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AN ADAPTIVE PID CONTROLLER BASED ON BAYESIAN THEORY

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ABSTRACT

One of the most popular trajectory-tracking controllers used in industry is the PID controller. The PID controller utilizes three types of gains and the tracking error in order to provide a control gain to a system. The PID gains may be tuned manually or using a number of different techniques. Under most operating conditions, only one set of PID gains are used. However, techniques exist to compensate for dynamic systems such as gain scheduling or basic time-varying functions. In this paper, an adaptive PID controller is presented based on Bayesian theory. The interacting multiple model (IMM) method, which utilizes Bayes' theorem and likelihood functions, is implemented on the PID controller to present an adaptive control strategy. The strategy is applied to a simulated electromechanical system, and the results of the proposed controller are compared with the standard PID method. Future work is also considered.

INTRODUCTION

Control theory is an integral part of many, if not all, engineering systems. It typically involves the modification of a system or environment in order to obtain a desired outcome. One of the most common scenarios is trajectory tracking or trajectory following. In this case, an engineer requires a system to follow a certain path, such as in robotic welding systems. The engineer knows what path needs to be followed by the robot, and this path is referred to as the desired trajectory. Utilizing sensors and measurements, and some knowledge of the system dynamics, a signal is sent from the output of the system to the controller; known as feedback. The controller compares the desired trajectory with the actual trajectory, and a corresponding control input is calculated. This input signal is used by the system to modify its output accordingly. Although drastically simplified, this is the basic principle behind control theory.

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The ability to control a mechanical or electrical system depends on the knowledge of the true states or parameters of interest. For example, consider a linear electromechanical system, where the kinematics states such as position, velocity, and acceleration are defined to be the states of interest. The state dynamics, or how the system operates with time, may be captured by using a state representation as follows:

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{1.1}$$

where x_k defines the system states, A is the linear system matrix, B is the input gain matrix, u_k is the corresponding input to the system, and w_k refers to the system noise present in the system. To understand the behaviour of a system, elements from the state vector need to be observed or measured. Sensors placed in the environment are used to measure the states of interest. A relationship exists between the measurements and the states, and may be defined as follows:

$$z_{k+1} = Cx_{k+1} + v_{k+1} \tag{1.2}$$

where z_k defines the measurements, C refers to the linear measurement matrix, and v_k refers to the measurement noise present in the sensors. It is assumed in that the system and measurement noises are modeled as Gaussian noise, with zero mean and covariance's Q_k and R_k, respectively as follows:

$$p(w_k) \sim \mathcal{N}(0, Q_k) \tag{1.3}$$

$$p(v_k) \sim \mathcal{N}(0, R_k) \tag{1.4}$$

Therefore, it is the role of a filter to extract knowledge of the true states typically from noisy measurements or observations made of the system, and form state estimates $\hat{x}_k$. The name 'filter' is appropriate since it removes unwanted noise from the signal. The concept of filter applies equally well to nonlinear systems and measurements, defined respectively by:

$$x_{k+1} = f(x_k, u_k) + w_k \tag{1.5}$$

$$z_{k+1} = h(x_{k+1}) + v_{k+1} \tag{1.6}$$

where f and h represent the nonlinear system and measurement models, respectively. The most popular and well-studied estimation method is the Kalman filter (KF), which was introduced in the 1960s [1, 2]. The KF yields a statistically optimal solution for linear estimation problems, as defined by (1.1) and (1.2), in the presence of Gaussian noise. The KF derivation was based on linear algebra, statics, and probability theory [3].

In this paper, an adaptive controller is presented based on Bayesian or probability theory. The interacting multiple model (IMM) method, which utilizes Bayes' theorem and likelihood functions, is implemented on the PID controller to present an adaptive control strategy. The strategy is applied to a simulated electromechanical system, and the results of the proposed controller are compared with the standard PID method. The KF and IMM methods are described in the following sections. The standard PID controller and the proposed adaptive PID controller are then discussed. An electromechanical system simulation is described and the results are discussed. The paper then concludes with a summary and statement on future work.

