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CONDITION MONITORING OF AN ELECTRO-HYDROSTATIC ACTUATOR USING THE DYNAMIC 2ND-ORDER SMOOTH VARIABLE STRUCTURE FILTER

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ABSTRACT

This paper introduces the dynamic 2nd-order Smooth Variable Structure Filter (Dynamic 2nd-order SVSF) method for the purpose of robust state estimation. Thereafter, it presents an application of this method for condition monitoring of an electro-hydrostatic actuator system. The SVSF-type filtering is in general designed based on the sliding mode theory; whereas the sliding mode variable is equal to the innovation (measurement error). In order to formulate the dynamic 2nd-order SVSF, a dynamic sliding mode manifold is defined such that it preserves the first and second order sliding conditions. This causes that the measurement error and its first difference are pushed toward zero until reaching the existence subspace. Hence, this filter benefits from the robustness and chattering suppression properties of the second order sliding mode systems. These help the filter to suppress the undesirable chattering effects without the need for approximation or interpolation that however reduces accuracy and robustness of the SVSF-type filtering. In order to investigate the performance of the dynamic 2nd-order SVSF for state estimation, it applies to an Electro-Hydrostatic Actuator (EHA) system under the normal and uncertain scenarios. Simulation results are then compared with ones obtained by other estimation methods such as the Kalman filter and the 1st-order SVSF method.

INTRODUCTION

State estimation is the process of extracting numeric values of states from inaccurate and uncertain measurements of the system. The main goal is minimizing the estimation error as well as maintaining robustness against the noise and modeling uncertainties [1,2]. Noise and perturbations inherently exist in the measurement process, and caused by instruments and environmental factors. System uncertainties are usually due to inaccuracy in modeling the process, discretization error, and variations of physical parameters. Rudolf Kalman introduced the Kalman filter in 1960 for linear filtering and prediction. It uses a linear dynamic model and sequential measurements of the system to produce state estimates by minimizing the state error covariance matrix. The Kalman-type filtering is primarily designed based on an exact knowledge of the system's model with known parameters. In real applications, there may be considerable uncertainties about the model structure, physical parameters, level of noise, and initial conditions. These uncertainties will affect the Kalman filter performance. In order to overcome such potential difficulties, the robust state estimation is proposed. The main objective of robust estimation is to design a fixed filter that limits the effect of modeling uncertainties and noise on the filter performance. The main robust state estimation methods found in the literature are the robust Kalman (or H_2) filter [3,4], and the H_∞ filter [5,6].

The Smooth Variable Structure Filter (SVSF) has recently been introduced by Habibi [7], as a novel model-based robust state estimation method. The corrective gain of the SVSF has an inherent switching action that guarantees convergence of the state estimates to within a region of the real values. The switching characteristic of the SVSF is due to the variable structure formulation of the discontinuous gain, which provides robustness to bounded uncertainties. The main drawback of the SVSF method for state estimation is chattering that is a non-deterministic high frequency oscillations in the state estimation trajectories resulting from the discontinuous action of the gain. Habibi applied the smoothing boundary layer (a saturation function) for suppressing chattering [8]. The saturation function interpolates the discontinuous action around the switching hyperplane. Outside the smoothing layer the discontinuous correction is fully applied to maintain stability, while inside the smoothing boundary layer the signum function is still applied. By approximating the switching function via the smoothing boundary layer, however the accuracy and robustness of the sliding mode system would be partially lost [8]. Gadsden [9] developed the SVSF concept by adding a state error covariance variable to the SVSF formulation and later introducing the optimal SVSF estimation method with a time-varying smoothing boundary layer [9].

The second order sliding mode concept may be used as an alternative to the smoothing boundary layer for chattering suppression. This concept leads to push the first and the second order time-derivatives of the sliding variable towards zero. Hence, along with preserving the main advantages of the standard sliding mode systems, it is capable of reducing the chattering effect considerably [8,10,11]. This paper presents the dynamic 2nd-order SVSF state estimation method as an extension to the 1st-order SFSF [7]. This method only applies to systems with linear state and measurement models. It is formulated based on a dynamic sliding mode manifold that preserves the first and second order sliding mode conditions during state estimation. The corrective gain of the dynamic 2nd-order SVSF is derived such that it satisfies the Lyapunov's second law of stability in discrete time. To verify the accuracy and robustness of this filter, it is applied to an EHA system for state estimation under the normal and uncertain conditions.

