Nonlinear Estimation Strategies Applied on an RRR Robotic Manipulator

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Abstract

Nonlinear estimation strategies are important for accurate and reliable control of robotic manipulators. This paper studies the application of estimation theory to a simple robotic manipulator. Two estimation techniques are considered; the classic extended Kalman filter (EKF), and the smooth variable structure filter (SVSF). The EKF is included to present a basic background in estimation techniques and the SVSF demonstrates an example of the state-of-the art. We simulate the SVSF applied to a dynamically modeled three-link (RRR) robotic manipulator. The results of the paper demonstrate that nonlinear estimation techniques such as the SVSF can be applied effectively. Suggestions for future estimation and robotics research are also provided.

Keywords: Estimation theory, Kalman filter, smooth variable structure filter, robotic manipulator

1. Introduction

Accurate and predictable control of mechanical and electrical systems is often dependent on precise knowledge of system states. Accurate knowledge of these states however is rarely directly available, and instead some sort of estimate is required. Typically estimates will be made based on available measurement data as well as knowledge of the system model. Estimation theory concerns itself with the various techniques with which accurate and robust estimates can be made. Some estimators have become to be termed "filters" due their ability to effectively extract the true signal from system and measurement noise.

Many estimation strategies are well suited for linear systems in the discrete time domain. Systems can be generalized into four main categories (with physical systems only existing in the latter three): (i) linear, (ii) approximately linear systems, (iii) medium to strongly non-linear systems, (iv) systems for whom non-linearity is the main characteristic[1]. For categories i. and ii. typical estimations strategies work well. For category iii. estimation strategies can be extended with the use of linearization techniques. Category iv. systems can present a significant challenge for any estimation or control system.

Robotic manipulators have become quite ubiquitous over the last several decades. Many significant advances in

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manufacturing and automation are due to effective implementation of robotic manipulators. Robotic manipulators have achieved great utility in their ability to perform repetitive tasks in a highly consistent manner. As such estimation techniques necessarily play a critical role in the performance of these systems.

In this paper, we shall consider a relatively simple planer RRR robotic manipulator. The dynamics of even such a simple system are clearly nonlinear. For context we give a basic overview of the classic estimation algorithm known as the Kalman Filter, and its nonlinear variant the extended Kalman filter (EKF). We then present the relatively new estimation technique known as smooth variable structure filter (SVSF). We demonstrate the latter's application to a simple RRR manipulator and note the results. This paper is organized as follows: Section 1 is the introduction. Section 2 we present a brief background to both the EKF and SFVF with references for further review. In Section 3 we present the system dynamics and basic control loop. In Section 4 we present simulation results. In section 5 we draw our conclusions and discuss future work.

2. The Kalman Filter and Smooth Variable Structure Filter

2.1. *The Kalman Filter*

For a linear system, state dynamics can be expressed in state representation form as follows[1]:

$$x_{k+1} = Ax_k + Bu_k + w_k \qquad (1)$$

$$z_{k+1} = Cx_{k+1} + v_{k+1} \qquad (2)$$

In (1), x_k refers to the system states. A is the linear system matrix, B is the input gain matrix, u_k is the input vector and w_k is the system noise. In (2) z_k is the measurement vector, C is the linear measurement matrix, and v_k represents the measurement noise.

The Kalman filter assumes that the system model is well known and linear, the initial states are known, and the measurement and system noise is Gaussian with zero mean and known respective covariance matrices.[1,2] The KF works as a predictor-corrector; the system model is used to obtain an *a priori* estimate of the states, whereupon measurements combined with the Kalman

Gain matrix are used to apply a correction term to create an updated *a posteriori* state estimate. Figure 2 illustrates the general concept.
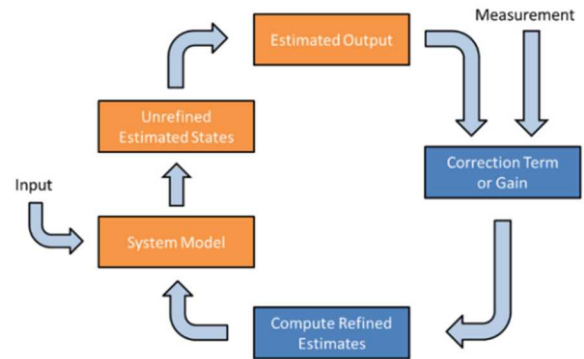

Fig 1. Predictor Estimator[1]

What makes the Kalman Filter so effective is the ability of an appropriately computed Kalman Gain matrix to optimally minimize the estimation error.

For a nonlinear system, equation's (1) and (2) become:

$$x_{k+1} = f(x_k, u_k) + w_k \qquad (3)$$

$$z_{k+1} = h(x_{k+1}) + v_{k+1} \qquad (4)$$

Where f and h are the nonlinear system and measurement models respectively.

