2013 IEEE Jordan Conference on Applied Electrical Engineering and Computing Technologies (AEECT)

The Toeplitz-Observability Smooth Variable Structure Filter

Mohammad A. Al-Shabi
Department of Mechatronics Engineering
Philadelphia University
Jerash, Jordan
maqas2002@yahoo.com

S. Andrew Gadsden
Department of Mechanical Engineering
McMaster University
Hamilton, Ontario, Canada
gadsden@mcmaster.ca

Saeid R. Habibi
Department of Mechanical Engineering
McMaster University
Hamilton, Ontario, Canada
habibi@mcmaster.ca

***Abstract*—The smooth variable structure filter (SVSF) is a recently proposed method that is used for estimation purposes, such as fault detection [1-2]. The SVSF demonstrates good results and robustness when it is applied to linear and non-linear systems that are fully measured. However, the results differ when some of the states are not measured. In this case, the SVSF is combined with the Luenberger method, which has some limitations. In this paper, a novel form of the SVSF is derived using the Observability and Toeplitz matrices. The benefits of the proposed method are demonstrated by using a computer simulation that involves an electro-hydrostatic actuator proposed in [3-5].**

***Keywords*—estimation; smooth variable structure filter; observability; Toeplitz; electro-hydrostatic actuator.**

I. INTRODUCTION

The estimation process is used to extract both states and parameters of a system from available measurements. This involves a mathematical algorithm that combines the prior and empirical knowledge. The former involves the prior information regarding the system; i.e. parameters. The latter involves with measuring the dynamic behavior of that system [6-8]. The estimation process could be used for filtering, smoothing, and prediction. All of these categories involve extracting a quantity of interest from measured signals. However, they differ in selecting the data from those signals, as filtering uses the data in the past up to and including the current time. Smoothing uses a segment that contains past, current and future points. Prediction uses the same data as filtering but produces the quantity of interest in the future [8].

The mathematical algorithm used for estimation is referred to as estimator. If the measured signal contains noise, then the estimator is referred to as a filter, as it needs to remove unwanted noise. In this paper, the smooth variable structure filter (SVSF) is considered due to its robustness, stability, and estimation performance [5-7]. The paper is organized as follows: the SVSF in its general form is discussed in Section II, while in Section III, the SVSF for linear-system with partially ranked measurement will be discussed. The novelty of this paper, which is the Observability/Toeplitz SVSF (OTSVSF), will be formulated and derived in Section IV. The application of the OTSVSF into an electro-hydrostatic actuator is discussed in Section V. Section VI contains the conclusion of this work and future research recommendations and directions.

Nomenclature

Italic-upper case letters are used to denote matrices and vectors, while their elements are denoted by italic lower case letters with subscripts i and/or j. The symbols $^{-1}$, ^ and ^{T} denote the matrix's inversion, estimation and transposition operators.

SYMBOL	COMMENTS	SIZE
$\lvert\;\rvert$	Absolute value.	
$\circ$	Schur product.	
$\mathbf{k}$	Time step value.	$\mathbf{1}\times\mathbf{1}$
$\mathbf{k}\vert\mathbf{k}-\mathbf{1}$	A priori estimate at time $\mathbf{k}$.	
$\mathbf{k}\vert\mathbf{k}$	A posteriori estimate at time $\mathbf{k}$.	
A	System matrix.	$\boldsymbol{n}\times\boldsymbol{n}$
B	Input matrix.	$\boldsymbol{n}\times\mathbf{1}$
Δ	Difference between actual and estimated values.	
e_i	Estimation error vector for i.	$\boldsymbol{m}\times\mathbf{1}$
γ	The SVSF's diagonal coefficient matrix.	$\boldsymbol{n}\times\boldsymbol{n}$
H	Output matrix.	$\boldsymbol{m}\times\boldsymbol{n}$
$I_{\mathbf{n}\times\mathbf{n}}$	Identity matrix with dimensions of $\boldsymbol{n}\times\boldsymbol{n}$.	$\boldsymbol{n}\times\boldsymbol{n}$
$K_{S_\mathbf{k}}, \boldsymbol{K}_{\boldsymbol{SS}_k}$	The SVSF's gain for system with fully and partially ranked measurement matrix, respectively.	$\boldsymbol{n}\times\mathbf{1}$
$\boldsymbol{K}_{\boldsymbol{e}_{\boldsymbol{TO}_k}}$	The proposed SVSF's gain.	$\boldsymbol{n}\times\mathbf{1}$
$\mathbf{m}$	Number of measurements.	$\mathbf{1}\times\mathbf{1}$
$\mathbf{n}$	Number of states.	$\mathbf{1}\times\mathbf{1}$
$\boldsymbol{O}$	The Observability matrix	$\boldsymbol{n}\times\boldsymbol{n}$
$sgn(a)$	Sign function of $\boldsymbol{a}$.	
$\mathbf{sgn}(\mathbf{a})$	Sign function of $\mathbf{a}$.	$\mathbf{1}\times\mathbf{1}$
T	The Toeplitz matrix.	$\boldsymbol{n}\times\mathbf{1}$
$\mathbf{T_s}$	Sampling time.	$\mathbf{1}\times\mathbf{1}$
$\mathbf{u}$	Input.	$\mathbf{1}\times\mathbf{1}$
$v, V_{\mathbf{max}}$	Measurement noise vector and its upper bound.	$\boldsymbol{m}\times\mathbf{1}$
$w, W_{\mathbf{max}}$	System noise vector and its upper bound.	$\boldsymbol{n}\times\mathbf{1}$
x	State vector.	$\boldsymbol{n}\times\mathbf{1}$
z	Output vector.	$\boldsymbol{m}\times\mathbf{1}$