THE DYNAMIC 2ND-ORDER SVSF METHOD

The dynamic 2nd-order SVSF is constructed in a predictor-corrector form (same as the Kalman filter). It has two main steps including the prediction and update. In the prediction step, the *a priori* state estimate $\hat{x}_{k+1|k}$ is predicted using knowledge of the system prior to step *k*. Furthermore, in the update step, the calculated *a priori* estimate is refined into the *a posteriori* state estimate $\hat{x}_{k+1|k+1}$. The corrective gain of the 2nd-order SVSF pushes the measurement error and its first difference to zero in finite time. The dynamic 2nd-order SVSF method applies to systems with a linear state and linear measurement models. Assume a linear system described by a discrete time state-space model as follows:

$$x_{k+1} = \hat{F} x_k + \hat{G} u_k + w_k, \tag{1}$$

where $x_k \in \mathbb{R}^{n\times 1}$ is the state vector, $u_k \in \mathbb{R}^{p\times 1}$ is the control vector, and $z_k \in \mathbb{R}^{m\times 1}$ is the measurement vector. $\hat{F} \in \mathbb{R}^{n\times n}$ is the estimated state matrix, $\hat{G} \in \mathbb{R}^{n\times p}$ is the estimated control matrix, $\hat{H} \in \mathbb{R}^{m\times n}$ is the estimated measurement matrix. The measurement model is linear as follows:

$$z_{k+1} = \hat{H} x_{k+1} + v_{k+1}, \tag{2}$$

where $z \in \mathbb{R}^{m\times 1}$ is the measurement vector, $v \in \mathbb{R}^{m\times 1}$ is the measurement noise, and $\hat{H} \in \mathbb{R}^{m\times n}$ is a positive diagonal or pseudo-diagonal measurement matrix.

It is assumed that the control vector $u \in \mathbb{R}^{p\times 1}$ is known and bounded such that:

$$|u_{i,k}| \le U_i; \quad i = 1,\ldots,n. \tag{3}$$

It is also assumed that vectors w_k and v_k are mutually independent white stochastic processes. They are bounded by $w_{\max}$ and $v_{\max}$ as their upper limits such that:

$$\begin{cases} |w_{i,k}| \le w_{\max}; & i = 1,\ldots,n, \\ |v_{i,k}| \le v_{\max}; & i = 1,\ldots,m. \end{cases} \tag{4}$$

The dynamic 2nd-order SVSF is presented as follows:

i. Prediction of the *a priori* state estimate based on the system's state model as:

$$\hat{x}_{k+1|k} = \hat{F} \hat{x}_{k|k} + \hat{G} u_k. \tag{5}$$

where $\hat{F}$ is an estimate of the state model F. The *a priori* measurement estimate is also calculated using the estimated state vector and the measurement model as:

$$\hat{z}_{k+1|k} = \hat{H} \hat{x}_{k+1|k}, \tag{6}$$

where $\hat{H}$ is an estimate of the measurement model *H*.

ii. Calculation of the *a posteriori* and *a priori* measurement error vectors, $e_{z_{k|k}} \in \mathbb{R}^{m\times 1}$ and $e_{z_{k+1|k}} \in \mathbb{R}^{m\times 1}$ respectively as:

$$e_{z_{k|k}} = z_k - \hat{H} \hat{x}_{k|k}, \tag{7}$$

$$e_{z_{k+1|k}} = z_{k+1} - \hat{H} \hat{x}_{k+1|k}. \tag{8}$$

iii. Calculation of the corrective gain for the dynamic 2nd-order SVSF $K_{k+1} \in \mathbb{R}^{n\times 1}$ as a function of the *a priori* and the *a posteriori* measurement errors as follows:

$$K_{k+1} = \hat{H}^{-1}\left[e_{z_{k+1|k}} - (\gamma + \Lambda_{k+1}) e_{z_{k|k}} + \gamma \Lambda_{k+1} e_{z_{k-1|k-1}}\right]. \tag{9}$$

where $\hat{H} \in \mathbb{R}^{m\times m}$ is a full measurement matrix, $\gamma = Diag(\gamma_{ii}) \in \mathbb{R}^{m\times m}$ is a diagonal matrix with positive entries such that $0 < \gamma_{ii} < 1$. It represents the convergence rate corresponding to each entry.

