# Optimal Portfolio Selection with Upper Bounds for Individual Securities

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## ABSTRACT

In this paper, we consider the problem of optimal portfolio selection with upper bound constraints on individual securities using a constant correlation model and a single index model. The results of our study, which are at variance with those arrived at by Elton, Gruber, and Padberg in an earlier study, indicate that their ranking criterion for portfolio selection is invalid. We have developed an algorithm which provides an optimal solution to the portfolio problem.

In recent years, Elton and Gruber [1981], and Elton, Gruber, and Padberg (hereafter EGP) [1976, 1977A, 1978A, 1978B, 1979] have established simple criteria for optimal portfolio selection using a variety of models, such as single index, multi-index, and constant correlation models. Their work represents an important advancement in mean-variance portfolio analysis, since for each of these models exact solutions to portfolio problems disallowing short sales of risky securities can be obtained directly via their simple ranking procedures. EGP [1977B] have also extended their analysis using a constant correlation model, as well as a single index model, to incorporate upper limits on investment in individual securities. Such an extension is particularly useful as institutions are often restricted by law, and individual investors by choice, from investing more than a certain fraction of funds in any one security in a portfolio.

Unfortunately, as pointed out below, there is a missing term in the EGP [1977B] expression of the Kuhn-Tucker conditions for optimality for each of the two models. Because of this, the ranking criterion that they use to select optimal portfolios is no longer valid. In fact, with the correct expression, their algorithm cannot be implemented at all. In view of this, we propose another algorithm in the present study.

In the following, the portfolio problem is treated separately for a constant correlation model in Section 1, and a single index model in Section 2. Considering that the variance-covariance structures of security returns as characterized by the two models are mathematically analogous, the algebraic forms of their solutions for the same portfolio problem are essentially equivalent. Since a common algorithm can be established to reach optimal portfolios, only one of the two cases needs to be presented in detail. We have chosen the constant correlation model as have EGP. Finally, we conclude the present study in Section 3.

## 1. Optimal Portfolio Selection Using a Constant Correlation Model

Following EGP, we first set up the optimization problem:

$$\max_{\{X_1,\ldots,X_n\}} \Theta = \frac{\bar{R}_p - R_f}{\sigma_p} = \frac{\sum_{i=1}^{n} X_i(\bar{R}_i - R_f)}{\left[\sum_{i=1}^{n}\sum_{j=1}^{n} X_i X_j \sigma_{ij}\right]^{1/2}}$$

subject to

$$g_i(X_1,X_2,\ldots,X_n) = X_i - C_i \sum_{j=1}^{n} X_j \leq 0,$$

and $$-X_i \leq 0,$$

for i = 1,2,...n.

Here, $\bar{R}_p$ is the expected return on the portfolio, $R_f$ is the risk-free rate of interest for lending and borrowing, $\sigma_p$ is the standard deviation of portfolio returns, $X_i$ is the fraction of funds invested in security i with short sales disallowed (i.e., $X_i \geq 0$), $\bar{R}_i$ is the expected return on security i, $\sigma_{ij}$ is the covariance of returns between security i and security j, n is the number of securities considered for inclusion in the portfolio, and $C_i$ is the upper bound for $X_i$. The Kuhn-Tucker conditions for optimality are

$$\frac{\partial\Theta}{\partial X_i} - \sum_{j=1}^{n} \frac{\partial g_j(X_1,X_2,\ldots,X_n)}{\partial X_i}\,\delta_j' + \mu_i' = 0, \qquad (1)$$

$$\left.\begin{aligned}
&X_i \le C_i \sum_{j=1}^{n} X_j, \\
&X_i \mu_i' = 0, \; (X_i - C_i \sum_{j=1}^{n} X_j)\delta_i' = 0, \\
&X_i \ge 0, \quad \mu_i \ge 0, \quad \delta_i' \ge 0,
\end{aligned}\right\} \qquad (2)$$

for $i = 1,2,\ldots,n$.

Here, $\delta_i'$ and $\mu_i'$ are Lagrange multipliers.

