The Cubature Smooth Variable Structure Filter Estimation Strategy Applied to a Quadrotor Controller

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ABSTRACT

Unmanned aerial systems (UAS) are becoming increasingly popular in industry, military, and social environments. An UAS that provides good operating performance and robustness to disturbances is often quite expensive and prohibitive to the general public. To improve UAS performance without affecting the overall cost, an estimation strategy can be implemented on the internal controller. The use of an estimation strategy or filter reduces the number of required sensors and power requirement, and improves the controller performance. UAS devices are highly nonlinear, and implementation of filters can be quite challenging. This paper presents the implementation of the relatively new cubature smooth variable structure filter (CSVSF) on a quadrotor controller. The results are compared with other state and parameter estimation strategies.

Keywords: Kalman filter, smooth variable structure filter, estimation strategies, quadrotor dynamics

1. INTRODUCTION

Although state and parameter estimation techniques have been in use for a long period of time, these strategies are growing significantly in popularity. This may be attributed to the development of industrial applications and processes, as well as other military and social environments. For example, robotic arms can replace humans during dangerous missions with improved operating performance. Furthermore, these devices can be used for repetitive and other difficult tasks. Most industries aim to improve overall efficiency and reduce production costs. Estimation techniques may be implemented in industrial applications in an effort to provide more accurate information in a timely manner. For example, estimation strategies can reduce sensor noise and estimate non-measurable states. This information may be implemented in the design and industrial process.

The field of estimation is extremely old, and is proposed to have started when estimates and predictions were made based on observations [1]. As per [2], 'estimation theory' was founded by Galileo in 1632, while Gauss in 1795 added the method of least squares to the estimation family [1]. In 1942, Wiener implemented his filter (Wiener Filter – WF) as a least square method that provided the first explicit solution for a stochastic process [3, 4, 5]. Kalman presented his filter, conveniently named the Kalman filter (KF), as a least square method implemented in a predictor-corrector form for linear system applications to compensate for white noise [6, 7].

Since the 1960s, researchers have been attempting to improve filter performance in terms of optimality, robustness, and stability [8, 9]. A number of estimation strategies exist, and include: the extended Kalman filter (EKF) [10, 11], the iterated extended Kalman filter (IKF) [12], the higher-order extended Kalman filter (HOEKF) [13], the sigma-point Kalman filter (SPKF) including the unscented Kalman filter (UKF) [14], the particle filter (PF) [4], Slotine's sliding mode observer [15], Walcott's sliding mode observer [16], Edward's sliding mode observer [17], the variable structure filter (VSF) [18], the extended variable structure filter (EVSF) [19], and the smooth variable structure filter (SVSF) [20, 21, 22]. These filters have been modified and/or combined to improve their performance for a variety of linear and nonlinear system and measurement applications.

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In this paper, a relatively new estimation strategy is applied on a quadrotor controller in an attempt to improve the overall performance and robustness. The combined cubature Kalman and smooth variable structure filter strategy (CSVSF) is applied, and the results are compared with the standard Kalman filter (KF) and the smooth variable structure filter (SVSF). The paper is organized as follows. The KF, CKF, and SVSF and their equations are summarized in section 2. The cubature smooth variable structure filter (CSVSF) is overviewed in section 3. In section 4, the quad-rotor scenario is described. The results of implementing the CSVSF are shown and compared with the standard CKF and SVSF. The paper is then concluded and future work is described.

2. ESTIMATION STRATEGIES

The Kalman Filter

The following equations form the core of the Kalman filter (KF) algorithm, and are used in an iterative fashion. Equations (2.1.1) and (2.1.2) define the a priori state estimate $\hat{x}_{k+1|k}$ based on knowledge of the system F and previous state estimate $\hat{x}_{k|k}$, and the corresponding state error covariance matrix $P_{k+1|k}$, respectively.

$$\hat{x}_{k+1|k} = f\left(\hat{x}_{k|k}, u_k\right) \tag{2.1.1}$$

$$P_{k+1|k} = FP_{k|k}F^T + Q_k \tag{2.1.2}$$

The Kalman gain K_{k+1} is defined by (2.1.3), and is used to update the state estimate $\hat{x}_{k+1|k+1}$ as shown in (2.1.4). The gain makes use of an innovation covariance S_{k+1}, which is defined as the inverse term found in (2.1.3).

$$K_{k+1} = P_{k+1|k}H^T\left(HP_{k+1|k}H^T + R_{k+1}\right)^{-1} \tag{2.1.3}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}\left(z_{k+1} - H\hat{x}_{k+1|k}\right) \tag{2.1.4}$$

The a posteriori state error covariance matrix $P_{k+1|k+1}$ is then calculated by (2.1.5), and is used iteratively, as per (2.1.2).

$$P_{k+1|k+1} = (I - K_{k+1}H)P_{k+1|k}(I - K_{k+1}H)^T + K_{k+1}R_{k+1}K_{k+1}^T \tag{2.1.5}$$

The derivation of the KF is well documented, with details available in [6, 3, 8]. The extended Kalman filter (EKF) is a natural extension of the KF method. However, the EKF may be used for nonlinear systems and measurements, unlike the KF. Nonlinear system or measurement equations may be linearized according to its Jacobian. The partial derivatives are used to compute linearized system and measurement matrices F and H, respectively found as follows [23]:

$$F_k = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}_{k|k}, u_k} \tag{2.1.6}$$

$$H_{k+1} = \left.\frac{\partial h}{\partial x}\right|_{\hat{x}_{k+1|k}} \tag{2.1.7}$$

Equations (2.1.6) and (2.1.7) essentially linearize the nonlinear system or measurement functions around the current state estimate [3].

