FACULTY OF BUSINESS

RESEARCH AND WORKING PAPER SERIES

APPROXIMATING THE STATISTICAL PROPERTIES OF ELASTICITIES DERIVED FROM TRANSLOG AND GENERALIZED LEONTIEF COST FUNCTIONS

By

Itzhak Krinsky*

A. Leslie Robb**

Working Paper No. 247

February, 1986

ER UNIVERSITY
eet West
rio, Canada L8S 4M4
6) 525-9140

APPROXIMATING THE STATISTICAL PROPERTIES OF ELASTICITIES DERIVED FROM TRANSLOG AND GENERALIZED LEONTIEF COST FUNCTIONS

By

Itzhak Krinsky*

A. Leslie Robb**

Working Paper No. 247

February, 1986

* Assistant Professor of Finance and Business Economics, Faculty of Business, McMaster University.

**Associate Professor of Economics, McMaster University.

Approximating the Statistical Properties of Elasticities Derived from Translog and Generalized Leontief Cost Functions+

Itzhak Krinsky and A. Leslie Robb

Empirical studies of consumer demand or of factor demand have now moved far beyond the Cobb-Douglas functional form and elasticities of interest are no longer estimated as parameters of the system. Instead, such elasticities are typically non-linear functions of the parameters that have been estimated and it is natural to want to be able to say something about the statistical properties of such elasticities. One way of dealing with this is to linearly approximate the elasticity formulas (in terms of the estimated parameters) and use classical statistical procedures to get approximations to the underlying variances. If $y=f(x)$ and x has a variance covariance matrix V, the linear approximation is given by: $\mathrm{Var}(y) = (\partial f/\partial x)V(\partial f/\partial x)$. The data needed for such an approximation are estimates of the parameters and of the associated variance-covariance matrix. Some of the earliest references that we have found to uses of this approximation technique in the elasticity context are to Griffin and Gregory (1976), Griffin (1977), and Fuss (1977), while the earliest references to the general method is to Klein (1953). Such an approximation procedure has also been recommended for the translog production function and cost function in recent papers by Alden L. Toevs (1980 and 1981).[1]

The appropriateness of such a procedure depends, of course, on the nature of the non-linearity of the underlying elasticity formulae. Little or no attention has been devoted to this issue heretofore. Here, we propose a simulation exercise to establish the empirical distributions of a set of factor demand elasticities and compare our simulation results with the linear approximation mentioned above. We start from the cost function and use either a translog (TL) or a generalized Leontief (GL) representation, the two most common functional forms in use these days.[2]

Given initial estimates of a parameter vector T, a corresponding variance-covariance matrix V, and a set of elasticity formulas

1) $E_i = f_i(T)$,

where E_i refers to the ith elasticity, and f_i indicates a non-linear function, we take random drawings for T from a multi-variate normal distribution with variance-covariance matrix V and mean $\bar{T}$.[3] For each drawing, we calculate the elasticities and thus generate an empirical distribution for each elasticity. By varying the number of drawings, one can generate an empirical distribution to any desired degree of accuracy.[4]

We study the two examples mentioned above, which we believe to be representative, to illustrate the procedure. We compare the means and variances of the empirical distributions so generated with the linear approximations mentioned earlier. We find that serious errors can be made in statements about the precision of elasticities based on the approximations for both functional forms employed.

The particular example we use is a three factor production process characterized by a cost function (homogeneous of degree one in factor prices) of the form $K=g(Q,P_1,P_2,P_3)$, where K refers to total cost, P_i to the price of the ith factor, and Q to output. The two share equations (the third is implied by the adding up restrictions) that are estimated simultaneously with the cost function, depend on the explicit functional form for g. For the TL cost function, the two share equations can be written as;

2) $S_1 = A_1 + B_{11}*\ell n\ (P_1/P_3) + B_{12}*\ell n\ (P_2/P_3) + C_1*\ell n\ Q$,

and,

3) $S_2 = A_2 + B_{21}*\ell n\ (P_1/P_3) + B_{22}*\ell n\ (P_2/P_3) + C_2*\ell n\ Q.$

where S_i refers to the ith share, and the A's, B's and C's are parameters. The GL cost function is:

4) $K = (\sum_{i}\sum_{j} B_{ij}*(P_iP_j)^{1/2})*Q*\ \mathrm{Exp}(\delta + \frac{\theta}{2}\ \ell n\ Q + \phi_1\ \ell n(P_1/P_3) + \phi_2\ \ell n(P_2/P_3))$

and the two share equations for the GL cost function are:

5) $S_1 = [B_{11}*P_1 + B_{12}*(P_2P_1)^{1/2} + B_{13}*(P_3P_1)^{1/2}]/ \sum_{i}\sum_{j} B_{ij}*\ (P_iP_j)^{1/2} + \phi_1*\ell n\ Q$

6) $S_2 = [B_{22}*P_2 + B_{21}*(P_2P_1)^{1/2} + B_{23}*(P_2P_3)^{1/2}]/ \sum_{i}\sum_{j} B_{ij}*(P_iP_j)^{1/2} + \phi_2*\ell n\ Q$

where, δ, θ, and the ϕ_i's are parameters.

