Combined Particle and Smooth Innovation Filtering for Nonlinear Estimation

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ABSTRACT

In this paper, a new state and parameter estimation method is introduced based on the particle filter (PF) and the sliding innovation filter (SIF). The PF is a popular estimation method, which makes use of distributed point masses to form an approximation of the probability distribution function (PDF). The SIF is a relatively new estimation strategy based on sliding mode concepts, formulated in a predictor-corrector format. It has been shown to be very robust to modeling errors and uncertainties. The combined method (PF-SIF) utilizes the estimates and state error covariance of the SIF to formulate the proposal distribution which generates the particles used by the PF. The PF-SIF method is applied on a nonlinear target tracking problem, where the results are compared with other popular estimation methods.

Keywords: Particle filter, sliding innovation filter, tracking, nonlinear estimation

1. INTRODUCTION

The surveillance, guidance, obstacle avoidance or tracking of a target often involves making use of measurements made of the target of interest [1]. In typical target tracking scenarios, a signal of a target is processed and output by sensors as a measurement and are subsequently used to relate to the target state. These measurements are often noise-contaminated [1]. Kinematic information such as the position, velocity and acceleration of a target are usually contained in the target state. A sequence of target state estimates that vary with time are referred to as a tracks, which consist of processed measurements. It is possible for multiple targets and measurements to yield multiple tracks, which can benefit from gating and data association techniques in order to classify and determine the source of measurements, associating them with the appropriate track. Multiple targets and measurements may yield multiple tracks. Gating and data association techniques help classify the source of measurements and associate them to the appropriate track [2, 3, 4]. The use of gating techniques introduces the benefit of helping to avoid extraneous measurements, which would otherwise result in instability in the estimation process, and ultimately, failure. Tracking filters are often used in a recursive manner to carry out the estimation of the target state.

The Kalman filter (KF), which was introduced in the 1960s, is considered to be the most popular and well-studied estimation strategy [2, 3]. The KF yields a statistically optimal solution for estimation problems that are linear in nature, as defined by (1.1) and (1.2), in the presence of Gaussian noise where $P(w_k) \sim \mathcal{N}(0, Q_k)$ and $P(v_k) \sim \mathcal{N}(0, R_k)$. A typical linear model is represented by the following equations:

$$x_{k+1} = Fx_k + Gu_k + w_k \tag{1.1}$$

$$z_{k+1} = Hx_{k+1} + v_{k+1} \tag{1.2}$$

A list of the nomenclature used throughout this paper is provided in the appendix of this paper. It is the goal of a filter to eliminate the effects that the system w_k and measurement v_k noise have on extracting the true state values x_k from the measurements z_k. The KF is formulated in a predictor-corrector manner, whereby the states are first estimated using the system model and termed as *a priori* estimates, or 'prior to' knowledge of

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the observations. A correction term is then added based on the innovation (also called residuals or measurement errors), thus forming the updated or *a posteriori* (meaning 'subsequent to' the observations) state estimates. Problems covering state and parameter estimation, signal processing, target tracking, fault detection and diagnosis, and even financial analysis, have been extensively researched as applications of the KF [1, 4, 5]. The driving force behind the KF's success is in virtue of the Kalman gain's optimality in minimizing the trace of the *a posteriori* state error covariance matrix. The computation of the trace is carried out as it represents the state error vector in the estimation process of the filter [6]. Due to page constraints, the KF estimation process and equations have been omitted from this paper as there is a plethora of readily available literature on the subject [3].

In real-world scenarios and applications, the dynamics of systems are often nonlinear. For such nonlinear systems, the posterior density encapsulating all the information about the current and former state cannot be described by a finite number of summary statistics. Consequently, one must be content with only an approximate filtering solution, and such examples of popular suboptimal nonlinear filters include the extended Kalman filter (EKF) [6], the unscented Kalman filter (UKF) [7], the particle filter (PF) [1], and the recently introduced cubature Kalman filter (CKF) [8]. Of the filters just mentioned, the CKF is reportedly the most numerically stable and accurate, and furthermore, does not require Jacobians and is thus applicable to a wide range of problems and applications [8].

