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EFFICIENCY, EFFECTIVENESS AND PROFITABILITY: AN INTERACTION MODEL

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AN INTERACTION MODEL

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Abstract

In order to analyze the interdependence of efficiency, profitability and effectiveness, an analytical model is developed which treats efficiency and profitability as functions of effectiveness. Since both efficiency and profitability in turn depend upon value and cost, the analytical nature of these functions are also examined. An example of a multiple server system in an office environment is used to demonstrate the applicability of this approach to the study of office productivity problems, but the technique may also be used in analyzing and optimizing organizational productivity in general.

(Productivity; Efficiency; Effectiveness; Office Models)

1. Introduction

There are many ways of measuring and describing organizational productivity. However, it is generally accepted that productivity or performance is not adequately described by only the organizational efficiency [5] (sometimes called total productivity or total factor productivity [16]). In a survey of executives and union leaders, Katzell and Yankelovich [10] found that a broad spectrum of statements could be suitable for describing productivity. The statements they used can also be organized into the three classifications of efficiency, effectiveness and job satisfaction. These three classifications are discussed by Harrison and Mumby [8]. The third classification, job satisfaction, could be said to affect productivity through

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its impact on the usual productivity dimensions of efficiency and effectiveness, although job satisfaction itself is not another dimension of productivity.

Technological change is the primary driving force behind productivity improvement [6, 25], although management innovation also plays a large role. Recent innovations such as the microprocessor have made available new technologies which may result in substantial productivity improvements in the white collar (office) employment sector. This labor-intensive sector has traditionally shown little productivity improvement relative to blue collar and agricultural occupations but it includes an ever-growing fraction of total employment [17], now approaching 50%. Because of the large potential impact of productivity improvement in this area, there is a great deal of interest in the rapid adoption of automation in the office.

The automation of office tasks has traditionally been in the highly structured clerical tasks, although there is no conclusive evidence [24] to show a decline in the total growth of office staff as the result of automation. In fact, the insatiable demands of management for more information has caused a substantial growth in office resource consumption which may have masked productivity improvement. Often the design [2] and the operation [15] of office information systems has been based primarily upon considerations of efficiency, where cost justification is related to the elimination of clerical staff. This may not lead to the most productive use of resources unless considerations of effectiveness and job satisfaction [7, 17] are allowed to play an equally important role in both the system design and implementation.

Ongoing system operation must also be monitored, evaluated and controlled through measurement plans in a manner which will encourage continuing attention to productivity. Tapscott [28] discusses a monitoring approach for office automation during testing and implementation phases. Peeples [19] describes a measurement plan for monitoring and controlling information system resources which weights effectiveness measures twice as heavily as efficiency measures, with good results in organizational productivity improvement. Kriebel and Raviv [12] describe an economic model for measuring aggregate computer systems productivity, incorporating both quality (effectiveness) and efficiency measures. Adam, Herschauer and Ruch [1] developed a general productivity monitoring and control procedure which also incorporates both efficiency and effectiveness measures. Effectiveness is included in their measurements only in the cost of correcting mistakes resulting in defective output. The interdependence of quality (effectiveness) and productivity has also been discussed by Shaw and Rapoor [26].

Clearly, there is general agreement that measures of efficiency and effectiveness are important in describing, measuring, and controlling organizational productivity. However, it is just as clear that these measures are interdependent and that this interdependency is not well understood. This paper will focus on the interdependence issue, and a conceptual model will be proposed and analyzed to bring out some general conclusions regarding the inter-relationships among efficiency, effectiveness, and a third measure of profitability. The general area of application will be oriented towards issues in office productivity, but the conclusions are applicable to organizational productivity issues in general.

2. An Efficiency and Profitability Model

The following conceptual model will be used in examining the relationships among efficiency, effectiveness, and profitability by developing the quantitative behavior of efficiency and profitability against effectiveness as the independent variable. First of all, effectiveness (often referred to as "quality" in physical production contexts) will be defined in the standard way [5] as a measure of how well a product or service approaches some performance criterion such as "zero defects", "zero turnaround time", etc. We will define effectiveness E to have a value of 1 when the objective is attained, and to have a value of 0 when effectiveness is as poor as possible in relation to that objective.

