

A Multi-Target Tracking Formulation of SVSF with the Joint Probabilistic Data Association Technique

Mina Attari

Centre for Mechatronics and
Hybrid Technology, McMaster
University, Hamilton, Canada
attarim@mcmaster.ca

S. Andrew Gadsden

Centre for Mechatronics and
Hybrid Technology, McMaster
University, Hamilton, Canada
gadsden@mcmaster.ca

Saeid R. Habibi

Centre for Mechatronics and
Hybrid Technology, McMaster
University, Hamilton, Canada
habibi@mcmaster.ca

ABSTRACT

Target tracking scenarios offer an interesting challenge for state and parameter estimation techniques. This paper studies a situation with multiple targets in the presence of clutter. In this paper, the relatively new smooth variable structure filter (SVSF) is combined with the joint probability data association (JPDA) technique. This new method, referred to as the JPDA-SVSF, is applied on a simple multi-target tracking problem for a proof of concept. The results are compared with the popular Kalman filter (KF).

I. INTRODUCTION

The purpose of multiple target tracking is to maintain true tracks using noisy measurements originated from true targets or the clutter. This environment interpretation has many applications in air traffic control, road vehicle tracking, medical image processing, and biology [1]. A recently investigated area of application for target tracking methods is in automotive industries. The ever increasing interest in intelligent vehicles broadens the use and development of multiple-target tracking algorithms in active automotive safety systems and advanced driver assistance system [2, 3, 4, 5].

In the situations where the tracking is handled in the presence of measurement origin uncertainty, one of the fundamental parts of target tracking methods is the data association algorithm, which differentiates the received measurements and categorizes them into target-originated and clutter-originated [6]. A comprehensive survey of several data association methods can be found in [1] and [7].

Probabilistic data association (PDA) is a widely used data association and tracking method [8, 9]. PDA is a type of ‘all-

neighbour’ data association methods, which assumes several feasible hypotheses for the measurement to track associations and then calculates the association probabilities for each of them [6]. However, PDA is a formulation for tracking single target in clutter and to use it for multiple targets, simply multiple copies of a similar filter are employed [6]. Moreover, PDA is derived with assumption that the tracks are initialized and, consequently, there should be some other algorithms taking care of track initiation [1, 10]. In [11], integrated probabilistic data association (IPDA) is proposed, which is basically a rederivation of PDA without the assumption of initialized tracks and therefore, provides both the data association and track existence probabilities [11]. An extension of PDA for multi-target tracking, where the targets are interfering, is the joint probabilistic data association (JPDA) [12]. In JPDA, the targets are clustered and then the association probabilities are calculated in a jointly manner across the targets in a cluster [9, 12]. A similar extension of IPDA for multiple targets, named joint integrated probabilistic data association (JIPDA), is suggested in [13].

The aforementioned association methods provide an association probability for each feasible hypothesis which is used to construct a combined innovation term. The combined innovation term substitutes the innovation term in Kalman filtering structure of these algorithms [9].

Kalman filter (KF) is the most well-known filtering strategy because of its optimal estimation properties for linear systems [14, 15]. Since its introduction in the 1960’s, there were some modifications to extent the formulation of KF for nonlinear systems and to cope with the issues of uncertainty and instability [16, 17]. In 2007, a recursive predictor-corrector filtering strategy

based on the sliding mode concept [18], named smooth variable structure filter (SVSF) was proposed [19]. Basically, the SVSF owes its stability to selecting a corrective gain in a way that in each step decreases the error in the estimated states [19]. In order to achieve this, a hyper-plane as a projection of true state trajectory is introduced and applying the corrective gain, the estimations are forced to go toward this region, and then remain in between [19]. The main characteristic of SVSF, which suggests it as a useful filter in systems with modeling uncertainty, is its robustness against this type of uncertainties [19].

Employing SVSF as the filtering strategy in target tracking algorithms is firstly proposed in [20] for single target tracking in clutter. This paper is an extension of that work for multiple targets in combination with JPDA algorithm.

