Two-Pass Smoother Based on the SVSF Estimation Strategy

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ABSTRACT

The smooth variable structure filter (SVSF) has seen significant development and research activity in recent years. It is based on sliding mode concepts, which utilizes a switching gain that brings an inherent amount of stability to the estimation process. In this paper, the SVSF is reformulated to present a two-pass smoother based on the SVSF gain. The proposed method is applied on an aerospace flight surface actuator, and the results are compared with the popular Kalman-based two-pass smoother.

Keywords: Minimum-variance, smoother, Kalman filter, smooth variable structure filter, aerospace actuator

1. INTRODUCTION

State and parameter estimation theory is an important field in mechanical and electrical engineering. As the name suggests, estimation strategies are used to predict, estimate, or smooth out important system state and parameters [1, 2]. Consider a simple example, a spring-damper-mass system. In order to accurately control and understand the dynamics of the system, an engineer or scientist must have an accurate representation of the spring constant, damper value, and system mass. If these values are not known with a degree of certainty, then the system will be modeled incorrectly and the dynamics will lead to instability or system failure [3, 4]. Estimation strategies are used to identify these state and parameter values. Filters use measurements and system information taken at time *t*, to estimate state values at time *t*. However, smoothers estimate the state of a system a time *t* using information before and after time *t*. The accuracy of a smoother is generally better than that of a filter since it makes use of more information for its estimate. As per [5, 6], there are three types of smoothers: fixed-interval, fixed-point, and fixed-lag. Fixed-interval smoothers are often used offline, and use all the measurements over a fixed interval to estimate the system states throughout the entire interval. Fixed-point smoothers estimate the state at a fixed time in the past. Fixed-lag smoothers estimate states at a fixed time interval at some point behind the current measurement (hence, lag).

The most popular estimation strategy was developed nearly 60 years ago, and is referred to as the Kalman filter (KF) [7]. The KF yields a statistically optimal solution to the linear estimation problem. The goal of the KF is to minimize the state error covariance, which is a measure of the estimation accuracy and is defined as the expectation of the state error squared [7]. The state error is defined as the difference between the true state value and the estimation state value. Although the KF yields a solution for linear estimation problems, it is based on a few strict assumptions: the system and measurement models must be known, the noise distribution is Gaussian, and the behavior is linear [4, 6]. If any of these assumptions are not held by the actual system, then the KF may yield inaccurate or unstable estimation results [8, 9]. Other KF-based solutions have been presented to overcome issues with nonlinearity, and include: the perturbation KF, the extended KF (EKF), the unscented KF (UKF), and the cubature KF (CKF) [8, 10, 11, 12]. Essentially, these solutions attempt to linearize or approximate the nonlinearities, with increasing degrees of complexity. For example, the EKF is a first-order Taylor series approximate, the UKF is equivalent to a second-order approximation, and the CKF is equivalent to a third-order approximation [8]. Although these methodologies are distinctly different and offer improved estimation accuracy, they still fall victim to modeling uncertainties and external, unwanted disturbances—which is often the case in real-world scenarios and estimation problems.

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Over the years, estimation strategies such as the H-infinity filter and the smooth variable structure filter (SVSF) have been introduced that can overcome modeling uncertainties and errors [13, 14, 15, 16, 17]. However, a trade-off often exists between accuracy and robustness. The SVSF was introduced in 2007, and was considered to be a sub-optimal filter albeit stable and robust [16]. It is based on sliding mode concepts that yields a switching-gain. This gain brings an inherent amount of stability to the estimation process, as the estimates are forced towards the true state trajectory. Improvements were made on this format and newer forms of the SVSF were introduced, and included covariance derivations, multiple-model formulations, a time-varying boundary layer solution, and combined KF-based derivations [8, 9, 11, 18]. A number of applications were considered, including target tracking, fault detection and diagnosis, system control, and basic state and parameter estimation examples [8]. Although the SVSF has shown significant improvement since its introduction, it remains a sub-optimal filter and has room for advancement.