The cost of operating the organization which produces the good or service will include all the usual input costs (labor, capital, and other operating expenses) associated with that good or service, but excluding the cost of raw material or component parts. In an office environment the raw material is data or information to be transformed in some way, or to be used in decision making. Normally, the relative cost of achieving an effectiveness of 1 will be very high, if it is attainable at all. The cost of producing the product or supplying the service will thus tend to rise sharply as the objective is approached closely. Conversely, we would expect a relatively low cost for producing a service or product with zero effectiveness. For the purpose of this discussion, we will therefore assume that, where the cost function C(E) exists, it will be a continuous positive, increasing convex function of effectiveness over the domain $E_{cmin} < E < E_{cmax}$. Also, $0 \leq E_{cmin} < E_{cmax} \leq 1$.

V(E), the value of a service or the value added to a product, may be measured directly if the product or service is supplied to the open market. The usual assumption is that the value added is the difference between the market price and the cost of the raw materials or component parts. In this discussion we will ignore such concepts as consumer's or producer's surplus and will assume that the value added to a product or the value of a service is the same to both producer and consumer. Since our primary concern is with white collar or office services, both producer and consumer are usually in the same firm and will therefore analyze cost and value from the same point of view. If the product or service is not supplied to the market but is supplied internally to other units of the same organization, then a transfer price must be negotiated between the supplier and the consumer, for internal accounting purposes. This transfer price, if realistically arrived at, could serve as the value of the product or service. Often the value added to internally transferred goods or services may be estimated by using the price of equivalent products or services available on the open market. The value of office services such as information services, and certainly the value of management services, is often very difficult to estimate. These services are necessary and vital to the operation of the organization but are normally treated as internal overhead functions for which the value may not normally be estimated directly. The effectiveness of such services is often measured in terms of accuracy, reliability, timeliness, adequacy, etc. The value of these services as a function of effectiveness may be determined in terms of a cognitive function of the information need of the consumers, or through a measure of consumer perceptions of value [4, 14]. Debons, Ramage, and O'Brien [3] indicate the resulting value function to be concave with respect to effectiveness (in their work, "effectiveness" is equivalent to value as defined here, and "attribute quality" is equivalent to effectiveness as used here). Land [13] also suggests a concave dependency of value on effectiveness (referred to as "quality" in his work). Keeney and Raiffa [11], in a discussion of utility preference for corporate objectives, found three types of functional dependencies of utility on effectiveness. They found that the utility of several objectives were monotonic concave functions of effectiveness, several were monotonic S-shaped, while one was non-monotonic concave. Although these objectives were not information-related *per se*, they give a reasonable indication of the general functional forms that may be expected in measuring value.

To allow for the uncertainty in the nature of the information value function, three possible functional dependencies of value on effectiveness will be conjectured. These include V(E) as a continuous positive increasing concave function, as a continuous positive increasing S-shaped function, or as a deadline function which has a discontinuous drop to zero below a certain effectiveness E_d for one of the previous value functions. These functions are discussed in more detail in the appendix. A fourth possibility is a non-monotonic positive concave function. However, this becomes a monotonic positive concave function if we consider only the domain up to the maximum value, which is normally of interest. The common characteristics for the continuous portions of these functions are that the slope of the value function is relatively flat near a maximum effectiveness of 1 and that value approaches zero as effectiveness approaches zero. The flatness of the value function near E=1 is a consequence of the fact that the consumer's concept of value tends to be indifferent to effectiveness as long as effectiveness is near the consumer's intended objective. This objective may, of course, lie below the theoretical objective E = 1.

The profitability P(E) per unit of production is given by

$$P(E) = V(E) - C(E).$$

For a profit maximizing operation,

$$dP(E)/dE = dV(E)/dE - dC(E)/dE = 0 \tag{1}$$

at the maximum profit where $E = E_p^*$. In economic terms this is equivalent to setting marginal value equal to marginal cost. In certain cases, the profit maximizing effectiveness E_p^* may also lie on a boundary or discontinuity in the value function in which case equation 1 may not apply The existence of local and global maxima for profitability, using the three types of value functions, is discussed in the appendix. Both profitability and efficiency functions will exhibit a global maximum value in the domain $E_{cmin} \leq E \leq E_{cmax}$.