Table 1 – Nomenclature

978-1-4799-2303-8/13/$31.00 ©2013 IEEE

II. The Smooth Variable Structure Filter For Systems With Fully Ranked Measurement Matrix

In 2007, Habibi presented the original form of the smooth variable structure filter (SVSF) [1]. This filter is a modified version of the variable structure [9] and the extended variable structure [10] filters (VSF and EVSF, respectively). The SVSF has also been extended further and developed in [11-14]. The SVSF is a predictor-corrector filter that is based on the SMC principles. It can be applied to both linear and nonlinear systems. In this research, its application into linear system is considered [1]. However, the findings may also be extended to nonlinear systems.

The process of the SVSF depends on the rank of the measurement matrix (number of independent measurements compared to the number of states). If the measurement matrix has partial rank (number of independent measurements < number of states), the SVSF's gain is calculated by using Luenberger's reduced order technique as discussed later in Section III.

If the system has a full rank measurement matrix (number of independent measurements = number of states), and it is described by equ. 1, then the process is found in Fig 1.

$$\mathbf{x}_k = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_{k-1}\mathbf{u}_{k-1} + \mathbf{w}_{k-1}$$
$$\mathbf{z}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k \tag{1}$$

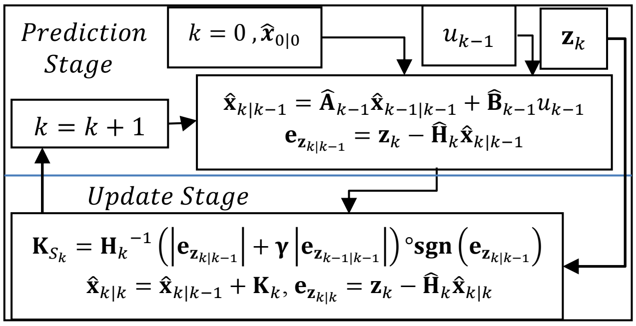

Fig 1: The Smooth Variable Structure Filter

III. The Smooth Variable Structure Filter For Systems With Partially Ranked Measurement Matrix

If the application consists of a system with a partially ranked matrix, then the reduced order algorithm of the SVSF discussed in [1] is used. This method is obtained using the Luenberger's method as follows:

- Assume the measurement matrix is linear, is time invariant and is defined as follows

$$\mathbf{H}_k = \mathbf{H} = [\mathbf{I}_{m\times m} \quad \mathbf{0}_{m\times(n-m)}] \tag{2}$$

- Defining a revised state vector Ξ_k as $\Xi_k = \begin{bmatrix}\mathbf{z}_k \\ \mathbf{y}_{2_k}\end{bmatrix}$, where $\mathbf{y}_{2_k}$ is the hidden states. The system of equ. 1 becomes (assuming no modeling error occurs):

$$\Xi_k = \mathbf{A}_{k-1}\Xi_{k-1} + \mathbf{B}_{k-1}\mathbf{u}_{k-1} + \mathbf{d}_{k-1}$$
$$\mathbf{d}_{k-1} = \begin{bmatrix}\mathbf{d}_{1_{k-1}} \\ \mathbf{d}_{2_{k-1}}\end{bmatrix} = \mathbf{w}_{k-1} - \mathbf{A}_{k-1}\begin{bmatrix}\mathbf{v}_{k-1} \\ \mathbf{0}_{(n-m)\times 1}\end{bmatrix} + \begin{bmatrix}\mathbf{v}_k \\ \mathbf{0}_{(n-m)\times 1}\end{bmatrix} \tag{3}$$