This paper proposes a new type of smoother, based on two-pass smoothing. It is derived conceptually similar to the two-pass smoother based on the KF; the main difference is found in the derivation of the gain. The goal of this is to provide a smoother with improved robustness to modeling uncertainties. The paper is organized as follows. The Kalman filter (KF) and smooth variable structure filter (SVSF) and their equations are summarized in section 2. The two-pass formulation of the KF smoother is introduced in section 3. The new two-pass smoother based on the SVSF is then introduced and summarized. In section 4, the aerospace actuator scenario is described. The results of implementing the two-pass KF-based smoother and two-pass SVSF-based smoother are shown and compared. The paper is then concluded and future work is described.

2. ESTIMATION STRATEGIES

The Kalman Filter

As mentioned earlier, the KF yields an optimal solution to the linear estimation problem [19]. It is formulated as a predictor-corrector, and is computed recursively. The following equations form the core of the KF algorithm, and are well published in literature [7, 1, 3]. The prediction stage consists of two equations. The a priori state estimate $\hat{x}_{k+1|k}$ is first calculated based on knowledge of the system F and previous state estimate $\hat{x}_{k|k}$. The corresponding state error covariance matrix $P_{k+1|k}$ is then calculated.

$$\hat{x}_{k+1|k} = F\hat{x}_{k|k} + Gu_k \tag{2.1.1}$$

$$P_{k+1|k} = FP_{k|k}F^T + Q_k \tag{2.1.2}$$

The next stage, referred to as the corrector or update stage, includes three equations, as follows. The predicted state-error covariance matrix $P_{k+1|k}$ is used in conjunction with the measurement model H and the measurement covariance matrix R_{k+1} to calculate the KF gain K_{k+1}. The inverse term found in the gain formulation is referred to as the innovation covariance S_{k+1}.

$$K_{k+1} = P_{k+1|k}H^T\left(HP_{k+1|k}H^T + R_{k+1}\right)^{-1} \tag{2.1.3}$$

The Kalman gain K_{k+1}, measurement z_{k+1}, and predicted state estimate $\hat{x}_{k+1|k}$, are used to update the state estimate $\hat{x}_{k+1|k+1}$.

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}\left(z_{k+1} - H\hat{x}_{k+1|k}\right) \tag{2.1.4}$$

The updated state error covariance matrix $P_{k+1|k+1}$ is then calculated based on the gain, the predicted state error covariance matrix, and the measurement covariance matrix.

$$P_{k+1|k+1} = (I - K_{k+1}H)P_{k+1|k}(I - K_{k+1}H)^T + K_{k+1}R_{k+1}K_{k+1}^T \tag{2.1.5}$$

The above five equations summarize the KF algorithm, and are used in an iterative fashion. The state estimates and covariances are initialized based on system and designer knowledge.

As mentioned earlier, the extended Kalman filter (EKF) is a natural extension of the KF method. The EKF formulation is used for nonlinear systems and measurements. Nonlinear system or measurement equations may be linearized according to its Jacobian, or its first-order Taylor series approximation. The partial derivatives are used to compute linearized system and measurement matrices F and H, respectively [20].

$$F_k = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}_{k|k},u_k} \tag{2.1.6}$$

$$H_{k+1} = \left.\frac{\partial h}{\partial x}\right|_{\hat{x}_{k+1|k}} \tag{2.1.7}$$

Equations (2.1.6) and (2.1.7) essentially linearize the nonlinear system or measurement functions around the current state estimate [1]. These matrices can then be used in place of the linear system and measurement matrices used by the KF. This comes at a loss of optimality; as such, the EKF yields a suboptimal solution to the nonlinear estimation problem [3]. However, for mildly-nonlinear estimation problems, the EKF provides a good solution.

The Smooth Variable Structure Filter

The SVSF is a model based approach, and may be applied on differentiable linear or nonlinear dynamic systems. The original form of the SVSF was presented in 2007, and did not include a covariance derivation. It has been shown to be stable and robust to bounded disturbances, modeling uncertainties and noise [14, 15]. An augmented form of the SVSF was presented in [9, 8], which proposed a strategy for obtaining an error covariance matrix for the filter. The estimation process is iterative and may be summarized by the following set of equations. The predicted state estimates $\hat{x}_{k+1|k}$ and the error covariance matrix $P_{k+1|k}$ are first calculated as per the KF strategy. Utilizing the predicted state estimates $\hat{x}_{k+1|k}$, the predicted measurements $\hat{z}_{k+1|k}$ and the measurement errors $e_{z,k+1|k}$ may be calculated, respectively as per follows:

$$\hat{z}_{k+1|k} = H\hat{x}_{k+1|k} \tag{2.2.1}$$

$$e_{z,k+1|k} = z_{k+1} - \hat{z}_{k+1|k} \tag{2.2.2}$$

The SVSF gain is calculated next as per (2.2.3), and is a function of: the a priori and the a posteriori measurement errors $e_{z,k+1|k}$ and $e_{z,k|k}$; the smoothing boundary layer widths ψ; the SVSF 'memory' or convergence rate γ; as well as the measurement matrix C. A complete explanation on how the gain K_{k+1} is derived is found in [16, 9]. The SVSF gain is defined as a diagonal matrix such that [8]:

$$K_{k+1} = C^{+} diag\left[\left(\left|e_{z_{k+1|k}}\right| + \gamma\left|e_{z_{k|k}}\right|\right) \circ sat\left(\bar{\psi}^{-1} e_{z_{k+1|k}}\right)\right] diag\left(e_{z_{k+1|k}}\right)^{-1} \tag{2.2.3}$$

The smoothing boundary layer term $\bar{\psi}$ in (2.2.3) is defined as:

$$\bar{\psi}^{-1} = \begin{bmatrix} \frac{1}{\psi_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{1}{\psi_m} \end{bmatrix} \tag{2.2.4}$$

where m is the number of measurements. This gain is used to calculate the updated state estimates $\hat{x}_{k+1|k+1}$ as well as the updated state error covariance matrix $P_{k+1|k+1}$, as per the KF strategy. Finally, the updated measurement estimate $\hat{z}_{k+1|k+1}$ and measurement errors $e_{z,k+1|k+1}$ are calculated, respectively as follows:

$$\hat{z}_{k+1|k+1} = C\hat{x}_{k+1|k+1} \tag{2.2.5}$$

$$e_{z,k+1|k+1} = z_{k+1} - \hat{z}_{k+1|k+1} \tag{2.2.6}$$

The above equations summarize the SVSF algorithm, and are used in an iterative fashion. The basic estimation concept of the SVSF is shown in the following figure. The SVSF process results in the state estimates converging to within a region of the state trajectory [16, 8]. Thereafter, it switches back and forth across the state trajectory within a region referred to as the existence subspace. This switching effect brings about an inherent amount of stability and robustness in the estimation process.

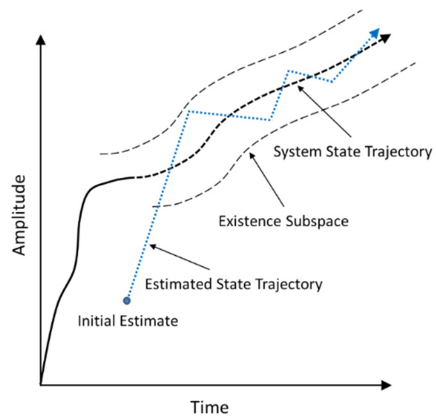
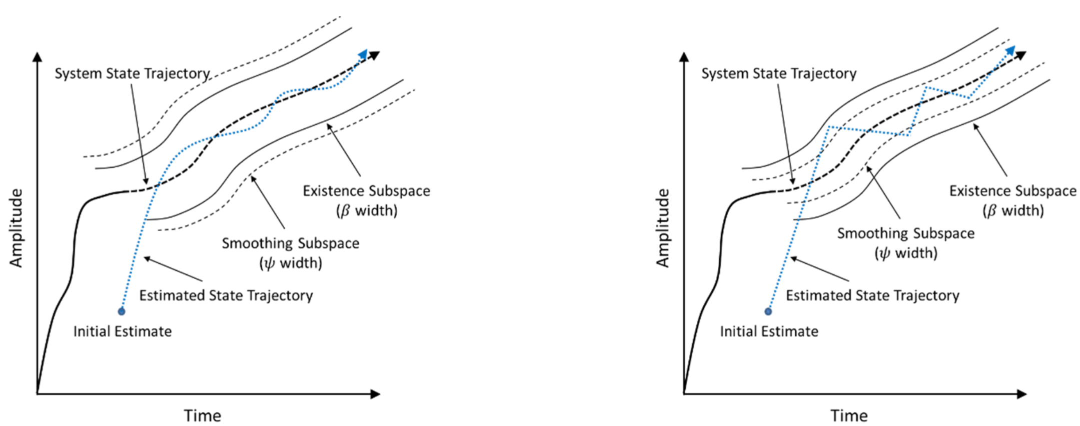


Figure 1. The above figure illustrates the SVSF estimation concept [8].