For a model where the correlation coefficient between all security returns is a positive constant $\rho$, the first Kuhn-Tucker condition given by equation (1) can be expressed as

$$Z_i = \left[\bar{R}_i - R_f - \rho\sigma_i \sum_{j=1}^{n} \sigma_j Z_j - \sigma_p(\delta_i' - \sum_{j=1}^{n} C_j \delta_j') + \sigma_p \mu_i'\right] \frac{1}{\sigma_i^2(1-\rho)}, \qquad (3)$$

where $Z_i = X_i(\bar{R}_p - R_f)/\sigma_p^2$. To simplify notation, let $\delta_i = \sigma_p \delta_i'/(1-\rho)$ and $\mu_i = \sigma_p \mu_i'/[\sigma_i^2(1-\rho)]$. Equation (3) becomes

$$Z_i = \frac{1}{\sigma_i(1-\rho)}\left(\frac{\bar{R}_i - R_f}{\sigma_i} - \phi\right) - \frac{1}{\sigma_i^2}\left(\delta_i - \sum_{j=1}^{n} C_j \delta_j\right) + \mu_i, \qquad (4)$$

where $\phi = \rho \sum_{j=1}^{n} \sigma_i Z_j$, and the complementarity conditions (2) can be stated as

$$\left.\begin{array}{l} Z_i \leq C_i \sum_{j=1}^{n} Z_j, \\ Z_i \mu_i = 0, \qquad (Z_i - C_i \sum_{j=1}^{n} Z_j)\delta_i = 0, \\ Z_i \geq 0, \quad \mu_i \geq 0, \quad \delta_i \geq 0, \end{array}\right\} \tag{5}$$

for $i = 1,2,\ldots,n$.

Following EGP, let us define N as the set of n securities, $K \subset N$ as the set of k securities in the optimal portfolio, and $H \subset K$ as the set of $\chi$ securities invested at their respective upper bounds. Then, using the complementarity conditions (5), we have

(i) $Z_i = C_i \sum_{j \in N} Z_j$ (or equivalently, $X_i = C_i$), $\delta_i \geq 0$, and $\mu_i = 0$ for $i \in H$,

(ii) $0 < Z_i < C_i \sum_{j \in N} Z_j$ (or equivalently, $0 < X_i < C_i$), $\delta_i = 0$, and $\mu_i = 0$, for $i \in K-H$, and

(iii) $Z_i = 0$ (or equivalently, $X_i = 0$), $\delta_i = 0$, and $\mu_i \geq 0$, for $i \in N-K$.

Multiplying both sides of equation (4) by $\sigma_i$, and summing over all $i \in K$, we obtain with the aid of (5),

$$\phi = \frac{1}{1+(k-1)\rho} \left[ \sum_{j \in K} \frac{\bar{R}_j - R_f}{\sigma_j} - (1-\rho)\left( \sum_{j \in H} \frac{\delta_j}{\sigma_j} - \sum_{j \in K} \frac{1}{\sigma_j} \sum_{j \in H} C_j \delta_j \right) \right] \tag{6}$$

The EGP version of the first Kuhn-Tucker condition has a term $\sigma_p(1 - C_i)\delta_i'$ instead of $\sigma_p(\delta_i' - \sum_{j=1}^{n} C_j\ \delta_j')$ in the square brackets on the right hand side of equation (3) above [see their equation (1') on p. 955]. What it amounts to is that $\sum_{\substack{j=1 \\ j\neq i}}^{n} C_j\delta_j'$ is missing from the expression. Based on the incomplete expression, they arrive at the ranking property that "If $(\bar{R}_i - R_f)/\sigma_i > (\bar{R}_j - R_f)/\sigma_j$ and security j is in the optimal portfolio, then security i is in the optimal portfolio as well" (p. 956). This property is important for their algorithm. After ranking and labelling all n securities in such a way that $(\bar{R}_j - R_f)/\sigma_j > (\bar{R}_{j+1} - R_f)/\sigma_{j+1}$, they initialize set K as $\{1,2,\ldots,k_o\}$ to satisfy $\sum_{j=1}^{k_o-1} C_j < 1$ and $\sum_{j=1}^{k_o} C_j > 1$, and add securities $k_o + 1$, $k_o + 2$, etc. sequentially to the set if required during the iterations. If the statement quoted above does not hold, it becomes possible that security $j\varepsilon K$ and security $i\varepsilon N-K$ for the optimal portfolio while $(\bar{R}_i - R_f)/\sigma_i > (\bar{R}_j - R_f)/\sigma_j$. Then, the portfolios produced via the EGP algorithm must sometimes be suboptimal.

