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STATE ESTIMATION AND FAULT DETECTION OF AN ELECTROHYDROSTATIC ACTUATOR

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ABSTRACT

The electrohydrostatic actuator (EHA) is an efficient type of linear actuator commonly found in aerospace applications. It consists of an external gear pump (fluid), an electric motor, a closed hydraulic circuit, a number of control valves and ports, and a linear actuator. An EHA, built for experimentation, is studied in this paper. Two types of estimation strategies, the popular Kalman filter (KF) and the smooth variable structure filter (SVSF), are applied to the EHA for kinematic state and parameter estimation. The KF strategy yields the statistical optimal solution to linear estimation problems. However, the KF becomes unstable when strict assumptions are violated. The SVSF is an estimation strategy based on sliding mode concepts, which brings an inherent amount of stability to the estimation process. Recent advances in SVSF theory include a time-varying smoothing boundary layer. This method, known as the SVSF-VBL, offers an optimal formulation of the SVSF as well as a method for detecting changes or faults in a system. In addition to the application of the KF and SVSF for state estimation, the SVSF-VBL is applied to the EHA for the purposes of fault detection. The EHA is operated under various operating conditions (normal, friction fault, leakage fault, and so on), and the experimental results are presented and discussed.

1.0 INTRODUCTION

State and parameter estimation theory is an important area of study, and spans a large number of fields ranging from control theory to financial analysis. Essentially, states are important values that are used to dynamically and mathematically model a system. For example, the kinematic states of an aircraft (position, velocity, and acceleration) may be used to accurately estimate and track the flight path of the aircraft. In an electrohydrostatic actuator (EHA), states like the effective bulk modulus of the hydraulic fluid, actuator position and velocity, and differential pressure, may be used to accurately model the EHA dynamics. Measurements are obtained from the environment in order to provide estimates of the system states and parameters. However, these measurements typically include unwanted signals and disturbances such as noise. It is the goal of an estimation strategy to minimize the effects of the disturbances in an effort to extract accurate state values. These state values may then be used to accurately control the system or track values of interests.

Introduced in the 1960s by Rudolph Kalman, the Kalman filter (KF) is one of the most popular state and parameter estimation strategies [1]. The filter is a predictor-corrector strategy, and makes use of a statistically optimal gain to correct predicted state estimates [2]. The KF is based on a number of strict assumptions. For example, the system and measurements functions must be linear and known, and the noise must be white and Gaussian-distributed [3]. If these assumptions are not followed, the KF may yield suboptimal or unstable estimates. For the case of nonlinear systems and measurements, the extended Kalman filter (EKF) may be implemented [4]. The EKF makes use of first-order Taylor series or Jacobian approximations to linearize the nonlinearities about a point of interest. The main extended Kalman filter equations are similar to the standard KF, except for the linearization of nonlinearities.

Another estimation strategy, which is growing in popularity, is referred to as the smooth variable structure filter (SVSF). It is a predictor-corrector method based on sliding mode concepts. Essentially, it uses a corrective gain to force state estimates to within a region referred to as the existence subspace. Once within this region, defined by the uncertainties present in the estimation process, the state estimates are forced to switch back and forth across the true state trajectory. This switching effect, similar to the sliding mode concept, brings an inherent amount of stability to the estimation process. Provided the system is bounded input and bounded output (BIBO) stable, the SVSF will always yield a stable estimate. The original formulation of the SVSF makes use of a fixed-width smoothing boundary layer, which is defined by the amount of uncertainties present (e.g., system and measurement noise, modeling error). New developments have led to a time-varying smoothing boundary layer (VBL), which yields a more accurate state estimate. A byproduct of the VBL is that the width may be used to determine the presence of system changes or faults. In some cases, the VBL widths may be used to identify the fault.

Fault detection strategies may be classified as signal-based or model-based. Signal-based strategies make use of system measurements which are analyzed and studied against benchmark data. Deviations from the norms can be used to identify system changes or faults. A popular signal-based method is the artificial neural network (ANN) strategy, and is a machine learning technique that can be used for advanced fault detection and identification. Model-based strategies, such as the interacting multiple model (IMM) or adaptive model-based method, as the name suggests, makes use of system equations and models to identify the system behaviour. For example, the IMM, makes use of Bayesian theory which assigns a probability to each system model or mode of operation. The IMM has been shown to work very well, and is one of the most popular model-based strategies.