KF ESTIMATION STRATEGY

The following five equations form the core of the KF algorithm, and are used in an iterative fashion. Equations (2.1) and (2.2) define the a priori state estimate $\hat{x}_{k+1|k}$ based on knowledge of the system A and previous state estimate $\hat{x}_{k|k}$, and the corresponding state error covariance matrix $P_{k+1|k}$, respectively.

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + Bu_k \tag{2.1}$$

$$P_{k+1|k} = AP_{k|k}A^T + Q_k \tag{2.2}$$

The Kalman gain K_{k+1} is defined by (2.3), and is used to update the state estimate $\hat{x}_{k+1|k+1}$ as shown in (2.4). The gain makes use of an innovation covariance S_{k+1}, which is defined as the inverse term found in (2.4).

$$K_{k+1} = P_{k+1|k}C^T\left(CP_{k+1|k}C^T + R_{k+1}\right)^{-1} \tag{2.3}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}\left(z_{k+1} - C\hat{x}_{k+1|k}\right) \tag{2.4}$$

The a posteriori state error covariance matrix $P_{k+1|k+1}$ is then calculated by (2.5), and is used iteratively, as per (2.2).

$$P_{k+1|k+1} = (I - K_{k+1}C)P_{k+1|k} \tag{2.5}$$

The derivation of the KF is well documented, with details available in [1, 2, 4]. The optimality of the KF comes at a price of stability and robustness. The basic KF assumptions do not always hold in real applications. If these assumptions are violated, the KF yields suboptimal results and can become unstable [5, 6]. Furthermore, the KF is sensitive to computer precision and the complexity of computations involving matrix inversions [7, 8].

INTERACTING MULTIPLE MODEL METHOD

In nature, many systems behave according to a number of different models (modes, or operating regimes) [9, 10]. For example, in target tracking, a target may travel straight (i.e., uniform motion) or turn (i.e., undergo a coordinated turn) [11]. Furthermore, a system may experience different types of noises (i.e., white or 'coloured') [7]. In these scenarios, it is desirable to implement adaptive estimation algorithms, which 'adapt' themselves to certain types of uncertainties or models in an effort to minimize the state estimation error [11]. One type of adaptive estimation technique includes the 'multiple model' (MM) algorithm [12]; which include the following: static MM [13], dynamic MM [11], generalized pseudo-Bayesian (GPB) [14, 15, 16, 17], and the interacting multiple model (IMM) [11, 18, 19]. For the MM methods, a Bayesian framework is used (i.e., probability based). Essentially, based on some prior probabilities of each model being correct (i.e., the system is behaving according a finite number of modes), the corresponding updated probabilities are calculated and implemented [11].

Throughout this paper, it will be assumed that all of the models are linear with the presence of Gaussian noise; however, nonlinear models could be used via linearization [11]. Each MM method requires estimation of the states and their corresponding probability. The most popular strategy that has been implemented in the MM framework remains the Kalman filter (KF), and is referred to as the IMM-KF [7]. The interacting multiple model (IMM) estimation algorithm is conceptually requires r number of filters (such as the KF) that operate in parallel [11]. The state estimate is calculated under each possible current model, with a mixed initial condition (i.e., a different combination of the previous model-conditioned estimates) [11]. Furthermore, according to and as presented in [11, 19], the input to the filter matched to M_j is obtained from an interaction of the r filters, which consists of the mixing of the estimates $\hat{x}_{i,k|k}$ and weightings $\mu_{i|j,k|k}$ (mixing probabilities). This is equivalent to merging taking place at the beginning of each estimation cycle, which limits the number of filters to r [19]. The IMM strategy has shown to be very effective, and is more computationally efficient than other multiple model algorithms [11]. The following figure helps to explain the IMM method more effectively.