- The estimated states (estimated measurement as well considering the measurement matrix is unity) is then obtained as the following:

$$\hat{\mathbf{z}}_{k|k-1} = \begin{bmatrix}\hat{\mathbf{z}}_{1_{k|k-1}} \\ \hat{\mathbf{z}}_{2_{k|k-1}}\end{bmatrix} = \hat{\mathbf{A}}_{k-1}\begin{bmatrix}\mathbf{z}_{k-1} \\ \hat{\mathbf{y}}_{2_{k-1|k-1}}\end{bmatrix} + \hat{\mathbf{B}}_{k-1}\mathbf{u}_{k-1} \tag{4}$$

- Subtracting equation (4) from (3) gives [1]:

$$\begin{bmatrix}\mathbf{e}_{\mathbf{z}_{k|k-1}} \\ \mathbf{e}_{\mathbf{y}2_{k|k-1}}\end{bmatrix} = \begin{bmatrix}\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1} \\ \mathbf{y}_{2_k} - \hat{\mathbf{y}}_{2_{k|k-1}}\end{bmatrix} \rightarrow$$
$$\begin{bmatrix}\mathbf{e}_{\mathbf{z}_{k|k-1}} \\ \mathbf{e}_{\mathbf{y}2_{k|k-1}}\end{bmatrix} = \hat{\mathbf{A}}_{k-1}\begin{bmatrix}\mathbf{0}_{m\times 1} \\ \mathbf{e}_{\mathbf{y}2_{k-1|k-1}}\end{bmatrix} + \mathbf{d}_{k-1} \tag{5}$$

Where $\hat{\mathbf{A}}_{k-1} = \begin{bmatrix}\hat{\mathbf{A}}_{11} \in \mathbb{R}^{m\times m} & \hat{\mathbf{A}}_{12} \in \mathbb{R}^{m\times(n-m)} \\ \hat{\mathbf{A}}_{21}\mathbb{R}^{(n-m)\times m} & \hat{\mathbf{A}}_{22} \in \mathbb{R}^{(n-m)\times(n-m)}\end{bmatrix}$ and $\mathbf{d}_{k-1}$ represents uncertainties and modeling errors and it is defined as follows:

$$\mathbf{d}_{k-1} = \Delta\mathbf{A}\Xi_{k-1} + \Delta\mathbf{B}\mathbf{u}_{k-1} + \mathbf{w}_{k-1} - \mathbf{A}_{k-1}\begin{bmatrix}\mathbf{v}_{k-1} \\ \mathbf{0}_{(n-m)\times 1}\end{bmatrix} + \begin{bmatrix}\mathbf{v}_k \\ \mathbf{0}_{(n-m)\times 1}\end{bmatrix} \tag{6}$$

The Luenberger method assumes that there are no uncertainties; $\mathbf{d}_{k-1} = \mathbf{0}$, Therefore, equ. (7) could be obtained by rearranging equ. (5) as follows (assume $M = \hat{\mathbf{A}}_{22}\hat{\mathbf{A}}_{12}^{-1}$):

$$\mathbf{e}_{\mathbf{y}2_{k-1|k-1}} = \hat{\mathbf{A}}_{12}^{-1}\mathbf{e}_{\mathbf{z}_{k|k-1}}$$
$$\mathbf{e}_{\mathbf{y}2_{k|k-1}} = \hat{\mathbf{A}}_{22}\hat{\mathbf{A}}_{12}^{-1}\mathbf{e}_{\mathbf{z}_{k|k-1}} \tag{7}$$

- Using the results of equ. 7, the gain is then modified to be as the following, [1]:

$$\mathbf{K}_{SS_k} = \begin{bmatrix}\left(\left|\mathbf{e}_{\mathbf{z}_{k|k-1}}\right| + \boldsymbol{\gamma}_1\left|\mathbf{e}_{\mathbf{z}_{k-1|k-1}}\right|\right)\circ\mathbf{sgn}\left(\mathbf{e}_{\mathbf{z}_{k|k-1}}\right) \\ \left(\left|M\mathbf{e}_{\mathbf{z}_{k|k-1}}\right| + \boldsymbol{\gamma}_2\left|\hat{\mathbf{A}}_{12}^{-1}\mathbf{e}_{\mathbf{z}_{k|k-1}}\right|\right)\circ\mathbf{sgn}\left(M\mathbf{e}_{\mathbf{z}_{k|k-1}}\right)\end{bmatrix} \tag{8}$$

- The rest of the process remain the same as in Fig 1.