Further details of the existence subspace are shown in Figs. 2a and 2b. The width represents the amount of uncertainties present in the estimation process, in terms of modeling errors or the presence of noise. As per [8], the width of the existence space β is a function of the uncertain dynamics associated with the inaccuracy of the internal model of the filter as well as the measurement model, and varies with time [16]. Once within the existence boundary subspace, the estimated states are forced (by the SVSF gain) to switch back and forth along the true state trajectory. High-frequency switching caused by the SVSF gain is referred to as chattering, and in most cases, is undesirable for obtaining accurate estimates [16]. However, the effects of chattering may be minimized by the introduction of a smoothing boundary layer ψ. The selection of the smoothing boundary layer width reflects the level of uncertainties in the filter and the disturbances. The effect of the smoothing boundary layer is shown in Fig. 2. When the smoothing boundary layer is defined larger than the existence subspace boundary, the estimated state trajectory is smoothed. However, when the smoothing term is too small, chattering remains due to the uncertainties being underestimated.



(a) Well-defined boundary layer (no chattering) (b) Poorly-defined boundary layer (chattering)

Figure 2. The above figures further illustrate the smoothing boundary layer concept [8].

3. FIXED-INTERVAL, TWO-PASS SMOOTHERS

The Fixed-Interval, Two-Pass Smoother

Smoothers (whether fixed-interval, -point, or -lag) may be derived from the KF model. In general, as per [2], the common methodology uses the KF for measurements up to each time step that the state needs to be estimated, combined with another algorithm. The second algorithm can be derived based on running the KF backward from the last measurement, to the measurement just past time *t*. The two independent estimates (forward and backward) can then be combined [2].

The two-pass smoother, also known as the RTS (Rauch-Tung-Striebel) smoother, is a popular type of smoother [1]. The standard KF estimate and covariance are computed in a forward pass, and the smoothed quantities are then computed in a backward pass [1]. The forward pass is similar to the standard KF, written as follows for completeness:

$$\hat{x}_{k|k-1} = F\hat{x}_{k-1|k-1} + Gu_k \tag{3.1.1}$$

$$P_{k|k-1} = FP_{k-1|k-1}F^T + Q_k \tag{3.1.2}$$

$$K_k = P_{k|k-1}H^T\left(HP_{k|k-1}H^T + R_k\right)^{-1} \tag{3.1.3}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k\left(z_k - H\hat{x}_{k|k-1}\right) \tag{3.1.4}$$

$$P_{k|k} = (I - K_kH)P_{k|k-1}(I - K_kH)^T + K_kR_kK_k^T \tag{3.1.5}$$

The backward pass is then performed using the state estimate and covariance values. The smoothed state estimate is first calculated as follows [21]:

$$\hat{x}_{k|n} = \hat{x}_{k|k} + A_k\left(\hat{x}_{k+1|n} - \hat{x}_{k+1|k}\right) \tag{3.1.6}$$

The supporting matrices are defined by [21]:

$$A_k = P_{k|k-1}\overline{F}_k^T P_{k+1|k}^T \tag{3.1.7}$$

$$\overline{F}_k = F - K_kH \tag{3.1.8}$$

The smoothed state error covariance becomes [21]:

$$P_{k|n} = P_{k|k} + A_k\left(P_{k+1|n} - P_{k+1|k}\right)A_k^T \tag{3.1.9}$$

Equations (3.1.1) through (3.1.9) summarize the two-pass smoother or the RTS algorithm. The forward pass includes the KF estimation strategy, and the backward pass computes the smoothed quantities based on the available system information and measurements. The process is computed iteratively.