The purpose of this paper is to demonstrate the results of applying a state estimator to an electrohydrostatic actuator (EHA). This state estimator, referred to as the SVSF-VBL, is a robust estimation strategy based on the smooth variable structure filter (SVSF) and sliding mode concepts. In addition to providing accurate state estimates, the SVSF-VBL is also able to detect system changes or faults. In some cases, the SVSF-VBL is able to identify the system changes or faults. The extended Kalman filter (EKF) and SVSF are described in Section 2. The experimental setup used in this paper and results are shown in Sections 3 and 4, respectively. The paper is then concluded in the last section.

2.0 ESTIMATION THEORY

This section summarizes the main equations of the extended Kalman filter (EKF) and smooth variable structure filter (SVSF). The recently proposed variable boundary layer (VBL) formulation of the SVSF is also described.

2.1 EXTENDED KALMAN FILTER

The prediction stage of the EKF begins with calculation of the state estimates and state error covariances respectively as follows [4]:

$$\hat{x}_{k+1|k} = f\left(\hat{x}_{k|k}, u_k\right) \tag{2.1.1}$$

$$P_{k+1|k} = \hat{F}_k P_{k|k} \hat{F}_k^T + Q_k \tag{2.1.2}$$

Note that the update stage is defined by the following set of equations [4]. The a priori measurement error or innovation is calculated as per (2.1.3). The measurement noise covariance matrix or innovation covariance is calculated as per (2.1.4). The KF gain is calculated by (2.1.5) and is used to provide a posteriori or updated state estimates as per (2.1.6). Equation (2.1.7) updates the a posteriori state error covariance matrix.

$$e_{z,k+1|k} = z_{k+1} - h\left(\hat{x}_{k+1|k}\right) \tag{2.1.3}$$

$$S_{k+1} = H_{k+1} P_{k+1|k} H_{k+1}^T + R_{k+1} \tag{2.1.4}$$

$$K_{k+1} = P_{k+1|k} H_{k+1}^T S_{k+1}^{-1} \tag{2.1.5}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} e_{z,k+1|k} \tag{2.1.6}$$

$$P_{k+1|k+1} = (I - K_{k+1} H_{k+1}) P_{k+1|k} \tag{2.1.7}$$

Partial derivatives are used for the first-order Taylor series approximations as follows:

$$F_k = \left.\frac{\partial f(x)}{\partial x}\right|_{x=\hat{x}_{k|k}, u_k} \tag{2.1.8}$$

$$H_{k+1} = \left.\frac{\partial h(x)}{\partial x}\right|_{x=\hat{x}_{k+1|k}} \tag{2.1.9}$$

The equations listed previously summarize the EKF process, and the process is repeated iteratively.

2.2 SMOOTH VARIABLE STRUCTURE FILTER

The smooth variable structure filter (SVSF) has been developed over the last ten years. The 'standard' SVSF was introduced in 2007, and makes use of sliding mode concepts and observer theory [5]. The main SVSF theory was advanced significantly in 2011 as described in [6]. The SVSF formulation is similar to the KF; however, the derivation of the gain is fundamentally different. The basic concept of the SVSF estimation process is shown in the following figure. Essentially, given some initial estimate, the state estimate is forced towards the true system state trajectory to within a region of the existence subspace. As previously mentioned, the existence subspace width is defined based on the amount of disturbances or uncertainties present in the estimation process (e.g., system and measurement noise). A switching gain is then applied which forces the state estimate to switch back and forth or chatter across the true state trajectory. This brings an inherent amount of stability to the estimation process.

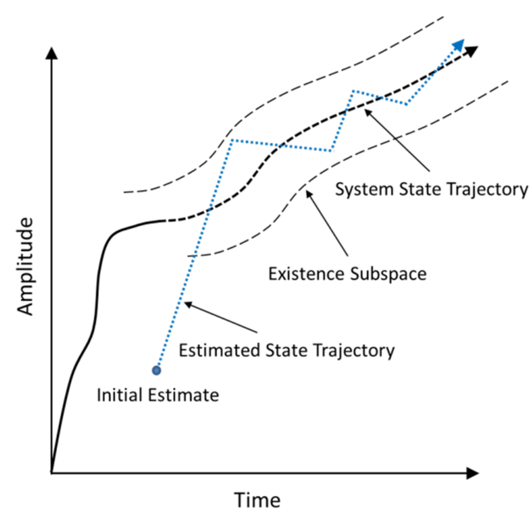

Figure 1. SVSF estimation concept as per [6].