The efficiency Y(E) of an organization's production operations on a per unit basis may be defined as [27]

$$Y(E) = V(E)/C(E)$$

At maximum efficiency E_Y^*, provided that the maximum does not lie at a boundary or discontinuity,

$$\frac{dY(E)}{dE} = \frac{V(E)}{C(E)} \left\{ \frac{dV(E)/dE}{V(E)} - \frac{dC(E)/dE}{C(E)} \right\} = 0 \qquad (2)$$

The relative positions of E_p^* and E_Y^* depend upon the profitability (or lack of it) in the organization's operations. There are three cases to be considered if interior maxima exist for both profitability and efficiency functions.

(a) If the cost curve lies entirely above the value curve, then the maximum profit (minimum loss) is negative. (This may occur frequently in the office, where the value of certain information outputs may be less than zero, but cross-subsidized by other information or physical outputs from the organization). From equation 2, dY(E)/dE is positive at the point of maximum profitability since $V(E_p^*) < C(E_p^*)$ and, from equation 1,

$$dV(E_p^*)/dE = dC(E_p^*)/dE.$$

Thus, the efficiency is an increasing function of E at $E = E_p^*$, and maximum efficiency is achieved at a higher effectiveness than for maximum profitability (i.e., $E_Y^* > E_p^*$).

(b) If the cost curve lies entirely above the value curve, but is tangent to it at one point, then this point will be E_p^* from equation 2, since

$$dV(E_p^*)/dE = dC(E_p^*)/dE$$

at the point of tangency. However, $V(E_p^*) = C(E_p^*)$ and thus $E_p^* = E_Y^*$ from equation 2. That is, maximum efficiency and maximum profitability occur at the same effectiveness ($E_Y^* = E_p^*$) only when the maximum profitability is zero.

(c) If the value curve lies above the cost curve over some region then the maximum profitability will occur in this region and, from equation 2, $dY(E)/dE$ is negative at the point of maximum profitability since $dV(E_p^*)/dE = dC(E_p^*)/dE$ and $V(E_p^*) > C(E_p^*)$. Thus the efficiency is decreasing at this point and the maximum efficiency is achieved at an effectiveness which is less than that for maximum profitability (i.e., $E_Y^* < E_p^*$).

3. Application

As a simple demonstration of the effectiveness model, let us assume that it is applied to a service system with S servers working in parallel. The demand stream is Poisson at mean rate λ, service time is exponential with mean rate μ and the service discipline is first-come-first-served. The effectiveness measure E will be assumed to be a function of the inverse of turnaround time W_S, or

$$E = \frac{1}{W_S+1} \qquad 1/\mu < W_S < \infty. \tag{3}$$

E varies from $E_{cmin} = 0$ at the maximum equilibrium load $\lambda_{max} = \mu S$- up to $E_{cmax} = \mu/(\mu + 1)$ at zero load, since $1/\mu$ is the turnaround time at zero load. Other effectiveness dimensions such as product quality will be assumed to be held constant, independent of system load.

The cost per unit processed will be given by

$$C(E) = \frac{C_f}{\mu S \rho} + C_v \tag{4}$$

where $\rho = \lambda/\mu S$, C_f is the fixed cost per unit time (labor, capital and other expenses) of operating the processing facility, and C_v is the variable cost per unit processed.

The expected time to process an item, W_S, is given by a well-known formula [9] which, when combined with equation (3), gives

$$E(\rho) = \{\frac{P_o(S\rho)^S}{S!(1-\rho)^2 S} + \mu + 1\}^{-1} \qquad (5)$$

where P_o is the probability that there are no jobs being processed by the system.

Inverting equation (5) by an inverse transformation ϕ^{-1} gives $\rho = \phi^{-1}(E)$ and substituting in equation (4) results in

$$C(E) = \frac{C_f}{\mu S \phi^{-1}(E)} + C_v. \qquad \begin{array}{l} 0 < E < \mu/(\mu+1) \\ S \geq 1 \end{array} \qquad (6)$$

C(E) is shown in the appendix to be a convex increasing function. Note that C(E) is undefined for $E \geq \mu/(\mu+1) = E_{cmax} +$.

