The Two-Pass Sliding Innovation Smoother

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ABSTRACT

The sliding innovation filter (SIF) is a newly developed filter that shares similar principles with sliding mode observers and variable structure techniques. The SIF is formulated as a predictor-corrector method that uses the innovation or measurement error as a switching hyperplane and forces the states to remain within a region of its state trajectory. In this brief paper, the SIF is reformulated as a two-pass smoother to reduce the effects of noise and improve the overall performance. The proposed method, known as the sliding innovation smoother (SIS), is applied on an aerospace flight surface actuator, and the results are compared to the original filter.
Keywords: two-pass, smoother, sliding innovation filter, aerospace actuator.

1. A BRIEF INTRODUCTION

Estimation is used to predict, estimate, filter and/or smooth dynamic states for the system under study. When the purpose of the estimation is to reduce the effect of the noise, the word “filter” becomes more applicable. Filters extract the system information from the available measurements while trying to reduce the effect of the noise that corrupts the signal. The last term “smoothing” is used to smooth the estimate and provides with more accurate results. The main difference between the filter and the smoother is that the former one uses the signal points up to and including the current data point, while the smoother uses an interval of data that includes the past, current and future data points [1-3].

The most famous filter in academia and industry is the Kalman filter (KF) and its versions. These filters are known to be model-based filters, where they use models that mimic the system and sensor [4-19]. Several works were developed to overcome the shortages of KF especially in the stability and robustness field. This led to the newly developed sliding innovation filter (SIF) [20].

A smoother, on the other hand, has three major types: fixed interval where the estimation is accomplished over fixed time intervals; fixed point where the smoother estimates the state at a certain time in the past; and, fixed-lag where the process estimates the state at the current point and a few data from the past [21].

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This brief paper is organized as follows. The SIF is introduced in Section 2. The proposed two-pass sliding innovation smoother (SIS) is summarize in Section 3. Section 4 describes the electro-hydrostatic actuator (EHA). The results are discussed in Section 5 and the paper is concluded Section 6.

2. THE SLIDING INNOVATION FILTER

The SIF is a predictor-corrector filter that uses the a priori estimation error as a hyper-plan and force the estimate to remain withing the state neighborhood. The filter is summarized in Table 1.

Table 1. The pseudocode for the SIF code, as per [20].

$k = 0 \rightarrow Initialize\ \hat{\mathbf{x}}_{0\|0}\ and\ \mathbf{P}_{0\|0}$
Start $k = k + 1$
$\hat{\mathbf{x}}_{i_{k\|k-1}} = \hat{\mathbf{f}}(\hat{\mathbf{x}}_{k-1\|k-1}, u_{k-1})$
$\hat{\mathbf{z}}_{k\|k-1} = \hat{\mathbf{h}}\left(\hat{\mathbf{x}}_{i_{k\|k-1}}\right)$
$e_{z_{k\|k-1}} = \mathbf{z}_k - \hat{\mathbf{z}}_{k\|k-1}$
$\mathbf{K}_k = \mathbf{H}^+ sat(\left\|e_{z_{k\|k-1}}\right\|, \Psi)$, where $sat\left(\left\|e_{z_{k\|k-1}}\right\|, \Psi\right) = \begin{Bmatrix} \left\|e_{z,1_{k\|k-1}}\right\|/\Psi_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \left\|e_{z,m_{k\|k-1}}\right\|/\Psi_m \end{Bmatrix}$
$\hat{\mathbf{x}}_{k\|k} = \hat{\mathbf{x}}_{k\|k-1} + \mathbf{K}_k\left(e_{z_{k\|k-1}}\right)$
Go back to Start

3. THE TWO-PASS SLIDING INNOVATION FILTER

The SIF is reformulated as a fixed-interval smoother which is summarized in Table 2.

Table 2. The pseudocode for the SIF code, as per [20].

