EFFICIENCY OF FOREIGN EXCHANGE MARKETS: AN EMPIRICAL STUDY USING MAXIMUM ENTROPY SPECTRAL ANALYSIS

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Introduction

The purpose of this paper is to test for the efficiency of foreign exchange markets in the weak-form sense.[1] Specifically, we will test the hypothesis that forward rates incorporate all the relevant information conveyed by past spot rates. Informational relevance is emphasized because, as we shall argue below, the extent to which past information is impounded in current prices is by itself neither necessary nor sufficient for determining market efficiency. Informational relevance, which is to be defined below, is equally crucial.

In what follows, Section I provides a conceptual framework for testing market efficiency. Following the direction suggested by this framework, the next three sections of the paper test the hypothesis that past spot rate information is impounded in the forward rate. Here the issue is not whether the information is relevant but, to repeat, whether the information is impounded in the forward rate. Since no one methodology is sufficiently definitive to assess the relationship between past spot rates and the forward rate, we will use three different approaches, two of which have already been suggested by the extant literature. Specifically, Section II employs the Fama [1976] approach while Section III utilizes Cornell's [1977] method. In addition to the Fama and Cornell techniques, Section IV describes a time-series approach which offers, as we shall see, a number of distinct advantages by comparison. Besides these latter advantages, we estimate the time-series model by the recently developed technique of maximum entropy spectral analysis which yields superior parameter estimates to the conventional Box-Jenkins [1976] approach. In particular, the parameters need not be estimated from a lengthy time-series thereby mitigating against the potential problem of structural change in the underlying process. Section V tests for the relevance of the information conveyed by past spot rates where relevance is defined in terms of a forecasting methodology. Section VI concludes the paper. An appendix briefly describes the maximum entropy method (MEM) of time-series analysis.

I. Market Efficiency: A Conceptual Approach

Markets are said to be efficient in the weak-form sense if current prices fully reflect all the information conveyed by past prices [eg. Fama 1970]. This definition is problematic because it abstracts from an important characteristic of information, namely, its relevance. The following schema illustrates the issue:

	Past Price Information Impounded	Past Price Information Not Impounded
Past Price Information Relevant	Market Efficient	Market Not Efficient
Past Price Information Irrelevant	Market Not Efficient	Market Efficient

Suppose in fact that past prices are relevant to market participants for making current decisions. Then presumably one would define the market to be efficient if the information conveyed by past prices is incorporated in current prices. An inefficient market would, in contradistinction, discard some if not all of this information. On the other hand, suppose the information conveyed by past prices is garbled or biased and hence irrelevant to market participants. Then, an efficient market would disregard this data in forming current prices whereas an inefficient market would impound at least some of this dysfunctional information.

The problem with this conceptual framework is its empirical implementation. How is informational relevance or irrelevance to be defined and operationalized? Given a definition of relevance, can it be distinguished empirically from the impounding of past information? Rather than trying to resolve these issues in general terms, we will frame our analysis in the context of our empirical results. First, we will show in Sections II, III and IV below that, independently of the test methodology, the information conveyed by past prices is simply not impounded in the forward rate. Therefore, only the right-hand side of the schema is relevant. Second, we will show in Section II that, for the data period and currencies under consideration, forward rates are unbiased estimates of future spot rates. Therefore, we can define informational relevance in terms of a decision making environment which is related to forecasting future spot rates. Specifically, past spot rate information is defined to be relevant if past spot rates alone yield a better prediction of future spot rates than does the forward rate. On the other hand, past spot rate information is irrelevant if the forward rate is a better predictor than past spot rates alone.

It is worth noting that our definition of information relevance is very much dependent on past spot rate information not being impounded in the forward rate. In other words we make no claims about the generality of our definition of informational relevance. To see why, suppose past spot rate information has been impounded in the forward rate. Then it is quite possible for the forward rate to yield better predictions (of future spot rates) than past spot rates and, nevertheless, past spot rate information is relevant. Indeed, the forward rate may yield better predictions because, in addition to other information, it has incorporated the relevant information conveyed by past spot rates. But, if the forward rate yields better predictions in spite of having disregarded past spot rate information, then one can reasonably claim that such past information is truly irrelevant. In fact, this will turn out to be the case. We will show that the information conveyed by past spot rates (i) is not impounded in the forward rate and (ii) is irrelevant. According to our conceptual framework then, foreign exchange markets are efficient. The remainder of the paper is devoted to proving these results.

II. The Fama Methodology

Fama [1976, p. 373] proposed a direct test of the hypothesis that forward rates impound the information conveyed by past rates without having to estimate the premium in the forward rate.[2] Briefly, Fama's test boils down to this. From term structure theory we know that

$$F_{t-1} = E_{t-1}(X_t) + L_t \tag{1}$$

so that

$$F_{t-1} - X_{t-1} = E_{t-1}(X_t) - X_{t-1} + L_t \tag{2}$$

where X_t is the spot rate at time t, F_t is the one period forward rate at t, L_t is the risk (liquidity) premium at time t, and $E_{t-1}(X_t)$ is the market expectation at time t-1 of the future spot rate X_t.[3] Although $E_{t-1}(X_t)$ is unobservable, if exchange markets are efficient, $E_{t-1}(X_t)$ should differ from X_t by a white noise error term (ε_t) only, so that equation (2) becomes