This method gives good results as long as the Luenberger conditions are valid. This limits the SVSF with the following:

- The SVSF becomes more likely as an observer rather than a filter. Therefore, the estimate becomes sensitive to noise amplitude.
- The entire method depends on the mapping function $(\hat{\mathbf{A}}_{12})$. This matrix needs to be invertible, and accurately estimated. Otherwise, the accuracy may be affected. However, the stability will not be affected as it depends on $\mathbf{e}_{\mathbf{z}_{k|k-1}}$ which is measured.
- The usage of the inversion operator. This increases the complexity of the calculation, may result in numerical instability, and may results is using a pseudo-inverse operator if the matrix $\hat{\mathbf{A}}_{12}$ is not square.

To improve the performance of this filter, a novel revised version of SVSF referred to as the Toeplitz/Observability SVSF is proposed in the next section.

IV. THE TOEPLITZ/OBSERVABILITY SMOOTH VARIABLE STRUCTURE

In this section, a novel form of the SVSF is developed. The ***Observability matrix*** is a mathematical tool that gives an indication of the possibility to extract the states uniquely from a finite number of measurements' data sets. Assuming a linear time-invariant with ***n*-states** and **one-measurement,** then the Observability matrix is defined as follows, [15-16]:

$$\mathbf{O} = [\mathbf{H}^T \quad \mathbf{A}^T\mathbf{H}^T \quad \dots \quad (\mathbf{A}^{n-1})^T\mathbf{H}^T]^T \tag{9}$$

If the Observability matrix is of full rank, then the system is *observable,* and the states can be uniquely extracted from measurements.

The ***Toeplitz matrix*** is a matrix that has zero elements on its diagonal and all the elements above it. Here it is referred to the matrix that shows the relation between an input segment to a measurement segment. Assuming a linear time-invariant with ***n*-states, one-measurement** and **one-input,** then the Toeplitz matrix is defined as follows, [14-15]:

$$\mathbf{T}_o = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ \mathbf{HB} & 0 & 0 & \cdots & 0 \\ \mathbf{HAB} & \mathbf{HB} & 0 & \cdots & 0 \\ \vdots & \vdots & & & \vdots \\ \mathbf{HA}^{n-2}\mathbf{B} & \mathbf{HA}^{n-3}\mathbf{B} & \dots & \mathbf{HB} & 0 \end{bmatrix} \tag{10}$$

Both of these matrices result from taking a segment of length n from the measurement, writing each measurement equation; i.e. equ. 1, in recursive form as the following:

$$z_k = \mathbf{H}\left(\mathbf{A}^k\mathbf{x}_0 + \sum_{i=0}^{k-1}\left(\mathbf{A}^{k-1-i}(\mathbf{B}u_i + \mathbf{w}_i)\right)\right) + v_k \tag{11}$$

and then staking them in a vector; i.e. $\begin{bmatrix} z_k \\ \vdots \\ z_{k+n-1} \end{bmatrix}$, as the following:

$$\begin{bmatrix} z_k \\ \vdots \\ z_{k+n-1} \end{bmatrix} = \mathbf{T}_o \begin{bmatrix} u_k \\ \vdots \\ u_{k+n-1} \end{bmatrix} + \begin{bmatrix} v_k \\ \vdots \\ v_{k+n-1} \end{bmatrix} + \mathbf{T}_w \begin{bmatrix} \mathbf{w}_k \\ \vdots \\ \mathbf{w}_{k+n-1} \end{bmatrix} + \mathbf{O}\mathbf{x}_k \tag{12}$$

Where $\mathbf{T}_w$ is another Toeplitz matrix that maps the system noise to the measurements.

By rearranging equ. 12, the states could be obtained as follows (assuming $G = \begin{bmatrix} v_k \\ \vdots \\ v_{k+n-1} \end{bmatrix} + \mathbf{T}_w \begin{bmatrix} \mathbf{w}_k \\ \vdots \\ \mathbf{w}_{k+n-1} \end{bmatrix}$):

$$\mathbf{x}_k = \mathbf{O}^{-1}\left(\begin{bmatrix} z_k \\ z_{k+1} \\ \vdots \\ z_{k+n-1} \end{bmatrix} - \mathbf{T}_o \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+n-1} \end{bmatrix} - G\right) \tag{13}$$

