Linear Estimation Strategies Applied to a Spring-Mass-Damper System

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***ABSTRACT*–** Spring-mass-damper (SMD) systems are considered a benchmark setup in vibration-based systems. The system is considered linear for certain ranges during an excitation. For a simple system, the model is considered a second order system with one of the following behaviours for an impulse/unit input: exponential, sinusoidal, polynomial, or combined from the previous performances. The performance depends highly on the values of the parameters including the values of the mass, spring constant, and viscous damper coefficient. In this brief paper, one of the new promising filtering techniques, referred to as the sliding innovation filter (SIF), is used to estimate the system trajectories including the position and velocity. The filter is known for being robust and stable when system parameters change, which makes it a suitable candidate when the SMD system crosses the ranges of its linearized model or one of the parameters changes significantly. To complicate the case, only one state is assumed to be measured, which is the position. In this paper, a revised formulation of the SIF with the Luenberger method is introduced for cases with fewer measurements than states. The results are compared with the well-known Kalman Filter (KF). The results demonstrate that the proposed filter works well with the presence of uncertainties.

***KEYWORDS*-** Luenberger method, KF, SMD, SIF, uncertainties.

1. INTRODUCTION

Model based filters use a model the mimics the system under study. The filter simulates the model with the given input signal that enters the system [1-10]. Some model based filters are of type predictor-corrector filters. These filters predict unrefined estimate from simulating the model under certain operation. This estimate includes different type of modeling uncertainties, noise, and disturbances. These are then reduced in the correction stage [11-20].

Based on the correction stage, the filters can be divided into two major groups: optimal filters, and robust and stable filters. The former depends on finding the optimal solution using one of the optimal methods like least squared error, and maximum likehood functions. The pioneer work in this field is the Kalman Filter (KF), and its variant [21-30]. On the other hand, the second group depends on one of the stability function, i.e. Lyapunov theorem, to obtain a robust, disturbance resistance, and stable estimate. The pioneer works of sliding mode observer (SMO) [31-40], the smooth variable structure filter (SVSF) [41-50], and the sliding innovation filter (SIF) [51-60] are worth to mention in this field. These filters define a hyperplane and then force the estimate to maintain within its zone. To overcome the limitation in these two groups, several works were developed where at least one method from each group were merged together. Other works merged a filter with other techniques like AI and Luenberger method [61-69]. This work belongs to the last category.

SIF is known to be the most recent robust and stable filter, where it was proposed in 2020. It can handle high molding uncertainties without becoming unstable. However, the filter suffers in the application where the number of measurement signals is less than the number of states. In such cases, the filter depends on the pseudoinverse of the measurement matrix and how deep is it connected to the hidden states. In this work, the SIF is applied to a spring-mass-damper (SMD) system with a single measurement signal that represents the position. The pseudoinverse of the measurement matrix is connected only to the first state, and hence, it is not connected to the hidden states. By combining the SIF with the Luenberger method, the measurement signal is linked to the hidden states. This improves the filter performance. The rest of the paper is organized as follows: Section 2 introduces the SIF, KF, and the proposed method SIF/L. Moreover, it shows the SMD model. The results of the proposed filter compared to SIF and KF are discussed and concluded in Section 3 and Section 4, respectively.

2. METHODOLOGY

2.1. SMD model

The SMD system has the form of:

$$m\ddot{x} + b\dot{x} + kx = f \tag{1}$$

2023 Advances in Science and Engineering Technology International Conferences (ASET) | 978-1-6654-5474-2/23/$31.00 ©2023 IEEE | DOI: 10.1109/ASET56582.2023.10180500

where m, b and k are the mass, viscos damper coefficient and spring constant, respectively. x and f are the system position and input, respectively. Assuming $x_1 = x$ and $x_2 = \dot{x}$ then the system is represented as:

$$\dot{x}_1 = x_2 \ and \ \dot{x}_2 = \frac{(f - bx_2 - kx_1)}{m} \tag{2}$$

Or in a matrix form as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f \tag{3}$$