$$F_{t-1} - X_{t-1} = X_t - X_{t-1} + L_t + \varepsilon_t \tag{3}$$

Now, and this is the crux of Fama's argument, assuming the risk premium L_t is uncorrelated with changes in past spot rates, equation (3) implies that the regression of $F_{t-1} - X_{t-1}$ on past spot rates will be the mirror image of $X_t - X_{t-1}$ regressed on past spot rates. That is, assuming the information contained in $X_t - X_{t-1}$ about past spot rates can be described by the autoregressive [AR(m)] model

$$X_t - X_{t-1} = c_0 + \sum_{i=1}^{m} a_i(X_{t-i} - X_{t-1-i}) + \mu_t \tag{4}$$

it necessarily follows, if forward rates utilize all the information conveyed by past spot rates, that

$$F_{t-1} - X_{t-1} = c_0' + \sum_{i=1}^{m} a_i(X_{t-i} - X_{t-1-i}) + \delta_t \tag{5}$$

where μ_t and δ_t are white noise terms. Whereas the parameters a_i should be identical in both equations, the intercept terms will be different unless the risk premium is zero.[4] Therefore, to test the hypothesis that the forward rate incorporates the past information, we need only compare the a_i parameter estimates from the regressions of equations (4) and (5).

The data employed to perform Fama's test, as well as future tests, are month-end and non-overlapping spot rates and one-month forward rates collected from the Wall Street Journal. The time period for the data was primarily from April 1973 to February 1981. This period is characterized by flexible exchange rate regimes for the currencies under consideration.[5] Table 1 summarizes the results from the regression of the current spot rate on changes in past spot rates for six foreign currencies. Three autoregressive schemes were used from AR(1) to AR(3). For three of the six currencies, regression coefficients are significantly different from zero on the basis of the t statistic. Specifically, the French franc and German mark are best described by an AR(1) process while the Canadian dollar is apparently an AR(2) process. For the remaining three currencies, none of the regression coefficients are significant. These results are in marked contrast to the parameter estimates obtained by regressing the forward rate on changes in past spot rates as summarized in Table 2. In the latter table, only the Canadian dollar and French franc have significant slope coefficients. But for neither one of these two currencies are the a_i parameter estimates in Tables 1 and 2 the same by any reasonable standard. Therefore, it appears that either past rates provide no information about the current spot rate and forward rate - this is so in the case of the pound, yen and Swiss franc - or the information proved by past spot rates is not incorporated in the forward rate - this is the case of the Canadian dollar, mark and French franc.

Further evidence that past information is not incorporated in the forward rate can be adduced by comparing Tables 2 and 3(a). The latter table represents summary statistics on the forward rate minus the current spot rate. Clearly, variations in $F_{t-1} - X_{t-1}$ over time are not captured by the regression of this dependent variable on past spot rate changes. Autocorrelations in both tables are both quite similar and large. Therefore, we conclude that current and past spot rates have no more impact on determining the forward rate than does the current spot rate alone.

Although our conclusion is predicated on the assumption that liquidity premia are uncorrelated wtih past spot rates, in fact the data appear to be characterized by the absence of liquidity premia altogether. Table 3(b) presents mean forward rate forecast errors. Not only are the t-statistics small but, with one exception, none of the autocorrelations are significant at the 5% level. Thus, the evidence is consistent with the absence of liquidity premia for all the currencies so that the forward rate is an unbiased estimator of the future spot rate.

III. Cornell's Methodology

Although superficially Cornell's and Fama's methodology appear to be similar, they are really quite distinct. Cornell's [1977, p. 306] approach is as follows. Subtracting X_t from both sides of equaton (1) yields

$$F_{t-1} - X_t = E_{t-1}(X_t) - X_t + L_t \tag{6}$$

Assuming L_t is uncorrelated with past spot rates, the regression $F_{t-1} - X_t$ on past spot rate changes will now focus on the relationship between the <u>forecast error</u> $E_{t-1}(X_t) - X_t$ and past changes in spot rates.[6] Moreover, if the market utilizes the information conveyed by past spot rates to estimate future spot rates, the forecast error should be uncorrelated with past spot rate changes. In other words, the regression of $F_{t-1} - X_t$ on changes in past spot rates should yield insignificant coefficients under such conditions.

Table 4 presents the regression of $F_{t-1} - X_t$ on past data where past data are modelled as autoregressive schemes of orders one through three. For three currencies, the pound, the yen and the Swiss franc, the regression coefficients are insignificant. On the other hand, nonzero coefficients were obtained for the Canadian dollar, French franc and German mark. This compares favourably with Cornell's own results for which only four out of seven currencies yielded insignificant coefficients.

While one might conclude from our results using Cornell's methodology (and from Cornell's own results) that, at least for some currencies, forward rates incorporate all the information conveyed by past spot rates, an alternative conclusion is tenable. Indeed, we believe that the evidence is also consistent with the hypothesis that forward rates do not incorporate the information contained in past spot rates. To see that Cornell's results and ours can be interpreted in the latter fashion, consider the following scenario. Suppose, as Cornell and we have shown, that there is no liquidity premium in the forward rate. In addition, suppose that the forward rate ignores all past spot rate information except for the current spot rate. Then F_{t-1} and the current spot rate X_{t-1} will differ by a white noise term at most. In other words, F_{t-1} is a proxy for X_{t-1} so that the regression of $F_{t-1} - X_t$ on changes in past spot rates will be identical (but of opposite sign) to the regression of $X_t - X_{t-1}$ on changes in past spot rates. Therefore, the fact that for over half the currencies in Cornell's study and ours, $F_{t-1} - X_t$ yielded insignificant coefficients in the regression, merely proxies the regression of $X_t - X_{t-1}$ on past spot rate changes. To see that our scenario is not farfetched, one need only compare our Table 1 and Table 4. Cornell's results are equally consistent with this rationalization. To illustrate, consider Cornell's regression results for the German mark [his Tables 3 and 4]. He obtains