The Cubature Kalman Filter

The cubature Kalman filter (CKF) was introduced in an effort to further improve upon the estimation accuracy; however, the strategy is still prone to failure caused by modeling uncertainties and disturbances [24]. The strategy uses cubature points to estimate nonlinearities present in the estimation process. The initial set of cubature points X are calculated based on the previous a posteriori state estimate $\hat{x}_{k|k}$, the previous a posteriori state covariance $P_{k|k}$, and the cubature-point set ξ_i:

$$X_{i,k|k} = \sqrt{P_{k|k}}\xi_i + \hat{x}_{k|k} \qquad i = 1,2,\dots,2n \tag{2.2.1}$$

These cubature points are then propagated through the system equation, as follows:

$$X^*_{i,k+1|k} = f(X_{i,k|k}, u_k) \qquad i = 1,2,\dots,2n \tag{2.2.2}$$

Next, the predicted state $\hat{x}_{k+1|k}$ and predicted state error covariance $P_{k+1|k}$ are calculated, respectively:

$$\hat{x}_{k+1|k} = \frac{1}{2n}\sum_{i=1}^{2n} X^*_{i,k+1|k} \tag{2.2.3}$$

$$P_{k+1|k} = \frac{1}{2n}\sum_{i=1}^{2n} X^*_{i,k+1|k}X^{*T}_{i,k+1|k} - \hat{x}_{k+1|k}\hat{x}^T_{k+1|k} + Q_{k+1} \tag{2.2.4}$$

The predicted cubature points $X_{i,k+1|k}$ are then evaluated based on the predicted stated $\hat{x}_{k+1|k}$ and predicted state error covariance $P_{k+1|k}$:

$$X_{i,k+1|k} = \sqrt{P_{k+1|k}}\xi_i + \hat{x}_{k+1|k} \qquad i = 1,2,\dots,2n \tag{2.2.5}$$

The predicted cubature points $X_{i,k+1|k}$ are then propagated through the measurements $Z_{i,k+1|k}$, and the corresponding predicted measurement is calculated $\hat{z}_{k+1|k}$, respectively as follows:

$$Z_{i,k+1|k} = h(X_{i,k+1|k}, u_{k+1}) \qquad i = 1,2,\dots,2n \tag{2.2.6}$$

$$\hat{z}_{k+1|k} = \frac{1}{2n}\sum_{i=1}^{2n} Z_{i,k+1|k} \tag{2.2.7}$$

In order to calculate the corresponding cubature Kalman gain W_{k+1}, the innovation covariance $P_{zz,k+1|k}$ and cross-covariance $P_{xz,k+1|k}$ matrices need to be evaluated, respectively as follows:

$$P_{zz,k+1|k} = \frac{1}{2n}\sum_{i=1}^{2n} Z_{i,k+1|k}Z^T_{i,k+1|k} - \hat{z}_{k+1|k}\hat{z}^T_{k+1|k} + R_{k+1} \tag{2.2.8}$$

$$P_{xz,k+1|k} = \frac{1}{2n}\sum_{i=1}^{2n} X_{i,k+1|k}Z^T_{i,k+1|k} - \hat{x}_{k+1|k}\hat{z}^T_{k+1|k} \tag{2.2.9}$$

The CKF gain may now be calculated as follows:

$$W_{k+1} = P_{xz,k+1|k}P^{-1}_{zz,k+1|k} \tag{2.2.10}$$

Finally, the updated states $\hat{x}_{k+1|k+1}$ and corresponding error covariance $P_{k+1|k+1}$ may be found:

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + W_{k+1}(z_{k+1} - \hat{z}_{k+1|k}) \tag{2.2.11}$$

$$P_{k+1|k+1} = P_{k+1|k} + W_{k+1}P_{zz,k+1|k}W^T_{k+1} \tag{2.2.12}$$

The CKF process may be summarized by the previous equations, and are repeated iteratively.