The Fixed-Interval, SVSF-Based Smoother

This paper introduces the fixed-interval formulation of the SVSF-based smoother, hereafter referred as to the variable structure smoother (VSS). It is based on the same approach as the RTS or two-pass smoother. The forward pass is essentially the SVSF estimation process, and is listed here for completeness:

$$\hat{x}_{k|k-1} = F\hat{x}_{k-1|k-1} + Gu_k \tag{3.2.1}$$

$$P_{k|k-1} = FP_{k-1|k-1}F^T + Q_k \tag{3.2.2}$$

$$e_{z,k|k-1} = z_k - H\hat{x}_{k|k-1} \tag{3.2.3}$$

$$K_k = C^+ diag\left[\left(\left|e_{z_{k|k-1}}\right| + \gamma\left|e_{z_{k-1|k-1}}\right|\right) \circ sat\left(\bar{\psi}^{-1}e_{z_{k|k-1}}\right)\right] diag\left(e_{z_{k|k-1}}\right)^{-1} \tag{3.2.4}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k\left(z_k - H\hat{x}_{k|k-1}\right) \tag{3.2.5}$$

$$P_{k|k} = (I - K_k H)P_{k|k-1}(I - K_k H)^T + K_k R_k K_k^T \tag{3.2.6}$$

$$e_{z,k|k} = z_k - H\hat{x}_{k|k} \tag{3.2.7}$$

The backward pass is then performed using the state estimate and covariance values. The smoothed state estimate is first calculated as follows [21]:

$$\hat{x}_{k|n} = \hat{x}_{k|k} + A_k\left(\hat{x}_{k+1|n} - \hat{x}_{k+1|k}\right) \tag{3.2.8}$$

The supporting matrices are defined by [21]:

$$A_k = P_{k|k-1}\overline{F}_k^T P_{k+1|k}^T \tag{3.2.9}$$

$$\overline{F}_k = F - K_k H \tag{3.2.10}$$

The smoothed state error covariance becomes [21]:

$$P_{k|n} = P_{k|k} + A_k\left(P_{k+1|n} - P_{k+1|k}\right)A_k^T \tag{3.2.11}$$

Equations (3.2.1) through (3.2.11) summarize the two-pass SVSF-based smoother or the variable structure smoother (VSS) algorithm. The forward pass includes the SVSF estimation strategy with a state error covariance matrix, and the backward pass computes the smoothed quantities based on the available system information and measurements. The process is computed iteratively.

4. COMPUTER EXPERIMENTS

Linear Aerospace Actuator

In this section, an electrohydrostatic actuator (EHA) is described. It is important to note that smoothers are typically used in applications that can have offline calculations or some estimation delay. However, this example uses computer simulations in order to allow a detailed investigation of the effects of the smoothers and parametric uncertainties. The EHA model is based on an actual prototype built for experimentation, and is commonly used as an aerospace flight surface actuator [16, 15]. The EHA has been modeled as a third-order linear system with state variables related to its position, velocity, and acceleration [16]. Initially, it is assumed that all three states have measurements associated with them (i.e., $C = I$). The sample time of the system is $T = 0.001\ s$, and the discrete-time state space system equation may be defined as follows [16]:

$$x_{k+1} = \begin{bmatrix} 1 & 0.001 & 0 \\ 0 & 1 & 0.001 \\ -557.02 & -28.616 & 0.9418 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0 \\ 557.02 \end{bmatrix} u_k \tag{4.1}$$

For this case, the corresponding measurement equation is defined by:

$$z_{k+1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x_{k+1} \tag{4.2}$$

The initial state values are set to zero. The system and measurement noises (w and v) are considered to be Gaussian, with zero mean and variances Q and R, respectively. The initial state error covariance $P_{0|0}$, system noise covariance Q, and measurement noise covariance R are defined respectively as follows:

$$P_{0|0} = 10Q \tag{4.3}$$

$$Q = \begin{bmatrix} 1\times 10^{-5} & 0 & 0 \\ 0 & 1\times 10^{-3} & 0 \\ 0 & 0 & 1\times 10^{-1} \end{bmatrix} \tag{4.4}$$

$$R = \begin{bmatrix} 1\times 10^{-4} & 0 & 0 \\ 0 & 1\times 10^{-2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4.5}$$

