The Luenberger sliding innovation filter for linear systems

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ABSTRACT

In this paper, the newly developed sliding innovation filter (SIF) is reformulated to accommodate the ability of extracting the hidden states. This is accomplished by using the well-known Luenberger technique, which is commonly used by observers. In this paper, the SIF is applied to a linear system, which has fewer measurements than states. The results show that the proposed filter extracts the hidden state with small RMSE, as low as 0.1, and small MAE, as low as 1.

Keywords: Luenberger method, SIF, performance.

1. INTRODUCTION

Filters are widely used in estimation applications. Their main purpose is to extract some valuable information from the available signals while overcoming the disturbances, uncertainties, and noise that they may contain [1-9]. This improves the quality of the controller and the dynamics performance of the system [10-20]. This work covers a new formulation for the sliding innovation filter (SIF) [21-29]. SIF is a model based filter that is derived from the sliding mode theory. It uses a model that represent the actual system and excites it with the system's input to obtain an unrefined estimate. Then it refines the estimate using a corrective gain that is derived from the Lyapunov stability theorem. Hence, the filter belongs to the robust filter types, i.e. the smooth variable structure filter [30-46] and Sliding mode observer [47-71].

Although, SIF is stable, its performance is not optimal. Moreover, the performance gets worse when disturbances and noise present. If the number of measured signal becomes fewer than the number of states, then the filter highly depends on the system and measurement matrices. If the non-measured states, i.e. hidden states, are not directly linked to the measured states, or the measurement, the filter fails to extract the required information. To overcome this, the filter was combined with other filters like the Regular [72-87], Extended [88-93], and Sigma-point Kalman filters [94-111]. However, the algorithm becomes more complicated, and the simulation time increases.

This work proposes a new form of SIF that is simple, yet efficient for certain applications. The proposed method combines the SIF with the Luenberger method [112-113]. The latter extracts the hidden states from the available measurement and feeds them to the SIF to do the filtering and maintain the stability and robustness of the process.

This brief paper is organized as follows. The SIF and the proposed method are introduced in Section 2. Section 3 discuss the application of the proposed method to a third order system. Section 4 concludes the paper and hint on the future works.

2. METHODOLOGY

2.1. Linear system model

The linear system can be represented in a matrix form as follow:

$$\mathbf{x}_k = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_{k-1}\mathbf{u}_{k-1} + \mathbf{w}_{k-1} \tag{1}$$

$$\mathbf{z}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k \tag{2}$$

Where $\mathbf{x}_k$ and $\mathbf{z}_k$ are the state and measurement vectors at time k. These represent the system dynamics and the output of the sensors that are used to measure these states, respectively. The system is defined by the matrices $\mathbf{A}_{k-1}$ and $\mathbf{B}_{k-1}$, which are called the system and the input matrices, respectively. The system's parameters are included in these matrices.

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The measurement matrix, $\mathbf{H}_k$, represent the model of the sensors and it includes their parameters. The signals obtained from system and sensors are subjected to disturbances, $\mathbf{w}_{k-1}$ and $\mathbf{v}_k$, respectively.

If the number of states is fewer than the number of measurements, then the system has hidden states. The idea of estimation is to extract all states from the available measurement signals while reducing the effect of $\mathbf{v}_k$. This paper discusses the use of Luenburger method combined with the SIF to extract the hidden states. We will assume that sensor model is linear and is defined as:

$$\mathbf{H}_k = [\mathbf{I}_{m\times m} \quad \mathbf{0}_{m\times(n-m)}] \tag{3}$$

Where m and n are the number of measurement and state signals, respectively. $\mathbf{I}$ is the identity matrix, and $\mathbf{0}$ is a matrix with zero elements.

2.2. SIF algorithm

The sliding Innovation filter consists of two steps:

1- **Prediction Stage**, where the a priori estimate and its measurement, $\hat{\mathbf{x}}_{k+1|k}$ and $\hat{\mathbf{z}}_{k+1|k}$, respectively, are calculated using the following equations:

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{A}_{k-1}\hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_{k-1}\mathbf{u}_{k-1} \tag{4}$$

$$\hat{\mathbf{z}}_{k|k-1} = \mathbf{H}_k\hat{\mathbf{x}}_{k|k-1} \tag{5}$$

2- **Update/Correction Stage**, where the a posteriori estimate and its measurements, $\hat{\mathbf{x}}_{k|k}$ and $\hat{\mathbf{z}}_{k|k}$, respectively, are calculated using the following

