# ALLOCATING A REPLENISHMENT ORDER AMONG A FAMILY OF ITEMS

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## ABSTRACT

Inventory control models which provide co-ordinated control (or joint replenishment) of families of items are very useful in practice. An important component of these models is the algorithm for allocating a total replenishment order among the items in the family. In this paper the allocation algorithm from a new class of co-ordinated control models is discussed. Inventory position is modelled as a diffusion process, and both continuous review and periodic review situations are considered. This new class of co-ordinated control models has been shown to outperform existing models (IBM's IMPACT Inventory Control Package).

## 1. INTRODUCTION

Inventory management is concerned with two basic questions:

1. How much to reorder, and
2. When to reorder.

If we define the cycle time for an individual item in the inventory as the time between successive reorders, then we must answer these two questions once each cycle for every item in the inventory. Many of the inventory control models that have been developed for such situations are so-called single-item or independent ordering models (since each item is considered independently of other items). Where an inventory contains many items this type of strategy requires a considerable number of individual orders and also overlooks potential savings associated with co-ordinating individual item replenishments. The savings may be in the form of quantity discounts, reduced freight rates, reduced ordering costs, etc. Co-ordinating item replenishments is also called co-ordinated control or joint replenishment in the inventory literature.

Little has been published on the topic of co-ordinated control (see, for example, pp. 494-529 of Peterson and Silver [15], page 337 of Brown [3], or Miltenburg and Silver [12], [13]). Co-ordinated control models usually include the following components; selection of a total reorder quantity, allocation of the reorder quantity among the items in the family, and calculation of reorder points. See Miltenburg [11], IBM [6], Silver [17] or Low and Waddington [9] for a discussion of specific co-ordinated control models.

In co-ordinated control the entire inventory is split into families of items. When an item reaches its reorder point a reorder is triggered for the specific family. The control model then examines the inventory position of <u>all</u> items in the family. After considering the cost of placing an order, the cost of carrying inventory, item demand rates, lead times and available discounts, the model selects a total reorder quantity (see Miltenburg and Silver [14]). This total reorder quantity must be allocated among the items in the inventory family.

Finally, inventory control models can be either continuous review or periodic review. In continuous review the inventory positions for all items are continuously monitored and as soon as one item reaches its reorder point a reorder is triggered. In periodic review, inventory position is checked periodically - once a week, once every ten days, etc. If, when the inventory is checked, an item has fallen to or below its reorder point then a reorder is triggered.

This paper will discuss the important topic of allocating the total reorder quantity among the items in an inventory family when inventory position is modelled as a diffusion process. Section 2 introduces the diffusion process and shows its use in inventory models, while section 3 discusses the allocation of stock among items in a family for both continuous and periodic review. This section also highlights computational considerations for allocation. Section 4 summarizes this research and outlines related results.

## 2. THE DIFFUSION PROCESS

In this paper we model the demand for each item as a diffusion process. The diffusion process has the important property that total demand over any finite period (for example, a replenishment lead time) has a normal distribution, a distribution commonly observed in practice. In addition, the diffusion process provides an analytically tractable framework for evaluating the expected time until a certain amount of stock is depleted, a key quantity needed in allocating a replenishment order among several items in a family. Akinniyi and Silver [1] used a diffusion model for a specific, single item, inventory control problem.

The diffusion process is a Markov process with continuous sample paths in continuous time. The Wiener Process models the diffusion process by taking the continuous limit of the random walk. The Wiener Process is described by either of two complex differential equations called the forward and backward diffusion equations (see, for example, pp. 203-225 of Cox and Miller [4]). Introducing an initial condition, namely that the process begins at the origin, and solving either differential equation gives

$$p(x,t) = \frac{1}{\sqrt{2\pi t}\,\sigma} \exp\left(-\frac{(x-\mu t)^2}{2\sigma^2 t}\right) \qquad \ldots(1)$$

the equation of the normal distribution probability density function, where;

$p(x,t)$ = Probability that the process is at location x at time t given that it starts from the origin,

$\mu$ = Mean drift parameter,

$\sigma$ = Standard deviation of the drift.

Then

$$p(x < b,t) = \int_{-\infty}^{b} \frac{1}{\sqrt{2\pi t}\,\sigma} \exp\left(-\frac{(x-\mu t)^2}{2\sigma^2 t}\right)dx$$

$$= \Phi\left(\frac{b-\mu t}{\sigma\sqrt{t}}\right) \qquad \ldots(2)$$

where

$\Phi(\ )$ = Left tail area of the unit normal distribution.

