Proceedings of the 25th CANCAM
London, Ontario, Canada, May 31 – June 4, 2015

STATE OF CHARGE ESTIMATION OF LI-ION BATTERIES USING THE DYNAMIC 2ND-ORDER SMOOTH VARIABLE STRUCTURE FILTER

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ABSTRACT

This paper presents a new approach for the robust state of charge (SOC) estimation of Lithium-ion (Li-ion) batteries. In this approach, the novel dynamic 2nd-order smooth variable structure filter (dynamic 2nd-order SVSF) applies to a Li-ion battery and generates robust SOC estimation under uncertain conditions. The dynamic 2nd-order SVSF is a model-based robust state estimation method that benefits from the robustness and chattering suppression of second-order sliding mode systems. To study the performance of Li-ion batteries, it is necessary to accurately estimate their SOC as a function of the operating time. The SOC estimation may be negatively affected by some factors including modeling imperfections, parametric uncertainties, and measurement noise relevant to the battery setup. To overcome or at least reduce effects of such factors on the SOC estimation, the robust state estimation is recommended that is insensitive to a wider range of noise and uncertainties. In this study, the robust characteristic of the dynamic 2nd-order SVSF method helps to accurately estimate the SOC of a Li-ion battery cell under uncertain and noisy conditions. The Li-ion cell is modeled using a simple first-order R-RC equivalent circuit model. The SOC estimation results are then compared to ones obtained by the extended Kalman filter (EKF) as a sub-optimal state estimator.

KEYWORDS: State of charge estimation, Li-ion battery, the 2nd-order smooth variable structure filter, robustness

INTRODUCTION

The Li-ion batteries have been extensively used in electric vehicles because of their high energy density, durability, safety, lack of hysteresis, and slow loss of charge when not in use. In order to improve their performance and increase their safety and efficiency, accurate management, monitoring, and control are required. For the battery management system, it is necessary to accurately estimate the SOC of the battery as a function of the operating time. The SOC presents the current battery capacity as a percentage of the maximum capacity. It has the same role as a fuel gage used in vehicles with an internal combustion engine. However note that in spite of the fuel gage that directly measures the amount of fuel in a fuel tank, the SOC cannot be measured. The SOC needs to be estimated using state and parameter estimation methods that extract the real-time value of the SOC based on the indirect, inaccurate and uncertain sensor measurements [1].

The SOC estimation may be affected by some factors including modeling imperfections, parametric uncertainties, and measurement noise. Noise and perturbations inherently exist in the measurement process, and caused by instruments and environmental factors. System uncertainties are usually due to inaccuracy in modeling the process, discretization error, and small variations of physical parameters from their nominal values. The Kalman filter is the most well-known method for state estimation that provides optimal state estimates by minimizing the state error covariance [2]. The main concern with the Kalman filter is that it is mainly designed based on an exact knowledge of the system's model with known parameters. In real applications, however, there may be considerable uncertainties about the model structure, physical parameters, level of noise, and initial conditions. These uncertainties may significantly degrade the Kalman filter's performance. To overcome or at least alleviate the effects of such factor on the SOC estimation, robust state estimation is proposed in which the main objective is to design a fix filter that is insensitive to a wider range of noise and modeling uncertainties. The main robust state estimation methods found in the literature are the robust Kalman (or H_2) filter [3], the H_∞ filter [4], and the smooth variable structure filter (SVSF) [5].

This paper presents a new approach for the robust SOC estimation of Li-ion batteries using the novel dynamic 2nd-order SVSF method [6]. The dynamic 2nd-order SVSF is a new model-based robust state estimation method that benefits from the robustness and chattering suppression characteristics of the second-order sliding mode systems. This method is used in this study in order to estimate the SOC of an experimental Li-ion battery cell under uncertain and noisy conditions. The Li-ion battery cell is modeled using a first-order R-RC equivalent circuit model. The SOC estimation results are then compared to ones obtained by the EKF method.