iv. Update the *a priori* state estimate into the *a posteriori* state estimate $\hat{x}_{k+1|k+1}$ such that:

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}. \tag{10}$$

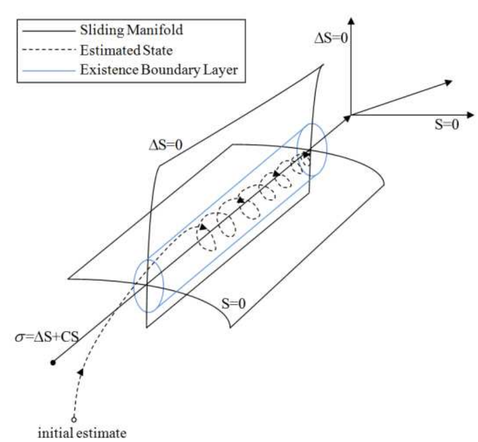

Figure 1. Main concept of the dynamic 2nd-order SVSF method

The dynamic 2nd-order SVSF is formulated based on the dynamic sliding mode theory introduced by Sira-Ramirez [10]. Here, the vector of sliding variables $S \in \mathbb{R}^{m\times 1}$ is defined as:

$$S_k = e_{z_{k|k}}, \tag{11}$$

According to (9), the corrective gain represents a second-order Markov process and updated using the measurement error values at different time steps. The main advantage of the dynamic 2nd-order SVSF over other approaches is the use of cut-off frequency coefficient within the corrective gin formulation. In this regard, a cut-off frequency coefficient is assigned to each measurement that filters out the unwanted chattering. This coefficient is formulated into the filter by defining a new dynamic sliding mode manifold as follows:

$$\sigma_k = \Delta S_k + CS_k \tag{12}$$

where $C \in \mathbb{R}^{m\times m}$ denotes the manifold's cut-off frequency. It is also corresponding to the slope of the sliding manifold (12) in a phase plane coordinated by S and ΔS. Since the sliding variable is equal to the *a posteriori* measurement error $S_k = e_{z_{k|k}}$, the difference of the sliding variable will be equal to the measurement error difference as: $\Delta S_k = e_{z_{k|k}} - e_{z_{k-1|k-1}}$. Hence, by defining the sliding manifold as $\sigma_k = \Delta S_k + CS_k$ and proving the stability of state estimates about it, it is ensured that the estimation error and its difference are decreasing in finite time. Figure 1 presents the dynamic 2nd-order SVSF concept using the linear sliding mode manifold.

STABILITY PROOF FOR THE DYNAMIC 2ND-SVSF

The Lyapunov's second law of stability may be used to provide the stability proof of the dynamic 2nd-order SVSF under the corrective gain given by (9).

Theorem 1: The dynamic 2nd-order SVSF method with the corrective gain of (9) is stable and preserves the first and second order sliding conditions under an ideal sliding regime.

Proof: Assume a positive-definite Lyapunov function as:

$$V_k = \sigma_{i,k}{}^2, \tag{14}$$

where $\sigma_{i,k} \in \mathbb{R}^{m\times m}$ is an element of the linear sliding manifold and defined as: $\sigma_{i,k} = \Delta s_{i,k} + c_{ii}\, s_{i,k}$. In addition, $s_{i,k} \in \mathbb{R}^{m\times m}$ and $\Delta s_{i,k} \in \mathbb{R}^{m\times 1}$ denote elements the sliding variable vector and its difference, respectively. The difference of the sliding variable is obtained using the backward difference operator given by: $\Delta s_{i,k} = s_{i,k} - s_{i,k-1}$. The dynamic 2nd-order SVSF under the gain (9) is stable if $\Delta V_{k+1} = V_{k+1} - V_k < 0$. Substituting the Lyapunov function into the last inequality, the difference of Lyapunov function is calculated as:

$$\Delta V_{k+1} = (\Delta s_{i,k+1} + c_{ii}\, s_{i,k+1})^2 - (\Delta s_{i,k} + c_{ii}\, s_{i,k})^2 \tag{15}$$

where $\Delta s_{i,k+1} = s_{i,k+1} - s_{i,k}$ and $\Delta s_{i,k} = s_{i,k} - s_{i,k-1}$. Substituting the above terms and rearranging them, ΔV_{k+1} is formulated as:

$$\begin{aligned}\Delta V_{k+1} &= (1+c_{ii})^2 s_{i,k+1}{}^2 - 2(1+c_{ii}) s_{i,k+1} s_{i,k} - s_{i,k-1}{}^2 \\ &\quad -2c_{ii}(1+c_{ii}) s_{i,k}{}^2 + 2(1+c_{ii}) s_{i,k}\, s_{i,k-1}.\end{aligned} \tag{16}$$

