A Study of Variable Structure and Sliding Mode Filters for Robust Estimation of Mechatronic Systems

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***Abstract*—In this paper, a study of estimation strategies based on variable structure and sliding mode theory is performed. The smooth variable structure filter (SVSF) and the new sliding innovation filter (SIF) are based on similar sliding mode concepts but with some notable differences. The relevant literature and background are explored and the SVSF and SIF estimation algorithms are presented. For comparison purposes, the two estimation strategies are applied on a mechatronic system. The results indicate that although both the SVSF and SIF provide robust estimates to faults, the SIF formulation provides slightly more accurate estimates while maintaining robustness, and is less computationally complex.**

Keywords—estimation theory, robustness, sliding innovation filter, smooth variable structure filter, variable structure

I. Introduction

Estimation theory is typically found in the field of signal processing and involves the use of filters to extract useful system information from noisy measurements [1]. For example, consider a mechatronic system found in aerospace such as a flight surface actuator. One of the most important states used for successful maneuvers of aircraft is the angle of the flap. A sensor is used to provide measurements of the angle; however, sensors are usually corrupted by unwanted signals such as measurement noise. It is the goal of an estimation method, such as the popular and well-known Kalman filter (KF), to extract the true state value from the measurement noise [2]. This value would then be used by the flight control computer to adjust control signals to ensure desired flight performance. The estimation method would also need to minimize the effects of system noise (e.g., natural flight surface vibrations or internal system vibrations).

Most estimation strategies found in the literature are based on statistical methods and Bayesian theory [1]. The KF offers the optimal solution to the linear estimation problem as it minimizes the state error covariance, which is a second-order moment that provides a means to approximate the state error. The state error is defined as the difference between the true state value and the estimated value. Although the KF is considered an optimal filter for linear systems, it is derived based on a number of strict assumptions such as the noise must be white (or zero mean and with Gaussian distributions), and the system and measurement dynamics must be known [3]. For nonlinear systems and measurements, a number of different forms of the KF have been presented. The two most common are the extended Kalman filter (EKF) which uses first-order Taylor series approximations, and the unscented Kalman filter (UKF), which uses the mathematical unscented transform to approximate the nonlinearities. The UKF is a type of sigma-point filter as a number of points are derived and propagated through the nonlinearities to provide a higher-order approximation when compared with the EKF [3]. The EKF and UKF methods are considered sub-optimal solutions to the nonlinear estimation problem as there is currently no known analytical solution [4, 5].

Although popular, KF-based methods suffer from stability and robustness issues [1]. For example, should the system model being used by the filter derive from the true system dynamics, the estimation results will deviate from the truth. This proves a challenge for sensitive control systems to ensure safe and reliable operation. A number of improvements have been suggested in literature to improve the robustness and stability of KF-based methods [6, 7, 8]. Some methods have looked at improving numerical stability through the use of square-root formulations [9, 10]. Others have looked at bounding the estimates and covariance values to ensure the estimates remain close to the true state values [11]. Some literature explored the use of modifying the system noise covariance or adding some degree of memory [4, 12]. For estimation problems which contain a lot of modeling uncertainty, the system noise covariance can be made artificially large to capture a wider-range of values [4, 12]. This has led to dual-estimation strategies where two KFs are run in parallel and small and large system noise covariances are implemented, respectively. The measurement error, also known as the innovation, could be used to determine which KF yields the better estimate. This strategy is considered a basic adaptive KF approach [13].

Sliding mode observers and variable structure approaches were introduced to improve the robustness of the estimation process [12]. These methods make use of a variable structure gain which essentially implements a boundary layer across the true state trajectory and switches back-and-forth across it. Although there are a number of other filters and strategies based on sliding mode and variable structure theory, this study will focus on the smooth variable structure filter (SVSF) and the sliding innovation filter (SIF) as they are derived using the same principles. The SVSF was introduced in 2007, and was

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formulated as a predictor-corrector filter based on sliding mode and variable structure techniques [14]. The estimation process for the SVSF is similar to the KF, except that it is considered a sub-optimal filter since its gain was derived for robustness as opposed to optimality [14]. The SVSF gain is a function of measurement errors and a switching term. This gain formulation allowed for a robust estimation strategy which reduced the negative effect of modeling uncertainties and disturbances.