$k = 0 \rightarrow Initialize\ \hat{\mathbf{x}}_{0\|0}\ and\ \mathbf{P}_{0\|0}$
Start $k = k + 1$
$\hat{\mathbf{x}}_{i_{k\|k-1}} = \hat{\mathbf{f}}(\hat{\mathbf{x}}_{k-1\|k-1}, u_{k-1})$
$\hat{\mathbf{z}}_{k\|k-1} = \hat{\mathbf{h}}(\hat{\mathbf{x}}_{k\|k-1})$
$e_{z_{k\|k-1}} = \mathbf{z}_k - \hat{\mathbf{z}}_{k\|k-1}$
$\mathbf{P}_{k\|k-1} = \mathbf{F}\mathbf{P}_{k\|k-1}\mathbf{F}^T + \mathbf{Q}_k$
$\mathbf{K}_k = \mathbf{H}^+ sat(\left\|e_{z_{k\|k-1}}\right\|, \Psi)$, where $sat\left(\left\|e_{z_{k\|k-1}}\right\|, \Psi\right) = \begin{Bmatrix} \left\|e_{z,1_{k\|k-1}}\right\|/\Psi_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \left\|e_{z,m_{k\|k-1}}\right\|/\Psi_m \end{Bmatrix}$
$\hat{\mathbf{x}}_{k\|k} = \hat{\mathbf{x}}_{k\|k-1} + \mathbf{K}_k\left(e_{z_{k\|k-1}}\right)$
$\hat{\mathbf{z}}_{k\|k} = \hat{\mathbf{h}}\left(\hat{\mathbf{x}}_{i_{k\|k}}\right)$
$\mathbf{P}_{k\|k} = (I - \mathbf{K}_k\mathbf{H})\mathbf{P}_{k\|k-1}(I - \mathbf{K}_k\mathbf{H})^T + \mathbf{K}_k\mathbf{R}_k\mathbf{K}_k^T$

$e_{z_{k\|k-1}} = \mathbf{z}_k - \hat{\mathbf{z}}_{k\|k}$
$\hat{\mathbf{x}}_{k\|n} = \hat{\mathbf{x}}_{k\|k} + \mathbf{P}_{k\|k-1}(\mathbf{F} - \mathbf{K}_k\mathbf{H})\mathbf{P}_{k+1\|k}^T(\hat{\mathbf{x}}_{k+1\|n} - \hat{\mathbf{x}}_{k+1\|k})$
$\mathbf{P}_{k\|n} = \mathbf{P}_{k\|k} + \mathbf{P}_{k\|k-1}(\mathbf{F} - \mathbf{K}_k\mathbf{H})\mathbf{P}_{k+1\|k}^T(\mathbf{P}_{k+1\|n} - \mathbf{P}_{k+1\|k})\mathbf{P}_{k+1\|k}(\mathbf{F}^T - \mathbf{H}^T\mathbf{K}_k^T)\mathbf{P}_{k\|k-1}^T$
Go back to Start

4. THE EHA SETUP AND STATE SPACE MODEL

The methods of Section 2 and Section 3 are applied on the EHA shown in Fig. 1 [20]. The EHA model equation is defined as follows:

$$x_{k+1} = \begin{bmatrix} 1 & 0.001 & 0 \\ 0 & 1 & 0.001 \\ -557.02 & -28.616 & 0.9418 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0 \\ 557.02 \end{bmatrix} u_k \quad (1)$$

There are measurements associated with each state.

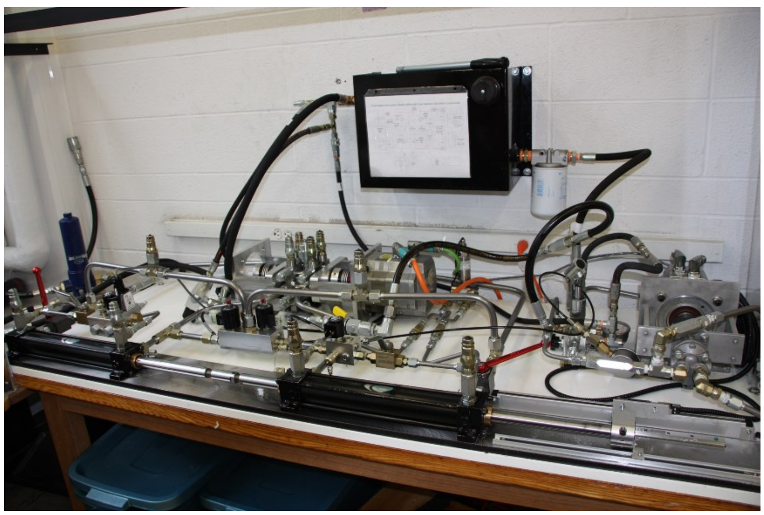

Figure 1. The EHA experimental setup [19].