In section 2, KF and SVSF filtering algorithms are briefly overviewed. The basic formulation of JPDA algorithm is provided in section 3. Section 4 introduces the JPDA-SVSF tracking algorithm. In section 5 a simple multiple-target tracking example is studied to get a comparison between JPDA-KF and JPDA-SVSF. The paper is concluded in section 6.

II. ESTIMATION STRATEGIES

The Kalman filter is the best estimator in MMSE sense [21]. Indeed, KF minimizes the trace of state covariance matrix [18, 21]. In its original form, KF is based on two models: system model ((2.1)) which describes the evolution of the states, and measurement model ((2.2)) which relates the measurements to states.

$$x(k+1) = Ax(k) + Bu(k) + v(k) \quad (2.1)$$

$$z(k) = Cx(k) + w(k) \quad (2.2)$$

where $v(k)$ and $w(k)$ are respectively zero mean white Gaussian process and measurement noises with covariance matrices $Q(k)$ and $R(k)$. The KF algorithm is based on recursive prediction and updating the estimated states and their corresponding error covariance. The prediction consists of the following steps:

$$\hat{x}(k+1|k) = A\hat{x}(k|k) + Bu(k) \quad (2.3)$$

$$P(k+1|k) = AP(k|k)A^T + Q(k) \quad (2.4)$$

The Kalman gain $K(k+1)$ is calculated and then used to obtain the updated states and covariance, as follows:

$$K(k+1) = P(k+1|k)C^T[CP(k+1|k)C^T + R(k+1)]^{-1} \quad (2.5)$$

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K(k+1)[z(k+1) - C\hat{x}(k+1|k)] \quad (2.6)$$

$$P(k+1|k+1) = [I - K(k+1)C]P(k+1|k) \quad (2.7)$$

The Kalman filter in the above form is only applicable on linear systems. A very popular extension of KF for nonlinear systems is Extended KF, which linearizes the nonlinear function using the Jacobian matrix and then uses the same algorithm as KF [17, 22].

The smooth variable structure filter (SVSF) is a relatively new state and parameter estimation technique based on sliding mode concepts [18]. The basic concept is shown in the following figure.

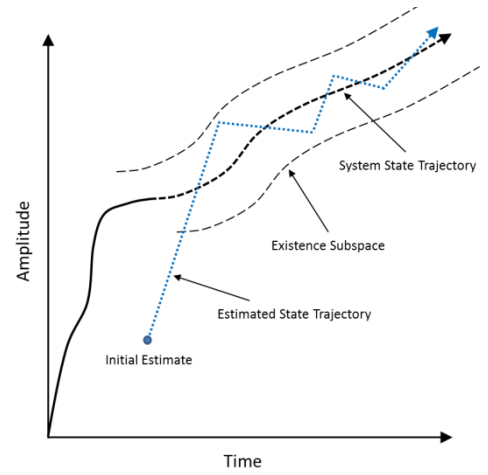


Figure 1. SVSF estimation concept [18].

The prediction stage of the SVSF is similar to the KF, and may be summarized by the following sets of equations.

$$\hat{x}(k+1|k) = A\hat{x}(k|k) + Bu(k) \quad (2.8)$$

$$P(k+1|k) = AP(k|k)A^T + Q(k) \quad (2.9)$$

Note that the SVSF may also be formulated to handle nonlinear system and measurement functions [18]. The a priori or predicted measurement error is also calculated by (2.10).

$$e_z(k+1|k) = z(k+1) - \hat{z}(k+1|k) \quad (2.10)$$

The SVSF gain is calculated as follows [18]:

$$K_{SVSF} = C^+ \text{diag} \left[(|e_z(k+1|k)|_{Abs} + \gamma |e_z(k|k)|_{Abs}) \circ \text{sat} \left(\frac{e_z(k+1|k)}{\psi_i} \right) \right] [diag(e_z(k+1|k))]^{-1} \quad (2.11)$$

As described in [18], the SVSF gain is a function of: the a priori and a posteriori measurement error vectors $e_{z,k+1|k}$ and $e_{z,k|k}$; the smoothing boundary layer widths ψ_i where i refers to the i^{th} width; the 'SVSF' memory or convergence rate γ with elements $0 < \gamma_{ii} \leq 1$; and the linear measurement matrix C . However, for numerical stability, it is important to ensure that one does not divide by zero in (2.11). This can be accomplished using a simple *if* statement with a very small threshold (i.e. 1×10^{-12}) [18].