The SVSF presented in [14] did not contain a state error covariance function which provides a measure of performance as well as can be used to expand the number of useful applications. In [2, 15], a new formulation of the SVSF was presented which included the derivation for a state error covariance function. This increased the number of useful applications, and allowed for the SVSF to be combined with the interacting multiple model (IMM) method for target tracking and fault detection and diagnosis purposes [16]. The proposed fault detection strategy, the SVSF-IMM, yielded higher detection rates for faults and minimized misclassifications of faults when compared with the KF-IMM. In [17], the chattering effects of the SVSF gain were used to detect faults. Since the SVSF gain is a function of the measurement error, this process was essentially the same as using the magnitudes of the innovation as others have proposed in literature and practice [1]. Another formulation of the SVSF for fault detection and diagnosis was presented in [18], and was found to yield good fault detection accuracy.

A higher-order form of the SVSF was proposed in [19]. The results of the proposed second-order SVSF indicated that it was more accurate than the standard SVSF, however it was more computationally expensive and complex. Another type of higher-order SVSF was presented in [20]. Due to the state error covariance derivation of the SVSF presented in [2, 15], the SVSF was reformulated for a number of different applications such as object target tracking [13, 21, 22]. The SVSF has also been applied to other robotics [23, 24], mechatronics [25], and aerospace estimation problems [26, 27, 28, 29] and the literature continues to expand.

The SIF is a new filter introduced in 2020, and is based on similar concepts to the SVSF [30]. Like the SVSF, it can be applied to both linear and nonlinear systems and measurements. However, the SIF utilizes a simpler gain structure which reduces the computational complexity [30]. This allows for faster online implementations and real-time embedding within sensors and chips. The results presented in [30] did not compare the new SIF with the SVSF.

In this paper, the SVSF and SIF are applied on a linear mechatronic system, and the results are compared. Since the SIF is closely related to similar concepts as the SVSF, their differences are important to share and discuss as it adds to the overall body of literature. The paper is organized as follows. For completeness, the SVSF equations are provided in Section 2, followed by the SIF equations in Section 3. The simulation results are provided and discussed in Section 4, followed by concluding remarks.

II. The Smooth Variable Structure Filter

In this section, the linear smooth variable structure filter (SVSF) estimation process with a covariance derivation is summarized. The SVSF may also be formulated to handle nonlinear systems and measurements. For this implementation, it is assumed that the linear systems and measurements are modelled using the following respective state space equations:

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{2.1}$$

$$z_{k+1} = Cx_{k+1} + v_{k+1} \tag{2.2}$$

where x refers to the states, z refers to the measurements, u refers to the system input, A refers to the system matrix, B refers to the input gain matrix, C refers to the measurement matrix, and k refers to the time step. The system and measurement noises are defined using w and v, respectively. Furthermore, the system and measurement noise are assumed to be zero mean with Gaussian covariance's Q and R, respectively.

The SVSF is formulated as a predictor-corrector estimator. During the prediction stage, the state estimates and state error covariances are predicted, as well as the predicted measurement error, respectively as follows:

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_k \tag{2.3}$$

$$P_{k+1|k} = AP_{k|k}A^T + Q_{k+1} \tag{2.4}$$

$$\tilde{z}_{k+1|k} = z_{k+1} - C\hat{x}_{k+1|k} \tag{2.5}$$

where the subscript $k|k$ refers to the updated value at the previous time step, and the subscript $k+1|k$ refers to the predicted value at the current time step.

The SVSF update stage includes the calculation of the SVSF gain, the updated state estimates, updated state error covariance, and the updated measurement error. These are respectively calculated using the following equations:

$$K_{k+1} = C^+\left(\left|\tilde{z}_{k+1|k}\right| + \gamma\left|\tilde{z}_{k|k}\right|\right) \circ sat\left(\tilde{z}_{k+1|k}/\psi\right) \tag{2.6}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}\tilde{z}_{k+1|k} \tag{2.7}$$

$$P_{k+1|k+1} = (I - K_{k+1}C)P_{k+1|k}(I - K_{k+1}C)^T \ldots$$
$$\ldots + K_{k+1}R_{k+1}K_{k+1}^T \tag{2.8}$$

$$\tilde{z}_{k+1|k+1} = z_{k+1} - C\hat{x}_{k+1|k+1} \tag{2.9}$$

where $+$ refers to the pseudoinverse, γ represents the SVSF 'memory' and is a value between 0 and 1, $\circ$ is the Schur product (element by element multiplication), sat refers to the saturation function (returns values between -1 and $+1$), ψ is the SVSF smoothing boundary layer width (and has a value for each measurement), and I refers to the identity matrix (of dimension n-by-n where n is the number of states).