In addition to the initial condition a boundary condition can be incorporated into the analysis. Specifying that the process begins at the origin and once the process reaches a point "a" it is absorbed (terminates) gives

$$p(x,t) = \frac{1}{\sqrt{2\pi t}\,\sigma} \left\{\exp\left[-\frac{(x-\mu t)^2}{2\sigma^2 t}\right] - \exp\left[\frac{2\mu a}{\sigma^2} - \frac{(x-2a-\mu t)^2}{2\sigma^2 t}\right]\right\}$$

$$p(x < b,t) = \int_{-\infty}^{b} p(x,t)\,dx \qquad \ldots(3)$$

$$= \Phi\left(\frac{b - \mu t}{\sigma\sqrt{t}}\right) - \exp\left(\frac{2\mu a}{\sigma^2}\right) \Phi\left(\frac{b - 2a - \mu t}{\sigma\sqrt{t}}\right) \qquad \ldots(4)$$

Another useful expression is the probability density function of the time (t) to absorption at the barrier "a". This is given by

$$g(t) = \frac{a}{\sqrt{2\pi t^3}\,\sigma} \exp\left[-\frac{(a - \mu t)^2}{2\sigma^2 t}\right] \qquad \ldots(5)$$

These Wiener Process results can be used in an inventory setting. Assume at time t = 0 a quantity of cycle stock "a", is available to meet demands. (If a particular item has an inventory position of "b" units of stock of which "s" units of stock is the reorder point then we define a= b-s as the cycle stock, stock designed to meet demand before the reorder is triggered.) For a continuous review situation the reorder point can be modelled as an absorbing barrier at "a". Once this cycle stock has been sold a reorder must be placed. That is, a continuous review situation can be modelled as a diffusion process beginning from the origin with an absorbing barrier located at "a". This is shown in Figure 1.

Similarly a periodic review situation can be modelled as a diffusion process beginning from the origin but without the boundary condition at "a", since under periodic review the total stock sold, x, can go beyond the level "a", before a reorder is triggered as Figure 2 illustrates. A reorder is triggered only if x exceeds "a" at a review time.

## 3. STOCK ALLOCATION

### 3.1 Introduction

In allocating stock one attempts to solve the following problem. A family consists of n items. One of these items has reached its reorder point and a joint replenishment for a specified dollar ($) amount has been placed. How, taking into account the present stock levels, should this reorder