In addition, the cost functions must satisfy the property $B_{ij} = B_{ji}$.

The own and cross price elasticities of factor demand for these models are given by;

7) $E_{kk} = (P_k/S_k)*(\partial S_k/\partial P_k) + (S_k - 1.0)$ and

8) $E_{kj} = (P_j/S_k) * (\partial S_k/\partial P_j) + S_j$

where E_{kj} refers to the elasticity of the kth factor with respect to the jth price.[5] These elasticity formulas are written in terms of the factor shares though these are endogenous and would need to be replaced by the appropriate share equations (e.g. 2 or 3 above for the TL case and 5 or 6 for the GL case) before calculation of the elasticities. The elasticities so calculated vary with the levels of the exogenous variables and one is required to pick a point of evaluation. Typically, the means of the data or some recent values (in the case of a time series) are chosen. For this paper, we work with the means.

For the TL cost function, the estimates of the 7 parameters of interest (equations 2 and 3 contain 8 parameters, but $B_{12} = B_{21}$) and the associated variance-covariance matrix for our particular example are given in Table 1.[6] We take these simply as a convenient example and make no particular claim for the estimates though we think them to be reasonably representative of such empirical models.

Table 2 reports a variety of statistics of interest related to 6 of the elasticities derived according to equations 7 and 8 above. We report on the three own-price elasticities and three of the cross-price elasticities (since E_{ij} is not equal to E_{ji}, there are, in fact, 6 cross-price elasticities, though 3 suffice for this example) calculated at the means of the data. The first column reports the elasticity calculated by inserting the parameter estimates into the elasticity formulas. The second column reports the average elasticity calculated over 1000 drawings of the parameter vector according to the variance-covariance matrix given in Table 1. Columns 3 through 7 report the 2.5, 5, 50, 95, and 97.5 percentile points according to the empirical distribution. Thus, for example, columns 3 and 7 can be interpreted as the 95 percent confidence limits for the elasticity, while column 5 can be interpreted as the median. Column 8 reports the standard deviation calculated from the empirical distribution while columns 9 and 10 report the mean and standard deviation calculated using the linear approximation mentioned earlier.[7]

For the GL cost function, the parameter information is recorded in Table 4 while the simulation results are recorded in Table 5.

The first thing one notices in Tables 2 and 5 is the dramatic differences between the standard deviations estimated by the linear approximation and those estimated from the simulations. Compared to the standard deviations derived from the simulations, the ones from the linear approximations frequently appear to be dramatically understated.[8] According to the simulation results, the standard deviations are frequently ten times too small and one is as much as a thousand times too small. The estimates of the means of the elasticities are much better, by comparison.[9]

Of course, the question arises whether the simulation results serve as a reasonable basis of comparison. If one accepts the estimates of the parameters of the model and the estimates of their variance-covariance matrix, then an infinite number of drawings should lead to the precise distribution of an elasticity measure. Additional experimentation reported elsewhere leads us to believe that our estimates from a thousand drawings should be sufficient to get a reasonably good estimate.[10]

In Tables 3 and 6 we report the results of an experiment designed to shed light on the issue of whether the standard deviations reported in Tables 2 and 5 are useful to the researcher interested in statistical testing. If the simulated distributions are approximately normal, then the standard deviations will be very useful in approximating confidence limits and the like. On the other hand, if the distributions are quite non-normal, then the standard deviations may not be very helpful in this regard. The columns in Table 3 and 6 report the difference between the upper and lower 2.5% tails (and 5% tails) and the mean, divided by the standard deviation (itself calculated from the simulation). If the distributions are normal these numbers should be $\pm$ 1.96 ($\pm$ 1.65). As is easily seen, in the case of the translog the numbers are reasonably close to this standard, though in the case of the GL cost function, there is less accord. Whether such distributions will be approximately normal more generally will, no doubt, depend on the particular example and the non-linearities involved. Anyone using this simuilation technique to find distributions will find it as easy to generate the confidence limits as to calculate the standard deviation and we would recommend it.

