PEIRCE THE LOGICIAN: PRAGMAT(IC)IST ANTICIPATIONS OF MODERN LOGIC

PEIRCE THE LOGICIAN: PRAGMAT(IC)IST ANTICIPATIONS OF MODERN LOGIC

By BRENT C. ODLAND, B.A., M.A.

A Thesis Submitted to the School of Graduate Studies in Partial Fulfilment of the Requirements for the Degree Doctor of Philosophy

McMaster University DOCTOR OF PHILOSOPHY (2025) Hamilton, Ontario (Philosophy)
TITLE: Peirce the Logician: Pragmat(ic)ist Anticipations of Modern Logic
AUTHOR: Brent C. Odland, B.A. (University of Calgary), MA. (University of Calgary)
SUPERVISOR: Professor Sandra Lapointe
NUMBER OF PAGES: x, 115

Lay Abstract

This dissertation discusses the contributions to the field of logic by the American philosopher Charles S. Peirce. It begins with a new interpretation of his universal categories–his three-category method of conceptual analysis–and argues that these are behind his most impressive anticipations of modern logic. After giving the interpretation of Peirce's category system, I demonstrate its involvement in Peirce's discovery of quantification theory, his arguments against psychologism, his philosophy of mathematics, and his delineation of the three branches of logic.

Abstract

This dissertation argues that each of Charles S. Peirce's most impressive anticipations of modern logic are rooted in his universal categories–his triadic method of conceptual analysis. In chapter one, I argue for a quasi-type-theoretic interpretation of the universal categories based on the medieval notions of first and second intentionality. In chapter two, I discuss Peirce's algebraic systems of logic and argue that his discovery of quantification theory was rooted in his triadic analysis of the types of signs. In chapter three, I discuss Peirce's architectonic classification of the sciences, which is also based on the universal categories, and demonstrate its involvement in his delineation of logic as a science distinct from psychology and mathematics. The final chapter is devoted to the three branches of logic borne out of Peirce's triadic method.

Acknowledgements

First of all, I would like to thank Sandra Lapointe for all of her support and guidance over the years. Had it not been for her and her scholarship, my understanding of logic during the period between Kant and Russell would have been more or less limited to Peirce's work, and I would not have been able to appreciate the broader contexts under which Peirce's studies in logic were conducted. I am also indebted to Sandra for including me in her research with The/La Collaborative which provided me invaluable opportunities to develop skills and conduct academic research outside of philosophy.

I would also like to thank my other committee members, Alex Klein and Megan Stotts, for all of their feedback and support on this project, from the early days of my Qualifying Exam to the final draft of the dissertation. I would also like to thank Alex for including me in events associated with the Bertrand Russell Research Center, and for all of our discussions about Peirce and James over the years. I would like to thank Megan also for all of her professional advice about academia and the academic job market over the years (thanks in this regard also go to Allauren Forbes, Eric Bayruns García, and Mark Johnstone).

I am grateful to Cheryl Misak for being a terrific external reviewer and providing excellent feedback on what I might revise when I try to publish this work as a book. I am also grateful to her for all of her scholarship on pragmatism. Her *Truth and the End of Inquiry* was the first book I ever read on Peirce and her presence at my defense was a dream come true.

I feel compelled to also express my gratitude to Mark Migotti, for introducing me to Peirce, and Richard Zach, for helping me get started in the history of logic, and everything else they have done to help me since starting an MA at the University of Calgary.

Finally, I wish to thank my friends and family. Thanks are due to Scott Metzger, Stephen Ross, Nathan Haydon, and Ian MacDonald, for countless discussions on Peirce, logic, and philosophy, in which I tried out many of the ideas expressed here for the first time. I'm grateful to my parents for all their support during my lamentably lengthy period of post-secondary education. Thanks also to my wife, Emma Ritchie, who bore with me through all of the highs and many lows of the whole process. I could not have done it without you.

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Declaration of Academic Achievement

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

Introduction

This dissertation has two primary influences. The first is Hilary Putnam's paper, "Peirce the Logician" (Putnam 1982). The second is Sandra Lapointe's introduction to *Logic from Kant to Russell*. From the former, I take the starting point for the content of this work as well as the first part of its title. The latter informs my historiography and provides historical context for Peirce's work on logic.

The introduction to Putnam's essay ends as follows:

> [I]n a very widely used text by a philosopher I admire enormously, I read that 'logic is an old subject and since 1879 it has been a great one' (Quine 1950). In short, logic only broke out of its long stagnation with Frege. In one pen stroke Quine dismisses the entire Boolean school–of which Peirce was, in a sense, the last and greatest figure.

The view that Putnam takes himself to be rebuting is unfortunately a common one. This view holds something like 'Logic began with Aristotle, then there was a long period of stagnation until quantification popped out of Frege's head, fully formed and without precedent.' Of course, this view is a vast oversimplification. It neglects huge swathes of the rich history of a subject as old as philosophy itself. Within that lacuna is the work on logic conducted by Boole and his followers, as exemplified by Peirce. One of the chief aims of this dissertation is to understand Peirce's role within this broader history and to provide context to help evaluate some of Putnam's claims. For instance, Putnam writes: "Frege did 'discover' the quantifier in the sense of having the rightful claim to priority. But Peirce and his students discovered it in the effective sense (297)." What Putnam is describing here is related to the issue of priority when it comes to the discovery of the predicate calculus. The issue is complicated for a handful of reasons. One is that, while Frege arrived at quantification theory in 1879, very few seem to have read Frege's work prior to its popularization at the hands of Bertrand Russell. Peirce, on the other hand, does not acquire full quantification theory until 1885, and even then the details are hazy (Brady 2000). Yet, the logical community seems to have been much more aware of Peirce in the 19th century, than they were of Frege (Putnam 1982). This is what Putnam means when he claims Peirce and his students to be the *effective* discoverers of quantification theory. Putnam also claims for Peirce, the invention of the nearest typographical antecedent of modern quantifier notation, as well as the distinction between first and second order logic, in anticipation of type theory.

While this dissertation is heavily inspired by both Putnam's paper and Lapointe's book, it also aims to fill some gaps left by each. Putnam's short paper does not go into any detail about Peirce's logical systems, so many of the claims he makes on Peirce's behalf remain unsubstantiated. For instance, he claims that the distinction between first and second order logic is due to Peirce, but he does not say when or where Peirce made this distinction (It was in 1885, in "On the Algebra of Logic: a Contribution to the Philosophy of Notation"). Putnam also makes some errors in his discussion of the historical material. For example, he claims O. H. Mitchell, Peirce's student at John Hopkins university, discovered quantification with Peirce in 1883 (294). While Mitchell's system does make for a vastly more expressive system than Boolean algebra, it falls short of full quantification theory (Brady 2000, 89). Full quantification theory isn't reached in the algebraic tradition until two years later, when Peirce publishes his 1885 paper. Likewise, while Lapointe and her contributors have made great strides forward in providing much needed contextualization for 19th century logic and the historical backdrop against which Peirce's work on logic occurs, they have mostly left out an important figure in the period of logical study between Kant and Russell: Peirce. Peirce is mentioned three times in the book: twice in Lapointe's introduction and once in Reck's chapter on Dedekind's logicism. I address the gap left by Putnam by substantiating his claims with proper textual evidence and by correcting historical inaccuracies. I address the gap left by *Logic from Kant to Russell* by expounding on Peirce's thought and role within the history of analytic philosophy and logic.

The view Putnam attributes to Quine above, is one example of "what historians of logic get wrong" (Lapointe 2018, 13). The proliferation of the predicate calculus in logic, and analytic philosophy more broadly, was so momentous that it appears to have warped our views on the overall history of the discipline. This is further exacerbated by the vast differences between how logic is understood now as opposed to how it was understood in the 19th century. Nowadays, logic is thought of mostly as a formal discipline replete with axiom systems and calculi. In the early 19th century, logic was dominated primarily by post-Kantian philosophers, like J. F. Fries and J. F. Herbart, who conceived of it as the study "of the form of the understanding" (21). They did not carve sharp disciplinary boundaries between logic and other fields, like psychology, epistemology, and anthropology. These borders were in a process of negotiation through the course of the 19th century. The appearance of new traditions, like the Boolean-algebraic, Logicist, and phenomenological traditions, are no doubt the cause of such negotiations. Logic was being radically rethought in the 19th century, and any interpretation which fails to keep this historical backdrop in mind would fail to do justice to Peirce's thought on the subject. Peirce was writing during the cusp of a watershed moment for logic, rooted in its immediate past but anticipating its future.

The fact that Peirce is frequently credited with 'anticipations' of aspects modern logic–from many-valued logic to possible world semantics–is the reason for the second part of my title. These 'anticipations' raise another thorny question about Peirce's work on logic: What was he up to? What was motivating his logical investigations? Due to his logicism, it is relatively straightforward to understand what was guiding Frege's inquiry. Peirce's overall project is much harder to unravel. This is due in part to the difficulty of accessing Peirce's writings as well as shifts in the understanding of logic in the 19th century. My answer to these questions, and the main claim I wish to argue in this project, is that Peirce's work in logic was spurred by his theory of conceptual analysis, his universal categories, and was motivated by a desire to extend the scope of logic to cover the complex array of relations and concepts that this analysis revealed. Initially, this involved various technical innovations that would increase the expressivity of Boolean style algebras. More specifically, Peirce's categorial analysis of signs provided him with the insights necessary to develop his general logic of relations. Later, an application of his categorial analysis of the kinds of sciences allows him to clarify and articulate the scope of logic that is distinct from related fields, like mathematics and psychology. In Peirce's mature thought, the categories also enable him to broaden the scope of logic far beyond current understandings of the subject, uniting the theory of reasoning with the theory of scientific discovery and progress. There logic becomes concerned not just with the theory of correct reasoning, but also with how reasoning can be fruitfully applied in inquiry. The categories are at the heart of each of these developments. Before giving an outline of the dissertation, I would like to discuss some biographical details about Peirce and his writings.

If there is one thing I would like my non-Peirce-scholar readers to understand from the outset, it is this: Peirce is a mess. His life was a mess. His *Nachlass* is a mess. And as a result, his scholarship is, at times, a bit of a mess. In my MA thesis, I motivated this issue with the following quote from a review of a collection of Peirce's writings:

> There is a story that when Peirce retired to Milford, he built an attic study in his house accessible only by ladder. When the creditors that plagued his later years came calling, Peirce retreated to uninterrupted philosophizing by climbing to his study and drawing up the ladder behind him.
>
> This story suggests a moral: the student of Peirce is often in a position like that of the bill collector. Access to Peirce can be difficult (Thayer 1967).

As the quote suggests, Peirce's life was filled with difficulties, especially in his later years. While he is remembered today as a philosopher and a logician, during his life he made his living primarily as a working scientist, conducting research for the government. Peirce began working for the U.S. Coast and Geodetic survey in 1859 to support his day to day needs while he devoted his free time to his work on abstract logic (Burch 2024). In 1865 he gave a series of lectures at Harvard on the logic of science. The series contains a lecture on Boole's calculus of logic, marking the beginning of a 20 year obsession with the system. His only brief academic appointment was as a lecturer at John Hopkins University, from 1879-1884. In 1883 he published a collection of essays by himself and his students, called *Studies in Logic by Members of the John Hopkins University*. It's most important contributions lie in the modifications of Boole's logical algebra contained in the papers composed by O. H. Mitchell, Christine Ladd-Franklin, and Peirce himself. However, it also contains some interesting work on logic machines and the logic of the Epicureans, written by Allan Marquand. Peirce was apparently fired from that position due to circumstances involving the divorce of his first wife and subsequent remarriage to a Roma women, named Juliette Annette Froissy (Burch 2024). Peirce's government work dried up in 1891 and he remained chronically underemployed until he died in 1914.

Peirce's troubles seem to have prevented him from participating in the scholarly community in the way he would have liked. He published a number of articles throughout his life but he never succeeded in publishing a book on his views on philosophy and logic in particular, despite several attempts. His *nachlass* contains numerous outlines for books, chapter drafts, and abandoned manuscripts, many quite substantial. In 1914, shortly after his death, Juliette Peirce sold his unpublished manuscripts[1] to Harvard where they survived under the care of Josiah Royce (Burch 2024). Royce died in 1916 and eventually care of the manuscripts passed to C. I. Lewis, who had been Royce's student (Murphey 2005). Eventually, "Charles Hartshorne, Paul Weiss, and Arthur Burks," were recruited to the task of publishing a collection of Peirce's manuscripts, and the *Collected papers of Charles Sanders Peirce* began to appear over the 1930s (Burch 2024). Their editorial decisions left a lot to be desired. As Burch puts it, "Often a single entry will consist of patches of writing from very different periods of Peirces intellectual life, and these patches might even be in tension or outright contradiction with each other (Burch 2024)." Still, the *Collected Papers* (henceforth, CP) remain the first point of access for much of Peirce's work, especially his mature philosophy. The difficulty of accessing Peirce's writings, as well as the questionable practices of the editors of the CP, have been alleviated somewhat by the efforts of the Peirce Edition Project in publishing a *Chronological Edition* of Peirce's writings. However, progress in these efforts has appeared to stall as a result of poor funding, and as a result, we only have seven volumes covering the period between 1857-1892 (Burch 2024). Seven volumes covering such a long period may sound like a lot but this is a small fraction of the projected 30 volume total (Misak 2016, 11). While the problem of accessing Peirce's writings is improving, we still have a long way to go.

Having addressed the issues surrounding Peirce scholarship, I now turn to give an overview of the dissertation. The claim I wish to argue for overall, is that Peirce's efforts in his logical inquiries were motivated and directed by his universal categories: firsts, seconds, and thirds. They are universal because Peirce seems to have thought that they could be used to draw fruitful distinctions between anything imaginable, though he uses more descriptive terms for them depending on the context. To that end, in chapter one I begin by giving an overview of Peirce's categories and the conceptual

1. The bulk of Peirce's surviving manuscripts were cataloged in Richard Robins *Annotated Catologue of the Papers of Charles S. Peirce* in 1967. Two years later, a trove of previously unknown papers were discovered in storage at Harvard by Carolyn Eisele, necessitating a supplement to the catalogue (Robin 1971, 37). Peirce scholars typically refer to the volume as the "Robin Catologue" and I follow this convention.

analysis they yield. I then argue for an interpretation of Peirce's categories that relates them to the medieval notions of first and second intentionality, in an attempt to explain the categories.

In chapter two, I begin by giving an account of Peirce's application of his categorial analysis in the domain of logic. First I discuss his term-proposition-argument triad. I then turn to his application of his icon-index-symbol triad to analyze the notational apparatus of Boolean style algebraic logic. I argue that this analysis is responsible for his development of the quantifiers. More specifically, I argue that O. H. Mitchell's use of indices in his notation led Peirce to discover the quantifiers, which allowed him to extend the scope of Boolean formalism to encompass a full logic of relations.

In chapter three, I take a turn towards Peirce's philosophy of logic and discuss how Peirce would have answered questions surrounding logic's scope in relation to other disciplines, in particular, mathematics and psychology. This discussion relies heavily on Peirce's use of his categories to construct an architectonic taxonomy of the sciences. I first argue that it would be a mistake to lump Peirce in with the logicists and intuitionists, despite some claims to the contrary. I then argue that it would be a mistake to say that Peirce would be sympathetic to the view known as psychologism. Here, Peirce's categories allow him to demarcate the scope of logic from related fields.

In chapter four, I wrap up discussion of Peirce's philosophy of logic by discussing his positive program for logic. This involves discussion of each of its branches in the architectonic. These branches are speculative grammar, logical critic, and methodeutic. I argue against interpreting these as straightforward analogues of the later distinction between syntax, semantics, and pragmatics, due to Morris and Carnap. In particular, I disagree with regarding methodeutic and pragmatics as analogues. I argue instead that with methodeutic, Peirce's intent is to extend the scope of logic far beyond current understandings by incorporating something like a theory of applied reasoning.

I take the centrality of the universal categories in each of these discussions to be evidence in favor of the claim expressed above.

Chapter 1

Peirce's Categories and Peirce's Logic

> "Just one more whacked out triadomaniac."
>
> Richard Rorty, "The Pragmatist's Progress," 93.

The first volume of what has likely become the most common introduction to the writings of Peirce, the *Essential Peirce*, begins with the brief and difficult 1867 essay, "On a New List of Categories." In the essay, Peirce debuts what has become likely the most perplexing set of notions in his philosophical work, his proposal that there are three universal categories: Firsts, Seconds, and Thirds. Difficulties surrounding these categories have earned Peirce the ire of at least one philosopher. Richard Rorty, commenting on his and Umberto Eco's former ambitions as "code-crackers," writes "[t]his ambition led me to waste my twenty-seventh and twenty-eighth years trying to discover the secret of Charles Sanders Peirce's esoteric doctrine of 'the reality of Thirdness' and thus of his fantastically elaborate semiotico-metaphysical 'System'. I imagined that a similar urge must have led the young Eco to the study of that infuriating philosopher" (Rorty 1992, 93). So, why would a text meant to introduce new readers to the philosophy of Peirce begin with an essay on such a notoriously difficult topic? Well, because these ideas are so central to all of Peirce's work that it is hard to get anywhere with his writing without some understanding of them, and his writings on logic (especially his later ones) are no exception. They buttress Peirce's account of the "three grades of clarity" that give rise to the pragmatic maxim, for which he is probably most famous for (Peirce 1992, §8, c. 1878). We can find them in Peirce's thesis that logic is meant to be concerned with three kinds of objects (terms, propositions, and arguments) and the three kinds of arguments (deduction, induction, and abduction). They even go on to inform his conception of the evolving universe, which I will argue is what some of Peirce's later systems of logic are intended to capture in subsequent chapters.

Rorty makes Peirce's category system sound like one of the most mystifying things in philosophy but this is hardly the case. One of the modest aims of this chapter is to show that Peirce's treatment of categories is not so "whacked-out" and confusing as it might appear. The underlying idea behind it is quite similar to more familiar doctrines in analytic philosophy, like the ontological aspects of Russell's logical atomism or multiple-relation theories, or Husserl's formal ontology, but with one important difference. The ontological commitment behind Russellian logical atomism is that the world consists of a plethora of independently existing entities exhibiting qualities in the form of monadic relations, all standing in a complex of other relations to each other. On the picture informed by Peirce's category theory, the world is composed of a complex network of relations between objects, minds, and representations (or signs, as he would probably prefer). Like Russell's logical atomism, Peirce's category theory is also a kind of method for analysis. The glaring difference between the two is that Peirce's theory does not presuppose that there are any fundamental components of reality or atoms; the analysis does not necessarily bottom out anywhere. This feature is in line with Peirce's anti-foundationalist commitments and his doctrine of synechism, the view that we ought to consider everything in the world as continuous with everything else. This results in a sometimes confusing recursive application of the categorical analysis, whereby Peirce will analyze something into a first, second, and third, and then, taking a different perspective, analyze that third further into a different set of firsts, seconds, and thirds. Appreciating the non-atomistic nature of Peirce's analysis can help make sense of these seeming peculiarities. Since there are no atoms, analysis can always be taken further or re-imagined in different contexts.

The most famous category systems in philosophy are likely those we associate with Aristotle and Kant. Aristotle's categories are intended to supply a system of genus of the highest order that can carve up everything that exists in the world. His approach is realist in nature since it is meant to give the highest categories of being, as opposed to language or thought. Kant's category system differs from Aristotle's in that it does not assume we have access to the fundamental divisions amongst the objects behind our perceptions of the world. Rather than carving up being, Kant's system is intended to give the categories of judgements necessary to our understanding of the world. So, while Aristotle's theory is meant to supply genera for reality, Kant's theory is more modestly meant to supply genera of thought. Peirce's theory lies somewhere in between the two. As John Deely puts it:

> Peirce's categorial scheme is neither a scheme designed to express exclusively what is there in the objective world prior to the scheme and independently of it, as Aristotle's was, nor is it a scheme designed to express exclusively necessary aspects of the mind's own working in developing discursively the content of experience, as Kant's was. Peirce's scheme is designed to express the mixture and interweave of mind-dependent and mind-independent relations which constitute human experience in its totality as a network of sign relations (Deely 2001, 662).

While his category system is inspired by both Kant and Aristotle, it is quite different from both of theirs. It differs from Aristotle in that it does not just provide categories for mind independent entities. Peirce's system is intended to supply genera for metaphysics, epistemology, logic, phenomenology, language, etc. Yet, what distinguishes him from Aristotle is that he believes there are different "versions" of the categories for each of these domains. It is in this sense that they are universal. However, unlike Kant, Peirce does not think that our theories should assume incognizable entities like things in themselves.

It might be wondered why Peirce, who preferred to be identified primarily as a logician, would spill so much ink about his categories. Logic, it could be argued, has to do with structure and this is certainly how Peirce understood it. When Peirce is articulating the various applications of his theory of categories to different domains, he takes himself to be working out the fundamental structure of reality in each of its domains, each corresponding to a specific "science": metaphysics, epistemology, logic, phenomenology, etc. He thought that his categories were of the utmost importance because he thought they could facilitate better reasoning about literally everything.

The primary aim of the present chapter is to advance an interpretation of Peirce's categories that will help clear up some confusions and lay the groundwork for later chapters. I argue that the categories have their basis in two organizing principles, one based on the number of constituents in each category and the other based on something similar to a type hierarchy. The first of these is explicit in Peirce's writings on the categories. The second requires some additional textual interpretation to point out. In section 1.1 I discuss the development of Peirce's categories and the first principle. In 1.2 I make an argument for the second principle. I conclude by briefly discussing Peirce's attempts to justify his category system.

1.1 The Universal Categories

Peirce was introduced to logic when he read his brothers copy of Whately's 1827 *Elements of Logic* at the age of twelve. He was introduced to philosophy more generally by reading Kant's *Critique of Pure Reason*, who he admired throughout his life and inspired his understanding of logic as involving a reflection on categories that went on to inform nearly all of his philosophical work. In 1898 he remarks "In the early sixties I was a passionate devotee of Kant, at least as regards the Transcendental Analytic in the *Critic of Pure Reason*. I believed more implicitly in the two tables of the Functions of Judgement and the Categories than if they had been brought down from Sinai" (Peirce 1982, xxiv–xxv). [1]

The inclusion of category systems in logic goes at least as far back as Aristotle's *Organon*, where they feature in the analysis of types of predication (Brumberg-Chaumont 2016, 19). In the *Critique of Pure Reason*, Kant's categories of human understanding are derived from his table of judgements, on the basis of a version of Aristotelean logic (Thomasson 2019; Rohlf 2024). According to Kant there are four kinds of judgements that are employed in our attempts to understand phenomena, and these four types correspond to quantity, quality, relation, and modality. Each of these kinds of judgement are then further broken down into three types. So, for example, with respect to quantity, there are judgements of unity, judgements of plurality, and judgements of totality.

1. Quoted in Fisch's introduction.

Quantity	**Quality**	**Relation**	**Modality**
Unity	Reality	Inherence and Subsistance	Possibility
Plurality	Negation	Causality and Dependence	Existence
Totality	Limitation	Community	Necessity

Figure 1.1: Kant's table of categories (B95/A70)

That there are exactly three kinds of judgement within each of Kant's broader categories seems to have been indicative to Peirce of a more general organizing principle underpinning categorization. When he introduces a list of categories for the first time in 1867[2], it will contain only three categories of conceptual judgement.[3] He comments on this in 1885:

> "It is remarkable that although the system of formal logic upon which Kant founded his list of categories was extremely imperfect, yet his categories themselves are at least highly important conceptions. They form four triads, and each triad involves the conceptions of first, second, and third. This shows that even when the method is not followed out with perfect accuracy, its results are not utterly without value. If in place of distinctions absolutely essential in logic we take those which approach that character, the categories obtained though not quite *a priori* will be highly primitive conceptions" (Peirce 1995, §34, 236).

In fact, when Peirce comes to his most general[4] characterization of the categories 19 years later (see (Peirce 1992, §18, c. 1886)), which he terms universal categories,[5] all reference to anything other than the number of elements that they unite fall away. Under that conception, the items sandwiched[6] between being and substance in figure 1.2, quality, relation and representation, will correspond respectively to Firstness, Secondness, and Thirdness.

2. There are interesting precursors to the 1867 paper. Peirce seems to have began with a set of categories based on the first, second, and third person voice, called 'I', 'It', and 'Thou.' See (Peirce 1982, §13 and 14).

3. He also recognizes his categories in Hegel's three moments or stages of thought (Ketner 1983, 77).

4. On Peirce's understanding the most general kind of categorization would be the one with the widest range of applicability.

5. The Universal form is just one instance of his three categories. There are plenty of other versions of it, i.e. categories of fact, of phenomena, of metaphysics, of logic, etc.

6. The original formulation in 1867 also includes being and substance. These fall away almost immediately and do not factor in to his proceeding discussions of the categories.