Equations (2.3) through (2.9) inclusively represent the SVSF estimation process for linear systems and measurements, and is repeated iteratively. Note that the process requires initialized state estimates and state error covariances. Note also that $\tilde{z}_{k|k}$ found in (2.6) refers to (2.9) from the previous time step.

The smoothing boundary layer ψ refers to the amount of uncertainties present in the estimation process. The greater the amount of uncertainties, such as noise, modeling uncertainties, and disturbances, the larger its defined value. The value, or width, may be set as a fixed value or can be made time-varying. Note also that if the boundary layer width is chosen to be too small, chattering or high-frequency switching may occur due to the uncertainties being underestimated.

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III. The Sliding Innovation Filter

In this section, the sliding innovation filter (SIF) estimation process is summarized. Note that the SIF is also formulated as a predictor-corrector estimator, and can be applied to both linear and nonlinear systems. However, since a linear mechatronic system is considered in Section IV, the linear SIF estimation process will be summarized. The SIF estimation concept is visualized in Fig. 1.

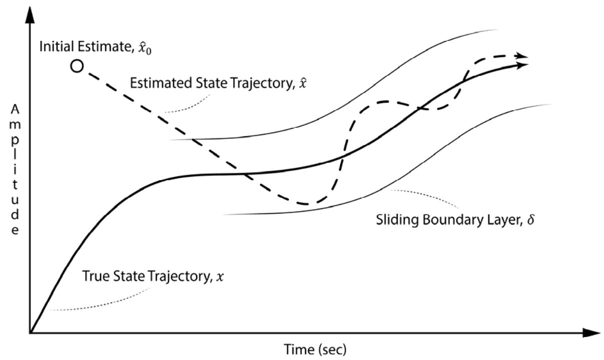

Figure 1. The SIF estimation process is summarized here [30]. The initial estimates are forced to converg along the true state trajectory, and switches back and forth due to the switching effect in the SIF gain. The values are bounded to within a region of the true trajectory by the sliding boundary layer width which is defined based on the amount of uncertainties.

Similar to the SVSF, the prediction stage includes calculating the predicted state estimates, the predicted state error covariance, and the predicted innovation, as per the following three equations, respectively:

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_k \tag{3.1}$$

$$P_{k+1|k} = AP_{k|k}A^T + Q_k \tag{3.2}$$

$$\tilde{z}_{k+1|k} = z_{k+1} - C\hat{x}_{k+1|k} \tag{3.3}$$

The update stage includes calculating the SIF gain, the updated state estimates, and the updated state error covariance, as per the following three equations, respectively:

$$K_{k+1} = C^{+}\overline{sat}\left(\left|\tilde{z}_{k+1|k}\right|/\delta\right) \tag{3.4}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}\tilde{z}_{k+1|k} \tag{3.5}$$

$$P_{k+1|k+1} = (I - K_{k+1}C)P_{k+1|k}(I - K_{k+1}C)^T \ldots$$
$$\ldots + K_{k+1}R_{k+1}K_{k+1}^T \tag{3.6}$$

where $\overline{sat}$ refers to the diagonal of the saturation term, and δ refers to the sliding boundary layer width. Equations (3.1) through (3.6) represent the SIF estimation process for linear systems and measurements defined by (2.1) and (2.2), respectively.

The notable difference between the SVSF and SIF estimation strategies are the gains defined by (2.6) and (3.4). The SVSF gain as defined in (2.6) requires two tuning parameters (γ and ψ), a Schur product calculation, and the updated measurement error from the previous time step. However, the SIF gain as defined in (3.4) requires only one tuning parameter (δ), and there are no values required from the previous time step. The SVSF and SIF gains are inherently robust to modeling uncertainties and disturbances due to the switching effects of the saturation function. However, the SIF gain is simpler and requires less computational power.

Finally, note that both the SVSF and SIF have stability proofs defined by Lyapunov theory [14, 30]. Their respective proofs of stability show that the measurement error must decrease with time for the methods to be considered stable. Both filters have their gains derived based on the proofs of stability, which allows for robust estimation processes.

