An Adaptive Smooth Variable Structure Filter based on the Static Multiple Model Strategy

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ABSTRACT

Estimation theory is an important field in mechanical and electrical engineering, and is comprised of strategies that are used to predict, estimate, or smooth out important system state and parameters. The most popular and well-studied estimation strategy was developed over 60 years ago, and is referred to as the Kalman filter (KF). The KF yields the optimal solution in terms of estimation error for linear, well-known systems. Other variants of the KF have been developed to handle modeling uncertainties, non-Gaussian noise, and nonlinear systems and measurements. Although KF-based methods typically work well, they lack robustness to uncertainties and external disturbances – which are prevalent in signal processing and target tracking problems. The smooth variable structure filter (SVSF) was introduced in an effort to provide a more robust estimation strategy. In an effort to improve the robustness and filtering strategy further, this paper introduces an adaptive form of the SVSF based on the static multiple model strategy.

Keywords: Smooth Variable Structure Filter, Static Multiple Models Estimator, Extended Kalman Filter

1. INTRODUCTION

State estimation is the process of extracting numeric values of state variables from uncertain and noisy measurement data [1]. The term state refers to a vector that changes over time by which are governed by equations that describe the dynamics of a system [2, 3]. There are two different types of estimators which include the state and parameter estimator. The purpose of the estimation is to minimize the parameter or state estimation error while simultaneously being robust to disturbances and noise [2]. Disturbances and noise are inherently present in the measurement process, and are caused by the sensor and environmental factors [2]. System uncertainties are usually caused by an inaccurate model, variations and nonlinearities in the physical system parameters.

Reliable estimates of state parameters is necessary for safely controlling electro-mechanical systems in real-time. When system dynamics are changed abruptly in the presence of faults, adaptive estimation strategies can be used to mitigate inaccurate estimation is the prediction stage. This paper introduces the use of a smooth variable structure filter (SVSF) in conjunction with a static multiple models estimator (SMM) applied to an electro-hydrostatic actuator (EHA) model with leakage faults.

Contributions to estimation theory date back to the 1500s from a several contributors of various backgrounds [2]. The first major contributor to this field was Thomas Bayes who introduced the Bayesian rule for statistical inference [2]. This provides the foundation for Bayesian estimation methods [2]. Carl Friedrich Gauss pioneered the study that produced an optimal estimate from noisy data. In 1975, he invented the famous least square estimation method to solve nonlinear estimation problems in mathematical astronomy [2].

Andrei Markov later introduced the Markov process and Markov chain theories of statistical methods and probability [2]. The Markov process describe the transition of random processes from one state to another between a finite number states [2]. Markov proved that the probability distribution of states may be calculated using its current distribution that contains the effects of all the past events of the system [2, 3]. In 1933, Andrei Kolmogorov published *Foundations of the Theory of Probability*, which introduced the modern foundations of probability theory. Sydney Chapman continued the research on the Markov processes and Kolmogorov independently presented the Chapman-Kolmogorov equations used for solving basic equations in the estimation field [2].

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Ronald Aylmer Fisher introduced the Fisher information matrix which represents the measure of the amount of information extracted from a sample of values with a given probability distribution [2]. In 1949, Norbert Wiener introduced the Wiener filter formulation for signal processing applications which reduces the signal noise when compared with an estimation of the desired noiseless signal [2]. Wiener and Kolmogorov laid the foundation of estimation theory that were later used to develop prediction, filtering, and smoothing theory [2]. Weiner's research eventually led to the derivation of an optimal estimator for continuous-time systems while Kolmogorov independently derived an optimal linear predictor for discrete-time systems [2]. Their research became famously known as the Wiener-Kolmogorov filter (WF), a predecessor to the Kalman filter [2].

2. ESTIMATION STRATEGIES

2.1 Kalman Filter

Based on the previously introduced estimation theories, Rudolf Kalman introduced a new approach to prediction problems and linear filtering in 1960. His work would eventually be referred to as the Kalman filter [2, 3]. The Kalman filter (KF) uses sequential discrete-time measurements from a linear system model with Gaussian noise to produce an optimal state estimate. Kalman and Bucy later developed a continuous version of the KF which later became referred to as the Kalman-Bucy filter [2, 4].