The SVSF prediction stage of the SVSF is defined as follows [6]. The state estimates and state error covariances are predicted as per (2.2.1) and (2.2.2), respectively. The a priori measurement error is defined by (2.2.3).

$$\hat{x}_{k+1|k} = f\left(\hat{x}_{k|k}, u_k\right) \tag{2.2.1}$$

$$P_{k+1|k} = \hat{F}_k P_{k|k} \hat{F}_k^T + Q_k \tag{2.2.2}$$

$$e_{z,k+1|k} = z_{k+1} - h\left(\hat{x}_{k+1|k}\right) \tag{2.2.3}$$

The SVSF gain is defined by (2.2.4) [6].

$$\begin{aligned} K_{k+1} &\\ = C^+ diag&\left[\left(\left|e_{z,k+1|k}\right|_{Abs} + \gamma\left|e_{z,k|k}\right|_{Abs}\right)\right. \\ \circ\, sat&\left.\left(\bar{\psi}^{-1} e_{z,k+1|k}\right)\right] diag\left(e_{z,k+1|k}\right)^{-1} \end{aligned} \tag{2.2.4}$$

The update stage is defined as follows [6]. The state estimates and the state error covariances are updated as per (2.2.5) and (2.2.6), respectively. The a posteriori or updated measurement error is defined by (2.2.7).

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} e_{z,k+1|k} \tag{2.2.5}$$

$$\begin{aligned} P_{k+1|k+1} = (I - K_{k+1}H_{k+1})P_{k+1|k}(I& \\ - K_{k+1}H_{k+1})^T& \\ + K_{k+1}R_{k+1}K_{k+1}^T& \end{aligned} \tag{2.2.6}$$

$$e_{z,k+1|k+1} = z_{k+1} - h\left(\hat{x}_{k+1|k+1}\right) \tag{2.2.7}$$

The SVSF gain is a function of a priori (predicted) and previous a posteriori (updated) measurement errors, SVSF 'memory' γ, and a smoothing boundary layer term ψ. The smoothing boundary layer term is used to reduce or smooth the chattering magnitude caused by the switching term (2.2.4).

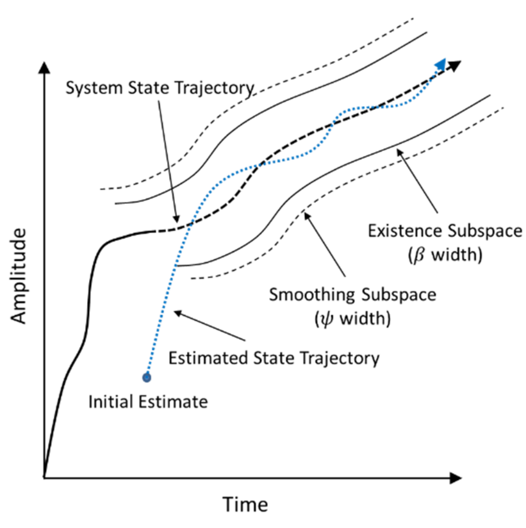

Figure 2. Smoothed estimated trajectory ($\psi \geq \beta$) [7].

The SVSF estimation process is further illustrated in Figs. 1-3. To reiterate, the existence subspace represents the amount of uncertainties present in the estimation process [8]. This value is defined in terms of modeling errors and noise. It is often tuned by trial and error based on designer knowledge (e.g., estimated amount of system or measurement noise). The width of the existence space β is a function of the uncertain dynamics associated with the inaccuracy of the internal model of the filter as well as the measurement model, and may vary with time [5]. In most cases, this value is not known exactly, but an upper bound may be selected based on designer knowledge.

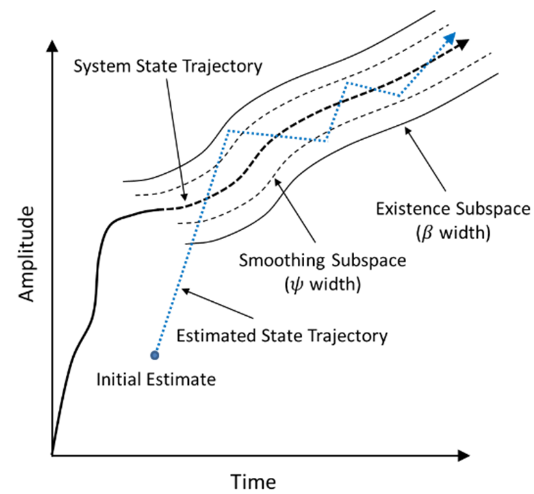

Figure 3. Presence of chattering effect ($\psi < \beta$) [7].

