A Comprehensive Comparison of Sigma-Point Kalman Filters Applied on a Complex Maneuvering Road

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ABSTRACT

In this paper, a comprehensive comparison is made of the following sigma-point Kalman filters: unscented Kalman filter (UKF), cubature Kalman filter (CKF), and the central difference Kalman filter (CDKF). A simulation based on a complex maneuvering road (an s-path) is used as a benchmark problem. This paper studies the response, stability, robustness, convergence, and computational complexity of the filters. Future work will look at implementing the methods on a robot built for experimentation.

Keywords: Sigma point, Kalman filter, unscented, cubature, central-difference, maneuvering.

1. A BRIEF INTRODUCTION

Model based filters as described in [1], [2], [3], [4], [5], [6] and [7], have been used widely as an efficient manner to obtain information from a noisy, measured signal. Moreover, they are used to extract the system states (description of the system behavior) that are not measured, or may be embedded in other measurements. For linear systems, the statistically optimal solution is known as the Kalman filter (KF), and has been well described in [6], [8], [9], [10], [11] and [12]. If the system is nonlinear, a linearization technique is used to make it applicable. If the technique involves the Taylor series approximation (TSA), the basic KF becomes the perturbation Kalman filter ([9], [13] and [14]), the extended Kalman filter (EKF) ([8], [15], [16] and [17]), and the iterated extended Kalman filter (IEKF) ([6], [15], [18], [19] and [20]). Another type of filter that uses higher orders is referred to as the higher order extended Kalman filter (HOEKF) ([15], [21], [22] and [23]). If the TSA technique is used and accurate results are required, more complicated structures need to be implemented. Different linearization approaches have been developed such as the unscented [15], cubature [24], and central difference [25] transformations. These transformations create sigma points that are used to linearize the model statistically; as such, they are called sigma-point (SP) filters ([1], [3] and [4]). The paper is organized as follows. A brief overview of the sigma-point Kalman filter (SPKF) is presented in Section 2. Three SPKFs are presented in Section 3, including: the unscented (UKF), the cubature (CKF), and the central difference (CDKF) Kalman filters. Section 4 describes the maneuvering system which is used as a benchmark for this work. The results of the paper are discussed in Section 5, and the paper is concluded in the final section.

2. THE SIGMA-POINT KALMAN FILTER

According to [3], [4], [15], [26], [27], [28], [29] and [30], the SPKFs linearize the nonlinear models statistically using weighted linear regression method. Certain points, known as sigma points, are drawn from known distributions around the state neighborhood. The SPKFs differ in the number of those points, their weights, and how to select them. The points are projected through the system model (each one separately), and the results are combined together using appropriate weights as shown in Figure 1.

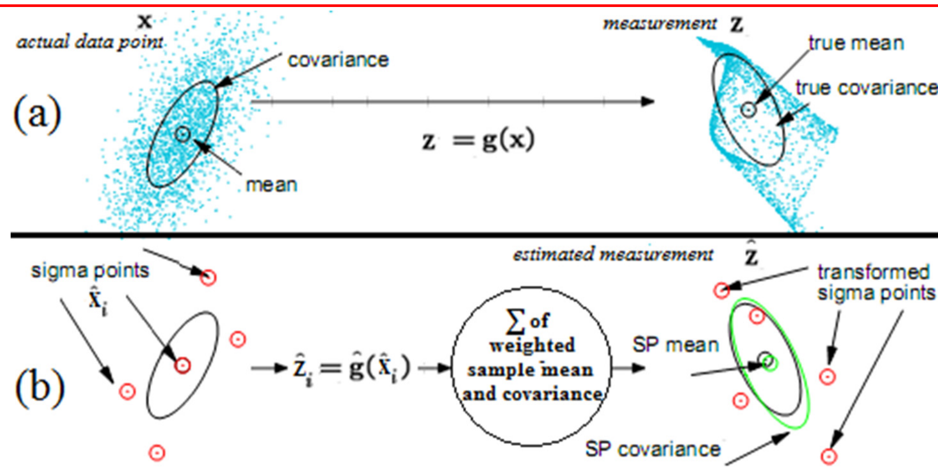

Figure 1. a) The actual system states and their nonlinear measurement, and b) The sigma-points KF estimates [26][30].

