

# A MULTIPLE MODEL-BASED SLIDING INNOVATION FILTER

Mohammad Al Shabi<sup>a</sup>, S. Andrew Gadsden<sup>b</sup>, Mamdouh El Haj Assad<sup>c</sup> and Bassam Khuwaileh<sup>d</sup>

<sup>a</sup> Department of Mechanical and Nuclear Engineering, University of Sharjah, PO Box 27272, Sharjah, UAE,  
malshabi@sharjah.ac.ae

<sup>b</sup> College of Engineering and Physical Sciences, University of Guelph, Guelph, Ontario, Canada, N1G 2W1,  
gadsden@uoguelph.ca

<sup>c</sup> Department of Sustainable & Renewable Energy Engineering, University of Sharjah, PO Box 27272, Sharjah,  
UAE, massad@sharjah.ac.ae

<sup>d</sup> Department of Mechanical and Nuclear Engineering, University of Sharjah, PO Box 27272, Sharjah, UAE,  
bkuwaileh@sharjah.ac.ae

## ABSTRACT

In this brief work, a novel filtering technique that combines the newly developed sliding innovation filter with a multiple model strategy is proposed. Introduced in 2020, the sliding innovation filter is a relatively new filter used for state and parameter estimation. Based on variable structure techniques, it shares the same principles with sliding mode observers. The filter is robust and stable under system modeling uncertainties. The proposed method multiple model-based sliding innovation filter is tested on an electrohydrostatic actuator (EHA) and the results are discussed.

**Keywords:** sliding innovation filter, multiple models, fault detection, robust estimation

## 1. INTRODUCTION

The Kalman filter (KF) is the most well-known filter that minimizes state and parameter estimation error. It is applicable on linear, known systems under the presence of white system and measurement noise. In order to make it applicable to nonlinear systems, the KF was extended to several forms including the extended, the unscented (UKF), the cubature (CKF), and the central difference KF (CDKF) [1-11]. However, these filters assume that the system is well-known. If the system changes its structure, the performance of the filter degrades significantly. In order to overcome this, several techniques have been introduced, such as using other type of filters or combining the previous filters with a more robust filter (e.g., sliding mode observer, the smooth variable structure filter, and the sliding innovation filter (SIF) [12-19]). Another approach is to use multiple models and fuse them statistically as in interacting multiple model (IMM) [20-23].

This very brief paper is organized as follows. The IMM, SIF, and KF are introduced in Section 2. The system under study and the simulation results are summarized in Section 3. The paper is then concluded in Section 4.

## 2. STRATEGIES IN USE

This section provides a general overview of the IMM strategy along with the UKF and SIF. Figure 1 shows the IMM structure in general. This algorithm is then combined with SIF or UKF. The differences between IMM-SIF and IMM-UKF can be summarized with two main points: the calculation of the a priori (predicted) states and covariance matrix, and calculation of the gain. These can be summarized by Table 1.

The IMM process depends on the mixing probabilities  $\mu_{i|j,k|k}$ , which is defined as the probability of the system switching from one mode to another mode.

