A Sliding Mode Controller Based on the Interacting Multiple Model Strategy

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ABSTRACT

Sliding mode controllers (SMCs) are a form of variable structure control, which utilizes a discontinuous switching plane along some desired trajectory. This plane is often referred to as a sliding surface, in which the objective is to keep the state values along this surface by minimizing the state errors (between the desired trajectory and the estimated or actual values). Ideally, if the state value is off or away from the surface, a switching gain would be used to push the state towards the sliding surface. This switching brings an inherent amount of stability; and as such, SMCs have become a popular control strategy for systems with modelling uncertainty. Furthermore, some systems behave according to a number of different models (or operating regimes). In these scenarios, it is desirable to implement adaptive estimation algorithms, which 'adapt' themselves to certain types of uncertainties or models in an effort to minimize the state estimation error and improve tracking performance. One type of adaptive estimation technique includes the multiple model (MM) algorithm and its interactive form (IMM). For the MM methods, a Bayesian or probability based framework is used. Essentially, based on some prior probabilities of each model being correct (i.e., the system is behaving according a finite number of modes) the corresponding updated probabilities are calculated. This paper introduces a new type of sliding mode controller based on the interacting multiple model strategy (SMC-IMM). The SMC-IMM method is applied on an electrohydrostatic actuator, and the results are compared with the standard SMC strategy.

1 Introduction

Variable structure systems with sliding mode control have gained international attention following Utkin's paper [1] and book [2]. Since then, there has been considerable progress in this field. Many applications have been reported, for example, an SMC-based controller was applied to a tracking problem with a three degree of freedom manipulator [3]. Simulation results demonstrate that SMC provides perfect tracking capabilities with minimum tracking error.

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A continuous SMC control strategy has been implemented in a Cartesian pneumatic robot in [4]. In this research, the performance of the SMC is compared to the linear proportional-velocity-acceleration (PVA) controller. The SMC provides an improvement over the PVA regarding trajectory tracking precision with minimal tuning effort. In addition, the discontinuous nature of the SMC helps in providing a measure of friction compensation. In this research, the importance of the boundary layer approach to eliminating chatter with SMC was also highlighted. A robust SMC and adaptive control strategy has been applied to a two-degrees of freedom robotic arm in [5]. The robotic arm was required to track a circular trajectory with a period of 4 sec. The adaptive control is used to estimate unknown disturbances and environmental factors while the SMC is used to overcome the dynamic system model uncertainties. Experimental results show the robustness of the proposed SMC to track trajectories with bounded errors. An SMC strategy with an integral switching surface in an electro-pneumatic rotary actuator has been proposed in [6]. First, a nonlinear model of the servo drive is attained, the developed model was found to be nonlinear with respect to the state variables and the control input. Accordingly, model linearization and transformation was carried out with respect to a new control input in order to apply the nonlinear discontinuous controller. Experimental results show the advantage of the SMC with integral switching surface over standard SMC in terms of tracking error.

A recently proposed robust state and parameter estimation strategy referred to as the variable structure filter (VSF) combined with the SMC strategy has been proposed in [7]. The proposed combined strategy is referred to as SMCF and it results in increased robustness in both, control and state estimation given bounded parametric uncertainties and noise. In addition, the proposed strategy can achieve high regulation rates or short settling time. The proposed strategy has been applied to a high precision electro-hydrostatic actuator (EHA) system. From the plant, some of the outputs can be physically measured and they might contain measurement uncertainties. These outputs are fed into the VSF to estimate the full set of internal system states. These estimated states may be integrated into a trajectory following sliding mode controller. The sliding mode controller uses information contained in the state estimates besides the reference input to produce a discontinuous control signal to attain trajectory following. One of the major considerations in the SMC and the VSF are their chattering phenomenon due to the discontinuous action. Chattering can be removed by using a smoothing boundary layer [2].

A fuzzy sliding mode controller for flight simulator servo system is presented in [8]. The stand-alone SMC provides strong robustness against parameter variations and external disturbances but the chattering produced by SMC limits its application to practical systems. In this research, by using fuzzy control, the chattering can be effectively reduced. At the same time, the optimal fuzzy reasoning is adopted in fuzzy control. Simulation and real system experiment results confirm the effectiveness of the proposed control method. An adaptive sliding mode control for regenerative braking in hybrid vehicles is presented in [9]. In this research, a nonlinear control strategy based on adaptive sliding-mode control (ASMC) is implemented to tackle the problem of engine torque control during regenerative braking mode. The ASMC-based controller combines the partially known inverse dynamic model of the engine. Numerical and experimental results show performance enhancement of the proposed strategy compared to a high-gain PID controller and to the conventional smooth sliding mode controller regarding tracking error, chattering elimination capability, and robustness to disturbance.