BEING,

Quality (Reference to a Ground),
Relation (Reference to a Correlate),
Representation (Reference to an Interpretant),

SUBSTANCE.

Figure 1.2: Peirce's table of categories in 1867

But Peirce's categories are perceived as notoriously difficult. Robert Burch, a well-established Peirce scholar, claims

> If Peirce had a general technical rationale for his triadism, Peirce scholars have not yet made it abundantly clear what this rationale might be... It is difficult to imagine even the most fervently devout of the passionate admirers of Peirce, of which there are many, saying that his account (or, more accurately, his various accounts) of the three universal categories is (or are) absolutely clear and compelling (Burch 2024).

This is partially due to the breadth of subject matter that Peirce applies them to. To get a sense of this breadth, and to set up the problem, we may examine the following collage of Peirce quotes (each separated by a semicolon) defining or giving examples of each category, composed by Atkins:

> "*Firstness*fleeting; a momentary feelingwithout even the recognition of it as a feeling; a moment divorced from the future (or anticipation of the future) and from the past that preceded it; it is without relation to anything else and so it is like a mere possibility; it is flash, like the immediate sensation of a color or the feeling of an emotion (not necessarily the emotion itself); the momentary, immediate hearing of a train whistle when awakened from a deep slumber.
> *Secondness*resistance; obstinance; reaction; effort; force; it is like the past, which considered in itself is unalterable; a fact that simply 'is'; *haecceity*; it is a struggleimagine pushing on a door and an unknown force resists, or a white mark drawn on a black chalkboardit is that 'tension' of forces or of color; a mutual action regardless of a Medium or Law of Action; surprise; the reaction of Ego and Non-Ego; brute force.
> *Thirdness*-transuasion; law; mediation; habit-taking; growth; representation; giving is a third, for that A gives B to C involves a habit or law

> in the transfer of property as it is not simply the case that A put down B and C picked it up; the signing relationship is a third for A (the sign) denotes some object, B, to some interpretant C; it is being *in futuro*, in mental forms, intentions and expectations" (Atkins 2006, 487–488).

Atkins's collage is confusing at first glance, since it is not immediately apparent what is motivating the above groupings. The explicit organizing principle behind the categories, and also the motive behind their names, has to do with the number of constituent elements involved in every First, Second, or Third. For a more straightforward example of these, we can turn to Peirce's categorization of relations. A monadic or unary relation would be a First, since it relates only one object. For the same reason, a dyadic relation is a Second and a triadic relation is a Third. A reaction is a second because it also involves two things, say a response to an event. Mediation involves three things, a mediator and two mediatees, so it is a third. However, it can be difficult to see how the organizing principle is operative in some of Peirce's triads. An example where it seems to fall short is his categories of consciousness: feeling (First), experience (Second), and thought (Third). At face value, it is difficult to see how the number of elements involved alone leads to this particular categorization. I will go on to suggest a second organizing principle, but first we must examine Peirce's attempts at defining the categories.

Let us begin with one of Peirce's more developed definitions of the categories from 1903:

> "Category the First is the Idea of that which is such as it is regardless of anything else. That is to say, it is a *Quality* of Feeling.
> Category the Second is the Idea of that which is such as it is as being Second to some First, regardless of anything else and in particular regardless of any *law*, although it may conform to a law. That is to say, it is *Reaction* as an element of the Phenomenon.
> Category the Third is the Idea of that which is such as it is as being a Third, or Medium, between a Second and its First. That is to say, it is *Representation* as an element of the Phenomenon" (Peirce 1998, §12).

As Peirce understands them, what he calls 'Firsts' are the most basic category. His rationale for calling them this is that they depend on nothing else for their being. In the 1867 version of the categories, Firsts are associated with qualities, though what he means is more like disembodied qualities. His favorite example is redness, abstracted away from any particular red objects. Later in 1903 when Peirce applies the categories in the domain of metaphysics, Firsts are considered pure possibilities because they are what they are without having to be actualized. If the possibility were actualized, then it would depend on an existent object and the result would no longer be a First. Firsts do not relate things and it is this essential unary character that makes them Firsts.

Seconds on the other hand, relate things, albeit in an arbitrary way, hence Peirce's addendum "regardless of any *law*" (Peirce 1998, §12). If there were some underlying law, principle, or reason that determined a Second, then that would be essential to its analysis, so the Second in question would also involve this third element, and effectively be a Third. If the relation has only two terms, then it is a Second. On Peirce's account, the counterpart of Secondness in its application to the domain of phenomenology is experience. So, if while I'm going about my day, I spot something red, then my experience of that red object is a Second. The counterpart of Secondness in the domain of metaphysics is actuality or existence. If someone accidentally spills red paint on a rock in the actual world, that red rock would be a Second. Just as Firsts are essentially unary, Seconds are meant to be essentially dyadic. This is why concepts like effort or resistance are considered Seconds. As Peirce understands them, efforts and resistances are always against something, that is relational and essentially dyadic.

While his understanding of Firsts and Seconds remains relatively unchanged throughout his lifetime, Peirce's understanding of Thirds appears to have been somewhat fluid and his definition of Thirdness shifted over time. But because Peirce applied his categorial analysis to multiple and varying domains, he may have been forced to stress different aspects of Thirdness. For instance, when he's discussing the categories in the domain of phenomenology, he uses slightly different language than he does when discussing the categories in metaphysics. To get at what the categories in general are, it is best to examine a few different examples to illustrate what they have in common.

Peirce defines Thirds as "the Idea of that which is such as it is as being [sic.] a Third, or Medium, between a Second and its First. That is to say, it is *Representation* as an element of the Phenomenon" (160). A Third brings two things together, specifically a First and a Second, into something new. He describes them as the "representational" element of phenomena because he understands representations as consisting of three things: whatever is being represented, the representation itself, and the interpretation of the representation. On Peirce's view, the thing being represented is a First while the representation itself is a Second, since it depends on something represented and hence involves a dyadic relation. The interpretation of the representation unites the whole complex into a Third. This is because it involves the interpretation of the representation as a representation of the object, so it involves three things.

Representations are not the only things that count as Thirds for Peirce. In his application of the categories to the domain of metaphysics, Thirdness is associated with necessity, or law, as Peirce usually puts it. Let's consider a rock that is red because it contains iron minerals that have oxidized. Here the First is the quality of redness, the Second is the actual red stones, and the Third is the law that determines that the stone and all others with a similar chemical make up turn red when certain conditions are realized. Thirds are always complexes involving three components and one of the three always plays some kind of mediating role between the other two. Peirce, somewhat confusingly, sometimes will also call that unifying element a Third, so the term refers to both the entire complex and its unifying element.

One difficulty with Peirce's categories is that he is not always so explicit about what each of their components are. In the 1903 definition, Peirce indicates that Seconds always emerge in relation to a First, and Thirds emerge by relating a First and a Second. In other places Peirce is less specific. Just months later in 1903, he writes:

> *Firstness* is that which is such as it is positively and regardless of anything else.
> *Secondness* is that which is as it is in a *Second* something's being as it is, regardless of any third.
> *Thirdness* is that whose being consists in its bringing about a Secondness (§20).

Here reference to Firsts is removed from the definition of Seconds and Thirds entirely. While some notion of Secondness typically remains in Peirce's subsequent definitions of Thirdness, this also is not consistent. In correspondence with Victoria Welby in 1904:

> I was long ago (1867) led, after only three or four years study, to throw all

> ideas into the three classes of Firstness, of Secondness, and of Thirdness. This sort of notion is as distasteful to me as to anybody; and for years, I endeavored to pooh-pooh and refute it; but it long ago conquered me completely. Disagreeable as it is to attribute such meaning to numbers, and to a triad above all, it is as true as it is disagreeable. The ideas of Firstness, Secondness, and Thirdness are simple enough. Giving to being the broadest possible sense, to include ideas as well as things, and ideas that we fancy we have just as much as ideas we do have, I should define Firstness, Secondness, and Thirdness thus:
>
> Firstness is the mode of being of that which is such as it is, positively and without reference to anything else.
>
> Secondness is the mode of being of that which is such as it is, with respect to a second but regardless of any third.
>
> Thirdness is the mode of being of that which is such as it is, in bringing a second and third into relation to each other (CP 8:328, MS [R] L463).[7]

In the second 1903 quote from (Peirce 1998, §12), Seconds are defined in terms of Firsts, and Thirds are defined in terms of Firsts and Seconds. In the second quote from the leter to Welby, Firsts are dropped from the definition of Seconds, and Thirds are defined in terms of *generating* Secondness.

Is there a definitive answer to the question of which definition should be considered as the most accurate representation of Peirce's views? If Peirce ever answered the question himself, it does not appear in any of his published works. However, it may be possible to approach an answer by considering the scope of what he thought he had achieved. In an 1885 letter to William James, discussing the categories, he writes "I have something very vast now... It is... an attempt to explain the laws of nature, to show their general characteristics and to trace them to their origin & predict new laws by the laws of the laws of nature" (Peirce 1992, 242). Peirce thought of his categories as a grand theory of everything (or at least something only slightly more

7. I will try to avoid using citations that are idiosyncratic to Peirce scholarship throughout this text wherever it is uncomplicated to do so. Unfortunately there is a large portion of his later work that is not published at all, or merely in fragments contained in the Collected Papers of Charles Sanders Peirce (CP). CP is a sometimes useful resource though some editorial decisions leave a lot to be desired. The citation CP X:xyz refers to the volume (X) and the paragraph (xyz) of that volume in CP. MS and any letters and digits after it refer to that items entry in the Robin Catalogue.

modest). Given their purported universality, we should expect claims made on their basis to be applicable to the highest levels of generality, but also fine grained enough to avoid being facile. The definition Peirce gives Welby seems to me more apt to meet these criteria because it is less restrictive about what kinds of things can be stitched together in Seconds and Thirds. Under that definitions, a Second might be made up of a Third and a First, or a Third might be made up of three Firsts, or a Second, a Third and a First, etc.

There are times where the less restrictive formulation may be advantageous, since it allows Thirds to be constituent members of other Thirds. For example, consider one of the paradigm examples of Thirdness: a gift (see CP 8:331-332). Gifts are considered Thirds because they involves some object, a giver, and a receiver. If anything is removed from that arrangement, then we no longer have a gift. Now, consider a re-gift. The arrangement is pretty much the same but the object being given has to be a Third, i.e., an object that was received by the giver from another giver. If the arrangement does not respect the thirdness of its constituent, then a gift would be indistinguishable from a re-gift and though the difference is slight they are clearly different kinds of things. The first definition would not be able to distinguish the two because it does not allow for Thirds to be constituents of other thirds, while the later definition would differentiate the two, avoiding clashes with our intuitions about such things. The less restrictive definition also avoids another counterintuitive result. Representations are also paradigmatic examples of Thirds, but a representation does not need to have exclusively Firsts or Seconds as their object, i.e. what is represented, as the more restrictive definition would imply. In principle, it ought to be the case that some of the things Peirce would describe as Thirds are capable of representation as well, like the laws of chemistry, for example. For this reasons we may wish to take Peirce's letter to Welby as the definition he would have preferred, had he been presented with our discussion.

Still, there seems to be more than one organizing principle behind Peirce's universal categories. By "organizing principle," I mean something like the rules by which Peirce's analysis draws distinctions between entities. We might call one of these the "quantitative" principle, since it draws distinctions on the basis of the quantity of constituent elements. There also appears to be another "qualitative" (for lack of a better term) distinction between each category, that seems to be based on something different than the former principle. The quantitative principle relies on each of the concepts being made up of an essential number of constituents, i.e. a gift would not be a gift if it doesn't have exactly three constituents. The bulk of the definitions we have examined so far are focused on that organizing principle. However, there are just as many definitions where the difference between the three categories seems to be based on something beyond the number of constituents involved:

> It seems, then, that the true categories of consciousness are: first, feeling, the consciousness which can be included with an instant of time, passive consciousness of quality, without recognition or analysis; second, consciousness of an interruption into the field of consciousness, sense of resistance, of an external fact, of another something; third, synthetic consciousness, binding time together, sense of learning, thought (Peirce 1995, 246).

The quantitative organizing principle behind the categories is fairly straightforward. The qualitative principle is not. In this application of Peirce's categorical distinctions to consciousness, it is difficult to see how the quantitative principle is operative. Roughly, the First in this arrangement is a feeling or sensation, the Second is experience, and the Third is thought. I take it that feelings are First here because they are the unanalyzable parts of consciousness. Experiences are Second because they are analyzable into something like expectation and actual result. Thoughts are Third because they compare experiences and produce new expectations. While there is likely some story that could be told from the quantitative perspective, it seems obvious that there is something more in the difference between a feeling and a thought than a couple of errant elements. A theory that based the difference between a sensation and a thought solely on the grounds of the number of elements composing them would leave something to be desired. So, what might this other organizing principle be?

1.2 An Interpretive Hypothesis

I would like to argue that the qualitative principle is something similar to a type hierarchy or the difference between levels of genera on a taxonomic tree. To my knowledge, I am the first to argue for this view. On this view, Firsts would be the lowest level of genera or the lowest type, with Thirds being the highest. The highest level will be made up of the lower level entities. So in the previous example, the lowest level of the elements of consciousness would be feelings, which make up the elements on the next rung i.e., experiences, which make up the highest order elements i.e., thoughts. I will make this more precise by discussing a notion Peirce inherited from the medievals, the distinction between First and Second intentions. The view I will ultimately arrive at is that Seconds are first intentional (or first order) concepts, Thirds are second intentional concepts (second order), and a new nought or null intentional order of concepts need to be introduced for Firsts.[8] I will provide evidence for this view by analyzing Peirce's explication of Thirdness contained in a letter to Victoria Welby.

The distinction between first and second intentions goes back at least as far as Aquinas and likely further:[9]

> What is first known (*prima intellecta*) are things outside the soul, the things which first draw the intellect to knowledge. But the intentions which follow on our mode of knowing are said to be secondly known (*secunda intellecta*); for the intellect comes to know them by reflecting on itself, by knowing what it knows and the mode of its knowing (Bobik 2016, 17).[10]

According to Bobik's interpretation, "first intentions are meanings or concepts derived from, or at least verified in, extramental, or real, things,"(17) while "second intentions are *concepts* about anything and everything involved in the human way of knowing"(56). Roughly, first intentional concepts are those which we apply to the actual, existent, objects of our experience, which would be seconds according to Peirce's scheme. Second intentions are the more general concepts that we use to think about first intentional concepts or other second intentional concepts. For example, "dog" as applied to the small creature currently sitting under the table in front of me would be a first intentional concept, while "canine" would be a second intentional concept

8. Interestingly, first and second intentions are also a foundational notion to Brentano in his 1874 *Psychology from an Empirical Standpoint*. Peirce's discussion of the notions begins a few years prior and as far as I know neither Peirce nor Brentano acknowledge the other, so it seems they came to the notion independently. It would be unsurprising if it turned out Peirce had read Brentano but his name does not appear in any of the entries of the *Collected Papers* or in the *Writings of Charles S. Peirce*. Peirce was aware of Brentano's student, Husserl, but the evidence suggests he never seriously read Husserl. I briefly speak to Peirce's reception of Husserl in chapter three.

9. Aquinas is preceded in this regard at least by the Arabic thinker Abu Nasr al-Farab (c. 872950/1), though his understanding of the topic appears to be less developed (Oschman 2018).

10. Originally from *De ente et essentia*, 1252-125, Bobik's translation.

which applies to the former. "Canidae" then would be a second order intention that applies to the latter. The distinction is present in other medieval figures, like William of Ockham[11], however he seems to credit the idea to Aquinas when he composed its definitions for *Baldwin's Dictionary of Philosophy and Psychology* (CP 2:548).

Peirce, so far as I'm aware, has never made the connection between his categories and the medieval notion explicit, which is why I have titled this section "An Interpretive Hypothesis." However, first and second intentions make an appearance in the very place where the categories debuted. In the 1867 article introducing them, he writes:

> I shall now show how the three conceptions of reference to a ground [(firsts)], reference to an object [(seconds)], and reference to an interpretant ([thirds]) are the fundamental ones of at least one universal science, that of logic. Logic is said to treat of second intentions as applied to first. It would lead me too far away from the matter in hand to discuss the truth of this statement; I shall simply adopt it as one which seems to me to afford a good definition of the subject-genus of this science. Now, second intentions are the objects of the understanding considered as representations, and the first intentions to which they apply are the objects of those representations. The objects of the understanding, considered as representations, are symbols, that is, signs which are at least potentially general. But the rules of logic hold good of any symbols, of those which are written or spoken as well as of those which are thought. They have no immediate application to likenesses or indices, because no arguments can be constructed of these alone, but do apply to all symbols (Peirce 1992, §1, 1867).

Here at least we can see Peirce making a connection between second intentions and some canonical examples of thirdness i.e., representations, symbols, and arguments.

Peirce also discusses the notion in an article in the same series as his "New List of Categories," but this time in the context of "The Logic of Mathematics."

> Equality is a relation of which identity is a species.
>
> If we were to leave equality without further defining it, then by the last scholium all the formal rules of arithmetic would follow from it. And

11. For discussion of Ockham's treatment of the topic, see (Spade 1981).

> this completes the central design of this paper, as far as arithmetic is concerned.
>
> Still it may be well to consider the matter a little further. Imagine, then, a particular case under Boole's calculus, in which the letters are no longer terms of first intention, but terms of second intention, and that of a special kind. Genus, species, difference, property, and accident, are the well-known terms of second intention. These relate particularly to the *comprehension* of first intentions; that is, they refer to different sorts of predication. Genus and species, however, have at least a secondary reference to the *extension* of first intentions.[12] Now let the letters, in the particular application of Boole's calculus now supposed, be terms of second intention which relate exclusively to the extension of first intentions(Peirce 1982, 68).

The purported aim of the source paper is to "is to show that there are certain general propositions from which the truths of mathematics follow syllogistically" (59). In the paper, which will be discussed in more detail in the next chapter, Peirce attempts to achieve his declared aim by showing that certain theorems of arithmetic are provable in a slightly modified Boolean calculus of logic. Here Peirce is supposing that the terms in such a calculus be second intentional concepts rather than the usual first intentional ones which refer only to objects and exploring the differences within this context between the Boolean notion of identity and the arithmetical notion of equality. The point of this discussion in the paper is to argue that when the terms of the logic are second intentional rather than first, the converse of the first quoted sentence is true. What is important for present purposes though is that the passages about second intentions in "Logic of Mathematics" are a precursor to the system of second order logic that Peirce discusses in his 1885 paper "On the Algebra of Logic: A Contribution to the Philosophy of Notation." Again, this paper will be discussed in detail in the next chapter, but it develops Peirce's account of quantification by presenting systems of propositional, first-, and second-order logic. However, rather than "first order" or

12. The distinction between "comprehension" and "extension" has to do with something like the range of predication of a term. The extension of a term is roughly the set of individuals or objects that it can truthfully refer to. The comprehension of a term is the set of other terms, qualities, or classes that it refers to. So, for example, the comprehension of "canidae" would be all of the subclasses of organisms it contains while its extension would be the number of individuals in each of those subclasses.

“second order” logic, Peirce calls these systems the “First-” and “Second-intentional Logic of Relatives,” with the propositional system being called “Non-relative Logic.” Peirce evoked the second-intentional system decades later in his explanation of his categories to Welby and Russell at the beginning of the 20th century.

Welby was a wealthy, self-educated Englishwoman, who maintained a vast correspondence with a number of prominent scholars in the 19th and early 20th century. She had an intense philosophical interest in issues surrounding language, like accounts of meaning and signification, and conducted her own independent scholarship in these areas.[13] Among her list of correspondents are not only Russell and Peirce, but also F. H. Bradley, William James, Cook Wilson, O.K Ogden, Mary Everest Boole, G. F. Stout, Giovanni Vailati, and Henri Bergson (Petrilli 2009). Her correspondence with Peirce began in 1903 when Welby sent a letter requesting that Peirce read her book, *What is Meaning?*, and provide her with comments if he was so inclined (Hardwick and Cook 1979, 2). Peirce wrote back that he had already received the book and was hoping to review it in *The Nation*. Peirce pointed out some of his own published work on meaning to Welby, including the *Illustrations of the Logic of Science* series where he develops his pragmatism. Peirce found a sympathetic reader in Welby and, as we have already seen, proceeded to expound on his categories and theory of representation to her in a 1904 letter. Welby shared this letter to her with a number of her other correspondents, including Russell. A year earlier Peirce had published his review of Welby’s book alongside Russell’s *Principles of Mathematics*, both of which are discussed in the letter. The relevant portions come when Peirce begins his explanation of thirdness:

> I now come to Thirdness. To me, who have for forty years considered the matter from every point of view that I could discover, the inadequacy of Secondness to cover all that is in our minds is so evident that I scarce know how to begin to persuade any person of it who is not already convinced of it. Yet I see a great many thinkers who are trying to construct a system without putting any thirdness into it. Among them are some of my best friends who acknowledge themselves indebted to me for ideas but have never learned the principal lesson. Very well. It is highly proper that Secondness should be searched to its very bottom. Thus only can the indispensibleness and irreducibility of thirdness be made out, although for

13. For example, see (**welby__iiisense__1896**; Welby 1893)

> him who has the mind to grasp it, it is sufficient to say that no branching of a line can result from putting one line on the end of another. My friend Schröder fell in love with my algebra of dyadic relations. The few pages I gave to it in my Note B in the 'Studies in Logic by Members of the Johns Hopkins University' were proportionate to its importance. His book is profound, but its profundity only makes it more clear that Secondness cannot compass Thirdness (He is careful to avoid ever saying that it can, but he does go so far as to say that Secondness is the more important. So it is, considering that Thirdness cannot be understood without Secondness. But as to its applications, it is so inferior to Thirdness as to be in that aspect quite in a different world.)... As to my algebra of dyadic relations Russell in his book which is superficial to nauseating me, has some silly remarks, about my "relative addition" etc. which are mere nonsense. He says, or Whitehead says, that the need for it seldom occurs. The need for it *never* occurs if you bring in the same mode of connection in another way. It is part of a system which does not bring in that mode of connection in any other way. In that system, it is indispensible. But let us leave Russell and Whitehead to work out their own salvation. The criticism which I make on that algebra of dyadic relations, with which I am by no means in love, though I think it is a pretty thing, is that the very triadic relations which it does not recognize it does itself employ. For every combination of relatives to make a new relative is a triadic relation irreducible to dyadic relations. Its *inadequacy* is shown in other ways, but in this way it is in a conflict with itself *if it be regarded*, as I never did regard it, *as sufficient for the expression of all relations*. My universal algebra of relations, with the subjacent indices and Σ and Π is susceptible of being enlarged so as to comprise everything (29–30).[14]

The part of this quote that leads to a connection between the categories and first and second intentions is the last sentence but I have to say quite a lot about Peirce's systems of logic to make that connection clear. Peirce's expression of ire towards Russell here is the result of this sentence from *Principles of Mathematics*:

> Peirce and Schröder have realized the great importance of the subject, but

14. I sincerely apologize for the length of this quote and regret Peirce's blatant disregard for paragraph breaks.

> unfortunately their methods, being based, not on Peano, but on the older Symbolic Logic derived (with modifications) from Boole, are so cumbrous and difficult that most of the applications which ought to be made are practically not feasible (B. Russell 1903, 24).

At the time Russell's *Principles* was released, Schröder's text, *Vorlesungen über die Algebra der Logik*, was likely the most widely read text on mathematical logic in academe. Throughout his work, Schröder expresses his gratitude towards Peirce for the development of quantification. The text at the heart of Peirce's influence on Schröder's work is the former's 1883 *Studies in Logic: By the Members of John Hopkins University.* The book is a collection of papers by Peirce and his students. In it, O. H. Mitchell makes some strides towards what we today would call "first order logic" and Peirce carries the ball a bit further in an appendix, called "Note B." However, Peirce views this system as incapable of expressing thirdness, which is why he says "the very triadic relations which it does not recognize it does itself employ" (Hardwick and Cook 1979, 30). It makes use of thirdness but is not capable of expressing it in general. To do that, we need Peirce's quantifiers, Π (universal) and Σ (existential), as well as his "subjacent indices." Now, both of Peirce's quantifiers make an appearance in "Note B" as well as in Schröder's *Vorlesungen.* Absent in both is any mention of "subjacent indices." Thus, it must be the inclusion of these that enables one to have a logic of thirdness.

The phrase "subjacent indices" only crops up a handful of times in Peirce's corpus: the first in his 1885 "On the Algebra of Logic" and a few subsequent times in his experiment reports for the U.S. coastal service. In the logic paper, the term "indices" comes up constantly but doesn't receive the "subjacent" prefix until the last section, called "Second-intentional Logic." While delivering a definition of the identity predicate, he writes:

> If we please, we can dispense with the token q, by using the index of a token and by referring to this in the Quantifier just as subjacent indices are referred to. That is to say, we may write
>
> $$1_{ij} = \Pi_x(x_i x_j + \overline{x}_i \overline{x}_j)$$
>
> (Peirce 1995, 185).