THE DYNAMIC 2ND-ORDER SVSF METHOD [6]

The dynamic 2nd-order SVSF is formulated in a predictor-corrector form. It has two main steps including the prediction and update. In the prediction step, the *a priori* state estimate is predicted using knowledge of the system prior to step *k*. In the update step, the calculated *a priori* estimate is refined into the *a posteriori* state estimate. The corrective gain of the dynamic 2nd-order SVSF pushes the innovation sequence and its first difference to zero in a finite time. This method applies to systems with a linear state and a linear measurement model. In order to describe this method, assume a stochastic system with a linear state model as:

$$x_{k+1} = \hat{F} x_k + \hat{G} u_k + w_k , \tag{1}$$

where $x_k \in \mathbb{R}^{n\times 1}$ is the state vector, $u_k \in \mathbb{R}^{p\times 1}$ is the control vector, and $z_k \in \mathbb{R}^{m\times 1}$ is the measurement vector. The measurement model is also linear and given by:

$$z_{k+1} = \hat{H} x_{k+1} + v_{k+1}, \tag{2}$$

where $z \in \mathbb{R}^{m\times 1}$ is the measurement vector, $v \in \mathbb{R}^{m\times 1}$ is the measurement noise, and $\hat{H} \in \mathbb{R}^{m\times n}$ is a measurement matrix.

The dynamic 2nd-order SVSF is presented as follows [6]:

i. Prediction of the *a priori* state estimate based on the system's state model as:

$$\hat{x}_{k+1|k} = \hat{F} \hat{x}_{k|k} + \hat{G} u_k . \tag{3}$$

where $\hat{F}$ is an estimate of the state model F. The *a priori* state estimate is calculated using the previous *a posteriori* state estimate $\hat{x}_{k|k}$. The *a priori* estimate of the measurement vector $\hat{z}_{k+1|k}$ is calculated using the estimated state vector and the measurement model as:

$$\hat{z}_{k+1|k} = \hat{H} \hat{x}_{k+1|k} , \tag{4}$$

where $\hat{H}$ is an estimate of the measurement model *H*.

ii. Calculation of the *a posteriori* and *a priori* measurement error, $e_{z_{k|k}} \in \mathbb{R}^{m\times 1}$ and $e_{z_{k+1|k}} \in \mathbb{R}^{m\times 1}$ respectively as:

$$e_{z_{k|k}} = z_k - \hat{H} \hat{x}_{k|k} , \tag{5}$$

$$e_{z_{k+1|k}} = z_{k+1} - \hat{H} \hat{x}_{k+1|k} . \tag{6}$$

iii. Calculation of the corrective gain for the dynamic 2nd-order SVSF $K_{k+1} \in \mathbb{R}^{n\times 1}$ as a function of the *a priori* and the *a posteriori* measurement errors as follows:

$$K_{k+1} = \hat{H}^{-1}\left[e_{z_{k+1|k}} - (\gamma + \Lambda_{k+1}) e_{z_{k|k}} + \gamma \Lambda_{k+1} e_{z_{k-1|k-1}}\right]. \tag{7}$$

where $\gamma = Diag(\gamma_{ii}) \in \mathbb{R}^{m\times m}$ is a diagonal matrix with positive entries such that $0 < \gamma_{ii} < 1$. It represents the convergence rate corresponding to each entry.

iv. Update the *a priori* state estimate into the *a posteriori* state estimate $\hat{x}_{k+1|k+1}$ such that:

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}. \tag{8}$$

In the dynamic 2nd-order SVSF method, the vector of sliding variables $S \in \mathbb{R}^{m\times 1}$ is defined as [6]:

$$S_k = e_{z_{k|k}} , \tag{9}$$

Following (7), the corrective gain represents a second-order Markov process and updated using the measurement error values at different time steps. The main advantage of the dynamic 2nd-order SVSF over other approaches is the use of a cut-off frequency coefficient within the corrective gain formulation. The cut-off frequency coefficient is assigned to each measurement that filters out unwanted chattering effects. This coefficient is formulated into the filter by defining a new dynamic sliding mode manifold that is given by:

$$\sigma_k = \Delta S_k + C S_k \tag{10}$$

where $C \in \mathbb{R}^{m\times m}$ denotes the manifold's cut-off frequency. Since the sliding variable is equal to $S_k = e_{z_{k|k}}$, the difference of the sliding variable is obtained by: $\Delta S_k = e_{z_{k|k}} - e_{z_{k-1|k-1}}$. Hence, by defining the sliding manifold as $\sigma_k = \Delta S_k + C S_k$ and proving the filter stability about it, it is ensured that the measurement error and its difference are decreasing in time.