In this paper, a new type of sliding mode controller based on the interacting multiple model strategy (SMC-IMM) is proposed. The SMC-IMM method is applied on an electrohydrostatic actuator, and the results are compared with the standard SMC strategy. The paper is organized as follows. Section 2 describes SMC in more detail, followed by a section on the IMM method. Section 4 provides an overview of the SMC-IMM method. The problem statement and results are provided in Section 5. The main findings of the paper are then summarized in the conclusion.

2 Sliding Mode Control Theory

Sliding mode control (SMC) is known for its ability to provide robustness and stability in the presence of uncertainties. Misawa proposed a discrete sliding mode control method for nonlinear systems with uncertainties that do not satisfy the matching condition [10]. Later in [11], this design was extended for linear systems and was reported to provide good results. Wang adopted Misawa's control technique to design a controller for an EHA prototype for trajectory tracking applications [12]. Using an accurate model of the EHA's nonlinear friction in the controller design, his results indicated that the controller was able to provide precise tracking while suppressing the chatter in acceleration and velocity profiles. In [13], Wang's controller is shown susceptible to high frequency limit cycle oscillations when the uncertainty associated with the friction characteristics increased. This oscillation is more noticeable in the acceleration profile. The simulation results clearly show that chattering can be eliminated by expanding the SMC boundary layer thickness, at the expense of reducing the positional precision [13].

Consider a single input linear dynamical system as follows:

$$x_{k+1} = \hat{F} x_k + \hat{G} u_k + w_k \tag{2.1}$$

where A and B are the system and input matrices respectively, x refers to the state vector, u is the system input, and w refers to uncertainties present in the system (e.g., modelling errors or system noise). The uncertainties w are assumed to be bound such that:

$$\gamma \geq |Cw| \tag{2.2}$$

In trajectory tracking mode, the objective is to force the system to follow a desired trajectory x_d. This objective can be restated as driving the tracking error ($e_k = x_{d,k} - x_k$) as close as possible to zero. A sliding manifold is defined as:

$$\Sigma = \{e_k | s_k = Ce_k = 0\} \tag{2.3}$$

where C is the sliding surface parameter vector, and with a smoothing boundary layer defined by:

$$\Psi = \{e_k || s_k| = |Ce_k| \leq \psi\} \tag{2.4}$$

In this paper, the control strategy will be based on Misawa’s SMC control structure [10,11]. The control input may be defined as follows [10,11]:

$$u_k = u_{eq,k} - (C\hat{G})^{-1} s_k + (C\hat{G})^{-1} K_c sat\left(\frac{s_k}{\psi}\right) \tag{2.5}$$

where u_{eq} refers to the equivalent control component, and the remainder is the switching control component. The follow is also defined:

$$u_{eq,k} = (C\hat{G})^{-1} C(x_{d,k+1} - \hat{F} x_k) \tag{2.6}$$

$$K_c = \gamma + 2\epsilon,\ \psi \geq \gamma + \epsilon \tag{2.7}$$

$$sat\left(\frac{s_k}{\psi}\right) = \begin{cases} +1\ if\ s > \psi \\ \frac{s}{\psi}\ if\ |s| \leq \psi \\ -1\ if\ s < \psi \end{cases} \tag{2.8}$$

where ϵ is an arbitrary positive constant. However, a major drawback in Misawa's SMC derivation is the assumption that the uncertainties w are bounded by a constant [10,11]. This assumption is not realistic since w is inherently dependent on the system states. Accordingly, a new gain calculation is required, where a variable gain may be used to compensate for the uncertainties. A variable gain and boundary layer were introduced in [14], and are defined as follows:

$$K_c = C\tilde{F}_{max}|e_k| + C\tilde{F}_{max}|x_{d,k}| + C\tilde{G}_{max} u_{max} + C v_{max} + 2\epsilon \tag{2.9}$$

$$\psi = C\tilde{F}_{max}|e_k| + C\tilde{F}_{max}|x_{d,k}| + C\tilde{G}_{max} u_{max} + C v_{max} + \epsilon \tag{2.10}$$

where $\tilde{F}_{max}$ and $\tilde{G}_{max}$ are the upper bounds on the uncertainties in the system matrix and the input matrix respectively, u_{max} is the maximum allowable input, and v_{max} is the maximum noise amplitude. In trajectory tracking, the system may be forced to follow some desired trajectory by implementing (2.5) through (2.10).

3 The Interacting Multiple Model Strategy

In nature, many systems behave according to a number of different models (modes, or operating regimes). For example, in target tracking, a target may travel straight (i.e., uniform motion) or turn (i.e., undergo a coordinated turn) [15]. Furthermore, a system may experience different types of noises (i.e., white or 'coloured') [16]. In these scenarios, it is desirable to implement adaptive estimation algorithms, which 'adapt' themselves to certain types of uncertainties or models in an effort to minimize the state estimation error [15]. One type of adaptive estimation technique includes the 'multiple model' (MM) algorithm [17]; which include the following: static MM [18], dynamic MM [15], generalized pseudo-Bayesian (GPB) [19,20,21,22], and the interacting multiple model (IMM) [15,23,24]. For the MM methods, a Bayesian framework is used (i.e., probability based). Essentially, based on some prior probabilities of each model being correct (i.e., the system is behaving according a finite number of modes), the corresponding updated probabilities are calculated and implemented [15].