The formula can be read as 'i and j are identical if, and only if, for all properties X, either X is true of i and j or X is false of i and j.' "Subjacent indices" might only refer to the variable printed slightly below the predicates. However, in the section on first-intensional logic these are simply referred to as "indices." They only receive the "subjacent" tag when they are attached to another variable which is bound to a quantifier, so it seems likely that the term refers to an entire complex, like $\Pi_x x_i$. Since that kind of predicate is only possible in a second-order system, and since the "second-intentional" system is the most radical departure from that of "Note B," it is reasonable to conclude that Peirce's second order logic is the one capable of expressing thirdness in general. It is by allowing quantification over predicates that the algebra is "enlarged so as to comprise everything" (Hardwick and Cook 1979, 30). And if second order logic is the logic of thirdness, then its subject, second-intentional concepts or predicates, must be thirds. This would make first-intentional concepts seconds.

The interpretation of Secondness as first-intentional concepts and Thirdness as second-intentional concepts is also consistent with Peirce's metaphysical construal of his categories, where he discusses them as modes of being. In that context, Secondness is associated with actuality and Thirdness is associated with law, natural or otherwise:

> A brute force, as, for example, an existent particle, on the other hand, is nothing for itself; whatever it is,... its being is actual, consists in action, is dyadic. That is what I call *existence*. A reason has its being in bringing other things into connexion with each other; its essence is to compose: it is triadic, and it alone has a real power (CP 6:343, 1907).

Given that first-intentional concepts are supposed to be those used to pick out or signify the objects of actual experience, it would make sense that these would be associated with existence. Peirce's description of Thirdness in this context also seems exemplary of second intensions, given that they are essentially the conceptual bridges between ideas.

But how are we to interpret Firstness on this picture? One aspect that differentiates Firstness from the other two categories is the notion of extension. Seconds refer to (or perhaps *are*) the ordinary objects of experience, and hence have those objects in their extension. Likewise, Thirds refer to other concepts and have those in their extension. This is apparently not the case when it comes to Firsts:

> The First is that whose being is simply in itself, not referring to anything

> nor lying behind anything (Peirce 1992, 248, c. 1887–88).
> The mode of being of that which is whatever it is positively and regardless of anything else... It is said that firstness is not attributed to simple qualities as embodied in subjects, but in the peculiar positive suchness which... makes each what it is and which is not affected by the embodiment of the quality (MS R 1147, c. 1901-1902).

In these two definitions, Firsts do not refer to anything outside of themselves, so in a way they do not really have extension, or at least they have no real objects in their extension. This makes sense in the context of the 1867 paper, where Peirce tells us we arrive at qualities (Firsts) by abstracting them away from facts or objects (seconds). Sometimes he refers to this kind of abstraction as "prescision": "The category of first can be prescinded [sic.] from second and third, and second can be prescinded from third. But second cannot be prescinded from first, nor third from second (CP 1: 353, c. 1885)." Since the act of prescision forms a First by essentially removing it from its object, it sets it a part from any object it might refer to.

Now, since there is no corresponding notion in the scholastic doctrine of first and second intentions, something new needs to be introduced to correspond to the extensionless Firsts. So, we might call Firsts, null-intentional concepts, Seconds, first-intentional concepts, and Thirds, second-intentional concepts. Peirce's remarks about prescinding between categories also seem evidence to support the claim that the second organizing principle must be something akin to a type hierarchy since it is apparently only possible in one direction. If this is correct, then there is also a rather pleasing symmetry between the categories and the systems of logic discussed in Peirce's 1885 paper, "On the Algebra of Logic: a Contribution to the Philosophy of Notation." These three systems are what we today would call propositional, first order, and second order logic but Peirce referred to them as Non-relational Algebra, First-intentional Logic, and Second-intentional Logic.

Category	**Intentionality**	**Extension**	**Logic**
Firsts	Null-intensional	None	Non-relational Algebra
Seconds	First-intensional	Objects	First-intentional Logic
Thirds	Second-intensional	Concepts	Second-intentional logic

Figure 1.3: Categories and Associated Logical Systems

1.3 Conclusion

Before turning to the application of the categories to the domain of logic, I would like to consider Peirce's arguments when it comes to vindicating his approach. Given how prevalent the categories are in Peirce's corpus, we might expect to find strong evidence for why we should accept them and the distinctions they lead to. Peirce seems to waffle on this, sometimes justifying them by appeals to the logic of relations, sometimes by phenomenological arguments where he seems to suggest they are evident on direct inspection of ordinary experience (Burch 2024). As far as the logic of relations goes, Peirce argues that every relation is reducible to either a monadic, dyadic, or triadic relation, and that these are irreducible to each other. Peirce makes this argument in (Peirce 1998, §12) in perceived response to Alfred Bray Kempe (Kempe 1886). This "reduction thesis" was eventually proven in (Burch 1991). The same result can be reached for monadic and dyadic relations, but this requires different assumptions (W. V. Quine 1937).

However, Peirce must not have been satisfied with such an argument as elsewhere he seems to suggest that he accepts the three categories provisionally, since he has been unable to refute them despite every effort to do so. This line of thought is observable in the above quotation from his letter to Lady Welby, as well as in the following from his failed 1902 application for a Carnegie Grant:

> Long after, when I had developed the only effective methods of doing the one thing and the other, that is, of rendering my apprehension clear and of finding the forms of combination of the categories, I ascertained that the latter was from the nature of things, not to be compassed by mere hard thinking, that it was necessary to wait for the compounds to make their appearance, and patiently to analyze them, until the list down to a certain point was complete. But, not then knowing this, after years of fruitless effort (I will not say they were wasted, since they gave me great training,) I said to myself, this list of categories, specious as it is, must be a delusion of which I must disabuse myself. Thereupon, I spent five years in diligently, yes, passionately, seeking facts which should refute my list. Never in my life have I been more thoroughly in earnest than I was in that long struggle. It was in vain. Everything that promised to refute the list, when carefully examined only confirmed it. The evidence became

> irresistible. Then that in which I had failed must be feasible.
> But it never proved so; and at length I learned why it could not prove so. To this solution I was guided by the very categories themselves. Then began the long work of collecting the compounds and analyzing them into the categories. This work is of its nature absolutely interminable. It involves a logical doctrine which can never be completed. But it was now worked up to the point at which the general method of research could be made evident to every mind (MS L75).

Here Peirce makes no mention of the logic of relations, phenomenology, or the unification of experience, which he previously used to justify his categories. Rather, he appears to suggest that the theory is more like a hypothesis that should be accepted on the basis of its fruitfulness in analysis and lack of contrary evidence. It seems to be a "the proof is in the pudding" style of defense, which is perhaps a bit weak. He also appears to give us a reason why we should not expect knock-down evidence supporting the theory: that the analytic method suggested by his category theory is interminable. When he mentions 'compounds,' presumably he means combinations of categorical elements, which can be analysed according to the theory again, perhaps under different forms or from different perspectives. All the possible combinations together with the fact that the triadic divisions seem always capable of being carried out further, creating new internal and external divisions among the subject matter under consideration, does seem to lend support to the idea that this would be an interminable process. Thus, Peirce seems to regard his category theory as providing a fruitful, though unprovable, method of analysis.

Chapter 2

Peirce's Symbolic Logic

I believe Peirce's categories are behind each of his most impressive innovations in logic in some way or another: from his anticipations of modern modal logic to his work on non-bivalent logic. The main aim of this chapter is to show their involvement in the development of his most well-known claim to fame in logic: his discovery of quantification theory. More specifically, I argue that the sign distinctions generated by his triadic method of conceptual analysis gave him the insights necessary to develop his general logic of relations. Of course, Peirce is overshadowed in these regards due to Frege's development of quantification in 1879, six years before Peirce. While Peirce does not *officially* attain full-blown quantifier logic until 1885, that work springs from a collection of studies on Boolean Algebraic logic dating back at least 15 years earlier. Peirce and Frege seem somehow to have been wholly unaware of each other, despite the significant overlap of their work and their contemporaneity. This is likely due in part to Frege's obscurity before the popularization of his work at the hands of Russell, as well as Peirce's intermittent participation in professional scholarship. There is evidence to suggest that Peirce at least glanced at Frege's name at some point, since *Begriffsschrift* appears in the bibliography of Christine Ladd-Franklin's contribution to *Studies in Logic by the Members of John Hopkins University*. However, that is the closest point of connection between the two scholars. While there is some dispute as to who holds priority here, I will resist the urge to weigh in. However, there are some interesting differences between Peirce and Frege's approach that I would like to point out at the outset.

While Frege's engagement with the overall history of logic is somewhat shallow, Peirce's work in the field is steeped in tradition. Peirce's formal work in logic is situated within the Boolean tradition but draws on notions deep in the field's past. In his discussions of logic, Peirce brings up Boole, De Morgan, Dedekind, and Peano, but also Avicenna (Ibn Sīnā), Ockham, Duns Scotus, Cartesian Logic, Mill, and Hamilton. This makes Peirce's work continuous with logic's past in ways that Frege's work is not (at least on some interpretations of Frege's work). The fact that Schröder's work, which makes extensive use of Peirce's quantifiers, was more widely read than Frege's, also makes Peirce's work continuous with what came after. Lowenheim, Skolem, and Peano worked with Peirce's notation and, prior to his collaboration with Russell, so did Whitehead (Brady 2000; Putnam 1982). None of this is to throw shade on Frege. His work on logic is certainly much more systematic than Peirce's. However, historian's obsession with Frege has had a warping effect on our views of logic in the 19th century that drives out discussion of other important historical trajectories, like the tradition Peirce was a part of but also that of Brentano's school for example. To find evidence of this obsession we can look to Quine, Dummett or Van Heijenoort:

> Logic is an old subject, and since 1879 it has been a great one (W. V. O. Quine 1966, vii).
>
> The publication of *Begriffsschrift* marked, as Quine says, the beginning of modern logic... It is astonishing because it has no predecessors: it appears to have been born from Frege's brain unfertilized by external influences (Dummett 1981, xxxv).
>
> Boole, De Morgan, and Jevons are regarded as the initiators of modern logic, and rightly so. The first phase, however, suffered from a number of limitations... Considered by itself, the period would, no doubt, leave its mark upon the history of logic, but it would not count as a great epoch. A great epoch in the history of logic did open in 1879, when Gottlob Frege's *Begriffsschrift* was published (Van Heijenoort 1967, vi).

Remarks such as these lead to a dramatic oversimplification of a complex and rich history.

In this chapter, I will argue that Peirce's invention of the quantifier is a result of applying one facet of his category theory, his division of signs, to logic. To this end, in section 2.1 I will discuss Peirce's categorical analysis of the elements of logic, such as terms, propositions, arguments, modes of inference, etc. In section 2.2 and 2.3, I argue that Peirce's categorization of signs that led to his development of quantification theory and give the history of this development. Peirce's invention of the quantifiers for first and second order logic was a result of his realization that indices, in the form of special logical pronouns, were needed to unite Boole's algebra of logic with Aristotelian syllogism and provide a general logic of relatives. I conclude with some brief reflections about Peirce's symbolic notation.

2.1 The Categories Applied to Logic

Peirce invented his category theory with multiple purposes in mind, from providing genera for phenomenology, metaphysics, theories of perception, etc. However, he thought the categories worked best when applied to logical analysis, i.e. when used to unearth the constituents in arguments and inferences. In 1903, he writes: "I dwell upon this rule of division, because, however it may be in other fields, in logic I am pretty sure that the divisions [it leads to are] the most important. At any rate, though I have never relied upon it, not seeing any clear reason why it should be so, and feeling sure that nothing in logic can be universally true without a reason, yet I have invariably found its suggestions useful. They certainly ought not to be neglected" (MS 464 or 465, pp 102-104). Here it may seem like Peirce is suggesting that the importance of his categories lies in their usefulness as a heuristic device for developing his logical theory. While it is certainly true that he thought this as well, I suspect his infatuation with them goes beyond their heuristic facility. He uses the term 'division' here because the categories are a framework for analysis. In fact, one of Peirce's earliest applications of this framework was an attempt to show how non-deductive forms of reasoning, i.e. induction and hypothesis, eventually converge on facts when properly applied.

Peirce applied his categorial analysis to logic as early as 1867. In *On a New List of Categories*, he begins by giving the three kinds of representations: icons, indices, and symbols. He tells us that logic is concerned with symbols, which come in a further triad: terms (which are Firsts), propositions (Seconds), and arguments (Thirds) (Peirce 1992, 8). Later, in 1878, he will also come to claim that there are three kinds of argument: hypothesis[1], induction, and deduction (186–199). However, Peirce does not give a more sustained treatment until he writes an outline for a book, *A Guess at*

1. He also refers to hypothesis as abduction or retroduction.

the Riddle, written between 1887-1888. The book, never published or completed, was one of Peirce's several attempts to articulate his "guess at the secret of the sphynx" (Peirce 1992, 242).[2] Houser claims that in 1886, Peirce believed he may have stumbled upon "the key to the secret of the universe" (242). He begins the usual way:

> Chapter 2. The triad of reasoning. Not touched. It is to be made as follows. 1. Three kinds of signs; as best shown in my last paper in the *Am. Jour. Math.* 2. Term, proposition, and argument, mentioned in my paper on a new list of categories. 3. Three kinds of argument, deduction, induction, hypothesis, as shown in my paper in *Studies in Logic* (245).

Each member of these triads is further analysed using the First-Second-Third framework. For instance, he goes on to give a triad of terms: "4. Three kinds of terms, absolute, relative and conjugative, as shown in my first paper on Logic of Relatives. There are various other triads which may be alluded to" (245). And a further triad of propositions:

> The dual divisions of logic result from a false way of looking at things absolutely. Thus, besides affirmative and negative, there are really probable enunciations, which are intermediate. So besides universal and particular there are all sorts of propositions of numerical quantity. For example, the particular proposition Some A is B, means At least one A is B. But we can also say At least 2 A's are B's... We pass from dual quantity, or a system of quantity such as that of Boolian algebra, where there are only two values, to plural quantities (245–246).[3]

The book would have been meant to trace out the categories throughout philosophy and the sciences, the second chapter of which was to be about the categories in logic. While the book never materialized, from this passage we can begin to see how his categorial analysis of logic manifests. Here Peirce even projects his categories onto the kinds of proposition his logic of relatives, which is equivalent to ordinary predicate logic, is intended to analyze: particular propositions are a kind of First, universals a kind of Third, with propositions about discrete multitudes being the interim Second.

2. The quoted text is from an 1886 letter to William James describing his category system.

3. The articles he refers to in this quote may be found in (Peirce 1995, 451–462), in the 1867 article just mentioned, in Note B of (Peirce 1883), and in §3 and 39 in (Peirce 1982), respectively.

Around 1892 (see CP 3:421), the term-proposition-argument triad receives a terminological update. He describes the change most clearly in 1904:

> In regard to its relation to its signified interpretant, a sign is either a Rheme, a Dicent, or an Argument. This corresponds to the old division Term, Proposition, & Argument, modified so as to be applicable to signs generally... A rheme is any sign that is not true or false, like almost any single word except 'yes' or 'no', which are almost peculiar to modern languages. A *proposition* as I use the term, is a dicent symbol. A dicent is not an assertion, but is a sign capable of being asserted. But an assertion is a dicent (MS L 463, 1904).

Here, term and proposition have been replaced with rhemes and dicents. The change is a result of Peirce's gradual shift in understanding of the relationship between logic and semeiotic, which he understands as the science of representation and signification.[4] Peirce believed the two were related as far back as 1867 (see (8)), but over time the relation shifted from overlap to wholesale identity (I discuss this in more detail in Chapter 4). Rhemes, as Peirce tells us here, are the same thing as words when they are considered independently of the sentences they might compose. We can also form a rheme by taking a proposition and replacing parts of it with blanks, as in '_____ is a butcher' or '_____ loves _____,' or even 'Cain _____ Abel.' Each of these is a kind of predicate for Peirce because they can each be true of an object or, in the last (somewhat odd) case, a relation.

> By a rheme, or predicate, will here be meant a blank form of proposition which might have resulted by striking out certain parts of a proposition, and leaving a blank in the place of each, the parts stricken out being such that if each blank were filled with a proper name, a proposition (however nonsensical) would thereby be recomposed (CP 4:560).

Formulations such as these led Cassius Keyser to claim Peirce's rhemes are an anticipation of Russell's notion of the propositional function (Douglas et al. 1941, 104).[5]

4. See (Ransdell 1981) for further discussion of the rheme-dicent-argument distinction. These notions are also treated extensively throughout (Bellucci 2017), which also discusses the relationship between semeiotics and logic.

5. Keyser was supervisor to Emil Post, another prominent mathematical logician in the early 20th century.

Peirce's 1892 comparison of rhemes "to a chemical atom or radicle with unsaturated bonds" reveals similarities between that concept and Frege's 'unsaturated' functions (CP 3:421).[6] But again, there is no known evidence that Peirce ever read Frege.

Rhemes on their own cannot be true or false because they don't assert anything, they simply are, hence they are firsts according to Peirce's category division. When all of the blanks in a rheme are filled in, we have a dicent, Peirce's new term for a proposition. Since they map onto truth or falsity, they are seconds according to the division, essentially involving two things: the proposition itself and its truth value. Arguments do not appear to have been updated in the same way as terms and propositions, but it is easy to recognize their thirdness, since they involve three things: dicent premises, a concluding dicent, and a reason for it's validity if it is a genuine argument. Peirce often calls these reasons 'leading principles,' and he understands these as something like a habit of mind whereby reasoners are able to move from true premises to true conclusions, something like a derivation rule.[7]

So, what does this update amount to? Peirce tells us that he makes the change to the old triad to "[modify it] so as to be applicable to signs generally," which suggests he thought the connotations of the previous terms too narrow (MS L 463, 1904). There are a couple of ways he might have understood this limitation. In the same passage, he also distinguishes dicents and assertions and this might give a clue to what he means here. He claims that all assertions are dicents, but not the other way around, and this is because dicents do not actually need to be uttered or written down to be a dicent. It's possible that Peirce is generalizing the terms to apply to what potentially could be written or uttered in addition to what is actually written, or uttered. The other, and perhaps more probable, possibility is that they are meant to generalize the notion of terms and propositions even further, to pick out non-linguistic analogues.[8] The triad is meant to be a division of a particular kind of signifying entity in Peirce's sign theory, i.e., symbols[9], and not all of these are linguistic in nature. The quoted

6. See (Heck and May 2013) for discussion of Frege's notion.

7. See the entry for 'leading principle' in (Baldwin 1960) or CP 2:589.

8. See (Stjernfelt 2022) for discussion of non-linguistic rhemes, dicents, and arguments.

9. Calling terms, propositions, and arguments symbols may be a bit imprecise when it comes to the later stages of Peirce's thought. Early on, as can be seen above, Peirce certainly thought this was the case but by 1904, he came to a different understanding. At that time, the rheme-dicent-argument triad and the icon-index-symbol triad have become part of a classificatory scheme involving kinds of signs. The former has to do with the ways signs are related to their interpretants while the latter has to do with the way that signs are related to the objects that they represent. I gloss over these details here to avoid lengthy digressions into Peirce's theory of signs.

material comes from Peirce's correspondence with Welby, and in it he suggests that she is "in danger of falling into some error in consequence of limiting [her] studies so much to language," which seems a point in favor of the latter interpretation.

The definition Peirce gives in Baldwin's dictionary may help to illuminate this point: "Symbol. A Sign (q.v.) which is constituted a sign *merely or mainly by the fact that it is used and understood as such*, whether the habit is natural or conventional, and without regard to the motives which originally governed its selection" (Baldwin 1960, my emphasis). Words and sentences are excellent examples of symbols, since they represent their objects simply in virtue of epistemic agents knowing what they are intended to mean, as opposed to representing objects by some other means, by say resembling their objects. But these are not the only examples of symbols. Suppose you and your roommate have an arrangement where you are to hang a tie or sock on the doorknob when you have company and don't wish to be disturbed. The tie on the doorknob in this case would be a symbol based on the mutual understanding you have of what it signifies. Seeing the tie on the doorknob indicating their wish for privacy could be interpreted as a dicent which, in propositional form, might assert something like "My room mate has company and does not wish to be disturbed." Thus, the updated terminology is an attempt to broaden the domain logic to signification more generally, beyond linguistic modes of signification, to include mental and natural signs as well. Locke and Condillac make a similar move in their writings on signs (Falkenstein and Grandi 2017). This interpretation is consistent with Peirce's late understanding of logic: "Now what is Logic? I early remarked that it is quite indifferent whether it be regarded as having to do with thought or with language, the wrapping of thought, since thought, like an onion, is composed of nothing but wrappings (Peirce 1998, §30)." In other words, it is immaterial to ask whether logic aimed towards thought in general or language, since language is nothing more than the packaging of thoughts. Thus, the rheme-dicent-argument trichotomy allows Peirce to articulate a scope for logic that goes beyond language and thought.

However, the most interesting consequence of Peirce's application of his category theory to logic actually comes from the uses he makes of his theory of signs in terms of icons, indices, and symbols in his analysis of various notational apparatuses used by logicians in the 19th century. Surprisingly, Peirce's advancement of Boolean algebra does not come from considerations concerning symbols, but rather through analysis as of how the other members of the sign triad are operative in logic. In particular, it is Peirce's analysis of indexes in logical notation that enabled to fully articulate first order quantification theory in 1885. Because of this analysis, Peirce is able to spot the importance of a particular kind of index in his logical notation, the variable. This together with his analysis of indefinite pronouns as another kind of index, led him to realize he could appreciate a fully general logic of relations by incorporating quantifiers and their compliment variables. The historical significance of this event cannot be overstated. Since Frege's writing remained obscure until his works were popularized by Russell, Peirce's 1885 work and those that lead up to it effectively amount to the first time quantification theory was put before the logical community (Putnam 1982). It is *effectively* the first time, because Frege's *Begriffsschrift* was not widely read or understood until much later on, when it was popularized at the hands of Russell.

2.2 The Categories Applied to Boole's Algebra

Arguably, a longstanding effort in Peirce's early career as a mathematical logician was to reconcile Boolean algebra with Aristotelian syllogism (Brady 2000, 17). He had noticed deficiencies in the expressive power of Boole's system as early as 1867, and attempted a solution in "On an improvement in Boole's calculus of logic." In the article, Peirce aimed to modify Boolean Algebra to express particular statements about arbitrary individuals, like 'some *a*.' This was intended to solve a problem for Boolean algebra, since it was incapable of handling valid forms of argument that involve arbitrary or singular individuals, which can occur within the context of syllogism. Peirce first tried to address this issue using an operation he calls 'logical subtraction':

> Boole does not make use of the operations here termed logical addition and subtraction. the advantages obtained by the introduction of them are three, viz. they give unity to the system; they greatly abbreviate the labor of working with it; and they enable us to express particular propositions. Let i be a class only determined to be such that only some one individual of the class a comes under it. Then $a - i, a$ is the expression for some a. Boole cannot properly express some a (Peirce 1985, 21).

In the quote as Peirce intended it, 'a' is intended to represent a class, '$-$' is logical subtraction (or set subtraction in modern terms), and ',' in the expression indicates the logical multiplication (intersection) of the two sets. So, $a - i, a$ should be taken to read 'the intersection of the set a less a singelton i and the set a' which unfortunately would denote everything in a except i.[10] That's a far cry away from 'some a.' Brady raises the possibility that the issue arises from a transcription error, suggesting that Peirce may have intended that the second a be negated (Brady 2000, 17). The result would better match what Peirce appears to believe he is expressing. However, the result is still inadequate because it only signifies a singleton set by Peirce's stipulation. Even if that were adequate, Peirce has only succeeded in selecting a class containing one element, when what is needed is to select the element itself (17). Still, the paper made an advancement by interpreting expressions which use logical addition in the inclusive rather than exclusive sense, which added symmetry to the system since it could now prove De Morgan's laws.

Peirce comes closer to uniting Boole's logic with the syllogism in 1870 in "Description of a notation for the logic of relatives" (DNLR), where he introduces the conceptual resources for a logic of relations for the first time. Specifically, Peirce's system incorporates three kinds of terms: absolute (First), simple relative (Second), and conjugative (Third). Plainly, this analysis of terms is based on his category theory. According to Peirce, the problem with Boole's system is that it is limited to the treatment of the first of there, i.e. 'absolute' terms.