THE LI-ION BATTERY MODELING

The experimental setup of the Li-ion battery is designed and built in the Centre for Mechatronics and Hybrid Technologies at McMaster University. The dynamics of the battery cell is simply modeled using a first-order R-RC equivalent circuit model. The first-order R-RC model is the simplest equivalent circuit model in which the battery dynamics are modeled using three elements including: 1) the open circuit voltage (OCV), 2) Internal resistance, and 3) capacitors [7]. Figure 1 shows a first-order R-RC model that is used in this study for modeling the Li-ion battery cell.

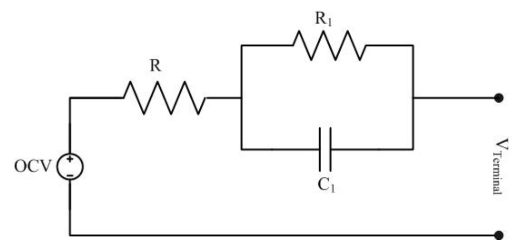

Figure 1. The first-order R-RC model of a battery cell

The first-order R-RC model in the state space from is [7]:

$$\begin{bmatrix} V_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} 1 - \dfrac{\Delta t}{R_1 C_1} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_k \\ z_k \end{bmatrix} + \begin{bmatrix} \dfrac{\Delta t}{C_1} \\ -\dfrac{\eta_i \Delta t}{C} \end{bmatrix} i_k , \tag{11}$$

$$y_k = OCV(z_k) - R i_k - V_k , \tag{12}$$

where V_k is the voltage across the capacitor C_1, z_k is the state of charge, C is the battery nominal capacity, R is the battery ohmic resistance, and η_i is the cell Columbic efficiency. In addition, $R_1 C_1$ represents the polarization time constant, and $OCV(z_k)$ represents the open circuit voltage as a function of the state of charge. The first-order R-RC model has two state variables that are the state of charge z_k and the voltage V_k. The input to the system is the current i_k, and the output *y* is the terminal voltage V_T. In this study, values of the physical parameters including R, R_1, C_1 are obtained using the Genetic algorithm optimization technique. The *OCV* is also formulated as a 9th-order polynomial function of the *SOC*. Note that its ten coefficients are calculated by firstly averaging the $OCV(z_k)$ curve for charging and discharging and then approximating it using a 9th-order polynomial function. Figure 2 presents a typical $OCV(z_k)$ of the under studied Li-ion polymer cell during a charging cycle.

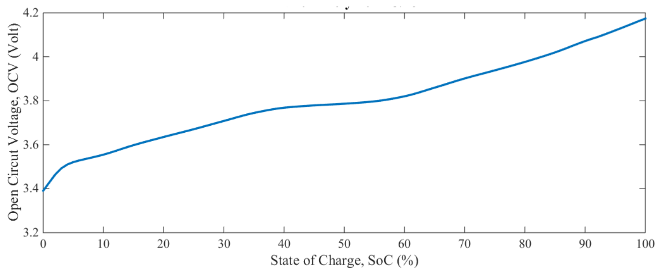

Figure 2. Experimental profile of OCV in terms of SoC

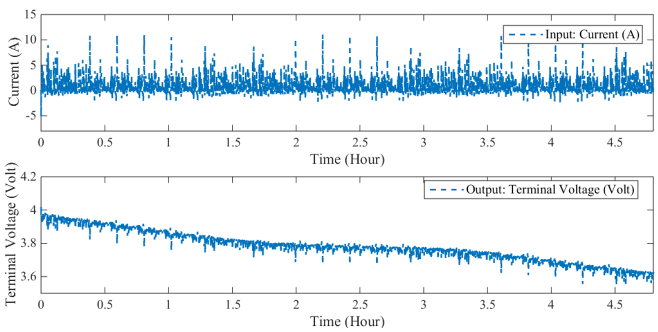

Figure 3. Profiles of the input current calculated from a UDDS cycle and the measured terminal voltage as the cell output

Figure 3 presents profiles of the input current i_k and the output terminal voltage V_T . The input profile is directly converted from a velocity profile of a mix of three benchmark driving schedules, namely: an Urban Dynamometer Driving Schedule (UDDS), a light duty drive cycle for high speed and high load (US06), and a Highway Fuel Economy Test (HWFET) [8]. The UDDS driving cycle is used to describe city driving conditions, and used to replicate the average speed, idle time, and number of stops for an average North American driver. The US06 cycle is a high acceleration, aggressive driving cycle, and the HWFET describes highway driving conditions with speeds below 60 miles/hour [8]. The three above mentioned driving cycles are shown in Figure 4. The pack current profiles from these driving cycles are scaled down to the cell-level while ignoring cell-to-cell balancing.