$$
\begin{aligned}
X_t - X_{t-1} &= \underset{(0.29)}{0.001} + \underset{(1.46)}{0.215}(X_{t-1} - X_{t-2}), \\
R^2 &= 0.051, \\
S(e) &= 0.0142, \\
\hat{\rho}_1(e) &= -0.02,\ \hat{\rho}_2(e) = 0.04,\ \hat{\rho}_3(e) = 0.08;
\end{aligned} \tag{7}
$$

and

$$
\begin{aligned}
F_{t-1} - X_t &= \underset{(-0.10)}{-0.000} - \underset{(-1.50)}{0.223}\ (X_{t-1} - X_{t-2}), \\
R^2 &= 0.053, \\
S(e) &= 0.0144, \\
\hat{\rho}_1(e) &= -0.01,\ \hat{\rho}_2(e) = 0.05,\ \hat{\rho}_3(e) = 0.06.
\end{aligned} \tag{8}
$$

By any reasonable standard the two regressions are identical but of opposite sign. In short, Cornell's methodology either implies that the market ignores past spot rates when pricing forward rates (as in the case of the

Canadian dollar) or it cannot reject the hypothesis that the market ignores past spot rates when pricing forward exchange because of the proxy relationship between X_{t-1} and F_{t-1} (as in the case of the yen).

IV. A Time-series Approach

(i) The Test

Insofar as the forward rate is an unbiased estimate of the future spot rate [as supported by the empirical evidence shown in Table 3(b)], the impact of past spot rates on the forward rate can be assessed by comparing the conditional expectations $E_{t-1}(X_t|X_{t-1})$ and $E_{t-1}(X_t|X_{t-1}, X_{t-2}, X_{t-3}, \ldots, X_{t-m-1})$ to the forward rate F_{t-1}. Here $E_{t-1}(X_t|X_{t-1}, \ldots, X_{t-m-1})$ is the period t-1 prediction of period t's spot rate given that market uses all past spot rate information in making its prediction. By contrast, $E_{t-1}(X_t|X_{t-1})$ is the period t-1 prediction of period t's spot rate where the market chooses to ignore information in past spot rates. Although these conditional forecasts are unobservable, they can be estimated. In particular, $E_{t-1}(X_t|X_{t-1})$ is assumed to be the current spot rate X_{t-1} while $E_{t-1}(X_t|X_{t-1}, \ldots, X_{t-m-1})$ is estimated to be $\hat{X}_t$ where

$$\hat{X}_t = X_{t-1} + \hat{a}_0 + \sum_{i=1}^{m} \hat{a}_i(X_{t-1} - X_{t-i-1}) \qquad (9)$$

The order m and the parameter estimates $\hat{a}_i$ in equation (9) are determined by the MEM (see the Appendix). The test is fairly obvious. If the deviation of F_{t-1} from $\hat{X}_t$ is less on average (in a way soon to be defined) than the deviation of F_{t-1} from X_{t-1}, then we will conclude that past spot rates must have had some influence on the pricing of forward exchange. Otherwise, we will conclude that forward rates ignore information conveyed by past spot rates.

The average deviations of F_{t-1} from $\hat{X}_t$ and X_{t-1} are calculated on the basis of a mean squared error metric. Specifically, the original time series of N monthly observations of spot rates, labelled $X_1, X_2 ..., X_N$ are grouped into N - h + 1 overlapping subseries of h terms each

$$\begin{array}{l} X_1, X_2, \ldots, X_h; \\ X_2, X_3, \ldots, X_{h+1}; \\ \vdots \\ X_{N-h+1}, X_{N-h+2}, \ldots, X_N. \end{array}$$

Each of these N-h+1 series is fitted to the AR(m) process

$$Z_t = a_0 + \sum_{i=1}^{m} a_i Z_{t-i} + \varepsilon_t \tag{10}$$

using the MEM where $Z_t = X_t - X_{t-1}$. In doing so, we allowed the model to be specified by the properties of the h terms of the particular subseries. Thus, the model specification could differ across all N-h+1 subseries for each currency so that instead of searching for an optimal model specification for the entire sample period, we permitted the model to be specified by the subseries data. Then, for each series ending at X_L, where L = h to N, we calculated $\hat{X}_{L+1}$ using equation (9). The deviations of $\hat{X}_{L+1}$ and X_L from the observed forward rate F_L are expressed as proportional deviations,

$$u_L = \frac{\hat{X}_{L+1} - F_L}{F_L}, \tag{11}$$

$$\text{and } v_L = \frac{X_L - F_L}{F_L} \tag{12}$$

so as to standardize the results for the different currencies. The mean squared deviations (MSD) based on these N-h+1 overlapping series are

$$\text{MSD}(u) = \frac{1}{N-h+1} \sum_{L=h}^{N} u_L^2, \tag{13}$$

$$\text{and MSD}(v) = \frac{1}{N-h+1} \sum_{L=h}^{N} v_L^2. \tag{14}$$

An observation of MSD(u) < MSD(v) is consistent with the interpretation that, over the sample period, the forward rate responds to the information in past spot rates. Such an interpretation is rejected if MSD(u) > MSD(v).