3. THE UNSCENTED, CUBATURE AND CENTER-DIFFERENCE KALMAN FILTERS

The Unscented Kalman Filter

The unscented Kalman filter is a SPKF that has been developed using unscented transformations as described in [15], [31], [32], and [33]. The following table summarizes the pseudocode of the UKF.

Table 1. The pseudocode for the unscented Kalman filter code, as per [1], [2], [15] and [34].

$k=0 \rightarrow Initialize\ \hat{\mathbf{x}}_{0\|0}\ and\ \mathbf{P}_{0\|0}$ Start $k=k+1$	////Comments //// n is the number of the states
$for\ \ i=0,1,\dots,2n$ $\hat{\mathbf{X}}_{i_{k-1\|k-1}}=\hat{\mathbf{x}}_{k-1\|k-1}+\begin{cases}0 & i=0\\ \left(\sqrt{n\mathrm{P}_{k-1\|k-1}}\right)_i^T & 1\le i\le n\\ -\left(\sqrt{n\mathrm{P}_{k-1\|k-1}}\right)_i^T & n+1\le i\le 2n\end{cases}$	//// draw the sigma points and their weights
$\hat{\mathbf{X}}_{i_{k\|k-1}}=\hat{\mathbf{f}}\left(\hat{\mathbf{X}}_{i_{k-1\|k-1}},u_{k-1}\right)$ end	//// propagate the points through the filter
$\hat{\mathbf{x}}_{k\|k-1}=\sum_{i=0}^{2n}\frac{1}{2n}\hat{\mathbf{X}}_{i_{k\|k-1}}$	//// combining the sigma points to obtain the a priori estimate
$\mathbf{P}_{k\|k-1}=\sum_{i=0}^{2n}\frac{1}{2n}\left(\hat{\mathbf{X}}_{i_{k\|k-1}}-\hat{\mathbf{x}}_{k\|k-1}\right)\left(\hat{\mathbf{X}}_{i_{k\|k-1}}-\hat{\mathbf{x}}_{k\|k-1}\right)^T+\mathbf{Q}_{k-1}$	//// calculating the a priori covariance matrix
$for\ i=0,1,\dots,2n$ $\hat{\mathbf{X}}_{i_{k\|k-1}}=\hat{\mathbf{x}}_{k\|k-1}+\begin{cases}0 & i=0\\ \left(\sqrt{n\mathrm{P}_{k\|k-1}}\right)_i^T & 1\le i\le n\\ -\left(\sqrt{n\mathrm{P}_{k\|k-1}}\right)_i^T & n+1\le i\le 2n\end{cases}$	//// Redefine the sigma point and their weight to obtain their a priori measurements
$\hat{\mathbf{Z}}_{i_{k\|k-1}}=\hat{\mathbf{g}}\left(\hat{\mathbf{X}}_{i_{k\|k-1}}\right)$ end $\hat{\mathbf{z}}_{k\|k-1}=\sum_{i=0}^{2n}\frac{1}{2n}\hat{\mathbf{Z}}_{i_{k\|k-1}}$	//// combining the sigma points' measurements to obtain the a priori measurement

Table 1. The pseudocode for the unscented Kalman filter code, as per [1], [2], [15] and [34] (continued).