5. SIMULATION RESULTS

The benchmark in Section 4 is used with two scenarios:

1. Case 1: No modeling uncertainties.
2. Case 2: With modeling uncertainties (as defined by (2)).

$$x_{k+1} = \begin{bmatrix} 1 & 0.001 & 0 \\ 0 & 1 & 0.001 \\ -240 & -28 & 0.9418 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0 \\ 557.02 \end{bmatrix} u_k \quad (2)$$

The results of the simulation are summarized in Tables 3 and 4, and illustrated by Figures 2 to 7. The results demonstrate that the smoothing process improves the KF and SIF results. However, it does not significantly improve the results when modelling uncertainties present (as defined by (2)). The root mean square error (RMSE) was reduced by more than 20% for Case 1, and less than 5% for Case 2.

Table 3. RMSE results for Case 1.

Filter	**RMSE**		
	Position (cm)	**Velocity (cm/s)**	**Acceleration (cm/s^2)**
KF	0.0078993	0.096544	2.6057
SIF	0.0086774	0.10621	8.1357
KF-Smoothed	0.0062699	0.071117	2.3317
SIF-Smoothed	0.0057723	0.10581	5.818

Table 4. RMSE results for Case 2.

Filter	**RMSE**		
	Position (cm)	**Velocity (cm/s)**	**Acceleration (cm/s^2)**
KF	0.3473	0.22396	3.3713
SIF	0.0088735	0.11169	5.3208
KF-Smoothed	0.35028	0.21213	6.094
SIF-Smoothed	0.0088735	0.11169	5.3208

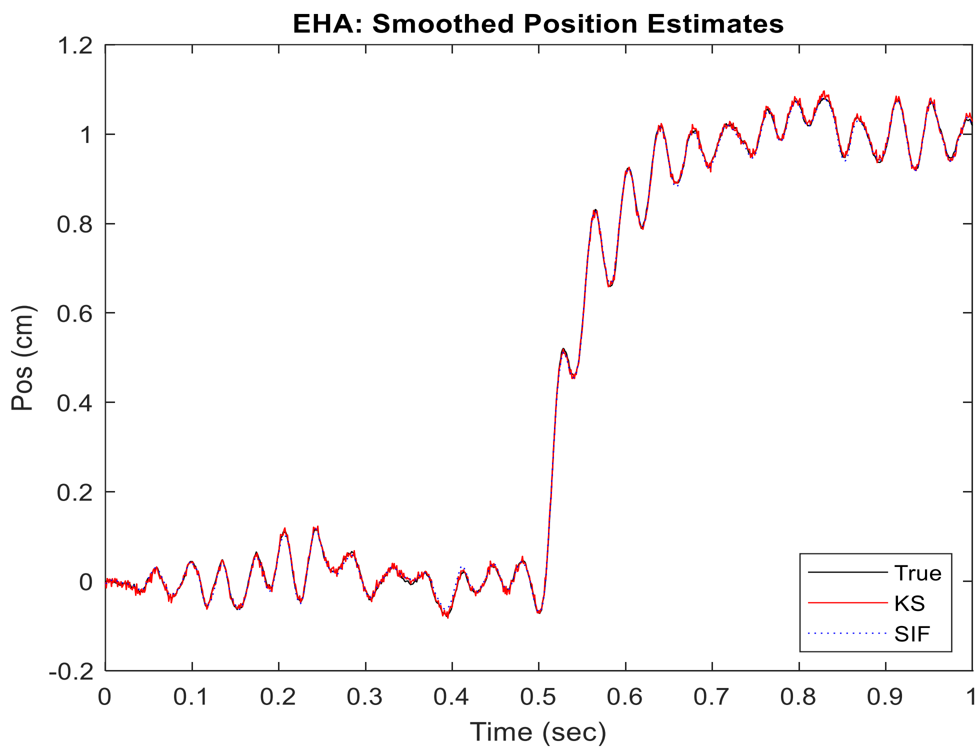

Figure 2. State 1 estimation for Case 1.