(ii) The Strengths of the Time-series Approach

Before presenting the empirical results of our test, we will briefly note what we believe to be the strengths of the time series approach by comparison to the Fama and Cornell methods. First, instead of arbitrarily positing a specific autoregressive process which is presumed to be invariant over time, we let the data "speak for itself". Specifically, the order m of the AR(m) model for the time series is chosen optimally using Akaike's

[1969] criterion of final prediction error. Also, the optimal order is permitted to change (although it may not) over the data period. Both of these flexibilities are absent from the Cornell and Fama methods. In their case, the order of the autogressive model is specified a priori. Also, they assume that the same model is operative over the entire sample period. Second, as explained in the appendix, the MEM for estimating the autoregressive parameters makes far fewer assumptions about the nature of the underlying process for periods outside of the data period than does the conventional Box-Jenkins approach. Finally, and perhaps most importantly, the time-series approach determines the response of the forward rate to past spot rate information in a far more direct fashion than the alternative methodologies. By this we mean that, in the time-series approach, past information is captured by the index $\hat{X}_t$ which can be directly compared to the forward rate. This is in contradistinction to the Fama and Cornell approaches in which the impact of past spot rate information is measured relative to the forecast error or $F_{t-1} - X_{t-1}$ rather than relative to the forward rate itself.

(iii) The Empirical Results

Following the procedure described above, the original time-series of N monthly observations of spot rates for each of six currencies were first divided into N-h+1 overlapping subseries of h terms each. For a given length N, the choice of the length of the base period h involves a trade-off between having sufficient data to fit the AR(m) processes for each subseries and having sufficient numbers of u_L's and v_L's (L = h to N) to compute MSD(u) and MSD(v). Different feasible values of h were tried, and we found the empirical evidence to be robust with respect to the choice of h. Thus, there is no need to report our results for all attempted values of h. Instead, we will report the results for h = 36 and 48 only.

Table 5 shows, for h = 36 and 48, the distribution of the optimal order m of the AR(m) processes for all subperiods and for each of the six currencies.[7] In the majority of cases, the time series of spot rate changes are best described as some AR(1) or AR(2) process. Furthermore, with the exception of the Canadian dollar, model specification is fairly robust to the length of the base period h. But even in the case of the Canadian dollar, most switches take place between AR(1) and AR(2) models.

Table 6 lists the computed values of MSD(u) and MSD(v) for h = 36 and 48. The results are unambiguous. In the case of all six currencies, MSD(u) is consistently greater than MSD(v). Thus, provided that the forward rate is an unbiased predictor of the future spot rate, the time-series analysis leads us to conclude that the market ignores information in past spot rates when pricing forward exchange.

V. The Relevance of Past Spot Rate Information

In Section I of this paper, we argued that past spot rate information is relevant---conditioned on the empirical result that past spot rate information is not impounded in the forward rate---provided past spot rates yield a better prediction of the future spot rate than does the forward rate. Otherwise, past spot rate information was defined to be irrelevant. Table 7 lists the mean squared deviations of two forecast error metrics for the base periods h = 36 and 48. The forecast error metric of the forward rate as a predictor of the future spot rate is

$$\frac{F_L - X_{L+1}}{X_{L+1}}$$

The mean squared deviation of this error metric is labelled MSD(F). The forecast error metric of past spot rate information as a predictor of the future spot rate is

$$\frac{\hat{X}_{L+1} - X_{L+1}}{X_{L+1}}$$

The mean squared deviation of latter is labelled MSD(MEM). Again, the results are quite unambiguous. In all cases, MSD(F) < MSD(MEM) so that the forward rate predicts better than past spot rates. Given our definition of information relevance, we conclude that the information conveyed by past spot rates is irrelevant.

VI. Conclusion

We have shown, utilizing a number of test methodologies, that the information conveyed by past spot rates is not impounded in the forward rate. But, as we noted, this result is neither necessary nor sufficient for determining the efficiency of foreign exchange markets. Equally crucial for judging the efficiency of foreign exchange markets is the relevance of past spot rate information for future decisions. Assuming that the informational relevance of past spot rates can be measured by usefulness in predicting future spot rates, we showed that past spot rate information is in fact irrelevant. Therefore, we are able to conclude that foreign exchange markets are efficient. However, as in any study of market efficiency, we temper our conclusions by noting that our results are conditioned on the underlying model. In particular, we have modelled past information by a maximum entropy univariate time series model using Akaike's criterion of final prediction error. Such a model may not be adequate for determining the informational content conveyed by past spot rates. On the other hand, it is worth remembering that this model has proved itself a viable alternative to standard Box-Jenkins models and that it has some very nice properties. (See for example, Van Den Bos 1971, Makhoul 1976). In particular, the order of the model is determined by the data instead of being specified a priori. The order of the model is allowed to change over the data period. The model is estimable from a reasonably short time series attenuating the problem of structural change in the underlying series. While these properties cannot validate the model, they enhance confidence in our conclusions.