Table 1. IMM-SIF versus IMM-UKF

	calculations of the a priori states and covariance matrix	calculate the gain
SIF	$\hat{x}_{j,k+1 k} = f_j(\hat{x}_{k k}, u_k)$ $P_{j,k+1 k} = A_j P_{k k} A_j^T + Q_k$ $S_{j,k+1 k} = C_j P_{j,k+1 k} C_j^T + R_{k+1}$	$K_{k+1}$ $= C^+ \text{diag}(\text{sat}( e_{1,z,k+1 k} , \psi))$
UKF	$X_{i,k k} = \hat{x}_{k k} \pm \left( \sqrt{(n + \kappa) P_{k k}} \right)_i$ $W_i = \frac{1}{2(n)}$ $X_{i,j,k+1 k} = f_j(X_{i,k k}, u_k)$ $\hat{x}_{j,k+1 k} = \sum_{i=0}^{2n} W_i \hat{X}_{i,j,k+1 k}$ $P_{k+1 k} = \sum_{i=0}^{2n} W_i (\hat{X}_{i,j,k+1 k} - \hat{x}_{j,k+1 k})(\hat{X}_{i,j,k+1 k} - \hat{x}_{j,k+1 k})^T$ $P_{zz,k+1 k} = \sum_{i=0}^{2n} W_i (C \hat{X}_{i,j,k+1 k} - C \hat{x}_{j,k+1 k})(C \hat{X}_{i,j,k+1 k} - C \hat{x}_{j,k+1 k})^T$ $P_{xz,k+1 k} = \sum_{i=0}^{2n} W_i (\hat{X}_{i,j,k+1 k} - \hat{x}_{j,k+1 k})(C \hat{X}_{i,j,k+1 k} - C \hat{x}_{j,k+1 k})^T$	$K_{k+1} = P_{xz,k+1 k} P_{zz,k+1 k}^{-1}$