In summary, however, the moral of the story is simple. Be wary of using linear approximations to get estimates of the dispersion of measures (e.g. elasticities) that are non-linear functions of random variables.

References

1. Appelbaum Elie, (1979), "On the Choice of Functional Forms", International Economic Review, 20, pp. 449-458.

2. Berndt, E.K. and M.S. Khaled, (1979), "Parametric Productivity Measurement and Choice Among Flexible Functional Forms", Journal of Political Economy, 87, pp. 1220-1245.

3. Freedman, D.A. and S.C. Peters, (1984), "Bootstrapping a Regression Equation: Some Empirical Results", Journal of the American Statistical Association, 79, pp. 97-106.

4. Fuss, Melvyn A., (1977), "The Demand for Energy in Canadian Manufacturing", Journal of Econometrics, 5, pp. 89-116.

5. Griffin, James M., (1977), "Long-Run Production Modelling with Pseudo Data: Electric Power Generation", Bell Journal of Economics, 8, pp. 112-127.

6. Griffin, James M. and Paul R. Gregory, (1976), "An Intercounty Translog Model of Energy Substitution Responses", American Economic Review, 66, pp. 845-857.

7. Klein, Lawrence R., (1953) A Textbook of Econometrics, Ross, Peterson and Co.

8. Krinsky Itzhak and A. Leslie Robb, (1985), "On Approximating the Statistical Properties of Elasticities", QSEP Research Report No. 146, McMaster University.

9. Toevs, Alden L., (1980), "Approximate Variance Formulas for the Elasticities of Substitution Obtained from Translog Production Functions", Economics Letters, 5, pp. 155-160.

10. ________________, (1982), "Approximate Variance Formula for the Elasticities of Substitution Obtained from Translog Cost Functions, Economic Letters, 10, pp. 107-113.

FOOTNOTES

+We would like to thank Mr. F. Mahmud and Maria Berruti for research assistance in this project, and Mike Veall, Gordon Anderson and anonymous referees for comments on an earlier draft.

1. Toevs calculates the explicit derivatives necessary for the linear approximation. The TSP ANALYZ command will, however, do the calculations for you. It presumably approximates these derivatives by a numerical procedure.

2. See, for example, Applebaum (1979) or Berndt and Khaled (1979).

3. The approximation methods, of course, also assume the distribution of T to be multi-variate normal. In this exercise, we use the IMSL routine GGNSM to make the drawings.

4. The simulation nature of this exercise shares the philosophy of the Bootstrap (see, for example, Freedman and Peters (1984)) but is, in fact, somewhat different. To appreciate this, note that the Bootstrap could be used to generate the distribution of the parameters (T) initially. The parameter distributions so generated could then be used in place of our multivariate normal to calculate elasticities. The estimation of the parameters and their distributions is prior to the issues we raise here, though these two can obviously be combined.

5. One could also calculate elasticities of substitution if they were the object of interest.

6. The example is taken from estimates made by our research assistant in some of his thesis work. The factor shares are .002, .220 and .777, respectively.

7. In fact, we have used the ANALYZ command in TSP to effect this. Note that the mean from the linear approximation differs from Column 1. In Column 1, the parameter estimates are inserted into the elasticity formula while in Column 9, the linear approximation is applied first.

8. Note, though, in the TL case the linear approximation of the standard deviation of the last elasticity differs from the simulated one by only about 15% while in the GL case, the penultimate elasticity is within 2%. Although in all the cases reported here, the standard deviations from the 1000 draws are larger than those calculated by the linear approximation, there is nothing to require this. In fact, in some additional work we have found examples where the results go the other way.

9. The careful reader may notice and wonder at the differences in the elasticities estimated from the two different functional forms. The sensitivity of such elasticities to functional form is not unknown. See, for example, the tables in Berndt and Khaled (1979). The differences here may be greater than one commonly encounters, but, we have not made serious efforts to properly handle autocorrelation and to find the best specifications of each form. The results that we are interested in here do not depend on such considerations, in any event.

10. We have experimented with many drawings of 1000 and found the results to be very close to those reported here. In fact, many fewer than 1000 drawings give much the same estimate as the larger numbers. See Krinsky and Robb (1985) for details.