The system is discretized as

$$\overbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_k}^{\mathbf{x}_k} = \overbrace{\begin{bmatrix} 1 & T \\ -\dfrac{kT}{m} & 1 - \dfrac{bT}{m} \end{bmatrix}}^{\mathbf{A}_{k-1}} \overbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{k-1}}^{\mathbf{x}_{k-1}} + \overbrace{\begin{bmatrix} 0 \\ \dfrac{T}{m} \end{bmatrix}}^{\mathbf{B}_{k-1}} \overbrace{f_{k-1}}^{\mathbf{u}_{k-1}}$$

$$\rightarrow \mathbf{x}_k = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_{k-1}\mathbf{u}_{k-1} \tag{4}$$

where T is the sampling time and k is the time index. The general form of (4) is represented as:

$$\mathbf{x}_k = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_{k-1}\mathbf{u}_{k-1} + \mathbf{w}_{k-1} \tag{5}$$

$$\mathbf{z}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k, \mathbf{H}_k = [1 \ 0] \tag{6}$$

where $\mathbf{A}_{k-1}, \mathbf{B}_{k-1}$ and $\mathbf{H}_k$ are the system, input and measurement matrices, respectively. $\mathbf{x}_k$ is the states and $\mathbf{z}_k$ is the measurement vectors at time k. From (6), only the first state is measured, which is the position. The vectors $\mathbf{w}_{k-1}$ and $\mathbf{v}_k$ represents the system and measurement noise vectors.

2.2. SIF algorithm

The SIF consists of two stages, one to obtain the unrefined estimates $\hat{\mathbf{x}}_{k|k-1}$ and its measurement $\hat{\mathbf{z}}_{k|k-1}$ and the second stage to obtain the refined estimates $\hat{\mathbf{x}}_{k|k}$ and its measurement $\hat{\mathbf{z}}_{k|k}$. The stages are as follows:

1- **Prediction Stage**,

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{A}_{k-1}\hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_{k-1}\mathbf{u}_{k-1} \tag{7}$$

$$\hat{\mathbf{z}}_{k|k-1} = \mathbf{H}_k\hat{\mathbf{x}}_{k|k-1} \tag{8}$$

2- **Update Stage**,

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{H}_k^{+}\left[Sat_{1,k}\left(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}\right)\right] \tag{9}$$

$$\hat{\mathbf{z}}_{k|k} = \mathbf{H}_k\hat{\mathbf{x}}_{k|k} \tag{10}$$

where:

$$Sat_{1,k} = \begin{cases} \dfrac{|\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}|}{\psi_{1,k}} & \left|\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}\right| < \psi_{1,k} \\ 1 & \left|\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}\right| \geq \psi_{1,k} \end{cases} \tag{11}$$

where $\psi_{1,k}$ is the boundary layer. From (10), the shortcoming of The SIF can be observed as the $\mathbf{H}_k^{+}$ is the pseudoinverse, which is in this case has a zero value for the second state. This means that the correction action will not refine the second state estimates. To fix this issue, the SIF is combined with the Luenberger method that maps the second state to the measurement.

2.3. Luenberger algorithm

The Luenberger method assumes that the input matrix $\mathbf{A}_{k-1}$ in SMD is divided as follows:

$$\mathbf{A}_{k-1} = \begin{bmatrix} A_{11} = 1 & A_{12} = T \\ A_{21} = -\frac{kT}{m} & A_{22} = 1 - \frac{bT}{m} \end{bmatrix} \tag{12}$$

And hence the error in the second state is mapped to the error in the first state as follows:

$$e_{z_2,k|k-1} = A_{22}A_{12}^{-1}\left(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}\right)$$

$$= \left(\frac{1}{T} - \frac{b}{m}\right)\left(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}\right) \tag{13}$$

The error in the second state compensates the missing information in the SIF gain. However, it worth to mention that the Luenberger method assumes an ideal system with no uncertainties or noise vectors. Therefore, the method by itself cannot reduce the noise effect and cannot handle the modeling uncertainties. It needs to be combined with a robust filter that can reduce the effect of them when they present.