With the motivation of further increasing the estimation accuracy of the PF for nonlinear systems, it has been proposed to combine the PF with both the EKF and the UKF [9, 10, 11]. As such, the extended particle filter (EPF) and the unscented particle filter (UPF) utilize the EKF and UKF estimates and covariances, respectively, to result in a formulation of the distribution used to generate the particles [3, 12]. In this paper, a new PF combination is introduced, utilizing the newly proposed sliding innovation filter (SIF) [13]. The proposed method is implemented to be applied on a nonlinear target tracking problem. The performance of the proposed SIF-based PF is examined alongside the popular EKF, UKF and PF, and a comparative analysis is provided of each approach.

2. ESTIMATION STRATEGIES

The Particle Filter

Many other names exist for the PF, such as Monte Carlo filters, interacting particle approximations [14], bootstrap filters [15], condensation algorithm [16], and survival of the fittest [17], to name just a few. Since its discovery in 1993, the PF has become a very popular method for tackling nonlinear estimation problems which can range from predicting chemical processes to target tracking, and even in financial econometrics [5]. The PF takes the Bayesian approach to dynamic state estimation, in which one attempts to accurately represent the probability distribution function (PDF) using values of interest [1].

The PF's name comes from its use of weighted particles or 'point masses' (2.3) that are distributed throughout the state space, forming an approximation of the PDF as in (2.4). These particles are utilized in recursive fashion, whereby new particles and importance weights are obtained, with the goal being to create a more accurate approximation of the PDF with the progression of time. Generally, as the number of particles implemented increases and becomes very large, there is a positive correlation with the accuracy of the approximated PDF in that its accuracy improves [1].

$$\{x_k^{(i)}, \omega_k^{(i)}\}_{i=1}^{N} \tag{2.3}$$

$$p_k(x|Z^k) \approx \sum_{i=1}^{N} \omega_k^{(i)} \delta(x - x_k^{(i)}) \tag{2.4}$$

In order to avoid the degeneracy problem associated with the PF, it is important to involve resampling of the particles. Resampling eliminates the particles with low weights, increasing the prevalence of those with high weights [1]. This procedure prohibits a few number of particles from having significant importance weights following a large number of recursions, known as the degeneracy problem. An increase in the accuracy of the PDF approximation is resulted from resampling, as the particles and weights are redistributed near those that are of higher weights, while eliminating those with lower weights.

A very popular form of the PF is the sequential importance resampling (SIR) algorithm, which is summarized by the following sets of equations [15]. The first equation draws samples or particles from the proposal distribution, here chosen as the state transition function:

$$x_k^{(i)} = f(x_{k-1}^{(i)}, u_k^{(i)}) \tag{2.5}$$

Next, the importance weights are updated, up to a normalizing constant, as follows:

$$\widetilde{\omega}_k^{(i)} = f(x_k^{(i)} | x_{k-1}^{(i)}) \cdot \omega_{k-1}^{(i)} \tag{2.6}$$

The normalized weights are then calculated for each particle:

$$\omega_k^{(i)} = \frac{1}{\sum_{i=1}^{N} \omega_k^{(i)}} \cdot \widetilde{\omega}_k^{(i)} \tag{2.7}$$

Finally, a constant known as the effective number of particles is calculated as shown in (2.8). Resampling is performed if the effective number of particles is lower than some design threshold.

$$N_{eff} = \frac{1}{N \cdot \sum_{i=1}^{N} (\omega_k^{(i)})^2} \tag{2.8}$$

The final PF estimate of the states is typically calculated as a weighted sum of the particles, as follows:

$$\hat{x}_k = \sum_{i=1}^{N} \omega_k^{(i)} x_k^{(i)} \tag{2.9}$$

The Sliding Innovation Filter

Similar to the Kalman filter (KF), the SIF is formulated as a predictor-corrector estimation method. The state estimates and state error covariances are first predicted using values obtained at the previous time step (or initialization), and then the state estimates and state error covariance are updated based on the measurements and correction term at the current time step. The correction term in this case is referred to as the SIF gain.