The Observability matrix should be invertible, otherwise equ. 13 is invalid and the states or their estimation cannot be obtained. Using equ. 13, an estimated state vector, $\hat{\mathbf{x}}_{TO_k}$, could be obtained as follows:

$$\hat{\mathbf{x}}_{TO_k} = \widehat{\mathbf{O}}^{-1}\left(\begin{bmatrix} z_k \\ z_{k+1} \\ \vdots \\ z_{k+n-1} \end{bmatrix} - \widehat{\mathbf{T}}_o \begin{bmatrix} u_k \\ u_{k+1} \\ \vdots \\ u_{k+n-1} \end{bmatrix}\right) \tag{14}$$

Where $\widehat{\mathbf{O}}$ and $\widehat{\mathbf{T}}_o$ are the estimated Observability and Toeplitz matrices, respectively. If the these matrices are well estimated, then the error in the estimated states, which will be referred to as the alternative measurement vector, is equal to the following:

$$\mathbf{x}_k - \hat{\mathbf{x}}_{TO_k} = -\widehat{\mathbf{O}}^{-1}([v_k \quad v_{k+1} \quad \cdots \quad v_{k+n-1}]^T + \mathbf{T}_w[\mathbf{w}_k \quad \mathbf{w}_{k+1} \quad \cdots \quad \mathbf{w}_{k+n-1}]^T) \tag{15}$$

According to [17], if the system is written in its Observability canonical form, and at least the first state is measured, then the Observability and the Toeplitz matrices are independent on the system parameters. The Observability canonical form satisfies the following conditions:

1- The input matrix has zero elements except for the last row.
2- The system matrix has the following form:

$$\mathbf{A}_k = \begin{bmatrix} 1 & T_s & 0 & \cdots & 0 \\ 0 & 1 & Ts & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & T_s \\ a_1 & a_2 & a_3 & \dots & a_n \end{bmatrix} \tag{16}$$

3- Each measurement is related to only one of the states.

In this paper, the system in Observability canonical form is considered. Therefore, the vector $\hat{\mathbf{x}}_{TO_k}$ is simply extracted from the measurement and the input, and it contains the vector $\mathbf{x}_k$'s information blurred with measurement and system noise, and their derivatives. Therefore, the vector $\hat{\mathbf{x}}_{TO_k}$ can be used to compensate the missing $(n-m)$ measurements for the SVSF as follows:

$$\hat{\mathbf{z}}_{TO_k} = \bar{\bar{\mathbf{H}}}_k\hat{\mathbf{x}}_{TO_k} \tag{17}$$

Where $\hat{\mathbf{z}}_{TO_k}$ is the alternative measurement vector and $\bar{\bar{\mathbf{H}}}_k$ is the unity matrix. This solve the issue in applying the SVSF into systems with partially ranked measurement matrix. The alternative measurement vector has dimensions of $\hat{\mathbf{z}}_{TO_k} \in \mathbb{R}^{n\times 1}$ (similar to measurement of a system with fully ranked measurement matrix).

The proposed SVSF has the same structure as in Fig 1. However, the gain is calculated as the following (assuming $\bar{\bar{\mathbf{H}}}_k = I_{n\times n}$):

$$\mathbf{K}_{\mathbf{e}_{TO_k}} = \left(\left|\mathbf{e}_{\mathbf{z}_{TO_{k|k-1}}}\right| + \boldsymbol{\gamma}\left|\mathbf{e}_{\mathbf{z}_{TO_{k-1|k-1}}}\right|\right)\circ\mathbf{sgn}\left(\mathbf{e}_{\mathbf{z}_{TO_{k|k-1}}}\right) \tag{18}$$

Where $\mathbf{e}_{\mathbf{z}_{TO_{a|b}}} = \hat{\mathbf{z}}_{TO_a} - \hat{\mathbf{z}}_{a|b}$.

V. THE APPLICATION OF TOEPLITZ/OBSERVABILITY SMOOTH VARIABLE STRUCTURE FILTER INTO HYDROSTATIC ACTUATOR

The Toeplitz/Observability SVSF has been tested on an electro-hydrostatic actuator proposed in [3-5]. The system is a third order system with the following system, input and measurement matrices:

$$A = \begin{bmatrix} 1 & T_s & 0 \\ 0 & 1 & T_s \\ 0 & -\omega_n{}^2 T_s & 1 - 2\xi\omega_n T_s \end{bmatrix} \qquad 19$$

$$B = [0 \quad 0 \quad BT_s]^T$$

$$H = [1 \quad 0 \quad 0]$$

Where $\omega_n = 360\ H_z$, $B = 30\ \frac{m}{sec \times rad}$ and $\xi = 0.4$. The sampling time is 0.001 sec. The measurement and the system noise signals are white noise with a noise-to-signal ratio of 5% with respect to the state amplitudes. The results obtained from the proposed method (TOSVSF) is compared to the original SVSF described in section III. The coefficient matrix γ has a value of $\gamma = 0.1 \times I_{3\times3}$. The input has as a signal is shown in Fig 2. The same simulation is performed again assuming that the model parameters are inaccurate and with values of ω_n, B and ξ are equal to $252\ H_z$, $21.5\ \frac{m}{sec \times rad}$ and 0.28, respectively. The modeling errors are almost 30% of their actual values.