> Now logical terms are of three grand classes. The first embraces those whose logical form involves only the conception of quality, and which therefore represent a thing simply as "a –." These discriminate objects in the most rudimentary way, which does not involve any consciousness of discrimination. They regard an object as it is in itself as *such (quale)*; for example, as horse, tree, or man. These are *absolute terms* (Peirce 1985, 365).

At the outset of the essay Peirce praises De Morgan and claims that his work allows for a "slight treatment" of relative terms (359). However, he claims that De Morgan's system leaves something to be desired and after complimenting the "singular beauty" of Boole's system, proposes to extend the latter to deal with relative terms alongside absolute ones (359). Peirce's move here is to merge the two systems and argue that logic needs to include simple relative and conjugative terms as well.

10. This inadequacy is mentioned in (Brady 2000, 16–17) and (Hailperin 1986, 122–123).

> The second class embraces terms whose logical form involves the conception of relation, and which require the addition of another term to complete the denotation. These discriminate objects with a distinct consciousness of discrimination. They regard an object as over against another, that is as relative; as father of, lover of, or servant of. These are *simple relative terms*. The third class embraces terms whose logical form involves the conception of bringing things into relation, and which require the addition of more than one term to complete the denotation. They discriminate not only with consciousness of discrimination, but with the consciousness of its origin. They regard an object as medium or third between two others, that is as conjugative; as giver of – to –, or buyer of – for – from –. These may be termed *conjugative terms* (Peirce 1985, 365).

In this context, Peirce's understanding of 'terms' is consistent with our previous discussion of rhemes.[11] Peirce understands them both as the Firsts in the division of symbols as well as the elements of the logical systems he is analyzing. While he has not yet updated 'term' to 'rheme' in 1870, the examples of conjugative terms that he gives are so similar to what he describes in 1892 and on wards that we might consider this an antecedent notion. The blanks in the examples are quite similar to the "unsaturated bonds" he refers to in (CP 3:421) and in that passage he is also discussing relative terms.

It is quite clear that his usual categorical analysis is being applied in his analysis of terms. Peirce spells this out himself:

> The conjugative term involves the conception of THIRD, the relative that of second of OTHER, the absolute term simply considers AN object. No fourth class of terms exists involving the conception of *fourth*, because when that of *third* is introduced, since it involves the conception of bringing objects into relation, all higher numbers are given at once, inasmuch as the conception of bringing objects into relation is independent of the number of members of the relationship (365).

The reason there does not need to be Fourths, Fifths, etc., seems to be that the Third type of term, as Peirce believes it properly construed, ranges over everything. Because

11. There are similarities between Peirce's distinction between absolute and relative or conjugative terms and Frege's distinction between arguments and functions. Bolzano also makes a similar distinction in his writings on logic.

of this, Peirce claims all higher order terms can be constructed out of complexes of Thirds. So, for example, a Fifth could be constructed using a Third whose constituents are another Third, and two Firsts, and a Fourth could be a Third whose members are a Second and two Firsts, and so on. Thus, once Peirce has three blanks or argument places that can be filled with other terms, that is enough for him to build terms and relations of any arity.

The system Peirce presents in DNLR introduces different type faces to distinguish each type of term, so I will use 'a, b, c, ...,' for absolutes, 'a, b, c, ...,' for simple relatives, and '$\mathfrak{a}$, $\mathfrak{b}$, $\mathfrak{c}$ ...,' for conjugatives. See figure 2.1 to compare how this differs from Peirce's original type face.

itself is made perfectly evident by the study of the logic of relatives. I shall denote absolute terms by the Roman alphabet, a, b, c, d, etc.; relative terms by italics, *a, b, c, d,* etc.; and conjugative terms by a kind of type called Madisonian, *a, b, c, d,* etc.

I shall commonly denote individuals by capitals, and generals by small letters. General symbols for numbers will be printed in black-letter, thus, $\mathfrak{a}$, $\mathfrak{b}$, $\mathfrak{c}$, $\mathfrak{d}$, etc. The Greek letters will denote operations.

Figure 2.1: Peirce's type faces for each kind of term.

In addition to terms Peirce's logic, just like that of Boole and Jevons, uses connectives that designate Boolean-type operations over terms. He uses multiplication '$(ab)c$' to indicate intersection and conjunction, and '$+_,$'[12] to indicate union and inclusive disjunction. Negation is expressed by '$\overline{a}$' for terms. '$\overline{(a +_, b)}$' is used when a connective falls within the scope of a negation. Peirce uses the sign of illation, '$\prec$', which is not present in Boole's or Jevons's system, as his primitive connective in the system, which can be understood as set inclusion or material implication. Peirce's use of the symbol for both operations earned he and Schröder criticism from Frege and Russell who perceived the dual usage as an ambiguity, though I know of no technical issue or error in reasoning that arises from this.[13]

In DNLR, Peirce primarily interprets the symbol '$\prec$' as set inclusion and '$\overline{\prec}$' as its negation. Set inclusion is an operation used to indicate that one set is contained

12. The comma here is actually a subscript, meant to differentiate the Boolean-style operation from ordinary addition.

13. Russell raises the issue in *Principles of Mathematics* (B. Russell 1903) and Frege does in*Die Grundlagen der Arithmetik* (Frege 1958). See (Sluga 1987) for more discussion on Frege's distaste with the Booleans.

in another or that an individual element is contained in a set. In modern notation, if we wish to express 'the set A is contained in the set B' we would write '$A \subset B$.' If we want to express that an individual element is included in 'B,' we would write '$a \in B$.' We can use this notation to express propositions, like 'all dolphins are mammals' or 'something is a dolphin' i.e., $D \subset M$ and '$\exists x(x \in D)$. With the apparatus available in 1870, Peirce can use his notation to give an equivalent expression of all of the formulae in this paragraph except the last, as I will go on to explain.

Peirce did not give anything like the rules of construction that we might ordinarily expect to see in a modern logic textbook. He also did not have any notion of well-formed formulae. The lack of systematic and organizational concepts such as these makes DNLR a rather messy affair. However, it isn't too difficult to imagine how this would have gone, if Peirce had been aware of such things:

1. All absolute terms, a, b, c, ..., relative terms, $a(...)$, $b(...)$, $c(...)$, ..., and conjugative terms, $\mathfrak{a}(...)$, $\mathfrak{b}(...)$, $\mathfrak{c}(...)$ ..., where the (...) are filled with absolute relative or conjugative terms, are well-formed formulae.

2. If ϕ is a well-formed formula, then so is $\overline{\phi}$.

3. If ϕ and ψ are well-formed formula, then so is $\phi \prec \psi$

4. Nothing else is a well-formed formula.

So, if p and q are absolute terms, r is a relative term, and $\mathfrak{f}$ and $\mathfrak{g}$ are conjugative terms, then: By 1, $r(\mathrm{p})$, $\mathfrak{f}(r(\mathrm{p})\mathrm{q})$, and $\mathfrak{f}(r(\mathrm{p})\mathfrak{g}(\mathrm{pq}))$ would all be well-formed formulae. By 2, $\overline{r(\mathrm{p})}$ is a well-formed formula. By 3, $\mathfrak{f}(r(\mathrm{p})\mathfrak{g}(\mathrm{pq})) \prec \mathfrak{f}(r(\mathrm{p})\mathrm{q})$ is also a well-formed formula.

Peirce does not use parentheses to group the correlates of relative or conjugative terms so, the ones in 1 and the previous examples are simply there to indicate that each has argument places and are not absolute terms. Absolute terms have no argument places, simple relatives have one, and conjugative terms have any number. Peirce follows De Morgan in interpreting his formulae according to a 'universe' which is quite similar to the contemporary notion of 'universe of discourse.' Using Peirce's universe in figure 2.2, we can look at some examples.

a. animal.
b. black.
f. Frenchman.
h. horse.
m. man.
p. President of the United States Senate.
r. rich person.
u. violinist.
v. Vice-President of the United States.
w. woman.

a. enemy.
b. benefactor.
c. conqueror.
e. emperor.
h. husband.
l. lover.
m. mother.
n. not.
o. owner.
s. servant.
w. wife.

$\mathfrak{g}$. giver to — of —.
$\mathfrak{w}$. winner over of — to — from —.
$\mathfrak{b}$. betrayer to — of —.
$\mathfrak{t}$. transferrer from — to —.

Figure 2.2: Peirce's assignment of terms in DNLR

'hw' would say something like 'a husband to a woman.' '$\mathfrak{b}$p$a$mf'[14] would mean something like 'betrayer to the president of a man who was the enemy of a Frenchman' but the conjugative terms can easily become convoluted. A more straightforward example, using set inclusion, would be '$f \prec m$, read 'all Frenchmen are men.'

Given the lack of parentheses together with the fact that conjugative terms can have simple relatives as well as other conjugative terms within their scope, Peirce thought there may be ambiguities, or at least difficulties, with keeping track of which correlate belongs to which conjugative or relative. While he could have gotten around the issue by using brackets, stipulating that conjugative terms be interpreted as ordered tuples, or employing a set of rules for reading order, these options did not occur to him. Instead, Peirce attaches marks of reference to the subscript of each relation that correspond to superscripts preceding its correlates. Thus,

$$\mathfrak{g}_{\dagger\ddagger}\,{}^{\dagger}l_{\|}\,{}^{\|}\mathrm{w}\,{}^{\ddagger}\mathrm{h}$$

says "giver of a horse to the lover of a woman."

Guided by his triadic method of analysis of terms, Peirce is able to come up with a system that is clearly much more expressive than Boole's. The system has the ability

14. Peirce's examples are equally ugly.

to express statements like "a head of a cow is a head of a mammal,"[15]

$$h_{\dagger}\,{}^{\dagger}\mathrm{c} \prec h_{\dagger}\,{}^{\dagger}\mathrm{m}.$$

The marks of reference included in these formula serve a role not entirely dissimilar to that of names in modern predicate logic notation. In fact, using this notation Peirce can express any quantifier free sentence of first order logic (Brady 2000, 37). This means that it would be capable of expressing any formula of our modern predicate calculus that does not contain quantifiers with bound variables '$\exists x$' or '$\forall x$.' For example, if we wanted to express "An owner of a horse is an owner of a saddle" in modern notation, we may write $O(h) \rightarrow O(s)$, while Peirce in 1870 would write $o\mathrm{h} \rightarrow o\mathrm{s}$. They are also interesting insofar as they arguably anticipate Peirce's eventual inclusion of quantifier-bound variables. The marks serve as something close to variables but at this stage they are free.

While DNLR is a success insofar as it increases the expressivity of Boolean Algebra by a large margin, it still falls short on achieving the expressivity of a fully general logic of relations. Their are straightforward examples of relations that Peirce cannot express with the apparatus of DNLR, for instance, 'someone loves everyone.' To achieve this, Peirce needed to look beyond his analysis of symbols and terms and that begins 13 years later. The paper is merely a stepping stone. All of its devices were abandoned in Peirce's subsequent writings on logic aside from the use of $\prec$ for material implication. DNLR coincides with the first printing of Peirce's father's *Linear Associative Algebra*, which the younger Peirce had been reading and editing. According to Brady, DNLR is largely the result of Peirce's noticing analogous applications between his father's work on algebra and logic (48). Peirce needed to step outside the algebraic tradition to fully articulate quantification and this wouldn't happen until he was inspired by his student, O. H. Mitchell, at the newly created Johns Hopkins university, to incorporate a different kind of sign into his notation, i.e. indices. While the diacritical marks he affixes to his terms are a step in this direction, they do not carry him far enough.

Peirce's logic of relations receives an update in the 1880 work, "On the algebra of logic," published in the *American Journal of Mathematics*, shortly after Peirce entered his appointment at John Hopkins in 1879. Little progress is made there on the problem of quantifiers. However, he does introduce quasi-natural deduction

15. Peirce does not give this example himself.

rules to his propositional logic based on $\prec$ (73). In doing so, he steps outside of the algebraic mold insofar as he applying inference rules rather than reducing equations. Peirce's discussion of these inference rules is rather informal (see (64–70) for a more rigorous reconstruction of what Peirce was trying to do). The rules are derived on the basis of Peirce's identification of inclusion, $\prec$, and the copula, $\therefore$, and what he calls "the principle of identity" (Peirce 1995, 173):

$$x \prec x. \tag{2.1}$$

x and all other lower case letters are treated as variables that can stand for any proposition. Now, on Peirce's understanding of the equivalence between $\prec$ and $\therefore$, he notices that

$$\begin{array}{r} x \\ y \\ \therefore z \end{array} \tag{2.2}$$

and

$$\begin{array}{r} x \\ \therefore y \prec z \end{array} \tag{2.3}$$

are equivalent argument forms. Because they are equivalent, it is valid to infer either from the other. The inference from 2.2 to 2.3 corresponds to what we might call Peirce's $\prec$-introduction rule, while the move in the other direction would be his $\prec$-elimination rule. Each rule involves moving propositions and introducing or eliminating $\prec$ on the other side of the copula in a way not dissimilar to Gentzen style systems (Brady 2000, 65). The lines preceding the $\therefore$ are taken to be premises joined by a conjunction and what follows are taken to be valid conclusions on their basis. While Peirce does not give step by step proofs using these rules himself, it is possible to show what that might look like. For example, I will prove a simple equivalence,

$$\{x \prec (y \prec z)\} \equiv \{y \prec (x \prec z)\}.$$

Which can be read as "'if x, then if y, then z' is equivalent to 'if y, then if x, then z,'" where x, y, and z can be taken to be any well-formed formula. Starting from

$$\{x \prec (y \prec z)\},$$

we apply $\prec$-elimination to obtain

$$x$$
$$\therefore y \prec z.$$

After another elimination step, we arrive at

$$x$$
$$y$$
$$\therefore z.$$

Then, by $\prec$-introduction we have

$$y$$
$$\therefore x \prec z.$$

Finally, after one more introduction step,

$$\{y \prec (x \prec z)\}$$

we have completed the first direction of the proof. The other direction is equally trivial and so similar to the first direction that I will not bother giving it here.

While Peirce's quasi-inference rules are an incredible anticipation of modern systems in their own right, the 1880 paper does nothing to further the goal of producing a more expressive formal language. Ideally, what he is looking for is a system capable of representing any arbitrary relation. This beyond the goal of uniting Boole's system with syllogistic logic, because even the syllogism is limited in the relations it can express. However, the paper did serve as the basis of a course on logic that Peirce delivered to Mitchell, Ladd-Franklin, and later Dewey.[16]

Peirce was so impressed with the work of some of his students that he edited and published a book containing some of their essays alongside two new papers of his own in 1883. The book is called *Studies in Logic by Members of the Johns Hopkins University*. Gilman, Marquand, Ladd-Franklin, and Mitchell, each contributed a paper to the volume. In the preface, Peirce discusses his students work and how it contributes to the broader projects pursued by Boolean's like himself, McColl, Jevons and others. Commenting on some of the differences between Ladd-Franklin and Mitchell's

16. Dewey does not seem to have liked the course or formal logic, as Houser reports he dropped it because it was "too mathematical" (Peirce 1990, lxi).

work–as published in *Studies in Logic*–and Boole's original system, Peirce notes:

> Mr. McColl and I find it to be absolutely necessary to add some new sign to express *existence*; for Boole's notation is only capable of representing that some description of thing does *not* exist, and cannot say that anything *does* exist... Miss Ladd and Mr. Mitchell also use two signs expressive of simple relations involving existence and non-existence; but in their choice of these relations they diverge both from McColl and me, and from one another.

Both Ladd-Franklin's and Mitchell's algebras do have mechanisms for expressing rudimentary quantified sentences, however, neither go very far beyond what Peirce can do in 1870.

Ladd-Franklin's paper is commonly mistaken as contributing a solution to a problem dating back to Aristotle, that of reducing all of the syllogisms to one (Uckelman 2021).[17] Interestingly, Ladd-Franklin includes *Begriffsschrift* and Schröder's review of it in her bibliography. To the best of my knowledge, this is the only evidence that we have that Peirce had ever come across Frege's name before. Mitchell, on the other hand, was trying to develop a method of carrying out inferences by regarding premises as a polynomial and reducing them to a conclusion (Dipert 1994). The method he employs is similar to the proof-theoretic technique known today as "resolution" in that both convert formulae to disjunctive normal form and proceed by eliminating contradictory terms (520). All in all, it is a successful proof procedure for propositional logic (Brady 2000, 76). While this is arguably the most remarkable feature of the paper, Peirce's admiration of Mitchell seems to have had little to do with this procedure.

Peirce was particularly enamored with Mitchell's paper, describing it as "epoch making" in 1903 and essentially credits him with the invention of the quantifier (CP 4:391). However, upon examining the paper, Peirce's enthusiasm might seem a bit overstated, given that he was not in a position to appreciate its anticipation of resolution. While Mitchell's paper is certainly an impressive step towards the quantifiers, it wasn't quite there yet. In the first part of the paper, Mitchell introduces his own notation for quantified statements, which include two quantifier-like operators, one

17. In actuality, she attempted to resolve an issue due to Jevons, called "the inverse logical problem involving three terms" (Jevons 1877, 137–138). She solved the problem with what she calls "the antilogism" (Ladd-Franklin 1983, 40, 50). See (Uckelman 2021) for more context and discussion.

universal, the other existential.[18] He correctly identifies the two as duals of one another, showing that they are inter-definable. Using these quantifier-like operators, he unites Boole's algebra with Aristotelian syllogism by giving expression to the entire square of opposition. However, the system he employs is not yet the general logic of relations that Peirce is looking for, and this for two reasons: 1) It is only capable of dealing with single-variable monadic predicate logic (Brady 2000, 79). 2) It lacks the mechanisms necessary to deal with quantifiers with mixed scope.

In the second part of the paper, Mitchell goes on to introduce a second new system of Boolean-style logic that does includes two quantified variables in a single formula. However, in that system the quantifiers range over two distinct universes of discourse, "the universe of class terms,... and ... the universe of time" (Mitchell 1883, 87). The former is the ordinary kind of universe of discourse familiar from De Morgan, Peirce, or a standard introductory logic course. The other distinct universe of discourse contains moments in time. When Mitchell uses doubly-quantified expressions, one quantifier ranges over the familiar class terms, while the other ranges over times, so each quantifier has a distinct scope. Taking the universe of class terms to contain a village and its inhabitants, and the other to be the moments of a summer, Mitchell can express "some of the Browns were at the seashore during the summer" but he cannot express "there is someone whom every one of the Browns loves" (87). Mitchell's logic is slightly more expressive than Peirce's in DNLR, since it can handle monadic predicate logic in addition to quantifier free formulae, but still falls short of the general logic of relations. His second system is closer to a primitive temporal logic than it is to full quantification theory (Brady 2000, 86).

Mitchell's systems, especially his temporal one, are certainly interesting but at first glance it is hard to see why Peirce thought they were "epoch making," though his reasons will become clear before the end of this chapter. He has not gotten much closer to the full logic of relations than Peirce did in 1870. While his temporal system might be construed as anticipating aspects of developments that would gain interest in

18. "Let F be any logical polynomial involving class terms and their negatives, that is, any sum of products (aggregates) of such terms. Then the following are respectively the forms of the universal and the particular propositions" : −

$$\text{All } U \text{ is } F, \text{ here denoted by } F_1,$$

$$\text{Some } U \text{ is } F, \text{ here denoted by } F_u$$

(Mitchell 1883, 74).

the middle of the 20th century, Peirce obviously would have no notion of this. So, why was he so impressed? The answer has to do with what Peirce understood to be the implications for his logic, given his analysis of signs as icons-indexes-symbols. While Peirce had this trichotomy of signs as far back as 1867, Mitchell's paper, through its quantifier like devices, appears to have suggested to him that it could be applied fruitfully to logic. The division of signs remained relatively unchanged between 1867 and 1883, so in principle, Peirce could have pursued this earlier. Why he didn't is anyone's guess.

In his 1885 essay, "On the Algebra of Logic: A Contribution to the Philosophy of Notation" (henceforth, CPN), published in the *American Journal of Mathematics*, Peirce tells us "The introduction of indices into the algebra of logic is the greatest merit of Mr. Mitchell's system. He writes F_1 to mean that the proposition F is true of every object in the universe, and F_u to mean that the same is true of some object (Peirce 1995, 164)." Peirce's understanding of Mitchell's epoch making contribution was the incorporation of another kind of sign into the algebraic notation used by the late Booleans. Indices are the Seconds of the sign triad, together with icons (Firsts) and symbols (Thirds). Peirce categorizes these signs according to how they represent their objects. On his understanding, Icons represent by resembling their objects, like how a footprint in the sand resembles my foot.[19] It might be difficult to see how icons bear the null-intentional property I described in chapter one. There is one illuminating passage in CPN on icons that may help:

> The third case is where a dual relation between the sign and its object is degenerate and consists in a mere resemblance between them. I call a sign which stands for something merely because it resembles it, an icon. Icons are so completely substituted for their objects as hardly to be distinguished from them. Such are the diagrams of geometry. A diagram, indeed, so far as it has a general signification, is not a pure icon; but in the middle part of our reasonings we forget that abstractness in great measure, and the diagram is for us the very thing. So in contemplating a painting, there is a moment when we lose the consciousness that it is not the thing, the distinction of the real and the copy disappears, and it is for the moment a pure dream, not any particular existence, and yet not general. At that moment we are contemplating an icon (Peirce 1995).

19. This example was suggested to me by Scott Metzger.

By this description, it seems that there is a sense in which icons become indistinguishable from what they represent. At the end of the quote Peirce insists on their abstract nature and tells us that, at least in some moments, icons do not necessarily represent any particular things. In this way, icons need not refer to anything, granting them the null-intentionality described in chapter one. The other signs are relatively unproblematic on that picture. Peirce understands indexes as representing their objects by direct indication, like how a pointing finger represents what is being pointed at. Symbols, according to Peirce, represent by way of established convention, the way words of ordinary language signify their objects.[20]

In CPN, Peirce uses these categories of signs to analyze logical notation.[21] There, Peirce describes indices as follows:

> The index asserts nothing; it only says 'There!' It takes hold of our eyes, as it were, and forcibly directs them to a particular object, and there it stops. Demonstrative and relative pronouns are nearly pure indices, because they denote things without describing them; so are the letters on a geometrical diagram, and the *subscript numbers* which in algebra distinguish one value from another without saying what those values are (Peirce 1995, 163, my emphasis).

An index represents its object by picking it out, highlighting it, or essentially indicating that it is the subject of a representation or of part of a representation. They are differentiated from icons, which represent their objects by resembling them, and symbols which represent their objects by a pre-established connection to them, usually a convention or some other kind of general rule. In practice, the three are rarely isolatable, so, for example, many symbolic representations will have an indexical component. Pronouns are an excellent example of this, since as parts of speech, they are symbols, but the way in which they denote objects are indexical, as Peirce understands them. Symbols, or tokens as Peirce calls them in CPN, are the most general representational element. In logic, class terms, like 'horses,' are examples of symbols. If I use h to denote this class, then h is an index to the symbol 'horses'.

20. In keeping with the broad scope Peirce wishes to reserve for logic and semeotic, Peirce later broadens the scope of symbols to accommodate natural signification. However, his broader definitions are exceptionally vague, even for Peirce. See (CP 2:304, 307) for examples.

21. For a more detailed and sustained treatment of the ways in which Peirce's sign theory and logic intersect, see (Bellucci 2017).

Mitchell's paper made it clear to Peirce that the notation that the Boolean's were previously using had no seamless way of accounting for indices in general. While the diacritical marks Peirce introduces in DNLR are a kind of index, there was no apparatus for dealing with pronouns, like 'all' or 'some,' in all of the ways they are used in relative propositions. For example, Peirce has roughshod ways of dealing with "All Frenchmen are men," but not "someone loves everyone." In CPN, Peirce argues that any logical notation that fails to incorporate all three kinds of signs will not be adequate to encompass the generality that logic is supposed to have:

> I have taken pains to make my distinction of icons, indices, and tokens clear, in order to enunciate this proposition: in a perfect system of logical notation signs of these several kinds must all be employed. Without tokens [or symbols] there would be no generality in the statements, for they are the only general signs; and generality is essential to reasoning... But tokens [symbols] alone do not state what is the subject of discourse; and this can, in fact, not be described in general terms; it can only be indicated. The actual world cannot be distinguished from a world of imagination by any description. Hence the need of pronouns and indices, and the more complicated the subject the greater the need of them... Indices are also required to show in what manner other signs are connected together. With these two kinds of signs alone any proposition can be expressed; but it cannot be reasoned upon, for reasoning consists in the observation that where certain relations subsist certain others are found, and it accordingly requires the exhibition of the relations reasoned with in an icon (163–164).