It can be shown that, when at least one security is invested at its upper bound, the statement quoted above will not hold. To see this, let us write equation (4) as

$$Z_i - y_i + \frac{\delta_i}{\sigma_i^2} - \mu_i = 0, \tag{7}$$

where

$$y_i = \frac{1}{\sigma_i(1-\rho)}\left(\frac{\bar{R}_i - R_f}{\sigma_i} - \phi\right) + \frac{1}{\sigma_i^2}\sum_{j=1}^{n} C_j\delta_j. \tag{8}$$

Since $\frac{1}{\sigma_i^2}\sum_{j=1}^{n} C_j\delta_j > 0$, $y_i$ can be of either sign even when $(\bar{R}_i - R_f)/\sigma_i < \phi$. Suppose that for securities i and j, $\phi > (\bar{R}_i - R_f)/\sigma_i > (\bar{R}_j - R_f)/\sigma_j$, $y_i < 0$, and $y_j > 0$. For security i, equation (7) gives

$$Z_i + \frac{\delta_i}{\sigma_i^2} - \mu_i < 0.$$

Using also the complementarity conditions (5), we have $Z_i = 0$, $\delta_i = 0$, and $\mu_i > 0$, implying that security i does not belong to the optimal portfolio. However, for security j, equation (7) gives

$$Z_j + \frac{\delta_j}{\sigma_j^2} - \mu_j > 0.$$

Then, $Z_j > 0$, $\delta_j > 0$, and $\mu_j = 0$, implying that security j belongs to the optimal portfolio in spite of the fact that $(\bar{R}_j - R_f)/\sigma_j < \phi$. This also demonstrates that, although $(\bar{R}_i - R_f)/\sigma_i > (\bar{R}_j - R_f)/\sigma_j$, security j but not security i belongs to the optimal portfolio.

In the following, we propose a different algorithm for the same portfolio problem. Combining equations (4) and (6) yields

$$Z_i = Z_i^* + \frac{\rho}{\sigma_i[1+(k-1)\rho]} \left( \sum_{j\varepsilon H} \frac{\delta_j}{\sigma_j} - \sum_{j\varepsilon K} \frac{1}{\sigma_j} \sum_{j\varepsilon H} C_j\delta_j \right)$$
$$- \frac{1}{\sigma_i^2} (\delta_i - \sum_{j\varepsilon H} C_j\delta_j) + \mu_i, \tag{9}$$

where

$$Z_i^* = \frac{1}{\sigma_i(1-\rho)} \left[ \frac{\bar{R}_i - R_f}{\sigma_i} - \frac{\rho}{1+(k-1)\rho} \sum_{j\varepsilon K} \frac{\bar{R}_j - R_f}{\sigma_j} \right] \tag{10}$$

Note that the solution of the optimal portfolio selection problem without upper bounds is $Z_i' = \max(Z_i^*, 0)$ (see EGP [1981, 1976, 1978B] for $i\varepsilon N$. Equation (9) can be written as

$$\begin{aligned} Z_i &= Z_i^* + \sum_{\ell\varepsilon H} \alpha_{i\ell}\delta_\ell + \mu_i, \\ &= Z_i^\dagger + \mu_i \end{aligned} \tag{11}$$

for all $i\varepsilon N$, where

$$\alpha_{i\ell} = \frac{\rho}{\sigma_i[1+(k-1)\rho]} \left[ \frac{1}{\sigma_\ell} - \left( \sum_{j\varepsilon K} \frac{1}{\sigma_j} \right) C_\ell \right] - \frac{1}{\sigma_i^2} (\Delta_{i\ell} - C_\ell), \tag{12}$$

$\Delta_{i\ell}$ being the Kronecker delta (i.e., $\Delta_{ii} = 1$ and $\Delta_{i\ell} = 0$ for $i \neq \ell$).

Once sets H and K are identified, values of $Z_i^*$ and $\alpha_{i\ell}$ for all $i\varepsilon N$ and

$\ell \varepsilon H$ can be computed directly using equations (10) and (12). Then, values of $\delta_\ell$ for all $\ell \varepsilon H$ in equation (11) can be computed by solving $\chi$ simultaneous equations

$$Z_i = C_i \sum_{j \varepsilon K} Z_j,$$

with $i \varepsilon H$, which are

$$\sum_{\ell \varepsilon H} (\alpha_{i\ell} - C_i \sum_{j \varepsilon K} \alpha_{j\ell})\delta_\ell = -(Z_i^* - C_i \sum_{j \varepsilon K} Z_j^*). \tag{13}$$

When all $\delta_\ell$'s are known, equation (11) will provide values of $Z_i$ and $\mu_i$ for all $i \varepsilon N$ with the aid of the complementarity conditions (5).

The task now is to determine which of the n securities belong to set H and which belong to set K. The procedure is as follows:

1. Determine initial set K using EGP's [1981, 1976, 1978B] ranking procedure for the optimal portfolio that disallows short sales of risky securities but has no upper bound on investment in individual securities.
2. Compute $Z_i^*$ for all $i \varepsilon N$ using equation (10).
3. Define $B_i = \max (C_i, D)$ for all $i \varepsilon N$, D being a parameter. Starting from D = 0, increase D until $\sum_{i \varepsilon K} B_i > 1$.
4. Determine set H so that $Z_i^* / \sum_{j \varepsilon K} Z_j^* > B_i$ for all $i \varepsilon H$. If H is null, go to step 11. Otherwise, continue.

