Development of a Second-Order Sliding Innovation Filter for an Aerospace System

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ABSTRACT

The sliding innovation filter is a new type of predictor-corrector estimation method. The strategy is used to estimate relevant states of interests and has been found to be robust to modeling uncertainties and disturbances. In this paper, a second-order formulation of the sliding innovation filter is presented to improve its estimation performance in terms of accuracy while maintaining robustness. The strategy is applied to an aerospace system that has been designed for experimentation. The results are compared with the well-known Kalman filter, and future work is considered.

Keywords: Estimation theory, sliding innovation filter, Kalman filter, aerospace systems, robustness, variable structure

1. BRIEF INTRODUCTION

Estimation theory plays an important role in a variety of fields, ranging from target tracking to controlling robots on other planets [1]. Estimation is the act of extracting knowledge of the states of a system from noisy measurements and unknown environments [2]. For example, the movement of an aircraft can be captured and predicted by the aircraft's position, velocity, and turn rate [3, 4]. With these so-called states of interest, we can predict the future position of the aircraft using a radar and some mathematical models that represent the aircraft motion.

The most well-known estimation strategy is the Kalman filter [1]. It offers an optimal solution, in terms of estimation error, for linear and known systems under the presence of white noise. White noise is stochastically defined as a measurement signal with zero mean and covariance that is normally distributed (also known as Gaussian). The KF has been studied extensively in literature [1, 5]. Furthermore, a number of variants have been derived to extend the KF to nonlinear systems and measurements; which allows for more useful application as most systems found in nature are indeed nonlinear [2]. The KF is derived according to a few strict assumptions: the system and measurement are linear and known, and the system and measurement noise are white. If these assumptions are not met, the KF may become unstable or yield unreliable estimates.

A number of methods have been developed to improve the stability and robustness of the KF and its variants, as well as combine the KF with control theory [1, 2, 6, 7]. One such method is the H_∞ filter, which utilizes boundaries based on the worst-case uncertainties to regulate the filtering gain which ensures the state estimates are bounded to within a region of the true state trajectory [8, 9]. Most recently, a variable structure-based estimation strategy known as the sliding innovation filter (SIF) was introduced [10]. The SIF is based on sliding mode and variable structure methods first introduced in the 1970s, and later utilized for filtering methods [11, 12, 13, 14, 15]. Like the KF, the SIF is a predictor-corrector method for linear systems and measurements, however its corrective gain allows for robust estimation of unknown systems and under the presence of external disturbances [2]. The SIF is considered sub-optimal, however is robust as compared to the restrictive assumptions of the KF. The SIF has been extended to nonlinear systems and measurements (i.e., ESIF) through the use of first-order Taylor series approximations, as per the extended Kalman filter (EKF). The results presented in demonstrate the effectiveness of the SIF and ESIF strategies for unknown systems and disturbances.

In this paper, we propose a second-order sliding innovation filter (SO-SIF or SIF2). The SIF2 is formulated base on the use of additional innovation or measurement error terms from the previous time step. The proposed SIF2 is applied on an aerospace system and compared with the standard SIF, as well as the well-known KF. In Section 2, the SIF equations are summarized. In Section 3, the proposed SIF2 estimation process is summarized. The aerospace system and results are provided in Section 4. Concluding remarks and future work are then discussed in Section 5.

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2. THE SLIDING INNOVATION FILTER

Similar to the KF, the prediction stage includes calculating the predicted or *a priori* ('before the fact') state estimates $\hat{x}_{k+1|k}$, the predicted state error covariance $P_{k+1|k}$, and the predicted innovation $\tilde{z}_{k+1|k}$ as per the following three equations, respectively:

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_k \tag{2.1}$$

$$P_{k+1|k} = AP_{k|k}A^T + Q_k \tag{2.2}$$

$$\tilde{z}_{k+1|k} = z_{k+1} - C\hat{x}_{k+1|k} \tag{2.3}$$

The update stage includes calculating the SIF gain K_{k+1}, the updated or *a posteriori* ('after the fact') state estimates $\hat{x}_{k+1|k+1}$, and the updated state error covariance $P_{k+1|k+1}$ as per the following three equations, respectively:

$$K_{k+1} = C^+\overline{sat}\left(\left|\tilde{z}_{k+1|k}\right|/\delta\right) \tag{2.4}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}\tilde{z}_{k+1|k} \tag{2.5}$$

$$P_{k+1|k+1} = (I - K_{k+1}C)P_{k+1|k}(I - K_{k+1}C)^T \dots$$
$$\dots + K_{k+1}R_{k+1}K_{k+1}^T \tag{2.6}$$

Note that C^+ refers to the pseudoinverse of the measurement matrix, $\overline{sat}$ refers to the diagonal of the saturation term, sat refers to the saturation of a value (yields a result between -1 and +1), $\left|\tilde{z}_{k+1|k}\right|$ refers to the absolute value of the innovation, δ refers to the sliding boundary layer width, and I refers to the identity matrix (of dimension n-by-n where n is the number of states). Equations (2.1) through (2.6) represent the SIF estimation process for linear systems and measurements.

As described in [10], the main difference between the KF and SIF strategies is in the structure of the gain. For the KF, the gain is derived as a function of the state error covariance, which offers optimality [1, 2]. However, for the SIF, the gain is based on the measurement matrix, the innovation, and a sliding boundary layer term. Although the state error covariance is not used to calculate the SIF gain, it still provides useful information as it represents the amount of estimation error in the filtering process. Figure 1 provides an overview of the SIF estimation concept. As described in [10], an initial estimate is pushed towards the sliding boundary layer which is defined based on the amount of uncertainties in the estimation process. Once inside the sliding boundary layer, the estimates are forced to switch about the true state trajectory by the SIF gain.

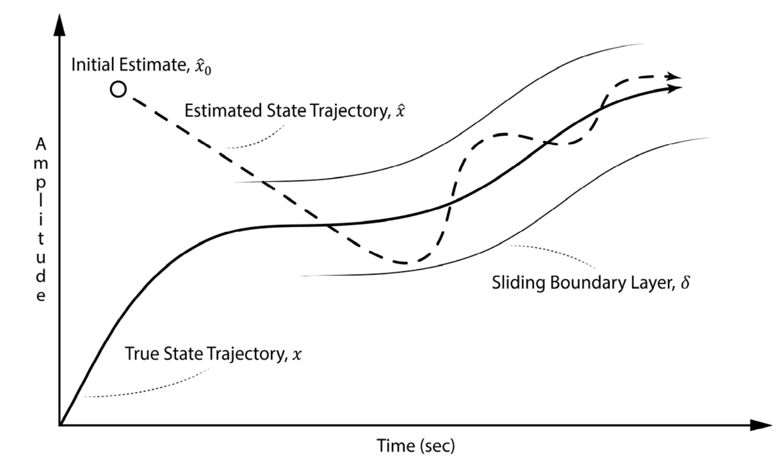

Figure 1. The sliding innovation filter (SIF) concept illustrating the effects of the switching gain and the sliding boundary layer used to maintain robustness and stability of estimates [10].

The state estimates are updated with their corresponding innovation and sliding boundary layer term. The SIF gain effectively acts as a switching term, which forces the measurement errors to be bounded towards the true state trajectory. The sliding boundary layer δ is defined as a function of the modeling uncertainty and noise present in the estimation process. The width can be tuned to obtain the desired estimation result. A method to set the width is also explained in [10]. Another starting point for tuning is to use the values of the measurement noise covariance. For example, $\delta = 10diag(R)$. The values can then be tuned by trial-and-error, grid search methods, or optimization techniques to reduce the estimation error. For the cases when there are fewer measurements than states ($m < n$), artificial measurements can be created based on existing measurements to create a full measurement matrix. The structure could also be modified as per a Luenberger observer or other strategies as per [16, 7]. This process would be required to estimate parameters of the system matrix using the SIF.