Appendix: The Maximum Entropy Method

Upon removing the mean value ϕ_0 from each term of the time series, the AR(m) process of equation (10) becomes

$$z_t = \sum_{i=1}^{m} \phi_i z_{t-i} + \varepsilon_i \tag{A1}$$

where $z_t = Z_t - \phi_0$. By rewriting equation (A1) in the autocorrelation form, the ϕ_i's can be solved theoretically via the Yule-Walker equations

$$\begin{bmatrix} \rho_0 & \rho_1 & \cdots & \rho_{m-1} \\ \rho_1 & \rho_0 & \cdots & \rho_{m-2} \\ \vdots & \vdots & & \vdots \\ \rho_{m-1} & \rho_{m-2} & & \rho_0 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_m \end{bmatrix} = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_m \end{bmatrix} \tag{A2}$$

However, solving for the ϕ_i's requires complete knowledge of the autocorrelations ρ_k, $k = 0, 1, \ldots, m$. But, with the exception of $\rho_0 = 1$, all other autocorrelations are unknown and can only be estimated. Conventionally, in estimating ρ_k, $k= 1,2, \ldots, m$, the time-series beyond the sample period is implicitly treated as if it were composed of zeros. Alternatively, equation (A2) can be transformed to the frequency domain, and the ϕ_i's can then be estimated spectrally. But this alternative approach imposes periodicity of the time-series beyond the sample period. Thus, relatively long time-series data are required to reduce the undesirable consequences of truncation in the time domain, or of periodic extension in the frequency domain. Unfortunately, the potential gain may be offset by possible structural change in the time series over a long sample period.

To avoid this problem, Burg [1967, 1968, 1975] has proposed that the estimator of autocorrelations ought to maximize the randomness of the time-series outside the sample period while remaining consistent with the auto-correlations based on the observed time-series data. Since he measures randomness by the entropy of information, Burg's approach is commonly called the Maximum Entropy Method (MEM).

The essence of the MEM is to find the power spectrum P(f) which maximizes entropy subject to the constraint that P(f) agrees with the N+1 measured values of the autocorrelation function. Formally, the problem is to

$$\underset{P(f)}{\text{MAXIMIZE}} \int_0^w \log P(f)df \tag{A3}$$

$$\text{SUBJECT TO} \int_0^w P(f)\cos(2\pi f\tau t')df = \rho(\tau) \tag{A4}$$

$$\tau = 0, 1, \ldots, N$$

where $\rho(\tau)$ is the autocorrelation function, t' is the sample period of the time series, and $w = 1/2t'$. Burg has shown that the solution to this problem can be obtained by minimizing the sum of squared residuals associated with forward and backward predictions. That is, given an n term time-series z_t and a set of parameter estimates ϕ_i^*, the fitted values z_t, defined by

$$\hat{z}_t = \sum_{i=1}^{m} \phi_i^* z_{t-i}, \qquad t = m+1,\ m+2,\ \ldots,\ n \tag{A5}$$

are called forward predictions for z_t. the backward predictions $\hat{\hat{z}}_t$ are obtained by reversing the time series so that

$$\hat{\hat{z}}_t = \sum_{i=1}^{m} \phi_i^* z_{t+i}, \qquad t = 1,\ 2,\ \ldots,\ n-m \tag{A6}$$

The MEM parameter estimates can be obtained by minimizing the sum of squared residuals

$$S_m = \sum_{t=m+1}^{n} (z_t - \hat{z}_t)^2 + \sum_{t=1}^{n-m} (z_t - \hat{\hat{z}}_t)^2. \tag{A7}$$

This is equivalent to solving for Φ^* in

$$(\hat{Z}'\hat{Z} + \hat{\hat{Z}}'\hat{\hat{Z}})\ \Phi^* = \hat{Z}'\hat{Y} + \hat{\hat{Z}}'\hat{\hat{Y}} \tag{A8}$$

where

$$\hat{Z} = \begin{bmatrix} z_m & z_{m+1} & \cdots & z_1 \\ z_{m+1} & z_m & \cdots & z_2 \\ \vdots & \vdots & & \vdots \\ z_{n-1} & z_{n-2} & & z_{n-m} \end{bmatrix}$$

$$\hat{Z} = \begin{bmatrix} z_2 & z_3 & \cdots & z_{m+1} \\ z_3 & z_4 & \cdots & z_{m+2} \\ \vdots & \vdots & & \vdots \\ z_{n-m+1} & z_{n-m+2} & \cdots & z_n \end{bmatrix}$$

$$\hat{Y} = [\ z_{m+1}\ z_{m+2}\ \cdots\ z_n\ ]'$$

$$\hat{\hat{Y}} = [\ z_1\quad z_2\quad \cdots\ z_{n-m}]'$$

$$\text{and } \phi^* = [\phi_1^*\quad \phi_2^*\quad \cdots\ \phi_m^*]'.$$

Regardless of whether or not a conventional method or the MEM is used to estimate the parameters in the AR(m) process, the order m must first be specified. The usual procedure for order selection is an iterative one involving model identification, parameter estimation, and fairly subjective diagnostic checking of the residual autocorrelations. Recently, an optimal order selection process has been suggested based on Akaike's [1969] criterion of final prediction error (FPE). For an n-point time-series fitted to an AR(m) process, the final prediction error is defined by

$$FPE_m = \frac{n+m+1}{n-m-1} P_m \tag{A9}$$

where P_m is the mean squared residual, which in the case of the MEM is given by

$$P_m = \frac{1}{2(n-m)} S_m^* \qquad (A10)$$

Here, S_m^* is the minimum sum of squared residuals defined by expression (A7). The optimal order m is the one which minimizes the Final Prediction Error. Barrodale and Erickson [1980a,b] have recently developed an algorithm for determining m and estimating ϕ_i, $i = 1, 2, \ldots, m$, of the AR(m) model. Their algorithm, which is efficient and numerically stable, was adopted for the present study.[8]

FOOTNOTES

[1] For a review of the literature on the efficiency of foreign exchange markets, see Kolhagen [1978] and Levich [1979 a,b]. It is fair to say that much of the past literature has concluded that foreign exchange markets (spot and forward) are efficient. However, more recent results by Hansen and Hodrick [1980] and Longworth [1981] have put the issue in doubt again.