Obviously, symbols are going to be required for any symbolic notational system. Indices are essential to any symbolic system because they connect symbols to the contents to be reasoned about. For example, the conventions that link the word 'lion' to the objects that the word denotes, must involve indexicality for Peirce. Indices are also needed to direct our attention to the precise objects being reasoned about, as well as to express the relations holding between symbols. These two are sufficient to express any proposition on Peirce's account, but reasoning requires the first representational element, i.e. the icon, according to Peirce, because he thinks reasoning requires the expectation that we can find the relations other than those stated in the premises but that still resemble them in form. In this way, Peirce understands conclusions as resemblances of premises. Admittedly, even I find Peirce's last point mysterious.

The most important insight that Peirce's analysis of the representational devices used in notation provided, for present purposes, is his understanding of a need to deal with indexicality in general. It isn't so much the case that indices, as Peirce understands them, are not at all present in Boole's system or any of the other precursors. For instance, Peirce considers variables of all types to be a kind of index, and these are clearly present in previous work on logic. Even in his 1847 work, Boole intends

$$x(1-y)=0$$

to be read as "All Xs are Ys".[22] The english sentence that Boole intends to represent obviously contains the kind of pronoun Peirce is identifying with indices, i.e. the word 'all.' The problem, rather, is that the pronoun itself is treated implicitly and it is not reflected in the notation. More specifically, the problem is that the complement of the indefinite pronoun is reflected in the notation, but the pronoun itself is not, and this is precisely the problem that needed to be overcome to give a general logic of relations. An additional index was needed.

As a result of this defect, quantification could only be expressed in so far as it had certain analogies with the operations of logical multiplication and addition rather than as a distinct operation on its own. As Peirce understands them, the quantifiers are straightforward analogues of the pronouns 'all' and 'some.' This is reflected in the way we today read quantified formulae like '$\forall x$...' or '$\exists x$...' as 'For all x...' and 'There is some x...', respectively.

Peirce's efforts prior to 1883 address the poor treatment of quantified statements to a certain extent but he didn't realize how fruitful the inclusion of additional indexical signs was until he saw Mitchell's application of this concept, 13 years after he first tried to unite Boole's algebra with De Morgan's logic. In an undated[23] manuscript comparing Schröder's notation to his own, Peirce writes:

22. Really the formula states that the intersection of the set of Xs and non-Ys is empty, which is somewhat different than the intended meaning though equivalent in this particular case.

23. The manuscript is undated but on the first page Peirce refers to "Prof. Schröder's first two volumes." I know of know other text he could be referring to other than the three volume set, *Vorlesungen über die Algebra der Logik*, which was published in 1890, 1891, and 1895. The second part of volume two was published in 1905 after Schröder's death. So, the manuscript could not have been written prior to 1891 and probably would not have been written earlier than 1895, when volume three came out.

> This simple device of indices was used by me in 1880, or earlier; but I never appreciated its importance until I saw the use made of it in 1883 by Prof. O. C. Mitchell, then my student. Nor did he see its powers until I showed them. The credit of the notation must be divided between us. Not only does this simple device remove the difficulty with regard to particulars but it also furnishes, at once, by attaching two or three or more indices to a letter (especially if we use as *quantifiers*, not merely Π and Σ, but Π' and Σ' where the multiplication or summation is to omit some one individual), it furnishes at once the best possible general algebra of relatives (MS 520, 2-3).

Now, Peirce was certainly not aware of meta-logical properties like expressive adequacy (completeness), or soundness and completeness, so his evaluation of the system as "the best possible general algebra of relatives" is more intuitive than it is rigorous.[24] Nonetheless, the result of this realization was dramatic. Immediately after Mitchell's paper, Peirce begins to use quantifiers in so familiar a form they likely do not need to be translated for modern readers. Π is the symbol Peirce uses for universal quantification while Σ is for existential quantification. In the same volume Mitchell's paper appears, Peirce includes his own "Note A" and "Note B" as appendixes to the volume. In "Note B" he gives

$$\Sigma_i \Sigma_j l_{ij} \text{ and } \Pi_i \Sigma_j l_{ij},$$

intended to be read as "something is the lover of something" and "everything is the lover of something" (Peirce 1883, 200–201). In modern notation, these would be written as

$$\exists x \exists y L(xy) \text{ and } \forall x \exists y L(xy).$$

This generalizes Mitchell's quantifiers beyond single variable quantification. However, Peirce does not fully articulate the system until CPN in 1885.

24. These properties did not enter into logical investigations until they started to be taken up by Husserl and Hilbert in 1901 (Hartimo 2018). Within the algebraic tradition they are taken up to some extent by Löwenheim in 1915 (Brady 2000, 169–196).

2.3 Peirce's Contribution to the Philosophy of Notation

CPN contains 4 sections. The first describes Peirce's analysis of signs and their application to logic generally, which we have already discussed. The second brings that analysis to Boole's system, which Peirce calls "non-relational logic." The third applies the same analysis to what Peirce calls "first-intentional logic" and the fourth to "second-intentional logic."

Peirce distinguishes Boole's algebra from the "first-intentional logic of relatives" as follows:

> The algebra of Boole affords a language by which anything may be expressed which can be said without speaking of more than one individual at a time. It is true that it can assert that certain characters belong to a whole class, but only such characters as belong to each individual separately. The logic of relatives considers statements involving two and more individuals at once. Indices are here required (Peirce 1995, 177).

The first kind of indices needed are free variables, or names, needed to pick out particular individuals. $x_i y_j$ says "x is true of the individual i while y is true of individual j (177–178)." z_{ij} says that i is in the z relation to j. We then need two more indices to express the pronouns 'all' and 'some.' Peirce uses

> Σ for *some*, suggesting a sum, and Π for *all*, suggesting a product. Thus $\Sigma_i x_i$ means that x is true of some one of the individuals denoted by i or
>
> $$\Sigma_i x_i = x_i + x_j + x_k + \text{ etc.}$$
>
> In the same way, $\Pi_i x_i$ means that x is true of all these individuals, or
>
> $$\Pi_i x_i = x_i x_j x_k, \text{ etc.}$$
>
> (180).

Peirce stresses that these operations are not precisely sum and product, he is merely suggesting an analogy in a way typical of the tradition he saw himself a part of, i.e.

the Boolean tradition. With the quantifiers in hand, Peirce can use them to bind variables in relations of any arity. Peirce here clearly understands how the order of the quantified variables effects the meaning of formulae, claiming

$$\Pi_i \Sigma_j l_{ij} b_{ij}$$

means 'everything is a lover and benefactor of something,' while

$$\Pi_i \Sigma_j l_{ij} b_{ji}$$

means 'everything loves some benefactor of itself.' He gives other examples demonstrating he knows the difference between a formula that begins with $\Pi_i \Sigma_j$ as opposed to $\Sigma_j \Pi_i$, as well. What Peirce has stumbled upon here, through his categorical analysis of signs, is a first order prenex predicate calculus that is just as expressive as the ordinary first order systems of today (Brady 2000, 127). Peirce does not give a full proof system for this logic but he is able to offer a handful of sound deduction rules.

Peirce also distinguishes this first order system from what he calls the "Second-intentional Logic." He argues for its necessity on grounds that the first order system is incapable of expressing certain kinds of relations, like identity. On Peirce's understanding, two objects can be said to be identical if, and only if, they have all of the same properties. Realizing that the identity relation isn't defineable in the first-intensional system, he attempts to extend the system to address this. To do so, he introduces a special token (or symbol), q, intended to signify "the relation of a quality, character, fact, or predicate to its subject (Peirce 1995, 185)." q_{ki} then would be read as something like "the quality k is true of i." Using this, the identity relation, 1_{ij} can be defined as

$$1_{ij} = \Pi_k (q_{ki} q_{kj} + \overline{q}_{ki} \overline{q}_{kj}).$$

However, it is also possible to define the relation without using the special token:

> If we please, we can dispense with the token q, by using the index of a token and by referring to this in the Quantifier just as subjacent indices are referred to. That is to say, we may write
>
> $$1_{ij} = \Pi_x (x_i x_j + \overline{x}_i \overline{x}_j)$$

(Peirce 1995, 185).

The formula can be read as 'i and j are identical if, and only if, for all properties X, either X is true of i and j or X is false of i and j.' By quantifying over predicates, Peirce has made the move from first order logic to second order logic. Thus, the second-intentional system is really a second order logic. The section on second-intensional logic is the shortest of CPN, so he does not investigate the system very far. He also does not explain his reasons for calling it a "second-intensional logic." After defining identity, in terms of Leibniz's law, Peirce investigates the properties of the special token, q, which is clearly intended to line up with the category of firsts. Following that, he introduces another special token, r, intended to represent "the conjoint relation of a simple relation, its relate and its correlate. That is, r_{jai} is to mean that i is in the relation a to j." These seem to correspond to seconds. He does not investigate the special properties of r. Rather, he shows how it can be used to define a finite collection, anticipating Dedekind's definition by three years (Brady 2000, 141). Thus, he seems to have been aware of the relevance that these logical systems could have for the work on the foundations of mathematics that was being carried out at the time.

But Peirce could not have done any of this had it not been for his insight that Boole-style algebra's were hitherto incapable of capturing all the ways in which indefinite pronouns can be used in natural language. His recognition of Mitchell's use of indices in his notation, together with his analysis of pronouns as indexical signs, are what afforded him this insight. He simply could not have come to understand the expressive deficit left by Boole's system in this way if he did not have his icon-index-symbol distinction. As such, Peirce's discovery of quantification theory is an upshot of his universal categories, by way of the analysis of signs that they suggest.

To sum up, Peirce's application of his analysis of signs to the notation of 19th century logical algebras led to his understanding and articulation of quantification theory. Through this analysis he realized that the best way to deal with 'all' and 'some' was to use indices to incorporate the pronouns into the notation directly. CPN represents the culmination of a nearly 20 year effort on Peirce's part to improve and generalize Boole's calculus of relatives. Given Frege's obscurity at the time, this was the first time a logic with quantifiers was presented to the logical community. It also contains, to my knowledge, the first use of the word 'quantifier' to describe those operations. It is also the first paper ever to draw a distinction between propositional, first order, and second order systems (Putnam 1982).

2.4 Conclusion

Above, I explained Peirce's categories and his triadic method of analysis generally. I explained his categorial analysis of the elements of logic. In the final section I showed how the application of his semiotic categories led to his invention of the predicate calculus. I claim, along with Putnam, that this was *effectively* the first time was put before the logical community.

Of course, it was not *actually* the first time. That was Frege in 1879, six years before CPN was published. I have not mentioned Frege very much because that story has been told many times before. The story of Frege's discovery of quantification theory is apparently quite short anyways: "It is astonishing because it has no predecessors : it appears to have been born from Frege's brain unfertilized by external influences (Dummett 1981, xxxv)." The infatuation with Frege that many historians of logic share is doubtlessly well-earned, but it has led to the neglect of other 19th century logicians, like Peirce, Schröder, Jevons, and McColl. Dummett claims that in *Begriffsschrift* "the modern notation of quantifiers and variables appears for the first time (xxxv)." A side by side comparison of Frege's, Peirce's, and our modern notation, may cast doubt on that claim:

$$\vdash\!\overset{\mathfrak{a}}{\smile}\!\!-\!\!\top\!\overset{\mathfrak{b}}{\smile}\!\!\top L(\mathfrak{a},\mathfrak{b})$$

$$\Pi_i \Sigma_j l_{ij}$$

$$\forall x \exists y L(xy)$$

I do not mean to wade into a priority dispute here. Frege discovered quantification first. However, the reason that the notation currently in use more clearly resembles Peirce's than Frege's is likely because the world first learned of quantification first through Schröder and Peano, who popularized Peirce's notation (Putnam 1982). Frege's work had to wait for Russell's appreciation before it gained notoriety. Still, Peirce and the other late Booleans are frequently neglected in historical scholarship.

Chapter 3

Peirce's Philosophy of Logic

> The study of logic, then, considered relatively to human knowledge, stands in as low a place as that of the humble rules of arithmetic, with reference to the vast extent of mathematics and their physical applications. Neither is the less important for its lowliness: but it is not everyone who can see that.
>
> ---
>
> Augustus De Morgan, *Formal Logic*.

In the 19th century, logic was on the cusp of overcoming a long standing identity crisis. The subject had not been disambiguated from some of the other non-natural sciences. Writers had a wide array of views as to its relation to language, mathematics, and psychology, to name a few. At the time, many thought logic was properly regarded as a branch of psychology. As such, it is not unusual to find bibliographies citing Boole alongside prominent German psychologists like Wilhelm Wundt.

This state of affairs is likely an outcome of the early modern period in philosophy, when Descartes and many others severely criticised the Aristotelian syllogistic logic that was so entrenched in the earlier scholastic period of western thought (Ariew 2014, 107–112). Their familiar criticism was that the deductive logic of the medieval schoolmen was useless because it could only serve to explain things already known.

Due to this perceived deficiency, Descartes argued that the old logic should be thought of as part of rhetoric rather than as a science of its own and sought to develop principles for a new "logic" which were more focused on right methods of inquiry (Ariew 2014, 111). Thus, logic became bound up with methodology. Formal deductive reasoning was brought somewhat back into the fold upon the publication of *La Logique ou lart de penser*, which epitomizes the dominant understanding of logic during this period, but it also bound up the subject with other psychological processes. That text, more commonly known as the *Port-Royal Logic*, was first written by Antoine Arnauld and Pierre Nicole in 1662 but it has been translated and republished many times over the years. In the 19th century, when Peirce was writing, it was still among the most widely read texts on logic ever written (Nelson and Buroker 2022).

While the *Port-Royal Logic* does leave a place for the old logic, its authors understanding of the subject is quite different from that of the scholastics as well as the present day. "Logic," Arnauld and Nicole claim, "is the art of directing reason aright in obtaining the knowledge of things, for the instruction both of ourselves and others. It consists in the reflections which have been made on the four principal operations of the mind: *conceiving (concevoir), judging, reasoning, and disposing (ordonner)*" (Arnauld and Nicole 1861). Here reasoning is only a part of what logic is about and when they discuss formal deductive methods, the authors stress that these are of little value. Arnauld and Nicole place the greatest emphasis on the last operation they mention, disposition, which they understand as methodology, by which they mean the study of methods for arriving at and presenting knowledge (Nelson and Buroker 2022). They also emphasize inductive reasoning over deductive. To present day readers the text might seem more like a 17th century textbook for a course on critical thinking rather than a treatise on logic. The basic structure and content of the text is repeated in a number of influential 19th century logic textbooks. One example is Mill's 1843 *System of Logic*. Another is Whatley's 1824 *Elements of Logic*, which was Peirce's first introduction to the subject. These texts are primarily aimed at improving the skills of students rather than providing theoretical reflections on logic. So, on the one hand, what we might call the "textbook tradition" in logic was alive and well in the 19th century. Many aspects of the textbook tradition's understanding of logic, especially Mill's text, also had a major impact on the German school of logic headed by Brentano (Brandl and Textor 2022; Tnsescu et al. 2022).[1]

1. See Lapointe's introduction to *Logic From Kant to Russell* for further contextualization of the

On the other hand, there was another tradition that was gaining prominence in the 19th century. 1847 saw the publication of both *The Mathematical Analysis of Logic, Being an Essay Towards a Calculus of Deductive Reasoning* by Boole and *Formal Logic: Or, The Calculus of Inference, Necessary and Probable* by De Morgan. Unlike Arnauld and Nicole, Boole and De Morgan are not bashful about the use of formalisms. Boole's 1847 work introduces his now famous algebra of logic, which was intended to provide foundations for the traditional Aristotelian logic and give a generalized propositional system. De Morgan's book makes great strides towards quantification theory and introduces his own notation for logic. These works were not intended to bolster the reasoning skills of students. They are complicated and technical texts intended to advance the theory of reasoning. They certainly were not intended for novices. These marked the beginning of an enduring mathematical tradition in logic that was further advanced in the 19th century by Jevons, McColl, Dedekind, Schröder, and Peano, to name a few. But the development of the new mathematical logic raised new questions surrounding the relation between mathematics and logic.

Boole opens his 1854 work as follows:

> The design of the following treatise is to investigate the fundamental laws of those operations of the mind by which reasoning is performed; to give expression of them in the symbolical language of a Calculus, and upon this foundation to establish the science of logic and construct its method; to make that method itself the basis of a general method for the application of the mathematical doctrine of Probablities; and, finally, to collect from the various elements of truth brought to view in the course of these inquiries some probable intimations concerning the nature and constitution of the human mind (Boole 1854, 1).

While the psychological pull of the Cartesian logic can still be felt here, the emphasis is placed on formal methods. On the one hand, on Boole's understanding logic is construed as an investigation into certain mental operations to uncover the laws of thought. On the other, Boole seems to think the best method to proceed with this investigation is by way of an algebraic calculus that provides the foundation for the systematic and theoretical study of logic. Boole spills more ink developing the calculus than he does the philosophical view, but it does tacitly paint a picture in which logic,

figures mentioned here and many other post-Cartesian logicians (Lapointe 2018, 1–27).

at least as a theoretical science, is founded upon mathematics. Later in the 19th century, Dedekind and Frege turn to logic to provide foundations for number and arithmetic, turning the foundational picture upside down. The view they championed came to be known as logicism, though it is controversial whether Dedekind can be accurately described as a logicist.[2]

Each of these traditions, the texbook and mathematical,[3] though not diametrically opposed, had a radically different understanding of what logic is or ought to be considered. This difference in perspective is reflected in their projects. The textbook tradition (which I take to encompass Arnauld and Nicole as well Mill, and Whately) seems to have regarded logic as primarily a tool to aid in critical thinking or inquiry, assigning the subject mostly instrumental value. The mathematical tradition (which I take to cover Boole, De Morgan, Peano, and Dedekind, but also dates back to other post-Kantian logicians, like Bolzano) was interested in the subject for its own sake.[4] Another aspect on which they differ is whether logic ought to be considered art or science. Arnauld and Nicole are firmly in the art camp. Whately and Mill think logic is part art and part science. On the other hand, the writers in the mathematical tradition are united in calling logic a science. While many of the mathematical logicians shared an interest in ampliative reasoning with the textbook tradition, you will find no mention of methodology in the texts of Boole, De Morgan, or Schröder. These differences I believe are symptomatic of a broader disagreement about logic that has carried on to this day. When asked “what is logic?” the textbook tradition would identify the subject with a set of skills, while the mathematical tradition would say that it is a formal science not totally unlike mathematics.

So, where does Peirce sit in relation to these issues? Given that he was an active contributor to Boole’s algebraic program early on in his career, it might be expected that he would fall squarely in the mathematical tradition. Indeed, what he is most famous for in logic is developing a logic of relations that could handle mixed scope quantification, independently of Frege. However, if we were to ask Peirce what he thinks logic consists in, the answer he would give would locate him somewhere in between the two traditions in many ways. Peirce read and appreciated the *Port Royal Logic* but he also had great admiration for the scholastic work on logic that the

2. See (Klev 2015; Reck 2019) for views in favor, (Benis-Sinaceur et al. 2015) for views against.

3. This short list is by no means exhaustive of all the traditions active during the periods I am discussing.

4. Or, in Dedekind or Peano’s case, for the foundations of mathematics.

Cartesians were so critical of. He was critical of the aspects of textbook traditionalists theories that would eventually be termed “psychologism” by Husserl and Frege. Yet he was happy to leave a place for other aspects of their theories in his systematization of logic, like methodology for instance. While he did a lot of work on mathematical logic, he was convinced that this was not the whole of the science. He was also interested in the processes of reasoning underlying scientific inquiry generally, and thought the study of such was also conducted under the banner of logic.

In this and the following chapter, I will show how Peirce would have answered the pressing questions of 19th century philosophy of logic. These questions are: what is the relation between logic and mathematics? What is the relation between logic and psychology? And how does logic relate to scientific inquiry more generally. Specifically, in this chapter I discuss what logic *is not*, for Peirce. The next chapter will give Peirce’s positive answers to these questions. I argue that it is not a foundation for mathematics nor a branch of psychology. I will do this by examining the place and branches logic occupies on Peirce’s theory of the architectonic structure of the sciences. This organizational structures organizes the sciences from the most formal to the most empirical. Since the purposes of the Peircean architectonic (and any architectonic) is classificatory, Peirce’s categories are reflected at every level. He first applies his categorial analysis to science in general, to obtain three broad groups of sciences. The analysis is then recursively applied to those groupings to reveal individual sciences, and then again to reveal branches of the individuals. Four branches of the architectonic are devoted to logic. It first appears as a branch of mathematics, termed mathematical logic. The next place is as a sub-discipline of philosophy, where it occupies a branch of the normative sciences. Section 1 is mainly expository and will discuss the architectonic in general before turning to logic specifically. This exposition is necessary to understand this and the following chapter because all of the distinctions Peirce makes in his philosophy of logic depend on his categorizations of the sciences. Section 2 will discuss logic’s relation to mathematics. Section 3 discusses Peirce’s views on psychologism.

3.1 The Architectonic

There are many places in Peirce’s corpus where Peirce shows how his categories may be found *within* diverse fields such as logic, psychology, phenomenology, etc. Examples of this appear in "A Guess at the Riddle" (1887-88) and "The Architecture of Theories" (1891) (Peirce 1992, §19 & 21). However, Peirce does not apply his categories to the classification of the sciences themselves (as opposed to the constituents of their theories) until slightly over a decade later. Atkins claims that he does not do so until 1903 (Atkins 2006, 483). However, Atkins's argument for this claim draws on a comparison between Peirce's 1902 classification of the sciences in *Minute Logic*[5] and his "Outline Classification of the Sciences" (henceforth, OCS) of 1903 (Peirce 1998, §18). The former classification is not informed by Peirce's triad of categories but is instead based on Louis Agassiz's work, *Essay on Classification*, and draws on zoological notions of classification to create a taxonomy of the sciences (Agassiz 1859). He does not here insist on making triadic divisions between the types of sciences he refers to. In the 1903 work, however, Peirce insists on triads from the outset, the first main division of science yielding sciences of discovery (associated with firstness), sciences of review (secondness, and this branch includes the kind of taxonomic work under discussion), and practical sciences (thirdness). Here Peirce claims to be drawing on Auguste Comte rather than Agassiz (Comte 1855). Thus, the classificatory schema presented in *Minute Logic* is not informed by the categories but the one in "Outline Classification of the Sciences" is.

However, there is another 1902 work where Peirce attempts to give a taxonomy of the sciences and this is in his application to the newly established Carnegie foundation requesting financial aid to complete a number of memoirs on logic (36, he claims in said application). This taxonomy was intended to take up the first of such memoirs. One might wonder why such a project might be included in a collection purportedly about logic. To this Peirce replies: "The relation of this present memoir to those which follow it in the series is that it gives, from a general survey of science, an idea of the place of logic among the sciences" (MS L75). The purpose of the architectonic classification of the sciences within the collection is, in keeping with Comte's aims, to illustrate "the relations of logical dependence" between the sciences and show how logic fits into those relations (MS L75A: 6-7). He is also motivated by a desire to establish logic as a science in its own right.

Peirce tells us his taxonomy is meant to be an improvement on Comte's, mainly in that it fills "the shocking omissions which Comte's rage against nonsense led him

5. This has been partially published in CP 2:1-202, 1:203-283, 7:362-387, 4:227-323, 1:575-584, and 6:349-352. Migotti and Odland have plans to publish a critical edition of the book, but the latter should probably finish his dissertation first.

to commit" and also by his intent to carry the divisions out as far as he possibly can based on how inquirers organize themselves into communities concerned with specific phenomena (Peirce 1976, 15). The omissions he refers to appear to be the areas of inquiry he groups under the title "Philosophy, or Cenoscopy." Cenoscopy is construed as the group of sciences which study ordinary or immediate experience. This includes categorics (another term he uses for what Husserl or Hegel would likely call phenomenology), the normative sciences, and metaphysics. Comte was especially hostile towards metaphysics, at one point claiming that the only utility of metaphysical conceptions is to bridge the gap between the theological and scientific stages of thought (Comte 1855, 28). There is no mention of "phenomenology" in the treatise and only one of the sciences Peirce groups under the "normative sciences" is mentioned. Comte's classification omits virtually all of what Peirce thinks of as the philosophical sciences. For instance, Comte does mention logic frequently throughout the text but not as a science in its own right, which is likely the main omission Peirce wanted to address.

As can be seen from Figure 3.1, the taxonomy presented in the Carnegie letters are clearly informed by Peirce's triadic category distinction, though this is not as thoroughgoing as it will be in the 1903 version in Figure 3.2. In both, the science of research is split into three classes of disciplines: Mathematics, cenoscopy and idioscopy[6]. On Peirce's understanding, the cenoscopic sciences are those concerned with ordinary experience (CP 1:239-241). Their investigations are informed by observing ordinary sense experience. Idioscopy, on the other hand, is understood as the group of sciences that conduct their investigations by augmenting the senses in some way, be it with a microscope, travel, or by special training (CP 1:239-241). The mathematical sciences are also based upon observations "in so far as it makes constructions in the imagination according to abstract precepts, and then observes these imaginary objects" (CP 1:240). Each of these groupings is associated with a member of the metaphysical triad of possibility-actuality-law mentioned in chapter 1. Mathematics studies abstract possibility. Cenoscopy studies existence or actuality. Idioscopy studies the realm of natural law. Mathematics is not given sub-divisions in the 1902 scheme in Figure 3.1, but cenoscopy is divided into a triad containing categorics, the normative sciences, and metaphysics. The normative sciences are further divided by 3, into esthetics, ethics, and logic. Idioscopy, or the special sciences, is not divided

6. Peirce gets these terms from Bentham (Bentham 1843, 83)

(A) Theoretical Science.