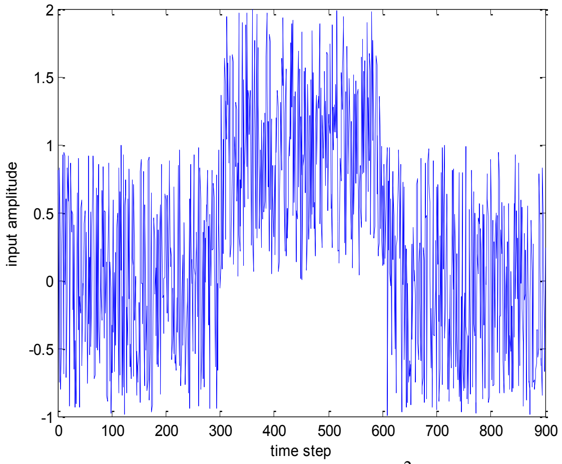

Fig 2: The input signal (rad/s^2).

The results of the Toeplitz/Observability SVSF compared to the original SVSF for a system model without uncertainties is presented in table 2 and the Figs 2, 3 and 4. These results show that both methods give similar good performance as long as the conditions in section III and IV are valid. However, once uncertainties are injected in the filter models, the Toeplitz/Observability SVSF gives better results than the original SVSF (the RMSE of x_1, x_2 and x_3 obtained by the Toeplitz/Observability SVSF are smaller than their corresponding values obtained by the original SVSF as shown in table 3). Both methods show robustness and stability behavior. These results are shown in Figs 5, 6 and 7. Comparing tables 2 and 3. Three distinct observations have been noticed:

RMSE	Original SVSF	Toeplitz /Observability SVSF
Position	2.6×10^{-10}	2.6×10^{-10}
Velocity	6.8×10^{-7}	4.6×10^{-7}
Acceleration	3.8×10^{-4}	8×10^{-4}

Table 2 – A comparison between the original SVSF compared to the newly proposed version when no modeling uncertainties occurred.

- The Toeplitz/Observability SVSF does not affect with modeling uncertainties and the performance remains the same.
- The Toeplitz/Observability SVSF and the original SVSF give the same performance for the first state, which is not sensitive to the modeling uncertainties.
- Increasing modeling uncertainties, increases the RMSE for the original SVSF although it remains stable.

RMSE	Original SVSF	Toeplitz /Observability SVSF
Position	2.6×10^{-10}	2.6×10^{-10}
Velocity	3.1×10^{-5}	4.6×10^{-7}
Acceleration	0.0314	0.0008

Table 3 – A comparison between the original SVSF compared to the newly proposed version when modeling uncertainties are injected.

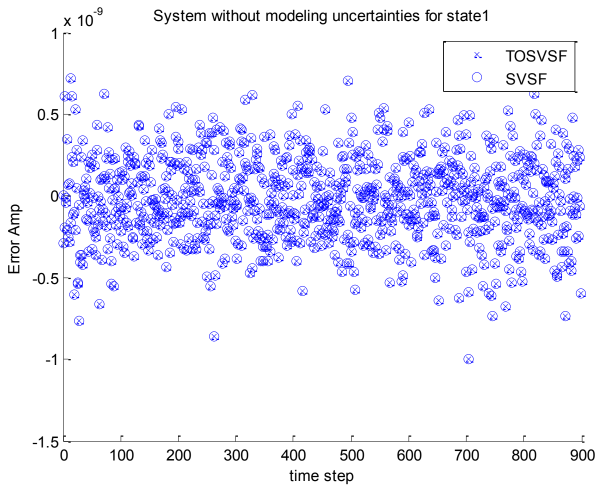

Fig 3: Error in estimating the first state while no modeling error is present (m).