The prediction stage includes calculating the predicted or *a priori* ('before the fact') state estimates $\hat{x}_{k+1|k}$, the predicted state error covariance $P_{k+1|k}$, and the predicted innovation $\tilde{z}_{k+1|k}$ as per the following three equations, respectively. Note that the nonlinear SIF process is the same as the nonlinear KF process, except that system and measurements equations are no longer linear.

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_k \tag{2.10}$$

$$P_{k+1|k} = AP_{k|k}A^T + Q_k \tag{2.11}$$

$$\tilde{z}_{k+1|k} = z_{k+1} - C\hat{x}_{k+1|k} \tag{2.12}$$

The update stage includes calculating the SIF gain K_{k+1}, the updated or *a posteriori* ('after the fact') state estimates $\hat{x}_{k+1|k+1}$, and the updated state error covariance $P_{k+1|k+1}$ as per the following three equations, respectively:

$$K_{k+1} = C^{+}\overline{sat}\left(\left|\tilde{z}_{k+1|k}\right|/\delta\right) \tag{2.13}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}\tilde{z}_{k+1|k} \tag{2.14}$$

$$P_{k+1|k+1} = (I - K_{k+1}C)P_{k+1|k}(I - K_{k+1}C)^T \ldots$$
$$\ldots + K_{k+1}R_{k+1}K_{k+1}^T \tag{2.15}$$

Note that C^{+} refers to the pseudoinverse of the measurement matrix, $\overline{sat}$ refers to the diagonal of the saturation term, sat refers to the saturation of a value (yields a result between -1 and +1), $\left|\tilde{z}_{k+1|k}\right|$ refers to the absolute value of the innovation, δ refers to the sliding boundary layer width, and I refers to the identity matrix (of dimension n-by-n where n is the number of states). Equations (2.10) through (2.15) represent the SIF estimation process for linear systems and measurements defined by (1.1) and (1.2), respectively.

The main difference between the KF and SIF strategies is in the structure of the gain. For the KF, the gain is derived as a function of the state error covariance, which offers optimality [18, 19]. However, for the SIF, the gain is based on the measurement matrix, the innovation, and a sliding boundary layer term. Although the state error covariance is not used to calculate the SIF gain, it still provides useful information as it represents the amount of estimation error in the filtering process. Figure 1 provides an overview of the SIF estimation concept. An initial estimate is pushed towards the sliding boundary layer which is defined based on the amount of uncertainties in the estimation process. Once inside the sliding boundary layer, the estimates are forced to switch about the true state trajectory by the SIF gain.

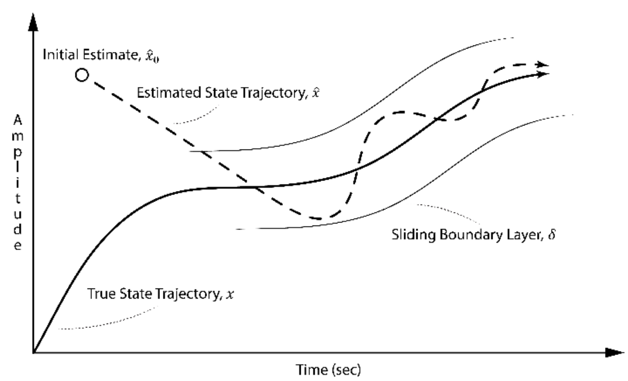

Figure 1. The above figure illustrates the SIF estimation concept.

The state estimates are updated with their corresponding innovation and sliding boundary layer term. The SIF gain effectively acts as a switching term, which forces the measurement errors to be bounded towards the true state trajectory. The sliding boundary layer δ is defined as a function of the modeling uncertainty and noise present in the estimation process. The width can be tuned to obtain the desired estimation result. Another starting point for tuning is to use the values of the measurement noise covariance. For example, $\delta = 10diag(R)$. The values can then be tuned by trial-and-error, grid search methods, or optimization techniques to reduce the estimation error.

For the cases when there are fewer measurements than states $(m < n)$, artificial measurements can be created based on existing measurements to create a full measurement matrix. The structure could also be modified as per a Luenberger observer or other strategies as per [20, 21]. This process would be required to estimate parameters of the system matrix using the SIF.