Throughout this paper, it will be assumed that all of the models are linear with the presence of Gaussian noise; however, nonlinear models could be used via linearization [15]. Each MM method requires estimation of the states and their corresponding probability. The most popular strategy that has been implemented in the MM framework remains the Kalman filter (KF), and is referred to as the IMM-KF [16]. The interacting multiple model (IMM) estimation algorithm is conceptually requires r number of filters (such as the KF) that operate in parallel [15]. The state estimate is calculated under each possible current model, with a mixed initial condition (i.e., a different combination of the previous model-conditioned estimates) [15]. Furthermore, according to and as presented in [15,24], the input to the filter matched to M_j is obtained from an interaction of the r filters, which consists of the mixing of the estimates $\hat{x}_{i,k|k}$ and weightings $\mu_{i|j,k|k}$ (mixing probabilities). This is equivalent to merging taking place at the beginning of each estimation cycle, which limits the number of filters to r [24]. The IMM strategy has shown to be very effective, and is more computationally efficient than other multiple model algorithms [15]. The following figure helps to explain the IMM method more effectively.

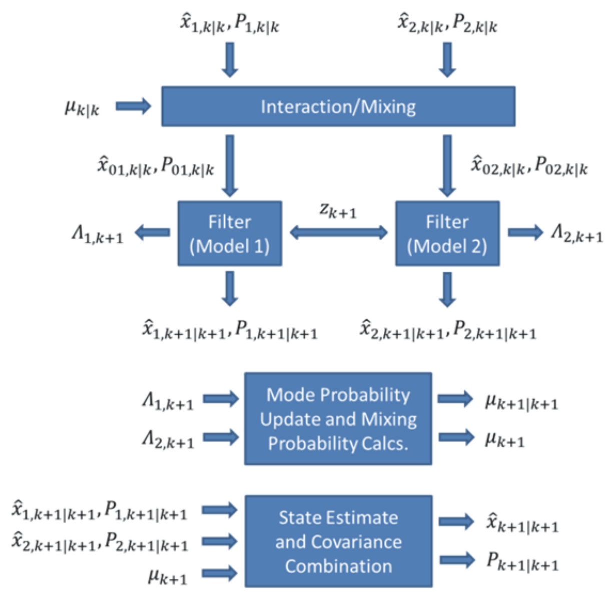

Figure 1: IMM estimator for two models (adapted from [15,25])

The IMM estimator consists of five main steps: calculation of the mixing probabilities, mixing stage, mode-matched filtering, mode probability update, and state estimate and covariance combination. The first step involves calculating the mixing probabilities (i.e., the probability of the system currently in mode i, and switching to mode j at the next step). These are calculated using the following two equations [15]:

$$\mu_{i|j,k|k} = \frac{1}{\bar{c}_j} p_{ij} \mu_{i,k} \tag{3.1}$$

$$\bar{c}_j = \sum_{i=1}^{r} p_{ij}\, \mu_{i,k} \tag{3.2}$$

The mixing probabilities $\mu_{i|j,k|k}$ are used in the mixing stage, next. In addition to the mixing probabilities, the previous mode-matched states $\hat{x}_{i,k|k}$ and covariance's $P_{i,k|k}$ are also used to calculate the mixed initial conditions (states and covariance) for the filter matched to M_j. The mixed initial conditions are found respectively as follows [15]:

$$\hat{x}_{0j,k|k} = \sum_{i=1}^{r} \hat{x}_{i,k|k}\mu_{i|j,k|k} \tag{3.3}$$

$$P_{0j,k|k} = \sum_{i=1}^{r} \mu_{i|j,k|k} \left\{P_{i,k|k} + \left(\hat{x}_{i,k|k} - \hat{x}_{0j,k|k}\right)\left(\hat{x}_{i,k|k} - \hat{x}_{0j,k|k}\right)^T\right\} \tag{3.4}$$

The next step involves mode-matched filtering, which involves using (3.3) and (3.4) as inputs to the filter matched to M_j. Each filter also uses the measurement z_{k+1} and input to the system u_k (if any). The likelihood functions are calculated for each mode-matched filter as follows [15]:

$$\Lambda_{j,k+1} = \mathcal{N}\left(z_{k+1}; \hat{z}_{j,k+1|k}, S_{j,k+1}\right) \tag{3.5}$$

Equation (3.5) may be solved by each filter as follows [15,16]:

$$\Lambda_{j,k+1} = \frac{1}{\sqrt{\left|2\pi S_{j,k+1}\right|_{Abs}}} exp\left(\frac{-\frac{1}{2} e_{j,z,k+1|k}^T e_{j,z,k+1|k}}{S_{j,k+1}}\right) \tag{3.6}$$