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \left[\mathbf{H}_k^+\left(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}\right)\right]^\circ sat\left(\left|\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}\right|, \mathbf{\Psi}_k\right) \tag{6}$$

$$\hat{\mathbf{z}}_{k|k} = \mathbf{H}_k\hat{\mathbf{x}}_{k|k} \tag{7}$$

Where $\mathbf{H}_k^+$ is the pseudoinverse vector of $\mathbf{H}_k$, $\mathbf{\Psi}_k$ is the boundary layer, $A°B$ is schur product that is done by multiplying each element of A with it corresponding element in B, and sat is the saturated function.

The SIF performance for fewer number of measurements compared to the number of states depends highly on the interconnectivity between the states through the matrix $\mathbf{A}_{k-1}$, and the mapping between the sensors and the hidden states through the matrix $\mathbf{H}_k^+$. If $\mathbf{H}_k$ is defined as in (3), then the filter cannot correct the values of the hidden states as they are not connect to the measurement. Hence, the filter performance degrades. To overcome this issue, the filter is reformulated using the Luenberger method to map the hidden states to the measurement.

2.3. Luenberger/SIF algorithm

The Luenberger method is used for observers rather than the filter, as it assumes no disturbances, neither uncertainties exist in the signals. The method can be explained and derived as follows [112-113]:

By subtracting (4) from (1) and assume $\mathbf{w}_{k-1}$ is zero, the following can be obtained:

$$\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1} = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_{k-1}\mathbf{u}_{k-1} - \mathbf{A}_{k-1}\hat{\mathbf{x}}_{k-1|k-1} - \mathbf{B}_{k-1}\mathbf{u}_{k-1} \tag{8}$$

$$\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1} = \mathbf{A}_{k-1}\left(\mathbf{x}_{k-1} - \hat{\mathbf{x}}_{k-1|k-1}\right) \rightarrow e_{x,k|k-1} = \mathbf{A}_{k-1}e_{x,k-1|k-1} \tag{9}$$

Where e_x represents the error in estimation.

Assuming the availability of imaginary sensors that measure the hidden states to be, $\mathbf{y}_k$, then a full rank measurement vector can be obtained, $\mathbf{Z}_k$, as follows:

$$\mathbf{Z}_k = \begin{bmatrix}\mathbf{z}_k \\ \mathbf{y}_k\end{bmatrix} \tag{10}$$

And the measurement matrices, $\mathbf{H}_k$ and $\widehat{\mathbf{H}}_k$ become identity matrices. In this case:

$$\mathbf{Z}_k = \mathbf{x}_k \tag{11}$$

And

$$\hat{\mathbf{Z}}_k = \hat{\mathbf{x}}_k \tag{12}$$

Substitute (11) and (12) in (9) yields:

$$e_{Z,k|k-1} = \mathbf{A}_{k-1} e_{Z,k-1|k-1} \tag{13}$$

Or

$$\begin{bmatrix} e_{z,k|k-1} \\ e_{y,k|k-1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} e_{z,k-1|k-1} \\ e_{y,k-1|k-1} \end{bmatrix} \tag{14}$$

Where

$\mathbf{A}_{11} \in \mathbb{R}^{m\times m}, \mathbf{A}_{12} \in \mathbb{R}^{m\times(n-m)}, \mathbf{A}_{21} \in \mathbb{R}^{(n-m)\times m}$ and $\mathbf{A}_{22} \in \mathbb{R}^{(n-m)\times(n-m)}$ are submatrices from the matrix $\mathbf{A}_{k-1}$.

Luenberger is assumed stable for such application if $e_{z,k-1|k-1}$ vanished. Then (14) becomes:

$$\begin{bmatrix} e_{z,k|k-1} \\ e_{y,k|k-1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} 0 \\ e_{y,k-1|k-1} \end{bmatrix} \tag{15}$$