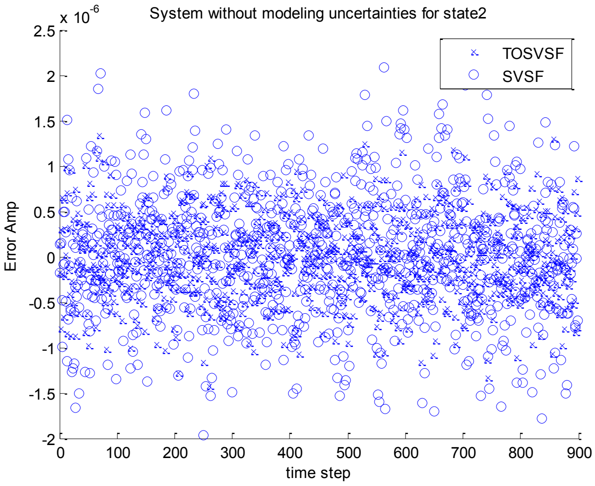

Fig 4: Error in estimating the second state while no modeling error is present (m/s).

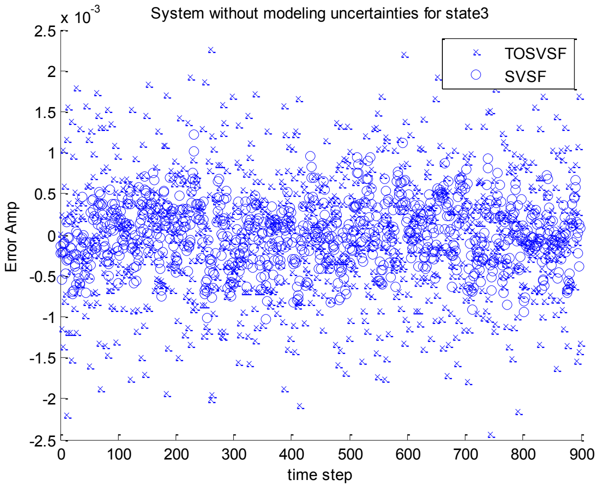

Fig 5: Error in estimating the third state while no modeling error is present (m/s^2).

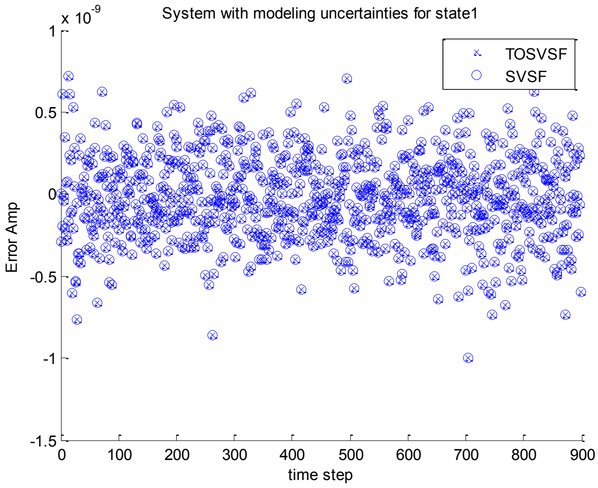

Fig 6: Error in estimating the first state when modeling error is present (m).

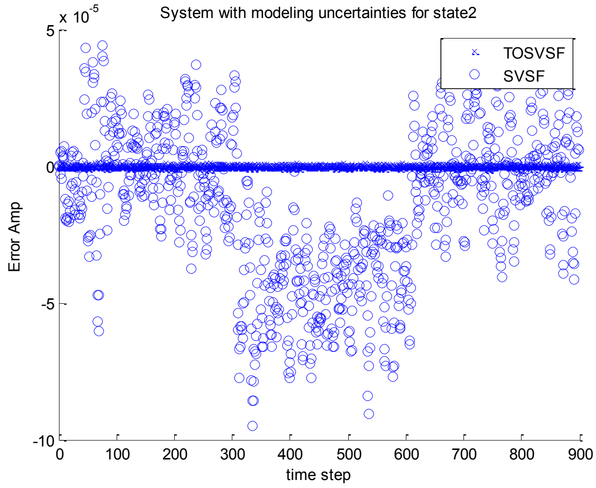

Fig 7: Error in estimating the second state when modeling error is present (m/s).