Some extensions to the KF formulation, such as linearization and approximation led to the extended Kalman filter (EKF) and the unscented Kalman filter (UKF), respectively [2]. These extensions allow the KF to be applied to nonlinear systems as well. Other advanced variants of the Kalman filter include the quadrature Kalman filter (QKF) [5, 6], mixture Kalman filter (MKF) [7], and the cubature Kalman filter (CKF) [2, 8]. Since its inception, the Kalman filter has had several monumental applications such as sensor filtering by NASA for the Apollo's guidance and navigation system and quickly became popular as the most practical method for state estimation due to its proven optimality [2, 3, 9].

The KF has numerous applications pertaining to parameter and state estimation, target tracking, signal processing, fault detection, and financial analysis [10, 11, 12, 13]. The KF's popularity stems from the optimality of the Kalman gain in minimizing the trace of the a posteriori state error covariance matrix [10, 11, 14, 15]. The trace represents the state error vector in the estimation process [11, 16]. Equations 2.1 through 2.5 form the basis of the KF estimation algorithm which is used in an iterative process. Equations (2.1) and (2.2) define the a priori state estimate $\hat{x}_{k+1|k}$ and the corresponding state error covariance matrix $P_{k+1|k}$, respectively [1].

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_k \tag{2.1}$$

$$P_{k+1|k} = AP_{k|k}A^T + Q_k \tag{2.2}$$

The Kalman gain K_{k+1} (1.3) is used to update the state estimate $\hat{x}_{k+1|k+1}$ as per (2.4). The gain makes use of an innovation covariance S_{k+1}, which is defined as the inverse term found in (2.3) [10].

$$K_{k+1} = P_{k+1|k}C^T\left[CP_{k+1|k}C^T + R_{k+1}\right]^{-1} \tag{2.3}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}\left[z_{k+1} - C\hat{x}_{k+1|k}\right] \tag{2.4}$$

The a posteriori state error covariance matrix $P_{k+1|k+1}$ is then calculated (2.5), and is used iteratively, as per (2.2) [10].

$$P_{k+1|k+1} = [I - K_{k+1}C]P_{k+1|k}[I - K_{k+1}C]^T + K_{k+1}R_{k+1}K_{k+1}^T \tag{2.5}$$

Several different methods have extended the original KF to nonlinear systems [10]. The extended Kalman filter (EKF) is the simplest and most popular extension [10, 11, 17, 18]. The EKF algorithm is virtually identical to the KF with the exception of linearizing the system using its Jacobian [10]. However, this linearization process introduces uncertainties that can cause inaccurate estimates and numerical instability [10, 11, 18].

2.2 Smooth Variable Structure Filter

The SVSF is an extension of the KF that adds robustness. The SVSF is stable and robust to disturbances, noise, and modeling uncertainties when given an upper bound on the level of unmodeled dynamics and noise [10, 11, 19, 20, 21]. The SVSF method is model-based and may be applied to differentiable linear or nonlinear dynamic equations [10, 11, 22, 23]. The basic estimation concept of the SVSF is shown in Figure 1.

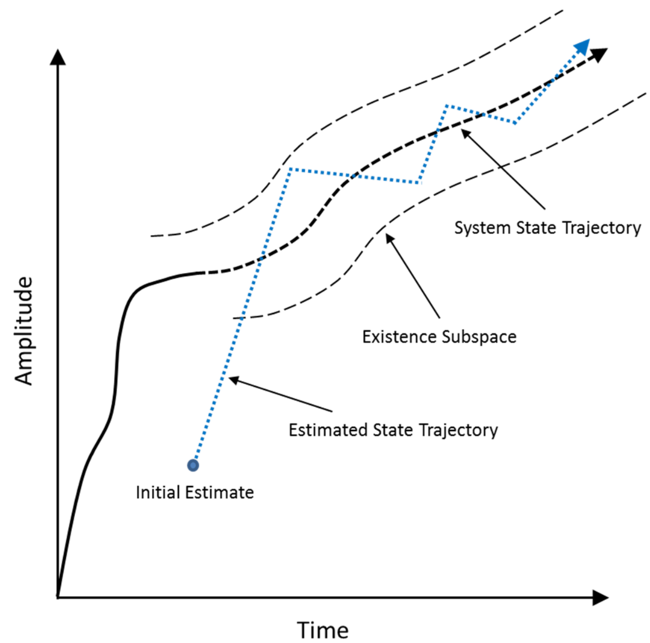

Figure 1. Standard SVSF estimation concept with existence subspace boundary layer.