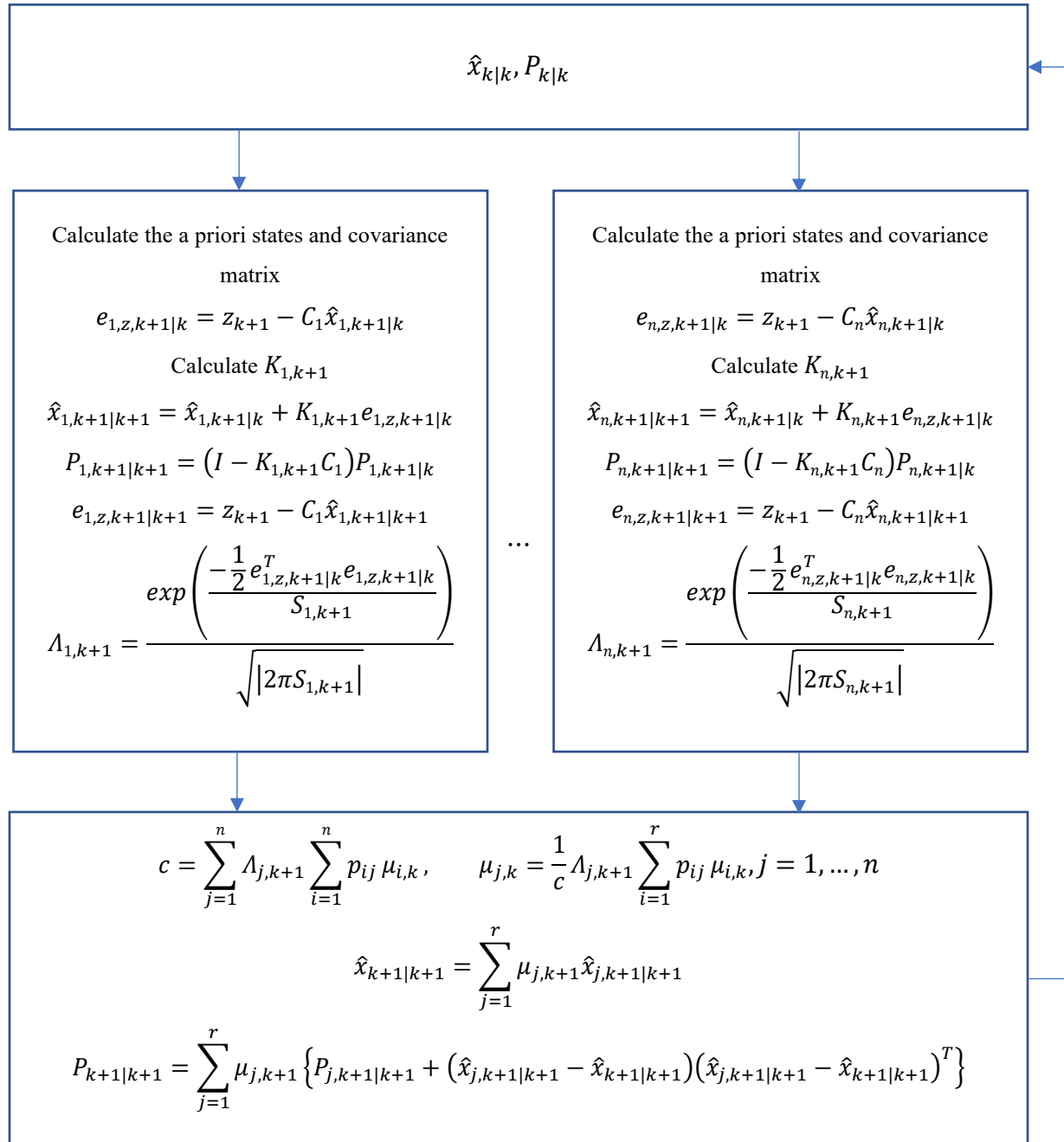


Fig. 1 IMM algorithm [24].

### 3. SYSTEM UNDER SCOPE AND SIMULATION RESULTS

#### 3.1 The Electrohydrostatic Actuator (EHA)

The EHA from [24] is used to test the proposed method IMM-SIF. The system is shown in Fig. 2 and summarized by the following state equation:

$$x_{k+1} = \begin{bmatrix} x_{1,k} + Tx_{2,k} \\ x_{2,k} + Tx_{3,k} \\ \left[1 - T \frac{a_2 V_0 + M\beta_e L}{MV_0}\right] x_{3,k} - T \frac{(A_E^2 + a_2 L)\beta_e}{MV_0} x_{2,k} \\ -T \frac{2a_1 V_0 x_{2,k} x_{3,k} + \beta_e L (a_1 x_{2,k}^2 + a_3)}{MV_0} \operatorname{sgn}(x_{2,k}) + T \frac{A_E \beta_e}{MV_0} u_k \\ \frac{1}{A_E} (a_2 x_{2,k} + (a_1 x_{2,k}^2 + a_3) \operatorname{sgn}(x_{2,k})) \end{bmatrix} \quad (1)$$

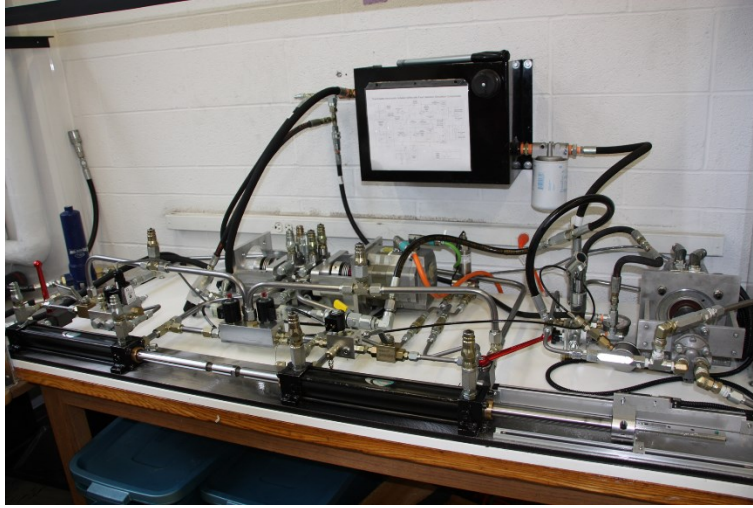


Fig 2. The EHA under study [24].

The system is studied under three different operating modes: normal mode, friction mode, and internal leakage mode. We assume that only the first and the fourth states are measured. The sample rate used in this simulation was  $T = 0.1 \text{ ms}$ . The results of the proposed method are shown in Fig 3. The proposed method was compared to IMM-UKF. The comparisons are shown in Figures 4 and 5, and Tables 2 and 3.

### 3.2 Results and Discussion

Both the IMM-SIF and IMM-UKF successfully detected the correct operating mode. The IMM-UKF strategy correctly identified the normal operation with a probability level of 70.28% while IMM-SIF had 93.26%. They obtained the leakage operation with the highest probability level of 80.33% for IMM-UKF and 96.27% for IMM-SIF. For the friction mode, the highest probability levels were 93.68% and 93.91% for the IMM-UKF and IMM-SIF, respectively. In this example, the IMM-SIF has better performance in detecting the correct operating mode.