2.4. SIF/Luenberger algorithm

By using the SIF combined with the Luenberger method, the system assumed that the states are measured, one with an actual measurement, and the other one derived from that measurement as in (13). The algorithm is modified as follows:

1- **Prediction Stage**,

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{A}_{k-1}\hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_{k-1}\mathbf{u}_{k-1} \tag{14}$$

$$\hat{\mathbf{z}}_{k|k-1} = [1 \ 0]\hat{\mathbf{x}}_{k|k-1} \tag{15}$$

2- **Update Stage**,

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \begin{bmatrix} Sat_{1,k}\left(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}\right) \\ Sat_{2,k}\left(\left(\frac{1}{T} - \frac{b}{m}\right)\left(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}\right)\right) \end{bmatrix} \tag{16}$$

$$\hat{\mathbf{z}}_{k|k} = \hat{\mathbf{x}}_{k|k} \tag{17}$$

Where

$$Sat_{2,k} = \begin{cases} \left|\dfrac{(m - Tb)(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1})}{Tm\psi_{2,k}}\right| & \left|\dfrac{(m - Tb)(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1})}{Tm}\right| < \psi_{2,k} \\ 1 & \left|\dfrac{(m - Tb)(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1})}{Tm}\right| \geq \psi_{2,k} \end{cases} \tag{18}$$

where $\psi_{2,k}$ is another boundary layer. Both boundary layers are considered design parameters that are needed to reduce the noise effects.

3. SIMULATION RESULTS

The KF, SIF and SIF/L are tested on the model of SMD represented by (1) to (6). The system has one measurement, which represents the first state. The algorithms are applied to two cases: with and without modeling uncertainties. The system and filters parameters are summarized by Table 1 for both cases. Monte Carlo Simulation is used to show the effectiveness of the method. The simulation is repeated 1000 times for each filter and for each case. Each iteration consists of different noise and measurement noise vectors. The filters and the system are excited by the input of Figure 1. The root mean squared error (RMSE) and the maximum absolute error (MAE) which are calculated as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{ns}\left(x_{Actual,i} - x_{Prediction,i}\right)^2}{ns}} \quad (19)$$

$$MAE = \max(|x_{Actual} - x_{Prediction}|) \quad (20)$$

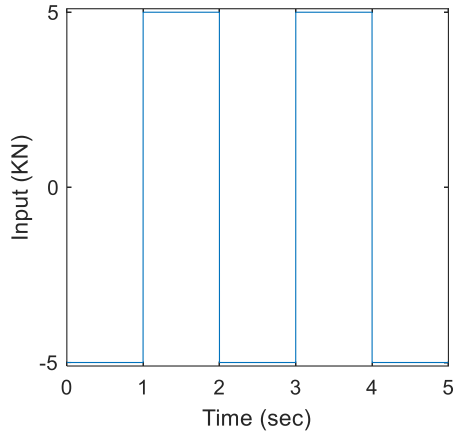
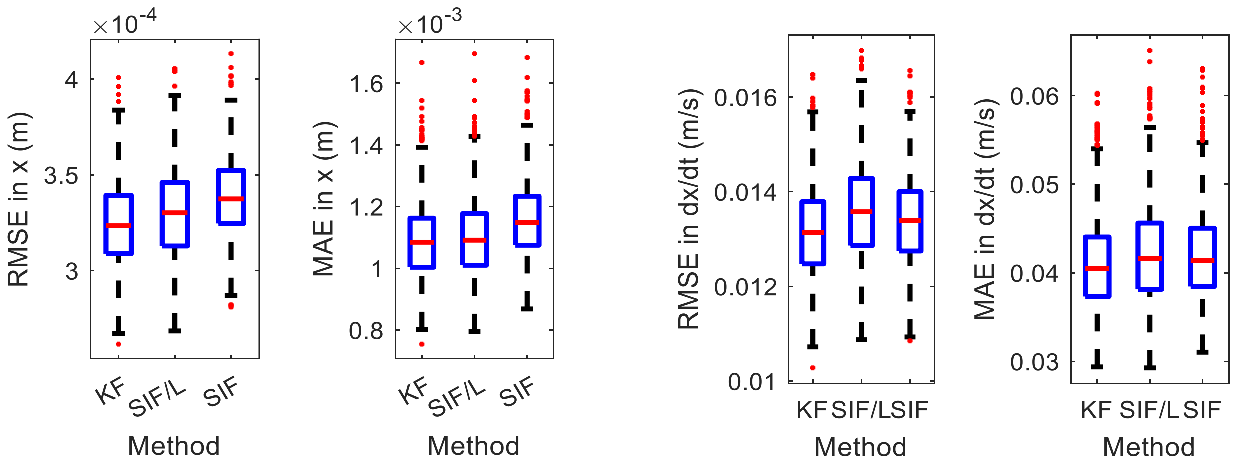