[2] Fama tested for efficiency in the U.S. Treasury Bill market rather than in the foreign exchange market but the methodology is the same.

[3] For a rigorous proof of equation (1) in the context of foreign exchange, see Grauer, Litzenberger and Stehle [1976], Mehra [1978], or Stulz [1981].

[4] It is worth noting, however, that $c_o' - c_o$ will reflect the liquidity premium only if (i) equation (4) correctly models the informational content of past spot rates <u>and</u> (ii) foward rates reflect all the information conveyed by past spot rates. In other words, if the a_i parameters in equations (4) and (5) are not identical then the differential in the intercept terms will not be the risk premium.

[5] The spot rate data are from April 1973 to February 1981 except in the case of the French franc for which the spot data begin April, 1974. The dates for the forward rates are March 1975 to February 1981 for the pound, the mark and the yen; March 1977 to February 1981 for the French and Swiss

francs; and September 1977 to February 1981 for the Canadian dollar. In all cases, we used the 3 p.m. Eastern time quote for the last business day of the month.

[6] Cornell's is really a forecast error methodology and does not deal directly with the impounding of past information. Nevertheless, we thought it useful to employ Cornell's technique to show how it differs from Fama's. More importantly, we want to show that Cornell's method can be interpreted as being quite consistent with the result that the market ignores the information conveyed by past spot rates when pricing forward exchange.

[7] We searched for optimal orders up to and including AR(10) but we failed to find a solution for our sample beyond AR(7).

[8] The numerical shortcomings of Burg's algorithm was first reported by Chen and Stegen [1974].

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Table 1

Regression Results for Changes in Spot Rates

$$X_t - X_{t-1} = c_0 + \sum_{i=1}^{m} a_i(X_{t-i} - X_{t-i-1}) \qquad m = 1, 2, 3$$

Currency	m	c_0 [$t(c_0)$]	a_1 [$t(a_1)$]	a_2 [$t(a_2)$]	a_3 [$t(a_3)$]	R^2	S(e)	$\hat{\rho}_1(e)$	$\hat{\rho}_2(e)$	$\hat{\rho}_3(e)$	No.
British Pound	1	−0.00288 (−0.38)	0.0066 (0.05)			0.000	0.06451	−0.00	0.04	−0.01	71
	2	−0.00288 (−0.37)	0.0075 (0.06)	0.0709 (0.56)		0.005	0.06483	−0.00	0.01	−0.02	71
	3	−0.00287 (−0.37)	0.0106 (0.08)	0.0708 (0.56)	−0.0358 (−0.28)	0.006	0.06527	−0.01	0.01	−0.00	71
Canadian Dollar	1	−0.00253 (−1.27)	−0.0421 (−0.26)			0.002	0.01259	−0.00	−0.18	−0.05	41
	2	−0.00332 (−1.70)	−0.0563 (−0.37)	−0.3225* (−2.10)		0.106	0.01207	−0.04	−0.03	−0.02	41
	3	−0.00377 (−1.85)	0.1001 (−0.61)	−0.3300* (−2.14)	−0.1345 (−0.83)	0.122	0.01212	−0.01	−0.01	0.02	41
French Franc	1	−0.00002 (−0.02)	−0.3298* (−2.33)			0.108	0.00759	0.03	0.07	0.04	47
	2	−0.00007 (−0.06)	0.2939 (−1.91)	0.1006 (0.62)		0.116	0.00764	−0.02	0.03	0.05	47
	3	−0.00019 (−0.17)	−0.3029 (−1.98)	0.1798 (1.03)	0.1967 (1.19)	0.144	0.00761	0.00	−0.01	−0.00	47
German Mark	1	0.00071 (0.35)	−0.2595* (−2.24)			0.068	0.01700	0.03	0.07	0.01	71
	2	0.00055 (0.27)	−0.2252 (−1.87)	0.1291 (1.03)		0.082	0.01699	−0.02	0.02	0.02	71
	3	0.00042 (0.20)	−0.2328 (−1.91)	0.1504 (1.15)	0.0775 (0.61)	0.087	0.01707	−0.00	0.01	−0.00	71
Japanese Yen	1	1.96×10^{-5} (1.06)	−0.0126 (−0.10)			0.000	1.538×10^{-4}	−0.00	−0.01	0.06	71
	2	2.01×10^{-5} (1.07)	−0.0132 (−0.11)	−0.0228 (−0.19)		0.001	1.549×10^{-4}	0.00	0.00	0.06	71
	3	1.65×10^{-5} (0.88)	−0.00117 (−0.10)	−0.0217 (0.18)	0.1926 (1.56)	0.036	1.533×10^{4}	−0.04	−0.00	0.01	71
Swiss Franc	1	0.00297 (0.73)	−0.1777 (−1.21)			0.031	0.02777	0.03	0.07	−0.01	47
	2	0.00236 (0.57)	−0.1487 (−0.99)	0.1501 (0.97)		0.052	0.02779	−0.00	0.01	−0.00	47
	3	0.00225 (0.53)	−0.1522 (−0.99)	0.1556 (0.97)	0.0279 (0.18)	0.052	0.02810	0.00	0.01	−0.01	47

* Significant at the 5% level

Note: S(e) ≡ standard error of regression;
$\rho_i(e)$ ≡ autocorrelations of the residuals for lag i;
No. ≡ number of observations.