By expanding the matrices, the followings are obtained:

$$e_{z,k|k-1} = A_{12} e_{y,k-1|k-1} \tag{16}$$

$$e_{y,k|k-1} = A_{22} e_{y,k-1|k-1} \tag{17}$$

Substitute (16) in (17) yields

$$e_{y,k|k-1} = A_{22} A_{12}^{-1} e_{z,k|k-1} \tag{18}$$

Equation (18) relates the error in the imaginary measurement vector to the actual measurement vector, which means that (6) is rewritten as:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \begin{bmatrix} \left[\mathbf{H}_k^{+}\left(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}\right)\right]^{\circ} sat\left(\left|\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}\right|, \mathbf{\Psi}_k\right) \\ \left[\mathbf{H}_k^{+} A_{22} A_{12}^{-1}\left(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}\right)\right]^{\circ} sat\left(\left|A_{22} A_{12}^{-1}\left(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}\right)\right|, \mathbf{\Psi}_k\right) \end{bmatrix} \tag{19}$$

$$\hat{\mathbf{z}}_{k|k} = \hat{\mathbf{x}}_{k|k} \tag{20}$$

The boundary layer can be adjusted to reduce the effect of the disturbances and uncertainties.

3. CASE STUDY

The method of section 2 is tested on a third order system that is defined by (1) and (2) assuming:

$$\mathbf{A}_{k-1} = \mathbf{A} = \begin{bmatrix} 1 & \tau & 0 \\ 0 & 1 & \tau \\ 0 & -\omega_n^2 \tau & 1 - 2\zeta\omega_n\tau \end{bmatrix} \tag{21}$$

$$\mathbf{B}_{k-1} = \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ b\tau \end{bmatrix} \tag{22}$$

Where ω_n , b, ξ and τ have values of $360\ \mathrm{H_z}$, $30\ \frac{\mathrm{m}}{\mathrm{sec}\times\mathrm{rad}}$, 0.4 and 0.001 sec, respectively [112]. The input is assumed to be multiple level random signal as shown in figure 1.

The results are obtained for applying SIF to the system, and they are illustrated by Fig. 2 and tables 1 and 2. Fig. (2-a), Fig. (2-c) and Fig. (2-e) show the estimation of the position, velocity and acceleration, respectively, while the error in their estimation are shown in Fig. (2-b), Fig. (2-d) and Fig. (2-f), respectively. Table 1 shows the root mean squared error and Table 2 shows the maximum absolute error in the results, which are calculated using the following:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{ns}\left(x_{Actual,i} - x_{Prediction,i}\right)^2}{ns}} \tag{23}$$

