-Tax Return Combinations on Stock in Ontario^a

Selected Federal Marginal Tax Rates

Holding Period (Years)	16%					34%				
	Capital Gain Rates	Dividend Yields				Capital Gain Rates	Dividend Yields			
		0%	5%	10%	15%		0%	5%	10%	15%
1	0%			11.48%	17.22%	0%			7.48%	11.23%
	5		10.15%	15.89	21.63	5		7.48%	11.23	14.97
	10	8.82%	14.56	20.30	26.04	10	7.48%	11.23	14.97	18.71
	15	13.22	18.96	24.70		15	11.23	14.97	18.71	
	20	17.63	23.37			20	14.97	18.71		
	25	22.04				25	18.71			
3	0%			11.48%	17.22%	0%			7.48%	11.23%
	5		10.20%	15.97	21.73	5		7.57%	11.35	15.13
	10	8.91%	14.70	20.48	26.26	10	7.66%	11.47	15.28	19.09
	15	13.42	19.23	25.03		15	11.60	15.44	19.28	
	20	17.97	23.79			20	15.60	19.47		
	25	22.54				25	19.66			
10	0%			11.48%	17.22%	0%			7.48%	11.23%
	5		10.34%	16.15	21.94	5		7.82%	11.67	15.51
	10	9.17%	15.05	20.89	26.71	10	8.17%	12.14	16.06	19.94
	15	13.93	19.83	25.69		15	12.61	16.62	20.57	
	20	18.75	24.66			20	17.19	21.21		
	25	23.61				25	21.86			

^aThe basis of these calculations is provided in Equation D-1 of the Appendix.

differences range from about 130 basis points up to about 730 basis points over the extremes of tax rates shown. This effect is reduced with longer holding periods, such as 10 years, where differences range from about 100 basis points up to about 680 basis points.

After-Tax Risk Premiums Provided by Tax Effects

The foregoing analysis has been based on examples which compare equal pre-tax investment returns of various types in terms of their after-tax equivalents without comment on the differential riskiness of these types of returns. It should be recognized, however, that an individual investor will not be indifferent between, say, a 15% dividend return and a 15% interest return. Since dividend or capital gain returns are generally considered to be riskier than interest returns, the investor should expect to be compensated for the additional risk. That compensation should be in the form of a higher after-tax return or risk premium from the riskier types of return.

From an inspection of Table 1, it would appear that the tax effects discussed previously provide such a risk premium. Note that returns from dividends or capital gains are higher than those from interest for a given tax bracket and a given holding period. The magnitudes of these risk premiums provided by the differential taxation of returns are illustrated in Table 4. In this table a risk premium is computed by subtracting the after-tax return from a 15% pre-tax interest return from the after-tax return either a 15% pre-tax dividend or a 15% pre-tax capital gain for a given tax bracket and a given holding period as shown in Table 1. Table 4 also shows similar computations of risk premiums for the combinations of returns shown in Table 2 relative to the interest return shown in Table 1.

Note that the risk premiums on dividends relative to interest

TABLE 4

Sample After-Tax Risk Premiums from Various 15% Pre-Tax Returns in Ontario

Holding Period (Years)	Selected Federal Marginal Tax Rates				
	16%	18%	23%	25%	34%
	<u>Dividend vs. Interest Returns</u>				
1	5.77%	5.55%	5.00%	4.77%	3.78%
5	5.49	5.24	4.63	4.39	3.36
10	5.45	5.20	4.59	4.35	3.31
15	5.42	5.16	4.54	4.30	3.26
	<u>Capital Gain vs. Interest Returns</u>				
1	1.77%	2.00%	2.56%	2.78%	3.78%
5	1.87	2.11	2.70	2.95	4.06
10	2.16	2.44	3.13	3.42	4.69
15	2.36	2.66	3.42	3.73	5.13
	<u>10% Dividend + 5% Capital Gain vs. Interest Returns</u>				
1	4.44%	4.37%	4.19%	4.11%	3.78%
5	4.33	4.22	4.00	3.92	3.59
10	4.38	4.31	4.12	4.06	3.75
15	4.43	4.35	4.18	4.13	3.86
	<u>5% Dividend + 10% Capital Gain vs. Interest Returns</u>				
1	3.11%	3.19%	3.37%	3.44%	3.73%
5	3.09	3.17	3.36	3.44	3.82
10	3.28	3.39	3.64	3.75	4.22
15	3.41	3.52	3.81	3.94	4.49

decrease with increasing tax rates and with increasing holding period. This would act as a relative incentive to invest in dividend-paying securities for individuals in lower tax brackets and for individuals in any tax brackets with relatively shorter investment holding periods.