3. COMBINED STRATEGY

Although the SIF is an estimation process that is sub-optimal, it has been shown in the literature to be robust and stable. Hence, it may prove auspicious to combine the accurate performances of the KF with the stability of the SIF; prior to combining it with the PF. An adaptive formulation of the SIF was presented in [22], where a time-varying delta formulation is derived. Consider the following:

$$\delta_{k+1} = S_{k+1}\left(CP_{k+1|k}C^T\right)^{-1}\left|\overline{\tilde{z}_{k+1|k}}\right| \tag{3.1}$$

Equation (3.1) represents the time-varying sliding boundary layer δ_{k+1} that is used by the proposed adaptive SIF. The width of δ_{k+1} is found to be a function of the innovation covariance matrix S_{k+1}, the measurement matrix C, the state error covariance matrix $P_{k+1|k}$, and the absolute magnitude of the innovation $\tilde{z}_{k+1|k}$. Note that (3.1) may be simplified even further as follows:

$$\delta_{k+1} = S_{k+1}(S_{k+1} - R_{k+1})^{-1}\left|\overline{\tilde{z}_{k+1|k}}\right| \tag{3.2}$$

The adaptive SIF strategy remains the same as the standard SIF strategy presented in Section 2, except that δ is no longer fixed and is calculated at each time step as per (3.2). Now, consider the following sets of figures to help describe the overall implementation of the adaptive SIF strategy; prior to combining it with the PF.

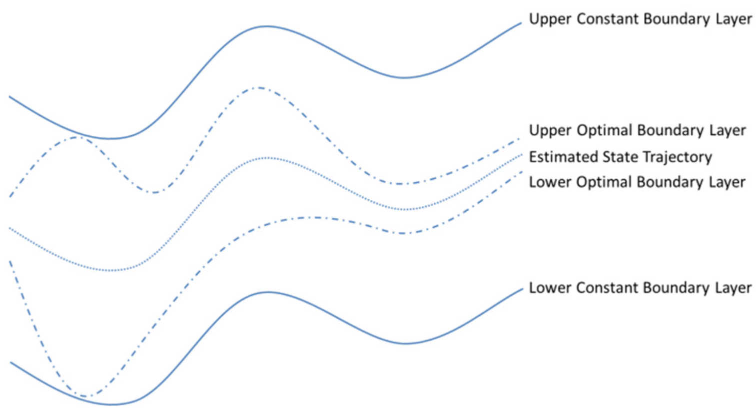

Figure 2. Boundary Layer Concept for the Well-Defined System Case [23]

Figure 2 illustrates the case when the constant smoothing boundary layer width used by the SIF is defined larger than the optimal smoothing boundary layer (i.e., a conservative choice) calculated by (3.2). The difference between the constant and upper layers leads to a loss in optimality for the SIF. Essentially, in this case, the KF gain should be used to obtain the best result (for the linear system and measurement case).

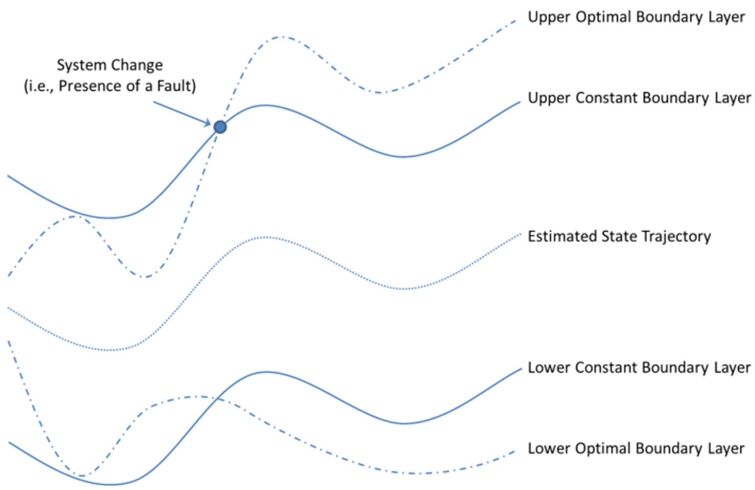

Figure 3. Presence of Fault or Poorly-Defined System Case [23]