$$MAE = \max(|x_{Actual} - x_{Prediction}|) \quad (24)$$

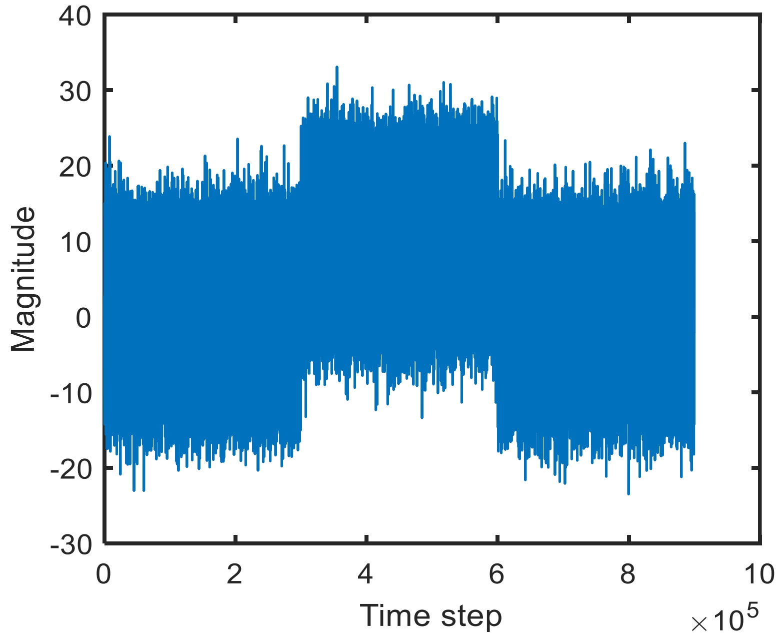

Figure 1. Input signal to the system

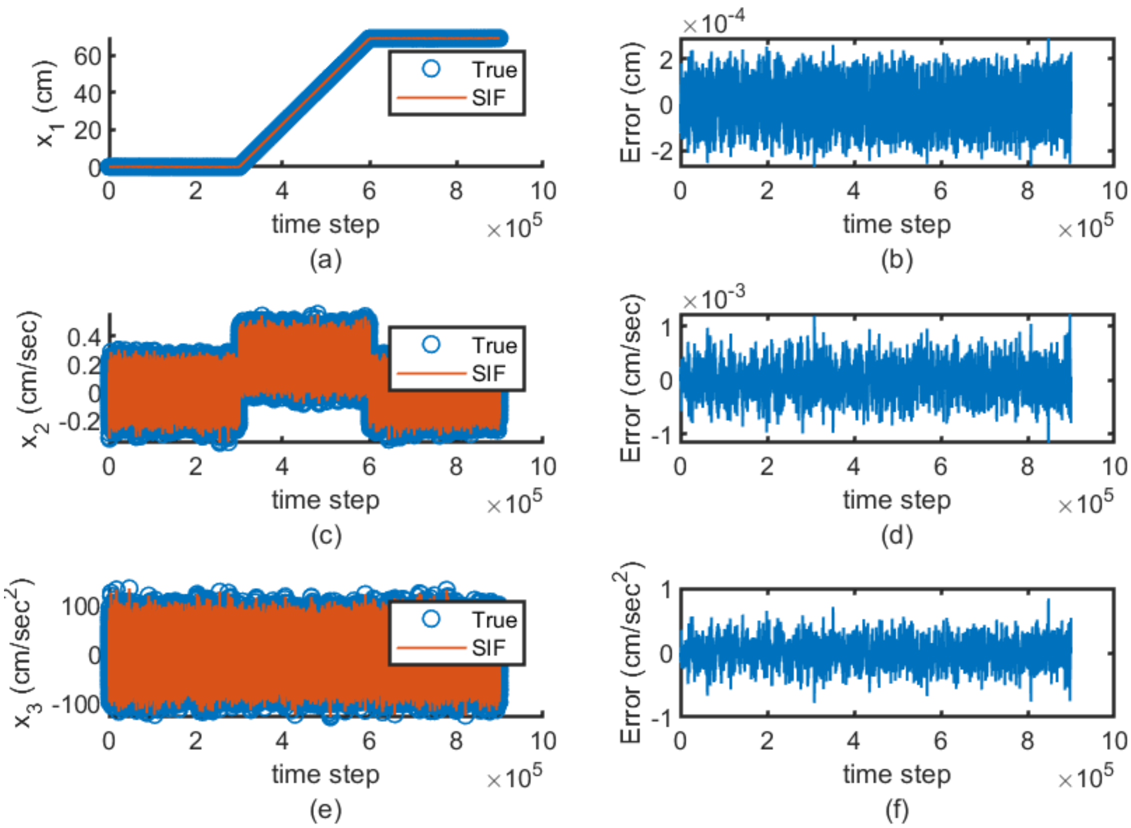

Figure 2. The results of the SIF compared to the true states, (a) position, (b) error in position, (c) velocity, (d) error in velocity, (e) acceleration and (f) error in acceleration

Table 1. RMSE of the SIF's results

	$RMSE$ in		
SIF	$x_1\ (cm)$	$x_2(cm/s)$	$x_3(cm/s^2)$
	7.8983×10^{-05}	1.27793×10^{-04}	9.7940×10^{-02}

Table 2. MAE of the SIF's results

	MAE in		
SIF	$x_1\ (cm)$	$x_2(cm/s)$	$x_3(cm)$
	2.756×10^{-04}	1.18×10^{-03}	8.043×10^{-01}

The results show that Luenberger/SIF is capable of extracting the hidden states with excellent performance. The highest RMSE is found in the acceleration state and it is less than 0.1 cm/s^2. This value is less than 0.1% of the maximum acceleration's amplitude. Similar results are found for RMSE in position and velocity estimations, where they are equal to 0.001% and 0.02%, respectively. The MAE is found to be 10 times the results of RMSE, where it has values less than 0.01%, 0.2% and 1% for position, velocity and acceleration estimates, respectively. These values are small and can be neglected.

4. CONCLUSION

In this article, the SIF is formulated and combined with Luenberger method. This gives the benefit of extracting the hidden states from the available measurement. The formulated filter maps the information in the measurement to the entire states. Moreover, the SIF keeps the filter stable. The results show that RMSE and MAE are in acceptable range, as the highest values of RMSE is less than 0.1% and for MAE is less than 1%. In future work, the filter will be tested using an experimental setup and the results will be compared to other filters.

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