On the other hand, the risk premium on capital gain relative to interest returns increases with increasing tax rates and with increasing holding periods. Thus, the tax system provides a relative incentive to individuals in higher tax brackets to invest in capital gains producing properties and for individuals in any tax bracket to hold such properties for longer periods. The risk premiums provided by the combined returns presented in the table show similar effects to those noted in the foregoing analysis with the effects being weighted by the relative proportion of dividends and capital gains in the combination.

Effects of the Budget and Draft Legislation on After-Tax Investment Returns

Three proposals in the November 12, 1981 Budget contained in the June 1982 draft legislation directly affect the calculation of after-tax returns under consideration in this paper. The first has already been discussed in presenting the calculation of after-tax returns from compound-interest bearing securities. This proposal reduces the benefits of tax-free compounding of interest throughout the holding period on such securities. The second is the proposal which reduces tax rates for individuals in six of the previous thirteen tax brackets. This proposal will, of course, affect all investment returns for individuals in these brackets. The third proposal reduces the dividend tax credit on dividends from taxable Canadian corporations from 37-1/2% to 34% of the dividend.

If data similar to those presented in Tables 1 and 2 are computed

under the pre-Budget legislation and compared with the data in Tables 1 and 2 which are based on the Budget proposals and draft legislation indicated, the effects of the Budget proposals can be observed. Table 5 presents these differences in after-tax returns for the same 15% pre-tax returns presented in Table 1. Note that returns from dividends for individuals in lower tax brackets are reduced by 78 basis points after taxes on a 15% pre-tax dividend due to the reduction in the dividend tax credit. However, this is offset for individuals in the higher tax brackets because of the reduction of the tax rates.

The only effect of the Budget and draft legislation on after-tax returns from capital gains results from the reduction of tax rates. The benefit of this reduction decreases with increasing holding periods when tax on capital gains become less important in present value terms. Also, the benefit of reduced taxes at the higher tax brackets is generally lower for capital gain returns than that benefit for dividend returns.

Finally, note how compounding interest returns are adversely affected by the Budget and draft legislation, primarily for individuals in the lower tax brackets, when holding periods exceed the three-year maximum tax-free compounding period proposed by the Budget. This effect is offset to some extent for individuals in the higher tax brackets which are reduced under the Budget. However, even for these individuals with longer holding periods, the loss of the compounding benefit has a greater effect than the reduction in tax rates. In general, under the Budget proposals dividends become relatively less attractive compared to interest or capital gains, except for the investor with very high tax brackets or very long holding periods.

A similar type of analysis can be done for the combinations of dividend and capital gain returns shown in Table 2. The results would reflect a combination of the effects noted for separate dividends and capital gains shown in Table 5. Thus, after-tax returns for individuals in lower tax brackets are adversely affected by the reduction in the dividend tax credit and the magnitude

TABLE 5

Effects of the 1981 Budget and 1982 Draft Legislation on Different
15% Pre-Tax Returns in Ontario

Holding Period (Years)	Selected Federal Marginal Tax Rates (Pre-Budget Rates Shown in Brackets)				34% (43%)
	16% (16%)	18% (18%)	23% (23%)	25% (28%)	
<u>Annual Dividend Returns</u>					
1	(.78)%	(.78)%	(.78)%	.22%	2.22%
3	(.78)	(.78)	(.78)	.22	2.22
5	(.78)	(.78)	(.78)	.22	2.22
10	(.78)	(.78)	(.78)	.22	2.22
15	(.78)	(.78)	(.78)	.22	2.22
<u>Compounding Capital Gain Returns</u>					
1	Ø%	Ø%	Ø%	.34%	1.00%
3	Ø	Ø	Ø	.31	.94
5	Ø	Ø	Ø	.28	.87
10	Ø	Ø	Ø	.22	.72
15	Ø	Ø	Ø	.18	.58
<u>Maximum Compounding Interest Returns</u>					
1	Ø%	Ø%	Ø%	.67%	2.00%
2	Ø	Ø	Ø	.66	2.01
3	Ø	Ø	Ø	.64	2.02
4	(.26)	(.28)	(.33)	.28	1.63
5	(.39)	(.43)	(.50)	.08	1.41
6	(.47)	(.51)	(.61)	(.05)	1.25
7	(.65)	(.71)	(.86)	(.34)	.92
8	(.77)	(.85)	(1.03)	(.53)	.67
9	(.85)	(.94)	(1.14)	(.67)	.48
10	(.99)	(1.10)	(1.35)	(.91)	.18
15	(1.43)	(1.57)	(1.96)	(1.67)	(.93)