The SVSF update equations are also very similar to the KF, and may be defined as follows:

$$\hat{x}(k+1|k+1) = \hat{x}(k+1|k) + K_{SVSF}e_z(k+1|k) \quad (2.12)$$

$$P(k+1|k+1) = [I - K(k+1)C]P(k+1|k) \quad (2.13)$$

However, note that the a posteriori or updated measurement error needs to be calculated as per (2.14). This value is used in the next time step.

$$e_z(k+1|k+1) = z(k+1) - \hat{z}(k+1|k+1) \quad (2.14)$$

III. JOINT PROBABILISTIC DATA ASSOCIATION PRINCIPLES

Originally, PDAF was formulated for tracking single targets in clutter. In PDAF, it is assumed that all the non-target originated received measurements are from clutter and are of a uniform distribution in the validation gate [8]. This assumption is violated in the presence of interfering targets [6]. The extension of PDAF to tackle this issue in tracking multiple targets in clutter is JPDAF [6]. In JPDAF, it is assumed that the number of initialized tracks is known and the density of state vector conditioned on past data is approximated by a Gaussian distribution as [12]:

$$p[x(k)|Z^{k-1}] = \mathcal{N}[x(k); \hat{x}(k|k-1), P(k|k-1)] \quad (3.1)$$

The JPDA and PDA algorithms utilize the same estimation equations. The difference is on the way the association probabilities are calculated [12, 6]. The association probabilities in PDA are calculated separately for each target, whereas in JPDA these probabilities are calculated in a jointly manner across the targets in a cluster [6]. In this sense, in JPDA algorithm the conditional probabilities of the following joint events are evaluated [6]:

$$\mathcal{H}(k) = \bigcap_{j=1}^{m(k)} \mathcal{H}_{j,t_j}(k) \quad (3.2)$$

where $\mathcal{H}_{j,t_j}(k)$ is the hypothesis that measurement j is originated from target t , $0 \leq j \leq m(k)$, $0 \leq t \leq T$, k is the time index, t_j is the target that measurement j is associated with, m_k is the number of measurements, and T is the number of targets [12]. The measurements at time k are named as z^j . Thus, the total available measurements at time k are $Z^k = \{z^1, \dots, z^{n_m}\} \cup Z^{k-1}$. Assuming the number of false measurements being from a Poisson distribution with spatial density λ , the joint association probabilities are calculated as (47) in [6]:

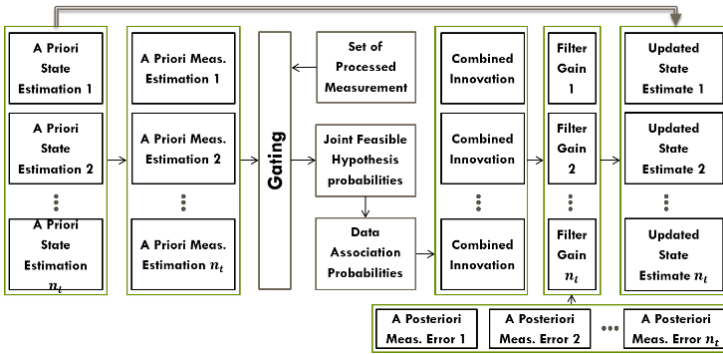


Figure 2. Schematic representation of JPDA-SVSF algorithm

$$P\{\mathcal{H}|Z^k\} =$$

$$c \prod_j \left\{ \lambda^{-1} L_{t_j}[z_j(k)] \right\}^{\tau_j} \prod_t (P_D^t)^{\delta_t} (1 - P_D)^{1-\delta_t} \quad (3.3)$$

where

$$L_{t_j}[z_j(k)] = \mathcal{N}[z_j(k); \hat{z}^{t_j}(k|k-1), S^{t_j}(k)] \quad (3.4)$$

and P_D^t is the detection probability of target t , τ_j and δ_t are respectively, the target detection and measurement association indicators [12].