TABLE 1: PARAMETER INFORMATION - TRANSLOG COST FUNCTION

PARAMETER	MEAN	VARIANCE-COVARIANCE MATRIX						
		A1	A2	B11	B22	B12	C1	C2
A1	0.943562	9.359E-03						
A2	0.016581	2.594E-05	9.926E-06					
B11	0.156399	-1.87E-03	-5.93E-06	4.726E-04				
B22	0.003571	1.715E-06	1.278E-06	-6.44E-08	3.055E-07			
B12	-0.005287	-3.08E-06	-5.66E-08	1.386E-06	1.678E-07	2.875E-07		
C1	-0.112353	-5.96E-04	-1.23E-06	6.799E-05	-2.12E-07	-1.46E-07	6.513E-05	
C2	0.002008	-2.33E-06	-1.32E-06	2.561E-07	-2.28E-07	-9.91E-08	2.306E-07	2.177E-07

TABLE 2: ELASTICITIES FROM THE TRANSLOG COST FUNCTION -- BASED ON 1000 DRAWINGS

ELASTICITY	CALCULATED FROM PARAMETER MEAN	MEAN ELASTICITY FROM 1000 DRAWS	LOWER TAIL 2.5%	LOWER TAIL 5%	MEDIAN	UPPER TAIL 5%	UPPER TAIL 2.5%
E11	-0.02171	-.02624	-.081614	-.074099	-.025305	.018037	.029091
E22	-0.44410	-.45982	-.625690	-.593510	-.460320	-.316080	-.295810
E33	-0.09098	-.10765	-.303350	-.276940	-.104290	.050763	.090956
E12	-0.00019	-.00019	-.001598	-.001332	-.000170	.000972	.001180
E13	0.02111	.02643	-.030017	-.018541	.025487	.073771	.082000
E23	0.48105	.48213	.203200	.242660	.479570	.716230	.773360

STANDARD DEVIATION FROM 1000 DRAWS	RESULTS FROM LINEAR APPROXIMATION MEAN	RESULTS FROM LINEAR APPROXIMATION ST. DEV.
.02751	-0.021344	0.002183
.08458	-0.443957	0.000060
.09793	-0.092633	0.002186
.00070	-0.000307	0.000064
.02764	0.021686	0.008474
.14432	0.481716	0.122543

TABLE 3: COMPARISON OF THE RESULTS FOR THE TRANSLOG COST FUNCTION WITH THE NORMAL DISTRIBUTION

ELASTICITY	-2.5%/ST.DEV.	-5%/ST.DEV.	+5/ST.DEV.	+2.5%/ST.DEV.
E11	-2.01	-1.74	1.61	2.01
E22	-1.96	-1.58	1.70	1.94
E33	-2.00	-1.73	1.62	2.03
E12	-2.01	-1.63	1.66	1.96
E13	-2.04	-1.63	1.71	2.01
E23	-1.93	-1.66	1.62	2.02

TABLE 4: PARAMETER INFORMATION - GENERALIZED LEONTIEF COST FUNCTION

PARAMETER	MEAN	VARIANCE-COVARIANCE MATRIX B12	B13	B22	B23	B33
B12	0.167355	0.175E-02				
B13	0.513360	0.535E-02	0.717E-01			
B22	-0.094983	-0.15E-03	0.737E-02	0.134E-00		
B23	0.050003	-0.38E-02	-0.69E-02	-0.91E-01	0.725E-01	
B33	5.601208	0.112E-01	0.656E-01	0.408E-01	-0.26E-01	0.565E-00
ϕ_1	-0.000866	0.200E-04	0.150E-03	0.100E-04	0.380E-03	0.140E-03
ϕ_2	0.000694	-0.20E-04	-0.13E-03	0.543E-07	-0.40E-04	-0.12E-03
B	1.022877	-0.17E-02	-0.23E-01	-0.75E-03	-0.56E-03	-0.52E-01
θ	-0.104499	0.330E-03	0.519E-02	0.300E-04	0.380E-03	0.127E-01

ϕ_1	ϕ_2	B	θ
0.103E-07			
-0.94E-08	0.867E-08		
-0.40E-04	0.300E-04	0.124E-01	
0.616E-07	-0.60E-07	-0.31E-02	0.790E-03