IV. Computer Experiments and Results

In this section, the KF, SVSF, and SIF are applied to a linear mechatronic system and the results are compared. In this case, a linear aerospace flight surface actuator, referred to as the electrohydrostatic actuator (EHA) is considered. The EHA is a type of mechatronic system that modifies the flight surface or flap of an aircraft to control aircraft flight motion. In the literature, the system has been modelled under two operating modes: normal conditions, and faulty conditions.

Similar to the state space equations defined in (2.1) and (2.2), the EHA system is defined respectively as follows:

$$x_{k+1} = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & T \\ -557 & -28.6 & 0.94 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0 \\ 557 \end{bmatrix} u_k + w_k \tag{4.1}$$

$$z_{k+1} = Cx_{k+1} + v_{k+1} \tag{4.2}$$

where the sample rate T is defined as $1\,ms$, C refers to the measurement matrix which is an identity matrix of dimension $m \times m$ or 3×3, and u is the controller input for the system (in this case, defined as Fig. 1) that drives the desired trajectory. The system and measurement noises (w and v) are normally distributed with zero mean and covariance's Q and R defined by (4.3) and (4.4), respectively.

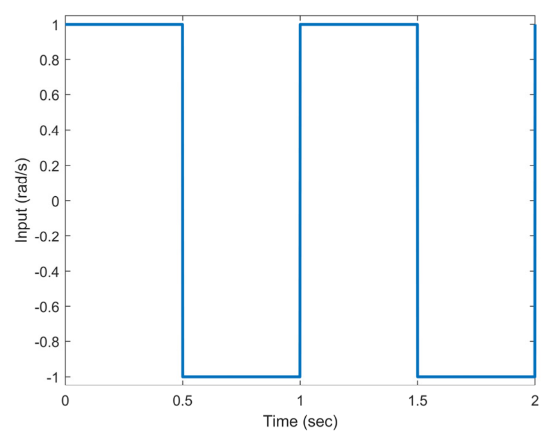

Figure 2. Control signal (rad/s) used as the input for the linear EHA simulation described by (5.1) to (5.4).

$$Q = diag([10^{-5} \quad 10^{-3} \quad 0.1]) \tag{4.3}$$

$$R = diag([10^{-4} \quad 10^{-2} \quad 1]) \tag{4.4}$$

The three states defined by the system (4.1) are kinematic EHA states, or position, velocity, and acceleration. Note that in this simulation, the states, measurements, and estimates were all set to zero. The initial state error covariance was set to $P_{0|0} = 10Q$. Furthermore, the boundary layers were tuned manually to yield the smallest estimation error, and were set to

$\psi = \delta = [0.05 \quad 1 \quad 0.5]^T$. The simulation was coded in the MATLAB environment.

The results of applying the KF, SVSF, and SIF strategies on the linear EHA are shown in Fig. 3. This plot shows the EHA position with time with the overlapping estimates. All of the filters were able to provide good estimates of the states. However, as expected, the KF provided the best results as it yields the optimal solution when the system and measurements are known. One measure of performance used in this study is the root mean squared error (RMSE) which is defined by (4.5) where n is the number of time steps. The results are summarized in Tab. I.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \hat{x}_i)^2}{n}} \tag{4.5}$$

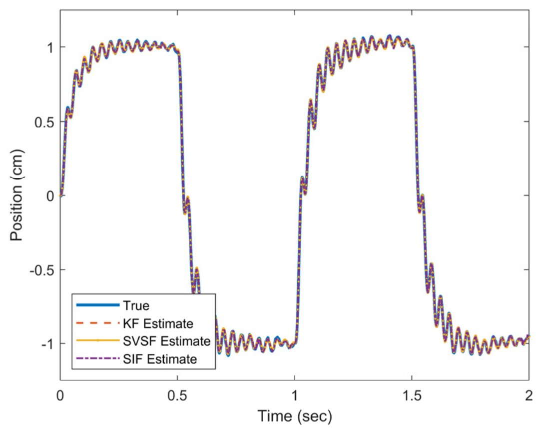

Figure 3. True and estimated position values for the linear EHA system example under normal operating conditions. The results are nearly identical so appear overlapping. However, note that for this case, the KF yields the best results which is expected since it is an optimal filter for known, linear systems and measurements.

TABLE I
EHA RMSE RESULTS: NORMAL CASE

State	KF	SVSF	SIF
Position (m)	3.83×10^{-3}	6.29×10^{-3}	5.92×10^{-3}
Vel. (m/s)	5.11×10^{-2}	6.38×10^{-2}	5.75×10^{-2}
Accel. (m/s^2)	9.00×10^{-1}	9.71×10^{-1}	9.62×10^{-1}

For completeness, the true and estimated state values for the velocity and acceleration are shown in Figs. 4 and 5.