Utilizing the likelihood functions from each filter, the mode probability may be updated by [15]:

$$\mu_{j,k} = \frac{1}{c}\Lambda_{j,k+1} \sum_{i=1}^{r} p_{ij}\, \mu_{i,k} \tag{3.7}$$

where the normalizing constant is defined as [15]:

$$c = \sum_{j=1}^{r} \Lambda_{j,k+1} \sum_{i=1}^{r} p_{ij}\, \mu_{i,k} \tag{3.8}$$

Finally, the overall state estimates (3.9) and covariance (3.10) are calculated. However, note that for this paper, one is mainly interested in utilizing (3.7) for determining the system behavior in an effort to improve controller accuracy.

$$\hat{x}_{k+1|k+1} = \sum_{j=1}^{r} \mu_{j,k+1} \hat{x}_{j,k+1|k+1} \tag{3.9}$$

$$P_{k+1|k+1} = \sum_{j=1}^{r} \mu_{j,k+1} \Big\{ P_{j,k+1|k+1} + \left(\hat{x}_{j,k+1|k+1} - \hat{x}_{k+1|k+1}\right)\left(\hat{x}_{j,k+1|k+1} - \hat{x}_{k+1|k+1}\right)^T \Big\} \tag{3.10}$$

Equations (3.1) through (3.10) summarize the IMM estimator strategy, and are used recursively. Note that (3.9) and (3.10) are used for output purposes only, and are not part of the algorithm recursions [15]. The IMM strategy has successfully been applied to a number of estimation problems [26]; ranging from target tracking in a traffic controller setting [27] to fault detection and diagnosis [28,29].

4 An SMC Based on the IMM Strategy

In this paper, it is proposed that combining the SMC method (Section 2) with the IMM strategy (Section 3) will improve the overall trajectory tracking accuracy, particularly in systems that are not well defined. The basic principle and concept of the SMC-IMM strategy may be summarized by the following figure.

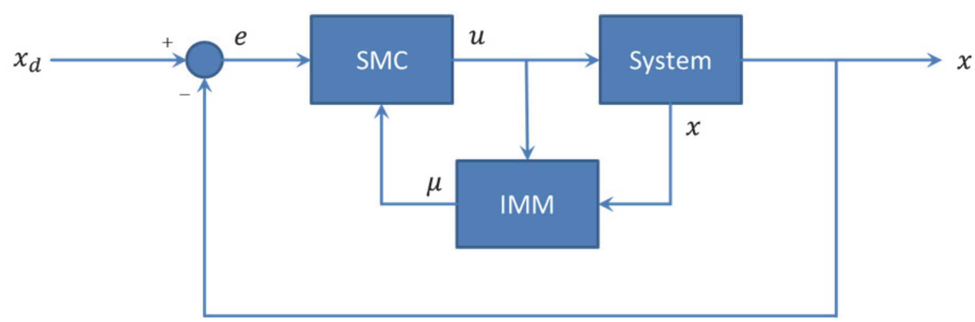

Figure 2: Proposed SMC-IMM strategy

In the SMC-IMM strategy, the SMC utilizes the tracking error e and the mode likelihood probability μ (a value between 0 and 1), to generate a 'weighted' system input u. The IMM requires the system input and the output from the system x (or z if using measurements) in order to calculate the mode probability as per (3.7). If the system is being measured, a Kalman filter (KF) or smooth variable structure filter (SVSF) may be implemented to reduce the effects of unwanted noise [25]. Essentially, the SMC-IMM strategy utilizes the IMM mode probabilities to formulate 'weighted' system and input matrices in an effort to capture the actual dynamics of the system. In this case, the controller input is modified as follows:

$$u_k = u_{eq,k} - \left(C\hat{G}_{\mu,k}\right)^{-1} s_k + \left(C\hat{G}_{\mu,k}\right)^{-1} K_c sat\left(\frac{s_k}{\psi}\right) \tag{4.1}$$

$$u_{eq,k} = \left(C\hat{G}_{\mu,k}\right)^{-1} C\left(x_{d,k+1} - \hat{F}_{\mu,k} x_k\right) \tag{4.2}$$

where $\hat{G}_{\mu,k} = \sum_{i=1}^{r} \mu_{i,k} \hat{G}_i$ and $\hat{F}_{\mu,k} = \sum_{i=1}^{r} \mu_{i,k} \hat{F}_i$. Recall that r refers to the number of operating modes (e.g., normal operation, and various levels of faults). Equations (4.1) and (4.2) are used with (2.9) and (2.10) to generate the system input required to force the system to follow the desired trajectory.