For more simplicity in calculations, let elements of the manifold's cut-off frequency matrix be defined as:

$$\lambda_{ii} = \frac{1}{1+c_{ii}}, \tag{17}$$

where $\Lambda = Diag(\lambda_{ii}) \in \mathbb{R}^{m\times m}$ is also a diagonal matrix.

Multiplying both sides of the corrective gain formulation (9) by $\hat{H}$ and rearranging:

$$e_{z_{k+1|k}} - \hat{H}\, K_{k+1} = (\gamma + \Lambda_{k+1}) e_{z_{k|k}} - \gamma \Lambda_{k+1} e_{z_{k-1|k-1}}. \tag{18}$$

Since the state estimates are updated using (10), namely $\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}$, it is possible to restated the gain as: $K_{k+1} = \hat{x}_{k+1|k+1} - \hat{x}_{k+1|k}$. Substituting this relation into (18) yields:

$$e_{z_{k+1|k}} - \hat{H}(\hat{x}_{k+1|k+1} - \hat{x}_{k+1|k}) = (\gamma + \Lambda_{k+1}) e_{z_{k|k}} - \gamma \Lambda_{k+1} e_{z_{k-1|k-1}}. \tag{19}$$

The *a priori* and the *a posteriori* measurement errors at time step *k* are obtained from (7) and (8) as: $e_{z_{k+1|k}} = z_{k+1} - \hat{H}\, \hat{x}_{k+1|k}$ and $e_{z_{k+1|k+1}} = z_{k+1} - \hat{H}\, \hat{x}_{k+1|k+1}$. Subtracting the *a priori* error from the *a posteriori* error leads to:

$$e_{z_{k+1|k+1}} - e_{z_{k+1|k}} = -\hat{H}(\hat{x}_{k+1|k+1} - \hat{x}_{k+1|k}). \tag{20}$$

Let restate equality (19) based on equation (20) as follows:

$$e_{z_{k+1|k+1}} = (\gamma + \Lambda_{k+1}) e_{z_{k|k}} - \gamma \Lambda_{k+1} e_{z_{k-1|k-1}}. \tag{21}$$

Since $s_k = e_{z_{k|k}}$, equality (21) may be expressed in terms of the sliding variable entries $s_{i,k}$ as:

$$s_{i,k+1} = (\gamma_{ii} + \lambda_{ii}) s_{i,k} - \gamma_{ii}\, \lambda_{ii}\, s_{i,k-1}. \tag{22}$$

In order to show negative definiteness of the Lyapunov candidate (14), let substitute equality (22) into the difference of the Lyapunov function (18) and expand the result as:

$$\begin{aligned}\Delta V_{k+1} &= (\gamma_{ii}{}^2 - 1)(1+\lambda_{ii})^2 s_{i,k}{}^2 + (\gamma_{ii}{}^2 - 1) s_{i,k-1}{}^2 \\ &\quad -2(\gamma_{ii}{}^2 - 1)(1+\lambda_{ii}) s_{i,k}\, s_{i,k-1}.\end{aligned} \tag{23}$$

Rearranging equality (23) results in:

$$\Delta V_{k+1} = (\gamma_{ii}{}^2 - 1)\left[(1+\lambda_{ii}) s_{i,k} - s_{i,k-1}\right]^2. \tag{24}$$

Since the convergence rate matrix $\gamma = Diag(\gamma_{ii}) \in \mathbb{R}^{m\times m}$ is defined such that $0 < \gamma_{ii} < 1$, it leads to $\Delta V_{k+1} < 0$ that indicates stability of the 2nd-order SVSF under the corrective gain (9). Since the Lyapunov function V_k is in terms of S_k and ΔS_k, it can be deduced from equation (24) with $\Delta V_{k+1} < 0$ that convergence is attained for the first and second order sliding conditions. □