$\mathbf{P}_{zz} = \sum_{i=0}^{2n} \frac{1}{2n} \left(\hat{\mathbf{Z}}_{i_{k\|k-1}} - \hat{\mathbf{z}}_{k\|k-1}\right)\left(\hat{\mathbf{Z}}_{i_{k\|k-1}} - \hat{\mathbf{z}}_{k\|k-1}\right)^T + \mathbf{R}_k$	//// Calculating the output's error covariance matrix
$\mathbf{P}_{xz} = \sum_{i=0}^{2n} \frac{1}{2n} \left(\hat{\mathbf{X}}_{i_{k\|k-1}} - \hat{\mathbf{x}}_{k\|k-1}\right)\left(\hat{\mathbf{Z}}_{i_{k\|k-1}} - \hat{\mathbf{z}}_{k\|k-1}\right)^T$	
$\mathbf{K}_k = \mathbf{P}_{xz}\,\mathbf{P}_{zz}^{\ -1}$	//// The correction gain
$\hat{\mathbf{x}}_{k\|k} = \hat{\mathbf{x}}_{k\|k-1} + \mathbf{K}_k\left(\mathbf{z}_k - \hat{\mathbf{z}}_{k\|k-1}\right)$	//// Updating the estimate and its covariance matrix
$\boldsymbol{P}_{k\|k} = \left(\mathbf{P}_{k\|k-1} - \mathbf{K}_k\,\mathbf{P}_{zz}\mathbf{K}_k^{\ T}\right)$	
Go back to Start	//// Repeat Stages

The Cubature Kalman Filter

The cubature Kalman filter (CKF) is derived by using the third-degree cubature rule to numerically approximate the Gaussian-weighted (W) integrals defined as per [24], [35] and [36]:

$$\int_R \mathbf{F}(x)W(x)dx \tag{3.1}$$

The filter has the same pseudocode of the UKF in Table 1. The differences may be summarized by how the covariance matrices will be calculated. These equations are as follows:

$$\mathbf{P}_{k|k-1} = \frac{1}{2n}\sum_{i=1}^{2n}\left(\hat{\mathbf{X}}_{i_{k|k-1}}\hat{\mathbf{X}}_{i_{k|k-1}}^T - \hat{\mathbf{x}}_{k|k-1}\hat{\mathbf{x}}_{k|k-1}^T\right) + \mathbf{Q}_{k-1} \tag{3.2}$$

$$\mathbf{P}_{zz} = \frac{1}{2n}\sum_{i=1}^{2n}\left(\hat{\mathbf{Z}}_{i_{k|k-1}}\hat{\mathbf{Z}}_{i_{k|k-1}}^T - \hat{\mathbf{z}}_{k|k-1}\hat{\mathbf{z}}_{k|k-1}^T\right) + \mathbf{R}_k \tag{3.3}$$

$$\mathbf{P}_{xz} = \frac{1}{2n}\sum_{i=1}^{2n}\left(\hat{\mathbf{X}}_{i_{k|k-1}}\hat{\mathbf{Z}}_{i_{k|k-1}}^T - \hat{\mathbf{x}}_{k|k-1}\hat{\mathbf{z}}_{k|k-1}^T\right) \tag{3.4}$$

The Central Difference Kalman Filter

The central difference Kalman filter (CDKF), described in [25], [37], [38], [39] and [40], is derived by replacing the derivatives of TSA with their numerical Stirling's polynomial interpolation forms (NSPI) as per [41]. It is defined as follows [42]:

$$\partial f^{(n)}(x) = \frac{1}{2}\left(f^{(n-1)}\left(x + \frac{T_s}{2}\right) - f^{(n-1)}\left(x - \frac{T_s}{2}\right)\right) \tag{3.5}$$

The resultant algorithm is similar to the SPKFs described earlier, and the pseudocode is summarized by the following table.

Table 2. Pseudocode of the sigma-point central difference Kalman filter for Gaussian distributions [39].