5. Replace $C_i$ by $B_i$ for all $i \varepsilon H$ in equations (12) and (13) and determine $\delta_i$ for all $i \varepsilon H$ using these equations.
6. Compute $Z_i^{\dagger}$ in equation (11) for all $i \varepsilon N$.
7. Determine a set K' such that $Z_i^{\dagger} > 0$ for $i \varepsilon K'$.
8. If $K \gtrless K'$, let $K = K'$, compute $Z_i^*$ for all $i \varepsilon N$ using equation (10), and go to step 5. Otherwise let $Z_i = Z_i^{\dagger}$ and $\mu_i = 0$ for all $i \varepsilon K$, and $Z_i = 0$ and $\mu_i = - Z_i^{\dagger}$ for all $i \varepsilon N-K$, and continue.
9. Determine a set H' such that $Z_i / \sum_{j \varepsilon K} Z_j > B_i$ for all $i \varepsilon H'$.
10. If $H \gtrless H'$, let $H = H'$ and go to step 5. Otherwise, continue.
11. If $B_i = \max (C_i, D) > C_i$ for any $i \varepsilon N$, decrease D to the lowest value that still maintains $\sum_{i \varepsilon K} B_i > 1$, and go to step 5. Otherwise, stop.

At the start of the above procedure, we initialize set K as the one corresponding to the optimal portfolio without upper bound constraints. If $\sum_{i \varepsilon K} C_i < 1$, the initial set K clearly does not have sufficient capacity to accommodate the funds to be distributed. Since the ranking hierarchy of securities cannot be used to select additional securities for set K for increasing its capacity, we choose to change each upper bound $C_i$ to $B_i = \max (C_i, D)$ for all $i \varepsilon N$. Here, D has been assigned the lowest value that makes $\sum_{i \varepsilon K} B_i > 1$. The initial set H contains all securities $i \varepsilon K$ with $Z_i^* / \sum_{j \varepsilon K} Z_j^* > B_i$, where $Z_i^*$'s correspond to the optimal portfolio without upper bound constraints.

It is obvious that the unused funds beyond the upper bounds for securities in set H must be redistributed among other securities. Intuitively, this can sometimes cause securities in set N-K (set N-H) to enter set K (set H), but not the reverse. Indeed, when the upper bounds become more restrictive, more unused funds will be available for redistribution among securities, and changes in sets K and H as the result of the redistribution of funds, if any, must be due to the addition of more securities to the two sets. Using this simple idea, we allow the two sets to grow by gradually imposing more restrictive upper bounds until the solution corresponding to our original optimization problem is reached. In the following, we provide further explanation and justification of our approach.

After initializing sets K and H in steps 1 to 4, we search for the optimal portfolio satisfying $0 \leq Z_i / \sum_{j \varepsilon N} Z_j \leq B_i$ for all $i \varepsilon H$ and $Z_i \geq 0$ for all $i \varepsilon N-H$. In this particular problem where set H is predetermined, there is no upper bound constraint on any security outside the intial set H. Following steps 5 to 8, we compute $\delta_i$ for all $i \varepsilon H$ and $Z_i$ and $\mu_i$ for all $i \varepsilon N$, and revise set K to be the one containing all securities $i \varepsilon N$ with $Z_i > 0$. Our criterion for revising set K can be justified on the grounds that a positive $Z_i$ indicates that security i should belong to the portfolio. Once a new set K is established, new values of $Z_i^*$ for all $i \varepsilon N$, $\delta_i$ for all $i \varepsilon H$, and $Z_i$ and $\mu_i$ for all $i \varepsilon N$ must be computed. Subsequent to the computations, set K is revised again using the same criterion. The procedure is repeated until there is no change in the set. It is worth noting that, although there is no restriction on the signs of $\delta_i$ for all $i \varepsilon H$ when we solve for $\delta_i$ using equations (12) and (13), their values should always be positive during each iterative step. Intuitively, a negative $\delta_i$, if obtained, would indicate that security i has been placed in set H where it does not belong. Since all securities we place in set H actually belong there, the possibility of obtaining any negative $\delta_i$ for all $i\varepsilon H$ can be ruled out.