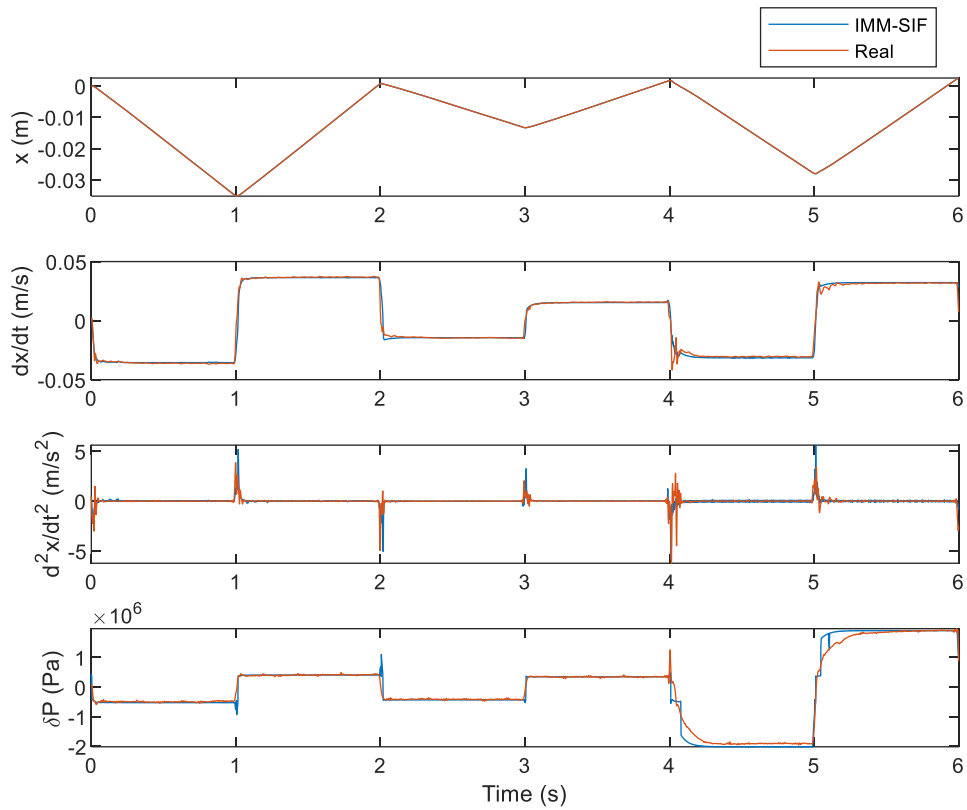


Figure 3. The experimental EHA states with time.

Table 2. IMM-UKF Mode Probability Results (Confusion Matrix)

		Actual Condition		
		Normal	Leakage	Friction
Predicted Condition	Normal	70.28 %	16.67 %	2.72 %
	Leakage	29.63 %	80.33 %	3.60 %
	Friction	0.09 %	3.00 %	93.68 %

Table 3. IMM-SIF Mode Probability Results (Confusion Matrix)

		Actual Condition		
		Normal	Leakage	Friction
Predicted Condition	Normal	93.26 %	3.06 %	2.80 %
	Leakage	6.16 %	96.27 %	3.29 %
	Friction	0.458%	0.66 %	93.91 %