2.3 THE SVSF-VBL

As per [5, 6], the inherent switching or chattering effect caused by the SVSF gain reduces the estimation accuracy. However, it drastically improves the robustness and stability to the presence of disturbances, and modeling uncertainties [5, 6]. In an effort to increase the estimation accuracy, a time-varying smoothing boundary layer formulation of the SVSF was developed in [6, 7] and is referred to as the SVSF-VBL.

The partial derivative of the a posteriori covariance (trace) with respect to the smoothing boundary layer term was found in order to obtain the VBL formulation. This is similar to the KF formulation and the method used to obtain a statistically optimal gain [1]. In this case, the smoothing boundary layer term is redefined and is considered to be a full square matrix, as opposed to one boundary layer assigned to each measurement [6]. The SVSF-VBL prediction stage is similar to (2.2.1) through (2.2.3). The time-varying smoothing boundary layer (VBL) is calculated using the following three equations [6]. The measurement or innovation covariance is calculated by (2.3.1). The error components and time-varying boundary layer are calculated by (2.3.2) and (2.3.3), respectively.

$$S_{k+1} = H_{k+1} P_{k+1|k} H_{k+1}^T + R_{k+1} \tag{2.3.1}$$

$$A_{k+1} = \left|e_{z,k+1|k}\right|_{Abs} + \gamma\left|e_{z,k|k}\right|_{Abs} \tag{2.3.2}$$

$$\psi_{k+1} = \left(\bar{A}_{k+1}^{-1} H_{k+1} P_{k+1|k} H_{k+1}^T S_{k+1}^{-1}\right)^{-1} \tag{2.3.3}$$

The SVSF-VBL gain is then calculated as per (2.3.4) and is used to update the state estimates (2.3.5) and state error covariance matrix (2.3.6). The a posteriori or updated measurement error is calculated as per (2.3.7).

$$K_{k+1} = H_{k+1}^{-1} \bar{A}_{k+1} \psi_{k+1}^{-1} \tag{2.3.4}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} e_{z,k+1|k} \quad (2.3.5)$$

$$P_{k+1|k+1} = (I - K_{k+1} H_{k+1}) P_{k+1|k} (I - K_{k+1} H_{k+1})^T + K_{k+1} R_{k+1} K_{k+1}^T \quad (2.3.6)$$

$$e_{z,k+1|k+1} = z_{k+1} - h(\hat{x}_{k+1|k+1}) \quad (2.3.7)$$

The SVSF-VBL process is very similar to the SVSF process, with the main difference being the calculation of the VBL (2.3.3) and the SVSF gain (2.3.4). The main disadvantage of the SVSF, compared with the SVSF-VBL, is the fact that a conservative fixed smoothing boundary layer is defined which reduces the overall estimation accuracy [5, 6]. The SVSF-VBL calculates a near-optimal value for the boundary layer, in an effort to improve the accuracy. As described by Figs. 4 and 5, an interesting byproduct of the optimal or time-varying boundary layer is the fact that the presence of system changes or faults may be detected.

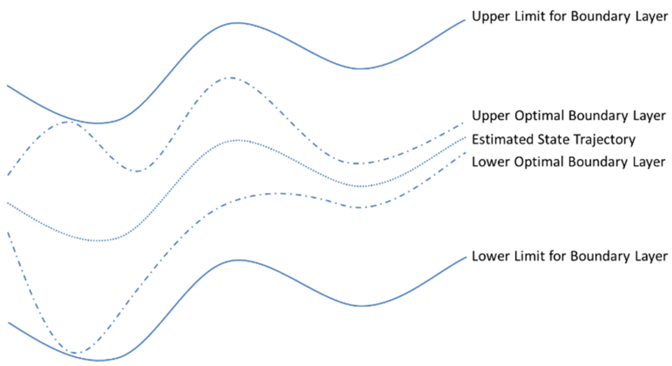

Figure 4. Well-defined time-varying boundary layer concept [7].