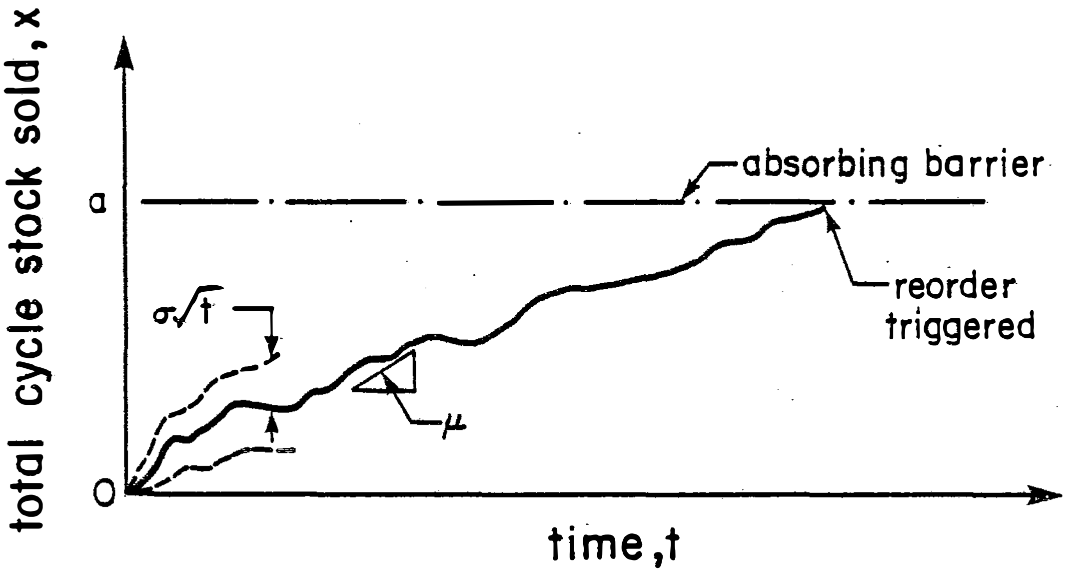



Figure 1 Continuous Review

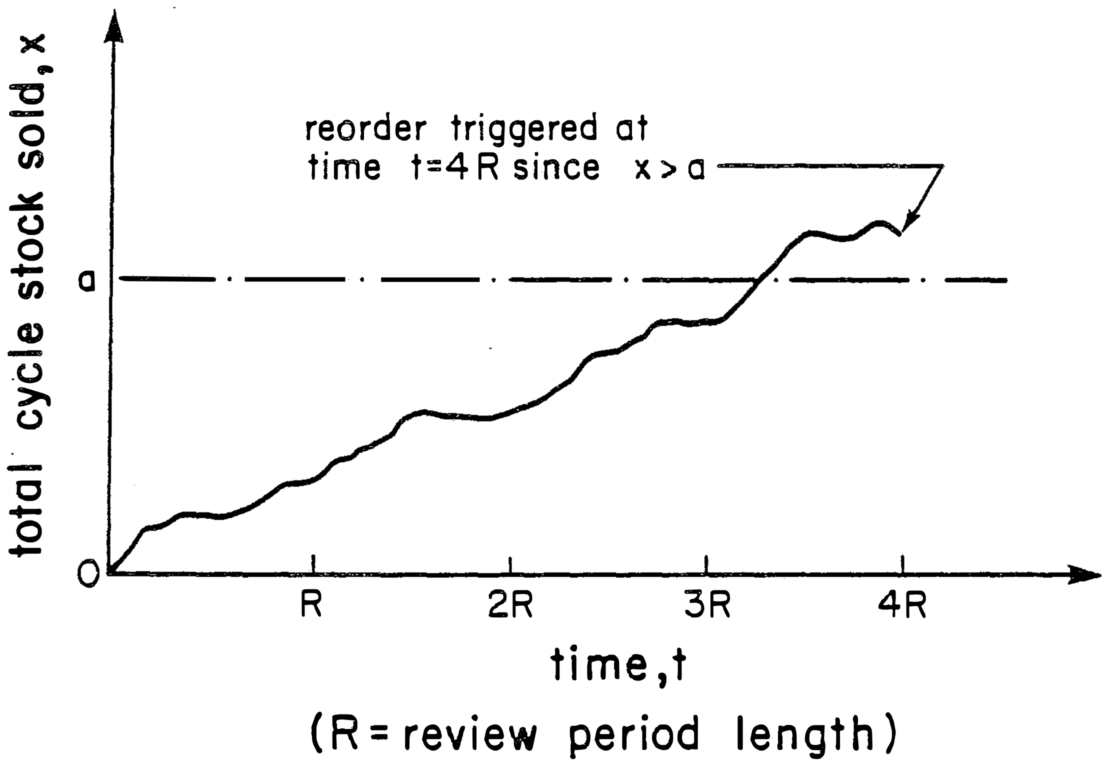



Figure 2 Periodic Review

quantity be divided among the n items? The strategy for allocating a replenishment among the items in a family must attempt to optimize some objective function. Low and Waddington [9] accurately describe the objectives as follows:

> "The ideal criterion of optimality would be to minimize the long-term average stocks and hence stockholding costs of a discount group (subject to service constraints), where the total replenishment quantity at each cycle...is dictated by discount offers (and inventory positions). However, the difficulties of sales forecasting for other than short periods would make such an approach impracticable and, therefore, only a single reorder cycle is usually considered. Three possible criteria suggest themselves;
>
> 1. Minimize the expected total stock remaining when a reorder is triggered.
> 2. Maximize the expected number of sales before the reorder is triggered.
> 3. Maximize the expected elapsed time before the reorder is triggered."

However it has been shown that these criteria are all equivalent to each other (see, for example, Mendelson et al. [10], or Karmarker [7]). That is, maximizing the expected elapsed time before the reorder is triggered is equivalent to maximizing the expected number of sales before a reorder is triggered, or minimizing the expected total stock remaining when the triggering occurs.