As per [22], for the standard SVSF estimation process, the 'memory' or convergence rate was set to $\gamma = 0.1$, and the limits for the smoothing boundary layer widths (diagonal elements) were defined as $\psi = [0.05 \quad 0.5 \quad 5]^T$. These parameters were selected based on the distribution of the system and measurement noises. For example, the limit for the smoothing boundary layer width ψ was set to 5 times the maximum system noise, or approximately equal to the measurement noise. The initial state estimates for the filters were defined randomly by a normal distribution, around the true initial state values x_0 and using the initial state error covariance $P_{0|0}$. Two different cases were studied in this section. The first case was considered 'normal', and the second included system modeling error half-way through the simulation. A total of 500 Monte Carlo runs were performed, and the results were averaged.

Simulation Results

The following three figures show the result of applying the Kalman-based smoother (labelled as KS) and the variable structure smoother (labelled as VSS). Both the KS and VSS were able to smooth out the kinematic states. However, the KS performed slightly better in terms of accuracy, as expected. The position RMSE for the KS and VSS were $0.0019\ m$ and $0.0023\ m$, respectively. The velocity RMSE for the KS and VSS were $0.0216\ m/s$ and $0.0269\ m/s$, respectively. Finally, the acceleration RMSE was closer, and was found for the KS and VSS as $0.3199\ m/s^2$ and $0.3202\ m/s^2$, respectively.

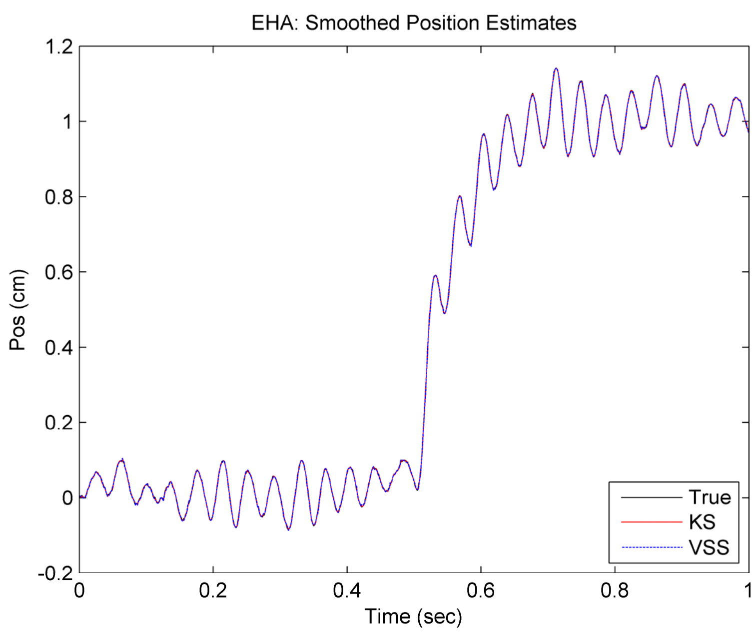


Figure 3. Smoothed position estimates of the EHA by the KS and VSS.

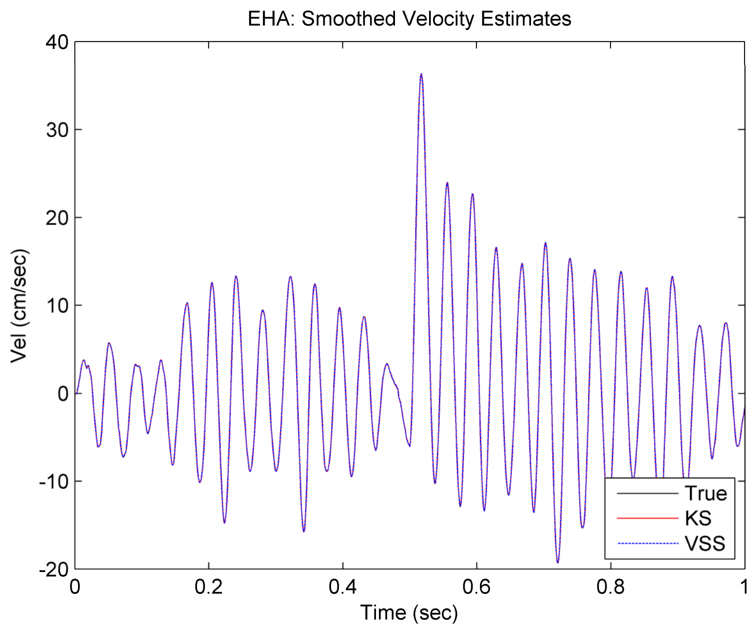


Figure 4. Smoothed velocity estimates of the EHA by the KS and VSS.