5 Simulation Problem and Results

5.1 Problem

In this paper, an electrohydrostatic actuator (EHA) was studied. An EHA is an emerging type of actuator typically used in the aerospace industry. EHAs are self-contained units comprised of their own pump, hydraulic circuit, and actuating cylinder [30]. The main components of an EHA include a variable speed motor, an external gear pump, an accumulator, inner circuitry check valves, a cylinder (or actuator), and a bi-directional pressure relief mechanism. A mathematical model for the EHA has been described in detail in [14,25]. For the purposes of this paper, only the main state space equations will be explored. The input to the system is the rotational speed of the pump ω_p, with typical units of rad/s. In this setup, the sample rate for this simulation was defined as $T = 0.1\ ms$. The state space equations are defined as follows:

$$x_{1,k+1} = x_{1,k} + T x_{2,k} + T w_{1,k} \tag{5.1}$$

$$x_{2,k+1} = x_{2,k} + T x_{3,k} + T w_{2,k} \tag{5.2}$$

$$\begin{aligned} x_{1,k+1} = {} & \left[1 - T\left(\frac{BV_0 + M\beta_e L}{MV_0}\right)\right] x_{3,k} - T\frac{(A^2 + BL)\beta_e}{MV_0} x_{2,k} \\ & - T\left[\frac{2B_2 V_0 x_{2,k} x_{3,k}}{MV_0} + \frac{\beta_e L\left(B_2 x_{2,k}^2 + B_0\right)}{MV_0}\right] sign(x_2, k) \\ & + T\frac{AD_p\beta_e}{MV_0} u_k + T w_{3,k} \end{aligned} \tag{5.3}$$

Note that A refers to the piston cross-sectional area, $B_{\#}$ represents the load friction present in the system, β_e is the effective bulk modulus (i.e., the 'stiffness' in the hydraulic circuit), D_p refers to the pump displacement, L represents the leakage coefficient, M is the load mass (i.e., weight of the cylinders), and V_0 is the initial cylinder volume. The values used to obtain a linear normal operating model are summarized in the appendix. Two more models were created based on a severe friction fault (the friction was increased 3 times) and a severe leakage fault (the leakage coefficient was increased 4 times). The normal, friction fault, and leakage fault system matrices are respectively defined as follows:

$$F_1 = \begin{bmatrix} 1 & 0.0001 & 0 \\ 0 & 1 & 0.0001 \\ 0 & -41.0258 & 0.6099 \end{bmatrix} \tag{5.4}$$

$$F_2 = \begin{bmatrix} 1 & 0.0001 & 0 \\ 0 & 1 & 0.0001 \\ 0 & -51.8627 & 0.2226 \end{bmatrix} \tag{5.5}$$

$$F_3 = \begin{bmatrix} 1 & 0.0001 & 0 \\ 0 & 1 & 0.0001 \\ 0 & -73.5364 & 0.6015 \end{bmatrix} \tag{5.6}$$

Note that all three input gain matrices remained the same, and were calculated as follows:

$$G = \begin{bmatrix} 0 \\ 0 \\ 0.0135 \end{bmatrix} \tag{5.7}$$

The important SMC parameters were defined by:

$$C = [2500 \quad 100 \quad 1] \tag{5.8}$$

$$\tilde{F}_{max} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 8.6695 & 0.3099 \end{bmatrix} \tag{5.9}$$

Note also that artificial system and measurement noise was added to the simulation problem to make it more challenging. The zero-mean Gaussian noise was generated using system and measurement noise covariance's Q and R which were diagonal matrices with elements equal to 1×10^{-6}. Furthermore, even when the system was operating 'normally', there was still a modelling error of 20% added for both controllers (SMC and SMC-IMM) to overcome. The desired position, velocity, and acceleration trajectories are shown in the following three figures.