$k = 0 \rightarrow Initialize\ \hat{\mathbf{x}}_{0\|0}\ and\ \mathbf{P}_{0\|0}$ Start $k = k + 1$ $for\ \ i = 0,1,\dots,2n$	////Comments
$\hat{\mathbf{X}}_{i_{k-1\|k-1}} = \hat{\mathbf{x}}_{k-1\|k-1} + \begin{cases} 0 & i = 0 \\ \left(\sqrt{3\mathbf{P}_{k-1\|k-1}}\right)_i^T & 1 \le i \le n \\ -\left(\sqrt{3\mathbf{P}_{k-1\|k-1}}\right)_i^T & n+1 \le i \le 2n \end{cases}$	//// draw the sigma points
$\hat{\mathbf{X}}_{i_{k\|k-1}} = \hat{\mathbf{f}}\left(\hat{\mathbf{X}}_{i_{k-1\|k-1}}, u_{k-1}\right)$	//// propagate the points through the filter
end $\hat{\mathbf{x}}_{k\|k-1} = \sum_{i=0}^{2n}\left[\hat{\mathbf{X}}_{i_{k\|k-1}} \times \begin{cases} \frac{3-n}{3} & i = 0 \\ \frac{1}{6} & i \ne 0 \end{cases}\right]$	//// combining the sigma points to obtain the a priori estimate
$\boldsymbol{E}_i = \hat{\mathbf{X}}_{i_{k\|k-1}} - \hat{\mathbf{X}}_{i+n_{k\|k-1}}, \boldsymbol{D}_i = \hat{\mathbf{X}}_{i_{k\|k-1}} + \hat{\mathbf{X}}_{i+n_{k\|k-1}} - 2\hat{\mathbf{X}}_{0_{k\|k-1}}$ $\mathbf{P}_{k\|k-1} = \sum_{i=1}^{n}\frac{1}{12}\boldsymbol{E}_i\boldsymbol{E}_i^T + \sum_{i=1}^{n}\frac{1}{18}(\boldsymbol{D}_i\boldsymbol{D}_i^T) + \mathbf{Q}_{k-1}$	//// calculating the a priori covariance matrix
$for\ i = 0,1,\dots,2n$ $\hat{\mathbf{X}}_{i_{k\|k-1}} = \hat{\mathbf{x}}_{k\|k-1} + \begin{cases} 0 & i = 0 \\ \left(\sqrt{3\mathbf{P}_{k\|k-1}}\right)_i^T & 1 \le i \le n \\ -\left(\sqrt{3\mathbf{P}_{k\|k-1}}\right)_i^T & n+1 \le i \le 2n \end{cases}$ $\hat{\mathbf{Z}}_{i_{k\|k-1}} = \hat{\mathbf{g}}\left(\hat{\mathbf{X}}_{i_{k\|k-1}}\right)$	//// Redefine the sigma point to obtain their a priori measurements
end $\hat{\mathbf{z}}_{k\|k-1} = \sum_{i=0}^{2n}\left[\hat{\mathbf{Z}}_{i_{k\|k-1}} \times \begin{cases} \frac{3-n}{3} & i = 0 \\ \frac{1}{6} & i \ne 0 \end{cases}\right]$	//// combining the sigma points' measurements to obtain the a priori measurement
$\boldsymbol{E}_i = \hat{\mathbf{Z}}_{i_{k\|k-1}} - \hat{\mathbf{Z}}_{i+n_{k\|k-1}}, \boldsymbol{D}_i = \hat{\mathbf{Z}}_{i_{k\|k-1}} + \hat{\mathbf{Z}}_{i+n_{k\|k-1}} - 2\hat{\mathbf{Z}}_{0_{k\|k-1}}$ $\mathbf{P}_{zz} = \sum_{i=1}^{n}\frac{1}{12}\boldsymbol{E}_i\boldsymbol{E}_i^T + \sum_{i=1}^{n}\frac{1}{18}(\boldsymbol{D}_i\boldsymbol{D}_i^T) + \mathbf{R}_k$ $\mathbf{P}_{xz} = \frac{1}{2}\sqrt{\frac{\mathbf{P}_{k\|k-1}}{3}}\left(\begin{bmatrix} \hat{\mathbf{Z}}_{1_{k\|k-1}}^T \\ \vdots \\ \hat{\mathbf{Z}}_{n_{k\|k-1}}^T \end{bmatrix} - \begin{bmatrix} \hat{\mathbf{Z}}_{1+n_{k\|k-1}}^T \\ \vdots \\ \hat{\mathbf{Z}}_{2n_{k\|k-1}}^T \end{bmatrix}\right)$	//// Calculating the output's error covariance matrix
$\mathbf{K}_k = \mathbf{P}_{xz}\,\mathbf{P}_{zz}^{\ -1}$	//// The correction gain
$\hat{\mathbf{x}}_{k\|k} = \hat{\mathbf{x}}_{k\|k-1} + \mathbf{K}_k\left(\mathbf{z}_k - \hat{\mathbf{z}}_{k\|k-1}\right)$ $\boldsymbol{P}_{k\|k} = \left(\mathbf{P}_{k\|k-1} - \mathbf{K}_k\,\mathbf{P}_{zz}\mathbf{K}_k^{\ T}\right)$	//// Updating the estimate and its covariance matrix
Go back to Start	//// Repeat Stages