The value function in this example will be assumed to be S-shaped, and the cumulative Weibull function is useful in modeling this case. Then

$$V(E) = a_o\{1 - \exp - ((E - a_1)/ a_2)^{a_3}\} \qquad a_1 \leq E \leq 1 \qquad (7)$$

$$= 0 \text{ elsewhere}$$

where a_o, a_1, a_2 and a_3 are the magnitude, origin, scale and shape parameters respectively. V(E) is non-decreasing over the domain. The function is convex over $0 \leq E \leq E_f$ and concave over $E_f < E \leq 1$. E_f, the inflection point, is given by

$$E_f = a_1 + a_2\{(a_3-1)/a_3\}^{1/a_3} \qquad a_3 > 1.$$

For $a_3 \leq 1$, the function is concave over the entire range. Figure 1 is a composite plot of the value, cost, profitability, efficiency and expected load or throughput for an example set of cost and value parameters, with a six server (S = 6) system. The parameters were selected to approximate a six person typing pool using to some extent the average office costs given by Panko [18].

The parameters used in this example are:

C_f = \$360/day μ = 10/day

C_v = \$0.25/document processed

a_o = \$20, $a_1 = 0$, $a_2 = 0.7$, $a_3 = 5$.

The value parameters are not based on real data, but are intended to reflect a value curve for a 2-3 page document. The maximum profitability (optimal operating point) of \$11.40 is achieved at $E_p^* = 0.862$ and the maximum efficiency of 2.58 is achieved at $E_y^* = 0.836$. The input load handled at optimal effectiveness is 50.2 documents per day, at a cost per document of approximately \$7.43, while the theoretical capacity is 60 per day. The expected turnaround time corresponding to E_p^* is 0.16 days which is 60% higher than the theoretical minimum of 0.10 days.

Automation in a service operation such as a typing pool is often justified on the basis of reducing costs by replacing labor with capital. The model was used to examine the economic impact of such a change, at a fixed expected load level of 50.2 documents which was found to be optimal in the above case. With the load level fixed, the total fixed cost C_f and the total service coapacity S were varied over a wide range for 1, 2, 4 and 6 servers. The envelope of the region where profitability is greater than or equal to the optimal profitability of \$11.40 for the six server case is plotted in Figure 2. Note that optimality is not achieved throughout the region shown. Rather, both profitability and effectiveness are at least as great as in the test case; the direction of increase of both these parameters is indicated on the drawing. The dashed lines on the lower region of the figure indicate the lower limit of the region where profitability is at least as great as the test case, but effectiveness less. For any one point in the region shown, the effectiveness increases as the number of servers decreases.