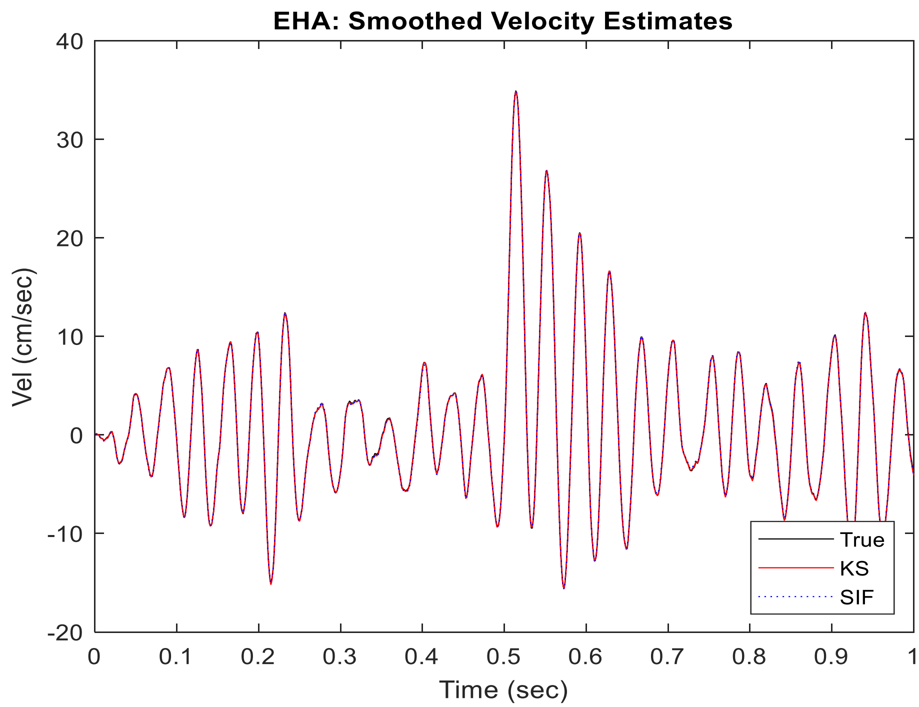

Figure 3. State 2 estimation for Case 1.

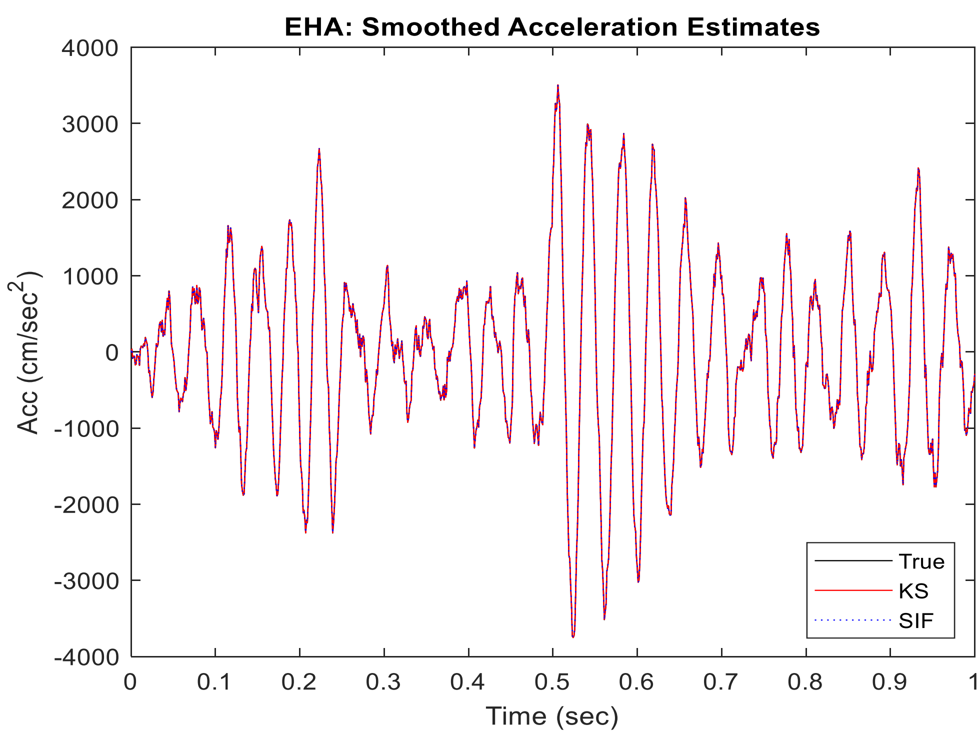

Figure 4. State 3 estimation for Case 1.