In this paper, using the diffusion process as a model for inventory position, an objective function (the expected elapsed time before the next reorder time) is developed for continuous review (section 3.2) and for periodic review (section 3.3). The allocation objective chosen is to maximize this expected elapsed time. Section 3.4 discusses computational considerations.

3.2 Allocation for Continuous Review

The probability that item i has not used up its cycle stock before time t is given from equation 4 as

$$p(x_i < a_i,t) = \Phi\left(\frac{a_i - \mu_i t}{\sigma_i\sqrt{t}}\right) - \exp\left(\frac{2\mu_i a_i}{\sigma_i^2}\right)\Phi\left(\frac{-a_i - \mu_i t}{\sigma_i\sqrt{t}}\right)$$

where

$\Phi(\ )$ = The left tail area of the unit normal distribution,

$a_i$ = The cycle stock for item i,

$\mu_i$ = Expected demand for item i per unit time,

$\sigma_i$ = Standard deviation of demand for item i per unit time.

The joint probability that no reorder has been triggered before time t for the entire family, assuming that individual item demands are independent of each other, is

$$P(t) = \prod_{i=1}^{n} P(x_i < a_i,\ t)$$

$$= \prod_{i=1}^{n} \left\{\Phi\left(\frac{a_i - \mu_i t}{\sigma_i\sqrt{t}}\right) - \exp\left(\frac{2\mu_i a_i}{\sigma_i^2}\right)\Phi\left(\frac{-a_i - \mu_i t}{\sigma_i\sqrt{t}}\right)\right\} \qquad \ldots(6)$$

The probability that a reorder is triggered before time t is given by F(t) the cumulative density function of t, where

$$F(t) = 1-P(t)$$

The expected value of t, the time to the reorder is

$$E(t) = \int_0^\infty [1 - F(t)]dt$$

$$= \int_0^\infty \prod_{i=1}^{n} \left\{\Phi\left(\frac{a_i - \mu_i t}{\sigma_i\sqrt{t}}\right) - \exp\left(\frac{2\mu_i a_i}{\sigma_i^2}\right)\Phi\left(\frac{-a_i - \mu_i t}{\sigma_i\sqrt{t}}\right)\right\} dt \qquad \ldots(7)$$

Equation 7 is the objective function for the allocation algorithm under continuous review. An allocation vector $a^*=(a^*_1,a^*_2,\dots,a^*_n)$ must be selected to maximize E(t) subject to

$$\sum_{i=1}^{n} a_i = a_{TOTAL}$$

where $a_{TOTAL}$ is the total stock available (from the family replenishment) to allocate to cycle stocks.

### 3.3 Allocation for Periodic Review

Let $E_{i,mR}$ be the event that an item i does not trigger a reorder at review time mR. The probability that item i has not triggered a reorder before review time t=mR is then

$$P(E_{i,mR}, E_{i,(m-1)R}, E_{i,(m-2)R},\dots,E_{i,R})$$

The joint probability that no reorder has been triggered before time t=mR for the entire family assuming that individual item demands are independent of each other, is

$$P(t) = \prod_{i=1}^{n} P(E_{i,mR}, E_{i,(m-1)R}, E_{i,(m-2)R}, \dots, E_{i,R}) \qquad \dots(8)$$

The probability that the first trigger occurs before time t=mR is given by F(t) the cumulative density function of t, where F(t)=1-P(t). The expected value of t, the time to the first trigger is

$$E(t) = \sum_{m=0}^{\infty} mR \cdot \text{prob}(t=mR)$$

$$= R \sum_{m=0}^{\infty} P(t)$$

$$= R[1 + \sum_{m=1}^{\infty} \{\prod_{i=1}^{n} P(E_{i,mR}, E_{i,(m-1)R}, E_{i,(m-2)R}, \dots, E_{i,R})\}] \qquad \dots(9)$$

Equation 9 is the objective function for the allocation algorithm under periodic review. An allocation vector $a^*=(a_1^*,a_2^*,\dots,a_n^*)$ must be selected to maximize $E(t)$, subject to

$$\sum_{i=1}^{n} a_i = a_{TOTAL}$$

where $a_{TOTAL}$ is the total stock available (from the family replenishment) to allocate to cycle stocks.