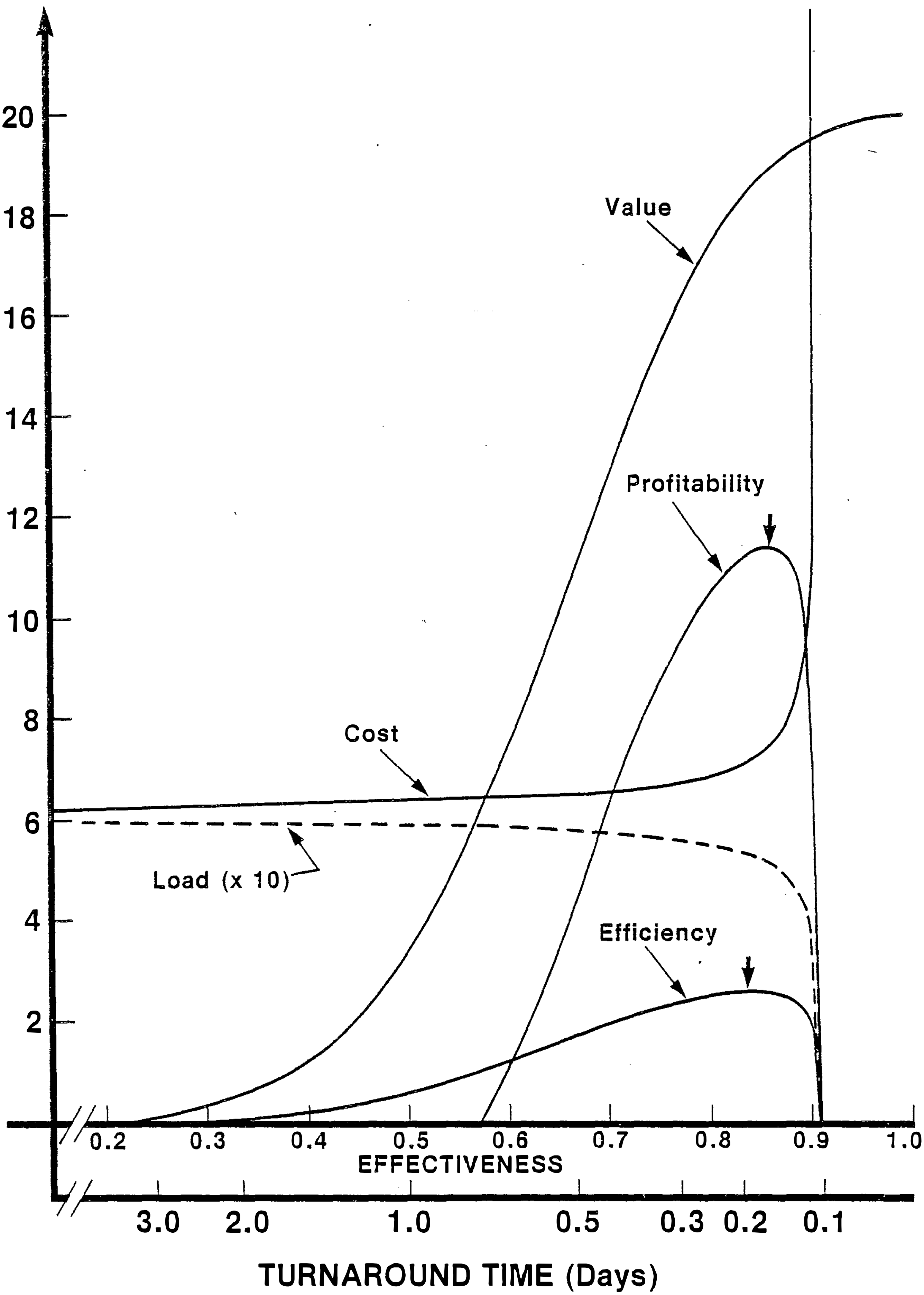

FIGURE 1. Functional dependencies on effectiveness, in a six server queueing system. Value, profitability and cost are in terms of dollars per unit output. Load is in unit job demand per day. The maximum profitability and efficiency are indicated by arrows.