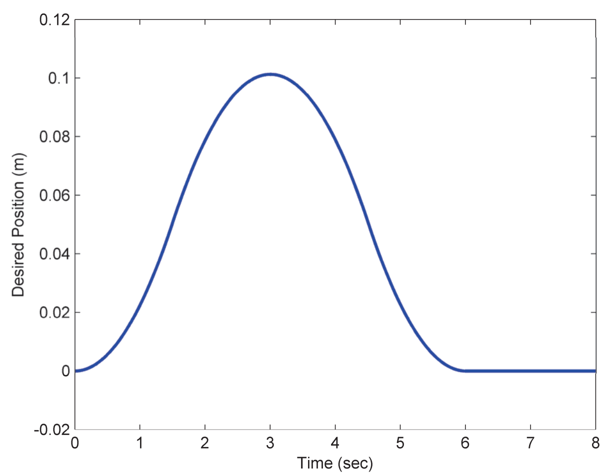

Figure 3: Desired EHA position trajectory

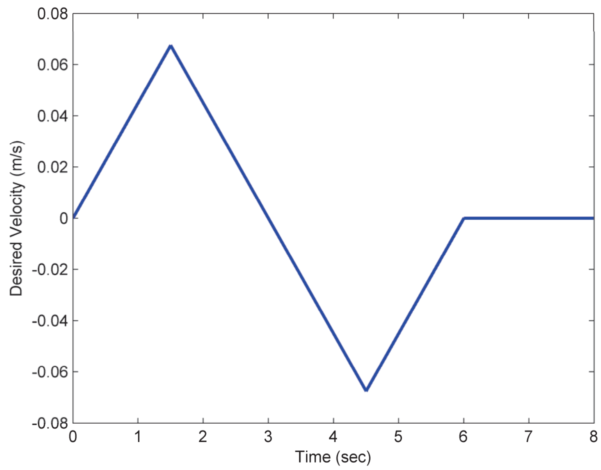

Figure 4: Desired EHA velocity trajectory

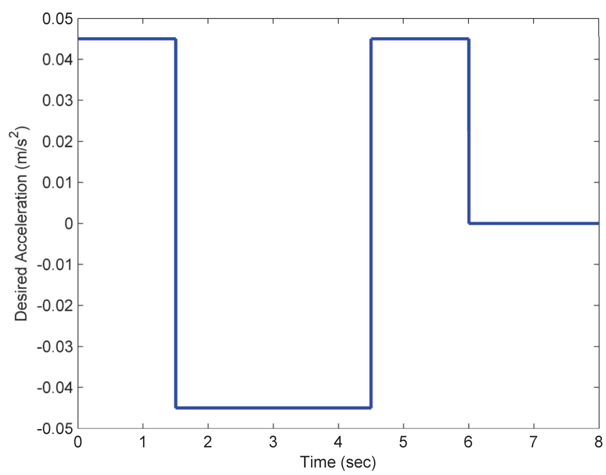

Figure 5: Desired EHA acceleration trajectory

Note that for the first 1.5 second, the system behaved normally. A friction fault was injected at 1.5 seconds and lasted for 1.5 seconds. At 3 seconds, a 1.5 second leakage fault was implemented. At 4.5 seconds, the leakage fault was removed and a friction fault was injected again. The system operated normally during the last two seconds.

5.2 Results

The results of implementing the standard SMC and proposed SMC-IMM on the EHA are shown in this subsection. As an example, the following figure shows the normal mode probability calculated by the IMM.

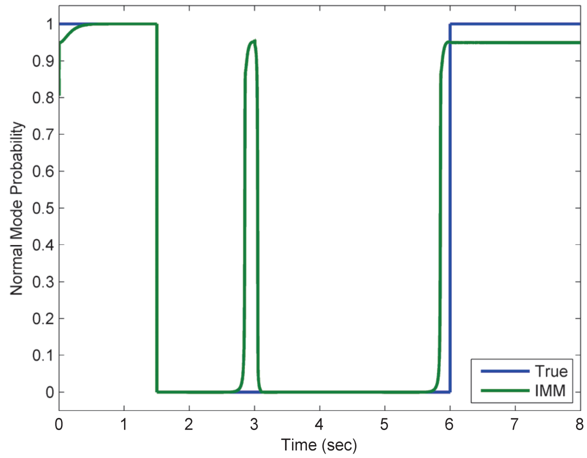

Figure 6: True and estimated normal mode probability

Note that the estimated normal mode probability closely follows the true normal mode probability. However, at around 3 seconds the IMM has trouble distinguishing between the three modes, and the normal mode is misclassified. This is most likely due to the fact that all three models behave similarly around this point in the trajectory. The following three figures represent the trajectory tracking (position, velocity, and acceleration) errors for the SMC and SMC-IMM methodologies.