However, as is shown in Miltenburg and Silver [13], the expression for the joint probability in equation 9 is very complex and cannot be simplified to facilitate routine calculations. In that research (on a related topic - the residual stock probability density function under periodic review) a simplifying assumption was made. The same assumption is made here, namely that

$$P(E_{j,t} | E_{j,(t+\ell)R}) = 1.0 \qquad \ell = 1,2,3,\ldots \qquad \ldots(10)$$

This implies that if an item j does not trigger a reorder at a review time $(t+\ell)R$ then, given this information, it is assumed that no reorder was triggered by this item at a prior review time. As Figure 3 shows this is equivalent to assuming that certain "paths" do not exist. This assumption increases in accuracy when the review period, R, is long.

Using this assumption, consider the following joint probabilities.

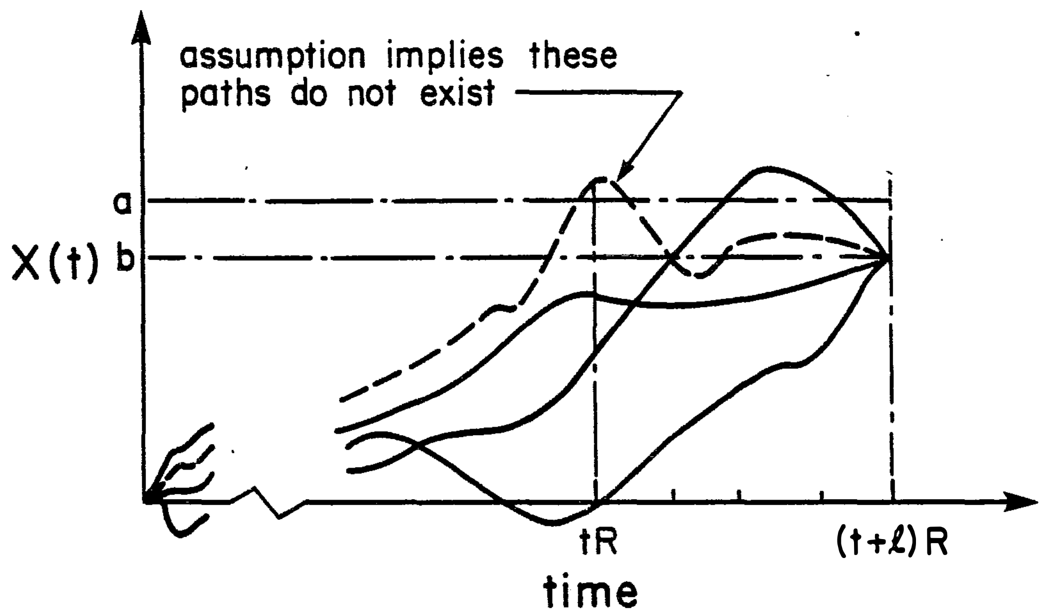
$$\begin{aligned} P(E_{j,R}, E_{j,2R}) &= P(E_{j,R} | E_{j,2R})\, P(E_{j,2R}) \\ &= 1.0 \times P(E_{j,2R}) \\ &= P(E_{j,2R}) \end{aligned}$$

$$\begin{aligned} P(E_{j,R}, E_{j,2R}, E_{j,3R}) &= P(E_{j,R} | E_{j,2R}, E_{j,3R})\, P(E_{j,2R}, E_{j,3R}) \\ &= 1.0 \times P(E_{j,2R}, E_{j,3R}) \\ &= P(E_{j,2R} | E_{j,3R})\, P(E_{j,3R}) \\ &= P(E_{j,3R}) \end{aligned}$$

In general

$$P(E_{j,R}, E_{j,2R}, E_{j,3R}, \ldots, E_{j,tR}) = P(E_{j,tR})$$

Equation 9 under this assumption can then be written as




—— possible paths to $X((t+\ell)R) < a$ where $X(tR) < a$
---- possible paths to $X((t+\ell)R) < a$ where $X(tR) \geq a$

Figure 3 Possible Paths to $X((t+\ell)R)<a$

$$E(t) = R[1 + \sum_{m=1}^{\infty} \{ \prod_{j=1}^{n} P(E_{j,mR}) \}]$$

$$= R[1 + \sum_{m=1}^{\infty} \{ \prod_{j=1}^{n} \Phi\left( \frac{a_j - \mu_j mR}{\sigma_j \sqrt{mR}} \right) \}] \qquad \ldots (11)$$

3.4 Computational Considerations

An allocation vector $a^* = (a_1^*, a_2^*, \ldots, a_n^*)$ must be selected to maximize E(t) as given by equation 7 for continuous review or equation 11 for periodic review.