Next, as described by step 9, we add to set H all securities $i\varepsilon K-H$ with $Z_i / \sum_{j\varepsilon K} Z_j > B_i$. With this new set H, we search for the optimal portfolio satisfying $0 < Z_i / \sum_{j\varepsilon N} Z_j < B_i$ for all $i\varepsilon H$ and $Z_i > 0$ for all $i\varepsilon N-H$. The constraints for the optimization problem here are more restrictive than those for the problem just solved because more securities are subject to their upper bound constraints. Using set K from the solution just obtained as the initial set K, we go through steps 5 to 8 repeatedly until the current optimization problem is solved.

Set H is revised again using the same criterion as described in step 9, and the whole procedure is repeated until we reach the optimal portfolio satisfying $0 < Z_i / \sum_{j\varepsilon N} Z_j < B_i$ for all $i\varepsilon N$. Unless $B_i = C_i$ for all $i\varepsilon N$, the above portfolio is not the one corresponding to our original optimization problem. To make the upper bounds more restrictive, we let $B_i = \max(C_i, D)$ for all $i\varepsilon N$ where D has been given the lowest value that maintains $\sum_{i\varepsilon K} B_i > 1$. Using sets K and H just obtained as the initial sets, and going through the same iterative procedure described above, we sequentially reach optimal portfolios satisfying progressively more restrictive constraints. After reaching the optimal portfolio satisfying $0 < Z_i / \sum_{j\varepsilon N} Z_i < B_i$ for all $i\varepsilon N$, each $B_i$ which is above $C_i$ is again reduced while maintaining $\sum_{i\varepsilon K} B_i > 1$. By making the upper bounds more restrictive from one iteration to the next, we will finally reach the optimal portfolio that satisfies $0 < Z_i / \sum_{j\varepsilon N} Z_j < C_i$ for all $i\varepsilon N$.

An illustrative example is shown in Table I. Here, among the 20 securities which have been labelled in such a way that security i is ranked no lower than security i+1, the optimal portfolio without the upper bound constraints has the first 6 securities in set K. Since $\sum_{i\varepsilon K} C_i = 0.65 < 1$, D is given an initial value of 0.17 which makes $\sum_{i\varepsilon K} B_i = 1.02 > 1$. The initial set H is {1,2} because $(Z_1^*/\sum_{j\varepsilon K} Z_j^*) > B_1$ and $(Z_2^*/\sum_{j\varepsilon K} Z_j^*) > B_2$. After going through steps 5 to 10 and using simultaneous equations (13) four times to determine $\delta_i$ for all $i\varepsilon H$, sets H and K become {1,2,...,5} and {1,2,...,7,10} respectively, and an optimal portfolio is reached.

The value of D is then reduced to 0.12 yielding $\sum_{i\varepsilon K} B_i = 1.02 > 1$, and the same iterative procedure is repeated using the same H and K just obtained as the initial sets. This time, calculations for $\delta_i$ for $i\varepsilon H$ have been performed four times yielding an optimal portfolio with H = {1,2,..., 6,10} and K = {1,2,...,7,10,14}. Finally, the value of D is reduced to 0.1 resulting in $B_i = C_i$ for all $i\varepsilon N$ and $\sum_{i\varepsilon K} B_i = 1.1 > 1$. After two repeated calculations for $\delta_i$ for $i\varepsilon H$, we reach the optimal portfolio as required. The corresponding sets H and K are respectively {1,2,...,6,10} and {1,2, ...,7,9,10,12,14}.

Note that in the optimal portfolio, security 11 $\varepsilon N-K$, while securities 12 and 14 $\varepsilon K$. Likewise, securities 11 and 13 $\varepsilon N-K$, while security 14 $\varepsilon K$. Such results clearly demonstrate that if $j\varepsilon K$ and $(\bar{R}_i - R_f)/\sigma_i > (\bar{R}_j - R_f)/\sigma_j$, it does not always follow that $i\varepsilon K$. To provide additional support to this argument, we have also sought the optimal portfolio using our algorithm for the example shown in Table I of EGP's [1977B] study. In that example, the 10 securities considered have already been ranked. While they obtain H = {1,2,3,4} and K = {1,2,3,4,5} with $X_1 = X_2 = 1/5$, $X_3 = X_5 =$

1/10, $X_4 = 2/5$, and $X_6$ to $X_{10} = 0$, our result differs from theirs in that $K = \{1,2,3,4,6\}$ and $X_6 = 1/10$ but $X_5 = 0$. Our result is found to satisfy the optimality conditions established above. To determine whether the EGP portfolio is indeed suboptimal we have computed $\Theta = (\bar{R}_p - R_f)/\sigma_p$. It turns out that while this ratio is 10.660 in their case, ours is 10.875.