Figure 3 illustrates the case when the optimal smoothing boundary layer is calculated to exist beyond the constant smoothing boundary layer. This typically occurs when there is modeling uncertainty (which leads to a loss in optimality) that exceed the limits of a constant smoothing boundary layer. The limits are set by the width of the existence subspace, which was discussed earlier. In a situation defined by Fig. 3, to ensure a stable estimate, the SIF gain should be used to update the state estimates. The sliding boundary layer widths are saturated at the constant values. This ensures a stable estimate, as defined by the proof of stability for the SIF [13]. Furthermore, to improve the SIF results, the averaged sliding boundary layers (for the well-defined system) can be used to set the constant boundary layer widths. Doing so provides a well-tuned existence subspace that yields more accurate estimates.

Next, to combine the aforementioned SIF strategy with the PF, a similar approach to formulating the EPF and UKF will be taken [12]. Essentially, the updated state estimates and state error covariance are used to formulate the proposal distribution used by the PF to generate the particles, such that:

$$x_k^{(i)} = q(\hat{x}_{k+1|k+1}, P_{k+1|k+1}) \tag{3.3}$$

Following the distribution of the particles, the PF continues as normal [1].

4. COMPUTER EXPERIMENTS

Tracking Scenario

The tracking problem being studied in this research is described in this section, which involves one of the most well studied aerospace applications: ballistic objects on reentry [1]. A ballistic target reentering the atmosphere is considered in the experimental simulations of this study, as described in [1]. The experimental setup for the ballistic target problem under examination is shown in the following figure.

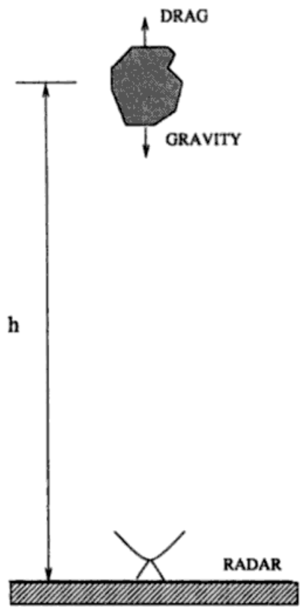

Figure 4. Ballistic Target Tracking Scenario [1]

Assuming that drag D and gravity g are the only forces acting on the object, the following differential equations govern its motion [1, 24]:

$$\dot{h} = v \tag{4.1}$$

$$\dot{v} = -\frac{\rho(h) g v^2}{2\beta} + g \tag{4.2}$$

$$\dot{\beta} = 0 \tag{4.3}$$

The state vector is defined as $x = [h \quad v \quad \beta]^T$, which refers to the target altitude, velocity, and ballistic coefficient, respectively. The air density ρ is modeled as follows:

$$\rho = \gamma e^{-\eta h} \tag{4.4}$$

where $\gamma = 1.754$ and $\eta = 1.49 \times 10^{-4}$. The discrete-time state equation is defined as follows [1]:

$$x_{k+1} = F x_k - G[D(x_k) - g] + w_k \tag{4.5}$$

With matrices F and G defined by:

$$F = \begin{bmatrix} 1 & -T & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4.6}$$

$$G = [0 \quad T \quad 0]^T \tag{4.7}$$

Furthermore, the function for drag $D(x_k)$ (the only nonlinear term) is defined by:

$$D(x_k) = \frac{g\rho(x_{k,1}) x_{k,2}^2}{2x_{k,3}} x_{k+1} = F x_k - G[D(x_k) - g] + w_k \tag{4.8}$$

As in [1], the system noise w_k is assumed to be zero-mean Gaussian with a covariance matrix Q defined by:

$$Q \approx \begin{bmatrix} q_1 \frac{T^3}{3} & q_1 \frac{T^2}{2} & 0 \\ q_1 \frac{T^2}{2} & q_1 T & 0 \\ 0 & 0 & q_2 T \end{bmatrix} \tag{4.9}$$