Figure 4 illustrates the case when a limit is imposed on the smoothing boundary layer width (a conservative value) and the time-varying (optimal) smoothing boundary layer per (3.22) follows within this limit. In the standard SVSF, the smoothing boundary layer width is made equal to the limit; such that the difference between the limit and the optimal variable boundary layers quantifies the loss in optimality.

Figure 5 illustrates the case when the optimal time-varying smoothing boundary layer is larger than the limit imposed on the smoothing boundary layer. This typically occurs when there is modeling uncertainty (which leads to a loss in optimality) or when the limit on the smoothing boundary layer is underestimated. The width of the smoothing boundary layer is directly related to the level of modeling uncertainties (by virtue of the errors), as well as the estimated system and measurement noise (captured by $P_{k+1|k}$ and S_{k+1}). As per [7], the VBL creates another indicator of performance for the SVSF: the widths may be used to determine the presence of modeling uncertainties, as well as detect any changes in the system.

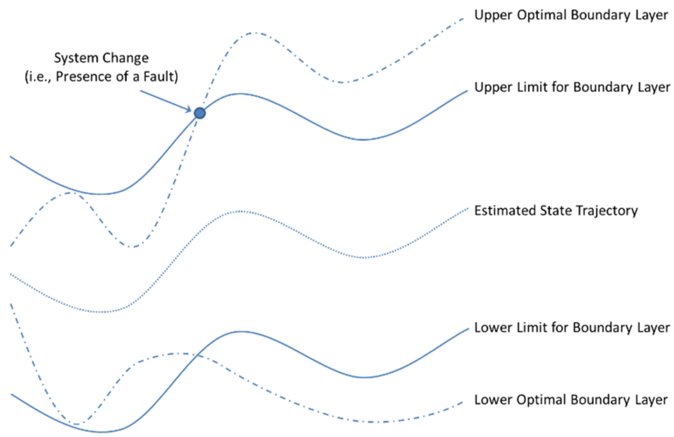

Figure 5. Presence of system changes or faults [7].

3.0 EXPERIMENTAL SETUP

The experimental setup used in this paper is known as an electrohydrostatic actuator (EHA). Asa per [5], EHAs are commonly used in aerospace and heavy-industry applications, and are becoming increasingly popular due to their high force-to-weight ratio. The EHA setup used in this paper is shown in Fig. 6. Further details on the EHA design and setup may be found in [9, 10, 11, 12].

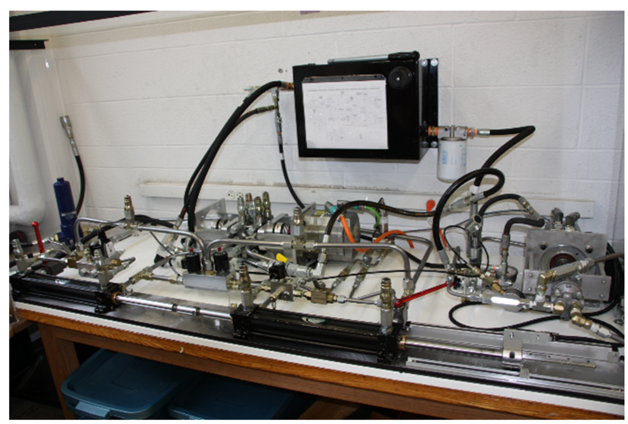

Figure 6. EHA experimental setup [13].

Two types of fault conditions were physically induced on this system: internal leakage and friction. To induce a friction fault, one axis was used as the driving mechanism while the second axis acted as a load. To implement internal leakage across the circuit, the first axis throttling valve is used (i.e., throttle blocking valve is open). The first axis throttling valve incurs cross-port leakage between both chambers of its corresponding cylinder. These two faults negatively affect the output of the EHA.