It is important to note that due to modeling uncertainties, noise, and switching imperfections, however the ideal second order sliding motion does not occur. This results in a real second order sliding mode regime in which the BIBO stability of the dynamic 2nd-order SVSF is ensured given bounded noise and modeling uncertainties. Satisfaction of the Lyapunov function (14) leads to: $|\sigma_{k+1}| < |\sigma_k|$. Since $\sigma_k = \Delta S_k + C\, S_k$, where $S_k = e_{z_{k|k}}$ and $\Delta S_k = \Delta e_{z_{k|k}}$, it is deduced that the measurement error and its difference are decreasing over time while $\sigma_k > \varepsilon_\sigma$. Due to measurement noise and modeling uncertainties, σ_k only decreases until it reaches the existence subspace bounded ε_σ. However, under an ideal sliding condition: $\sigma_k = 0$. The corrective gain (9) is a linear combination of the *a priori* and the *a posteriori* measurement error terms. It is a second-order Markov process and causes that the dynamic 2nd-order SVSF updates the *a posteriori* error $e_{z_{k+1|k+1}}$ based on the available information of $e_{z_{k|k}}$ and $e_{z_{k-1|k-1}}$. Having access to higher amount of information increases smoothness and robustness of the filter for state estimation.

COMPARATIVE ANALYSIS USING AN ELECTRO-HYDROSTATIC ACTUATOR (EHA) SYSTEM

In order to study the performance of the dynamic 2nd-order SVSF for state estimation, it is applied to an EHA model (as presented in Figure 2). Its performance is then compared to other estimation methods such as the Kalman filter, and the 1st-order SVSF. Two scenarios are considered for comparisons that are the normal condition with a known model but including white noise, and a faulty condition with a large degree of modeling uncertainties. The EHA system is described by a discrete third-order model. The three state variables include the actuator position $x_1 = x$, velocity $x_2 = dx_1/dt$, and acceleration $x_3 = d^2x_1/dt^2$, with position being the only measurable state [7]. The linear state and measurement model of the EHA are given by (1) and (2), respectively. Numeric values of the state, control and measurement matrixes are equal to [7]:

$$\hat{F} = \begin{bmatrix} 1 & 0.001 & 0 \\ 0 & 1 & 0.001 \\ -557.02 & -28.616 & 0.9418 \end{bmatrix}, \quad \hat{G} = \begin{bmatrix} 0 \\ 0 \\ 557.02 \end{bmatrix}, \quad \hat{H} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}. \qquad (25)$$

w_k and v_k denote the process uncertainties and measurement noise. They are multivariate white normal random vectors with the mean of zero and standard deviation vectors equal to [7]:

$$w_{std} = \begin{bmatrix} 0.05 & 0.1 & 0.1 \end{bmatrix}^T, \quad v_{std} = \begin{bmatrix} 0.05 \end{bmatrix}. \qquad (26)$$

In order to apply the dynamic 2nd-order SVSF to states that are not measured directly, it is combined with the Luenberger's observer [7]. In simulation, the corrective gain is calculated for the case with the convergence rate equal to $\gamma = [0.5]$. In simulation, it is assumed that the initial state error covariance for the Kalman filter and the dynamic 2nd-order SVSF are equal. For both strategies, the process noise, measurement noise and the initial error covariance are respectively obtained as: $Q = diag([1 \quad 10 \quad 100])$, and $P_0 = 20Q$. Furthermore, $R = 0.1cm^2$ is obtained by calculating variance of the innovation signal for a time period. For the 1st-order SVSF [7], the width of the smoothing boundary layer is set to $\psi = \begin{bmatrix} 5 & 5 & 5 \end{bmatrix}^T \times v_{std}$, where v_{std} is the standard deviation of the measurement noise. To compare the robustness characteristic of these three methods, a large degree of uncertainties is injected into the model by changing the state matrix after 0.5 sec of simulation to [7]:

$$\hat{F}_2 = \begin{bmatrix} 1 & 0.001 & 0 \\ 0 & 1 & 0.001 \\ -240 & -28 & 0.9418 \end{bmatrix}. \qquad (27)$$

The input to the EHA system is a random signal with the amplitude in the range of -1 to 1, superimposed on a step input that occurs at 0.5 sec. The initial values of states are assumed to be zero and the sampling time for discretization is 0.001 sec. Simulations are performed using the MATLAB and under the 10^3 Monte-Carlo runs. Tables 1 to 3 compare a number of numerical performance indicators generated from the three estimation methods for the above mentioned normal and uncertain EHA models.