of this effect depends on the relative weighting of dividends in the total return. Also, after-tax returns for individuals in the higher tax brackets are increased due to the reduction in the rates for those tax brackets.

Equivalent Pre-Tax Rates of Return

To this point, pre-tax returns from various types of investments have been reduced to their after-tax equivalents depending on the investor's marginal tax rate and holding period for investments. Once the returns from investments under consideration have been converted to their after-tax equivalents a choice can be made to maximize after-tax return whether that return is from a single type of return or from a combination of types of return. If the various types of return are equated based on their after-tax amounts, it is possible to convert each type of return back to its pre-tax equivalent returns for, perhaps, more direct comparisons of quoted market rates.

Consider, for example, the after-tax equivalents presented in Table 2 for a security producing a 5% dividend and a 10% expected annual capital gain for a total pre-tax return of 15%. Given an investor in a particular tax bracket with a specified investment holding period, how much return on a pre-tax basis would the investor require on other possible securities to be in the same after-tax position as the foregoing 15% return from dividends and capital gains?

This question can be answered with data such as those presented in Table 6. It should be emphasized that all of the returns shown in the body of the table are pre-tax equivalents of the 15% pre-tax return on the stock used as an example. Note that differences in pre-tax return equivalents can be significantly different ranging up to almost 950 basis points above the pre-tax 15% return for interest taxed annually for a

TABLE 6

Sample Pre-Tax Equivalents to 15% Pre-Tax Return on Stock in Ontario^a

(5% Dividend Taxed Annually + 10% Capital Gain Compounded Annually)

Holding Period (Years)		Selected Federal Marginal Tax Rates		
		16%	25%	34%
1	After-Tax Return Other Investment Returns	14.56%	12.89%	11.23%
	Interest (taxed annually)	19.07%	20.46%	22.60%
	Interest (maximum compounding)	19.07	20.46	22.60
	Dividends (taxed annually)	12.68	13.60	15.00
	Capital gain (taxed annually)	16.51	15.82	15.00
	Capital gain (compounding)	16.51	15.82	15.00
	3	After-Tax Return Other Investment Returns	14.70%	13.09%
Interest (taxed annually)		19.26%	20.79%	23.09%
Interest (maximum compounding)		18.57	19.55	21.09
Dividends (taxed annually)		12.81	13.81	15.33
Capital gain (taxed annually)		16.67	16.07	15.33
Capital gain (compounding)		16.41	15.68	14.84
10		After-Tax Return Other Investment Returns	15.05%	13.62%
	Interest (taxed annually)	19.72%	21.61%	24.43%
	Interest (maximum compounding)	19.07	20.41	22.41
	Dividends (taxed annually)	13.11	14.36	16.22
	Capital gain (taxed annually)	17.07	16.71	16.22
	Capital gain (compounding)	16.17	15.35	14.47

^aThe basis of these calculations is provided in Equations D2 to D5 of the Appendix.

person in a high tax bracket with a relatively long holding period.

Of course, other combinations of returns, other tax brackets and other holding periods can be compared in this manner. In fact, the analysis can be done for any set of data within the parameters specified by the comparisons shown in Table 6.

Conclusions

This paper has developed a framework for the analysis of investment returns on an equivalent tax-adjusted basis through the presentation of a series of examples. With the aid of computer programs developed for this purpose, the analysis can be extended to consider any federal and provincial tax rate combination, any rate of return from any available investment generating single-type returns or combination-type returns involving taxable Canadian dividends, capital gains or interest and similar income from property for any holding period of concern. The analysis can be extended to consider almost any conversion of after-tax return so computed into a pre-tax equivalent for comparison of investment return data quoted in the capital markets. Thus, many more specific cases, beyond those used as examples in this paper, can be evaluated.