The following state space equations (related to its position, velocity, and acceleration) represent the EHA dynamics [13]:

$$x_{1,k+1} = x_{1,k} + T x_{2,k} \quad (4.1)$$

$$x_{2,k+1} = x_{2,k} + T x_{3,k} \quad (4.2)$$

$$x_{3,k+1} = \left[1 - T\frac{a_2 V_0 + M\beta_e L}{MV_0}\right] x_{3,k} - T\frac{(A_E^2 + a_2 L)\beta_e}{MV_0} x_{2,k} - T\frac{2a_1 V_0 x_{2,k} x_{3,k} + \beta_e L\left(a_1 x_{2,k}^2 + a_3\right)}{MV_0} sgn\left(x_{2,k}\right) + T\frac{A_E \beta_e}{MV_0} u_k \tag{4.3}$$

As per [10], the differential pressure of the EHA may be determined based on the actuator friction, modeled as a second-order quadratic function related to the actuator velocity:

$$\Delta P A_E = a_2 \dot{x} + (a_1 \dot{x}^2 + a_3) sgn(\dot{x}) \tag{4.4}$$

The sample rate of the EHA system is $T = 0.1\, ms$. The corresponding EHA parameter values for each case may be found in [6]. Based on the EHA model, there are four important states and parameters: actuator position, velocity, acceleration, and differential pressure. In this study, the actuator position and differential pressure are measured, and the velocity and acceleration were extrapolated from the position, such that the measurement matrix was an identity matrix of size four.

For the EKF, the SVSF, and SVSF-VBL, the initial state estimate and state error covariance matrix were defined as follows:

$$\hat{x}_{0|0} = [0 \quad 0 \quad 0 \quad 0]^T \tag{4.6}$$

$$P_{0|0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} \tag{4.7}$$

The system and measurement noise covariance's Q and R were based on previous work on the EHA [10], and tuning:

$$Q = diag([10^{-12} \quad 10^{-10} \quad 10^{-9} \quad 10^{-3}]) \tag{4.8}$$

$$R = diag([10^{-12} \quad 10^{-9} \quad 10^{-6} \quad 5 \times 10^{3}]) \tag{4.9}$$

In an effort to minimize the estimation error, the SVSF convergence rate was set to $\gamma = 0.1$, and the layers set to $\psi = [3.5 \times 10^{-3} \quad 1 \times 10^4 \quad 1 \times 10^6 \quad 1 \times 10^9]^T$.

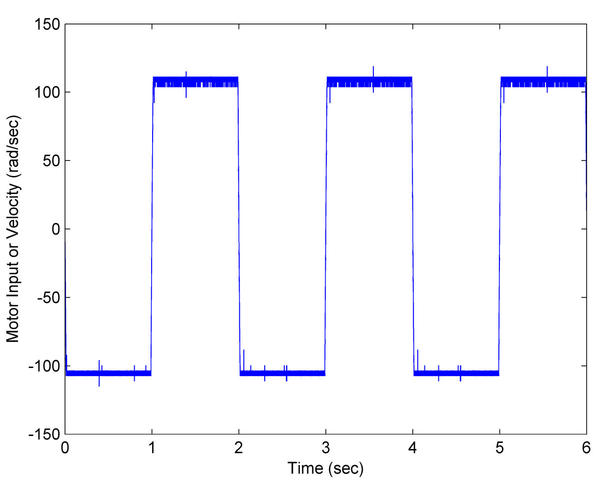

Figure 7. Motor input (velocity) used to actuate the EHA.

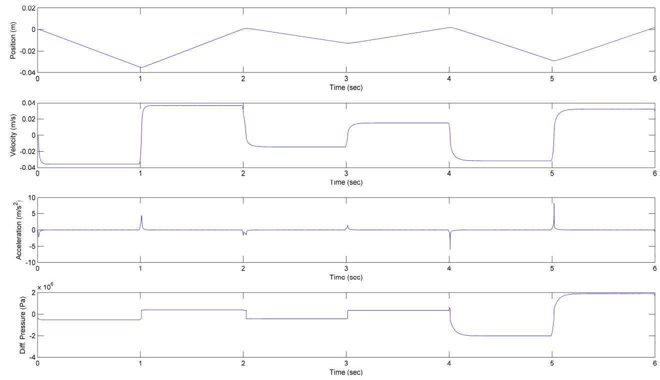

Figure 8. EHA state trajectory.

The scenario that was studied involves the EHA operating normally for 2 seconds, followed by a leakage fault for 2 seconds, and then a friction fault for the last 2 seconds. The input into the system is shown in Fig. 7, and the corresponding EHA trajectory and differential pressure are shown in Fig. 8.

4.0 EXPERIMENTAL RESULTS

Three estimation strategies were applied on the EHA for state estimation: the EKF, SVSF, and SVSF-VBL. The root mean square error (RMSE) for the estimation process is summarized in Table 1. As predicted, the SVSF-VBL was able to provide the best result in terms of estimation accuracy. The EKF and SVSF also performed well, with the SVSF performing slightly better when estimating the actuator position and differential pressure. This is most likely due to the fact that these two states were measured directly from the environment. The velocity and acceleration were obtained based on differentiation of the position, which introduced unwanted noise and errors.