In order to compare these estimators, their RMSE, as well as the bias and STD of their state estimation error are calculated and compared. The RMSE indicator is calculated as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \hat{x}_i)^2}{n}}, \qquad (28)$$

where x_i denotes the actual state value, $\hat{x}_i$ denotes the estimated state value, and n is the number of time steps. The actual state values are obtained by solving state trajectories of the EHA system with state matrices. Furthermore, the bias and the STD of the state estimation error are obtained as follows:

$$Bias = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{x}_i). \qquad (29)$$

$$STD = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(e_{x,i} - \bar{e}_{x,i})^2}. \qquad (30)$$

Figure 2. The electro-hydrostatic actuator (EHA) prototype [7]

Table 1 presents the root mean squared error (RMSE) value of the state estimation error $e_{x_{k|k}}$ for both normal and uncertain conditions. Further to Table 1, the Kalman filter produces the most accurate state estimates in terms of the RMSE for the normal model, followed by the dynamic 2nd-order SVSF and the 1st-order SVSF. This is because for a known model the Kalman filter is optimal in terms of the RMSE. Under the uncertain case, the dynamic 2nd-order SVSF produces more accurate state estimates in terms of the RMSE. This accuracy is due to preserving the first and second order sliding conditions that increases its robustness versus modeling uncertainties.

Table 1: Comparison between RMSE indices of the estimation methods

	Kalman Filter		1st-order SVSF		Dynamic 2nd-SVSF	
RMSE	**Normal**	**Uncertain**	**Normal**	**Uncertain**	**Normal**	**Uncertain**
Position	0.010	0.018	0.011	0.016	0.011	0.014
Velocity	1.046	34.660	1.061	19.507	1.055	12.494
Accel.	160.240	2206.06	172.31	1471.53	163.91	1315.18

Table 2: Comparison between Bias indices of the estimation methods

	Kalman Filter		1st-order SVSF		Dynamic 2nd-SVSF	
Bias	**Normal**	**Uncertain**	**Normal**	**Uncertain**	**Normal**	**Uncertain**
Position	-1.5×10^{-4}	-7.3×10^{-3}	-2.6×10^{-4}	-4.5×10^{-4}	-1.7×10^{-4}	-2.9×10^{-4}
Velocity	-0.0019	9.63	-0.0048	3.78	-0.0027	1.63
Accel.	9.84	36.32	10.04	26.86	9.98	18.76

Table 3: Comparison between STD indices of the estimation methods

	Kalman Filter		1st-order SVSF		Dynamic 2nd-SVSF	
STD	**Normal**	**Uncertain**	**Normal**	**Uncertain**	**Normal**	**Uncertain**
Position	0.0963	0.30	0.0105	0.0709	0.0075	0.0358
Velocity	1.59	30.29	1.22	17.95	1.10	12.63
Accel.	195.55	2867.9	186.16	1823.9	161.37	1374.60

Note that satisfying the second order sliding condition instead of using the smoothing boundary layer is the main reason why the dynamic 2nd-order SVSF is more accurate than the 1st-order SVSF for both normal and uncertain cases. In the 1st-order SVSF chattering is alleviated by defining a smoothing boundary layer in a vicinity of the sliding hyperplane. In this context, the signum function is replaced with a smoother function such as the saturation function. This however approximates the sliding motion in a close vicinity of the sliding hyperplane and reduces the ultimate accuracy and robustness of the SVSF-type filtering. The second order sliding condition not only removes the need for approximation, but also alleviates higher degrees of chattering.

Table 2 compares state estimates in terms of the bias (mean of the state estimation error) for both the normal and uncertain conditions. Table 3 compares the state estimates in terms of the STD of the state estimation error. For the normal case, the Kalman filter produces the smallest bias, followed by the dynamic 2nd-order SVSF and the 1st-order SVSF. But for the uncertain case, the dynamic 2nd-order SVSF generates the smallest bias, followed by the 1st-order SVSF and the Kalman filter. Following Table 3, the dynamic 2nd-order SVSF has the smallest values pertaining to the STD of the state estimation error $e_{x,k|k}$. The Kalman filter has the best performance in the normal case with no uncertainties and Gaussian noise, followed by the dynamic 2nd-order SVSF. For the case with uncertainties, the dynamic 2nd-order SVSF has the best performance, followed by the 1st-order SVSF.