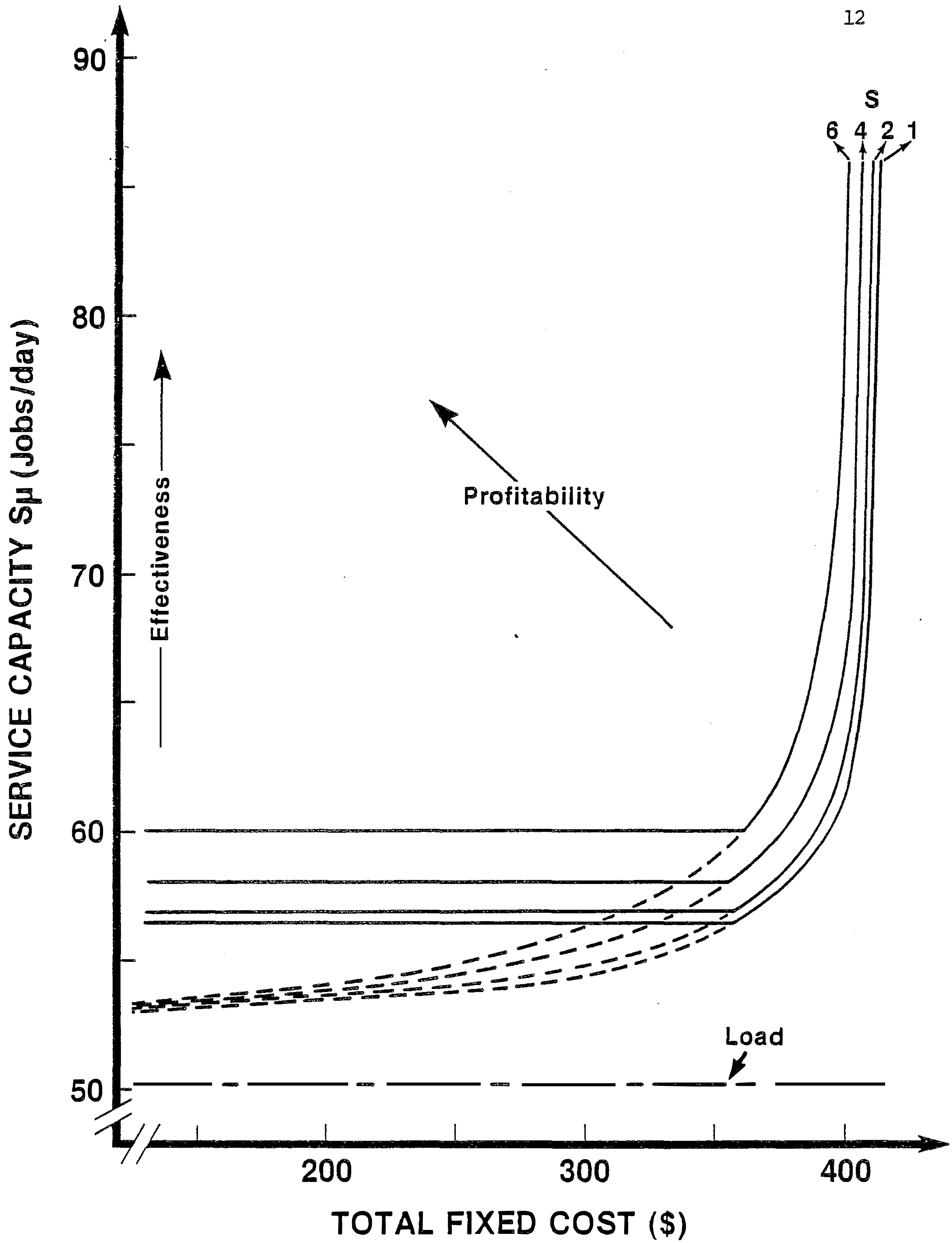

FIGURE 2. The envelope of profitability and effectiveness which meets or exceeds the optimal profitability for the six server system shown in Figure 1, for a number of S server systems, as a function of total fixed cost and system service capacity. Effectiveness and profitability increase in the directions indicated.

Thus, for example with four servers at a total service capacity of 60(μ = 15), effectiveness is higher than it is for six servers (μ = 10). It is possible to achieve this result and at the same time to meet or exceed the six server test case profitability out to a total fixed cost of $375 per day as shown in the Figure. Some caution is advised in the interpretation of these results, because server availability has not been included in this simple model and this could have a significant effect on the relative performance of systems with differing numbers of servers.

4. Conclusions

The following conclusions are based upon the assumed conceptual model, which appears to be fairly representative of reality.

The first general conclusion that can be drawn from this model is that, when the effectiveness of a given organization's operation is relatively low, it is possible to improve efficiency and profitability simultaneously by improving effectiveness. This agrees with a published report on a relatively large information systems operation, where Peeples [19] accomplished improvements in both efficiency (utilization) and effectiveness which very likely led to improvements in profitability. This is possible only when $E < \min (E_y^*, E_p^*)$. For $\min (E_p^*, E_y^*) \leq E < \max (E_p^*, E_y^*)$, one or the other of profitability or efficiency will be reduced while the other one increases. This region is familiar to many information systems managers who must balance customer demands for greater effectiveness and hence (sometimes) profitability with the need to maintain high efficiency, through a negotiation or tradeoff procedure. For $E \geq \max (E_p^*, E_y^*)$, both efficiency and effectiveness are reduced as E increases, and small gains in effectiveness are purchased at a very high cost (e.g., a very small average response time in a time sharing system may require a substantial addition of costly hardware).

A second conclusion is that it is very unlikely that efficiency can be maximized simultaneously with profitability. As discussed in section 2, this can only occur when maximum profitability is zero. If maximum profitability is greater than zero, then optimal effectiveness E_p^* (for maximum profitability) is greater than E_y^*. For maximum profitability less than zero, $E_p^* < E_y^*$. Although the latter case is never considered in standard works in economics on the firm level, (a private firm with negative profitability will not survive) it can obviously happen at the sub-firm or organizational level, where an unprofitable organization within the firm is subsidized by profitable operations elsewhere. This is more likely to occur with overhead functions such as office organizations.