The SVSF estimation process is similar to the KF, but presents a novel method of gain calculation [10, 11, 24]. The predicted state estimates $\hat{x}_{k+1|k}$ and state error covariance $P_{k+1|k}$ are first calculated as per (2.1) and (2.2). Utilizing the predicted state estimates $\hat{x}_{k+1|k}$, the corresponding predicted measurements $\hat{z}_{k+1|k}$ and measurement errors $e_{z,k+1|k}$ may be calculated:

$$\hat{z}_{k+1|k} = C\hat{x}_{k+1|k} \tag{2.6}$$

$$e_{z,k+1|k} = z_{k+1} - \hat{z}_{k+1|k} \tag{2.7}$$

The SVSF gain is a function of the following: 1) *a priori* and the *a posteriori* measurement errors $e_{z,k+1|k}$ and $e_{z,k|k}$; 2) the smoothing boundary layer widths ψ; 3) and the 'SVSF' memory or convergence rate γ. The SVSF gain K_{k+1} is defined as follows [10, 11, 14, 25]:

$$K_{k+1} = C_k^+ diag\left[\left(\left|e_{z_{k+1|k}}\right| + \gamma\left|e_{z_{k|k}}\right|\right) \circ sat\left(\bar{\psi}^{-1} e_{z_{k+1|k}}\right)\right] diag\left(e_{z_{k+1|k}}\right)^{-1} \tag{2.8}$$

where $\circ$ signifies Schur (or element-by-element) multiplication and the superscript $+$ refers to the pseudoinverse of a matrix [10]. The saturation function of (2.8) is defined by the following:

$$sat\left(\bar{\psi}^{-1} e_{z_{k+1|k}}\right) = \begin{cases} 1, & e_{z_i,k+1|k}/\psi_i \geq 1 \\ e_{z_i,k+1|k}/\psi_i, & -1 < e_{z_i,k+1|k}/\psi_i < 1 \\ -1, & e_{z_i,k+1|k}/\psi_i \leq -1 \end{cases} \tag{2.9}$$

where $\bar{\psi}^{-1}$ is a diagonal matrix constructed from the elements of the smoothing boundary layer vector ψ:

$$\bar{\psi}^{-1} = \begin{bmatrix} \frac{1}{\psi_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{1}{\psi_m} \end{bmatrix} \tag{2.10}$$

The state estimates $\hat{x}_{k+1|k}$ and state error covariance matrix $P_{k+1|k}$ are updated respectively as per (2.4) and (2.5) [10]. Finally, the updated measurement estimate $\hat{z}_{k+1|k+1}$ and measurement errors $e_{z,k+1|k+1}$ are calculated, and are used in later iterations:

$$\hat{z}_{k+1|k+1} = C\hat{x}_{k+1|k+1} \tag{2.11}$$

$$e_{z,k+1|k+1} = z_{k+1} - \hat{z}_{k+1|k+1} \tag{2.12}$$

The existed subspace denoted by the dotted black line shown in Figure 2.1 represents the level of uncertainty present in the estimation process, in terms of modeling errors or the presence of noise [10]. The width of the existence space β is a function of the uncertain dynamics associated with the inaccuracy of the internal model of the filter as well as the measurement model, and varies with time [10, 11, 25]. While this value is not precisely known, an upper bound may be selected based on *a priori* knowledge [10]. The estimated state trajectory is smoothed when the smoothing boundary layer is defined larger than the existence subspace boundary [10]. If the smoothing term is too small, however, chattering oscillations persist due to the uncertainties being underestimated [10].