To carry out the estimation, the marginal association probabilities are needed. These probabilities are obtained from joint probabilities (3.3) by summing over all joint hypotheses in which the marginal hypothesis of interest happens as (51) in [6]:

$$\begin{aligned} \beta_{jt}(k) &= P\{\mathcal{H}_{jt}(k)|Z^k\} \\ &= \sum_{\mathcal{H}:\mathcal{H}_{jt} \in \mathcal{H}} P\{\mathcal{H}(k)|Z^k\} \end{aligned} \quad (3.5)$$

These probabilities are used to make the combined innovation for each target.

IV. FORMULATION OF THE JPDA-SVSF

This section is a generalization of the method introduced in [20]. Here, we propose a novel formulation of SVSF for multi-target tracking in clutter based on JPDA method. Fig. 2 illustrates a schematic presentation of the method, referred to as SVSF-JPDA. The JPDA-SVSF algorithm is outlined as follows.

A. Gating Step

A validation gate is constructed around the predicted measurement of each track, based on the statistical distance, as follows ((34) in [6]):

$$\mathcal{V}_t(k, \gamma) = \{z: [z - \hat{z}_t(k|k-1)]' S_t(k)^{-1} [z - \hat{z}_t(k|k-1)] \leq \vartheta\} \quad (4.1)$$

where ϑ is the gate threshold corresponding to the gate probability, and $S_t(k)$ is the covariance of the innovation for each track. Then, the feasible hypotheses are determined and target detection and measurement association indicators are obtained [12].

B. Prediction Step

This step provides the prediction of states and measurements for each track, using the state and measurement models, and then a priori state error covariance [18, 20].

$$\hat{x}_t(k|k-1) = A_t(k-1)\hat{x}_t(k-1|k-1) \quad (4.2)$$

$$P_t(k|k-1) = A_t(k-1)P_t(k-1|k-1)A_t'(k-1) + Q_t(k-1) \quad (4.3)$$

$$z_t(k|k-1) = C_t(k-1)\hat{x}_t(k|k-1) \quad (4.4)$$

The marginal association probabilities of (3.5) are used to calculate the combined innovation for each track as:

$$\tilde{z}_t(k) = \sum_{i=1}^{m(k)} \beta_{it}(k) \tilde{z}_{it}(k) \quad (4.5)$$

The a priori measurement error of each track is set to be equal to the corresponding combined innovation:

$$e_{zt}(k|k-1) = \tilde{z}_t(k) \quad (4.6)$$

C. State Update Step

In this step, the SVSF gain is calculated for each track and is used to update the states [18, 20].

$$\hat{x}_t(k|k-1) = A_t(k-1)\hat{x}_t(k-1|k-1) \quad (4.7)$$

$$\hat{x}_t(k|k) = \hat{x}_t(k|k-1) + K_t(k)e_{zt}(k|k-1) \quad (4.8)$$

$$\begin{aligned} & K_t(k) \\ &= C_t^+ \text{diag}[(|e_{zt}(k|k-1)|_{Abs} \\ &+ \gamma_t |e_{zt}(k-1|k-1)|_{Abs}) \\ &\circ \text{sat}\left(\frac{e_{zt}(k|k-1)}{\psi_t}\right)] [\text{diag}(e_{zt}(k|k-1))]^{-1} \end{aligned} \quad (4.9)$$