The Smooth Variable Structure Filter

The SVSF estimation process is iterative and may be summarized by the following set of equations. The predicted state estimates $\hat{x}_{k+1|k}$ and the error covariance matrix $P_{k+1|k}$ are first calculated as per the KF strategy. Utilizing the predicted state estimates $\hat{x}_{k+1|k}$, the predicted measurements $\hat{z}_{k+1|k}$, and the measurement errors $e_{z,k+1|k}$ may be calculated by (2.3.1) and (2.3.2) respectively.

$$\hat{z}_{k+1|k} = H\hat{x}_{k+1|k} \tag{2.3.1}$$

$$e_{z,k+1|k} = z_{k+1} - \hat{z}_{k+1|k} \tag{2.3.2}$$

The SVSF gain is a function of: the a priori and the a posteriori measurement errors $e_{z,k+1|k}$ and $e_{z,k|k}$; the smoothing boundary layer widths ψ; the SVSF 'memory' or convergence rate γ; as well as the measurement matrix C. Refer to [20, 21] for a complete explanation on how the gain K_{k+1} is derived. The SVSF gain is defined as a diagonal matrix such that [7]:

$$K_{k+1} = C^{+}diag\left[\left(\left|e_{z_{k+1|k}}\right| + \gamma\left|e_{z_{k|k}}\right|\right) \circ sat\left(\bar{\psi}^{-1}e_{z_{k+1|k}}\right)\right] diag\left(e_{z_{k+1|k}}\right)^{-1} \tag{2.3.3}$$

The smoothing boundary layer term $\bar{\psi}$ in (2.3.3) is defined as:

$$\bar{\psi}^{-1} = \begin{bmatrix} \frac{1}{\psi_1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{1}{\psi_m} \end{bmatrix} \tag{2.3.4}$$

where m is the number of measurements. This gain is used to calculate the updated state estimates $\hat{x}_{k+1|k+1}$ as well as the updated state error covariance matrix $P_{k+1|k+1}$, as per the KF strategy. Finally, the updated measurement estimate $\hat{z}_{k+1|k+1}$ and measurement errors $e_{z,k+1|k+1}$ are calculated, and are used in later iterations:

$$\hat{z}_{k+1|k+1} = C\hat{x}_{k+1|k+1} \tag{2.3.5}$$

$$e_{z,k+1|k+1} = z_{k+1} - \hat{z}_{k+1|k+1} \tag{2.3.6}$$

The SVSF process results in the state estimates converging to within a region of the state trajectory [20, 7]. Thereafter, it switches back and forth across the state trajectory within a region referred to as the existence subspace [7]. This switching effect brings about an inherent amount of stability and robustness in the estimation process, as will be demonstrated in the simulation [25].

3. CUBATURE SMOOTH VARIABLE STRUCTURE FILTER

This paper utilizes the cubature smooth variable structure filter (CSVSF) in an effort to yield accurate state estimates while maintaining robustness to modeling errors and external disturbances. Two SVSF filters will be used as in Fig. 1. The first filter is the primary one and iteratively estimates the states. That is done using a smoothing boundary layer with zero width. After that, the CKF is used in a secondary loop to refine the estimates by a time-varying smoothing boundary layer that was previously derived in [26] as:

$$\mathbf{\Psi}_{tv_k} = \left(\mathbf{P}_{\mathbf{zz}_{1,k|k-1}} \circ \mathbf{I}_{nxn}\right)\left(\left(\mathbf{P}_{\mathbf{zz}_{1,k|k-1}} - \mathbf{R}_k\right) \circ \mathbf{I}_{nxn}\right)^{-1}\left|\mathbf{e}_{\mathbf{z},\mathbf{1}_{k|k-1}}\right| \tag{3.1}$$

where $\mathbf{e}_{\mathbf{z},\mathbf{1}_{k|k-1}}$ and $\mathbf{P}_{\mathbf{zz}_{1,k|k-1}}$ are the a priori output's estimation error for the first filter and its covariance matrix, respectively. The latter is obtained from equation 2.2.8. According to Fig. 1, the CKF is used to obtain the boundary layer and it is not recursive. That guarantees the system stability and reduces the computational time. Moreover, it solves the inversion operator ill conditions.

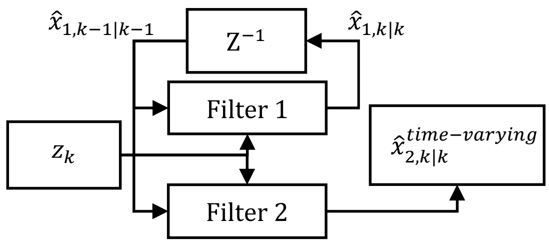

Figure 1. The CKF-SVSF II structure [26].

4. COMPUTER EXPERIMENTS

Quadroter Dynamics

Quadroters (also referred to as quad-copters) consist of four rotors in a cross-configuration. Each rotor produces an upward thrust against its own weight. Figure 2 shows the (x, y, z) frame attached to the body B of a quadrotor. The frame defines the Euler angles φ, θ, ψ of the UAV. The fixed (X, Y, Z) Earth frame E is used to define the global position and orientation of the flying vehicle.