2.3 Static Multiple Models Estimator

The SMM assumes that the systems follows one of r possible models M^1, M^2, … , $M^r$and uses weights, μ_k^j, at time k associated with model M^j to combine the state estimates of each model [26]. The weights are initially uniformly distributed and subsequent weights are calculated by the following [26]:

$$\mu_k^j = \frac{p(z_k|M^j)\mu_{k-1}^j}{\sum_{i=1}^r p(z_k|M^i)\mu_{k-1}^i} \tag{2.13}$$

The likelihood of measurement z_k given M^j is defined by the following [26]:

$$p(z_k\big|M^j) = \frac{1}{\sqrt{2\pi\sigma_j^2}} exp\frac{-(z_k - \hat{z}_{k|k-1})^2}{2\sigma_j^2} \tag{2.14}$$

$$\sigma_j^2 = C_k^j P_{k|k-1}^j C_k^{j^T} + (\sigma_z^2)^j \tag{2.15}$$

where $\sigma_j^2$is the variance of the measured state for model M^j and $\hat{z}_{k|k-1}$ is the predicted measurement at time k for model M^j [26]. The adaptive state estimated is calculated using the weighted sum of the estimated produced by models as per (2.16) [26]:

$$\hat{x}_{k|k} = \sum_{j=1}^r \mu_k^j\, \hat{x}_{k|k}^j \tag{2.16}$$

The adaptive covariance is calculated in a similar fashion:

$$P_{k|k} = \sum_{j=1}^r \mu_k^j \left[P_{k|k}^j + \left(\hat{x}_{k|k}^j - \hat{x}_{k|k}\right)\left(\hat{x}_{k|k}^j - \hat{x}_{k|k}\right)^T\right] \tag{2.17}$$

3. ELECTRO HYDROSTATIC ACTUATOR

The EHA modeled in this paper was designed and manufactured at the Centre for Mechatronics and Hybrid Technology at McMaster University [1]. The EHA is a self-contained hydraulic system and is composed of several components including a symmetric linear actuator, variable-speed servomotor, a bi-directional gear pump, a pressure relief valve, an accumulator, connecting tubes, and safety circuits for fault simulations [1]. A variable-speed brushless DC electric motor drives the bi-directional gear pump and forces oil into the cylinder. The gear pump adjusts the actuation performance by changing the fluid flow rate [1]. An accumulator is used to avoid cavitation and to collect the case drain leakage from the gear pump [1]. The control variable of the EHA is the input voltage to the motor that regulates the direction and speed of the pump [1]. This results in controlling the value of the fluid flow rate in the outer circuit and adjusts the position of the piston [1].

The EHA system is described by four state variables which include the actuator position $x_1 = x$, velocity $x_2 = \dot{x}$, acceleration $x_3 = \ddot{x}$, and differential pressure across the actuator $x_4 = P_1 - P_2$ [1]. The physical modeling approach was used to obtain the nonlinear state-space equations in discrete-time described as follows [1]:

$$x_{1,k+1} = x_{1,k} + Tx_{2,k} \tag{3.1}$$

$$x_{2,k+1} = x_{2,k} + Tx_{3,k} \tag{3.2}$$

$$x_{3,k+1} = 1 - \left[T\frac{a_2V_0 + M\beta_eL}{MV_0}\right]x_{3,k} - T\frac{\left({A_E}^2 + a_2L\right)\beta_e}{MV_0}x_{2,k} + T\frac{A_E\beta_e}{MV_0} - T\frac{2a_1V_0x_{2,k}x_{3,k} + \beta_eL\left(a_1{x_{2,k}}^2 + a_3\right)}{MV_0}sgn(x_{2,k}) \tag{3.3}$$

$$x_{4,k+1} = \frac{a_2}{A_E}x_{2,k} + \frac{\left(a_1{x_{2,k}}^2 + a_3\right)}{A_E}sgn\left(x_{2,k}\right) + \frac{M}{A_E}x_{3,k} \tag{3.4}$$

The input of the system is given by [1]:

$$u = D_p\omega_p - sgn(P_1 - P_2)Q_{L0} \tag{3.5}$$

where ω_p is the pump speed. The numeric values of the parameters in the state space equations are presented in Table 1.