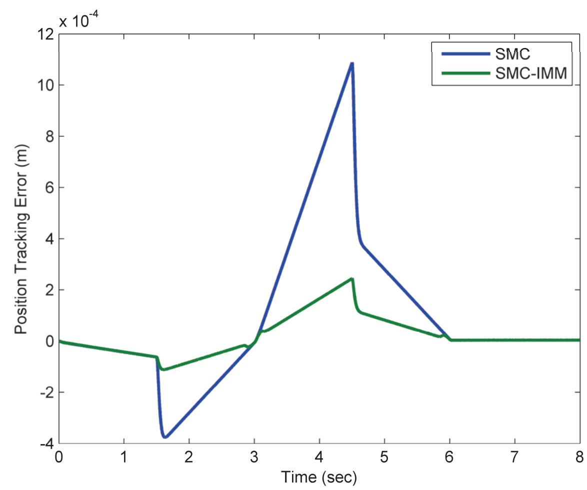

Figure 7: Position tracking error for SMC and SMC-IMM

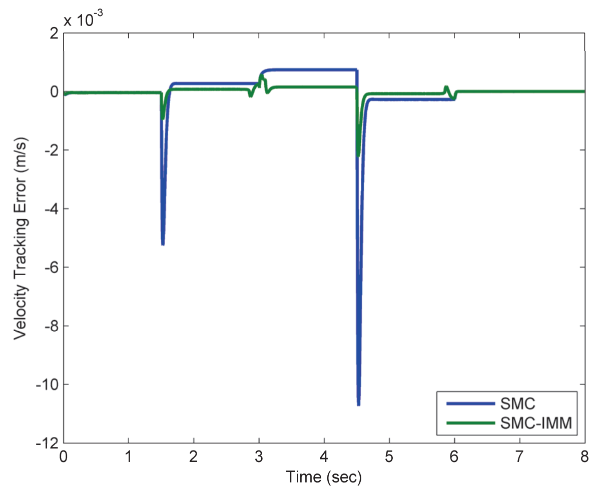

Figure 8: Velocity tracking error for SMC and SMC-IMM

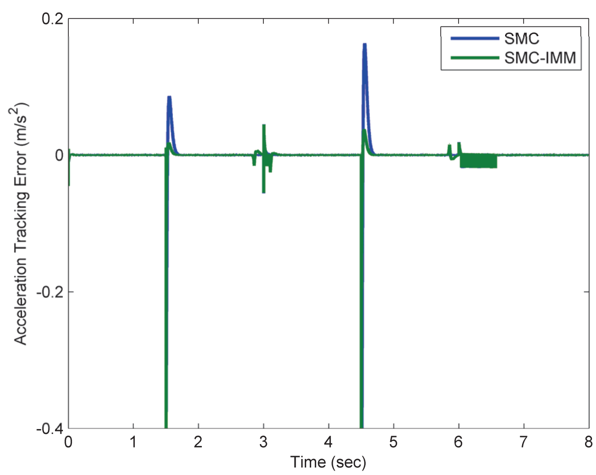

Figure 9: Acceleration tracking error for SMC and SMC-IMM

As shown in the above figures, the SMC-IMM strategy is able to overcome the modelling uncertainties, and improves the trajectory tracking accuracy by nearly 4 times when compared with the standard SMC. Furthermore, the introduction of system changes (i.e., faults) causes the tracking error to spike. The magnitude of this error is considerably smaller with the proposed strategy, which makes for a smoother controller motion in the presence of faults.

6 Conclusion

In this paper, a new type of sliding mode controller based on the interacting multiple model strategy (SMC-IMM) was proposed. The SMC-IMM method was applied on an electrohydrostatic actuator, and the results were compared with the standard SMC strategy. As demonstrated in the paper, the proposed SMC-IMM method was found to provide more accurate trajectory following when compared with the standard SMC strategy. In fact, the tracking error was reduced by over 4 times, which is a significant improvement. Future work will involve: implementation of the method on an EHA system in real-time, utilization of filtering strategies to remove noise, and the development of the proof of stability.

7 Appendix

The following is a list of important EHA parameters and their corresponding values.

Table 1. Important EHA Parameter Values

Parameter	Physical Significance	EHA Model Value
A	Piston Area	$1.52 \times 10^{-3}\ m^2$
D_P	Pump Displacement	$6.876 \times 10^{-7}\ m^3/rad$
M	Load Mass	$7.376\ kg$
V_0	Initial Cylinder Volume	$2.1789 \times 10^{-4}\ m^3$
x_0	Maximum Cylinder Stroke	$0.14335\ m$
β_e	Effective Bulk Modulus	$2.1 \times 10^8\ Pa$
B	Friction Damping Term	$28{,}569\ Ns/m$
B_0	Second Friction Term	$25\ Ns/m$
B_2	Third Friction Term	$0\ Ns/m$
L	Leakage Coefficient	$2.903 \times 10^{-11}\ Nm/s$

8 References

[1] V. Utkin, "Variable Structure Systems with Sliding Modes," *IEEE Transactions on Automatic Control*, vol. 2, pp. 212-222, 1977.

[2] V. Utkin, *Sliding Modes and Their Application in Variable Structure Systems*, 1st ed. Moscow, U.S.S.R.: Mir (English), 1978.

[3] F. Harashima and J. H. H. Xu, "Tracking Control of Robot Manipulators Using Sliding Mode," *IEEE Transactions on Power Electronics*, pp. 169-176, 1987.

[4] X. Yang, "Continuous Sliding Mode Control of a Cartesian Pneumatic Robot," Queen's University, Kingston, Ontario, Master's Thesis 2001.

[5] A. R. R. J. Lanzon, "Trajectory/Force Control for Robotic Manipulation Using Sliding Mode and Adaptive Control," in *American Control Conference*, San Diego, California, 1999, pp. 1940-1944.

[6] M. Bouri and D. Thomasset, "Sliding Control of an Electropneumatic Actuator Using an Integral Switching Surface," *IEEE Transactions on Control Systems Technology*, vol. 9, pp. 368-375, 2002.

[7] S. Wang, R. Burton, and S. R. Habibi, "Sliding Mode Controller and Filter Applied to an Electrohydraulic Actuator System," *ASME Journal of Dynamic Systems, Measurement and Control*, vol. 133, no. 2, 2011.