3. THE SECOND-ORDER SLIDING INNOVATION FILTER

In an effort to improve the accuracy of the SIF, a second-order SIF (SIF2) is proposed in this paper. The SIF2 estimation process is nearly identical to the SIF, except for the gain defined by (3.5). The SIF2 gain is derived based on [17] and the requirement to also calculate the updated innovation as per (3.8). The gain defined by (3.5) is derived using a Lyapunov function defined by:

$$M_{k+1} = \tilde{z}_{k+1|k+1} \circ \tilde{z}_{k+1|k+1} + \Delta\tilde{z}_{k+1|k+1} \circ \Delta\tilde{z}_{k+1|k+1} \tag{3.1}$$

where $\Delta\tilde{z}_{k+1|k+1} = \tilde{z}_{k+1|k+1} - \tilde{z}_{k|k}$ and $\circ$ is the Schur product (element-by-element multiplication). Based on Lyapunov theory, the estimation process may be considered stable if $\Delta M_{k+1} < 0$. Using these definitions and SIF equations, the new gain (3.5) is defined. The SIF2 prediction stage includes calculating the predicted state estimates $\hat{x}_{k+1|k}$, the predicted state error covariance $P_{k+1|k}$, and the predicted innovation $\tilde{z}_{k+1|k}$ as per the following three equations, respectively:

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_k \tag{3.2}$$

$$P_{k+1|k} = AP_{k|k}A^T + Q_k \tag{3.3}$$

$$\tilde{z}_{k+1|k} = z_{k+1} - C\hat{x}_{k+1|k} \tag{3.4}$$

The SIF2 update stage includes calculating the SIF2 gain K_{k+1}, the updated state estimates $\hat{x}_{k+1|k+1}$, the updated state error covariance $P_{k+1|k+1}$, and the updated innovation $\tilde{z}_{k+1|k+1}$, as per the following equations, respectively:

$$K_{k+1} = C^+\overline{sat}\left(\left|\tilde{z}_{k+1|k}/\delta - \tilde{z}_{k|k}/(2\delta)\right|\right) \tag{3.5}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}\tilde{z}_{k+1|k} \tag{3.6}$$

$$P_{k+1|k+1} = (I - K_{k+1}C)P_{k+1|k}(I - K_{k+1}C)^T \ldots$$
$$\ldots + K_{k+1}R_{k+1}K_{k+1}^T \tag{3.7}$$

$$\tilde{z}_{k+1|k+1} = z_{k+1} - C\hat{x}_{k+1|k+1} \tag{3.8}$$

Equations (3.2) through (3.8) represent the SIF2 estimation process for linear systems and measurements. Note that the updated innovation (3.8) is used in the next iteration as per (3.5).

4. COMPUTER EXPERIMENT AND RESULTS

In this section, the well-known KF, SIF, and proposed SIF2 are applied on a linear system with noise. As per [10], the studied system is a type of aerospace flight surface actuator, referred to as the electrohydrostatic actuator (EHA). It has been well-studied and presented in literature [18, 19, 20]. A simplified linear EHA model was formulated in state space where the states of interest refer to position, velocity, and acceleration [2, 21]. The model parameters were found through experimentation of an EHA [21, 22]. The linear form of the system and measurements are described using the following state space equations [21]:

$$x_{k+1} = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & T \\ -557 & -28.6 & 0.94 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0 \\ 557 \end{bmatrix} u_k + w_k \tag{4.1}$$

$$z_{k+1} = Cx_{k+1} + v_{k+1} \tag{4.2}$$

where the sample rate T is defined as $1\ ms$, k is the time step, C refers to the measurement matrix which in this case is an identity matrix of dimension $m \times m$ or 3×3, and u is the controller input for the system (a square wave of amplitude

1.5 rad/s and frequency 2π) that drives the desired trajectory. The system and measurement noises (w and v) are normally distributed with zero mean and covariance's Q and R defined by (4.3) and (4.4), respectively.

$$Q = diag([10^{-4} \quad 10^{-2} \quad 1]) \tag{4.3}$$

$$R = diag([10^{-3} \quad 10^{-1} \quad 10]) \tag{4.4}$$

The initial state values, measurements, and estimates were set to zero. The initial state error covariance values were set to $P_{0|0} = 10Q$. The sliding boundary layer width was manually tuned to yield the smallest estimation error, and was found for this simulation to be $\delta = [0.05 \quad 1 \quad 0.5]$. The simulation was coded in MATLAB.