In order to generate the current profile from the velocity profile, a mid-size battery electric vehicle (BEV) model as shown in Figure 5 was modified from an existing hybrid vehicle model [9]. The BEV model consists of a lithium-ion battery pack, a vehicle speed controller, vehicle dynamic model, DC electric motor, and DC-DC converter. The model was developed in Simulink using Simscape library components. The driving range of the BEV is approximately 200 km per full charge.

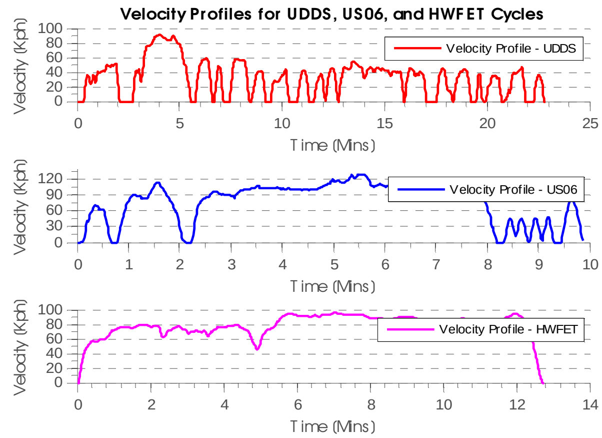

Figure 4. Velocity profiles for the UDDS (upper), US06 (middle), and HWFET (lower) Cycles [8]

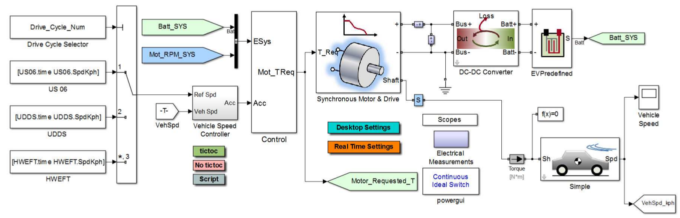

Figure 5. All-electric mid-size sedan simulation model in Simscape (Adopted from [9])

The parametric vector that needs to be optimized using the Genetic algorithm is: $\theta = [\mathrm{R}, \mathrm{R}_1, \mathrm{C}_1]$. Modeling and estimation tasks are performed using the MATLAB-SIMULINK real-time environment. Figure 6 shows a block-diagram of the Li-ion battery model in SIMULINK. Table 1 also presents numeric values of the physical and optimized parameters.

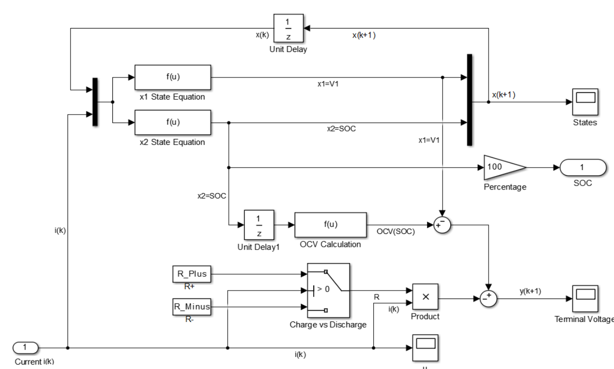

Figure 6. The Li-ion battery model in SIMULINK

Table 1: Numeric values of physical and optimized parameters

Parameter	Numeric Value
Nominal capacity, *C*	19440 (A.s)
Cell Columbic efficiency, η	1
Modeling capacity, C_1	3860.14 (A.s)
Modeling resistance, R_1	0.0049 (Ω)
Internal Resistance, *R*	0.0096 (Ω)
Initial state of charge	80 %
Sampling time, Δt	0.1 (s)