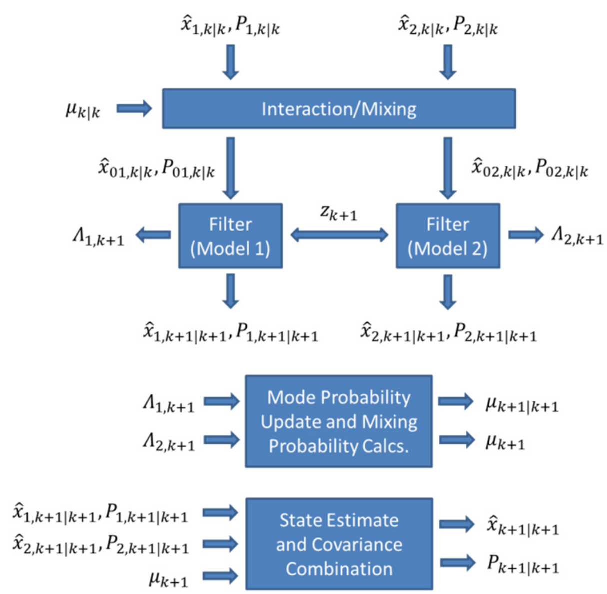

Figure 1. IMM estimator for two models (adapted from [3, 11]).

The IMM estimator consists of five main steps: calculation of the mixing probabilities, mixing stage, mode-matched filtering, mode probability update, and state estimate and covariance combination. The first step involves calculating the mixing probabilities (i.e., the probability of the system currently in mode i, and switching to mode j at the next step). These are calculated using the following two equations [11]:

$$\mu_{i|j,k|k} = \frac{1}{\bar{c}_j} p_{ij} \mu_{i,k} \tag{3.1}$$

$$\bar{c}_j = \sum_{i=1}^{r} p_{ij}\, \mu_{i,k} \tag{3.2}$$

The mixing probabilities $\mu_{i|j,k|k}$ are used in the mixing stage, next. In addition to the mixing probabilities, the previous mode-matched states $\hat{x}_{i,k|k}$ and covariance's $P_{i,k|k}$ are also used to calculate the mixed initial conditions (states and covariance) for the filter matched to M_j. The mixed initial conditions are found respectively as follows [11]:

$$\hat{x}_{0j,k|k} = \sum_{i=1}^{r} \hat{x}_{i,k|k} \mu_{i|j,k|k} \tag{3.3}$$

$$P_{0j,k|k} = \sum_{i=1}^{r} \mu_{i|j,k|k} \left\{ P_{i,k|k} + \left(\hat{x}_{i,k|k} - \hat{x}_{0j,k|k}\right)\left(\hat{x}_{i,k|k} - \hat{x}_{0j,k|k}\right)^T \right\} \tag{3.4}$$

The next step involves mode-matched filtering, which involves using (3.3) and (3.4) as inputs to the filter matched to M_j. Each filter also uses the measurement z_{k+1} and input to the system u_k (if any). The likelihood functions are calculated for each mode-matched filter as follows [11]:

$$\Lambda_{j,k+1} = \mathcal{N}\left(z_{k+1}; \hat{z}_{j,k+1|k}, S_{j,k+1}\right) \tag{3.5}$$

Equation (3.5) may be solved by each filter as follows [11, 7]:

$$\Lambda_{j,k+1} = \frac{1}{\sqrt{\left|2\pi S_{j,k+1}\right|_{Abs}}} exp\left(\frac{-\frac{1}{2} e_{j,z,k+1|k}^T e_{j,z,k+1|k}}{S_{j,k+1}} \right) \tag{3.6}$$

Utilizing the likelihood functions from each filter, the mode probability may be updated by [11]:

$$\mu_{j,k} = \frac{1}{c} \Lambda_{j,k+1} \sum_{i=1}^{r} p_{ij}\, \mu_{i,k} \tag{3.7}$$

where the normalizing constant is defined as [11]:

$$c = \sum_{j=1}^{r} \Lambda_{j,k+1} \sum_{i=1}^{r} p_{ij}\, \mu_{i,k} \tag{3.8}$$

Finally, the overall state estimates (3.9) and covariance (3.10) are calculated. However, note that for this paper, one is mainly interested in utilizing (3.7) for determining the system behavior in an effort to improve controller accuracy.