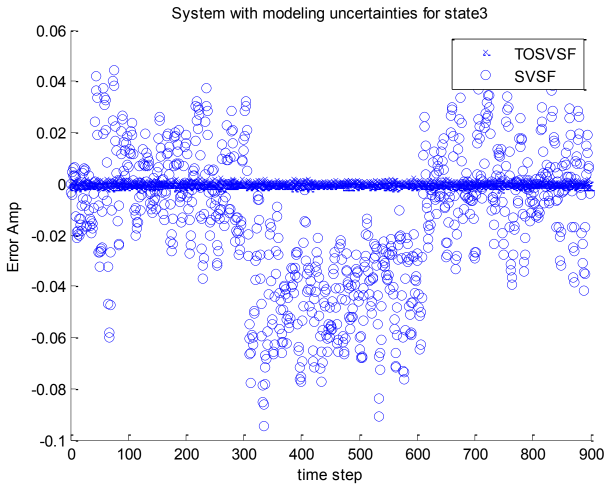

Fig 8: Error in estimating the third state when modeling error is present (m/s^2).

VI. CONCLUSION AND FUTURE WORK

This paper is concerned with the development of robust estimation techniques for their application to fault detection. Due to its robustness, the SVSF was considered. The original SVSF has some limitations when applied to systems that have measurement matrices without full rank. A novel algorithm referred to as the Toeplitz/Observability SVSF is proposed in this paper to overcome these limitations, and is referred to as the TOSVSF. If the system is described in its Observability canonical form, the measurement matrix is known and at least the first state is measured, then the proposed method gives a very good performance. It is independent of the modeling errors that may be present. The Toeplitz/Observability SVSF was tested on an electro-hydrostatic actuator and compared to the original SVSF. It demonstrated superior performance in the presence of modeling uncertainties. Future work will include the application of the proposed filter on an experimental setup.

REFERENCES

[1] Habibi, S. (2007). “The Smooth Variable Structure Filter”. Proceedings of the IEEE, 95(5), 1026 - 1059.

[2] Habibi, S., Burton, R., & Chinniah, Y. (2002). “Estimation Using a New Variable Structure Filter”. Proceedings of the American Control Conference, 4, pp. 2937 - 2942. Anchorage, AK, United states.

[3] Habibi, S., Burton, R., & Sampson, E. (2006). “High Precision Hydrostatic Actuation Sytems for Micro and Nanomanipulation of Heavy Loads”. Transactions of the ASME, 128, 778 - 787.

[4] Habibi, S., & Burton, R. (2007).” Parameter identification for a high-performance hydrostatic actuation system using the variable structure filter concept”. Journal of Dynamic Systems, Measurement and Control, Transactions of the ASME, 129(2), 229 - 235.

[5] Habibi, S., & Goldenberg, A. (2000).” Design of a New High-Performance ElectroHydraulic Actuator”. IEEE/ASME Transactions on Mechatronics, 5(2), 158 - 164.

[6] Van Der Heidan, F., Duin, R., de Ridder, D., & Tax, C. (2004).” Classification, Parameter Estimation And State Estimation – An Engineering Approach Using MATLAB”. England: John Wiley & Sons, Ltd.

[7] Grewal, M., & Andrews, A. (2001). “Kalman Filtering: Theory and Practice Using Matlab” (Second Edition ed.). USA: John Wiley & Sons Inc.

[8] Haykin, S. (2002). “Adaptive Filtering Theory”. Prentice Hall.

[9] Habibi, S., & Burton, R. (2003). “The Variable structure Filter”. Journal of Dynamic Systems, Measurement, and Control, 125, 287 - 293.

[10] Habibi, S. (2006). “The Extended Variable Structure Filter”. Journal of Dynamic Systems, Measurement and Control, Transactions of the ASME, 128(2), 341 - 351.

[11] Gadsden, S. A. (2011). “Smooth Variable Structure Filtering: Theory and Applications”, McMaster University, Doctoral dissertation, 2011.

[12] Gadsden, S. A., & Habibi, S. R. (2013). “A New Robust Filtering Strategy for Linear Systems”. ASME Journal of Dynamic Systems, Measurement and Control, 135(1).

[13] Gadsden, S. A., Song. Y., & Habibi, S. R. (2013). “Novel Model-Based Estimators for the Purposes of Fault Detection and Diagnosis”. IEEE/ASME Transactions on Mechatronics, 18(4).

[14] Al-Shabi, M., Gadsden, S. A., & Habibi, S. R. (2013). “Kalman Filtering Strategies Utilizing the Chattering Effects of the Smooth Variable Structure Filter”. Signal Processing, 93(2).

[15] Hsu, H. (1995). “Signals And Systems”. Schaum’s Series, McGRAW-HILL.

[16] Kailath, T. (1980). “Linear Systems”. Prentice-Hall, In., Englewood Cliffs.

[17] Al-Shabi, M. (2011). “The General Toeplitz/Observability Smooth Variable Structure Filter”, McMaster University, Doctoral dissertation, 2011.