Table 2

The Reaction of Forward Rates to Pat Spot Rates

$$F_{t-1} - X_{t-1} = c_0' + \sum_{i=1}^{m} a_i(X_{t-i} - X_{t-i-1}) \qquad m = 1, 2, 3$$

Currency	m	c_0' [$t(c_0')$]	a_1 [$t(a_1)$]	a_2 [$t(a_2)$]	a_3 [$t(a_3)$]	R^2	S(e)	$\hat{\rho}_1(e)$	$\hat{\rho}_2(e)$	$\hat{\rho}_3(e)$	No.
British Pound	1	−0.00585* (−7.95)	0.0026 (0.21)			0.001	0.00620	0.76	0.28	0.11	71
	2	−0.00585* (−7.94)	0.0027 (0.22)	0.0111 (0.92)		0.013	0.00620	0.75	0.25	0.10	71
	3	−0.00585* (−7.95)	0.0017 (0.14)	0.0111 (0.92)	0.0123 (1.01)	0.028	0.00620	0.73	0.24	0.09	71
Canadian Dollar	1	−0.00033 (−1.91)	−0.0274 (−2.00)			0.093	0.00108	0.50	0.08	−0.07	41
	2	−0.00033 (−1.85)	−0.0274 (−1.97)	0.0006 (0.04)		0.093	0.00109	0.50	0.08	−0.07	41
	3	−0.00035 (−1.91)	−0.0301* (−2.03)	0.0001 (−0.01)	−0.0083 (−0.56)	0.100	0.00110	0.51	0.08	−0.07	41
French Franc	1	0.00001 (0.06)	−0.0288* (−2.33)			0.108	0.00066	0.66	0.26	0.11	47
	2	0.00002 (0.24)	−0.0414* (−3.31)	−0.0353* (−2.66)		0.231	0.00062	0.59	0.22	0.15	47
	3	0.00004 (0.48)	−0.0400* (−3.36)	−0.0477* (−3.50)	−0.0309* (−2.39)	0.322	0.00059	0.66	0.25	0.15	47
German Mark	1	0.00168* (11.61)	−0.0072 (−0.86)			0.011	0.00122	0.83	0.35	0.17	71
	2	0.00169* (11.79)	−0.0107 (−1.25)	−0.0133 (−1.50)		0.042	0.00121	0.85	0.36	0.20	71
	3	0.00170* (11.69)	−0.0104 (−1.20)	−0.0143 (−1.54)	−0.0035 (−0.39)	0.045	0.00121	0.86	0.37	0.20	71
Japanese Yen	1	1.27×10^{-5}* (6.24)	−0.0052 (−0.39)			0.002	0.170×10^{-4}	0.88	0.37	0.21	71
	2	1.29×10^{-5}* (6.28)	−0.0055 (−0.41)	−0.0111 (−0.83)		0.012	0.170×10^{-4}	0.89	0.38	0.21	71
	3	1.29×10^{-5}* (6.17)	−0.0055 (−0.41)	−0.0110 (−0.82)	0.0028 (0.20)	0.013	0.172×10^{-4}	0.89	0.37	0.21	71
Swiss Franc	1	0.00394* (13.29)	−0.0115 (−1.07)			0.025	0.00202	0.63	0.26	0.11	47
	2	0.00401* (13.49)	−0.0145 (−1.34)	−0.0157 (−1.41)		0.067	0.00200	0.64	0.25	0.11	47
	3	0.00402* (13.23)	−0.0142 (−1.28)	−0.0163 (−1.41)	−0.0027 (−0.23)	0.068	0.00202	0.64	0.25	0.10	47

* Significant at the 5% level

Table 3(a)

Summary Statistics for the Forward Rate Minus the

Current Spot Rate $F_{t-1} - X_{t-1}$

Currency	Mean	Std. dev.	t - Stat	$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	No.
British Pound	-0.00585	0.00615	-8.01	0.76	0.28	0.12	71
Canadian Dollar	-0.00027	0.00112	-1.52	0.53	0.08	-0.09	41
French Franc	0.43×10^{-5}	0.00069	0.04	0.78	0.28	0.11	47
German Mark	0.00167	0.00121	11.61	0.85	0.34	0.17	71
Japanese Yen	1.26×10^{-5}	1.69×10^{-5}	6.28	0.88	0.37	0.21	71
Swiss Franc	0.00391	0.00203	13.23	0.69	0.27	0.11	47

Table 3(b)

Summary Statistics for the Forward Rate

Minus the Subsequently Observed

Spot Rate $F_{t-1} - X_t$

Currency	Mean	Std. dev.	t - Stat	$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	No.
British Pound	-0.00297	0.06393	-0.39	-0.00	0.03	-0.02	71
Canadian Dollar	-0.00217	0.01276	1.09	-0.00	-0.18	-0.05	41
French Franc	0.00004	0.00817	0.04	-0.27	0.11	0.03	47
German Mark	0.00107	0.01766	0.51	-0.25*	0.10	-0.00	71
Japanese Yen	-0.167×10^{-5}	1.56×10^{-4}	-0.36	0.01	0.01	0.07	71
Swiss Franc	0.00142	0.02850	0.34	-0.15	0.10	-0.00	47

* Significant at the 5% level

Table 4

The Reaction of the Forecast Error to Past Spot Rates

$$F_{t-1} - X_t = b_0 + \sum_{i=1}^{m} b_i (X_{t-1} - X_{t-i-1}) \qquad m = 1, 2\ 3$$