While the nature of most of the tax effects examined in this paper is generally known, the magnitude of these effects is usually not considered. To the extent that this magnitude is significant, the effects should be examined if appropriate investment decisions which will maximize the investor's after-tax return are to be made. It has been shown that such investment decisions can be based on a ranking of returns from alternative investments under consideration and a quantification of the after-tax premium for risk provided by the tax effects examined.

Appendix: Formulae For Computation of After-Tax Investment Returns

A. Interest

I = pre-tax rate of return from interest

I_T = after-tax rate of return from interest

T_F = the individual's marginal federal tax rate

T_P = the provincial rate of personal tax applicable to the individual

N = number of years in the investor's holding period

J = number of years tax on interest can be deferred through compounding

L = number of complete periods of J years in the investor's holding period

M = number of years in the investors holding period in excess of L multiples
of J years = $N - JL$

1. Interest Taxed Annually

$$I_T = I [1 - T_F(1+T_P)] \quad \text{A-1}$$

2. Interest With Tax Deferred For J Years

$$I_T = \left[\left[\left((1+I)^J - 1 \right) (1 - T_F(1+T_P)) + 1 \right]^L \cdot \left[\left((1+I)^M - 1 \right) (1 - T_F(1+T_P)) + 1 \right]^{1/N} - 1 \right] \quad \text{A-2}$$

B. Dividends

D = pre-tax rate of return from Canadian-source dividends taxed annually

D_T = after-tax rate of returns from Canadian-source dividends taxed annually

T_F = the individual's marginal federal tax rate

T_P = the provincial rate of personal tax applicable to the individual

X = dividend gross-up as a fraction

C = dividend tax credit as a fraction of the gross-up

$$D_T = D [1 - [(1+X)T_F - CX](1+T_P)] \quad \text{B-1}$$

C. Capital Gains

G = pre-tax rate of return from capital gains

G_T = after-tax rate of returns from capital gains

T_F = the individual's marginal federal tax rate

T_P = the provincial rate of personal tax applicable to the individual

N = number of years in the investor's holding period

1. Capital Gains Taxed Annually

$$G_T = G[1 - .5T_F(1+T_P)] \quad \text{C-1}$$

2. Capital Gains With Tax Deferred For N Years

$$G_T = [(1+G)^N [1 - .5T_F(1+T_P)] + .5T_F(1+T_P)]^{1/N} - 1 \quad \text{C-2}$$

D. Stock

D = pre-tax rate of return from Canadian-source dividends taxed annually

G = pre-tax rate of return from capital gains

T_F = the individual's marginal federal tax rate

T_P = the provincial rate of personal tax applicable to the individual

X = dividend gross-up as a fraction

C = dividend tax credit as a fraction of the gross-up.

T_D = effective tax rate on Canadian-source dividends

$$= [(1+X)T_F - CX](1+T_P)$$

T_G = effective tax rate on capital gains

$$= .5 T_F(1+T_P)$$

N = number of years in the investor's holding period

S = pre-tax rate of return from stock

$$= D + G$$

S_T = after-tax rate of return from stock

V = annual growth in investment value

$$= D(1-T_D) + G$$

1. After-Tax Rate of Return From Stock With Pre-tax Annual Dividend Rate of D and Compound Capital Gain of G

$$S_T = [(1+V)^N - T_G[(1+V)^N - 1 - \sum_{n=0}^{N-1} D(1-T_D)(1+V)^n]]^{1/N} - 1 \quad D-1$$

2. Pre-Tax Equivalents to An After-Tax Rate of Return From Stock of S_T^*

$$\text{Pre-Tax Interest Rate (Annual)} = \frac{S_T}{1 - T_F(1+T_P)} \quad D-2$$

$$\text{Pre-Tax Dividend Rate (Annual)} = \frac{S_T}{1 - [(1+X)T_F - CX](1+T_P)} \quad D-3$$

$$\text{Pre-Tax Capital Gain (Annual)} = \frac{S_T}{1 - .5T_F(1+T_P)} \quad D-4$$

$$\text{Pre-Tax Capital Gain (Compound)} = \left[\frac{(1+S_T)^N - .5T_F(1+T_P)}{1 - .5T_F(1+T_P)} \right]^{1/N} - 1 \quad D-5$$

*The pre-tax interest equivalent with maximum compounding is solved by iteration.

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