- (I) Science of Research.
    - (a) Mathematics.
    - (b) Philosophy, or Cenoscopy.
        - (i) Categorics.
        - (ii) Normative Science.
            - (a) Esthetics.
            - (b) Ethics.
            - (c) Logic.
        - (iii) Metaphysics.
    - (c) Idioscopy, or Special Science.
        - (i) Psychognosy.
            - (a) Nomological, or General Psychology.
            - (b) Classificatory.
                - (1) Linguistics.
                - (2) Critics.
                - (3) Ethnology.
            - (c) Descriptive.
                - (1) Biography.
                - (2) History.
                - (3) Archeology.
        - (ii) Physiognosy.
            - (a) Nomological, or General Physics.
                - (1) Dynamics.
                    - (I) Of particles.
                    - (II) of Aggregations.
                - (2) Elaterics and Thermotics.
                - (3) Optics and Electrics.
            - (b) Classificatory.
                - (1) Crystallography.
                - (2) Chemistry.
                - (3) Biology.
            - (c) Descriptive.
                - (1) Astronomy.
                - (2) Geognosy.
- (II) Science of Review, or Synthetic Philosophy (Humboldt's Cosmos; Comte's Phil. Positive).

(B) Practical Science, or the Arts.

Figure 3.1: Architectonic in 1902 Carnegie Application

- (A) Science of Discovery
  - (I) Mathematics
    - (a) Mathematics of Logic
    - (b) Mathematics of Discrete Series
    - (c) Mathematics of Continua and Pseudo-continua
  - (II) Philosophy
    - (a) Phenomenology
    - (b) Normative Sciences
      - (i) Esthetics
      - (ii) Ethics
      - (iii) Logic
        - (a) Speculative Grammar
        - (b) Critic
        - (c) Methodeutic
    - (c) Metaphysics
      - (i) General Metaphysics or Ontology
      - (ii) Psychical or Religious Metaphysics
      - (iii) Physical Metaphysics
  - (III) Idioscopy
    - (a) Physical Sciences
      - (i) Nomological or General Physics
      - (ii) Classificatory Physics
      - (iii) Descriptive Physics
    - (b) Psychical or Human Sciences
      - (i) Nomological Psychics or Psychology
      - (ii) Classificatory Psychics or Ethnology
      - (iii) Descriptice Psychics or History
- (B) Science of Review
- (C) Practical Science

Figure 3.2: Architectonic in 1903 “An Outline Classification of the Sciences” (Many of the further divisions of the idioscopic sciences are suppressed).

into a triad but its branches, and their branches, are. The compositional structure described in connection to the second organizing principle in chapter one is also present in these applications, as will become apparent through the course of our discussion. For example, ethics applies notions from esthetics in various contexts, and logic applies normative ethical notions within one quite specific context (more on these in chapter 4). So it seems Peirce's categories did factor into his architectonic prior to the 1903 "Outline Classification of the Sciences."

The way the taxonomic arrangement is organized in both 1902 and 1903 is such that the sciences that are lower on the tree are logically dependent on the sciences above them. Thus, logic is dependent upon mathematics in some regard just as idioscopy is dependent upon cenoscopy. However, it is not totally clear what Peirce means by "logical dependence," since Peirce is neither a foundationalist in his epistemology, nor a reductionist in his philosophy of science. So, Peirce must not be using the phrase in a way that is consistent with the way it would be understood in debates on contemporary foundationalist epistemology. Peirce famously argues against Descartes' foundationalist approach to epistemology in his early cognition series, published over 1868-69.[7] In that series Peirce criticizes Cartesian style epistemology for replacing "The multiform argumentation of the" scholastics with "a single thread of inference depending often upon inconspicuous premises" (EP1:28). He claims that instead

> Philosophy ought to imitate the successful sciences in its methods, so far as to proceed only from tangible premises which can be subjected to careful scrutiny, and to trust rather to the multitude and variety of its arguments than to the conclusiveness of any one. Its reasoning should not form a chain which is no stronger than its weakest link, but a cable whose fibres may be ever so slender, provided they are sufficiently numerous and intimately connected (EP1:29).[8]

Contrast this picture of knowledge with Descartes' given in *Principles of Philosophy*:

> Thus all philosophy is like a tree, the roots of which are metaphysics, the trunk is physics, and the branches that emerge from this trunk are all the

7. For discussion of Peirce's anti-foundationalism as pertaining to his theory of perception, see (Rosenthal 1994).

8. For extended discussion of this metaphor and how it relates to Peirce's philosophy of science, see (Haack 2020).

> other sciences, which are reduced to three main ones, namely medicine, mechanics and morality, I mean the highest and most perfect morality, which, presupposing a complete knowledge of the other sciences, I mean is the last degree of wisdom (IXb 14). (Ainsi toute la philosophie est comme un arbre, dont les racines sont la métaphysique, le tronc est la physique, et les branches qui sortent de ce tronc sont toutes les autres sciences, qui se réduisent à trois principales, à savoir la médecine, la mécanique et la morale, j'entends la plus haute et la plus parfaite morale, qui, présupposant une entière connaissance des autres sciences, est le dernier degré de la sagesse.)

Extended discussion of Peirce's anti-foundationalist and anti-Cartesian doctrines are beyond the scope of this chapter. Suffice it to say, the "relations of logical dependence" highlighted in Peirce's architectonic are not the one-sided relations we would expect in a foundationalist theory. On Peirce's picture, the relations between the sciences are much more nuanced and mutually supporting. The sciences that appear closest to the "roots" of Peirce's taxonomy are there not because they are fundamental to what comes after, but because the types of experience that they depend on are more universal or accessible under normal circumstances. They are more universal on the lower rungs because they are wider in scope and rely on less specialized methods. Sciences further from the "roots" rely on special experiences, occurring mostly in the context of scientific experimentation or observation, like viewing an object from a telescope or witnessing a dissection. The relations between the sciences on Peirce's multiform-cable like structure are therefore not ones of logical dependence or epistemic priority.

Peirce's anti-foundationalism informs both the 1902 and 1903 versions of the architectonic. Not much changes substantially in the jump from the presentation of the architectonic of the 1902 Carnegie letters and its presentation in 1903's "An Outline Classification of the Sciences." There are four differences: 1) The main branches of science in general have become the triad of sciences of discovery, sciences of review, and practical sciences, making the taxonomy follow the categories even more closely. All three of these were on the previous list but two of them have moved up an order. 2) Logic has been given subdivisions. 3) Metaphysics has been given new subdivisions and one of its subdivisions has also been given subdivisions. And 4) Mathematics has been given subdivisions. Some of the subdivisions of the special sciences are divided further as well, but this is immaterial for our purposes and so, these are suppressed in figure 3.2. The changes that matter for logic are 2 and 4.

A look at any of Peirce's taxonomic arrangements of his architectonic will reveal that logic is always fairly close to the root, however it is closest in the 1903 version when the phrase 'logic' appears not only as a branch in the normative sciences but as a part of the first branch of mathematics. Logic occupies two distinct spaces in the architectonic: as a branch of mathematics, but also as a branch of the normative sciences, which are a branch of cenoscopy or philosophy. In OCS, Peirce calls the mathematical branch "Mathematics of Logic" but elsewhere he refers to this science as mathematical or formal logic. The logic within the normative sciences, on the other hand, are typically referred to as "logic proper." Logic proper is placed in the normative sciences because Peirce understands it as an investigation into the norms governing correct reasoning. I discuss logic proper in more detail in Chapter 4. I discuss it in this chapter only to show how it is differentiated from mathematics. While mathematical logic is not part of logic proper, which is the logic contained in the normative sciences, it is nonetheless closely related to logic. Unsurprisingly, Peirce regards pure mathematics in general to be closely related to logic: "But seeing that pure mathematics is so close to logic, that eminent mathematicians class it as a branch of logic, it is hard to see how one can deny pure scientific worth to logic and yet accord such worth to pure mathematics" (Peirce 1976, 35). The eminent mathematician he is referring to here is Dedekind, who he mentions just prior in the manuscript. Peirce does not agree with the view that Mathematics is a part of logic. He brings the view up here in an attempt to convince his reviewers of the importance of the science he's proposing to write his memoirs on. In order to understand the precise relationship between logic and mathematics for Peirce, we need to understand the relationship between mathematics and the other two kinds of sciences of discovery.

3.2 Logic and Mathematics

As presented in OCS, the three kinds of sciences of discovery are mathematics, philosophy (previously cenoscopy), and idioscopy. As mentioned above, these are associated with the three modes of being, possibility, actuality, and "law that will govern facts about the future" (CP 1:23). In Peirce's view mathematical deduction is not concerned with actuality or existence, nor laws. Mathematics is only concerned with what is possible[9], by way of consistent (or at least non-trivial), deductive representation:

> Mathematics, in general, is the science of the logical possibility & impossibility of hypotheses (MS [R] 470:130-132, 1903).
>
> Mathematics is the science of hypotheses, the science of what is supposable. Supposable does not mean directly imaginable, it means what makes sense (MS [R] 458:3-4, 1903).
>
> Mathematics merely traces out the consequences of hypotheses without caring whether they correspond to anything real or not. It is purely deductive, and all necessary inference is mathematics, pure or applied. Its hypotheses are suggested by any of the other sciences, but its assumption of them is not a scientific act (Ketner 1983, 70, 1904).

The abstract structures entertained by the mathematician might come from sciences concerned with laws or actuality, but mathematics is just about starting from a hypothesis or theory and working out its conclusions, without considering whether the theory or results actually map onto anything in the real world. Cenoscopy and idioscopy, on the other hand, are concerned with actuality and law:

> Science of Discovery is either, I. Mathematics; II. Philosophy; or III. Idioscopy.
>
> Mathematics studies what is and what is not logically possible, without making itself responsible for its actual existence. Philosophy is *positive science*, in the sense of discovering what really is true; but it limits itself to so much of truth as can be inferred from common experience. Idioscopy embraces all the special sciences, which are principally occupied with the accumulation of new facts (CP 1:183-187).

Philosophy is concerned with the facts that can be gleaned from ordinary experience and common sense, while idioscopy generates facts that are or will be true by way of experience augmented with special tools and theories. His distinction between mathematics, philosophy, and idioscopy is strikingly similar to Kant's distinction between formal logic, transcendental logic, and special logic, discussed in (Lapointe 2018, 18–

9. It is possible that Peirce's understanding of mathematics is inspired by Leibniz's notion of *Mathesis Universalis*.

19). This is one way in which whatever logic happens to occur in mathematics is distinct from the logic proper of the normative sciences: logic proper is concerned with actual reasonings while mathematical logic is concerned with possible reasoning, whether it maps on to the actual or not.

- (A) Science of Discovery
  - (I) Mathematics
    - (a) Mathematics of Logic
    - (b) Mathematics of Discrete Series
    - (c) Mathematics of Continua and Pseudo-continua
  - (II) Philosophy
    - (a) Phenomenology
    - (b) Normative Sciences
      - (i) Esthetics
      - (ii) Ethics
      - (iii) Logic
        - (a) Speculative Grammar
        - (b) Critic
        - (c) Methodeutic

Figure 3.3: 1903 divisions containing logic.

Another important difference between the mathematics of logic and logic proper is the role logic plays in each science. In mathematics, logic is the means by which mathematicians deduce facts about the abstract entities they investigate. In logic proper, reasoning is the object under investigation rather than the means of its expedience, which is different from the role it plays in mathematics. As Haack puts it,

> Adopting a dictum of his father's, he characterises mathematics as 'the science which draws necessary conclusions'; logic, by contrast, is 'the science of drawing necessary conclusions'. So mathematics is a 'prelogical science'. Logic *studies* what mathematics *does*; and mathematics doesn't need the support of logic, which supplies the theory of validity of its arguments, for those arguments are 'more evident than any such theory could be' [CP 2:120] (Haack 1979, 39).

Remarks such as these have led some to believe that Peirce's view places him in opposition to the logicists, giving priority to mathematics rather than logic (Haack 1979;

Dea 2006). I believe this view is correct. Peirce does give priority to mathematics over logic but only in a much more modest methodological sense. However, it would not be correct to think that because of this view on the relationship between mathematics and logic, we can lump Peirce in with Brouwer and the intuitionists as some seem to claim (Patin 1957; Mayo-Wilson 2011).

Some have attributed to Peirce a weak form of logicism (Haack 1993). Others disagree (Houser 1993). In an 1867 work entitled "Upon the Logic of Mathematics," Peirce might be seen as flirting with a quasi-logicist position when he claims:

> The object of the present paper is to show that there are certain general propositions from which the truths of mathematics follow syllogistically, and that these propositions may be taken as definitions of the objects under the consideration of the mathematician without involving any assumption in reference to experience or intuition. That there actually are such objects in experience or pure intuition is not in itself a part of pure mathematics (Peirce 1985, §5).

Peirce's claim that mathematical objects can be defined syllogistically seems on track towards something like logicism. This might be seen to place logic and mathematics on the same level of generality. The story Peirce seems to be telling here is that there are general propositions of logic from which certain truths of arithmetic follow. The paper begins by giving definitions of logical objects and operations from Boole's calculus alongside a definition of arithmetical addition and multiplication. He then presents some theorems governing the logical identity and shows that certain truths of arithmetic can be derived from them together with the Boolean logical operators. Towards the end of the paper, Peirce claims that these results suggest that logical identity is a subspecies of mathematical equality. If Peirce had any kind of a foundational structure in mind here, that claim would place mathematics in the foundational position instead of logic. However, he goes on to draw a distinction between logics of "first intention" and those of "second intention." Each of these "refer to different sorts of predication" (68). Logics of first intention predicate over individuals. Logics of second intention predicate over the extensions of expressions of first intension, i.e. they predicate over predicates.[10] For logics of second intension, logical identity would

10. Peirce's distinction here might be seen as an anticipation of the distinction between first and second order logic (Martin 1965; Putnam 1982). However, the distinction actually goes back to at least the medievals so, if there is an anticipation, it is unclear that it is Peirce's alone.

become class identity rather than identity between individual objects. And from this perspective, Peirce tells us that equality would be a subspecies of second intension logical identity, placing logic in the foundational position. While the purported aim of the paper is to make the relation between mathematics and logic clearer, Peirce leaves the issue in a bit of a snarl.

Peirce would eventually call the paper "the worst [he] ever published" but whether this is in connection with the ideas expressed above is difficult to say (CP 4:333). Peirce did express a certain amount of disagreement with "Dedekind's Logicism," though neither he nor Dedekind used the term:[11]

> The philosophical mathematician, Dr. Richard Dedekind, holds mathematics to be a branch of logic. This would not result from my father's definition, which runs, not that mathematics is the science of *drawing* necessary conclusions which would be deductive logic but that it is the science which *draws* necessary conclusions. It is evident, and I know as a fact, that he had this distinction in view. At the time when he thought out this definition, he, a mathematician, and I, a logician, held daily discussions about a large subject which interested us both; and he was struck, as I was, with the contrary nature of his interest and mine in the same propositions. The logician does not care particularly about this or that hypothesis or its consequences, except so far as these things may throw a light upon the nature of reasoning. The mathematician is intensely interested in efficient methods of reasoning, with a view to their possible extension to new problems; but he does not, *quâ* mathematician, trouble himself minutely to dissect those parts of this method whose correctness is a matter of course (CP 4:239).

So, Peirce does disagree with at least Dedekind's logicism, but this seems to be on grounds of the role deductive reasoning plays in each science, rather than what comes first in an epistemological foundationalist doctrine. He would also disagree with a foundationalist doctrine that put mathematics before logic, but this is because he is not a foundationalist. "Let us look upon science the science of today as a living thing. What characterizes it generally, from this point of view, is that the thoroughly established truths are labeled and put upon the shelves of each scientists mind, where

11. See (Reck 2019) for discussion of Dedekind's view and its context.

they can be at hand when there is occasion to use things arranged, therefore, to suit his special convenience while science itself, the living process, is busied mainly with conjectures, which are either getting framed or getting tested" (Peirce 1998, 129). For Peirce, science is a living and constantly evolving thing. Developments in mathematics might call for revisions or new developments in logic proper or nomological physics, and vice versa. He assigns names to what we might think of as discrete sciences, but really they are all continuous with and borrow frequently from each other. The sciences near the top have that place because those below borrow from them most frequently, not because they provide the necessary foundation for everything below.

Some have taken Peirce's comments about the relation between mathematics and logic to be indicative of substantial agreement with intuitionism (Patin 1957). Intuitionism about mathematics is opposed to Platonism about mathematical objects and other forms of mathematical realism. It was initiated by the mathematician L. E. J. Brouwer in reaction to the formalist views espoused by Hilbert. It holds that mathematical objects are dependent on the mental constructions that validate them and these are not necessarily constant across time (Iemhoff 2024). The connection is thought to be bolstered by Peirce's puzzling remarks about the principle of excluded middle (PEM), which Brouwer vehemently rejected: " anything is general in so far as the principle of excluded middle does not apply to it" (CP 5:448). This remark could be taken as evidence that Peirce does not think the principle of excluded middle holds in general, just as Brouwer did. However, Peirce's limitations on the principle of excluded middle are not nearly so global. They also involve a somewhat idiosyncratic understanding of the principle that relies on an ambiguity between propositions with a negative quality and those with a negative quantity (Lane 1997). Peirce's remarks about PEM are reserved for what he calls "general propositions," by which he means propositions of the form 'All P are Q.' Stating PEM for such propositions could take one of two forms:

1. 'Either all P are Q or it is not the case that all P are Q,' on the one hand, or
2. 'Either all P are Q or all P are not Q,' on the other.

In 1, the scope of the negation is distributed over the whole of the second disjunct. In 2, the negation is built into the predicate of the second disjunct. When Peirce denies that PEM holds for general propositions, it is 2 that he has in mind. As evidence to support this interpretation, Lane gives the following passage: "By the Principle of

Excluded middle (or of excluded third,) is always meant the principle that no pair of mutually contradictory *predicates* are both false of any individual subject" (MS 611, 1908, emphasis mine). Thus, according to Peirce's understanding, PEM involves negative quality rather than quantity. So, while the intuitionistic denial of PEM is directed at 1, Peirce's much weaker denial is directed at 2. Peirce does not deny PEM in the form that we would typically state it today. He also does not think that logic is wholly a part of mathematics, as at least the intuitionist of Heyting's dialogue "Disputation" claims: "Logic is a part of mathematics, and can by no means serve as a foundation for it" (Heyting 1966, 6).

Peirce does give priority to mathematics over logic but only in a much more modest methodological sense. Mathematicians use deductive reasoning to prove theorems and logicians study the reasoning of mathematicians. Mathematics is prior because mathematicians give logicians reasonings to study. And both make use of the "mathematics of logic" but with different aims in mind.

> "[Logic] is a normative science. It thus has a strongly mathematical character, at least in its methodeutic division; for here it analyzes the problem of how, with given means, a required end is to be pursued. This is, at most, to say that it has to call in the aid of mathematics; that it has a mathematical branch. But so much may be said of every science. There is a mathematical logic, just as there is a mathematical optics and a mathematical economics. Mathematical logic is formal logic. Formal logic, however developed, is mathematics. Formal logic, however, is by no means the whole of logic, or even its principal part. It is hardly to be reckoned as a part of logic proper" (CP 4:240).

Logic and mathematics are intimately connected yet here Peirce seems to draw sharp partitions between the two. So, what is mathematical logic? In the Carnegie Application, in which this topic was intended to be the entire subject matter of the second memoir, Peirce tells us that "[t]his is that mathematics which distinguishes only two different values, and is of great importance for logic" (Peirce 1976, 18). So, when Peirce refers to the 'mathematics of logic' or 'mathematical logic,' what he means is the logical algebras developed by Boole (Boole 1854, 1847) and De Morgan (De Morgan 1847), but also William Jevons (Jevons 1890), Hugh MacColl (MacColl 1906), Schröder (Schröder 1890), and Peirce himself (to name just a few). Presumably, this would also include portions of Peirce's later experiments in three-valued logic, where he extends this kind of algebra to three values (see Figure 3.4). It is somewhat puzzling that Peirce claims that this subject is "*hardly* to be reckoned as a part of logic proper," given that a large portion of his own work would fall into this category and he always considered himself primarily a logician, rather than a mathematician. It is also unclear where the boundary between mathematical logic and logic proper would be (if there even is one given his fluid understanding of the sciences). For example, his work on the algebra of relations and developing truth tables[12] seem clearly within the domain of mathematical logic. But what of his system of existential graphs and their transformation rules which analyse reasonings in such a way as to give us "a moving picture of thought" (CP 4-8)? It is possible to give essentially mathematical demonstrations of validity in the latter but its principal purpose is to analyse reasoning for its own sake. So, its purpose is not primarily instrumental as the mathematics of logic is when considered from within mathematics. Perhaps this comment need not be taken that seriously and we may regard this boundary as fluid. As far as I can tell, this is the only place where Peirce more or less carves mathematical logic out of logic proper. In any case, if we are to draw a sharp boundary between the two it would be on the same basis that mathematics is distinguished from philosophy on the grounds that one is concerned with possibility, and the other, actuality: "But, indeed, the difference between the two sciences is far more than that between two points of view. Mathematics is purely hypothetical: it produces nothing but conditional propositions. Logic, on the contrary, is categorical in its assertions" (CP 4:240).

To sum up, logic does not provide foundations for logic for Peirce. Nor does mathematics provide foundations for, or wholly incorporate logic. Peirce's understanding of the relationship between the two is actually much more intricate than any kind of foundationalist account of the sciences could express. Having disambiguated Peirce's views on this subject, I now turn to do the same for another subject that vexed 19th century philosophers and logicians: The relationship between logic and psychology.

12. See (Anellis 2012) for discussion of Peirce's role in the development of the truth table.

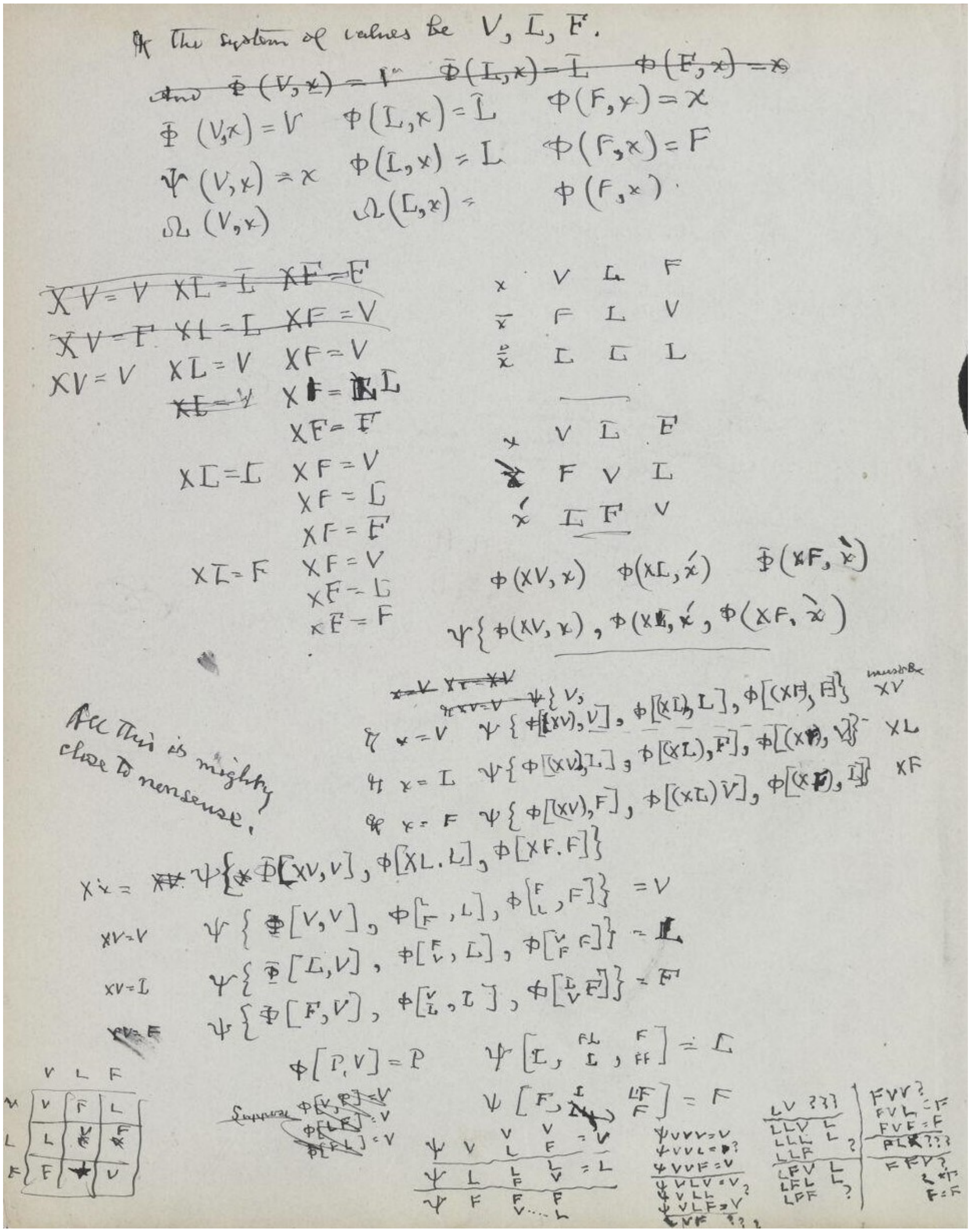

Figure 3.4: A page from Peirce's experiments in his Logic Notebook (MS 339)

3.3 Peirce and Psychologism

While foundational questions concerning the relationship between mathematics and logic were emerging relatively late in Peirce's life, another foundational issue concerning logic was being hotly debated much earlier. These debates figured largely around the relationship between psychology and philosophy broadly, but were particularly heated when logic was the object of consideration. The view at the center of the debate has come to be called "psychologism," though it was not initially named so when Peirce began his writings on the issue.[13] As Kusch points out, there have been many different definitions of the doctrine in the recent history of philosophy and there is likely no universally accepted definition (Kusch 1995, 23–25). A more extreme definition would go something like this: "Psychologism is the position that psychology is the most fundamental branch of science, and that all other sciences and all other disciplines are special branches of psychology" (Richards 1980, 19–20). If one's psychologism is localized to logic, then it would simply hold that logic is a branch of psychology. The term is typically used in a pejorative sense and the extreme version of the doctrine has few, if any, defenders. Yet in recent years, psychologism has become bound up with philosophical naturalism, defended in opposition to armchair philosophy (Kusch 1995, 11).