Note that the parameters $q1$ and $q2$ respectively control the amount of system noise in the target dynamics and the ballistic coefficient [1]. As shown in Figure 4, a radar is positioned on the ground below the target. The measurement equation in this scenario is defined by:

$$z_k = Hx_k + v_k \tag{4.10}$$

where it is assumed that two measurements are available, such that:

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{4.11}$$

In the tracking scenario involved in this study, the initial states are defined as follows: $x_{1,0} = 61{,}000\ m$, $x_{2,0} = 3{,}048\ m/s$, and $x_{3,0} = 19{,}161\ kg/ms^2$. Other notable parameters were defined as: $q_1 = 10^4$, $q_2 = 10$, $T = 0.1\ sec$, $R = diag([10^4 \quad 10^3])$, and $g = 9.81\ m/s^2$.

As per the earlier SIF discussion, it is required to transform (4.11) into a square matrix (i.e., identity), such that an 'artificial' measurement is created. A number of methods exist, such as the reduced order or Luenberger's approach, which are presented in [25, 26, 27]. Consider a system model involving phase variables. It is possible to derive a third 'artificial' measurement $y_{3,k}$ based on the available measurements ($z_{1,k}$ and $z_{2,k}$). In (4.11), the ballistic coefficient measurement is not available. If the system model (4.5) is known with complete confidence, then it is possible to derive an artificial measurement for the ballistic coefficient from the first two measurements. Hence, consider the following from (4.5):

$$y_{3,k} = \frac{Tg\gamma z_{2,k}^2}{2\left(z_{2,k+1} - z_{2,k} + Tg\right)e^{-\eta z_{1,k}}} \tag{4.12}$$

The accuracy of (4.12) depends on the sampling rate T. Applying (4.12) allows a measurement matrix equivalent to the identity matrix. The estimation process would continue as in the previous section, where a full measurement matrix was available. Note however that the artificial acceleration measurement would be delayed one time step. Furthermore, note that the artificial measurement would have to be initialized (i.e., 0 is a typical value). Equation (4.12) essentially propagates the known measurements through the system model to obtain the artificial ballistic coefficient measurement. It is conceptually similar to the method presented in [27] and creates a full measurement matrix.

The initial state estimates $\hat{x}_0$ were set 10% away from the true values x_0. The initial state error covariance matrix was set to $P_0 = 10Q$. The following figure shows the object altitude and estimates over time.

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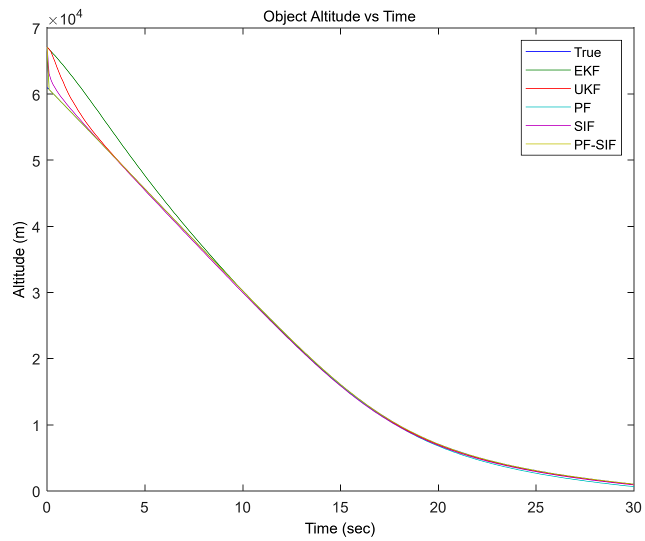