A number of methods can be used to obtain an estimate of $a^*$ (calculus, search algorithms, etc.). Calculus would require solving n simultaneous equations, given by;

$$\frac{\partial E(t)}{\partial a_1} = 0, \frac{\partial E(t)}{\partial a_2} = 0, \ldots, \frac{\partial E(t)}{\partial a_n} = 0$$

Unless the expressions for E(t) can be simplified this approach is impractical even for small n due to the difficulty of differentiating and simplifying the product of a number of cumulative Normal distributions. Efforts to simplify either equation 7 or 11 were unsuccessful.

However E(t), as defined by equation 7 or equation 11 is a concave function of $a = (a_1, a_2, \ldots, a_n)$. That is, if we increase $a_j$ while holding $a_k$ ($k = 1,2,\ldots,n \quad k \neq j$) constant and plot this against E(t) we will obtain curves of the form shown in Figure 4. Recall that we wish to maximize this concave function subject to the capacity constraint; $\sum_i a_i = a_{TOTAL}$.

Maximizing a concave objective function (or equivalently minimizing a convex function) subject to a resource constraint is a common problem in operations research. It is known as the Distribution of Effort Problem in the literature.

A very popular solution method for the Distribution of Effort Problem is the "Incremental Solution Technique." (See, for example, Galil and Megiddo

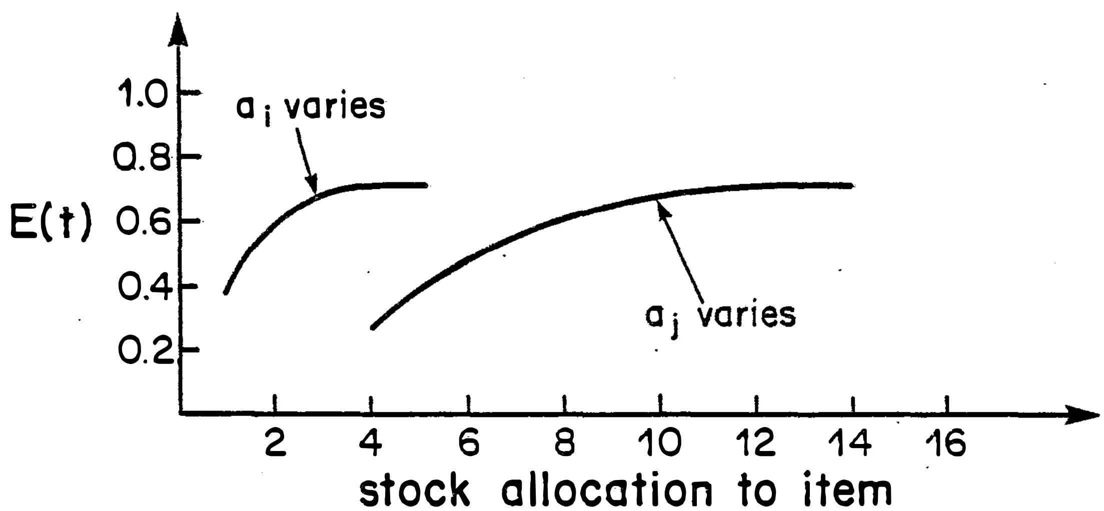



Figure 4 Relationship Between E(t) and Stock Allocation (Under Continuous Review, equation 7)

[5].) Consider the allocation of stock among the items in the family as a multi-stage decision process in which the total stock to be allocated is increased by one dollar at each stage. The extra dollar is added to the stock of one of the items in such a way that at each stage the best decision as to where the extra dollar should be added is made. For example, suppose the optimum allocation of $40 between the four items of a particular family is $4, $8, $11 and $17. The optimum allocation of $41 may be obtained by answering the question; If one further dollar must be ordered, to which item should it be allocated? In this case the optimum allocation of $41 must be one of the four alternatives (5,8,11,17), (4,9,11,17), (4,8,12,17) or (4,8,11,18) and may be determined by evaluating E(t) (equation 7 or 11) for the four cases (see also, Low and Waddington [9]).

Notice that in our allocation algorithm we are allocating dollars ($) of stock rather than units. Because we are using the diffusion process to model a situation where there are a large number of small demands occurring frequently, the error introduced when adjusting the final allocation to integer numbers of units should be small.