The updated state covariance associated with each track is calculated as (42) in [6]:

$$\begin{aligned} P_t(k|k) &= \beta_{0t}(k)P_t(k|k-1) \\ &+ [1 - \beta_{0t}(k)]P_t^*(k|k) \\ &+ \tilde{P}_t(k) \end{aligned} \quad (4.10)$$

where $P_t^*(k|k)$ is the SVSF covariance matrix computed by [18]:

$$\begin{aligned} P_t^*(k|k) &= [I - K_t(k)C_t(k)]P_t(k|k-1)[I \\ &- K_t(k)C_t(k)]' \\ &+ K_t(k)R_t(k)K_t'(k) \end{aligned} \quad (4.11)$$

and $\tilde{P}_t(k)$ is an added uncertainty because of the associations uncertainties, as [8]:

$$\begin{aligned} \tilde{P}_t(k) &= K_t(k) \left[\sum_{i=1}^{m(k)} \beta_{it}(k) \tilde{z}_{it}(k) \tilde{z}_{it}'(k) - \right. \\ &\left. \tilde{z}_t(k) \tilde{z}_t'(k) \right] K_t'(k) \end{aligned} \quad (4.12)$$

A posteriori measurement error for each track is calculated in the same manner of [20], as:

$$e_{zt}(k|k) = [I - C_t(k)K_t(k)]e_{zt}(k|k-1) \quad (4.13)$$

V. ESTIMATION PROBLEM AND RESULTS

A. Problem Setup

A simple near constant velocity model is implemented as per [24]. There are four states in total, related to the target's position and velocity (x and y directions), defined as follows: $x = [\xi \ \eta \ \dot{\xi} \ \dot{\eta}]$. Note that ξ and η are the position in two Cartesian directions, and $\dot{\xi}$ and $\dot{\eta}$ are the corresponding velocities. This model assumes that the accelerations of the target between two sequential samples are constant and are drawn from a discrete-time zero mean white noise. The near constant velocity model is defined as follows:

$$x(k+1) = Ax(k) + Bv(k) \quad (5.1)$$

where the system and process noise gain matrices are defined by:

$$A = \begin{bmatrix} 1 & 0 & T_s & 0 \\ 0 & 1 & 0 & T_s \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5.2)$$

$$B = \begin{bmatrix} T_s^2/2 & 0 \\ 0 & T_s^2/2 \\ T_s & 0 \\ 0 & T_s \end{bmatrix} \quad (5.3)$$

The white acceleration noise is defined as follows:

$$Q = \text{cov}\{v(k)\} = \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix} \quad (5.4)$$

The measurement function, matrix, and noise covariance are defined respectively as follows:

$$z(k) = Cx(k) + w(k) \quad (5.5)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (5.6)$$

$$R = \text{cov}\{w(k)\} = \begin{bmatrix} \sigma_w^2 & 0 \\ 0 & \sigma_w^2 \end{bmatrix} \quad (5.7)$$

The JPDA-KF and JPDA-SVSF algorithms were implemented on two scenarios, with three targets under the presence of clutter. The parameter values used for the simulations are $T_s = 0.5$ s and $P_D = 0.9$. The clutter is assumed to have a spatial uniform distribution, and the number of cluttered measurements is generated by a Poisson's distribution of $\lambda = 10^{-4}$. The process noise variance is $\sigma_v^2 = 1^2$, and the measurement noise variance is $\sigma_w^2 = 3^2$.

B. Estimation Results

For the normal scenario, three targets are tracked and clutter occurs at $T = 229$ sec and $T = 294$ sec. The total simulation length is 300 sec. For a well-defined smoothing boundary layer (i.e., implementing the time-varying boundary layer presented in [18]), the JPDA-SVSF is able to match the performance of the JPDA-KF. This is shown in the following figures.

The RMSE errors for this scenario were computed across the three targets and are shown in the following table. Note that, for this case, the two filters yielded the same results.