Table 1. RMSE Results for the Estimation Strategies

Strategy	$x_1\ (m)$	$x_2\ (m/s)$	$x_3\ (m/s^2)$	$x_4\ (Pa)$
EKF	$5.13x10^{-4}$	$2.80x10^{-3}$	$3.32x10^{-1}$	$1.64x10^{4}$
SVSF	$5.02x10^{-4}$	$2.81x10^{-3}$	$3.51x10^{-1}$	$1.25x10^{4}$
SVSF-VBL	$4.81x10^{-4}$	$2.14x10^{-3}$	$3.09x10^{-1}$	$1.01x10^{4}$

The following four figures illustrate the values of the time-varying smoothing boundary layers. Figures 9 and 10 are the results for when the SVSF-VBL operates using the normal system model. As demonstrated, the values are distinctly different depending on how the EHA behaves. For example, notice the changes of VBL widths every two seconds, which aligns with the varying operating modes. This technique may be used to identify modeling uncertainties or system changes during the estimation process. Figure 11 shows the VBL results for the position state, when the SVSF-VBL uses the leakage model to model the EHA system. Figure 12 shows the VBL results for the differential pressure, when the SVSF-VBL uses the friction model. Notice these results are also unique, and provides a good indicator for system changes or faults.

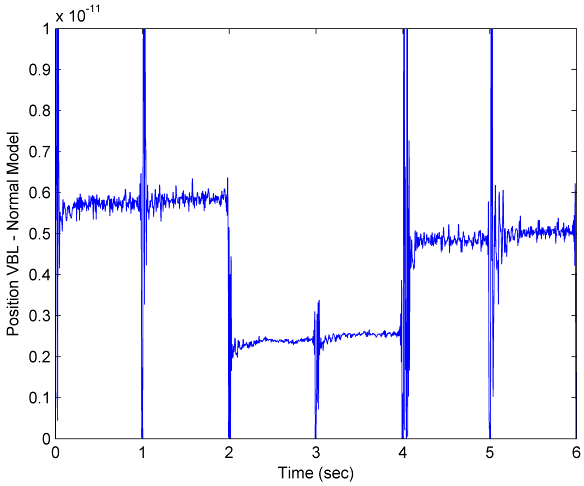

Figure 9. Time-varying boundary layer for the position state (normal model).

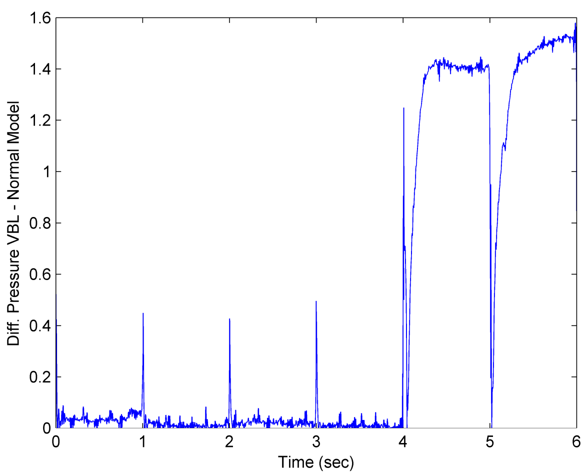

Figure 10. Time-varying boundary layer for the fourth state (normal model).

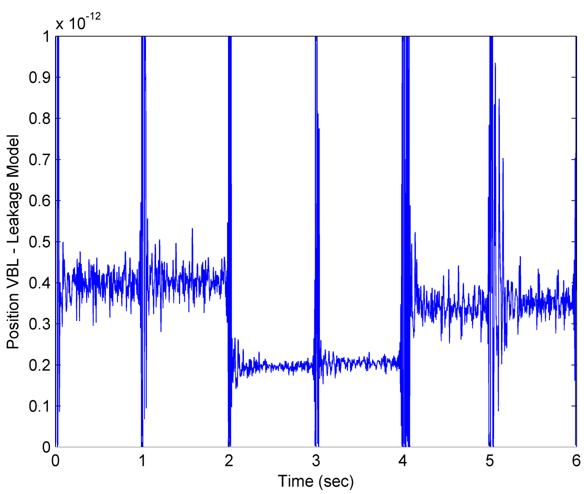

Figure 11. Time-varying boundary layer for the position state (leakage model).