## 2. Optimal Portfolio Selection Using the Single Index Model

The standard version of the single index model [Sharpe, 1963] can be written as

$$\left.\begin{aligned} R_i &= \alpha_i + \beta_i I + \varepsilon_i, \\ \text{and} \quad I &= \alpha_{n+1} + \varepsilon_{n+1}, \end{aligned}\right\} \tag{14}$$

for $i = 1,2,\ldots,n$, where $R_i$ and I are respectively random returns on security i and on a market index, the $\alpha$'s and $\beta$'s are parameters, and the $\varepsilon$'s are random noise. For this model each $\varepsilon_i$, for $i = 1,2,\ldots n+1$, has a zero mean and a finite non-zero variance $\upsilon_i^2$. Here, to simplify notation, $\upsilon_{n+1}^2$ is labelled as $\tau^2$. It is further assumed that cov $(\varepsilon_i, \varepsilon_j) = 0$ for $i,j = 1,2, \ldots,n+1$ and $i \neq j$.

The first Kuhn-Tucker condition given by equation (1) can be written as

$$Z_i = \frac{1}{\upsilon_i^2}\left[\bar{R}_i - R_f - \tau^2\beta_i \sum_{j=1}^{n} \beta_j Z_j - \sigma_p(\delta_i' - \sum_{j=1}^{n} C_j\delta_j') + \sigma_p\mu_i'\right], \tag{15}$$

where $Z_i = X_i(\bar{R}_p - R_f)/\sigma_p^2$. Letting $\delta_i = \sigma_p\delta_i'$ and $\mu_i = \sigma_p\mu_i'/\upsilon_i^2$, we have

$$Z_i = \frac{\beta_i}{\upsilon_i^2}\left(\frac{\bar{R}_i - R_f}{\beta_i} - \phi\right) - \frac{1}{\upsilon_i^2}\left(\delta_i - \sum_{j=1}^{n} C_j\delta_j\right) + \mu_i, \tag{16}$$

where $\phi = \tau^2 \sum_{j=1}^{n} \beta_j Z_j$, which is similar to equation (4) for the constant correlation model. The complementarity conditions (5) also apply here. In equation (16),

$$\phi = \frac{\tau^2}{1 + \tau^2 \sum_{j\varepsilon K} (\beta_j/\upsilon_j^2)} \left[ \sum_{j\varepsilon K} \frac{\beta_j(\bar{R}_j - R_f)}{\upsilon_j^2} - \sum_{j\varepsilon H} \frac{\beta_j \delta_j}{\upsilon_j^2} + \sum_{j\varepsilon K} \frac{\beta_j}{\upsilon_j^2} \sum_{j\varepsilon H} C_j \delta_j \right], \tag{17}$$

where sets H and K, and implicitly set N, are defined as before.

A comparison between equation (16) and EGP's [1977B] equation (25) on p. 962 reveals a missing term in their version analogous to the case described in Section 1 above. The term $\frac{1}{\upsilon_i^2}\left(\delta_i - \sum_{j=1}^{n} C_j \delta_j\right)$ on the right hand side of our equation (16) is $\delta_i$ in their version. Based on their equation (25), they state that "If $(\bar{R}_i - R_f)/\beta_i \geq (\bar{R}_k - R_f)/\beta_k$, $B_i > 0$, $\beta_k > 0$ and security k is in the optimal portfolio, then so is security i. If $(\bar{R}_i - R_f)/\beta_i \leq (\bar{R}_k - R_f)/\beta_k$, $\beta_i < 0$, $\beta_k < 0$ and security k is in the optimal portfolio, then so is security i" (p. 962). For the same reason given earlier in the constant correlation model, such a statement cannot be considered valid, and therefore their algorithm will not always provide optimal solutions.

The striking similarity between the expressions for $Z_i$ in equations (4) and (16) allows us to use the same algorithm developed earlier for the single index model. To do this, let us combine equations (16) and (17) to obtain

$$Z_i = Z_i^* + \frac{\beta_i \tau^2}{\upsilon_i^2 \left[1 + \tau^2 \sum_{j\varepsilon K} (\beta_j^2/\upsilon_j^2)\right]} \left[ \sum_{j\varepsilon H} \frac{\beta_j \delta_j}{\upsilon_j^2} - \sum_{j\varepsilon K} \frac{\beta_j}{\upsilon_j^2} \sum_{j\varepsilon H} C_j \delta_j \right]$$
$$- \frac{1}{\upsilon_i^2} (\delta_i - \sum_{j\varepsilon H} C_j \delta_j) + \mu_i, \tag{18}$$

where

$$Z_i^* = \frac{\beta_i}{\upsilon_i^2} \left\{ \frac{\bar{R}_i - R_f}{\beta_i} - \frac{\tau^2 \sum_{j\varepsilon K} \left[\beta_j (\bar{R}_j - R_f)/\upsilon_j^2\right]}{1 + \tau^2 \sum_{j\varepsilon K} (\beta_j^2/\upsilon_j^2)} \right\}. \tag{19}$$