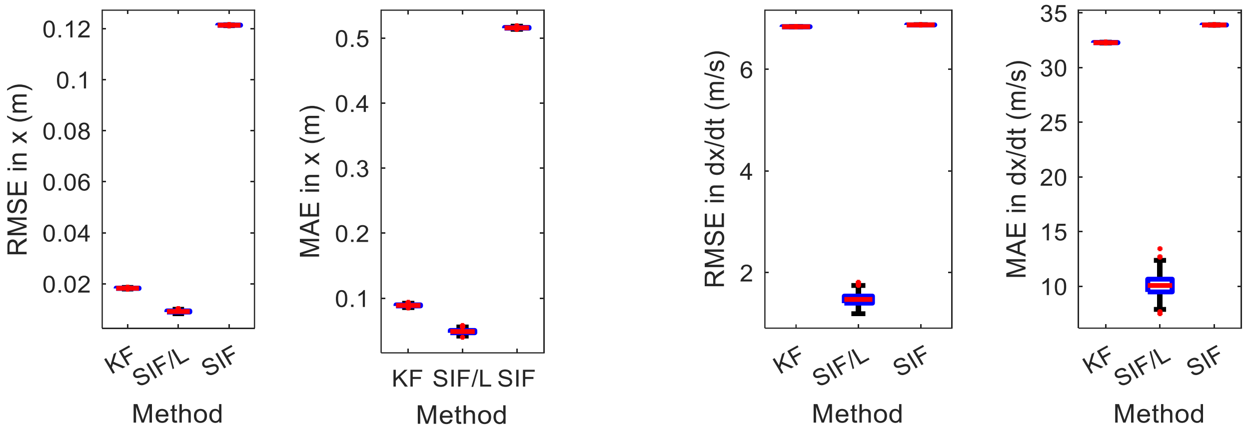
Figure 1. Input signal to the system

Table 1. Simulation parameters for system and filters

Parameter	Actual System	Filter with no uncertainties	Filter with uncertainties
System Matrix, $\mathbf{A}_{k-1}$	$\begin{bmatrix} 1 & 0.001 \\ -2 & 0.98 \end{bmatrix}$	$\begin{bmatrix} 1 & 0.001 \\ -2 & 0.98 \end{bmatrix}$	$\begin{bmatrix} 1 & 0.001 \\ 0 & 1 \end{bmatrix}$
Input system, $\mathbf{B}_{k-1}$	$\begin{bmatrix} 0 \\ 2 \times 10^{-4} \end{bmatrix}$	$\begin{bmatrix} 0 \\ 2 \times 10^{-4} \end{bmatrix}$	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
$\psi_{1,k}$ for SIF		1	10
$\psi_{1,k}$ for SIF/L		10	1
$\psi_{2,k}$ for SIF/L		5×10^5	5×10^2
Q for KF		$\begin{bmatrix} 5.77 \times 10^{-7} & 0 \\ 0 & 5.75 \times 10^{-6} \end{bmatrix}$	$\begin{bmatrix} 5.77 \times 10^{-4} & 0 \\ 0 & 5.75 \times 10^{-3} \end{bmatrix}$
R for KF		0.0057	0.0057