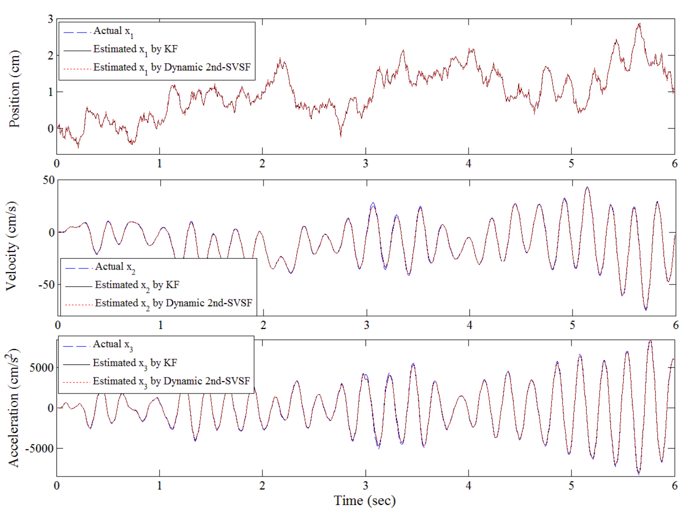

Figure 3. State estimation using three estimators for normal EHA system

Figure 3 presents the actual and estimated state trajectories using the Kalman filter and the dynamic 2nd-order SVSF for the EHA under the normal condition. Figure 4 compares these trajectories using the Kalman filter and the dynamic 2nd-order SVSF for the EHA with modeling uncertainties. The position's estimation error signals obtained from the dynamic 2nd-order SVSF and the Kalman filter are presented in Figure 5. It is deduced from Figure 5 that the dynamic 2nd-order SVSF produces the smoothest state estimates with the smallest variation for both normal and uncertain cases.

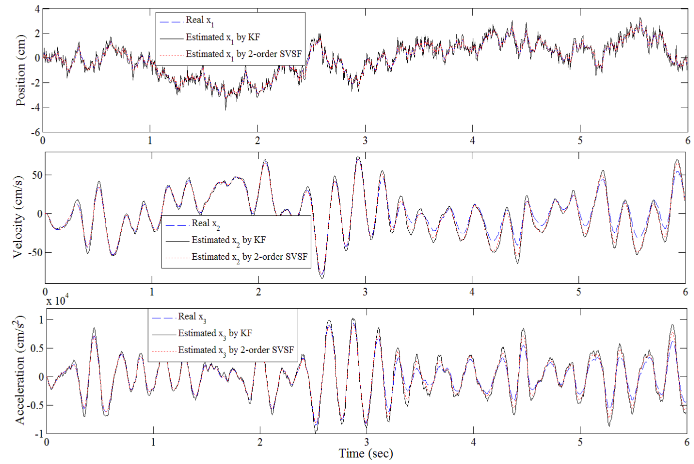

Figure 4. State estimation using three estimators for uncertain EHA system

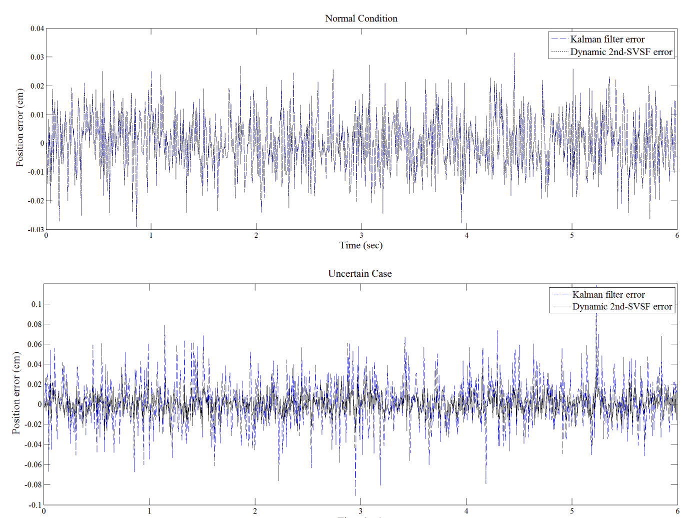

Figure 5. Position estimation error by Kalman filter and dynamic 2nd-SVSF

Figure 6 presents the phase portrait of the measurement error and its first difference for the normal and uncertain EHA systems using the dynamic 2nd-order SVSF. As demonstrated, in both cases the measurement error and its difference are decreasing in time until they reach the existence subspace. Figure 7 also presents profiles of the sliding variable s and the dynamic sliding manifold σ for both the normal and uncertain cases. In both cases, σ is decreasing in time until it reaches the existence subspace such that $|\sigma| \le \varepsilon_{\sigma}$. Figures 6 and 7 illustrate convergence of the dynamic 2nd-order SVSF under the dynamic sliding manifold given bounded noise and uncertainties.