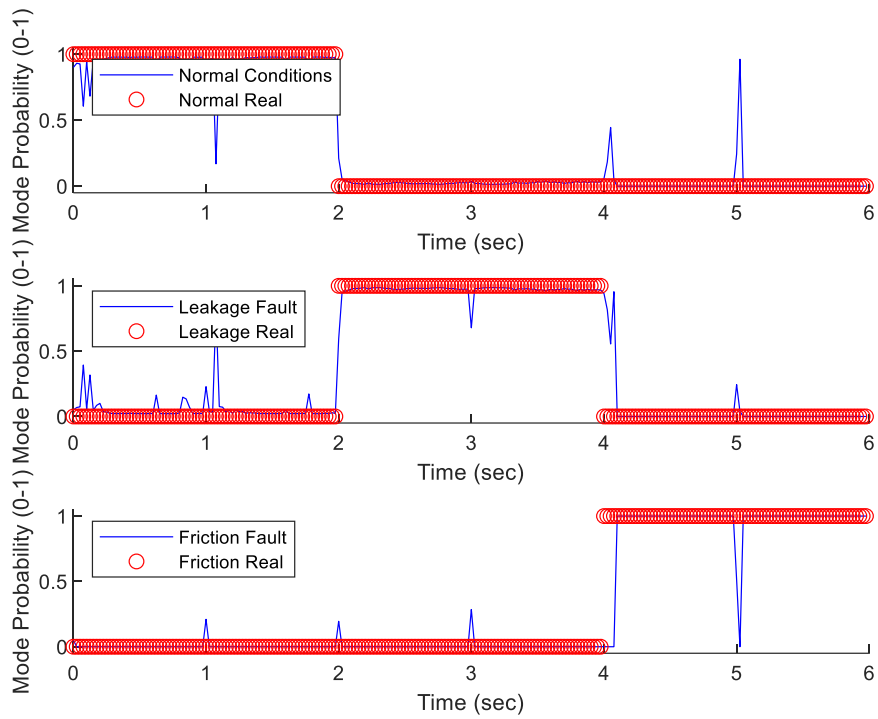


Fig 4. Calculated mode probabilities over time for the EHA using IMM-SIF.

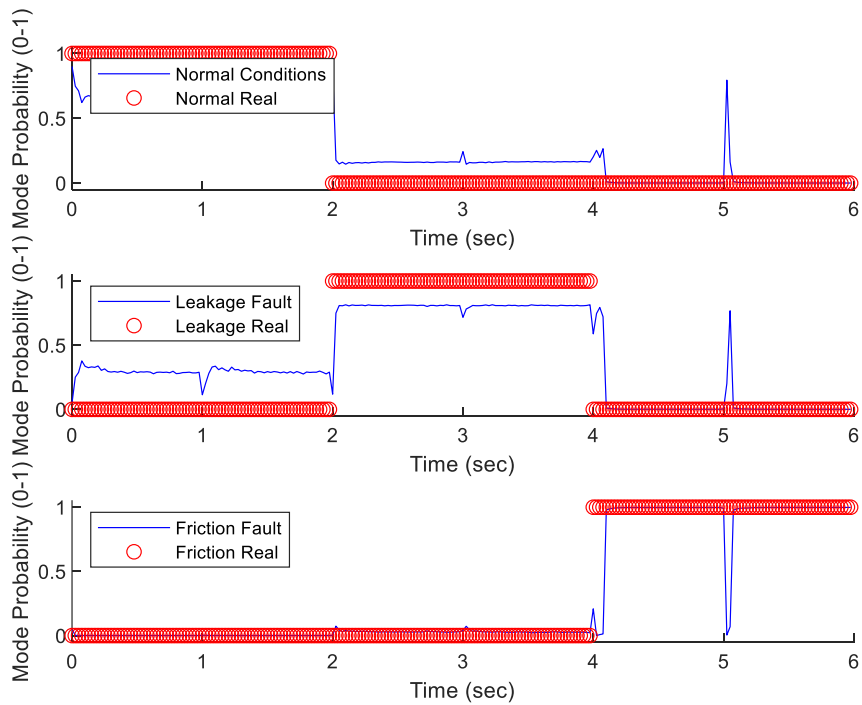


Fig 5. Calculated mode probabilities over time for the EHA using IMM-UKF.

## 4. CONCLUSIONS

In this very brief work, both the SIF and KF were combined with the IMM technique. The methods were used to estimate the states during faulty conditions of an electrohydrostatic actuator. The results demonstrated that the IMM-SIF was robust during fault conditions and predicted the correct operating mode with a higher probability compared with the IMM-UKF. For future work, an experimental setup will be used to verify the results and a more comprehensive study and comparison will be completed.