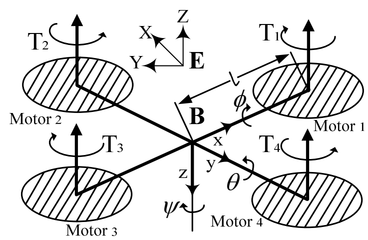

Figure 2. Schematic diagram of a quadrotor [27]

Reference [26] presented a detailed nonlinear dynamics model of the RAVEN Quadrotor. The authors of this work have developed a discrete-time dynamics model for the same platform as described below.

$$\begin{bmatrix} x_{k+1} \\ \dot{x}_{k+1} \\ y_{k+1} \\ \dot{y}_{k+1} \\ z_{k+1} \\ \dot{z}_{k+1} \\ \phi_{k+1} \\ p_{k+1} \\ \theta_{k+1} \\ q_{k+1} \\ \psi_{k+1} \\ r_{k+1} \end{bmatrix} = \begin{bmatrix} x_k + T\dot{x}_k \\ \dot{x}_k + \frac{T}{m}(S\phi_k C\psi_k - C\phi_k S\theta_k S\psi_k)u_k \\ y_k + T\dot{y}_k \\ \dot{y}_k - \frac{T}{m}(S\phi_k S\psi_k + C\phi_k S\theta_k C\psi_k)u_k \\ z_k + T\dot{z}_k \\ \dot{z}_k + \frac{T}{m}(C\phi_k C\theta_k)u_k - Tg \\ \phi_k + T\big(p_k + \theta_k(q_k S\phi_k + r_k C\phi_k)\big) \\ p_k + T\big(q_k r_k \alpha_1 - \alpha_2 q_k + \alpha_3 \delta_{roll_k}\big) \\ \theta_k + T(q_k C\phi_k - r_k S\phi_k) \\ q_k + T\big(p_k r_k \alpha_4 + \alpha_5 r_k + \alpha_6 \delta_{pitch_k}\big) \\ \psi_k + \frac{T(q_k S\phi_k + r_k C\phi_k)}{C\theta_k} \\ r_k + T\big(p_k q_k \alpha_7 + \alpha_8 \delta_{yaw_k}\big) \end{bmatrix} + V_k + v_{k+1} \quad (4.1)$$

where ($\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$), ($\dot{x}$, $\dot{y}$ and $\dot{z}$) are the body's angular rates and linear velocities respectively, expressed relative to the fixed frame. (p, q and r) are the body's angular rates expressed relative to the body frame. u_k, δ_{roll}, δ_{pitch} and δ_{yaw} are system inputs.

As per [28], to replace unavailable real flight-test data, the authors deployed the developed discrete time model in a dynamics simulation run and generated simulated motion states and their derivatives of a general quadrotor. The values of the parameters necessary for the dynamics simulation were provided by [26]. To mimic the expected noise in sensor measurements, an artificial white additive noise was introduced. The noise signals have maximum values of about 5% of the corresponding states' actual values. The covariance matrix has a size of 12 × 12, so it will not be listed here. However, the diagonal elements of that matrix have the following values:

$$R_{diag} = \begin{bmatrix} 8\times10^{-4}, 8{,}6\times10^{-4}, 7{,}7\times10^{-4}, 8{,}8\times10^{-4}, \\ 7\times10^{-12}, 8\times10^{-4}, 8\times10^{-12}, 6\times10^{-4}, 6\times10^{-12} \end{bmatrix} \quad (4.2)$$

The input values used for the quadrotor simulation are summarized in the following series of figures.

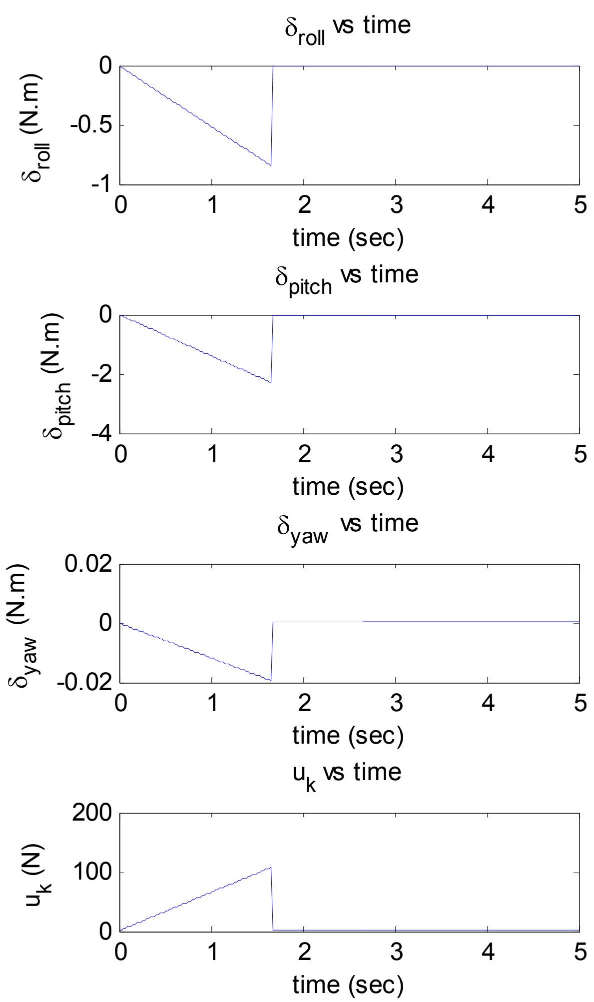

Figure 3. Input values for the quadrotor simulation [28].