Table 1. RMSE Estimation Results – Normal Case

	Car #1		Car #2		Car #3	
	JPDA KF	JPDA SVSF	JPDA KF	JPDA SVSF	JPDA KF	JPDA SVSF
ξ	10.09	10.31	9.92	10.15	12.38	12.57
η	10.16	10.39	10.33	10.55	5.70	6.08
$\dot{\xi}$	4.57	5.21	4.57	5.12	4.61	5.17
$\dot{\eta}$	4.59	5.13	4.60	5.02	4.55	5.11

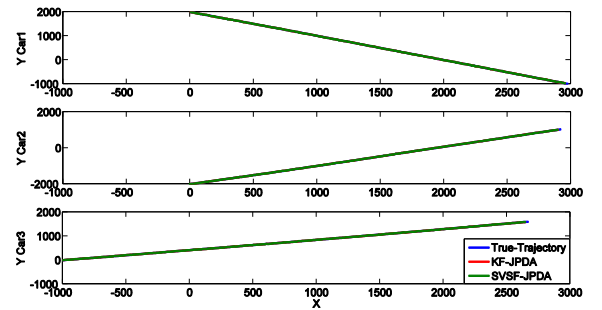


Figure 3. JPDA-KF and JPDA-SVSF estimation results (normal case) for the three targets.

The second scenario looked at the case of modeling errors or uncertainties. To further investigate the robustness of the proposed JPDA-SVSF strategy, modeling uncertainty is injected at $t = 75$ s for a duration of 10 sampling times into the simulation in the form of changes in the state transition matrix of the model. Fig. 3 shows the simulation results for the level of modeling uncertainty of 3% ($A_{unc} = 1.034$), where JPDA-SVSF provides a more stable estimate, as the results did not diverge from the true state trajectory.

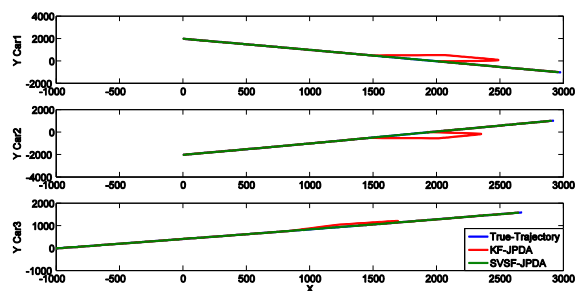


Figure 4. JPDA-KF and JPDA-SVSF estimation results (error case) for the three targets.

The RMSE under this scenario was recalculated, and is shown in the following table. The position RMSEs of JPDA-SVSF method are considerably smaller than of JPDA-KF method.

Table 2. RMSE Estimation Results –uncertainty of 3%

	Car #1		Car #2		Car #3	
	JPDA KF	JPDA SVSF	JPDA KF	JPDA SVSF	JPDA KF	JPDA SVSF
ξ	86.88	10.31	75.21	10.23	25.79	12.49
η	21.03	10.29	28.65	10.57	18.45	6.24
ξ	5.88	5.91	5.43	5.41	5.61	5.77
η	5.46	5.55	4.98	5.01	5.49	5.61

Furthermore, increasing the modeling uncertainty causes the JPDA-KF strategy to fail. While due to the unique switching action of the SVSF, the JPDA-SVSF method maintained its tracking capability and was able to provide a good estimate with up to 8% uncertainty in the system matrix.

VI. CONCLUSION

This paper introduced a new multi-target tracking strategy referred to as the JPDA-SVSF. A multi target tracking simulation was studied to compare the well-studied JPDA-KF algorithm with the JPDA-SVSF. Both methods were able to perform a good target tracking in normal cases. However, the proposed JPDA-SVSF outperforms JPDA-KF in the case of modeling uncertainty in the system matrix, and yielded a more robust estimation method. Future work will build upon the results of this paper, and will study more challenging multi-target tracking scenarios, including interfering targets. Also, some unique characteristics of SVSF method, such as extra indicators of performance will be formulated to improve the data association probabilities.

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