[8] Y. Wu, W. Le, and Y. Liu, "Fuzzy Sliding Mode Control Based on Optimal Fuzzy Reasoning and Its Application on Simulator Turntable," in *7th International Conference on System Simulation and Scientific Computing*, 2008, p. 765.

[9] A. Fazeli, M. Zeinali, and A. Khajepour, "Application of Adaptive Sliding Mode Control for Regenerative Braking Torque Control," *ASME Dynamic Systems and Control Division*, vol. 1, no. 11, 2011.

[10] E. Misawa, "Discrete Time Sliding Mode Control for Nonlinear Systems with Unmatched Uncertainties and Uncertain Control Vector," *ASME Journal of Dynamic Systems, Measurement and Control*, vol. 119, no. 3, pp. 503-512, 1997.

[11] E. Misawa, "Discrete Time Sliding Mode Control: The Linear Case," *ASME Journal of Dynamic Systems, Measurement and Control*, vol. 119, no. 4, pp. 819-21, 1997.

[12] S. Wang, S. R. Habbi, and R. Burton, "Sliding Mode Control for a Model of an Electrohydraulic Actuator System with Discontinuous Nonlinear Friction," in *American Control Conference*, 2006, p. 38.

[13] M. A. El Sayed and S. R. Habibi, "Multiple Sliding Mode Control for an Electrohydrostatic Actuator System," in *ASME Fluid Power and Motion Control Symposium*, Bath, UK, 2010, pp. 353-364.

[14] M. A. El Sayed, "Multiple Inner-Loop Control of an Electrohydrostatic Actuator," McMaster University, Hamilton, Ontario, Ph.D. Thesis 2012.

[15] Y. Bar-Shalom, X. Rong Li, and T. Kirubarajan, *Estimation with Applications to Tracking and Navigation*. New York: John Wiley and Sons, Inc., 2001.

[16] M. S. Grewal and A. P. Andrews, *Kalman Filtering: Theory and Practice Using MATLAB*, 3rd ed. New York: John Wiley and Sons, Inc., 2008.

[17] X. R. Li, *Advances in Aerospace Systems Dynamics and Control Systems (Volume 76)*, C. T. Leondes, Ed. New York, USA: Academic Press, 1996.

[18] D. T. Magill, "Optimal Adaptive Estimation of Sampled Stochastic Processes," *IEEE Transactions on Automatic Control*, vol. AC-10, pp. 434-439, 1965.

[19] G. A. Ackerson and K. S. Fu, "On State Estimation in Switching Environments," *IEEE Transactions on Automatic Control*, vol. AC-15, no. 1, pp. 10-17, January 1970.

[20] A. G. Jaffer and S. C. Gupta, "Recursive Bayesian Estimation with Uncertain Observation," *IEEE Transactions on Information Theory*, vol. IT-17, pp. 614-616, September 1971.

[21] A. G. Jaffer and S. C. Gupta, "Optimal Sequential Estimation of Discrete Processes with Markov Interrupted Observations," *IEEE Transactions on Automatic Control*, vol. AC-16, pp. 417-475, October 1971.

[22] C. B. Chang and M. Athans, "State Estimation for Discrete Systems with Switching Parameters," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-14, no. 5, pp. 418-425, May 1978.

[23] H. A. P. Blom, "An Efficient Filter for Abruptly Changing Systems," in *23rd IEEE Conference on Decision and Control*, Las Vegas, Nevada, December, 1984.

[24] H. A. P. Blom and Y. Bar-Shalom, "The Interacting Multiple Model Algorithm for Systems with Markovian Switching Coefficients," *IEEE Transactions on Automatic Control*, vol. AC-33, no. 8, pp. 780-783, August 1988.

[25] S. A. Gadsden, "Smooth Variable Structure Filtering: Theory and Applications," Department of Mechanical Engineering, McMaster University, Hamilton, Ontario, Ph.D. Thesis 2011.

[26] E. Mazor, A. Averbuch, Y. Bar-Shalom, and J. Dayan, "Interacting Multiple Model Methods in Target Tracking: A Survey," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 34, no. 1, pp. 103-123, January 1998.

[27] H. Wang, T. Kirubarajan, and Y. Bar-Shalom, "Precision Large Scale Air Traffic Surveillance Using an IMM Estimator with Assignment," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 35, no. 1, pp. 255-266, January 1999.

[28] Y. M. Zhang and X. R. Li, "Detection and Diagnosis of Sensor and Actuator Failures Using IMM Estimator," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 34, pp. 1293-1313, October 1998.

[29] S. Kim, J. Choi, and Y. Kim, "Fault Detection and Diagnosis of Aircraft Actuators Using Fuzzy-Tuning IMM Filter," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 44, no. 3, pp. 940-952, July 2008.

[30] S. R. Habibi, "The Smooth Variable Structure Filter," *Proceedings of the IEEE*, vol. 95, no. 5, pp. 1026-1059, 2007.