Again, the solution of the optimal portfolio selection problem without the upper bound constraints is max $(Z_i^*, 0)$ (see EGP [1981, 1976, 1978B]) for all $i\varepsilon N$. Equation (18) can also be written as equation (11) for all $i\varepsilon N$ where

$$\alpha_{i\ell} = \frac{\beta_i \tau^2}{\upsilon_i^2 [1 + \tau^2 \sum_{j\varepsilon K} (\beta_j^2/\upsilon_j^2)]} \left[ \frac{\beta_\ell}{\upsilon_\ell^2} - \left( \sum_{j\varepsilon K} \frac{\beta_j}{\upsilon_j^2} \right) C_\ell \right]$$
$$- \frac{1}{\upsilon_i^2} (\Delta_{i\ell} - C_\ell), \tag{20}$$

$\Delta_{i\ell}$ being the Kronecker delta. Since the same algorithm as described in Section 1 for the constant correlation model also applies to this case, there is no need to duplicate the description here.

## 3. Conclusion

In this paper, we consider the problem of optimal portfolio selection with upper bounds for individual securities using a constant correlation model and a single index model. Once the upper bounds are imposed, unused funds beyond the bounds must be redistributed among other securities, and this can sometimes cause the addition of more securities to the portfolio. However, unlike what is shown in the Elton, Gruber, and Padberg [1977B] study on the same topic, we demonstrate that, in selecting additional securities to the portfolio, one cannot depend on the ranking hierarchy based on the excess-return-to-risk ratios of securities. To see this, suppose that security i is ranked higher than security j, but neither belong to the optimal portfolio without the upper bound constraints. If security j is in the optimal portfolio with upper bounds, it does not follow that security i must also be there. We show that the Elton, Gruber, and Padberg statement that security i must also be in the optimal portfolio does not always hold because of a missing term in their expression of the first Kuhn-Tucker condition. As their algorithm for optimal portfolio selection with upper bounds is based on the preservation of ranking properties carried over from the same problem without upper bounds, it is natural that the portfolio reached is not always optimal.

In this paper, a different algorithm is developed. The iteration starts with the optimal portfolio without upper bound constraints, determined via Elton, Gruber, and Padberg's [1981, 1976, 1978B] ranking procedure. Once this is done, the subsequent iterations that are required to identify securities in the optimal portfolio as well as those at their upper bounds will no longer rely on the ranking arrangement of securities. To illustrate, a numerical example showing the iterative steps is presented in Section 1. The approach here is clearly a better alternative to the less formal approach of selecting optimal portfolios by simply redistributing unused funds beyond upper bounds among remaining securities without fully utilizing the Kuhn-Tucker conditions.

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TABLE 1: Illustrative Example of Optimal Portfolio Selection with Upper Bound Constraints Using a Constant Correlation Model ($\rho = 0.4$)