SOC ESTIMATION USING THE DYNAMIC 2ND- SVSF

In this section, the dynamic 2nd-order SVSF and the extended Kalman filter (EKF) methods are applied for the SOC estimation based on the provided Li-ion model and the captured input-output UDDS data where there exist noise and modeling uncertainties. The SOC estimation results are then compared with the actual SOC data generated from the experimental setup model. Comparison are made in terms of the root mean square error (RMSE). Figure 7 presents a block-diagram of the dynamic 2nd-order SVSF method built in SIMULINK. For the dynamic 2nd-order SVSF method, the convergence rate γ is set to 0.5, and the cut-off frequency matrix Λ is designed to be a diagonal matrix with elements equal to 0.4. For the EKF, the process noise covariance Q and the measurement noise covariance R are respectively set to:

$$Q=\begin{bmatrix}10^3 & 0 & 0\\ 0 & 10^4 & 0\\ 0 & 0 & 5\times10^3\end{bmatrix},\quad R=\left[3\times10^{-1}\right]. \tag{13}$$

Figure 8 compares the SOC estimation profile using the dynamic 2nd-order SVSF method with the actual profile. Table 2 also compares the RMSE values obtained by the EKF and the dynamic 2nd-order SVSF method. It demonstrates the higher accuracy of the dynamic 2nd-order SVSF over the EKF method under an uncertain experimental condition.

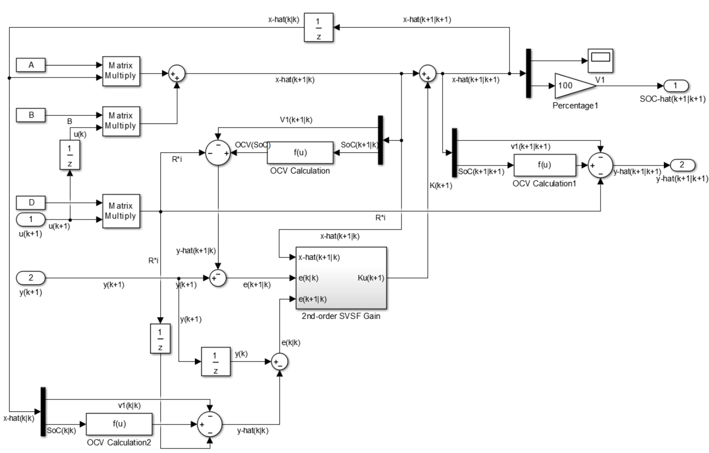

Figure 7. The dynamic 2nd-order SVSF in SIMULINK

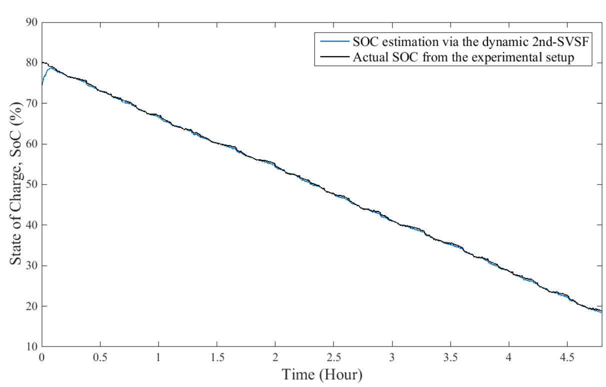

Figure 8. Actual and estimated SOC estimation profiles

Table 2: RMSE values obtained by different estimators

Estimator	RMSE for SOC Estimation
Extended Kalman filter	0.24
Dynamic 2nd-order SVSF	0.20

CONCLUSIONS

This paper introduces a new SOC estimation method that benefits from the dynamic 2nd-order SVSF as a robust state estimation technique. This method applies to an experimental Li-ion battery setup and the SOC estimation results are then compared with ones obtained by the EKF method. The input-output data set is provided from a mix of real-world, benchmark driving cycle. Estimation results show that the SOC estimation obtained by the dynamic 2nd-order SVSF precisely follows the actual SOC profile. The dynamic 2nd-order SVSF also produces a more accurate SOC estimate with a smaller RMSE in comparison to the EKF SOC estimate.

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