According to the received view, logical psychologism was first explicitly promoted by Mill, who once wrote:

> [Logic is] not a Science distinct from, and co-ordinate with Psychology. So far as it is a science at all, it is a part, or branch, of Psychology; differing from it, on the one hand as the part differs from the whole, and on the other, as an Art differs from a Science. Its theoretical grounds are wholly borrowed from Psychology, and include as much of that science as is required to justify its rules of art (Mill 1979, 359).

It was taken up by some German logicians, like Erdmann, Lipps, and Sigwart (Kusch 1995, 3), and perhaps Brentano and some of his followers (Huemer 2019). It was then summarily executed by Frege, in *Grundlagen der Arithmetik* (1884) and *Grundgesetze der Arithmetik* (1893), and again by Husserl in *Logische Untersuchungen* (1900). Of course, this story is a vast oversimplification. Even Mill's status as a psychologistic

13. The term appears to have been introduced in 1870 (Kusch 2024).

thinker is somewhat contentious (Godden 2005, 115). I cannot give an account of the history of psychologism or the vast number of philosophers who have been accused of it here. Instead I will focus on Peirce's arguments against psychologism, which may constitute an anticipation of Frege and Husserl's criticisms. Despite his arguments against psychologism, Peirce too stands amongst the accused, so I will also address charges of psychologism against him.

It might seem strange to those of us familiar with the formal logic of the present day to find commentators calling the subject a branch of psychology. However, when Mill was writing, logic was not considered a wholly formal discipline. While it included the theory of inference, theories of judgment and concepts were also thought to be a part of logic. Since inferences, judgments, and concepts are all kinds of mental entities, and psychology was thought to be the science that studies mental entities and processes, it is understandable why someone would make such a pronouncement. And it is against this background that Peirce makes his first criticisms against regarding logic as a psychological enterprise. "An Unpsychological View of Logic" (1865) opens as follows:

> A certain growth may be traced in the conception of logic held by the body of philosophers of different times. At first, it was a mere art of wrangling. Then it was an organum of discovery. Then it was a science of the mind. Now it is the law which rules the products of the mind without showing the mental processes themselves. The human elements of the conception have thus been gradually eliminated; first the selfish element, then the personal element, finally the psychological element. It is surprising how little the definition now usually given namely that it is the science of the forms of thought in general, has to do with the mind. The instances of Geometry and Algebra suffice to show how little a science of Kantian Forms need have to do with any introspection. *Thought in general* too is a very different thing from a thought in the mind. In the first place, it does not exist in the individual mind but is common to you and me. In the second place it is prescinded from intuition and so reduced to an *ens rationis* which is one of Kant's four species of nothing (Peirce 1982, 310–311).

Here we find Peirce celebrating what he perceives as a progressive removal of subjective and mental elements from the study of logic. He claims that logic is no more the study of mental processes than algebra or geometry is. He goes on to blame psychological accounts of logic for a number of difficulties and confusions facing logicians of his time:

> All advanced logicians would probably agree that the greatest stumbling-blocks in the way of logical research always have been and are now psychological difficulties. Thus the question of the thorough-going quantification of the predicate depends upon how we think. The doubtful matter of reasoning in two quantities, and all the questions of the day, have the same character. Finally it is from certain doctrines of psychology that one school of logicians has been led to deny explicitly and another implicitly the fundamental principle of contradiction. If therefore it could be shown that the conditions of valid argument have no more to do with how we think than geometry has with how we intuite [sic.], so that logic should be extricated once and forever from all the entanglements of introspection, we might hope that its future progress however slow would be as free from the impediments of controvertible doctrine as that of mathematics itself (311).

When he mentions the principle of non-contradiction, the schools he likely has in mind are Kant's and Hegel's. Priest has argued that dialetheism was implicit in Kant's doctrine and explicit in Hegel's (Priest 2019). Hegel argued against the principle of non-contradiction because he believed contradictions can occur in *ordinary experience* (56–57). But therein lies Peirce's disagreement with psychological understandings of logic. For him, ordinary experience is not the subject-matter of logic. Logic is not about how any particular mind works either. He presents an argument against such views in a thought experiment.

> Is this, after all, ridiculous? Suppose that in an undecipherable inscription of a long-extinct people a syllogism be contained. Is this syllogism, this argument any the less correct or fallacious because, nobody being able to read it, nobody can think it? Such an inscription is like a flower in a desert. Has this not colour because nobody can see its colour? It is true that to call that a colour which cannot be seen is a sort of fiction; but it is a fiction which is purified from its fictitious element completely as soon as we add that it cannot be seen. Without this kind of fiction, not only

> modern mathematics would be impossible; but philosophy, itself, would be deprived of all its terms. What are such words as blueness, hardness, loudness, but fictions of this kind? It has been said that these "abstract names" denote qualities and connote nothing. But it seems to me the phrase "denoted object" is nothing but a roundabout expression for a thing. What else is a thing but that which a *perception* or *sign* stands for (Peirce 1982, 310–311)?

For Peirce the subject-matter of logic is signs, defined according to the icon-indices-symbols triad. While it is true that these are involved in thinking and reasoning, when studied by the logician they are abstracted (prescinded) from those particular contexts. This is what Peirce means when he uses the phrase "thought in general." By Peirce's reasoning, if logic was about the mental operations of any individual in particular, then the undecipherable argument would be no argument at all, since it would no longer be possible for any individual to 'think it.' Peirce clearly thinks it would still count as an argument even if it could no longer enter the mind of any individual.

A year later (1866), psychological conceptions of logic are no longer vestiges of a bygone era that were slowly being rejected by the logical community, but a camp opposed to formalism:

> We have, at present, Formal logicians and Anthropological logicians. Anthropological logicians think that Logic must be founded upon a knowledge of human nature and requires a constant reference to the facts of human nature. Formal logicians believe that logic can be learned merely by the comparison of the products of thinking. For my part, while admitting that the greater array of talent is upon the side of anthropologists, I agree myself with the formal logicians (361).

While he uses the term "anthropological logicians," with post-Kantian logicians like J. F. Fries in mind, his remarks about basing logic on human elements seem to indicate he has the same ideas in mind. This is further clarified when we find out that Mill, the usual suspect, is among those Peirce places in the anthropological camp:

> Let us suppose, for example, that the opinion of James Mill be adopted that all inference arises merely from the association of ideas. Here is one

> of those psychological facts which Stuart Mill thinks has a great bearing upon logic. But would such a fact make any argument good which we had hitherto supposed to be bad? Not at all. Does it make any argument bad which we had hitherto supposed to be good? Not at all. Does it alter the degree or character of the belief we ought to repose in any inference, when certain premisses have been admitted? Not at all. Does it afford any new characteristic by which we can distinguish good reasoning from bad or what we should believe in one mode from what we should believe in another? It does not. Then I say that however true and important the discovery may be, it has nothing to do with logic whatsoever (362).

At this stage in Peirce's thought, logic is conceived as a purely classificatory since its only function is to classify inferences as good or bad. Its only goal is to delineate bad arguments from good ones and in that capacity, psychological considerations don't seem to matter. While I know of no place where Peirce uses the precise term, "psychologism," it seems clear that early Peirce had reservations similar to Frege and Husserl when he delivers arguments such as these.

Peirce's disdain for psychological or anthropological conceptions of logic appear to be constant throughout his career, though his ire seems to have shifted from Mill. In 1906, Peirce attributes psychologism to Husserl:[14]

> How many writers of our generation (if I must call names, in order to direct the reader to further acquaintance with a generally described character let it be in this case the distinguished Husserl), after underscored protestations that their discourse shall be of logic exclusively and not by any means of psychology (almost all logicians protest that on file), forthwith become intent upon those elements of the process of thinking which seem to be special to a mind like that of the human race, as we find it, to too great neglect of those elements which must belong, as much to any one as to any other mode of embodying the same thought.

Peirce seems to be suggesting here that Husserl's attempts to excise psychology from his understanding of logic were unsuccessful.[15] However, Peirce does not give any

14. For an in depth account of the connections between Peirce's work and Husserl's, see (Pietarinen and Shafiei 2019).
15. Pietarinen interprets this passage the same way in (Pietarinen 2013, 146).

indication as to which text of Husserl's he is thinking of, so it is difficult to say what his motivations were or whether he has Husserl right. Peirce also disdains the anthropological account of logic as a natural history of thought in Dewey's *Studies in Logical Theory* on similar grounds in a 1904 letter:

> But I must say to you that your style of reasoning about reasoning has, to my mind, the usual fault that when men touch on this subject, they seem to think that no reasoning can be too loose, that indeed there is a merit in such slipshod arguments as they themselves would not dream of using in any other branch of science. You propose to substitute for the Normative Science which in my judgment is the greatest need of our age a "Natural History" of thought or of experience. Far be it from me to do anything to hinder a man's finding out whatever kind of truth he is on the way to finding out. But I do not think anything like a natural history can answer the terrible need that I see of checking the awful waste of thought, of time, of energy, going on, in consequence of men's not understanding the theory of inference... The effect of teaching that such a Natural History can take the place of a normative science of thought must be to render the rules of reasoning lax; and in fact I find you and your students greatly given over to what to me seems like a debauch of loose reasoning. Chicago hasn't the reputation of being a moral place; but I should think that the effect of living there upon a man like you would be to make you feel all the more the necessity for Dyadic distinctions, Right and Wrong, Truth and Falsity. These are only to be kept up by self control. Now just as Moral Conduct is Self-controlled conduct so Logical Thought is Moral, or Self-controlled, thought. The Germans have always been in favor of giving thought the rein. What is taught in German Universities bespeaks only the fashion of the day. No doubt a slow evolutionary process will gradually bring them round to the truth. But that is the Wild Oats doctrine applied to thought. It involves unspeakable waste... There are three sciences according to me to which Logic ought to appeal for principles, because they do not depend upon Logic. They are Mathematics, Phenomenology, and Ethics. There are several sciences to which logicians often make appeal by arguments which would be circular if they rose to the degree of correctness necessary to that kind of fallacy. They are Metaphysical Philosophy, Psychology,

> Linguistics (of which they barely know that of the Aryan Languages, and not Gaelic which does not ordinarily give a sentence a subject nominative), History, etc (CP 8: 239-240).

Peirce's reception of the book was harsh. He repeats earlier points about not founding logic upon psychology, or indeed any of the special or idioscopic sciences, which psychology would count as a part of. Logic is methodologically independent to the idioscopic sciences, hence he thinks founding logic upon any of them will be viciously circular.[16] His criticisms are repeated in his anonymous review in the nation where he also gives the names of who he has in mind when he mentions German logicians:

> Those whom we may as roughly call the German school of logicians, meaning such writers as Christoph Sigwart, Wundt, Schuppe, Benno Erdmann, Julius Bergmann, Glogau, Husserl, etc., are engaged upon problems which must be acknowledged to underlie the others, but attack them in a manner which the exact logicians regard as entirely irrelevant, because they make truth, which is a matter of fact, to be a matter of a way of thinking or even of linguistic expression (CP 8: 189).

Peirce perceives the projects of Dewey and this "German School" to be in opposition to his exact logic.

Hopefully I have provided enough evidence to show that Peirce would have regarded himself as an anti-psychologistic logician. It is surprising then, to find him accused of lapsing into that position, which he clearly regarded as a stumbling block for logic in both his early and later period. According to Kasser, who also documents many of Peirce's other anti-psychological remarks, Peirce is charged with psychologism by Murray Murphey and Christopher Hookway (Kasser 1999). Unlike Kasser, I find it difficult to interpret Murphey and Hookway in precisely the same way.[17] However, Peirce has also been attributed with a form of psychologism much more

16. He makes this same argument in the Carnegie application: "It is almost universally held that logic is a science of thought (so far as it is a science at all), that thought is a modification of consciousness, and that consciousness is the object of the science of psychology. The effect of this, were it perceived, is to make logic logically dependent upon the very one of all the special sciences which most stands in logical need of a science of logic" (MS L75, D 233).

17. Murphey does not mention the term psychologism in his book and the passage Kasser cites does not seem to me to be about logic (Murphey 1961, 326). Hookway does use the term in his book but usually in connection with the anti-psychological arguments mentioned above. He questions whether Peirce lapses into psychologism but seems to answer in the negative (Hookway 1985, 36).

explicitly by Susan Haack. In *Philosophy of Logics*, she distinguishes between three forms of psychologism:

1. logic is descriptive of mental processes (it describes how we *do*, or perhaps how we *must*, think)

2. logic is prescriptive of mental processes (it prescribes how we *should* think

3. logic has nothing to do with mental processes (Haack 1978, 238).

She describes these positions as, strong psychologism, weak psychologism, and anti-psychologism. She attributes the first to Kant, the second to Peirce, and the third to Frege. Haack's understanding of what makes ones notion of logic psychologistic seems to rest on the extent to which it has anything whatsoever to do with thought. The strength of ones psychologism on this view then has to do with the manner in which logic investigates thought. This can be descriptive (strong), prescriptive (weak), or presumably both (perhaps very strong?). While "there is indeed no consensus among philosophers as to what psychologism actually amounts to," I'm not sure that I agree with Haack that prescriptive accounts of logic are psychologistic (Kusch 1995, 6). After all, there are normative ethical theories that have at least something to do with thought, but accusations of psychologism are not typically laid at the feet of those who espouse them (or at least not for reasons having to do with their ethics). For example, Kant's moral philosophy has quite a bit to do with thought, but he is not normally accused of psychologism on the basis of his ethical theory. While it is perhaps unclear whether the normative status of logic has anything to do with psychologism, Haacks view at least comes apart from Peirce's understanding of what we now call "psychologism," which seems best reflected by position 1.

It is certainly true that Peirce thought that logic is normative (more on this shortly). If Haack is right that having a normative view of logic is indicative of weak psychologism, then Peirce is indeed weakly psychologistic. However, characterizing him as a psychologistic thinker, even a weak one, is hard to square with remarks of his like this one: "Logic has nothing at all to do with operations of the understanding, acts of the mind, or facts of the intellect" (Peirce 1982, 164, 1865). The question this raises is whether it is possible to conceive of logic as normative in regards to thought while simultaneously having nothing to do with psychology. To this question, Peirce would answer in the affirmative and appeal to his notion of "thought in general," as described above. Haack, however, appears to assume the negative. She takes the following passage to be descriptive of the weak psychologism she attributes to Peirce:

> At the same time that this process of inference, or the spontaneous development of belief, is continually going on within us, fresh peripheral excitations are also continually creating new belief-habits. Thus, belief is partly determined by old beliefs and partly by new experience. Is there any law about the mode of the peripheral excitations? The logician maintains that there is, namely, that they are all adapted to an end, that of carrying belief, in the long run, toward certain predestinate conclusions which are the same for all men. This is the faith of the logician. This is the matter of fact, upon which all maxims of reasoning repose. In virtue of this fact, what is to be believed at last is independent of what has been believed hitherto, and therefore has the character of reality. Hence, if a given habit, considered as determining an inference, is of such a sort as to tend toward the final result, it is correct; otherwise not. Thus, inferences become divisible into the valid and the invalid; and thus logic takes its reason of existence (1880, CP 3: 161).

The passage Haack cites as support for weak psychologism in Peirce is connected to his doubt-belief theory of inquiry. Peirce's most influential papers on that subject are also the ones Kasser claims are typically regarded as psychologistic; "Most Peirce scholars hold that his most influential papers, 'The Fixation of Belief' and 'How to Make Our Ideas Clear,' both published in the late 1870's, are thoroughly psychologistic" (Kasser 1999, 501–502). While it is true that here Peirce is discussing mental entities like beliefs and habits in the same breath as "the faith of the logician," it is difficult to definitively say that remarks such as this should be taken as evidence of psychologism, especially when there are many passages when Peirce explicitly comes out against that way of thinking. What's more is that Peirce eventually criticized the paper containing that passage: "The whole of these two parts is bad, first, because it does not treat the subject from the point of view of pure mathematics, as it should have done; and second because the fundamental propositions are not made out" (1903, CP 3: 154, endnote). Hence, even if Peirce did intend to found logic upon psychological grounds at this point in 1880, it seems he repudiated himself for this.

Since my focus is primarily on Peirce's logic and not his pragmatism, I have discussed "The Fixation of Belief" and "How to Make our Ideas Clear" very little. In the two, Peirce describes the doubt and belief theory of inquiry and uses it to ground his pragmatic maxim. In short, he holds that doubt is an uncomfortable state of being and when we find ourselves in it, we are stimulated to conduct an inquiry so that we may replace our doubt with belief, a more comfortable state of being. Now, certain methods of inquiring are going to produce beliefs that are more resistant to recalcitrant experiment. For example, if I base my beliefs on empirical evidence, they are less likely to be cast into doubt than if I base my beliefs on astrological predictions. Peirce uses this theory to demonstrate the superiority of what he calls 'the scientific method' over the methods of 'tenacity, authority,' or the '*a priori* method.' Peirce is certainly invoking psychological concepts in both those articles. The problem is that it is not at all clear that he is invoking them to found a theory of logic. Rather, it seems he is invoking them to state his theory of inquiry, his theory of meaning (pragmatism), and his theory of truth. While all of those things seem to be logic adjacent, I see no evidence that Peirce is attempting to ground his theory of logic in them. When he seeks to ground logic, he seems to consistently do so in his theory of signs or representation. So, was Peirce a psychologistic thinker? This likely depends on one's definition of psychologism. He certainly would not have identified himself that way. And he certainly did not think that logic was a branch of psychology, nor did he consider any other science an extension of psychology.

3.4 Conclusion

In summary, despite Haack's charges, Peirce vehemently rejected psychologistic treatments of logic. He was an early adopter of the anti-psychologistic view of logic, and he maintained that view to the end. Logic for Peirce, is not a branch of psychology.

He also maintained logic was not a foundation for mathematics, arguing against Dedekind's view. Peirce was neither a logicist, nor was he an intuitionist. His anti-foundationalist epistemological views likely run a foul of both of these views. While there are likely aspects of each that Peirce would have had sympathies for, fitting him into either mold would require a lot of force.

Chapter 4

Logic Proper

So far, I have said a lot about what logic *isn't* for Peirce: It is not a foundation for mathematics, nor is it a branch of psychology. In this chapter, I will turn to Peirce's positive program for logic and discuss how it relates to scientific inquiry more generally.

Logic as a sub-discipline of the normative sciences has three branches of its own: speculative grammar, logical critic, and methodeutic. Speculative grammar has some rough correspondence with the study of syntax.[1] It is the study of the general conditions under which arrangements of signs have significance. Likewise, logical critic has rough correspondence to semantics. It is the study of the conditions under which signs or representations may be regarded as true as well as the theory of valid inference. It has been claimed that Peirce's tripartite division of logic as a normative science anticipates the later distinction between syntax, semantics, and pragmatics (see p. xxxi of Houser's introduction in (Peirce 1992)). While it does seem that Morris was inspired by Peirce in making this distinction and suggesting it to Carnap, I shall argue that methodeutic, sometimes termed speculative rhetoric, does not straightforwardly correspond to pragmatics. Instead, I believe methodeutic is Peirce's attempt to maintain a place in logic for the study of scientific methodology, which was an important aspect of the Cartesian and textbook tradition's conception of logic, as discussed in the introduction of the previous chapter.

In section 4.1 I pick up where the last chapter left off and discuss Peirce's under-

1. See (Bellucci 2017) for a more complete account of speculative grammar and its relation to logic.

standing of the relation between Logic and other sciences, specifically the normative sciences and semeiotic. Section 4.2 will clarify the three main branches of the logic of the normative sciences and I will argue that it would be a mistake to regard methodeutic as a straightforward analogue for what we today call pragmatics. This runs contrary to a claim made by Houser in the introduction to *Essential Peirce: Volume 1* I will close by briefly returning to the relationship between logic and Peirce's semeiotics.

4.1 Logic, the Normative Sciences, and Semeiotic

Just as mathematics is related to all the other sciences in some respect, so too is logic, because every science is going to involve some kind of reasoning, be it deductive, inductive, abductive, or, more likely, some combination of the three. But logic bears a much more interesting relationship with two sciences that come prior in the architectonic:

> Logic has to define its aim; and in doing so is even more dependent upon ethics, or the philosophy of aims, by far, than it is, in the methodeutic branch, upon mathematics. We shall soon come to understand how a student of ethics might well be tempted to make his science a branch of logic; as, indeed, it pretty nearly was in the mind of Socrates. But this would be no truer a view than the other. Logic depends upon mathematics; still more intimately upon ethics; but its proper concern is with truths beyond the purview of either (CP 4:240, c. 1902).

In Peirce's view, logic is dependent upon ethics even more so than is the case with mathematics. Peirce's understanding of ethics is closely related to Aristotle's. "*Ethics* is that normative science which studies the conditions of that excellence which may or may not belong to voluntary action in its relation to its purpose" (Peirce 1976, 192). Logic is like ethics in this regard, but restricted to a specific domain of action, i.e., thought. And both ethics and logic depend on the other prior normative science: Esthetics. "Esthetics is the science of ideals, or of that which is objectively admirable without any ulterior reason. I am not well acquainted with this science; but it ought to repose on phenomenology" (CP 1:191, c. 1903). So, esthetics is the science that studies the good or desirable ends, without relation to actions of any kind. Ethics is informed by esthetics which kinds of things are good and then studies the ways these are brought about. Logic then, is informed by ethics and esthetics insofar as it studies how we can reach a certain good, i.e. truth, by way of a particular kind of action, i.e. thinking (Burks 1943, 190–191). More specifically, the specific "good" logic studies is that of truth preserving inference. Because of this, Peirce conceived of logic as having a normative role in inquiry (Finley 2024). It might be wondered why Peirce considered logic a normative science, especially given recent disagreements about the normative status of logic (G. Russell 2020; Steinberger 2022). However, ascribing logic a normative or prescriptive status is one of the ways that Peirce distinguished his view of logic from the psychologistic conception of other authors, though Haack may find this move unsuccessful. Logic conceived as a branch of psychology or anthropology would be a descriptive endeavor. It would describe the way that humans actually reason rather than investigating the right ways of reasoning. This notion is involved in his critique of Sigwart's *Logik*, which he objects to on his understanding that it is reliant on psychological notions like feelings of logicality (Poggiani 2012). So logic is dependent and entangled with, mathematics in part, and even more so on the other normative sciences.[2]

On the architectonic, Logic Proper occupies the third branch of the normative sciences. As I've already mentioned, the logic as a normative sciences has three branches of its own: speculative grammar, logical critic, and methodeutic (characterized by Peirce initially as speculative rhetoric). Those familiar with Peirce will likely be aware that these are also the three branches of Peirce's theory of signs, or semeiotic (his preferred spelling). Peirce's understanding of logic is deeply entangled with his semeiotic. In 1865, he regarded logic as a branch of semeiotics:

> [W]e have now established three species of representations: *copies*, *signs*, and *symbols*; of the last of which only logic treats. A second approximation to a definition of it then will be, the science of symbols in general and as such. But this definition is still too broad; this might indeed, form the definition of a certain science which would be a branch of Semiotic or the general science of representations which might be called symbolistic, and of this Logic would be a species (Peirce 1982, 173).