$$\hat{x}_{k+1|k+1} = \sum_{j=1}^{r} \mu_{j,k+1} \hat{x}_{j,k+1|k+1} \tag{3.9}$$

$$P_{k+1|k+1} = \sum_{j=1}^{r} \mu_{j,k+1} \left\{ P_{j,k+1|k+1} + \left(\hat{x}_{j,k+1|k+1} - \hat{x}_{k+1|k+1}\right)\left(\hat{x}_{j,k+1|k+1} - \hat{x}_{k+1|k+1}\right)^T \right\} \tag{3.10}$$

Equations (3.1) through (3.10) summarize the IMM estimator strategy, and are used recursively. Note that (3.9) and (3.10) are used for output purposes only, and are not part of the algorithm recursions [11]. The IMM strategy has successfully been applied to a number of estimation problems [20], ranging from target tracking in a traffic controller setting [21] to fault detection and diagnosis [22, 23].

PID CONTROLLER

The PID controller is one of the post popular control strategies used in industry [24, 25]. The controller makes use of system feedback (typically a state of interest such as position or pressure). The system feedback is compared with the desired state trajectory and an error signal is created. The error signal is used in conjunction with three types of gains: proportional, integral, and derivative (or PID). The proportional gain is multiplied with the error signal, the integral gain is multiplied with the integral of the error signal, and the derivative gain is multiplied by the derivative of the error signal. The three gain multiplications are summed and the corresponding signal is used as an input to the system. The PID controller input is defined as follows:

$$u_k = K_P e_k + K_I \int e_k + K_D \frac{de}{dt} \tag{4.1}$$

In most cases, the input to the system will cause the system to behave according to the desired state trajectory. However, modeling uncertainties, noise, and external disturbances can cause the PID controller to fail [3]. In general, increasing the proportional gain causes the system to respond faster but may introduce unwanted overshoot and oscillations. Increasing the integral gain typically reduces the steady-state error (since it increases the system order by adding a pole to the system). Increasing the derivative gain may reduce unwanted overshoot but usually slows down the system response. Due to significant interaction among the gains, tuning manually can be challenging. This fact has encouraged the development of various tuning rules and methods [3].

PROPOSED PID-IMM CONTROLLER

In this paper, it is proposed that combining the PID controller with the IMM strategy will improve the overall trajectory tracking accuracy, particularly in systems that are not well defined. The basic principle and concept of the adaptive PID or PID-IMM strategy may be summarized by the following figure.

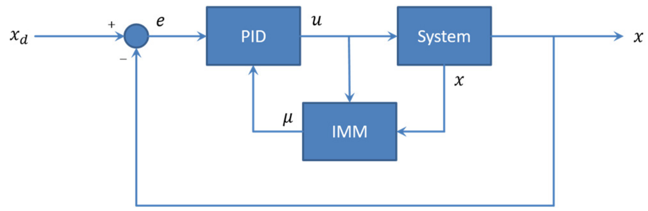

Figure 2. Proposed adaptive PID control strategy.

In the PID-IMM strategy, the PID utilizes the tracking error e and the mode likelihood probability μ (a value between 0 and 1), to generate a 'weighted' system input u. The IMM requires the system input and the output from the system x (or z if using measurements) in order to calculate the mode probability as per (3.7). If the system is being measured, a Kalman filter (KF) or smooth variable structure filter (SVSF) may be implemented to reduce the effects of unwanted noise [3]. Essentially, the adaptive PID strategy utilizes the IMM mode probabilities to formulate 'weighted' system and input matrices in an effort to capture the actual dynamics of the system.