Simulation Results

This section describes the results of implementing the three estimation strategies (CKF, SVSF, CSVSF) on the quadrotor dynamics. The following series of figures show the important state estimates.

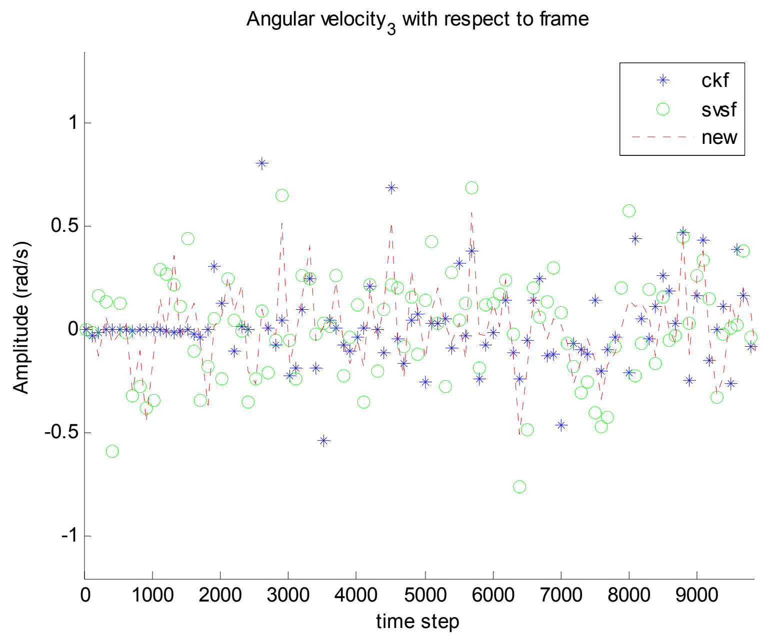

Figure 4. Estimated roll angular velocity with the respect to the frame (no modeling error).

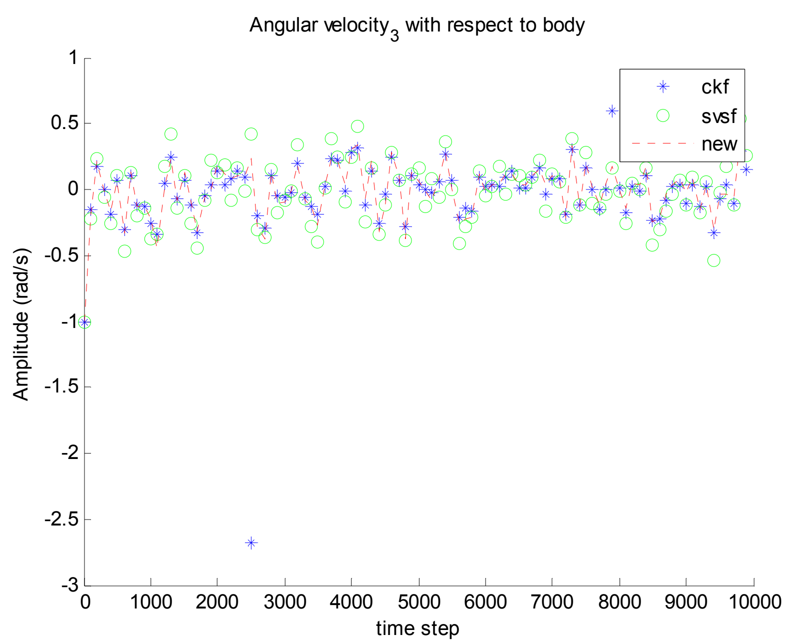

Figure 5. Estimated roll angular velocity with the respect to the body (no modeling error).

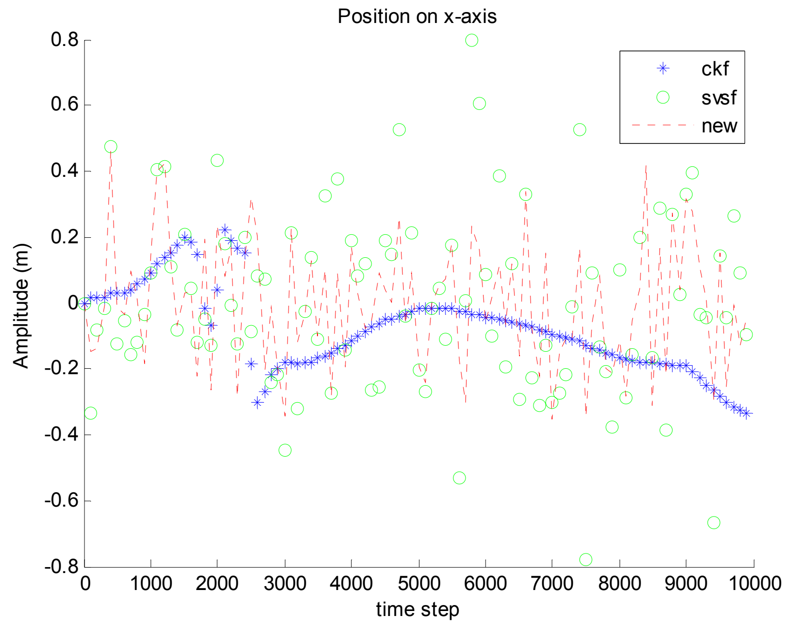

Figure 6. Estimated values for the x-position.