The results of applying the KF, SIF, and SIF2 strategies on the linear EHA are shown in Figure 2. As expected, since the system is linear and well-known, the KF yields better results in terms of root mean square error (RMSE) under normal operating conditions. However, the results appear nearly identical on the plot. Note that RMSE is defined by (4.5) where n is the number of time steps. The results are summarized in Table 1.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \hat{x}_i)^2}{n}} \tag{4.5}$$

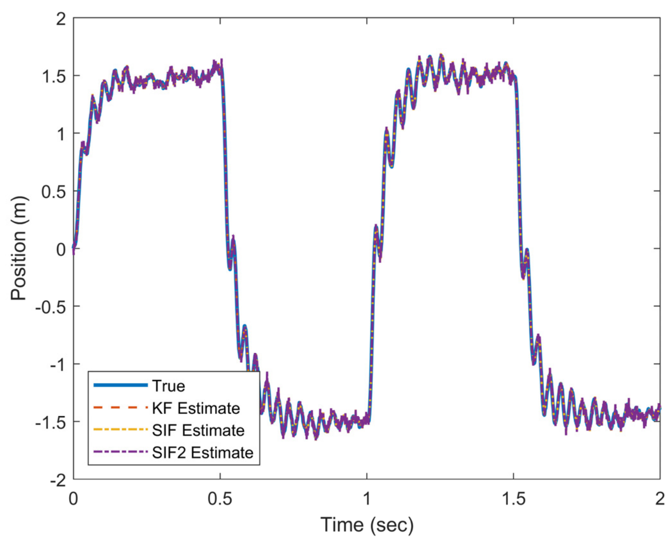

Figure 2. The results of applying the KF, SIF, and SIF2 estimation strategies to the simulated EHA under normal operating conditions are shown here. Note the lines appear to be nearly overlapping because the results are very similar at this scale.

Table 1. RMSE results for the normal EHA scenario.

State	KF	SIF	SIF2
Position (m)	0.0117	0.0301	0.0287
Velocity (m/s)	0.1541	0.2385	0.2349
Acceleration (m/s^2)	2.9091	3.2033	3.2024

Consider the case when the system has a fault injected half-way through the simulation (at $t = 1$ sec). In this case, the linear system state equation used by the filters is changed [10]:

$$x_{k+1} = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & T \\ -240 & -28 & 0.94 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0 \\ 557 \end{bmatrix} u_k + w_k \tag{4.6}$$

The results of the modeling uncertainty and its effects on the filters are shown in Figures 3 and 4. The model mismatch at 1 second causes the KF to deviate from the true state trajectory, yielding poor estimates of the true position. The SIF and SIF2 were still able to perform relatively well, and was bounded to the true state trajectory due to the switching effects of the gains.