Table 1. Numeric values of the EHA parameters

Parameter	Description	Parameter Values
A_E	Piston Area	1.52×10^{-3} m^2
D_p	Pump Displacement	5.57×10^{-7} m^3/rad
L	Leakage Coefficient	4.78×10^{-12} m^3/(s×Pa)
M	Load Mass	7.376 kg
Q_{L0}	Flow Rate Offset	2.41×10^{-6} m^3/s
V_0	Initial Cylinder Volume	1.08×10^{-3} m^3
β_e	Effective Bulk Modulus	2.07×10^{8} Pa
a_1	Friction Coefficient	6.589×10^{4}
a_2	Friction Coefficient	2.144×10^{3}
a_3	Friction Coefficient	436

4. SIMULATION RESULTS AND DISCUSSION

In this section, the proposed algorithm (SVSF-SMM) is applied for state estimation on the EHA. The EHA was simulated over 9 seconds with a 0.5 Hz square wave pump speed input. Computer simulations were used to investigate the effect of parametric uncertainties in the system. The purpose of this example to demonstrate the efficacy of the SVSF-SMM estimation process in comparison to the classical SVSF and EKF. The SVSF-SMM algorithm also demonstrates robustness in the presence of modeling errors.

The modeling errors are introduced to the system as leakage faults. The numerical values of the leakage coefficients and flow rate offsets are presented in Table 2. A minor leakage is introduced to the system at t = 3 sec and a major leakage is introduced at t = 6 sec.

Table 2. Numeric values of the leakage coefficients and flow rate offsets

Condition	**Leakage (L)**	**Flow Rate Offset (Q_{L0})**
Normal	4.78×10^{-12} $m^3/(s \times Pa)$	2.41×10^{-6} m^3/s
Minor Leakage	2.52×10^{-11} $m^3/(s \times Pa)$	1.38×10^{-5} m^3/s
Major Leakage	6.01×10^{-11} $m^3/(s \times Pa)$	1.47×10^{-5} m^3/s

The EKF and SVSF methods performed comparatively for EHA differential pressure estimation. However, the EKF performed better than the SVSF for position and velocity estimation, but considerably worse for acceleration estimation. Further tuning of the SVSF boundary layer widths may be required for the SVSF to outperform the EKF for all state estimate. The SVSF-SMM significantly improved estimation for the SVSF and vastly outperformed the EKF for velocity estimation and acceleration. The root-mean squared error (RMSE) of the simulation is shown in Table 3.

The estimates for each state are shown in Figures 2, 3, 4, and 5. A minor leakage fault is introduced at 3 seconds and a major leakage fault was introduced at 6 seconds. Figure 2 shows that the position of the EHA piston was accurately estimated by all three filters (EKF, SVSF, SVSF-SMM). For velocity estimation, the error increased slightly for the EKF and significantly for the SVSF when the minor leakage fault was introduced as shown in Figure 3. The major leakage exacerbated the error further due to an even greater deviation from the initial assumed system model. Figure 4, however, depicts the SVSF outperforming the EKH for acceleration estimation. The SVSF-SMM process was able to accurately track the EHA for all four states even in the presence of leakage faults.

Table 3. RMSE computer experiment results

Filter	**Position (m)**	**Velocity (m/s)**	**Acceleration (m/s^2)**	**Differential Pressure (Pa)**
EKF	0.00017642	0.0014599	1.3433	0.0010086
SVSF	0.00029707	0.0091964	0.28991	0.0010076
SVSF-SMM	0.00018335	0.0000801	0.00727	0.0010076