(a) *(b)*

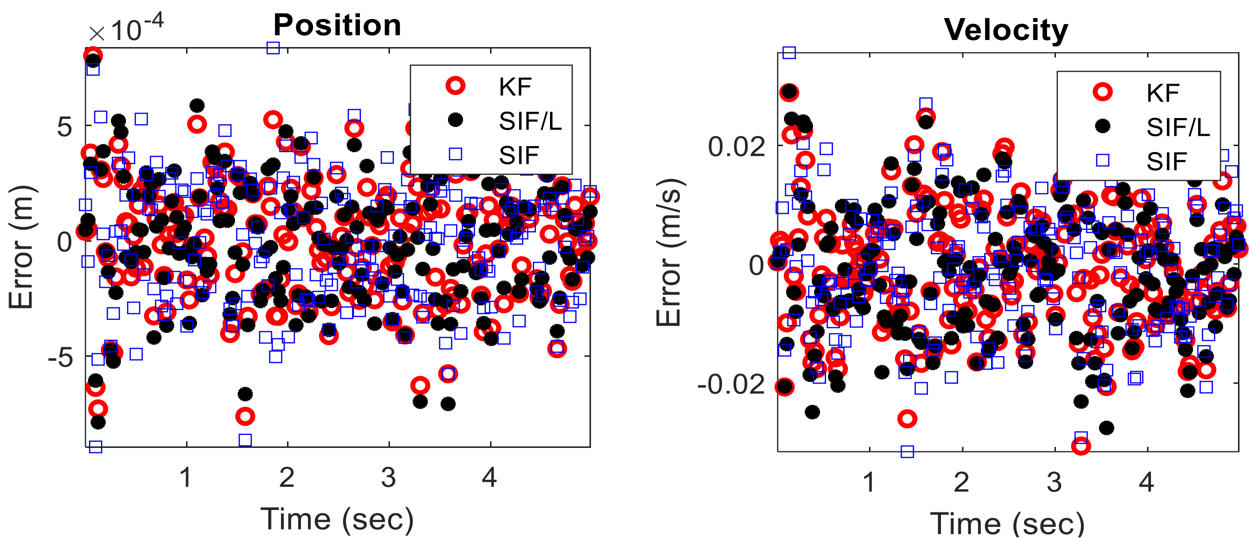


(c) *(d)*

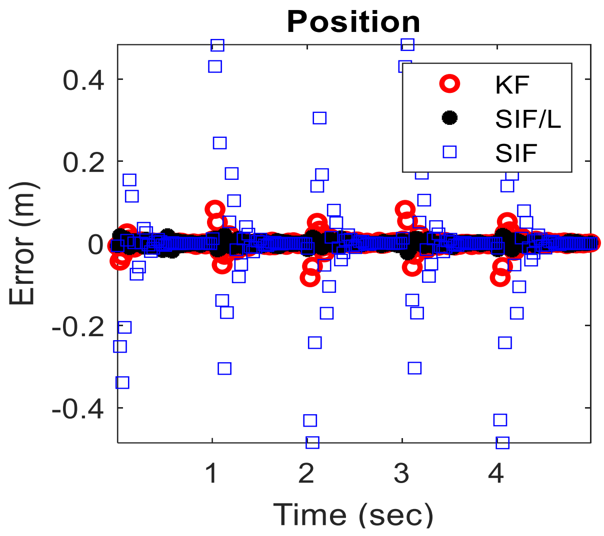
Figure 2. The RMSE and MAE of the Monte Carlo Simulation for (a) first and (b) second states for the without uncertainties case, and (c) first and (d) second states for the with uncertainties case.