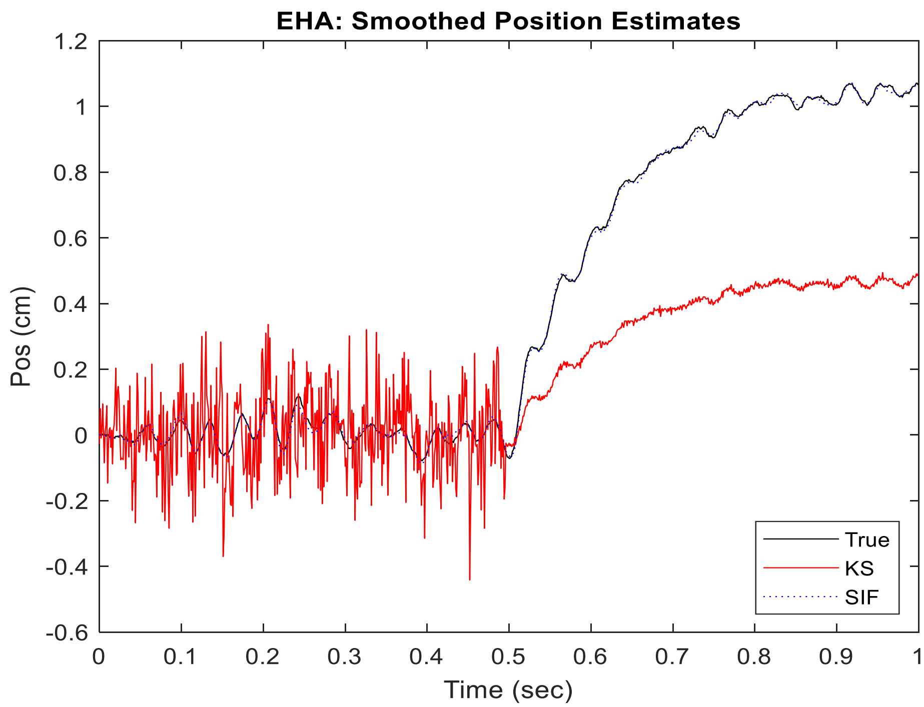

Figure 5. State 1 estimation for Case 2.

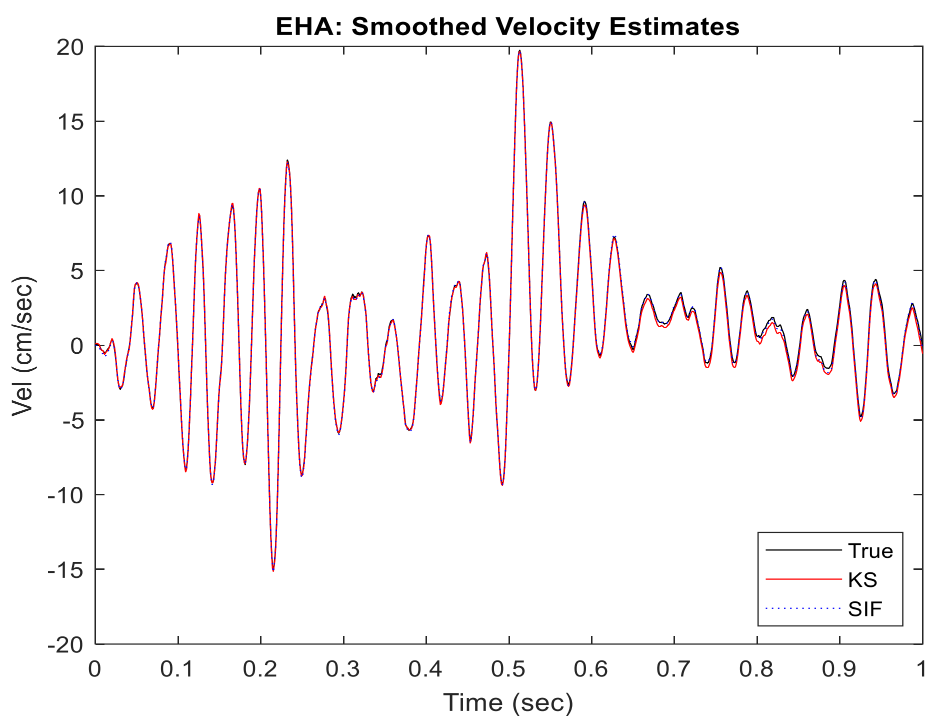

Figure 6. State 2 estimation for Case 2.