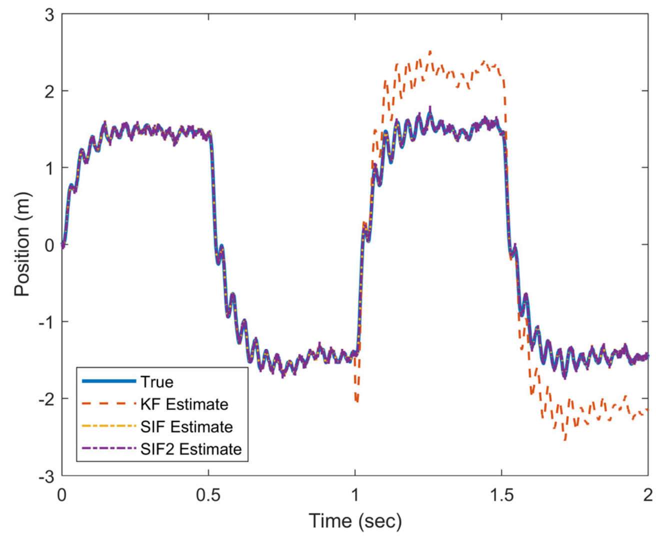

Figure 3. The results of applying the KF, SIF, and SIF2 estimation strategies to the simulated EHA under faulty operating conditions are shown here. The KF fails to yield a good estimate at the injection of a fault mid-way through the simulation.

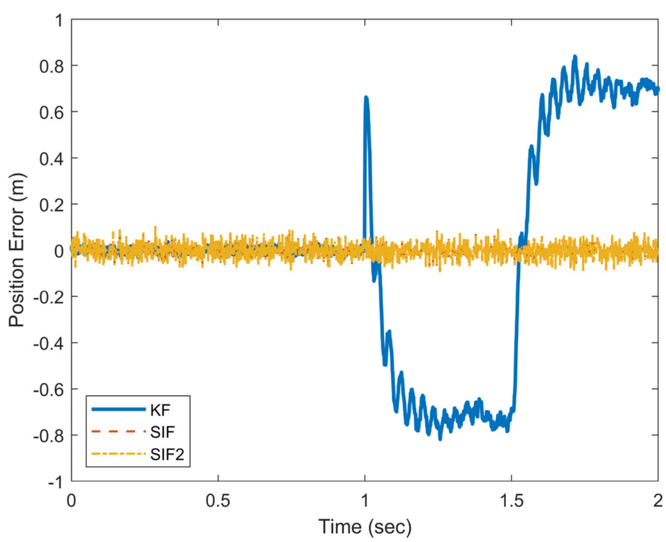

Figure 4. The position errors of the three filtering strategies for the simulated EHA under faulty operating conditions are shown here. Note the KF's significant position error at the onset of the fault (mid-way through the simulation).

The RMSE results for the faulty case are shown in Table 2. The SIF and SIF2 perform slightly worse than the normal case. However, the KF is unable to recover from the modeling uncertainty and yields poor performance. This was expected as the KF is derived based on the assumption that the system is known. In this case, the SIF and SIF2 yielded nearly the same result (around less than 1% difference).

Table 2. RMSE results for the faulty EHA scenario.

State	KF	SIF	SIF2
Position (m)	0.4623	0.0369	0.0362
Velocity (m/s)	5.2127	0.3142	0.3088
Acceleration (m/s^2)	26.786	3.9328	3.9316

5. CONCLUSION

In this brief paper, a second-order formulation of the sliding innovation filter (SO-SIF or SIF2) was presented to improve the SIF estimation performance in terms of accuracy. The strategy was applied to an aerospace system that was simulated in MATLAB. The results were compared with the well-known Kalman filter (KF) and the standard SIF. Under normal operating conditions, the KF yielded the best results in terms of estimation accuracy. This was expected since the KF is the optimal filter for linear, known systems with white noise. The SIF2 provided only marginally better results when compared with the SIF. Under faulty conditions, the KF failed to perform well, which was expected as the system used by the estimator was no longer ‘known’ with complete accuracy (i.e., one of the KF assumptions for optimality failed). The SIF and SIF2 still performed well as they are robust to modeling uncertainties. The SIF2 only yielded slightly better results than the SIF. Based on this simulation, it was determined that the second-order formulation of the SIF (SIF2) did not offer substantially better results to warrant a more complicated gain or the requirement to save the updated innovation error for each iteration. The standard SIF yields good estimation results and maintains robustness, and by comparison, is a simpler estimation process. In this case, the SIF is recommended over the SIF2. However, future work will look at modifications to the SIF2 to see if higher-order formulations warrant a more complicated estimation process. Additionally, other literature in signal processing will be explored and reported upon, as well as compared with the SIF.

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