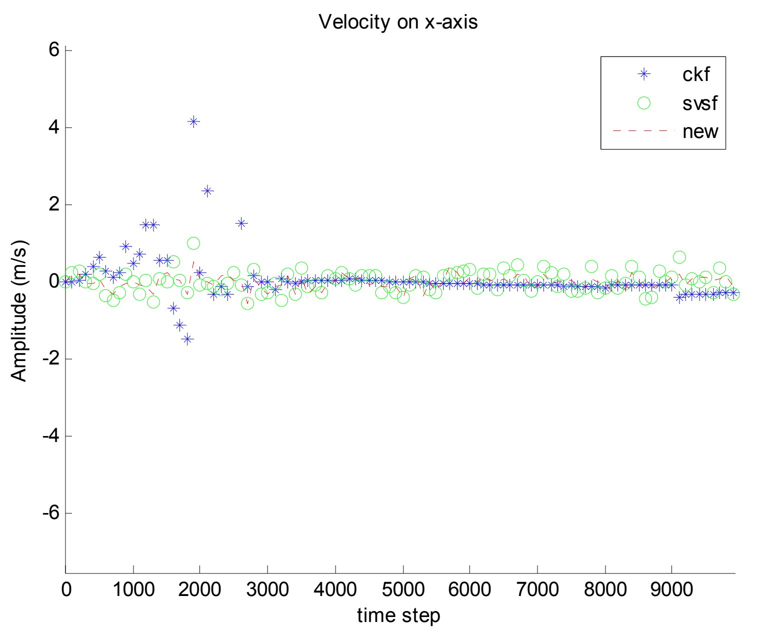

Figure 7. Estimated values for the x-velocity.

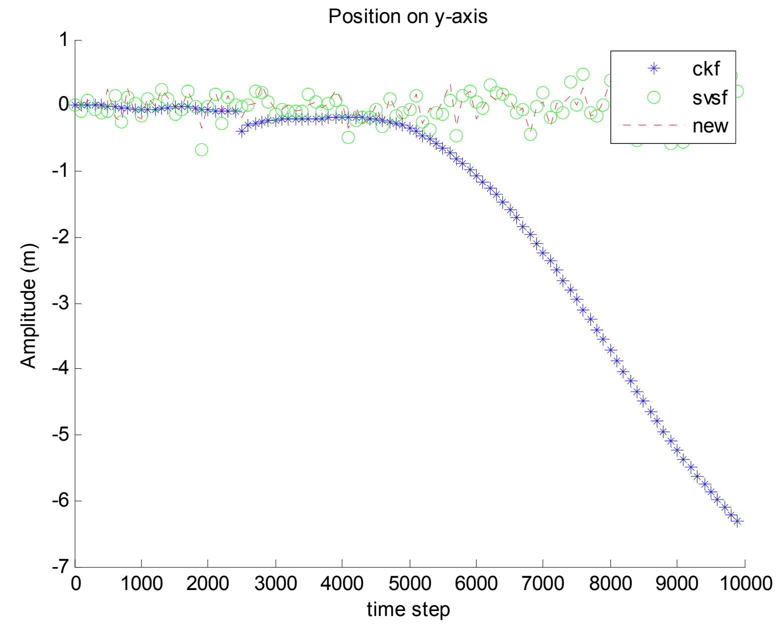

Figure 8. Estimated values for the y-position.

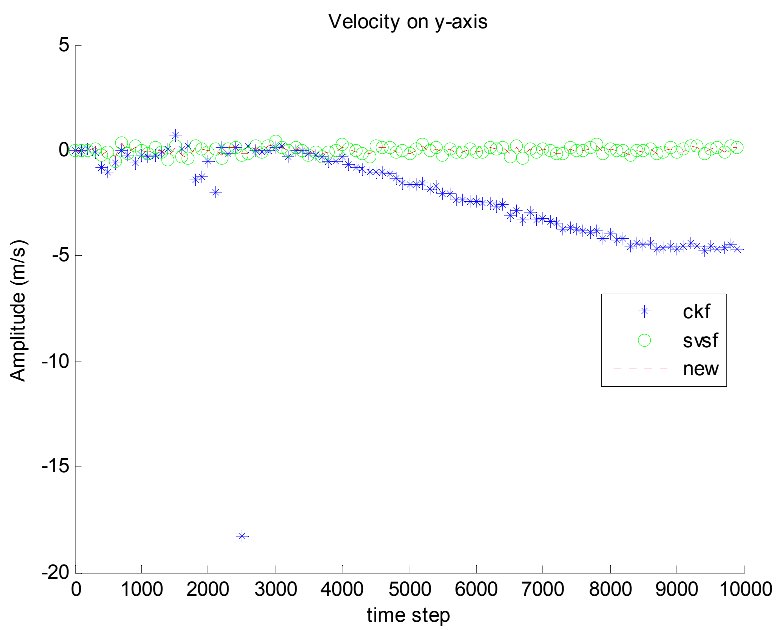

Figure 9. Estimated values for the y-velocity.