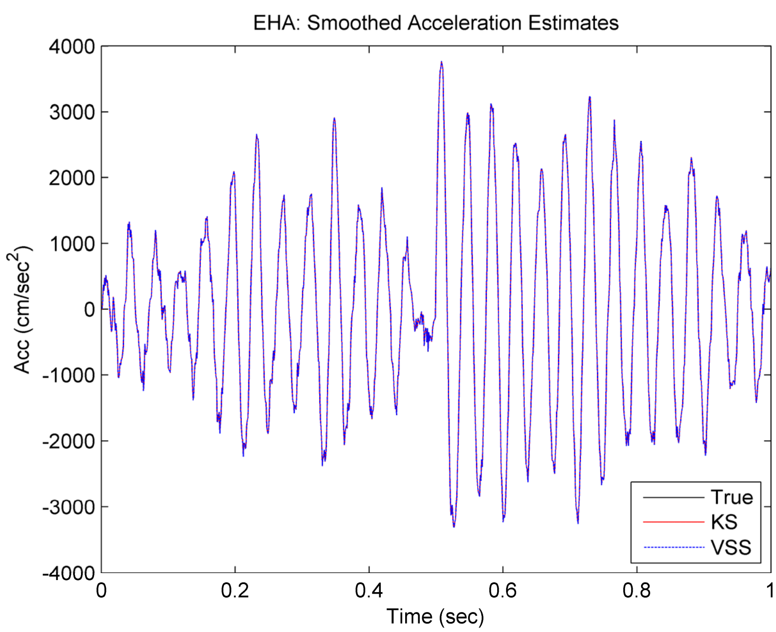


Figure 5. Smoothed acceleration estimates of the EHA by the KS and VSS.

As per [16], consider the introduction of modeling error or uncertainty, such that the system used by the smoothers is modified (4.6) at 0.5 seconds. The model changes at this point to coincide with the input step, to exaggerate the effects of modeling uncertainty. The following three figures show the result of applying the KS and VSS on the system with modeling uncertainty half-way through the simulation. The KS failed to yield a good position estimate, however was able to follow (somewhat) the velocity and acceleration trajectories. The VSS was able to overcome the uncertainties and yielded accurate and robust state estimates.

$$x_{k+1} = \begin{bmatrix} 1 & 0.001 & 0 \\ 0 & 1 & 0.001 \\ -240 & -28 & 0.9418 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0 \\ 557.02 \end{bmatrix} u_k \tag{4.6}$$

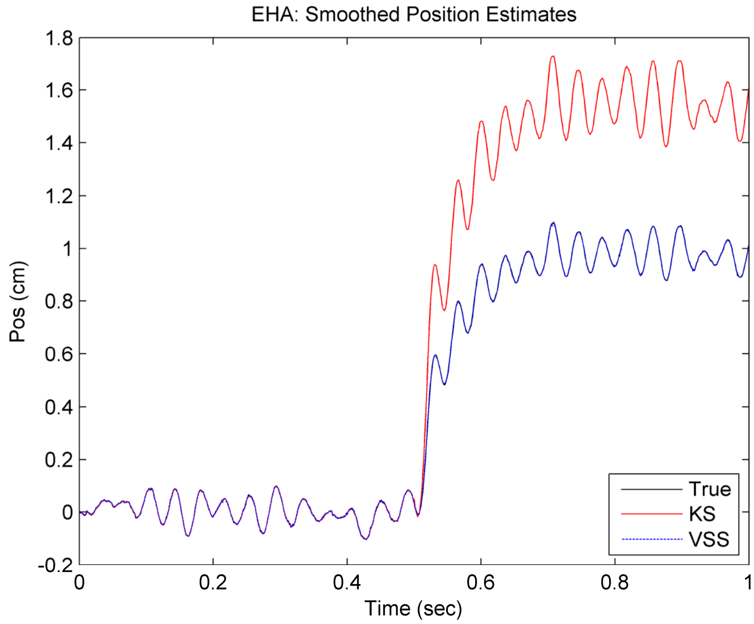


Figure 6. Smoothed position estimates of the EHA by the KS and VSS (with uncertainties).