A third conclusion is that exchanging capital for labor costs by automation and thus increasing the speed at which individuals can work, has a complex effect on profitability, effectiveness and efficiency. This effect can be examined by means of the multiple server model proposed here, or by similar models which reflect the relationships among these measures.

Finally, the successful use of the conceptual model proposed here depends upon the ability to estimate cost and value as a function of effectiveness. Of these two measures, value is the most difficult to quantify. In general, value will depend upon more than one attribute of effectiveness. In the simple example discussed in this paper, effectiveness was assumed to have only one attribute, turnaround time. In a more complete study, another attribute would certainly be error rate. In any general study of information value, one would expect to find up to four or five effectiveness attributes for each type of information. This complicates the measurement process for studies of office productivity However, techniques have been developed for the general measurement of utility [11] which also have a great deal of promise in the area of information value measurement.

APPENDIX

A. The Nature of P(E) and Y(E)

P(E), or profitability, is defined as

$$P(E) = V(E) - C(E)$$

Y(E), or efficiency, is defined as

$$Y(E) = V(E)/C(E)$$

In general, C(E) will be assumed to be a continuous positive, increasing, convex function with finite first derivative over the interval (E_{cmin}, E_{cmax}). For V(E), the following cases will be considered:

i) V(E) is a continuous non-negative increasing concave function with finite first derivative over (0,1).

ii) V(E) is a continuous non-negative increasing S-shaped function on (0,1), with finite first derivative. It is convex over $(0,E_f]$ and proper concave over $(E_f,1)$, where E_f is the point of inflection.

iii) The function V(E) behaves according to either of cases (i) or (ii) on $[E_d, 1)$ where E_d is the deadline effectiveness, but V(E) = 0 on $(0,E_d)$.

Since the domain of C(E) is a sub-domain of V(E), then we will consider the nature of P(E) and Y(E) over the domain of C(E) only.

Case (i)

P(E) = V(E) - C(E) is concave since V(E) and -C(E) are concave, and the sum of two concave functions is concave [21]. P(E) will therefore have exactly one (global) maximum E_p^* on (E_{cmin}, E_{cmax}). Consider the slope of the efficiency curve,

$$\frac{dY(E)}{dE} = \frac{V(E)}{C(E)} \left\{\frac{d\ln V(E)}{dE} - \frac{d\ln C(E)}{dE}\right\}.$$

Since V(E) and C(E) are respectively concave and convex functions on (E_{cmin}, E_{cmax}), then so are lnV(E) and lnC(E), respectively. lnV(E) and lnC(E) are both positive increasing functions with finite second derivatives. Since the derivative of a concave function is monotone decreasing and the derivative of a convex function is monotone increasing, then there can be at most one point in $(E_{cmin}\ E_{cmax})$ where $dY(E)/dE = 0$. If this point exists, it must be the global maximum E_Y^* since $dY(E)/dE$ is positive below that point and negative above it. If no such interior point exists, then the global maximum may be at either end of the region. The function Y(E) cannot be shown to be concave in general, although it does possess only one maximum.

Case (ii)

The remarks for case (i) also hold in this case for the region (E_f, E_{cmax}); but P(E) and Y(E) may each have either one or no local maximum within this region. In the lower region $(E_{cmin}, E_f]$, P(E) is the difference between two positive increasing convex functions. P(E) may have a local maximum or minimum at $E = E_{cmin} +$. No general statement may be made about the existence of local maxima or minima in the interior of the lower region.

In the region $(E_{cmin}, E_f]$, Y(E) is the ratio of two increasing convex functions, and no general statement may be made about the presence of a local maximum or minimum within this region. However, it will have either a local maximum or minimum at $E = E_{cmin} +$.

Case (iii)

Remarks made for cases (i) and (ii) also apply in this case over the region above the discontinuity $[E_d, E_{cmax})$. If a maximum does not occur in this region then the global maximum may be either $E=E_d$ or $E_{cmax}-$. For $E < E_d$, $V(E) = 0$ and $P(E) = -C(E)$. Thus P(E) will have a local maximum at $E = 0 +$ but it will not be the global maximum. P(E) will have a local minimum at $E = E_d -$. For (E_{cmin}, E_d), $Y(E) = 0$ identically, and its maximum must occur at $E \geq E_d$.