| Input Data | | | | | Solution without Upper Bounds | Solution for D = 0.17 | | | | | Solution for D = 0.12 | | | | | Solution for D = 0.10 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $i$ | $C_i$ | $\bar{R}_i - R_f$ | $\sigma_i$ | $\frac{\bar{R}_i - R_f}{\sigma_i}$ | $X_i^{\dagger}$ | $B_i$ | $Z_i$ | $X_i$ | $\delta_i$ | $\mu_i$ | $B_i$ | $Z_i$ | $X_i$ | $\delta_i$ | $\mu_i$ | $B_i$ | $Z_i$ | $X_i$ | $\delta_i$ | $\mu_i$ |
| 1 | 0.10 | 15 | 4 | 3.750 | 0.241 | 0.17 | 0.264 | 0.170 | 4.862 | 0 | 0.12 | 0.198 | 0.120 | 8.176 | 0 | 0.10 | 0.167 | 0.100 | 9.592 | 0 |
| 2 | 0.10 | 11 | 3 | 3.667 | 0.293 | 0.17 | 0.264 | 0.170 | 4.697 | 0 | 0.12 | 0.198 | 0.120 | 7.434 | 0 | 0.10 | 0.167 | 0.100 | 8.505 | 0 |
| 3 | 0.10 | 22 | 6 | 3.667 | 0.147 | 0.17 | 0.264 | 0.170 | 1.940 | 0 | 0.12 | 0.198 | 0.120 | 6.805 | 0 | 0.10 | 0.167 | 0.100 | 9.096 | 0 |
| 4 | 0.10 | 18 | 5 | 3.600 | 0.162 | 0.17 | 0.264 | 0.170 | 2.832 | 0 | 0.12 | 0.198 | 0.120 | 6.855 | 0 | 0.10 | 0.167 | 0.100 | 8.678 | 0 |
| 5 | 0.15 | 13 | 4 | 3.250 | 0.115 | 0.17 | 0.264 | 0.170 | 1.529 | 0 | 0.15 | 0.248 | 0.150 | 4.050 | 0 | 0.15 | 0.251 | 0.150 | 4.920 | 0 |
| 6 | 0.10 | 15 | 5 | 3.000 | 0.042 | 0.17 | 0.178 | 0.114 | 0 | 0 | 0.12 | 0.198 | 0.120 | 1.855 | 0 | 0.10 | 0.167 | 0.100 | 3.678 | 0 |
| 7 | 0.15 | 16 | 6 | 2.667 | 0 | 0.17 | 0.040 | 0.026 | 0 | 0 | 0.15 | 0.109 | 0.066 | 0 | 0 | 0.15 | 0.142 | 0.085 | 0 | 0 |
| 8 | 0.10 | 18 | 8 | 2.250 | 0 | 0.17 | 0 | 0 | 0 | 0.071 | 0.12 | 0 | 0 | 0 | 0.028 | 0.10 | 0 | 0 | 0 | 0.006 |
| 9 | 0.20 | 13 | 6 | 2.167 | 0 | 0.20 | 0 | 0 | 0 | 0.098 | 0.20 | 0 | 0 | 0 | 0.030 | 0.20 | 0.003 | 0.002 | 0 | 0 |
| 10 | 0.10 | 4 | 2 | 2.000 | 0 | 0.17 | 0.015 | 0.010 | 0 | 0 | 0.12 | 0.198 | 0.120 | 1.296 | 0 | 0.10 | 0.167 | 0.100 | 2.083 | 0 |
| 11 | 0.10 | 9 | 5 | 1.800 | 0 | 0.17 | 0 | 0 | 0 | 0.222 | 0.12 | 0 | 0 | 0 | 0.128 | 0.10 | 0 | 0 | 0 | 0.085 |
| 12 | 0.30 | 5 | 3 | 1.667 | 0 | 0.30 | 0 | 0 | 0 | 0.325 | 0.30 | 0 | 0 | 0 | 0.087 | 0.30 | 0.001 | 0.001 | 0 | 0 |
| 13 | 0.20 | 10 | 6 | 1.667 | 0 | 0.20 | 0 | 0 | 0 | 0.237 | 0.20 | 0 | 0 | 0 | 0.168 | 0.20 | 0 | 0 | 0 | 0.135 |
| 14 | 0.20 | 3 | 2 | 1.500 | 0 | 0.20 | 0 | 0 | 0 | 0.401 | 0.20 | 0.105 | 0.064 | 0 | 0 | 0.20 | 0.272 | 0.162 | 0 | 0 |
| 15 | 0.25 | 3 | 3 | 1.000 | 0 | 0.25 | 0 | 0 | 0 | 0.695 | 0.25 | 0 | 0 | 0 | 0.457 | 0.25 | 0 | 0 | 0 | 0.369 |
| 16 | 0.10 | 7 | 7 | 1.000 | 0 | 0.17 | 0 | 0 | 0 | 0.371 | 0.12 | 0 | 0 | 0 | 0.318 | 0.10 | 0 | 0 | 0 | 0.292 |
| 17 | 0.25 | 3 | 4 | 0.750 | 0 | 0.25 | 0 | 0 | 0 | 0.682 | 0.25 | 0 | 0 | 0 | 0.541 | 0.25 | 0 | 0 | 0 | 0.483 |
| 18 | 0.15 | 4 | 6 | 0.667 | 0 | 0.17 | 0 | 0 | 0 | 0.515 | 0.15 | 0 | 0 | 0 | 0.446 | 0.15 | 0 | 0 | 0 | 0.413 |
| 19 | 0.10 | 1 | 3 | 0.333 | 0 | 0.17 | 0 | 0 | 0 | 1.066 | 0.12 | 0 | 0 | 0 | 0.828 | 0.10 | 0 | 0 | 0 | 0.739 |
| 20 | 0.15 | 1 | 4 | 0.250 | 0 | 0.17 | 0 | 0 | 0 | 0.890 | 0.15 | 0 | 0 | 0 | 0.749 | 0.15 | 0 | 0 | 0 | 0.691 |

$^{\dagger} X_i = Z_i / \sum_{j \in K} Z_j$

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