To demonstrate the robustness of the variable structure and sliding mode strategies, consider the case when the system has a fault injected half-way through the simulation (at $t = 1$ sec). In this faulty case, the linear system state equation used by the system is changed as follows:

$$x_{k+1} = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & T \\ -240 & -28 & 0.94 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0 \\ 557 \end{bmatrix} u_k + w_k \tag{4.6}$$

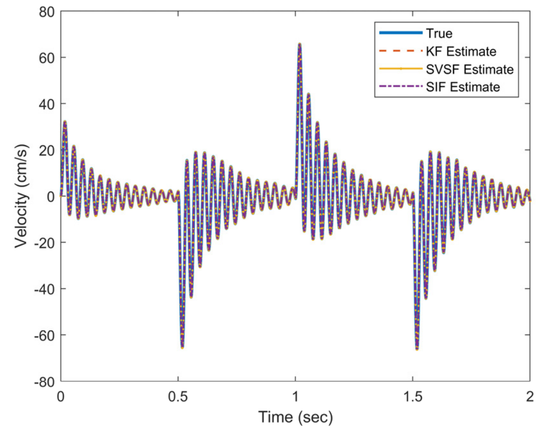

Figure 4. True and estimated velocity values for the linear EHA system example under normal operating conditions. The results are nearly identical so appear overlapping.

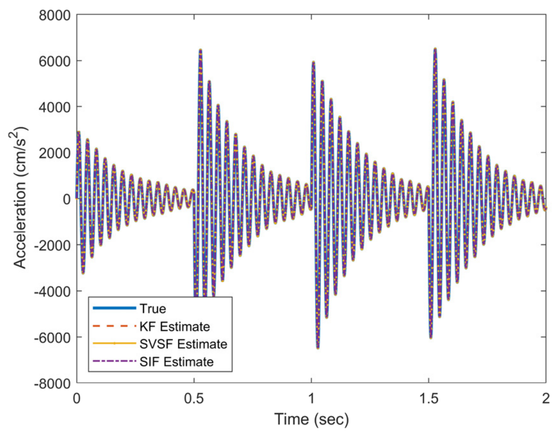

Figure 5. True and estimated acceleration values for the linear EHA system example under normal operating conditions. The results are nearly identical so appear overlapping.

Note that the filters do not use (4.6) but still use (4.1) to define the system dynamics. This causes model-mismatch or modeling uncertainties to occur. The results of the modeling uncertainty and its effects on estimating the EHA position are shown in Fig. 6. At one second, the fault is injection into the system. The model mismatch causes the KF to deviate from the true state trajectory, yielding poor estimates of the true position. However, both the SVSF and SIF were able to perform relatively well, and were bounded to the true state trajectory. This was due to the inherent stability caused by the switching effects and the defined boundary layers within their respective gains.

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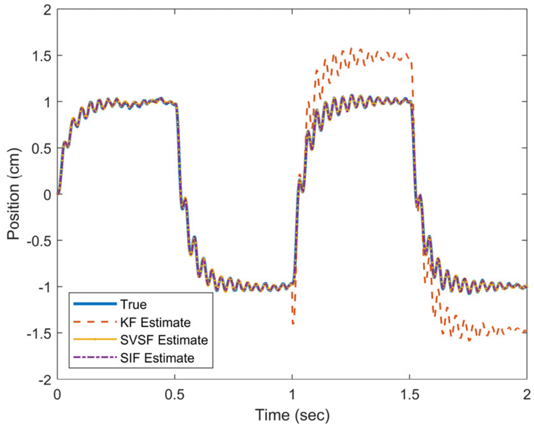

Figure 6. True and estimated position values for the linear EHA system example under faulty operating conditions. The negative effects of modeling uncertainity on the KF is clearly apparent after one second. The SVSF and SIF were robust to the model mismach and yielded relatively good estimates.