4. MATHEMATICAL MODELS FOR TARGET TRACKING

The complex maneuvering tracking problem is based on a generic air traffic control (ATC) scenario found in [43]. A sensor stationed at the origin provides direct position only measurements, with a very large standard deviation of 1,000 m in each coordinate. As shown in the following figure, a vehicle starts from an initial position of [25,000 m, 10,000 m] at time $t = 0\ s$, and drives (speeding) westward at 120 m/s for 125 s. The vehicle then begins a coordinated turn for a period of 90 s at a rate of $1^{\circ}/s$. It then travels southward at 120 m/s

for 125 s, followed by another coordinated turn for 30 s at $3°/s$. Finally, it continues to drive westward. The behaviour of targets may be modeled by two different modes: uniform motion (UM) which involves a straight path with a constant speed and course, and maneuvering which includes turning [43]. In this case, maneuvering will refer to a coordinated turn (CT) model, where a turn is made at a constant turn rate and speed. The uniform motion model used for this target tracking problem is given by (4.1) [43].

$$x_{k+1} = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} \frac{1}{2}T^2 & 0 \\ 0 & \frac{1}{2}T^2 \\ T & 0 \\ 0 & T \end{bmatrix} w_k \tag{4.1}$$

The state vector of the vehicle may be defined as follows:

$$x_k = [\xi_k \quad \eta_k \quad \dot{\xi}_k \quad \dot{\eta}_k]^T \tag{4.2}$$

The first two states refer to the position along the x-axis and y-axis, respectively, and the last two states refer to the velocity along the x-axis and y-axis, respectively. The sampling time used in this simulation was 5 seconds. When using the CT model, the state vector needs to be augmented to include the turn rate, as shown in (4.3) [43]. The CT model may be considered nonlinear if the turn rate of the vehicle is not known. Note that a left turn corresponds to a positive turn rate, and a right turn has a negative turn rate. This sign convention follows the commonly used trigonometric convention (the opposite is true for navigation convention) [43]. As per [43], the CT model is given by (4.4).

$$x_k = [\xi_k \quad \eta_k \quad \dot{\xi}_k \quad \dot{\eta}_k \quad \omega_k]^T \tag{4.3}$$

$$x_{k+1} = \begin{bmatrix} 1 & 0 & \frac{sin\omega_k T}{\omega_k} & -\frac{1-cos\omega_k T}{\omega_k} & 0 \\ 0 & 1 & \frac{1-cos\omega_k T}{\omega_k} & \frac{sin\omega_k T}{\omega_k} & 0 \\ 0 & 0 & cos\omega_k T & -sin\omega_k T & 0 \\ 0 & 0 & sin\omega_k T & cos\omega_k T & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} \frac{1}{2}T^2 & 0 & 0 \\ 0 & \frac{1}{2}T^2 & 0 \\ T & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & T \end{bmatrix} w_k \tag{4.4}$$

Note that the measurements of the state vector (4.3) are m, m, m/s, m/s, and rad/s respectively. Since the radar stationed at the origin provides direct position measurements only, the measurement equation may be formed linearly as follows:

$$z_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} x_k + v_k \tag{4.5}$$

Equations (4.1) through (4.5) were used to generate the true state values of the trajectory and the radar measurements for this target tracking scenario.