### Appendix

Table 6. List of Nomenclature and Corresponding Definition

$f$	Nonlinear system	$e_z$	Innovation vector
$x$	State vector	$\psi$	SIF smoothing boundary layer width
$z$	Measurement vector	$\mu_j$	Mode probabilities
$A$	linearized system matrix	$\mu_{ij}$	Mixing probabilities
$C$	Measurement matrix	$\Lambda_j$	Likelihood function
$K$	Filter gain matrix	$diag[a]$	Diagonal of some value $a$
$P$	State error covariance matrix	$sat()$	Saturation function
$P_{xz}$	Cross-covariance matrix	$ a $	Absolute value of $a$
$P_{zz}$	Innovation covariance matrix	$T$	Sample rate
$Q$	System noise covariance matrix	$+$	Pseudoinverse of a non-square matrix
$R$	Measurement noise covariance matrix	$\sim$	Error or difference of some value
$S$	Innovation covariance matrix	$\wedge$	Estimated values

### References

- Gadsden, S.A., Song, Y and Habibi, S. R. Novel Model-Based Estimators for the Purposes of Fault Detection and Diagnosis. (2013), IEEE/ASME Transactions on Mechatronics, 18, no. 4, pp. 1237-1249.
- Lee, A.S., Andrew Gadsden, S., Al-Shabi, M. Application of nonlinear estimation strategies on a magnetorheological suspension system with skyhook control. (2020) IEMTRONICS 2020 - International IOT, Electronics and Mechatronics Conference, Proceedings, art. no. 9216390. DOI: 10.1109/IEMTRONICS51293.2020.9216390
- Hill, E., Lee, A.S., Gadsden, S.A., Al-Shabi, M. Intelligent estimation strategies applied to a flight surface actuator. (2018) Proceedings - 2017 IEEE 5th International Symposium on Robotics and Intelligent Sensors, IRIS 2017, 2018-January, pp. 98-103. DOI: 10.1109/IRIS.2017.8250105
- Al-Shabi, M., Hatamleh, K., Al Shaer, S., Salameh, I., Gadsden, S.A. A comprehensive comparison of sigma-point Kalman filters applied on a complex maneuvering road. (2016) Proceedings of SPIE - The International Society for Optical Engineering, 9842, art. no. 98421I. DOI: 10.1117/12.2224233
- Al-Shabi, M., Gadsden, S.A., Habibi, S.R. Kalman filtering strategies utilizing the chattering effects of the smooth variable structure filter. (2013) Signal Processing, 93 (2), pp. 420-431. DOI: 10.1016/j.sigpro.2012.07.036
- Al-Shabi, M., Cataford, A., Gadsden, S.A. Quadrature Kalman filters with applications to robotic manipulators. (2018) Proceedings - 2017 IEEE 5th International Symposium on Robotics and Intelligent Sensors, IRIS 2017, 2018-January, pp. 117-124. DOI: 10.1109/IRIS.2017.8250108