TABLE II
EHA RMSE RESULTS: FAULTY CASE

State	KF	SVSF	SIF
Position (m)	3.06×10^{-1}	6.42×10^{-3}	6.03×10^{-3}
Vel. (m/s)	3.44	6.67×10^{-2}	5.89×10^{-2}
Accel. (m/s^2)	17.69	9.98×10^{-1}	9.97×10^{-1}

The RMSE results for the faulty case are shown in Tab. II. Under the presence of a fault, the SVSF and SIF perform slightly worse than the normal case, which is to be expected. However, the KF was unable to recover from the modeling uncertainty and yielded significantly worse performance which would have a significant impact on aircraft flight performance. This was expected as the KF is derived based on the assumption that the system is known, and when that assumption is violated the KF yields poor and sometimes unstable results. For completeness, the true and estimated state values for the velocity and acceleration under faulty conditions are shown in Figs. 7 and 8.

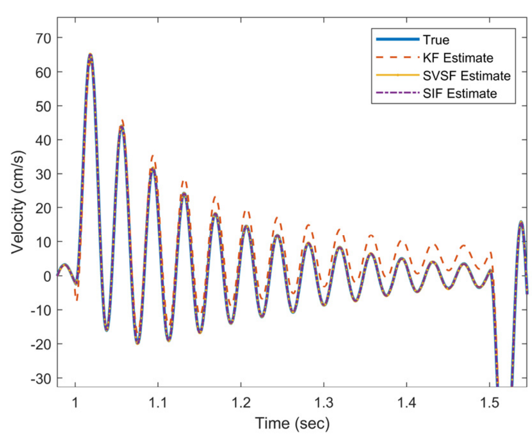

Figure 7. True and estimated velocity values for the linear EHA system example under faulty operating conditions. Note that the shown results are between about 1 and 1.5 seconds to illustrate the KF deviations.

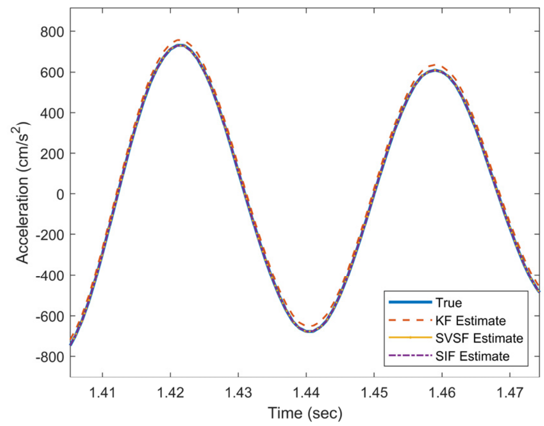

Figure 8. True and estimated acceleration values for the linear EHA system example under faulty operating conditions. Note that the results were zoomed-in to illustrate the KF deviations.

Figure 9 further illustrates the presence of the modeling uncertainty injected at 1 second in the simulation. The KF was unable to recover from the system change, whereas the SVSF and SIF were still able to provide a good estimate. This further demonstrates the robustness of the SVSF and SIF for modeling uncertainties.

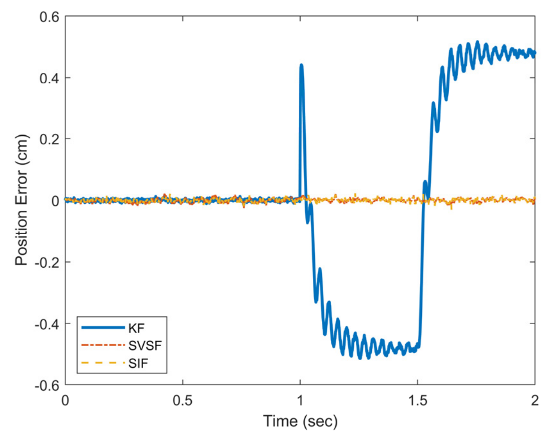

Figure 9. Position estimation error for the KF, SVSF, and SIF as shown in the faulty case (i.e., modeling uncertainty).

V. CONCLUSIONS

In this paper, a study of estimation strategies based on variable structure and sliding mode theory was performed. The smooth variable structure filter (SVSF) and the new sliding innovation filter (SIF) are based on similar sliding mode concepts but with some notable differences. The main difference is in the formulation of the gain and requirement of information from previous time steps. The SVSF and SIF gains are inherently robust to modeling uncertainties and disturbances due to the switching effects of the saturation function. A linear mechatronic system was studied where the SVSF and SIF were compared with the well-known Kalman filter (KF). The results indicate that although both the SVSF and SIF provide robust estimates to faults, the SIF formulation provides slightly more accurate estimates while maintaining robustness. Furthermore, the SIF estimation process is determined to be simpler and less computationally complex.

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