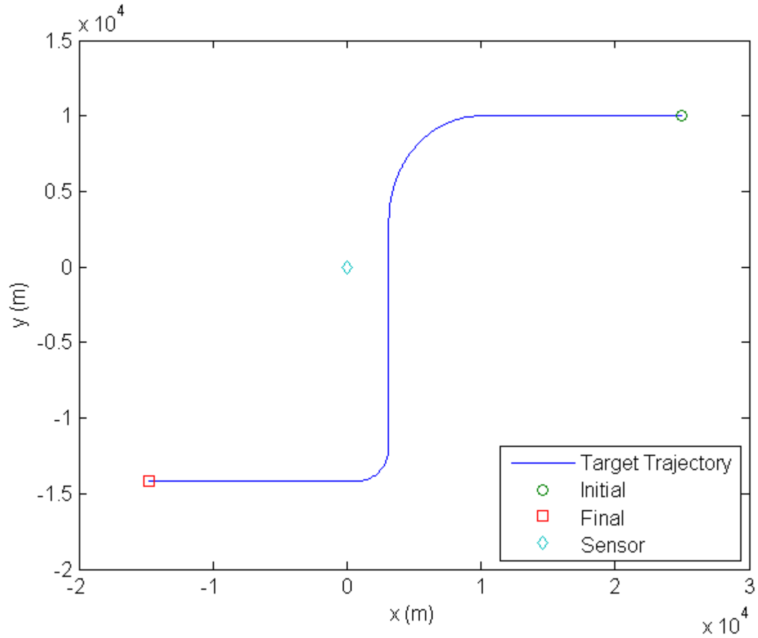

Figure 2. Target tracking scenario (with system noise not yet added) [43].

5. SIMULATION RESULTS

The first two states refer to the position along the x-axis and y-axis, respectively, and the last two states refer to the velocity along the x-axis and y-axis, respectively. The sampling time used in this simulation was 5 seconds. When using the CT model, the state vector needs to be augmented to include the turn rate, as shown in (4.3) [43]. The CT model may be considered nonlinear if the turn rate of the vehicle is not known. Note that a left turn corresponds to a positive turn rate, and a right turn has a negative turn rate. This sign convention follows the commonly used trigonometric convention (the opposite is true for navigation convention) [43]. As per [43], the CT model is given by (4.4).

The following four figures show the results of the tracking scenario as outlined in this paper.

Table 3. RMSE results for the uniform motion model case.

Filter \ RMSE	Position	Velocity	Omega
CDKF	2,093	59.2	0.55
UKF	2,101	59.4	0.55
CKF	2,650	87.3	0.55

Table 4. RMSE results for the coordinated turn model case.

Filter \ RMSE	Position	Velocity	Omega
CDKF	1,546	85.1	0.57
UKF	1,841	86.0	0.56
CKF	1,890	128.5	0.89

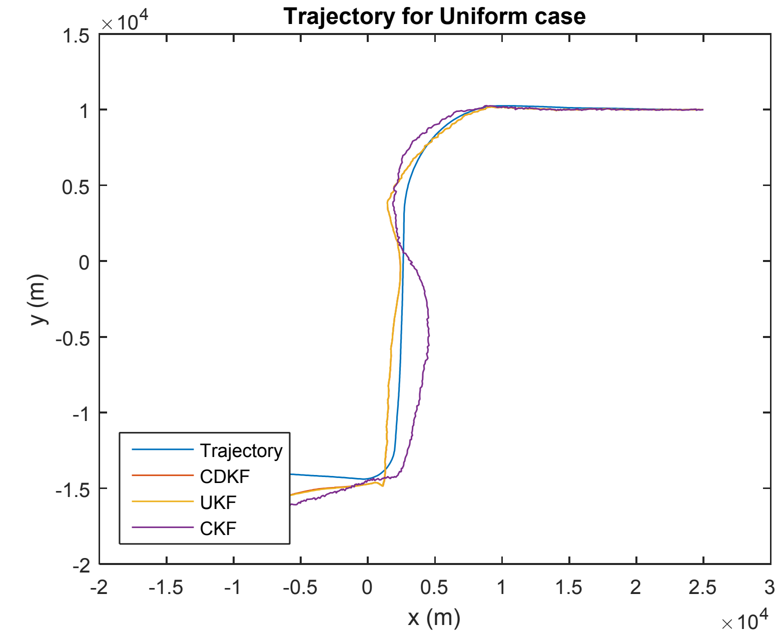

Figure 3. Target tracking simulation results for the uniform motion model case.