Figure 5. Object Altitude and Estimates

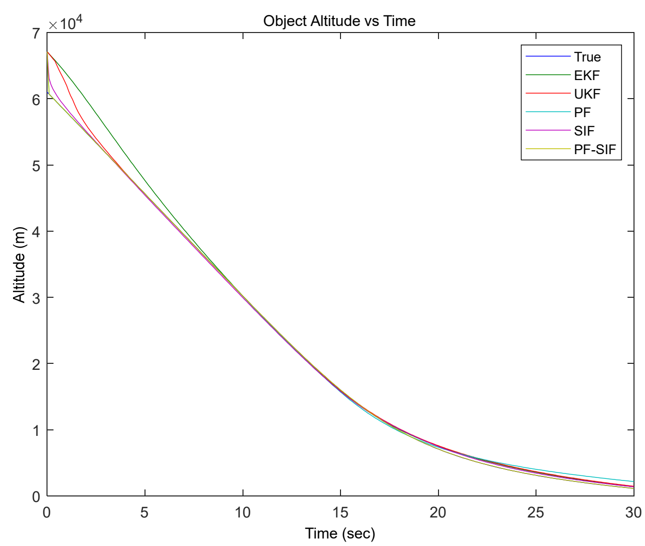

Figure 6. Object Altitude and Estimates with the Presence of Modeling Error at 15 seconds

Simulation Results

In Table 1 below, the summarized results of the RMSE are shown for each filter applied to the tracking scenario defined in earlier sections of this paper (note: these results are repeatable). Overall, the proposed PF-SIF algorithm provides the best result in terms of estimation accuracy. The PF performs very well, with the exception of the ballistic coefficient (it fails to provide a good estimate). The SIF also performed well. The EKF provided the worst estimate, most likely due to the slower convergence rate. An interesting result occurs when one introduces modeling errors into the system model (4.5). As an example, in an effort to demonstrate the robustness of the PF-SIF and SIF to modeling uncertainties, consider the case when the gravity coefficient is doubled. The following figure shows the implications of modeling error being introduced at 15 seconds during the tracking scenario. The filters begin to diverge from the true state trajectory, with the PF being the furthest at 30 seconds. The RMSE for this case was calculated for each filter, and is shown in the following table.

Table 1. RMSE of the Tracking Scenario

Filter	**Altitude** (m)	**Velocity** (m/s)	**Ballistic** (kg/ms^2)
EKF	1,902	286	1,916
UKF	957	64.5	2,329
PF	425	29.3	23,674
SIF	443	87.6	1,846
PF-SIF	335	20.2	1,641

Table 2. RMSE of Tracking with Modeling Errors

Filter	**Altitude** (m)	**Velocity** (m/s)	**Ballistic** (kg/ms^2)
EKF	1,918	306	1,917
UKF	1,166	116	3,382
PF	619	88.9	39,814
SIF	472	124	1,992
PF-SIF	348	38.8	2,101

It is interesting to note that the PF-SIF estimates remained relatively insensitive to the added modeling error. The combination of the SIF and the PF improved the overall accuracy and stability of the PF estimation strategy.

5. CONCLUSIONS

In this paper, a new state and parameter estimation based on the combination of the PF and the SIF was introduced. The combined method (PF-SIF) utilizes the estimates and state error covariance of the SIF to formulate the proposal distribution which generates the particles used by the PF. The PF-SIF method was applied on a nonlinear target tracking problem. The results of this tracking scenario demonstrate the improved performance of the combined methodology. Future research work will involve studying other nonlinear estimation problems, and analyzing how the PF-SIF performs.

APPENDIX

Table 3. List of Nomenclature

Parameter	Definition
x	State vector or values
z	Measurement (system output) vector or values
y	Artificial measurement vector or values
u	Input to the system
w	System noise vector
v	Measurement noise vector
F	Linear system transition matrix
G	Input gain matrix
H	Linear measurement (output) matrix
K	Filter gain matrix (i.e., KF or SIF)
P	State error covariance matrix
Q	System noise covariance matrix
R	Measurement noise covariance matrix
S	Innovation covariance matrix
e	Measurement (output) error vector
$diag(a)$ or $\bar{a}$	Defines a diagonal matrix of some vector *a*
$sat(a)$	Defines a saturation of the term *a*
δ	SIF boundary layer width
$\lvert a \rvert$	Absolute value of some parameter *a*
$E\{\cdot\}$	Expectation of some vector or value
T	Sample time, or transpose of some vector or matrix
^	Estimated vector or values
$x_k^{(i)}$	Particles used by the PF
$\omega_k^{(i)}$	Importance weights used by the PF
N_{eff}	Effective threshold for the PF
$k+1\lvert k$	A priori time step (i.e., before applied gain)
$k+1\lvert k+1$	A posteriori time step (i.e., after update)

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