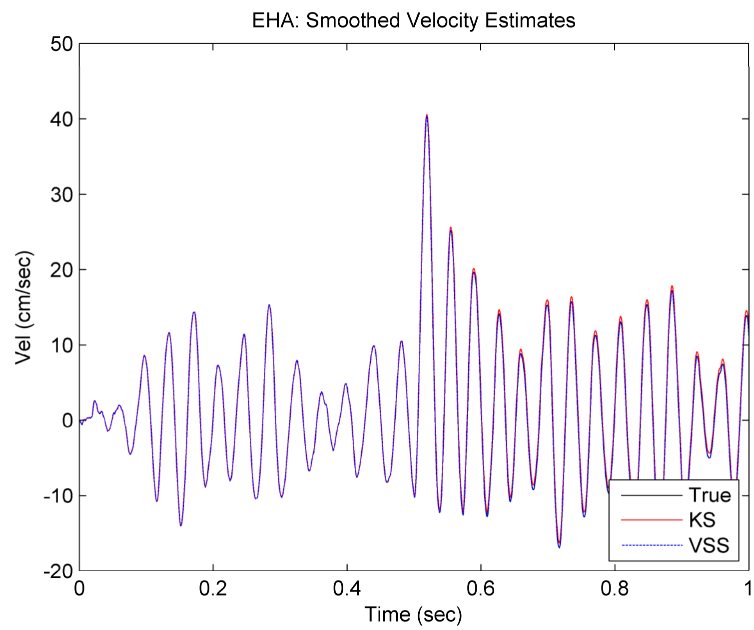


Figure 7. Smoothed velocity estimates of the EHA by the KS and VSS (with uncertainties).

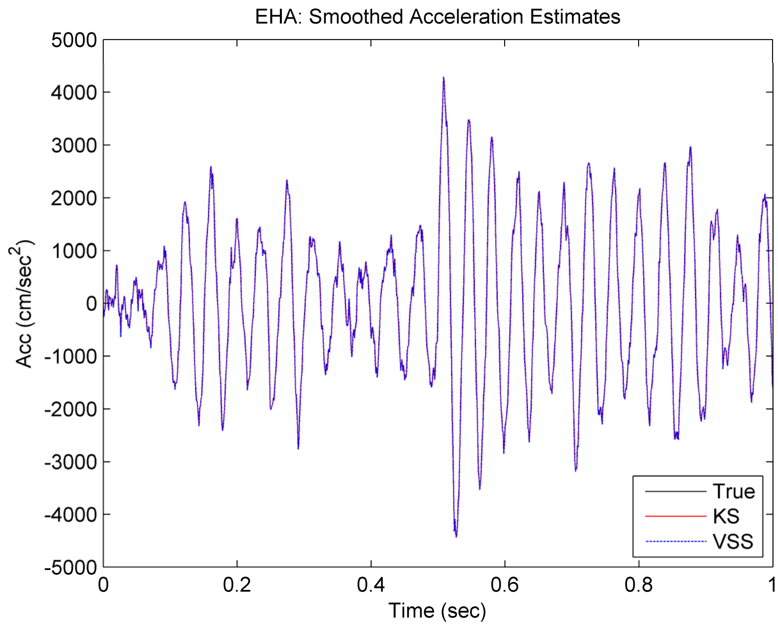


Figure 8. Smoothed acceleration estimates of the EHA by the KS and VSS (with uncertainties).

5. CONCLUSIONS

This paper introduced a two-pass, SVSF-based smoother for the purpose of state and parameter estimation. The proposed algorithm was applied to a linear flight surface actuator, and was compared with the popular KF-based smoother. For the computer experiment, under normal conditions, both the two-pass smoother and the proposed variable structure smoother (VSS) performed well. During the presence of modeling uncertainties, the VSS was able to overcome inaccuracies and yield a stable solution. Future work will look at implementing the other types of smoothers, and will include applications to real-world data and measurements.

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