The EKF follows the basic procedure of the KF. When computing the predicted and updated state error covariance matrices, where the nonlinear equations cannot be used, the strategy of the EKF is to linearize f and h around the current state estimate by means of the Jacobian. While this algorithm is effective, it does come at the price of optimality as well as robustness. Linearization can fail to account for some modeling uncertainties which can lead to instability[1]. More in depth treatment of the Kalman filter can be found in the literature.[3,4,5]

2.2. *The Smooth Variable Structure Filter*

The smooth variable structure filter is a relatively recent development, appearing in 2007. Also formulated as a predictor corrector, it is based on variable structure theory as well as sliding mode concepts.[1,6]. The basic idea is to use a switching gain to drive estimates to within a defined boundary of the true states - termed the existence subspace.[1] The SVSF can be applied to both linear and

nonlinear systems. Important prerequisites of the SVSF is that the system in question be differentiable as well as observable.[7] The basic SVSF concept is illustrated in Figure 2 below. An in-depth treatment of the concepts can be found in the reference literature.[1,6,7,8,10,11,12]
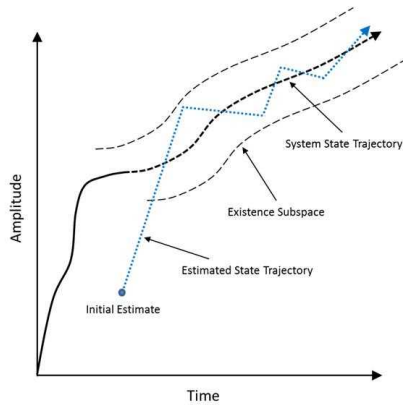

Fig. 2. SVSF Concept Diagram[1]

3. RRR Manipulator Dynamics

The basic configuration of an RRR robotic manipulator appears in Figure 3 below. Note the important parameters include the rod masses m_i, center of gravity lengths l_i, and overall lengths L_i. From these the moments of inertias J_i can be computed. θ_i are the joint variables. For all parameters i=1,2,3.
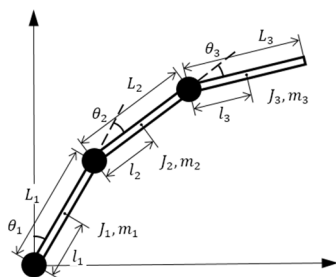

Fig. 3. RRR Planer Manipulator

The generalized equation for a frictionless robotic arm can shown to be[9]:

$$M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + B(\dot{\boldsymbol{\theta}}, \boldsymbol{\theta})\dot{\boldsymbol{\theta}} + G(\boldsymbol{\theta}) = \boldsymbol{u} \qquad (5)$$

Where $\boldsymbol{\theta}$ refers to the joint variable matrix with n degrees of freedom. In the case of an RRR manipulator n=3. M is the inertia matrix, V is the Coriolis vector, G is the gravity vector, and $\boldsymbol{u}$ is the control input vector.

4. Simulation

For simulation we consider a simplified RRR manipulator with unitary masses and moments of inertia. Reference inputs are simple sinusoids. A simple PD controller is used to drive the system inputs. A SVSF filter is applied to the measurement feedback loop. In this simulation the SVSF operates with full measurements.
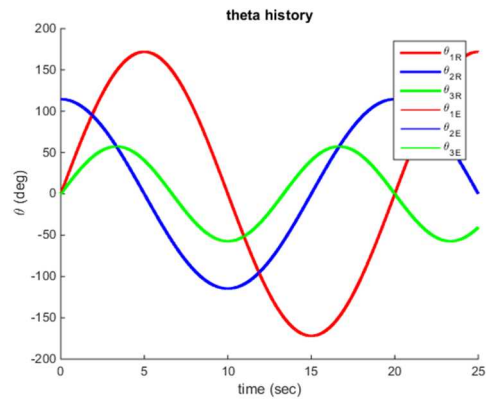

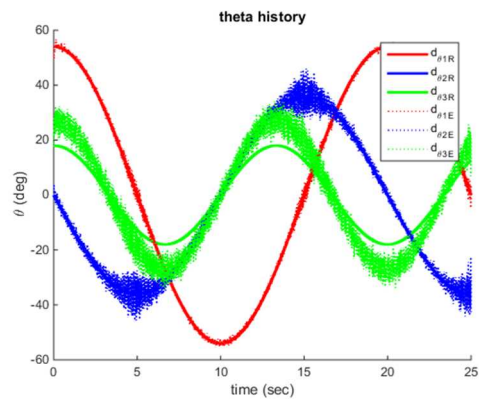Fig. 7. Angle Input and Response
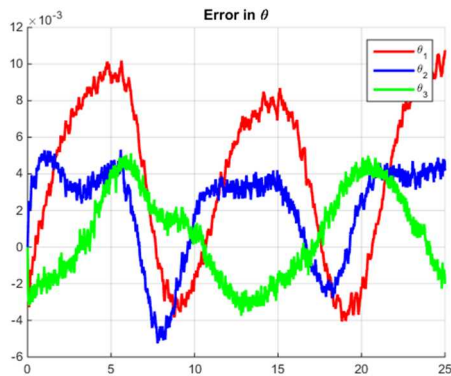

Fig. 8. Velocity Input and Response

Fig. 10. Angle Errors

5. Conclusions and Future Work

Estimation theory has profound utility to a wide variety of control problems. Robotic manipulators in their requirement for precise and consistent control represent an important application of accurate and robust estimation techniques. Filters such as the SVSF are particularly suited for robotic manipulators as manipulator dynamics are by necessity almost always nonlinear. The results demonstrate that the SVSF can be effectively applied to an idealized robotic arm. For future work, more realistic model conditions can be considered, and the SVSF as well as other estimation strategies can be assessed for both robustness and accuracy.

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