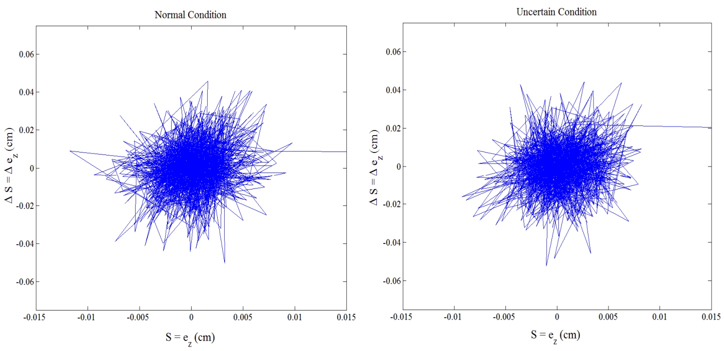

Figure 6. Phase portrait of the position error and its first difference generated by the dynamic 2nd-SVSF method

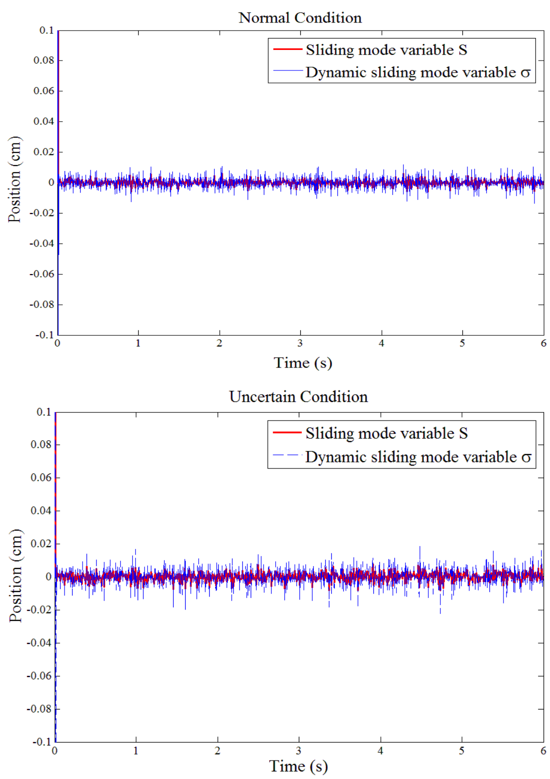

Figure 7. Profiles of the sliding mode variable and the dynamic sliding manifold generated by the dynamic 2nd-SVSF method

CONCLUSION

This paper introduces the dynamic 2nd-order SVSF state estimation method based on defining a linear dynamic sliding mode manifold. This manifold is defined in terms of the sliding variable and its first difference, where the sliding variable represents the *a posteriori* measurement error (innovation). Hence, by reaching the dynamic sliding manifold, the first and the second order sliding mode conditions are satisfied. The corrective gain of the dynamic 2nd-order SVSF is obtained such that it ensures reaching the sliding mode manifold in a finite time. The Lyapunov's second law of stability is then used in order to prove the stability and convergence of the dynamic 2nd-order SVSF method under the proposed corrective gain. The linear sliding manifold introduces a cut-off frequency coefficient matrix into the filter formulation that alleviates the undesirable chattering effect. In order to compare accuracy, robustness, and smoothness of the dynamic 2nd-order SVSF method, it applies to an EHA system under the normal and faulty scenarios. Simulation results are then compared with other state estimation methods such as the Kalman filter and the 1st-order SVSF method in terms of RMSE, Bias, and standard Deviation of the estimation error. Simulation results demonstrate that under the normal condition, the Kalman filter produces the most accurate state estimates with smallest RMSE. Besides, under the uncertain condition, the dynamic 2nd-order SVSF produces the most accurate estimates with smallest STD and bias, followed by the 1st-order SVSF. This confirms the superior performance of the dynamic 2nd-order SVSF for state estimation under uncertain faulty situations.

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