TABLE 5: ELASTICITIES FROM THE GENERALIZED LEONTIEF COST FUNCTION -- BASED ON 1000 DRAWINGS

ELASTICITIES

ELASTI-CITY	CALCULATED FROM PARAMETER MEAN	MEAN ELASTICITY FROM 1000 DRAWS	LOWER TAIL		MEDIAN	UPPER TAIL		STANDARD DEVIATION FROM 1000 DRAWS	RESULTS FROM LINEAR APPROXIMATION	
			2.5%	5%		5%	2.5%		MEAN	ST. DEV.
E11	-1.3020	-1.3039	-1.3882	-1.3676	-1.3013	-1.2476	-1.2386	.03797	-1.301966	0.031785
E22	-0.63192	-.57153	-.81418	-.79189	-.63759	-.23104	.05239	.32092	-0.631930	0.074926
E33	0.05794	.06398	-.01559	.00162	.06091	.13565	.15603	.04335	0.057889	0.039191
E12	0.02320	.02329	.01421	.01552	.02336	.03075	.03254	.00473	0:023231	0.002990
E13	0.26143	.25907	.16725	.18173	.26147	.32858	.33712	.04441	0.261357	0.046311
E23	0.24621	.23543	-.07104	-.00762	.24658	.46294	.50444	.17409	0.246194	0.126761

TABLE 6: COMPARISON OF THE RESULTS FOR THE GENERALIZED LEONTIEFF COST FUNCTION WITH NORMAL DISTRIBUTION

ELASTICITY	-2.5%/ST.DEV.	-5%/ST.DEV.	+5/ST.DEV.	+2.5%/ST.DEV.
E11	-2.22	-1.68	1.48	1.72
E22	-0.76	-0.69	1.06	1.94
E33	-1.84	-1.45	1.65	2.12
E12	-1.92	-1.65	1.58	1.96
E13	-2.07	1.74	1.57	1.76
E23	-1.76	-1.40	1.31	1.55

Faculty of Business

McMaster University

WORKING PAPERS - RECENT RELEASES

221. Joseph B. Rose and Gary N. Chaison, "The State of the Unions: United States and Canada", April 1984.

222. Naresh C. Agarwal, "Economic Costs of Employment Discrimination", May 1984.

223. R.J. Adams, "Estimating the Extent of Collective Bargaining in Canada", May 1984.

224. Robert G. Cooper, "New Product Strategies: What Distinguishes The Top Performers", May 1984.

225. Norman P. Archer, "Selection of a Simulation Language for an MBA Modeling and Simulation Course", June 1984.

226. Robert F. Love, William G. Truscott, John H. Walker, "A Facility Location Decision: Thibodeau-Finch Transport Ltd.", June 1984.

227. R.J. Adams, "Two Policy Approaches to Labour-Management Decision Making at the Level of the Enterprise: A Comparison of the Wagner Model and Statutory Works Councils", July 1984.

228. George Saunders, "Employment and the Productivity Slowdown 1958 to 1980", September 1984.

229. George Saunders, "Industrial Relations in Canada", September 1984.

230. George Saunders, "Wages and Interest Arbitration: A Case Study of the Canadian Federal Public Service", September 1984.

231. Norman P. Archer, "Optimal Control of an Ornstein-Uhlenbeck Diffusion Process, with Applications", November 1984.

232. Robert G. Cooper, "Selecting Winning New Product Projects: The Newprod System", January 1985.

233. R.J. Adams, "Industrial Relations and the Economic Crisis Canada Moves Towards Europe", January 1985.

234. G. John Miltenburg and Itzhak Krinsky, "Evaluating Flexible Manufacturing Systems", January 1985.

235. G.J. Miltenburg, "The Transient Behaviour of Transfer Lines With Buffer Inventories", February 1985.

236. Paul D. Dowling and Robert F. Love, "An Evaluation of the Dual as a Lower Bound in Facilities Location Problems", March 1985.

237. Min Basadur, George B. Graen and Terri A. Scandura, "Improving Attitudes Toward Creative Problem Solving Among Manufacturing Engineers", March 1985.

238. Christopher K. Bart and Roderick E. White, "Managing To Make Business Unit Strategies Work", May 1985.

239. Harish C. Jain, "Sexual Harassment: Issues and Policies", June 1985.

240. George Steiner, "Searching In 2-Dimensional Partial Orders", July 1985.

241. Norman P. Archer, "Measuring the productivity of office organizations", August 1985.

242. P.L. Abad, "Coordination of Pricing, Advertising, Production and Work-Force Level Decisions in a Functionally Decentralized Firm", September 1985.

243. Robert G. Cooper, "Defining the New Product Strategy", January 1986.

244. R.G. Cooper and E.J. Kleinschmidt, "An Investigation into the New Product Process: Steps, Deficiencies, and Impact", January 1986.

245. Elko J. Kleinschmidt, "Does Foreign Ownership Matter in the Export Behaviour of Exporting Industrial Firms in Canada?", January 1986.

246. John C.C. Macintosh, "Reporting to Employees: Establishing the Demand by Employees for Information", February 1986.