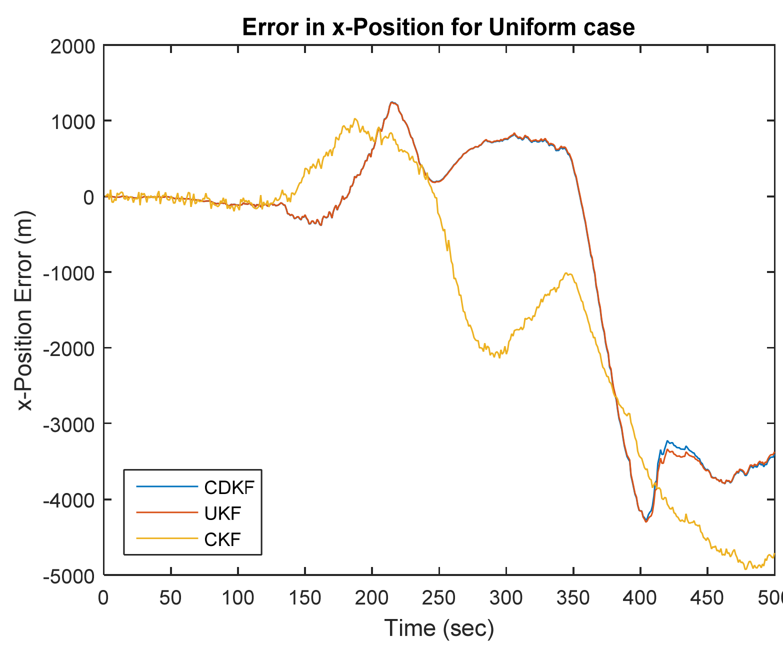

Figure 4. Position estimation errors for the uniform motion model case.

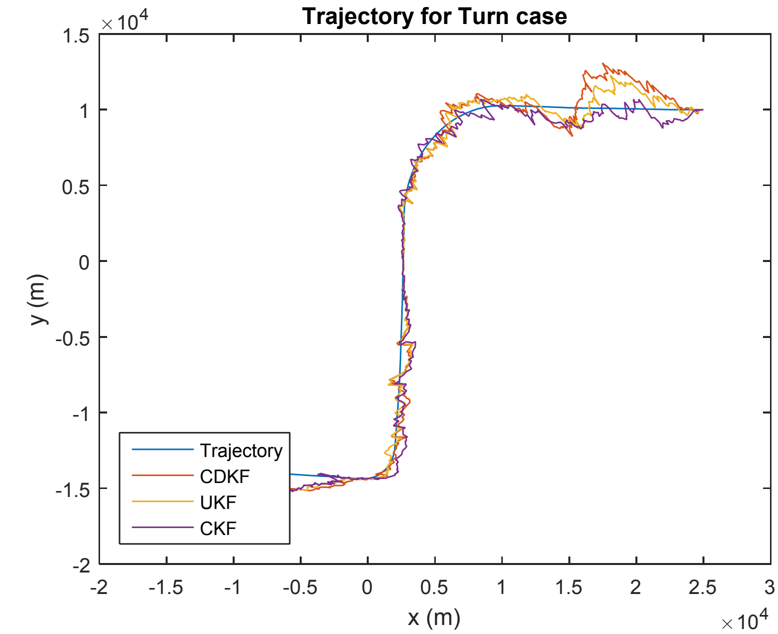

Figure 5. Target tracking simulation results for the coordinated turn model case.