2. Interestingly, while Peirce seems to think that the relation to the other normative sciences is more important, he seems to have spilled quite a bit more ink over the relation between logic and mathematics.

Later in life however, instead of logic being merely a branch of semeiotic, he came to equate the two: "Logic, in its general sense, is, as I believe I have shown, only another name for *semiotic* ($\sigma\eta\mu\epsilon\iota\omega\tau\iota\kappa\eta$), the quasi-necessary, or formal, doctrine of signs" (CP 2.227, MS 798, 1897). The previous quote comes from an unidentified and incomplete fragment of a manuscript, so it is difficult to say what Peirce's demonstration amounted to. However, elsewhere in the manuscript, Peirce describes the interpretation of signs as an inferential and experimental process, which is likely the reason he came to identify the two sciences. While this likely seems strange to contemporary readers, the incorporation of signs into his logical theory is simply another way in which Peirce is building upon the work of the scholastics. Signs play a central role in many philosophical discussions of the medieval period, but it was Ockham who put them at the center of his logic (Meier-Oeser 2011). This theoretical orientation is one way in which Peirce's understanding of logic might be said to be broader than contemporary understanding, since it makes his version of syntax the study of how signs are meaningful, rather than what counts as a well formed formula or meaningful linguistic construction.

The triad of speculative grammar, logical critic, and methodeutic, which furnish the branches of logic proper and semeiotic are yet another example of Peirce's category system at work in the architectonic. It should be fairly obvious from our discussion in this and the preceeding chapter that Peirce constructs his architectonic by way of a recursive application of his categories. At every stage there is a first component, concerned with what there is, a second, concerned with how those things relate, and a third, which employs the first two in a more dynamic or at least specific context. The process goes like this: he begins with a singular concept, science, and divides this into a group of sciences considered independently (sciences of discovery), a group which reacts to and relates sciences (sciences of review), and a group of sciences that relate other sciences to particular ends (the practical sciences). In the sciences of discovery, we have a group of sciences that considers possibility or the purely hypothetical (mathematics), a group concerned with possibilities that have been actualized (philosophy or cenoscopy), and a group of sciences that look for patterns or habits in which possibilities become actualized (idioscopy). At each stage it is more or less the same, we have a science that lays out the raw materials independent of other considerations, a science that connects those materials to each other or something new, and a science that applies the previous two to something else. This is pronounced again in the normative sciences: there is a science of goods, a science of achieving goods, and a science of using reasoning to achieve truth. The same pattern occurs when Peirce divides logic into its principal branches, and these are the subject of the following section.

4.1.1 The branches of logic and what logic is

It is often remarked that Peirce's conception of logic is much broader than ours today (see (Atkin 2015, 23) or (Shin 2022) for examples of these kinds of claims). It is not clear precisely how much broader though. One way it might be broader is that Peirce's study of logic is concerned not only with deductive logic, but with inductive and abductive logic as well. But while it may be true that the bulk of work on logic today focuses on deduction, this does not entail that the theory of inductive reasoning is not at all part of logic. It might also be more broad if you take the logic of today to be completely exhausted by mathematical logic. But there are entire journals devoted to philosophical logic and even informal logic that might make that view seem a form of professional gate keeping. Furthermore, despite Peirce's potentially conflicting claims that mathematical logic is hardly part of logic proper and that mathematical logic is of great importance to logic, mathematical logic is not excised from the field entirely. Mathematical logic is part of logic, even if it is only so far as we might say real analysis is part of physics. So, in what way is Peirce's logic broader than ours? According to Atkin, logic for Peirce also includes philosophy of language, philosophy of science and even epistemology as well. While these fields are all connected to logic (and it is important to remember that all fields are connected for Peirce), this notion might make Peirce's understanding of logic seem vague or unspecific. This is not the case, as I shall argue, and in this section I will lay out what logic is for Peirce and precisely which way his conception of logic differs from today's.

Peirce gives many varied definitions of logic throughout his life, some quite consistent with the current picture and some that seem to go a bit beyond. Consider, for example:

1. "Logic is the Theory of Reasoning. Its main business is to ascertain the conditions upon which the just strength of reasoning depends" (Peirce and Bisanz 2016, 35, c. 1897).

2. "Logic proper is the theory of reasoning. That is to say, it is the study which aims

to ascertain what must be the perceptible relations between possible facts in order that the knowledge that certain ones are true may warrant us in assuming that certain others are not true" (Peirce and Bisanz 2016, 43, c. 1897).

3. "'Dyalectica,' says the logical text-book of the middle ages, 'est ars artium et scientia scientiarum, ad omnium aliarum scientiarum methodorum principia viam habens [It is the art of arts and the science of sciences, leading to the principles of the methods of all other sciences.],' and although the logic of our day must naturally be utterly different from that of the Plantagenet epoch, yet this general conception that it is the art of devising methods of research, the method of methods, is the true and worthy idea of the science" (Peirce 1990, 378, c. 1882).

4. "The ultimate purpose of the logician is to make out the theory of how knowledge is advanced" (Peirce 1998, 256, c. 1903).

1 and 2 match up quite closely to the received view of what logic is. It is the study of correct inference, of how we may move from true (or probable) premises, to true (or probable) conclusions, of valid (or cogent) arguments. These two, however, contrast sharply with 3 and 4, where it seems the chief occupation of logic is methodological and has to do with how we can use what we know to learn about what we do not. 1 claims the main business of logic is the study of correct reasoning, and 4 claims instead that it is to understand how knowledge is advanced. So, which is it? The answer, of course, is both for Peirce, and this is precisely where Peirce's conception of logic goes beyond what most of us today think logic is about: For Peirce logic is not only about correct reasoning, but also *how to do things with reason*. But it will become clearer how these two aims fit together after careful consideration of the branches of logic. In addition to the logic conceived as a branch of mathematics, there are three other branches of logic proper. One which gives the objects of concern in logic, another which studies the ways those objects can be related to each other, and one which applies studies from the first two to particular aims. In OCS, Peirce names these Speculative Grammar, Critic (sometimes called Logical Critic, Speculative Critic, or Critical Logic), and Methodeutic (Speculative Rhetoric):

> Logic, which began historically, and in each individual still begins, with the wish to distinguish good and bad reasonings, develops into a general

> theory of signs. Its three departments are the physiological, or Speculative Grammar; its classificatory part, judging particularly what reasoning is good and what bad, or Logical Critic; and finally, Methodeutic, or the principles of the production of valuable courses of research and exposition (272, c. 1903).

1 and 2 fall within the aims of one of the second of these branches, while 3 and 4 fall within the third.

- (A) Science of Discovery
    - (I) Mathematics
        - (a) Mathematics of Logic
        - (b) Mathematics of Discrete Series
        - (c) Mathematics of Continua and Pseudo-continua
    - (II) Philosophy
        - (a) Phenomenology
        - (b) Normative Sciences
            - (i) Esthetics
            - (ii) Ethics
            - (iii) Logic
                - (a) Speculative Grammar
                - (b) Critic
                - (c) Methodeutic

Figure 4.1: 1903 divisions containing logic.

Peirce calls speculative grammar the physiological branch because it consists in an analysis of the constituent parts of reasonings and how they can be combined in meaningful ways. It is called speculative grammar because it is intended to be a general theoretical syntax. In Peirce's words, it is "the Theory of the general conditions under which a representamen may embody a Meaning; this has been known as Speculative Grammar" (MS 1345, c. 1896). As it turns out, I have already given an account of speculative grammar in the first section of chapter two, where I discuss his term-proposition-argument triad along side the rheme-dicent-argument triad. When Peirce is carving representations into icons, indices, and symbols, or rhemes, dicents, and arguments, he is engaging in speculative grammar. This part of Peirce's conception of logic is present in modern treatments in the form of rules for the construction of formulae, which identify the constituents of propositions and give rules for constructing meaningful ones. However, because Peirce's theory generalises that notion, speculative grammar also encroaches on philosophy of language[3], which is distinct from but connected to logic as it is currently understood.

Once we know what makes arguments meaningful, we can begin the work of classifying them according to their validity. This is the branch of logic Peirce calls logical critic: "That part of logic, that is, of *logica docens*, which, setting out with such assumptions as that every assertion is either true or false, and not both, and that some propositions may be recognized to be true, studies the constituent parts of arguments and produces a classification of arguments such as is above described, is often considered to embrace the whole of logic; but a more correct designation is Critic" (Baldwin 1960, c. 1902). As Peirce says, this branch might more typically be regarded as the whole of logic, but this view is mistaken at least in so far as we only identify meaningful propositions or other representations as true or false. This is why Peirce thinks the first branch ought to be regarded as part of logic as well. Logical critic is also reliant on mathematical or formal logic, as can be gleaned from his remarks in the Carnegie Application:

> I shall here show that no less than thirteen different methods of *establishing* logical truth are in current use today and mostly without any principle of choice and in a deplorably uncritical manner. I shall show that the majority of these methods are quite inadmissible, and that of the remainder all but one should be restricted to one department of logic. The one universally valid method is that of mathematical demonstration (Peirce 1976, 21, c. 1902, my emphasis).[4]

Here Peirce tells us that the way in which argument validity is established has to be through mathematical proof, likely by way of mapping actual arguments onto logically valid forms as established through mathematical logic. Peirce certainly does not widen the scope of logic beyond current understanding here.

Where Peirce does appear to widen the scope of logic is in the third branch, but there are some familiar faces here as well:

3. The kind of representations dealt with in speculative grammar need not be linguistic, see (Stjernfelt 2022).

4. In this passage Peirce distinguishes *establishing* from *discovering*, in part because discovering truths is very different from establishing argument validity.

> Of Methodeutic. The first business of this memoir is to develope a precise conception of the nature of methodeutical logic. In methodeutic, it is assumed that the signs considered will conform to the conditions of critic, and be true. But just as critical logic inquires whether and how a sign corresponds to its intended *ultimate* object, the reality; so methodeutic looks to the purposed *ultimate* interpretant and inquires what conditions a sign must conform to, in order to be pertinent to the purpose. Methodeutic has a special interest in Abduction, or the inference which starts a scientific hypothesis. For it is not sufficient that a hypothesis should be a justifiable one. Any hypothesis which explains the facts is justified critically. But among justifiable hypotheses we have to select that one which is suitable for being tested by experiment. There is no such need of a subsequent choice after drawing deductive and inductive conclusions. Yet although methodeutic has not the same special concern with them, it has to develop the principles which are to guide us in the investigation of proofs, those which are to govern the general course of an investigation, and those which determine what problems shall engage our energies (62, c. 1902).

Methodeutic is essentially a form of applied logic. Abduction is an interesting form of inference whereby we move from observed facts to a hypothesis that would explain them. Methodeutic has a special interest in this form of reasoning because Peirce takes this kind of inference to establish theories which can be tested through experimentation, which he construes broadly to include work in mathematics, because it is through these theories that knowledge is advanced. But the study of correct formulation of hypotheses is not the only business of methodeutic.

As Peirce hints at above, not all hypotheses are created equal, and the selection of ones that are operationizable in light of experimentation is also within the domain methodeutic. In connection with this notion, the passage above goes on to state that "[t]wo other problems of methodeutic which the old logics usually made almost its only business are, first, the principles of definition, and of rendering ideas clear; and second, the principles of classification" (62, c. 1902). When he mentions clarity of ideas and principles of definitions here, he is referring to what came to be known as the pragmatic maxim, which can be thought of as a principle of defining concepts in such away that they have some kind of observable or truth functional content. It is important that our scientific concepts conform to this principle, otherwise the theories that we construct out of them would be incapable of proof, which is another domain of methodeutic. What Peirce means by proof is “is either mathematical demonstration; a probable deduction of so high probability that no real doubt remains; or an inductive, i.e., experimental, proof” (Baldwin 1960, 782). So, it includes both the kind of proof that justifies theories in the idioscopic sciences as well as what Peirce called the theory of provability,[5] which coincides to some extent with what we would today call proof theory. Thus, in its third branch, logic seems also to encroach on the philosophy of science.

So, Peirce’s understanding of the science is indeed broader than current conceptions, but in very specific ways. Furthermore, reasoning is at center stage in every branch of logic. The first establishes what sort of things can be reasoned about, i.e., meaningful representations. The second establishes good reasonings and bad ones and is essentially the core of logic. The third is about how we can successfully use reasoning to increase our knowledge. This combines principles of hypothesis formulation, definition and classification, as well as evidentiary support, through either deductive or empirical means. The third is where it might be said Peirce takes the greatest liberties in defining the scope of logic, but it is essentially all about setting up our conceptual frameworks in a way that we can use them to reason successfully. So, there is a sense in which Peirce’s logic does go a way afield, but it never really goes off topic. The areas of study that Peirce’s logic does encroach on, philosophy of language and of science, are intimately connected to and reliant upon logic, especially for the early analytic philosophers who were so enamored with the new logic that Peirce helped to create. Peirce’s three branches of logic also have interesting precedents in the scholastic trivium and are also likely what inspired Charles Morris’s syntax, semantics, and pragmatics distinction, which was later taken up by Carnap and others.

While Peirce’s methodeutic may have inspired Morris’s understanding of pragmatics, I do not believe there is a straightforward correspondence between the two. Yet, in his introduction to *Essential Peirce: Volume 1*, Nathan houser writes: “These three branches [(speculative grammar, logical critic, and methodeutic)] correspond more of less to Carnap’s syntactics-semantics-pragmatics triad, which he learned from Morris, who had probably derived it from Peirce (Peirce 1992, xxxi–xxxii).” Given that

5. Which Peirce at one time claimed to have made great strides in “by exploring the connections between logic and topology” (Burch 2022).

speculative grammar and syntactics are both concerned with designating proper or meaningful constructions, it seems fair to say there is a correspondence between the two. Likewise for semantics and logical critic, since both are concerned with truth and validity. However, to say the same is true of methodeutic and pragmatics seems a bit of a stretch. Carnap introduces the term into his logical theory in his short *Foundations of Logic and Mathematics* in a section on the analysis of language:

> We see these in the following example: (1) the action, state, and environment of a man who speaks or hears, say, the German word blau; (2) the word blau as an element of the German language (meant here as a specified acoustic [or visual] design which is the common property of the many sounds produced at different times, which may be called the tokens of that design); (3) a certain property of things, viz., the color blue, to which this manand German-speaking people in generalintends to refer... We shall call pragmatics the field of all those investigations which take into consideration the first component, whether it be alone or in combination with the other components (Carnap 1937, 146).[6]

In other words, for Carnap, pragmatics is the study of language in use. Even with semantics and syntactics, there is a major difference between Peirce's understanding and Carnap's, since Peirce understands these as ranging over all kinds of representations, not merely linguistic ones. However, the most marked difference seems to be between pragmatics and methodeutic.

Methodeutic is another of Peirce's technical terms which changed names throughout the course of his scholarship. In his 1867 "On a New List of Categories," Peirce refers to the science that he'll later call "methodeutic" as "formal rhetoric." In later writings, he'll refer to the science as either "speculative rhetoric" or "methodeutic," but it is clear that he regards the two terms as synonymous. In some passages, where Peirce describes the science under the label "speculative rhetoric," it does bear a family resemblance to what Morris and Carnap mean by "pragmatics." For example, in 1904, he writes:

> Let us cut short such objections by acknowledging at once... a universal art of rhetoric, which shall be the general secret of rendering signs

6. Carnap goes on to credit the notion to Peirce and Morris.

> effective, including under the term "sign" every picture, diagram, natural cry, pointing finger, wink, knot in ones handkerchief, memory, dream, fancy, concept, indication, token, symptom, letter, numeral, word, sentence, chapter, book, library, and in short whatever, be it in the physical universe, be it in the world of thought, that, whether embodying an idea of any kind (and permit us throughout to use this term to cover purposes and feelings), or being connected with some existing object, or referring to future events through a general rule, causes something else, its interpreting sign, to be determined to a corresponding relation to the same idea, existing thing, or law (Peirce 1998, 326).

Peirce's description of the science as one that studies what makes signs effective could be construed as something akin to the study of how language can be used to effect the world around us, though he clearly has something broader than language in mind. He goes on to describe the field he has in mind as

> a *speculative rhetoric*, the science of the essential conditions under which a sign may determine an interpretant sign of itself and of whatever it signifies, or may, as a sign, bring about a physical result. Yes, a physical result; for though we often speak with just contempt of "mere" words, inasmuch as signs by themselves can exert no brute force, nevertheless it has always been agreed, by nominalist and realist alike, that general ideas are words,–or ideas or signs of some sort. Now, by whatever machinery it may be accomplished, certain it is that somehow and in some true and proper sense general ideas do produce stupendous physical effects... If it be objected that it is not the general ideas, but the men who believe in them, that cause the physical events, the answer is that it is the ideas that prompt men to champion them, that inspire those champions with courage, that develop their characters, and that confer upon them a magical sway over other men (326).

Here the science Peirce is defending does seem to involve some notion of how words (or more broadly, signs or even ideas) are used to produce actions. It is likely passages similar to this one that inspired Morris's notion of pragmatics. Passages like this one are also likely behind Houser's tacit conflation of methodeutic (or speculative rhetoric) with pragmatics. However, this passage is not representative of the whole of what Peirce means by methodeutic.

Unfortunately, Peirce never gave a systematic account of methodeutic so the details surrounding it are sketchy. However, from the remarks that are available to us it seems Peirce had in mind something considerably broader than the study of language in use. For comparison, let us examine Peirce's 1902 discussion of the subject:

> All this brings us close to Methodeutic, or Speculative Rhetoric. The practical want of a good treatment of this subject is acute. It is not expected that any general doctrine shall teach men much about methods of solving problems that are familiar to them. But in problems a little remote from those to which they are accustomed, it is remarkable how not merely common minds, but those of the very highest order, stumble about helplessly... Many persons will think that there are other ways of acquiring skill in the art of inquiry which will be more instructive than the logical study of the theory of inquiry. That may be; I shall not dispute it; for it would carry me far beyond the confines of my province. I only claim that however much one may learn in other ways of the method of attacking an unfamiliar problem, something may be added to that knowledge by considering the general theory of how research must be performed. At the same time, it is this theory itself, for itself, which will here be the principal object...Time was when a theorem could constitute a considerable contribution to mathematical science. But now new theorems are turned out wholesale. A single treatise will contain hundreds of them. Nowadays methods alone can arrest attention strongly; and these are coming in such flocks that the next step will surely be to find *a method of discovering methods.* This can only come from a theory of the method of discovery. In order to cover every possibility, this should be founded on a general doctrine of methods of attaining purposes, in general; and this, in turn, should spring from a still more general doctrine of the nature of teleological action, in general (CP 2:105-108).

What Peirce appears to be describing is a science that studies the methods of generating new truths or a systematic study of scientific methodology. This is a far cry away from the study of language in use. While what Morris and Carnap mean by pragmatics might fall under Peirce's broader notion to some extent, it appears to be a narrow facet of what he originally intended. Rather, it seems more akin to the study of logic in use, which involves words in addition to a host of other kinds of signs. It is possible that the field of study Peirce is describing might encompass pragmatics as Carnap and Morris understand it. However, Peirce's methodeutic seems to have much more in common with what Arnauld and Nicole call "*ordonner*" than it does with pragmatics. Even in the passages raised in the previous paragraph, Peirce will go on to link speculative rhetoric to Francis Bacon's *Novum Organum*, which occupies a philosophical space much closer to *Port Royal Logic* than to notions of pragmatics.

4.2 Conclusion

I will now restate the answers to the questions given in the introduction to this chapter as well as the previous one. These questions were:

1. What is the relation between logic and mathematics?
2. What is the relation between logic and psychology?
3. And how does logic relate to scientific inquiry more generally (or, put simply, what is logic)?

For Peirce, logic is the normative science that studies good reasoning. This includes the study and classification of the parts of reasoning, i.e. terms, propositions, arguments, relations, proper names, etc, the classification of inferences or arguments, and the application of reasoning to the furthering of knowledge as it converges upon truth. Each of these is informed by and relies heavily upon Peirce's category theory.

Logic is related to all subsequent sciences that appear on the architectonic, as all of these make use of at least some form of reasoning, be it in the formulation of hypotheses and theories, or conferring support for these in either deductive or probabilistic form. Logic is preceded by mathematics in the architectonic and relies on mathematical reasoning to furnish some of its content. Logic is not epistemically prior to mathematics in any foundational sense, nor is the opposite the case.

Logic precedes psychology on Peirce's architectonic, and while logic is certainly involved in the study of psychology on his view, the converse does not hold. Logic is also reliant upon the other two normative sciences, esthetics, which supplies for logic its end in truth, and ethics, because logic consists in a goodness in a particular kind of action, i.e., thought.

Conclusion

To summarize, in chapter 1, I gave an overview of Peirce's categories and argued for their basis on two organizing principles. I then argued for an interpretation of Firsts, Seconds, and Thirds as null-intentional concepts, first-intentional concepts, and second-intentional concepts respectively.

In chapter 2, I discuss Peirce's various applications of his universal categories to logic. First I discussed his term-proposition-argument triad and its terminological update in the rheme-dicent-argument triad. I then discussed his application of the icon-index-symbol triad to the notation of Boolean algebra. I argued that it was O. H. Mitchell's use of indices in his notation that led Peirce to discover the quantifiers, which allowed him to extend the scope of Boolean formalism to encompass a full logic of relations.

In chapter 3, I began with discussion of the state of logic in the 19th century, and discussed how Peirce would have dealt with the issues vexing logicians and philosophers at the time by appealing to his architectonic classification of the sciences. I discussed Peirce's views on the relation between mathematics and logic, and argued that his views were distinct from those of the logicists and intuitionists. I closed that chapter by discussing Peirce's views on psychologism and argued that it would be a mistake to claim he was sympathetic to the doctrine. Here, Peirce's categories allow him to demarcate the scope of logic from related fields.

In chapter 4, I closed the discussion started in the previous chapter by discussing Peirce's positive program for logic. I discussed it's place within the architectonic, locating it within the normative sciences, which are themselves conceived of as branches of philosophy. I then turn to the branches of logic proper itself and argue that it would be a mistake to think of speculative grammar, logical critic, and methodeutic as straightforward analogues of syntax, semantics, and pragmatics, despite claims to the contrary. This is because what Peirce means by methodeutic seems to have much more in common with what earlier thinkers mean by methodology than it does with what Carnap and Morris mean by pragmatics. With methodeutic, Peirce is able to extend the scope of logic far beyond current understandings by incorporating something like a theory of applied reasoning.

I hope that the centrality of the categories in each of these chapters provides enough evidence to show that Peirce's logical endeavors and his many anticipations of modern logic were motivated and facilitated by his universal categories. Unfortunately, I have had to leave out a large number of Peirce's anticipations. The bulk of these anticipations occur within Peirce's late system of logic, his existential graphs. The existential graphs contain Peirce's modal and multi-modal logics, as well as his derivation rules for propositional logic and first order logic. In the future, I would like to extend this work to cover those as well. My main aim in conducting this research has been an attempt to answer the question of what Peirce's philosophical motivations were behind his often surprising anticipations of modern logic. In the introduction I said

> My answer to these questions, and the main claim I wish to argue in this project, is that Peirce's work in logic was spurred by his theory of conceptual analysis, his universal categories, and was motivated by a desire to extend the scope of logic to cover the complex array of relations and concepts that this analysis revealed.

I believe the above chapters are sufficient to demonstrate the truth of this claim. In the future, I would like to take this a step further and show how the categories motivate the other anticipations of modern logic just mentioned. Peirce's late system of formal logic was developed in concert with another upshot of his category theory, his hypothetical cosmology. By this theory, Peirce held that the universe was evolving from a state of pure possibility that is progressively being actualized towards a state where everything is governed by law. This, of course, involves the metaphysical gloss of his categories, the possibility-actuality-law triad. In (Odland 2021), I argue that Peirce's experiments with three valued logic were connected to an attempt to extend logic to capture his hypothetical cosmology. Peirce's work on logic in his later period became increasingly entangled with this theory. My suspicion is that Peirce's modal logic is similarly motivated, but those suspicions will have to wait for future efforts.

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