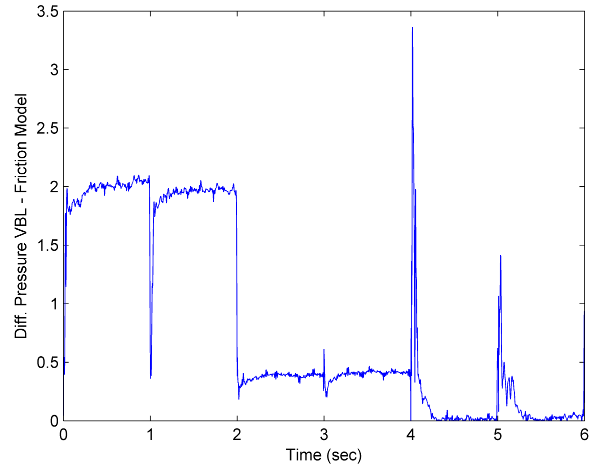

Figure 12. Time-varying boundary layer for the fourth state (friction model).

5.0 CONCLUSIONS

The purpose of this paper was to demonstrate the results of applying a state estimator to an electrohydrostatic actuator (EHA). This state estimator, referred to as the SVSF-VBL, is a robust estimation strategy based on the smooth variable structure filter (SVSF) and sliding mode concepts. In addition to providing accurate state estimates, the SVSF-VBL is also able to detect system changes or faults. The results are compared with the popular EKF and the standard SVSF. It was determined that the SVSF-VBL was able to provide very accurate state estimates while also detecting system changes or faults. Future work will involve studying up to nine different EHA conditions, including combined degrees of faults.

6.0 REFERENCES

[1] R. E. Kalman, "A New Approach to Linear Filtering and Prediction Problems," *Journal of Basic Engineering, Transactions of ASME,* vol. 82, pp. 35-45, 1960.

[2] B. D. O. Anderson and J. B. Moore, Optimal Filtering, Englewood Cliffs: NJ: Prentice-Hall, 1979.

[3] A. Gelb, Applied Optimal Estimation, Cambridge, MA: MIT Press, 1974.

[4] M. S. Grewal and A. P. Andrews, Kalman Filtering: Theory and Practice Using MATLAB, 3 ed., New York: John Wiley and Sons, Inc., 2008.

[5] S. R. Habibi, "The Smooth Variable Structure Filter," *Proceedings of the IEEE,* vol. 95, no. 5, pp. 1026-1059, 2007.

[6] S. A. Gadsden, "Smooth Variable Structure Filtering: Theory and Applications," Hamilton, Ontario, 2011.

[7] S. A. Gadsden and S. R. Habibi, "A New Robust Filtering Strategy for Linear Systems," *ASME Journal of Dynamic Systems, Measurements and Control,* vol. 135, no. 1, January 2013.

[8] S. A. Gadsden, M. Al-Shabi and S. R. Habibi, "A Review of Smooth Variable Structure Filters: Recent Advances in Theory and Applications," in *2013 IEEE Jordan Conference on Applied Electrical Engineering and Computing Technologies (AEECT)*, Amman, Jordan, 2013.

[9] S. R. Habibi and R. Burton, "Parameter Identification for a High Performance Hydrostatic Actuation System using the Variable Structure Filter Concept," *ASME Journal of Dynamic Systems, Measurement, and Control,* 2007.

[10] Y. A. Chinniah, "Fault Detection in the Electrohydraulic Actuator Using the Extended Kalman Filter," Saskatoon, Saskatchewan, 2004.

[11] M. El Sayed and S. R. Habibi, "Multiple Sliding Mode Control for Electro-Hydraulic Actuation (EHA) Systems," *ASME Journal of Dynamic Systems, Measurement and Control,* 2011 (Article Submitted).

[12] K. McCullough, "Design and Characterization of a Dual Electro-Hydrostatic Actuator," Hamilton, Ontario, 2011.

[13] S. A. Gadsden, Y. Song and S. R. Habibi, "Novel Model-Based Estimators for the Purposes of Fault Detection and Diagnosis," *IEEE/ASME Transactions on Mechatronics,* vol. 18, no. 4, 2013.