7. Al-Shabi, M. Sigma-point Smooth Variable Structure Filters applications into robotic arm. (2017) 2017 7th International Conference on Modeling, Simulation, and Applied Optimization, ICMSAO 2017, art. no. 7934865. DOI: 10.1109/ICMSAO.2017.7934865
8. Al-Shabi, M., Bani-Yonis, M., Hatamleh, K.S. The sigma-point central difference smooth variable structure filter application into a robotic arm. (2015) 12th International Multi-Conference on Systems, Signals and Devices, SSD 2015, art. no. 7348201. DOI: 10.1109/SSD.2015.7348201
9. Al-Shabi, M., Gadsden, S.A., Wilkerson, S.A. The cubature smooth variable structure filter estimation strategy applied to a quadrotor controller. (2015) Proceedings of SPIE - The International Society for Optical Engineering, 9474, art. no. 94741I. DOI: 10.1117/12.2181250
10. Al-Shabi, M., Hatamleh, K.S. The unscented smooth variable structure filter application into a robotic arm. (2014) ASME International Mechanical Engineering Congress and Exposition, Proceedings (IMECE), 4B. DOI: 10.1115/IMECE2014-40118
11. Gadsden, S.A., Al-Shabi, M., Arasaratnam, I., Habibi, S.R. Combined cubature Kalman and smooth variable structure filtering: A robust nonlinear estimation strategy. (2014) Signal Processing, 96 (PART B), pp. 290-299. DOI: 10.1016/j.sigpro.2013.08.015
12. Gadsden, S. A. and Kirubarajan, T. Development of a Variable Structure-Based Fault Detection and Diagnosis Strategy Applied to an Electromechanical System. (2017) SPIE Signal Processing, Sensor/Information Fusion, and Target Recognition XXVI, Anaheim, California.
13. Andrew Gadsden, S., Al-Shabi, M. The Sliding Innovation Filter. (2020) IEEE Access, 8, art. no. 9096294, pp. 96129-96138. DOI: 10.1109/ACCESS.2020.2995345
14. Avzayesh, M., Abdel-Hafez, M.F., Al-Masri, W.M.F., Al-Shabi, M., El-Hag, A.H. A Hybrid Estimation-Based Technique for Partial Discharge Localization. (2020) IEEE Transactions on Instrumentation and Measurement, 69 (11), art. no. 9104966, pp. 8744-8753. DOI: 10.1109/TIM.2020.2999165
15. Spotts, I., Brodie, C.H., Gadsden, S.A., Al-Shabi, M., Collie, C.M. Comparison of nonlinear filtering techniques for photonic systems with blackbody radiation. (2020) Applied Optics, 59 (30), pp. 9303-9312. DOI: 10.1364/AO.403484
16. Andrew Gadsden, S., Al-Shabi, M. A study of variable structure and sliding mode filters for robust estimation of mechatronic systems. (2020) IEMTRONICS 2020 - International IOT, Electronics and Mechatronics Conference, Proceedings, art. no. 9216381. DOI: 10.1109/IEMTRONICS51293.2020.9216381
17. Gadsden, S.A., Al-Shabi, M., Kirubarajan, T. Two-pass smoother based on the SVSF estimation strategy. (2015) Proceedings of SPIE - The International Society for Optical Engineering, 9474, art. no. 947409. DOI: 10.1117/12.2177256
18. Gadsden, S.A., Al-Shabi, M., Kirubarajan, T. Square-root formulation of the SVSF with applications to nonlinear target tracking problems. (2015) Proceedings of SPIE - The International Society for Optical Engineering, 9474, art. no. 947408. DOI: 10.1117/12.2177226
19. Al-Shabi, M., Hatamleh, K.S., Gadsden, S.A., Soudan, B., Elnady, A. Robust nonlinear control and estimation of a PRRR robot system. (2019) International Journal of Robotics and Automation, 34 (6), pp. 632-644. DOI: 10.2316/J.2019.206-0160
20. Aly, S., El Fouly, R., and Braka, H. Extended Kalman filtering an dinteracting multiple model for tracking maneuvering targets in sensor networks. (2009) in IEEE 17th Workshop on Intelligent Solution in Embedded Systems.
21. Wang, X., and Syrmos, V. Interacting multiple particle filters for fault diagnosis of nonlinear systems. (2008) in American Control Conference, Seattle.
22. Guo, R., Qin, Z., Li, Q., and Chen, J. Interacting multiple model particle-type filtering approaches to ground target tracking. (2008) Journal of Computers, vol. 3, no. 7, pp. 23-29.
23. Gadsden, S. A., Habibi, S. R. and Kirubarajan, T. A novel interacting multiple model method in target tracking. (2010) in 13th Conference on Information Fusion, Edinburg, UK.
24. S. A. Gadsden, "Smooth Variable Structure Filtering: Theory and Applications," Hamilton, Ontario, 2011.