Figure 2 (a) and (b) show the Monte Carlo Simulation results for first and second states for the without uncertainties case, while (c) and (d) are for the simulation with uncertainties case. Figure 3 (a) and (b) show the performance of the filters on the first and second states of the case of no modeling uncertainties, while (c) and (d) are the performance for the case of modeling uncertainties present. These results are reflected in the Table 2 for the without uncertainties case, and table 3 for the with uncertainties case.

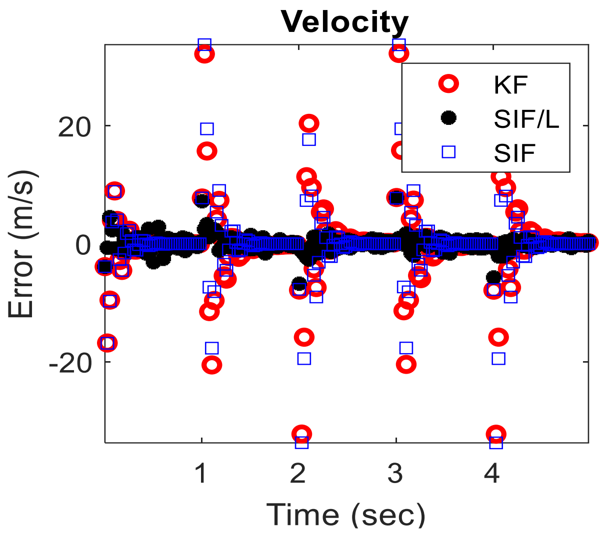
The results in Figure 2 and Table 2 show that the KF, SIF and SIF/L have similar performance with some superiority to KF as expected. These results are for the system with no uncertainties present. That means that the application fits with KF criteria and hence, the best performance is achieved by KF. Once uncertainties present, SIF/L shows the superior performance with RMSE of 0.0085 and 1.19 for the first and second states, respectively. Comparing these values to KF and SIF, it can be found that it is 53%, and 83% for the first state, and 30% and 83% for the second states, respectively. The MAE of SIF/L has value of 0.0436 and 9.41 for the first and second states, respectively. Comparing these values to KF and SIF, it can be found that it is 51%, and 71% for the first state, and 92% and 72% for the second states, respectively.



(a) *(b)*



(c)



(d)

Figure 3. The performance of the (a) first and (b) second states for the without uncertainties case, and (c) first and (d) second states for the with uncertainties case.

Table 2. RMSE and MAE for the without uncertainties case

	$RMSE$ in		MAE in	
	$x_1\ (m)$	$x_2(m/s)$	$x_1\ (m)$	$x_2(m/s)$
KF	2.61×10^{-04}	1.03×10^{-02}	1.1×10^{-03}	3.35×10^{-02}
SIF/L	2.71×10^{-04}	1.09×10^{-02}	1.1×10^{-03}	3.57×10^{-02}
SIF	2.87×10^{-04}	1.09×10^{-02}	1.1×10^{-03}	3.70×10^{-02}

Table 3. RMSE and MAE for the with uncertainties case

	$RMSE$ in		MAE in	
	$x_1\ (m)$	$x_2(m/s)$	$x_1\ (m)$	$x_2(m/s)$
KF	1.82×10^{-02}	6.84×10^{00}	8.87×10^{-02}	3.23×10^{01}
SIF/L	8.50×10^{-03}	1.19×10^{00}	4.36×10^{-02}	9.41×10^{00}
SIF	1.21×10^{-01}	6.87×10^{00}	5.17×10^{-01}	3.39×10^{01}

4. CONCLUSION

In this brief article, a modified version of the SIF is tested on a SMD system. The proposed filter, referred to as the SIF/L, is a combination of the SIF and Luenberger method. It extracts the hidden states with a performance that competes with the KF and SIF, especially when modeling uncertainties are present. The RMSE and MAE of SIF/L for the worst case scenario of the first state are 0.0085 and 0.0436, respectively, and for the second state 1.19 and 9.41, respectively. For future work, the proposed filter will be investigated for higher-order systems, and will be applied to nonlinear experimental systems.

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