ELECTROMECHANICAL SYSTEM AND RESULTS

In this paper, an electromechanical system based on a type of aerospace actuator was studied [26]. An electrohydrostatic actuator (EHA) is typically used in the aerospace industry for aircraft maneuvering by controlling flight surfaces. EHAs are self-contained units comprised of their own pump, hydraulic circuit, and actuating cylinder. The main components of an EHA include a variable speed motor, an external gear pump, an accumulator, inner circuitry check valves, a cylinder (or actuator), and a bi-directional pressure relief mechanism. A mathematical model for the EHA has been described in detail in [27]. For the purposes of this paper, only the main state space equations will be explored. The input to the system is the rotational speed of the pump ω_p, with typical units of rad/s. In this setup, the sample rate for this simulation was defined as $T = 1\,ms$. The state space equations are defined as follows:

$$x_{1,k+1} = x_{1,k} + Tx_{2,k} + Tw_{1,k} \tag{6.1}$$

$$x_{2,k+1} = x_{2,k} + Tx_{3,k} + Tw_{2,k} \tag{6.2}$$

$$\begin{aligned} x_{1,k+1} &= \left[1 - T\left(\frac{BV_0 + M\beta_e L}{MV_0}\right)\right] x_{3,k} - T\frac{(A^2 + BL)\beta_e}{MV_0} x_{2,k} \\ &\quad - T\left[\frac{2B_2V_0x_{2,k}x_{3,k}}{MV_0} + \frac{\beta_e L\left(B_2x_{2,k}^2 + B_0\right)}{MV_0}\right] sign(x_2, k) \\ &\quad + T\frac{AD_p\beta_e}{MV_0} u_k + Tw_{3,k} \end{aligned} \tag{6.3}$$

Note that A (in this case) refers to the piston cross-sectional area, $B_{\#}$ represents the load friction present in the system, β_e is the effective bulk modulus (i.e., the 'stiffness' in the hydraulic circuit), D_p refers to the pump displacement, L represents the leakage coefficient, M is the load mass (i.e., weight of the cylinders), and V_0 is the initial cylinder volume. The values used to obtain a linear normal operating model are summarized in the appendix. Two more models were created based on a severe friction fault (the friction was increased 3 times) and a severe leakage fault (the leakage coefficient was increased 4 times). The normal, friction fault, and leakage fault system matrices (A_1, A_2, and A_3) are respectively defined as follows:

$$A_1 = \begin{bmatrix} 1 & 0.001 & 0 \\ 0 & 1 & 0.001 \\ 0 & -41.0258 & 0.6099 \end{bmatrix} \tag{6.4}$$

$$A_2 = \begin{bmatrix} 1 & 0.001 & 0 \\ 0 & 1 & 0.001 \\ 0 & -51.8627 & 0.2226 \end{bmatrix} \tag{6.5}$$

$$A_3 = \begin{bmatrix} 1 & 0.001 & 0 \\ 0 & 1 & 0.001 \\ 0 & -73.5364 & 0.6015 \end{bmatrix} \tag{6.6}$$

All three input gain matrices remained the same, and were calculated as follows:

$$B = \begin{bmatrix} 0 \\ 0 \\ 0.0135 \end{bmatrix} \tag{6.7}$$

Note also that artificial system and measurement noise was added to the simulation problem to make it more challenging. The zero-mean Gaussian noise was generated using system and measurement noise covariance's Q and R which were diagonal matrices with elements equal to 1×10^{-6}.

The desired position, velocity, and acceleration trajectories are shown in the following three figures. Note that for the first 4 seconds, the system behaved normally. A friction fault was injected at 4 seconds and lasted for 4 seconds. At 8 seconds, the friction fault was remove and a 2 second leakage fault was implemented.

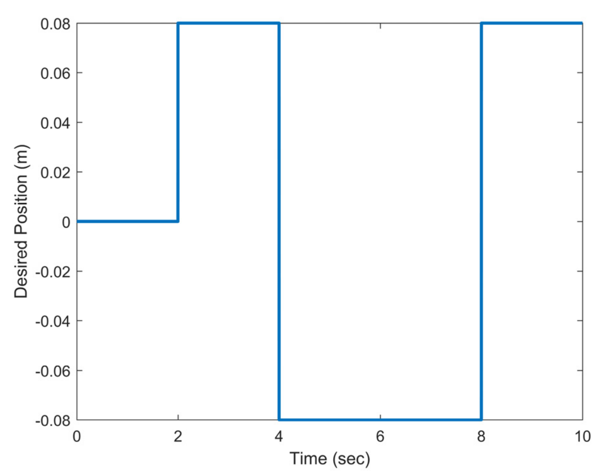

Figure 3. Desired EHA position trajectory.