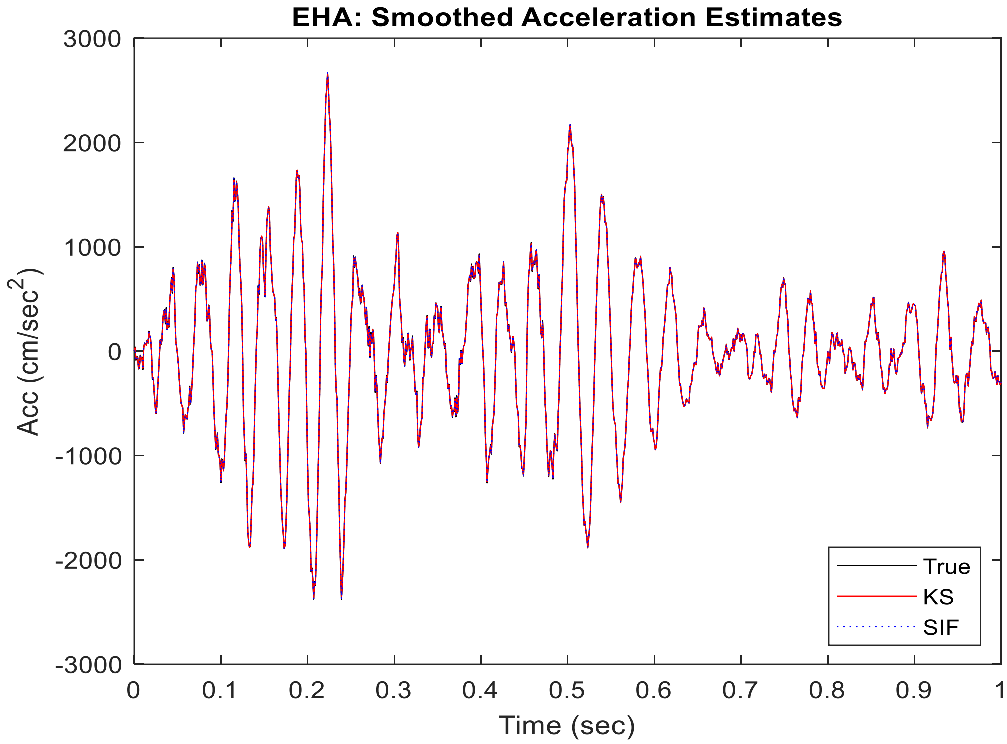

Figure 7. State 3 estimation for Case 2.

6. CONCLUSIONS

In this brief paper, the two-pass smoother algorithm is utilized with SIF and KF. The proposed SIS was applied to the EHA system. The results showed improvement on the estimated states. Moreover, the SIF and SIS demonstrated robustness to modeling uncertainties, unlike the KF and KF-smoothed. Future work will consider other versions of smoothers and a more comprehensive study as applied on the experimental setup.

7. APPENDIX

The following table summarizes the main nomenclature used in this paper.

Table 5. List of nomenclature.

$\cdot^{-1}$ $\cdot^{T}$	Inverse, and transpose, respectively.	$\mathbf{K}_X$	The correction gain of the filter X.
$\mathbf{e_m}$	The estimation error vectors in **m**.	m, n	Number of measurements and states, respectively.
$\mathbf{f}(.)$	The system's model function.	$\mathbf{P}$	The error covariance matrix.
$\mathbf{h}(.)$	The sensor's model function.	$\mathbf{Q}$	The process noise covariance matrix.
i, j	Subscripts used to identify elements.	$\mathbf{R}$	The measurements noise covariance matrix.
k	Time step value.	T_s	Sampling time, and is equal to $0.001\ sec$.

$k\|k-1$	The a priori value at time k.		$\mathbf{x}$	The state vector.
$k\|k$	The a posteriori value at time k.		$\mathbf{z}$	The output vector.

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