Rather than start our solution technique from a = (0,0,...,0) and incrementally allocate the entire order quantity until we get our final optimum allocation $a^* = (a_1^*, a_2^*, \ldots, a_n^*)$, we make use of the following solution characteristic. It has been shown (Mendelson et al. [9]) that, as the total order quantity to be allocated becomes large, then the individual item allocations become proportional to the individual item mean demand rates. If the total order quantity to be allocated is smaller, then items with low demand rates receive relatively higher allocations while high demand rate items receive relatively lower allocations. The reason for this is that in trying to maximize E(t), equations 7 and 11 take advantage of the feature that allocating an additional dollar of stock to a low demand rate item can greatly increase the expected time to stockout for that item while deleting a dollar of stock from a high demand rate item results in only a small decrease in the expected time to stockout for that item.

This feature is illustrated by columns (2), (4) and (5) of Table 1. Columns 2 and 3 show the mean demand ($/year) and the standard deviation of the demand for each of the 5 items in a family. If stock allocations are made on the basis of mean demand rates the allocation shown in column 4 results. If we use equation 7 and allocate to maximize E(t), then the allocation shown in column 5 results. This allocation gives E(t) = 3.48 years while allocating on the basis of mean demand rates results in E(t) = 3.38 years. Had $a_{TOT}$ the total quantity to be allocated been very large (say $a_{TOT}$ = $400), then allocating to maximize E(t) and allocating on the basis of mean demand rates would result in the same allocation. Column 6 illustrates a useful feature. If we allocate 80% of $a_{TOT}$ on the basis of mean demand rates and allocate the remaining 20% on the basis of maximizing E(t), then this results in the same allocation as allocating 100% of $a_{TOT}$ on the basis of maximizing E(t). A threshold percentage (here 80%) of the total order quantity is allocated to each item on the basis of mean demand rates, with the remainder being allocated to take advantage of family characteristics to maximize E(t). (The only time that problems can arise is if we try to allocate m units of stock among n items where $m \backsimeq n$. However this situation rarely occurs in inventory problems.) Note that this allocation procedure (column 6) took only 1.03 CPU seconds of computer time (IBM 4350 computer with FORTRAN G compiler).

Many problems were run using the above approach. That is, the first x% of $a_{TOT}$ is allocated on the basis of mean demand rates and the remaining (100-x)% is allocated on the basis of maximizing E(t) using equation 7 (for continuous review) or equation 11 (for periodic review) and the incremental

TABLE 1

Allocation of $a_{TOT}$ = $195

| i (1) | $\mu_i$ (2) | $\sigma_i$ (3) | Allocations 1 (4) | Allocations 2 (5) | Allocations 3 (6) |
|---|---|---|---|---|---|
| 1 | 10.49 | 7.34 | 52 | 46 | 42 + 4 = 46 |
| 2 | 9.80 | 6.86 | 49 | 49 | 39 + 10 = 49 |
| 3 | 8.18 | 5.73 | 41 | 42 | 33 + 9 = 42 |
| 4 | 7.51 | 5.26 | 37 | 39 | 30 + 9 = 39 |
| 5 | 3.14 | 2.2 | 16 | 19 | 12 + 7 = 19 |
| | | | 195 | 195 | 156 + 39 = 195 |

solution technique. It was found that using x = 80 gave the optimum allocation for 5 item and 10 item families with very little computational effort.

## 4. SUMMARY AND EXTENSIONS

Using the diffusion process as a model for inventory position, an algorithm for allocating a total replenishment order among the items in a family was developed for a co-ordinated control inventory problem. Both continuous and periodic review situations were discussed. Computational considerations were also presented.

The allocation concepts discussed in this paper, along with other co-ordinated control model components (see Miltenburg and Silver [12], [13], and Miltenburg [11]) have been coded in FORTRAN to form an inventory control package. Extensive comparison tests with IBM's IMPACT inventory system show that this new control package outperforms IMPACT both in terms of average costs per unit time and in terms of providing the service level specified by the user (see Miltenburg [11]). IMPACT, like many commercial packages, allocates a total replenishment order, among the items in a family, based on mean demand rates. Hence each item is allocated the same time supply of inventory. The results in this paper represent an improvement over this simplistic allocation scheme and are responsible for a part of the superior performance of the new inventory control system.

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