Currency	m	b_0 [$t(b_0)$]	b_1 [$t(b_1)$]	b_2 [$t(b_2)$]	b_3 [$t(b_3)$]	R^2	S(e)	$\hat\rho_1(e)$	$\hat\rho_2(e)$	$\hat\rho_3(e)$	No.
British Pound	1	−0.00297	−0.0040			0.000	0.06439	−0.00	0.03	−0.02	71
		(−0.39)	(−0.03)								
	2	−0.00297	−0.0048	−0.0598		0.003	0.06476	−0.00	0.01	−0.02	71
		(−0.39)	(−0.06)	(−0.48)							
	3	−0.00298	−0.0089	−0.0597	0.0481	0.005	0.06517	−0.01	0.01	−0.00	71
		(−0.38)	(−0.07)	(−0.47)	(0.38)						
Canadian Dollar	1	0.00220	0.0147			0.000	0.01792	0.01	−0.18	−0.05	41
		(1.07)	(0.09)								
	2	0.00299	0.0289	0.3231		0.099	0.01242	−0.02	−0.03	−0.02	41
		(1.49)	(0.18)	(2.05)							
	3	0.00342	0.0700	0.3301*	0.1263	0.113	0.01249	−0.00	−0.02	0.02	41
		(1.63)	(0.42)	(2.07)	(0.75)						
French Franc	1	0.00003	0.3010*			0.085	0.00790	0.06	0.09	0.05	47
		(0.02)	(2.05)								
	2	0.00009	0.2525	−0.1359		0.098	0.00793	−0.00	0.04	0.06	47
		(0.08)	(1.58)	(−0.80)							
	3	0.00023	0.2629	−0.2275	−0.2275	0.134	0.00786	0.02	0.00	0.01	47
		(0.20)	(1.66)	(−1.26)	(−1.33)						
German Mark	1	0.00096	0.2524*			0.063	0.01722	0.03	0.08	0.01	71
		(0.47)	(2.15)								
	2	0.00114	0.2145	−0.1424		0.080	0.01719	−0.01	0.02	0.02	71
		(0.56)	(1.76)	(−1.12)							
	3	0.00128	0.2225	−0.1647	−0.0811	0.085	0.01727	0.00	0.01	0.00	71
		(0.62)	(1.81)	(−1.25)	(−0.63)						
Japanese Yen	1	-0.69×10^{-5}	0.0074			0.000	1.574×10^{-4}	0.02	0.01	0.07	71
		(−0.37)	(0.06)								
	2	-0.71×10^{-5}	0.0077	0.0118		0.000	1.585×10^{-4}	0.02	0.01	0.07	71
		(−0.37)	(0.06)	(0.09)							
	3	-0.36×10^{-5}	0.0063	0.0107	−0.1899	0.033	1.571×10^{-4}	−0.01	0.01	0.02	71
		(−0.19)	(0.05)	(0.09)	(−1.51)						
Swiss Franc	1	0.00097	0.1662			0.026	0.02843	0.04	0.09	−0.00	47
		(0.23)	(1.10)								
	2	0.00164	0.1342	−0.1658		0.050	0.02840	0.01	0.02	0.00	47
		(0.39)	(0.87)	(−1.05)							
	3	0.00176	0.1381	−0.1719	−0.0306	0.051	0.02872	0.02	0.02	−0.00	47
		(0.41)	(0.88)	(−1.05)	(−0.19)						

* Significant at the 5% level

Table 5

The Distribution of the Optimal Order m for the AR(m) Processes

(i) h = 36

Currency	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)	AR(6)	AR(7)	Total
British Pound	55	4	1	0	0	0	0	60
Canadian Dollar	15	25	1	0	1	0	0	42
French Franc	39	7	0	1	0	0	1	48
German Mark	51	6	2	0	0	1	0	60
Japanese Yen	49	7	3	1	0	0	0	60
Swiss Franc	37	8	1	2	0	0	0	48

(ii) h = 48

British Pound	39	6	2	1	0	0	0	48
Canadian Dollar	26	14	1	1	0	0	0	42
French Franc	28	5	3	0	0	0	0	36
German Mark	35	12	1	0	0	0	0	48
Japanese Yen	41	2	3	1	1	0	0	48
Swiss Franc	37	11	0	0	0	0	0	48

Table 6

MSD(u) and MSD(v) Calculations

Currency	Base Period	No. of Observations	MSD(v) $\times 10^4$	MSD(u) $\times 10^4$
British Pound	36	60	.199	.880
	48	48	.081	.642
Canadian Dollar	36	42	.018	.550
	48	42	.018	.288
French Franc	36	48	.101	2.283
	48	36	.080	1.868
German Mark	36	60	.176	1.300
	48	48	.215	1.212
Japanese Yen	36	60	.233	.519
	48	48	.289	.419
Swiss Franc	36	48	.555	1.351
	48	48	.555	1.050

Table 7

MSD(F) and MSD (MEM) Calculations

Currency	Base Period	No. of Observations	MSD(F) × 10^4	MSD(MEM) × 10^4
British Pound	36	59	10.32	11.04
	48	47	10.27	11.40
Canadian Dollar	36	41	22.09	26.17
	48	41	22.09	22.34
French Franc	36	47	12.63	14.29
	48	35	15.67	17.01
German Mark	36	59	12.74	13.88
	48	47	15.34	16.76
Japanese Yen	36	59	13.38	15.12
	48	47	16.24	18.06
Swiss Franc	36	47	24.73	28.08
	48	47	24.73	26.76