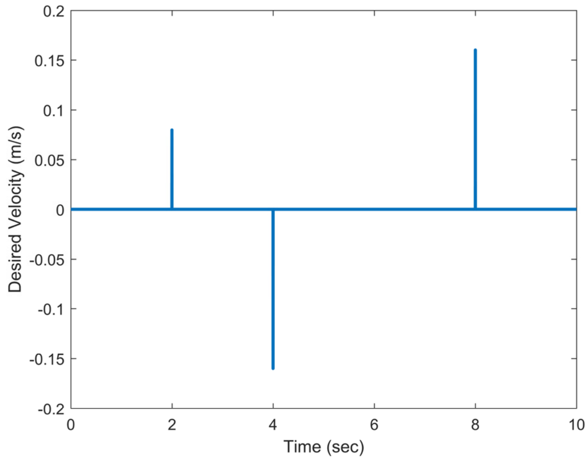

Figure 4. Desired EHA velocity trajectory.

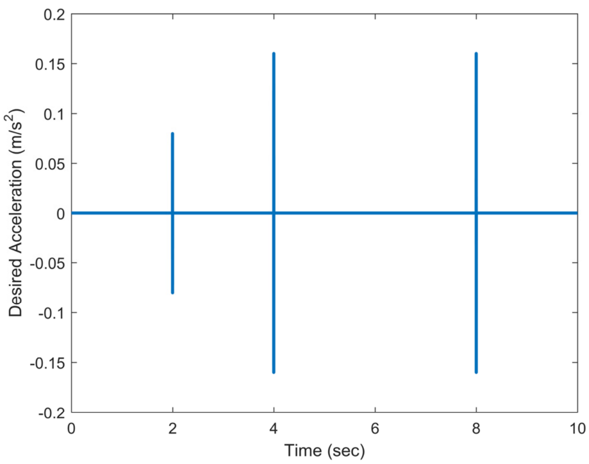

Figure 5. Desired EHA acceleration trajectory.

Three different sets of PID gains were tuned (manually) for each operating mode, as shown in the following table. The gains were tuned such that rise time was maximized and overshoot was minimized. For the standard PID controller, only the 'normally' tuned sets of gains were used since the controller had no knowledge of the operating mode. The results of the tuned PID controller applied on the EHA simulation are shown in Figure 6. After 1,000 simulations, the average root mean square error (RMSE) of the PID controller was $0.0115\ m$. The first four seconds was tracked very well. At four seconds, the EHA overshot the change in trajectory, and was relatively slow to respond to the change at eight seconds.

Table 1. PID values for EHA operating modes.

Gain	Normal	Friction Fault	Leakage Fault
Proportional	100	120	500
Integral	10	10	10
Derivative	10	40	25

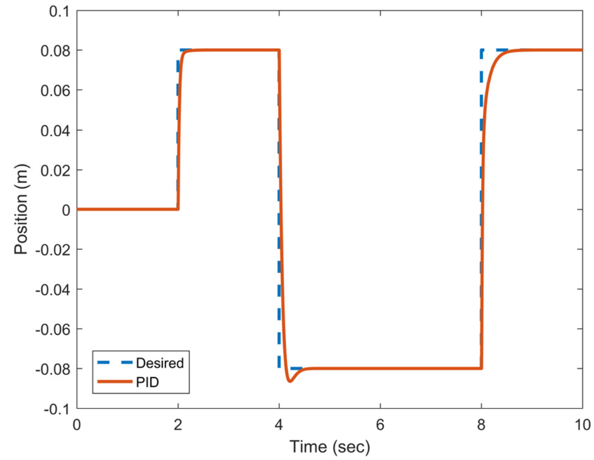

Figure 6. Simulation results with tuned PID controller.