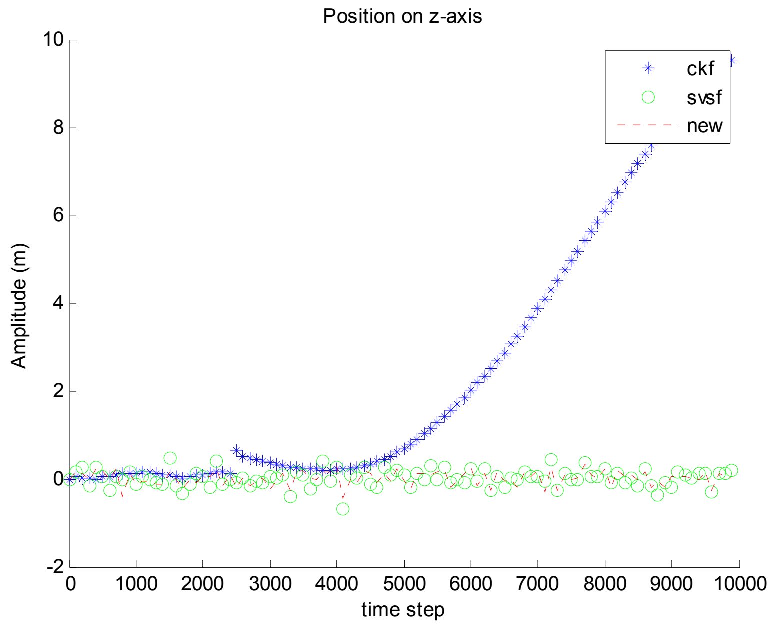

Figure 10. Estimated values for the z-position.

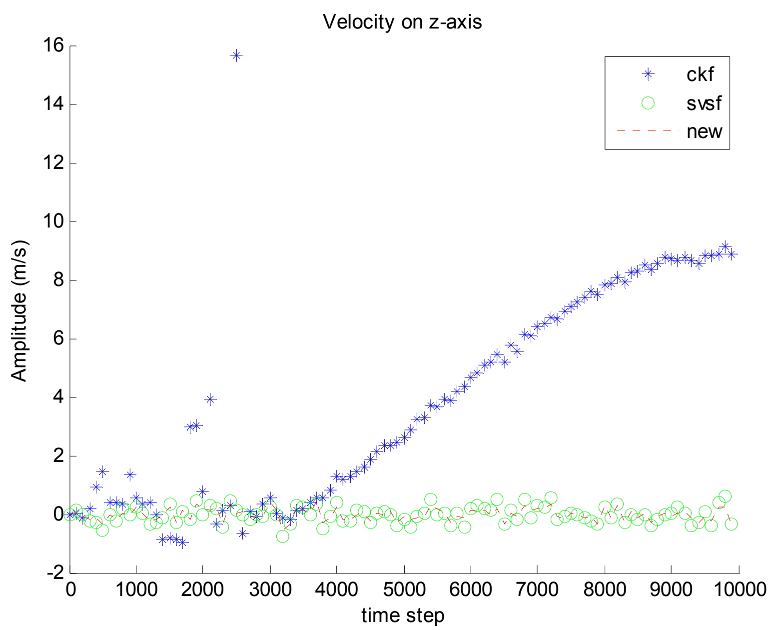

Figure 11. Estimated values for the z-velocity.

As demonstrated by the simulation results, the CKF yielded good estimates for some of the states. However, at the presence of modeling uncertainties, the CKF failed to yield correct estimates and diverged from the true state trajectory. The SVSF was able to overcome the modeling uncertainties. The combined strategy, referred to as the CSVSF yielded improved estimation accuracy and robustness to external disturbances and modeling uncertainties.

5. CONCLUSIONS

To improve UAS performance without affecting the overall cost, an estimation strategy can be implemented on the internal controller. This paper presented the implementation of the relatively new cubature smooth variable structure filter (CSVSF) on a quadrotor controller. The results were compared with the standard CKF and SVSF. It was found that the CKF yielded good estimation results when the model is well known and without the presence of external disturbances. The SVSF yielded sub-optimal estimation results however was robust and stable. The combined strategy was both accurate and stable, making it an ideal estimation strategy for quadrotors and other nonlinear systems. Future work includes implementing the CSVSF on real-life data and benchmark problems.

REFERENCES

[1] T. Kailath, "A View of Three Decades of Linear Filtering Theory," *IEEE Transactions on Information Theory,* vol. 20, no. 2, pp. 146-181, 1974.

[2] M. Al-Shabi, "The General Toeplitz/Observability SVSF," Hamilton, Ontario, 2011.