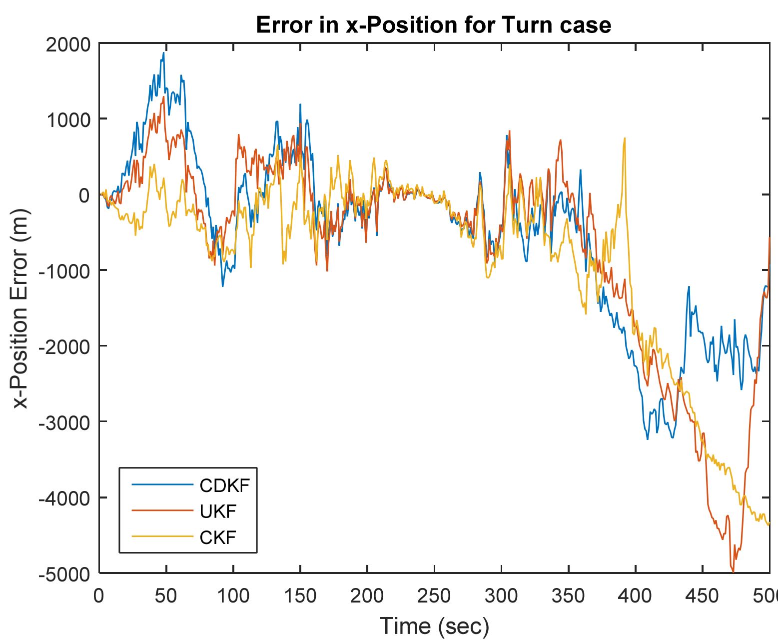

Figure 6. Position estimation errors for the coordinated turn model case.

6. CONCLUSIONS

In this paper, a comparison of the following sigma-point Kalman filters were made: unscented Kalman filter (UKF), cubature Kalman filter (CKF), and the central difference Kalman filter (CDKF). A simulation based on a complex maneuvering road (an s-path) was used as a benchmark problem. The results of the simulation demonstrate that all three filters were able to successfully tracking the maneuvering vehicle. When using the uniform motion model, the CDKF yielded the best tracking results, followed by the UKF and CKF, respectively. When implementing the coordinated turning model, the CDKF also yielded the best tracking results. In general, when the coordinated turn model was used by the filters, the estimation results were found to be more accurate. This was expected since higher-order models were used to predict and update the state estimates and covariances. Future work will look at studying these filters as applied on an experimental robot setup.

7. APPENDIX

The following table summarizes the main nomenclature used in this paper.

Table 5. List of nomenclature.

$\cdot^{-1}\ \cdot^{T}$	Inverse, and transpose, respectively.	$\mathbf{P}_{\mathrm{zz}}$	The output's error covariance matrix.
$(\mathbf{a})_i$	The i row of **a.**	$\mathbf{P}$	The error covariance matrix.
$\mathbf{e}_{\mathbf{m}}$	The estimation error vectors in **m**.	q	The number of the sigma points.
$\mathbf{f}(.)$	The system's model function.	$\mathbf{Q}$	The process noise covariance matrix.
$\mathbf{g}(.)$	The sensor's model function.	$\mathbf{R}$	The measurements noise covariance matrix.
i, j	Subscripts used to identify elements.	$\sum$	The summation operator.
$\mathbf{I}_{n\times n}$	The identity matrix with dimensions of $\mathrm{n} \times \mathrm{n}$.	T_s	Sampling time, and is equal to $0.001\ sec$.
k	Time step value.	$\mathbf{v}, \mathbf{w}$	The measurement and system noise, respectively.
$k\|k-1$	The a priori value at time k.	W_i	The assigned weight.
$k\|k$	The a posteriori value at time k.	$\mathbf{x}$	The state vector.
$\mathbf{K}_X$	The correction gain of the filter X.	$\mathbf{z}$	The output vector.
m, n	Number of measurements and states, respectively.	$\mathbf{X}_i$ and $\mathbf{Z}_i$:	The estimate and its measurement for the i^{th} sigma point, respectively.
$\mathbf{P}_{\mathrm{xx}}$	The state's error covariance matrix.		

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