As shown in Figure 2, the adaptive PID strategy makes use of the mode probabilities calculated by the IMM strategy. The mode probabilities are weighted against the gains used by the PID controller. For example, if the EHA was detected to operate normally at 80% and with friction at 20%, the proportional gain used by the PID would be 104. For this simulation, the IMM yielded very strong operating modes, as shown in Figure 7. The operating mode probabilities (out of 1) were initialized at 0.8, 0.1, and 0.1. The results of applying the proposed adaptive PID controller is shown in Figure 8. After 1,000 simulations, the average root mean square error (RMSE) of the adaptive PID controller was $0.0089\ m$, a tracking improvement of about 35%. As illustrated also in Figure 9, the adaptive PID provides significantly improved tracking performance. Utilizing the mode probabilities greatly improved the results.

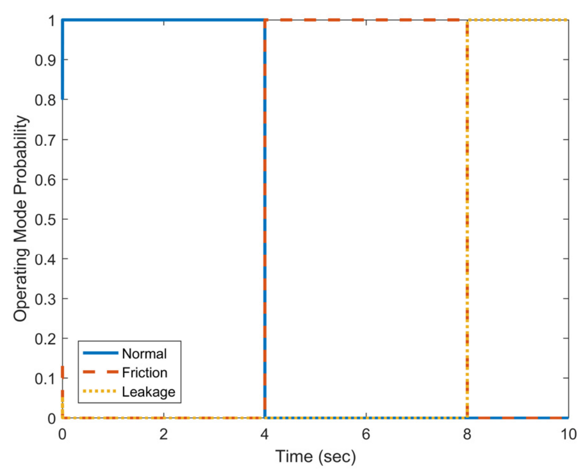

Figure 7. Operating mode probabilities calculated by the IMM strategy.

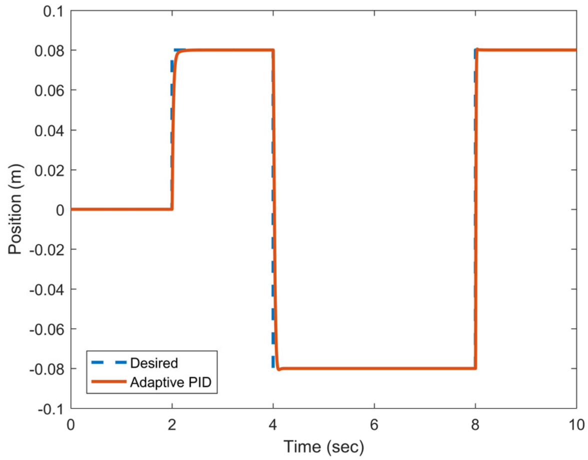

Figure 8. Simulation results with adaptive PID controller.

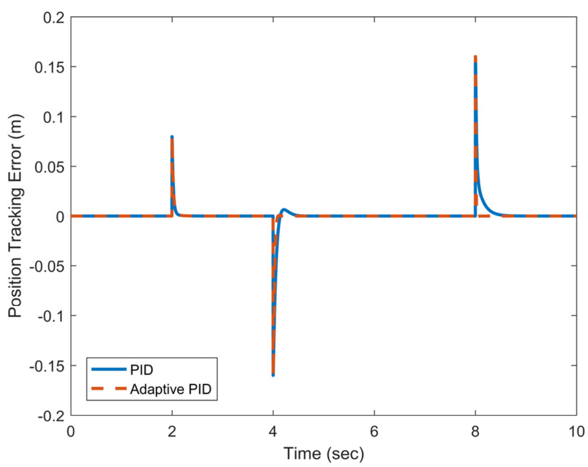

Figure 9. Position tracking error of PID and proposed adaptive PID controllers.

CONCLUSIONS AND FUTURE WORK

This paper proposed an adaptive PID controller based on Bayesian theory. The interacting multiple model (IMM) method, which utilizes Bayes' theorem and likelihood functions, was implemented on the PID controller to present an adaptive control strategy. The strategy was applied to a simulated electromechanical system. When compared with the standard PID controller, the proposed adaptive PID controller improved the tracking performance by approximately 35%, as well as the overall system response. Future research will be more comprehensive and provide a more in-depth study. The adaptive controller will be applied to an experimental EHA, and will be compared with other robust control strategies.

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