[3] B. D. O. Anderson and J. B. Moore, Optimal Filtering, Englewood Cliffs, NJ: Prentice-Hall, 1979.

[4] Y. Bar-Shalom, X. Rong Li and T. Kirubarajan, Estimation with Applications to Tracking and Navigation, New York: John Wiley and Sons, Inc., 2001.

[5] Y. Chan, A. Hu and J. Plant, "A Kalman Filter Based Tracking Scheme with Input Estimation," *IEEE Transactions on Aerospace and Electronic Systems,* vol. 15, pp. 237-244, 1979.

[6] R. E. Kalman, "A New Approach to Linear Filtering and Prediction Problems," *Journal of Basic Engineering, Transactions of ASME,* vol. 82, pp. 35-45, 1960.

[7] S. A. Gadsden, "Smooth Variable Structure Filtering: Theory and Applications," Ph.D. Thesis, McMaster University, Hamilton, Ontario, 2011.

[8] A. Gelb, Applied Optimal Estimation, Cambridge, MA: MIT Press, 1974.

[9] M. S. Grewal and A. P. Andrews, Kalman Filtering: Theory and Practice Using MATLAB, 3 ed., New York: John Wiley and Sons, Inc., 2008.

[10] D. Simon, Optimal State Estimation: Kalman, H-Infinity, and Nonlinear Approaches, Wiley-Interscience, 2006.

[11] G. Welch and G. Bishop, "An Introduction to the Kalman Filter," 2006.

[12] J. Hyland, "An Iterated-Extended Kalman Filter Algorithm for Tracking Surface and Sub-Surface Targets," in *Oceans Conference and Exhibition*, 2002.

[13] D. Bayard and B. Kang, "A High-Order Kalman Filter for Focal Plane Calibration for NASA's Space Infrared Telescope Facility (SIRTF)," in *AIAA Guidance, Navigation and Control Conference*, 2003.

[14] S. J. Julier, J. K. Ulhmann and H. F. Durrant-Whyte, "A New Method for Nonlinear Transformation of Means and Covariances in Filters and Estimators," *IEEE Transactions on Automatic Control,* vol. 45, pp. 472-482, March 2000.

[15] J. Slotine, J. Hedrick and E. Misawa, "On Sliding Observers for Nonlinear Systems," *ASME Journal of Dynamic Systems, Measurement and Control,* vol. 109, no. 3, pp. 245-252, 1987.

[16] B. Walcott and S. Zak, "State Observation of Nonlinear Uncertain Dynamical Systems," *IEEE Transactions on Automatic Control,* vol. 32, no. 2, pp. 166-170, 1987.

[17] C. Edwards, S. Spurgeon and R. Patton, "Sliding Mode Observers for Fault Detections," *Automatica,* vol. 36, pp. 541-553, 2000.

[18] S. R. Habibi and R. Burton, "The Variable Structure Filter," *Journal of Dynamic Systems, Measurement, and Control (ASME),* vol. 125, pp. 287-293, September 2003.

[19] S. R. Habibi and R. Burton, "Parameter Identification for a High Performance Hydrostatic Actuation System using the Variable Structure Filter Concept," *ASME Journal of Dynamic Systems, Measurement, and Control,* 2007.

[20] S. R. Habibi, "The Smooth Variable Structure Filter," *Proceedings of the IEEE,* vol. 95, no. 5, pp. 1026-1059, 2007.

[21] S. A. Gadsden and S. R. Habibi, "A New Form of the Smooth Variable Structure Filter with a Covariance Derivation," in *IEEE Conference on Decision and Control*, Atlanta, Georgia, 2010.

[22] S. A. Gadsden, S. R. Habibi and T. Kirubarajan, "Kalman and Smooth Variable Structure Filters for Robust Estimation," *IEEE Transactions on Aerospace and Electronic Systems,* vol. 50, no. 2, 2014.

[23] S. Haykin, Kalman Filtering and Neural Networks, New York: John Wiley and Sons, Inc., 2001.

[24] S. A. Gadsden, M. Al-Shabi, I. Arasaratnam and S. R. Habibi, "Combined Cubature Kalman and Smooth Variable Structure Filtering: A Robust Estimation Strategy," *Signal Processing,* vol. 96, no. B, 2014.

[25] S. A. Gadsden, S. R. Habibi and T. Kirubarajan, "A Novel Interacting Multiple Model Method for Target Tracking," in *13th International Conference on Information Fusion*, Edinburgh, UK, 2010.

[26] M. Al-Shabi, K. Hatamleh and A. Asad, "UAV dynamics model parameters estimation techniques: A comparison study," in *Applied Electrical Engineering and Computing Technologies (AEECT)*, Amman, 2013.

[27] K. Shojaie, K. Ahmadi and A. Shahri, "Effects of Iteration in Kalman Filter Family for Improvement of Estimation Accuracy in Simultaneous Localization and Mapping," in *IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, Zurich, 2007.

[28] J. P. e. a. How, "Real-time indoor autonomous vehicle test environment," *Control Systems,* vol. 28, no. 2, pp. 51-64, 2008.