B. Cost Function Convexity

Suppose that the task input to a service organization with S parallel servers ($S \geq 1$) is a Poisson stream at rate λ per unit time. Each server in the organization works independently, with a service distribution time which is exponential with mean μ, and the service discipline is first-come-first-served. Then the expected total time W_S spent in the service system is well known [9] to be

$$W_1 = 1/\mu(1-\rho) \qquad S = 1 \tag{8}$$

$$W_S = \frac{P_o(S\rho)^S}{S\mu S!(1-\rho)^2} + \frac{1}{\mu} \qquad S > 1 \tag{9}$$

where $\rho = \lambda/S\mu$, and

the probability of no busy servers is

$$P_o = 1/\left\{\sum_{n=0}^{S-1} \frac{(S\rho)^n}{n!} + \frac{(S\rho)^S}{S!(1-\rho)}\right\} \tag{10}$$

Let us suppose that the effectiveness E of the organization is proportional to the reciprocal of the expected service time. That is:

$$E = \frac{1}{W_S+1} \tag{11}$$

Then, since $0 < W_S < \infty$ in general, we have

$$0 < E < 1.$$

However, since the minimum W_S in this specific case is $1/\mu$, this sets the domain of E as $0 < E < \mu/(\mu+1)$. That is, $E_{cmin} = 0$ and $E_{cmax} = \mu/(\mu+1)$. Also suppose that the cost C per task processed is given by

$$C = \frac{C_f}{\mu S\rho} + C_v \tag{12}$$

Here, C_f is the total fixed cost per unit time of operating the service organization and C_v is the variable cost per task processed. Then, since $0 < \rho < 1$, we have

$$\frac{C_f}{\mu S} + C_V < C < \infty$$

The result shown in the following is that C(E) is convex.

The convexity of C(E) can be shown directly for S = 1. Combining equations 8, 11 and 12, we have

$$C(E) = \frac{C_f(1-E)}{S\{\mu - (\mu+1)E\}} + C_V \qquad 0 < E < \mu/(\mu+1) \qquad S = 1$$

The second derivative of this function is positive over the indicated domain and hence C(E) is convex.

In the case S > 1, the function C(E) cannot be derived directly. However, convexity can be shown indirectly. First of all, equation 9 can be combined with equation 11 and re-written as

$$E(\rho) = \frac{1}{W_s+1} = \mu/\{\frac{P_o(S\rho)^S}{S!(1-\rho)^2 S} + 1 + \mu\} \qquad 0 < \rho < 1 \qquad (13)$$

The reciprocal of the first factor in the denominator,

$$\frac{S!(1-\rho)^2 S}{P_o(S\rho)^S}$$

is a sum of products of non-negative decreasing convex functions, which by [21, 22] is also convex and decreasing. The reciprocal of a convex function is concave [23], and since a concave function plus a constant is also concave, the denominator in equation (13) is an increasing concave function. Again from [23] the reciprocal of a concave function is convex, making E(ρ) a decreasing convex function.

Inverting E by the inverse transformation ϕ^{-1} gives the functional dependence of ρ as $\rho = \phi^{-1}(E)$. Note that

$$d\phi^{-1}/dE = \frac{1}{dE/d\phi^{-1}}$$

Since E(ρ) is convex, then $dE/d\phi^{-1}$ is an increasing function of ρ [20].The reciprocal of an increasing function is a decreasing function, thus making $d\phi^{-1}/dE$ a decreasing function and $\rho = \phi^{-1}(E)$ is therefore concave. Since $E(\rho)$ is a decreasing function, so also is $\phi^{-1}(E)$. As before, the domain is $0 < E < \mu/(\mu+1)$ and the range is $1 > \phi^{-1}(E) > 0$.

Substituting the transformed ρ into equation (12) gives

$$C(E) = \frac{C_f}{\mu S \phi^{-1}(E)} + C_v. \qquad \begin{array}{l} 0 < E < \mu/(\mu+1) \\ S \geq 1 \end{array}$$

Again using [23] and [20] and the concavity of $\phi^{-1}(E)$, $C(E)$ is a convex increasing function of E, which was to be shown.

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