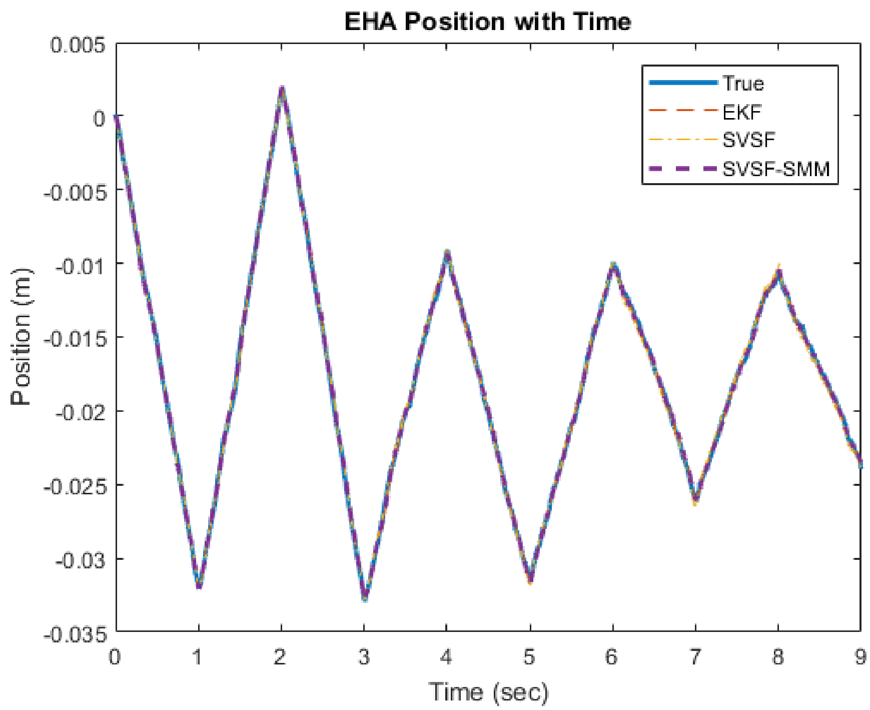

Figure 2. Position estimates for the EHA computer experiment

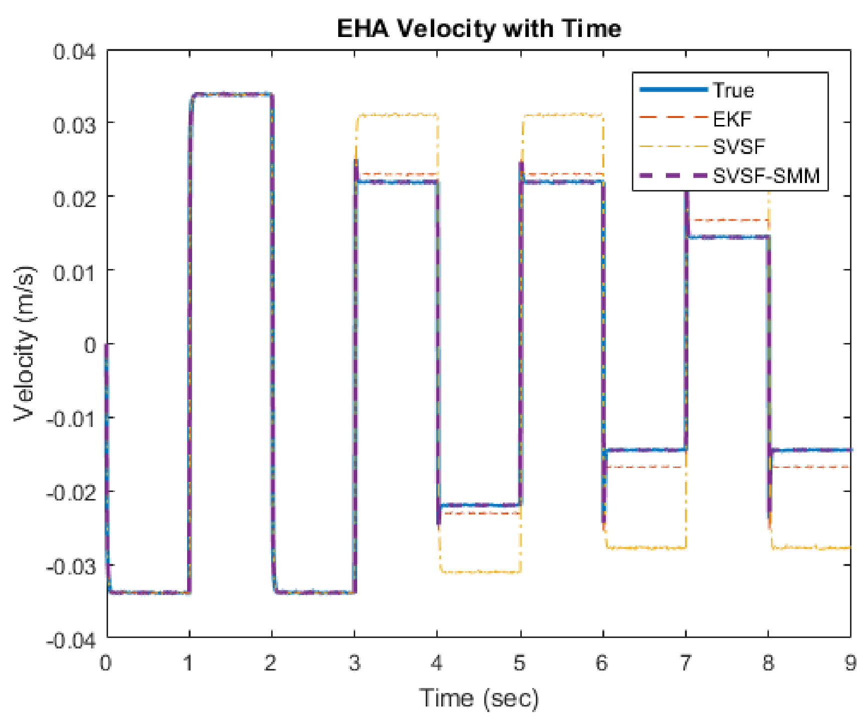

Figure 3. Velocity estimates for the EHA computer experiment

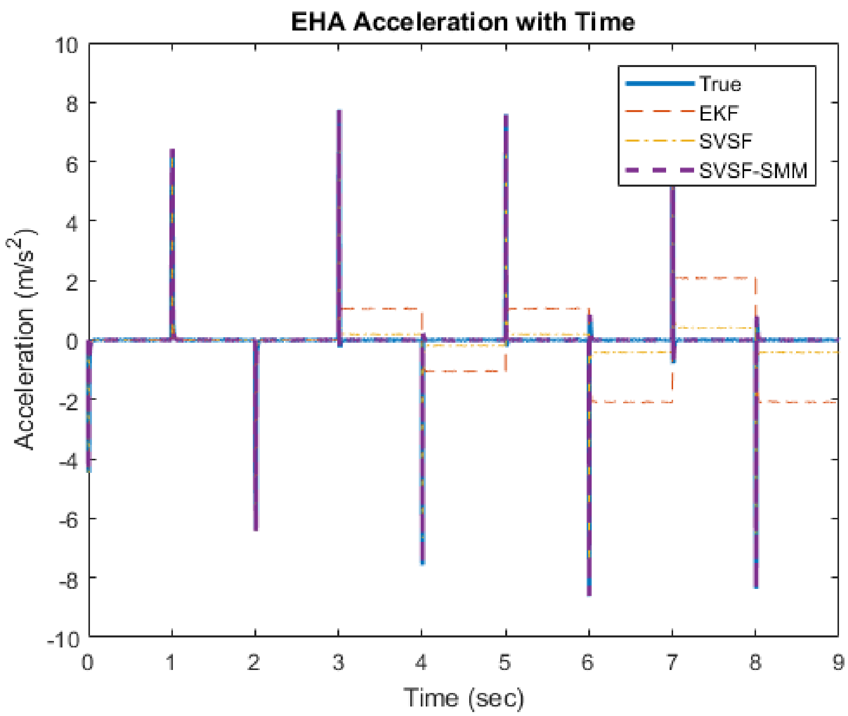

Figure 4. Acceleration estimates for the EHA computer experiment

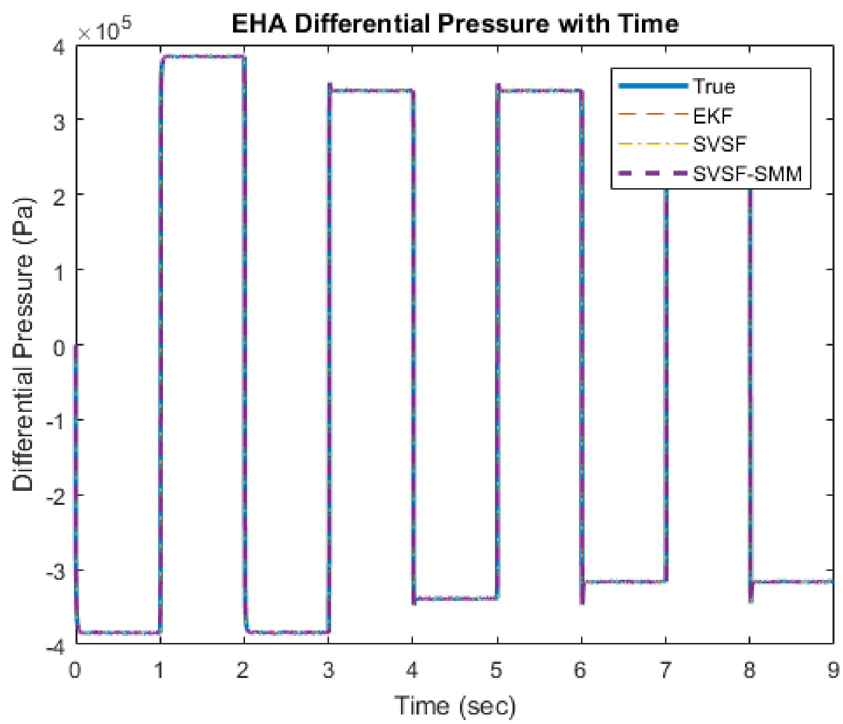

Figure 5. Position Estimates for the EHA computer experiment

5. CONCLUSIONS

This paper introduced the combination of the SVSF and SMM estimation strategies to produce a more robust filter in the presence of modeling uncertainties such as spontaneous leakage faults. The SVSF-SMM performs well for this particular EHA model due to two main factors. The system parameters of the different leakage modes vary significantly and the system noise covariance and measurement noise covariance are sufficiently small. Thus, there is minimal overlap between the *a priori* predictions of each leakage mode. This paper demonstrates that the addition of SMM to the SVSF method improves state estimation for a system with multiple modes. Future work will include application of the SVSF-SMM to the physical system.

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